Notes on Globalization and Monetary Policy (Rules): *A User's Guide*[*]

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[*][VERY PRELIMINARY AND INCOMPLETE. DO NOT QUOTE WITHOUT PERMISSION.] This is still very much work in progress, so any comments or suggestions are more than welcome. Please report any mistakes, typos, and inconsistencies you find.

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Preface

These notes were originally prepared for a course on Topics of International Macroeconomics and Monetary Theory at the University of Alicante (Spain) on December 17-19, 2007. Several colleagues at the Federal Reserve Bank of Dallas deserve special credit for pointing out errors and typos, and for helpful discussions on the material. My understanding of Monetary Theory in an Open Economy environment has greatly benefited from the invaluable advice of Charles Engel and Rodolfo Manuelli. I would also like to thank Mark Wynne and Dallas Fed President Richard Fisher for their encouragement, and the grad students and faculty at the University of Alicante for many helpful discussions. All of them deserve great recognition, but bear no responsibility for the mistakes that remain in the text.

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"By way of background, two developments fundamentally distinguish modern monetary theory from its predecessors: first, the embedding of forward-looking (rational) expectations and microeconomic-based theories of wage/price formation into classical monetary models, and, second, the use of optimal monetary policy rules to describe and analyze policy" (Taylor, 2006, p. 1).

1 Introduction[1]

A milestone in the international business cycle literature came with the work of Backus, Kehoe and Kydland (1992, 1994, 1995). The new open economy macro literature (NOEM) was launched by Obstfeld and Rogoff (1995, 2002) shortly after that. It can be argued that its precursors go back at least to Svensson and van Wijnbergen (1989), who worked out a model with NOEM features as an open economy development of Blanchard and Kiyotaki (1987). The recent treatise on the theory of monetary policy by Woodford (2003) is a common reference point for the NOEM literature too, even if it is concerned primarily with closed economies. Woodford (2003) has formalized the modern view that monetary policy is relevant even in a cashless economy.

The purpose of these notes is to briefly introduce the benchmark NOEM model and to discuss the role for monetary policy in it. I study the neoclassical synthesis which incorporates commonly-used frictions in the literature (i.e., monopolistic competition, staggered prices and pricing-to-market) into an otherwise simplified version of the standard two-country international business cycle model. The analysis is conducted largely on theoretical grounds. I focus my attention primarily on the dynamics of the real and nominal exchange rates, the current account and inflation.

[1] As Galí and Gertler (2007) note, "Goodfriend and King employ the term 'New Neoclassical Synthesis', while Woodford uses 'NeoWicksellian'. At the insistence of a referee, in our (Galí and Gertler's) 1999 paper with Richard Clarida, we used 'New Keynesian'. The latter term has probably become the most popular, though it does not adequately reflect the influence of real business cycle theory".

The analysis is framed around a number of stylized facts that are thought to be robust in major industrialized countries for the post-Bretton Woods period and characteristic of open economies (see, e.g., Obstfeld and Rogoff, 2000). My presentation also emphasizes differences with the closed-economy and the frictionless models, and explores the conduct of monetary policy in an open environment.

The Extent of these Notes. Truth in advertisement, I should also acknowledge that these notes are by no-means exhaustive of all topics and models in international macroeconomics/finance and monetary economics. The scope of these notes is limited, and it is worth noticing that:

◦ Frequency: I discuss fundamentally, if not exclusively, the real business cycle frequency. Most of the data discussed is expressed at quarterly frequency, and the NOEM model is designed with that reference in mind. In spite of its importance, I do not discuss the foreign exchange microstructure and the high frequency data. I do not address the long-run growth trends either.

◦ Structure:

· Two-country[2], mostly symmetric model. The issue of multiple countries arises only whenever necessary to make an important point. I avoid small open economies altogether (see, e.g., Galí and Monacelli, 2005).

· Discrete-time model. Discrete-time seems appropriate to describe fluctuations at the quarterly frequency.

· Representative agent model with infinite-horizon consumers. Alternatively, the demand-side could be developed from the Yaari (1965), Blanchard (1985) perpetual youth model with overlapping generations, where consumers are uncertain about their life length. Under the representative agent framework, life-cycle wealth distribution effects are overlooked[3].

· Linear-in-labor technologies, no capital. This specification has become popular in the literature. I abstract from capital accumulation in the problem of the firm. For an analysis of the impact of capital investment, see Chari, Kehoe and McGrattan (2002) and Martínez-García and Sondergaard (2008).

· Staggered price adjustment à la Calvo (1983). I concentrate on nominal rigidities in prices, and do not introduce wage rigidities unless otherwise required. This model assumes that firms choose their prices optimally, while the timing of their price changes is exogenous (time-dependent pricing). I do not discuss state-dependent pricing in spite of its increasing relevance in the debate about international pricing. For more details on state-dependent pricing, see Dotsey, King and Wolman (1999) and Landry (2007)[4].

· Money-in-the-utility-function for households. This 'trick' introduces a demand for money. It is based on the idea that the liquidity service of money increases the 'utility' of households by reducing their transaction costs. Alternatively, I could use a cash-in-advance constraint. No further attempts to introduce

[2] Most of the time I use the bilateral relationships between the U.S. and the Euro-zone as a point of reference.

[3] Wealth distribution effects may be quite important. However, it is not well understood how: (a) monetary policy (and more specifically short-term nominal interest rates) affect the distribution of wealth, and (b) how significative are these wealth effects for the 'objectives' of the monetary authority. More research needs to be done along these lines. See di Giorgio and Nisticò (2007) for an interesting discussion on the topic.

[4] Time-depent pricing often makes the model more tractable. However, it implies that firms do not adjust the frequency of their price changes in response to macroeconomic conditions. If we believe that price changes are tied to inflation, these simplification can be very limiting for monetary policy analysis. Bakhshi, Kahn *et al.* (2007) show that the state-dependent Phillips curve encompasses the Phillips curve based on Calvo-type price setting as a special. Their equation relates inflation to lagged inflation, expected future inflation, current and expected future real marginal costs and current and past variations in the distribution of price vintages. I explore in this notes the special case, and defer to Bakhshi *et al.* (2007) for the extension.

a micro-funded demand for money are discussed. The overlapping generations model could offer a natural way to do so, but that goes beyond the scope of these notes.

· Free floating exchange rate regime. I recognize that in most parts of the world other exchange rate regimes are in place (like pegs, target zones, etc.). However, the free floating characterization best describes the current regime in the U.S. and most industrialized countries during the post-Bretton Woods period. Therefore, this regime is taken as given and little consideration is given to other choices.

· Zero-inflation, zero-trade balance steady state. The equilibrium conditions are linearized around the zero-inflation, zero-trade balance steady state. I abstract from the implications of an inflationary steady state (see, e.g., Bakhshi, Burriel, *et al.*, 2007) and other relevant features like policy regime switches.

· Ignore popular short-cuts and other features. I choose to ignore rule-of-thumb agents. I also keep the rational expectations assumptions and impose that all agents have access to the same information set and know the objective probabilities. For details on the impact of sticky information on the dynamics of inflation, see Mankiw and Reis (2002, 2006).

◦ Analysis: I explore the analytic solution whenever one exists, and the first-order effects otherwise. Log-linearization has become commonplace in international macro, and is the technique most often used to study the dynamics of the model around the steady state. I explore the macroeconomic effects of monetary policy on the economy based on conventional Taylor-type interest rate rules too.

The Concept of Openness. Globalization in its most rudimentary of meanings is just openness to trade (whether in goods, inputs or assets). The NOEM literature, therefore, concerns itself with the effects that openness has on the domestic economy. As we go along, I will compare the open economy to the autarky allocation to stress this point. I try to make this comparison as clear and fair as possible. But, how do we quantify the effects of globalization anyway?

First, the magnitude and effects of openness cannot be unequivocally determined by the volumes of trade. It is possible to write models in which countries are fully open to trade (without frictions) and yet optimally decide not to exchange anything. We cannot infer from the lack of trade that the economies are closed. Most importantly, we cannot infer that openness is irrelevant. Let me give a simple example for illustrative purposes. Suppose that production of a certain variety is controlled by a domestic monopolist. The local government decides to eliminate all trade barriers and foreign competitors appear to challenge the incumbent domestic monopolist. It is possible to visualize conditions under which a subgame perfect equilibrium has the domestic incumbent defeating the challenger by cutting prices aggressively. Trade does not occur, and yet the impact of greater openness on prices is quite significative.

Second, openness is a relative concept best defined in opposition to a closed economy. Its effects, however, are conditional on a certain characterization of the market structure and the economy. The trade volumes are an endogenous response that certainly depends on the market structure, but also on other features of the model (preferences, technologies, taxes, trading costs, etc.). A decline in trade volumes alone cannot be shown as proof of decreasing openness or argued as evidence of greater insulation from foreign shocks.

Third, there is no guarantee that monetary policy is welfare-enhancing or that monetary policy is more or less 'effective' with greater openness. It is always possible to write a frictionless model where money is neutral, but consumption prices vary because of the choice of a policy rule. But, in this case monetary policy is not welfare-enhancing whether the economy is open or closed because it has no real effects. Whether monetary policy is welfare-enhancing critically depends on the market structure preventing an optimal allocation from occurring naturally. Nominal rigidities and other frictions are necessary to make sense of monetary policy. How to conduct monetary policy optimally in an open economy is still an open question, but it very much hinges upon the set of frictions that one assumes.

The benchmark NOEM model posits a market structure characterized by costless and instantaneous trading in goods markets in both countries[5], complete asset markets, and fully segregated markets for inputs (particularly, labor). Openness operates fundamentally on the goods market and the asset markets, while frictions on both markets are expected to account for a number of stylized facts and puzzles. The literature emphasizes the importance of the supply-side, and has developed a framework to do so based on nominal rigidities, monopolistic competition and the effects of local-currency pricing (henceforth, LCP) and producer-currency pricing (henceforth, PCP). It has also given much attention to the role of monetary policy on the demand-side.

In summary, the increased mobility and interdependence of the world's financial markets and goods markets, may be due to improved information technology, the reduction in government barriers to doing business internationally, preference changes, financial innovations, etc. The interest on globalization driven by a trend for increased trade across countries may decay in the future if these patterns reverse themselves, even though that reversal may have little to do with market openness *per se*. But openness, or lack thereof, is a topic that has accompanied mankind since the dawn of time and it will continue to preoccupy scholars long after these notes have come out of fashion. Naturally, the only thing left to do is to devote ourselves to the subject as best as we can given the tools at our disposal in hopes that our efforts will improve our general understanding of economics.

Small-Scale vs. Large-Scale Models. The international business cycle literature has developed many different variants of the workhorse model of openness. Calibration and estimation are often used to help us understand how these models behave. They are wildly used these days, although with varying degrees of success. Large-scale variants of the model have become popular approximation tools to understand the complex features of a highly interconnected global economy. Estimated and calibrated large-scale open economy models are now instrumental for policy analysis, in place of large-scale VARs. As the reasoning goes, large scale models are more 'flexible' than purely econometric models for policy analysis because they endogenously account for the response of agents to changes in monetary policy (the Lucas' critique).

The complexity of these large-scale models makes them very difficult to handle effectively, and very difficult to estimate in practice. In reality, they are like a large 'black box'. So it is not always obvious how to interpret the results, or even the lessons we learn regarding the contribution that particular friction (or specific modelling assumption) impresses on the impact of monetary shocks (or other shocks for that matter) on the macro variables of interest. The workhorse model discussed in these notes is a small-scale model, and presents a much more stylized description of the economy. However, it has also the advantage that it isolates a small number of frictions. This makes possible the analysis of the transmission channels embedded in the economy and the quantification of the role they play.

Even monetary policy analysis becomes more tractable in this setting. A better understanding of the key elements that drive the dynamics of the economy gives us a better chance to understand how monetary policy operates and what are the dominant transmission channels that we should care about. Estimation and

[5] See Obstfeld and Rogoff (2000) for a discussion of the role that costly international trading can play.

calibration of these models is always an option. Nonetheless, we cannot ignore the stylized characterization of the economy. Estimation in particular may reflect the internal tensions of a model too stylized to capture all features of the data. As best as I can, I attempt to find analytic or numerical (if necessary) solutions. I use a first-order log-linearization whenever appropriate to capture the first-order effects of the model in a simpler linearized rational expectations framework. First-order approximations are useful to understand the implications of the model, but sometimes they may be insufficient to capture the richness of the building blocks of the economy.

2 The Six Fundamental Puzzles in International Macroeconomics

At a basic level, I attempt to model and understand the basic primitives of an economy. The model of reference is the frictionless international real business cycles model (or a simplified version of it), and the benchmark for the NOEM literature adds some key frictions (nominal rigidities, pricing-to-market and monopolistic competition). I also discuss the role of frictions in the asset markets. The ultimate goal is to obtain a unified framework that captures a number of relevant stylized facts of the external sector. Having a unified framework gives us a null hypothesis to think about issues of international macroeconomics and finance with a point of reference. Parsimony is always preferred in modelling.

The stylized facts that inform my discussion, at least partially, are summarized in the form of six puzzles. These six puzzles come with the seal of approval of Obstfeld and Rogoff (2000a). The six puzzles, which I briefly describe here, are viewed as tell-tale signs of modelling failures and require deeper thought. However, a model that can replicate these stylized facts is by no means a guarantee that it describes the economy in a way that is meaningful or relevant for policy analysis. For example, a VAR model may describe well the time series properties of certain macroeconomic variables. However, it may not be very useful for policy analysis because of the Lucas' critique. A DSGE model may be observationally equivalent up to a first-order approximation to another one with different features. Replicating the dynamics of a number of macroeconomic variables with this approximation may not suffice to disentangle the true sources of the effects we care about. That makes policy analysis more obscure, rather than clear.

The six stylized facts described by Obstfeld and Rogoff (2000a) are:

◦ *The Home Bias in Trade Puzzle.* The evidence in McCallum (1995) and Evans (2003) suggests that trade within a country is substantially greater than across the border and, therefore, international goods markets appear to be quite segmented. The main focus of these notes are the international finance puzzles. However, it is worth noticing that there are other related real trade puzzles. For example, Trefler's (1995) 'missing trade' puzzle arising because the imputed factor content of trade does not seem to reflect comparative advantage, or the empirical failure of the Heckscher-Ohlin prediction of factor-price equalization. The home bias in trade should be viewed as part of this larger set of trade anomalies.

◦ *The Savings-Investment (or Feldstein-Horioka) Puzzle.* Feldstein and Horioka (1980) demonstrate that long period averages of national savings rates are highly correlated with the same averages of domestic investment rates among OECD countries. Obstfeld and Rogoff (2000a) show that the correlations have decreased over time, but they remain surprisingly higher than one might expect in a world with fully integrated asset markets.

◦ *The Home Bias in Equity Portfolios Puzzle.* French and Poterba (1991) observe that investors maintain a perplexing preference for home assets in spite of the rapid expansion of international asset markets experienced over the past decades. Over time, portfolios have become slightly more diversified. But the pervasive effects of the bias remain and have been documented across many (small and large) economies. See Lewis (1999) for a detailed survey of the literature.

◦ *The International Consumption Correlations Puzzle.* The evidence on consumption growth correlations is hard to reconcile with the often much higher correlations predicted in models with complete international asset markets. Under complete asset markets, country-specific risks are easily pooled and shared across countries making consumption growth less dependent on them. As noted by Obstfeld and Rogoff (2000a), the consumption correlation puzzle is intimately connected to the Feldstein-Horioka and the home equity bias puzzles. "Given that the most transparent of market means of consumption smoothing -debt and equity trade- are far less operative across borders than within them, it should not come as any great surprise that international consumption correlations are low".

Backus, Kehoe and Kydland (1992) note that, moreover, international output growth rates appear to be more correlated than consumption. Backus and Smith (1993) show that fluctuations in consumption ratios and those in bilateral real exchange rates are highly correlated whenever international asset markets are complete, and yet there is little empirical evidence for this relationship in the data. Ultimately, consumption correlation puzzles play a prominent role in distinguishing among alternative models, so the discussion often revolves around them.

◦ *The Purchasing Power Parity Puzzle.* Rogoff (1996) shows how tenuous the connection is between nominal exchange rates and consumption price indexes resulting into highly volatile and persistent real exchange rates[6]. The slow mean reversion of the real exchange rate, however, does not appear to depend on the contribution of nontraded goods as documented most strikingly by Engel (1999) and Chari, Kehoe and McGrattan (2002).

There are also other stylized facts to keep in mind while exploring this puzzle. Goldberg and Knetter (1997) survey the empirical literature and conclude that the pass-through of nominal exchange rates relative to international prices is much faster at the importer level than at the consumer level. They also provide evidence of pricing-to-market behavior (and price discrimination). Obstfeld and Rogoff (2000b) argue that terms of trade worsen in response to an exchange rate depreciation. This fact, however, is hard to account for with very sluggish import prices. The power purchasing parity puzzle is, therefore, a broad reflection of the trouble that conventional general equilibrium models have to rationalize the observed patterns of international pricing.

◦ *The Exchange Rate Disconnect Puzzle.* This puzzle refers to the weak relationships found empirically between the exchange rate and most macro variables of interest, except for long horizons. Meese and Rogoff (1983) show that standard present-value models of the nominal exchange rate are regularly beaten by a naive random walk when forecasting nominal exchange rates at short and medium horizons. Baxter and Stockman (1989) and Flood and Rose (1995) argue about the neutrality of the exchange rate regime based on the fact that transitions to floating exchange rates tend to generate increases in nominal and real exchange rate variability without corresponding changes in the fundamentals. All these papers are manifestations of the same puzzle, anyway. They point the difficulties of explaining exchange rate dynamics in terms of

[6] As a note of caution, estimates of the persistence of real exchange rates can be missleading. In the presence of transaction costs or other nonlinearities, the real exchange rate can move slowly within a band and more quickly outside of it.

macroeconomic fundamentals as general equilibrium models attempt to do.

The power purchasing parity puzzle and the exchange rate disconnect puzzle (the so-called pricing puzzles) are the two sides of the same coin. They focus our attention on the price behavior, including the dynamic covariation between prices and other macro variables. However, as Obstfeld and Rogoff (2000a) already indicate, they also are more prevalent in the short to medium run, while all other puzzles (the so-called quantity puzzles) are more important in the long run. In these notes, let that be clear, I concentrate most of the discussion on the pricing puzzles.

2.1 Stylized Facts on Inflation, the Output Gap, the Current Account and the Exchange Rates

[TBA]

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3 The Workhorse Open Economy Macro Model

I specify a stochastic, two-country general equilibrium model in the spirit of Clarida, Galí and Gertler (2002), Obstfeld and Rogoff (2002), Chari, Kehoe and McGrattan (2002), and Woodford (2007), among others. The home country has a mass of n identical households, the foreign country (denoted with an asterisk) has a mass of $1 - n$ identical households. There is also a continuum of monopolistically competitive firms located in each country, which produce a differentiated, tradable good. Firms in the home country produce varieties in the range $[0, n]$, while foreign firms produce in the range $(n, 1]$. The population size of each country matches with the range of produced varieties. Hence, the number of firms in each country equals the number of households.

Under complete asset markets, households in both countries have access to a full set of Arrow-Debreu securities (or state-contingent bonds) which are quoted in both the domestic and foreign currencies. Under incomplete asset markets, households in both countries have access to two uncontingent bonds (each quoted in a different currency). A frictionless goods market is modelled with flexible prices and no trading costs. Frictions in the goods market are introduced in the form of nominal price stickiness à la Calvo (1983) and pricing-to-market. Deviations from purchasing power parity (henceforth, PPP) occur because nominal prices are sticky in the local currency of the buyer, and firms can effectively price discriminate across markets.

Both countries are essentially symmetric, although shocks are not always perfectly correlated. I allow for two important deviations from symmetry. I assume a home bias in consumption preferences (see Warnock, 2003), and I conjecture that the degree of price stickiness may vary depending on whether the firm is located in one country or the other[7].

3.1 Preliminary Concepts

The *law of one price* (henceforth, LOOP) states that barring frictional or complicating factors such as tariffs, taxes, and transportation costs, the price of an internationally traded good in one country should be equal to

[7] For other types of asymmetry in the degree of price rigidity, see G. Benigno (2004) and Martínez-García (2007).

the identical price in another country, once the price is expressed in the same currency. *Absolute purchasing power parity* (henceforth, absolute PPP) extends the concept to a common basket of goods. It simply states that a basket of goods that are frictionlessly and instantaneously traded must have an identical price in every country once the corresponding price index is expressed in the same currency.

If the basket of consumption goods contains some goods that are non-tradeable, some goods for which trading costs are non-negligible or the composition of the basket differs across countries, absolute PPP would likely fail even if the LOOP holds for all tradable goods. It is quite common to compare the CPI across countries expressed in the same currency with the help of the nominal exchange rate, and the evidence suggests that absolute PPP fails. However, this is neither proof that the LOOP fails nor a proper comparison precisely because CPI baskets are not homogeneous across countries and often contain goods that are not internationally traded[8].

Relative Purchasing Power Parity (henceforth, relative PPP) states that a basket of goods must have the same aggregate price in each pair of countries, if the price indexes are expressed in the same currency, up to a constant. Absolute PPP can be viewed as a special case of relative PPP. The LOOP may fail to hold because there is only limited pass-through. But if the pass-through rate is constant and identical for all goods, then the aggregation over all goods displays a similar pattern. Differences in prices may also be due to distribution costs or mark-ups that are country specific, but that should help explain differences in consumption prices and not so much in import prices (particularly the distribution costs). Furthermore, as noted before, pricing differences across CPIs could also be due to trading costs and compositional effects.

Systematic differences in those components of the observable price can explain the failure of absolute PPP. However, if those differences are constant over time, then relative PPP should still hold. (*See Mario Crucini for references...*).

Fluctuations of the Real Exchange Rate under the LOOP. The real exchange rate, RS_t, can be computed as the nominal exchange rate, S_t, times the ratio of the CPI in the foreign country, P_t^*, over the CPI in the domestic country, P_t, i.e.

$$RS_t \equiv \frac{S_t P_t^*}{P_t}.$$

If relative PPP holds, the real exchange rate should be equal to a constant, and equal to one if PPP is absolute. The empirical evidence suggests that relative PPP fails (at least in the short- and medium-run), and fluctuations of the real exchange rate are very volatile and persistent. Not surprisingly, the interpretation of these real exchange rate movements has become one of the most relevant topics in the international macro literature these days. Most researchers emphasize the presence of nominal rigidities and local-currency pricing in the goods market as an explanation for the failure of the LOOP[9], based on the existing evidence that suggests most goods adjust their prices infrequently while changes in the nominal exchange rate occur almost continually.

[8] In these notes, I abstract from nontraded goods entirely. However, I shall explore deviations from absolute PPP due to differences in the underlying basket of consumption goods across countries.

[9] I follow the same logic in these notes. However, as noted in my prior observations, fluctuations of the real exchange rate may occur even if the LOOP holds. This point is of great importance because it affects how we think about monetary monetary policy. If nominal rigidities are the primary channel, then optimal monetary policy may have real effects and a role to play in response to relative price distortions. If the LOOP holds and the real exchange rate fluctuates purely due to compositional effects in the basket of goods or trading costs, optimal monetary policy has no real effects in the model.

However, the failure of the LOOP is not required to induce fluctuations on the real exchange rate and violations of PPP. For simplicity, let me assume that prices are fully flexible, then:

Compositional Differences. Warnock's (2003) model of consumption bias in preferences gives us a natural way to do that. I can use the definition of the real exchange rate and the CPI indexes implied by the workhorse model, which I describe later, to argue that,

$$\begin{aligned} RS_t &= \frac{S_t\left[\xi^*\left(P_t^{H*}\right)^{1-\sigma}+(1-\xi^*)\left(P_t^{F*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\xi\left(P_t^{H}\right)^{1-\sigma}+(1-\xi)\left(P_t^{F}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \\ &= \frac{\left[\xi^*\left(P_t^{H}\right)^{1-\sigma}+(1-\xi^*)\left(P_t^{F}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\xi\left(P_t^{H}\right)^{1-\sigma}+(1-\xi)\left(P_t^{F}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \neq 1. \end{aligned} \tag{3.1.1}$$

In fact, under flexible prices. it follows that terms of trade are $ToT_t \equiv \frac{P_t^F}{S_t P_t^{H*}} = \frac{P_t^F}{P_t^H}$. Hence, it is possible to express the real exchange rate as a function of terms of trade,

$$RS_t = \begin{cases} \left[1+(\xi^*-\xi)\frac{1-(ToT_t)^{1-\sigma}}{\xi+(1-\xi)(ToT_t)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}, \text{ if } \xi^*\neq\xi, \\ 1, \text{ if } \xi^*=\xi. \end{cases} \tag{3.1.2}$$

Absolute and relative PPP fail in this setting, even if the LOOP holds, except in the case where the terms of trade endogenously turn out to be time invariant or preferences are identical across countries. In the long-run, however, it is likely to find support for relative PPP assuming that the real exchange rate and the terms of trade converge towards a steady state.

Trading Costs. Let me assume 'iceberg' shipping costs as in Obstfeld and Rogoff (2000a) such that for every unit of home and foreign good shipped abroad, only a fraction $1-\tau^c$ arrives at the foreign and domestic shores respectively. These trading costs are identical for all varieties and across countries. I denote P_t^H and P_t^F the domestic price of the home and foreign good, while P_t^{H*} and P_t^{F*} denote the corresponding foreign prices. Then, if markets are competitive and prices fully flexible, arbitrage would imply that,

$$\begin{aligned} P_t^{F*} &= \frac{1}{1-\tau^c}\frac{1}{S_t}P_t^F, \\ P_t^H &= (1-\tau^c)\, S_t P_t^{H*}. \end{aligned}$$

For any $0 \leq \tau^c < 1$, I can use again the definition of the real exchange rate and the CPI indexes implied by the workhorse model to show that,

$$\begin{aligned} RS_t &= \frac{S_t\left[\xi^*\left(P_t^{H*}\right)^{1-\sigma}+(1-\xi^*)\left(P_t^{F*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\xi\left(P_t^{H}\right)^{1-\sigma}+(1-\xi)\left(P_t^{F}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \\ &= \frac{1}{1-\tau^c}\frac{\left[\xi^*\left(P_t^{H}\right)^{1-\sigma}+(1-\xi^*)\left(P_t^{F}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\xi\left(P_t^{H}\right)^{1-\sigma}+(1-\xi)\left(P_t^{F}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \neq 1, \end{aligned} \tag{3.1.3}$$

which can be expressed as a function of terms of trade, $ToT_t \equiv \frac{P_t^F}{S_t P_t^{H*}} = (1-\tau^c)\frac{P_t^F}{P_t^H}$, as follows,

$$RS_t = \begin{cases} \frac{1}{1-\tau^c}\left[1+(\xi^*-\xi)\frac{1-\left(\frac{1}{1-\tau^c}ToT_t\right)^{1-\sigma}}{\xi+(1-\xi)\left(\frac{1}{1-\tau^c}ToT_t\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}}, \text{ if } \xi^* \neq \xi, \\ \frac{1}{1-\tau^c}, \text{ if } \xi^* = \xi. \end{cases} \tag{3.1.4}$$

Absolute and relative PPP fail in this setting too, even though the LOOP holds. However, relative PPP (but not absolute PPP) is a possible outcome if terms of trade are time invariant or preferences are identical across countries. This is no longer true if I assume that the trading costs are either time-varying or different across varieties.

Non-traded Goods. Non-traded varieties can be viewed as a special case where the 'iceberg' shipping costs are equal to one, i.e. $\tau^c = 1$. Therefore, prices abroad for domestic non-traded varieties are always infinity and demand for them is zero. A similar statement can be made regarding the domestic prices of foreign non-traded varieties. I denote $P_t^{H,T}$ and $P_t^{H,NT}$ the domestic price of the traded and non-traded home goods, and $P_t^{F,T}$ the domestic price of the traded foreign good. Similarly for the foreign prices $P_t^{F,T*}$, $P_t^{F,NT*}$ and $P_t^{H,T*}$. Then, if markets are competitive and prices fully flexible, arbitrage would imply that,

$$\begin{aligned} P_t^{F,T*} &= \frac{1}{1-\tau^c}\frac{1}{S_t}P_t^{F,T}, \\ P_t^{H,T} &= (1-\tau^c)\, S_t P_t^{H,T*}. \end{aligned}$$

A simple and straightforward extension of the consumption price indexes in the workhorse model allows me to write the real exchange rate as,

$$\begin{aligned} RS_t &= \frac{S_t\left[\xi^{T*}\left(P_t^{H,T*}\right)^{1-\sigma}+\xi^{NT*}\left(P_t^{F,NT*}\right)^{1-\sigma}+\left(1-\xi^{T*}-\xi^{NT*}\right)\left(P_t^{F,T*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\xi^{T}\left(P_t^{H,T}\right)^{1-\sigma}+\xi^{NT}\left(P_t^{H,NT}\right)^{1-\sigma}+\left(1-\xi^{T}-\xi^{NT}\right)\left(P_t^{F,T}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \\ &= \frac{1}{1-\tau^c}\frac{\left[\xi^{T*}\left(P_t^{H,T}\right)^{1-\sigma}+\xi^{NT*}\left((1-\tau^c)S_t P_t^{F,NT*}\right)^{1-\sigma}+\left(1-\xi^{T*}-\xi^{NT*}\right)\left(P_t^{F,T}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\xi^{T}\left(P_t^{H,T}\right)^{1-\sigma}+\xi^{NT}\left(P_t^{H,NT}\right)^{1-\sigma}+\left(1-\xi^{T}-\xi^{NT}\right)\left(P_t^{F,T}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}} \neq 1. \end{aligned} \tag{3.1.5}$$

Hence, if I redefine terms of trade for tradable goods only as $ToT_t^T \equiv \frac{P_t^{F,T}}{S_t P_t^{H,T*}} = (1-\tau^c)\frac{P_t^{F,T}}{P_t^{H,T}}$, it can be said that,

$$RS_t = \begin{cases} \frac{1}{1-\tau^c}\left[1+\frac{\left(\xi^{T*}-\xi^T\right)\left(1-\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}\right)+\xi^{NT}\left(\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}-\left(P_t^{NT,T}\right)^{1-\sigma}\right)-\xi^{NT*}\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}\left(1-\left(P_t^{NT,T*}\right)^{1-\sigma}\right)}{\xi^T+\left(1-\xi^T\right)\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}-\xi^{NT}\left(\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}-\left(P_t^{NT,T}\right)^{1-\sigma}\right)}\right]^{\frac{1}{1-\sigma}}, \\ \quad \text{if } \xi^{T*} \neq \xi^T \text{ and } \xi^{NT*} \neq \xi^{NT}, \\ \frac{1}{1-\tau^c}\left[1+\frac{\left(\xi^{T*}-\xi^T\right)\left(1-\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}\right)+\xi^{NT}\left(\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}\left(P_t^{NT,T*}\right)^{1-\sigma}-\left(P_t^{NT,T}\right)^{1-\sigma}\right)}{\xi^T+\left(1-\xi^T\right)\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}-\xi^{NT}\left(\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}-\left(P_t^{NT,T}\right)^{1-\sigma}\right)}\right]^{\frac{1}{1-\sigma}}, \\ \quad \text{if } \xi^{T*} \neq \xi^T \text{ and } \xi^{NT*} = \xi^{NT}, \\ \frac{1}{1-\tau^c}\left[1+\frac{\xi^{NT}\left(\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}\left(P_t^{NT,T*}\right)^{1-\sigma}-\left(P_t^{NT,T}\right)^{1-\sigma}\right)}{\xi^T+\left(1-\xi^T\right)\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}-\xi^{NT}\left(\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{1-\sigma}-\left(P_t^{NT,T}\right)^{1-\sigma}\right)}\right]^{\frac{1}{1-\sigma}}, \\ \quad \text{if } \xi^{T*} = \xi^T \text{ and } \xi^{NT*} = \xi^{NT}, \end{cases} \tag{3.1.6}$$

where $P_t^{NT,T} \equiv \frac{P_t^{H,NT}}{P_t^{H,T}}$ and $P_t^{NT,T*} \equiv \frac{P_t^{F,NT*}}{P_t^{F,T*}}$ denote the ratio of prices between nontraded and traded domestic and foreign goods quoted in their respective currencies. In the case where the elasticity of intratemporal substitution between the home and foreign bundles is one, i.e. $\sigma = 1$, the expression for the real exchange rate takes the well-known form of,

$$RS_t = \frac{1}{1-\tau^c}\left[\left(\frac{1}{1-\tau^c}ToT_t^T\right)^{\xi^T+\xi^{NT}-\xi^{T*}} \frac{\left(P_t^{NT,T*}\right)^{\xi^{NT*}}}{\left(P_t^{NT,T}\right)^{\xi^{NT}}}\right], \tag{3.1.7}$$

which depends exclusively on $P_t^{NT,T}$ and $P_t^{NT,T*}$ if $\xi^{T*} = \xi^T + \xi^{NT}$. Absolute and relative PPP fail in this setting once again, even though the LOOP holds. However, not even relative PPP except in the unlikely event when terms of trade and the ratios of nontraded to traded goods are time invariant.

Even though fluctuations can arise from any of these channels, the evidence shows that consumption-based real exchange rates fluctuate a lot over the short to medium horizon. Seemingly more than can be accounted for with fluctuations of the terms of trade, the trading costs or non-traded goods[10]. This has been interpreted as concurring evidence that violations of the LOOP are necessary to explain important features of the data, and has supported a strong interest on nominal rigidities and other goods market frictions (pricing-to-market) in the international macro literature. These notes build on the same idea.

3.2 The Households' Problem

For tractability, I abstract from a number of relevant features like capital and investment (see, e.g., Chari, Kehoe and McGrattan, 2002, or Martínez-García and Sondergaard, 2008) and durable goods (see, e.g., Engel and Wang, 2007). All consumption is in terms of perishable consumption goods. The lifetime utility for the representative household in the home country is additively separable in aggregate consumption, C_t, real money balances, $\frac{M_t^d}{P_t}$, and labor supply, L_t^s. The representative domestic household maximizes[11],

$$\sum_{\tau=0}^{\infty}\beta^\tau \mathbb{E}_t\left[\frac{1}{1-\gamma}\left(C_{t+\tau}\right)^{1-\gamma} + \frac{\chi}{1-\zeta}\left(\frac{M_{t+\tau}^d}{P_{t+\tau}}\right)^{1-\zeta} - \frac{\kappa}{1+\varphi}\left(L_{t+\tau}^s\right)^{1+\varphi}\right], \tag{3.2.1}$$

where $\beta \in (0,1)$ is the subjective intertemporal discount factor[12]. The inverse of the intertemporal elasticity of substitution, the risk aversion on real balances and the inverse of the Frisch elasticity of labor supply satisfy that $\gamma > 0$ ($\gamma \neq 1$), $\zeta > 0$ ($\zeta \neq 1$) and $\varphi > 0$, respectively. The coefficients χ and κ are nonnegative.

[10] Trading costs are thought to be slow-moving and more relevant for the long-run horizon. See Engel (1999) and Chari, Kehoe and McGrattan (2002) for a discussion of the importance of non-traded goods fluctuations on the real exchange rate.

[11] Epstein-Zin preferences provide an alternative where the coefficient of relative risk aversion is separated from the inverse of the intertemporal elasticity of sustitution. It is known that the intertemporal elasticity of substitution appears in a first-order approximation of these preferences, while the coefficient of relative risk-aversion only appears starting with a second-order approximation (Tallarini, 2000?). Given that I rely on a first-order approximation of the model, I do not need to add the more complex Epstein-Zin preferences. However, I interpret the parameter γ purely as the inverse elasticity of intertemporal substitution.

[12] For a similar version of the workhorse model with external habits, see Martínez-García (2007).

Similarly, the lifetime utility of the representative foreign household is,

$$\sum\nolimits_{\tau=0}^{\infty}\beta^{\tau}\mathbb{E}_t\left[\frac{1}{1-\gamma}\left(C_{t+\tau}^{*}\right)^{1-\gamma}+\frac{\chi}{1-\zeta}\left(\frac{M_{t+\tau}^{d*}}{P_{t+\tau}^{*}}\right)^{1-\zeta}-\frac{\kappa}{1+\varphi}\left(L_{t+\tau}^{s*}\right)^{1+\varphi}\right]. \tag{3.2.2}$$

Hence, the lifetime utility is symmetric in both countries.

The Budget Constraints. In each period t the world economy confronts one of infinitely many events $\omega_t \in \Omega$ in the state space Ω. The initial realization ω_0 is given, h_t denotes the history of events up to time t, and $\mu\left(\omega_{t+1} \mid h_t\right) \equiv \mu_t\left(\omega_{t+1}\right)$ the conditional probability at time t. The representative household in the home country allocates his wealth between three different asset types: domestic currency, M_t^d; either a set of Arrow-Debreu securities or an uncontingent bond expressed in the domestic currency, $B_t^H\left(\omega_{t+1}\right)$ or B_t^H; and, either a set of Arrow-Debreu securities or an uncontingent bond quoted in the foreign currency, $B_t^F\left(\omega_{t+1}\right)$ or B_t^F.

Similarly, the foreign household demands foreign currency, M_t^d, a set of Arrow-Debreu securities or a bond expressed in the domestic currency, $B_t^{H*}\left(\omega_{t+1}\right)$ or B_t^{H*}, and a set of Arrow-Debreu securities or a bond quoted in the foreign currency, $B_t^{F*}\left(\omega_{t+1}\right)$ or B_t^{F*}. As a convention, the time subscript indicates the period at which the security is issued and sold to the households. Also each household only holds money issued in the local currency. Local money is issued exclusively by the local monetary authority and assumed to be a one-period liability on their balance sheet.

Analogous to Woodford (2007), domestic money at time t promises a riskless nominal net return (in units of the domestic currency) of i_t^m payable at time $t+1$. Similarly, foreign money promises a nominal net return of i_t^{m*}. The current monetary regime suggests that $i_t^m = i_t^{m*} = 0$. However, I discuss the monetary policy implications of these administered rates later on because they can be interpreted as a modelling device to allow the monetary authority to regulate the liquidity supply in the markets (and, therefore, the price level) without necessarily changing the short-term nominal interest rates.

Complete Asset Markets. Arrow-Debreu securities (or claims) are modelled as one-period contingent bonds in zero net-supply. At each period t they involve the promise to pay one unit of the local currency if state ω_{t+1} occurs and 0 otherwise[13]. The domestic-currency price of one unit of the domestic contingent bond at time t is denoted $Q\left(\omega_{t+1} \mid h_t\right) \equiv Q_t\left(\omega_{t+1}\right)$, while the foreign-currency price of one unit of the foreign contingent bond is $Q^*\left(\omega_{t+1} \mid h_t\right) \equiv Q_t^*\left(\omega_{t+1}\right)$. The continued existence of a full set of one-period Arrow-Debreu securities suffices to complete the asset markets recursively. If this contingent bonds are available in the asset markets and asset trading is otherwise frictionless, I say that the economy has 'complete asset markets'.

The domestic household maximizes its lifetime utility in (3.2.1) subject to the sequence of budget con-

[13] Instead, money is defined as an asset that promises a regulated nominal interest rate of $1+i_t^m$ (or $1+i_t^{m*}$) tomorrow in exchange for one unit of the currency today. The reason is timing. What matters today is the amount of money held today (not the interest rate promise for tomorrow), because only the currently held money provides liquidity services to the households. If I wanted to define money as a claim to one unit of the currency tomorrow, then I would have to consider that today only a discounted amount (given by the regulated interest rates) is liquid money available for trade and change the utility specification accordingly.

straints,

$$\begin{aligned} & P_t C_t + M_t^d + \int_{\omega_{t+1}\in\Omega} Q_t\left(\omega_{t+1}\right) B_t^H\left(\omega_{t+1}\right) + S_t \int_{\omega_{t+1}\in\Omega} Q_t^*\left(\omega_{t+1}\right) B_t^F\left(\omega_{t+1}\right) \\ & \leq \left(1+i_{t-1}^m\right) M_{t-1}^d + B_{t-1}^H\left(\omega_t\right) + S_t B_{t-1}^F\left(\omega_t\right) + W_t L_t^s + \Pi_t - T_t, \end{aligned} \tag{3.2.3}$$

where W_t is the domestic nominal wage, P_t is the domestic consumption price index or CPI, S_t is the nominal exchange rate, T_t is the nominal lump-sum tax (or transfer) from the domestic government, and Π_t are nominal domestic profits. Similarly, the foreign household maximizes its lifetime utility in (3.2.2) subject to the sequence of budget constraints,

$$\begin{aligned} & P_t^* C_t^* + M_t^{d*} + \tfrac{1}{S_t}\int_{\omega_{t+1}\in\Omega} Q_t\left(\omega_{t+1}\right) B_t^{H*}\left(\omega_{t+1}\right) + \int_{\omega_{t+1}\in\Omega} Q_t^*\left(\omega_{t+1}\right) B_t^{F*}\left(\omega_{t+1}\right) \\ & \leq \left(1+i_{t-1}^{m*}\right) M_{t-1}^{d*} + \tfrac{1}{S_t} B_{t-1}^{H*}\left(\omega_t\right) + B_{t-1}^{F*}\left(\omega_t\right) + W_t^* L_t^{s*} + \Pi_t^* - T_t^*, \end{aligned} \tag{3.2.4}$$

where W_t^* is the foreign nominal wage, P_t^* is the foreign CPI, T_t^* is the nominal lump-sum tax (or transfer) received from the foreign government, and Π_t^* are nominal foreign profits.

I assume that there is no trade in either domestic or foreign shares. Moreover, I also impose a strict home bias in portfolios by giving sole ownership of the local firms to the local households. For asset markets to be complete it is not necessary to have each Arrow-Debreu claim quoted in both currencies. One of these two claims is redundant. In fact, most models with complete asset markets only have securities quoted in one of the currencies. However, having both of them is very convenient to compare the 'complete' versus the 'incomplete' asset markets.

Incomplete Asset Markets. I still maintain that trading in assets is frictionless, but I assume instead that households only have access to uncontingent bonds to define an alternative economy with 'incomplete asset markets'. That is, households can only invest their wealth in two bonds (each one of them quoted in a different currency). The domestic bond promises one unit of the domestic currency at time $t+1$ (independently of the event ω_{t+1}) in exchange for $Q_t \equiv \frac{1}{1+i_t}$ units of the domestic currency at time t. Similarly, the foreign bond promises one unit of the foreign currency tomorrow in exchange for $Q_t^* \equiv \frac{1}{1+i_t^*}$ units of the foreign currency today. The one-period riskless nominal interest rates in the domestic and foreign country are i_t and i_t^*, respectively.

Given my description of the asset markets with two sets of Arrow-Debreu claims quoted in each currency, it naturally follows that the incomplete asset market model with uncontingent bonds is equivalent to imposing the following restrictions on the demand,

$$\begin{aligned} B_t^H\left(\omega_{t+1}\right) &= B_t^H,\ B_t^F\left(\omega_{t+1}\right) = B_t^F, \\ B_t^{H*}\left(\omega_{t+1}\right) &= B_t^{H*},\ B_t^{F*}\left(\omega_{t+1}\right) = B_t^{F*}, \end{aligned}$$

for all $\omega_{t+1} \in \Omega$. In other words, the demand for bonds is uncontingent because that's the nature of the assets traded. In this situation, the uncontingent bonds are not sufficient in general to complete the asset markets (unless there are only two states of nature in each period or the assets markets are irrelevant as in Cole and Obstfeld, 1991, and Corsetti and Pesenti, 2001), and are not redundant. This structure is particularly convenient because it makes easier to visualize the difference between complete and incomplete asset markets.

The returns on the uncontingent bonds can be replicated with Arrow-Debreu claims. Therefore, the prices of the uncontingent bonds must satisfy that,

$$\int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1}) = Q_t \equiv \frac{1}{1+i_t},\ \int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1}) = Q_t^* \equiv \frac{1}{1+i_t^*}. \tag{3.2.5}$$

The sequence of budget constraints for the domestic and foreign households can be re-written in the following terms,

$$P_tC_t + M_t^d + \frac{1}{1+i_t}B_t^H + S_t\frac{1}{1+i_t^*}B_t^F \leq \left(1+i_{t-1}^m\right)M_{t-1}^d + B_{t-1}^H + S_tB_{t-1}^F + W_tL_t^s + \Pi_t - T_t \tag{3.2.6}$$

$$P_t^*C_t^* + M_t^{d*} + \frac{1}{S_t}\frac{1}{1+i_t}B_t^{H*} + \frac{1}{1+i_t^*}B_t^{F*} \leq \left(1+i_{t-1}^{m*}\right)M_{t-1}^{d*} + \frac{1}{S_t}B_{t-1}^{H*} + B_{t-1}^{F*} + W_t^*L_t^{s*} + \Pi_t^* - T_t^* \tag{3.2.7}$$

This type of asset market structure is often referred to in the literature as a *bond economy.*

Remark 1 *I can write the budget constraints of the domestic and foreign households (equations* (3.2.3) – (3.2.4)*) in the following form,*

$$\begin{aligned} &P_tC_t + \Delta_tM_t^d + \int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1})A_t^H(\omega_{t+1}) + S_t\int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1})B_t^F(\omega_{t+1}) \\ &\leq A_{t-1}^H(\omega_t) + S_tB_{t-1}^F(\omega_t) + W_tL_t^s + \Pi_t - T_t, \end{aligned} \tag{3.2.8}$$

$$\begin{aligned} &P_t^*C_t^* + \Delta_t^*M_t^{d*} + \tfrac{1}{S_t}\int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1})B_t^{H*}(\omega_{t+1}) + \int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1})A_t^{F*}(\omega_{t+1}) \\ &\leq \tfrac{1}{S_t}B_{t-1}^{H*}(\omega_t) + A_{t-1}^{F*}(\omega_t) + W_t^*L_t^{s*} + \Pi_t^* - T_t^*, \end{aligned} \tag{3.2.9}$$

where $A_{t-1}^H(\omega_t) \equiv \left(1+i_{t-1}^m\right)M_{t-1}^d + B_{t-1}^H(\omega_t)$ *and* $A_{t-1}^{F*} \equiv \left(1+i_{t-1}^m\right)M_{t-1}^{d*} + B_{t-1}^{F*}(\omega_t)$ *represent the total value of nominal wealth in the local currency at the beginning of period* t*. The interest rate differential between a riskless, one-period, non-monetary asset and money (which yields liquidity services) is,*

$$\Delta_t \equiv \frac{i_t - i_t^m}{1+i_t},\ \Delta_t^* \equiv \frac{i_t^* - i_t^{m*}}{1+i_t^*}.$$

It is evident from (3.2.8) – (3.2.9) *that these terms measure the opportunity cost of holding part of one's wealth in monetary form.*

Preference for Varieties and the Price Indexes. The home and foreign consumption bundles of the domestic household, C_t^H and C_t^F, are aggregated by means of a CES preference index as,

$$C_t^H = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n C_t(h)^{\frac{\theta-1}{\theta}}dh\right]^{\frac{\theta}{\theta-1}},\ C_t^F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}}\int_n^1 C_t(f)^{\frac{\theta-1}{\theta}}df\right]^{\frac{\theta}{\theta-1}}, \tag{3.2.10}$$

while aggregate consumption, C_t, is defined with another CES preference index as,

$$C_t = \begin{cases} \left[\xi^{\frac{1}{\sigma}}\left(C_t^H\right)^{\frac{\sigma-1}{\sigma}} + (1-\xi)^{\frac{1}{\sigma}}\left(C_t^F\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},\ \text{if } \sigma>0,\ \sigma\neq 1, \\ \frac{\left(C_t^H\right)^{\xi}\left(C_t^F\right)^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}},\ \text{if } \sigma=1. \end{cases} \tag{3.2.11}$$

The elasticity of substitution across varieties produced within a country is $\theta > 1$, the elasticity of intratemporal substitution between the home and foreign bundles of varieties is $\sigma > 0$, and the share of the home goods in the preference of the domestic consumer is ξ. The superscripts H and F refer to purchases of goods produced in the home and foreign country, respectively. The lower case h and f indicate domestic and foreign varieties, respectively.

Similarly, the representative foreign household has preferences defined by,

$$C_t^{H*} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C_t^*(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \; C_t^{F*} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 C_t^*(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}, \tag{3.2.12}$$

$$C_t^* = \begin{cases} \left[(\xi^*)^{\frac{1}{\sigma}} \left(C_t^{H*}\right)^{\frac{\sigma-1}{\sigma}} + (1-\xi^*)^{\frac{1}{\sigma}} \left(C_t^{F*}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \text{ if } \sigma > 0, \; \sigma \neq 1, \\ \frac{\left(C_t^{H*}\right)^{\xi^*} \left(C_t^{F*}\right)^{1-\xi^*}}{(\xi^*)^{\xi^*} (1-\xi^*)^{1-\xi^*}}, \text{ if } \sigma = 1. \end{cases} \tag{3.2.13}$$

Preferences are identical for households in both countries, except for the share of home and foreign goods in the preference of each consumer. The parameter ξ^* denotes the share of domestic goods for the foreign household. These definitions allow for home bias in consumption as in Warnock (2003). A convenient simplification is to assume that $\xi = \xi^* = n$.

The domestic CPI, P_t, is defined as the minimum expenditure needed to buy one unit of the consumption index, C_t. Similarly for P_t^* and C_t^*. Under standard results on functional separability, the indexes which correspond to this specification of preferences are[14],

$$P_t = \begin{cases} \left[\xi \left(P_t^H\right)^{1-\sigma} + (1-\xi) \left(P_t^F\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \text{ if } \sigma > 0, \; \sigma \neq 1, \\ \left(P_t^H\right)^{\xi} \left(P_t^F\right)^{1-\xi}, \text{ if } \sigma = 1, \end{cases} \tag{3.2.14}$$

$$P_t^* = \begin{cases} \left[\xi^* \left(P_t^{H*}\right)^{1-\sigma} + (1-\xi^*) \left(P_t^{F*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \text{ if } \sigma > 0, \; \sigma \neq 1, \\ \left(P_t^{H*}\right)^{\xi^*} \left(P_t^{F*}\right)^{1-\xi^*}, \text{ if } \sigma = 1, \end{cases} \tag{3.2.15}$$

and

$$P_t^H = \left[\frac{1}{n} \int_0^n P_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, \; P_t^F = \left[\frac{1}{1-n} \int_n^1 P_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}, \tag{3.2.16}$$

$$P_t^{H*} = \left[\frac{1}{n} \int_0^n P_t^*(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, \; P_t^{F*} = \left[\frac{1}{1-n} \int_n^1 P_t^*(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}, \tag{3.2.17}$$

where P_t^H and P_t^F are the price sub-indexes for the home- and foreign-produced bundles of goods in units of the home currency. Similarly for P_t^{H*} and P_t^{F*}. Home and foreign households have identical tastes and, therefore, the respective price indexes are identical. Moreover, I define the terms of trade in both countries as,

$$ToT_t \equiv \frac{P_t^F}{S_t P_t^{H*}}, \; ToT_t^* \equiv \frac{S_t P_t^{H*}}{P_t^F} = \frac{1}{ToT_t}, \tag{3.2.18}$$

[14] Clarida, Galí and Gertler (2002) only explore the case where the elasticity of intratemporal substitution between the home and foreign bundles of varieties is $\sigma = 1$. But preferences are not scaled and, therefore, the CPI must be pre-multiplied by $\left[n^n (1-n)^{1-n}\right]^{-1}$.

and the real exchange rate as,

$$RS_t \equiv \frac{S_t P_t^*}{P_t}, \tag{3.2.19}$$

where S_t denotes the nominal exchange rate.

I represent the relative price in each country by[15],

$$RP_t \equiv \frac{P_t^F}{P_t^H},\ RP_t^* \equiv \frac{P_t^{H*}}{P_t^{F*}}. \tag{3.2.20}$$

The relative price, RP_t, represents the value of imported goods (quoted in the domestic market) relative to the value of the domestic good supplied locally. This ratio is the 'local market' cost of replacing one unit of imports with one unit of the domestically-produced good. Instead, terms of trade ToT_t represents the value of imported goods (quoted in the domestic market) relative to the value of the domestic good exported to the foreign market, but expressed in units of the local currency. This ratio measures the 'foreign market' cost of replacing one unit of imports with one unit of exports. Similarly for RP_t^* and ToT_t^*.

The Demand Function for each Variety. Each household decides how much to allocate to the different varieties of home and foreign goods. Given the structure of preferences, the solution to the sub-utility maximization problem implies that the home and foreign households' demands for each variety are given by,

$$C_t(h) = \frac{1}{n}\left(\frac{P_t(h)}{P_t^H}\right)^{-\theta} C_t^H,\ C_t^*(h) = \frac{1}{n}\left(\frac{P_t^*(h)}{P_t^{H*}}\right)^{-\theta} C_t^{H*},\ \text{if } h \in [0,n], \tag{3.2.21}$$

$$C_t(f) = \frac{1}{1-n}\left(\frac{P_t(f)}{P_t^F}\right)^{-\theta} C_t^F,\ C_t^*(f) = \frac{1}{1-n}\left(\frac{P_t^*(f)}{P_t^{F*}}\right)^{-\theta} C_t^{F*},\ \text{if } f \in (n,1], \tag{3.2.22}$$

while the demands for the bundles of home and foreign goods are simply equal to,

$$C_t^H = \xi\left(\frac{P_t^H}{P_t}\right)^{-\sigma} C_t,\ C_t^{H*} = \xi^*\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma} C_t^*, \tag{3.2.23}$$

$$C_t^F = (1-\xi)\left(\frac{P_t^F}{P_t}\right)^{-\sigma} C_t,\ C_t^{F*} = (1-\xi^*)\left(\frac{P_t^{F*}}{P_t^*}\right)^{-\sigma} C_t^*. \tag{3.2.24}$$

These equations indicate that the domestic demand for each variety, whether domestic or foreign, is a function of total domestic consumption. Similarly, the foreign demand for each variety, whether domestic or foreign, is a function of total foreign consumption.

