Innovation, Firm Dynamics, and International Trade*

Andrew Atkeson† and Ariel Burstein‡

October 2006

Abstract

We present a simple general equilibrium model of the decisions of firms to innovate and to engage in international trade. We use the model to analyze the impact of a reduction in international trade costs on firms' innovative activity. In our model, monopolistically competitive firms invest managerial time to lower their marginal cost of production. We first show analytically that if all firms export with equal intensity, then a reduction in international trade costs has no impact at all on firms' investments in innovation. We then analyze the impact of a reduction in marginal trade costs on firms' innovation in a quantitative version of the model in which only a fraction of firms choose to export. Here, a reduction in trade costs leads to a reallocation of investment in innovation from firms that do not export to firms that do export. This reallocation of innovative investment reinforces existing patterns of comparative advantage and leads to an amplified response of trade volumes and output over time. Nevertheless, quantitatively, the welfare gains from a reduction in trade costs are small since investments in innovation require current resources and take a long time to pay off.

*Preliminary and incomplete.

†UCLA, Federal Reserve Bank of Minneapolis, and NBER.

‡UCLA and NBER.

1. Introduction

Over the past several decades, there has been a striking growth in the share of international trade in output, both for the US economy and for the world as a whole. With this expansion of international trade, firms in each country now have the opportunity to sell to a wider market and they face at the same time increased competition from firms abroad. How has this expansion of opportunities for international trade changed firms' incentives to innovate?

In this paper, we present a simple general equilibrium model of the decisions of firms to innovate and to engage in international trade. In our model, a continuum of firms produce differentiated products and engage in monopolistic competition. Owners of these firms make decisions to start new firms, to engage in innovative activity in existing firms, to enter into the business of exporting abroad, and to close down unpromising firms, and these decisions shape the model's implications for both for industry dynamics and corresponding aggregate levels of productivity, output, and international trade. We show that our model, despite its simplicity, reproduces quantitatively many salient features of the US data on firm dynamics and export behavior. We then use this model to assess the impact of a reduction in the costs of international trade on firms' innovative activity and the associated changes in firm dynamics and corresponding levels of aggregate productivity, output and international trade both in the short and the long run.

The basic elements of our model are as follows. To start a new firm, the owners of this firm must hire a fixed amount of managerial labor time as a sunk investment. This investment yields, one period later, a new firm capable of producing a new product. This firm has a randomly drawn initial productivity indexing its marginal cost of production. Once this new firm is created and its productivity is realized, the owners of the firm must make several decisions identical to those that they must make for any continuing firm. All firms, whether newly created or continuing, start each period with a random productivity draw. This productivity draw is the only state variable characterizing the firm. Once this productivity draw is realized, the owners of each firm decide whether to operate the firm in the current period or to exit. If the owners decide to exit, the opportunity to operate this firm is lost permanently. To operate the firm, the owners must hire a fixed amount of managerial labor time as a fixed cost of production, and then they hire workers to produce output at the firm's current marginal cost and to sell that output at a price marked up over marginal cost. If the owners of the firm wish to export some of their output, they must hire a fixed amount of managerial time as a fixed cost of exporting, and exported output is also subject to an iceberg-type marginal cost of exporting denominated in goods. Additional managerial labor time can also be hired to engage in innovative activity that results in a higher expected increment to the productivity of the firm in the next period. The incentive to innovate within the firm, then, stems from the incentive to reduce marginal costs to increase future profits.

In this model, changes in productivity at the firm level correspond to changes in the marginal costs of production and hence changes in the optimal size of the firm. Thus, our model of innovation at the firm level is a model of the determinants of firm growth and the steady-state distribution of employment across firms. The assumption that the fruits of innovative activity are stochastic in our model means that our model can account for simultaneous growth and decline and entry and exit of firms in steady-state.

We start our analysis of the impact of a reduction in the costs of international trade on firm-level innovation with a stark analytical result. We show in our model that if all firms export (so the fixed costs of international trade are zero), then, in general equilibrium, a decrease in the marginal costs of international trade have no impact at all on firms' innovation decisions and hence no impact on aggregate productivity and output over an above the direct impact of this trade cost reduction of the volume of trade and production.

