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Risk Measurement Illiquidity Distortions

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Risk Measurement Illiquidity Distortions

by

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Abstract

We examine the effects of smoothed hedge fund returns on standard deviation, skewness, and kurtosis of return and on correlation of returns using an MA(2)-GARCH(1,1)-skewed-t representation instead of the traditional MA(2) model employed in the literature. We present evidence that our proposed representation is more consistent with the behavior of hedge fund returns than the traditional MA(2) representation and that that traditional method tends to overstate the degree of smoothing observed in hedge fund returns. We examine methods for correcting the distortive effects of smoothing using our representation.

Keywords: return smoothing, GARCH, skewed-t.

JEL code: G12

1. Introduction

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Hedge-fund returns are characterized by certain stylized facts. Among them, return distributions are often skewed and leptokurtic, individual hedge fund returns may exhibit volatility clustering where periods of high, or low, volatility persist, and monthly returns of individual hedge funds are often positively serially correlated, a phenomenon known as smoothed returns.

Several writers have examined this phenomenon. Among these are Asness, Krail, and Liew (2002); Aragon (2007); Kosowski, Naik, and Teo (2007); Jagannathan, Malakov, and Novikov (2010); and Titman and Tiu (2010), and Lietchty, Huang, and Rossi (2012). In their paper on return smoothing, Getmansky, Lo, and Makarov (2004), GLM hereafter, argue that hedge fund monthly returns are smoothed because of the effects of various forms of illiquidity. For example, for its month-end books, a hedge fund may average broker estimates of an illiquid asset's value. Brokers may estimate the month-end value as a markup at an assumed rate from the prior month's value. This and the practice of averaging the estimates produces smoothed returns.

GLM propose an MA(2) model of smoothed returns, which we refer to henceforth as the traditional MA(2) model because it is widely cited in the literature. This paper takes GLM as a point of departure. We replace their traditional MA(2) model of smoothed returns with an MA(2)-GARCH(1,1)-skewed-t model and present evidence that it provides a better representation of smoothed returns. We find that the traditional MA(2) model exaggerates the distortive effects of return smoothing on statistical measures.

It is established in the literature that with smoothed returns, standard deviation using reported returns is understated so that the Sharpe ratio and information ratio are overstated. It is also established that measures of skewness and kurtosis are also distorted by smoothing as is correlation of returns between funds. Using our proposed method, we present an empirical comparison of these risk measurement distortions.

The next section discusses the different styles of investment management used by hedge funds and how those styles affect illiquidity. The section after that describes the data used in our analysis. The section following that examines our computational methods. The section after that examines the impact of smoothed hedge fund returns on measured standard deviation, skewness, and kurtosis of individual hedge fund return streams, and on correlation of returns between hedge funds. The final section presents conclusions.

2. Hedge Fund Styles and Return Smoothing

The effects of illiquidity may vary with the style of investment management employed by a hedge fund. Hedge Fund Research, Inc. (HFR hereafter) separates hedge fund management styles into four categories: equity, event driven, macro, and relative value strategies.

Equity hedge funds maintain long and short positions in equity and equity derivative securities. They employ both quantitative and fundamental techniques and may be diversified or focused on specific sectors of the equity market. They often employ leverage and may invest in companies with a range of market capitalizations. Equity hedge funds often invest in listed equities. However, equity funds investing in small unlisted companies could be subject to illiquidity effects and exhibit return smoothing.

Event driven funds invest in companies involved in capital restructuring. These include mergers, tender offers, and buybacks. They may invest in corporate equities and bonds as well as in related derivatives. As in the case with equity funds, illiquidity effects may be absent in cases where event driven funds invest in listed assets, but some event driven funds may invest in idiosyncratic instruments that are illiquid.

Macro hedge funds employ investment strategies which seek to exploit the effects of variations in macroeconomic variables. They employ a variety of methods and operate in equity, fixed income, currency, and commodity markets. Many macro funds implement their strategies in futures markets, these markets are generally liquid, and thus these funds do not typically exhibit return smoothing. However, some macro funds may implement their strategies in less liquid markets, in particular, foreign equity and fixed income markets.

Relative value funds seek to take advantage of pricing discrepancies between related securities. They employ a variety of techniques, and may invest in various types of assets including stocks, bonds, currencies, and related derivatives of these asset types. Because they focus on exploiting mispricing of related assets, relative value funds often invest in liquid markets where bid-ask spreads are small and assets can be traded in sufficient volume to open and close positions quickly, but relative value funds may at times take positions in illiquid markets to take advantage of large price discrepancies.

3. Data

Hedge fund monthly return data come from the Hedge Fund Research, Inc. database. The period for evaluating portfolio performance is from January 1994 to December 2016. We combine both live and dead, or graveyard, funds to give us a more comprehensive understanding of the issue of return smoothing. To facilitate the MA(2)-GARCH(1,1)-skewed-t estimation, we restrict our selection of hedge funds to those with assets under management of at least \$10 million at their peak and continuous reporting of returns of at least 5 years. This gives us 7,389 funds in our study.

4. Computational Methods

GLM employ an underlying MA(2) model of monthly hedge fund returns as follows:

(1)
$$
\begin{aligned}
&R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2} \\
&\theta_0 + \theta_1 + \theta_2 = 1 \\
&0 \leq \theta_i \leq 1
\end{aligned}
$$

where R_t^o is the observed, i.e., the reported, monthly return in period t, R_t is the underlying actual, or true, return in t which is not observed, and the θ_i are parameters. Return smoothing is present in data where $0 \le \theta_i < 1$. GLM propose to estimate the MA(2) model using maximum likelihood estimation (MLE) while relaxing the constraint $0 \le \theta_i \le 1$. They define the de-meaned observed return $X_t = R_t^o - \mu$. Then:

(2) $$\begin{aligned} X_t &= \theta_0 a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2} \\ \theta_0 &+ \theta_1 + \theta_2 = 1 \\ a_t &= R_t - \mu \\ a_t &\sim \text{iid } N(0, \sigma^2) \end{aligned}$$

Equation (2) is what we refer to as the traditional MA(2) model

Our paper is rooted in a simple observation that a key assumption in the traditional MA(2) model, that the a_t have an iid normal distribution, is not always the case in reality. It is not uncommon for returns to exhibit skewness, kurtosis, and volatility clustering. For example, Figure 1 below shows the residuals, i.e., the a_t in equation (2), for one of the funds in our dataset obtained through the traditional MA(2) method. The fund is classified as an event driven fund specializing in merger arbitrage. The distribution of the residuals, which should be iid normal, exhibits kurtosis, and it is highly skewed. Figure 2 shows the autocorrelation function plot of the absolute residuals in the traditional MA(2) model for the same fund. The autocorrelations at lags of 1 and 2 fall outside the 95% confidence interval for zero autocorrelation as computed from Bartlett's formula (see Box, Jenkins, and Reinsel, 1994). We observe properties like these frequently in hedge fund return streams in our database.