The First-Order Conditions of the Households' Problem. Under complete asset markets, the (interior) optimal allocation of consumption expenditures in each country over time and across states implies

[15] Terms of trade and relative prices are identical and the real exchange rate is equal to one only if the LOOP holds in both countries. Since the LOOP condition fails in the workhorse model, fluctuations of the real exchange rate arise and the distinction between terms of trade and relative prices matters.

that,

$$Q_t(\omega_{t+1}) = \beta \left(\frac{C_{t+1}(\omega_{t+1})}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}(\omega_{t+1})} \mu_t(\omega_{t+1}), \tag{3.2.25}$$

$$Q_t^*(\omega_{t+1}) = \beta \left(\frac{C_{t+1}^*(\omega_{t+1})}{C_t^*} \right)^{-\gamma} \frac{P_t^*}{P_{t+1}^*(\omega_{t+1})} \mu_t(\omega_{t+1}), \tag{3.2.26}$$

$$\frac{S_t}{S_{t+1}(\omega_{t+1})} \frac{Q_t^*(\omega_{t+1})}{Q_t(\omega_{t+1})} = 1, \tag{3.2.27}$$

which must hold for all $\omega_{t+1} \in \Omega$. These are the intertemporal conditions that characterize the optimal allocation of resources, and perfect international risk-sharing. The combination of equations (3.2.25) – (3.2.27) results in the following equilibrium condition expressed compactly (and without reference to the state for ease of notation) as,

$$\frac{RS_{t+1}}{RS_t} = \left(\frac{C_{t+1}^*}{C_t^*} \frac{C_t}{C_{t+1}} \right)^{-\gamma}, \tag{3.2.28}$$

where the real exchange rate is defined in (3.2.19).

Backward recursion allows me to express the real exchange rate as follows,

$$RS_t = \upsilon \left(\frac{C_t^*}{C_t} \right)^{-\gamma}, \tag{3.2.29}$$

where $\upsilon \equiv \frac{S_0 P_0^*}{P_0} \left(\frac{C_0^*}{C_0} \right)^{\gamma}$ is a constant that depends on the initial conditions[16]. This equation is central to the so-called *Backus-Smith Puzzle* (also known as *the real exchange rate-relative consumption anomaly*) and is at the core of what the *international consumption correlations puzzle* is all about. The puzzle arises because the complete asset markets model clearly ties the real exchange rate to the relative consumption, while the empirical evidence suggests that the correlation between the two is weak and often negative. The choice of a functional form for the preferences is obviously not trivial in this result. However, most of the literature has argued that the empirical evidence is symptomatic of lack of international risk-sharing and has concentrated on theories of asset market incompleteness or frictions in asset trading to seek an explanation.

Under incomplete asset markets, I obtain a conventional set of stochastic Euler equations,

$$\frac{1}{1+i_t} = Q_t = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right], \tag{3.2.30}$$

$$\frac{1}{1+i_t^*} = Q_t^* = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} \frac{P_t^*}{P_{t+1}^*} \right], \tag{3.2.31}$$

$$\mathbb{E}_t \left[\frac{S_t}{S_{t+1}} \frac{m_{t,t+1}^*}{m_{t,t+1}} \right] = 1, \tag{3.2.32}$$

which is equivalent to taking conditional expectations on both sides of (3.2.25) – (3.2.27) and re-arranging

[16] In a symmetric world and after a convenient normalization of the steady state, which I discuss later, home and foreign consumption are identical and the real interest rate is equal to one. Hence, if the initial conditions correspond to those of the normalized steady state, the constant υ in the above expression is equal to one.

terms. Here, I define the one-period ahead intertemporal marginal rates of substitution (IMRS) as,

$$m_{t,t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_t}{P_{t+1}}, \tag{3.2.33}$$

$$m^*_{t,t+1} \equiv \beta \left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma} \frac{P^*_t}{P^*_{t+1}}. \tag{3.2.34}$$

It holds true that the IMRS and the price of the Arrow-Debreu securities are related as follows,

$$m_{t,t+1} \equiv m_t\left(\omega_{t+1}\right) = \frac{Q_t\left(\omega_{t+1}\right)}{\mu_t\left(\omega_{t+1}\right)},\ m^*_{t,t+1} \equiv m^*_t\left(\omega_{t+1}\right) = \frac{Q^*_t\left(\omega_{t+1}\right)}{\mu_t\left(\omega_{t+1}\right)},$$

for all $\omega_{t+1} \in \Omega$. In other words, the price of an Arrow-Debreu claim can be interpreted as the product of the IMRS at a given state of nature times the conditional probability of that event actually occurring. If investors were risk-neutral, then they would be willing to buy insurance through Arrow-Debreu claims at a cost equal to the conditional probability of each possible event. The IMRS, therefore, could be viewed as the premium (or discount) that investors are ready to pay over the risk-neutral price of a claim in order to be indifferent between buying the claim and 'tolerating the risk'.

Remark 2 *Simple algebra gives me the familiar decomposition,*

$$\mathbb{E}_t\left[\frac{S_{t+1}}{S_t}\right] = \mathbb{E}_t\left[\frac{m^*_{t,t+1}}{m_{t,t+1}}\right] = \left(\frac{1+i_t}{1+i^*_t}\right) \underbrace{\mathbb{E}_t\left[m_{t,t+1}\right]\mathbb{E}_t\left[\frac{1}{m_{t,t+1}}\right]}_{\textit{Convexity term}} + \underbrace{\mathbb{CV}_t\left[m^*_{t,t+1}, \frac{1}{m_{t,t+1}}\right]}_{\textit{FX premium}}. \tag{3.2.35}$$

A variant of the uncovered interest rate parity (UIP) condition *would hold in levels if the convexity term is equal to one and the FX premium equal to zero. To be more precise, what equation* (3.2.35) *shows is that the UIP condition does not hold in general. This feature is qualitatively consistent with the empirical evidence, but gets lost in a first-order approximation.*

Finally, the equilibrium conditions of the households' problem also include a pair of stable money demand and a pair of labor supply functions which can be expressed as,

$$\chi\left(\frac{M^d_t}{P_t\left(C_t\right)^{\frac{\gamma}{\zeta}}}\right)^{-\zeta} = \Delta_t, \tag{3.2.36}$$

$$\chi\left(\frac{M^{d*}_t}{P^*_t\left(C^*_t\right)^{\frac{\gamma}{\zeta}}}\right)^{-\zeta} = \Delta^*_t, \tag{3.2.37}$$

$$\frac{W_t}{P_t} = \kappa\left(C_t\right)^{\gamma}\left(L^s_t\right)^{\varphi}, \tag{3.2.38}$$

$$\frac{W^*_t}{P^*_t} = \kappa\left(C^*_t\right)^{\gamma}\left(L^{s*}_t\right)^{\varphi}, \tag{3.2.39}$$

plus the appropriate no-Ponzi games, transversality conditions and the budget constraint of both representative households. Equations (3.2.36) – (3.2.39) are the intratemporal equilibrium conditions. The labor supply and the money demand functions in this framework do not depend on the assumption of complete asset markets. These first-order conditions are the same whether households have access to a full set of

Arrow-Debreu securities or just a pair of one-period riskless bonds.

The labor supply function is crucial to determine the marginal costs faced by firms. The money demand function is of interest for the conduct of monetary policy. Therefore, I describe these conditions more carefully in later subsections.

Balanced Growth. Let me abstract from all stochastic uncertainty, or monetary policy regime changes that would shift the opportunity cost of holding money. Moreover, I require that in the market sector the labor-augmenting productivity, g_t, grows at an exogenous and constant net rate of g. In other words, $L_t^s\,(= L_t)$ units of time produce $g_t L_t$ units of labor for production purposes.

Chari, Kehoe and McGrattan (2002) also assume that the non-market sectors grow at the same rate g. Adapting their idea to this model implies that technical progress raises the effort of time allocated to market production, so that $L_t^s\,(= L_t)$ units of time produce $g_t L_t$ units of labor output as well as $g_t L_t$ units of labor disutility. In fact, technical progress raises the time allocated for transaction or liquidity services (which is a function of real balances), such that $\frac{M_t^d}{P_t}\left(= \frac{M_t}{P_t}\right)$ units of real balances buy $(g_t)^z \frac{M_t}{P_t}$ of transaction services[17].

Given the specification of preferences adopted in these notes, whenever consumption, C_t, and real balances, $\frac{M_t}{P_t}$, grow at the same rate as g_t, but labor L_t is constant, then it must hold true in equilibrium that,

$$\chi\,(1+g)^{z(1-\zeta)t}\left(\frac{(1+g)^t}{(1+g)^{\frac{\gamma}{\zeta}t}}\right)^{-\zeta} = \Delta_t,$$
$$\frac{W_t}{P_t} = \kappa\,(L^s)^{\varphi}\,(1+g)^{(1+\varphi)t}\,(1+g)^{\gamma t},$$

which follows naturally from the same first-order conditions as equations (3.2.36) and (3.2.38). Similarly for the foreign country. Along the balanced growth path, real wages must grow at the same rate as g_t and the opportunity cost of holding money should be constant (unless a policy regime is possible). Hence, ensuring a balanced growth in this model with additively separable utility functions requires that $\gamma = -\varphi$ and $\gamma = \zeta - z\,(1-\zeta)$. These parametric conditions are most often not satisfied in the literature (specially the restriction on the inverse of the Frisch elasticity of labor supply[18]).

3.3 The Firms' Problem

Each firm supplies the home and foreign markets. I impose competitive markets for inputs (labor), that are fully segregated across countries. That is, cross-border labor migrations are not feasible. Frictions in the goods market are modelled with nominal price stickiness à la Calvo (1983) and pricing-to-market. Firms set prices in the local currency (LCP pricing) and, consequently, invoice exports in the currency of the importer.

[17] The utility function used by Chari, Kehoe and McGrattan (2002) is not additively separable in real balances. However, they make the sensible assumption that $z = 0$.

[18] Chari, Kehoe and McGrattan (2002) propose that leisure utility instead of labor disutility appear in the utility function as,

$$\frac{\kappa}{1-\varphi}\,(1-L_t^s)^{1-\varphi}.$$

The parametric restriction for balanced growth path with this alternative utility is $\gamma = \varphi$. Balanced growth is easier to reconcile with this restriction than the one used in these notes.

Furthermore, firms engage in third-degree price discrimination across markets and enjoy monopolistic power in their own variety. Re-selling must be precluded so that the optimal pricing policy is not reversed by re-sellers exploiting the arising arbitrage opportunities in the goods market[19].

These assumptions require a degree of international market segmentation which prevents the equalization of prices across borders, and opens up an important channel for deviations from the LOOP (and, therefore, from absolute PPP). These notes focus on fluctuations in real exchange rates arising solely from deviations of the LOOP on traded goods or home bias in consumption (see Warnock, 2003). I abstract from non-traded goods altogether to be consistent with the evidence documented by Engel (1999) and Chari, Kehoe and McGrattan (2002)[20]. I also sidestep distribution costs on tradable goods even though these assumption adds a non-tradeable component to local prices that could explain the pricing differences across markets. I also do without the iceberg-type trading costs proposed by Obstfeld and Rogoff (2000a).

The Technology. With probability $\alpha \in [0,1]$, at time t the domestic firm producing variety h is forced to maintain its previous period prices in the domestic and foreign markets[21]. With probability $(1-\alpha)$, the firm receives a signal to optimally reset each price. Firm $h \in [0,n]$ produces a differentiated (and tradable) variety with a linear-in-labor technology, i.e.

$$Y_t^s(h) = A_t L_t^d(h), \tag{3.3.1}$$

where A_t is the domestic productivity shock. Analogously, the probability of re-setting prices for the foreign firm $f \in (n,1]$ is given by $(1-\alpha^*)$, and the linear-in-labor technology is,

$$Y_t^{s*}(f) = A_t^* L_t^{d*}(f), \tag{3.3.2}$$

where A_t^* is the foreign productivity shock.

The labor force is homogenous within a country and immobile across borders, and the national labor markets are perfectly competitive. Hence, wages equalize in each country but not necessarily across countries, i.e. $W_t(h) = W_t$ for all $h \in [0,n]$, $W_t^*(f) = W_t^*$ for all $f \in (n,1]$, and usually $W_t \neq W_t^*$. Otherwise, only the firms with the largest wages would be supplied in a competitive labor market. As a consequence, the nominal cost function is multiplicatively separable in output and the nominal unit-cost is equal to the nominal

[19] Costly reselling seems more plausible for perishable goods, which would make price discrimination across markets more effective (consistently with the model). However, as noted by Engel and Wang (2007), most of the trade across countries is in durable goods and this has important implications for the behavior of imports and exports, as well as for other macroeconomic variables. The open question is: Why don't we observe more arbitrage in the international pricing of durable goods?

[20] This evidence, however, is not undisputed. For instance, Crucini, Telmer and Zachariadis (2005) use retail goods and services prices between all EU countries to conclude that "good-by-good measures of cross-sectional price dispersion are negatively related to the tradeability of the good, and positively related to the share of non-traded inputs required to produce the good". They also argue that their findings are supportive of a model where the retail goods sold to consumers include a significative share of non-traded input (which e.g. could be interpreted as distribution services).

[21] The Calvo contract can be interpreted as a fixed price offer with a constant revision probability per period. Under the Calvo specification, the probability of setting a new price, $1-\alpha$, is the same for all firms and is independent of the time elapsed since the last price change. Hence, the average time under fixed prices is equal to $\frac{1}{1-\alpha}$.

marginal cost. I define the nominal marginal costs after taxes (or subsidies) as,

$$MC_t \equiv \left(\frac{(1+\phi_t) W_t}{A_t} \right), \; MC_t^* \equiv \left(\frac{(1+\phi_t^*) W_t^*}{A_t^*} \right), \tag{3.3.3}$$

where ϕ_t and ϕ_t^* are a pair of taxes (or subsidies) charged by the fiscal authority in each country in order to influence wage costs and the conditions in the labor market. I discuss these labor taxes in more detail when I describe the role of the government later on.

The Net Discounted Profits. Households are charged a different price for the same variety in each country, but they still face a constant price within a country for all units of output purchased. Re-selling across borders is either banned or infeasible due to high costs. A domestic firm h has to choose the price charged domestically (in units of the domestic currency), $\widetilde{P}_t(h)$, and the price charged abroad (in units of the foreign currency), $\widetilde{P}_t^*(h)$. The objective is to maximize the expected discounted value of its net profits subject to a demand constraint in each goods market,

$$\sum_{\tau=0}^{\infty} \mathbb{E}_t \left\{ \alpha^\tau m_{t,t+\tau} \left[\begin{array}{c} n\widetilde{Y}_{t,t+\tau}^d(h) \left(\widetilde{P}_t(h) - \frac{(1+\phi_{t+\tau})W_{t+\tau}}{A_{t+\tau}} \right) + \\ +(1-n)\widetilde{Y}_{t,t+\tau}^{d*}(h) \left(S_{t+\tau}\widetilde{P}_t^*(h) - \frac{(1+\phi_{t+\tau})W_{t+\tau}}{A_{t+\tau}} \right) \end{array} \right] \right\}, \tag{3.3.4}$$

where $m_{t,t} = 1$ and $m_{t,t+\tau} \equiv \beta^\tau \left(\frac{C_{t+\tau}}{C_t}\right)^{-\gamma} \frac{P_t}{P_{t+\tau}}$ is an extension of the one-period ahead IMRS introduced in (3.2.33). I derive the demand for variety h in the home and foreign markets by combining equations (3.2.21) − (3.2.24). As a result, $\widetilde{Y}_{t,t+\tau}^d(h)$ and $\widetilde{Y}_{t,t+\tau}^{d*}(h)$ indicate the per capita demand for any variety h at home and abroad respectively, given that prices $\widetilde{P}_t(h)$ and $\widetilde{P}_t^*(h)$ remain unchanged between time t and $t+\tau$, i.e.

$$\widetilde{Y}_{t,t+\tau}^d(h) = \frac{\xi}{n} \left(\frac{\widetilde{P}_t(h)}{P_{t+\tau}^H} \right)^{-\theta} \left(\frac{P_{t+\tau}^H}{P_{t+\tau}} \right)^{-\sigma} C_{t+\tau}, \tag{3.3.5}$$

$$\widetilde{Y}_{t,t+\tau}^{d*}(h) = \frac{\xi^*}{n} \left(\frac{\widetilde{P}_t^*(h)}{P_{t+\tau}^{H*}} \right)^{-\theta} \left(\frac{P_{t+\tau}^{H*}}{P_{t+\tau}^*} \right)^{-\sigma} C_{t+\tau}^*. \tag{3.3.6}$$

Similarly, I characterize the problem of the foreign firm with the objective of maximizing the expected discounted value of its net profits subject to a demand constraint,

$$\sum_{\tau=0}^{\infty} \mathbb{E}_t \left\{ (\alpha^*)^\tau m_{t,t+\tau}^* \left[\begin{array}{c} n\widetilde{Y}_{t,t+\tau}^d(f) \left(\frac{1}{S_{t+\tau}}\widetilde{P}_t(f) - \frac{(1+\phi_{t+\tau}^*)W_{t+\tau}^*}{A_{t+\tau}^*} \right) + \\ +(1-n)\widetilde{Y}_{t,t+\tau}^{d*}(f) \left(\widetilde{P}_t^*(f) - \frac{(1+\phi_{t+\tau}^*)W_{t+\tau}^*}{A_{t+\tau}^*} \right) \end{array} \right] \right\}, \tag{3.3.7}$$

where $m_{t,t}^* = 1$ and $m_{t,t+\tau}^* \equiv \beta \left(\frac{C_{t+\tau}^*}{C_t^*}\right)^{-\gamma} \frac{P_t^*}{P_{t+\tau}^*}$ is an extension of the one-period IMRS introduced in (3.2.34).

I obtain that $\widetilde{Y}^{d}_{t,t+\tau}(f)$ and $\widetilde{Y}^{d*}_{t,t+\tau}(f)$ describe the demand constraints of the foreign firms as,

$$\widetilde{Y}^{d}_{t,t+\tau}(f) = \frac{1-\xi}{1-n}\left(\frac{\widetilde{P}_t(f)}{P^F_{t+\tau}}\right)^{-\theta}\left(\frac{P^F_{t+\tau}}{P_{t+\tau}}\right)^{-\sigma} C_{t+\tau}, \tag{3.3.8}$$

$$\widetilde{Y}^{d*}_{t,t+\tau}(f) = \frac{1-\xi^*}{1-n}\left(\frac{\widetilde{P}^*_t(f)}{P^{F*}_{t+\tau}}\right)^{-\theta}\left(\frac{P^{F*}_{t+\tau}}{P^*_{t+\tau}}\right)^{-\sigma} C^*_{t+\tau}, \tag{3.3.9}$$

given that prices $\widetilde{P}_t(h)$ and $\widetilde{P}^*_t(h)$ remain unchanged between time t and $t+\tau$.

Finally, the per capita profits of the domestic firms can be expressed in the domestic currency as,

$$\Pi_t = \frac{1}{n}\left[\int_0^n \left[P_t(h)\, nC_t(h) + S_t P^*_t(h)(1-n)\, C^*_t(h)\right] dh - (1+\phi_t)\, W_t \int_0^n L^d_t(h)\, dh\right], \tag{3.3.10}$$

while the per capita profits of the foreign firms quoted in the foreign currency are,

$$\Pi^*_t = \frac{1}{1-n}\left[\int_n^1 \left[\frac{1}{S_t}P_t(f)\, nC_t(f) + P^*_t(f)(1-n)\, C^*_t(f)\right] df - (1+\phi^*_t)\, W^*_t \int_n^1 L^{d*}_t(f)\, df\right]. \tag{3.3.11}$$

The labor demand functions of the domestic firms, $L^d_t(h)$ for all $h \in [0,n]$, and the foreign firms, $L^{d*}_t(f)$ for all $f \in (n,1]$, are implicit in the cost function and can be derived using Shephard's lemma. The labor markets are fully segmented across countries, so the market clearing conditions merely require that,

$$nL^s_t = \int_0^n L^d_t(h)\, dh, \tag{3.3.12}$$

$$(1-n)\, L^{s*}_t = \int_n^1 L^{d*}_t(f)\, df. \tag{3.3.13}$$

where L^s_t is the labor supply of the representative domestic household and L^{s*}_t is the labor supply of the representative foreign household.

The First-Order Conditions. *The Dixit-Stiglitz Pricing Equations.* In the polar case where all prices are adjusted in each period (flexible prices and frictionless trading in goods, i.e. $\alpha = 0$), the pricing equations imply that,

$$P^H_t = P_t(h) = \frac{\theta}{\theta-1}\left(\frac{(1+\phi_t)\, W_t}{A_t}\right) = P^*_t(h)\, S_t = P^{H*}_t S_t, \tag{3.3.14}$$

$$P^{F*}_t = P^*_t(f) = \frac{\theta}{\theta-1}\left(\frac{(1+\phi^*_t)\, W^*_t}{A^*_t}\right) = P_t(f)\,\frac{1}{S_t} = P^F_t\,\frac{1}{S_t}, \tag{3.3.15}$$

which is the outcome expected under monopolistic competition. These formulas are often known as the Dixit-Stiglitz pricing equations. The LOOP, then, holds in each variety and the pricing decision in either market is equal to a mark-up times the nominal marginal cost (whenever expressed in units of the same currency). The gross mark-up, $\frac{\theta}{\theta-1}$, is clearly a function of the elasticity of substitution across varieties, i.e. θ. Often the literature assumes that the elasticity of substitution across varieties ought to be greater than one. This parametric assumption is meant to insure that the gross mark-up is always above one, as we would expect it to be.

The Optimal Pricing Equations. The necessary and sufficient first-order conditions for the domestic firm producing variety h give me the following pair of price-setting formulas,

$$\widetilde{P}_t(h) = \frac{\theta}{\theta-1} \frac{\sum_{\tau=0}^{\infty} \alpha^{\tau} \mathbb{E}_t \left[m_{t,t+\tau} \widetilde{Y}^d_{t,t+\tau}(h) \left(\frac{(1+\phi_{t+\tau}) W_{t+\tau}}{A_{t+\tau}} \right) \right]}{\sum_{\tau=0}^{\infty} \alpha^{\tau} \mathbb{E}_t \left[m_{t,t+\tau} \widetilde{Y}^d_{t,t+\tau}(h) \right]}, \quad (3.3.16)$$

$$\widetilde{P}^*_t(h) = \frac{\theta}{\theta-1} \frac{\sum_{\tau=0}^{\infty} \alpha^{\tau} \mathbb{E}_t \left[m_{t,t+\tau} \widetilde{Y}^{d*}_{t,t+\tau}(h) \left(\frac{(1+\phi_{t+\tau}) W_{t+\tau}}{A_{t+\tau}} \right) \right]}{\sum_{\tau=0}^{\infty} \alpha^{\tau} \mathbb{E}_t \left[m_{t,t+\tau} \widetilde{Y}^{d*}_{t,t+\tau}(h) S_{t+\tau} \right]}. \quad (3.3.17)$$

Under proper aggregation rules (using the law of large numbers), the price sub-indexes under sticky prices on domestic varieties, P^H_t and P^{H*}_t, are

$$P^H_t = \left[\alpha \left(P^H_{t-1} \right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}_t(h) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (3.3.18)$$

$$P^{H*}_t = \left[\alpha \left(P^{H*}_{t-1} \right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}^*_t(h) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (3.3.19)$$

Equations (3.3.18)$-$(3.3.19) are a convenient way to reformulate (3.2.16). The price-setting rule is symmetric for all firms who can re-optimize at time t. The lagged term reflects the aggregate behavior of all domestic firms who cannot re-set prices. Equations (3.3.16) $-$ (3.3.17) can also be re-expressed as,

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \alpha^{\tau} m_{t,t+\tau} \widetilde{Y}^d_{t,t+\tau}(h) \left(\frac{\widetilde{P}_t(h)}{P^H_{t+\tau}} - \frac{\theta(1+\phi_{t+\tau})}{\theta-1} \left(\frac{W_{t+\tau}}{P_{t+\tau} A_{t+\tau}} \right) \frac{P_{t+\tau}}{P^H_{t+\tau}} \right) \right] = 0, \quad (3.3.20)$$

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \alpha^{\tau} m_{t,t+\tau} \widetilde{Y}^{d*}_{t,t+\tau}(h) \left(\frac{\widetilde{P}^*_t(h)}{P^{H*}_{t+\tau}} \frac{S_{t+\tau} P^{H*}_{t+\tau}}{P^H_{t+\tau}} - \frac{\theta(1+\phi_{t+\tau})}{\theta-1} \left(\frac{W_{t+\tau}}{P_{t+\tau} A_{t+\tau}} \right) \frac{P_{t+\tau}}{P^H_{t+\tau}} \right) \right] = 0. \quad (3.3.21)$$

One way to interpret these equations is that a weighted average of current and future deviations of the Dixit-Stiglitz pricing rule is expected to be equal to zero.

Similar conditions, pricing rules and price sub-indexes hold for the foreign firms. The first-order conditions for the foreign firm producing variety f give me the following price-setting formulas,

$$\widetilde{P}_t(f) = \frac{\theta}{\theta-1} \frac{\sum_{\tau=0}^{\infty} (\alpha^*)^{\tau} \mathbb{E}_t \left[m^*_{t,t+\tau} \widetilde{Y}^d_{t,t+\tau}(f) \left(\frac{(1+\phi^*_{t+\tau}) W^*_{t+\tau}}{A^*_{t+\tau}} \right) \right]}{\sum_{\tau=0}^{\infty} (\alpha^*)^{\tau} \mathbb{E}_t \left[m^*_{t,t+\tau} \widetilde{Y}^d_{t,t+\tau}(f) \frac{1}{S_{t+\tau}} \right]}, \quad (3.3.22)$$

$$\widetilde{P}^*_t(f) = \frac{\theta}{\theta-1} \frac{\sum_{\tau=0}^{\infty} (\alpha^*)^{\tau} \mathbb{E}_t \left[m^*_{t,t+\tau} \widetilde{Y}^{d*}_{t,t+\tau}(f) \left(\frac{(1+\phi^*_{t+\tau}) W^*_{t+\tau}}{A^*_{t+\tau}} \right) \right]}{\sum_{\tau=0}^{\infty} (\alpha^*)^{\tau} \mathbb{E}_t \left[m^*_{t,t+\tau} \widetilde{Y}^{d*}_{t,t+\tau}(f) \right]}, \quad (3.3.23)$$

while the price sub-indexes under sticky prices on foreign varieties, P_t^F and P_t^{F*}, are

$$P_t^F = \left[\alpha^* \left(P_{t-1}^F\right)^{1-\theta} + (1-\alpha^*) \left(\widetilde{P}_t(f)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{3.3.24}$$

$$P_t^{F*} = \left[\alpha^* \left(P_{t-1}^{F*}\right)^{1-\theta} + (1-\alpha^*) \left(\widetilde{P}_t^*(f)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{3.3.25}$$

Equations (3.3.24) – (3.3.25) are a convenient way to reformulate (3.2.17). Naturally, equations (3.3.22) – (3.3.23) can be conveniently re-written in the following terms,

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\alpha^*)^\tau m_{t,t+\tau}^* \widetilde{Y}_{t,t+\tau}^d(f) \left(\frac{\widetilde{P}_t(f)}{P_{t+\tau}^F} \frac{P_{t+\tau}^F}{S_{t+\tau} P_{t+\tau}^{F*}} - \frac{\theta\left(1+\phi_{t+\tau}^*\right)}{\theta-1} \left(\frac{W_{t+\tau}^*}{P_{t+\tau}^* A_{t+\tau}^*}\right) \frac{P_{t+\tau}^*}{P_{t+\tau}^{F*}}\right)\right] = (3.3.26)$$

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\alpha^*)^\tau m_{t,t+\tau}^* \widetilde{Y}_{t,t+\tau}^{d*}(f) \left(\frac{\widetilde{P}_t^*(f)}{P_{t+\tau}^{F*}} - \frac{\theta\left(1+\phi_{t+\tau}^*\right)}{\theta-1} \left(\frac{W_{t+\tau}^*}{P_{t+\tau}^* A_{t+\tau}^*}\right) \frac{P_{t+\tau}^*}{P_{t+\tau}^{F*}}\right)\right] = (3.3.27)$$

with the same interpretation as (3.3.20) – (3.3.21). It is worthwhile to always remember that in the workhorse model the prices of the varieties traded in the goods markets may be sticky, but the nominal exchange rate is still a purely flexible price (floats freely).

Clarida, Galí and Gertler (2002) assume PCP and complete pass-through. Then, the LOOP holds in their model. Furthermore, in the absence of other features like home bias in preferences or trading costs, their model also implies absolute PPP. Instead, the workhorse model explored in these notes relies on LCP pricing as emphasized among others by Devereux and Engel (2003). Local-currency pricing is, however, not sufficient to guarantee that the LOOP fails. Some degree of nominal rigidities is also necessary for that to happen. A research topic of interest which I don't cover here is the endogenous choice of local currency vs. producer currency pricing (*references to be added*). It is also important to document how prevalent is the local-currency pricing behavior of firms and price stickiness. Widespread local-currency pricing behavior is documented among others by Knetter (1993) and Gopinath and Rigobon (2007).

Nominal Wages and Aggregate Output. Wages are a fundamental part of the marginal cost faced by the firms in the context of this model as seen in (3.3.3). Wages are determined in a segmented, but otherwise competitive, labor market. The labor market clearing conditions can be expressed as follows (see also equations (3.3.12) – (3.3.13)),

$$nL_t^s = \int_0^n L_t^d(h)\, dh = \frac{1}{A_t} \int_0^n Y_t^s(h)\, dh, \tag{3.3.28}$$

$$(1-n) L_t^{s*} = \int_n^1 L_t^{d*}(f)\, df = \frac{1}{A_t^*} \int_n^1 Y_t^{s*}(f)\, df, \tag{3.3.29}$$

where the second equality comes from the fact that technologies are linear in labor as described by equations (3.3.1) – (3.3.2). I define aggregate output in either the domestic or foreign country as $nY_t = \int_0^n Y_t^s(h)\, dh$

and $(1-n)\,Y_t^* = \int_n^1 Y_t^{s*}(f)\,df$[22]. Using the labor supply function derived from the first-order conditions of the household in equations (3.2.38) – (3.2.39), I can argue that real wages should be equal to,

$$\frac{W_t}{P_t} = \kappa\,(A_t)^{-\varphi}\,(C_t)^{\gamma}\,(Y_t)^{\varphi}, \tag{3.3.30}$$

$$\frac{W_t^*}{P_t^*} = \kappa\,(A_t^*)^{-\varphi}\,(C_t^*)^{\gamma}\,(Y_t^*)^{\varphi}. \tag{3.3.31}$$

These equilibrium conditions define how wages and marginal costs evolve in the economy. Notice, however, that real wages as well as consumption and output are all determined endogenously.

Equations (3.2.21) – (3.2.24) determine the demand function for each variety. That coupled with the market clearing condition at the variety level allows me to calculate the aggregate output as follows,

$$\begin{aligned} nY_t &= \int_0^n Y_t^s(h)\,dh = \int_0^n \left[nC_t(h) + (1-n)\,C_t^*(h)\right]dh \\ &= n\underbrace{\left[\int_0^n \left(\frac{P_t(h)}{P_t^H}\right)^{-\theta} dh\right]}_{\text{relative price dispersion}}\left(\frac{\xi}{n}\left(\frac{P_t^H}{P_t}\right)^{-\sigma} C_t\right) + (1-n)\underbrace{\left[\int_0^n \left(\frac{P_t^*(h)}{P_t^{H*}}\right)^{-\theta} dh\right]}_{\text{relative price dispersion}}\left(\frac{\xi^*}{n}\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma} C_t^*\right) \end{aligned} \tag{3.3.32}$$

$$\begin{aligned} (1-n)\,Y_t^* &= \int_n^1 Y_t^{s*}(f)\,df = \int_n^1 \left[nC_t(f) + (1-n)\,C_t^*(f)\right]df \\ &= n\underbrace{\left[\int_n^1 \left(\frac{P_t(f)}{P_t^F}\right)^{-\theta} df\right]}_{\text{relative price dispersion}}\left(\frac{1-\xi}{1-n}\left(\frac{P_t^F}{P_t}\right)^{-\sigma} C_t\right) + (1-n)\underbrace{\left[\int_n^1 \left(\frac{P_t^*(f)}{P_t^{F*}}\right)^{-\theta} df\right]}_{\text{relative price dispersion}}\left(\frac{1-\xi^*}{1-n}\left(\frac{P_t^{F*}}{P_t^*}\right)^{-\sigma} C_t^*\right) \end{aligned} \tag{3.3.33}$$

In equilibrium, aggregate output is affected by a measure of relative price dispersion as can be see from the equations above. Relative price dispersion and the failure of the LOOP are the consequence of having introduced nominal rigidities in the model. However, relative price dispersion would appear in the model whether firms relied on LCP or PCP pricing, while the LOOP is violated only if firms price-to-market (LCP pricing). I will also show the relative price dispersion wedges are of second-order importance for aggregate output in the model. Put it differently, these wedges do not enter into a first-order approximation of the aggregate output equations in (3.3.32) and (3.3.33).

If prices were fully flexible, aggregate output in a (symmetric) equilibrium should be,

$$nY_t = \int_0^n Y_t^s(h)\,dh = n\left(\frac{\xi}{n}\left(\frac{P_t^H}{P_t}\right)^{-\sigma} C_t\right) + (1-n)\left(\frac{\xi^*}{n}\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma} C_t^*\right), \tag{3.3.34}$$

$$(1-n)\,Y_t^* = \int_n^1 Y_t^{s*}(f)\,df = n\left(\frac{1-\xi}{1-n}\left(\frac{P_t^F}{P_t}\right)^{-\sigma} C_t\right) + (1-n)\left(\frac{1-\xi^*}{1-n}\left(\frac{P_t^{F*}}{P_t^*}\right)^{-\sigma} C_t^*\right) \tag{3.3.35}$$

In fact, given that the LOOP holds under flexible prices (see (3.3.14) – (3.3.15)), it is possible to re-write

[22] Equations (3.3.28) – (3.3.29), naturally, imply that

$$Y_t = A_t L_t^s,\ Y_t^* = A_t^* L_t^{s*}.$$

Aggregate output is also a linear function of aggregate labor.

these equations as,

$$nY_t = \int_0^n Y_t^s(h)\,dh = \left(\frac{P_t^H}{P_t}\right)^{-\sigma}\left[n\left(\frac{\xi}{n}C_t\right)+(1-n)\left(\frac{\xi^*}{n}\left(\frac{1}{RS_t}\right)^{-\sigma}C_t^*\right)\right], \tag{3.3.36}$$

$$(1-n)\,Y_t^* = \int_n^1 Y_t^{s*}(f)\,df = \left(\frac{P_t^{F*}}{P_t^*}\right)^{-\sigma}\left[n\left(\frac{1-\xi}{1-n}(RS_t)^{-\sigma}C_t\right)+(1-n)\left(\frac{1-\xi^*}{1-n}C_t^*\right)\right] \tag{3.3.37}$$

In a symmetric equilibrium with flexible prices, the relative price of all varieties is identical and price dispersion has no role in aggregate output. In this sense, it can be said that relative price dispersion at the variety level defines the degree of real-side distortions on aggregate output and the impact of misallocated expenditures caused by the presence of nominal rigidities. These real-side effects, in turn, give a different motivation to monetary policy: money is no longer neutral, at least in the short-run.

Equations (3.3.32) – (3.3.33) tie the aggregate output in both countries to aggregate consumption as well as optimal prices. Hence, nominal wages are endogenously determined by the consumption savings decision of the households (see (3.2.25) – (3.2.27) or (3.2.30) – (3.2.32)), the optimal pricing decision of the firms (see (3.3.16) – (3.3.17) and (3.3.22) – (3.3.23), or (3.3.14) – (3.3.15)), and the corresponding price indexes (see (3.2.14) – (3.2.17)).

3.4 The Fiscal and Monetary Policy

In the spirit of the positive theory of monetary and fiscal policy of Mankiw (1987), the government chooses tax rates and interest rates to minimize the welfare loss associated with the underlying distortions of the economy. In Woodford's (2003) benchmark model the economy is closed, and asset markets are complete. The only two distortions are nominal rigidities and monopolistic competition. The government, therefore, requires at least two instruments to be effective: a labor subsidy (fiscal policy) and a short-term interest rate (monetary policy). As I will argue in this section, the workhorse model of openness is by no means so straightforward for policy-makers.

The monetary and fiscal policies of the domestic government must satisfy a consolidated budget constraint of the following form,

$$n\left[P_t G_t + B_{t-1} - \frac{B_t}{1+i_t}\right] = R_t, \tag{3.4.1}$$

where G_t denotes per capita real government expenditure, B_t is the per capita nominal amount of debt issued at time t which promises one unit of the local currency tomorrow, and R_t is the nominal total revenues raised by the domestic government. All variables are expressed in per capita terms by convenience, except for total government revenues. I abstract completely from different debt maturities and, for simplicity, I impose that the government does not issue bonds[23] and that government spending (whether for consumption purposes or for investment) is zero. That is, I assume that $G_t = B_t = 0$ for all $t \geq 1$ and $B_0 = 0$.

[23] I characterize government bonds as uncontingent claims. I could alternatively re-write equation (3.4.1) assuming that bonds are contingent, but only for the case where asset markets are complete. For the purposes of these notes, this distinction is of lesser importance because I keep the assumption that all traded assets are in zero-net supply independently of whether they are government-issued or not and independently of whether they are contingent or uncontingent. I recognize that there are not many examples of contingent government bonds to speak off. However, what truly matters is whether households can write contingent or uncontingent lending contracts between themselves (and whether these contracts are open to international investors, that is foreign households).

Fiat money is an unbacked asset that serves as a unit of account. Each currency serves as the numeraire in the country where it is issued, and the nominal exchange rate is the relative price between the two currencies[24]. Money promises $1 + i_t^m$ units of the local currency in period $t + 1$ in exchange for one unit of the local currency in period t. The government raises a lump-sum tax on household and a proportional tax on the wage receipts. Therefore, the total revenue is,

$$R_t = n \left[\underbrace{T_t + \phi_t W_t L_t^s}_{\text{direct taxation per capita}} + \underbrace{\left(M_t - \left(1 + i_{t-1}^m\right) M_{t-1}\right)}_{\text{seigniorage revenues per capita}} \right], \tag{3.4.2}$$

where M_t is the per capita money supply, and L_t^s the per capita labor supply. Taxes on households and wages imply that $T_t > 0$ and $\phi_t > 0$, while a households transfer and a wage subsidy occur if $T_t < 0$ and $\phi_t < 0$.

Similarly, the foreign government consolidated budget constraint can be expressed as follows,

$$(1 - n) \left[P_t^* G_t^* + B_{t-1}^* - \frac{B_t^*}{1 + i_t^*} \right] = R_t^*, \tag{3.4.3}$$

$$R_t^* = (1 - n) \left[\underbrace{T_t^* + \phi_t^* W_t^* L_t^{s*}}_{\text{direct taxation per capita}} + \underbrace{\left(M_t^* - \left(1 + i_{t-1}^{m*}\right) M_{t-1}^*\right)}_{\text{seigniorage revenues per capita}} \right], \tag{3.4.4}$$

where G_t^* denotes per capita real government expenditure, B_t^* is the per capita nominal amount of debt issued at time t which promises one unit of the foreign currency tomorrow, and R_t^* are the nominal total revenues raised by the foreign government. M_t is the per capita money supply, L_t^s the per capita labor supply, while T_t and ϕ_t are the household and wage taxes (or subsidies) respectively. I also assume that $G_t^* = B_t^* = 0$ for all $t \geq 1$ and $B_0^* = 0$.

Total revenues are the sum of the receipts from direct taxation and seigniorage. First, I ignore the possibility of independent budgets for the monetary authority and the fiscal authority. Arguably, the fiscal authority balances its own budget without taking into account seigniorage revenues. There may be additional budgetary restrictions. Nonetheless, the joint consolidated budget of the fiscal authority and the monetary authority must satisfy the constraints described above. Second, I summarize all possible instruments of fiscal policy into a lump-sum tax and a wage tax (or a wage subsidy). There may be many other taxes as well as a role to be played by government spending and borrowing. However, these instruments suffice for the purposes of the model and my focus on monetary policy. They are also key ingredients in Woodford's (2003) closed economy model.

While Taylor (1993) introduced one of the staples of monetary theory, Woodford (2003) set the standard of reference. The aim of monetary and fiscal policy in Woodford (2003) is always the frictionless, competitive, and flexible-price allocation. Policy is understood as a set rules implemented by the policy-makers with the aim of driving the economy towards the frictionless allocation. Whether policy rules are optimal or not depends to a great extent on the options available to policy-makers, and the welfare costs associated with a

[24] In a pure float regime, monetary policy is set independently from developments in the foreign exchange market. Here, I assume that the exchange rate floats freely and I do not discuss the role that monetary policy may have under a 'managed' exchange rate regime. For a discussion of alternative policy regimes (target zones, fixed exchange rates, etc.) and their implications for exchange rate volatility, see for instance Jeanne and Rose (2002) (*add other references!*).

policy choice intended to approximate the frictionless allocation. The literature has placed great emphasis on the properties of certain rules and their impact on the dynamics of the economy. For instance, short-term interest rate Taylor rules have become mainstream for monetary policy analysis and very helpful to forecast the behavior of this policy instrument.

Here, I also focus my attention on policy rules. I devote little time to the discussion of alternatives (particularly for the fiscal authority) or their optimality (which would require me to take a stand on the elusive 'objective function' of the policy-makers).

Fiscal Policy Rules. Woodford (2003) works with a closed-economy framework where there are two basic frictions, nominal rigidities and monopolistic competition. The choice of a labor tax/subsidy in either country is simple,

$$\phi_t = \phi_t^* = \frac{1-\mu}{\mu} = \frac{-1}{\theta}, \tag{3.4.5}$$

where $\mu \equiv \frac{\theta}{\theta-1}$ is the mark-up charged by the monopolistic firms and θ is the elasticity of substitution across varieties produced within a country. The fiscal authority, therefore, subsidizes labor wages to eliminate the mark-up. If prices were fully flexible, firms would set them equal to their marginal costs. Hence, monetary policy is left with the task of dealing with the distortion introduced by nominal rigidities alone. In other words, the monetary authority is entrusted with the responsibility of driving the economy towards the allocation that would prevail if prices were flexible and firms perfectly competitive.

The lump-sum taxes/transfers on households, T_t and T_t^*, simply guarantee that the government budget constraints are satisfied in each period. If seigniorage revenues offset the cost of the wage subsidy, the lump-sum transfer re-distributes whatever is left to households. If seigniorage revenues are insufficient, then the lump-sum tax will raise enough resources. This tax/transfer scheme on households is purely non-distortionary. Such idea naturally carries over to a world populated by two countries which are open to trade in some markets. I adopt it in these notes without much more comment.

However, the simplicity of this fiscal policy rule is somewhat misleading. If the elasticity of substitution across varieties is time-varying (because preferences are not well-approximated with CES aggregators) or the mark-ups are time-varying for some other reason, then the subsidies may have to be time-varying too. Other assumptions that may complicate the role of fiscal policy if lifted are the assumption of segregated national labor markets, flexible and competitive wages, symmetric firms and households, etc. If mark-ups were time-varying and firm (or industry) specific, that would require the fiscal authority to have in place a very complex time-varying and firm (or industry) specific labor subsidy scheme. But evidence of such subsidies is hard to find in the data.

The Quantity Theory of Money. The money market is fully segregated across countries and the market clearing conditions require that,

$$nM_t = nM_t^d, \tag{3.4.6}$$

$$(1-n)\, M_t^* = (1-n)\, M_t^{d*}, \tag{3.4.7}$$

where M_t and M_t^* are the money supply in the domestic and foreign country, respectively. The money market clearing condition in equations (3.4.6) – (3.4.7) combined with the money demand functions in

(3.2.36) − (3.2.37) indicate that in equilibrium,

$$M_t = \left(\frac{1}{\chi}\frac{i_t - i_t^m}{1+i_t}\right)^{-\frac{1}{\varsigma}} P_t \left(C_t\right)^{\frac{\gamma}{\varsigma}}, \tag{3.4.8}$$

$$M_t^* = \left(\frac{1}{\chi}\frac{i_t^* - i_t^{m*}}{1+i_t^*}\right)^{-\frac{1}{\varsigma}} P_t^* \left(C_t^*\right)^{\frac{\gamma}{\varsigma}}, \tag{3.4.9}$$

These equations imply that a version of the *quantity theory of money* holds in the model, i.e.

$$P_t Y_t = V_t M_t,\ V_t \equiv \left(\frac{1}{\chi}\frac{i_t - i_t^m}{1+i_t}\right)^{\frac{1}{\varsigma}} \left(\frac{\left(C_t\right)^{\frac{\gamma}{\varsigma}}}{Y_t}\right)^{-1}, \tag{3.4.10}$$

$$P_t^* Y_t^* = V_t^* M_t^*,\ V_t^* \equiv \left(\frac{1}{\chi}\frac{i_t^* - i_t^{m*}}{1+i_t^*}\right)^{\frac{1}{\varsigma}} \left(\frac{\left(C_t^*\right)^{\frac{\gamma}{\varsigma}}}{Y_t^*}\right)^{-1}, \tag{3.4.11}$$

where V_t and V_t^* are the corresponding money velocities in the domestic and foreign country[25]. From these equations, it follows immediately that of the three monetary instruments available to the domestic monetary authority, i.e. (M_t, i_t, i_t^m), one of them is redundant. Similarly, for the instruments of the foreign monetary authority. Woodford's (2003) implicit argument is that having money or not is completely irrelevant for modelling purposes as long as money itself is non-distortionary[26]. Relevant monetary policy questions can be addressed without having to refer to the money markets at all.

Monetary Policy Rules. Monetary policy relies on two instruments in practice: the money supply, M_t, or the administered interest rate on money, i_t^m, and the short-term nominal interest rate, i_t. Woodford (2007) recognizes that variations in the money supply and variations in the rate of interest paid on money are essentially redundant. This follows immediately from the money market clearing conditions. Any policy objective that can be achieved by varying the interest rate paid on money can alternatively be attained through an appropriate adjustment of the money supply. As Woodford (2007) notes, however, there may be practical advantages to the use of the administered interest rate. For example, calculating changes in the administered rate can be more straightforward than guessing the size of open market operations required to achieve the same effect of pumping liquidity into the market (particularly, in the presence of disturbances

[25] The money velocity is a function of the short-term nominal interest rate, the administed rate on money, and a weighted ratio of per capita total consumption over per capita total output. Introducing a cash-in-advance constraint in the model instead of money-in-the-utility function, also introduces a similar version of the *quantity theory of money*. The difference often arises because the implied equation for the velocity of money is not usually the same.

[26] If $i_t^m = i_t^{m*} = 0$ and the monetary authorities set i_t and i_t^*, the amount of money held in equilibrium is endogenously determined by equations (3.4.8) − (3.4.9). As I will show, consumption and CPIs can be determined without any direct reference to money in the workhorse model. Here money holdings are non-distortionary, and therefore can be ignored while exploring all other macro variables of interest in the model.

to the money demand function that the model is not capturing)[27][28].

The distinction between the short-term interest rate, i_t, and the regulated money rate, i_t^m, is intended to reflect the fact that the monetary authority often has control over how much liquidity is available other than by altering the short-term interest rate. This is pretty clear from the money market clearing condition. The supply of money can be restricted or augmented by simply adjusting the regulated rate without changing the short-term interest rate. Most of the recent literature has ignored the role of money entirely and has assumed that $i_t^m = 0$. I keep this option open because it may prove to be helpful to pin down the price level.

Monetary policy is determined by the choice of a target rule on the short-term nominal interest rate in the tradition of Taylor (1993) as advocated in Woodford's (2003, 2007) research. Then, the monetary authority simply promises to supply the money market with as much currency as it is required given a (constant and zero) administered rate. Money is supplied locally by the monetary authority and demanded locally too. Money supply evolves over time, generates seigniorage revenues, and is used to finance the fiscal authority's budget. Seigniorage revenues are non-distortionary because the fiscal authority does not set its optimal wage subsidy on the basis of these revenues and whatever is distributed to the households comes in the form of lump-sum transfers (moreover, the utility function is additively separable in real balances).

The Taylor rule is often defined as the trademark of modern monetary policy. I assume that the monetary authorities set short term nominal interest rates according to Taylor (1993) type rules,

$$\frac{1+i_t}{1+\bar{i}} = \frac{Z_t}{Z}\left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\rho}\left[\left(\Pi_t\right)^{\psi_1}\left(\frac{Y_t}{\overline{Y}_t}\right)^{\psi_2}\right]^{1-\rho}, \tag{3.4.12}$$

$$\frac{1+i_t^*}{1+\bar{i}^*} = \frac{Z_t^*}{Z^*}\left(\frac{1+i_{t-1}^*}{1+\bar{i}^*}\right)^{\rho^*}\left[\left(\Pi_t^*\right)^{\psi_1^*}\left(\frac{Y_t^*}{\overline{Y}_t^*}\right)^{\psi_2^*}\right]^{1-\rho^*}, \tag{3.4.13}$$

where Z_t and Z_t^* are the monetary policy shocks, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ are the (gross) CPI inflation rates, Y_t and Y_t^* are the per capita output levels, and $\overline{Y}_t$ and $\overline{Y}_t^*$ are the per capita potential output levels. The ratios $\frac{Y_t}{\overline{Y}_t}$ and $\frac{Y_t^*}{\overline{Y}_t^*}$ are the output gaps in levels for the domestic and foreign country, respectively. Potential output is defined as the output level that would prevail in the economy if all frictions could be eliminated, that is in a frictionless economy with competitive firms and flexible prices. This index specification of the Taylor rule takes the standard form once it is log-linearized. This monetary policy index captures both a smoothing term and a policy component.

Fiscal policy takes care of the mark-up charged by firms, so monetary policy becomes the tool of choice to offset the 'distortions' caused by nominal rigidities. Disparities in relative prices arise due to price stickiness. In fact, the more rapid inflation grows, the larger are the relative price differences between

[27] I could re-interpret $1+i_t^m$ as the rate of utilization of liquid money in the economy. This could account for depreciation, because bills and coins get lost all the time, or be used to model a liquidity crisis. In this sense, a liquidity crisis occurs because money acquired today disappears from circulation. A liquidity crisis could be handled exclusively by pumping liquidity into the markets when it occurs and taking it out when it recedes, without having to adjust the short-term nominal interest rate. It would be interesting to endogeneize this story and show under which circunstances money gets 'stored' by households and, therefore, disapears from circulation.

[28] Alternatively, I could argue that the instruments of monetary policy are the money supply and the administered interest rate on money. The money market clearing conditions, then, show how the interest rate controlled by the policy-makers influences the short-term nominal interest rate in the economy. A model with assets of different maturities would also allow me to intepret the linkages between monetary policy and the yield curve more broadly.

otherwise symmetric firms. The resulting misallocation of expenditures is thought to be costly in terms of welfare for the consumers. Price stability, therefore, guarantees that relative prices differences are contained and that the misallocation of expenditures is limited.

With perfect foresight and no uncertainty, it should be possible to set the nominal interest rate in such a way that prices are stable over time. Therefore, the distortion introduced by nominal rigidities is completely eliminated. That coupled with the fiscal policy suffices to attain the desired optimal allocation. With rational expectations and uncertainty, there is no simple way to implement a monetary policy that guarantees price stability in every state of nature, except perhaps on some very special cases. For instance, whenever there are only two states of nature and the monetary authority is allowed to use two instruments, i_t and i_t^m. Hence, Taylor rules cannot guarantee price stability at every state of the world. However, Taylor rules are a convenient specification for monetary policy because: (a) they set their objective to be price stability to force inflation and output to revert to their desired targets[29], and (b) they explicitly account for a trade-off between inflation and the output gap to balance the benefits of price stability against the costs of attaining that goal[30].

Finally, it is worth commenting on the monetarist view that sound monetary policy should be grounded in stable money supply growth rules as advocated by Friedman (1960). In Poole's (2008) own words we must recognize that: "If the money-demand function were stable, a central bank that pursued such an interest rate policy would find that the growth rate of the money stock would fall when inflation rose above target. Similarly, under the Taylor rule money growth would tend to rise when inflation fell below the target. If a central bank is successful in maintaining a relatively low rate of inflation, the resulting average growth in the money stock will be relatively low and stable. Hence, a central bank following an appropriately specified Taylor rule with an invariant inflation target would induce the pattern of money growth that the monetarists argued was required for a nominal anchor in the economy. The Taylor rule reconciles somewhat different theoretical approaches of economists who emphasize the money stock in their analysis of monetary policy and those who emphasize interest rates in their analysis."

An 'Open' Discussion about Monetary Policy and Openness. My previous discussion is essentially borrowed from the literature on closed-economies. It follows the treatment in Woodford (2003, chap. 2, sec. 3.3) and Woodford (2007) based on the simple idea that fiscal and monetary policy should be implemented in the presence of frictions in such a way that the effects of those frictions are minimized (to approximate the outcome of the frictionless allocation). It is not immediately obvious, however, how to extend this logic to a two-country economy with nominal rigidities, monopolistic firms and LCP. The role of fiscal policy is still to eliminate the distortion caused by the mark-up. However, the role of monetary policy becomes more obscure and complex.

