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Abstract

We extend a standard taxable income model with its typical functional-form assumptions to account for nonlinear budget sets. We propose a new method to estimate taxable income elasticity that is more policy relevant than the typically estimated elasticity based on linearized budget sets. Using U.S. data from the NBER tax panel for 1979–1990 and differencing methods, we estimate an elasticity of 0.75 for taxable income and 0.20 for broad income. These estimates are higher than those obtained by specifications based on linearization. Our approach offers a new way to address the problem of endogenous observed marginal tax rates.

Keywords: taxable income, nonlinear budget sets, panel data

JEL classification: D11, H24, J22

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1. Introduction

The responsiveness of taxable income to tax rate changes is widely recognized as an important research question in public finance. Following the work of Feldstein (1995, 1998), a large body of literature has emerged regarding estimation of the elasticity of taxable income with respect to the marginal net-of-tax rate at an observed taxable income.[1] The standard method adopted in much of the previous literature is to linearize the budget set at the observed taxable income and regress the change in the log of taxable income on the change in the log of observed marginal net-of-tax rate. This approach views tax reforms as an exogenous source of tax rate variation. The main econometric difficulty is that budget sets are nonlinear and an individual's observed marginal tax rate is mechanically a function of income; therefore, the change in marginal net-of-tax-tax rate is endogenous to the change in taxable income.

Since the influential work of Gruber and Saez (2002), the taxable income literature has primarily addressed the endogeneity of change in the log of the observed marginal net-of-tax rate by instrumenting it with change in the log of the predicted marginal net-of-tax rate, keeping real taxable income fixed at the base-year level. However, this instrumental variable remains endogenous if income is mean reverting or if the income distribution is widening due to factors unrelated to tax rate changes. The literature has addressed these concerns by controlling for base-year income (Auten and Carroll, 1999; Saez, 2003; Gruber and Saez, 2002). Weber (2014), however, recently showed that including controls for base-year income does not ensure instrument validity. She demonstrated that even alternative income-based instrumental variables proposed in the previous literature (e.g., Carroll, 1998; Kopczuk, 2005; Blomquist and Selin, 2010) have similar drawbacks. Instead, she argued that instrumental variables based on income lagged multiple years are potentially valid.

In addition to concerns related to instrument validity, an important simplification in the previous literature is the implicit assumption that behavior under linearized budget sets closely approximates that when budget sets are nonlinear. While optimal income choices under both settings are the same if budget sets are convex, the estimated impact of tax changes can significantly differ if individuals change tax brackets. While the search for a valid instrumental variable to overcome the endogeneity of the observed marginal tax rate has been the focal point of the literature, estimation of the impact of tax policy leading to complicated budget set changes has received far less attention.

As a backdrop for a more concrete discussion of our contributions, we start by noting that in the standard empirical double-log specification of the taxable income function, the estimated elasticity with respect to the marginal net-of-tax rate corresponds to the elasticity of substitution parameter in a quasi-linear utility function when individuals face linear budget sets. In the presence of nonlinear budget sets, however, this elasticity of substitution (henceforth linear elasticity) only imperfectly captures the responsiveness of taxable income to tax rate changes. Blomquist et al. (2011, 2014) showed that a policy-specific taxable income elasticity can be defined with respect to changes in the entire nonlinear budget set by rotating the budget frontier. More generally, nonlinear budget sets could be characterized by several policy-relevant parameters not endogenous to income unlike the observed marginal net-of-tax rate. Different

[1] The net-of-tax rate is one minus the tax rate. See Saez et al. (2012) for a review of the literature.

elasticity concepts could be defined for these parameters (henceforth nonlinear elasticities). Building on their work, we define a nonlinear consumption net-of-tax elasticity corresponding to changing a flat consumption tax, the most natural analog of the linear elasticity. We show that this nonlinear elasticity is higher than the linear elasticity.

We then investigate the standard approach to estimating elasticity involving the observed marginal net-of-tax rate for linearized budget sets when budget sets are nonlinear. We show that the estimated elasticity is a downward-biased estimate of the linear elasticity if there is preference heterogeneity. Additionally, a valid instrumental variable can help overcome this bias. We argue, however, that the structural (instrumental variable) estimate of the linear elasticity cannot be used to consistently simulate policy effects of tax changes, while the reduced-form estimate of taxable income on the instrumental variable yields a policy-relevant nonlinear elasticity.

Our main contribution is to extend the benchmark model with quasi-linear utility to incorporate nonlinear budget sets. We derive the taxable income function that depends on a weighted average of the marginal net-of-tax rates at every income level with the probability density function of unconditional income as weights. Since our specification accounts for the entire budget set, we circumvent the issues related to the observed marginal net-of-tax rate being endogenous to taxable income, such as mean reversion and widening income distribution, plaguing specifications based on linearized budget sets. A key advantage of our approach is that even while it accounts for nonlinear budget sets, it can be estimated using panel data with differencing methods, much like the previous literature. The estimated (weighted) average marginal net-of-tax rate elasticity also has a direct policy-relevant interpretation as it could be interpreted as the nonlinear consumption net-of-tax elasticity.

From the perspective of our taxable income function, specifications based on linearization that include just the observed marginal net-of-tax rate and no other marginal net-of-tax rates are valid only when budget sets are linear. In addition to the previously known sources of endogeneity of the observed marginal net-of-tax rate related to mean reversion and widening income distribution, the omission of the remaining marginal net-of-tax rates is a previously unrecognized source of endogeneity. This is because observed and remaining marginal net-of-tax rates are very likely correlated. A valid instrumental variable for consistent estimation of linear elasticity is significantly more demanding. The instrumental variable not only must be correlated with the observed marginal net-of-tax-rate, but also should be orthogonal to the omitted marginal net-of-tax-rates to overcome the omitted variable bias.

We also present an empirical application of our methodology on U.S. data from the NBER tax panel for 1979-1990 exploiting exogenous variation in tax reforms. This data set has been used several times previously for estimating the linear elasticity (e.g., Gruber and Saez, 2002).

We find an estimated nonlinear elasticity of 0.75 for taxable income and 0.20 for broad income when applying the average marginal net-of-tax rate. The estimated elasticities are statistically significant at the one percent level. By comparison, the standard instrumental variable linearization method yields a reduced-form estimate of the nonlinear elasticity at the base-year income of 0.27 for taxable income and 0.13 for broad income. Accounting for nonlinear budget sets therefore increases the estimated elasticity. Unlike the standard method, our estimated elasticity for the average marginal net-of-tax rate is insensitive to controlling for base-year income, indicating that these estimates are unaffected by bias due to mean reversion

or widening income distribution. This result supports the view that tax rate variation arising from tax reform is exogenous after appropriately accounting for nonlinear budget sets.

We are not the first to explain for nonlinear budget sets. Such methods have a well-established tradition in the labor supply literature that uses hours of work as the outcome variable. Burtless and Hausman (1978) and Hausman (1985) proposed a method in which estimation is carried out using maximum likelihood. Dagsvik (1994) and Hoynes (1996), among others, proposed an alternative method that discretizes the outcome variable and focuses on estimating parameters of the utility function. Blomquist and Newey (2002) developed a method allowing nonparametric estimation with least squares. Ongoing work by Blomquist et al. (2011, 2014) extends this method to estimation of taxable income. This paper differs from this literature by using the same arguably strong functional-form assumptions regarding the utility function and the probability density function of income (that we show was used implicitly before) as in the previous taxable income literature that linearized budget sets. By making these assumptions, we can provide a simple, transparent, and intuitive method that is amenable to use of panel data and differencing methods and that we can identify the bias due to linearization (and isolate it from the bias due to functional-form, for instance).