Figure 1. Histogram of the residuals for fund #4 in the traditional MA(2) model

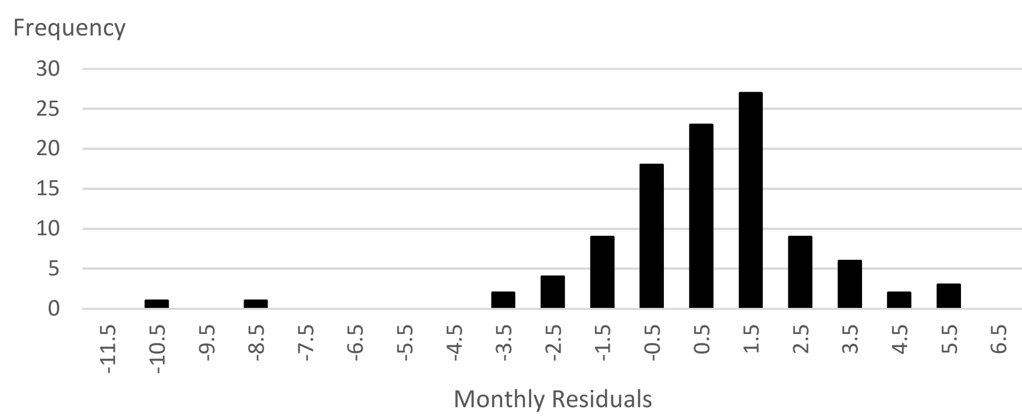
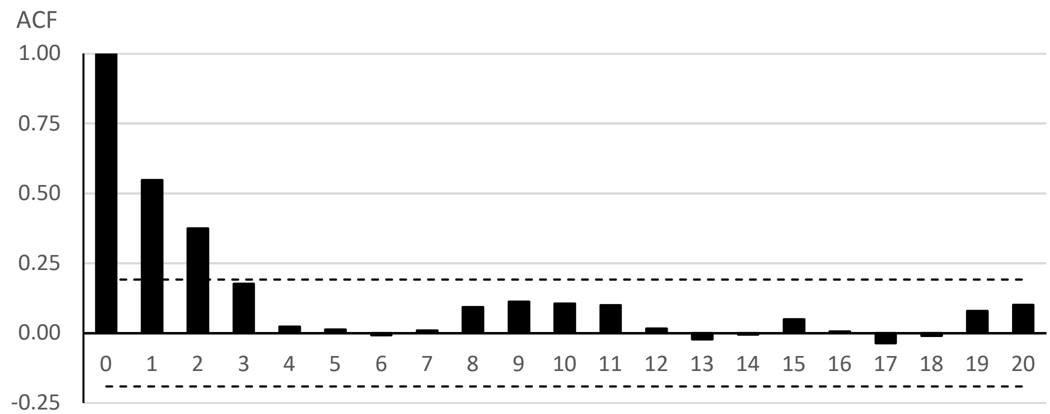


Figure 2. Autocorrelation function plot of the absolute residuals for Fund #4 in the traditional MA(2) model and the bands for the 95% confidence interval for zero autocorrelation



Our goal is to deal simultaneously with the effects of skewness, kurtosis, and volatility clustering in monthly hedge fund returns. To that end, we propose an MA(2)-GARCH(1,1)-skewed-t model where the skewed-t part of the model deals with kurtosis and skewness effects and the GARCH part of the model simultaneously deals with volatility clustering. The model is as follows:

$$\text{(3)} \quad \begin{aligned} &X_t = \theta_0 a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2} \\ &\theta_0 + \theta_1 + \theta_2 = 1 \\ &a_t = R_t - \mu = \sigma_t \epsilon_t \\ &\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{aligned}$$

where the term ϵ_t has a skewed-t distribution defined in Lambert and Laurent (2001) as follows. Suppose that x has a Student-t distribution with probability density function (PDF) g and degrees of freedom υ. Kurtosis is controlled by υ. Skewness is introduced by constructing the function g^* such that:

$$\text{(4)} \quad g^*(x|\xi, \upsilon) = \begin{cases} g(x\,\xi) \text{ if } x < 0 \\ g\left(\frac{x}{\xi}\right) \text{ if } x \geq 0 \end{cases}$$

The positive parameter ξ controls skewness. We normalize g^* by simple scaling so that it becomes a PDF, i.e., so that its density sums to 1. Then, the mean of the PDF is:

$$\text{(5)} \quad E(x|\xi, \upsilon) = \frac{\Gamma[(\upsilon-1)/2]\sqrt{\upsilon-2}}{\sqrt{\pi}\Gamma(\upsilon/2)}\left\{\xi - \frac{1}{\xi}\right\} = m$$

and the variance is:

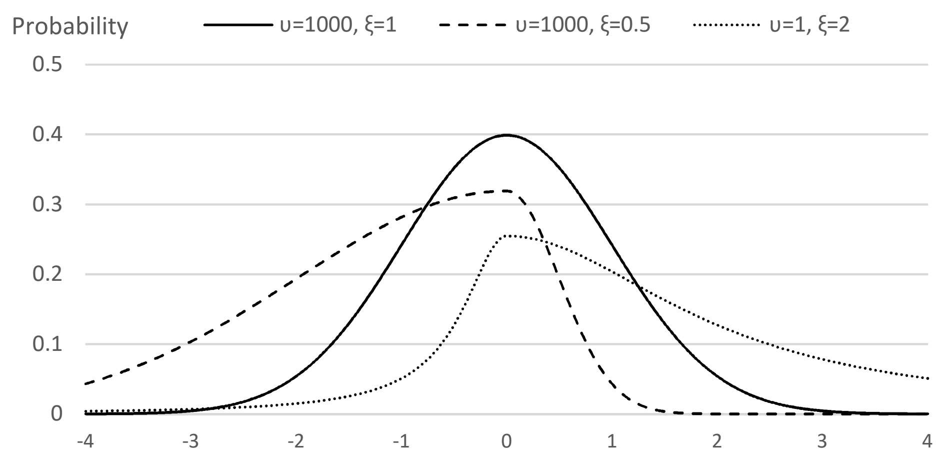
(6) $$V(x|\xi, \upsilon) = \xi^2 + \frac{1}{\xi^2} - 1 - m^2 = s^2.$$

A variate $y = \frac{x-m}{s}$ is constructed. From the preceding, its PDF $f(y|\xi, \upsilon)$ is:

(7) $$f(y|\xi, \upsilon) = \frac{2s}{\xi + 1/\xi}\left\{g[\xi(sy+m)|\upsilon]I_{(-\infty,0)}\left[y + \frac{m}{s}\right] + g[\frac{sy+m}{\xi}|\upsilon]I_{(0,\infty)}\left[y + \frac{m}{s}\right]\right\}$$

where I is inverse Γ. By construction, i.e., since $y = \frac{x-m}{s}$, the mean of y is equal to 0, and the variance is equal to 1. The variate ϵ_t is distributed as $f(\epsilon_t|\xi, \upsilon)$. This is our MA(2)-GARCH(1,1)-skewed-t model. In the chart below, we see that alternative PDFs can be constructed with the skewed-t model by varying υ and ξ.