[29] In this sense, monetary policy complements fiscal policy by trying to force the solution of the model back to its optimal allocation.

[30] It is not obvious, however, that the specification of the Taylor rule should depend only on the domestic output gap. In a closed economy under certain assumptions (see Woodford, ????), the output gap is proportional to the real marginal costs. Therefore, tracking the output gap serves a dual purpose: it contributes to keep inflation under control and it also gives a measure of how much 'pain' they are willing to accept to pursue the goal of price stability. In an open economy as simple as that posited by Clarida, Galí and Gertler (2002) the relevant measure of real marginal costs is a mixture of domestic and foreign output gaps. The question is: Should the local monetary authority target the output gap of the other country to keep inflation in check?

If trade is allowed between these two economies it is not clear that a Taylor reaction function for the interest rate that targets price stability and the output gap would suffice. For example:

- Domestic output gap should not be the only measure of real marginal costs tracked by the domestic monetary authority, because the foreign output gap also has value as proxy for foreign real marginal costs (and hence imported inflation).

- The potential output in both countries changes as a result of trade and depends on external factors too. As a result, measuring the potential for the domestic economy can be more complex.

- Market clearing requires that domestic consumption be equal to domestic output in a closed economy, but this equality does not hold true for an open economy. If the monetary authority cares about the welfare of the domestic households, it could be better to balance the benefits of pursuing price stability against the costs on a consumption gap rather than on the output gap.

Furthermore, even after taking away the mark-up with fiscal policy, the distortions of the workhorse model of openness are more than the relative price distortions introduced by nominal rigidities. First of all, there is an implicit assumption that all firms are specialized in their own varieties and can neither relocate nor tap into an international input market (inputs are non-traded across countries). It is not obvious to me that specialization in this type of models is due to the competitive advantage of one country over the other. In fact, it seems to me that if it can be shown that specialization is not due to competitive advantage, a model build around this assumption could lead to an allocation that is not optimal. Hence, if monetary policy is used to 'push' the allocation closer to its optimal level, it could come into conflict with its goal of price stability.

Secondly, the workhorse model also implies that firms rely on pricing-to-market (LCP). In this context, movements in the nominal exchange rate -which is assumed to be a flexible price- can induce large mis-allocations of expenditure across countries because the LOOP fails to hold. It is not clear how monetary policy should respond to this type of relative price distortions in the external sector (if at all). This is just another worry that policy-makers should have because of openness and international pricing behavior, which is completely alien to the closed-economy literature. I will re-evaluate the impact of monetary policy in light of my subsequent findings on the impact it has on inflation, the output gap, the exchange rates and the trade balance.

Finally, a common variant of the workhorse model allows for frictions in the asset markets to be considered. This is the case of the *bond economy* described in these notes. But, what monetary policy is or ought to be if asset markets are incomplete is not clear to me. Incomplete markets implies that households in each country have only limited opportunities to pool risks among themselves. Potentially models could be written in which the fiscal and monetary authorities can provide some degree of insurance for households. Is there a trade-off between the risk-sharing effects of monetary policy and the goal of price stability? Is the structure of the assets markets relevant for the purpose of conducting monetary policy?

Remark 3 After-thoughts on Monetary Policy and LCP. *The frictions in the workhorse model operate solely on the supply-side in the goods market being: monopolistic competition, nominal rigidities and pricing-to-market.*

- Clarida, Galí and Gertler (2002) assume PCP and complete pass-through. This implies that firms choose one price for the local market, which may remain unchanged for a number of periods. Then, they quote the price in the exports market as the local market price expressed in units of the foreign currency.

Export prices change every period with the nominal exchange rate, even if local prices are kept unchanged. There is strong evidence of pervasive LCP and evidence that complete pass-through fails both at the aggregate level as well as at a disaggregated level. One of the implications of LCP under sticky prices is that PPP fails, the real exchange rate is no longer constant, and the nominal exchange rate pass-through is incomplete. These notes intend to explore precisely those features in greater detail.

- A model where firms set prices under LCP introduces more distortions than a simpler model of nominal rigidities (e.g., Clarida, Galí and Gertler, 2002). These distortions affect directly the external sector. The monetary and fiscal authorities, therefore, have to be concerned about it. Even if the monetary authority succeeds in keeping prices relatively stable over time, there is no guarantee that the distortions introduced by the combination of nominal rigidities and pricing-to-market are effectively diminished. It is possible that the consumption basket in one country becomes relatively cheaper than in another country for a while, even if the composition is identical, due to a low exchange rate pass-through. If that was the case, the external sector of the economy who is 'overvalued' would be suffering the ultimate consequences. The misallocation of expenditure in the external sector would generate a welfare loss for one country over the other.

- In principle, it is possible to conjecture that a measure of (net) trade gap would take into account whether the external sector is 'depressed' or not, then it is possible to use monetary policy to respond against these distortions. However, how do we define this measure of trade gap in relation to the goal of price stability? Do asset prices like the nominal or real exchange rate signal the size of the distortion? Should these asset prices be added to the monetary policy rules? Should monetary policy be concerned at all with the external sector specifically beyond its impact on aggregate output?...

3.5 The Resource Constraint

The structure of asset markets has obvious implications for the degree of risk-sharing that can be attained across countries and for financing the trade balance. But the results presented in this section are general enough, so I focus on the complete asset specification (unless otherwise noted). If I aggregate the budget constraint of the domestic household in (3.2.3) with the consolidated domestic government budget constraint in (3.4.1) – (3.4.2), I obtain,

$$\begin{aligned} & P_t C_t + \left(M_t^d - M_t\right) + \int_{\omega_{t+1}\in\Omega} Q_t\left(\omega_{t+1}\right) B_t^H\left(\omega_{t+1}\right) + S_t \int_{\omega_{t+1}\in\Omega} Q_t^*\left(\omega_{t+1}\right) B_t^F\left(\omega_{t+1}\right) \\ & \quad \leq \left(1+i_{t-1}^m\right)\left(M_{t-1}^d - M_{t-1}\right) + B_{t-1}^H\left(\omega_t\right) + S_t B_{t-1}^F\left(\omega_t\right) + \Pi_t + \left(1+\phi_t\right) W_t L_t^s. \end{aligned} \tag{3.5.1}$$

Similarly, it follows that if I aggregate the budget constraint of the foreign household in (3.2.4) with the consolidated foreign government budget constraint in (3.4.3) – (3.4.4), I obtain,

$$\begin{aligned} & P_t^* C_t^* + \left(M_t^{d*} - M_t^*\right) + \tfrac{1}{S_t} \int_{\omega_{t+1}\in\Omega} Q_t\left(\omega_{t+1}\right) B_t^{H*}\left(\omega_{t+1}\right) + \int_{\omega_{t+1}\in\Omega} Q_t^*\left(\omega_{t+1}\right) B_t^{F*}\left(\omega_{t+1}\right) \\ & \quad \leq \left(1+i_{t-1}^{m*}\right)\left(M_{t-1}^{d*} - M_{t-1}^*\right) + \tfrac{1}{S_t} B_{t-1}^{H*}\left(\omega_t\right) + B_{t-1}^{F*}\left(\omega_t\right) + \Pi_t^* + \left(1+\phi_t^*\right) W_t^* L_t^{s*}. \end{aligned} \tag{3.5.2}$$

Equations (3.5.1) and (3.5.2) hold with equality in equilibrium. Using the money market clearing conditions described in (3.4.6) – (3.4.7), the definitions of per capita profits distributed to the representative household in each country as given by (3.3.10) and (3.3.11), and the labor market clearing conditions in equations

(3.3.12) – (3.3.13), I immediately infer that,

$$\begin{aligned} & P_t C_t + \int_{\omega_{t+1}\in\Omega} Q_t\left(\omega_{t+1}\right) B_t^H\left(\omega_{t+1}\right) + S_t \int_{\omega_{t+1}\in\Omega} Q_t^*\left(\omega_{t+1}\right) B_t^F\left(\omega_{t+1}\right) \\ & \quad = B_{t-1}^H\left(\omega_t\right) + S_t B_{t-1}^F\left(\omega_t\right) + \tfrac{1}{n}\int_0^n \left[P_t\left(h\right) n C_t\left(h\right) + S_t P_t^*\left(h\right)\left(1-n\right) C_t^*\left(h\right)\right] dh, \end{aligned} \tag{3.5.3}$$

$$\begin{aligned} & P_t^* C_t^* + \tfrac{1}{S_t}\int_{\omega_{t+1}\in\Omega} Q_t\left(\omega_{t+1}\right) B_t^{H*}\left(\omega_{t+1}\right) + \int_{\omega_{t+1}\in\Omega} Q_t^*\left(\omega_{t+1}\right) B_t^{F*}\left(\omega_{t+1}\right) \\ & \quad = \tfrac{1}{S_t} B_{t-1}^{H*}\left(\omega_t\right) + B_{t-1}^{F*}\left(\omega_t\right) + \tfrac{1}{1-n}\int_n^1 \left[\tfrac{1}{S_t} P_t\left(f\right) n C_t\left(f\right) + P_t^*\left(f\right)\left(1-n\right) C_t^*\left(f\right)\right] df. \end{aligned} \tag{3.5.4}$$

Equations (3.5.3) and (3.5.4) can easily be adapted to reflect that only trade in uncontingent bonds is possible, as posited in the incomplete asset markets setting discussed in these notes.

Real and Nominal Goods Market Clearing Conditions. Given the fact that all firms are committed to satisfy the household demands at the given prices, the market clearing condition for any given variety in either location can be computed as,

$$Y_t\left(h\right) = n C_t\left(h\right) + \left(1-n\right) C_t^*\left(h\right), \ \forall h \in \left[0, n\right], \tag{3.5.5}$$

$$Y_t^*\left(f\right) = n C_t\left(f\right) + \left(1-n\right) C_t^*\left(f\right), \ \forall f \in \left(n, 1\right]. \tag{3.5.6}$$

The demand functions for each variety of goods are analyzed in equations (3.2.21) – (3.2.24). The implicit assumption that preferences over different varieties within a bundle (either domestic or foreign) are identical implies that the nominal market clearing conditions for each bundle of varieties can be aggregated as follows,

$$\begin{aligned} PPI_t^H Y_t &= \frac{1}{n}\int_0^n \left[P_t\left(h\right) n C_t\left(h\right) + S_t P_t^*\left(h\right)\left(1-n\right) C_t^*\left(h\right)\right] dh \\ &= \frac{1}{n}\left[\frac{1}{n}\int_0^n \left(\frac{P_t\left(h\right)}{P_t^H}\right)^{1-\theta} dh\right] n P_t^H C_t^H + \frac{1}{n}\left[\frac{1}{n}\int_0^n \left(\frac{P_t^*\left(h\right)}{P_t^{H*}}\right)^{1-\theta} dh\right]\left(1-n\right) S_t P_t^{H*} C_t^{H*}, \end{aligned} \tag{3.5.7}$$

$$\begin{aligned} PPI_t^{F*} Y_t^* &= \frac{1}{1-n}\int_n^1 \left[\frac{1}{S_t} P_t\left(f\right) n C_t\left(f\right) + P_t^*\left(f\right)\left(1-n\right) C_t^*\left(f\right)\right] df \\ &= \frac{1}{1-n}\left[\frac{1}{1-n}\int_n^1 \left(\frac{P_t\left(f\right)}{P_t^F}\right)^{1-\theta} df\right] n \frac{1}{S_t} P_t^F C_t^F + \frac{1}{1-n}\left[\frac{1}{1-n}\int_n^1 \left(\frac{P_t^*\left(f\right)}{P_t^{F*}}\right)^{1-\theta} df\right]\left(1-n\right) P_t^{F*} C_t^{F*}, \end{aligned} \tag{3.5.8}$$

where Y_t are the units of output produced per firm in the home country, Y_t^* are the units of output produced per firm in the foreign country, and PPI_t^H and PPI_t^{F*} are the corresponding producer price indexes (PPI) expressed in the local currency of the domestic and foreign country, respectively.

The expressions within brackets in the previous equations are a measure of relative price dispersion across firms within a given location. From the definitions of the price indexes for the bundles of domestic and foreign varieties in (3.2.16) – (3.2.17), it immediately follows that these measures of dispersion aggregate to one. Therefore, I obtain that,

$$n PPI_t^H Y_t = n P_t^H C_t^H + \left(1-n\right) S_t P_t^{H*} C_t^{H*}, \tag{3.5.9}$$

$$\left(1-n\right) PPI_t^{F*} Y_t^* = n \frac{1}{S_t} P_t^F C_t^F + \left(1-n\right) P_t^{F*} C_t^{F*}, \tag{3.5.10}$$

while the market clearing condition for each country can be expressed in real terms as,

$$nY_t = nC_t^H + (1-n)\,C_t^{H*}, \tag{3.5.11}$$

$$(1-n)\,Y_t^* = nC_t^F + (1-n)\,C_t^{F*}. \tag{3.5.12}$$

The producer price indexes (PPIs) are identical to the consumption price sub-indexes for each bundle of goods, i.e. $PPI_t^H = P_t^H$ and $PPI_t^{F*} = P_t^{F*}$, whenever the LOOP holds. Otherwise, the PPI can be derived as,

$$PPI_t^H = \frac{nC_t^H}{nC_t^H + (1-n)\,C_t^{H*}} P_t^H + \frac{(1-n)\,C_t^{H*}}{nC_t^H + (1-n)\,C_t^{H*}} S_t P_t^{H*}, \tag{3.5.13}$$

$$PPI_t^{F*} = \frac{nC_t^F}{nC_t^F + (1-n)\,C_t^{F*}} \frac{1}{S_t} P_t^F + \frac{(1-n)\,C_t^{F*}}{nC_t^F + (1-n)\,C_t^{F*}} P_t^{F*}. \tag{3.5.14}$$

In other words, each PPI is a weighted sum of the aggregate prices of the bundle of goods in both the local and the exports markets expressed in the nominal currency of the producer. The weights are the shares of output produced to be sold in each market.

Using the demand functions for each variety of goods in equations (3.2.21) – (3.2.24) one more time, I re-write the nominal market clearing conditions in (3.5.9) – (3.5.10) in terms of aggregate consumption as follows,

$$nPPI_tY_t = n\xi \left(\frac{P_t^H}{P_t}\right)^{1-\sigma} P_tC_t + (1-n)\,\xi^* \left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} S_tP_t^*C_t^*, \tag{3.5.15}$$

$$(1-n)\,PPI_t^*Y_t^* = n\,(1-\xi) \left(\frac{P_t^F}{P_t}\right)^{1-\sigma} \frac{1}{S_t} P_tC_t + (1-n)\,(1-\xi^*) \left(\frac{P_t^{F*}}{P_t^*}\right)^{1-\sigma} P_t^*C_t^*, \tag{3.5.16}$$

while the market clearing conditions are expressed in real terms as,

$$nY_t = n\xi \left(\frac{P_t^H}{P_t}\right)^{1-\sigma} C_t + (1-n)\,\xi^* \left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} C_t^*, \tag{3.5.17}$$

$$(1-n)\,Y_t^* = n\,(1-\xi) \left(\frac{P_t^F}{P_t}\right)^{1-\sigma} C_t + (1-n)\,(1-\xi^*) \left(\frac{P_t^{F*}}{P_t^*}\right)^{1-\sigma} C_t^*. \tag{3.5.18}$$

Naturally, the sum of the real aggregate output in both countries equals the sum of the real aggregate consumption,

$$nY_t + (1-n)\,Y_t^* = nC_t + (1-n)\,C_t^*, \tag{3.5.19}$$

which follows from the market clearing conditions above. I can also derive an alternative characterization of the PPI computed in terms of the CPI of each country which takes up the form,

$$PPI_t = \frac{n\xi\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} C_t}{n\xi\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} C_t + (1-n)\xi^*\left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} C_t^*} P_t + \frac{(1-n)\xi^*\left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} C_t^*}{n\xi\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} C_t + (1-n)\xi^*\left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} C_t^*} S_t P_t^*,$$

$$PPI_t^* = \frac{n(1-\xi)\left(\frac{P_t^F}{P_t}\right)^{1-\sigma} C_t}{n(1-\xi)\left(\frac{P_t^F}{P_t}\right)^{1-\sigma} C_t + (1-n)(1-\xi^*)\left(\frac{P_t^{F*}}{P_t^*}\right)^{1-\sigma} C_t^*} \frac{1}{S_t} P_t + \frac{(1-n)(1-\xi^*)\left(\frac{P_t^{F*}}{P_t^*}\right)^{1-\sigma} C_t^*}{n(1-\xi)\left(\frac{P_t^F}{P_t}\right)^{1-\sigma} C_t + (1-n)(1-\xi^*)\left(\frac{P_t^{F*}}{P_t^*}\right)^{1-\sigma} C_t^*} P_t^*. \tag{3.5.20}$$

If absolute PPP were to hold, which in the context of the workhorse model requires the LOOP to hold and no home bias in consumption (i.e., $\xi = \xi^*$), then the local PPI corresponds exactly to the local CPI. In other words, it is true that $PPI_t = P_t$ and $PPI_t^* = P_t^*$.

This long digression on the differences between the CPI and the PPI, or consumption-based and producer-based prices, is meant to reinforce the idea that it is necessary to distinguish between the nominal resource constraint of each country and the market clearing condition in real terms. It is also important to realize that PPI and CPI may not be equal to each other. This is a subject that has received some attention specially in policy circles because not everybody agrees that targeting the CPI is the best strategy for monetary policy. I do not cover the discussion in this notes for lack of space. These comments also reinforce the idea that pricing-to-market introduces pricing differences that would not be present if the only pricing friction resulted from the reliance of firms on Calvo contracts[31].

Finally, this discussion is meant to simplify our understanding of the nominal resource constraints and to point out (or emphasize) that only relative prices can have an impact on the real allocation.

The Current Account, the Capital Account and the Trade Balance. Following on the previous results, it is possible to argue that the nominal resource constraint in each country is equal to,

$$\begin{aligned} &\int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1}) B_t^H(\omega_{t+1}) + S_t \int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1}) B_t^F(\omega_{t+1}) = B_{t-1}^H(\omega_t) + S_t B_{t-1}^F(\omega_t) + \\ &\quad + P_t \frac{1}{n}\left[\xi\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} nC_t + \xi^*\left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} RS_t(1-n)C_t^* - nC_t\right], \end{aligned} \tag{3.5.21}$$

$$\begin{aligned} &\frac{1}{S_t}\int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1}) B_t^{H*}(\omega_{t+1}) + \int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1}) B_t^{F*}(\omega_{t+1}) = \frac{1}{S_t} B_{t-1}^{H*}(\omega_t) + B_{t-1}^{F*}(\omega_t) + \\ &\quad + P_t^* \frac{1}{1-n}\left[(1-\xi)\left(\frac{P_t^F}{P_t}\right)^{1-\sigma} \frac{1}{RS_t} nC_t + (1-\xi^*)\left(\frac{P_t^{F*}}{P_t^*}\right)^{1-\sigma}(1-n)C_t^* - (1-n)C_t^*\right], \end{aligned} \tag{3.5.22}$$

where the real exchange rate is defined as in equation (3.2.19). These pair of equations in per capita terms hold true independently of whether the prices are *flexible* or *sticky*. The per capita current account equations

[31] Remember also the distinction between terms of trade, ToT_t and ToT_t^*, and relative prices, RP_t and RP_t^*, mentioned in sub-section (3.2) (definitions (3.2.18) and (3.2.20)).

in (3.5.21) and (3.5.22) can be expressed more conveniently as,

$$\begin{aligned}
&\underbrace{P_t \frac{1}{n}\left[\xi\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} nC_t + \xi^*\left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma}(1-n)\, RS_t C_t^* - nC_t\right] +}_{\text{Nominal Trade Balance of Country } H} \\
&\quad \underbrace{+\left[B_{t-1}^H(\omega_t) - \int_{\omega_{t+1}\in\Omega} Q_{t-1}(\omega_t)\, B_{t-1}^H(\omega_t)\right] + S_t\left[B_{t-1}^F(\omega_t) - \int_{\omega_{t+1}\in\Omega} Q_{t-1}^*(\omega_t)\, B_{t-1}^F(\omega_t)\right]}_{\text{Nominal Service Balance of Country } H} \\
&= \underbrace{\begin{array}{l}\left[\int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1})\, B_t^H(\omega_{t+1}) - \int_{\omega_{t+1}\in\Omega} Q_{t-1}(\omega_t)\, B_{t-1}^H(\omega_t)\right] + \\ +S_t\left[\int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1})\, B_t^F(\omega_{t+1}) - \int_{\omega_{t+1}\in\Omega} Q_{t-1}^*(\omega_t)\, B_{t-1}^F(\omega_t)\right],\end{array}}_{\text{Nominal Capital (and Financial) Account of Country } H}
\end{aligned} \tag{3.5.23}$$

and

$$\begin{aligned}
&\underbrace{P_t^* \frac{1}{1-n}\left[(1-\xi)\left(\frac{P_t^F}{P_t}\right)^{1-\sigma} n\frac{1}{RS_t} C_t + (1-\xi^*)\left(\frac{P_t^{F*}}{P_t^*}\right)^{1-\sigma}(1-n)\, C_t^* - (1-n)\, C_t^*\right] +}_{\text{Nominal Trade Balance of Country } F} \\
&\quad \underbrace{+\frac{1}{S_t}\left[B_{t-1}^{H*}(\omega_t) - \int_{\omega_{t+1}\in\Omega} Q_{t-1}(\omega_t)\, B_{t-1}^{H*}(\omega_t)\right] + \left[B_{t-1}^{F*}(\omega_t) - \int_{\omega_{t+1}\in\Omega} Q_{t-1}^*(\omega_t)\, B_{t-1}^{F*}(\omega_t)\right]}_{\text{Nominal Service Balance of Country } F} \\
&= \underbrace{\begin{array}{l}\frac{1}{S_t}\left[\int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1})\, B_t^{H*}(\omega_{t+1}) - \int_{\omega_{t+1}\in\Omega} Q_{t-1}(\omega_t)\, B_{t-1}^{H*}(\omega_t)\right] + \\ +\left[\int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1})\, B_t^{F*}(\omega_{t+1}) - \int_{\omega_{t+1}\in\Omega} Q_{t-1}^*(\omega_t)\, B_{t-1}^{F*}(\omega_t)\right].\end{array}}_{\text{Nominal Capital (and Financial) Account of Country } F}
\end{aligned} \tag{3.5.24}$$

Equations (3.5.23) and (3.5.24) are expressed in nominal terms using their respective local currencies as numeraire. The first term inside brackets in the left-hand side is the nominal *trade balance* of each country. The second term inside brackets in the left-hand side is the *service balance*. The sum of the trade balance and the service balance is the *current account*. The terms in the right-hand side represent the *capital account*, that is the inflow or outflow of capital. A current account deficit (surplus) automatically equates to a capital account surplus (deficit).

The current account of the balance of payments (BoP) is the sum of the trade balance (exports minus imports of goods and services) and the services balance (which includes net factor incomes such as interests and dividends, and net transfer payments such as foreign aid). The trade balance is typically the most important part of the current account. The BoP in the official statistics often includes an accounting item to balance net errors and omissions which, for obvious reasons, must be ignored in the model.

The official reserves (including foreign exchange reserves, official gold reserves, and IMF Special Drawing Rights, all denominated in foreign currency) should also be added to the BoP. In these notes I abstract from foreign reserves entirely, which means that the monetary authority looses a policy instrument often used for interventions in the foreign exchange market. This is consistent with my focus on a free-floating exchange rate regime. As noted previously, I also abstract from the possibility that private agents would demand foreign currency. Hence, money supply does not enter neither the capital account nor the financial services account in the workhorse model.

The World Nominal Trade Balance. The Arrow-Debreu securities can be simply interpreted as borrowing and lending short-term contracts between the households in both countries, since the government neither borrows nor lends in the model. The market clearing condition for the Arrow-Debreu securities requires that,

$$\begin{aligned} nB_t^H(\omega_{t+1}) + (1-n) B_t^{H*}(\omega_{t+1}) &= nB_t^s(\omega_{t+1}) = 0, && (3.5.25) \\ nB_t^F(\omega_{t+1}) + (1-n) B_t^{F*}(\omega_{t+1}) &= (1-n) B_t^{s*}(\omega_{t+1}) = 0, && (3.5.26) \end{aligned}$$

where $B_t^s(\omega_{t+1})$ and $B_t^{s*}(\omega_{t+1})$ are the per capita supply of each claim. All claims are in zero net supply. The asset market clearing conditions must hold true for all period t and for all events in the space $\omega_{t+1} \in \Omega$. Furthermore, the intertemporal first-order condition on the household's problem given in equation (3.2.27) tells me that in equilibrium $Q_t(\omega_{t+1}) = Q_t^*(\omega_{t+1}) \frac{S_t}{S_{t+1}(\omega_{t+1})}$. Let me aggregate the current account equations of both countries as given by (3.5.21) and (3.5.22) across all households, transform the equations to be quoted in units of the same currency with the help of the nominal exchange rate, and add them up. Then, it must follow that,

$$\begin{aligned} &\int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1}) \left[nB_t^H(\omega_{t+1}) + (1-n) B_t^{H*}(\omega_{t+1})\right] + S_t \int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1}) \left[nB_t^F(\omega_{t+1}) + (1-n) B_t^{F*}(\omega_{t+1})\right] \\ &= \left[nB_{t-1}^H(\omega_t) + (1-n) B_{t-1}^{H*}(\omega_t)\right] + S_t \left[nB_{t-1}^F(\omega_t) + (1-n) B_{t-1}^{F*}(\omega_t)\right] + \\ &+ P_t \left(\xi \left(\frac{P_t^H}{P_t}\right)^{1-\sigma} nC_t + \xi^* RS_t \left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} (1-n) C_t^* - nC_t\right) + \\ &+ P_t \left((1-\xi) \left(\frac{P_t^F}{P_t}\right)^{1-\sigma} nC_t + (1-\xi^*) RS_t \left(\frac{P_t^{F*}}{P_t^*}\right)^{1-\sigma} (1-n) C_t^* - (1-n) RS_t C_t^*\right). \end{aligned} \tag{3.5.27}$$

Alternatively, I can simply write that,

$$\begin{aligned} &P_t \left(\xi \left(\frac{P_t^H}{P_t}\right)^{1-\sigma} nC_t + \xi^* RS_t \left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} (1-n) C_t^* - nC_t\right) + \\ &+ S_t P_t^* \left((1-\xi) \frac{1}{RS_t} \left(\frac{P_t^F}{P_t}\right)^{1-\sigma} nC_t + (1-\xi^*) \left(\frac{P_t^{F*}}{P_t^*}\right)^{1-\sigma} (1-n) C_t^* - (1-n) C_t^*\right) = 0, \end{aligned} \tag{3.5.28}$$

which implies that the nominal trade balance surplus in one country has to be equal to the nominal trade balance deficit in the other country whenever expressed in the same currency (as expected).

In other words, the nominal trade balance of the entire world has to be equal to zero when expressed in the same currency. I can re-express this equation one more time as follows,

$$\left(\frac{PPI_t}{P_t} nY_t - nC_t\right) + RS_t \left(\frac{PPI_t^*}{P_t^*} (1-n) Y_t^* - (1-n) C_t^*\right) = 0, \tag{3.5.29}$$

while in real terms it holds true that[32],

$$\left(nY_t + (1-n)\,Y_t^*\right) - \left(nC_t + (1-n)\,C_t^*\right) = 0, \tag{3.5.30}$$

from the goods market clearing conditions in (3.5.17)−(3.5.18). These pair of equations shows that differences in the equilibrium conditions between the nominal and the real values arise from fluctuations in the real exchange rate and from discrepancies between PPIs and CPIs. Both of which are the result of home bias in preferences and violations of the LOOP due to sticky prices and LCP pricing.

There is much more to be said about the capital account and the 'valuation channel' (*add references!!*). This topic has certainly generated a great deal of interest in the literature. Finding ways to determine the portfolio allocations in a model is not a trivial matter (see Evans and Hnatkovska, 2005, 2007). Furthermore, the complexity of the assets traded in the real world must be taken into account. However, the basic principle still carries over. Once all assets (whether bonds or stocks or money) are expressed in the same currency and added up worldwide, an asset for one agent is a liability for another one, so they cancel each other out in the aggregate. Stocks are supplied in certain amounts, but unless ownership changes the optimal decision of the firms, forces a relocation of production, or makes them somehow more productive (to mention just a few possibilities), it is unlikely that it will affect the world-wide real trade balance. And it definitely has no effect on the world-wide nominal trade balance.

It is very difficult to pin down the portfolio that is implied by the model (unless asset markets are irrelevant or there are only two possible state-events). However, by Walras' law it is not necessary to ensure that both national goods markets clear. Since equation (3.5.29) holds, it becomes obvious that only one of the current account equations in (3.5.21) − (3.5.22) must be verified. Asset prices can have a value for policy-makers, because they may convey information about the underlying shocks hitting the economy. Only analysis of asset prices, but not portfolio allocations, is required for the purposes of these notes.

Remark 4 *If I combine equation* (3.5.29) *with the market clearing condition for goods in* (3.5.30)*, it must follow that,*

$$\begin{aligned}
\left(RS_t - 1\right) nC_t &= -\left[\frac{PPI_t}{P_t} - RS_t\right] nY_t - \left[RS_t \frac{PPI_t^*}{P_t^*} - RS_t\right](1-n)\,Y_t^*, \\
\left(RS_t - 1\right)(1-n)\,C_t^* &= \left[\frac{PPI_t}{P_t} - 1\right] nY_t + \left[RS_t \frac{PPI_t^*}{P_t^*} - 1\right](1-n)\,Y_t^*.
\end{aligned}$$

These expressions become a tautological identity if the LOOP holds and preferences on consumption are symmetric across countries. They simply say that 0 *must be equal to* 0*. However, in the workhorse model I find that* $RS_t \neq 1$*,* $PPI_t \neq P_t$ *and* $PPI_t^* \neq P_t^*$*. Hence, I can express the relationship between the aggregate*

[32] This is a condition that holds true no matter what assets are available to trade, or which nominal rigidities are present, and so on. In the end, nominal output must be equal to nominal consumption in the world. This expression, however, reflects some of the simplifications of the workhorse model. Arguably, the government may also demand a share of each variety of goods. The elasticity of intratemporal substitution between the home and foreign bundles of varieties, σ, may be different for the government. In that case, it could be possible that the joint response to variations in the prices of the bundles differs from that expected given the household's elasticity. (A finer point should be made if elasticity of substitution across varieties produced within a country θ differs.) In any event, integrating the government demand does not appear to change the structure of this equilibrium condition radically. On the other hand, the complexity of adding either durable goods or investment goods to the mix of international trade is particularly important given the share of these goods in total trade, but is not considered here either.

consumption and output in both countries in the following terms,

$$\begin{bmatrix} nC_t \\ (1-n)\, C_t^* \end{bmatrix} = \frac{1}{RS_t - 1} \begin{bmatrix} -\left[\frac{PPI_t}{P_t} - RS_t\right] & -\left[RS_t \frac{PPI_t^*}{P_t^*} - RS_t\right] \\ \left[\frac{PPI_t}{P_t} - 1\right] & \left[RS_t \frac{PPI_t^*}{P_t^*} - 1\right] \end{bmatrix} \begin{bmatrix} nY_t \\ (1-n)\, Y_t^* \end{bmatrix}.$$

External conditions matter for the behavior of the local economy.

The Current Account Equation with Incomplete Asset Markets. From equation (3.5.21) and the intertemporal first-order condition in (3.2.27), it follows that the domestic nominal current account equation can be expressed as,

$$\int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1})\, B_t(\omega_{t+1}) = B_{t-1}(\omega_t) + P_t \frac{1}{n}\left[\xi \left(\frac{P_t^H}{P_t}\right)^{1-\sigma} nC_t + \xi^* \left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} RS_t (1-n)\, C_t^* - nC_t\right], \tag{3.5.31}$$

where I define the net borrowing as $B_t(\omega_{t+1}) \equiv B_t^H(\omega_{t+1}) + S_{t+1}(\omega_{t+1})\, B_t^F(\omega_{t+1})$. Alternatively, I could also redefine the current account equation for the foreign country in (3.5.22) by noting that foreign net borrowing can be expressed as $B_t^*(\omega_{t+1}) \equiv \frac{1}{S_{t+1}(\omega_{t+1})} B_t^{H*}(\omega_{t+1}) + B_t^{F*}(\omega_{t+1})$. That is,

$$\begin{aligned} &\int_{\omega_{t+1}\in\Omega} Q_t^*(\omega_{t+1})\, B_t^*(\omega_{t+1}) \\ &\quad = B_{t-1}^*(\omega_t) + P_t^* \tfrac{1}{1-n}\left[(1-\xi)\left(\tfrac{P_t^F}{P_t}\right)^{1-\sigma} \tfrac{1}{RS_t} nC_t + (1-\xi^*)\left(\tfrac{P_t^{F*}}{P_t^*}\right)^{1-\sigma} (1-n)\, C_t^* - (1-n)\, C_t^*\right]. \end{aligned} \tag{3.5.32}$$

In the case where asset market are incomplete, the domestic current account equation can be expressed as,

$$\left[1 + \left(\frac{S_t}{S_{t+1}} \frac{1+i_t}{1+i_t^*} - 1\right)\alpha_t\right] \frac{1}{1+i_t} B_t = B_{t-1} + P_t \frac{1}{n}\left[\xi\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} nC_t + \xi^*\left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma} RS_t (1-n)\, C_t^* - nC_t\right], \tag{3.5.33}$$

where $B_t \equiv B_t^H + S_{t+1} B_t^F$ and $\alpha_t \equiv \frac{S_{t+1} B_t^F}{B_t}$. In turn, the foreign current account equation is equivalent to,

$$\begin{aligned} &\left[1 + \left(\tfrac{S_{t+1}}{S_t} \tfrac{1+i_t^*}{1+i_t} - 1\right)\alpha_t^*\right] \tfrac{1}{1+i_t^*} B_t^* \\ &\quad = B_{t-1}^* + P_t^* \tfrac{1}{1-n}\left[(1-\xi)\left(\tfrac{P_t^F}{P_t}\right)^{1-\sigma} \tfrac{1}{RS_t} nC_t + (1-\xi^*)\left(\tfrac{P_t^{F*}}{P_t^*}\right)^{1-\sigma} (1-n)\, C_t^* - (1-n)\, C_t^*\right], \end{aligned} \tag{3.5.34}$$

where $B_t^* \equiv \frac{1}{S_{t+1}} B_t^{H*} + B_t^{F*}$ and $\alpha_t^* \equiv \frac{B_t^{H*}}{S_{t+1} B_t}$.

The Real Exports, Real Imports and Real Net Exports. The aggregate imports of the domestic country in real terms are defined from equation (3.2.24) as follows,

$$IM_t = nC_t^F = (1-\xi)\left(\frac{P_t^F}{P_t}\right)^{-\sigma} nC_t,$$

while the aggregate exports of the domestic country in real terms are derived from (3.2.23) as,

$$EX_t = (1-n) C_t^{H*} = \xi^* \left(\frac{P_t^{H*}}{P_t^*} \right)^{-\sigma} (1-n) C_t^*.$$

In a two-country model it must be the case that aggregate imports (exports) of the domestic country correspond to aggregate exports (imports) of the foreign country. In other words, $IM_t = EX_t^*$ and $EX_t = IM_t^*$. There is no need to emphasize that point further. Naturally, the real net exports or real trade balance per capita in the domestic country can be computed as,

$$nTB_t^R = EX_t - IM_t = \xi^* \left(\frac{P_t^{H*}}{P_t^*} \right)^{-\sigma} (1-n) C_t^* - (1-\xi) \left(\frac{P_t^F}{P_t} \right)^{-\sigma} nC_t. \tag{3.5.35}$$

The domestic aggregate real trade balance, nTB_t^R, is equal to the aggregate real trade balance of the foreign country with the opposite sign. The nominal trade balance in (3.5.23) for the home country is, by comparison,

$$\begin{aligned} nTB_t &= P_t \left[\xi \left(\frac{P_t^H}{P_t} \right)^{1-\sigma} nC_t + \xi^* \left(\frac{P_t^{H*}}{P_t^*} \right)^{1-\sigma} (1-n) RS_t C_t^* - nC_t \right] \\ &= S_t P_t^{H*} \left[\xi^* \left(\frac{P_t^{H*}}{P_t^*} \right)^{-\sigma} (1-n) C_t^* \right] - P_t \left(1 - \xi \left(\frac{P_t^H}{P_t} \right)^{1-\sigma} \right) nC_t \\ &= S_t P_t^{H*} \left[\xi^* \left(\frac{P_t^{H*}}{P_t^*} \right)^{-\sigma} (1-n) C_t^* \right] - P_t^F \left[(1-\xi) \left(\frac{P_t^F}{P_t} \right)^{-\sigma} nC_t \right], \end{aligned} \tag{3.5.36}$$

and comes also expressed in per capita terms. The third equality follows from manipulating the domestic CPI formula in (3.2.14). As noted before, the aggregate nominal trade balance in the domestic country, nTB_t, is equal to the aggregate nominal trade balance of the foreign country with the opposite sign. The nominal trade balance can be combined with (3.5.15) to obtain that,

$$nTB_t = nPPI_t Y_t - nP_t C_t. \tag{3.5.37}$$

Usually, the definition of the real trade balance in units of the consumption good that we compute is $\frac{TB_t}{P_t}$.

3.6 The Six Puzzles: Revisited

By now, it should be evident that the workhorse model either avoids or exogenously imposes certain features designed to side-step the modelling hurdles of a fully integrated theory of the six puzzles. The model is naturally positioned to become the benchmark for the international correlations puzzle, the PPP puzzle and to some extent the exchange rate disconnect puzzle.

The assumption of home bias in preferences proposed by Warnock (2003) allows me to exogenously replicate the real trade patterns noted in the *home bias in trade puzzle*. It certainly allows me to regulate the amount of imports and exports in steady state to account for the observation that cross-border trade seems to be quite limited. I have already pointed out that these preference specification introduces compositional effects that can explain fluctuations of the real exchange rate even if the LOOP condition holds. Some compositional effects may be relevant to explain movements in real exchange rates and a pervasive bias in trade. However, this is unlikely to be the whole story on both accounts.

The workhorse model also assumes that there are neither capital investment goods nor durable goods. Moreover, the production function is linear in labor. This makes the model ill-equipped to convincingly account for the *savings-investment (or Feldstein-Horioka) puzzle*. It also limits the ability of the model to replicate certain features of the net exports and relative prices (e.g., Martínez-García and Sondergaard, 2008) and exports and imports (e.g., Engel and Wang, 2007). Moreover, it eliminates certain margins of great interest like investing in capital to move production intertemporally.

Finally, the assumption that portfolios of the local firms are solely owned by the local households imposes exogenously a strict home bias that is consistent with the *home bias in equity portfolios puzzle*. The model still allows for either contingent or uncontingent bonds to be traded internationally. Hence, it can say something about the degree of home bias in bonds instead (under incomplete asset markets). Nonetheless, the model cannot account endogenously for and explain the overwhelming evidence of insufficient portfolio diversification.

The question that we should always keep in mind is to what extent these simplifications and exogenous restrictions are also distorting our understanding of the three puzzles around which much of the current international business cycle literature has developed. It is not obvious to me that these simplifications are just 'innocuous'.

4 The Frictionless Allocation

Here, I re-write the basic building blocks of the model by assuming that the economy is fully flexible, frictionless and competitive. The result characterizes the frictionless allocation and, more concretely, identifies the potential output of each country, i.e. $\overline{Y}_t$ and $\overline{Y}_t^*$. Potential output is necessary to define the output gap measure that enters into the determination of the monetary policy rules in (3.4.12) – (3.4.13). In order to notationally distinguish this frictionless equilibrium I denote the variables with an upper bar. Productivity shocks are unaffected by the presence of frictions in the model or lack thereof, hence they do not require a different notation.

Using the equilibrium conditions in the labor market as discussed in (3.3.30) – (3.3.31), I express the marginal cost in both countries in the following compact form under flexible prices,

$$\frac{\theta-1}{\theta} = \frac{\overline{MC}_t}{\overline{P}_t^H} = \left(1+\overline{\phi}_t\right)\left(\frac{\overline{W}_t}{\overline{P}_t^H A_t}\right) = \left(1+\overline{\phi}_t\right)\left(\kappa\left(A_t\right)^{-\varphi-1}\left(\overline{C}_t\right)^{\gamma}\left(\overline{Y}_t\right)^{\varphi}\left(\frac{\overline{P}_t}{\overline{P}_t^H}\right)\right), \tag{4.1}$$

$$\frac{\theta-1}{\theta} = \frac{\overline{MC}_t^*}{\overline{P}_t^{F*}} = \left(1+\overline{\phi}_t^*\right)\left(\frac{\overline{W}_t^*}{\overline{P}_t^{F*} A_t^*}\right) = \left(1+\overline{\phi}_t^*\right)\left(\kappa\left(A_t^*\right)^{-\varphi-1}\left(\overline{C}_t^*\right)^{\gamma}\left(\overline{Y}_t^*\right)^{\varphi}\left(\frac{\overline{P}_t^*}{\overline{P}_t^{F*}}\right)\right). \tag{4.2}$$

Fiscal policy is optimally implemented and completely eliminates the distortion caused by the mark-up as in (3.4.5), i.e. $\overline{\phi}_t = \overline{\phi}_t^* = \frac{-1}{\theta}$. Alternatively, I could have assumed that prices are competitive and equal to marginal costs. Therefore, each individual firm charges a price equal to marginal costs and, by symmetry, the same is true in the aggregate. The labor market equilibrium conditions can be naturally reduced to,

$$\kappa\left(A_t\right)^{-\varphi-1}\left(\overline{C}_t\right)^{\gamma}\left(\overline{Y}_t\right)^{\varphi}\left(\frac{\overline{P}_t}{\overline{P}_t^H}\right) = 1, \tag{4.3}$$

$$\kappa\left(A_t^*\right)^{-\varphi-1}\left(\overline{C}_t^*\right)^{\gamma}\left(\overline{Y}_t^*\right)^{\varphi}\left(\frac{\overline{P}_t^*}{\overline{P}_t^{F*}}\right) = 1. \tag{4.4}$$

These pair of equations, however, depends on consumption, output and prices (all of which are endogenously determined).

By assuming that fiscal policy is optimal or prices are competitive, I eliminate the mark-up which is the distortion introduced by monopolistic competition. The economy behaves as if it was populated by perfectly competitive firms. I also assume complete asset markets to avoid the distortions due to incompleteness. As a result, the perfect international risk-sharing condition under complete asset markets described in equation (3.2.29) tells me that,

$$\upsilon\overline{C}_t = \left(\overline{RS}_t\right)^{\frac{1}{\gamma}}\overline{C}_t^*, \tag{4.5}$$

where $\upsilon = 1$ after normalization of the steady state. Moreover, I assume that prices are fully flexible, which immediately implies that the LOOP holds (independently of whether firms price-to-market or not). Eliminating this friction, however, does not mean the real exchange rate should be constant because I still maintain the assumption that households have home bias on consumption. The real exchange rate can be related to terms of trade as in equation (3.1.2),

$$\overline{RS}_t = \frac{\left[\xi^* + (1-\xi^*)\left(\overline{ToT}_t\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\left[\xi + (1-\xi)\left(\overline{ToT}_t\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}. \tag{4.6}$$

The model under these conditions is, therefore, equivalent to one where prices are fully flexible, firms are perfectly competitive, and asset markets are complete.

If I take the ratio of aggregate output in both countries as given by equations (3.3.36) − (3.3.37), it follows that,

$$\begin{aligned}
\frac{n\overline{Y}_t}{(1-n)\overline{Y}_t^*} &= \frac{\left(\frac{\overline{P}_t^H}{\overline{P}_t}\right)^{-\sigma}\left[n\left(\frac{\xi}{n}\overline{C}_t\right) + (1-n)\left(\frac{\xi^*}{n}\left(\frac{1}{\overline{RS}_t}\right)^{-\sigma}\overline{C}_t^*\right)\right]}{\left(\frac{\overline{P}_t^{F*}}{\overline{P}_t^*}\right)^{-\sigma}\left[n\left(\frac{1-\xi}{1-n}\left(\overline{RS}_t\right)^{-\sigma}\overline{C}_t\right) + (1-n)\left(\frac{1-\xi^*}{1-n}\overline{C}_t^*\right)\right]} \\
&= \left(\frac{\overline{P}_t^H}{\overline{P}_t^{F*}}\frac{\overline{P}_t^*}{\overline{P}_t}\right)^{-\sigma}\left[\frac{n\left(\frac{\xi}{n}\frac{1}{\upsilon}\left(\overline{RS}_t\right)^{\frac{1}{\gamma}}\right) + (1-n)\left(\frac{\xi^*}{n}\left(\overline{RS}_t\right)^{\sigma}\right)}{n\left(\frac{1-\xi}{1-n}\frac{1}{\upsilon}\left(\overline{RS}_t\right)^{-\sigma+\frac{1}{\gamma}}\right) + (1-n)\left(\frac{1-\xi^*}{1-n}\right)}\right] \\
&= \left(\frac{\overline{RS}_t}{\overline{ToT}_t}\right)^{-\sigma}\left[\frac{n\left(\frac{\xi}{n}\frac{1}{\upsilon}\left(\overline{RS}_t\right)^{\frac{1}{\gamma}}\right) + (1-n)\left(\frac{\xi^*}{n}\left(\overline{RS}_t\right)^{\sigma}\right)}{n\left(\frac{1-\xi}{1-n}\frac{1}{\upsilon}\left(\overline{RS}_t\right)^{-\sigma+\frac{1}{\gamma}}\right) + (1-n)\left(\frac{1-\xi^*}{1-n}\right)}\right],
\end{aligned} \tag{4.7}$$

where the second equality comes from the perfect international risk-sharing condition in (3.2.29). Equations (4.6) − (4.7) for the real exchange rate and the terms of trade can be naturally interpreted as two non-linear functions of the ratio of aggregate output between the two countries. Generically, I can say that,

$$\overline{RS}_t = R\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right), \tag{4.8}$$

$$\overline{ToT}_t = T\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right). \tag{4.9}$$

In other words, both the real exchange rate and the terms of trade are functions of the output ratio. Notice also that based on the structure of the consumption price indexes in (3.2.14) – (3.2.15), I can argue that the following price ratios,

$$\frac{\overline{P}_t^H}{\overline{P}_t} = \left[\frac{1}{\xi + (1-\xi)\left(\frac{\overline{P}_t^F}{\overline{P}_t^H}\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} = \left[\frac{1}{\xi + (1-\xi)\left(\overline{TOT}_t\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} = X\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right), \tag{4.10}$$

$$\frac{\overline{P}_t^{F*}}{\overline{P}_t^*} = \left[\frac{1}{\xi^*\left(\frac{\overline{P}_t^{H*}}{\overline{P}_t^{F*}}\right)^{1-\sigma} + (1-\xi^*)}\right]^{\frac{1}{1-\sigma}} = \left[\frac{1}{\xi^*\left(\frac{1}{\overline{ToT}_t}\right)^{1-\sigma} + (1-\xi^*)}\right]^{\frac{1}{1-\sigma}} = X^*\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right), \tag{4.11}$$

are a function of terms of trade and therefore also connected to the aggregate output ratio.

Using this information about the structure of the economy and the aggregate output equations in (3.3.36) – (3.3.37), I can re-write the aggregate consumption in terms of aggregate output (appropriately using the perfect risk-sharing condition in (3.2.29)),

$$n\overline{Y}_t = \left(X\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right)\right)^{-\sigma}\left[n\left(\frac{\xi}{n}\right) + (1-n)\left(\frac{\xi^*}{n}\upsilon\left(R\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right)\right)^{\sigma-\frac{1}{\gamma}}\right)\right]\overline{C}_t, \tag{4.12}$$

$$(1-n)\,\overline{Y}_t^* = \left(X^*\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right)\right)^{-\sigma}\left[n\left(\frac{1-\xi}{1-n}\frac{1}{\upsilon}\left(R\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right)\right)^{-\sigma+\frac{1}{\gamma}}\right) + (1-n)\left(\frac{1-\xi^*}{1-n}\right)\right]\overline{C}_t^*. \tag{4.13}$$

These equations clearly imply that the aggregate consumption in equilibrium follows from,

$$\overline{C}_t = C\left(\overline{Y}_t, \overline{Y}_t^*\right), \tag{4.14}$$

$$\overline{C}_t^* = C^*\left(\overline{Y}_t, \overline{Y}_t^*\right), \tag{4.15}$$

which are functional forms that depend exclusively on the output in both countries. The model shows that aggregate consumption is a function of output in both countries and this relationship can be derived easily from the goods market clearing conditions. Finally, if I replace the equations for optimal aggregate consumption (i.e., $C\left(\overline{Y}_t, \overline{Y}_t^*\right)$ and $C^*\left(\overline{Y}_t, \overline{Y}_t^*\right)$) and the equations for the price ratios (i.e., $X\left(\frac{n\overline{Y}_t}{(1-n)\overline{Y}_t^*}\right)$ and $X^*\left(\frac{n\overline{Y}_t}{(1-n)\overline{Y}_t^*}\right)$) inside the equilibrium labor market clearing conditions in (4.3) – (4.4), I immediately derive a pair of equilibrium conditions that link output in both countries to productivity shocks,

$$\kappa \left(A_t\right)^{-\varphi-1} \left(C\left(\overline{Y}_t, \overline{Y}_t^*\right)\right)^{\gamma} \left(\overline{Y}_t\right)^{\varphi} = X\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right), \tag{4.16}$$

$$\kappa \left(A_t^*\right)^{-\varphi-1} \left(C^*\left(\overline{Y}_t, \overline{Y}_t^*\right)\right)^{\gamma} \left(\overline{Y}_t^*\right)^{\varphi} = X^*\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right). \tag{4.17}$$

This is a system of two equations in two unknowns that can be resolved.

Finding a solution for the system in (4.16) – (4.17) implies, in general, that output in each country is a function of productivity shocks in both countries. To be more precise, the equilibrium level of employment varies (nonlinearly) with the productivity shocks in both countries and, as a result, local output also responds to the foreign productivity shocks. The formulas take a highly nonlinear form, which I do not derive here explicitly, but nevertheless convey two important ideas. First, that monetary policy is neutral under flexible prices, perfect competition and complete asset markets. So, money affects the nominal variables (wages and prices), but it has no real-side effects (employment and output). Second, openness has a direct effect on the real-side of the economy because, unlike in the case of autarky, it implies that equilibrium employment and output respond to foreign shocks.

I define the functional forms for output and employment generically as,

$$\overline{Y}_t = \overline{Y}\left(A_t, A_t^*\right),\ \overline{L}_t = \overline{L}\left(A_t, A_t^*\right), \tag{4.18}$$

$$\overline{Y}_t^* = \overline{Y}^*\left(A_t, A_t^*\right),\ \overline{L}_t^* = \overline{L}^*\left(A_t, A_t^*\right), \tag{4.19}$$

and I assume that these expressions represent and reflect the true 'potential' of the economy (in terms of output and employment). If monetary policy targets deviations of output from its potential, as posited by the Taylor rule, it can no longer ignore the foreign productivity shocks (except in special cases as in Corsetti and Pesenti, 2001). Therefore, it becomes ever more clear that monetary policy cannot be conducted looking at domestic sources or shocks in isolation.

A Special Case: Corsetti and Pesenti (2001). As in Corsetti and Pesenti (2001), let me assume that the preferences are logarithmic in aggregate consumption (i.e., $\gamma = 1$) and that the elasticity of substitution across bundles of goods is equal to one (i.e., $\sigma = 1$), then I argue that the real exchange rate and terms of trade can be expressed as,

$$\overline{RS}_t = R\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right) = \left(\overline{ToT}_t\right)^{\xi-\xi^*}, \tag{4.20}$$

$$\overline{ToT}_t = T\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right) = \left[\frac{n\left(\frac{1-\xi}{1-n}\frac{1}{\upsilon}\right) + (1-n)\left(\frac{1-\xi^*}{1-n}\right)}{n\left(\frac{\xi}{n}\frac{1}{\upsilon}\right) + (1-n)\left(\frac{\xi^*}{n}\right)}\right]\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right). \tag{4.21}$$

In other words, both the real exchange rate and the terms of trade are very simple functions of the output ratio properly adjusted to take into account the home bias in consumption. If the elasticity of substitution across bundles of goods is equal to one (i.e., $\sigma = 1$), then the consumption price indexes in (3.2.14) – (3.2.15) become,

$$\frac{\overline{P}_t^H}{\overline{P}_t} = X\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right) = \left(\frac{1}{\overline{ToT}_t}\right)^{1-\xi}, \tag{4.22}$$

$$\frac{\overline{P}_t^{F*}}{\overline{P}_t^*} = X^*\left(\frac{n\overline{Y}_t}{(1-n)\,\overline{Y}_t^*}\right) = \left(\overline{ToT}_t\right)^{\xi^*}. \tag{4.23}$$

Using this information about the structure of the economy and the aggregate output equations in (3.3.36) – (3.3.37), I can naturally re-write the aggregate consumption in terms of aggregate output (appropriately using the perfect risk-sharing condition in (3.2.29)) as,

$$\overline{C}_t = C\left(\overline{Y}_t, \overline{Y}_t^*\right) = \frac{\left(n\overline{Y}_t\right)^{\xi}\left((1-n)\,\overline{Y}_t^*\right)^{1-\xi}}{\upsilon\left(n\left(\frac{\xi}{n}\frac{1}{\upsilon}\right)+(1-n)\left(\frac{\xi^*}{n}\right)\right)^{\xi}\left(n\left(\frac{1-\xi}{1-n}\frac{1}{\upsilon}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)\right)^{1-\xi}}, \tag{4.24}$$

$$\overline{C}_t^* = C^*\left(\overline{Y}_t, \overline{Y}_t^*\right) = \frac{\left(n\overline{Y}_t\right)^{\xi^*}\left((1-n)\,\overline{Y}_t^*\right)^{1-\xi^*}}{\left(n\left(\frac{\xi}{n}\frac{1}{\upsilon}\right)+(1-n)\left(\frac{\xi^*}{n}\right)\right)^{\xi^*}\left(n\left(\frac{1-\xi}{1-n}\frac{1}{\upsilon}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)\right)^{1-\xi^*}}. \tag{4.25}$$

In this particular example, the aggregate consumption is proportional to the aggregate world output computed using the consumption aggregator. This reflects the fact that each household consumes a constant fraction of the total output produced locally and a different (but still constant) fraction of the output produced abroad. However, this is only a special case. In general, the model shows that aggregate consumption ought to be a more complex function of output in both countries.