The rest of the paper is organized as follows. The next section provides the theoretical analysis. Section 3 describes the empirical specifications and presents the data. Section 4 reports the elasticity estimates. The last section concludes.

2. Theoretical model

2.1 Model with linear budget set

We start with a theoretical framework where the individual chooses (c, y) to maximize utility $u(c, y)$ subject to a budget constraint, $c(y)$, where c is consumption that equals the net after-tax income and y is taxable income. The budget set is the area below the budget constraint.

Assuming a quasi-linear utility function will yield the double-log specification of the taxable income function estimated in the previous literature. With linear budget sets, the decision problem is:

$$\max_{c,y} u = -\frac{\beta_l^i \exp\left(-\frac{\alpha_l^i}{\beta_l^i}\right)}{\beta_l^i + 1} y^{\frac{\beta_l^i + 1}{\beta_l^i}} + c \quad \text{s.t.} \tag{1}$$

$$c = \tau y + c_0. \tag{2}$$

Superscript i indexes individuals and subscript l denotes parameters estimated when budget sets are linear. α_l^i and β_l^i are both parameters of the utility function. β_l^i is the elasticity of substitution with respect to the taxable income parameter. τ is the slope of the budget frontier (i.e., consumption derivative with respect to taxable income, c_y), which is also the marginal net-of-tax rate. c_0 is the intercept and equals net income from remaining residual sources and is assumed to be exogenous.

The taxable income choice y^* for an interior solution is given by the first-order condition. Solving for y^* yields:

$$\ln y^* = \alpha_l^i + \beta_l^i \ln \tau. \tag{3}$$

With quasi-linear utility, the optimal choice of taxable income does not depend on the intercept and the income effect is, therefore, zero. Note that $\beta_l^i = d \ln y^* / d \ln \tau$ is the elasticity of taxable income with respect to the marginal net-of-tax rate in a linear budget set (henceforth linear taxable income elasticity).

We now generalize the model in two ways. First, in order to capture differences in taste for work, we introduce two preference heterogeneity errors, e_1 and e_2, and let α_l and β_l be population-average parameters. Second, because individuals may not be able to precisely fine-tune their taxable income due to hours restrictions or availability of a limited set of taxable income options, we allow for optimization and measurement errors, e_3.[2] The error terms enter according to:

$$\alpha_l^i = e_1 \alpha_l, \tag{4}$$

$$\beta_l^i = e_2 \beta_l, \tag{5}$$

$$y = e_3 y^*. \tag{6}$$

We assume that the sources of error vary between individuals, have a mean of one, and are statistically independent conditional on the budget set. We omit the i index on them for notational simplicity. The expected taxable income can then be written as:

$$E(\ln y) = \alpha_l + \beta_l \ln \tau. \tag{7}$$

Eq. (7) shows that accounting for preference heterogeneity and optimization errors according to Eqs. (4) to (6) still leads to a double-log specification. Therefore, the models estimated in the previous literature are generally consistent with the presence of such complications and could be used to estimate the population-average elasticity $\beta_l = E(d \ln y^* / d \ln \tau) = E(d \ln y / d \ln \tau)$.

Letting z be a component of the preference heterogeneity terms e_1 and e_2 that can be accounted for, we only require independence of the error terms conditional on z. In the empirical differencing estimation setting that we later apply, individual-specific heterogeneity is differenced away, relaxing the independence assumption on the error terms considerably.

The taxable income functions in Eqs. (3) and (7) are obtained analytically by solving for y^* from the first-order condition that depends on a marginal tax rate that is constant across income levels and, therefore, not endogenous to income. In nonlinear budget sets, there are many marginal tax rates and the observed marginal tax rate is endogenous to income. We present an iterative procedure to derive the taxable income function that is useful for handling this complication later when we extend the model to account for nonlinear budget sets. The procedure involves increasing y in steps and checking at each step whether the first-order condition holds using the relevant marginal tax rate (which is constant in linear budget sets). Using j to denote the running variable index, the taxable income function can then be expressed as:

$$E(\ln y) = E(\ln y^*) = \iiint \ln y_j \, f\big(\ln y_j \mid \ln \tau, e_1, e_2\big) f(e_1) f(e_2) d\big(\ln y_j\big) de_1 de_2, \tag{8}$$

[2] e_3 is the product of optimization and measurement errors.

$$f\left(\ln y_j \,\middle|\, \ln\tau\,, e_1, e_2\right) = 1\left(\ln y_j = e_1\alpha_l + e_2\beta_l \ln\tau\right). \tag{9}$$

where $f\left(\ln y_j \mid \ln\tau\,, e_1, e_2\right)$ is a conditional probability density function, and 1(.) is an indicator function. Note that, because of the independence assumption on the error terms, we could make use of $f\left(\ln y_j\,, e_1, e_2 \mid \ln\tau\right) = f\left(\ln y_j \mid \ln\tau\,, e_1, e_2\right) f(e_1) f(e_2)$ in deriving Eq. (8).

For Eq. (8) to equal the double-log specification in Eq. (7), we require:

$$\int \ln y_j\, f\left(\ln y_j \mid \ln\tau\,, e_1, e_2\right) d\left(\ln y_j\right) = \int \left(e_1\alpha_l + e_2\beta_l \ln\tau\right) f\left(\ln y_j\right) d\left(\ln y_j\right). \tag{10}$$

This clarifies the exact implicit functional-form assumption on the probability density function of income needed to obtain the double-log specification of the taxable income function applicable to linear budget sets. We show in Subsection 2.4 the implications of making a similar functional-form assumption when budget sets are nonlinear.

2.2 Nonlinear budget set elasticity

Real-world tax and transfer systems, being progressive, lead to piecewise-linear budget sets. A work-around is to linearize the budget set at the observed taxable income to obtain the observed marginal net-of-tax rate and estimate the linear taxable income elasticity assuming a linear budget set. A rationale for this procedure is that the optimal choice on the linearized budget set is the same as the optimal choice on the nonlinear budget set if preferences and budget set are convex (Hausman, 1985; Mofitt, 1990). Varying the observed marginal net-of-tax rate of the linearized and nonlinear budget sets, therefore, has the same effect on behavior as long as the individual stays within the same tax bracket after the variation.

Assume for simplicity that the nonlinear budget frontier is differentiable.[3] The first-order condition in Eq. (3) from the optimization problem in Eqs. (1) and (2) with the linearized budget set continues to be both sufficient and necessary for an optimum. A complication arises when the budget set is nonlinear, the marginal net-of-tax rate is a function of taxable income:

$$\ln\tau\,(\ln y) = \ln c_y(\ln y). \tag{11}$$ [4]

The linearized marginal net-of-tax rate needed to obtain the taxable income in Eq. (3) is therefore, not constant; because it depends on taxable income, it is endogenous. In this setting, the chosen taxable income, y^*, and marginal net-of-tax rate, τ^*, are the solutions to the equation system given by Eqs. (3) and (11).[5]

An important question is whether the elasticity of substitution in the utility function, β_l^i, remains an interesting parameter when budget sets are nonlinear. Under linear budget sets, β_l^i conveniently represents the elasticity of taxable income with respect to a unique marginal net-of-tax rate that is a budget-set parameter that tax policy can directly control. In nonlinear budget sets, both chosen taxable income and marginal tax rates are outcome variables that tax policy

[3] Most budget sets are piece-wise linear, with the budget frontier not differentiable. Individuals may then want to locate at kinks, which are the main complications with nonlinear budget sets. The linearization approach ignores such complications. Assuming that the nonlinear budget frontier is smooth simplifies the analysis and still illustrates the essential complications with nonlinear budget sets.