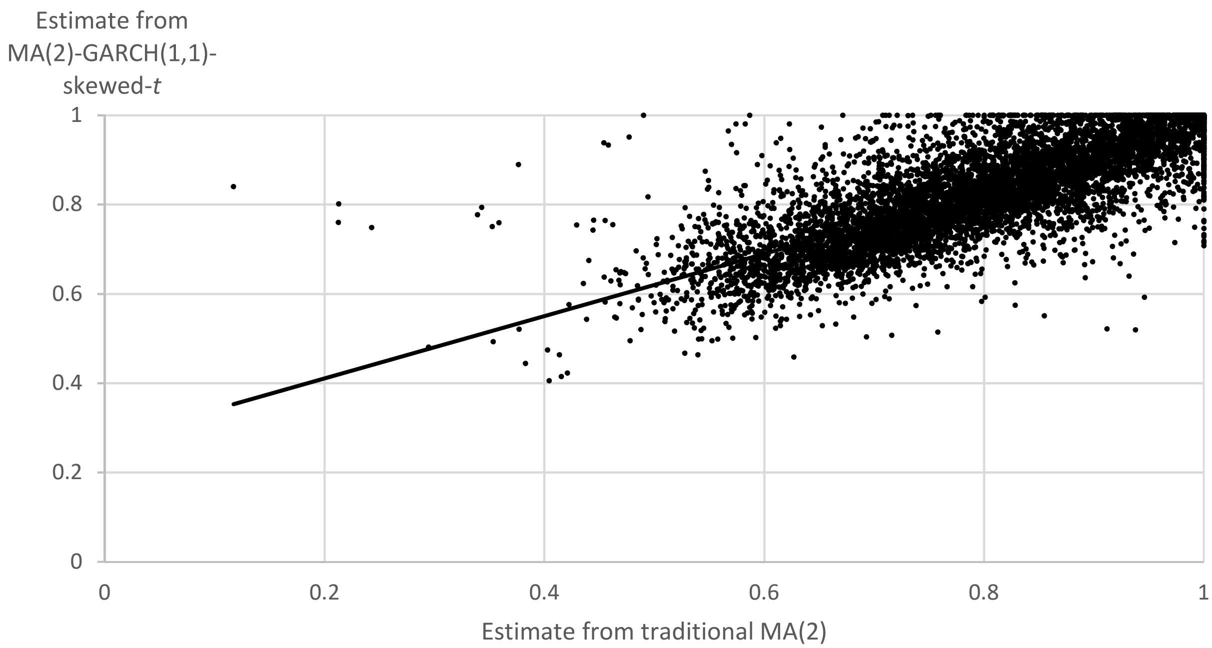
Figure 3. Alternative PDFs for the skewed-t model.



We note in passing that, in our experiments where we employ MLE without the constraint that $0 \leq \theta_i \leq 1$, the estimated θ_i are sometimes negative in value and thus are not consistent with what would be expected in a reasonable interpretation of observed data. Therefore, we employ the constraint.

In the chart below, we see the scatter plot of the θ_0 estimated by the traditional MA(2) method and the MA(2)-GARCH(1,1)-skewed-t method. The chart also shows the fitted linear trend for the two estimates of θ_0. It is evident in the chart that the two models generate very different values for θ_0; points are scattered widely around the trend line indicating that the choice of the model, either the traditional MA(2) model or the MA(2)-GARCH(1,1)-skewed-t model, does matter. As for the trend line itself, it passes close to the (1,1) coordinate. In either model, a value of θ_0 equal to 1 indicates that return smoothing is absent. However, along the trend line, as values of θ_0 generated by the traditional MA(2) method decline, indicating that the degree of smoothing is increasing, values of θ_0 from the MA(2)-GARCH(1,1)-skewed-t model decline much less. The traditional MA(2) method, which does not take into consideration skewness, kurtosis, and volatility clustering effects, tends to magnify the degree of smoothing—especially for low values of θ_0–relative to our MA(2)-GARCH(1,1)-skewed-t proposal, which incorporates those effects.

Figure 4. Scatter plot of θ_0 estimates



For our 7,389 hedge funds, 5,398 of them have returns statistically significantly different from a normal distribution at the 0.05 significance level according to the Jarque and Bera (1987) statistic, which is asymptotically distributed as a chi-square random variable with two degrees of freedom. This is a strong indication that the traditional MA(2) method with its normality assumption is inappropriate for modeling hedge fund returns. In Table 1 below, we list the 10 funds with the lowest values of θ_0 estimated from the traditional MA(2) model and ranked in increasing order of θ_0 to illustrate the difference between the two estimation processes. The table shows the estimated θ_i using both the traditional MA(2) model and the MA(2)-GARCH(1,1)-skewed-t model and presents the Jarque and Bera statistics for the residuals ϵ_t from the traditional MA(2) model. The p values of the Jarque and Bera statistics indicate that the normality assumption of the traditional MA(2) method is rejected. And, again, the low values of θ_0 generated by the traditional MA(2) model indicate a substantial degree of smoothing in hedge fund returns while the higher respective values of θ_0 generated by the MA(2)-GARCH(1,1)-skewed-t method indicate that return smoothing is much less pronounced.

Table 1. Comparison of the skewed-t model and the traditional MA(2) model

Traditional MA(2)			MA(2)-GARCH(1,1)-skewed-t			Jarque-Bera (p-value)
θ_0	θ_1	θ_2	θ_0	θ_1	θ_2	
0.117	0.365	0.518	0.840	0.000	0.160	930.41 (0.000)
0.212	0.575	0.212	0.760	0.240	0.000	3,930.78 (0.000)
0.213	0.575	0.213	0.802	0.198	0.000	3,915.16 (0.000)
0.243	0.266	0.492	0.749	0.118	0.133	1,851.35 (0.000)
0.295	0.245	0.460	0.481	0.302	0.218	4,500.70 (0.000)
0.339	0.423	0.238	0.777	0.223	0.000	398.23 (0.000)
0.343	0.657	0.000	0.794	0.153	0.054	11,228.91 (0.000)
0.353	0.647	0.000	0.751	0.249	0.000	2,854.24 (0.000)
0.353	0.248	0.399	0.493	0.318	0.188	271.74 (0.000)
0.359	0.000	0.641	0.760	0.020	0.220	204.52 (0.000)

To test whether the GARCH(1,1)-skewed-t model is better than the GARCH(1,1) model employed individually, a log-likelihood ratio test is applied where the null model, i.e. the GARCH(1,1) model, is a special case of the GARCH (1,1)-skewed-t model. A p-value of less than 0.05 indicates that the GARCH(1,1)-skewed-t model fits the data significantly better than the null model. HFR produces a monthly index for each of its four strategies. The results for the four indexes is as follows:

Table 2. p-values for HFR strategy indexes

Equity	Event Drive	Macro	Relative Value
0.1788	0.0010	0.2688	0.0000

Thus, for the HFR indexes, the GARCH(1,1)-skewed-t model is significantly better than the null model for event driven and relative value funds.