The intuition for this result in our model is straightforward. Innovative activity is costly. It required the use of managerial time and this time is a scarce resource. Managers can be employed to manage and innovate in existing firms or they can be employed to start new firms. If all firms export, holding everything else fixed, a reduction in the costs of international trade serves to increase the size of the market and hence the profit opportunities for all firms. Hence, the incentives to hire managers to innovate in existing firms to reduce the marginal costs of production rises because now the benefits of any reduction in marginal costs can be spread over a larger market. But the same is true for the incentives to hire managers to start new firms. Holding everything else fixed, a reduction in international trade costs increases the profits to be earned by newly created firms. In general equilibrium, the incentives to innovate in existing firms and to start new firms rise by the same amount and the managerial wage simply rises to leave the allocation of managers across these two activities unchanged. We show that the same result holds in a version of our model in which managers and workers are perfect substitutes so that labor may be used either to manage and innovate in existing firms, to start new firms, or to work as labor in existing firms for the same reason — the returns to all of these activities all rise by the same proportion in response to a reduction in the costs of international trade.

Our analytical results echo the findings of Eaton and Kortum (2001). They find the same result that changes in the costs of international trade have no impact on innovative effort in a different context. Theirs is a model of quality ladders embedded in a multi-country Ricardian model of international trade, with a research sector that produces new ideas randomly across goods. The intuition for their result is different from ours. In their model, only the firm with the highest productivity produces each product. A reduction in trade costs expands the market available to that leading firm and thus, holding everything else fixed, this increases a firm's incentive to innovate to increase the probability its becoming the firm with the highest productivity. There is a second countervailing effect as well. This reduction in trade costs also increases the probability that a foreign firm will become the leading firm and take over the domestic market. Holding everything else fixed, this effect reduces a firm's incentives to innovate. In their model, these two effects cancel out exactly. Our result differs from theirs in that in our model innovation is directed toward reducing the firm's marginal cost of producing a specific product and firms do not compete over technological leadership in producing any particular product. All of the effects in our model of foreign competition on the profits of an individual firm come through the effects of foreign competition on aggregate prices and wages.

We then move to the quantitative part of our analysis. A quantitative analysis is necessary because our analytical result that changes in the costs of trade have no impact on firm level innovation only hold if all firms have equal exposure to opportunities for international trade. In the data, however, firms vary greatly in their participation in international trade — exports are highly concentrated among large firms. Here we build on the insight of Melitz (2003). He observed that, in the presence of fixed costs of trade, only the more productive (and hence larger) firms choose to export. In this case, changes in the marginal costs of trade lead in general equilibrium to an expansion in the profits of exporting firms and a contraction in the profits of firms that do not export. Hence, as Melitz showed, this reduction in trade costs leads immediately to a reallocation of labor away from small firms that do not export toward large firms that do. In our model, this reduction in trade costs also leads in equilibrium to an increase in innovation incentives for exporting firms and a decrease in innovation incentives for firms that do not export. Hence, over time, this reduction in trade costs leads to an amplification of the initial reallocation effects of a decrease in trade costs — exporting firms grow over time and increase their exports while firms that do not export shrink. We use our model to assess the quantitative magnitude of these effects.

Our model is closely related to several papers in the literature. If we assume that the optimal choice of innovative activity within the firm does not respond substantially to variation in the incentives to innovate, then our model is a variant of Hopenhayn's (1992) model in which firms' experience exogenous random shocks to their productivity. In this case, our model is specifically an open economy version of the model of firm dynamics in Luttmer (2006) and hence is quite similar to Irarrazabal and Opromolla (2006). Our inclusion of fixed and marginal costs of exporting also follows Melitz (2003), Ruhl (2004), Chaney (2005), Ghironi and Melitz (2005), Alessandria and Choi (2006), and Corsetti et. al. (2005) among others. We study such a version of our model as a benchmark to show that it can reproduce many of the same features of the US data on firm dynamics and export decisions studied in these papers. One of the most important of these features is Gibrat's Law — the observation that, at least for large firms, firm growth is independent of firm size. As discussed in detail in Luttmer (2006), corresponding to this observation is the observation that the right tail of the distribution of firm sizes is roughly Pareto. We also reproduce data on the fraction of firms that export, their average export intensity, and the rate at which they change status from exporting to not exporting as documented in Bernard et. al. (2005)