[4] The nonlinear budget constraint comparable to Eq. (2) is $c = c(y)$. We express the budget set in terms of a marginal net-of-tax rate function and take logs of the variables here because this is the function relevant for elasticities.

[5] The nonlinear budget set model in Burtless and Hasusman (1978) and Hausman (1985) is equivalent to our Eqs. (3) and (11) for the case with piecewise-linear budget sets.

cannot control. Instead, there are many policy-relevant marginal tax rates, and therefore, many ways to vary the marginal net-of-tax rate function. Policy-relevant nonlinear elasticities may be defined with respect to any subset of these rates.

We note that linear taxable income elasticity corresponds to a tax schedule alteration where marginal net-of-tax rates are changed proportionally at every income level. Such an overall tax schedule change that proportionally rotates the budget frontier is possible also when the tax schedule is nonlinear, as Blundell and Shephard (2012) note. Such rotation can be thought of as capturing the modification in the budget set due to the change in a flat consumption tax and is best describe as a nonlinear consumption net-of-tax elasticity and can be expressed as follows:

$$\beta_{nl}^{i} = \frac{d \ln y^{*}}{d \ln \tau\,(.)}, \;\; \beta_{nl} = E\left(\beta_{nl}^{i}\right). \qquad (12)^{6}$$

This concept of nonlinear elasticity is the most natural analog of linear elasticity. Note that, it collapses to the linear elasticity when the marginal-net-of-tax-rate is constant, i.e., $\ln \tau\,(.) = \ln \tau$.

The consumption net-of-tax elasticity is similar in spirit to the nonlinear elasticity defined by Blomquist et al. (2011, 2014) that rotates the budget frontier upward in absolute terms.[7] Besides providing a policy-relevant overall measure of tax rate effects, the consumption net-of-tax elasticity also corresponds to a parameter in the taxable income function with nonlinear budget sets, as we show in Subsection 2.4. For now, we note that we could directly estimate the consumption net-of-tax elasticity by regressing (the change in) taxable income on (the change in) the consumption net-of-tax rate if the (change in) consumption tax is exogenous.

We now investigate how the nonlinear elasticity β_{nl} relates to the linear elasticity β_l when budget sets are nonlinear. Plugging Eq. (11) into Eq. (3) and differentiating with respect to $\ln \tau(.)$ assuming that $\ln \tau(.)$ is differentiable, and solving for β_{nl}^{i} yields:

$$\beta_{nl}^{i} = \frac{\beta_{l}^{i}}{1 - \beta_{l}^{i} \ln \tau_{\ln y}} \leq \beta_{l}^{i}. \qquad (13)^{8}$$

Our nonlinear elasticity is lower than the linear elasticity when the tax system is progressive since $\ln \tau_{\ln y} < 0$ in this case. The intuition is that increasing marginal net-of-tax rates has a direct positive effect on income when the budget set is linearized. However, the individual may enter new tax brackets with higher marginal tax rates (or lower marginal net-of-tax rates), leading to a subsequent counteracting negative effect on income. The difference between the

[6] More generally, let $\ln \tau = \ln \tau\,(\ln y\,; \boldsymbol{\theta})$ where $\boldsymbol{\theta}$ is a parameter vector characterizing the budget set. A nonlinear elasticity for tax policy $d\boldsymbol{\theta}$ could be defined as $d \ln y / d\boldsymbol{\theta}$. The consumption net-of-tax elasticity is $d \ln y / d\theta_1$ where $\ln \tau = \ln \tau\,(\ln y\,; \boldsymbol{\theta}_{-1}) + \theta_1$ where $\boldsymbol{\theta} = (\theta_1, \boldsymbol{\theta}_{-1})$.

[7] Their absolute rotation corresponds to changing a linear local tax. In linear budget sets, absolute or relative rotations are identical up to a scaling factor. In nonlinear budget sets, this is not the case. An absolute rotation has theoretically attractive features by keeping the intercept income effects of any linearized budget sets fixed. However, generalizing a *constant* linear elasticity leads to a nonlinear elasticity that corresponds to a relative rotation.

[8] Eqs. (3) and (11) yield $\ln y^{*} = \alpha_{l}^{i} + \beta_{l}^{i} \ln \tau\,(\ln y^{*})$. Differentiating with respect to $\ln \tau\,(.)$ yields $\beta_{nl}^{i} = \beta_{l}^{i} + \beta_{l}^{i} \ln \tau_{\ln y}\, \beta_{nl}^{i}$. Rearraging yields Eq. (13).

linear and nonlinear elasticity is positively related to the linear elasticity and the progressiveness of the tax system.[9]

2.3 The linearization approach

We now investigate the standard approach to estimating elasticity with respect to the observed marginal net-of-tax rate for linearized budget sets when those budget sets are actually nonlinear. Consider, for example, that all the variation in taxable income is generated by an exogenous variation in a consumption tax. The association between the chosen taxable income and marginal-net-of tax rate can then be expressed as follows:

$$\frac{d\ln y^*}{d\ln\tau^*}\Big|_{\ln\tau(.)\text{ varies}} = \frac{\ln y^*{}_{\ln\tau(.)}}{\ln\tau^*{}_{\ln\tau(.)}} = \frac{\dfrac{\beta_l^i}{1-\beta_l^i\ln\tau_{\ln y}}}{\dfrac{1}{1-\beta_l^i\ln\tau_{\ln y}}} = \beta_l^i. \qquad (14)^{10}$$

The marginal net-of-tax elasticity in Eq. (3) using the observed marginal net-of-tax rate and linearized budget sets, therefore, equals the elasticity of substitution in the utility function when budget sets are linear. Intuitively, as an individual enters tax brackets with higher tax rates, there is a negative effect on the chosen marginal net-of-tax rate (denominator in Eq. (14)) that is the same size as the counteracting effect on the chosen taxable income. The two effects cancel out. It can be shown that any other type of exogenous budget set variation can be used to estimate the linear elasticity.[11]

The association between taxable income and the observed marginal net-of-tax rate may also be affected by preference heterogeneity in addition to budget set variation. Allowing for preference heterogeneity according to Eq. (5), the association due to such heterogeneity is:

$$\frac{d\ln y^*}{d\ln\tau^*}\Big|_{e_2\text{ varies}} = \frac{\ln y^*{}_{e_2}}{\ln\tau^*{}_{e_2}} = \frac{\dfrac{\beta_l\ln\tau}{1-e_2\beta_l\ln\tau_{\ln y}}}{\dfrac{\ln\tau_{\ln y}\,\beta_l\ln\tau}{1-e_2\beta_l\ln\tau_{\ln y}}} = \frac{1}{\ln\tau_{\ln y}} < 0 \qquad (15)^{12}$$

The intuition is that, due to the progressivity of the tax system, individuals with a high taste for work choose higher incomes and lower observed marginal net-of-tax rates, leading to a negative association. The association between y^* and τ^* observed in the data typically reflects a mixture of the variations in preferences and budget sets, whereas we are interested in sorting out the effect of preferences to isolate the impact of tax rates.

