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Abstract

We present a theory of targeted search, where people with a finite information processing capacity search for a match. Our theory explicitly accounts for both the quantity and the quality of matches. It delivers a unique equilibrium that resides in between the random matching and the directed search outcomes. The equilibrium that emerges from this middle ground is inefficient relative to the constrained Pareto allocation. Our theory encompasses the outcomes of the random matching and the directed search literature as limiting cases.

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1 Introduction

Searching for a match – be it for a spouse, an employer, a college or a restaurant – involves distinguishing among various alternatives. Identifying suitable candidates is a costly and time-consuming process. When the welfare of both sides of the market relies on distinguishing among available options, it is important to understand whether the sides are coordinating search efforts in an efficient manner, i.e. to produce a sufficient number of high quality matches.

The literature on search and matching does not consider agents' ability to distinguish among options; nor does it take into account the associated costs. Instead, existing models either postulate that matching is a result of luck of the draw,[1] or that all agents are able to identify their best partners.[2] These built-in structures are the reason why the literature has little to say about the determinants of the quantity and the quality of matches.

In this paper, we propose a model that accounts for both the quantity and the quality of matches. It does so by incorporating two key assumptions. First, we model the costs associated with the process of distinguishing among potential matches. Second, we allow for two-sided heterogeneity, which motivates both sides of the market to actively search.

The outcome of our model is a unique equilibrium that resides *in between* the outcomes of random matching and optimal assignment and is inefficient relative to the constrained Pareto allocation. Moreover, our model encompasses the outcomes of the random matching and the directed search literature as limiting cases.

Building on rational inattention theory put forward by Sims (2003), we identify costly information processing as the modeling device that can rationalize both the quantity and the quality of matches. When information processing is costly, people can identify their best matches only partially; they optimally target those prospective matches that are expected to render a higher payoff. This is why we call it a theory of *targeted* search: agents cannot identify their best match because it is costly, so they target their more

[1] Random matching models postulate that if there are two workers looking for a job, each one has equal chances of getting it, irrespective of their characteristics.

[2] Directed search models postulate a process of competitive search, introduced by Moen (1997), which allows each agent to perfectly identify her best match. Since the outcomes of directed search models typically allow agents to pair with their best matches in equilibrium, we broadly refer to the outcomes of directed search models as the optimal assignment.

promising matches. In the extreme, if information processing costs tend to infinity, then no information is processed and we obtain the random matching outcome. If processing information costs nothing, then the outcome is that of the optimal assignment model. Although the outcomes are constrained efficient in both limiting cases, the equilibria that emerge from the middle ground are inefficient.

Even though our theory applies to many different markets, we focus on the labor market throughout the paper. We first present a two-sided search model where workers and firms are subject to information processing constraints. In this model agents can send or process only one application. Then, we relax this assumption and allow workers and firms to send and process multiple applications. In this latter case, the cost function does not depend only on the cost of processing information, but also on the physical cost of sending an application. In both cases, a match is formed if it is mutually beneficial and the surplus from the match is split between the two parties through ex-post Nash bargaining.

For each case, our analysis consists of two parts. First, we construct a matching market equilibrium and find the conditions for its existence and uniqueness. We provide the necessary and sufficient conditions that characterize the equilibrium strategies in a closed form. Second, we compare the equilibria of the decentralized economy to the constrained efficient allocation.

The role of information-processing constraints in our model is threefold. First, they generate endogenous delays in matching, as the search involves balancing the cost and the precision of information about prospective matches. As a result, some participants will not find partners immediately.

Second, costly information processing can produce a partially targeted distribution of attention. This distribution places a greater probability on the matches that promise a higher return.

Third, information-processing constraints are crucial to obtaining a unique equilibrium. The rationale behind this finding lies in the presence of complementarities among the search strategies of market participants. The complementarities in our model are different from search externalities studied by Diamond (1982) where an increase in the number of participants makes it easier for one side of the market and more difficult for the other side to find a match. In our model, complementarity arises because if one person targets another, the other has an incentive to reciprocate.

In the optimal assignment model, complementarities between search strategies are strong and lead to a multiplicity of equilibria;[3] whereas complementarities are absent in random matching models. We show that an increase in information costs makes the search strategies of market participants less complementary and eliminates the multiplicity of equilibria.

As for efficiency, we show that both physical costs of sending an application and information-processing costs imply a lower equilibrium number of matches than is socially optimal. However, while physical costs can only affect the quantity of matches, information-processing costs coupled with the heterogeneity of market participants are essential for the inefficient quality of matches that our model generates.

The intuition behind the inefficiency is as follows: When an agent decides to process more information, she is choosing a more targeted strategy. This increases the probability of ending up with a more promising match and lowers the probability of being paired with a bad match. Because of the complementarities in our model, high types are more likely to assign a greater probability to other high types. Low types would then invest less time and effort processing information about this participant and target somebody else. Market participants are unable to fully internalize this positive externality. Agents on both sides of the market fail to appropriate all social benefits of their actions, and, as a result, in equilibrium the quantity and the quality of matches are both inefficiently low.

With endogenous information choice as the driving force of matching patterns, our model is well suited to study a host of real-life matching markets where people typically have limited time and cognitive ability to process information. Roth and Sotomayor (1990) and Sönmez and Ünver (2010) provide examples of such markets. Moreover, for many markets equilibrium outcomes are neither pure random matching nor optimal assignment, as documented in the empirical literature.[4] Our model can be a useful tool for analyzing these markets.

From a theoretical standpoint, the paper contributes to the search and matching literature by providing a framework that produces equilibrium outcomes between random matching and directed search, as opposed to nesting them. Examples of models that nest directed search and random matching are Menzio (2007) and Lester (2011).

[3] Many models of directed search, e.g. Shimer (2005), introduce additional assumptions in order to select one of those equilibria.

[4] An extensive, albeit necessarily non-exhaustive, list of examples from the empirical literature is presented in Section 3.

Finally, the paper contributes to the literature on directed search and coordination frictions. The directed search paradigm generally predicts efficient static equilibrium outcomes.[5] In contrast, our targeted search model does not appear to possess a market mechanism that can implement the constrained efficient allocation.

The paper proceeds as follows: Section 2 outlines the theoretical framework. We first discuss a one-shot version of the two-sided search model and then we move to a model of two-sided search with endogenous search intensity. For each model we first find conditions under which there exists a unique equilibrium, and then establish that equilibria in both models are socially suboptimal. Section 3 discusses available extensions of the model and its applicability to different markets. Section 4 provides a conclusion. Proofs and extensions in the Appendix.

2 Theoretical Framework

We consider an environment where a number of heterogeneous participants are searching for a match. We model the search process building on elements of information theory and the rational inattention literature. We assume that each agent can choose how much information to gather about potential matches. Given that information processing is costly, agents optimally choose a search strategy: a distribution of attention over all possible matches.[6]

When agents have infinite information processing capacity, i.e. costs of acquiring information are zero, they can perfectly identify mutual best matches, and the outcome of the model is that of the classical assignment model. In this case, each agent focuses her attention on a single counter-party, and the optimal strategy is infinitely precise. Likewise, when agents have no information processing capacity, i.e. costs of acquiring information are infinite, their attention is distributed uniformly over all possible matches, and the

[5]This paradigm was developed by Moen (1997) and has been extended in many directions. Among others, Shi (2002) considers the case of heterogeneity, Shimer (2005) accommodates for coordination failures, and Kircher (2009) incorporates simultaneous search. In these models of directed search, the decentralized equilibrium is constrained efficient. There are a few exceptions, such as Guerrieri (2008) and Anderson and Smith (2010), that do generate inefficient outcomes in dynamic settings.

[6]Equivalently, we could describe agents as receiving costly signals about potential matches and choosing not only the precision of these signals, but the whole probability distribution.

equilibrium outcome is random matching.

Our framework represents the middle ground connecting these two polar cases. When information processing capacity is finite, i.e. we assume that agents have finite costs of acquiring information, agents choose how to optimally distribute their attention. Our specification implies that it is costly to divert attention both *towards* a particular agent as well as *away from* a particular agent, which together form the notion of targeted search.

To gain intuition on how the information technology works, we start by considering a case where both sides of the market search actively, each worker can send one application and each firm can process one application. In Section 2.2 we relax this assumption and allow workers and firms to send and process multiple applications. In each case we show conditions for existence and uniqueness of the equilibrium, characterize the equilibrium in closed form, and check for efficiency of the equilibrium. We find that the equilibrium is generally inefficient because two sides of the market cannot simultaneously appropriate the whole surplus of the match.[7]

2.1 Two-sided one shot model

There are N workers indexed by $x \in \{1, ..., N\}$, who are actively applying to firms, and M firms indexed by $y \in \{1, ..., M\}$, which are actively searching for applicants. A match between worker x and firm y generates a surplus $f(x, y)$. If a firm and a worker match, the surplus is split between them by ex-post unilateral bargaining and we normalize the outside option of the worker and firm to zero. We denote the wage that the worker receives by $w_x(y)$ and the profit that the firm gets by $\pi_y(x)$. The surplus, wage and profit generated by any potential (x, y) match are known ex-ante to worker x and firm y.[8]

[7]In the appendix we analyze the case where participants on one side of the market actively search, while participants on the other side are passively waiting. We show that in that case a social planner can restore efficiency if she is willing to allocate the whole surplus of each match to the party that actively searches.

[8]The meeting process can be thought of as having two stages. In the first stage, links between workers and firms are formed. In the second stage they bargain over the surplus. Furthermore, we are not placing any restrictions on the surplus function. One could think of the special case where workers and firms are of high and low types, and the combinations of types determine the surplus.

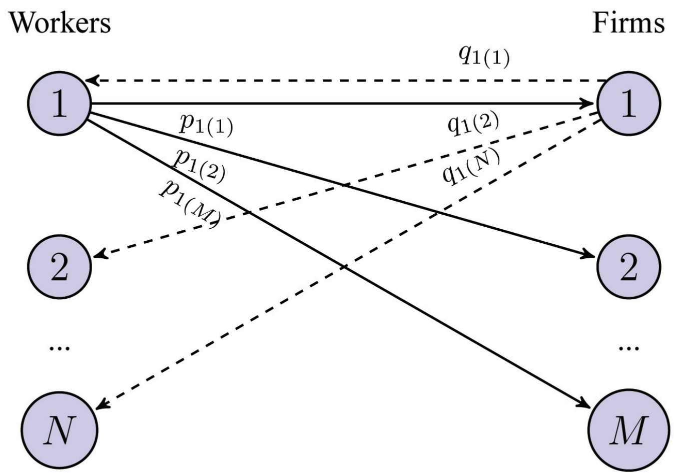

Figure 1. Strategies of workers and firms

Each worker chooses an application strategy which we denote $p_x(y)$. This strategy is the probability that worker x applies to firm y, and it represents the worker's distribution of attention. Each worker rationally chooses his strategy (i.e. the probability of sending an application to firm y) while facing a trade-off between a higher payoff and a higher cost of processing information. Likewise we denote the strategy of the a firm $q_y(x)$. It represents the probability of firm y considering an application of worker x, and it is the firm's distribution of attention. Similarly to workers, we assume that firms can rationally choose these strategies given potential gains and costs of search.

Figure 1 illustrates the strategies of firms and workers. Consider worker $x = 1$. The solid arrows show how she assigns a probability of sending an application to each firm y $(p_1(y))$. Similarly, dashed arrows show the probability that firm $y = 1$ assigns to processing an application from each worker x. As mentioned earlier, these probabilities constitute the distribution of attention $p_x(y)$ for workers and the distribution of attention $q_y(x)$ for firms. Once these are selected, each worker and each firm will make one draw from their respective distribution to determine which firm they will send an application to in the case of the workers and which application to look at in the case of the firms. A match is formed between worker x and firm y if and only if: 1) according to the worker's draw from $p_x(y)$, worker x applies to firm y; 2) according to the firm's draw from $q_y(x)$, firm y accepts applications from worker x; and 3) their payoffs are non-negative.

Since negative payoffs lead to de facto zero payoffs due to absence of a match, we can assume that all payoffs are non-negative:

$$f(x, y) \geq 0, \qquad \pi_y(x) \geq 0, \qquad w_x(y) \geq 0.$$

The worker's cost of searching is denoted by $c_x(\kappa_x)$. This cost is a function of the amount of information processed by worker x measured in bits, κ_x. Likewise, we denote a firm's cost of searching by $c_y(\kappa_y)$, where the cost is a function of the firm's information capacity, denoted κ_y.

Since information is freely available but costly to process, both workers and firms optimally choose how much effort they want to exercise in searching for their counterparts. We can think about these optimal strategies as a way of partitioning the set of potential matches into subsets and assigning to each subset a probability that reflects the likelihood of choosing somebody within that subset. Firms and workers simultaneously turn their attention to subsets that, according to the information processed, have greater probabilities of providing them with a better match. The choice of a subset is reflected in the choice of a distribution $p_x(y)$ for a worker and $q_y(x)$ for a firm. A random draw from the optimal strategy $p_x(y)$ determines the firm that worker x sends the application to. Similarly, a random draw from $q_y(x)$ determines the worker whose application firm y processes.

We denote $m_w(x, y)$ the equilibrium matching rate faced by worker x when applying to firm y. It represents the worker's perception of the probability that firm y is interested in worker x. Similarly, we denote $m_f(y, x)$ the equilibrium matching rate faced by firm y when considering worker x. As matching rates are equilibrium objects, they are assumed to be common knowledge to participating parties, and equal to the distribution of attention of the counter-party.

Each worker x chooses a strategy $p_x(y)$ to maximize his expected net income flow:

$$Y_x = \sum_{y=1}^{M} w_x(y)\, m_w(x, y)\, p_x(y) - c_x(\kappa_x) \to \max_{p_x(y)}.$$

The worker gets his expected wage from a match with firm y conditional on matching with that firm. The probability of a match between worker x

and firm y is the product of the probability that worker x sends an application to firm y and the probability that firm y is interested in worker x.