To get a sense of whether the outcomes above are indicative of outcomes for individual funds within the four strategy classifications, we randomly selected 10 individual funds from each of the four groups and conducted the log-likelihood ratio test for each of them. Ordered by descending p-values within each strategy group, the results are as follows:

Table 3. p-values for selected funds within each HFR strategy group

Equity	Event Driven	Macro	Relative Value
0.3126	0.4981	0.8439	0.0547
0.2555	0.1678	0.7102	0.0452
0.1516	0.1342	0.5035	0.0047
0.1389	0.1105	0.4908	0.0045
0.1286	0.0188	0.2122	0.0013
0.0605	0.0046	0.1620	0.0000
0.0597	0.0033	0.1469	0.0000
0.0267	0.0023	0.0245	0.0000
0.0101	0.0005	0.0134	0.0000
0.0018	0.0000	0.0004	0.0000

Thus, we see that individual outcomes vary within each of the four strategy groups.

5. Smoothed Returns and Distortion of Statistical Measures

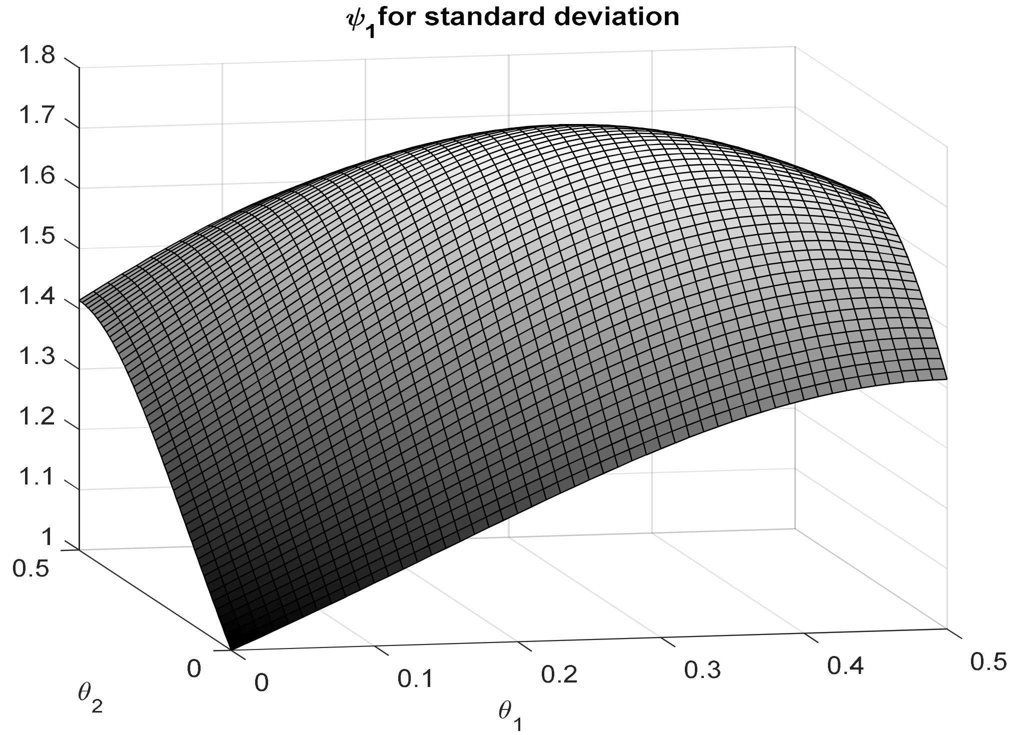
Return Smoothing and Standard Deviation of Return

Geltner (1991) shows that for the moving average model, i.e., equation (1), the standard deviation $\mathrm{SD^o}$ of the reported returns R_t^o and the standard deviation SD of the actual returns R_t are related as follows:

(8) $$\mathrm{SD} = \psi_1\, \mathrm{SD^o}$$
$$\psi_1 = 1/(\sum \theta_i^2)^{1/2}$$

The term ψ_1 is greater than 1 where $0 \leq \theta_i < 1$; i=0,1,2, i.e., where returns are smoothed, so that the standard deviation of reported returns is less than the standard deviation of the actual returns. The distortive effect of return smoothing, i.e., the value of ψ_1, is greatest where $\theta_0 = \theta_1 = \theta_2 = 1/3$. In that case, $\psi_1 = 1.732$. The understatement of standard deviation computed from reported returns means that the Sharpe ratio and the information ratio computed from reported returns are higher than they would be using actual returns. GLM find that large distortions of standard deviation are frequently encountered in hedge fund data. The chart below shows ψ_1 plotted as functions of θ_1 and θ_2 ($\theta_0 = 1 - \theta_1 - \theta_2$, of course).

Figure 5. Distortion factor ψ_1 for standard deviation under return smoothing



Return Smoothing and Higher Statistical Moments

Higher statistical moments are also distorted by smoothing (Cavenaile, Coën, and Hübner, 2011). Let SK, SK^o, K, and K^o denote, respectively, skewness of actual returns, skewness of observed returns, excess kurtosis of actual returns, and excess kurtosis of observed returns. Then:

(9) $SK = \psi_2 \, SK^o$

$$\psi_2 = (\sum \theta_i^2)^{3/2} / (\sum \theta_i^3)$$

(10) $K = \psi_3 \, K^o$

$$\psi_3 = (\sum \theta_i^2)^2 / (\sum \theta_i^4)$$

Of course, in the absence of return smoothing, $\psi_2 = \psi_3 = 1$, but these distortion factors are elevated where returns are smoothed and may be well above 1. Again, the distortive effects of return smoothing are greatest for both skewness and excess kurtosis where $\theta_0 = \theta_1 = \theta_2 = 1/3$. In that case, $\psi_2 = 1.732$ and $\psi_3 = 3$. The charts below show ψ_2 and ψ_3 plotted as functions of θ_1 and θ_2.