Using an instrumental variable that depends only on exogenous variation in the tax schedule represents an empirical approach to isolate and estimate the effect of taxes on taxable income. For a consumption-tax based instrumental variable, Eq. (14) shows that the direct

[9] Blomquist et al. (2011, 2014) also showed that their nonlinear elasticity is lower than the linear elasticity when the budget frontier is piecewise linear and quasi-concave. They also postulated that welfare effects are more closely related to a nonlinear elasticity than the linear elasticity.

[10] Writing the relationship between $\ln y^*$ and $\ln\tau^*$ as $\ln y^* = \gamma\ln\tau^*$, we see that $d\ln y^*/d\ln\tau^* = \gamma$ is the regression coefficient. The numerator in Eq. (14) is given by Eq. (13). Eqs. (3) and (11) yield $\ln\tau^* = \ln\tau\left(\alpha_l^i + \beta_l^i\ln\tau^*\right)$. Differentiating with respect to $\ln\tau\,(.)$ yields $\ln\tau^*{}_{\ln\tau(.)} = 1 + \ln\tau_{\ln y}\,\beta_l^i\ln\tau^*{}_{\ln\tau(.)}$. Rearranging yields the denominator in Eq. (14).

[11] Following footnote 5, we could show that $d\ln y^*/d\ln\tau^*|_{\boldsymbol{\theta}\,varies} = \ln y^*{}_{\boldsymbol{\theta}}/\ln\tau^*{}_{\boldsymbol{\theta}} = \beta_l^i$.

[12] The derivation follows the same procedure as outlined in footnotes 7 and 9.

reduced-form effect of the instrumental variable on taxable income provides the nonlinear consumption net-of-tax elasticity. Reduced-form estimates of nonlinear elasticities are, however, specific to the type of tax-schedule variation upon which the instrumental variable is based. Eq. (14) also shows that the denominator is the first-stage effect and the ratio between the reduced-form and first-stage effects yields the structural estimate of the linear elasticity. Unlike the reduced form estimates of the nonlinear elasticity, the instrumental variable estimate of the linear elasticity is biased if the instrumental variable is weak.

Introducing optimization or measurement errors causes additional complications. Not only would the observed taxable income differ from the optimal taxable income choice, but the marginal net-of-tax rate at the observed taxable income, used to proxy the rate at the optimal choice, would contain an error. For the estimation of a nonlinear elasticity, the error only enters the dependent variable and, therefore, does not lead to bias. In the case of linear elasticity, however, the error enters an independent variable and, without use of an instrumental variable uncorrelated with the errors, would induce a bias (e.g., attenuation if the error is normally distributed). In the next subsection, we discuss an additional problem that arises when trying to estimate the linear elasticity consistently using the linearization approach.

Even if the linear elasticity can be consistently estimated, it is less useful for simulating taxable income responses to tax rate changes. With individual heterogeneity, we can only estimate the population-average linear taxable income elasticity in Eq. (7) and not the individual-specific elasticities in Eq. (3) needed for predictions. To evaluate a consumption tax reform, we are, e.g., interested in $\beta_{nl} = E\left(\beta_l^i / \left(1 - \beta_l^i \ln \tau^i{}_{\ln y}\right)\right) \neq E\left(\beta_l / \left(1 - \beta_l \ln \tau^i{}_{\ln y}\right)\right)$.

2.4 Taxable income function with nonlinear budget sets

In this subsection, we extend the outlined benchmark model with the quasi-linear utility function in Eq. (1) to incorporate nonlinear budget sets. Rather than linearizing budget sets, we derive a structural taxable income function that accounts for nonlinear budget sets. The only additional assumption needed is a modification of the functional-form assumption on the probability density function of income in Eq. (10). Replacing the linear budget constraint in Eq. (2) with the nonlinear budget constraint in Eq. (3), and allowing for preference heterogeneity and optimization errors according to Eqs (4) to (6), the first-order condition and the optimal taxable income can now be written as:

$$\ln y^* = e_1 \alpha_l + e_2 \beta_l \ln \tau^* \tag{16}$$

where τ^* is the slope of the linearized budget set at the chosen y^*.[13]

We can derive the taxable income function using the theoretical iterative procedure described previously for linear budget sets, which consists of increasing y in steps and at each step checking whether the first-order condition underlying Eq. (16) holds. This is feasible because at each fixed y_j, τ_j is exogenous if the budget set is exogenous. Given the independence assumptions on the error terms, we then obtain:

[13] Kink points can be handled by modifying the first-order condition so that we require $FOC > 0$ at $y^* - \epsilon$ and $FOC < 0$ at $y^* + \epsilon$. Corner solutions can be handled in a similar manner. In Eq. (16), we could replace $\tau^* = \tau(y^*)$ by $\tau^* = \tau\left(y = \lim_{\epsilon \to 0+} y^* + \epsilon\right)$ or $\tau^* = \tau\left(y = \lim_{\epsilon \to 0-} y^* + \epsilon\right)$. It can be shown in a more fullfledged nonlinear budget set model that the two specifications bound the true estimate, where the bounds decrease with the spread of the optimization errors. In practice, both specifications yield almost identical estimates.

$$E(\ln y) = E(\ln y^*) = \iiint \ln y_j \, f\big(\ln y_j \mid \ln \tau_j, e_1, e_2\big) f(e_1) f(e_2) d\big(\ln y_j\big) de_1 de_2, \tag{17}$$

$$f\big(\ln y_j \,\big|\, \ln \tau_j, e_1, e_2\big) = 1\big(\ln y_j = e_1 \alpha_l + e_2 \beta_l \ln \tau_j\big), \tag{18}$$

which is a straightforward generalization of the expression for linear budget sets in Eqs. (8) and (9). The only difference is that τ_j varies with y_j. Making a functional-form assumption similar to Eq. (10) we get:

$$\int \ln y_j \, f\big(\ln y_j \mid \ln \tau_j, e_1, e_2\big) d\big(\ln y_j\big) = \int \big(e_1 \alpha_{nl} + e_2 \beta_{nl} \ln \tau_j\big) f\big(\ln y_j\big) d\big(\ln y_j\big), \tag{19}$$

which yields the taxable income function:

$$E(\ln y) = \alpha_{nl} + \beta_{nl} \int \ln \tau_j \, f\big(\ln y_j\big) d\big(\ln y_j\big). \tag{20}$$

This taxable income function depends on a weighted average of marginal net-of-tax rates on the budget frontier rather than the marginal net-of-tax rate at a single point. The weight is the probability density function of the unconditional income. For estimation, we could discretize the budget set and numerically integrate it over points on the budget frontier. We could approximate the probability density function by the sample probability distribution.[14] Since our specification depends on the entire budget set, we circumvent a main source of endogeneity of the observed marginal net-of-tax rate to income plaguing specifications based on linearized budget sets.

Note that, unlike linear budget sets that require just two parameters to characterize (τ and c_0), the taxable income function with nonlinear budget set depends on the function $\ln \tau(.)$ consisting of a large number of parameters. Nevertheless, estimation remains tractable because the taxable income function is an integral over a two-dimensional term (τ_j and j). Compared with the linear budget set case (that only depends on τ), the dimensionality in the nonlinear case increases only by one. This is because the first-order conditions are the same at every point on the budget frontier and the optimality at every point only depends on the marginal net-of-tax rate at that point.