The worker incurs a search cost, $c_x(\kappa_x)$, which depends on the amount of information he processed, defined as follows:

$$\kappa_x = \sum_{y=1}^{M} p_x(y) \log_2 \frac{p_x(y)}{1/M}, \tag{1}$$

where the worker's strategy must satisfy $\sum_{y=1}^{M} p_x(y) = 1$ and $p_x(y) \geq 0$ for all y.

Our definition of information, κ_x, represents the relative entropy between a uniform prior $\{1/M\}$ over firms and the posterior strategy, $p_x(y)$. Shannon's relative entropy can be interpreted as the reduction in the uncertainty of finding a job that the worker can achieve by choosing his distribution of attention. This definition is a special case of Shannon's channel capacity where information structure is the only choice variable.[9] Thus, our assumption is a special case of a uniformly accepted definition of information tailored to our problem.

Similarly, firm y chooses her strategy $q_y(x)$ to maximize her expected income flow:

$$Y_y = \sum_{x=1}^{N} \pi_y(x) m_f(y,x) q_y(x) - c_y(\kappa_y) \to \max_{q_y(x)}.$$

The firm profits from a match with worker x conditional on matching with that worker and pays the cost of search. The search cost on the firm's side also depends on the amount of information processed:

$$\kappa_y = \sum_{x=1}^{N} q_y(x) \log_2 \frac{q_y(x)}{1/N}, \tag{2}$$

where the firm's strategy must satisfy $\sum_{x=1}^{N} q_y(x) = 1$ and $q_y(x) \geq 0$ for all x.

[9] See Thomas and Cover (1991), Chapter 2.

Definition 1. *A* matching equilibrium *is a set of strategies of workers,* $\{p_x(y)\}_{x=1}^{N}$*, and firms,* $\{q_y(x)\}_{y=1}^{M}$*, and matching rates* $\{m_w(x,y)\}_{x,y=1}^{N,M}$ *and* $\{m_f(y,x)\}_{x,y=1}^{N,M}$ *such that:*

1) strategies solve problems of the workers and the firms;

2) matching rates satisfy equilibrium conditions:

$$m_f(y,x) = p_x(y), \qquad m_w(x,y) = q_y(x). \tag{3}$$

Theorem 1. *A matching equilibrium exists.*

Proof. Note that if we substitute the matching rates (3) into the payoffs of workers and firms we can express the model as a normal-form game. The equilibrium of the matching model can be interpreted as a Nash equilibrium of this game. The set of distributions mapping compact sets into compact sets is a lattice under the natural ordering. Hence, all the results for lattices described by Vives (1990) apply to it. Since cross-derivatives of objective functions in our case are all non-negative, this game is super-modular. Hence there exists a Nash equilibrium. □

Theorem 2. *The matching equilibrium is unique, if*

a) cost functions are non-decreasing and convex;

b) $\left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^*(y)} > w_x(y)\, p_x^*(y)$*;*

c) $\left.\frac{\partial c_y(\kappa_y)}{\partial \kappa_y}\right|_{q_y^*(x)} > \pi_y(x)\, q_y^*(x)$*.*

Proof. The payoffs of all firms and workers are continuous in their strategies. They are also concave in these strategies when cost functions are (weakly) increasing and convex in information capacities. "Diagonal dominance" conditions (b) and (c) guarantee that the Hessian of the game is negative definite along the equilibrium path. Then, by the generalized Poincare-Hopf index theorem of Simsek, Ozdaglar and Acemoglu (2007), the equilibrium is unique. □

Note that the assumptions we make to prove uniqueness are by no means restrictive. The assumption that cost functions are non-decreasing and convex is a natural one. The additional "diagonal dominance" conditions in our case can be interpreted as implying that the marginal cost of information processing should be sufficiently high for the equilibrium to be unique. If these conditions don't hold, then there can be multiple equilibria. This is a well-known outcome of the assignment model, which is a special case of our model under zero marginal information costs.

The result of Theorem 2 is intuitive. There are two motives for worker x to target firm y. The first motive is that firm y may pay a higher wage compared with other firms. The second motive is that firm y may have a greater probability to reciprocate. The payoff of the worker depends on the product of the wage and the probability of reciprocation. While the wage (profit) motive does not depend on equilibrium strategies, the reciprocation motive does. When costs of information are very low, firms (and workers) are able to place a high probability of contacting one counter-party and exclude all others. As a result, when informaiton costs are extremely low, the reciprocation motive dominates. It does not matter what wage worker x will get from a match with firm y if the firm chooses not to consider worker x. When the reciprocation motive dominates, multiplicity of equilibria is a natural outcome. In the extreme, any pairing of agents is an equilibrium since nobody has an incentive to deviate from any mutual reciprocation.

As information costs increase, distributions of attention become less precise as it is increasingly costly to target a particular counter-party. That is, information processing constraints dampen the reciprocation motive and the wage motive starts playing a bigger role. At some threshold level of information costs each agent will be exactly indifferent between following the reciprocation motive and seeking a better match. This level of costs is precisely characterized by the "diagonal dominance" conditions of Theorem 2. They require the reciprocation motive, characterized by the off-diagonal element of the Hessian of the game, to be lower than the wage (profit) motive, captured by the diagonal element. Above the threshold the unique equilibrium has the property that each agent places a greater probability on the counter-party that promises a higher payoff, i.e. the wage (profit) motive dominates.

When cost functions are non-decreasing and convex, it is easy to verify that first-order conditions are necessary and sufficient conditions for equilibrium. Rearranging the first order conditions for the worker and the firm, we obtain:

$$p_x^*(y) = \exp\left(\frac{w_x(y)\, q_y^*(x)}{\frac{1}{\ln 2}\left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^*}}\right) \Big/ \sum_{y'=1}^{M} \exp\left(\frac{w_x(y')\, q_{y'}^*(x)}{\frac{1}{\ln 2}\left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^*}}\right),$$

$$q_y^*(x) = \exp\left(\frac{\pi_y(x)\, p_x^*(y)}{\frac{1}{\ln 2} \left.\frac{\partial c_y(\kappa_y)}{\partial \kappa_y}\right|_{q_y^*}}\right) / \sum_{x'=1}^{N} \exp\left(\frac{\pi_y(x')\, p_{x'}^*(y)}{\frac{1}{\ln 2} \left.\frac{\partial c_y(\kappa_y)}{\partial \kappa_y}\right|_{q_y^*}}\right). \tag{4}$$

These necessary and sufficient conditions for equilibrium cast the optimal strategy of worker x and firm y in the form of a best response to optimal strategies of firms and workers respectively.

Equilibrium conditions (4) have an intuitive interpretation. They predict that the higher the worker's private gain from matching with a firm, the greater the probability of applying to that firm. Similarly, the greater the probability that a firm considers a particular worker, the greater the probability that that worker applies to the firm. Overall, workers target better firms, i.e. firms that promise higher expected private gains, by placing greater probabilities on those firms. Firms are naturally sorted in each worker's strategy by the probability of applying to each firm. The strategies of firms have the same properties due to the symmetry of the problem.

In equilibrium, a firm's strategy is a best response to the strategies of workers, and a worker's strategy is a best response to the strategies of firms. Theorem 2 predicts that an increase in information costs reduces the complementarities between search strategies of workers and firms. Once costs of information are sufficiently high, the intersection of best responses leads to a unique equilibrium. Note that, by the nature of the index theorem used in the proof of uniquness, it is enough to check diagonal dominance conditions locally in the neighborhood of the equilibrium. There is no requirement for them to hold globally. This suggests a simple way of finding equilibria of our model in most interesting cases. We first need to find one solution to the first-order conditions (4) and then check that diagonal dominance conditions are satisfied.

Now, consider the properties of equilibria for two limiting cases. First, as the marginal costs of processing information go to zero, application and consideration strategies become more and more precise. In the limit, in every equilibrium each worker places a unit probability on a particular firm, and that firm responds with a unit probability of considering that worker. Each equilibrium of this kind implements a stable matching of the classical assignment problem.

Second, consider the opposite case when marginal costs go to infinity. In this case, optimal strategies of firms and workers approach a uniform distribution. This unique equilibrium implements the standard uniform random matching assumption extensively used in the literature. Thus, the assignment model and the random matching model are special cases of our model, when costs of information are either very low or very high.

Efficiency In order to evaluate the efficiency of the equilibrium we compare the solution of the decentralized problem to a social planner's solution. We assume that the social planner maximizes the total surplus of the economy, which is a utilitarian welfare function. In order to achieve a social optimum, the planner can choose the strategies of workers and firms. If no costs of processing information were present, the planner would always choose to match each worker with the job that produces the highest surplus. The socially optimal strategies of workers would be infinitely precise.

To study the constrained efficient allocation we impose upon the social planner the same information processing constraints that we place on workers and firms. Thus, the social planner maximizes the following welfare function:

$$W = \sum_{x=1}^{N}\sum_{y=1}^{M} f(x,y)\, p_x(y)\, q_y(x) - \sum_{x=1}^{N} c_x(\kappa_x) - \sum_{y=1}^{M} c_y(\kappa_y)$$

subject to information constraints (1-2) and to the constraints that $p_x(y)$ and $q_y(x)$ are well-defined probability distributions.

Under the assumption of increasing convex cost functions, the social welfare function is concave in the strategies of workers and firms. Hence, first-order conditions are necessary and sufficient conditions for a maximum. Rearranging and substituting out Lagrange multipliers, we arrive at the following characterization of the social planner's allocation:

$$\begin{aligned}
p_x^o(y) &= \exp\left(\frac{f(x,y)\, q_y^o(x)}{\frac{1}{\ln 2}\left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^o}}\right) \Big/ \sum_{y'=1}^{M} \exp\left(\frac{f(x,y')\, q_{y'}^o(x)}{\frac{1}{\ln 2}\left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^o}}\right), \\
q_y^o(x) &= \exp\left(\frac{f(x,y)\, p_x^o(y)}{\frac{1}{\ln 2}\left.\frac{\partial c_y(\kappa_y)}{\partial \kappa_y}\right|_{q_y^o}}\right) \Big/ \sum_{x'=1}^{N} \exp\left(\frac{f(x',y)\, p_{x'}^o(y)}{\frac{1}{\ln 2}\left.\frac{\partial c_y(\kappa_y)}{\partial \kappa_y}\right|_{q_y^o}}\right).
\end{aligned} \tag{5}$$

The structure of the social planner's solution is very similar to the structure of the decentralized equilibrium given by (4). From the workers' perspective, the only difference between the two strategies is that the probability of applying to a firm depends on the social gain from a match rather than on her private gain. Notice that the same difference holds from the perspective of the firm. Thus, it is socially optimal for both firms and workers to consider the total surplus, while in the decentralized equilibrium they only consider their private gains.

This result is reminiscent of goods with positive externalities where the producer undersupplies the good if she is not fully compensated by the marginal social benefits that an additional unit of the good would provide to society. As we shall see in greater detail in the following subsections, in our model, additional search effort exerted by an individual worker or firm has a positive externality on the whole matching market.

For instance, when a worker chooses to increase her search effort, she can better identify her preferable matches. As a consequence, the firms she targets will benefit (through an increase in the personal matching rate), and the firms that she does not target, will also be better off as her more targeted strategy will help them exclude her from their search (through a decrease in the personal matching rate). Nevertheless, in this environment the worker can not appropriate all the social benefits (the surplus of a match) she provides to society when increasing her search effort. The worker only gets her bargained share of the surplus. The same statement is true for the firms. This failure of the market to fully compensate both firms and workers with their social marginal products leads to under-supply of search effort by both sides in the decentralized equilibrium.

Because the social gain is always the sum of private gains, there is no feasible way of splitting the surplus such that it implements the social optimum. When information costs are finite and positive, a socially optimal equilibrium has to satisfy the following conditions simultaneously:

$$\pi_y(x) = f(x,y), \qquad w_x(y) = f(x,y).$$

In the presence of heterogeneity, these optimality conditions can only hold in equilibrium if the surplus is zero, as private gains have to add up to the total surplus, $\pi_y(x) + w_x(y) = f(x,y)$. Therefore, we have just proven the following theorem:

Theorem 3. *The matching equilibrium is socially inefficient for any split of the surplus if all of the following hold:*

1) cost functions are increasing and convex;

2) $f(x,y) > 0$ for some (x,y);

3) $f(x,y) \neq f(x,y')$ *for some* x, y *and* y';
4) $f(x,y) \neq f(x',y)$ *for some* y, x *and* x';
5) $0 < \left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^*} < \infty$;
6) $0 < \left.\frac{\partial c_y(\kappa_y)}{\partial \kappa_y}\right|_{q_y^*} < \infty$.

Proof. See Appendix A. □

The first two conditions are self-explanatory; the case when all potential matches yield zero surplus is a trivial case of no gains from matching. Conditions 5 and 6 state that marginal costs of information have to be finite and positive in the neighborhood of the equilibrium. When costs of information are zero, the best equilibrium of the assignment model is socially optimal. When costs of information are very high, the random matching outcome is the best possible outcome. For all intermediate values of costs the decentralized equilibrium is socially inefficient.

Conditions 3 and 4 together require heterogeneity to be two-sided. If heterogeneity is one-sided, i.e. condition 3 or condition 4 is violated, then the allocation of attention towards the homogeneous side of the market will be uniform. In this case, search becomes one-sided and equilibrium allocations are efficient contingent on one side having all the bargaining power.

One notable property of the equilibrium is that, by considering only fractions of the total surplus in choosing their strategies, workers and firms place lower probabilities on applying to their best matches. This implies that in equilibrium, attention of workers and firms is more dispersed and the number of matches is lower than is socially optimal.

Another way of thinking about the inefficient quantity of matches is to consider the reduction in strategic complementarities. To illustrate these complementarities consider the case of a firm that chooses its strategy under the assumption that all workers implement socially optimal application strategies. Because the firm only considers its private gains from matching with a worker, the firm's optimal response would be to pay less attention to (target less accurately) the best workers than it is socially optimal. In a second step, taking as given these strategies of firms, workers will be dis-incentivized not only by the fact that they consider fractions of the total gains from a match, but also by the fact that firms pay less attention to them than it is socially optimal. These complementary dis-incentives will lower the probabilities of workers applying to their best match. Iterating in this way on strategies of workers and firms, at each step we get a reduction in the probability of applying to the best matches. As a result, agents will target their more promising matches instead of the best possible matches.