Finally, if I replace the equations for optimal aggregate consumption (i.e., $C\left(\overline{Y}_t, \overline{Y}_t^*\right)$ and $C^*\left(\overline{Y}_t, \overline{Y}_t^*\right)$) and the equations for the price ratios (i.e., $X\left(\frac{n\overline{Y}_t}{(1-n)\overline{Y}_t^*}\right)$ and $X^*\left(\frac{n\overline{Y}_t}{(1-n)\overline{Y}_t^*}\right)$) inside the equilibrium labor market clearing conditions in (4.3) – (4.4), I immediately derive an explicit pair of equilibrium conditions that link output in both countries to productivity shocks,

$$n\overline{Y}_t = \left[\frac{\upsilon}{\kappa}n^{\varphi}\left(n\left(\frac{\xi}{n}\frac{1}{\upsilon}\right)+(1-n)\left(\frac{\xi^*}{n}\right)\right)\right]^{\frac{1}{1+\varphi}} A_t, \tag{4.26}$$

$$(1-n)\,\overline{Y}_t^* = \left[\frac{1}{\kappa}(1-n)^{\varphi}\left(n\left(\frac{1-\xi}{1-n}\frac{1}{\upsilon}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)\right)\right]^{\frac{1}{1+\varphi}} A_t^*. \tag{4.27}$$

This is a very special case in which aggregate output only depends on the local productivity shock. Another way of interpreting the result is by saying that labor in the economy is constant, i.e.

$$n\overline{L}_t = \left[\frac{\upsilon}{\kappa}n^{\varphi}\left(n\left(\frac{\xi}{n}\frac{1}{\upsilon}\right)+(1-n)\left(\frac{\xi^*}{n}\right)\right)\right]^{\frac{1}{1+\varphi}}, \tag{4.28}$$

$$(1-n)\,\overline{L}_t^* = \left[\frac{1}{\kappa}(1-n)^{\varphi}\left(n\left(\frac{1-\xi}{1-n}\frac{1}{\upsilon}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)\right)\right]^{\frac{1}{1+\varphi}}. \tag{4.29}$$

Openness has only a limited impact. It only varies the size of the constant labor employment and the aggregate output level because output must be produced now to satisfy the foreign demand. In this context, potential output depends only on domestic shocks and, therefore, monetary policy can be conducted as usual: by targeting a measure of potential output that is purely domestic, and depends on domestic shocks only.

The Nominal Variables of the Frictionless Economy. For the purpose of these notes, it is not necessary to derive explicitly the CPI for each country. From my previous derivations, I know that aggregate consumption in both countries can be expressed as a function of domestic and foreign productivity shocks. In fact, the demand for each variety and for the entire bundle of goods in each country can be obtained from equations (3.2.21) − (3.2.24), i.e.

$$\overline{C}_t(h) = \frac{1}{n}\overline{C}_t^H,\ \overline{C}_t^*(h) = \frac{1}{n}\overline{C}_t^{H*},\ \text{if } h \in [0,n],\ \overline{C}_t(f) = \frac{1}{1-n}\overline{C}_t^F,\ \overline{C}_t^*(f) = \frac{1}{1-n}\overline{C}_t^{F*},\ \text{if } f \in (n,1], \tag{4.30}$$

$$\overline{C}_t^H = \xi\left(X\left(\frac{n\overline{Y}(A_t,A_t^*)}{(1-n)\overline{Y}^*(A_t,A_t^*)}\right)\right)^{-\sigma}\overline{C}(A_t,A_t^*),\ \overline{C}_t^{H*} = \xi^*\left(\frac{X\left(\frac{n\overline{Y}(A_t,A_t^*)}{(1-n)\overline{Y}^*(A_t,A_t^*)}\right)}{R\left(\frac{n\overline{Y}(A_t,A_t^*)}{(1-n)\overline{Y}^*(A_t,A_t^*)}\right)}\right)^{-\sigma}\overline{C}^*(A_t,A_t^*), \tag{4.31}$$

$$\overline{C}_t^F = (1-\xi)\left(R\left(\frac{n\overline{Y}(A_t,A_t^*)}{(1-n)\overline{Y}^*(A_t,A_t^*)}\right)X^*\left(\frac{n\overline{Y}(A_t,A_t^*)}{(1-n)\overline{Y}^*(A_t,A_t^*)}\right)\right)^{-\sigma}\overline{C}(A_t,A_t^*), \tag{4.32}$$

$$\overline{C}_t^{F*} = (1-\xi^*)\left(X^*\left(\frac{n\overline{Y}(A_t,A_t^*)}{(1-n)\overline{Y}^*(A_t,A_t^*)}\right)\right)^{-\sigma}\overline{C}^*(A_t,A_t^*), \tag{4.33}$$

where the equations $X(\cdot)$, $X^*(\cdot)$ and $R(\cdot)$ were defined before. Naturally, there is no price dispersion among the local varieties and price dispersion across bundles of domestic and foreign goods is entirely due to differences in preferences (that is, due to home bias in consumption).

Hence, I only need to specify a pair of monetary policy rules and a pair of Euler equations in order to close down the model. The monetary policy functions involve the standard zero administered rate on money, i.e. $\overline{i}_t^m = \overline{i}_t^{m*} = 0$, and Taylor rules in the spirit of equations (3.4.12) − (3.4.13)[33]. The Euler equations in (3.2.30) − (3.2.31) are derived after aggregating the price of each Arrow-Debreu security. In summary, I have the following system,

$$\frac{1+\overline{i}_t}{1+\overline{i}} = \frac{Z_t}{Z}\left(\frac{1+\overline{i}_{t-1}}{1+\overline{i}}\right)^{\rho}\left(\overline{\Pi}_t\right)^{(1-\rho)\psi_1}, \tag{4.34}$$

$$\frac{1+\overline{i}_t^*}{1+\overline{i}^*} = \frac{Z_t^*}{Z^*}\left(\frac{1+\overline{i}_{t-1}^*}{1+\overline{i}^*}\right)^{\rho^*}\left(\overline{\Pi}_t^*\right)^{(1-\rho^*)\psi_1^*}, \tag{4.35}$$

$$\frac{1}{1+\overline{i}_t} = \beta\mathbb{E}_t\left[\left(\frac{\overline{C}(A_{t+1},A_{t+1}^*)}{\overline{C}(A_t,A_t^*)}\right)^{-\gamma}\frac{1}{\overline{\Pi}_{t+1}}\right], \tag{4.36}$$

$$\frac{1}{1+\overline{i}_t^*} = \beta\mathbb{E}_t\left[\left(\frac{\overline{C}^*(A_{t+1},A_{t+1}^*)}{\overline{C}^*(A_t,A_t^*)}\right)^{-\gamma}\frac{1}{\overline{\Pi}_{t+1}^*}\right], \tag{4.37}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ and $\Pi_{t+1}^* \equiv \frac{P_{t+1}^*}{P_t^*}$. The nominal interest rate and the CPI level also involve a lag.

[33] I already noted that monetary policy is neutral. Therefore, targeting output is a pointless exercise.

Therefore, an initial condition for both variables is required (i.e., i_0 and i_0^*, and P_0 and P_0^*, respectively). The money market clearing conditions given by (3.4.8) – (3.4.9), i.e.

$$\overline{M}_t = \left(\frac{1}{\chi}\frac{\overline{i}_t}{1+\overline{i}_t}\right)^{-\frac{1}{\zeta}} \overline{P}_t \left(\overline{C}\left(A_t, A_t^*\right)\right)^{\frac{\gamma}{\zeta}}, \tag{4.38}$$

$$\overline{M}_t^* = \left(\frac{1}{\chi}\frac{\overline{i}_t^*}{1+\overline{i}_t^*}\right)^{-\frac{1}{\zeta}} \overline{P}_t^* \left(\overline{C}^*\left(A_t, A_t^*\right)\right)^{\frac{\gamma}{\zeta}}, \tag{4.39}$$

can be used afterwords to determine the money supplied by the monetary authority in equilibrium.

Alternatively, I could rely on a money supply growth rule in the spirit of Friedman to replace the Taylor rules. The Euler equations in (3.2.30) – (3.2.31) and the money market clearing conditions given by (3.4.8) – (3.4.9) are all that is needed to pin down the CPI level, assuming that the supply growth rule does not depend on other variables. However, these are only two possibilities among infinitely many rules that would induce different paths for inflation without no real consequences. For instance, the monetary authorities could also set the interest rates to satisfy that,

$$\frac{1}{1+\overline{i}_t} = \beta\mathbb{E}_t\left[\left(\frac{\overline{C}\left(A_{t+1}, A_{t+1}^*\right)}{\overline{C}\left(A_t, A_t^*\right)}\right)^{-\gamma}\right], \tag{4.40}$$

$$\frac{1}{1+\overline{i}_t^*} = \beta\mathbb{E}_t\left[\left(\frac{\overline{C}^*\left(A_{t+1}, A_{t+1}^*\right)}{\overline{C}^*\left(A_t, A_t^*\right)}\right)^{-\gamma}\right], \tag{4.41}$$

promise a zero administered rate for money, and limit the supply of money to the amounts given by the following rule,

$$\overline{M}_t = \left(\frac{1}{\chi}\frac{\overline{i}_t}{1+\overline{i}_t}\right)^{-\frac{1}{\zeta}} \left(\overline{C}\left(A_t, A_t^*\right)\right)^{\frac{\gamma}{\zeta}}, \tag{4.42}$$

$$\overline{M}_t^* = \left(\frac{1}{\chi}\frac{\overline{i}_t^*}{1+\overline{i}_t^*}\right)^{-\frac{1}{\zeta}} \left(\overline{C}^*\left(A_t, A_t^*\right)\right)^{\frac{\gamma}{\zeta}}, \tag{4.43}$$

in order to ensure that $\overline{P}_t = \overline{P}_t^* = 1$ for all t. The CPI price level and inflation may fluctuate in response to real shocks (not just monetary shocks) as seen in these examples, however, it is entirely determined by the monetary policy rule. Even the sensitivity to real shocks depends on the choice of monetary policy.

Once the CPI is determined, then I immediately derive the price of each Arrow-Debreu security from the intertemporal first-order conditions of the household in (3.2.25) – (3.2.26). Noting that in equilibrium both labor and output are a function of productivity shocks only, I can infer nominal wages from the first-order conditions on labor supply given by (3.2.38) – (3.2.39) (or from equations (3.3.30) – (3.3.31)), i.e.

$$\overline{W}_t = \kappa\left(\overline{C}\left(A_t, A_t^*\right)\right)^{\gamma}\left(\overline{L}\left(A_t, A_t^*\right)\right)^{\varphi}\overline{P}_t, \tag{4.44}$$

$$\overline{W}_t^* = \kappa\left(\overline{C}^*\left(A_t, A_t^*\right)\right)^{\gamma}\left(\overline{L}^*\left(A_t, A_t^*\right)\right)^{\varphi}\overline{P}_t^*, \tag{4.45}$$

The pricing for each variety and for the bundle of goods in each country comes from the optimal pricing equations in (3.3.14) – (3.3.15), i.e.

$$\overline{P}_t^H = \overline{P}_t(h) = \left(\frac{\overline{W}_t}{A_t}\right) = \overline{P}_t^*(h)\,\overline{S}_t = \overline{P}_t^{H*}\overline{S}_t,\ \text{if } h \in [0,n], \tag{4.46}$$

$$\overline{P}_t^{F*} = \overline{P}_t^*(f) = \left(\frac{\overline{W}_t^*}{A_t^*}\right) = \overline{P}_t(f)\,\frac{1}{\overline{S}_t} = \overline{P}_t^F\frac{1}{\overline{S}_t},\ \text{if } f \in (n,1], \tag{4.47}$$

assuming the optimal fiscal policies, i.e. $\overline{\phi}_t = \overline{\phi}_t^* = \frac{-1}{\theta}$, to ensure that prices are equal to marginal costs.

Finally, the real exchange rate described in (4.6) is known to be a function of the productivity shocks. Using the definition of the real exchange rate it is possible to write the nominal exchange rate as the product of this function and the ratio of the CPIs in both countries[34], i.e.

$$\overline{S}_t = R\left(\frac{n\overline{Y}(A_t, A_t^*)}{(1-n)\,\overline{Y}^*(A_t, A_t^*)}\right)\frac{\overline{P}_t}{\overline{P}_t^*}.$$

Even in an elementary model like this one, monetary policy is not the only source of fundamental shocks driving the nominal exchange rate. As a matter of fact, it is arguably possible to write a model where the monetary policy is such that CPIs are very stable in both countries. The outcome of that model would be that very little of the fluctuations of the nominal exchange rate has to do with monetary shocks in either country. Hence, the nominal exchange rate mostly reflects the impact of productivity shocks. This could be related to the *exchange rate disconnect puzzle* since it gives little role to monetary policy shocks in the nominal exchange rate, but it certainly does not help us explain the high volatility observed in the data. The crucial lesson here, anyway, is that real shocks can affect the nominal exchange rate because PPP fails. However, the sensitivity of the nominal exchange rate to real shocks depends on what determines the failure of PPP. In other words, the effects due to non-traded goods or LOOP violations may be very different.

Remark 5 *Money is neutral in a frictionless economy. CPI inflation is induced by the monetary authority's willingness to interfere with the nominal interest rate. Although, monetary policy can act as a conduit that allows real-shocks (and probably demand and fiscal shocks too) to have an impact on the price level and inflation, it is still true that the domestic inflation is determined and can be regulated by the domestic monetary policy. Openness to trade on goods and assets has no effect whatsoever on the ability of the domestic monetary authority to control domestic inflation. In any event, it is not clear why the monetary authority would find it useful to induce any other outcome rather than simple* price stability*.*

Ultimately, monetary policy controls the price level and this is what the workhorse model reflects. In certain situations, for example, whenever prices are sticky and firms rely on pricing-to-market policies, the price level can effect the relative prices of different goods within a country and the relative prices of the same goods across borders. Therefore, this makes possible for monetary policy to have real effects. Under flexible prices, however, there is no 'other' theory that would allow us to explain how monetary policy can use its influence on the price level to distort relative prices. What would be needed is a theory of frictions that makes adjustments (in quantities) costly, a theory of financial intermediation frictions, etc. But, naturally, if monetary policy cannot manipulate relative prices, it cannot have real effects either.

[34] This nominal exchange rate should be consistent with the intertemporal first-order condition of the households' problem in either (3.2.27) or (3.2.32).

5 The Irrelevance of Asset Markets

The irrelevance of asset markets is a result often cited, but one that needs further exploration. Cole and Obstfeld (1991) are credited with having pointed out this result, which has been subsequently studied by Corsetti and Pesenti (2001, 2005a, 2005b, 2006). It states that under certain assumptions, the structure of the asset markets has no impact on the allocation of the economy. I have demonstrated already that if prices are flexible and firms are competitive, then monetary policy is neutral. Now, I shall demonstrate that in certain cases asset markets are irrelevant. Therefore, financial openness may have no impact on the allocation and, therefore, should not enter as a consideration in monetary policy decisions.

I assume that the preferences are logarithmic (and additively separable) in consumption, i.e. $\gamma = 1$, and the elasticity of substitution between the domestic and foreign bundle of goods is equal to one, i.e. $\sigma = 1$. I know from the demand functions in equations (3.2.23) – (3.2.24) that,

$$P_t^H C_t^H = \xi P_t C_t,\ P_t^{H*} C_t^{H*} = \xi^* P_t^* C_t^*, \tag{5.1}$$

$$P_t^F C_t^F = (1-\xi) P_t C_t,\ P_t^{F*} C_t^{F*} = (1-\xi^*) P_t^* C_t^*, \tag{5.2}$$

which naturally implies that the nominal value of the demand for households in both countries has to satisfy that,

$$nP_t^H C_t^H + nP_t^F C_t^F = nP_t C_t, \tag{5.3}$$

$$(1-n) P_t^{H*} C_t^{H*} + (1-n) P_t^{F*} C_t^{F*} = (1-n) P_t^* C_t^*, \tag{5.4}$$

while the aggregate income described in equations (3.5.9) – (3.5.10) is equal to,

$$nPPI_t^H Y_t = n\xi P_t C_t + (1-n) \xi^* S_t P_t^* C_t^*, \tag{5.5}$$

$$(1-n) PPI_t^{F*} Y_t^* = n(1-\xi) \frac{1}{S_t} P_t C_t + (1-n)(1-\xi^*) P_t^* C_t^*. \tag{5.6}$$

Hence, the aggregate nominal trade balance expressed in the same currency should be equal to,

$$nTB_t = nPPI_t^H Y_t - nP_t C_t, \tag{5.7}$$

$$-\frac{(1-n) TB_t^*}{S_t} = (1-n) PPI_t^{F*} Y_t^* - (1-n) P_t^* C_t^*, \tag{5.8}$$

where TB_t and TB_t^* are nominal quantities in per capita terms. Equation (3.5.29) clearly establishes a relationship between the nominal trade balance in both countries which is given by,

$$(nPPI_t Y_t - nP_t C_t) + S_t ((1-n) PPI_t^* Y_t^* - (1-n) P_t^* C_t^*) = 0, \tag{5.9}$$

or simply by,

$$nTB_t = (1-n) TB_t^*. \tag{5.10}$$

I can use the nominal trade balance as expressed in equations (5.7) – (5.8) and the aggregate income value derived in equations (5.5) − (5.6) to write that,

$$
\begin{aligned}
nP_tC_t + nTB_t &= nPPI_t^H Y_t = n\xi P_tC_t + (1-n)\,\xi^* S_t P_t^* C_t^*, && (5.11)\\
(1-n)\,P_t^*C_t^* - \frac{nTB_t}{S_t} &= (1-n)\,PPI_t^{F*} Y_t^* = n\,(1-\xi)\,\frac{1}{S_t}P_tC_t + (1-n)\,(1-\xi^*)\,P_t^*C_t^*. && (5.12)
\end{aligned}
$$

Taking the ratio of both equations,

$$
\frac{nP_tC_t + nTB_t}{(1-n)\,S_tP_t^*C_t^* - nTB_t} = \frac{n\xi P_tC_t + (1-n)\,\xi^* S_tP_t^*C_t^*}{n\,(1-\xi)\,P_tC_t + (1-n)\,(1-\xi^*)\,S_tP_t^*C_t^*}, \tag{5.13}
$$

and re-arranging terms appropriately, I derive the following expression,

$$
\begin{aligned}
nTB_t = &\left(\tfrac{n\xi P_tC_t + (1-n)\xi^* S_tP_t^*C_t^*}{nP_tC_t + (1-n)S_tP_t^*C_t^*}\right)(1-n)\,S_tP_t^*C_t^* - \\
&- \left(\tfrac{n(1-\xi)P_tC_t + (1-n)(1-\xi^*)S_tP_t^*C_t^*}{nP_tC_t + (1-n)S_tP_t^*C_t^*}\right) nP_tC_t.
\end{aligned} \tag{5.14}
$$

Let me assume the polar case where the *asset markets are complete*. Under complete asset markets, the perfect international risk-sharing condition given by (3.2.29) holds, so I can write the domestic nominal trade balance in the following terms,

$$
nTB_t = \left[\left(\frac{n\xi + (1-n)\,\xi^*\upsilon}{n + (1-n)\,\upsilon}\right)(1-n)\,\upsilon - \left(\frac{n\,(1-\xi) + (1-n)\,(1-\xi^*)\,\upsilon}{n + (1-n)\,\upsilon}\right) n\right] P_tC_t, \tag{5.15}
$$

where $\upsilon = 1$ after normalization of the steady state. Further algebra allows me to compute the following formula for the nominal trade balance,

$$
nTB_t = \left[(1-n)\,\xi^* - n\,(1-\xi)\right] P_tC_t. \tag{5.16}
$$

The way the model is written, it follows that the nominal trade balance is equal to zero if and only if,

$$
(1-n)\,\xi^* = n\,(1-\xi) \Longleftrightarrow \xi = 1 - \left(\frac{1-n}{n}\right)\xi^*. \tag{5.17}
$$

This condition is at the center of the irrelevance result.

I use two particular specifications of preferences here. One specification where $\xi = \xi^* = n$, which I call the symmetric preference. The other specification where $\xi = 1 - \left(\frac{1-n}{n}\right)\xi^*$, which I call the home bias in consumption preference. Notice that if the population size is identical in both countries, the home bias in consumption reduces to the well-known case where $\xi = 1 - \xi^*$. Both specifications of preferences satisfy the condition (5.17) by construction and, therefore, this guarantees that the nominal trade balance is equal to zero in equilibrium every period.

Let me assume the polar opposite case where the *asset markets are fully separated and no cross-country asset trading is permitted*. This case is also discussed in Woodford (2007). I do not take a stand on the number of assets available within each local market, so there may be less than a full set of Arrow-Debreu securities. If asset trading does not occur in the international markets, then the nominal trade balance must be equal to zero in every period. International borrowing and lending is simply impossible. Based on equation (5.14), I can argue that,

$$\begin{aligned}&\left(\tfrac{n\xi P_t C_t+(1-n)\xi^* S_t P_t^* C_t^*}{nP_t C_t+(1-n)S_t P_t^* C_t^*}\right)(1-n)\, S_t P_t^* C_t^* - \\ &-\left(\tfrac{n(1-\xi)P_t C_t+(1-n)(1-\xi^*)S_t P_t^* C_t^*}{nP_t C_t+(1-n)S_t P_t^* C_t^*}\right) nP_t C_t = 0,\end{aligned} \tag{5.18}$$

and naturally it follows that,

$$\left(n\xi C_t + (1-n)\,\xi^* RS_t C_t^*\right) RS_t\,(1-n)\,C_t^* = \left(n\,(1-\xi)\,C_t + (1-n)\,(1-\xi^*)\,RS_t C_t^*\right) nC_t. \tag{5.19}$$

Simple algebra tells me that the real exchange rate can be pin down as a second-order equation,

$$\xi^*\left((1-n)\,C_t^*\right)^2 (RS_t)^2 + \left[\xi - (1-\xi^*)\right]\left((1-n)\,C_t^*\right)(nC_t)\,RS_t - (1-\xi)\,(nC_t)^2 = 0. \tag{5.20}$$

If the well-known specification for home bias in consumption applies (i.e., $\xi = (1-\xi^*)$) and the population size is identical across countries, the real exchange rate must satisfy the following equation,

$$RS_t = \frac{C_t}{C_t^*}, \tag{5.21}$$

which corresponds exactly with the perfect risk-sharing condition in (3.2.29) whenever $\gamma = 1$. More generally, I observe that the solution to the second-order equation on the real exchange rate takes the following form,

$$RS_t = \left[\frac{-\left[\xi-(1-\xi^*)\right] + \sqrt[2]{\left[\xi-(1-\xi^*)\right]^2 + 4\xi^*\,(1-\xi)}}{2\xi^*}\right]\frac{nC_t}{(1-n)\,C_t^*},$$

where only the positive root gives a solution that is consistent with the real exchange rate being positive. I suppose that the preference parameters satisfy the condition in (5.17), then it follows that the real exchange rate is equal to,

$$RS_t = \left[\frac{-\left(\frac{2n-1}{n}\right) + \sqrt[2]{\left(\frac{2n-1}{n}\right)^2 - 4\left(\frac{1-n}{n}\right)}}{2}\right]\frac{nC_t}{(1-n)\,C_t^*} = \frac{C_t}{C_t^*}, \tag{5.22}$$

which again coincides with the real exchange rate that would prevail if the asset markets were complete and both countries were able to perfectly share their risks.

The point of these calculations is that an optimal allocation can be attained with no trade at all in assets. Trading in the goods market suffices to attain the optimal allocation. In fact, the allocation attained should be the same independently of whether I allow for unrestricted asset trading internationally and have a complete set of Arrow-Debreu securities to do so or I impose that international trade is infeasible and the local asset markets have only a limited number of assets to trade on. The same logic applies to other intermediate cases where international asset trading is possible, but asset markets are not complete (as in these notes). It is always possible to attain the perfect risk-sharing allocation without having to resort to the asset markets at all. Therefore, the availability of more or less assets should not make a difference.

Let me review the implicit and explicit assumptions of these exercise. First, I have explicitly required logarithmic (and additively separable) preferences on consumption, i.e. $\gamma = 1$. Second, I have required that the elasticity of substitution between the domestic and the foreign bundle of goods be equal to one in both countries (Cobb-Douglas aggregator), i.e. $\sigma = 1$. Third, I have to impose that the combination of the population size in each country and the shares of local consumption satisfy that $(1-n)\xi^* = n(1-\xi)$. Forth, implicitly I have made the assumption that the initial condition on the net foreign asset position of each country be equal to zero. Obviously, if I assume that one country starts as a net debtor (and the other as a net creditor), the trade balance can not be zero from that point onwards because the debtor country needs to either trade goods with the other country to repay its debts or simply needs to borrow more.

The irrelevance of the asset market structure is a very powerful an interesting result and it has been widely exploited to simplify the model in cases where asset trading or the nominal trade balance are deemed of secondary importance for the questions being asked. Of course, in this particular case, increased financial openness has no consequences whatsoever for the determination of the aggregate demand under any monetary policy rule. Woodford (2007, p. 8) argues that "the fact that complete irrelevance is possible (and does not even require an 'extreme' preference specification) indicates that the effects of financial globalization need not be large". This irrelevance result is a very special case, convenient as it may be, so I would caution from reading too much into Woodford's (2007) line of argument. Financial globalization is a very complex phenomenon that it is not well-understood yet. These notes try to shed some light on them. For the more advanced readers, I would suggest to start with the papers of Evans and Hnatkovska (2005, 2007).

Remark 6 *As relevant as portfolio allocations can be in order to address certain specific questions, like what explains the fact that some countries have very large capital account surpluses (for example, Spain and the U.S.) while others do not, the issue is of secondary importance in these notes. What matters is how the set of available assets affects the risk-sharing opportunities of households and their consumption-savings margin. And how those decisions are reflected on the asset prices. Beyond that, as I will discuss later, all that is needed to close the model is to specify the relevant pricing equations. In this sense, portfolio allocations could be derived endogenously as a 'residual' after having determined the optimal allocation of resources first. Following in the spirit of Evans and Hnatkovska (2005, 2007) that only requires a second-order approximation of the asset pricing equations and either the optimal allocation of resources or a number of conditions that would characterize it appropriately.*

6 The Deterministic Steady State[35]

6.1 The Zero-Inflation Steady State

I require that all shocks be evaluated at their unconditional mean in steady state. The steady state productivity shocks are denoted, A and A^*, while the monetary policy shocks are denoted, Z and Z^*. I also conjecture the existence of a (symmetric) deterministic steady state in which prices, consumption and the nominal exchange rate are constant, i.e.

$$P_{t+1} = P_t = \overline{P},\ P^*_{t+1} = P^*_t = \overline{P}^*, \quad (6.1.1)$$

$$C_{t+1} = C_t = \overline{C},\ C^*_{t+1} = C^*_t = \overline{C}^*, \quad (6.1.2)$$

$$S_{t+1} = S_t = \overline{S}. \quad (6.1.3)$$

[35] The initial conditions of the model at time $t = 0$ should naturally correspond to the steady state values.

I often refer to this steady state as the *zero-inflation steady state*.

Given my steady state conjecture and the Euler equations in (3.2.31) and (3.2.32), it follows that the steady state nominal interest rate in both countries is identical and equal to the inverse of the rate of the subjective intertemporal discount factor,

$$\beta = \frac{1}{1+\overline{i}} = \frac{1}{1+\overline{i}^*}, \tag{6.1.4}$$

which holds true for the case of incomplete asset markets. Under complete asset markets, I can also argue based on the intertemporal first-order conditions in (3.2.25) – (3.2.26), that the Arrow-Debreu security prices are,

$$\begin{aligned} \overline{Q}(\omega) &= \beta\overline{\mu}(\omega), \ \frac{1}{1+\overline{i}} = \int_{\omega\in\Omega} \overline{Q}(\omega) = \beta, & (6.1.5) \\ \overline{Q}^*(\omega) &= \beta\overline{\mu}(\omega), \ \frac{1}{1+\overline{i}^*} = \int_{\omega\in\Omega} \overline{Q}^*(\omega) = \beta, & (6.1.6) \end{aligned}$$

which is consistent with my findings on the steady state interest rate. The invariant distribution function is denoted $\overline{\mu}(\omega)$. The assumption that the nominal exchange rate is constant satisfies also the intertemporal first-order conditions in either (3.2.27) or (3.2.32).

From the firm's first-order conditions in (3.3.16) – (3.3.17) and (3.3.22) – (3.3.23), I obtain that the LOOP holds in steady state[36], i.e.

$$\begin{aligned} \overline{\widetilde{P}}(h) &= \overline{S}\overline{\widetilde{P}}^*(h) = \frac{\theta}{\theta-1}\left(\frac{\left(1+\overline{\phi}\right)\overline{W}}{A}\right), & (6.1.7) \\ \frac{1}{\overline{S}}\overline{\widetilde{P}}(f) &= \overline{\widetilde{P}}^*(f) = \frac{\theta}{\theta-1}\left(\frac{\left(1+\overline{\phi}^*\right)\overline{W}^*}{A^*}\right). & (6.1.8) \end{aligned}$$

The fiscal policy rule in this model is time-invariant, as described in (3.4.5). Furthermore, it cancels out the effect of the mark-up charged by firms and results in a competitive Dixit-Stiglitz price-setting rule for all varieties in steady state, i.e.

$$\begin{aligned} \overline{\widetilde{P}}(h) &= \overline{S}\overline{\widetilde{P}}^*(h) = \left(\frac{\overline{W}}{A}\right), & (6.1.9) \\ \frac{1}{\overline{S}}\overline{\widetilde{P}}(f) &= \overline{\widetilde{P}}^*(f) = \left(\frac{\overline{W}^*}{A^*}\right), & (6.1.10) \end{aligned}$$

where the price equals the marginal cost. In other words, the pricing behavior of these monopolistic competitors is observationally equivalent to the behavior of perfectly competitive firms. The steady state price

[36] The IMRS implied by the pricing functions generalizes the expressions in (3.2.33) – (3.2.34). In any event, it can be shown that in steady state $\overline{m}_{t,t+\tau} = \overline{m}^*_{t,t+\tau} = \beta^\tau$ for all $\tau \geq 1$.

sub-indexes in equations (3.3.18) – (3.3.19) and (3.3.24) – (3.3.25) simply become,

$$\overline{P}^{H} = \overline{\widetilde{P}}(h),\ \overline{P}^{H*} = \overline{\widetilde{P}}^{*}(h), \tag{6.1.11}$$
$$\overline{P}^{F} = \overline{\widetilde{P}}(f),\ \overline{P}^{F*} = \overline{\widetilde{P}}^{*}(f), \tag{6.1.12}$$

and, naturally,

$$\overline{P}^{H} = \overline{SP}^{H*},\ \overline{P}^{F} = \overline{SP}^{F*}.$$

The LOOP holds also at the level of the aggregate bundles of goods produced at home and abroad. Using the price indexes in equations (3.2.14) and (3.2.15) evaluated at their steady state, I also infer that the real exchange rate should be equal to,

$$\overline{P} = \left[\frac{\xi + (1-\xi)\left(\overline{ToT}\right)^{1-\sigma}}{\xi^{*} + (1-\xi^{*})\left(\overline{ToT}\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \overline{SP}^{*} \Rightarrow \overline{RS} \equiv \frac{\overline{SP}^{*}}{\overline{P}} = \left[\frac{\xi^{*} + (1-\xi^{*})\left(\overline{ToT}\right)^{1-\sigma}}{\xi + (1-\xi)\left(\overline{ToT}\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}},$$

where $\overline{ToT} \equiv \frac{\overline{P}^{F}}{\overline{SP}^{H*}} = \frac{\overline{P}^{F}}{\overline{P}^{H}}$ denotes the steady state terms of trade. Absolute PPP holds in steady state (and the real exchange rate equals to one) only if preferences on consumption are identical across countries, i.e. $\xi = \xi^{*}$, or the terms of trade are equal to one in steady state. Relative PPP holds in steady state, but not absolute PPP, whenever the preferences differ across countries due to home-bias in consumption and terms of trade are not equal to zero. In the workhorse model, the difference between the long-run price of a basket of goods in either country expressed in terms of the same currency is a function of the steady state value of the terms of trade.

These findings prove that, if a zero-inflation steady state exists as conjectured, the LOOP holds and the pricing decisions of firms are equivalent to those of (symmetric) competitive firms under flexible prices. The steady state is also observationally equivalent whether asset markets are complete or not, as I shall discuss shortly. In other words, the steady state for any variant of the workhorse model that I explore here (with nominal rigidities, LCP pricing and/or incomplete asset markets) is equivalent to the steady state of the frictionless allocation described in subsection 4. That explains the notational convention of describing all endogenous steady state variables by dropping the time subscript and marking them with an 'upper bar' (as in the frictionless model).

The Other Real Steady State Variables. The analytic results reported in subsection 4 allow me to argue that the steady state real exchange rate, i.e. $\overline{RS}$, the steady state terms of trade, i.e. $\overline{TOT}$, the steady state total consumption, i.e. $\overline{C}$ and $\overline{C}^{*}$, the steady state output, i.e. $\overline{Y}$ and $\overline{Y}^{*}$, and the steady state employment, i.e. $\overline{L}$ and $\overline{L}^{*}$, are all functions of the steady state productivity shocks in both countries (and the structural parameters of the model). Taking that as given, I can infer real wages from the first-order conditions on labor supply given by (3.2.38) – (3.2.39), or from the labor market clearing conditions discussed in (3.3.30) – (3.3.31), i.e.

$$\frac{\overline{W}}{\overline{P}} = \kappa (A)^{-\varphi} \left(\overline{C}(A, A^{*})\right)^{\gamma} \left(\overline{Y}(A, A^{*})\right)^{\varphi}, \tag{6.1.13}$$
$$\frac{\overline{W}^{*}}{\overline{P}^{*}} = \kappa (A^{*})^{-\varphi} \left(\overline{C}^{*}(A, A^{*})\right)^{\gamma} \left(\overline{Y}^{*}(A, A^{*})\right)^{\varphi}, \tag{6.1.14}$$

noting that in steady state both labor and output are functions of the steady state productivity shocks only.

The demand for each variety and for the entire bundle of goods in each country can be obtained from equations (3.2.21) – (3.2.24) as,

$$\overline{C}(h) = \frac{1}{n}\overline{C}^{H}(A,A^{*}),\ \overline{C}^{*}(h) = \frac{1}{n}\overline{C}^{H*}(A,A^{*}),\ \text{if } h \in [0,n], \tag{6.1.15}$$

$$\overline{C}(f) = \frac{1}{1-n}\overline{C}^{F}(A,A^{*}),\ \overline{C}^{*}(f) = \frac{1}{1-n}\overline{C}^{F*}(A,A^{*}),\ \text{if } f \in (n,1], \tag{6.1.16}$$

and

$$\begin{aligned} \overline{C}^{H} &= \xi\left(\tfrac{1}{\xi+(1-\xi)\left(\overline{TOT}(A,A^{*})\right)^{1-\sigma}}\right)^{\frac{-\sigma}{1-\sigma}}\overline{C}(A,A^{*}),\\ \overline{C}^{H*} &= \xi^{*}\left(\tfrac{1}{\xi^{*}+(1-\xi^{*})\left(\overline{ToT}(A,A^{*})\right)^{1-\sigma}}\right)^{\frac{-\sigma}{1-\sigma}}\overline{C}^{*}(A,A^{*}), \end{aligned} \tag{6.1.17}$$

$$\begin{aligned} \overline{C}^{F} &= (1-\xi)\left(\tfrac{1}{\xi\left(\overline{TOT}(A,A^{*})\right)^{\sigma-1}+(1-\xi)}\right)^{\frac{-\sigma}{1-\sigma}}\overline{C}(A,A^{*}),\\ \overline{C}^{F*} &= (1-\xi^{*})\left(\tfrac{1}{\xi^{*}\left(\overline{ToT}(A,A^{*})\right)^{\sigma-1}+(1-\xi^{*})}\right)^{\frac{-\sigma}{1-\sigma}}\overline{C}^{*}(A,A^{*}). \end{aligned} \tag{6.1.18}$$

The pricing for each variety and for the bundle of goods in each country comes from the optimal pricing equations in (3.3.14) – (3.3.15) and can be expressed relative to the CPI, by manipulating the indexes in (3.2.14) and (3.2.15), as follows,

$$\frac{\overline{P}^{H}}{\overline{P}} = \left[\frac{1}{\xi+(1-\xi)\left(\overline{TOT}(A,A^{*})\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}},\ \frac{\overline{P}^{F}}{\overline{P}} = \left[\frac{1}{\xi\left(\overline{TOT}(A,A^{*})\right)^{\sigma-1}+(1-\xi)}\right]^{\frac{1}{1-\sigma}}, \tag{6.1.19}$$

$$\frac{\overline{P}^{F*}}{\overline{P}^{*}} = \left[\frac{1}{\xi^{*}\left(\overline{ToT}(A,A^{*})\right)^{\sigma-1}+(1-\xi^{*})}\right]^{\frac{1}{1-\sigma}},\ \frac{\overline{P}^{H*}}{\overline{P}^{*}} = \left[\frac{1}{\xi^{*}+(1-\xi^{*})\left(\overline{ToT}(A,A^{*})\right)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \tag{6.1.20}$$

These prices ratios are, not surprisingly, purely determined by real variables even in steady state.

The Nominal Steady State Variables. I should point out that the zero administered rates on money, i.e. $\overline{i}^{m} = \overline{i}^{m*} = 0$, the Taylor rules in the spirit of equations (3.4.12) – (3.4.13), and the Euler equations in (3.2.30) – (3.2.31) are not sufficient to pin down the steady state price level. Although the system is consistent with the conjecture that the inflation should be zero in steady state. To be more specific, the Euler equations imply that,

$$\frac{1}{1+\overline{i}} = \frac{1}{1+\overline{i}^{*}} = \beta, \tag{6.1.21}$$

while the Taylor rules ensure that,

$$\frac{1+\overline{i}}{1+\overline{i}} = \frac{Z}{Z}\left(\frac{1+\overline{i}}{1+\overline{i}}\right)^{\rho}\left[\left(\overline{\Pi}\right)^{\psi_{1}}\left(\frac{\overline{Y}}{\overline{Y}}\right)^{\psi_{2}}\right]^{1-\rho} \Leftrightarrow \overline{\Pi} = 1, \tag{6.1.22}$$

$$\frac{1+\overline{i}^{*}}{1+\overline{i}^{*}} = \frac{Z^{*}}{Z^{*}}\left(\frac{1+\overline{i}^{*}}{1+\overline{i}^{*}}\right)^{\rho^{*}}\left[\left(\overline{\Pi}^{*}\right)^{\psi_{1}^{*}}\left(\frac{\overline{Y}^{*}}{\overline{Y}^{*}}\right)^{\psi_{2}^{*}}\right]^{1-\rho^{*}} \Leftrightarrow \overline{\Pi}^{*} = 1. \tag{6.1.23}$$

This means that by enforcing a Taylor rule the long-run inflation target of the monetary authority becomes the steady state level. By construction, therefore, price stability is guaranteed in the steady state. However, this is not sufficient to determine the price level in steady state. It only guarantees that the price level is unchanged in steady state. Under price stability, the Euler equations also imply that the steady state nominal and real interest rates[37] are determined by the rate of time preference, β.

The determination of the price level may seem of little practical value since the log-linearized version of the workhorse model that I explore in these notes depends on the inflation dynamics and not the price level itself. Relative prices do matter, however, but those are determined by real shocks in the steady state. However, as it turns out neither the nominal wages nor the nominal exchange rate (or any other nominal price) can be determined in steady state without knowing the price level first. Hence, I cannot simply ignore the issue.

The steady state version of the money market clearing conditions given by (3.4.8) – (3.4.9) characterizes the real balances in real terms. It can also be re-interpreted as a function for the steady state CPI level, i.e.

$$\overline{P} = \left(\frac{1-\beta}{\chi}\right)^{\frac{1}{\zeta}} \left(\overline{C}\left(A, A^*\right)\right)^{-\frac{\gamma}{\zeta}} \left[\left(1 - \frac{\beta}{1-\beta}\overline{i}^m\right)^{\frac{1}{\zeta}} \overline{M}\right], \tag{6.1.24}$$

$$\overline{P}^* = \left(\frac{1-\beta}{\chi}\right)^{\frac{1}{\zeta}} \left(\overline{C}^*\left(A, A^*\right)\right)^{-\frac{\gamma}{\zeta}} \left[\left(1 - \frac{\beta}{1-\beta}\overline{i}^{m*}\right)^{\frac{1}{\zeta}} \overline{M}^*\right], \tag{6.1.25}$$

which has been re-arranged to reflect that the monetary authority can alter the price level in steady state by either manipulating the steady state money supply or by changing the administered rate on money[38]. For simplicity, however, I make the assumption that the administered rates are always zero and, naturally, they are zero in steady state too. While in the short-run money supply is endogenously determined by the model after the monetary authority sets a Taylor interest rate rule, in the long-run (that is, in steady state) it is not. As a result, the monetary authority can set the long-run money supply and determine the price level in steady state.

Moreover, what this entails for monetary policy is that: (a) the short-term interest rate that the monetary authority tries to manipulate in response to short-term deviations of the inflation rate must be consistent with its long-run rate (which in the context of the model is completely independent of policy choice), and (b) the long-run inflation rate is zero, but the CPI level depends on the long-run money supply of the economy. Therefore, if monetary policy is going to determine the price level in the long-run it has to be the case that a second instrument (for instance, money supply) be used to determine the price level.

In other words, there is a case to be made as to why the monetary authority may care about the long-run trends of money supply. For instance, the ECB still cares about the long-run trends of the money aggregates.

[37] The steady state interest rates and the inflation are isolated from the steady state value of the shocks to the Taylor rule.

[38] If the money supply when the administered rate is zero is denoted as $\overline{M}^0$ and $\overline{M}^{0*}$, the money supply that implies the same price level whenever the administered rate is different than zero must satisfy that

$$\overline{M}^0 = \left(1 - \frac{\beta}{1-\beta}\overline{i}^m\right)^{\frac{1}{\zeta}} \overline{M}^{\overline{i}^m},$$

$$\overline{M}^{0*} = \left(1 - \frac{\beta}{1-\beta}\overline{i}^{m*}\right)^{\frac{1}{\zeta}} \overline{M}^{\overline{i}^{m*}}.$$

Imagine an economy that lies on a steady state. If money supply is increased because the monetary authority decides to allow more liquidity in the markets than its average trend it may create inflation and a transition towards a new steady state with higher prices. If money is not neutral, this may come at a cost. In any event, letting the long-run money supply loose introduces unnecessary noise in the system.

Remark 7 *Interestingly, the monetary authority controls the nominal interest rates in the short-run, but cannot control the long-run interest rates. In steady state, those depend on the time preference of households. Monetary policy commonly relies on a Taylor interest rule to deal with the inflation dynamics (and to some extent with the fluctuations over the business cycle) and this rule, in turn, endogenously defines the short-run path of the money supply. In the long-run, however, the Taylor rule does not constraint the price level or the money supply, and the steady state interest rates cannot be controlled by policy-makers. Hence, policy-makers could still exercise some control over the price level by regulating the money supply (but only over the long-run).*

Remark 8 After-thoughts on Monetary Policy and the Long-Run Money Supply. *From my previous discussion on the subject, it seems that the CPI is both insulated from the steady state shocks to the Taylor rule and indeterminate. The nominal money supply cannot be determined independently from the price level, therefore there are as many price levels as possible choices for the money supply. This is in part due to the fact that setting the administered rate on money to zero is realistic, but does not help me pin down the money supply. I resist, however, the idea that I need to introduce a new rule for the administered rate on money. It seems too 'convenient' given that in reality I do not observe such rules being implemented on a systematic basis. Therefore, something needs to be done regarding the money supply directly.*

As I have noted, the Taylor rule combined with the Euler equation and the consumption functions suffices to determine the inflation rate in a forward looking model. To be more precise, it should be possible to determine the price level if some initial conditions on the interest rates and the CPI are given. My conjecture is that the initial state should be equal to the steady state of the economy. I already find that the steady state interest rates are a function of the time preference rate, β, and therefore they are independent and out of the control for the monetary authority. In the context of this model, the monetary authority sets the short-term interest rate but cannot alter the long-term rates. The initial condition for the CPI level offers the monetary authority an opportunity to operate over the long-run. From the market clearing conditions on money, I know that a given initial condition on the price level is equivalent to a given initial condition on the money supply.

In a purely monetarist fashion, I assume that the monetary authority sets a regime for the long-run money supply. In other words, the monetary authority sets an initial condition for the money supply. Then, the economy may get hit by monetary shocks (to the Taylor rule) as well as by real shocks. In the long-run, the model should converge towards a steady state in which the price level is determined by the initial condition on money. The monetary authority is responsible for setting the money supply regime and changes in it would necessarily cause inflation and a subsequent re-adjustment. Remember that since the administered rate on money is set to zero, the money supply is endogenously determined in the short-run. Money supply is not also a control variable in the short-run, but the choice of the initial conditions is necessary to pin down the price level and influences the steady state that the economy reaches.

My derivations suggest that the steady state outcome is consistent with the notion of long-run neutrality of money. Monetary variables have only effects on the price level (and other nominal variables), but not *on real output or employment. However, in the short-run the presence of nominal rigidities (as well as other frictions) in the workhorse model means that monetary variables can have real effects that have to be accounted for.*

The Steady State of the External Sector. Using the definition of the real exchange rate in (3.2.19), which is a function of the steady state values of the productivity shock in both countries (as implied in subsection 4), it is possible to write the nominal exchange rate as the product of this function and the ratio of the CPIs in both countries, i.e.

$$\overline{S} = \overline{RS}\left(A, A^{*}\right)\frac{\overline{P}}{\overline{P}^{*}}, \tag{6.1.26}$$

which is consistent with the intertemporal first-order condition of the households' problem in either (3.2.27) or (3.2.32). I can also use the domestic current account equation in (3.5.31) for the complete asset markets case and equation (3.5.33) for the incomplete asset markets case in order to pin down the real amounts of borrowing and lending, i.e.

$$\int_{\omega\in\Omega}\overline{Q}\left(\omega\right)\overline{B}\left(\omega\right) = \overline{B}\left(\omega\right)+\overline{P}\left[\frac{\xi}{n}\left(\frac{\overline{P}^{H}}{\overline{P}}\right)^{1-\sigma}n\overline{C}+\frac{\xi^{*}}{n}\left(\frac{\overline{P}^{H*}}{\overline{P}^{*}}\right)^{1-\sigma}\overline{RS}\left(1-n\right)\overline{C}^{*}-\overline{C}\right], \tag{6.1.27}$$

$$\frac{1}{1+\overline{i}}\overline{B} = \overline{B}+\overline{P}\left[\frac{\xi}{n}\left(\frac{\overline{P}^{H}}{\overline{P}}\right)^{1-\sigma}n\overline{C}+\frac{\xi^{*}}{n}\left(\frac{\overline{P}^{H*}}{\overline{P}^{*}}\right)^{1-\sigma}\overline{RS}\left(1-n\right)\overline{C}^{*}-\overline{C}\right], \tag{6.1.28}$$

where I define the net borrowing as either $\overline{B}\left(\omega\right)\equiv\overline{B}^{H}\left(\omega\right)+\overline{S}\left(\omega\right)\overline{B}^{F}\left(\omega\right)$ or $\overline{B}\equiv\overline{B}^{H}+\overline{SB}^{F}$. These steady state equations can be re-written as follows,

$$\begin{aligned}\beta\left[\int_{\omega\in\Omega}\overline{\mu}\left(\omega\right)\frac{\overline{B}(\omega)}{\overline{P}}\right]-\frac{\overline{B}(\omega)}{\overline{P}} &= \frac{\xi}{n}\left(\frac{1}{\xi+(1-\xi)\left(\overline{TOT}(A,A^{*})\right)^{1-\sigma}}\right)n\overline{C}\left(A,A^{*}\right)+\\ &+\frac{\xi^{*}}{n}\left(\frac{1}{\xi^{*}+(1-\xi^{*})\left(\overline{ToT}(A,A^{*})\right)^{1-\sigma}}\right)\overline{RS}\left(A,A^{*}\right)\left(1-n\right)\overline{C}^{*}\left(A,A^{*}\right)-\overline{C}\left(A,A^{*}\right),\end{aligned} \tag{6.1.29}$$

$$\begin{aligned}\left(\beta-1\right)\frac{\overline{B}}{\overline{P}} &= \frac{\xi}{n}\left(\frac{1}{\xi+(1-\xi)\left(\overline{TOT}(A,A^{*})\right)^{1-\sigma}}\right)n\overline{C}\left(A,A^{*}\right)+\\ &+\frac{\xi^{*}}{n}\left(\frac{1}{\xi^{*}+(1-\xi^{*})\left(\overline{ToT}(A,A^{*})\right)^{1-\sigma}}\right)\overline{RS}\left(A,A^{*}\right)\left(1-n\right)\overline{C}^{*}\left(A,A^{*}\right)-\overline{C}\left(A,A^{*}\right),\end{aligned} \tag{6.1.30}$$

using the already known results that link steady state consumption, terms of trade and the real exchange rate to the steady state level of the productivity shocks in both countries.

There are a few comments that deserve to be noted explicitly here:

First, I have considered all along that the deterministic steady state should correspond to that of the frictionless allocation independently of whether the model in its full complexity has nominal rigidities and/or incomplete international asset markets. Naturally, if a deterministic steady state does exist, whether the workhorse model has multiple Arrow-Debreu securities traded internationally or just one bond is completely irrelevant. As can be seen from the current account equation: whatever the optimal trade balance is for the steady state frictionless economy, it can be implemented in steady for an economy where only one asset is available. Because the steady state is deterministic there is no uncertainty and, therefore, having more assets is irrelevant. In this steady state, the outcome should be observationally equivalent to that under complete asset markets[39].

Second, I argue that the domestic current account equation is all that is needed to describe the resource allocation across countries thanks to Walras' Law. However, the portfolio allocation is indeterminate in this setting. Since in steady state all Arrow-Debreu securities look the same, it seems plausible to assume that $\frac{\overline{B}(\omega)}{\overline{P}} = \frac{\overline{B}}{\overline{P}}$. It seems also natural to think that if markets are incomplete with only two bonds, these two bonds are indistinguishable in steady state (after being expressed in the same currency). Therefore, one possible steady state allocation would have both countries trading in just one bond and $\overline{\alpha} \equiv \frac{\overline{SB}^F}{\overline{B}}$ taking either the value of 0 or the value of 1. However, any value in between could be a portfolio steady state too. But it is not necessary to take a stand about that in these notes because we are concerned with the resource allocation, not the portfolio allocation. And, as I discuss in these notes, portfolio allocations are not needed to pin down the resource allocations of the workhorse model.

Third, independent of the portfolio structure, the only thing that really matters in steady state is the net borrowing that can be attained in real terms. Alternatively, I could say that the only fixture of importance is the domestic trade balance which can be computed as,

$$\begin{aligned} \frac{\overline{TB}}{\overline{P}} = \frac{\xi}{n}\left(\frac{1}{\xi+(1-\xi)\left(\overline{TOT}(A,A^*)\right)^{1-\sigma}}\right) n\overline{C}\left(A,A^*\right) + \\ + \frac{\xi^*}{n}\left(\frac{1}{\xi^*+(1-\xi^*)\left(\overline{ToT}(A,A^*)\right)^{1-\sigma}}\right)\overline{RS}\left(A,A^*\right)(1-n)\overline{C}^*\left(A,A^*\right) - \overline{C}\left(A,A^*\right). \end{aligned} \tag{6.1.31}$$

Not surprisingly, the real trade balance is a function of the steady state productivity shock levels in both countries. I can determine the steady state of real wages, real balances and the real trade balance (or the real net borrowing). However, obtaining the steady state for nominal wages, the nominal trade balance, the nominal exchange rate or even the price of domestic and foreign goods requires that I pin down the steady state level of the CPI in both countries. Hence, the relevance of my previous discussion on the subject.