Note also that the estimated parameters, α_{nl} and β_{nl} in Eqs. (19) and (20), are not equal to α_l and β_l in Eqs. (16) and (18), unless budget sets are linear. The expected nonlinear budget set function nests the expected linear budget set function. β_{nl} is a nonlinear elasticity with respect to the (weighted) average marginal net-of-tax rate and could be used to simulate the effect of tax reform on taxable income based on reform-induced changes in average marginal net-of-tax rates. β_{nl} can also be interpreted as the nonlinear consumption net-of-tax elasticity in Eq. (12—a unit increase in the log of marginal net-of-tax rates at every income level results in a unit increase in the weighted average of log of marginal net-of-tax rates.

In the presence of nonlinear budget sets, preference heterogeneity, and optimization errors, the specification of the taxable income function in Eq. (7) based on linearization (including just the observed marginal net-of tax rate)is essentially misspecified. Eq. (20) shows that marginal net-of-tax rates at income levels other than the observed level also affect taxable income. In addition to the previously known sources of endogeneity of the observed marginal

[14] We set $\tau_j = \big(c_{j+1} - c_j\big) / \big(y_{j+1} - y_j\big)$ in the numerical integration.

net-of-tax rate related to mean reversion and widening income distribution, the omission of the remaining marginal net-of-tax rates is a previously unrecognized source of endogeneity. The observed and remaining marginal net-of-tax rates are very likely correlated. A valid instrumental variable for consistent estimation of the linear elasticity is significantly more demanding. The instrumental variable not only needs to be correlated with the observed marginal net-of-tax-rate, but also should be orthogonal to the omitted marginal net-of-tax-rates to overcome the omitted variable bias.[15]

The proper way to obtain a projection of the linear elasticity would be to estimate the population-average nonlinear elasticity in Eq. (20) and then to derive and make use of the equivalent of Eq. (13), accounting for optimization errors that apply to population-average elasticities. By focusing on nonlinear elasticities in the next empirical sections, we avoid this complication.

3. Empirical estimation

3.1 Regression specifications

It is possible to estimate the taxable income function in Eq. (20) using a single cross-section. However, the independence assumption of the budget set is unlikely to hold because budget sets typically correlate with demographic variables that may also affect taxable income. To the extent that these variables are individual-specific, their effects can be differenced away if panel data is available. We could then estimate the basic specification:

$$\Delta \ln y_{i,t} = \alpha_{i,t} + \beta_{nl} \sum_j \Delta \ln \tau_{j,i,t} \Pr(\ln y_j) + \varepsilon_{i,t}, \tag{21}$$

where i index individuals, t index year, Δ is the difference operator, $\alpha_{i,t}$ is a constant, and $\varepsilon_{i,t}$ is an idiosyncratic error term. In our application, j indexes 200 income intervals where the first 199 each cover \$1,000, and the 200th covers the open interval above \$199,000. $\Pr(\ln y_j)$ is the observed probability of taxable income, y, being in interval j. $\tau_{j,i,t}$ is the average marginal net-of-tax rate in j.

In our baseline specification, we use three-year differences. Obviously, identification requires at least one tax reform that differently affected individuals' budget sets. To the extent that these tax reforms lead to exogenous budget set changes, the nonlinear elasticity with respect to the (weighted) average marginal net-of-tax rate, which also could be interpreted as the consumption net-of-tax elasticity, could be consistently estimated. In our empirical application, we stack differences from eight different years and make use of multiple tax reforms. Because we have single filers (singles and couples that choose to file separately) and joint filers (couples), we also control for filing status.

[15] The omitted-variables problem is also present for the reduced-form estimate of any instrumental variable on taxable income representing an estimated nonlinear elasticity. Consider the reduced-form estimate of a first-dollar marginal net-of-tax rate instrumental variable on taxable income, for instance. Such an estimate may not provide a consistent estimate of the taxable income response to a tax reform that varies the first-dollar marginal net-of-tax rate if the first-dollar marginal net-of-tax rate variable correlates with other marginal net-of-tax rates. Inflating the reduced-form estimate by the first-stage estimate to obtain the structural estimate of the linear elasticity, however, additionally inflates the bias.

Note that constant relative changes in taxable income over years and between individuals — due to, e.g., productivity growth — are accounted for by the intercept term in Eq. (21). To remove any potential correlation between the timing of the different tax reforms and differences in productivity growth between years, we also include year dummies.

By comparison, the standard method is to linearize the budget set at the observed taxable income and estimate the effect of the change in the log of the observed marginal net-of-tax rate on the change in log taxable income. The endogenous change in marginal net-of-tax rate is instrumented by the predicted change in marginal net-of-tax rate at the base-year income level. Our specification in Eq. (21) is similar to the structural equation, but replaces the change in log of observed marginal net-of-tax rate by the change in log of the weighted average of marginal net-of-tax rates. We, thus, avoid the endogeneity of the observed marginal net-of-tax rate to income.

As discussed in Subsection 2.3, nonlinear elasticities correspond to reduced-form effects of instrumental variables on taxable income. Our nonlinear elasticity of the average marginal net-of-tax rate is therefore more comparable to the reduced-form estimate of the nonlinear elasticity with respect to the marginal net-of-tax rate at the base-year income than to the structural estimate of the linear elasticity in the standard linearization specification. We therefore compare our estimates with estimates from the following standard reduced-form linearization specification:

$$\Delta \ln y_{i,t} = \alpha_{i,t} + \beta_{nl} \Delta \ln \tau_{y,i,t} + \varepsilon_{i,t}, \tag{22}$$

where the marginal net-of-tax rate, $\tau_{y,i,t}$, is evaluated at the base-year income level, y. Comparing our specification that accounts for nonlinear budget sets in the estimation in Eq. (21), with this reduced-form specification, we use changes in marginal net-of-tax rates averaged over income levels rather than at a single income level. The standard specification also usually includes year dummies. Any difference between estimated elasticities from Eqs. (21) and (22) reflects the bias from not accounting for nonlinear budget sets in the estimation, although both provide elasticities of taxable income with respect to nonlinear budget sets.

An issue with the standard instrumental variable ($\Delta \ln \tau_{y,i,t}$), based on the observed base-year income, is that it is not valid because observed base-year income may be directly correlated with taxable income. This is the case if there is a temporary component of taxable income causing mean reversion. In that case, individuals with high income in one year tend to revert toward the mean in the next year. Another issue is heterogeneous productivity growth between individuals with different background characteristics who also have different income levels. Income inequality trends, such as a widening income distribution over time, are an example of such an issue.

The standard method to address both mean reversion and heterogeneous income growth is to control for either the log base-year income or its spline. Because the instrumental variable and the spline both rely on variation in base-year income, identification of the marginal net-of-tax rate parameter could be problematic and could possibly depend heavily on functional-form assumptions. The availability of data spanning several reforms may help identification by providing variation in the change in marginal net-of-tax rate at the same income level for different base years. Of course, this only helps if the effect of base-year income is the same for different base years.

Weber (2014) investigated whether the standard instrumental variable and other related instrumental variables that are functions of taxable income are valid. She found that they cannot overcome the mean reversion problem, even when they include a base-year income control function. She showed, however, that income lagged several years could be used to construct instrumental variables that are valid in the limit, because the temporary component of taxable income diminishes over time. As for the issue of heterogeneous productivity growth, including an income control function is more promising, although again, base-year income is endogenous. Weber suggested using longer income lags.

Because our independent variable is not a function of income, there is no automatic correlation with factors that correlate with income in any year. To the extent that tax reforms are exogenous and not correlated with income, there is no need to include an income control function.