Figure 6. Distortion factor ψ_2 for skewness under return smoothing

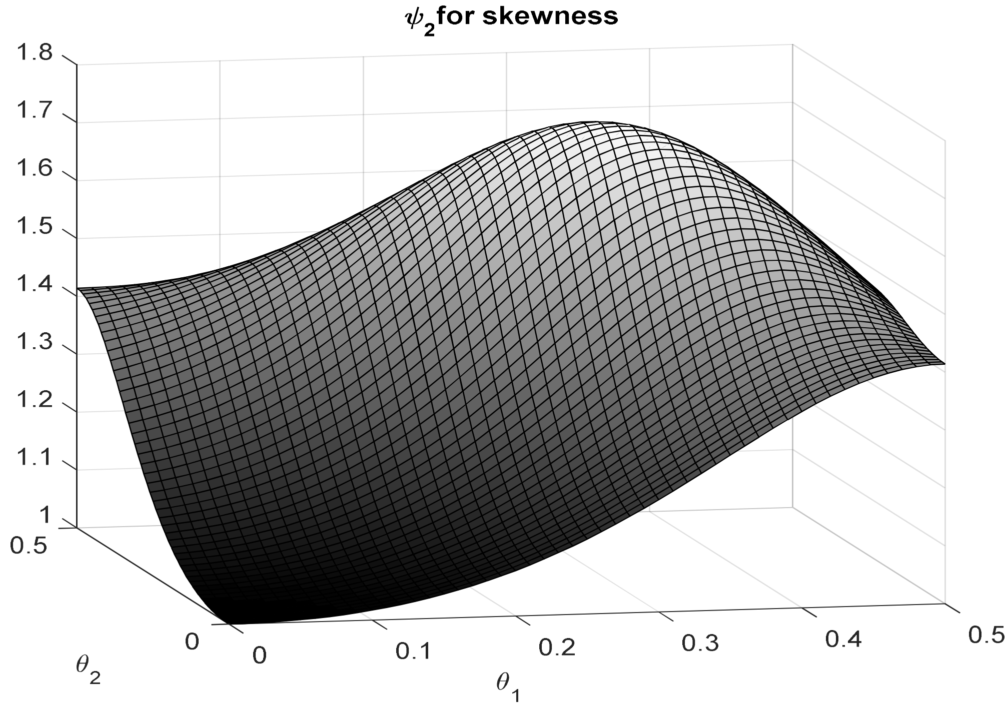
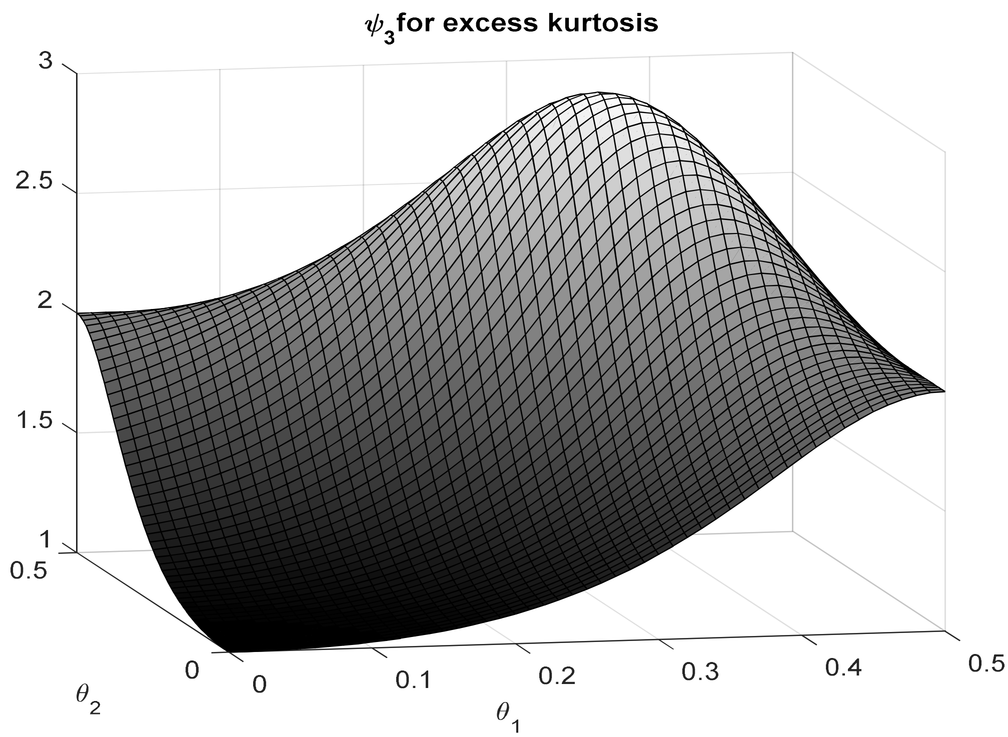


Figure 7. Distortion factor ψ_3 for excess kurtosis under return smoothing



Return Smoothing and Correlation of Returns

Let x_t^o and y_t^o be the observed returns of two hedge funds, let x_t and y_t be the respective actual returns, and suppose again that the two return streams are subject to MA(2) smoothing so that:

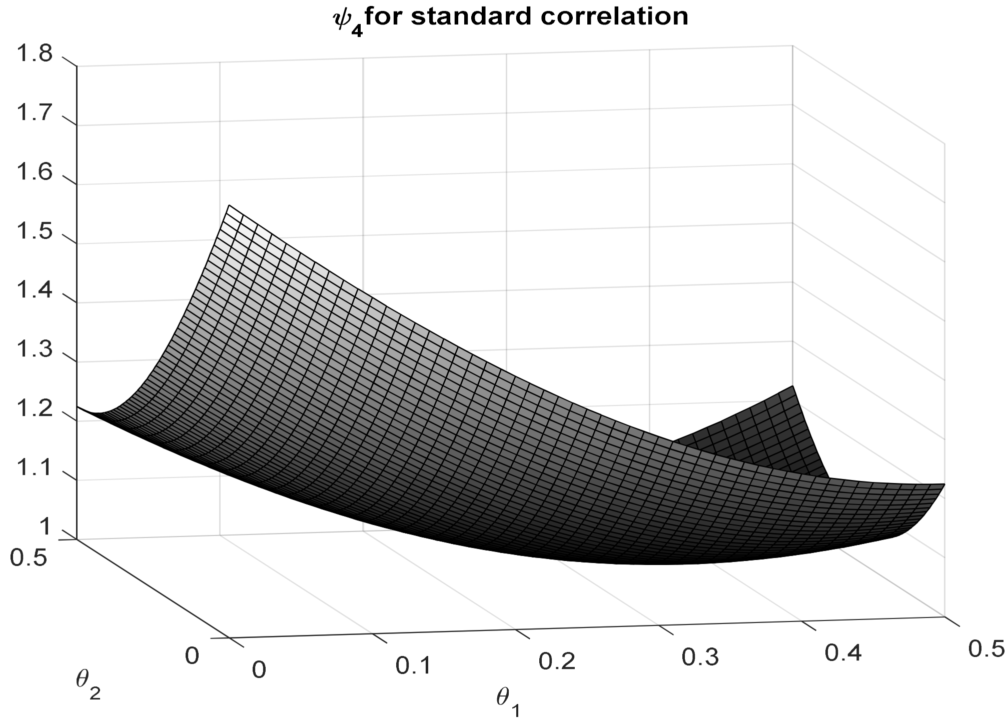
(11) $$x_t^o = \theta_{x,0}x_t + \theta_{x,1}x_{t-1} + \theta_{x,2}x_{t-2}$$
$$y_t^o = \theta_{y,0}y_t + \theta_{y,1}y_{t-1} + \theta_{y,2}y_{t-2}$$
$$\sum \theta_{x,i} = 1$$
$$\sum \theta_{y,i} = 1$$