The multiplied effects of considering only private gains by workers and firms reinforce each other through strategic best responses of workers to firms and firms to workers. Thus, we have uncovered a major source of inefficiency in the matching process. Information processing constraints weaken strategic complementarities between strategies of workers and firms. By the same token they reduce synergies from cooperation and lead to an under-supply of search effort. As a consequence, firms and workers fail to fully internalize the gains from coordination.

The inefficiency that arises in the two-sided model can in principle be corrected by a central planner. This can be done by promising both workers and firms that they will get the whole surplus of each match and then collecting lump-sum taxes from both sides of the market to cover the costs of the program. Nevertheless, in order to do so, the planner himself would need to acquire extensive knowledge about the distribution of the surplus, which is costly. We leave this direction of research for future work.

2.2 Two-sided model with endogenous search intensity

In this subsection, we consider an extension of our two-sided search model where firms and workers can simultaneously choose their distribution of attention and search intensity. Through this extension we want to characterize the role that information processing constraints play in determining the efficiency of the quantity and the quality of matches. As we shall see, while a model where search is costly generates an inefficiency in the quantity of matches irrespective of the source of search costs, a model with information-processing constraints can rationalize the inefficiency in both quality and quantity.

By optimally choosing a search intensity, workers decide how many applications to submit per unit of time, and firms choose how many applications to process per unit of time. Here, we consider a period of time comparable in length to the time period allotted for the decision of whether to accept or reject an offer of employment. We choose a sufficiently short period of time such that firms' and workers' choices of search intensity capture their decisions concerning the frequency of sending an application in the case of workers, and of processing an application in the case of firms. This assumption helps us abstract from issues of multilateral bargaining[10] by making it highly unlikely for a firm to receive more than one application in a period of time.

To account for the endogenous choice of search intensity by workers, we extend the strategy of each worker $p_x(y)$ to allow for the additional event of not sending an application. We denote the extended strategy of a worker of type x by $\hat{p}_x(y)$ on $y = \{0, 1, ..., M\}$, where $y = \{1, ..., M\}$ denotes events in which the worker sends an application to a firm of type y and $y = 0$ denotes the event in which the worker does not send an application. As before, the elements of the strategy must be non-negative and sum up to one. Each element of the strategy $\hat{p}_x(y)$ with a positive index denotes the probability of worker x sending an application to firm y. The complementary probability $\hat{p}_x(0) = 1 - \sum_{y=1}^{M} \hat{p}_x(y)$ is the probability that the worker does not send an application. We denote the partial sum $\sum_{y=1}^{M} \hat{p}_x(y) = \alpha_x$ and refer to it as the search intensity since it refers to the probability of sending an application. Thus, search intensity α_x is a deterministic function of the strategy $\hat{p}_x(y)$. We denote the normalized probability distribution $p_x(y) = \hat{p}_x(y)/\alpha_x$.

As in the previous subsection, worker x chooses his strategy $\hat{p}_x(y)$ to maximize his expected income flow:

$$Y_x = \sum_{y=1}^{M} w_x(y)\, m_w(x, y)\, \hat{p}_x(y) - c_x(\alpha_x, \kappa_x) \to \max_{\hat{p}_x(y)}.$$

Now, we assume that the cost of search $c_x(\alpha_x, \kappa_x)$ has two components:

$$c_x(\alpha_x, \kappa_x) = \alpha_x \frac{\chi_x}{\phi_x + 1} (\alpha_x)^{\phi_x} + \alpha_x \theta_x \kappa_x, \tag{6}$$

[10] In a recent paper Gautier and Holzner (2013) study ex-post wage competition on a network where firms and workers form multiple links in the first stage and negotiate wages subject to coordination frictions in the second stage. They show that conditional on a network, a simple ex-post wage mechanism can implement the efficient matching pattern. Our paper describes a specific network formation game that uses but does not place any restrictions on the outcome of ex-post negotiations. Thus, our main results should naturally extend to a framework where ex-post multilateral bargaining is efficient, as described by Gautier and Holzner (2013).

where χ_x, ϕ_x, θ_x are non-negative parameters. The first component of (6) represents the physical cost of sending an application as it is an increasing function of search intensity, α_x. We assume that increasing the frequency of sending applications implies a weak increase in the physical cost per application. The second component of (6) is the linear cost of processing κ_x bits of information conditional on choosing to send an application, α_x. The amount of information, κ_x, is defined as follows:

$$\kappa_x = \sum_{y=1}^{M} p_x(y) \log_2 \frac{p_x(y)}{1/M}. \tag{7}$$

As before, our definition of information, κ_x, represents the relative entropy between a uniform prior $\{1/M\}$ over all firms and the posterior normalized distribution of attention, $p_x(y)$.

Similarly to the worker, firm y chooses her strategy $\hat{q}_y(x) = \gamma_y q_y(x)$, where γ_y is the firm's search intensity, to maximize her expected income flow:

$$Y_y = \sum_{x=1}^{N} \pi_y(x)\, m_f(y,x)\, \hat{q}_y(x) - c_y(\gamma_y, \kappa_y) \to \max_{\hat{q}_y(x)}.$$

Like the worker, the firm's cost includes a physical cost of considering applications and an information cost:

$$c_y(\gamma_y, \kappa_y) = \gamma_y \frac{\chi_y}{\phi_y + 1} (\gamma_y)^{\phi_y} + \gamma_y \theta_y \kappa_y \tag{8}$$

where χ_y, ϕ_y, θ_y are non-negative parameters. The amount of information, κ_y, processed by firm y is defined as follows:

$$\kappa_y = \sum_{x=1}^{N} q_y(x) \log_2 \frac{q_y(x)}{1/M}. \tag{9}$$

Definition 2. *A* matching equilibrium *is a set of strategies of workers, $\{\hat{p}_x(y)\}_{x=1}^{N}$, and firms, $\{\hat{q}_y(x)\}_{y=1}^{M}$, and matching rates $\{m_w(x,y)\}_{x,y=1}^{N,M}$ and $\{m_f(y,x)\}_{x,y=1}^{N,M}$ such that:*

1) strategies solve problems of the workers and the firms;

2) matching rates satisfy equilibrium conditions:

$$m_f(y,x) = \hat{p}_x(y), \qquad m_w(x,y) = \hat{q}_y(x). \tag{10}$$

Theorem 4. *A matching equilibrium exists.*

Proof. Note that if we substitute the matching rates (10) into the payoffs of workers and firms we can express the model as a normal-form game. The equilibrium of the matching model can be interpreted as a Nash equilibrium of this game. The set of distributions mapping compact sets into compact sets is a lattice under the natural ordering. Hence, all the results for lattices described by Vives (1990) apply to it. Since cross-derivatives of objective functions in our case are all non-negative, this game is super-modular.

$$\frac{\partial^2 Y_x\left(\hat{p}_x, \hat{q}_y\right)}{\partial \hat{p}_x \partial \hat{q}_y} = w_x\left(y\right) \geq 0 \qquad \frac{\partial^2 Y_y\left(\hat{q}_y, \hat{p}_x\right)}{\partial \hat{q}_y \partial \hat{p}_x} = \pi_y\left(x\right) \geq 0$$

Hence there exists a Nash equilibrium. □

When cost functions are non-decreasing and convex, it is easy to verify that first-order conditions are necessary and sufficient conditions for equilibrium. Rearranging the first order conditions for the worker and the firm, we obtain:

$$\begin{aligned}
p_x^*\left(y\right) &= \exp\left(\frac{w_x\left(y\right) q_y^*\left(x\right)\gamma_y^*}{\theta_x/\ln 2}\right) / \sum_{y'=1}^{M} \exp\left(\frac{w_x\left(y'\right) q_{y'}^*\left(x\right)\gamma_{y'}^*}{\theta_x/\ln 2}\right), \\
\left(\alpha_x^*\right)^{\phi_x} &= \frac{\theta_x/\ln 2}{\chi_x} \ln\left(\sum_{y=1}^{M} \exp\left(\frac{w_x\left(y\right) q_y^*\left(x\right)\gamma_y^*}{\theta_x/\ln 2}\right)/M\right), \\
q_y^*\left(x\right) &= \exp\left(\frac{\pi_y\left(x\right) p_x^*\left(y\right)\alpha_x^*}{\theta_y/\ln 2}\right) / \sum_{x'=1}^{N} \exp\left(\frac{\pi_y\left(x'\right) p_{x'}^*\left(y\right)\alpha_{x'}^*}{\theta_y/\ln 2}\right), \\
\left(\gamma_y^*\right)^{\phi_y} &= \frac{\theta_y/\ln 2}{\chi_y} \ln\left(\sum_{x=1}^{N} \exp\left(\frac{\pi_y\left(x\right) p_x^*\left(y\right)\alpha_x^*}{\theta_y/\ln 2}\right)/N\right).
\end{aligned} \tag{11}$$

These sufficient conditions for equilibrium cast the optimal strategy of worker x and firm y in the form of a best response to optimal strategies of firms and workers respectively.

Theorem 5. *The matching equilibrium is unique, if*

a) costs parameters θ_x, θ_y, χ_x, χ_y are non-negative, costs of applications are convex: $\phi_x \geq 0$, $\phi_y \geq 0$.

b) "Diagonal dominance" conditions are satisfied along the equilibrium path:

$$\left|\frac{\partial^2 Y_x\left(\hat{p}_x,\hat{q}_y\right)}{\partial\hat{p}_x\partial\hat{p}_x}\right|_{\hat{p}_x^*,\hat{q}_y^*} > \left|\frac{\partial^2 Y_x\left(\hat{p}_x,\hat{q}_y\right)}{\partial\hat{p}_x\partial\hat{q}_y}\right|_{\hat{p}_x^*,\hat{q}_y^*}$$
$$\left|\frac{\partial^2 Y_y\left(\hat{q}_y,\hat{p}_x\right)}{\partial\hat{q}_y\partial\hat{q}_y}\right|_{\hat{p}_x^*,\hat{q}_y^*} > \left|\frac{\partial^2 Y_y\left(\hat{q}_y,\hat{p}_x\right)}{\partial\hat{q}_y\partial\hat{p}_x}\right|_{\hat{p}_x^*,\hat{q}_y^*}$$

Proof. The payoffs of all firms and workers are continuous in their strategies. They are also concave in these strategies when cost functions are (weakly) increasing and convex in information capacities. "Diagonal dominance" conditions guarantee that the Hessian of the game is negative definite along the equilibrium path. Then, by the generalized Poincare-Hopf index theorem of Simsek, Ozdaglar and Acemoglu (2007), the equilibrium is unique. □

As before, the assumptions we make to prove uniqueness are not restrictive. The assumption that cost functions are non-decreasing and convex is a natural one. Diagonal dominance conditions require costs of search to be high enough for equilibrium to be unique. More specifically, the diagonal dominance conditions can be rewritten as follows:

$$\frac{\theta_x}{\ln 2}\frac{\alpha_x^*}{p_x^*\left(y\right)}\left(1-p_x^*\left(y\right)\right)+\chi_x\phi_x\left(\alpha_x^*\right)^{\phi_x-1} > w_x\left(y\right),$$
$$\frac{\theta_y}{\ln 2}\frac{\gamma_y^*}{q_y^*\left(x\right)}\left(1-q_y^*\left(x\right)\right)+\chi_y\phi_y\left(\gamma_y^*\right)^{\phi_y-1} > \pi_y\left(x\right).$$

In the general case, high information costs, or high physical costs, or a combinaiton of the two guarantees uniqueness of equilibrium. If these conditions are violated, then there can be multiple equilibria. As in Section 2.1, by the nature of the index theorem used in the proof, it is enough to check diagonal dominance conditions locally in the neighborhood of the equilibrium. Hence, to find the equilibrium of the model we need to solve the first-order conditions (11) and then check that diagonal dominance conditions are satisfied for this solution.

Like in the one-shot model, equilibrium conditions (11) have an intuitive interpretation. They predict that the higher the worker's private gain from matching with a firm, the greater the probability of applying to that firm. Similarly, the greater the probability that a firm considers a particular worker, the greater the probability that that worker applies to the firm. As workers target better firms, firms are naturally sorted in each worker's strategy by the probability of applying to each firm.

Efficiency In order to evaluate the efficiency of the equilibrium we compare the solution of the decentralized problem to a social planner's solution. In order to achieve a social optimum, the planner can choose strategies of workers and firms. We assume that the social planner maximizes the total surplus of the economy. The social planner takes into account the costs of search borne by workers and firms. Thus, the social planner maximizes the following welfare function:

$$W = \sum_{x=1}^{N}\sum_{y=1}^{M} f(x,y)\,\hat{p}_x(y)\,\hat{q}_y(x) - \sum_{x=1}^{N} c_x(\alpha_x,\kappa_x) - \sum_{y=1}^{M} c_y(\gamma_y,\kappa_y),$$

subject to cost functions (6,8).