It is, of course, still possible that tax reforms may correlate with other groups-specific trends. For example, reforms may target groups with certain background characteristics, which may have different income trends. This could be addressed by including demographic control variables. We do not have such variables. Base-year income could, however, be a proxy for such heterogeneity and solve the issue to the extent that such controls correlate with income. We include a base-year income control function to check the sensitivity of our estimates and to assess the exogeneity of the tax reforms assumption. We also include such control functions in our standard reduced-form linearization specification.

3.2 Data

We use data from the NBER panel of tax returns over the 1979-1990 period, also known as the Continuous Work History File, which is also the data used by Gruber and Saez (2002) and Weber (2014). The data contains detailed administrative information on tax and income variables but does not contain any demographic background variables. See Gruber and Saez's paper for a detailed description of the data set.

We construct the individual budget sets by computing marginal net-of-tax rates at different income levels using NBER-TAXSIM. We vary earnings and capital income in steps of $1,000, keeping income from other sources fixed. Deductions/itemizations that vary between individuals are accounted for in the construction of the budget sets.

We employ two of the most common measures of taxable income used in the previous literature: actual taxable income (almost exactly as technically defined in the tax forms) and broad income. Broad income is an extensive definition of gross income and includes, among other things, wage income, interest income, dividends, and business income. We exclude capital gains, however. Taxable income is broad income minus a number of deductions. We use the definition in 1990 and include all adjustments that can be computed from the data for all sample years. Our definitions are consistently defined over the entire sample period and are identical to the ones used by Gruber and Saez (2002).

As discussed by Gruber and Saez (2002), one margin of behavioral responses to tax reforms is a shift income between different sources, e.g., between taxable income and other components of broad income. To the extent that composition of income is endogenous to the constructed budget sets, the taxable income estimates would be biased. The broad income

estimates should, however, be less problematic in this regard. The taxable income estimates do, however, include the effects of tax avoidance, which is also a margin of interest for policy and welfare evaluations.[16]

Our sample selection criteria are similar to the ones in Gruber and Saez (2002). We drop filers that change filing status and observations with abnormally large (top and bottom 1 percent of the sample) changes in the weighted average marginal net-of-tax rate since these observations are more likely to be driven by variable construction errors. We also drop observations where we could not compute either taxable income or broad income. However, we do not truncate our data from below, unlike Gruber and Saez. We use the log of income plus one as the outcome variable to enable inclusion of observations that involve individuals with zero income. Sample statistics can be found in the Appendix.

4. Results

4.1 Taxable income

In Table 1, we report estimates of the nonlinear taxable income elasticities in Eqs. (21) and (22). In the first rows, we report the estimates of the change in log of the (weighted) average marginal net-of-tax rate as defined in Eq. (21). This is the specification that fully accounts for nonlinear budget sets. In the last rows, we report the estimates of the change in log of the predicted marginal net-of-tax rate at the base-year income level. This is the reduced-form estimate of taxable income on the instrumental variable in the standard linearization method. In Column (1), no control variables are included. In subsequent columns, we successively add a dummy for filing status, year, log base-year income, and a 10-piece spline in log base-year income.

Table 1. Estimates of nonlinear elasticities of taxable income

	(1)	(2)	(3)	(4)	(5)
Change in log of average marginal net-of-tax rate	0.559**	0.600**	0.756**	0.742**	0.748**
	(0.056)	(0.056)	(0.059)	(0.054)	(0.052)
Change in log of predicted marginal net-of-tax rate	-0.146**	-0.110**	-0.082**	0.358**	0.269**
	(0.025)	(0.025)	(0.026)	(0.023)	(0.023)
Filing status	No	Yes	Yes	Yes	Yes
Year dummies	No	No	Yes	Yes	Yes
Log base income	No	No	No	Yes	No
Spline log base income	No	No	No	No	Yes

Notes: Each cell is a nonlinear elasticity estimate from one regression. The change in log taxable income is the outcome variable. 3-year differences are used. The spline in base-year income contains ten pieces. Each regression contains 51,392 observations. * $p<0.05$; ** $p<0.01$.

Our estimated elasticity of taxable income with respect to the average marginal net-of-tax rate that accounts for nonlinear budget sets in the estimation is 0.56 (when no other covariates are

[16] To properly investigate income shifting effects would require a model that allows for several choice variables. The dimensionality of the budget set would then be the number of choice variables plus one.

included) and around 0.75 (after controlling for filing status and year effects). The estimated elasticities are all statistically significant at the 1 percent level, and they do not change much when we additionally control for base-year income, either in the form of log base-year income or a spline in log base-year income. The evidence, therefore, does not contradict that the tax variation between individuals provided by tax reforms, as represented by the change in log of average marginal net-of-tax rate, is exogenous. An elasticity of 0.75 implies that a 1 percent increase in marginal net-of-tax rates at every income level that could be carried out, e.g., by decreasing the tax rate of a flat consumption tax, leads to an increase in taxable income of 0.75 percent.

By comparison, the estimated elasticity of taxable income with respect to the marginal net-of-tax rate at the base year income based on linearized budget sets (in the estimation) is negative in Column (1). This likely reflects a correlation at least partly caused by preference heterogeneity, as shown by Eq. (15). Controlling for base-year income turns the estimated elasticity positive, indicating that mean reversion and/or heterogeneous productivity growth induce a bias for the standard instrumental variable. Also, the exact estimated elasticity is sensitive to the functional form of the base-year income control. The specification with the spline corresponds to the reduced-form estimates of the standard method and produces an elasticity of 0.27. Again, all estimated elasticities are statistically significant at the 1 percent level. A comparison of the two sets of estimated elasticities indicates that accounting for nonlinear budget sets in the estimation increases the estimated elasticity by almost a factor of three.

Our linearized budget set estimates are comparable to those from Gruber and Saez's (2002) on the same data but a slightly different sample. Their estimated taxable income elasticity estimate of 0.40 from the specification that includes a spline, is comparable to our estimates in Column (5). There are three sources of difference between our estimated elasticites and their's. First, and most importantly, they estimate the structural equation and therefore obtain the linear elasticity. Second they weight their regression by taxable income. And finally, their sample differs slightly from ours. Their reduced-form estimate (based on our estimations since they did not report this estimate) would have been 0.21 and statistically significant at the 1 percent level, which is close to 0.27 that we obtain. This translates into a first-stage estimate around 0.53.[17]

Weber (2014) used instrumental variables based on longer income lags on the same data. She found a linear taxable income elasticity between 0.86 and 1.36 in most specifications. Because she used several instrumental variables, there are several first-stage and reduced-form estimates (one for each instrumental variable), and there is no single reduced-form estimate to which we can compare. Rescaling her estimates by Gruber and Saez's first-stage estimate (0.53 based on our estimations) results in a nonlinear elasticity between 0.46 and 0.72, which would still be smaller than our estimated elasticity.[18]

To illustrate our estimates and the influence of base-year taxable income, we plot the change in log taxable income variable against the two different measures of the change in the log of marginal net-of-tax rates in Figure 1. We also plot the budget set variables against log

[17] The structural estimate (0.40) equals the reduced-form estimate (0.21) divided by the first-stage estimate (0.53) according to Eq. (14).
[18] The computations are done in the same way as described in footnote 17.

base-year income in Figure 2. We use both a local polynomial and a linear fit to the data in Figure 1, whereas we only use the local polynomial fit in Figure 2.