Then, the correlation $\mathrm{corr}[x_t, y_t]$ of the actual returns and the correlation $\mathrm{corr}[x_t^o, y_t^o]$ of the observed returns are related as follows (Geltner, 1991):

(12) $$\mathrm{corr}[x_t, y_t] = \psi_4 \, \mathrm{corr}[x_t^o, y_t^o]$$
$$\psi_4 = \left(\sum \theta_{x,i}^2 \sum \theta_{y,i}^2\right)^{1/2} / \left(\sum \theta_{x,i}\, \theta_{y,i}\right)$$

Where the respective smoothing parameters are the same for both funds, i.e., where $\theta_{x,i} = \theta_{y,i}; i = 0,1,2$, we have $\psi_4 = 1$. However, where the smoothing parameters are different, we have $\psi_4 > 1$. The value of ψ_4 can be quite large. In fact, it is infinity where, say, $\theta_{x,0} = \theta_{y,2} = 1, \theta_{x,1} = \theta_{x,2} = \theta_{y,1} = \theta_{y,2} = 0$. The chart below shows ψ_4 as a function of $\theta_{y,1}$ and $\theta_{y,2}$ where $\theta_{x,0} = \theta_{x,1} = \theta_{x,2} = 1/3$.

Figure 8. Distortion factor ψ_4 for correlation under return smoothing



Distortion Comparisons of the Two Methods

We saw above that the results obtained from the MA(2)-GARCH(1,1)-skewed-t model and the traditional MA(2) model differ the most where θ_0 is small in value. In the charts below, we present the histograms of the ψ_i for the 1,000 hedge funds in our database with the lowest θ_0 as calculated by the traditional MA(2) model. The comparative results in the histograms show that the two methods produce very different outcomes. The traditional MA(2) method ignores skewness, kurtosis, and volatility clustering effects, and the result is that it magnifies the degree of distortion caused by return smoothing.