6.2 The Normalization of the Steady State[40]

I shall impose a number of normalizations that make the linearization exercise much easier. It is worthwhile to be explicit about these choices, because they are very common in the literature but not always trivial. First, I assume that the long-run level for the monetary shocks to the Taylor rule is equal to one. In other words, without loss of generality $Z = Z^* = 1$. I could assume that the long-run money supply (i.e., the initial condition on the money supply regime) needed to pin down the price level is also equal to one, i.e. $\overline{M} = \overline{M}^* = 1$. On the other hand, the steady state of the workhorse model corresponds to the steady state of the frictionless model. In that scenario, money is neutral and it seems reasonable to assume that the consumption good plays the role of numeraire (as it is often done in the RBC literature), i.e. $\overline{P} = \overline{P}^* = 1$. For this normalization to hold, the money supply regime implemented by the monetary authority must

[39] I have shown that in steady state asset market incompleteness and nominal rigidities coupled with LCP pricing don't matter at all. The steady state of any model that combines either one of these frictions in the goods market and the assets market is the same as the steady state that can be found for the case of complete asset markets, flexible prices and competitive firms. Therefore, the steady state follows directly from the results that I already discussed in subsection 4.

[40] Under this normalization of the steady state, it can be verified that $\upsilon \equiv \frac{S_0 P_0^*}{P_0}\left(\frac{C_0^*}{C_0}\right)^{\gamma} = 1$.

satisfy that,

$$\overline{M} = \left(\frac{\chi}{1-\beta}\right)^{\frac{1}{\varsigma}} \left(\overline{C}\right)^{\frac{\gamma}{\varsigma}}, \tag{6.2.1}$$

$$\overline{M}^{*} = \left(\frac{\chi}{1-\beta}\right)^{\frac{1}{\varsigma}} \left(\overline{C}^{*}\right)^{\frac{\gamma}{\varsigma}}, \tag{6.2.2}$$

while assuming that $\overline{i}^{m} = \overline{i}^{m*} = 0$. This comes from the money market clearing conditions. I adopt this particular normalization, which seems more natural to me, in these notes.

Second, I make the assumption that the steady state productivity shocks are chosen to ensure that terms of trade in steady state are equal to one, i.e. $\overline{ToT} = 1$. I know from (4.6) and (4.7) that, if the steady state terms of trade are equal to one, then the real exchange rate is also equal to one,

$$\overline{RS} = 1. \tag{6.2.3}$$

Furthermore, the ratio of aggregate output in both countries is given by,

$$\frac{n\overline{Y}}{(1-n)\,\overline{Y}^{*}} = \left[\frac{n\left(\frac{\xi}{n}\right) + (1-n)\left(\frac{\xi^{*}}{n}\right)}{n\left(\frac{1-\xi}{1-n}\right) + (1-n)\left(\frac{1-\xi^{*}}{1-n}\right)}\right], \tag{6.2.4}$$

which implies that it is a constant fraction. Using this information about the structure of the economy and the aggregate output equations in (3.3.36) – (3.3.37), I can re-write the steady state consumption in terms of aggregate output (appropriately using the perfect risk-sharing condition in (3.2.29)) as,

$$n\overline{Y} = \left[n\left(\frac{\xi}{n}\right) + (1-n)\left(\frac{\xi^{*}}{n}\right)\right]\overline{C}, \tag{6.2.5}$$

$$(1-n)\,\overline{Y}^{*} = \left[n\left(\frac{1-\xi}{1-n}\right) + (1-n)\left(\frac{1-\xi^{*}}{1-n}\right)\right]\overline{C}^{*}. \tag{6.2.6}$$

Naturally, it follows that these relationships are consistent with the ratio described above only if consumption is perfectly shared across countries,

$$\overline{C} = \overline{C}^{*}. \tag{6.2.7}$$

Given that the real exchange rate is equal to one, this result also follows directly from the perfect risk-sharing condition in (3.2.29). The money supply used to normalize the price level to one in steady state is analogously equalized, i.e. $\overline{M} = \overline{M}^{*}$.

The expressions above show that aggregate consumption in each country is proportional to the local aggregate output. From the equilibrium labor market clearing conditions in (4.3) – (4.4), I immediately derive a pair of steady state conditions that link output in both countries to productivity shocks under this normalization, i.e.

$$\kappa\,(A)^{-\varphi-1}\left(\overline{C}\right)^{\gamma}\left(\overline{Y}\right)^{\varphi} = 1 \Leftrightarrow \kappa\,(A)^{-\varphi-1}\left(n\overline{Y}\right)^{\gamma}\left(\overline{Y}\right)^{\varphi} = \left(n\left(\frac{\xi}{n}\right) + (1-n)\left(\frac{\xi^{*}}{n}\right)\right)^{\gamma}, \tag{6.2.8}$$

$$\kappa\,(A^{*})^{-\varphi-1}\left(\overline{C}^{*}\right)^{\gamma}\left(\overline{Y}^{*}\right)^{\varphi} = 1 \Leftrightarrow \kappa\,(A^{*})^{-\varphi-1}\left((1-n)\,\overline{Y}^{*}\right)^{\gamma}\left(\overline{Y}^{*}\right)^{\varphi} = \left(n\left(\frac{1-\xi}{1-n}\right) + (1-n)\left(\frac{1-\xi^{*}}{1-n}\right)\right)^{\gamma} \tag{6.2.9}$$

This is a system of two equations in two unknowns that can be resolved. If I re-write everything appropriately, I obtain that,

$$n\overline{Y} = \left[\frac{n^{\varphi}}{\kappa}\left(n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^{*}}{n}\right)\right)^{\gamma}\right]^{\frac{1}{\gamma+\varphi}}(A)^{\frac{1+\varphi}{\gamma+\varphi}}, \tag{6.2.10}$$

$$(1-n)\overline{Y}^{*} = \left[\frac{(1-n)^{\varphi}}{\kappa}\left(n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^{*}}{1-n}\right)\right)^{\gamma}\right]^{\frac{1}{\gamma+\varphi}}(A^{*})^{\frac{1+\varphi}{\gamma+\varphi}}. \tag{6.2.11}$$

If I take the ratio of both expressions and re-write everything according to equation (6.2.4), it should follow that,

$$\frac{n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^{*}}{n}\right)}{n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^{*}}{1-n}\right)} = \frac{n\overline{Y}}{(1-n)\overline{Y}^{*}} = \frac{n^{\frac{\varphi}{\gamma+\varphi}}\left(n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^{*}}{n}\right)\right)^{\frac{\gamma}{\gamma+\varphi}}(A)^{\frac{1+\varphi}{\gamma+\varphi}}}{(1-n)^{\frac{\varphi}{\gamma+\varphi}}\left(n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^{*}}{1-n}\right)\right)^{\frac{\gamma}{\gamma+\varphi}}(A^{*})^{\frac{1+\varphi}{\gamma+\varphi}}}, \tag{6.2.12}$$

and this implies that the ratio of the steady state productivity shocks is,

$$\frac{A}{A^{*}} = \left(\frac{1-n}{n}\right)^{\frac{\varphi}{1+\varphi}}\left[\frac{n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^{*}}{n}\right)}{n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^{*}}{1-n}\right)}\right]^{\frac{\varphi}{1+\varphi}}. \tag{6.2.13}$$

This is the most critical condition on the normalization of the steady state productivity shocks because it is needed to guarantee that the terms of trade and the real exchange rate are both equal to one.

This normalization is very convenient to determine the basic log-linearized set of equilibrium conditions that has become so popular in the international macro literature. However, this normalization is not innocuous because it constraints the parameters of the model and may force us to rely on a fundamental difference between the two countries. In other words, it is not always possible to assume that the steady state productivity level in both countries is identical. So even if I normalize the steady state productivity in the foreign country to be equal to one, I cannot make the same normalization for the domestic country. Let me suppose that preferences are identical across countries, i.e. $\xi = \xi^{*}$, then the restriction in (6.2.13) still implies that,

$$\frac{A}{A^{*}} = \left(\frac{1-n}{n}\right)^{\frac{2\varphi}{1+\varphi}}\left[\frac{\xi}{1-\xi}\right]^{\frac{\varphi}{1+\varphi}}. \tag{6.2.14}$$

This means that identical preferences are not sufficient to restore the conventional view that the productivity shocks in each country are identical in steady state. Instead, I would need also that,

$$\xi = \frac{n^{2}}{(1-n)^{2}+n^{2}} = \frac{n^{2}}{1+2n^{2}-2n}. \tag{6.2.15}$$

The assumption that $\xi = \xi^{*} = n = \frac{1}{2}$ is consistent with this restriction.

Instead, let me consider the possibility that the following restriction applies on the parameters of consumption bias, i.e.

$$(1-n)\,\xi^{*} = n\,(1-\xi).$$

This is the same condition derived for the irrelevance of asset markets. Then, it is possible to re-write the normalization condition on the steady state productivity levels as,

$$\frac{A}{A^*} = \left(\frac{n}{1-n}\right)^{-\frac{\varphi}{1+\varphi}}. \tag{6.2.16}$$

This equation indicates that under a particular specification of the home bias parameters it should follow that the differences in the productivity levels in steady state are inversely related to the relative population size of both countries. This condition is consistent with equal productivity levels, i.e. $A = A^* = 1$, if I assume that either $n = \frac{1}{2}$ (equal population sizes) or $\varphi = 0$ (linear labor disutility).

Other Implications of the Normalization. Assuming this normalization of the productivity shocks turns out to be very convenient. In the standard normalization that I propose in these notes the price level is equalized across countries by construction. If I explore now the alternative where $\overline{M} = \overline{M}^* = 1$, then by looking at the money market clearing conditions in (3.4.8) – (3.4.9) I conclude that,

$$1 = \overline{M} = \left(\frac{1}{\chi}\frac{\overline{i}}{1+\overline{i}}\right)^{-\frac{1}{\zeta}} \overline{P}\left(\overline{C}\right)^{\frac{\gamma}{\zeta}} = \left(\frac{1-\beta}{\chi}\right)^{-\frac{1}{\zeta}} \overline{P}\left(\overline{C}\right)^{\frac{\gamma}{\zeta}}, \tag{6.2.17}$$

$$1 = \overline{M}^* = \left(\frac{1}{\chi}\frac{\overline{i}}{1+\overline{i}}\right)^{-\frac{1}{\zeta}} \overline{P}^*\left(\overline{C}^*\right)^{\frac{\gamma}{\zeta}} = \left(\frac{1-\beta}{\chi}\right)^{-\frac{1}{\zeta}} \overline{P}^*\left(\overline{C}^*\right)^{\frac{\gamma}{\zeta}}. \tag{6.2.18}$$

Given that $\overline{C} = \overline{C}^*$, it follows that,

$$\frac{\overline{P}}{\overline{P}^*} = \frac{\overline{M}}{\overline{M}^*} = 1. \tag{6.2.19}$$

In a very intuitive fashion, this shows that the long-run differences in price levels across countries can only be due to long-run differences in the money supply regime under which each country's monetary authority is operating. Given this simplification of the initial condition on money supply I know that the CPI price level must be equalized across countries, i.e. $\overline{P} = \overline{P}^*$.

Since the real exchange rate is equal to one given the normalization of the steady state productivity shocks that I explore here, then, it must be the case that the nominal exchange rate is equal to one too, i.e.

$$\overline{S} = \frac{\overline{P}}{\overline{P}^*} = \frac{\overline{M}}{\overline{M}^*} = 1.$$

In principle the nominal exchange rate reflects the different steady state values (or the initial conditions) of the money supply. It could be said that the nominal exchange rate is a purely monetary phenomenon in the long-run, while this is not the case in the short-run if PPP fails. However, it must be noted that this is true under the particular normalization of the productivity shocks that I discussed before[41]. In any event, it is obvious that if the monetary authority switches towards a different regime with a different long-run level of money supply, the long-run CPI indexes as well as the nominal exchange rate will reflect the change.

Given the optimal Dixit-Stiglitz pricing rule in steady state in equations (6.1.9) – (6.1.12), the labor market clearing conditions in steady state in equations (6.1.13) – (6.1.14), and the fact that $\overline{P} = \overline{P}^H$ and

[41] Another normalization in which the real exchange rate is different than one and the consumption levels do not equalize across countries would result in a long-run nominal exchange rate that depends on steady state money supply but also on steady state productivity levels.

$\overline{P}^{*} = \overline{P}^{F*}$ whenever $\overline{TOT} = 1$, I obtain that,

$$1 = \frac{\overline{P}^{H}}{\overline{P}} = \frac{\overline{W}}{\overline{P}A} = \kappa \left(A\right)^{-\varphi-1} \left(\overline{C}\right)^{\gamma} \left(\overline{Y}\right)^{\varphi}, \tag{6.2.20}$$

$$1 = \frac{\overline{P}^{F*}}{\overline{P}^{*}} = \frac{\overline{W}^{*}}{\overline{P}^{*}A^{*}} = \kappa \left(A^{*}\right)^{-\varphi-1} \left(\overline{C}^{*}\right)^{\gamma} \left(\overline{Y}^{*}\right)^{\varphi}. \tag{6.2.21}$$

Using equations (6.2.10) – (6.2.11) to pin down the steady state levels of output, it should follow that aggregate consumption is equal to,

$$\overline{C} = \left(\frac{1}{\kappa}\right)^{\frac{1}{\gamma}} \left(\overline{Y}\right)^{-\frac{\varphi}{\gamma}} \left(A\right)^{\frac{1+\varphi}{\gamma}} = \left(\frac{1}{\kappa}\right)^{\frac{1}{\gamma+\varphi}} \left(\frac{1}{n}\right)^{-\frac{\varphi}{\gamma+\varphi}} \left(n\left(\frac{\xi}{n}\right) + (1-n)\left(\frac{\xi^{*}}{n}\right)\right)^{-\frac{\varphi}{\gamma+\varphi}} \left(A\right)^{\frac{1+\varphi}{\gamma+\varphi}}, \tag{6.2.22}$$

$$\overline{C}^{*} = \left(\frac{1}{\kappa}\right)^{\frac{1}{\gamma}} \left(\overline{Y}^{*}\right)^{-\frac{\varphi}{\gamma}} \left(A^{*}\right)^{\frac{1+\varphi}{\gamma}} = \left(\frac{1}{\kappa}\right)^{\frac{1}{\gamma+\varphi}} \left(\frac{1}{1-n}\right)^{-\frac{\varphi}{\gamma+\varphi}} \left(n\left(\frac{1-\xi}{1-n}\right) + (1-n)\left(\frac{1-\xi^{*}}{1-n}\right)\right)^{-\frac{\varphi}{\gamma+\varphi}} \left(A^{*}\right)^{\frac{1+\varphi}{\gamma+\varphi}}. \tag{6.2.23}$$

At any rate, consumption must be identical in both countries given the conditions I impose to normalize the steady state productivity levels. Considering that $\overline{TOT} = 1$, the demand of each bundle of goods given by (6.1.17) – (6.1.16) is simply,

$$\overline{C}(h) = \frac{1}{n}\overline{C}^{H},\ \overline{C}^{*}(h) = \frac{1}{n}\overline{C}^{H*},\ \text{if } h \in [0, n], \tag{6.2.24}$$

$$\overline{C}(f) = \frac{1}{1-n}\overline{C}^{F},\ \overline{C}^{*}(f) = \frac{1}{1-n}\overline{C}^{F*},\ \text{if } f \in (n, 1], \tag{6.2.25}$$

$$\overline{C}^{H} = \xi\overline{C},\ \overline{C}^{H*} = \xi^{*}\overline{C}^{*},\ \overline{C}^{F} = (1-\xi)\,\overline{C}^{*},\ \overline{C}^{F*} = (1-\xi^{*})\,\overline{C}^{*}, \tag{6.2.26}$$

which is the same allocation that would prevail if I assume that the aggregator for the domestic and foreign bundle of goods is of the Cobb-Douglas type. Given the money market clearing conditions, I know that a transformation of the consumption expenditure of each country must satisfy that,

$$\overline{P}\left(\overline{C}\right)^{\frac{\gamma}{\zeta}} = \left(\frac{1-\beta}{\chi}\right)^{\frac{1}{\zeta}} \overline{M}, \tag{6.2.27}$$

$$\overline{P}^{*}\left(\overline{C}^{*}\right)^{\frac{\gamma}{\zeta}} = \left(\frac{1-\beta}{\chi}\right)^{\frac{1}{\zeta}} \overline{M}^{*}, \tag{6.2.28}$$

which applies generically for any normalization of the money supply. Therefore, it should follow that the steady CPI levels are,

$$\overline{\widetilde{P}}(h) = \overline{P}^{H} = \overline{P} = \left(\frac{1-\beta}{\chi}\right)^{\frac{1}{\zeta}} \left(\frac{1}{\kappa}\right)^{-\frac{\gamma}{\zeta}\frac{1}{\gamma+\varphi}} \left(\frac{1}{n}\right)^{\frac{\gamma}{\zeta}\frac{\varphi}{\gamma+\varphi}} \left(n\left(\frac{\xi}{n}\right) + (1-n)\left(\frac{\xi^{*}}{n}\right)\right)^{\frac{\gamma}{\zeta}\frac{\varphi}{\gamma+\varphi}} \left(A\right)^{-\frac{\gamma}{\zeta}\frac{1+\varphi}{\gamma+\varphi}} \overline{M}, \tag{6.2.29}$$

$$\overline{\widetilde{P}}^{*}(f) = \overline{P}^{F*} = \overline{P}^{*} = \left(\frac{1-\beta}{\chi}\right)^{\frac{1}{\zeta}} \left(\frac{1}{\kappa}\right)^{-\frac{\gamma}{\zeta}\frac{1}{\gamma+\varphi}} \left(\frac{1}{1-n}\right)^{\frac{\gamma}{\zeta}\frac{\varphi}{\gamma+\varphi}} \left(n\left(\frac{1-\xi}{1-n}\right) + (1-n)\left(\frac{1-\xi^{*}}{1-n}\right)\right)^{\frac{\gamma}{\zeta}\frac{\varphi}{\gamma+\varphi}} \left(A^{*}\right)^{-\frac{\gamma}{\zeta}\frac{1+\varphi}{\gamma+\varphi}} \overline{M}^{*}, \tag{6.2.30}$$

which are equal to one given the normalization I have introduced for the initial condition on money supply and the restrictions on the steady state productivity in each country.

Finally, I can take another look at the real trade balance per capita given in (6.1.31) and conclude that,

$$\frac{\overline{TB}}{\overline{P}} = \left[\xi^* (1-n) - (1-\xi)\, n\right] \frac{1}{n} \overline{C},$$

which is equal to zero in the special case in which preferences are identical across countries and $\xi = \xi^* = n$ or, more generally, whenever,

$$(1-n)\,\xi^* = n\,(1-\xi) \Longleftrightarrow \xi = 1 - \left(\frac{1-n}{n}\right)\xi^*.$$

This is exactly the same condition that I derived for the irrelevance of the asset markets. This means that the normalization proposed ensures that international trading of assets is irrelevant in steady state without requiring that the intertemporal elasticity of substitution, $\frac{1}{\gamma}$, and the elasticity of substitution across domestic and foreign bundles, σ, be equal to one. In other words, this goes beyond what I discussed initially because if the trade balance is not equal to zero, then at least one internationally-traded asset is still required in equilibrium or the solution would be restricted to satisfy autarky. On the contrary, this normalization makes the nominal and real trade balance equal to zero, so the allocation (in steady state at least) is not distorted by the asset availability for international trading or lack thereof.

The portfolio allocation follows from the structure of the asset markets and the steady state real trade balance derived above,

$$\beta \left[\int_{\omega\in\Omega} \overline{\mu}(\omega)\, \frac{\overline{B}(\omega)}{\overline{P}}\right] - \frac{\overline{B}(\omega)}{\overline{P}} = \frac{\overline{TB}}{\overline{P}}, \tag{6.2.31}$$

$$(\beta - 1)\,\frac{\overline{B}}{\overline{P}} = \frac{\overline{TB}}{\overline{P}}. \tag{6.2.32}$$

The portfolio allocation in steady state remains indeterminate. I could argue that without uncertainty, the amount invested in different Arrow-Debreu security claims should be equal, i.e. $\overline{B}(\omega) = \overline{B}$. However, in this normalization, asset markets turn out to be irrelevant, so it naturally follows that the net borrowing and lending in each available asset is simply equal to zero, i.e. $\overline{B}(\omega) = \overline{B} = 0$.

7 The Linearized Equilibrium Conditions

In order to explore the first-order effects of the different shocks on the dynamics of the economy, the literature has traditionally relied on log-linearizations. Here I do the same, I log-linearize the equilibrium conditions around the deterministic zero-inflation steady state (see also King, Plosser and Rebelo, 1988). I approximate all variables in logs, and I denote $\widehat{x}_t \equiv \ln X_t - \ln \overline{X}$ the deviation of a variable in logs from its steady state. For more details on the technique of log-linearization applied to an international macro model, see Martínez-García (2007) and its corresponding companion technical appendix.

From this point on, I shall assume that fiscal policy is always optimal, i.e. $\phi_t = \phi_t^* = \frac{-1}{\theta}$. I take as given that fiscal policy entirely eliminates the mark-up distortion, but do not discuss the subject any further. However, I should point out that the workhorse model is very simple in its structure and, therefore, has time-invariant mark-ups as well as fiscal policy. In this case, neither the mark-up nor the fiscal policy terms have any impact on the short-run dynamics. The mark-up and the fiscal policy only enter in the specification of the long-run steady state.

7.1 Some Preliminaries

Let me start by describing the log-linearization of the domestic and foreign CPI indexes defined in (3.2.13) and (3.2.15). Given the normalized steady state, it must hold true that $\overline{P}^H = \overline{P}^F$ and $\overline{P}^{H*} = \overline{P}^{F*}$. Hence, taking a first-order approximation of the indexes I derive that,

$$\widehat{p}_t \approx \xi \widehat{p}_t^H + (1-\xi)\widehat{p}_t^F, \tag{7.1.1}$$

$$\widehat{p}_t^* \approx \xi^* \widehat{p}_t^{H*} + (1-\xi^*)\widehat{p}_t^{F*}. \tag{7.1.2}$$

I define the domestic and foreign inflation rates in deviations as $\widehat{\pi}_t \equiv \widehat{p}_t - \widehat{p}_{t-1}$ and $\widehat{\pi}_t^* \equiv \widehat{p}_t^* - \widehat{p}_{t-1}^*$, respectively.

I define the real exchange rate as $RS_t \equiv \frac{S_t P_t^*}{P_t}$ (as in (3.2.19)) and describe the relative prices in the home and foreign country respectively as $RP_t \equiv \frac{P_t^F}{P_t^H}$ and $(RP_t^*)^{-1} \equiv \frac{P_t^{F*}}{P_t^{H*}}$ (like in (3.2.20)). Therefore, it immediately follows that the RS equation is equal to,

$$\begin{aligned}\widehat{rs}_t &= \widehat{s}_t + \widehat{p}_t^* - \widehat{p}_t \\ &\approx \widehat{s}_t + \left(\xi^*\widehat{p}_t^{H*} + (1-\xi^*)\widehat{p}_t^{F*}\right) - \left(\xi\widehat{p}_t^H + (1-\xi)\widehat{p}_t^F\right).\end{aligned} \tag{7.1.3}$$

And, the RP equations take the form of,

$$\widehat{rp}_t = \widehat{p}_t^F - \widehat{p}_t^H, \tag{7.1.4}$$

$$\widehat{rp}_t^* = -\left(\widehat{p}_t^{F*} - \widehat{p}_t^{H*}\right). \tag{7.1.5}$$

I denote world relative prices, $\widehat{rp}_t^W$, and the discrepancy on relative prices across countries, $\widehat{rp}_t^R$, as follows,

$$\widehat{rp}_t^W \equiv \xi\widehat{rp}_t - (1-\xi^*)\widehat{rp}_t^*, \tag{7.1.6}$$

$$\widehat{rp}_t^R \equiv \widehat{rp}_t + \widehat{rp}_t^*. \tag{7.1.7}$$

Hence, the RP equations can also be expressed in terms of $\widehat{rp}_t^W$ and $\widehat{rp}_t^R$ as,

$$\widehat{rp}_t = \frac{1}{\xi-\xi^*+1}\widehat{rp}_t^W + \frac{1-\xi^*}{\xi-\xi^*+1}\widehat{rp}_t^R, \tag{7.1.8}$$

$$\widehat{rp}_t^* = -\frac{1}{\xi-\xi^*+1}\widehat{rp}_t^W + \frac{\xi}{\xi-\xi^*+1}\widehat{rp}_t^R. \tag{7.1.9}$$

I define domestic and foreign terms of trade respectively as $ToT_t \equiv \frac{P_t^H}{S_t P_t^{H*}}\frac{P_t^F}{P_t^H}$ and $ToT_t^* \equiv \frac{1}{ToT_t} = \frac{S_t P_t^{F*}}{P_t^F}\frac{P_t^{H*}}{P_t^{F*}}$ (like in (3.2.18)). Then, I can infer that,

$$\widehat{tot}_t \equiv \left(\widehat{p}_t^H - \widehat{s}_t - \widehat{p}_t^{H*}\right) + \widehat{rp}_t = \left(\widehat{p}_t^F - \widehat{s}_t - \widehat{p}_t^{H*}\right), \tag{7.1.10}$$

$$\widehat{tot}_t^* \equiv \left(\widehat{s}_t + \widehat{p}_t^{F*} - \widehat{p}_t^F\right) + \widehat{rp}_t^* = -\widehat{tot}_t, \tag{7.1.11}$$

which in turn allows me to re-write the real exchange rate as,

$$\begin{aligned} \widehat{rs}_t &\approx \xi \widehat{rp}_t - (1-\xi^*)\, \widehat{rp}_t^* - \left[\widehat{p}_t^F - \widehat{s}_t - \widehat{p}_t^{H*}\right] \\ &= \widehat{rp}_t^W - \widehat{tot}_t. \end{aligned} \tag{7.1.12}$$

These calculations show that movements in the real exchange rate can be thought as the result of differences between world relative prices and domestic terms of trade.

7.2 The Demand-Side in the Goods Markets

The *IS* Equations. The linearization of the Euler equations in (3.2.30) – (3.2.31) applies to the model under complete or incomplete asset markets and characterizes the consumption-savings decisions of the households. This is a crucial margin of choice. I obtain the following system of two linearized Euler equations,

$$\gamma \mathbb{E}_t\left[\Delta \widehat{c}_{t+1}\right] \approx \widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right], \tag{7.2.1}$$

$$\gamma \mathbb{E}_t\left[\Delta \widehat{c}_{t+1}^*\right] \approx \widehat{i}_t^* - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right], \tag{7.2.2}$$

which are conventionally denoted the IS^H and IS^F equations, respectively. The *ex ante* Fisher equation requires that the model-based implicit real interest rate be equal to,

$$\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right], \tag{7.2.3}$$

$$\widehat{r}_t^* \equiv \widehat{i}_t^* - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right]. \tag{7.2.4}$$

The expected growth rate of consumption is proportional to this measure of the real interest rate. The constant of proportionality is the intertemporal elasticity of substitution, $\frac{1}{\gamma}$. Using the linearized CPIs in (7.1.1) – (7.1.2), I obtain that,

$$\gamma \mathbb{E}_t\left[\Delta \widehat{c}_{t+1}\right] \approx \widehat{i}_t - \mathbb{E}_t\left[\xi \widehat{\pi}_{t+1}^H + (1-\xi)\, \widehat{\pi}_{t+1}^F\right], \tag{7.2.5}$$

$$\gamma \mathbb{E}_t\left[\Delta \widehat{c}_{t+1}^*\right] \approx \widehat{i}_t^* - \mathbb{E}_t\left[\xi^* \widehat{\pi}_{t+1}^{H*} + (1-\xi^*)\, \widehat{\pi}_{t+1}^{F*}\right], \tag{7.2.6}$$

where $\widehat{\pi}_{t+1}^H \equiv \widehat{p}_{t+1}^H - \widehat{p}_t^H$ and $\widehat{\pi}_{t+1}^F \equiv \widehat{p}_{t+1}^F - \widehat{p}_t^F$.

In the literature it is often preferred to express the consumption-savings margin in terms of the world *IS* equation, IS^W, and the relative *IS* equation, IS^R. These two equations describe the dynamics of world consumption defined as a weighted average of domestic and foreign consumption, i.e. $\widehat{c}_t^W \equiv n\widehat{c}_t + (1-n)\,\widehat{c}_t^*$, and the dynamics of relative consumption, i.e. $\widehat{c}_t^R \equiv \widehat{c}_t - \widehat{c}_t^*$. They are sufficient to describe domestic and foreign consumption because it can be easily shown that,

$$\widehat{c}_t \equiv \widehat{c}_t^W + (1-n)\,\widehat{c}_t^R, \tag{7.2.7}$$

$$\widehat{c}_t^* \equiv \widehat{c}_t^W - n\widehat{c}_t^R. \tag{7.2.8}$$

These dynamics can be derived by computing $n \cdot (7.2.1) + (1-n) \cdot (7.2.2)$ and $(7.2.1) - (7.2.2)$, respectively.

The world IS equation, IS^W, is obtained as,

$$\gamma \mathbb{E}_t \left[\Delta \widehat{c}_{t+1}^W \right] \approx \widehat{i}_t^W - \mathbb{E}_t \left[\widehat{\pi}_{t+1}^W \right], \tag{7.2.9}$$

where $\widehat{i}_t^W \equiv n\widehat{i}_t + (1-n)\,\widehat{i}_t^*$ and $\widehat{\pi}_{t+1}^W \equiv n\widehat{\pi}_{t+1} + (1-n)\,\widehat{\pi}_{t+1}^*$. The relative IS equation, IS^R, is obtained as,

$$\gamma \mathbb{E}_t \left[\Delta \widehat{c}_{t+1}^R \right] \approx \widehat{i}_t^R - \mathbb{E}_t \left[\widehat{\pi}_{t+1}^R \right], \tag{7.2.10}$$

where $\widehat{i}_t^R \equiv \widehat{i}_t - \widehat{i}_t^*$ and $\widehat{\pi}_{t+1}^R \equiv \widehat{\pi}_{t+1} - \widehat{\pi}_{t+1}^*$. I interpret the argument on the right-hand side of (7.2.10) as the difference between Fisher's real interest rate at home and abroad.

Monetary policy affects the demand-side of the economy because it directly affects the nominal interest rate and, arguably, also the *ex ante* real rate. A monetary policy based on a Taylor rule can be thought as a relationship that links the *ex ante* real interest rate to inflation, inflation expectations and the output gap. In doing so, it has the potential to affect the consumption path of households. However, I caution from reading too much into this. The reality is that every market has two sides, a demand-side and a supply-side. The monetary authority may be able to influence the demand (and the expenditure allocation towards consumption), but the endogenous equilibrium can only be determined once I add the supply-side of the economy.

Imagine, for example, a simple situation in which the economy is fully segregated from the rest of the world (autarky) and prices are flexible. Without frictions money is neutral, and it has no impact on the real output. By the market clearing conditions, however, everything that the economy produces is consumed by the local households (since the economy is closed and the aggregate savings are zero). The Euler equations are still satisfied, but that does not mean the consumption path in equilibrium changes in response to the interest rate policy rule. In this case, only nominal variables adjust in this context and they do so to ensure that the resource allocation is unchanged. In other words, inflation reacts to changes in the nominal interest rate to make sure that the households are satisfied with the consumption they already have.

The *UIP* Condition. The UIP equation, *UIP*, comes from the log-linearization of the intertemporal first-order condition in (3.2.29) (or (3.2.27)) if markets are complete, and from the log-linearization of the first-order condition in (3.2.32) if markets are incomplete. Under complete asset markets, I obtain that the relative consumption must be proportional to the real exchange rate, i.e.

$$\widehat{rs}_t \approx \gamma \widehat{c}_t^R. \tag{7.2.11}$$

Given this particular equilibrium condition, clearly equation (7.2.10) becomes somewhat redundant in the computation of the equilibrium path. Nonetheless, equation (7.2.10) still has an interesting reading because it implies that,

$$\widehat{i}_t - \mathbb{E}_t \left[\widehat{\pi}_{t+1} \right] \approx \widehat{i}_t^* - \mathbb{E}_t \left[\widehat{\pi}_{t+1}^* \right] + \mathbb{E}_t \left[\Delta \widehat{rs}_{t+1}^R \right]. \tag{7.2.12}$$

In other words, the *ex ante* real interest rates are equalized across countries whenever asset markets are complete, but only if PPP holds. This, obviously, puts a constraint on the behavior of consumption on both countries, but it also shows that violations of PPP can be important to interpret different consumption paths across countries.

The *UIP* condition in its more conventional form holds up to a first-order approximation, and it can be expressed as follows,

$$\mathbb{E}_t\left[\Delta\widehat{s}_{t+1}\right] \approx \widehat{i}_t - \widehat{i}_t^* \equiv \widehat{i}_t^R, \tag{7.2.13}$$

where the spread in nominal interest rates reflects movements in the expected exchange rate. Using the definition of the real exchange rate in (7.1.3), I can write a variant of the UIP equation in terms of the real exchange rate as,

$$\mathbb{E}_t\left[\Delta\widehat{rs}_{t+1}\right] \approx \widehat{i}_t^R - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^R\right], \tag{7.2.14}$$

One way to read this is that the wedge between the real interest rate in the domestic and foreign country is determined by the real exchange rate, and this is true up to a first-order approximation independently of whether markets are complete or not. Another way to look at this is by replacing the *ex ante* real interest rate differential using equation (7.2.10), i.e.

$$\mathbb{E}_t\left[\Delta\widehat{rs}_{t+1}\right] \approx \gamma\mathbb{E}_t\left[\Delta\widehat{c}_{t+1}^R\right]. \tag{7.2.15}$$

This equation reflects how the risk-sharing opportunities change whenever I switch from a world from complete markets to a world with incomplete assets markets (to a bond economy). Under complete asset markets, the relative marginal utility of consumption across countries is equalized across all possible states of nature and I end up with (7.2.11). Under incomplete asset markets, the relationship only holds in expectations because all traded bonds are uncontingent. Hence, I end up with equation (7.2.15) instead.

Equations (7.2.11) and (7.2.15), coupled with the definition of the real exchange rate as given by (7.1.3), are going to be crucial in the determination of the exchange rates. In any event, it is worth noticing that this is the only (log-linearized) equilibrium condition for which there is a difference between the incomplete asset markets model and the complete asset markets model. It could be said that the asset market structure is relevant up to a first-order as long as it 'changes' the risk-sharing opportunities available to households, so naturally is in the risk-sharing equations where that shows up.

The *MP* Equations. A simple log-linearization of the Taylor indexes described in equations (3.4.12) – (3.4.13) gives me the MP equations, MP^H and MP^F, as,

$$\begin{aligned} \widehat{i}_t &\approx \rho\widehat{i}_{t-1} + (1-\rho)\left[\psi_1\widehat{\pi}_t + \psi_2\left(\widehat{y}_t - \widehat{\overline{y}}_t\right)\right] + \widehat{z}_t, & (7.2.16)\\ \widehat{i}_t^* &\approx \rho^*\widehat{i}_{t-1}^* + (1-\rho^*)\left[\psi_1^*\widehat{\pi}_t^* + \psi_2^*\left(\widehat{y}_t^* - \widehat{\overline{y}}_t^*\right)\right] + \widehat{z}_t^*, & (7.2.17)\end{aligned}$$

where $\left(\widehat{y}_t - \widehat{\overline{y}}_t\right)$ and $\left(\widehat{y}_t^* - \widehat{\overline{y}}_t^*\right)$ represent the domestic and foreign output gap, respectively, $\widehat{z}_t$ denotes the domestic monetary policy shock, and $\widehat{z}_t^*$ the foreign monetary policy shock. I will derive a characterization of the potential output of the economy in subsection (7.7). Suffice to say for now that the approximation implies that,

$$\begin{aligned} \widehat{\overline{y}}_t &\approx \frac{\partial\overline{Y}(A,A^*)}{\partial A}\frac{A}{\overline{Y}(A,A^*)}\widehat{a}_t + \frac{\partial\overline{Y}(A,A^*)}{\partial A^*}\frac{A^*}{\overline{Y}(A,A^*)}\widehat{a}_t^*, & (7.2.18)\\ \widehat{\overline{y}}_t^* &\approx \frac{\partial\overline{Y}^*(A,A^*)}{\partial A}\frac{A}{\overline{Y}^*(A,A^*)}\widehat{a}_t + \frac{\partial\overline{Y}^*(A,A^*)}{\partial A^*}\frac{A^*}{\overline{Y}^*(A,A^*)}\widehat{a}_t^*, & (7.2.19)\end{aligned}$$

where $\widehat{a}_t$ and $\widehat{a}_t^*$ denote the domestic and foreign productivity shocks, respectively. These equations show that potential output in either country is a function of both productivity shocks. On one hand, it indicates that potential output can be affected by the productivity shock of the other country (if international trade in goods is feasible). On the other hand, it shows that the monetary authority has no influence whatsoever on potential output. This should not be surprising since the potential output of the economy is the output that would prevail in a frictionless economy, which is precisely the type of economy where money is neutral.

The Taylor monetary policy rules also suggest that errors in the estimation of potential output could be perceived as monetary policy shocks and misinterpreted as monetary shocks, while in reality they are truly supply (or real) shocks. For example, let me suppose that the domestic monetary authority believes the economy to be closed or the foreign productivity shocks to be unimportant and estimates the domestic potential output as $\widehat{\overline{y}}_t \approx \frac{\partial \overline{Y}(A,A^*)}{\partial A} \frac{A}{\overline{Y}(A,A^*)} \widehat{a}_t$. Depending on the sign of $\frac{\partial \overline{Y}(A,A^*)}{\partial A} \frac{A}{\overline{Y}(A,A^*)}$ and the realization of the foreign shock, the output gap may be either overestimated or underestimated by the monetary authority. That means the interest rates might be either too high or too low relative to what the correctly-specified Taylor rule would prescribe in order to drive the economy closer to the frictionless allocation. Hence, this specification error can be detrimental for the goals of the policy-maker.

Let me suppose now, as an alternative example, that the monetary authority follows the Taylor rule prescribed in (7.2.16) − (7.2.17), but an econometrician does not know how to correctly estimate the potential output that is being targeted. Then, the differences between the econometrician's conjecture and the real output gap of the economy will be interpreted as monetary policy shocks. While, in fact, these estimation errors should be functionally related to productivity shocks, because potential output only depends on real shocks. Because these output gap errors are misconstrued as monetary policy shocks, econometrician's may use them to conclude that monetary policy shocks are correlated with real shocks, even though there is no evidence of that. These errors could also lead the econometrician to argue that Taylor rules have real effects, even when there is little of that (if anything at all) and money is close to neutral.

Monetary Policy and the Nominal Exchange Rate. The nominal exchange rate is often characterized with a combination of the *UIP* equation in (7.2.13) and the *MP* equations described in (7.2.16) − (7.2.17). Let me define two state-dependent factors to capture the smoothing term and the idiosyncratic shock in the Taylor rule as,

$$\widehat{i}_t^n = \widehat{z}_t + \rho \widehat{i}_{t-1}, \tag{7.2.20}$$

$$\widehat{i}_t^{n*} = \widehat{z}_t^* + \rho \widehat{i}_{t-1}^*. \tag{7.2.21}$$

These variables summarize the way in which monetary policy moves autonomously from its policy goals. The impact of monetary policy on the determination of the nominal exchange rate becomes easier to grasp if I make the assumption that the reaction functions of the monetary authority are identical in both countries (i.e., $\rho = \rho^*$, $\psi_1 = \psi_1^*$, and $\psi_2 = \psi_2^*$). If I combine the *UIP* equation and the *MP* equations, it follows immediately that,

$$\mathbb{E}_t \left[\Delta \widehat{s}_{t+1}\right] \approx \widehat{i}_t - \widehat{i}_t^* = \widehat{i}_t^n - \widehat{i}_t^{n*} + (1-\rho) \left[\psi_1 \left(\widehat{\pi}_t - \widehat{\pi}_t^*\right) + \psi_2 \left(\left(\widehat{y}_t - \widehat{\overline{y}}_t\right) - \left(\widehat{y}_t^* - \widehat{\overline{y}}_t^*\right)\right)\right]. \tag{7.2.22}$$

For expositional purposes, I follow Woodford (2007) and conjecture that the real exchange rate is equal to one (which requires the LOOP to hold and equal preferences on consumption, i.e. $\xi = \xi^*$, in the context of the workhorse model). If that is the case, then I can re-write the inflation differential in terms of the nominal exchange rate alone, and obtain that,

$$\mathbb{E}_t\left[\Delta\widehat{s}_{t+1}\right] \approx \widehat{i}_t^n - \widehat{i}_t^{n*} + (1-\rho)\left[\psi_1\Delta\widehat{s}_t + \psi_2\left(\left(\widehat{y}_t - \widehat{\overline{y}}_t\right) - \left(\widehat{y}_t^* - \widehat{\overline{y}}_t^*\right)\right)\right]. \tag{7.2.23}$$

This equation is a version of the often-cited and widely-used present-value model of the nominal exchange rate.

Under the assumption that $(1-\rho)\psi_1 > 1$, this equation has a unique bounded solution for the depreciation of the nominal exchange rate which can be expressed as[42],

$$\Delta\widehat{s}_t \approx \sum_{j=0}^{\infty}\left(\frac{1}{(1-\rho)\psi_1}\right)^{j+1}\left[\mathbb{E}_t\left(\widehat{i}_{t+j}^{n*} - \widehat{i}_{t+j}^n\right) + (1-\rho)\psi_2\left(\mathbb{E}_t\left(\widehat{y}_{t+j}^* - \widehat{\overline{y}}_{t+j}^*\right) - \mathbb{E}_t\left(\widehat{y}_{t+j} - \widehat{\overline{y}}_{t+j}\right)\right)\right].$$

In very simple terms, this equation shows how the nominal exchange rate depreciates (i.e., $\uparrow \Delta\widehat{s}_t$) in response to either monetary policy shocks or a broadening of the output gap differential across countries. In both cases, either current or future expected changes generate a depreciation. The monetary authority may have an impact on the size of the output only if money has real effects. However, even if that is the case, a successful monetary policy should imply that output fluctuates around its potential over time. Therefore, if all economic agents anticipate a successful monetary policy, the contribution of the output gap to the determination of the nominal exchange rate is likely to be 'small'[43]. Therefore, it seems that the most crucial linkage between monetary policy and the nominal exchange rate comes from the direct impact it has on the stochastic term $\widehat{i}_t^{n*} - \widehat{i}_t^n$.

Remark 9 *The failure of the UIP condition is well-known and has been well-documented across a variety of countries. There may be a good reason for this condition to fail: after all, the linearization presented in these notes completely ignores all second-order effects and therefore it is biased. However, it is still considered puzzling that in many circumstances we observe that the nominal exchange rate depreciates even when the domestic interest rate goes down. Here I have presented a very simple structure to help us understand why that could happen. Putting it together with a Taylor rule, I see that what matters for the exchange rate is not the interest rate itself, but the monetary shocks. If the domestic interest rate decreases but the policy shock is positive (in other words, the interest rates are kept above what the Taylor rule prescribes), then the nominal exchange rate appreciates. Otherwise it will depreciate. This is true only if the future expectations of the monetary policy shock remain unchanged.*

A more complex dynamic is set in motion if a certain policy shock today also alters the expectations about monetary policy in the future. In any event, it is perfectly possible to rationalize a depreciation of the nominal exchange rate simultaneously with a cut in the nominal interest rate. In this sense, I don't think we should be surprised to find those patterns in the data. This little example, however, is based on the fact that the real exchange rate is equal to one (absolute PPP holds). Understanding how the real exchange rate

[42] Notice that a change in $\widehat{i}_t^{n*} - \widehat{i}_t^n$ at time t can only come from a differential policy shock, i.e. $\widehat{z}_t^* - \widehat{z}_t$.

[43] A simpler version of this logic is studied from an econometric standpoint by Rossi (2007).

behaves, therefore, it is essential to uncover the true impact that monetary policy shocks have on the nominal exchange rate. My suspicion is that the nominal exchange rate is probably more sensitive to monetary policy shocks in economies where the degree of pass-through is lower than one, but that remains an open question.

7.3 The Supply-Side in the Goods Markets

The Phillips curve has long served as a useful description of the inflation dynamics. In the workhorse model, the Phillips curve is explicitly derived from the pricing decisions of firms. One advantage of this new approach is that because the relationship has a structural interpretation, it is possible to infer the implications for the transmission of inflation of every shock. The Phillips curve is no longer a mere 'black box' that summarizes certain empirical regularities. But if there are structural changes in the economy, such as the move to a low-inflation environment witnessed since the 1990s, the price-setting behavior of firms is likely to change and with it the inflation dynamics.

From a policy perspective, therefore, two important issues arise. First, how sensitive are the short-run inflation dynamics to such shifts in the economic environment? Particularly whenever the structure of the Phillips curve is tied to the ad hoc characterization of price stickiness proposed by Calvo (1983). Second, how well does a Phillips curve based on the assumption of unchanged price-setting behavior of firms (including infrequent price changes at constant rates and pricing-to-market) describe the inflation dynamics of an economy where this behavior is thought to be evolving and naturally adapting to new circumstances?

The $\boldsymbol{AS}$ Equations[44]**.** The AS equations, AS^{H}, AS^{H*}, AS^{F} and AS^{F*}, come from a model with Calvo-style price-setting firms and LCP pricing. The pair of equations AS^{H} and AS^{H*} is obtained from the log linearization of the optimal price-setting rules, equations (3.3.20) and (3.3.21), and the home and foreign price sub-indexes of the domestic bundle, equations (3.2.16) and (3.2.17). Similarly, I derive the pair AS^{F} and AS^{F*} from the log linearization of the foreign firms' first-order conditions, equations (3.3.26) and (3.3.27), and the price sub-indexes of the foreign bundle of goods, equations (3.2.16) and (3.2.17).

The Optimal Pricing in the Domestic Market for the Domestic Firm. I can log-linearize the optimal pricing equation in (3.3.20) as in Martínez-García (2007). Accordingly, in steady state the Dixit-Stiglitz pricing rule holds, then the log-linearization around the steady state can be expressed as,

$$\mathbb{E}_t \left\{ \sum\nolimits_{\tau=0}^{\infty} (\beta\alpha)^{\tau} \left[\widehat{f}^{H}_{t+\tau} - \widehat{g}^{H}_{t+\tau} \right] \right\} \approx 0, \tag{7.3.1}$$

where the approximations are as follows,

$$\widehat{f}^{H}_{t+\tau} \equiv \widehat{\widetilde{p}}_t (h) - \widehat{p}^{H}_{t+\tau}, \tag{7.3.2}$$

$$\widehat{g}^{H}_{t+\tau} \equiv \left(\widehat{w}_{t+\tau} - \widehat{p}_{t+\tau} - \widehat{a}_{t+\tau}\right) + \left(\widehat{p}_{t+\tau} - \widehat{p}^{H}_{t+\tau}\right). \tag{7.3.3}$$

I can re-write the optimal pricing equation simply as,

$$\mathbb{E}_t \left\{ \sum\nolimits_{\tau=0}^{\infty} (\beta\alpha)^{\tau} \left[\left(\widehat{\widetilde{p}}_t (h) - \widehat{p}^{H}_{t+\tau}\right) - \left(\widehat{w}_{t+\tau} - \widehat{p}_{t+\tau} - \widehat{a}_{t+\tau}\right) - \left(\widehat{p}_{t+\tau} - \widehat{p}^{H}_{t+\tau}\right) \right] \right\} \approx 0. \tag{7.3.4}$$

[44] An important feature of the workhorse model is that while technologies are symmetric, the nominal side (pricing contracts) and the consumption preferences are asymmetric. This shows up on the aggregate supply (or AS) equations.

Notice that I can re-express the price indexes $\widehat{p}^{H}_{t+\tau}$ and $\widehat{p}^{H*}_{t+\tau}$ respectively as $\widehat{p}^{H}_{t+\tau} = \widehat{p}^{H}_{t-1} + \sum_{i=0}^{\tau} \widehat{\pi}^{H}_{t+i}$ and $\widehat{p}^{H*}_{t+\tau} = \widehat{p}^{H*}_{t-1} + \sum_{i=0}^{\tau} \widehat{\pi}^{H*}_{t+i}$. Hence, the optimal pricing equation becomes,

$$\widehat{\widetilde{p}}_{t}(h) - \widehat{p}^{H}_{t-1} \approx (1-\beta\alpha)\,\mathbb{E}_t\left\{\sum_{\tau=0}^{\infty}(\beta\alpha)^{\tau}\left[\sum_{i=0}^{\tau}\widehat{\pi}^{H}_{t+i} + \left(\widehat{w}_{t+\tau} - \widehat{p}^{H}_{t+\tau} - \widehat{a}_{t+\tau}\right)\right]\right\}. \tag{7.3.5}$$

In other words, the difference between the price charged by a firm that can reset prices and the average price of all the firms who cannot re-optimize is given by a weighted function of current and future inflation expectations and the marginal costs.

The nominal marginal cost net of (constant) labor subsidies from (3.3.3) can be expressed as,

$$\widehat{mc}_t \equiv \left(\widehat{w}_t - \widehat{a}_t\right), \tag{7.3.6}$$

$$\widehat{mc}^{*}_t \equiv \left(\widehat{w}^{*}_t - \widehat{a}^{*}_t\right). \tag{7.3.7}$$

Furthermore, after a little bit of algebra, I also find out that,

$$\sum_{\tau=0}^{\infty}(\beta\alpha)^{\tau}\left(\sum_{i=0}^{\tau}\widehat{\pi}^{H}_{t+i}\right) = \left(\sum_{j=0}^{\infty}(\beta\alpha)^{j}\right)\left(\sum_{\tau=0}^{\infty}(\beta\alpha)^{\tau}\,\widehat{\pi}^{H}_{t+\tau}\right) = \frac{1}{1-\beta\alpha}\left[\sum_{\tau=0}^{\infty}(\beta\alpha)^{\tau}\,\widehat{\pi}^{H}_{t+\tau}\right]. \tag{7.3.8}$$

Therefore, the optimal pricing equation can be generically written as,

$$\widehat{\widetilde{p}}_{t}(h) - \widehat{p}^{H}_{t-1} \approx \mathbb{E}_t\left[\sum_{\tau=0}^{\infty}(\beta\alpha)^{\tau}\,\widehat{\pi}^{H}_{t+\tau}\right] + (1-\beta\alpha)\,\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}(\beta\alpha)^{\tau}\left(\widehat{mc}_{t+\tau} - \widehat{p}^{H}_{t+\tau}\right)\right], \tag{7.3.9}$$

which is clearly a function of current and future marginal costs. I can easily re-write the above expression under rational expectations as follows,

$$\widehat{\widetilde{p}}_{t}(h) - \widehat{p}^{H}_{t-1} \approx \widehat{\pi}^{H}_{t} + (1-\beta\alpha)\left(\widehat{mc}_t - \widehat{p}^{H}_{t}\right) + \beta\alpha\mathbb{E}_t\left(\widehat{\widetilde{p}}_{t+1}(h) - \widehat{p}^{H}_{t}\right). \tag{7.3.10}$$

Equation (7.3.9) is the forward-looking (no-bubbles) solution to this equation.

The Optimal Pricing in the Foreign Market for the Domestic Firm. I can log-linearize the optimal pricing equation in (3.3.21) as in Martínez-García (2007). Accordingly, in steady state the Dixit-Stiglitz pricing rule holds, then the log-linearization around the steady state can be expressed as,

$$\mathbb{E}_t\left\{\sum_{\tau=0}^{\infty}(\beta\alpha)^{\tau}\left[\widehat{f}^{H*}_{t+\tau} - \widehat{g}^{H*}_{t+\tau}\right]\right\} \approx 0, \tag{7.3.11}$$

where the approximations are as follows,

$$\widehat{f}^{H*}_{t+\tau} \equiv \widehat{\widetilde{p}}^{*}_{t}(h) - \widehat{p}^{H*}_{t+\tau} + \left(\widehat{s}_{t+\tau} + \widehat{p}^{H*}_{t+\tau} - \widehat{p}^{H}_{t+\tau}\right), \tag{7.3.12}$$

$$\widehat{g}^{H*}_{t+\tau} \equiv \left(\widehat{w}_{t+\tau} - \widehat{p}_{t+\tau} - \widehat{a}_{t+\tau}\right) + \left(\widehat{p}_{t+\tau} - \widehat{p}^{H}_{t+\tau}\right). \tag{7.3.13}$$

I can re-write the optimal pricing equation simply as,

$$\mathbb{E}_t\left\{\sum_{\tau=0}^{\infty}(\beta\alpha)^{\tau}\left[\left(\widehat{\widetilde{p}}^{*}_{t}(h) - \widehat{p}^{H*}_{t+\tau}\right) + \left(\widehat{s}_{t+\tau} + \widehat{p}^{H*}_{t+\tau} - \widehat{p}^{H}_{t+\tau}\right) - \left(\widehat{w}_{t+\tau} - \widehat{p}_{t+\tau} - \widehat{a}_{t+\tau}\right) - \left(\widehat{p}_{t+\tau} - \widehat{p}^{H}_{t+\tau}\right)\right]\right\} \approx 0. \tag{7.3.14}$$

Notice that I can re-express the price indexes $\widehat{p}^{H}_{t+\tau}$ and $\widehat{p}^{H*}_{t+\tau}$ respectively as $\widehat{p}^{H}_{t+\tau} = \widehat{p}^{H}_{t-1} + \sum_{i=0}^{\tau} \widehat{\pi}^{H}_{t+i}$ and $\widehat{p}^{H*}_{t+\tau} = \widehat{p}^{H*}_{t-1} + \sum_{i=0}^{\tau} \widehat{\pi}^{H*}_{t+i}$. Hence, the optimal pricing equation becomes,

$$\widehat{\widetilde{p}}^{*}_{t}(h) - \widehat{p}^{H*}_{t-1} \approx (1-\beta\alpha)\,\mathbb{E}_t\left\{\sum_{\tau=0}^{\infty} (\beta\alpha)^{\tau}\left[\sum_{i=0}^{\tau} \widehat{\pi}^{H*}_{t+i} + \left(\widehat{w}_{t+\tau} - \widehat{p}^{H*}_{t+\tau} - \widehat{s}_{t+\tau} - \widehat{a}_{t+\tau}\right)\right]\right\}. \quad (7.3.15)$$

In other words, the difference between the price charged by a firm that can reset prices and the average price of all the local firms that cannot re-optimize is given by a weighted function of current and future inflation expectations and the marginal costs.