Under the assumption of increasing convex cost functions, the social welfare function is concave in the strategies of workers and firms. Hence, first-order conditions are necessary and sufficient conditions for a maximum. Rearranging, we arrive at the following characterization of the social planner's allocation:

$$\begin{aligned} p_x^o(y) &= \exp\left(\frac{f(x,y)\,q_y^o(x)\,\gamma_y^o}{\theta_x/\ln 2}\right) / \sum_{y'=1}^{M} \exp\left(\frac{f(x,y')\,q_{y'}^o(x)\,\gamma_{y'}^o}{\theta_x/\ln 2}\right), \\ (\alpha_x^o)^{\phi_x} &= \frac{\theta_x/\ln 2}{\chi_x} \ln\left(\sum_{y=1}^{M} \exp\left(\frac{f(x,y)\,q_y^o(x)\,\gamma_y^o}{\theta_x/\ln 2}\right)/M\right), \end{aligned}$$

$$\begin{aligned} q_y^o(x) &= \exp\left(\frac{f(x,y)\,p_x^o(y)\,\alpha_x^o}{\theta_y/\ln 2}\right) / \sum_{x'=1}^{N} \exp\left(\frac{f(x',y)\,p_{x'}^o(y)\,\alpha_{x'}^o}{\theta_y/\ln 2}\right), \\ \left(\gamma_y^o\right)^{\phi_y} &= \frac{\theta_y/\ln 2}{\chi_y} \ln\left(\sum_{x=1}^{N} \exp\left(\frac{f(x,y)\,p_x^o(y)\,\alpha_x^o}{\theta_y/\ln 2}\right)/N\right). \end{aligned} \quad (12)$$

Like in the one-shot model, the structure of the social planner's solution is very similar to the structure of the decentralized equilibrium given by (11). From the workers' perspective, the only difference between the two strategies is that the probability of applying to a firm depends on the social gain from a match rather than on her private gain. The same difference holds from the perspective of the firm. Thus, it is socially optimal for both firms and workers to consider the total surplus, while in the decentralized equilibrium they only consider their private gains.

The positive externality now works along both the intensive and extensive margins of search. When a worker chooses to target more accurately a type of firm that is a better match for him, this benefits the firms he targets but does not affect anybody else. A more targeted strategy requires both more information to be processed and more applications to be filled out. Thus, a more targeted strategy implies an increase in both physical and information costs. But the matching market does not compensate the worker with all the social benefits she provides to society by increasing her search effort. The matching market compensates the worker with only a fraction of those benefits (the bargained share of the surplus). This failure of the market to fully compensate the worker with his social marginal product leads to undersupply of search effort both along the intensive and the extensive margins.

Our main qualitative result can be stated as follows. When search is two-sided, and search is costly, in a decentralized equilibrium, both sides of the market supply an insufficient amount of search effort compared with the social optimum. As we illustrate with our numerical example, this applies to both margins of search.

2.3 Quantity, quality and efficiency of matches

In the one-shot model of Section 2.1, the cost function of the agents depends only on the costs of processing information. When we introduce search intensity in Section 2.2, the cost function has two components; the physical cost of sending an application and the cost of processing information to decide where to send it. In this Section, we illustrate the differential effect of these two types of costs on the quantity and quality of matches. First, we demonstrate that a model with physical costs of sending an application also generates an inefficient equilibrium, but it only generates an inefficiency in the quantity of matches and not in their quality. Second, we illustrate what happens when there are only information costs.

Consider the case where only physical costs are present, but there are no information costs ($\theta \to 0$). In this case, nothing restricts the agents from identifying their best matches but they choose not to contact them frequently enough, as a result of physical costs. This leads to a suboptimal equilibrium matching rate. Thus, physical costs of search imply an inefficiently low quantity of matches without a loss in their quality.

To see this more clearly, we consider the model presented in Section 2.2, and constrain the number of participants to a single worker and a single firm. In this case the problem of the worker and the firm are the following:

$$
\begin{aligned}
w\gamma\alpha - \frac{\chi_w}{\phi_w + 1}\alpha^{\phi_w+1} &\to \max_{\alpha}, \\
\pi\alpha\gamma - \frac{\chi_f}{\phi_f + 1}\gamma^{\phi_f+1} &\to \max_{\gamma}.
\end{aligned}
$$

These problems illustrate the positive externality that agents have on each other when choosing their search intensities. If a firm were to increase γ, it would directly increase the expected income of the worker. In turn, if the worker were to increase α, it would benefit the firm. The first-order conditions of the worker and the firm, by equalizing the marginal private gains and losses from an additional unit of search intensity, lead to a unique solution:

$$
\begin{aligned}
\alpha^* &= \left(\frac{\pi}{\chi_f}\left(\frac{w}{\chi_w}\right)^{\phi_f}\right)^{\frac{1}{\phi_w\phi_f - 1}}, \\
\gamma^* &= \left(\left(\frac{\pi}{\chi_f}\right)^{\phi_w}\frac{w}{\chi_w}\right)^{\frac{1}{\phi_w\phi_f - 1}}.
\end{aligned}
$$

In contrast, the planner's problem takes into account the overall welfare by maximizing:

$$
f\alpha\gamma - \frac{\chi_w}{\phi_w + 1}\alpha^{\phi_w+1} - \frac{\chi_f}{\phi_f + 1}\gamma^{\phi_f+1} \to \max_{\alpha,\gamma},
$$

and implies the following optimal levels of search effort:

$$
\begin{aligned}
\alpha^o &= \left(\frac{f}{\chi_f}\left(\frac{f}{\chi_w}\right)^{\phi_f}\right)^{\frac{1}{\phi_w\phi_f - 1}}, \\
\gamma^o &= \left(\left(\frac{f}{\chi_f}\right)^{\phi_w}\frac{f}{\chi_w}\right)^{\frac{1}{\phi_w\phi_f - 1}}.
\end{aligned}
$$

The planner's problem equalizes the marginal costs of search with the marginal social gains, which include both the private gains and the external effects of agents' actions. The consequence of this is a higher level of search intensity for both workes and firms, and a higher aggregate matching rate. More precisely, since the social gain, f, is always greater than private gains, w and π, when cost functions are sufficiently convex, $\phi_w > 1$, $\phi_f > 1$, the number of matches in equilibrium, $\alpha^*\gamma^*$, is suboptimal, i.e. lower than $\alpha^o\gamma^o$. As we just saw from the problem of the worker and the firm, the inefficiency comes from the fact that an increase in α benefits the firm, and an increase in γ benefits the worker. However, the firm does not internalize the externality that it generates on the worker's welfare and vice versa. As a result, both the worker and the firm supply an inefficiently low amount of search effort, but once the match is formed its quality is unaffected.

To illustrate this more generally, and to compare with the model presented in Section 2.1 (where only information processing is costly), we next assume that there are two types of workers and two types of firms; one worker and one firm of each type. For illustrative purposes, we envision that skill sets of workers and firms are complementary, such that type 1 worker better fits type 1 firm and type 2 worker better fits type 2 firm. This is captured by the following surplus matrix:

$$f = \begin{bmatrix} 1 & 0.1 \\ 0.1 & 1 \end{bmatrix}$$

We assume that if a worker and a firm meet, they split the surplus in equal proportions. We use the first-order conditions of the model to compute the decentralized equilibria and social optimum for different values of the information cost parameter, θ, and for different values of the convexity of the cost function, ϕ. We assume that both physical and information costs are identical across workers and firms. Note that when condition (b) of Theorem 5 is violated, the decentralized model displays multiple equilibria. However, in this simple numerical example we can bound the equilibria by computing the best and worst outcomes.

For each case, we compute the welfare function, W, defined earlier, the expected number of matches, EM, the average match quality, Q, the average information capacity, $E\kappa$, and the average search intensity, $E\alpha$ (note that firms and workers are symmetric), defined as follows:

$$EM = \sum_{x=1}^{N}\sum_{y=1}^{M} \hat{p}_x(y)\,\hat{q}_y(x)\,,$$

$$Q = \sum_{x=1}^{N}\sum_{y=1}^{M} f(x,y)\,\hat{p}_x(y)\,\hat{q}_y(x) \Bigg/ \sum_{x=1}^{N}\sum_{y=1}^{M} \hat{p}_x(y)\,\hat{q}_y(x),$$

$$E\kappa = \left(\sum_{x=1}^{N}\kappa_x + \sum_{y=1}^{M}\kappa_y\right) \Bigg/ (N+M).$$

$$E\alpha = \left(\sum_{x=1}^{N}\alpha_x + \sum_{y=1}^{M}\gamma_y\right) \Bigg/ (N+M).$$

Figures 2 and 3 show the results for the competitive equilibrium (CE) and the social optimum (PO) for the case where there are only physical costs of sending an application (Figure 2) and for the case where there are only information processing costs (Figure 3).

Figure 2 illustrates the case where there are only physical costs of sending an application ($\theta \to 0$). In this case, the best competitive equilibrium matches type 1 worker with type 1 firm, and type 2 worker with type 2 firm. The choice of search intensity by each worker and each firm imposes a positive externality on their best matches. This leads to an inefficient choice of contact rates by both firms and workers. However, only the quantity of matches is affected, as only best matches are contacted in equilibrium.

Figure 3 shows that the complementarity between search strategies of workers and firms leads to multiple competitive equilibria when the costs of processing information are low enough so as to violate the diagonal dominance condition. Below the threshold of $\theta = 0.14$, the worst equilibrium matches type 1 worker with type 2 firm, while the best equilibrium matches type 1 worker with type 1 firm. All equilibria except the best one vanish above the cost threshold. Figure 3 also shows that an increase in information costs leads to a decline in both quantity and quality of matches in all equilibria and in the socially optimal allocation. The positive externalities imposed by search strategies of workers and firms play a key role in the constrained inefficiency of equilibria and in the decline of the matching rate.

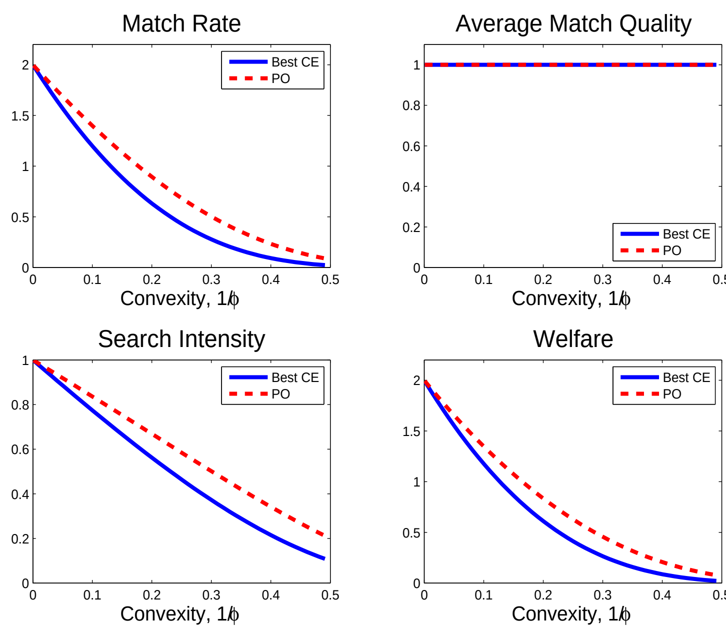

Figure 2. Two-sided Model with Information Costs Absent, $(\theta \to 0)$

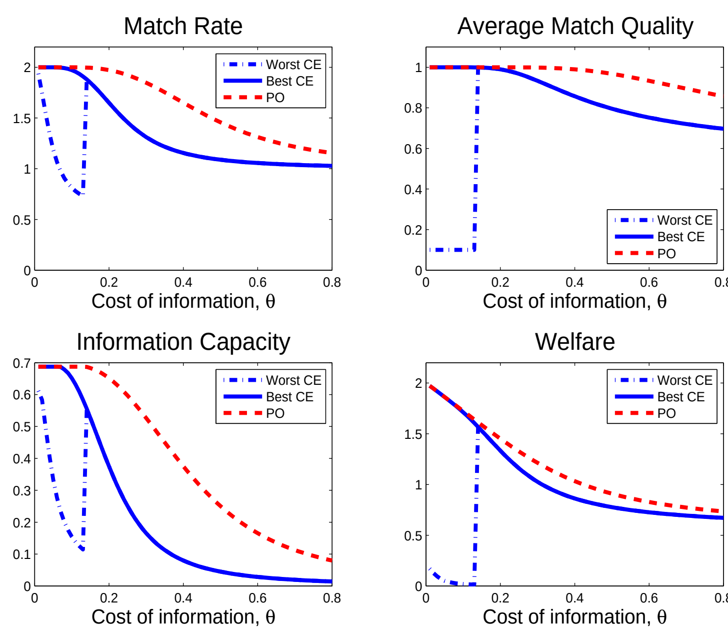

Figure 3. Two-sided Model with Maximum Intensity, $(\phi \to \infty)$

3 Extensions and applications

In this section we start by describing possible extensions of the model. Then we discuss how different markets can be modeled using the different versions of our framework. We point out the relationship between different properties of markets and the corresponding choice of model.

3.1 Extensions

The model presented in Section 2 is revealing but parsimonious. In the Appendix we extend our model to a continuous-time framework with a continuum of workers and firms, multiple applications and a more realistic meeting protocol. The general model allows market participants to choose how many counter-parties to contact and accounts for the possibility of repeated interactions. The meeting protocol modulates congestion and can lead to constant returns to scale. We show the assumptions that are necessary to derive a constant-returns-to-scale matching function and to solve the model in close form. All the qualitative results from Section 2 carry through to this richer environment. We show that under similar conditions the equilibrium of the general model is unique and conjecture that it is generally inefficient.

In addition, we consider a one-sided search model, where one side actively searches, and the other idly awaits to be contacted. In the one-sided model, the number of matches is unaffected by changes in costs of information, because each worker sends a single application which is accepted with a constant probability. For all values of information costs, the competitive equilibrium is unique, but constrained inefficient. Because workers get only a fraction of the surplus, they put a socially suboptimal amount of effort into search. As a consequence, they do not target their best matches well enough.

Although this does not lead to a reduction in the number of matches, average match quality suffers. As processing information becomes more costly, the quality of matches falls, which leads to a decline in welfare. The inefficiency in the model can be solved if the actively searching side of the market appropriates the whole surplus of the match.

3.2 Applications

Modeling search frictions with information-processing constraints might be a useful representation of many types of markets where the equilibrium outcome is neither random matching nor mutual best matches as predicted by the classical assignment model. We can categorize these markets according to the interaction of six main factors.

The first factor is the number of participants on each side of the market. The larger the number of participants is the higher is the effective cost of information. An asymmetry in the number of participants will determine if search is one-sided or two-sided. For instance, if the number of participants on one side of the market is restricted to one or two, then the cost of search for the other side is relatively low, and search is one-sided. One example of a one-sided search effort is grocery shopping, while search in the commercial loans market is an example of two-sided search.