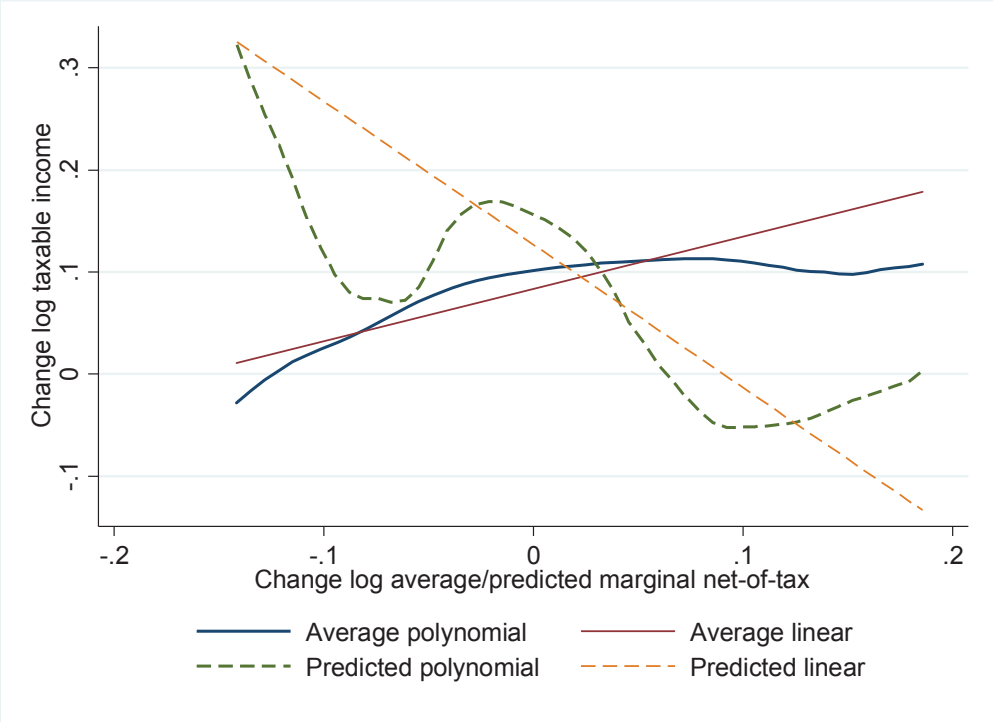

Figure 1. Graphs of the outcome variable against the marginal net-of-tax rate variables

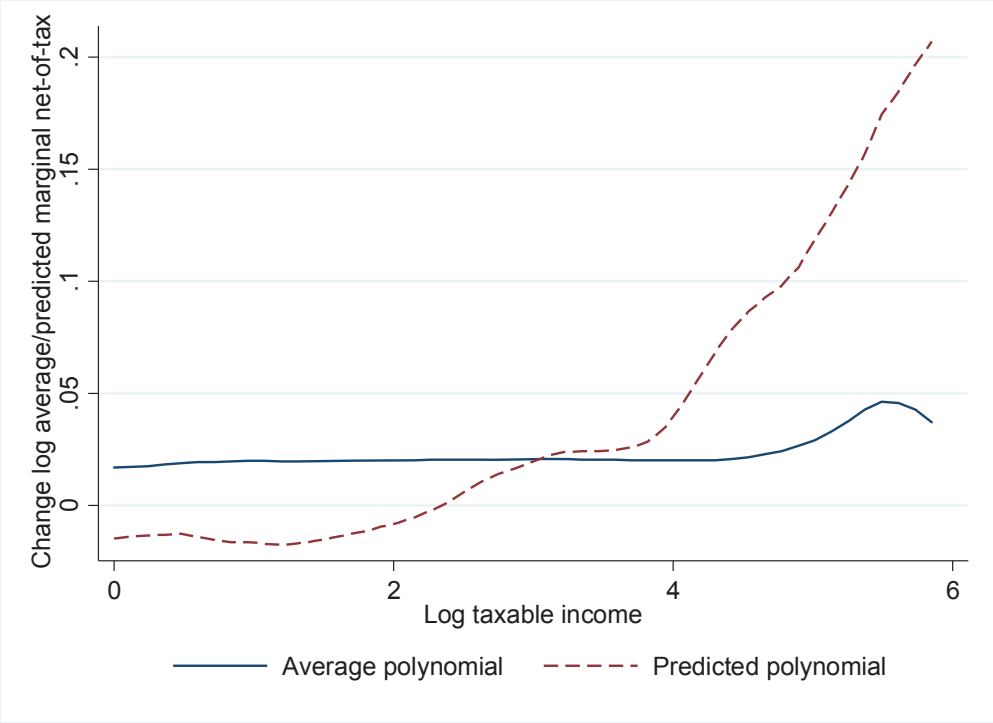

Figure 2. Graphs of marginal the net-of-tax variables against base-year income

Figure 1 shows that the correlation between the outcome variable and the change in average marginal net-of-tax rate is positive, whereas the correlation between the outcome variable and the change in predicted marginal net-of-tax rate is negative. In Figure 2, base-year income is uncorrelated with the change in average marginal net-of-tax rate, supporting the view that variation in this change induced by tax reforms is exogenous. Base-year income is, however, positively correlated with the change in predicted marginal net-of-tax rate. This suggests that the tax reforms in the 1980's gave individuals with higher base-year income a larger predicted marginal net-of-tax rate decrease. To sort out the effect of the change in predicted marginal net-of-tax rate at the base-year income from the direct effects of base-year income, therefore, relies on properly controlling for the independent effects of base-year income. The endogeneity of the change in marginal net-of tax rate at the base-year income, of course, does not necessarily indicate that variation provided by tax reforms are endogenous. More likely, it is a result of base-year income being endogenous.

4.2 Broad income

It is commonly believed that the taxable income elasticity largely captures the effect of income shifting. We therefore report nonlinear elasticity estimates for broad income in Table 2, which is organized similarly to Table 1. The estimated broad income elasticity that accounts for nonlinear budget sets in the estimation is in the region of 0.20 and still quite insensitive to including a base-year income control function. By comparison, the estimated elasticity based on linearized budget sets is around 0.13 and is sensitive to including the control function. All estimates are statistically significant at the 1 percent level. This reinforces our earlier result that accounting for nonlinear budget sets increases the estimated elasticity.

Table 2. Estimates of nonlinear elasticities of broad income

	(1)	(2)	(3)	(4)	(5)
Change in log of average marginal net-of-tax rate	0.126**	0.145**	0.173**	0.202**	0.199**
	(0.027)	(0.026)	(0.028)	(0.027)	(0.027)
Change in log of predicted marginal net-of-tax rate	-0.035**	-0.017	-0.032**	0.147**	0.133**
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)
Filing status	No	Yes	Yes	Yes	Yes
Year dummies	No	No	Yes	Yes	Yes
Log base income	No	No	No	Yes	No
Spline log base income	No	No	No	No	Yes

Notes: Each cell is a nonlinear elasticity estimate from one regression. The change in log broad income is the outcome variable. 3-year differences are used. The spline in base-year income contains ten pieces. Each regression contains 51,392 observations. * $p<0.05$; ** $p<0.01$.

In comparison, the estimated linear elasticity in Gruber and Saez (2002) were 0.12 and statistically insignificant. That corresponds to a statistically insignificant nonlinear elasticity of 0.06. Weber (2014) on the other hand obtained an estimated linear elasticity between 0.48 and 0.70 that translates to a nonlinear elasticity between 0.26 and 0.37. Therefore, our estimates lie within the range of the estimated elasticities in these two studies.