The nominal marginal cost net of (constant) labor subsidies from (3.3.3) can be expressed as,

$$\widehat{mc}_t \equiv \left(\widehat{w}_t - \widehat{a}_t\right), \quad (7.3.16)$$

$$\widehat{mc}^{*}_t \equiv \left(\widehat{w}^{*}_t - \widehat{a}^{*}_t\right). \quad (7.3.17)$$

Furthermore, after a little bit of algebra, I also find out that,

$$\sum_{\tau=0}^{\infty} (\beta\alpha)^{\tau}\left(\sum_{i=0}^{\tau} \widehat{\pi}^{H*}_{t+i}\right) = \left(\sum_{j=0}^{\infty} (\beta\alpha)^{j}\right)\left(\sum_{\tau=0}^{\infty} (\beta\alpha)^{\tau}\,\widehat{\pi}^{H*}_{t+\tau}\right) = \frac{1}{1-\beta\alpha}\left[\sum_{\tau=0}^{\infty} (\beta\alpha)^{\tau}\,\widehat{\pi}^{H*}_{t+\tau}\right]. \quad (7.3.18)$$

Therefore, the optimal pricing equation can be generically written as,

$$\widehat{\widetilde{p}}^{*}_{t}(h) - \widehat{p}^{H*}_{t-1} \approx \mathbb{E}_t\left[\sum_{\tau=0}^{\infty} (\beta\alpha)^{\tau}\,\widehat{\pi}^{H*}_{t+\tau}\right] + (1-\beta\alpha)\,\mathbb{E}_t\left[\sum_{\tau=0}^{\infty} (\beta\alpha)^{\tau}\left(\widehat{mc}_{t+\tau} - \widehat{p}^{H*}_{t+\tau} - \widehat{s}_{t+\tau}\right)\right], \quad (7.3.19)$$

which is clearly a function of current and future marginal costs. I can easily re-write the above expression under rational expectations as follows,

$$\widehat{\widetilde{p}}^{*}_{t}(h) - \widehat{p}^{H*}_{t-1} \approx \widehat{\pi}^{H*}_{t} + (1-\beta\alpha)\left(\widehat{mc}_t - \widehat{p}^{H*}_{t} - \widehat{s}_t\right) + \beta\alpha\mathbb{E}_t\left(\widehat{\widetilde{p}}^{*}_{t+1}(h) - \widehat{p}^{H*}_{t}\right). \quad (7.3.20)$$

Equation (7.3.19) is the forward-looking (no-bubbles) solution to this equation.

The Optimal Pricing in the Domestic Market for the Foreign Firm. I can log-linearize the optimal pricing equation in (3.3.26) as in Martínez-García (2007). Accordingly, in steady state the Dixit-Stiglitz pricing rule holds, then the log-linearization around the steady state can be expressed as,

$$\mathbb{E}_t\left\{\sum_{\tau=0}^{\infty} (\beta\alpha^{*})^{\tau}\left[\widehat{f}^{F}_{t+\tau} - \widehat{g}^{F}_{t+\tau}\right]\right\} \approx 0, \quad (7.3.21)$$

where the approximations are as follows,

$$\widehat{f}^{F}_{t+\tau} \equiv \widehat{\widetilde{p}}_{t}(f) - \widehat{p}^{F}_{t+\tau} + \left(\widehat{p}^{F}_{t+\tau} - \widehat{p}^{F*}_{t+\tau} - \widehat{s}_{t+\tau}\right), \quad (7.3.22)$$

$$\widehat{g}^{F}_{t+\tau} \equiv \left(\widehat{w}^{*}_{t+\tau} - \widehat{p}^{*}_{t+\tau} - \widehat{a}^{*}_{t+\tau}\right) + \left(\widehat{p}^{*}_{t+\tau} - \widehat{p}^{F*}_{t+\tau}\right). \quad (7.3.23)$$

I can re-write the optimal pricing equation simply as,

$$\mathbb{E}_t\left\{\sum_{\tau=0}^{\infty} (\beta\alpha^{*})^{\tau}\left[\left(\widehat{\widetilde{p}}_{t}(f) - \widehat{p}^{F}_{t+\tau}\right) + \left(\widehat{p}^{F}_{t+\tau} - \widehat{p}^{F*}_{t+\tau} - \widehat{s}_{t+\tau}\right) - \left(\widehat{w}^{*}_{t+\tau} - \widehat{p}^{*}_{t+\tau} - \widehat{a}^{*}_{t+\tau}\right) - \left(\widehat{p}^{*}_{t+\tau} - \widehat{p}^{F*}_{t+\tau}\right)\right]\right\} \approx 0. \quad (7.3.24)$$

Notice that I can re-express the price indexes $\widehat{p}_{t+\tau}^{F}$ and $\widehat{p}_{t+\tau}^{F*}$ respectively as $\widehat{p}_{t+\tau}^{F}=\widehat{p}_{t-1}^{F}+\sum_{i=0}^{\tau}\widehat{\pi}_{t+i}^{F}$ and $\widehat{p}_{t+\tau}^{F*}=\widehat{p}_{t-1}^{F*}+\sum_{i=0}^{\tau}\widehat{\pi}_{t+i}^{F*}$. Hence, the optimal pricing equation becomes,

$$\widehat{\widetilde{p}}_{t}(f)-\widehat{p}_{t-1}^{F}\approx(1-\beta\alpha^{*})\,\mathbb{E}_{t}\left\{\sum_{\tau=0}^{\infty}(\beta\alpha^{*})^{\tau}\left[\sum_{i=0}^{\tau}\widehat{\pi}_{t+i}^{F}+\left(\widehat{w}_{t+\tau}^{*}-\widehat{p}_{t+\tau}^{F}+\widehat{s}_{t+\tau}-\widehat{a}_{t+\tau}^{*}\right)\right]\right\}. \tag{7.3.25}$$

In other words, the difference between the price charged by a firm that can reset prices and the average price of all the local firms that cannot reset their prices is given by a weighted function of current and future inflation expectations and the marginal costs.

The nominal marginal cost net of (constant) labor subsidies from (3.3.3) can be expressed as,

$$\widehat{mc}_{t}\equiv(\widehat{w}_{t}-\widehat{a}_{t}), \tag{7.3.26}$$
$$\widehat{mc}_{t}^{*}\equiv(\widehat{w}_{t}^{*}-\widehat{a}_{t}^{*}). \tag{7.3.27}$$

Furthermore, after a little bit of algebra, I also find out that,

$$\sum_{\tau=0}^{\infty}(\beta\alpha^{*})^{\tau}\left(\sum_{i=0}^{\tau}\widehat{\pi}_{t+i}^{F}\right)=\left(\sum_{j=0}^{\infty}(\beta\alpha^{*})^{j}\right)\left(\sum_{\tau=0}^{\infty}(\beta\alpha^{*})^{\tau}\widehat{\pi}_{t+\tau}^{F}\right)=\frac{1}{1-\beta\alpha^{*}}\left[\sum_{\tau=0}^{\infty}(\beta\alpha^{*})^{\tau}\widehat{\pi}_{t+\tau}^{F}\right]. \tag{7.3.28}$$

Therefore, the optimal pricing equation can be generically written as,

$$\widehat{\widetilde{p}}_{t}(f)-\widehat{p}_{t-1}^{F}\approx\mathbb{E}_{t}\left[\sum_{\tau=0}^{\infty}(\beta\alpha^{*})^{\tau}\widehat{\pi}_{t+\tau}^{F}\right]+(1-\beta\alpha^{*})\,\mathbb{E}_{t}\left[\sum_{\tau=0}^{\infty}(\beta\alpha^{*})^{\tau}\left(\widehat{mc}_{t+\tau}^{*}-\widehat{p}_{t+\tau}^{F}+\widehat{s}_{t+\tau}\right)\right], \tag{7.3.29}$$

which is clearly a function of current and future marginal costs. I can easily re-write the above expression under rational expectations as follows,

$$\widehat{\widetilde{p}}_{t}(f)-\widehat{p}_{t-1}^{F}\approx\widehat{\pi}_{t}^{F}+(1-\beta\alpha^{*})\left(\widehat{mc}_{t}^{*}-\widehat{p}_{t}^{F}+\widehat{s}_{t}\right)+\beta\alpha^{*}\mathbb{E}_{t}\left(\widehat{\widetilde{p}}_{t+1}(f)-\widehat{p}_{t}^{F}\right). \tag{7.3.30}$$

Equation (7.3.29) is the forward-looking (no-bubbles) solution to this equation.

The Optimal Pricing in the Foreign Market for the Foreign Firm. I can log-linearize the optimal pricing equation in (3.3.27) as in Martínez-García (2007). Accordingly, in steady state the Dixit-Stiglitz pricing rule holds, then the log-linearization around the steady state can be expressed as,

$$\mathbb{E}_{t}\left\{\sum_{\tau=0}^{\infty}(\beta\alpha^{*})^{\tau}\left[\widehat{f}_{t+\tau}^{F*}-\widehat{g}_{t+\tau}^{F*}\right]\right\}\approx 0, \tag{7.3.31}$$

where the approximations are as follows,

$$\widehat{f}_{t+\tau}^{F}\equiv\widehat{\widetilde{p}}_{t}^{*}(f)-\widehat{p}_{t+\tau}^{F*}, \tag{7.3.32}$$
$$\widehat{g}_{t+\tau}^{F}\equiv\left(\widehat{w}_{t+\tau}^{*}-\widehat{p}_{t+\tau}^{*}-\widehat{a}_{t+\tau}^{*}\right)+\left(\widehat{p}_{t+\tau}^{*}-\widehat{p}_{t+\tau}^{F*}\right). \tag{7.3.33}$$

I can re-write the optimal pricing equation simply as,

$$\mathbb{E}_{t}\left\{\sum_{\tau=0}^{\infty}(\beta\alpha^{*})^{\tau}\left[\left(\widehat{\widetilde{p}}_{t}^{*}(f)-\widehat{p}_{t+\tau}^{F*}\right)-\left(\widehat{w}_{t+\tau}^{*}-\widehat{p}_{t+\tau}^{*}-\widehat{a}_{t+\tau}^{*}\right)-\left(\widehat{p}_{t+\tau}^{*}-\widehat{p}_{t+\tau}^{F*}\right)\right]\right\}\approx 0. \tag{7.3.34}$$

Notice that I can re-express the price indexes $\widehat{p}^{F}_{t+\tau}$ and $\widehat{p}^{F*}_{t+\tau}$ respectively as $\widehat{p}^{F}_{t+\tau} = \widehat{p}^{F}_{t-1} + \sum_{i=0}^{\tau} \widehat{\pi}^{F}_{t+i}$ and $\widehat{p}^{F*}_{t+\tau} = \widehat{p}^{F*}_{t-1} + \sum_{i=0}^{\tau} \widehat{\pi}^{F*}_{t+i}$. Hence, the optimal pricing equation becomes,

$$\widehat{\widetilde{p}}^{*}_{t}(f) - \widehat{p}^{F*}_{t-1} \approx (1-\beta\alpha^{*})\,\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\beta\alpha^{*})^{\tau} \left[\sum_{i=0}^{\tau} \widehat{\pi}^{F*}_{t+i} + \left(\widehat{w}^{*}_{t+\tau} - \widehat{p}^{F*}_{t+\tau} - \widehat{a}^{*}_{t+\tau} \right) \right] \right\}. \tag{7.3.35}$$

In other words, the difference between the price charged by a firm that can reset prices and the average price of all the local firms who cannot reset their prices is given by a weighted function of current and future inflation expectations and the marginal costs.

The nominal marginal cost net of (constant) labor subsidies from (3.3.3) can be expressed as,

$$\widehat{mc}_t \equiv (\widehat{w}_t - \widehat{a}_t), \tag{7.3.36}$$

$$\widehat{mc}^{*}_t \equiv (\widehat{w}^{*}_t - \widehat{a}^{*}_t). \tag{7.3.37}$$

Furthermore, after a little bit of algebra, I also find out that,

$$\sum_{\tau=0}^{\infty} (\beta\alpha^{*})^{\tau} \left(\sum_{i=0}^{\tau} \widehat{\pi}^{F*}_{t+i} \right) = \left(\sum_{j=0}^{\infty} (\beta\alpha^{*})^{j} \right) \left(\sum_{\tau=0}^{\infty} (\beta\alpha^{*})^{\tau} \widehat{\pi}^{F*}_{t+\tau} \right) = \frac{1}{1-\beta\alpha^{*}} \left[\sum_{\tau=0}^{\infty} (\beta\alpha^{*})^{\tau} \widehat{\pi}^{F*}_{t+\tau} \right]. \tag{7.3.38}$$

Therefore, the optimal pricing equation can be generically written as,

$$\widehat{\widetilde{p}}^{*}_{t}(f) - \widehat{p}^{F*}_{t-1} \approx \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\alpha^{*})^{\tau} \widehat{\pi}^{F*}_{t+\tau} \right] + (1-\beta\alpha^{*})\,\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\alpha^{*})^{\tau} \left(\widehat{mc}^{*}_{t+\tau} - \widehat{p}^{F*}_{t+\tau} \right) \right], \tag{7.3.39}$$

which is clearly a function of current and future marginal costs. I can easily re-write the above expression under rational expectations as follows,

$$\widehat{\widetilde{p}}^{*}_{t}(f) - \widehat{p}^{F*}_{t-1} \approx \widehat{\pi}^{F*}_{t} + (1-\beta\alpha^{*}) \left(\widehat{mc}^{*}_{t} - \widehat{p}^{F*}_{t} \right) + \beta\alpha^{*}\mathbb{E}_t \left(\widehat{\widetilde{p}}^{*}_{t+1}(f) - \widehat{p}^{F*}_{t} \right). \tag{7.3.40}$$

Equation (7.3.39) is the forward-looking (no-bubbles) solution to this equation.

The Price Sub-Indexes of Home and Foreign Bundles. In a symmetric equilibrium under sticky prices, the aggregate price sub-indexes in (3.2.16) – (3.2.17) can be expressed as follows,

$$P^{H}_{t} = \left[\alpha \left(P^{H}_{t-1} \right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}_t(h) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{7.3.41}$$

$$P^{H*}_{t} = \left[\alpha \left(P^{H*}_{t-1} \right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}^{*}_t(h) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{7.3.42}$$

$$P^{F}_{t} = \left[\alpha^{*} \left(P^{F}_{t-1} \right)^{1-\theta} + (1-\alpha^{*}) \left(\widetilde{P}_t(f) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{7.3.43}$$

$$P^{F*}_{t} = \left[\alpha^{*} \left(P^{F*}_{t-1} \right)^{1-\theta} + (1-\alpha^{*}) \left(\widetilde{P}^{*}_t(f) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{7.3.44}$$

which accounts for the fact that a proportion of firms maintains their prices while another fraction of firms re-optimizes and uses a symmetric pricing rule. In a zero-inflation steady state, it must hold that $\overline{P}^{H} = \overline{\widetilde{P}}(h)$, $\overline{P}^{H*} = \overline{\widetilde{P}}^{*}(h)$, $\overline{P}^{F} = \overline{\widetilde{P}}(f)$ and $\overline{P}^{F*} = \overline{\widetilde{P}}^{*}(f)$. Hence, the log-linear approximation of these price sub-indexes becomes simply,

$$\widehat{p}_t^H \approx \alpha \widehat{p}_{t-1}^H + (1-\alpha)\widehat{\widetilde{p}}_t(h), \tag{7.3.45}$$

$$\widehat{p}_t^{H*} \approx \alpha \widehat{p}_{t-1}^{H*} + (1-\alpha)\widehat{\widetilde{p}}_t^*(h), \tag{7.3.46}$$

$$\widehat{p}_t^F \approx \alpha^* \widehat{p}_{t-1}^F + (1-\alpha^*)\widehat{\widetilde{p}}_t(f), \tag{7.3.47}$$

$$\widehat{p}_t^{F*} \approx \alpha^* \widehat{p}_{t-1}^{F*} + (1-\alpha^*)\widehat{\widetilde{p}}_t^*(f). \tag{7.3.48}$$

A straightforward manipulation of these equations tells me that the difference between the optimal pricing rules, $\widehat{\widetilde{p}}_t(h)$ and $\widehat{\widetilde{p}}_t^*(h)$, and the price sub-indexes, $\widehat{p}_t^H$ and $\widehat{p}_t^{H*}$, is proportional to the inflation rate in logs, i.e.

$$\left[\widehat{\widetilde{p}}_t(h) - \widehat{p}_{t-1}^H\right] \approx \left(\frac{1}{1-\alpha}\right)\widehat{\pi}_t^H,\ \left[\widehat{\widetilde{p}}_t^*(h) - \widehat{p}_{t-1}^{H*}\right] \approx \left(\frac{1}{1-\alpha}\right)\widehat{\pi}_t^{H*}. \tag{7.3.49}$$

Similarly, the difference between the optimal pricing rule, $\widehat{\widetilde{p}}_t(f)$ and $\widehat{\widetilde{p}}_t^*(f)$, and the price sub-indexes, $\widehat{p}_t^F$ and $\widehat{p}_t^{F*}$, is proportional to the inflation rate in logs, i.e.

$$\left[\widehat{\widetilde{p}}_t(f) - \widehat{p}_{t-1}^F\right] \approx \left(\frac{1}{1-\alpha^*}\right)\widehat{\pi}_t^F,\ \left[\widehat{\widetilde{p}}_t^*(f) - \widehat{p}_{t-1}^{F*}\right] \approx \left(\frac{1}{1-\alpha^*}\right)\widehat{\pi}_t^{F*}. \tag{7.3.50}$$

The Aggregate-Supply Equations for the Domestic Firm: $\boldsymbol{AS^H}$ and $\boldsymbol{AS^{H*}}$ The aggregate supply equations can be derived from the optimal pricing rules in (7.3.10) and (7.3.20), and the aggregation rules in (7.3.49). If I combine these results, I obtain the dynamics of inflation on domestically-produced goods in the model as,

$$\left(\frac{1}{1-\alpha}\right)\widehat{\pi}_t^H \approx \widehat{\pi}_t^H + (1-\beta\alpha)\left(\widehat{mc}_t - \widehat{p}_t^H\right) + \beta\alpha\left(\frac{1}{1-\alpha}\right)\mathbb{E}_t\left(\widehat{\pi}_{t+1}^H\right), \tag{7.3.51}$$

$$\left(\frac{1}{1-\alpha}\right)\widehat{\pi}_t^{H*} \approx \widehat{\pi}_t^{H*} + (1-\beta\alpha)\left(\widehat{mc}_t - \widehat{p}_t^{H*} - \widehat{s}_t\right) + \beta\alpha\left(\frac{1}{1-\alpha}\right)\mathbb{E}_t\left(\widehat{\pi}_{t+1}^{H*}\right), \tag{7.3.52}$$

or, more compactly,

$$\widehat{\pi}_t^H \approx \beta\mathbb{E}_t\left(\widehat{\pi}_{t+1}^H\right) + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\left(\widehat{mc}_t - \widehat{p}_t^H\right), \tag{7.3.53}$$

$$\widehat{\pi}_t^{H*} \approx \beta\mathbb{E}_t\left(\widehat{\pi}_{t+1}^{H*}\right) + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\left(\widehat{mc}_t - \widehat{p}_t^{H*} - \widehat{s}_t\right). \tag{7.3.54}$$

These equations are central in my derivations of the Phillips curve for both economies.

The Aggregate-Supply Equations for the Foreign Firm: $\boldsymbol{AS^F}$ and $\boldsymbol{AS^{F*}}$ The aggregate supply equations can be derived from the optimal pricing rules in (7.3.30), and (7.3.40), and the aggregation rules in (7.3.50). If I combine these results, I obtain the dynamics of inflation on foreign-produced goods in the model as,

$$\left(\frac{1}{1-\alpha^*}\right)\widehat{\pi}_t^F \approx \widehat{\pi}_t^F + (1-\beta\alpha^*)\left(\widehat{mc}_t^* - \widehat{p}_t^F + \widehat{s}_t\right) + \beta\alpha^*\left(\frac{1}{1-\alpha^*}\right)\mathbb{E}_t\left(\widehat{\pi}_{t+1}^F\right), \tag{7.3.55}$$

$$\left(\frac{1}{1-\alpha^*}\right)\widehat{\pi}_t^{F*} \approx \widehat{\pi}_t^{F*} + (1-\beta\alpha^*)\left(\widehat{mc}_t^* - \widehat{p}_t^{F*}\right) + \beta\alpha^*\left(\frac{1}{1-\alpha^*}\right)\mathbb{E}_t\left(\widehat{\pi}_{t+1}^{F*}\right), \tag{7.3.56}$$

or, more compactly,

$$\widehat{\pi}_t^F \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^F \right) + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*} \left(\widehat{mc}_t^* - \widehat{p}_t^F + \widehat{s}_t \right), \tag{7.3.57}$$

$$\widehat{\pi}_t^{F*} \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^{F*} \right) + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*} \left(\widehat{mc}_t^* - \widehat{p}_t^{F*} \right). \tag{7.3.58}$$

These equations are central in my derivations of the Phillips curve for both economies too.

The Relative *AS* Equations. It follows from the two domestic aggregate supply curves in (7.3.53) and (7.3.54) that,

$$\widehat{\pi}_t^H - \widehat{\pi}_t^{H*} \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^H - \widehat{\pi}_{t+1}^{H*} \right) + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \left[\widehat{s}_t - \left(\widehat{p}_t^H - \widehat{p}_t^{H*} \right) \right]. \tag{7.3.59}$$

It follows from the two domestic aggregate supply curves in (7.3.53) and (7.3.54) that,

$$\widehat{\pi}_t^F - \widehat{\pi}_t^{F*} \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^F - \widehat{\pi}_{t+1}^{F*} \right) + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*} \left[\widehat{s}_t - \left(\widehat{p}_t^F - \widehat{p}_t^{F*} \right) \right]. \tag{7.3.60}$$

In either case the structure of the pricing equations is such that marginal cost -whatever that might be- completely drops out of the equation. This explains why pricing differences in the workhorse international macro model are closely linked to the nominal exchange rate, and why the standard model has such a hard time explaining the low degree of pass-through found in the data.

The Aggregate *AS* Equations: The Inflation Dynamics. The inflation dynamics in the domestic country can be derived by applying the aggregation rule in (7.1.1) to equations (7.3.53) and (7.3.57). Simple algebra gives me the following expression,

$$\widehat{\pi}_t \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1} \right) + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \xi \left(\widehat{mc}_t - \widehat{p}_t^H \right) + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*} (1-\xi) \left(\widehat{mc}_t^* - \widehat{p}_t^F + \widehat{s}_t \right). \tag{7.3.61}$$

Whenever $\alpha = \alpha^*$ (symmetric Calvo contracts), it follows that the inflation dynamics take a more conventional form,

$$\widehat{\pi}_t \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1} \right) + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \left[\xi \widehat{mc}_t + (1-\xi) \widehat{mc}_t^* - \widehat{p}_t + (1-\xi) \widehat{s}_t \right]. \tag{7.3.62}$$

This shows that the direct effect on inflation from fluctuations of the nominal exchange rate depends on the proportion of foreign goods in the domestic consumption basket, ξ, as well as the degree of price stickiness, α, and the rate of time preference, β.

The inflation dynamics in the foreign country can be derived by applying the aggregation rule in (7.1.2) to equations (7.3.54) and (7.3.58). Simple algebra gives me the following expression,

$$\widehat{\pi}_t^* \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^* \right) + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \xi^* \left(\widehat{mc}_t - \widehat{p}_t^{H*} - \widehat{s}_t \right) + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*} (1-\xi^*) \left(\widehat{mc}_t^* - \widehat{p}_t^{F*} \right). \tag{7.3.63}$$

Whenever $\alpha = \alpha^*$ (symmetric Calvo contracts), it follows that the inflation dynamics take their more conventional form,

$$\widehat{\pi}_t^* \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^* \right) + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \left[\xi^* \widehat{mc}_t + (1-\xi^*) \widehat{mc}_t^* - \widehat{p}_t^* - \xi^* \widehat{s}_t \right]. \tag{7.3.64}$$

This shows that the direct effect on inflation from fluctuations of the nominal exchange rate depends on the proportion of domestic goods in the foreign consumption basket, ξ^*, as well as the degree of price stickiness, α, and the rate of time preference, β.

7.4 The Labor Market Equilibrium Conditions and Marginal Costs

The *LM* Equations. The LM equations, LM^H and LM^F, are easily derived from the first-order conditions of the households' problem in (3.2.38) – (3.2.39), the labor market clearing conditions in (3.3.12) – (3.3.13), and the linear-in-labor technologies in (3.3.1)–(3.3.2). Log-linearizing the labor supply functions in (3.2.38)–(3.2.39) it follows that,

$$\begin{aligned} \widehat{w}_t - \widehat{p}_t &\approx \gamma\widehat{c}_t + \varphi\widehat{l}_t^s, && (7.4.1) \\ \widehat{w}_t^* - \widehat{p}_t^* &\approx \gamma\widehat{c}_t^* + \varphi\widehat{l}_t^{s*}. && (7.4.2) \end{aligned}$$

The combination of the labor market clearing conditions and the linear-in-labor technologies in (3.3.28) – (3.3.29) (or in (3.3.12) – (3.3.13) and (3.3.1) – (3.3.2)) can be log-linearized as follows,

$$\begin{aligned} \widehat{y}_t &\approx \widehat{a}_t + \widehat{l}_t^s, && (7.4.3) \\ \widehat{y}_t^* &\approx \widehat{a}_t^* + \widehat{l}_t^{s*}. && (7.4.4) \end{aligned}$$

Naturally, the combination of all these results helps me characterize both the path of real wages in the domestic and foreign country. If I put all these equations together, it should follow that,

$$\begin{aligned} \widehat{w}_t - \widehat{p}_t &\approx \gamma\widehat{c}_t + \varphi\widehat{y}_t - \varphi\widehat{a}_t, && (7.4.5) \\ \widehat{w}_t^* - \widehat{p}_t^* &\approx \gamma\widehat{c}_t^* + \varphi\widehat{y}_t^* - \varphi\widehat{a}_t^*. && (7.4.6) \end{aligned}$$

These pair are the LM^H and LM^F equations of the model, and they are both instrumental in the determination of the marginal costs faced by firms in the economy.

The *Y* Equations. The output equations, Y^H and Y^F, are easily derived from the aggregate goods market clearing conditions in (3.3.32) – (3.3.33). Almost mechanically, the aggregate market clearing conditions can be log-linearized to obtain that[45],

$$\begin{aligned} \widehat{y}_t \approx{} & \frac{n\left(\frac{\xi}{n}\right)}{n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^*}{n}\right)}\left[-\theta\left[\int_0^n \left(\widehat{p}_t(h) - \widehat{p}_t^H\right) dh\right] - \sigma\left(\widehat{p}_t^H - \widehat{p}_t\right) + \widehat{c}_t\right] + \\ & + \frac{(1-n)\left(\frac{\xi^*}{n}\right)}{n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^*}{n}\right)}\left[-\theta\left[\int_0^n \left(\widehat{p}_t^*(h) - \widehat{p}_t^{H*}\right) dh\right] - \sigma\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) + \widehat{c}_t^*\right], \end{aligned} \tag{7.4.7}$$

$$\begin{aligned} \widehat{y}_t^* \approx{} & \frac{n\left(\frac{1-\xi}{1-n}\right)}{n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)}\left[-\theta\left[\int_n^1 \left(\widehat{p}_t(f) - \widehat{p}_t^F\right) df\right] - \sigma\left(\widehat{p}_t^F - \widehat{p}_t\right) + \widehat{c}_t\right] + \\ & + \frac{(1-n)\left(\frac{1-\xi^*}{1-n}\right)}{n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)}\left[-\theta\left[\int_n^1 \left(\widehat{p}_t^*(f) - \widehat{p}_t^{F*}\right) df\right] - \sigma\left(\widehat{p}_t^{F*} - \widehat{p}_t^*\right) + \widehat{c}_t^*\right]. \end{aligned} \tag{7.4.8}$$

[45] The log-linearization is attained based on the normalization of the steady state that I discussed before in subsection 6.1.

Let me point out that the relative price dispersion has no first-order impact on the production in either country. This is because, up to a first-order approximation it follows that,

$$\int_0^n \left(\widehat{p}_t(h) - \widehat{p}_t^H\right) dh = \int_0^n \left(\widehat{p}_t^*(h) - \widehat{p}_t^{H*}\right) dh = \int_n^1 \left(\widehat{p}_t(f) - \widehat{p}_t^F\right) df = \int_n^1 \left(\widehat{p}_t^*(f) - \widehat{p}_t^{F*}\right) df = 0. \quad (7.4.9)$$

Therefore, I can re-write the domestic and foreign output as follows,

$$\widehat{y}_t \approx \eta\left[-\sigma\left(\widehat{p}_t^H - \widehat{p}_t\right) + \widehat{c}_t\right] + (1-\eta)\left[-\sigma\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) + \widehat{c}_t^*\right], \quad (7.4.10)$$

$$\widehat{y}_t^* \approx \eta^*\left[-\sigma\left(\widehat{p}_t^F - \widehat{p}_t\right) + \widehat{c}_t\right] + (1-\eta^*)\left[-\sigma\left(\widehat{p}_t^{F*} - \widehat{p}_t^*\right) + \widehat{c}_t^*\right], \quad (7.4.11)$$

where $\eta \equiv \frac{n\left(\frac{\xi}{n}\right)}{n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^*}{n}\right)}$ and $\eta^* \equiv \frac{n\left(\frac{1-\xi}{1-n}\right)}{n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)}$. Notice that $\eta = \eta^* = n$ whenever $\xi = \xi^*$, while $\eta = \xi$ and $\eta^* = \xi^*$ if $(1-n)\xi^* = n(1-\xi)$.

Per capita output can also be conveniently re-written if I use the log-linearization of the CPI in (7.1.1) and (7.1.2) as[46],

$$\widehat{y}_t \approx -\sigma\left[\eta(1-\xi)\left(\widehat{p}_t^H - \widehat{p}_t^F\right) + (1-\eta)(1-\xi^*)\left(\widehat{p}_t^{H*} - \widehat{p}_t^{F*}\right)\right] + \left[\eta\widehat{c}_t + (1-\eta)\widehat{c}_t^*\right], \quad (7.4.12)$$

$$\widehat{y}_t^* \approx \sigma\left[\eta^*\xi\left(\widehat{p}_t^H - \widehat{p}_t^F\right) + (1-\eta^*)\xi^*\left(\widehat{p}_t^{H*} - \widehat{p}_t^{F*}\right)\right] + \left[\eta^*\widehat{c}_t + (1-\eta^*)\widehat{c}_t^*\right], \quad (7.4.13)$$

or, based on the log-linearization of the definition of relative prices,

$$\widehat{y}_t \approx \sigma\left[\eta(1-\xi)\widehat{rp}_t - (1-\eta)(1-\xi^*)\widehat{rp}_t^*\right] + \left[\eta\widehat{c}_t + (1-\eta)\widehat{c}_t^*\right], \quad (7.4.14)$$

$$\widehat{y}_t^* \approx \sigma\left[-\eta^*\xi\widehat{rp}_t + (1-\eta^*)\xi^*\widehat{rp}_t^*\right] + \left[\eta^*\widehat{c}_t + (1-\eta^*)\widehat{c}_t^*\right], \quad (7.4.15)$$

where

$$\widehat{rp}_t = \widehat{p}_t^F - \widehat{p}_t^H, \quad (7.4.16)$$

$$\widehat{rp}_t^* = -\left(\widehat{p}_t^{F*} - \widehat{p}_t^{H*}\right). \quad (7.4.17)$$

The Y^H and Y^F equations of the model are also instrumental in the determination of the marginal costs faced by firms in the economy.

Remark 10 *I have argued that the dispersion in relative prices that affects the output levels supplied by each firm is of second-order importance as far as output is concerned. Making this claim comes natural from my derivations, and has become part of the jargon used in the literature. I think, however, that it is a somewhat misleading statement based on partial analysis. It does not take into account the endogeneity of the right-hand side variables and the general equilibrium effects of price stickiness that pop-up (even in a first-order approximation) from a fully-specified DSGE model. In the workhorse model, this is quite easy to understand. On one hand, it is true that relative price dispersion across firms occurs in the presence of price-stickiness, but it drops out of the output equation in a first-order approximation. On the other hand,*

[46] It is necessary to check that these formulas are consistent with the approximation of the market clearing condition in (3.5.19). I do not check this consistency condition here, but I will discuss it later in these notes when applied to the frictionless economy (subsection 7.7).

price stickiness has a first-order impact on the pricing decision of firms and, therefore, alters both the path of the price sub-indexes and aggregate consumption. Hence, the effects of price stickiness will also influence the output levels that are ultimately attained by both countries. The impact is there, and can be measured by the size of the output gap!

In other words, within the context of the workhorse model, stickiness matters for the short-run dynamics even in a first-order sense.

The Marginal Costs. The combination of the LM and Y equations allows me to characterize the marginal cost functions that are relevant for the pricing decisions of firms and for the dynamics of inflation. If I put all these equations together, I can re-write the nominal marginal costs as follows,

$$\begin{aligned} &\widehat{mc}_t - \widehat{p}_t^H \equiv (\widehat{w}_t - \widehat{a}_t) - \widehat{p}_t^H \\ &\quad \approx \gamma\widehat{c}_t + \varphi\eta\left[-\sigma\left(\widehat{p}_t^H - \widehat{p}_t\right) + \widehat{c}_t\right] + \varphi(1-\eta)\left[-\sigma\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) + \widehat{c}_t^*\right] - (1+\varphi)\widehat{a}_t + \widehat{p}_t - \widehat{p}_t^H, \end{aligned} \tag{7.4.18}$$

$$\begin{aligned} &\widehat{mc}_t^* - \widehat{p}_t^{F*} \equiv (\widehat{w}_t^* - \widehat{a}_t^*) - \widehat{p}_t^{F*} \\ &\quad \approx \gamma\widehat{c}_t^* + \varphi\eta^*\left[-\sigma\left(\widehat{p}_t^F - \widehat{p}_t\right) + \widehat{c}_t\right] + \varphi(1-\eta^*)\left[-\sigma\left(\widehat{p}_t^{F*} - \widehat{p}_t^*\right) + \widehat{c}_t^*\right] - (1+\varphi)\widehat{a}_t^* + \widehat{p}_t^* - \widehat{p}_t^{F*}, \end{aligned} \tag{7.4.19}$$

Alternatively, using the log-linearization of the definition of relative prices, I can re-label the marginal cost functions as follows,

$$\begin{aligned} \widehat{mc}_t - \widehat{p}_t^H &\approx \gamma\widehat{c}_t + \varphi\widehat{y}_t + (1-\xi)\widehat{rp}_t - (1+\varphi)\widehat{a}_t \\ &\approx \gamma\widehat{c}_t + \varphi\sigma\left[\eta(1-\xi)\widehat{rp}_t - (1-\eta)(1-\xi^*)\widehat{rp}_t^*\right] + \varphi\left[\eta\widehat{c}_t + (1-\eta)\widehat{c}_t^*\right] + (1-\xi)\widehat{rp}_t - (1+\varphi)\widehat{a}_t, \end{aligned} \tag{7.4.20}$$

$$\begin{aligned} \widehat{mc}_t^* - \widehat{p}_t^{F*} &\approx \gamma\widehat{c}_t^* + \varphi\widehat{y}_t^* + \xi^*\widehat{rp}_t^* - (1+\varphi)\widehat{a}_t^* \\ &\approx \gamma\widehat{c}_t^* + \varphi\sigma\left[-\eta^*\xi\widehat{rp}_t + (1-\eta^*)\xi^*\widehat{rp}_t^*\right] + \varphi\left[\eta^*\widehat{c}_t + (1-\eta^*)\widehat{c}_t^*\right] + \xi^*\widehat{rp}_t^* - (1+\varphi)\widehat{a}_t^*. \end{aligned} \tag{7.4.21}$$

Based on this equation, it clearly follows that the marginal cost is a function of relative prices and aggregate consumption. More generally, I can infer that,

$$\begin{aligned} &\widehat{mc}_t - \widehat{p}_t^H \approx \gamma\widehat{c}_t + \varphi\left[\eta\widehat{c}_t + (1-\eta)\widehat{c}_t^*\right] + \varphi\sigma\left[\eta(1-\xi)\widehat{rp}_t - (1-\eta)(1-\xi^*)\widehat{rp}_t^*\right] + (1-\xi)\widehat{rp}_t - \\ &\quad - (1+\varphi)\widehat{a}_t, \end{aligned} \tag{7.4.22}$$

$$\begin{aligned} &\widehat{mc}_t - \widehat{p}_t^{H*} - \widehat{s}_t \approx \gamma\widehat{c}_t + \varphi\left[\eta\widehat{c}_t + (1-\eta)\widehat{c}_t^*\right] + \varphi\sigma\left[\eta(1-\xi)\widehat{rp}_t - (1-\eta)(1-\xi^*)\widehat{rp}_t^*\right] + \widehat{tot}_t - \xi\widehat{rp}_t - \\ &\quad - (1+\varphi)\widehat{a}_t, \end{aligned} \tag{7.4.23}$$

$$\begin{aligned} &\widehat{mc}_t^* - \widehat{p}_t^F + \widehat{s}_t \approx \gamma\widehat{c}_t^* + \varphi\left[\eta^*\widehat{c}_t + (1-\eta^*)\widehat{c}_t^*\right] + \varphi\sigma\left[-\eta^*\xi\widehat{rp}_t + (1-\eta^*)\xi^*\widehat{rp}_t^*\right] - \widehat{tot}_t - (1-\xi^*)\widehat{rp}_t^* - \\ &\quad - (1+\varphi)\widehat{a}_t^*, \end{aligned} \tag{7.4.24}$$

$$\begin{aligned} &\widehat{mc}_t^* - \widehat{p}_t^{F*} \approx \gamma\widehat{c}_t^* + \varphi\left[\eta^*\widehat{c}_t + (1-\eta^*)\widehat{c}_t^*\right] + \varphi\sigma\left[-\eta^*\xi\widehat{rp}_t + (1-\eta^*)\xi^*\widehat{rp}_t^*\right] + \xi^*\widehat{rp}_t^* - \\ &\quad - (1+\varphi)\widehat{a}_t^*. \end{aligned} \tag{7.4.25}$$

Therefore, the inflation dynamics of the price sub-indexes in (7.3.53), (7.3.54), (7.3.57) and (7.3.58) are determined as follows,

$$\widehat{\pi}_t^H \approx \beta \mathbb{E}_t\left[\widehat{\pi}_{t+1}^H\right] + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\left[\begin{array}{c} \gamma\widehat{c}_t + \varphi\left(\eta\widehat{c}_t + (1-\eta)\widehat{c}_t^*\right) + \\ +\varphi\sigma\left(\eta(1-\xi)\widehat{rp}_t - (1-\eta)(1-\xi^*)\widehat{rp}_t^*\right) + (1-\xi)\widehat{rp}_t - (1+\varphi)\widehat{a}_t \end{array}\right], \tag{7.4.26}$$

$$\widehat{\pi}_t^{H*} \approx \beta \mathbb{E}_t\left[\widehat{\pi}_{t+1}^{H*}\right] + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\left[\begin{array}{c} \gamma\widehat{c}_t + \varphi\left(\eta\widehat{c}_t + (1-\eta)\widehat{c}_t^*\right) + \\ +\varphi\sigma\left(\eta(1-\xi)\widehat{rp}_t - (1-\eta)(1-\xi^*)\widehat{rp}_t^*\right) + \widehat{tot}_t - \xi\widehat{rp}_t - (1+\varphi)\widehat{a}_t \end{array}\right], \tag{7.4.27}$$

$$\widehat{\pi}_t^F \approx \beta \mathbb{E}_t\left[\widehat{\pi}_{t+1}^F\right] + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*}\left[\begin{array}{c} \gamma\widehat{c}_t^* + \varphi\left(\eta^*\widehat{c}_t + (1-\eta^*)\widehat{c}_t^*\right) + \\ +\varphi\sigma\left(-\eta^*\xi\widehat{rp}_t + (1-\eta^*)\xi^*\widehat{rp}_t^*\right) - \widehat{tot}_t - (1-\xi^*)\widehat{rp}_t^* - (1+\varphi)\widehat{a}_t^* \end{array}\right], \tag{7.4.28}$$

$$\widehat{\pi}_t^{F*} \approx \beta \mathbb{E}_t\left[\widehat{\pi}_{t+1}^{F*}\right] + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*}\left[\begin{array}{c} \gamma\widehat{c}_t^* + \varphi\left(\eta^*\widehat{c}_t + (1-\eta^*)\widehat{c}_t^*\right) + \\ +\varphi\sigma\left(-\eta^*\xi\widehat{rp}_t + (1-\eta^*)\xi^*\widehat{rp}_t^*\right) + \xi^*\widehat{rp}_t^* - (1+\varphi)\widehat{a} \end{array}\right]. \tag{7.4.29}$$

This system of four equations suffices to characterizes the dynamics of inflation in both the domestic and foreign economies when combined with the log-linearized CPI indexes in (7.1.1) and (7.1.2). Once again, the structure of these equations is the same independently of whether the asset markets are complete or incomplete in the model.

7.5 The Equilibrium Trade Balance

The *TB* Equation. The real trade balance equation, TB, comes from the log-linearization of equation (3.5.37) by parts, which holds whether asset markets are complete or incomplete, i.e.

$$\frac{TB_t}{P_t} = \frac{\xi}{n}\left(\frac{P_t^H}{P_t}\right)^{1-\sigma} nC_t + \frac{\xi^*}{n}\left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma}(1-n)\, RS_t C_t^* - C_t.$$

What I do is I log-linearize $n\frac{PPI_t}{P_t}Y_t$ and nC_t, and I define the log-linear approximation of the real trade balance as the difference. Then, it follows immediately that,

$$\begin{array}{c} \widehat{tb}_t - \widehat{p}_t \equiv \frac{\xi^*(1-n)}{\xi n+\xi^*(1-n)}\widehat{rs}_t + (1-\sigma)\frac{\xi n}{\xi n+\xi^*(1-n)}\left(\widehat{p}_t^H - \widehat{p}_t\right) + \\ +(1-\sigma)\frac{\xi^*(1-n)}{\xi n+\xi^*(1-n)}\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) - \frac{\xi^*(1-n)}{\xi n+\xi^*(1-n)}\widehat{c}_t^R, \end{array} \tag{7.5.1}$$

where $\widehat{c}_t^R \equiv \widehat{c}_t - \widehat{c}_t^*$. I use the normalization of the steady state discussed in subsection 6.1, which implies that $(1-n)\xi^* = n(1-\xi)$ ensures a zero trade balance in steady state, in order to simplify the equation above as,

$$\widehat{tb}_t - \widehat{p}_t \equiv (1-\xi)\widehat{rs}_t + (1-\sigma)\left(\xi\left(\widehat{p}_t^H - \widehat{p}_t\right) + (1-\xi)\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right)\right) - (1-\xi)\widehat{c}_t^R. \tag{7.5.2}$$

Finally, if I use the log-linearization of the CPI in (7.1.1) and (7.1.2), I am able to write the trade balance more compactly as,

$$\widehat{tb}_t - \widehat{p}_t \equiv (1-\eta)\widehat{rs}_t + (\sigma-1)\left(\eta(1-\xi)\widehat{rp}_t - (1-\eta)(1-\xi^*)\widehat{rp}_t^*\right) - (1-\eta)\widehat{c}_t^R, \tag{7.5.3}$$

where the composite parameter is $\eta \equiv \frac{n\left(\frac{\xi}{n}\right)}{n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^*}{n}\right)}$ and the relative exchange prices are defined as,

$$\begin{aligned} \widehat{rp}_t &= \widehat{p}_t^F - \widehat{p}_t^H, \\ \widehat{rp}_t^* &= -\left(\widehat{p}_t^{F*} - \widehat{p}_t^{H*}\right). \end{aligned}$$

In this equation, $\widehat{tb}_t$ measures the per capita nominal trade balance, and $\widehat{rp}_t^W \equiv \eta\xi\left(1-\xi\right)\widehat{rp}_t - \left(1-\eta\right)\xi\left(1-\xi^*\right)\widehat{rp}_t^*$ tracks the world relative exchange prices[47].

It also shows that the trade balance expressed in real terms (in units of the domestic consumption basket of goods) depends on consumption in both countries as well as on changes in the real exchange rate and world relative exchange prices. If PPP holds, then expenditure switching across countries comes exclusively from fluctuations of the world relative exchange prices. If PPP does not hold, then, expenditure switching will no longer depend on the world relative exchange prices alone.

7.6 The Money Market Equilibrium Conditions

The *MM* Equations. The MM equations, MM^H and MM^F, are easily derived from the money-market clearing conditions in (3.4.8) – (3.4.9). Log-linearizing these equations it follows that[48],

$$\widehat{m}_t - \widehat{p}_t \approx \frac{\gamma}{\zeta}\widehat{c}_t - \frac{1}{\zeta}\left(\frac{\beta\left(1+\overline{i}^m\right)}{1-\beta\left(1+\overline{i}^m\right)}\right)\left(\widehat{i}_t - \widehat{i}_t^m\right), \tag{7.6.1}$$

$$\widehat{m}_t^* - \widehat{p}_t^* \approx \frac{\gamma}{\zeta}\widehat{c}_t^* - \frac{1}{\zeta}\left(\frac{\beta\left(1+\overline{i}^{m*}\right)}{1-\beta\left(1+\overline{i}^{m*}\right)}\right)\left(\widehat{i}_t^* - \widehat{i}_t^{m*}\right), \tag{7.6.2}$$

For the most part, in these notes I assume for simplicity that $\overline{i}^m = \overline{i}^{m*} = 0$ and $\widehat{i}_t^m = \widehat{i}_t^{m*} = 0$ for all t. This reduces somewhat the complexity of the problem, since it gives me a trivial rule for the administered rate on money. It also ensures that money is endogenously determined given certain initial conditions.

In their more general form, these equations say that the real balances are cointegrated with the spread between the nominal interest rate and the administered rate on money, and with aggregate consumption (instead of aggregate output as in Cagan's money demand functions). Taking the difference between (7.6.1) and (7.6.2) and using the definition of the real exchange rate is easy to derive an expression for the nominal interest rate spread across both countries as,

$$\left(\frac{(1-\beta)\left(1+\overline{i}^m\right)}{1-\beta\left(1+\overline{i}^m\right)}\right)\left(\widehat{i}_t - \widehat{i}_t^m\right) - \left(\frac{(1-\beta)\left(1+\overline{i}^{m*}\right)}{1-\beta\left(1+\overline{i}^{m*}\right)}\right)\left(\widehat{i}_t^* - \widehat{i}_t^{m*}\right) \approx \zeta\left(\frac{1-\beta}{\beta}\right)\left[-\widehat{m}_t^R + \frac{\gamma}{\zeta}\widehat{c}_t^R + \widehat{p}_t^R\right], \tag{7.6.3}$$

[47] Whenever $\widehat{rp}_t^W > 0$, then foreign-produced goods are relatively more expensive and world demand is shifted towards home-produced goods.

[48] The approximation is calculated relative to gross interest rates, not relative to net interest rates.

or, simply,

$$\widehat{i}_t - \left(\frac{1-\beta\left(1+\overline{i}^m\right)}{1-\beta\left(1+\overline{i}^{m*}\right)}\right)\left(\frac{1+\overline{i}^{m*}}{1+\overline{i}^m}\right)\widehat{i}_t^* \approx \widehat{i}_t^m - \left(\frac{1-\beta\left(1+\overline{i}^m\right)}{1-\beta\left(1+\overline{i}^{m*}\right)}\right)\left(\frac{1+\overline{i}^{m*}}{1+\overline{i}^m}\right)\widehat{i}_t^{m*} + \zeta\left(\frac{1-\beta\left(1+\overline{i}^m\right)}{\beta\left(1+\overline{i}^m\right)}\right)\left[-\widehat{m}_t^R + \frac{\gamma}{\zeta}\widehat{c}_t^R + \widehat{p}_t^R\right] \tag{7.6.4}$$

where $\widehat{c}_t^R \equiv \widehat{c}_t - \widehat{c}_t^*$, $\widehat{m}_t^R \equiv \widehat{m}_t - \widehat{m}_t^*$, and $\widehat{p}_t^R \equiv \widehat{p}_t - \widehat{p}_t^* = \widehat{s}_t - \widehat{rs}_t$. Assuming merely that $\overline{i}^m = \overline{i}^{m*}$, this implies that,

$$\widehat{i}_t^R \approx \widehat{i}_t^{Rm} + \zeta\left(\frac{1-\beta\left(1+\overline{i}^m\right)}{\beta\left(1+\overline{i}^m\right)}\right)\left[-\widehat{m}_t^R + \frac{\gamma}{\zeta}\widehat{c}_t^R + \widehat{p}_t^R\right], \tag{7.6.5}$$

where $\widehat{i}_t^R \equiv \widehat{i}_t - \widehat{i}_t^*$, and $\widehat{i}_t^{Rm} \equiv \widehat{i}_t^m - \widehat{i}_t^{m*}$. In words, this tells me that the interest rate differential is inversely proportional to the money supply differential, to relative consumption and to the relative CPIs across countries.

7.7 The Frictionless Allocation for Potential Output

The labor market equilibrium conditions of the frictionless allocation in equations (4.3) and (4.4) can be easily log-linearized as,

$$-\left(1+\varphi\right)\widehat{a}_t + \gamma\widehat{\overline{c}}_t + \varphi\widehat{\overline{y}}_t - \left(\widehat{\overline{p}}_t^H - \widehat{\overline{p}}_t\right) \approx 0, \tag{7.7.1}$$

$$-\left(1+\varphi\right)\widehat{a}_t^* + \gamma\widehat{\overline{c}}_t^* + \varphi\widehat{\overline{y}}_t^* - \left(\widehat{\overline{p}}_t^{F*} - \widehat{\overline{p}}_t^*\right) \approx 0. \tag{7.7.2}$$

Similarly, the perfect international risk-sharing condition under complete asset markets described in equation (3.2.29) can be log-linearized to take the form,

$$\widehat{\overline{c}}_t \approx \frac{1}{\gamma}\widehat{\overline{rs}}_t + \widehat{\overline{c}}_t^*. \tag{7.7.3}$$

Re-labelling equation (7.1.3) I can express the log-linearized real exchange rate as,

$$\widehat{\overline{rs}}_t \approx \widehat{\overline{s}}_t + \left(\xi^*\widehat{\overline{p}}_t^{H*} + (1-\xi^*)\,\widehat{\overline{p}}_t^{F*}\right) - \left(\xi\widehat{\overline{p}}_t^H + (1-\xi)\,\widehat{\overline{p}}_t^F\right), \tag{7.7.4}$$

while the relevant definitions of terms of trade that apply to the frictionless model are $\widehat{\overline{tot}}_t \equiv \widehat{\overline{p}}_t^F - \widehat{\overline{p}}_t^H$ and $\widehat{\overline{tot}}_t^* \equiv \widehat{\overline{p}}_t^{H*} - \widehat{\overline{p}}_t^{F*} = -\widehat{\overline{tot}}_t$. The assumption of the frictionless model is that prices are fully flexible, so the LOOP holds at the variety level and, due to symmetry, also at the level of locally-produced bundles of goods. Hence, manipulating equation (7.1.3) I can re-write the log-linearized real exchange rate as a function of the terms of trade,

$$\begin{aligned}\widehat{\overline{rs}}_t &\equiv \widehat{\overline{p}}_t^* + \widehat{\overline{s}}_t - \widehat{\overline{p}}_t \\ &\approx (\xi-\xi^*)\left(\widehat{\overline{p}}_t^F - \widehat{\overline{p}}_t^H\right) = (\xi-\xi^*)\,\widehat{\overline{tot}}_t.\end{aligned} \tag{7.7.5}$$

In other words, up to a first-order approximation, the real exchange rate is unaffected by the elasticity of substitution across bundles, but depends upon the assumption of home bias in consumption.