The second factor is the degree of heterogeneity among participants. If participants on one side of the market are equally valuable to participants on the other side, then all matches are equally beneficial. In this case, only the homogeneous side of the market will actively search. However, if agents on both sides of the market are heterogeneous, search is two-sided. An example of a homogeneous good leading to one-sided search is the consumer electricity market.

The third factor is whether both demand per buyer and supply per seller are limited. If a product can be produced by the same supplier in unrestricted quantities, then it can satisfy any demand and has no incentive to actively search for customers. In this case, search is one-sided. Credit cards are an example of unlimited supply, while the marriage market has symmetric limitations. Restaurants can accommodate a finite number of eaters which places them in the middle of the spectrum.

The fourth factor that affects search is the period of time for which a potential match remains beneficial, i.e. the durability of the surplus. Durability effectively lowers the costs of information processing by making search less urgent. The housing rental market is an example of durable surplus. The typical search period is long enough to find out all the possible options. While the market for human organs is a case where a delay makes a match obsolete.[11]

The fifth factor is the structure of information flow. Our model applies to markets where information flow is unrestricted on either side of the market. The only restriction posed is on the capacity of participants to process information about the other side. This assumption is compatible with markets

[11]See Roth, Sönmez and Ünver(2007) for the case of kidney exchange.

where participants on one side of the market endogenously choose to process more information than on the other. For instance, a student applying for public school in the U.S. may choose to process more information about under-chosen schools in his district than his peers to increase the odds of being selected.[12] This endogenously chosen information advantage agrees with the set-up of our model.

In contrast, our assumption of unrestricted information flow makes our model not directly applicable to markets where asymmetric information arises from ex-ante restrictions on information flow on either side of the market. For instance, markets for used goods and mortgage loans[13] are better captured by models of private information. However, our model can be nested into models of search with asymmetric information.[14]

The sixth factor is the degree of centralization in the market. By centralization we mean a situation when an organization or a platform facilitates search by structuring the information flow and setting the rules for interaction. Our model describes markets where the degree of centralization is fairly low. This structure encompasses a number of markets ranging from labor markets to education and health care.[15] In contrast, our model does not directly apply to markets where the degree of centralization is fairly high as in the case of football bowls, college admissions, market for physicians, and two-sided platform markets.[16] Specifically, in two-sided market models the platform acts both as a coordination device and as a mechanism of surplus transfers. Our model can be used to study the optimal degree of centralization and the social efficiency of pricing schemes in these markets. We leave the study of the optimal design of centralization in two-sided search environments for future research.

[12]See Schneider, Teske and Marschall (2000).

[13]Used goods markets are described in, e.g. Lewis (2011), while mortgage loans are documented in Woodward and Hall (2010).

[14]Such as the work of Guerrieri, Shimer and Wright (2010) and Guerrieri and Shimer (2012).

[15]For instance, the efficiency of the labor market for new economists is discussed in Coles, Cawley, Levine, Niederle, Roth, Siegfried (2010). For the case of the education market see, e.g., Ballou (2010). A study of the medicare advantage program is conducted by Brown, Duggan, Kuziemko and Woolston (2011).

[16]See Frechette, Roth and Ünver(2010) for the case of football bowls. College admissions are studied by Roth and Sotomayor (1989), the market for physicians by, e.g., Roth and Peranson (1999). Two-sided and multi-sided market models have been developed by Rochet and Tirole (2003) and Weyl (2010).

While most of the matching markets we mention above would not be well described by either random matching or classical assignment, they rest comfortably within the predictions of our model of targeted search.

4 Conclusion

We presented a matching model where participants have finite information-processing capacity. We established that such a model delivers an equilibrium that resides in between the outcomes of random matching models and optimal assignment models. Furthermore, if we assume information-processing costs to be high (infinite in the limit) the outcome of our model is observationally equivalent to that of a random matching model; whereas if information-processing costs are zero, our model reproduces the outcome of the optimal assignment model.

We showed that our model with information-processing constraints can produce a unique equilibrium. The uniqueness result comes from the fact that information-processing constraints weaken the complementarities in search strategies of participants on both sides of the market. By contrast, strong complementarities in search strategies usually generate multiplicity in directed search models.

Finally, we show that in our model the equilibrium quantity and quality of matches is generally inefficient. The inefficiency result comes from the interplay of information-processing constraints and two-sided heterogeneity. By contrast, as long as the Hosios condition holds, the random matching equilibrium is unique and efficient while in an optimal assignment model efficiency is achieved by refinement. Our model of targeted search fills the middle ground between these two cases, generating a unique inefficient equilibrium, that can potentially explain the quantity and quality of matches observed in different markets.

References

[1] **Anderson, A. and L. Smith, (2010)**. "Dynamic Matching and Evolving Reputations." *Review of Economic Studies*, 77(1), pp. 3-29.

[2] **D. Ballou, (1996).** "Do Public Schools Hire the Best Applicants?" *Quarterly Journal of. Economics*, 111(1), (February), pp. 97-133.

[3] **Brown, J., Duggan, M., Kuziemko, I. and W. Woolston, (2011).** "How does Risk-selection Respond to Risk-adjustment? Evidence from the Medicare Advantage Program." NBER Working Paper No. 16977.

[4] **Burdett, K., Shi, S. and R. Wright, (2001).** "Pricing and Matching with Frictions." *Journal of Political Economy*, 109(5), pp. 1060-1085.

[5] **Butters, G. R., (1977).** "Equilibrium distributions of sales and advertising prices." *Review of Economic Studies*, 44(3), pp. 465-491.

[6] **Coles, P., Cawley, J., Levine, P. B., Niederle, M. Roth, A. E. and J. J. Siegfried, (2010).** "The Job Market for New Economists: A Market Design Perspective." *Journal of Economic Perspectives*, 24(4), pp. 187–206.

[7] **Davis, S. J., Faberman, R. J. and J. C. Haltiwanger, (2013).** "The Establishment-Level Behavior of Vacancies and Hiring." *Quarterly Journal of Economics*, 128(2), pp. 581–622.

[8] **Diamond, P. A., (1982).** "Wage Determination and Efficiency in Search Equilibrium." *Review of Economic Studies*, 49(2), pp. 217-227.

[9] **Eeckhout, J. and P. Kircher, (2010).** "Sorting and Decentralized Price Competition." *Econometrica*, 78(2), pp. 539–574.

[10] **Frechette, G. R., Roth, A.E. and M. U. Ünver, (2007).** "Unraveling Yields Inefficient Matchings: Evidence from Post-Season College Football Bowls." *RAND Journal of Economics*, 38(4), pp. 967–982.

[11] **Gautier, P. A. and C. L. Holzner, (2013).** "Simultaneous Search and Network Efficiency." mimeo.

[12] **Guerrieri, V. (2008).** "Inefficient Unemployment Dynamics under Asymmetric Information." *Journal of Political Economy*, 116(4), pp. 667-708.

[13] **Guerrieri, V., Shimer, R. and R. Wright (2010).** "Adverse Selection in Competitive Search Equilibrium." *Econometrica*, 78(6), pp. 1823-1862.

[14] **Guerrieri V. and R. Shimer, (2012).** "Markets with Multidimensional Private Information." manuscript.

[15] **Hall, R. E. (1979).** "A Theory of the Natural Unemployment Rate and the Duration of Employment." *Journal of Monetary Economics*, 5, pp. 153-169.

[16] **Hosios, A. J. (1990).** "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies*, 57(2), pp. 279–98.

[17] **Kircher P. (2009)**. "Efficiency of Simultaneous Search." *Journal of Political Economy*, 117(5), pp. 861-913.

[18] **Lagos, R. (2000).** "An Alternative Approach to Search Frictions." *Journal of Political Economy*, 108(5), pp. 851–873.

[19] **Lester, B. (2011).** "Information and Prices with Capacity Constraints," *American Economic Review*, 101(4), pp. 1591-1600.

[20] **Lewis, G. (2011).** "Asymmetric Information, Adverse Selection and Seller Disclosure: The Case of eBay Motors," *American Economic Review*, 101(4), pp. 1535-1546.

[21] **Menzio, G. (2007).** "A Theory of Partially Directed Search," *Journal of Political Economy*, 115(5), pp. 748–769.

[22] **Moen, E. R. (1997).** "Competitive Search Equilibrium." *Journal of Political Economy*, 105(2), pp. 385–411.

[23] **Mortensen D.T., (2005).** "Wage Dispersion: Why Are Similar Workers Paid Differently?" The *MIT Press, 2005.*

[24] **Pissarides C.A., (1979).** "Job Matchings with State Employment Agencies and Random Search." *Economic Journal*, 89(356), pp. 818-833.

[25] **Rochet J.-C. and J. Tirole, (2003).** "Platform Competition in Two-Sided Markets." *Journal of the European Economic Association*, 1(4), pp. 990-1029.

[26] **Roth, A.E., and E. Peranson, (1999).** "The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design." *American Economic Review*, 89(4), pp. 748-780.

[27] **Roth, A. E., Sönmez, T. and M. U. Ünver (2007).** "Efficient Kidney Exchange: Coincidence of Wants in a Structured Market." *American Economic Review*, 97(3), pp. 828–851.

[28] **Roth, A. E. and M. Sotomayor, (1989).** "The College Admissions Problem Revisited." *Econometrica*, 57(3), pp. 559-570.

[29] **Roth, A. E. and M. Sotomayor, (1990).** "Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis." Econometric Society Monograph Series, Cambridge University Press, 1990.

[30] **Sahin, A., Song, J., Topa, G. and G. Violante, (2013).** "Mismatch Unemployment." Federal Reserve Bank of New York Staff Reports 566.

[31] **Schneider, M., Teske, P, and M. Marschall, (2000).** "Choosing Schools: Consumer Choice and the Quality of American Schools." Princeton, NJ: Princeton University Press.

[32] **Shi, S., (2002).** "A directed search model of inequality with heterogeneous skills and skill-biased technology," *Review of Economic Studies*, 69, pp. 467-491.

[33] **Shimer, R. and L. Smith, (2000).** "Assortative Matching and Search." *Econometrica*, 68(2), pp. 343-370.

[34] **Shimer, R. (2005).** "The Assignment of Workers to Jobs in an Economy with Coordination Frictions." *Journal of Political Economy*, 113, pp. 996-1025.

[35] **Sims, C.A. (2003).** "Implications of Rational Inattention." *Journal of Monetary Economics*, 50(3), pp. 665-690.

[36] **Simsek, A. Ozdaglar, A. and D. Acemoglu. (2007).** "Generalized Poincare-Hopf Theorem for Compact Nonsmooth Regions," *Mathematics of Operations Research*, 32(1), pp. 193-214.

[37] **Sönmez, T. and M. U. Ünver, (2010).** "Matching, Allocation, and Exchange of Discrete Resources," in J. Benhabib, A. Bisin, and M. Jackson (eds.), *Handbook of Social Economics*, Elsevier.

[38] **Stevens, M. (2007).** "New Microfoundations for the Aggregate Matching Function." *International Economic Review*, 48(3), pp. 847-868.

[39] **Thomas, C. and J. Cover, (1991)**. "Elements of Information Theory". John Wiley & Sons, Inc.

[40] **Vives, X. (1990).** "Nash Equilibrium with Strategic Complementarities." *Journal of Mathematical Economics*, 19, pp. 305-321.

[41] **Weyl E. G. (2010).** "A Price Theory of Multi-Sided Platforms." *American Economic Review*, 100(4), pp. 1642-1672.

[42] **Woodward, S. and R.E. Hall (2012).** "Diagnosing Consumer Confusion and Sub-Optimal Shopping Effort: Theory and Mortgage-Market Evidence." *American Economic Review*, 102(7), pp. 3249-3276.

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Appendix A: Proof of Theorem 3

The proof proceeds in 3 steps.

Step 1. Under the assumption of increasing convex cost functions, both individual payoff functions and the social welfare function are concave in the strategies of workers and firms. Hence, first-order conditions are necessary and sufficient conditions for a maximum.

Step 2. We denote by CEFOC the first-order conditions of the decentralized equilibrium and by POFOC the first-order conditions of the social planner. In formulae:

$$\text{POFOC}_{q_y(x)}: \qquad f(x,y)\,\tilde{p}_x(y) - \left.\frac{\partial c_y(\tilde{\kappa}_y)}{\partial \tilde{\kappa}_y}\right|_{\tilde{q}_y(x)} \frac{1}{\ln 2}\left(\ln \frac{\tilde{q}_y(x)}{1/N} + 1\right) - \tilde{\lambda}_y = 0$$

$$\text{POFOC}_{p_x(y)}: \qquad f(x,y)\,\tilde{q}_y(x) - \left.\frac{\partial c_x(\tilde{\kappa}_x)}{\partial \tilde{\kappa}_x}\right|_{\tilde{p}_x(y)} \frac{1}{\ln 2}\left(\ln \frac{\tilde{p}_x(y)}{1/N} + 1\right) - \tilde{\lambda}_x = 0$$

$$\text{CEFOC}_{q_y(x)}: \qquad \pi_y(x)\,p_x(y) - \left.\frac{\partial c_y(\kappa_y)}{\partial \kappa_y}\right|_{q_y(x)} \frac{1}{\ln 2}\left(\ln \frac{q_y(x)}{1/N} + 1\right) - \lambda_y = 0$$

$$\text{CEFOC}_{p_x(y)}: \qquad w_x(y)\,q_y(x) - \left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x(y)} \frac{1}{\ln 2}\left(\ln \frac{p_x(y)}{1/N} + 1\right) - \lambda_x = 0$$

For the equilibrium to be socially efficient we need to have the following:

$$\tilde{p}_x(y) = p_x(y) \quad \text{for all } x, y$$

$$\tilde{q}_y(x) = q_y(x) \quad \text{for all } x, y$$

Step 3. By contradiction, imagine that the two conditions above hold. Then, by construction,

$$\left.\frac{\partial c_y(\tilde{\kappa}_y)}{\partial \tilde{\kappa}_y}\right|_{\tilde{q}_y(x)} = \left.\frac{\partial c_y(\kappa_y)}{\partial \kappa_y}\right|_{q_y(x)} = a_y$$

and

$$\left.\frac{\partial c_x\left(\tilde{\kappa}_x\right)}{\partial \tilde{\kappa}_x}\right|_{\tilde{p}_x(y)}=\left.\frac{\partial c_x\left(\kappa_x\right)}{\partial \kappa_x}\right|_{p_x(y)}=a_x .$$

Denote them a_y and a_x respectively.