Our approach to modelling innovation by the firm is similar to that pioneered by Ericson and Pakes (1995). Here it is an investment of managerial labor time in innovation that is required to advance the productivity of the firm. One challenge we face in making our model quantitative is that on putting some quantitative discipline on our assumed innovative investment cost function. We face two particular challenges here: first that of modelling how the costs of innovation scale with firm size, and second with modelling how sensitive optimal innovative effort is to changes in the incentives to innovate. We choose the form of the innovation investment cost function so that our model can reproduce key features of the data on firm growth. In particular, since changes in the productivity of the firm imply changes in optimal firm size, these investments in innovation within the firm affect the expected growth rate of the firm in terms of employment and profits. We choose a particular parameterization of the innovative investment cost function so that our model has the potential to reproduce Gibrat's Law: the observation that for large firms the expected growth rate of the firm is independent of firm size. In this parameterization, the cost in terms of managerial time of the innovative activity required to produce a given expected growth rate of the firm scales proportionately with the size of the firm (holding fixed the decision to export). We show that this parameterization of the innovative investment cost function leads to Gibrat's Law for large firms in our model because the incentives to innovate within the firm also scale with the size of the firm measured in terms of labor or profits (again holding fixed the decision to export). Hence, in our baseline model, for large firms, the costs and benefits of innovative activity both scale proportionally with firm size and thus, in equilibrium, innovation leads to firm growth rates that are independent of firm size. Our model's implications for firm growth rates is quite sensitive to this choice of scaling of the costs of innovation. For example, we show that one natural alternative investment cost function, one in which the costs of advancing the labor productivity of the firm by a fixed percentage scales with the current level of labor productivity within the firm, is inconsistent with Gibrat's Law. Since, for a firm that faces elastic demand, profits grow faster than productivity, this alternative scaling of the innovative investment cost function would lead in our model to the implication that the optimal growth rate of firms grew with firm size.

We use the fact that, in our model, the returns to innovation for small firms do not scale with firm size in the same way that they do with large firms to put some discipline on our assumed responsiveness of innovative effort to changes in the incentives to innovate. The returns to innovation for smaller firms in our model do not scale with the current productivity of the firm, as they do for large firms, for three reasons. The first reason is that firms that are too small to find it profitable to export face a smaller market than those that do, and hence have a smaller increase in profits from a given reduction in marginal cost. The second reason is that exporting firms that are relatively small face the risk of leaving the export market due to negative productivity shocks (and hence avoid paying the fixed cost). The possibility of exercising this option gives them less inventive to innovate. The third reason is that firms that have the lowest productivity levels face a greater risk of exit due to negative productivity shocks and hence have an even smaller incentive to innovate. In our model, if we make innovative investment too responsive to variation in the incentives to innovate, the implications of the model for firm dynamics and the distribution of firm size are counterfactual. Specifically, the deviations from Gibrat's Law for small firms and firms that do not export implied by the model grow too large to be plausible in light of the available data on firm dynamics.

In our quantitative exercises, we study a once and for all change in international trade costs that lead in equilibrium to a 10% increase in the export intensity of firms that choose to export. To help put this figure in perspective, in version of our model in which all firms export, this change in international trade costs would also lead to a 10% change in the ratio of exports to GDP. In our quantitative model, we find three main results.

The first of these is that this once and for all reduction in trade costs leads to an increase in the volume of trade substantially larger than 10%. As discussed in Melitz (2003), this reduction in marginal trade costs induces more firms to pay the fixed cost of exporting. This and the fact that exporters are larger than non-exporters leads to an amplified response of trade to trade costs on impact. In our baseline parameterization, for example, the ratio of exports to GDP increases by 14.5% rather than 10%. In parameterizations of our model in which the amount of innovative investment is responsive to changes in the incentives to innovate, increased innovation by exporting firms amplifies patterns of comparative advantage in the long run and the response of trade to this trade cost reduction is even larger — 22% to 90% or more. Thus our model can produce a substantial difference between the short-run and long-run elasticity of trade volumes to changes in trade costs.