Given that the LOOP holds under flexible prices (see equations (3.3.14) – (3.3.15)), it is possible to re-write the output equations for both countries as in (3.3.36) – (3.3.37). After re-labelling those equations to take account of the fact that I am referring to the frictionless allocation, I log-linearize them around the normalized steady state to obtain that,

$$\widehat{\overline{y}}_t \approx -\sigma\left(\widehat{\overline{p}}_t^H - \widehat{\overline{p}}_t\right) + \sigma(1-\eta)\widehat{\overline{rs}}_t + \eta\widehat{\overline{c}}_t + (1-\eta)\widehat{\overline{c}}_t^*, \tag{7.7.6}$$

$$\widehat{\overline{y}}_t^* \approx -\sigma\left(\widehat{\overline{p}}_t^{F*} - \widehat{\overline{p}}_t^*\right) - \sigma\eta^*\widehat{\overline{rs}}_t + \eta^*\widehat{\overline{c}}_t + (1-\eta^*)\widehat{\overline{c}}_t^*, \tag{7.7.8}$$

where $\eta \equiv \frac{n\left(\frac{\xi}{n}\right)}{n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^*}{n}\right)}$ and $\eta^* \equiv \frac{n\left(\frac{1-\xi}{1-n}\right)}{n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)}$. It must be noted, once again, that the condition on the home bias parameters applied for the irrelevance of asset markets, i.e. $n(1-\xi) = (1-n)\xi^*$, implies here that $\eta \equiv \xi$ and $\eta^* \equiv \xi^*$. Taking the difference between the output level in both countries, I can say that,

$$\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^* \approx -\sigma\left(\left(\widehat{\overline{p}}_t^H - \widehat{\overline{p}}_t\right) - \left(\widehat{\overline{p}}_t^{F*} - \widehat{\overline{p}}_t^*\right)\right) + \sigma(1-(\eta-\eta^*))\widehat{\overline{rs}}_t + (\eta-\eta^*)\left(\widehat{\overline{c}}_t - \widehat{\overline{c}}_t^*\right) \tag{7.7.9}$$

$$= -\sigma\left(\left(\widehat{\overline{p}}_t^H - \widehat{\overline{p}}_t^{F*} - \widehat{\overline{s}}_t\right) - \left(\widehat{\overline{p}}_t - \widehat{\overline{p}}_t^* - \widehat{\overline{s}}_t\right)\right) + \sigma(1-(\eta-\eta^*))\widehat{\overline{rs}}_t + (\eta-\eta^*)\left(\widehat{\overline{c}}_t - \widehat{\overline{c}}_t^*\right) \tag{7.7.10}$$

Using the terms of trade and the real exchange rate definitions and recalling that under flexible prices the LOOP holds (and $\widehat{\overline{p}}_t^{F*} - \widehat{\overline{s}}_t = \widehat{\overline{p}}_t^F$), I can re-write the difference above as,

$$\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^* \approx -\sigma\left(\widehat{\overline{rs}}_t - \widehat{\overline{tot}}_t\right) + \sigma(1-(\eta-\eta^*))\widehat{\overline{rs}}_t + (\eta-\eta^*)\left(\widehat{\overline{c}}_t - \widehat{\overline{c}}_t^*\right). \tag{7.7.11}$$

Using equations (7.7.3) and (7.7.5), I derive the following expressions for the real exchange rate and terms of trade[49],

$$\widehat{\overline{rs}}_t \approx \left[\sigma\left(\frac{1-(\xi-\xi^*)}{\xi-\xi^*}\right) + \sigma(1-(\eta-\eta^*)) + \frac{1}{\gamma}(\eta-\eta^*)\right]^{-1}\left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^*\right), \tag{7.7.12}$$

$$\widehat{\overline{tot}}_t \approx \left[\sigma(1-(\xi-\xi^*)) + \sigma(\xi-\xi^*)(1-(\eta-\eta^*)) + \frac{1}{\gamma}(\xi-\xi^*)(\eta-\eta^*)\right]^{-1}\left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^*\right), \tag{7.7.13}$$

which indicates that both relative prices are proportional to the output differential across countries.

Notice that these equations simplify further if I assume that $n(1-\xi) = (1-n)\xi^*$,

$$\widehat{\overline{rs}}_t \approx \left[\frac{\xi-\xi^*}{\sigma\left(1-(\xi-\xi^*)^2\right) + \frac{1}{\gamma}(\xi-\xi^*)^2}\right]\left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^*\right), \tag{7.7.14}$$

$$\widehat{\overline{tot}}_t \approx \left[\frac{1}{\sigma\left(1-(\xi-\xi^*)^2\right) + \frac{1}{\gamma}(\xi-\xi^*)^2}\right]\left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^*\right), \tag{7.7.15}$$

where these formulas depend on the home bias parameters but also on a weighted average of the elasticity of substitution across bundles of goods, σ, and the intertemporal elasticity of substitution, $\frac{1}{\gamma}$. The intertemporal

[49] These calculations do not depend on the assumptions I made regarding the technology used in this economy. Therefore, the same kind of relationships should hold also if I had a production techology that uses other inputs rather than labor alone, specially capital.

elasticity of substitution, however, only plays a role in determining the sensitivity of the real exchange rate to the cross-country output differentials whenever I assume home bias in consumption, i.e. $\xi \neq \xi^*$. Based on the structure of the consumption price indexes in (3.2.14) – (3.2.15), and the log-linearization in (7.1.1) and (7.1.2), I can argue that the following price ratios,

$$\widehat{\overline{p}}_t^H - \widehat{\overline{p}}_t \approx (1-\xi)\left(\widehat{\overline{p}}_t^H - \widehat{\overline{p}}_t^F\right) = -(1-\xi)\widehat{\overline{tot}}_t, \tag{7.7.16}$$

$$\widehat{\overline{p}}_t^{F*} - \widehat{\overline{p}}_t^* \approx -\xi^*\left(\widehat{\overline{p}}_t^{H*} - \widehat{\overline{p}}_t^{F*}\right) = \xi^*\widehat{\overline{tot}}_t, \tag{7.7.17}$$

are also related to terms of trade.

Then, I combine all these results in order to write domestic and foreign output as,

$$\begin{aligned}
\widehat{\overline{y}}_t &\approx \sigma(1-\xi)\widehat{\overline{tot}}_t + \sigma(1-\eta)\widehat{\overline{rs}}_t + \eta\widehat{\overline{c}}_t + (1-\eta)\widehat{\overline{c}}_t^* \\
&\approx \sigma\left[\frac{1-\xi}{\xi-\xi^*} + (1-\eta)\right]\widehat{\overline{rs}}_t - (1-\eta)\left(\widehat{\overline{c}}_t - \widehat{\overline{c}}_t^*\right) + \widehat{\overline{c}}_t, \qquad (7.7.18)\\
\widehat{\overline{y}}_t^* &\approx -\sigma\xi^*\widehat{\overline{tot}}_t - \sigma\eta^*\widehat{\overline{rs}}_t + \eta^*\widehat{\overline{c}}_t + (1-\eta^*)\widehat{\overline{c}}_t^* \\
&\approx -\sigma\left[\frac{\xi^*}{\xi-\xi^*} + \eta^*\right]\widehat{\overline{rs}}_t + \eta^*\left(\widehat{\overline{c}}_t - \widehat{\overline{c}}_t^*\right) + \widehat{\overline{c}}_t^*. \qquad (7.7.19)
\end{aligned}$$

Using one more time the perfect risk-sharing condition derived in (7.7.3), I can re-write the domestic and foreign consumption in terms of domestic and foreign output as,

$$\widehat{\overline{y}}_t \approx \left[\sigma\left(\frac{1-\xi}{\xi-\xi^*} + (1-\eta)\right) - \frac{1}{\gamma}(1-\eta)\right]\widehat{\overline{rs}}_t + \widehat{\overline{c}}_t, \tag{7.7.20}$$

$$\widehat{\overline{y}}_t^* \approx -\left[\sigma\left(\frac{\xi^*}{\xi-\xi^*} + \eta^*\right) - \frac{1}{\gamma}\eta^*\right]\widehat{\overline{rs}}_t + \widehat{\overline{c}}_t^*, \tag{7.7.21}$$

and

$$\begin{aligned}
\widehat{\overline{c}}_t &\approx \widehat{\overline{y}}_t - \left[\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^*} + (1-\eta)\right) - \frac{1}{\gamma}(1-\eta)}{\sigma\left(\frac{1-(\xi-\xi^*)}{\xi-\xi^*}\right) + \sigma(1-(\eta-\eta^*)) + \frac{1}{\gamma}(\eta-\eta^*)}\right]\left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^*\right) \\
&= \left[\frac{\sigma\left(\frac{\xi^*}{\xi-\xi^*} + \eta^*\right) + \frac{1}{\gamma}(1-\eta^*)}{\sigma\left(\frac{1}{\xi-\xi^*} - (\eta-\eta^*)\right) + \frac{1}{\gamma}(\eta-\eta^*)}\right]\widehat{\overline{y}}_t + \left[\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^*} + (1-\eta)\right) - \frac{1}{\gamma}(1-\eta)}{\sigma\left(\frac{1}{\xi-\xi^*} - (\eta-\eta^*)\right) + \frac{1}{\gamma}(\eta-\eta^*)}\right]\widehat{\overline{y}}_t^*, \qquad (7.7.22)\\
\widehat{\overline{c}}_t^* &\approx \widehat{\overline{y}}_t^* + \left[\frac{\sigma\left(\frac{\xi^*}{\xi-\xi^*} + \eta^*\right) - \frac{1}{\gamma}\eta^*}{\sigma\left(\frac{1-(\xi-\xi^*)}{\xi-\xi^*}\right) + \sigma(1-(\eta-\eta^*)) + \frac{1}{\gamma}(\eta-\eta^*)}\right]\left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^*\right) \\
&= \left[\frac{\sigma\left(\frac{\xi^*}{\xi-\xi^*} + \eta^*\right) - \frac{1}{\gamma}\eta^*}{\sigma\left(\frac{1}{\xi-\xi^*} - (\eta-\eta^*)\right) + \frac{1}{\gamma}(\eta-\eta^*)}\right]\widehat{\overline{y}}_t + \left[\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^*} + (1-\eta)\right) + \frac{1}{\gamma}\eta}{\sigma\left(\frac{1}{\xi-\xi^*} - (\eta-\eta^*)\right) + \frac{1}{\gamma}(\eta-\eta^*)}\right]\widehat{\overline{y}}_t^*. \qquad (7.7.23)
\end{aligned}$$

Assuming that $n(1-\xi) = (1-n)\xi^*$, the equations can be further simplified as,

$$\begin{aligned}
\widehat{\overline{c}}_t &\approx \widehat{\overline{y}}_t - \left[\frac{\sigma(1-\xi)(1+(\xi-\xi^*)) - \frac{1}{\gamma}(1-\xi)(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right) + \frac{1}{\gamma}(\xi-\xi^*)^2} \right] \left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^* \right) \\
&= \left[\frac{\sigma\xi^*(1+(\xi-\xi^*)) + \frac{1}{\gamma}(1-\xi^*)(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right) + \frac{1}{\gamma}(\xi-\xi^*)^2} \right] \widehat{\overline{y}}_t + \left[\frac{\sigma(1-\xi)(1+(\xi-\xi^*)) - \frac{1}{\gamma}(1-\xi)(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right) + \frac{1}{\gamma}(\xi-\xi^*)^2} \right] \widehat{\overline{y}}_t^*, \qquad (7.7.24) \\
\widehat{\overline{c}}_t^* &\approx \widehat{\overline{y}}_t^* + \left[\frac{\sigma\xi^*(1+(\xi-\xi^*)) - \frac{1}{\gamma}\xi^*(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right) + \frac{1}{\gamma}(\xi-\xi^*)^2} \right] \left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^* \right) \\
&= \left[\frac{\sigma\xi^*(1+(\xi-\xi^*)) - \frac{1}{\gamma}\xi^*(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right) + \frac{1}{\gamma}(\xi-\xi^*)^2} \right] \widehat{\overline{y}}_t + \left[\frac{\sigma(1-\xi)(1+(\xi-\xi^*)) + \frac{1}{\gamma}\xi(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right) + \frac{1}{\gamma}(\xi-\xi^*)^2} \right] \widehat{\overline{y}}_t^*, \qquad (7.7.25)
\end{aligned}$$

which tells me that the consumption in each country is a weighted function of the output produced in both countries (weighted according to the households' preferences).

Remark 11 Consistency with the market clearing conditions. *I know that the sum of the real aggregate output in both countries equals the sum of the real aggregate consumption which follows from the market clearing conditions in* (3.5.19)*. Hence, after log-linearizing it shall follow that,*

$$n\widehat{\overline{c}}_t + (1-n)\widehat{\overline{c}}_t^* \approx \frac{\overline{Y}}{n\overline{Y} + (1-n)\overline{Y}^*} n\widehat{\overline{y}}_t + \frac{\overline{Y}^*}{n\overline{Y} + (1-n)\overline{Y}^*}(1-n)\widehat{\overline{y}}_t^*, \qquad (7.7.26)$$

where it follows from the steady state derivations in (6.2.10) *and* (6.2.11) *that*

$$\begin{aligned}
n\overline{Y} &= \left[\frac{n^\varphi}{\kappa} \left(\frac{\xi}{\eta} \right)^\gamma \right]^{\frac{1}{\gamma+\varphi}} (A)^{\frac{1+\varphi}{\gamma+\varphi}}, \\
(1-n)\overline{Y}^* &= \left[\frac{(1-n)^\varphi}{\kappa} \left(\frac{n\left(\frac{1-\xi}{1-n}\right)}{\eta^*} \right)^\gamma \right]^{\frac{1}{\gamma+\varphi}} (A^*)^{\frac{1+\varphi}{\gamma+\varphi}}, \\
n\overline{Y} + (1-n)\overline{Y}^* &= \left[\frac{n^\varphi}{\kappa} \left(\frac{\xi}{\eta} \right)^\gamma \right]^{\frac{1}{\gamma+\varphi}} (A)^{\frac{1+\varphi}{\gamma+\varphi}} + \left[\frac{(1-n)^\varphi}{\kappa} \left(\frac{n\left(\frac{1-\xi}{1-n}\right)}{\eta^*} \right)^\gamma \right]^{\frac{1}{\gamma+\varphi}} (A^*)^{\frac{1+\varphi}{\gamma+\varphi}}.
\end{aligned}$$

The normalization, applied to any choice of the consumption bias parameters, requires that

$$A = 1, \; A^* = \left(\frac{n}{1-n} \right)^{\frac{\varphi}{1+\varphi}} \left[\frac{\frac{n\left(\frac{1-\xi}{1-n}\right)}{\eta^*}}{\frac{n\left(\frac{\xi}{n}\right)}{\eta}} \right]^{\frac{\varphi}{1+\varphi}} = \left[\left(\frac{n}{1-n} \right)^2 \frac{\eta}{\eta^*} \frac{(1-\xi)}{\xi} \right]^{\frac{\varphi}{1+\varphi}},$$

as per (6.2.13)*. Therefore, it follows that,*

$$
\begin{aligned}
n\overline{Y} &= \left[\frac{n^{\varphi}}{\kappa}\left(\frac{\xi}{\eta}\right)^{\gamma}\right]^{\frac{1}{\gamma+\varphi}}, \\
(1-n)\,\overline{Y}^{*} &= \left[\frac{n^{\varphi}}{\kappa}\left(\frac{\xi}{\eta}\right)^{\gamma}\right]^{\frac{1}{\gamma+\varphi}}\left(\frac{n}{1-n}\frac{\eta}{\eta^{*}}\frac{1-\xi}{\xi}\right), \\
n\overline{Y}+(1-n)\,\overline{Y}^{*} &= \left[\frac{n^{\varphi}}{\kappa}\left(\frac{\xi}{\eta}\right)^{\gamma}\right]^{\frac{1}{\gamma+\varphi}}\left(1+\frac{n}{1-n}\frac{\eta}{\eta^{*}}\frac{1-\xi}{\xi}\right).
\end{aligned}
$$

As a result I can express the market clearing condition becomes,

$$
n\widehat{\overline{c}}_t+(1-n)\,\widehat{\overline{c}}_t^{*}\approx\frac{\eta^{*}(1-n)\,\xi}{\eta^{*}(1-n)\,\xi+\eta n(1-\xi)}\widehat{\overline{y}}_t+\frac{\eta n(1-\xi)}{\eta^{*}(1-n)\,\xi+\eta n(1-\xi)}\widehat{\overline{y}}_t^{*}.
$$

Notice that often I impose that $n(1-\xi)=(1-n)\,\xi^{*}$ *on the home bias parameter, which further implies that* $\eta\equiv\xi$ *and* $\eta^{*}\equiv\xi^{*}$*. In this case, it must be true that*

$$
\frac{n}{1-n}\frac{\eta}{\eta^{*}}\frac{1-\xi}{\xi}=\frac{n}{1-n}\frac{1-\xi}{\xi^{*}}=1,
$$

so $n\overline{Y}=(1-n)\,\overline{Y}^{*}$*, and*

$$
n\widehat{\overline{c}}_t+(1-n)\,\widehat{\overline{c}}_t^{*}\approx\frac{1}{2}\widehat{\overline{y}}_t+\frac{1}{2}\widehat{\overline{y}}_t^{*}.
$$

If I aggregate equations (7.7.22) *and* (7.7.23) *accordingly, it shall follow that,*

$$
n\widehat{\overline{c}}_t+(1-n)\,\widehat{\overline{c}}_t^{*}\approx\left[\frac{\sigma\left(\frac{\xi^{*}}{\xi-\xi^{*}}+\eta^{*}\right)+\frac{1}{\gamma}\left(n(1-\eta^{*})-(1-n)\,\eta^{*}\right)}{\sigma\left(\frac{1}{\xi-\xi^{*}}-(\eta-\eta^{*})\right)+\frac{1}{\gamma}(\eta-\eta^{*})}\right]\widehat{\overline{y}}_t+\left[\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^{*}}+(1-\eta)\right)+\frac{1}{\gamma}\left((1-n)\,\eta-n(1-\eta)\right)}{\sigma\left(\frac{1}{\xi-\xi^{*}}-(\eta-\eta^{*})\right)+\frac{1}{\gamma}(\eta-\eta^{*})}\right]\widehat{\overline{y}}_t^{*}. \tag{7.7.27}
$$

This is a non-trivial matter, because it implies that in order for the approximation to be consistent with the market clearing condition, the elasticity of substitution across bundles of domestic and foreign goods, σ*, and the intertemporal elasticity of substitution,* $\frac{1}{\gamma}$*, must satisfy the following two parametric conditions,*

$$
\frac{\sigma\left(\frac{\xi^{*}}{\xi-\xi^{*}}+\eta^{*}\right)+\frac{1}{\gamma}\left(n(1-\eta^{*})-(1-n)\,\eta^{*}\right)}{\sigma\left(\frac{1}{\xi-\xi^{*}}-(\eta-\eta^{*})\right)+\frac{1}{\gamma}(\eta-\eta^{*})}=\frac{\eta^{*}(1-n)\,\xi}{\eta^{*}(1-n)\,\xi+\eta n(1-\xi)}, \tag{7.7.28}
$$

$$
\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^{*}}+(1-\eta)\right)+\frac{1}{\gamma}\left((1-n)\,\eta-n(1-\eta)\right)}{\sigma\left(\frac{1}{\xi-\xi^{*}}-(\eta-\eta^{*})\right)+\frac{1}{\gamma}(\eta-\eta^{*})}=\frac{\eta n(1-\xi)}{\eta^{*}(1-n)\,\xi+\eta n(1-\xi)}. \tag{7.7.29}
$$

Since there are two restrictions and two degrees of freedom (the parameters σ *and* $\frac{1}{\gamma}$*), it could happen that no solution exists, that only a solution exists that functionally relates these two elasticities to the home bias parameters or that these equations are satisfied for any possible choice of the elasticities. I will check here only the conditions applying to the special case where* $n(1-\xi)=(1-n)\,\xi^{*}$*, the general case is left as an* *exercise for the reader. Given that parametric restriction,*

$$
\begin{aligned}
\frac{\sigma\left(\frac{\xi^{*}}{\xi-\xi^{*}}+\xi^{*}\right)+\frac{1}{\gamma}\left(n\left(1-\xi^{*}\right)-(1-n)\,\xi^{*}\right)}{\sigma\left(\frac{1}{\xi-\xi^{*}}-\left(\xi-\xi^{*}\right)\right)+\frac{1}{\gamma}\left(\xi-\xi^{*}\right)} &= \frac{1}{2}, \\
\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^{*}}+(1-\xi)\right)+\frac{1}{\gamma}\left((1-n)\,\xi-n\,(1-\xi)\right)}{\sigma\left(\frac{1}{\xi-\xi^{*}}-\left(\xi-\xi^{*}\right)\right)+\frac{1}{\gamma}\left(\xi-\xi^{*}\right)} &= \frac{1}{2}.
\end{aligned}
$$

A little bit of manipulation and it follows that,

$$
\begin{aligned}
\frac{\sigma\left(\frac{\xi^{*}}{\xi-\xi^{*}}+\xi^{*}\right)+n\frac{1}{\gamma}\left(\xi-\xi^{*}\right)}{\sigma\left(\frac{1}{\xi-\xi^{*}}-\left(\xi-\xi^{*}\right)\right)+\frac{1}{\gamma}\left(\xi-\xi^{*}\right)} &= \frac{1}{2}\Longleftrightarrow\frac{1}{\gamma}\left[n-\frac{1}{2}\right]\left(\xi-\xi^{*}\right)=\sigma\frac{1}{2}\left[\frac{1-2\xi^{*}}{\xi-\xi^{*}}-\xi-\xi^{*}\right], \\
\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^{*}}+(1-\xi)\right)+(1-n)\,\frac{1}{\gamma}\left(\xi-\xi^{*}\right)}{\sigma\left(\frac{1}{\xi-\xi^{*}}-\left(\xi-\xi^{*}\right)\right)+\frac{1}{\gamma}\left(\xi-\xi^{*}\right)} &= \frac{1}{2}\Longleftrightarrow\frac{1}{\gamma}\left[n-\frac{1}{2}\right]\left(\xi-\xi^{*}\right)=\sigma\frac{1}{2}\left[\frac{1-2\xi^{*}}{\xi-\xi^{*}}-\xi-\xi^{*}\right].
\end{aligned}
$$

In other words, I end up with one additional parametric constraint which requires that,

$$
\frac{1}{\gamma}\left[n-\frac{1}{2}\right]\left(\xi-\xi^{*}\right)^{2}=\sigma\frac{1}{2}\left[1-2\xi^{*}-\left(\xi+\xi^{*}\right)\left(\xi-\xi^{*}\right)\right]=\sigma\frac{1}{2}\left[1-2\xi^{*}-\xi^{2}+\left(\xi^{*}\right)^{2}\right]=\sigma\frac{1}{2}\left[\left(1-\xi^{*}\right)^{2}-\xi^{2}\right],
$$

or

$$
\frac{1}{\gamma}\left[n-\frac{1}{2}\right]\left(\xi-\frac{n}{1-n}\left(1-\xi\right)\right)^{2}=\sigma\frac{1}{2}\left[\left(1-\frac{n}{1-n}\left(1-\xi\right)\right)^{2}-\xi^{2}\right], \tag{7.7.30}
$$

using that $n\left(1-\xi\right)=\left(1-n\right)\xi^{*}$. In general, this implies that for internal consistency with the market clearing condition, I cannot pick an arbitrary choice for the two elasticities. At the same time, I can also show that if I choose both countries to be symmetric in size, i.e. $n=\frac{1}{2}$, then the left- and the right-hand side terms of this restriction become zero and that allows me the possibility of an unrestricted selection of both elasticities. Making both countries symmetric was also a necessary condition in the standard normalization to be able to assume that the steady state productivity levels in both countries are identical and equal to one.

Mindful of the implicit restriction on the elasticity of substitution across bundles of domestic and foreign goods, σ, and the intertemporal elasticity of substitution, $\frac{1}{\gamma}$, I proceed ahead to derive the implicit relationship between output and productivity shocks. For that, I only have to replace consumption, prices and terms of trade in the log-linearized equilibrium labor market clearing conditions derived in (7.7.1) – (7.7.2) to easily obtain the following system of two equations in two unknowns,

$$
\begin{aligned}
&-\left(1+\varphi\right)\widehat{a}_{t}+\gamma\left[\left(\frac{\sigma\left(\frac{\xi^{*}}{\xi-\xi^{*}}+\eta^{*}\right)+\frac{1}{\gamma}(1-\eta^{*})}{\sigma\left(\frac{1}{\xi-\xi^{*}}-(\eta-\eta^{*})\right)+\frac{1}{\gamma}(\eta-\eta^{*})}\right)\widehat{\overline{y}}_{t}+\left(\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^{*}}+(1-\eta)\right)-\frac{1}{\gamma}(1-\eta)}{\sigma\left(\frac{1}{\xi-\xi^{*}}-(\eta-\eta^{*})\right)+\frac{1}{\gamma}(\eta-\eta^{*})}\right)\widehat{\overline{y}}_{t}^{*}\right]+ \\
&\quad+\varphi\widehat{\overline{y}}_{t}+\left[\frac{(1-\xi)}{\sigma(1-(\xi-\xi^{*})(\eta-\eta^{*}))+\frac{1}{\gamma}(\xi-\xi^{*})(\eta-\eta^{*})}\right]\left(\widehat{\overline{y}}_{t}-\widehat{\overline{y}}_{t}^{*}\right)\approx 0,
\end{aligned} \tag{7.7.31}
$$

$$
\begin{aligned}
&-\left(1+\varphi\right)\widehat{a}_{t}^{*}+\gamma\left[\left(\frac{\sigma\left(\frac{\xi^{*}}{\xi-\xi^{*}}+\eta^{*}\right)-\frac{1}{\gamma}\eta^{*}}{\sigma\left(\frac{1}{\xi-\xi^{*}}-(\eta-\eta^{*})\right)+\frac{1}{\gamma}(\eta-\eta^{*})}\right)\widehat{\overline{y}}_{t}+\left(\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^{*}}+(1-\eta)\right)+\frac{1}{\gamma}\eta}{\sigma\left(\frac{1}{\xi-\xi^{*}}-(\eta-\eta^{*})\right)+\frac{1}{\gamma}(\eta-\eta^{*})}\right)\widehat{\overline{y}}_{t}^{*}\right]+ \\
&\quad+\varphi\widehat{\overline{y}}_{t}^{*}-\left[\frac{\xi^{*}}{\sigma(1-(\xi-\xi^{*})(\eta-\eta^{*}))+\frac{1}{\gamma}(\xi-\xi^{*})(\eta-\eta^{*})}\right]\left(\widehat{\overline{y}}_{t}-\widehat{\overline{y}}_{t}^{*}\right)\approx 0,
\end{aligned} \tag{7.7.32}
$$

or, expressed more compactly,

$$\begin{aligned}&\left[\varphi+\left(\tfrac{\gamma\sigma(\xi^*+(\xi-\xi^*)\eta^*)+(\xi-\xi^*)(1-\eta^*)+(1-\xi)}{\sigma(1-(\xi-\xi^*)(\eta-\eta^*))+\frac{1}{\gamma}(\xi-\xi^*)(\eta-\eta^*)}\right)\right]\widehat{\overline{y}}_t+\\&\quad+\left[\tfrac{\gamma\sigma((1-\xi)+(\xi-\xi^*)(1-\eta))-(\xi-\xi^*)(1-\eta)-(1-\xi)}{\sigma(1-(\xi-\xi^*)(\eta-\eta^*))+\frac{1}{\gamma}(\xi-\xi^*)(\eta-\eta^*)}\right]\widehat{\overline{y}}_t^*\approx(1+\varphi)\,\widehat{a}_t,\end{aligned}\tag{7.7.33}$$

$$\begin{aligned}&\left[\tfrac{\gamma\sigma(\xi^*+(\xi-\xi^*)\eta^*)-(\xi-\xi^*)\eta^*-\xi^*}{\sigma(1-(\xi-\xi^*)(\eta-\eta^*))+\frac{1}{\gamma}(\xi-\xi^*)(\eta-\eta^*)}\right]\widehat{\overline{y}}_t+\\&\quad+\left[\varphi+\left(\tfrac{\gamma\sigma((1-\xi)+(\xi-\xi^*)(1-\eta))+(\xi-\xi^*)\eta+\xi^*}{\sigma(1-(\xi-\xi^*)(\eta-\eta^*))+\frac{1}{\gamma}(\xi-\xi^*)(\eta-\eta^*)}\right)\right]\widehat{\overline{y}}_t^*\approx(1+\varphi)\,\widehat{a}_t^*.\end{aligned}\tag{7.7.34}$$

The solution to this system of equations gives me the following pair of formulas,

$$\begin{aligned}\widehat{\overline{y}}_t&\approx\lambda_a\widehat{a}_t+\lambda_{a^*}\widehat{a}_t^*,\\\widehat{\overline{y}}_t^*&\approx\lambda_a^*\widehat{a}_t+\lambda_{a^*}^*\widehat{a}_t^*,\end{aligned}$$

where the coefficients are defined as follows,

$$\begin{aligned}\lambda_a&\equiv\frac{1+\varphi}{\gamma+\varphi}\left[\begin{array}{c}\frac{\gamma\xi^*+\sigma\gamma^2+\varphi\xi^*\eta^*-\sigma\gamma^2\xi^*+\sigma\gamma\varphi+\gamma\xi\eta+\xi\eta\varphi-\gamma\eta\xi^*-\xi\varphi\eta^*-\eta\varphi\xi^*-\sigma\gamma^2\xi\eta}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}+\\+\frac{\sigma\gamma^2\eta\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}\end{array}\right],\\\lambda_{a^*}&\equiv\left(\sigma-\frac{1}{\gamma}\right)\frac{1+\varphi}{\gamma+\varphi}\left[\frac{\xi^*+\xi\eta-\eta\xi^*-1}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}\right],\\\lambda_a^*&\equiv\left(\sigma-\frac{1}{\gamma}\right)\frac{1+\varphi}{\gamma+\varphi}\left[\frac{-\xi^*+\xi^*\eta^*-\xi\eta^*}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}\right],\\\lambda_{a^*}^*&\equiv\frac{1+\varphi}{\gamma+\varphi}\left[\begin{array}{c}\frac{\gamma-\gamma\xi^*+\gamma\xi^*\eta^*+\varphi\xi^*\eta^*+\sigma\gamma^2\xi^*+\sigma\gamma\varphi+\xi\eta\varphi-\gamma\xi\eta^*-\xi\varphi\eta^*-\eta\varphi\xi^*-\sigma\gamma^2\xi^*\eta^*}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}+\\+\frac{\sigma\gamma^2\xi\eta^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}\end{array}\right].\end{aligned}$$

These two equations characterize the potential output of the economy.

Let me assume that the usual restriction on the home bias parameters is imposed, i.e. $n\,(1-\xi)=(1-n)\,\xi^*$. Then, operating on (7.7.1) – (7.7.2), I can obtain the following system of equations,

$$\begin{aligned}&-(1+\varphi)\,\widehat{a}_t+\gamma\left[\left(\tfrac{\sigma\xi^*(1+(\xi-\xi^*))+\frac{1}{\gamma}(1-\xi^*)(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right)\widehat{\overline{y}}_t+\left(\tfrac{\sigma(1-\xi)(1+(\xi-\xi^*))-\frac{1}{\gamma}(1-\xi)(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right)\widehat{\overline{y}}_t^*\right]+\\&\quad+\varphi\widehat{\overline{y}}_t+\left(\tfrac{1-\xi}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right)\left(\widehat{\overline{y}}_t-\widehat{\overline{y}}_t^*\right)\approx0,\end{aligned}\tag{7.7.35}$$

$$\begin{aligned}&-(1+\varphi)\,\widehat{a}_t^*+\gamma\left[\left(\tfrac{\sigma\xi^*(1+(\xi-\xi^*))-\frac{1}{\gamma}\xi^*(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right)\widehat{\overline{y}}_t+\left(\tfrac{\sigma(1-\xi)(1+(\xi-\xi^*))+\frac{1}{\gamma}\xi(\xi-\xi^*)}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right)\widehat{\overline{y}}_t^*\right]+\\&\quad+\varphi\widehat{\overline{y}}_t^*-\left(\tfrac{\xi^*}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right)\left(\widehat{\overline{y}}_t-\widehat{\overline{y}}_t^*\right)\approx0,\end{aligned}\tag{7.7.36}$$

or, expressed more compactly,

$$\begin{aligned} &\left[\varphi+\left(\frac{\gamma\sigma\xi^*(1+(\xi-\xi^*))+(1-\xi^*)(\xi-\xi^*)+(1-\xi)}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right)\right]\widehat{\overline{y}}_t+ \\ &\quad+\left[\frac{\gamma\sigma(1-\xi)(1+(\xi-\xi^*))-(1-\xi)(\xi-\xi^*)-(1-\xi)}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right]\widehat{\overline{y}}_t^* \approx (1+\varphi)\,\widehat{a}_t, \end{aligned} \tag{7.7.37}$$

$$\begin{aligned} &\left[\frac{\gamma\sigma\xi^*(1+(\xi-\xi^*))-\xi^*(\xi-\xi^*)-\xi^*}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right]\widehat{\overline{y}}_t+ \\ &\quad+\left[\varphi+\left(\frac{\gamma\sigma(1-\xi)(1+(\xi-\xi^*))+\xi(\xi-\xi^*)+\xi^*}{\sigma\left(1-(\xi-\xi^*)^2\right)+\frac{1}{\gamma}(\xi-\xi^*)^2}\right)\right]\widehat{\overline{y}}_t^* \approx (1+\varphi)\,\widehat{a}_t^*. \end{aligned} \tag{7.7.38}$$

The solution to this system of equations gives me the following pair of formulas,

$$\begin{aligned} \widehat{\overline{y}}_t &\approx \lambda_a\widehat{a}_t+\lambda_{a^*}\widehat{a}_t^*, \\ \widehat{\overline{y}}_t^* &\approx \lambda_a^*\widehat{a}_t+\lambda_{a^*}^*\widehat{a}_t^*, \end{aligned}$$

where the coefficients are defined as follows,

$$\begin{aligned} \lambda_a &\equiv \frac{1+\varphi}{\gamma+\varphi}\left[\begin{array}{c} \frac{\varphi(\xi^*)^2+\gamma\xi^*+\sigma\gamma^2+\gamma\xi^2+\xi^2\varphi-\sigma\gamma^2\xi^*-\sigma\gamma^2\xi^2+\sigma\gamma\varphi-\gamma\xi\xi^*-2\xi\varphi\xi^*-\sigma\gamma\xi^2\varphi}{\gamma+\varphi(\xi^*)^2+\xi^2\varphi+\sigma\gamma\varphi-2\xi\varphi\xi^*-\sigma\gamma\xi^2\varphi-\sigma\gamma\varphi(\xi^*)^2+2\sigma\gamma\xi\varphi\xi^*}+ \\ +\frac{\sigma\gamma^2\xi\xi^*-\sigma\gamma\varphi(\xi^*)^2+2\sigma\gamma\xi\varphi\xi^*}{\gamma+\varphi(\xi^*)^2+\xi^2\varphi+\sigma\gamma\varphi-2\xi\varphi\xi^*-\sigma\gamma\xi^2\varphi-\sigma\gamma\varphi(\xi^*)^2+2\sigma\gamma\xi\varphi\xi^*} \end{array}\right], \\ \lambda_{a^*} &\equiv \left(\sigma-\frac{1}{\gamma}\right)\frac{1+\varphi}{\gamma+\varphi}\left[\frac{(\xi-1)(\xi-\xi^*+1)}{\gamma+\varphi(\xi^*)^2+\xi^2\varphi+\sigma\gamma\varphi-2\xi\varphi\xi^*-\sigma\gamma\xi^2\varphi-\sigma\gamma\varphi(\xi^*)^2+2\sigma\gamma\xi\varphi\xi^*}\right], \\ \lambda_a^* &\equiv \left(\sigma-\frac{1}{\gamma}\right)\frac{1+\varphi}{\gamma+\varphi}\left[\frac{\xi^*(\xi-\xi^*+1)}{-\gamma-\varphi(\xi^*)^2-\xi^2\varphi-\sigma\gamma\varphi+2\xi\varphi\xi^*+\sigma\gamma\xi^2\varphi+\sigma\gamma\varphi(\xi^*)^2-2\sigma\gamma\xi\varphi\xi^*}\right], \\ \lambda_{a^*}^* &\equiv \frac{1+\varphi}{\gamma+\varphi}\left[\begin{array}{c} \frac{\gamma+\gamma(\xi^*)^2+\varphi(\xi^*)^2-\gamma\xi^*+\xi^2\varphi+\sigma\gamma^2\xi^*+\sigma\gamma\varphi-\sigma\gamma^2(\xi^*)^2-\gamma\xi\xi^*-2\xi\varphi\xi^*-\sigma\gamma\xi^2\varphi}{\gamma+\varphi(\xi^*)^2+\xi^2\varphi+\sigma\gamma\varphi-2\xi\varphi\xi^*-\sigma\gamma\xi^2\varphi-\sigma\gamma\varphi(\xi^*)^2+2\sigma\gamma\xi\varphi\xi^*}+ \\ +\frac{\sigma\gamma^2\xi\xi^*-\sigma\gamma\varphi(\xi^*)^2+2\sigma\gamma\xi\varphi\xi^*}{\gamma+\varphi(\xi^*)^2+\xi^2\varphi+\sigma\gamma\varphi-2\xi\varphi\xi^*-\sigma\gamma\xi^2\varphi-\sigma\gamma\varphi(\xi^*)^2+2\sigma\gamma\xi\varphi\xi^*} \end{array}\right]. \end{aligned}$$

Using the structure of the linear-in-labor technology functions, it is possible to approximate aggregate output as,

$$\begin{aligned} \widehat{\overline{y}}_t &\approx \widehat{\overline{l}}_t+\widehat{a}_t, \\ \widehat{\overline{y}}_t^* &\approx \widehat{\overline{l}}_t^*+\widehat{a}_t^*, \end{aligned}$$

therefore, the equilibrium level of employment follows easily from here.

I already argued, while discussing the irrelevance of asset markets, that under certain special conditions (that is, if $\sigma=\gamma=1$, $n(1-\xi)=(1-n)\xi^*$ and the debt in the initial conditions is zero), the foreign shocks have no direct impact on the domestic output. That was true because under those conditions the nominal trade balance was equal to zero. However, in my previous calculations appears an interesting fact which is true independently of the choice of home bias parameters. If the elasticity of substitution across domestic and foreign bundles is identical to the intertemporal elasticity of substitution, i.e.

$$\sigma=\frac{1}{\gamma},$$

then local employment and output will only depend on local productivity shocks up to a first-order approximation. Arguably, this is a knife-edge case. However, these two parameters have very important implications for the way labor reacts to shocks coming from abroad. In fact, it can be said that these two parameters regulate whether the wealth effects or the substitution effects dominate on the employment decisions (and, hence, how that translates into the equilibrium output level). This can be seen from the fact that the sign of $\left(\sigma - \frac{1}{\gamma}\right)$ determines the sign of λ_{a^*} and λ_a^*.

In summary, if monetary policy targets deviations of output from its potential, as posited by the Taylor rule, it can no longer ignore the foreign productivity shocks except whenever $\sigma = \frac{1}{\gamma}$.

7.8 The Linearized Equilibrium Conditions: Revisited

The workhorse model is build around a log-linearized set of equilibrium conditions. It has been shown that asset prices, the nominal interest rate and the nominal exchange rate particularly, can be pin down with the current framework. Moreover, this first-order approximation is sufficient to close the model and determine the relevant allocations based on a small number of Euler equations, pricing equations and asset pricing equations. For instance, I derive aggregate output, exports and imports as well as the trade balance based on these equations only.

However, the portfolio allocation is indeterminate up to a first-order approximation. As I have noted several times already the trade balance and the capital account can be determined by relying exclusively on these linearized equilibrium conditions. However, the same cannot be said for the composition of assets in each country's portfolio. One way to attain a determinate portfolio allocation would be to obtain a second-order approximation around the asset pricing equations. This is precisely the approach that Evans and Hnatkovska (2005, 2007) follow. Portfolio determination is crucial for our macro models to be able to say something interesting on the *home bias in equity portfolios puzzle* or regarding the role of financial globalization.

It seems important to emphasize that asset prices, not portfolio quantities, is all that is needed to determine the trade balance and the net flows of capital and assets (including returns) in this model. In other words, portfolio allocations are only of second-order importance to help us understand the movements in the current account. On the other hand, the asset market structure is quite significative in this model because it determines the risk-sharing opportunities available and, therefore, how efficiently can households pool their risks and trade.

Remark 12 *What does this mean for monetary policy? The asset market structure influences decisively the risk-sharing that occurs across countries. Therefore, it changes the financial environment in which monetary policy is conducted and through which it is implemented. It also introduces a potential role for monetary policy to provide an allocation with better risk pooling across countries (insurance), which is not clear whether it conflicts or not with the objective of price stability.*

Woodford (2007, p. 8) boldly argues that "the effects of financial globalization need not be large". I would argue that portfolio allocations are of second-order importance for the determination of those variables (primarily inflation and output/consumption) that are thought to be of relevance for the monetary authority. Up to a first-order effect, only certain asset prices like the short-term nominal rates (and the yield curve), the consumption-saving decision of households and the pricing decisions of firms matter in the context of the workhorse model. However, the asset market structure cannot be ignored by policy-makers as exemplified by the distinction between a complete asset markets economy and a 'bond economy' in these notes.

Finally, it is worth always keeping an open mind on these topics and financial globalization is no exception. The portfolio composition can be ignored up to a first-order approximation because all wealth effects in the decision of agents (particularly consumers) can be summarized with the short-term nominal interest rate[50]. *In a slightly more complex model with borrowing constraints, differential trading costs across asset types or segmented markets and different degrees of asset substitutability, the claim that portfolio allocations are of second-order importance may no longer be true. Certainly, more research needs to be done along these lines.*

[50] Conceptually, this irrelevance of the portfolio allocation reminds me of the capital structure irrelevance principle (also known as the Modigliani-Miller theorem). The principle states that, in the absence of taxes, bankruptcy costs, and asymmetric information, and in an efficient market, the value of a firm is unaffected by whether the firm's capital is raised by issuing stock or selling debt. It does not matter what the firm's dividends are either. (*More comment on this later!*)

8 (Tentative) Concluding Remarks

First, monetary policy is neutral if prices are flexible, firms are competitive and asset markets are complete. In this context, it makes little sense for the monetary authority to worry about output fluctuations over the business cycle because it is outside the set of things monetary policy can actually accomplish. In the presence of nominal rigidities in the goods markets (and possible other frictions), monetary policy may have real effects because the price level can be manipulated to alter some relative prices. Monetary policy can play a role in dampening the relative price distortions embedded and implied by these frictions themselves. However, even if monetary policy was fully effective in eliminating these distortions, its real effects may only be modest. After controlling for other sources of heterogeneity across firms, large real distortions depend on the existence of very large relative price differences.

In Calvo's (1983) world with nominal rigidities, these relative price disparities are only possible if inflation is quite sizeable. But it is unlikely that firms will make their pricing decisions only sporadically if inflation (or some other state of nature) is pushing the relative prices too far apart. The firms being hurt by the resulting misallocation of spending would endogenously re-price their goods -and will do so frequently- to mitigate the decline in the expenditure share they get and, therefore, will keep relative price differences 'contained'. That's what logic dictates. Therefore, I should not anticipate a very large role for these real effects. If large relative price differences do exist, that may have to be explained by some other feature that this simple model of sticky price contracts cannot either capture or approximate sensibly.

Second, independently of the magnitude of the real effects of monetary policy, one of the underlying themes of these notes is how to conduct monetary policy in this environment with nominal rigidities. I would argue that, at least conceptually, the Taylor-type policy rule should be 'robust' to model uncertainty. If you believe that the frictionless model best describes the economy, then you must believe that output is most of the time at its 'potential'. Therefore, on the basis of a simple Taylor rule, you should set the short-term interest rate (or fix the money supply) to determine the inflation rate. Monetary policy only affects nominal variables, and the target of price stability seems a perfectly reasonable criteria. If you believe that nominal rigidities (or other frictions) are pushing the economy away from its potential, then a Taylor rule has real effects and can be used to force the economy to revert to the frictionless allocation. Inflation targeting would ensure that relative price distortions are minimized and, in doing so, it will reduce the welfare loss for consumers.

Third, price stability, therefore, is the ultimate goal of the monetary authority whether you believe that money is neutral or not. The discrepancy in the implementation of the Taylor policy rule arises because economists do not seem to agree on the potential output of the economy (even if they agree to adopt exactly the same policy rule). Conventional measures of output gap make critical assumptions on what potential output is likely to be. However, it is not necessarily obvious that observable output is not equal (or closely matches) the potential output. This is where model uncertainty becomes a contentious issue, and this is why most of the policy debate seems to move these days around the challenge of how to best compute a measure of the output gap.

Forth, as I have also discussed in the first-order approximation of the dynamics of the workhorse model, the conduct of monetary policy on the basis of policy rules depends also on how aggressively monetary policy responds to inflation. One way to think about anchored expectations is to say that monetary policy is implemented to bring the economy out of the indeterminacy region. If monetary policy is neutral this is less of an issue, because inflation has no direct impact on relative prices and has no real effects. If, instead, relative price distortions arise due to the presence of nominal rigidities, then inflation expectations that are not well-anchored can be a problem. If inflation spirals out of control because monetary policy has become too lax, then the relative price distortion may hurt the real economy. In the presence of model uncertainty, therefore, it seems best to be aggressive in targeting price stability.

Fifth, as I have shown in these notes, inflation and prices are an essentially monetary phenomenon. I am talking here about the consumption price level, not the effects on relative prices. In the frictionless economy, where I have analytical solutions, it is clear that consumption prices will be influenced by monetary shocks. However, it is also obvious that consumption prices are affected by other shocks to the economy (including productivity shocks). It is necessary to dispel the myth that lower inflation and more stable prices necessarily implies that monetary policy has become 'better' or 'tighter'. It is perfectly possible to be running a loose monetary policy while (either domestic or foreign) productivity shocks are pushing inflation down. In fact, that might even be the correct policy prescription to maintain price stability without the perils of deflation. In other words, inflation reflects more than just the effects of monetary shocks.

The problem with this situation is that we may get accustomed to run a loose monetary policy. Then, it may be costly to re-set monetary policy towards a tighter phase whenever the inflation downward bias caused by productivity shocks wanes down or disappears. We may keep running a loose monetary policy for too long and, hence, end up feeding the inflation 'monster' in the process.