It then follows that:

$$\begin{aligned}
f(x,y)\,\tilde{p}_x(y)-\tilde{\lambda}_y &= \left.\frac{\partial c_y\left(\tilde{\kappa}_y\right)}{\partial \tilde{\kappa}_y}\right|_{\tilde{q}_y(x)} \frac{1}{\ln 2}\left(\ln \frac{\tilde{q}_y(x)}{1/N}+1\right) \\
&= \left.\frac{\partial c_y\left(\kappa_y\right)}{\partial \kappa_y}\right|_{q_y(x)} \frac{1}{\ln 2}\left(\ln \frac{q_y(x)}{1/N}+1\right) \\
&= \pi_y(x)\,p_x(y)-\lambda_y
\end{aligned}$$

i.e. $f(x,y)\,p_x(y)-\tilde{\lambda}_y=\pi_y(x)\,p_x(y)-\lambda_y$ for all x and y. We can use the first-order conditions of the firms to derive the formulas for λ and $\tilde{\lambda}$:

$$(i) \qquad N\exp\left(1+\frac{\tilde{\lambda}_y}{a_y/\ln 2}\right)=\sum_{x=1}^{N}\exp\left(\frac{f(x,y)p_x(y)}{a_y/\ln 2}\right)$$

$$(ii) \qquad N\exp\left(1+\frac{\lambda_y}{a_y/\ln 2}\right)=\sum_{x=1}^{N}\exp\left(\frac{\pi_y(x)p_x(y)}{a_y/\ln 2}\right)$$

$$(iii) \qquad \left(f(x,y)-\pi_y(x)\right)p_x(y)=\tilde{\lambda}_y-\lambda_y \qquad \text{for all } x$$

Jointly (i) (ii) and (iii) imply:

$$\frac{\sum_{x'=1}^{N}\exp\left(\frac{f(x',y)p_{x'}(y)}{a_y/\ln 2}\right)}{\sum_{x'=1}^{N}\exp\left(\frac{\pi_y(x')p_{x'}(y)}{a_y/\ln 2}\right)}=\frac{\exp\left(\frac{f(x,y)p_x(y)}{a_y/\ln 2}\right)}{\exp\left(\frac{\pi_y(x)p_x(y)}{a_y/\ln 2}\right)} \qquad \text{for all } x$$

Hence,

$$\frac{\exp(f(x,y)p_x(y))}{\exp(\pi_y(x)p_x(y))}=\frac{\exp(f(x',y)p_{x'}(y))}{\exp(\pi_y(x')p_{x'}(y))} \qquad \text{for all } x \text{ and } x'.$$

Therefore, either:

$$\text{a) } f(x,y) = \pi_y(x) \text{ for all } x \text{ or}$$

$$\text{b) } f(x',y) = f(x'',y) \text{ and } \pi_y(x') = \pi_y(x'') \text{ for all } x' \text{ and } x'';$$

Similarly from workers' first-order conditions it follows that either :

$$\text{c) } f(x,y) = w_x(y) \text{ for all } y \text{ or}$$

$$\text{d) } f(x,y') = f(x,y'') \text{ and } w_x(y') = w_x(y'') \text{ for all } y' \text{ and } y''$$

Cases b) and d) have been ruled out by the assumptions of the theorem. Cases a) and b) jointly imply that $\pi_y(x) = w_x(y) = f(x,y) = w_x(y) + \pi_y(x)$ which leads to a contradiction $\pi_y(x) = w_x(y) = f(x,y) = 0$.

Appendix B: One-sided model

Here we consider a one-sided model where buyers are searching for sellers and buyers face information processing constraints. We assume that sellers can satisfy demands from any finite number of buyers simultaneously. Sellers are assumed to satisfy the demand of an individual buyer with an exogenously given seller-specific probability q_y. The strategy of a buyer, denoted $p_x(y)$, represents the probability of buyer x contacting to seller y. It is also the buyer's distribution of attention. We assume that each buyer can rationally choose his strategy facing a trade-off between a higher payoff and a higher cost of processing information.

A buyer's cost of searching is given by $c_x(\kappa_x)$. This cost is a function of the amount of information processed by a buyer measured in bits, κ_x. Once the optimal distribution $p_x(y)$ is chosen, each buyer draws from it to determine which seller to contact.[17]

Buyer x chooses a strategy $p_x(y)$ to maximize his expected income flow:

$$Y_x = \sum_{y=1}^{M} w_x(y)\, p_x(y)\, q_y - c_x(\kappa_x) \to \max_{p_x(y)}$$

[17]As in the two-sided one-shot model, we assume that each buyer sends a single application.

We normalize the outside option of the buyer to zero. The buyer receives his expected wage in a match with seller y conditional on matching with that seller. He also incurs a search cost, which depends on the information processing capacity defined as follows:

$$\kappa_x = \sum_{y=1}^{M} p_x(y) \log_2 \frac{p_x(y)}{1/M} \tag{13}$$

where the buyer's strategy must satisfy $\sum_{y=1}^{M} p_x(y) = 1$ and $p_x(y) \geq 0$ for all y. Our definition of information, κ_x, represents the relative entropy between a uniform prior $\{1/M\}$ over sellers and the posterior strategy, $p_x(y)$. Shannon's relative entropy can be interpreted as the reduction of uncertainty that the buyer can achieve by choosing his distribution of attention. This definition is a special case of Shannon's channel capacity when information structure is the only choice variable.[18] Thus, our assumption is a special case of a uniformly accepted definition of information tailored to our problem.

Definition 3. *A matching* equilibrium *of the one-sided matching model is a set of strategies of buyers, $\{p_x(y)\}_{x=1}^{N}$, which solve their optimization problems.*

Theorem 6. *If the cost functions are non-decreasing and convex, the one-sided matching model has a unique equilibrium.*

Proof. The payoffs of all buyers are continuous in their strategies. They are also concave in these strategies when cost functions are (weakly) increasing and convex in information capacities. Hence, each problem has a unique solution. □

When in addition the cost functions are differentiable, it is easy to verify that first-order conditions are necessary and sufficient conditions for equilib-

[18]See Thomas and Cover (1991), Chapter 2.

rium.[19] Rearranging the first order conditions for the buyer, we obtain:

$$p_x^*(y) = \exp\left(\frac{w_x(y)\,q_y}{\frac{1}{\ln 2}\left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^*}}\right) \Big/ \sum_{y'=1}^{M} \exp\left(\frac{w_x(y')\,q_{y'}}{\frac{1}{\ln 2}\left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^*}}\right). \tag{14}$$

This is an implicit relationship as p_x^* appears on both sides of the expression. If cost functions are linear functions of the amount of information, κ_x, then the derivatives on the right hand side are independent of p_x^*, and the relationship becomes explicit.

Note that the assumptions we use to prove uniqueness are by no means restrictive. The assumption that cost functions are non-decreasing and convex is a natural one. Most of the literature on information processing assumes that either the cost function is linear, or there is a capacity constraint on processing information, which implies a vertical cost function after a certain amount of information has been processed. Our assumption incorporates both of these as special cases.

The equilibrium condition (14) has an intuitive interpretation. It predicts that the higher is the buyer's expected gain from matching with a seller, the greater is the probability of contacting that seller. Thus, sellers are naturally sorted in each buyer's strategy by probabilities of contacting those sellers.

It is worth noting the equilibrium properties for two limiting cases. First, consider the case when marginal costs of processing information go to zero. In this case, the strategies become more and more focused. In the limit, each buyer places a unit probability on a single seller. Second, consider the opposite case when marginal costs go to infinity. In this case, the difference between probabilities of applying to different sellers shrinks. In the limit, optimal strategies of buyers approach a uniform distribution.

[19] Taking derivatives of the Lagrangian function corresponding to the problem of buyer x, we obtain for all y:

$$w_x(y)\,q_y - \left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^*} \frac{1}{\ln 2}\left(\ln \frac{p_x^*(y)}{1/M} + 1\right) = \lambda_x$$

We can invert this first-order condition to characterize the optimal strategy:

$$p_x^*(y) = \frac{1}{M}\exp\left(\frac{w_x(y)q_y - \lambda_x}{\frac{1}{\ln 2}\left.\frac{\partial c_x(\kappa_x)}{\partial \kappa_x}\right|_{p_x^*}} - 1\right).$$

Our model generates a continuum of possible outcomes, which reside in between these two special cases. For intermediate values of costs the search strategies of buyers are distributed among all sellers and optimally skewed towards their best matches.

Efficiency In order to evaluate the efficiency of the equilibrium we compare the solution of the decentralized problem to a social planner's solution. We assume that the social planner maximizes the total surplus of the economy, which is a utilitarian welfare function. In order to achieve a social optimum, the planner can choose buyers' strategies. If no costs of processing information were present, the planner would always choose to match each buyer with the seller that produces the highest surplus. The socially optimal strategies of buyers would be infinitely precise.

To study the constrained efficient allocation we impose upon the social planner the same information processing constraints that we place on the buyers. Thus, the social planner maximizes the following welfare function:

$$W = \sum_{x=1}^{N} \sum_{y=1}^{M} f(x,y)\, p_x(y)\, q_y - \sum_{x=1}^{N} c_x(\kappa_x)$$

subject to the information constraint (13) and to the constraint that the $p_x(y)$'s are well-defined probability distributions. Under the assumption of increasing convex cost functions, the social welfare function is concave in the strategies of buyers. Hence, first-order conditions are sufficient conditions for a maximum. Rearranging and substituting out Lagrange multipliers, we arrive at the following characterization of the social planner's allocation:

$$p_x^o(y) = \exp\left(\frac{f(x,y)\, q_y}{\frac{1}{\ln 2} \left. \frac{\partial c_x(\kappa_x)}{\partial \kappa_x} \right|_{p_x^o}} \right) \Big/ \sum_{y'=1}^{M} \exp\left(\frac{f(x,y')\, q_y}{\frac{1}{\ln 2} \left. \frac{\partial c_x(\kappa_x)}{\partial \kappa_x} \right|_{p_x^o}} \right). \tag{15}$$

The first observation to make is that the structure of the social planner's solution is very similar to the structure of the decentralized equilibrium. Second, from the buyers' perspective, the only difference between the centralized and decentralized equilibrium strategies is that the probability of applying to a seller depends on the social gain from a match rather than on the private gain. Thus, it is socially optimal to consider the whole expected surplus when determining the socially optimal strategies, while in the decentralized equilibrium buyers only consider their private gains.

To decentralize the socially optimal outcome the planner needs to give all of the surplus to the buyers, $w_x(y) = f(x, y)$, effectively assigning them a bargaining power of 1. Note that, if the planner could choose the probability that a seller accepts a worker, q_y, he would also set it to 1.

The only special cases, when the outcome is always efficient are the limiting cases discussed earlier. When costs of information are absent, the equilibrium of the model is socially optimal. When costs of information are very high, the random matching outcome is the best possible outcome. For all intermediate values of costs, the decentralized equilibrium is socially efficient contingent on the buyer having all the bargaining power.

Figure 4 shows the results for the competitive equilibrium (CE) and the social optimum (PO) for the one-sided model for the numerical example presented in the main body of the paper. In the one-sided model, the number of matches is unaffected by changes in costs of information, because each buyer sends contacts a single seller which is accepted with a constant probability. For all values of information costs, the competitive equilibrium is unique, but constrained inefficient. Because buyers get only half of the surplus, they put a socialy suboptimal amount of effort into search. As a consequence, they do not target their best matches well enough. Although this does not lead to a reduction in the number of matches, average match quality suffers. As processing information becomes more costly, the quality of matches falls, which leads to a decline in welfare.

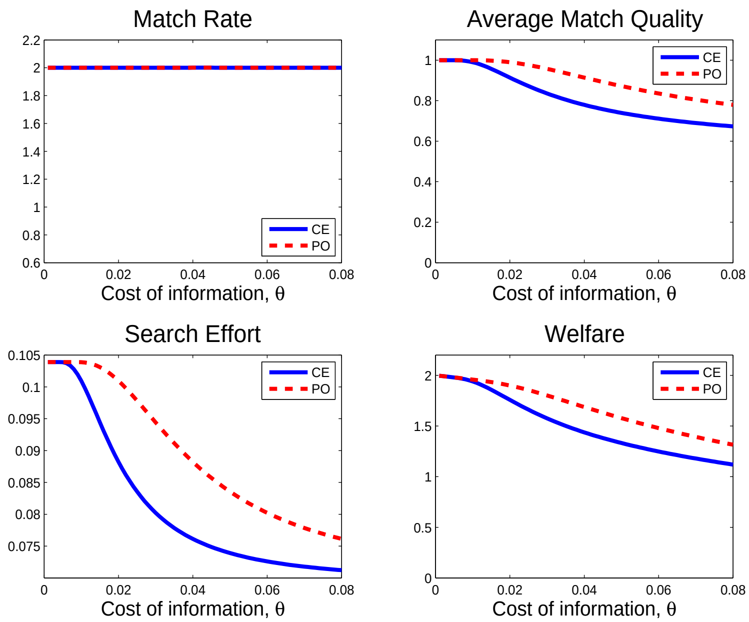


Figure 4. One-sided Model

Appendix C: Extended model

Primitives

Let worker and firm types be continuously distributed on compact measurable sets X and Y. Let there be a measure $u(x)$ of workers of each type $x \in X$ and a measure $v(y)$ of firms of each type $y \in Y$. Workers and firms search for each other in order to match. Like before, a match between a worker of type x and firm y generates a surplus $f(x, y)$. If a firm and a worker match, the surplus is split between the worker and the firm in a such a way that the worker gets a wage $w_x(y)$ and the firm gets a profit $\pi_y(x)$. The surplus, wage and profit conditional on types are common knowledge.