The second result is that the changes in output induced by these changes in trade costs are significantly smaller than the response of trade. As discussed in Melitz (2003), a reduction in trade costs induces on impact a reallocation of labor from less productive to more productive firms. In our quantitative model, this effect is essentially zero (0.02%). Over the long term, however, the changes in innovation patterns induced by the change in trade costs lead to more substantial changes in output, on the order of 100 times the impact effect (or 2%). We show that the main factor holding back the output gains from this reduction in trade costs is the constraint on managerial time — as increased trade induces increased innovative investment in existing firms, managers are drawn out of alternative activities such as starting new firms. Thus, while a reduction in trade costs does lead to a big increase in the average productivity of firms, it also leads to a reduction in the number of firms. To illustrate this point, we compute output gains in a version of our model in which the costs of investment in innovation are rebated to households lump sum and we find long-run output gains that are 10 times larger than before.

The third result is that consideration of endogenous innovation does not substantially alter the welfare implications of a reduction in international trade costs. This finding follows because innovation is an investment — the long-run productivity gains that result from increased innovation require an investment of current resources — and, in our model, the output gains from this investment come only slowly. We show in particular that the transition in our economy from one steady-state to another takes a lot of time.

Our paper complements the work of Klette and Kortum (2004) and Lentz and Mortensen (2006) in constructing a model of innovation that abstracts from strategic interactions across firms and is also consistent with data on firm dynamics. While their framework is a quality ladders model a la Grossman and Helpman (1991) where firms engage in undirected innovation and their dynamics are governed by creative destruction, ours is a model in which monopolistically competitive firms engage in innovation to shape the stochastic process of their production cost. Our form of innovation resembles that in Aghion et. al. (2003, 2005, 2006), their focus being on the impact of product competition on innovation.

The paper is organized as follows. Section 2 presents our model and section 3 characterize the equilibrium. Section 4 describes the analytic result that if all firms export then a change in trade costs has no effect on the innovation decisions the firm. Section 5 discusses why the model can generate departures (or not) from Gibrat's law. Section 5 describes how we calibrate the model to match salient features of the US data on firm dynamics and export behavior. Section 6 discusses how we can discipline the choice of responsiveness of innovative effort to changes in the incentives to innovate using the model's departures from Gibrat's law. Section 7 presents the model's aggregate implications to a decline in marginal trade costs, both across steady states and along the transition. Section 8 concludes.

2. The Model

Time is discrete and labelled $t = 0, 1, 2, \ldots$. There are two countries: home and foreign. Variables pertaining to the foreign country are denoted with a star. Households in each country are comprised of two types of agents: workers and managers. There are measure L workers and measure 1 managers in the representative household in each country. Both types of agents are each endowed with one unit of time per period that can be devoted to work. There is a single final good consumed in each country and a continuum of differentiated intermediate goods produced in each country. The measure of intermediate goods produced in each country is determined endogenously.

Households the home country have preferences of the form

$$\sum_{t=0}^{\infty} \beta^t \log(C_t),$$

where C_t is the consumption of the home final good at date t. Households in the foreign country have preferences of the same form over consumption of the foreign final good C_t^*. Each household in the home country faces an intertemporal budget constraint of the form

$$\sum_{t=0}^{\infty} Q_t \left(P_t C_t - W_t L - W_{mt} \right) \leq \bar{W} \tag{2.1}$$

where Q_t are intertemporal prices, W_t is the workers' wage in the home country, W_{mt} is the managers' wage, P_t is the price of the home final good, C_t is the household's consumption, and $\bar{W}$ is the initial stock of assets held by the household. Households in the foreign country face similar budget constraints with the same intertemporal prices Q_t and wages, prices, and assets all labelled with stars.

Intermediate goods are differentiated products each produced by heterogeneous firms indexed by z, which indicates their productivity. A firm in the home country with productivity index z has productivity equal to $\exp(z)^{1/(\rho-1)}$ and produces output $y_t(z)$ with labor $l_t(z)$ according to the CRS production technology

$$y_t(z) = \exp(z)^{1/(\rho-1)} l_t(z).$$

Production requires n_f units of managerial labor time during the period t as a fixed cost of production.