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Appendix

A Summary of the Equilibrium Conditions

A.1 The Nonlinear Dynamics

A.1.1 The Flexible Price, Complete Asset Markets Model

The Demand-Side in the Goods Market:

IS^H
$$\overline{Q}_t\left(\omega_{t+1}\right)=\beta\left(\frac{\overline{C}_{t+1}(\omega_{t+1})}{\overline{C}_t}\right)^{-\gamma}\frac{\overline{P}_t}{\overline{P}_{t+1}(\omega_{t+1})}\mu_t\left(\omega_{t+1}\right),\quad\int_{\omega_{t+1}\in\Omega}\overline{Q}_t\left(\omega_{t+1}\right)=\frac{1}{1+\overline{i}_t},$$

IS^F
$$\overline{Q}^*_t\left(\omega_{t+1}\right)=\beta\left(\frac{\overline{C}^*_{t+1}(\omega_{t+1})}{\overline{C}^*_t}\right)^{-\gamma}\frac{\overline{P}^*_t}{\overline{P}^*_{t+1}(\omega_{t+1})}\mu_t\left(\omega_{t+1}\right),\quad\int_{\omega_{t+1}\in\Omega}\overline{Q}^*_t\left(\omega_{t+1}\right)=\frac{1}{1+\overline{i}^*_t},$$

UIP
$$\frac{\overline{S}_{t+1}(\omega_{t+1})}{\overline{S}_t}=\frac{\overline{Q}^*_t(\omega_{t+1})}{\overline{Q}_t(\omega_{t+1})},$$

MP^H
$$\frac{1+\overline{i}_t}{1+\overline{i}}=\frac{Z_t}{Z}\left(\frac{1+\overline{i}_{t-1}}{1+\overline{i}}\right)^{\rho}\left[\overline{\Pi}_t^{\psi_1}\left(\frac{\overline{Y}_t}{\overline{\overline{Y}}_t}\right)^{\psi_2}\right]^{1-\rho},\quad\overline{i}^m_t=0,$$

MP^F
$$\frac{1+\overline{i}^*_t}{1+\overline{i}^*}=\frac{Z^*_t}{Z^*}\left(\frac{1+\overline{i}^*_{t-1}}{1+\overline{i}}\right)^{\rho^*}\left[\overline{\Pi}_t^{*\psi^*_1}\left(\frac{\overline{Y}^*_t}{\overline{\overline{Y}}^*_t}\right)^{\psi^*_2}\right]^{1-\rho^*},\quad\overline{i}^{m*}_t=0,$$

The Supply-Side in the Goods Market:

AS^H
$$\overline{P}^H_t=\overline{P}_t\left(h\right)=\frac{\theta}{\theta-1}\left(\frac{(1+\overline{\phi}_t)\overline{W}_t}{A_t}\right),$$

AS^{H*}
$$\overline{P}^H_t=\overline{P}_t\left(h\right)=\overline{P}^*_t\left(h\right)\overline{S}_t=\overline{P}^{H*}_t\overline{S}_t,$$

$$\overline{P}_t=\begin{cases}\left[\xi\left(\overline{P}^H_t\right)^{1-\sigma}+\left(1-\xi\right)\left(\overline{P}^F_t\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, & \text{if } \sigma>0,\ \sigma\neq1,\\ \left(\overline{P}^H_t\right)^{\xi}\left(\overline{P}^F_t\right)^{1-\xi}, & \text{if } \sigma=1,\end{cases}$$

AS^F
$$\overline{P}^{F*}_t=\overline{P}^*_t\left(f\right)=\frac{\theta}{\theta-1}\left(\frac{(1+\overline{\phi}^*_t)\overline{W}^*_t}{A^*_t}\right),$$

AS^{F*}
$$\overline{P}^{F*}_t=\overline{P}^*_t\left(f\right)=\overline{P}_t\left(f\right)\frac{1}{\overline{S}_t}=\overline{P}^F_t\frac{1}{\overline{S}_t},$$

$$\overline{P}^*_t=\begin{cases}\left[\xi^*\left(\overline{P}^{H*}_t\right)^{1-\sigma}+\left(1-\xi^*\right)\left(\overline{P}^{F*}_t\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, & \text{if } \sigma>0,\ \sigma\neq1,\\ \left(\overline{P}^{H*}_t\right)^{\xi^*}\left(\overline{P}^{F*}_t\right)^{1-\xi^*}, & \text{if } \sigma=1,\end{cases}$$

FP^H
$$\overline{\phi}_t=\frac{-1}{\theta},$$

FP^F
$$\overline{\phi}^*_t=\frac{-1}{\theta},$$

Other Auxiliary Equilibrium Conditions:

CA
$$\overline{TB}_t=\overline{P}_t\left[\frac{\xi}{n}\left(\frac{\overline{P}^H_t}{\overline{P}_t}\right)^{1-\sigma}n\overline{C}_t+\frac{\xi^*}{n}\left(\frac{\overline{P}^{H*}_t}{\overline{P}^*_t}\right)^{1-\sigma}\overline{RS}_t\left(1-n\right)\overline{C}^*_t-\overline{C}_t\right],$$

MM^H
$$\overline{M}_t=\left(\frac{1}{\chi}\frac{\overline{i}_t-\overline{i}^m_t}{1+\overline{i}_t}\right)^{-\frac{1}{\zeta}}\overline{P}_t\left(\overline{C}_t\right)^{\frac{\gamma}{\zeta}},$$

MM^F
$$\overline{M}^*_t=\left(\frac{1}{\chi}\frac{\overline{i}^*_t-\overline{i}^{m*}_t}{1+\overline{i}^*_t}\right)^{-\frac{1}{\zeta}}\overline{P}^*_t\left(\overline{C}^*_t\right)^{\frac{\gamma}{\zeta}},$$

LM^H
$$\frac{\overline{W}_t}{\overline{P}_t}=\kappa\left(A_t\right)^{-\varphi}\left(\overline{C}_t\right)^{\gamma}\left(\overline{Y}_t\right)^{\varphi},$$

LM^F
$$\frac{\overline{W}^*_t}{\overline{P}^*_t}=\kappa\left(A^*_t\right)^{-\varphi}\left(\overline{C}^*_t\right)^{\gamma}\left(\overline{Y}^*_t\right),$$

Y^H
$$n\overline{Y}_t=\left(\frac{\overline{P}^H_t}{\overline{P}_t}\right)^{-\sigma}\left[n\left(\frac{\xi}{n}\overline{C}_t\right)+\left(1-n\right)\left(\frac{\xi^*}{n}\left(\frac{1}{\overline{RS}_t}\right)^{-\sigma}\overline{C}^*_t\right)\right],$$

Y^F
$$\left(1-n\right)\overline{Y}^*_t=\left(\frac{\overline{P}^{F*}_t}{\overline{P}^*_t}\right)^{-\sigma}\left[n\left(\frac{1-\xi}{1-n}\left(\overline{RS}_t\right)^{-\sigma}\overline{C}_t\right)+\left(1-n\right)\left(\frac{1-\xi^*}{1-n}\overline{C}^*_t\right)\right],$$

Other Definitions:

RS
$$\overline{RS}_t\equiv\frac{\overline{S}_t\overline{P}^*_t}{\overline{P}_t},$$

TOT
$$\overline{ToT}_t\equiv\frac{\overline{P}^F_t}{\overline{S}_t\overline{P}^{H*}_t},\quad\overline{ToT}^*_t\equiv\frac{\overline{S}_t\overline{P}^{H*}_t}{\overline{P}^F_t}=\frac{1}{\overline{ToT}_t},$$

RP
$$\overline{RP}_t\equiv\frac{\overline{P}^F_t}{\overline{P}^H_t},\quad\overline{RP}^*_t\equiv\frac{\overline{P}^{H*}_t}{\overline{P}^{F*}_t}.$$

A.1.2 The Sticky Price, Complete Asset Markets Model

	The Demand-Side in the Goods Market:
$\mathbf{IS^H}$	$Q_t(\omega_{t+1}) = \beta\left(\frac{C_{t+1}(\omega_{t+1})}{C_t}\right)^{-\gamma}\frac{P_t}{P_{t+1}(\omega_{t+1})}\mu_t(\omega_{t+1}),\ \int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1}) = \frac{1}{1+i_t},$
$\mathbf{IS^F}$	$Q^*_t(\omega_{t+1}) = \beta\left(\frac{C^*_{t+1}(\omega_{t+1})}{C^*_t}\right)^{-\gamma}\frac{P^*_t}{P^*_{t+1}(\omega_{t+1})}\mu_t(\omega_{t+1}),\ \int_{\omega_{t+1}\in\Omega} Q^*_t(\omega_{t+1}) = \frac{1}{1+i^*_t},$
UIP	$\frac{S_{t+1}(\omega_{t+1})}{S_t} = \frac{Q^*_t(\omega_{t+1})}{Q_t(\omega_{t+1})},$
$\mathbf{MP^H}$	$\frac{1+i_t}{1+\overline{i}} = \frac{Z_t}{Z}\left(\frac{1+i_{t-1}}{1+\overline{i}}\right)^{\rho}\left[\Pi_t^{\psi_1}\left(\frac{Y_t}{\overline{Y}_t}\right)^{\psi_2}\right]^{1-\rho},\ i^m_t = 0,$
$\mathbf{MP^F}$	$\frac{1+i^*_t}{1+\overline{i}^*} = \frac{Z^*_t}{Z^*}\left(\frac{1+i^*_{t-1}}{1+\overline{i}^*}\right)^{\rho^*}\left[\Pi_t^{*\psi^*_1}\left(\frac{Y^*_t}{\overline{Y}^*_t}\right)^{\psi^*_2}\right]^{1-\rho^*},\ i^{m*}_t = 0,$
	The Supply-Side in the Goods Market:
$\mathbf{AS^H}$	$\widetilde{P}_t(h) = \frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}\alpha^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}\widetilde{Y}^d_{t,t+\tau}(h)\left(\frac{(1+\phi_t)W_{t+\tau}}{A_{t+\tau}}\right)\right]}{\sum_{\tau=0}^{\infty}\alpha^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}\widetilde{Y}^d_{t,t+\tau}(h)\right]},\ P^H_t = \left[\alpha\left(P^H_{t-1}\right)^{1-\theta} + (1-\alpha)\left(\widetilde{P}_t(h)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$
$\mathbf{AS^{H*}}$	$\widetilde{P}^*_t(h) = \frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}\alpha^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(h)\left(\frac{(1+\phi_t)W_{t+\tau}}{A_{t+\tau}}\right)\right]}{\sum_{\tau=0}^{\infty}\alpha^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(h)S_{t+\tau}\right]},\ P^{H*}_t = \left[\alpha\left(P^{H*}_{t-1}\right)^{1-\theta} + (1-\alpha)\left(\widetilde{P}^*_t(h)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$
	$P_t = \begin{cases} \left[\xi\left(P^H_t\right)^{1-\sigma} + (1-\xi)\left(P^F_t\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},\ \text{if } \sigma > 0,\ \sigma \neq 1, \\ \left(P^H_t\right)^{\xi}\left(P^F_t\right)^{1-\xi},\ \text{if } \sigma = 1, \end{cases}$
$\mathbf{AS^F}$	$\widetilde{P}_t(f) = \frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}(\alpha^*)^{\tau}\mathbb{E}_t\left[m^*_{t,t+\tau}\widetilde{Y}^d_{t,t+\tau}(f)\left(\frac{(1+\phi^*_t)W^*_{t+\tau}}{A^*_{t+\tau}}\right)\right]}{\sum_{\tau=0}^{\infty}(\alpha^*)^{\tau}\mathbb{E}_t\left[m^*_{t,t+\tau}\widetilde{Y}^d_{t,t+\tau}(f)\frac{1}{S_{t+\tau}}\right]},\ P^F_t = \left[\alpha^*\left(P^F_{t-1}\right)^{1-\theta} + (1-\alpha^*)\left(\widetilde{P}_t(f)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$
$\mathbf{AS^{F*}}$	$\widetilde{P}^*_t(f) = \frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}(\alpha^*)^{\tau}\mathbb{E}_t\left[m^*_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(f)\left(\frac{(1+\phi^*_t)W^*_{t+\tau}}{A^*_{t+\tau}}\right)\right]}{\sum_{\tau=0}^{\infty}(\alpha^*)^{\tau}\mathbb{E}_t\left[m^*_{t,t+\tau}\widetilde{Y}^{d*}_{t,t+\tau}(f)\right]},\ P^{F*}_t = \left[\alpha^*\left(P^{F*}_{t-1}\right)^{1-\theta} + (1-\alpha^*)\left(\widetilde{P}^*_t(f)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$
	$P^*_t = \begin{cases} \left[\xi^*\left(P^{H*}_t\right)^{1-\sigma} + (1-\xi^*)\left(P^{F*}_t\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},\ \text{if } \sigma > 0,\ \sigma \neq 1, \\ \left(P^{H*}_t\right)^{\xi^*}\left(P^{F*}_t\right)^{1-\xi^*},\ \text{if } \sigma = 1, \end{cases}$
$\mathbf{FP^H}$	$\phi_t = \frac{-1}{\theta},$
$\mathbf{FP^F}$	$\phi^*_t = \frac{-1}{\theta},$
	Other Auxiliary Equilibrium Conditions:
CA	$TB_t = P_t\left[\frac{\xi}{n}\left(\frac{P^H_t}{P_t}\right)^{1-\sigma} nC_t + \frac{\xi^*}{n}\left(\frac{P^{H*}_t}{P^*_t}\right)^{1-\sigma} RS_t(1-n)C^*_t - C_t\right],$
$\mathbf{MM^H}$	$M_t = \left(\frac{1}{\chi}\frac{i_t - i^m_t}{1+i_t}\right)^{-\frac{1}{\zeta}} P_t(C_t)^{\frac{\gamma}{\zeta}},$
$\mathbf{MM^F}$	$M^*_t = \left(\frac{1}{\chi}\frac{i^*_t - i^{m*}_t}{1+i^*_t}\right)^{-\frac{1}{\zeta}} P^*_t(C^*_t)^{\frac{\gamma}{\zeta}},$
$\mathbf{LM^H}$	$\frac{W_t}{P_t} = \kappa(A_t)^{-\varphi}(C_t)^{\gamma}(Y_t)^{\varphi},$
$\mathbf{LM^F}$	$\frac{W^*_t}{P^*_t} = \kappa(A^*_t)^{-\varphi}(C^*_t)^{\gamma}(Y^*_t),$
$\mathbf{Y^H}$	$nY_t = n\left(\frac{\xi}{n}\left[\int_0^n\left(\frac{P_t(h)}{P^H_t}\right)^{-\theta}dh\right]\left(\frac{P^H_t}{P_t}\right)^{-\sigma}C_t\right) + (1-n)\left(\frac{\xi^*}{n}\left[\int_0^n\left(\frac{P^*_t(h)}{P^{H*}_t}\right)^{-\theta}dh\right]\left(\frac{P^{H*}_t}{P^*_t}\right)^{-\sigma}C^*_t\right),$
$\mathbf{Y^F}$	$(1-n)Y^*_t = n\left(\frac{1-\xi}{1-n}\left[\int_n^1\left(\frac{P_t(f)}{P^F_t}\right)^{-\theta}df\right]\left(\frac{P^F_t}{P_t}\right)^{-\sigma}C_t\right) + (1-n)\left(\frac{1-\xi^*}{1-n}\left[\int_n^1\left(\frac{P^*_t(f)}{P^{F*}_t}\right)^{-\theta}df\right]\left(\frac{P^{F*}_t}{P^*_t}\right)^{-\sigma}C^*_t\right),$
	Other Definitions:
RS	$RS_t \equiv \frac{S_t P^*_t}{P_t},$
TOT	$ToT_t \equiv \frac{P^F_t}{S_t P^{H*}_t},\ ToT^*_t \equiv \frac{S_t P^{H*}_t}{P^F_t} = \frac{1}{ToT_t},$
RP	$RP_t \equiv \frac{P^F_t}{P^H_t},\ RP^*_t \equiv \frac{P^{H*}_t}{P^{F*}_t}.$

A.1.3 The Sticky Price, Incomplete Asset Markets Model

The Demand-Side in the Goods Market:

$\mathbf{IS^H}$
$$\frac{1}{1+i_t}=\beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_t}{P_{t+1}}\right],$$

$\mathbf{IS^F}$
$$\frac{1}{1+i_t^*}=\beta\mathbb{E}_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}\frac{P_t^*}{P_{t+1}^*}\right],$$

$\mathbf{UIP}$
$$\mathbb{E}_t\left[\frac{S_{t+1}}{S_t}\right]=\mathbb{E}_t\left[\frac{\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}\frac{P_t^*}{P_{t+1}^*}}{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_t}{P_{t+1}}}\right],$$

$\mathbf{MP^H}$
$$\frac{1+i_t}{1+\overline{i}}=\frac{Z_t}{Z}\left(\frac{1+i_{t-1}}{1+\overline{i}}\right)^{\rho}\left[\Pi_t^{\psi_1}\left(\frac{Y_t}{\overline{Y}_t}\right)^{\psi_2}\right]^{1-\rho},\ i_t^m=0,$$

$\mathbf{MP^F}$
$$\frac{1+i_t^*}{1+\overline{i}^*}=\frac{Z_t^*}{Z^*}\left(\frac{1+i_{t-1}^*}{1+\overline{i}^*}\right)^{\rho^*}\left[\Pi_t^{*\psi_1^*}\left(\frac{Y_t^*}{\overline{Y}_t^*}\right)^{\psi_2^*}\right]^{1-\rho^*},\ i_t^{m*}=0,$$

The Supply-Side in the Goods Market:

$\mathbf{AS^H}$
$$\widetilde{P}_t(h)=\frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}\alpha^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}\widetilde{Y}_{t,t+\tau}^{d}(h)\left(\frac{(1+\phi_t)W_{t+\tau}}{A_{t+\tau}}\right)\right]}{\sum_{\tau=0}^{\infty}\alpha^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}\widetilde{Y}_{t,t+\tau}^{d}(h)\right]},\ P_t^H=\left[\alpha\left(P_{t-1}^H\right)^{1-\theta}+(1-\alpha)\left(\widetilde{P}_t(h)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

$\mathbf{AS^{H*}}$
$$\widetilde{P}_t^*(h)=\frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}\alpha^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}\widetilde{Y}_{t,t+\tau}^{d*}(h)\left(\frac{(1+\phi_t)W_{t+\tau}}{A_{t+\tau}}\right)\right]}{\sum_{\tau=0}^{\infty}\alpha^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}\widetilde{Y}_{t,t+\tau}^{d*}(h)S_{t+\tau}\right]},\ P_t^{H*}=\left[\alpha\left(P_{t-1}^{H*}\right)^{1-\theta}+(1-\alpha)\left(\widetilde{P}_t^*(h)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

$$P_t=\begin{cases}\left[\xi\left(P_t^H\right)^{1-\sigma}+(1-\xi)\left(P_t^F\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, & \text{if } \sigma>0,\ \sigma\neq 1,\\ \left(P_t^H\right)^{\xi}\left(P_t^F\right)^{1-\xi}, & \text{if } \sigma=1,\end{cases}$$

$\mathbf{AS^F}$
$$\widetilde{P}_t(f)=\frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}(\alpha^*)^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}^*\widetilde{Y}_{t,t+\tau}^{d}(f)\left(\frac{(1+\phi_t^*)W_{t+\tau}^*}{A_{t+\tau}^*}\right)\right]}{\sum_{\tau=0}^{\infty}(\alpha^*)^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}^*\widetilde{Y}_{t,t+\tau}^{d}(f)\frac{1}{S_{t+\tau}}\right]},\ P_t^F=\left[\alpha^*\left(P_{t-1}^F\right)^{1-\theta}+(1-\alpha^*)\left(\widetilde{P}_t(f)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

$\mathbf{AS^{F*}}$
$$\widetilde{P}_t^*(f)=\frac{\theta}{\theta-1}\frac{\sum_{\tau=0}^{\infty}(\alpha^*)^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}^*\widetilde{Y}_{t,t+\tau}^{d*}(f)\left(\frac{(1+\phi_t^*)W_{t+\tau}^*}{A_{t+\tau}^*}\right)\right]}{\sum_{\tau=0}^{\infty}(\alpha^*)^{\tau}\mathbb{E}_t\left[m_{t,t+\tau}^*\widetilde{Y}_{t,t+\tau}^{d*}(f)\right]},\ P_t^{F*}=\left[\alpha^*\left(P_{t-1}^{F*}\right)^{1-\theta}+(1-\alpha^*)\left(\widetilde{P}_t^*(f)\right)^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

$$P_t^*=\begin{cases}\left[\xi^*\left(P_t^{H*}\right)^{1-\sigma}+(1-\xi^*)\left(P_t^{F*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, & \text{if } \sigma>0,\ \sigma\neq 1,\\ \left(P_t^{H*}\right)^{\xi^*}\left(P_t^{F*}\right)^{1-\xi^*}, & \text{if } \sigma=1,\end{cases}$$

$\mathbf{FP^H}$
$$\phi_t=\frac{-1}{\theta},$$

$\mathbf{FP^F}$
$$\phi_t^*=\frac{-1}{\theta},$$

Other Auxiliary Equilibrium Conditions:

$\mathbf{CA}$
$$TB_t=P_t\left[\frac{\xi}{n}\left(\frac{P_t^H}{P_t}\right)^{1-\sigma}nC_t+\frac{\xi^*}{n}\left(\frac{P_t^{H*}}{P_t^*}\right)^{1-\sigma}RS_t(1-n)C_t^*-C_t\right],$$

$\mathbf{MM^H}$
$$M_t=\left(\frac{1}{\chi}\frac{i_t-i_t^m}{1+i_t}\right)^{-\frac{1}{\zeta}}P_t(C_t)^{\frac{\gamma}{\zeta}},$$

$\mathbf{MM^F}$
$$M_t^*=\left(\frac{1}{\chi}\frac{i_t^*-i_t^{m*}}{1+i_t^*}\right)^{-\frac{1}{\zeta}}P_t^*(C_t^*)^{\frac{\gamma}{\zeta}},$$

$\mathbf{LM^H}$
$$\frac{W_t}{P_t}=\kappa(A_t)^{-\varphi}(C_t)^{\gamma}(Y_t)^{\varphi},$$

$\mathbf{LM^F}$
$$\frac{W_t^*}{P_t^*}=\kappa(A_t^*)^{-\varphi}(C_t^*)^{\gamma}(Y_t^*),$$

$\mathbf{Y^H}$
$$nY_t=n\left(\frac{\xi}{n}\left[\int_0^n\left(\frac{P_t(h)}{P_t^H}\right)^{-\theta}dh\right]\left(\frac{P_t^H}{P_t}\right)^{-\sigma}C_t\right)+(1-n)\left(\frac{\xi^*}{n}\left[\int_0^n\left(\frac{P_t^*(h)}{P_t^{H*}}\right)^{-\theta}dh\right]\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma}C_t^*\right),$$

$\mathbf{IS^F}$
$$(1-n)Y_t^*=n\left(\frac{1-\xi}{1-n}\left[\int_n^1\left(\frac{P_t(f)}{P_t^F}\right)^{-\theta}df\right]\left(\frac{P_t^F}{P_t}\right)^{-\sigma}C_t\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\left[\int_n^1\left(\frac{P_t^*(f)}{P_t^{F*}}\right)^{-\theta}df\right]\left(\frac{P_t^{F*}}{P_t^*}\right)^{-\sigma}C_t^*\right),$$

Other Definitions:

$\mathbf{RS}$
$$RS_t\equiv\frac{S_tP_t^*}{P_t},$$

$\mathbf{TOT}$
$$ToT_t\equiv\frac{P_t^F}{S_tP_t^{H*}},\ ToT_t^*\equiv\frac{S_tP_t^{H*}}{P_t^F}=\frac{1}{ToT_t},$$

$\mathbf{RP}$
$$RP_t\equiv\frac{P_t^F}{P_t^H},\ RP_t^*\equiv\frac{P_t^{H*}}{P_t^{F*}}.$$

A.2 The Deterministic Steady State

	The Demand-Side in the Goods Market:
IS^H	$\overline{Q}(\omega)=\beta\overline{\mu}(\omega)\,,\ \int_{\omega\in\Omega}\overline{Q}(\omega)=\frac{1}{1+\overline{i}}=\beta,$
IS^F	$\overline{Q}^{*}(\omega)=\beta\overline{\mu}(\omega)\,,\ \int_{\omega\in\Omega}\overline{Q}^{*}(\omega)=\frac{1}{1+\overline{i}^{*}}=\beta,$
UIP	$\frac{\overline{S}(\omega)}{\overline{S}}=\frac{\overline{Q}^{*}(\omega)}{\overline{Q}(\omega)}=1,\ \frac{\overline{S}}{\overline{S}}=\frac{\int_{\omega\in\Omega}\overline{Q}^{*}(\omega)}{\int_{\omega\in\Omega}\overline{Q}(\omega)}=1,$
MP^H	$\overline{\Pi}=1,\ \overline{i}^{m}=0,\ \overline{M}=\left(\frac{\chi}{1-\beta}\right)^{\frac{1}{\zeta}}\left(\overline{C}\right)^{\frac{\gamma}{\zeta}},$
MP^F	$\overline{\Pi}^{*}=1,\ \overline{i}^{m*}=0,\ \overline{M}^{*}=\left(\frac{\chi}{1-\beta}\right)^{\frac{1}{\zeta}}\left(\overline{C}^{*}\right)^{\frac{\gamma}{\zeta}},$
	The Supply-Side in the Goods Market:
AS^H	$\overline{P}^{H}=\overline{P}(h)=\frac{\theta}{\theta-1}\left(\frac{\left(1+\overline{\phi}\right)\overline{W}}{A}\right),$
AS^{H*}	$\overline{P}^{H}=\overline{P}(h)=\overline{P}^{*}(h)\,\overline{S}=\overline{P}^{H*}\overline{S},$
	$\overline{P}=\begin{cases}\left[\xi\left(\overline{P}^{H}\right)^{1-\sigma}+(1-\xi)\left(\overline{P}^{F}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, & \text{if } \sigma>0,\ \sigma\neq 1,\\ \left(\overline{P}^{H}\right)^{\xi}\left(\overline{P}^{F}\right)^{1-\xi}, & \text{if } \sigma=1,\end{cases}$
AS^F	$\overline{P}^{F*}=\overline{P}^{*}(f)=\frac{\theta}{\theta-1}\left(\frac{\left(1+\overline{\phi}^{*}\right)\overline{W}^{*}}{A^{*}}\right),$
AS^{F*}	$\overline{P}^{F*}=\overline{P}^{*}(f)=\overline{P}(f)\,\frac{1}{\overline{S}}=\overline{P}^{F}_{t}\frac{1}{\overline{S}},$
	$\overline{P}^{*}=\begin{cases}\left[\xi^{*}\left(\overline{P}^{H*}\right)^{1-\sigma}+(1-\xi^{*})\left(\overline{P}^{F*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, & \text{if } \sigma>0,\ \sigma\neq 1,\\ \left(\overline{P}^{H*}\right)^{\xi^{*}}\left(\overline{P}^{F*}\right)^{1-\xi^{*}}, & \text{if } \sigma=1,\end{cases}$
FP^H	$\overline{\phi}=\frac{-1}{\theta},$
FP^F	$\overline{\phi}^{*}=\frac{-1}{\theta},$
	Other Auxiliary Equilibrium Conditions:
CA	$\overline{TB}=\overline{P}\left[\frac{\xi}{n}\left(\frac{\overline{P}^{H}}{\overline{P}}\right)^{1-\sigma}n\overline{C}+\frac{\xi^{*}}{n}\left(\frac{\overline{P}^{H*}}{\overline{P}^{*}}\right)^{1-\sigma}\overline{RS}\,(1-n)\,\overline{C}^{*}-\overline{C}\right]$ $=\left[\xi^{*}(1-n)-(1-\xi)\,n\right]\frac{1}{n}\overline{C},$
MM^H	$\overline{M}=\left(\frac{1}{\chi}\frac{\overline{i}-\overline{i}^{m}}{1+\overline{i}}\right)^{-\frac{1}{\zeta}}\overline{P}\left(\overline{C}\right)^{\frac{\gamma}{\zeta}},$
MM^F	$\overline{M}^{*}=\left(\frac{1}{\chi}\frac{\overline{i}^{*}-\overline{i}^{m*}}{1+\overline{i}^{*}}\right)^{-\frac{1}{\zeta}}\overline{P}^{*}\left(\overline{C}^{*}\right)^{\frac{\gamma}{\zeta}},$
LM^H	$\frac{\overline{W}}{\overline{P}}=\kappa\,(A)^{-\varphi}\left(\overline{C}\right)^{\gamma}\left(\overline{Y}\right)^{\varphi},$
LM^F	$\frac{\overline{W}^{*}}{\overline{P}^{*}}=\kappa\,(A^{*})^{-\varphi}\left(\overline{C}^{*}\right)^{\gamma}\left(\overline{Y}^{*}\right),$
Y^H	$n\overline{Y}=\left(\frac{\overline{P}^{H}}{\overline{P}}\right)^{-\sigma}\left[n\left(\frac{\xi}{n}\overline{C}\right)+(1-n)\left(\frac{\xi^{*}}{n}\left(\frac{1}{\overline{RS}}\right)^{-\sigma}\overline{C}^{*}\right)\right],$
Y^F	$(1-n)\,\overline{Y}^{*}=\left(\frac{\overline{P}^{F*}}{\overline{P}^{*}}\right)^{-\sigma}\left[n\left(\frac{1-\xi}{1-n}\left(\overline{RS}\right)^{-\sigma}\overline{C}\right)+(1-n)\left(\frac{1-\xi^{*}}{1-n}\overline{C}^{*}\right)\right],$
	Other Definitions:
RS	$\overline{RS}\equiv\frac{\overline{SP}^{*}}{\overline{P}},$
TOT	$\overline{ToT}\equiv\frac{\overline{P}^{F}}{\overline{SP}^{H*}},\ \overline{ToT}^{*}\equiv\frac{\overline{SP}^{H*}}{\overline{P}^{F}}=\frac{1}{\overline{ToT}},$
RP	$\overline{RP}\equiv\frac{\overline{P}^{F}}{\overline{P}^{H}},\ \overline{RP}^{*}\equiv\frac{\overline{P}^{H*}}{\overline{P}^{F*}}.$

A.3 The Log-Linearized Dynamics (A First-Order Approximation)

A.3.1 The Flexible Price, Complete Asset Markets Model

	Potential Output:
$\mathbf{Y^H}$	$\widehat{\overline{y}}_t \approx \lambda_a \widehat{a}_t + \lambda_{a^*} \widehat{a}_t^*,$
$\mathbf{Y^F}$	$\widehat{\overline{y}}_t^* \approx \lambda_a^* \widehat{a}_t + \lambda_{a^*}^* \widehat{a}_t^*,$
	Potential Employment:
$\mathbf{L^H}$	$\widehat{\overline{l}}_t \approx \widehat{\overline{y}}_t - \widehat{a}_t,$
$\mathbf{L^{H*}}$	$\widehat{\overline{l}}_t^* \approx \widehat{\overline{y}}_t^* - \widehat{a}_t^*,$
	Potential Consumption:
$\mathbf{C^H}$	$\widehat{\overline{c}}_t \approx \left[\frac{\sigma(\xi^*+\eta^*(\xi-\xi^*))+\frac{1}{\gamma}(1-\eta^*)(\xi-\xi^*)}{\sigma(1-(\eta-\eta^*)(\xi-\xi^*))+\frac{1}{\gamma}(\eta-\eta^*)(\xi-\xi^*)}\right] \widehat{\overline{y}}_t + \left[\frac{\sigma((1-\xi)+(1-\eta)(\xi-\xi^*))-\frac{1}{\gamma}(1-\eta)(\xi-\xi^*)}{\sigma(1-(\eta-\eta^*)(\xi-\xi^*))+\frac{1}{\gamma}(\eta-\eta^*)(\xi-\xi^*)}\right] \widehat{\overline{y}}_t^*,$
$\mathbf{C^F}$	$\widehat{\overline{c}}_t^* \approx \left[\frac{\sigma(\xi^*+\eta^*(\xi-\xi^*))-\frac{1}{\gamma}\eta^*(\xi-\xi^*)}{\sigma(1-(\eta-\eta^*)(\xi-\xi^*))+\frac{1}{\gamma}(\eta-\eta^*)(\xi-\xi^*)}\right] \widehat{\overline{y}}_t + \left[\frac{\sigma((1-\xi)+(1-\eta)(\xi-\xi^*))+\frac{1}{\gamma}\eta(\xi-\xi^*)}{\sigma(1-(\eta-\eta^*)(\xi-\xi^*))+\frac{1}{\gamma}(\eta-\eta^*)(\xi-\xi^*)}\right] \widehat{\overline{y}}_t^*,$
	Real Exchange Rate and Terms of Trade:
$\mathbf{RS}$	$\widehat{\overline{rs}}_t \approx \left[\frac{\xi-\xi^*}{\sigma(1-(\xi-\xi^*))+\sigma(\xi-\xi^*)(1-(\eta-\eta^*))+\frac{1}{\gamma}(\xi-\xi^*)(\eta-\eta^*)}\right] \left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^*\right),$
$\mathbf{TOT}$	$\widehat{\overline{tot}}_t \approx \left[\frac{1}{\sigma(1-(\xi-\xi^*))+\sigma(\xi-\xi^*)(1-(\eta-\eta^*))+\frac{1}{\gamma}(\xi-\xi^*)(\eta-\eta^*)}\right] \left(\widehat{\overline{y}}_t - \widehat{\overline{y}}_t^*\right),$
	Parameters:
	$\lambda_a \equiv \frac{1+\varphi}{\gamma+\varphi} \left[\frac{\gamma\xi^*+\sigma\gamma^2+\varphi\xi^*\eta^*-\sigma\gamma^2\xi^*+\sigma\gamma\varphi+\gamma\xi\eta+\xi\eta\varphi-\gamma\eta\xi^*-\xi\varphi\eta^*-\eta\varphi\xi^*-\sigma\gamma^2\xi\eta+\sigma\gamma^2\eta\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}\right],$
	$\lambda_{a^*} \equiv \left(\sigma - \frac{1}{\gamma}\right) \frac{1+\varphi}{\gamma+\varphi} \left[\frac{\xi^*+\xi\eta-\eta\xi^*-1}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}\right],$
	$\lambda_a^* \equiv \left(\sigma - \frac{1}{\gamma}\right) \frac{1+\varphi}{\gamma+\varphi} \left[\frac{-\xi^*+\xi^*\eta^*-\xi\eta^*}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}\right],$
	$\lambda_{a^*}^* \equiv \frac{1+\varphi}{\gamma+\varphi} \left[\frac{\gamma-\gamma\xi^*+\gamma\xi^*\eta^*+\varphi\xi^*\eta^*+\sigma\gamma^2\xi^*+\sigma\gamma\varphi+\xi\eta\varphi-\gamma\xi\eta^*-\xi\varphi\eta^*-\eta\varphi\xi^*-\sigma\gamma^2\xi^*\eta^*+\sigma\gamma^2\xi\eta^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}{\gamma+\varphi\xi^*\eta^*+\sigma\gamma\varphi+\xi\eta\varphi-\xi\varphi\eta^*-\eta\varphi\xi^*+\sigma\gamma\xi\varphi\eta^*+\sigma\gamma\eta\varphi\xi^*-\sigma\gamma\varphi\xi^*\eta^*-\sigma\gamma\xi\eta\varphi}\right],$
	$\eta \equiv \frac{n\left(\frac{\xi}{n}\right)}{n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^*}{n}\right)},\ \eta^* \equiv \frac{n\left(\frac{1-\xi}{1-n}\right)}{n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)},$
	Parameter Restrictions:
	$\frac{\sigma\left(\frac{\xi^*}{\xi-\xi^*}+\eta^*\right)+\frac{1}{\gamma}(n(1-\eta^*)-(1-n)\eta^*)}{\sigma\left(\frac{1}{\xi-\xi^*}-(\eta-\eta^*)\right)+\frac{1}{\gamma}(\eta-\eta^*)} = \frac{\eta^*(1-n)\xi}{\eta^*(1-n)\xi+\eta n(1-\xi)},$
	$\frac{\sigma\left(\frac{1-\xi}{\xi-\xi^*}+(1-\eta)\right)+\frac{1}{\gamma}((1-n)\eta-n(1-\eta))}{\sigma\left(\frac{1}{\xi-\xi^*}-(\eta-\eta^*)\right)+\frac{1}{\gamma}(\eta-\eta^*)} = \frac{\eta n(1-\xi)}{\eta^*(1-n)\xi+\eta n(1-\xi)}.$

A.3.2 The Sticky Price, Complete Asset Markets Model

	The Demand-Side in the Goods Market:
$\mathbf{IS^H}$	$\gamma \mathbb{E}_t \left[\Delta \widehat{c}_{t+1} \right] \approx \widehat{i}_t - \mathbb{E}_t \left[\widehat{\pi}_{t+1} \right],$
$\mathbf{IS^F}$	$\gamma \mathbb{E}_t \left[\Delta \widehat{c}^*_{t+1} \right] \approx \widehat{i}^*_t - \mathbb{E}_t \left[\widehat{\pi}^*_{t+1} \right],$
$\mathbf{UIP}$	$\widehat{rs}_t \approx \gamma \widehat{c}^R_t,$
$\mathbf{MP^H}$	$\widehat{i}_t \approx \rho \widehat{i}_{t-1} + (1-\rho) \left[\psi_1 \widehat{\pi}_t + \psi_2 \left(\widehat{y}_t - \widehat{\overline{y}}_t \right) \right] + \widehat{z}_t, \ \widehat{i}^m_t = 0,$
$\mathbf{MP^F}$	$\widehat{i}^*_t \approx \rho^* \widehat{i}^*_{t-1} + (1-\rho^*) \left[\psi^*_1 \widehat{\pi}^*_t + \psi^*_2 \left(\widehat{y}^*_t - \widehat{\overline{y}}^*_t \right) \right] + \widehat{z}^*_t, \ \widehat{i}^{m*}_t = 0,$
	The Supply-Side in the Goods Market:
$\mathbf{AS^H}$	$\widehat{\pi}^H_t \approx \beta \mathbb{E}_t \left[\widehat{\pi}^H_{t+1} \right] + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \left[\begin{array}{c} \gamma \widehat{c}_t + \varphi \left(\eta \widehat{c}_t + (1-\eta) \widehat{c}^*_t \right) + \\ + \varphi\sigma \left(\eta (1-\xi) \widehat{rp}_t - (1-\eta)(1-\xi^*) \widehat{rp}^*_t \right) + (1-\xi) \widehat{rp}_t - (1+\varphi) \widehat{a}_t \end{array} \right],$
$\mathbf{AS^{H*}}$	$\widehat{\pi}^{H*}_t \approx \beta \mathbb{E}_t \left[\widehat{\pi}^{H*}_{t+1} \right] + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \left[\begin{array}{c} \gamma \widehat{c}_t + \varphi \left(\eta \widehat{c}_t + (1-\eta) \widehat{c}^*_t \right) + \\ + \varphi\sigma \left(\eta (1-\xi) \widehat{rp}_t - (1-\eta)(1-\xi^*) \widehat{rp}^*_t \right) + \widehat{tot}_t - \xi \widehat{rp}_t - (1+\varphi) \widehat{a}_t \end{array} \right],$
	$\widehat{p}_t \approx \xi \widehat{p}^H_t + (1-\xi) \widehat{p}^F_t,$
$\mathbf{AS^F}$	$\widehat{\pi}^F_t \approx \beta \mathbb{E}_t \left[\widehat{\pi}^F_{t+1} \right] + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*} \left[\begin{array}{c} \gamma \widehat{c}^*_t + \varphi \left(\eta^* \widehat{c}_t + (1-\eta^*) \widehat{c}^*_t \right) + \\ + \varphi\sigma \left(-\eta^* \xi \widehat{rp}_t + (1-\eta^*) \xi^* \widehat{rp}^*_t \right) - \widehat{tot}_t - (1-\xi^*) \widehat{rp}^*_t - (1+\varphi) \widehat{a}^*_t \end{array} \right],$
$\mathbf{AS^{F*}}$	$\widehat{\pi}^{F*}_t \approx \beta \mathbb{E}_t \left[\widehat{\pi}^{F*}_{t+1} \right] + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*} \left[\begin{array}{c} \gamma \widehat{c}^*_t + \varphi \left(\eta^* \widehat{c}_t + (1-\eta^*) \widehat{c}^*_t \right) + \\ + \varphi\sigma \left(-\eta^* \xi \widehat{rp}_t + (1-\eta^*) \xi^* \widehat{rp}^*_t \right) + \xi^* \widehat{rp}^*_t - (1+\varphi) \widehat{a} \end{array} \right],$
	$\widehat{p}^*_t \approx \xi^* \widehat{p}^{H*}_t + (1-\xi^*) \widehat{p}^{F*}_t,$
	Other Auxiliary Equilibrium Conditions:
$\mathbf{TB}$	$\widehat{tb}_t - \widehat{p}_t \equiv (1-\eta) \widehat{rs}_t + (\sigma - 1) \left(\eta (1-\xi) \widehat{rp}_t - (1-\eta)(1-\xi^*) \widehat{rp}^*_t \right) - (1-\eta) \widehat{c}^R_t,$
$\mathbf{MM^H}$	$\widehat{m}_t - \widehat{p}_t \approx \frac{\gamma}{\zeta} \widehat{c}_t - \frac{1}{\zeta} \left(\frac{\beta\left(1+\overline{i}^m\right)}{1-\beta\left(1+\overline{i}^m\right)} \right) \left(\widehat{i}_t - \widehat{i}^m_t \right),$
$\mathbf{MM^F}$	$\widehat{m}^*_t - \widehat{p}^*_t \approx \frac{\gamma}{\zeta} \widehat{c}^*_t - \frac{1}{\zeta} \left(\frac{\beta\left(1+\overline{i}^{m*}\right)}{1-\beta\left(1+\overline{i}^{m*}\right)} \right) \left(\widehat{i}^*_t - \widehat{i}^{m*}_t \right),$
$\mathbf{LM^H}$	$\widehat{w}_t - \widehat{p}_t \approx \gamma \widehat{c}_t + \varphi \widehat{y}_t - \varphi \widehat{a}_t,$
$\mathbf{LM^F}$	$\widehat{w}^*_t - \widehat{p}^*_t \approx \gamma \widehat{c}^*_t + \varphi \widehat{y}^*_t - \varphi \widehat{a}^*_t,$
$\mathbf{Y^H}$	$\widehat{y}_t \approx -\sigma \left[\eta \left(\widehat{p}^H_t - \widehat{p}_t \right) + (1-\eta) \left(\widehat{p}^{H*}_t - \widehat{p}^*_t \right) \right] + \left[\eta \widehat{c}_t + (1-\eta) \widehat{c}^*_t \right], \ \eta \equiv \frac{n\left(\frac{\xi}{n}\right)}{n\left(\frac{\xi}{n}\right) + (1-n)\left(\frac{\xi^*}{n}\right)},$
$\mathbf{Y^F}$	$\widehat{y}^*_t \approx -\sigma \left[\eta^* \left(\widehat{p}^F_t - \widehat{p}_t \right) + (1-\eta^*) \left(\widehat{p}^{F*}_t - \widehat{p}^*_t \right) \right] + \left[\eta^* \widehat{c}_t + (1-\eta^*) \widehat{c}^*_t \right], \ \eta^* \equiv \frac{n\left(\frac{1-\xi}{1-n}\right)}{n\left(\frac{1-\xi}{1-n}\right) + (1-n)\left(\frac{1-\xi^*}{1-n}\right)},$
	Other Definitions:
$\mathbf{RS}$	$\widehat{rs}_t = \widehat{s}_t + \widehat{p}^*_t - \widehat{p}_t,$
$\mathbf{TOT}$	$\widehat{tot}_t = \widehat{p}^F_t - \widehat{s}_t - \widehat{p}^{H*}_t, \ \widehat{tot}^*_t = \widehat{s}_t + \widehat{p}^{H*}_t - \widehat{p}^F_t = -\widehat{tot}_t,$
$\mathbf{RP}$	$\widehat{rp}_t = \widehat{p}^F_t - \widehat{p}^H_t, \ \widehat{rp}^*_t = -\left(\widehat{p}^{F*}_t - \widehat{p}^{H*}_t \right).$

A.3.3 The Sticky Price, Incomplete Asset Markets Model

	The Demand-Side in the Goods Market:
$\mathbf{IS^H}$	$\gamma\mathbb{E}_t\left[\Delta\widehat{c}_{t+1}\right] \approx \widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right],$
$\mathbf{IS^F}$	$\gamma\mathbb{E}_t\left[\Delta\widehat{c}^*_{t+1}\right] \approx \widehat{i}^*_t - \mathbb{E}_t\left[\widehat{\pi}^*_{t+1}\right],$
UIP	$\mathbb{E}_t\left[\Delta\widehat{rs}_{t+1}\right] \approx \gamma\mathbb{E}_t\left[\Delta\widehat{c}^R_{t+1}\right],$
$\mathbf{MP^H}$	$\widehat{i}_t \approx \rho\widehat{i}_{t-1} + (1-\rho)\left[\psi_1\widehat{\pi}_t + \psi_2\left(\widehat{y}_t - \widehat{\overline{y}}_t\right)\right] + \widehat{z}_t,\ \widehat{i}^m_t = 0,$
$\mathbf{MP^F}$	$\widehat{i}^*_t \approx \rho^*\widehat{i}^*_{t-1} + (1-\rho^*)\left[\psi^*_1\widehat{\pi}^*_t + \psi^*_2\left(\widehat{y}^*_t - \widehat{\overline{y}}^*_t\right)\right] + \widehat{z}^*_t,\ \widehat{i}^{m*}_t = 0,$
	The Supply-Side in the Goods Market:
$\mathbf{AS^H}$	$\widehat{\pi}^H_t \approx \beta\mathbb{E}_t\left[\widehat{\pi}^H_{t+1}\right] + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\left[\begin{array}{c}\gamma\widehat{c}_t + \varphi\left(\eta\widehat{c}_t + (1-\eta)\,\widehat{c}^*_t\right) + \\ +\varphi\sigma\left(\eta\,(1-\xi)\,\widehat{rp}_t - (1-\eta)\,(1-\xi^*)\,\widehat{rp}^*_t\right) + (1-\xi)\,\widehat{rp}_t - (1+\varphi)\,\widehat{a}_t\end{array}\right],$
$\mathbf{AS^{H*}}$	$\widehat{\pi}^{H*}_t \approx \beta\mathbb{E}_t\left[\widehat{\pi}^{H*}_{t+1}\right] + \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\left[\begin{array}{c}\gamma\widehat{c}_t + \varphi\left(\eta\widehat{c}_t + (1-\eta)\,\widehat{c}^*_t\right) + \\ +\varphi\sigma\left(\eta\,(1-\xi)\,\widehat{rp}_t - (1-\eta)\,(1-\xi^*)\,\widehat{rp}^*_t\right) + \widehat{tot}_t - \xi\widehat{rp}_t - (1+\varphi)\,\widehat{a}_t\end{array}\right],$
	$\widehat{p}_t \approx \xi\widehat{p}^H_t + (1-\xi)\,\widehat{p}^F_t,$
$\mathbf{AS^F}$	$\widehat{\pi}^F_t \approx \beta\mathbb{E}_t\left[\widehat{\pi}^F_{t+1}\right] + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*}\left[\begin{array}{c}\gamma\widehat{c}^*_t + \varphi\left(\eta^*\widehat{c}_t + (1-\eta^*)\,\widehat{c}^*_t\right) + \\ +\varphi\sigma\left(-\eta^*\xi\widehat{rp}_t + (1-\eta^*)\,\xi^*\widehat{rp}^*_t\right) - \widehat{tot}_t - (1-\xi^*)\,\widehat{rp}^*_t - (1+\varphi)\,\widehat{a}^*_t\end{array}\right],$
$\mathbf{AS^{F*}}$	$\widehat{\pi}^{F*}_t \approx \beta\mathbb{E}_t\left[\widehat{\pi}^{F*}_{t+1}\right] + \frac{(1-\alpha^*)(1-\beta\alpha^*)}{\alpha^*}\left[\begin{array}{c}\gamma\widehat{c}^*_t + \varphi\left(\eta^*\widehat{c}_t + (1-\eta^*)\,\widehat{c}^*_t\right) + \\ +\varphi\sigma\left(-\eta^*\xi\widehat{rp}_t + (1-\eta^*)\,\xi^*\widehat{rp}^*_t\right) + \xi^*\widehat{rp}^*_t - (1+\varphi)\,\widehat{a}\end{array}\right],$
	$\widehat{p}^*_t \approx \xi^*\widehat{p}^{H*}_t + (1-\xi^*)\,\widehat{p}^{F*}_t,$
	Other Auxiliary Equilibrium Conditions:
TB	$\widehat{tb}_t - \widehat{p}_t \equiv (1-\eta)\,\widehat{rs}_t + (\sigma-1)\left(\eta\,(1-\xi)\,\widehat{rp}_t - (1-\eta)\,(1-\xi^*)\,\widehat{rp}^*_t\right) - (1-\eta)\,\widehat{c}^R_t,$
$\mathbf{MM^H}$	$\widehat{m}_t - \widehat{p}_t \approx \frac{\gamma}{\zeta}\widehat{c}_t - \frac{1}{\zeta}\left(\frac{\beta\left(1+\overline{i}^m\right)}{1-\beta\left(1+\overline{i}^m\right)}\right)\left(\widehat{i}_t - \widehat{i}^m_t\right),$
$\mathbf{MM^F}$	$\widehat{m}^*_t - \widehat{p}^*_t \approx \frac{\gamma}{\zeta}\widehat{c}^*_t - \frac{1}{\zeta}\left(\frac{\beta\left(1+\overline{i}^{m*}\right)}{1-\beta\left(1+\overline{i}^{m*}\right)}\right)\left(\widehat{i}^*_t - \widehat{i}^{m*}_t\right),$
$\mathbf{LM^H}$	$\widehat{w}_t - \widehat{p}_t \approx \gamma\widehat{c}_t + \varphi\widehat{y}_t - \varphi\widehat{a}_t,$
$\mathbf{LM^F}$	$\widehat{w}^*_t - \widehat{p}^*_t \approx \gamma\widehat{c}^*_t + \varphi\widehat{y}^*_t - \varphi\widehat{a}^*_t,$
$\mathbf{Y^H}$	$\widehat{y}_t \approx -\sigma\left[\eta\left(\widehat{p}^H_t - \widehat{p}_t\right) + (1-\eta)\left(\widehat{p}^{H*}_t - \widehat{p}^*_t\right)\right] + \left[\eta\widehat{c}_t + (1-\eta)\,\widehat{c}^*_t\right],\ \eta \equiv \frac{n\left(\frac{\xi}{n}\right)}{n\left(\frac{\xi}{n}\right)+(1-n)\left(\frac{\xi^*}{n}\right)},$
$\mathbf{Y^F}$	$\widehat{y}^*_t \approx -\sigma\left[\eta^*\left(\widehat{p}^F_t - \widehat{p}_t\right) + (1-\eta^*)\left(\widehat{p}^{F*}_t - \widehat{p}^*_t\right)\right] + \left[\eta^*\widehat{c}_t + (1-\eta^*)\,\widehat{c}^*_t\right],\ \eta^* \equiv \frac{n\left(\frac{1-\xi}{1-n}\right)}{n\left(\frac{1-\xi}{1-n}\right)+(1-n)\left(\frac{1-\xi^*}{1-n}\right)},$
	Other Definitions:
RS	$\widehat{rs}_t = \widehat{s}_t + \widehat{p}^*_t - \widehat{p}_t,$
TOT	$\widehat{tot}_t = \widehat{p}^F_t - \widehat{s}_t - \widehat{p}^{H*}_t,\ \widehat{tot}^*_t = \widehat{s}_t + \widehat{p}^{H*}_t - \widehat{p}^F_t = -\widehat{tot}_t,$
RP	$\widehat{rp}_t = \widehat{p}^F_t - \widehat{p}^H_t,\ \widehat{rp}^*_t = -\left(\widehat{p}^{F*}_t - \widehat{p}^{H*}_t\right).$

B Description of the Dataset

I identify the United States with the home country and the 12 member country Euro-zone (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain) with the foreign country. I also report some data for the United Kingdom for comparison purposes. Whenever possible I use data sources comparable to those of Chari, Kehoe and McGrattan (2002), and similarly collect all quarterly data spanning the post-Bretton Woods period of 1973:I-2006:III (for a total of 135 observations per series). The sample period considered ends up before Slovenia became a member of the Euro-zone in January 2007. All data (except nominal prices, interest rates and exchange rates) is seasonally adjusted.

The Euro currency was introduced in non-physical form (travellers' checks, electronic transfers, banking, etc.) and the bilateral exchange rates where locked for all participating countries on January 1, 1999 (on June 19, 2000 for Greece). The new notes and coins did not circulate until January 1st, 2002. Whenever available, I rely on aggregate data computed by the OECD methodology to preclude exchange rate movements from affecting the real variables. I also construct a synthetic GDP-weighted series for the U.S. Dollar/Euro nominal exchange rate and the EURIBOR prior to 2001 based on country-by-country data. More details can be found in the dataset's companion description file.

Data Series. I collect information on real output (rgdp), real consumption (rcons), consumer price indexes (cpi), nominal exchange rates (ner), employment (emp), short-term nominal interest rates (i), population size (n), U.S. terms of trade (tot), and money with zero maturity (mzm).

∘ Real output (rgdp). Data at quarterly frequency, transformed to millions of national currency, and seasonally adjusted. Source: OECD's *Main Economic Indicators*, and OECD's *Quarterly National Accounts*.

∘ Real consumption (rcons). Data at quarterly frequency, transformed to millions of national currency, and seasonally adjusted. Source: OECD's *Main Economic Indicators*, and OECD's *Quarterly National Accounts*.

∘ Consumer price indexes (cpi). Data at quarterly frequency, expressed in percentages, and not seasonally adjusted. Source: OECD's *Main Economic Indicators*.

∘ Nominal exchange rate (ner). Data at quarterly frequency, transformed to be quoted as U.S. Dollars per National Currency (US$/National Currency), and not seasonally adjusted. Sources: Board of Governors of the Federal Reserve System, IMF's *International Financial Statistics*, and OECD's *Annual National Accounts*.

∘ Employment (emp). Data at quarterly frequency, expressed in thousands of employees and self-employed individuals, and seasonally adjusted. Source: OECD's *Economic Outlook*.

∘ (Working-age) Population between 15-64 years old (pop): Data at quarterly frequency, expressed in thousands of individuals, and seasonally adjusted. Source: OECD's *Economic Outlook*.

∘ Money market interest rates at 3-month maturity (i): Data at quarterly frequency, expressed in percentages, and not seasonally adjusted. Source: Eurostat.

∘ U.S. terms of trade (for all goods and services) = U.S. import deflator (for all goods and services) / U.S. export deflator (for all goods and services) (tot): Data at quarterly frequency, expressed in percentages, and not seasonally adjusted. Source: OECD's *Economic Outlook*.

∘ Money with zero maturity (mzm): Data at quarterly frequency, expressed in millions of the national currency, and seasonally adjusted. Source: Board of Governors of the Federal Reserve System, Bank of

England, and European Central Bank.

Updating Procedure. The real output (rgdp), real consumption (rcons), employment (emp), and the money with zero maturity (mzm) are expressed in per capita terms dividing each one of these series by the population size (n). I compute the ratio of CPI indexes, $P_t^R \equiv \left(\frac{P_t^*}{P_t}\right)^{-1}$, the real exchange rate, $RS_t \equiv \frac{S_t P_t^*}{P_t}$, and the consumption differential across countries, $C_t^R \equiv \frac{C_t}{C_t^*}$, from the dataset described before. I express all variables in logs, except the nominal short-term interest rates. I also multiply all data by 100, except the nominal short-term interest rates which come already in percentages. Finally, all series are Hodrick-Prescott (H-P) filtered to eliminate their underlying trend. I use the H-P smoothing parameter at 1600 for my quarterly dataset.

Tables

Table 1: The Standard Deviations and First-order Autocorrelations of the Exchange Rates.

Historical Statistics: Standard Deviations									
Foreign Country	$\sigma(\widehat{c})$	$\sigma(\widehat{c}^{*})$	$\sigma(\widehat{c}^{R})$	$\sigma(\widehat{p})$	$\sigma(\widehat{p}^{*})$	$\sigma(-\widehat{p}^{R})$	$\sigma(\widehat{rs})$	$\sigma(\widehat{s})$	$\sigma(\widehat{tot})$
U.K.	1.236	1.535	1.568	1.373	1.744	1.544	7.533	7.795	2.709
Euro12	1.236	1.044	1.232	1.373	0.908	1.287	7.683	7.989	2.709
Historical Statistics: First-Order Autocorrelations									
Foreign	$\rho(\widehat{c})$	$\rho(\widehat{c}^{*})$	$\rho(\widehat{c}^{R})$	$\rho(\widehat{p})$	$\rho(\widehat{p}^{*})$	$\rho(-\widehat{p}^{R})$	$\rho(\widehat{rs})$	$\rho(\widehat{s})$	$\rho(\widehat{tot})$
U.K.	0.866	0.788	0.748	0.933	0.823	0.774	0.823	0.843	0.817
Euro12	0.866	0.822	0.800	0.933	0.918	0.928	0.848	0.859	0.817

The standard deviations and first-order autocorrelations of the exchange rates, the CPI prices, the terms of trade and the per capita consumption in the U.S., the U.K. and the Euro-zone (12). Quarterly sample: 1973:I-2006:III. Sources: OECD and FRB/FRBNY.

This table reports the statistics after each series has been expressed in logs, multiplied by 100 and H-P filtered (smoothing parameter=1600). The superscript "*" denotes the foreign country, while no superscript indicates the U.S. The superscript "R" denotes a relative variable, computed as the ratio of the U.S. value over the foreign value.

Table 2: The Cross-correlation of the Exchange Rates.

Historical Statistics: The Correlation Matrix for the US-UK.									
	$\widehat{c}$	$\widehat{c}^*$	$\widehat{c}^R$	$\widehat{p}$	$\widehat{p}^*$	$-\widehat{p}^R$	$\widehat{rs}$	$\widehat{s}$	$\widehat{tot}$
$\widehat{c}$	1	0.376	0.421	−0.770	−0.499	0.121	−0.188	−0.206	−0.447
$\widehat{c}^*$		1	−0.683	−0.193	−0.543	−0.441	0.320	0.397	0.074
$\widehat{c}^R$			1	−0.419	0.138	0.528	−0.462	−0.551	−0.425
$\widehat{p}$				1	0.531	−0.289	0.209	0.260	0.595
$\widehat{p}^*$					1	0.657	0.103	−0.031	0.192
$-\widehat{p}^R$						1	−0.070	−0.266	−0.312
$\widehat{rs}$							1	0.980	0.481
$\widehat{s}$								1	0.527
$\widehat{tot}$									1

The cross-correlation of the exchange rates, the CPI prices, the terms of trade and the per capita consumption in the U.S. and the U.K. Quarterly sample: 1973:I-2006:III. Sources: OECD and FRB/FRBNY.

This table reports the statistics after each series has been expressed in logs, multiplied by 100 and H-P filtered (smoothing parameter=1600). The superscript "*" denotes the U.K., while no superscript indicates the U.S. The superscript "R" denotes a relative variable, computed as the ratio of the U.S. value over the U.K. value.

Historical Statistics: The Correlation Matrix for the US-Euro12.									
	$\widehat{c}$	$\widehat{c}^*$	$\widehat{c}^R$	$\widehat{p}$	$\widehat{p}^*$	$-\widehat{p}^R$	$\widehat{rs}$	$\widehat{s}$	$\widehat{tot}$
$\widehat{c}$	1	0.426	0.642	−0.770	−0.560	0.427	−0.007	−0.076	−0.447
$\widehat{c}^*$		1	−0.420	−0.138	−0.681	−0.334	0.266	0.309	0.164
$\widehat{c}^R$			1	−0.656	0.015	0.711	−0.233	−0.338	−0.587
$\widehat{p}$				1	0.423	−0.769	−0.176	−0.045	0.595
$\widehat{p}^*$					1	0.254	−0.492	−0.514	−0.004
$-\widehat{p}^R$						1	−0.160	−0.314	−0.637
$\widehat{rs}$							1	0.987	0.275
$\widehat{s}$								1	0.367
$\widehat{tot}$									1

The cross-correlation of the exchange rates, the CPI prices, the terms of trade and the per capita consumption in the U.S. and the Euro-zone (12). Quarterly sample: 1973:I-2006:III. Sources: OECD and FRB/FRBNY.

This table reports the statistics after each series has been expressed in logs, multiplied by 100 and H-P filtered (smoothing parameter=1600). The superscript "*" denotes the Euro-zone (12), while no superscript indicates the U.S. The superscript "R" denotes a relative variable, computed as the ratio of the U.S. value over the Euro-zone value.