We assume that the worker and the firm face relatively general search costs of the forms:

$$c_x(\alpha_x, \kappa_x) = \chi_x \frac{\alpha_x^{\phi_x+1}}{\phi_x + 1} + \alpha_x \theta_x \kappa_x$$

$$c_y\left(\gamma_y,\kappa_y\right)=\chi_y\frac{\gamma_y^{\phi_y+1}}{\phi_y+1}+\gamma_y\theta_y\kappa_y$$

Each search cost has two components. The first component represents a convex cost of processing applications, which depends only on the numbers of applications, $\alpha\left(x\right)$ and $\gamma\left(y\right)$. The second component is the cost of processing information. It is proportional to the number of applications. Following notation of the one-shot model, $\kappa_w\left(x\right)$ and $\kappa_f\left(y\right)$ are amounts of information per application processed by firms and workers. Both are measured in bits. Agent-specific parameters, denoted $\theta_w\left(x\right)$ and $\theta_f\left(y\right)$, stand for marginal costs of processing information in dollars per bit.

Denote the equilibrium matching rate faced by the worker of type x when applying to a firm of type y as $m_w\left(x,y\right)$. Similarly, we denote the matching rate faced by firm y when considering worker of type x as $m_f\left(y,x\right)$. The worker maximizes his expected income flow:

$$Y_x=\int_Y w_x\left(y\right)m_w\left(x,y\right)p_x\left(y\right)\alpha_x dy-c_x\left(\alpha_x,\kappa_x\right)$$

with respect to his search intensity α_x and allocation of attention $p_x\left(y\right)$. The worker x gets his expected wage conditional on matching with a firm of type y net of the cost of search. The search cost depends on the amount of information processed by the worker, defined as follows:

$$\kappa_x=\int_Y p_x\left(y\right)\log_2\frac{p_x\left(y\right)}{v\left(y\right)/\int_Y v\left(y\right)dy}dy \qquad (16)$$

where $p_x\left(y\right)$ is a probability distribution that satisfies the usual assumptions:

$$\int_Y p_x\left(y\right)dy=1,\qquad p_x\left(y\right)\geq 0. \qquad (17)$$

The firm also maximizes her expected income flow:

$$Y_y = \int_X \pi_y(x)\, m_f(y, x)\, q_y(x)\, \gamma_y dx - c_y(\gamma_y, \kappa_y)$$

with respect to her search intensity γ_y and allocation of attention $q_y(x)$. Firm y gets a profit conditional on matching with a worker of type x net of the cost of search. The search cost depends on the amount of information processed by the worker, defined as follows:

$$\kappa_y = \int_X q_y(x) \log_2 \frac{q_y(x)}{u(x) / \int_X u(x)\, dx} dx \tag{18}$$

where $q_y(x)$ is a probability distribution that also satisfies the usual assumptions:

$$\int_X q_y(x)\, dx = 1, \qquad q_y(x) \geq 0. \tag{19}$$

Meeting protocol

We extend the telephone line meeting protocol of Stevens (2007) to allow for two-sided heterogeneity as shown in Figure 5. We assume that out of the stock of $u(x)$ workers of type x, $\alpha_x u(x)$ are sending applications, while the rest are enjoying leisure/waiting. The expected number of applications sent by worker of type x to firm of type y is $p_x(y)\, \alpha_x u(x)$.

Out of the stock of $v(y)$ firms of type y, $v_p(y)$ spend time processing applications. Before knowing the type of worker they are facing, firms choose applications from which types of workers to pay attention to, and how quickly to respond. Upon receiving an application from worker of type x, the firm processes on average γ_y applications and accepts the application with probability $q_y(x)$.

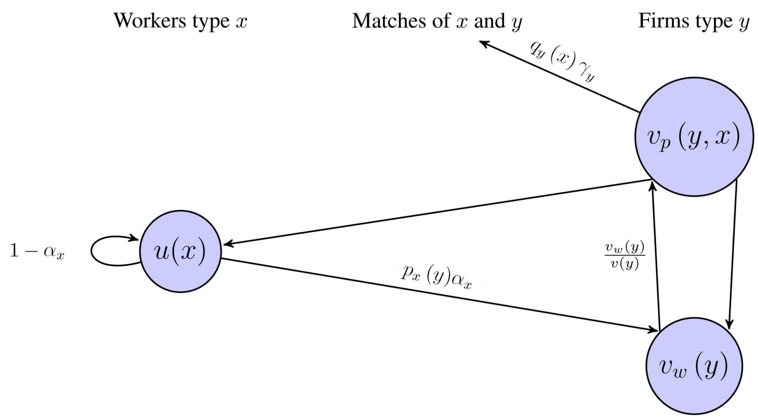


Figure 5. Meeting Protocol

We denote $v_p(x,y)$ the stock of firms of type y processing applications from workers of type x. A fraction γ_y of them transition to the waiting state per period. The total outflow to the waiting state is $\gamma_y v_p(y)$. Those firms that accepted the application hire the worker and are replaced by a copy of them in the waiting pool. Those which rejected the application start waiting for another application to arrive. In a stationary equilibrium, the inflow of firms into the processing pool equals the outflow:

$$u_s(x,y)\frac{v_w(y)}{v(y)} = \gamma_y v_p(x,y).$$

Using the accounting identity for the number of firms of type y, we can solve for the numbers of firms in each state. Then, the equilibrium number of matches for each pair of types equals:

$$m(x,y) = \gamma_y v_p(x,y)\, q_y(x) = \frac{p_x(y)\,\alpha_x u(x)\, q_y(x)\,\gamma_y v(y)}{\int_X \left(v(y)\,\gamma_y q_y(x') + p_{x'}(y)\,\alpha_{x'} u(x')\right) dx'}$$

The personal meeting rates arising from this meeting protocol are computed as follows:

$$\mu_w(x,y) = \frac{m(x,y)}{q_y(x)\,p_x(y)\,\alpha_x\gamma_y}\frac{1}{u(x)} \qquad \mu_f(y,x) = \frac{m(x,y)}{q_y(x)\,p_x(y)\,\alpha_x\gamma_y}\frac{1}{v(y)}$$

Equilibrium

Definition 4. *An equilibrium matching process is a set of strategies of workers* $\{p_x(y), \alpha_x\}$ *and firms* $\{q_y(x), \gamma_y\}$, *matching rates* $m_f(y,x)$ *and* $m_w(x,y)$ *such that:*

1) strategies solve problems of the workers and firms;

2) matching rates satisfy steady-state equilibrium conditions:

$$m_w(x,y) = q_y(x)\,\gamma_y \mu_w(x,y), \qquad m_f(y,x) = p_x(y)\,\alpha_x \mu_f(x,y).$$

We can simplify the definition of equilibrium and cast it into a Bayesian Nash equilibrium by redefining strategies of firms and workers. We introduce the following notation:

$$\hat{p}_x(y) = \alpha_x p_x(y), \qquad \hat{q}_y(x) = \gamma_y q_y(x)$$

Utilizing this notation, the workers' and firms' problems can be rewritten as an unconstrained maximization problems with payoffs:

$$Y_x(\hat{p}_x, \hat{q}) = \left[\begin{array}{c} \int_Y w_x(y)\,\mu_w(x,y)\,\hat{q}_y(x)\,\hat{p}_x(y)\,dy - \chi_x \frac{\left(\int_Y \hat{p}_x(y)dy\right)^{\phi_x+1}}{\phi_x+1} \\ -\frac{\theta_x}{\ln 2}\int_Y \hat{p}_x(y) \ln \frac{\hat{p}_x(y)/\int_Y \hat{p}_x(y')dy'}{v(y)/\int_Y v(y')dy'} dy \end{array} \right]$$

$$Y_y(\hat{q}_y, \hat{p}) = \left[\begin{array}{c} \int_X \pi_y(x)\,\mu_f(x,y)\,\hat{p}_x(y)\,\hat{q}_y(x)\,dx - \chi_y \frac{\left(\int_X \hat{q}_y(x)dx\right)^{\phi_y+1}}{\phi_y+1} \\ -\frac{\theta_y}{\ln 2}\int_X \hat{q}_y(x) \ln \frac{\hat{q}_y(x)/\int_X \hat{q}_y(x')dx'}{u(x)/\int_X u(x')dx'} dx \end{array} \right]$$

where equilibrium meeting rates are taken as given. These payoffs can be analyzed and optimized using standard techniques borrowed from the calculus of variations. We leave out the technical details and proofs for now. We introduce the following assumptions:

A1. Type sets $x \in X$ and $y \in Y$ are compact.

A2. Action sets $\hat{p}_x \in [0,P]$ and $\hat{q}_y \in [0,Q]$ are compact, i.e. P and Q are finite.

A3. $w_x(y)\,\mu_w(x,y) \geq 0$ and $\pi_y(x)\,\mu_f(x,y) \geq 0$ for all x and y.

A4. Costs parameters θ_x, θ_y, χ_x, χ_y are non-negative. Costs of applications are convex: $\phi_x \geq 0$, $\phi_y \geq 0$.

A5. "Diagonal dominance" conditions are satisfied along the equilibrium path:

$$\left|\frac{\partial^2 Y_x\left(\hat{p}_x, \hat{q}_y\right)}{\partial \hat{p}_x \partial \hat{p}_x}\right|_{\hat{p}_x^*, \hat{q}_y^*} > \left|\frac{\partial^2 Y_x\left(\hat{p}_x, \hat{q}_y\right)}{\partial \hat{p}_x \partial \hat{q}_y}\right|_{\hat{p}_x^*, \hat{q}_y^*}$$

$$\left|\frac{\partial^2 Y_y\left(\hat{q}_y, \hat{p}_x\right)}{\partial \hat{q}_y \partial \hat{q}_y}\right|_{\hat{p}_x^*, \hat{q}_y^*} > \left|\frac{\partial^2 Y_y\left(\hat{q}_y, \hat{p}_x\right)}{\partial \hat{q}_y \partial \hat{p}_x}\right|_{\hat{p}_x^*, \hat{q}_y^*}$$

Assumptions A1-A2 postulate that types and actions lie on compact domains, while Assumption A3 states that matching is profitable for both parties. Assumption A4 requires information processing costs to be non-negative. This assumption is important for uniqueness of equilibrium since information-processing constraints lower the perceived degree of complementarities between search efforts of workers and firms. Finally, A.5 guarantees that we have a contraction mapping of the best response functions.

Theorem 7. *Under assumptions A1, A2 and A3 Nash Equilibria exist.*

Proof. The proof is achieved in three steps and follows Vives (1990): (a) The set of all measurable functions mapping a compact set into a compact set is a lattice under the natural ordering. (b) The game is supermodular since the cross-derivatives of the objective functions are all non-negative.

$$\frac{\partial^2 Y_x\left(\hat{p}_x, \hat{q}_y\right)}{\partial \hat{p}_x \partial \hat{q}_y} = w\left(x, y\right) \mu_w\left(x, y\right) \qquad \frac{\partial^2 Y_y\left(\hat{q}_y, \hat{p}_x\right)}{\partial \hat{q}_y \partial \hat{p}_x} = \pi\left(x, y\right) \mu_f\left(x, y\right)$$

(c) In a supermodular game on a lattice Nash equilibria exist. □

Lemma 1. *Under A1 and A2 Y_x and Y_y are continuous in $\hat{p}_x$ and $\hat{q}_y$ respectively.*

Proof. All the integrands are continuously differentiable with respect to strategies, and all the integrals are taken over compact sets. □

Lemma 2. *Under assumptions A1, A2 and A4 Y_x and Y_y are concave in $\hat{p}_x$ and $\hat{q}_y$ respectively.*

Proof. Using the previous lemma, it remains to verify that the second variational derivatives are everywhere non-positive. That is indeed the case under assumption A4. □

Theorem 8. *Under A.1, A.2, A.3, A.4 the first-order conditions are necessary and sufficient conditions for equilibrium.*

Proof. This theorem is a direct consequence of the previous two lemmas and assumption A3. □

Theorem 9. *Under assumptions A.1, A.2, A.3, A.4, A.5 the matching process has a unique Nash equilibrium.*

Proof. Diagonal dominance conditions guarantee that the Hessian of the game is negative definite along the equilibrium path. It follows from lemmas 1 and 2 that the payoff functionals are continuous and concave. Then, the generalized Poincare-Hopf index theorem of Simsek, Ozdaglar and Acemoglu (2007) implies that the equilibrium is unique. □

The first-order conditions can be simplified and rewritten using the original notation to yield distributions of attention and search intensities for both firms and workers. We only report the necessary and sufficient conditions here:

$$p_x^*(y) = \frac{v(y)\exp\left(\frac{\ln 2}{\theta_x} g_x(y)\right)}{\int_Y v(y')\exp\left(\frac{\ln 2}{\theta_x} g_x(y')\right) dy'}$$

$$\alpha_x^* = \left[\frac{1}{\ln 2}\frac{\theta_x}{\chi_x}\ln\frac{\int_Y v(y)\exp\left(\frac{\ln 2}{\theta_x} g_x(y)\right) dy}{\int_Y v(y)\, dy}\right]^{\frac{1}{\phi_x}}$$

$$q_y^*(x) = \frac{u(x)\exp\left(\frac{\ln 2}{\theta_y} g_y(x)\right)}{\int_X u(x')\exp\left(\frac{\ln 2}{\theta_y} g_y(x')\right) dx'}$$

$$\gamma_y^* = \left[\frac{1}{\ln 2}\frac{\theta_y}{\chi_y}\ln\frac{\int_X u(x)\exp\left(\frac{\ln 2}{\theta_y} g_y(x)\right) dx}{\int_X u(x)\, dx}\right]^{\frac{1}{\phi_y}}$$

where private gains of workers and firms are defined as follows:

$$g_x(y) = w_x(y)\,\mu_w(x,y)\,q_y^*(x)\,\gamma_y^*$$

$$g_y(x) = \pi_y(x)\,\mu_f(x,y)\,p_x^*(y)\,\alpha_x^*$$

Like in the one shot model, equilibrium allocations of attention have an intuitive interpretation. The higher agents' expected private gains from matching with each other, the greater the probabilities of applying/processing applications. Firms and workers are naturally ordered in probabilities of allocating attention to each other. In equilibrium, firms' strategies are best responses to strategies of workers, and workers' strategies are best responses to strategies of firms. The strategies of firms and workers have similar properties due to the symmetry of the problem.