The output of this firm can be used in the production of the home final good, with the quantity of this domestic absorption denoted $a_t(z)$. Alternatively, some of this output can be exported to the foreign country to be used in the production of the foreign final good. International trade here is subject to both fixed and iceberg type costs of exporting. The fixed cost of exporting is in terms of managerial time and is random and i.i.d. over time for each firm. Each period, the firm draws a random fixed cost of exporting which is denoted n_x and denominated in the units of managerial time in period t the firm must hire to export any output that period. This fixed cost is drawn from a distribution G_x. The iceberg type marginal cost of exporting is in terms of goods. The firm must export $Da_t^*(z)$ units of output, with $D \geq 1$, to have $a_t^*(z)$ units of output arrive in the foreign country for use in the production of the foreign final good.

Let $\xi_t(z, n_x) \in \{0, 1\}$ be an indicator of the export decision of home firms with productivity z and fixed cost n_x (it is 1 if the firm exports and 0 otherwise). Let $x_t(z) = \int \xi_t(z, n_x) dG_x$ be the fraction of home firms with productivity index z that export any output at all. Define ξ_t^*, x_t^* in the same manner. Then feasibility requires that

$$a_t(z) + x_t(z) D a_t^*(z) = y_t(z)$$

and that $n_f + \int \xi_t(z, n_x) n_x dG_x$ units of managerial time be employed.

A firm in the foreign country with productivity index z has the same production technology, with output denoted $y_t^*(z)$, labor $l_t^*(z)$, and domestic absorption $b_t^*(z)$. Exports to the home country are subject to both fixed and marginal costs and hence feasibility requires that

$$x_t^*(z) D b_t(z) + b_t^*(z) = y_t^*(z)$$

and that $n_f + \int \xi_t^*(z, n_x) n_x dG_x$ units of managerial time be employed.

The home final good is produced from home and foreign intermediate goods with a constant returns production technology of the form

$$Y_t = \left[\int a_t(z)^{1-1/\rho} dM_t(z) + \int x_t^*(z) b_t(z)^{1-1/\rho} dM_t^*(z) \right]^{\rho/(\rho-1)} \tag{2.2}$$

and production of the foreign final good is given by

$$Y_t^* = \left[\int x_t(z) a_t^*(z)^{1-1/\rho} dM_t(z) + \int b_t^*(z)^{1-1/\rho} dM_t^*(z) \right]^{\rho/(\rho-1)} \tag{2.3}$$

where $M_t(z)$ is the measure of operating firms in the home country with productivity less than or equal to z and $M_t^*(z)$ the corresponding measure in the foreign country. Note that the final goods firm takes the export decisions $x_t^*(z)$ and $x_t(z)$ as given.

The final good in the home country is produced by competitive firms that choose output Y_t and inputs $a_t(z)$ and $b_t(z)$ subject to (2.2) to maximize profits taking prices P_t, $p_{at}(z)$, $p_{bt}(z)$, export decisions $x_t(z)$, $x_t^*(z)$, and measures of operating intermediate goods firms M_t and M_t^* as given. Standard arguments give that equilibrium prices must satisfy

$$P_t = \left[\int p_{at}(z)^{1-\rho} dM_t(z) + \int x_t^*(z) p_{bt}(z)^{1-\rho} dM_t^*(z) \right]^{1/(1-\rho)} \tag{2.4}$$

and quantities

$$\frac{a_t(z)}{Y_t} = \left(\frac{p_{at}(z)}{P_t}\right)^{-\rho} \text{ and } \frac{b_t(z)}{Y_t} = \left(\frac{p_{bt}(z)}{P_t}\right)^{-\rho}. \tag{2.5}$$

Analogous equations hold for prices in the foreign country

$$P_t^* = \left[\int x_t(z) p_{at}^*(z)^{1-\rho}\, dM_t(z) + \int p_{bt}^*(z)^{1-\rho}\, dM_t^*(z)\right]^{1/(1-\rho)} \tag{2.6}$$

and quantities

$$\frac{a_t^*(z)}{Y_t^*} = \left(\frac{p_{at}^*(z)}{P_t^*}\right)^{-\rho} \text{ and } \frac{b_t^*(z)}{Y_t^*} = \left(\frac{p_{bt}^*(z)}{P_t^*}\right)^{-\rho}. \tag{2.7}$$