Figure 9. Histogram of ψ_1

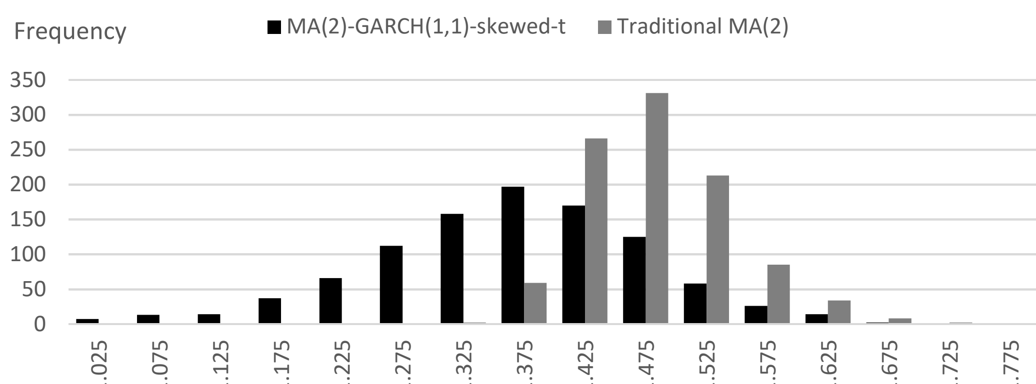


Figure 10. Histogram of ψ_2

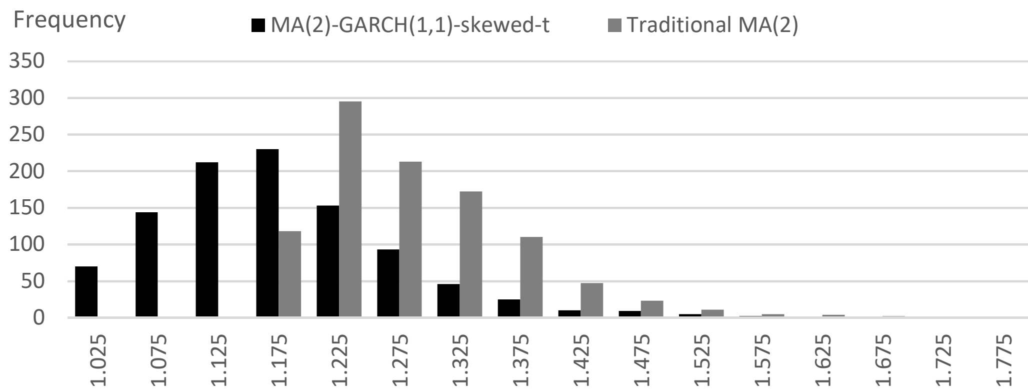


Figure 11. Histogram of ψ_3

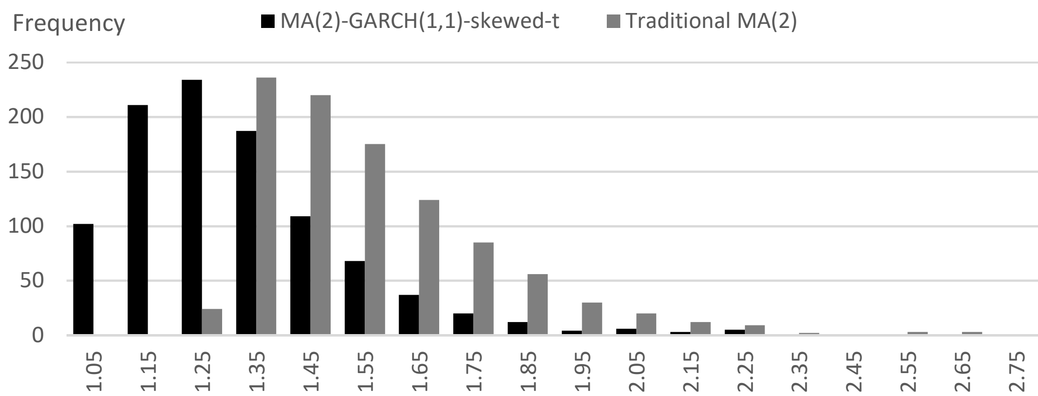
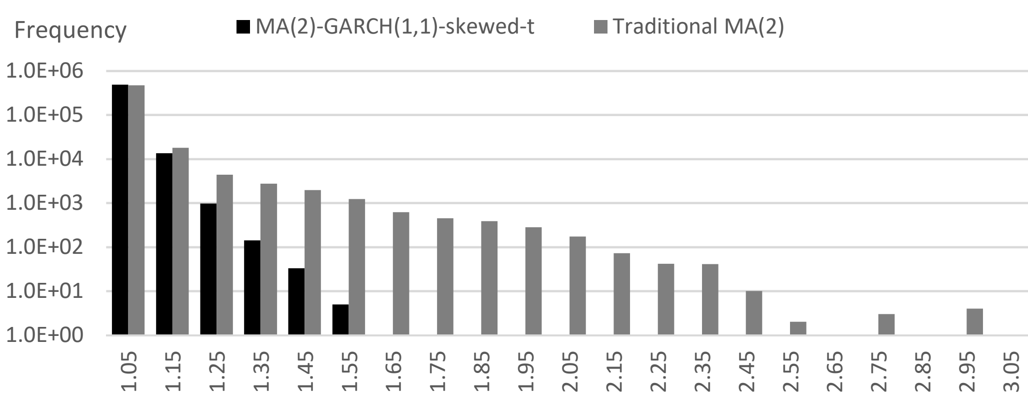


Figure 12. Histogram of ψ_4



6. Conclusion

It is established in the literature that hedge fund return smoothing causes distortions in hedge fund risk measures such as standard deviation, skewness, and excess kurtosis of returns and that it distorts measures of correlation of returns. We replace the traditional MA(2) model employed in the literature with an MA(2)-GARCH(1,1)-skewed-t model to deal with the distortions encountered in return smoothing. We provide evidence that our representation is more consistent with the behavior of hedge fund returns than the traditional MA(2) representation and that the traditional MA(2) representation tends to overstate the degree of smoothing observed in hedge fund returns and the distortive effects of smoothing on statistical measures. The construction of our indicators of return distortion, i.e., the ψ_i, is consistent with our analytical framework, and we suggest that such measures may be useful to analysts and policymakers who require correct assessment of risk in hedge fund space.

References

Aragon, George O. (2007), "Share Restrictions and Asset Pricing: Evidence from the Hedge Fund Industry," *Journal of Financial Economics*, 83 (1): 33-58.

Asness, Clifford S., Robert J. Krail, and John M. Liew (2001), "Do Hedge Funds Hedge?" *Journal of Portfolio Management*, 28 (1): 6-19.

Box, George E.P., Gwilym M. Jenkins, and Gregory C. Reinsel (1994), *Time Series Analysis: Forecasting and Control,* 3rd ed., Englewood Cliffs, NJ: Prentice Hall.

Cavenaile, Laurent, Alain Coën, and Georges Hübner (2011), "The Impact of Illiquidity and Higher Moments of Hedge Fund Returns on Their Risk-Adjusted Performance and Diversification Potential," *The Journal of Alternative Inv*estments, 13 (4): 9-29.

Geltner, David M. (1991), "Smoothing in Appraisal-Based Returns," *Journal of Real Estate Finance and Economics*, 4 (3): 327-345.

Getmansky, Mila, Andrew W. Lo, and Igor Makarov (2004), "An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns," *Journal of Financial Economics*, 74 (3): 529-610.

Jagannathan, Ravi, Alexey Malakhov, and Dmitry Novikov (2010), "Do Hot Hands Exist Among Hedge Fund Managers? An Empirical Evaluation," *Journal of Finance*, 65 (1): 217-55.

Jarque, Carlos M. and Anil K. Bera (1987), "A Test of Normality of Observations and Regression Residuals," *International Statistical Review*, 55 (2): 163-72.

Kosowski, Robert, Naryan Y. Naik, and Melvyn Teo (2007), "Do Hedge Funds Deliver Alpha? A Bayesian and Bootstrap Approach," *Journal of Financial Economics*, 84 (1): 229-64.

Lambert, Philippe and Sébastien Laurent (2001), "Modeling Financial Time Series Using GARCH-Type Models and a Skewed Student Density," Mimeo, Université de Liège.

Lietchty, John, Jing-Zhi Huang, and Marco Rossi (2012), "Return Smoothing and Its Implications for Performance Analysis of Hedge Funds," available at SSRN: http://ssrn.com/anstract=1363957.

Titman, Sheridan and Cristian Tiu (2011), "Do the Best Hedge Funds Hedge?" *The Review of Financial Studies*, 24: 123-67.