The rich structure of heterogeneity in costs, surpluses and types is fully taken into account by all agents in the model. Relatively unrestrictive conditions for uniqueness allow us to accomodate a rich class of matching models with different structures of fundamentals. Each element of this rich structure of fundamentals potentially has an impact on matching rates between all type pairs, which in turn affect the number and quality of matches in equilibrium. Therefore, this model can be extremely useful for understanding the consequences of heterogeneity for the aggregate matching function.

Note, that neither existence nor uniqueness of equilibrium relies on supermodularity of the surplus function. Therefore, assortative matching (in expected terms) needs not be an equilibrium outcome of the model. Thus, our model can generate a rich structure of equilibrium outcomes and has a potential to speak to the rich empirical literature on the determinants of wages.

The social planner's problem

Similarly to the one-shot model, we assume that the social planner maximizes the total surplus of the economy subject to the the same constraints that we place on workers and firms in equilibrium. Note that under the aforementioned assumptions the resulting conditions for social optimality are the same as for equilibrium, except social gains are defined as follows:

$$g_x^o(y) = f(x,y)\,\mu_w(x,y)\,q_y^o(x)\,\gamma_y^o - \phi(y)\,,$$

$$g_y^o(x) = f(x,y)\,\mu_f(x,y)\,p_x^o(y)\,\alpha_x^o - \phi(y)\,,$$

where $\phi(y)$ is a term that only depends on the firm types.

Conjecture 1. *The equilibrium is socially inefficient under assumptions A1-A4 and if all of the following hold:*
1) $0 < \theta_x < \infty$
2) $0 < \theta_y < \infty$
3) $f(x, y) > 0$ *for some* (x, y)
4) a non-zero measure of heterogeneity is present on both sides.

The conjecture has a similar intuition to the one shot model. It is not feasible to achieve the social optimum, because to do that the planner needs to promise private gains that violate the resource constraint. This result is crucial for understanding the magnitude of potential inefficiencies in the matching process. For that it is useful to compute the aggregate number of equilibrium matches. Note that in this framework the matching rate, an analog of the matching function, can be computed as:

$$M = \int_X \int_Y \frac{q_y(x) p_x(y) \alpha_x u(x) \gamma_y v(y)}{v(y) \gamma_y + \int_X p_{x'}(y) \alpha_{x'} u(x') dx'} dx dy$$

Simplifying assumptions

To facilitate quantitative explorations of the properties of equilibrium outcomes and the size of inefficiency we make several auxiliary assumptions.

A6 Workers and firms are distributed uniformly: $u(x) = U$, $v(y) = V$.

A7 Workers are identical: $\theta_x = \theta_w$, $\chi_x = \chi_w$, $\phi_x = \phi_w$.
Firms are identical: $\theta_y = \theta_f$, $\chi_y = \chi_f$, $\phi_y = \phi_f$.

A8 Workers and firms are placed on connected unit intervals:

$$X = [0, 1], \qquad Y = [0, 1].$$

A9 Match surplus and Nash bargaining weights depend on distance, $d(x, y)$, only:

$$f(x, y) = f(d(x, y)), \qquad w(d) = \beta(d) f(d), \qquad \pi(d) = (1 - \beta(d)) f(d).$$

where $d(x, y) = \min\{|x - y|, 1 - x + y, 1 - y + x\} \in \left[0, \frac{1}{2}\right]$.

Thus, we place workers and firms on connected unit intervals and define the surplus of each match as a function of the distance between types. Firm and worker types are symmetric. Symmetry and uniformity simplify the analysis substantially. Conditional on assumptions A6-A9, all match-specific variables become distance-specific, all firm- or worker-specific variables lose this dependence. Therefore, the solution to the model can be rewritten as follows:

$$\alpha^* = \left[\frac{1}{\ln 2}\frac{\theta_w}{\chi_w}\ln 2\int_0^{\frac{1}{2}}\exp\left(\frac{\ln 2}{\theta_w}\frac{V\gamma^*}{V\gamma^*+\alpha^*U}w\left(d\right)q^*\left(d\right)\right)dd\right]^{\frac{1}{\phi_w}}$$

$$\gamma^* = \left[\frac{1}{\ln 2}\frac{\theta_f}{\chi_f}\ln 2\int_0^{\frac{1}{2}}\exp\left(\frac{\ln 2}{\theta_f}\frac{\alpha^*U}{V\gamma^*+\alpha^*U}\pi\left(d\right)p^*\left(d\right)\right)dd\right]^{\frac{1}{\phi_f}}$$

$$p^*\left(d\right) = \frac{\exp\left(\frac{\ln 2}{\theta_w}\frac{V\gamma^*}{V\gamma^*+\alpha^*U}w\left(d\right)q^*\left(d\right)\right)}{2\int_0^{\frac{1}{2}}\exp\left(\frac{\ln 2}{\theta_w}\frac{V\gamma^*}{V\gamma^*+\alpha^*U}w\left(d'\right)q\left(d'\right)\right)dd'}$$

$$q^*\left(d\right) = \frac{\exp\left(\frac{\ln 2}{\theta_f}\frac{\alpha^*U}{V\gamma^*+\alpha^*U}\pi\left(d\right)p^*\left(d\right)\right)}{2\int_0^{\frac{1}{2}}\exp\left(\frac{\ln 2}{\theta_f}\frac{\alpha^*U}{V\gamma^*+\alpha^*U}\pi\left(d'\right)p^*\left(d'\right)\right)dd'}$$

Socially optimal allocations are similar, with the exception that private gains $w\left(d\right)$ and $\pi\left(d\right)$ are replaced by social gains, $f\left(d\right)$. Therefore, it is straightforward to see that no bargaining weights can help achieve the socially optimal allocation, unless $w\left(d\right)=\pi\left(d\right)=f\left(d\right)$, which is not feasible. The matching rate in this case equals:

$$M = 2\frac{\alpha U\gamma V}{\alpha U+\gamma V}\int_0^{\frac{1}{2}}p\left(d\right)q\left(d\right)dd$$

The matching function takes the form of a constant-returns-to-scale matching function with a constant elasticity of substitution between unemployed workers and vacant firms. In practice, it can be approximated by a CES or Cobb-Douglas function. Parameters of this function are fully endogenous. They are determined exclusively by the distribution of surplus and by costs of search. The solution to this matching process is easily computable using standard optimization algorithms. It also allows for a closed-form solution under additional assumptions, which we describe next.

Closed-Form Solution

We proceed to a closed-form solution by adding assumptions that cost functions, numbers of workers and bargaining powers are also symmetric:

A10 $\chi_f = \chi_w = \chi, \qquad \theta_w = \theta_f = \theta, \qquad \phi_w = \phi_f = \phi,$
$U = V$ and $\pi(d) = w(d) = \frac{1}{2} f(d)$.

In this case, the solution is symmetric with $p(d) = q(d)$ and $\alpha = \gamma$, and can be solved in closed-form:

$$p^*(d) = \frac{1}{A^*} \exp\left(-W\left(-\frac{1}{A^*}\frac{\ln 2}{4\theta} f(d)\right)\right) \qquad\qquad \alpha^* = \left[\frac{\theta}{\ln 2}\frac{1}{\chi}\ln A^*\right]^{\frac{1}{\phi}}$$

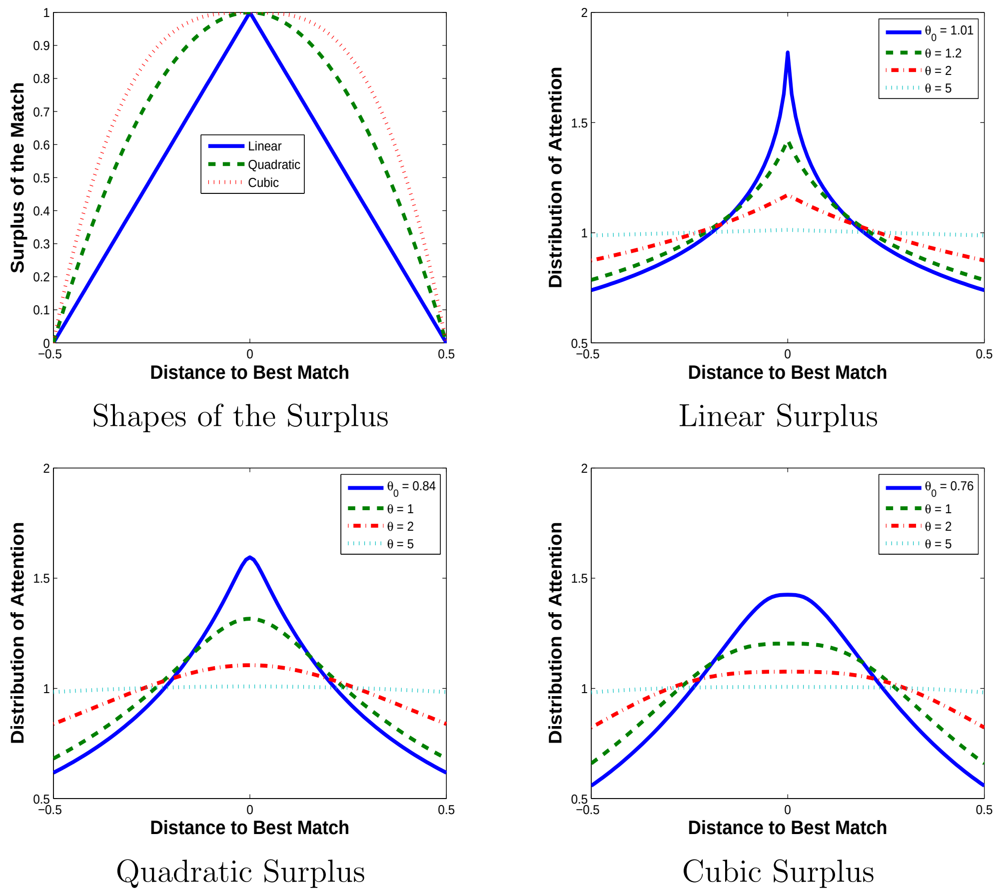
where $W(y)$ is the real branch of the Lambert-W function, defined as the solution to $y = We^W$ for $W(y) \geq -1$, and A^* is a normalizing constant that makes sure that the distribution of attention integrates to one. The planner's allocation has a similar form with both workers and firms assuming they will get the whole surplus instead of a half. Assuming the existence of an upper bound, F, on the surplus function, the equilibrium is unique if:

$$\theta \geq \theta_0 = \frac{Fe\ln 2}{8\int_0^{\frac{1}{2}} \exp\left(-W\left(-\frac{f(d)}{Fe}\right)\right) dd}$$

Constraints on costs of information illustrate that a high enough cost is necessary to weaken the strategic complementarity between strategies of workers and firms. The intuition behind the lower bounds is that, for $\theta < \theta_0$, the marginal cost is smaller than the marginal benefit of information:

$$\frac{F}{A}\frac{\ln 2}{4\theta} > \frac{1}{e} > \frac{\ln p(d)}{p(d)}$$

For lower costs of information, the strategic complementarities dominate. One solution to the problem in this case is the solution to the assignment model, characterized by infinitely precise strategies described by the Dirac-delta function, $p(d) = \delta(d)$. There is a multiplicity of other infinitely precise strategies that are also equilibria.



Shapes of the Surplus | Linear Surplus

Quadratic Surplus | Cubic Surplus

Figure 6. Shapes of the Surplus and Distributions of Attention

In Figure 6 we plot distributions of attention for three shapes of the surplus $f(d) = 1 - (2d)^p$ for different values of costs above their limiting values. For different values of curvature, $p = \{1, 2, 3\}$, the limiting values of costs for equilibria to be unique equal $\theta_0 = \{1.00, 0.83, 0.75\} * \ln 2$. The matching rate in these cases also has a closed-form solution:

$$M = U\alpha^* \int_0^{\frac{1}{2}} (p^*(d))^2 \, dd$$

For each of the aforementioned surplus functions the matching function is strictly decreasing in the cost of information as illustrated in the Figure 7 for the case $\phi \to \infty$. Figure 7 helps quantify losses in efficiency due to existence of strategic complementarities. For the symmetric economy the efficient outcome is equivalent to the equilibrium outcome under the assumption that cost of information is reduced in half. Figure 7 shows that, for intermediate values of costs, the number of lost matches in equilibrium can reach 50% compared with the social planner's allocation.

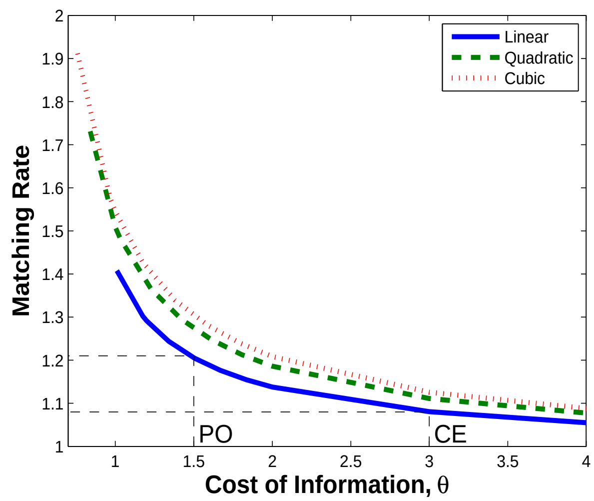


Figure 7. Matching Efficiency