Intermediate goods firms in each country are monopolistically competitive. A home firm with productivity $\exp(z)^{1/(\rho-1)}$ and fixed cost n_x that chooses to produce in the current period faces a static profit maximization problem of choosing labor input $l_t(z, n_x)$, prices $p_{at}(z)$, $p_{at}^*(z)$, quantities $a_t(z), a_t^*(z)$, and whether or not to export $\xi_t(z, n_x)$ given its productivity and fixed export cost, to maximize current period profits taking as given wages for workers and managers W_t, W_{mt}, and prices and output of the final good in both countries P_t, P_t^*, Y_t, and Y_t^*. This problem is written

$$\begin{aligned} \Pi_t(z) \;=\; & \max_{l(n_x), p_a, p_a^*, a, a^*, \xi(n_x)\in[0,1]} p_a a + \int \xi(n_x) dG_x p_a^* a^* \\ & -W_t \int l(n_x) dG_x - W_{mt} \int \xi(n_x) n_x dG_x - W_{mt} n_f \end{aligned} \tag{2.8}$$

subject to

$$a + \xi(n_x) D a^* = \exp(z)^{\frac{1}{\rho-1}} l(n_x)$$

and demand functions

$$a = \left(\frac{p_a}{P_t}\right)^{-\rho} Y_t \text{ and } a^* = \left(\frac{p_a^*}{P_t^*}\right)^{-\rho} Y_t^*.$$

Note that our definition of current profits here, $\Pi_t(z)$, includes the fixed cost of exporting if the firm chooses to export ($\xi W_{mt} n_x$) and the fixed cost of operating ($W_{mt} n_f$). Note also that it is not necessary to index prices p_a, p_a^* or quantities a and a^* by n_x. Clearly the optimal choices of these variables does not depend of the level of the fixed cost of exports.

Productivity at the firm level evolves over time depending both on idiosyncratic productivity shocks hitting the firm and on the level of investment in productivity improvements undertaken within the firm. We model the evolution of firm productivity as follows. At the beginning of each period t, every existing firm has probability δ of exiting exogenously and corresponding probability $1 - \delta$ of surviving to produce. After this exogenous shock is realized, surviving firms have the option of exiting at this time or of choosing to produce. A firm that chooses to produce, has current productivity $\exp(z)^{1/(\rho-1)}$, and that invests $H(z,p)$ units of managerial time in improving its productivity in the current period t, has probability p of having productivity $\exp(z+s)^{1/(\rho-1)}$ and probability $1-p$ of having productivity $\exp(z-s)^{1/(\rho-1)}$ in the next period $t+1$. We assume that $H(z,p)$ is increasing and convex in p. We will impose more structure on this function in Sections 5, 6, and 7.

Note that if the time period is small, this binomial productivity process approximates a geometric Brownian motion in continuous time (as in Luttmer 2006) in which the firm controls the drift of this process through investments of managerial time.

To see this approximation, assume that in continuous time z followed a Brownian motion of the form $dz_t = \alpha_t(z)dt + \sigma W (dt)^{1/2}$, where W is a standard Brownian motion. Here, $\alpha_t(z)$ denotes the expected change in z per unit time and σ^2 the variance of z per unit time. Letting $\Delta \leq 1$ denote the length of each time unit within a time period t, s and $p_t(z)$ should be chosen so that $z_{t+\Delta t} - z_t$ is distributed normal with mean $\alpha_t(z)\Delta t$ and variance $\sigma^2 \Delta t$. To do that, we set

$$s = \sigma\sqrt{\Delta t}$$

and

$$p_t(z) = \frac{1}{2}\left[1 + \frac{\alpha_t(z)}{\sigma}\sqrt{\Delta t}\right].$$

With these choices $Ez_{t+\Delta t} - z_t = \alpha_t(z)\Delta t$ and $Var\left(z_{t+\Delta t} - z_t\right) = \sigma^2\Delta t - \alpha_t(z)^2\left(\Delta t\right)^2$. Note that this formula works exactly when $\alpha_t(z) = 0$ (no drift) and it is approximate if the time interval is small (so $(\Delta t)^2$ is really small). Note also that for any fixed Δ, we may potentially have to worry about firms choosing corner solutions for $p_t(z) \in [0,1]$. In our computations below, we choose Δ small enough that these constraints do not bind.