4.3 Difference length

The responsiveness of taxable income to changes in tax rates may differ in the short-run and the long-run. On the one hand, it may take some time for individuals to react to tax changes rendering larger long-run effects. On the other hand, temporal income shifting between years (e.g., between December one year and January the following year) produces only short-run effects. In Table 3, we report nonlinear elasticity estimates that account for nonlinear budget sets in the estimation for taxable income and broad income using different difference lengths. We use observations with the same base-year, which keeps the number of observations constant.

Table 3. Nonlinear net-of-tax elasticity with different difference lengths

	(1) 1-year differences	(2) 2-year differences	(3) 3-year differences
Taxable income	1.250**	0.788**	0.748**
	(0.048)	(0.046)	(0.052)
Broad income	0.215**	0.143**	0.199**
	(0.022)	(0.022)	(0.027)

Notes: Each cell is a nonlinear consumption net-of-tax elasticity estimate from one regression. The change in log income is the outcome variable. All specifications include controls for filing status, year dummies, and a spline in log base-year income. Each regression contains 51,392 observations. * $p<0.05$; ** $p<0.01$.

Table 3 shows that the estimated taxable income elasticity decreases with difference length, suggesting that there are some short-run effects. The estimated broad income elasticity decreases first between the specifications with one- and two-year differences, but increases between the specifications with two- and three-year differences, suggesting that there are some effects given a longer response time.

5. Conclusion

We started with a standard taxable income model with a quasi-linear utility function. The estimated elasticity of taxable income involving the marginal net-of-tax rate corresponds to the parameter of the elasticity of substitution in the utility function when individuals face linear budget sets. We showed that, when allowing for nonlinear budget sets, the nonlinear consumption net-of-tax elasticity is a direct policy-relevant extension of the linear elasticity. We then showed that using linearized budget sets to estimate the linear elasticity when budget sets are actually nonlinear produces a downward bias unless a valid instrumental variable is used. We argued, however, that that the structural (instrumental variable) estimate of linear elasticity cannot be used to consistently simulate policy effects of tax changes, while the reduced-form estimate of taxable income on the instrumental variable yields a policy-relevant nonlinear elasticity.

We extended the benchmark model to account for nonlinear budget sets and derived the taxable income function that depends on a weighted average of the marginal net-of-tax rates at every income level with the probability density function of unconditional income as weights.

The estimated average marginal net-of-tax rate elasticity could also be interpreted as the nonlinear consumption net-of-tax elasticity. Since our specification accounts for the entire budget set, it circumvents the endogeneity of the observed marginal net-of-tax rate to taxable income, such as mean reversion and widening income distribution, plaguing specifications based on linearized budget sets. We also highlighted a new source of endogeneity in such specifications arising from omission of remaining marginal net-of-tax rates at income levels other than the observed income level.

In our empirical application, we estimated the nonlinear elasticity of the average marginal net-of-tax rate using U.S. data from the NBER tax panel for 1979–1990 and differencing methods. We found an estimated nonlinear elasticity of 0.75 for taxable income and 0.20 for broad income. By comparison, the standard instrumental variable linearization method yielded a reduced-form estimate of the nonlinear elasticity of 0.27 for taxable income and 0.13 for broad income with respect to base-year income. Accounting for nonlinear budget sets, therefore, increased the estimated elasticity. Unlike the standard method, our estimated elasticity with respect to the average marginal net-of-tax rate was insensitive to controlling for base-year income, indicating that these estimates were unaffected by bias due to mean reversion or widening income distribution.

References

Auten, G., Carroll, R., 1999. The effect of income taxes on household income. Review of Economics and Statistics, 81, 681-693.

Blomquist, S., Kumar, A., Liang, C.-Y., Newey, W., 2011. Nonparametric estimation of taxable income function. Manuscript.

Blomquist, S., Kumar, A., Liang, C.-Y., Newey, W., 2014. Individual heterogeneity, nonlinear budget sets, and taxable income. Uppsala Center for Fiscal Studies Working Paper 2014:1, Uppsala University.

Blomquist, S., Newey, W., 2002. Nonparametric estimation with nonlinear budget sets. Econometrica 70, 2455-2480.

Blomquist, S., Selin, H., 2010. Hourly wage rate and taxable labor income responsiveness to changes in marginal tax rates. Journal of Public Economics 94, 878-889.

Blundell, R., Shephard, A., 2012. Employment, hours of work and the optimal taxation of low income families. Review of Economic Studies 72, 481-510.

Burtless, G., Hausman, J., 1978. The effect of taxation on labor supply: evaluating the Gary negative income tax experiment. Journal of Political Economy 86, 1103-1130.

Carroll, R., 1998. Do taxpayers really respond to changes in tax rates? Evidence from the 1993 act. Office of Tax Analysis Working Paper 78, U.S. Department of Treasure, Washington D.C.

Dagsvik, J., 1994. Discrete and continuous choice, max-stable processes and independence from irrelevant attributes. Econometrica 62, 1179-1205.

Feldstein, M., 1995. The effect of marginal tax rates on taxable income: a panel study of the 1986 tax reform act. Journal of Political Economy 103, 551-572.

Feldstein, M., 1999. Tax avoidance and the deadweight loss of the income tax. Review of Economics and Statistics 81, 674-80.

Gruber, J., Saez, E., 2002. The elasticity of taxable income: evidence and implications. Journal of Public Economics 84, 1-32.

Hausman, J., 1985. The econometrics of nonlinear budget sets. Econometrica 53, 1255-1282.

Hoynes, H., 1996. Welfare transfers in two-parent families: labor supply and welfare participation under AFDC-UP. Econometrica 64, 295-332.

Kopczuk, W., 2005. Tax bases, tax rates and the elasticity of reported income. Journal of Public Economics 89, 2093-2119.

Liang, C.-Y., 2012. Nonparametric structural estimation of labor supply in the presence of censoring. Journal of Public Economics 96, 89-103.

Moffitt, R., 1990. The Econometrics of kinked budget constraints. Journal of Economic Perspectives 4, 119–139.

Saez, E., 2003. The effect of marginal tax rates on income: a panel study of “bracket creep”. Journal of Public Economics 87, 1231-1258.

Saez, E., Slemrod, J., Giertz, S., 2012. The elasticity of taxable income with respect to marginal tax rates: a critical review. Journal of Economic Literature 50, 3-50.

Weber, C., 2014. Toward obtaining a consistent estimate of the elasticity of taxable income using difference-in-differences. Journal of Public Economics 117, 90-103.

Appendix

Table A1. Sample statistics

Variables	Mean	Std. dev.	Min	Max
Change log taxable income	0.092	0.692	-4.022	4.275
Change log broad income	0.040	0.329	-2.265	2.885
Change log average marginal net-of tax rate	0.020	0.055	-0.142	0.186
Change log predicted marginal net-of tax rate	0.017	0.124	-3.005	4.902
Log base-year taxable income	2.905	0.937	0.000	6.520
Log base-year broad income	3.571	0.606	1.752	6.596
Log base-year average marginal net-of-tax rate	-0.431	0.061	-0.859	-0.175
Log base-year predicted marginal net-of-tax rate	-0.312	0.152	-3.297	0.877
Base-year taxable income	25.367	24.108	0.000	677.657
Base-year broad income	41.657	29.345	4.768	730.991
Base-year average marginal net-of-tax rate	0.655	0.039	0.424	0.847
Base-year predicted marginal net-of-tax rate	0.739	0.093	0.037	2.404

Notes: Taxable income and broad income are in USD at the 1990 price level.