Figure 11. Histogram of ψ_4

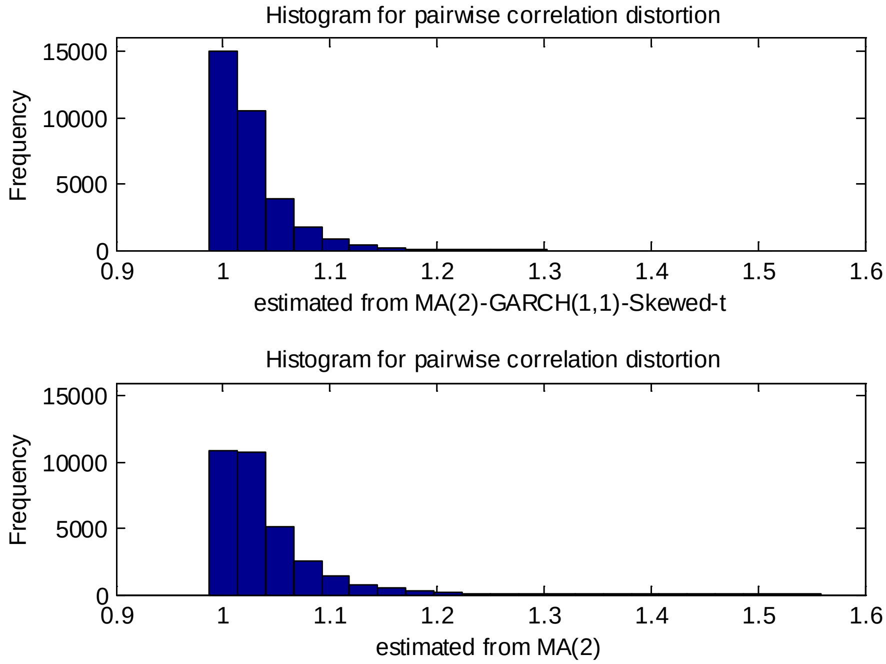


Figure 12. Comparison of cross-sectional volatility

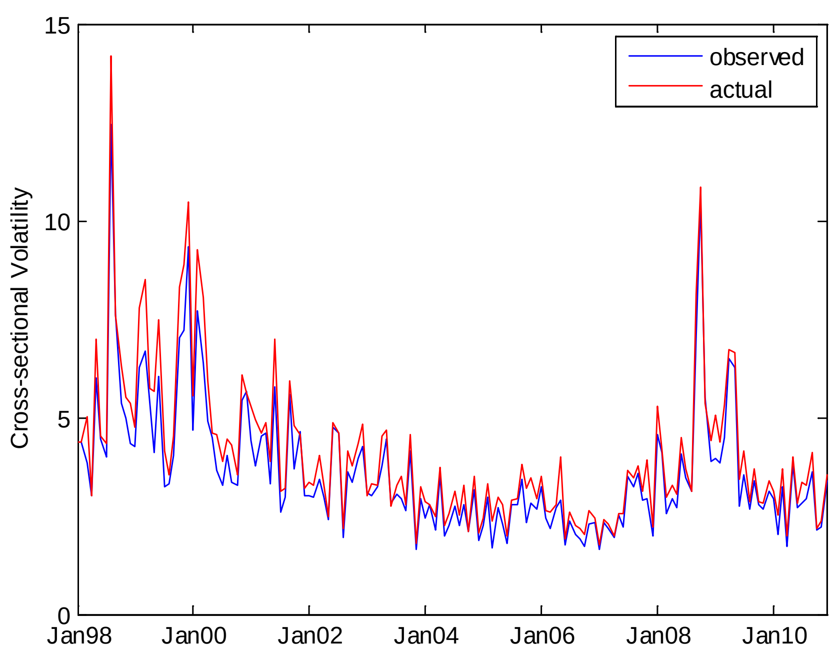
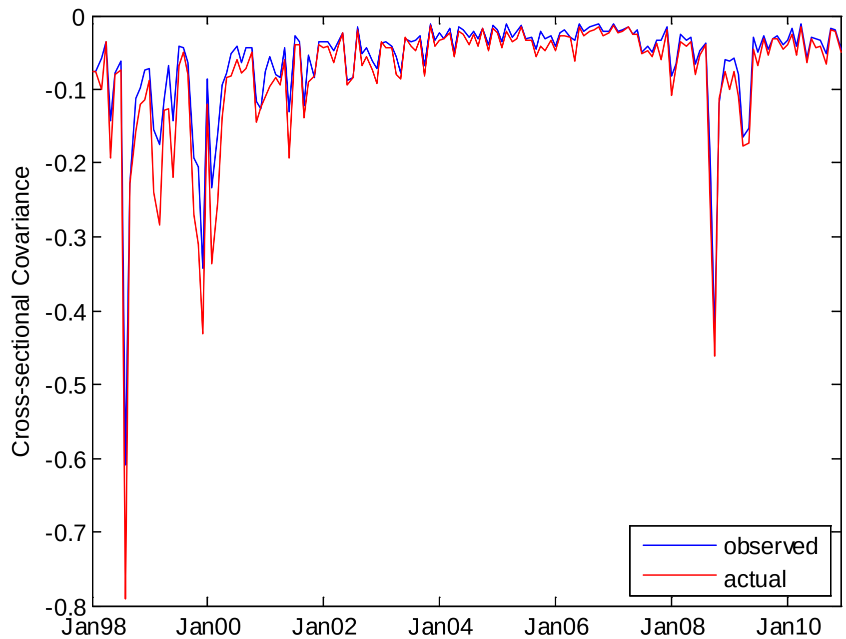


Figure 13. Comparison of cross-sectional covariance



Thus, in general using our method, standard deviation, skewness and excess kurtosis all show some distortions for most of the 256 funds in our study, while the distortions tend to be less than that estimated from the traditional method. The cross-sectional risk measures are affected to a lesser extent, and it would suffice to use only the observed return in this case. Most importantly, there are three big spikes in cross-sectional volatility and three big negative spikes in cross-sectional covariance corresponding to the Long Term Capital Management crisis in 1998, the internet bubble burst in 2000, and the recent financial crisis in 2008.

6. Conclusion

It is established in the literature that hedge fund return smoothing causes distortions in hedge fund statistics. We propose a new MA(2)-GARCH(1,1)-skewed-t model to incorporate the skewness, kurtosis and heteroscedasticity effects often encountered in hedge fund returns. In addition, it has been reported in the literature that return smoothing causes standard deviation of reported returns to understate true volatility and that smoothing distorts correlation of returns. Skewness and excess kurtosis are understated using smoothed returns. In addition, we find that some statistics such as cross-sectional volatility and covariance are not substantially distorted by illiquidity and so they may serve for risk-measurement purposes without correcting for illiquidity effects.

The construction of our indicators of hedge fund illiquidity, i.e., the ψ, is consistent with our analytical framework, and we suggest that such measures may be useful to policy-makers who require correct assessment of risk in hedge fund space.

References

Adrian, T., 2007, “Measuring Risk in the Hedge Fund Sector” Federal Reserve Bank of New York, Current Issues in Economics and Finance, Volume 13, Number 3.

Aragon, G., 2007, “Share restrictions and asset pricing: evidence from the hedge fund industry,” *Journal of Financial Economics*, 83, 33-58.

Asness, C., R. Krail and J. Liew, 2002, “Do hedge funds hedge?” *Journal of Portfolio Management*, 28, 6-19.

Cavenaile, L., A. Coën, and G. Hübner, 2011, “The impact of illiquidity and higher moments of hedge fund returns on their risk-adjusted performance and diversification potential,” *The Journal of Alternative Inv*estments, Spring 2011, vol. 13, no. 4, pp. 9-29.

Geltner, D.M., 1991, “Smoothing in appraisal-based returns,” *Journal of Real Estate Finance and Economics*, 4, 327-345.

Getmansky, M., A.W. Lo and I. Makarov, 2004, “An econometric model of serial correlation and illiquidity in hedge fund returns,” *Journal of Financial Economics*, 74, 529-610.

Jagannathan, R., A. Malakhov and D. Novikov, 2010, “Do hot hands exist among hedge fund managers? An empirical evaluation,” *Journal of Finance*, 65, 217-55.

Jarque, C.M., and A.K. Bera, 1987, “A test of normality of observations and regression residuals,” *International Statistical Review*, 55, 163-72.

Kosowski, R., N.Y. Naik and M. Teo, 2007, “Do hedge funds deliver alpha? A Bayesian and bootstrap approach,” *Journal of Financial Economics*, 84, 229-64.

Lambert, P. and S. Laurent, 2001, “Modeling financial time series using GARCH-type models and a skewed student density,” Mimeo, Université de Liège.

Lo, A.W., 2010, *Hedge funds, an analytic perspective*, Princeton University Press.

Titman, S. and C. Tiu, 2011, “Do the best hedge funds hedge?” *The Review of Financial Studies*, 24, 123-67.