With this evolution of firm productivity, the decision of a home firm to operate or to exit is a dynamic decision. The firm continues to operate as long as the expected discounted present value of profits is non-negative and it exits otherwise. The expected discounted present value of profits satisfies the Bellman equation

$$V_t(z) = \max\left[0, V_t^o(z)\right] \tag{2.9}$$

where

$$V_t^o(z) = \max_{p\in[0,1]} \Pi_t(z) - W_{mt}H(z,p) + (1-\delta)\frac{Q_{t+1}}{Q_t}\left[pV_{t+1}(z+s) + (1-p)V_{t+1}(z-s)\right]. \quad (2.10)$$

Let $p_t(z)$ denote the optimal choice of investment in improving productivity in the problem (2.10).

Note here that $V_t^0(z)$ is the value of choosing to operate the firm in period t. The firm exits if this value falls below zero. Since $V_t^0(z)$ is strictly increasing in z, it is clear that at each date t, the decision of firms to operate (2.9) follows a cutoff rule with firms with productivity above a cutoff $\bar{z}_t$ choosing to operate and firms with productivity below that cutoff exiting.

New firms are created with investments of managerial time. Investment of n_e units of managerial time in period t yield a new firm in period $t+1$ with initial productivity $\exp(z)^{1/(1-\rho)}$ drawn from a distribution over z given by G. In any period in which there is entry of new firms, free entry requires that

$$W_{mt}n_e = (1-\delta)\frac{Q_{t+1}}{Q_t}\int V_{t+1}(z)dG. \quad (2.11)$$

Let M_{et} denote the measure of new firms entering in period t. The analogous Bellman equation holds for the foreign firms as well.

Feasibility requires that for the final good

$$C_t = Y_t \quad (2.12)$$

in the home country and the analogous constraint holds in the foreign country. The feasibility constraint on labor in the home country is given by

$$\int\int l_t(z,n_x)dG_x dM_t(z) = L \quad (2.13)$$

and likewise in the foreign country. The feasibility constraint on managerial time in the home country is

$$M_{et}n_e + \int\left(n_f + \int \xi_t(z,n_x)n_x dG_x + H(z,p_t(z))\right)dM_t(z) = 1. \quad (2.14)$$

The evolution of $M_t(z)$ over time is given in the obvious way by the decisions of operating firms to invest in their productivity $p_t(z)$, the exogenous probability of exit δ, and the endogenous exit decision indexed by the cutoff $\bar{z}_t$.

We assume that the households in each country own those firms that initially exist at date 0. Thus we require that the initial assets of the households in both countries adds up to the total value of these firms

$$\bar{W} + \bar{W}^* = \int V_0(z) dM_0(z) + \int V_0^*(z) dM_0^*(z) \tag{2.15}$$

An equilibrium in this economy is a collection of sequences of prices and wages $\{Q_t, P_t, P_t^*, W_t, W_t^*, W_{mt}, W_{mt}^*, p_{at}(z), p_{at}^*(z), p_{bt}(z), p_{bt}^*(z)\}$ a collection of sequences of quantities $\{Y_t, Y_t^*, C_t, C_t^*, a_t(z), a_t^*(z), b_t(z), b_t^*(z), l_t(z, n_x), l_t^*(z, n_x)\}$ initial assets $\bar{W}, \bar{W}^*$, and a collection of sequences of firm value functions and profit, exit, export, and investment decisions $\{V_t(z), V_t^*(z), V_t^o(z), V_t^{o*}(z), \Pi_t(z), \Pi_t^*(z), \bar{z}_t, \bar{z}_t^*, \xi_t(z, n_x), \xi_t^*(z, n_x), x_t(z), x_t^*(z), p_t(z), p_t^*(z)\}$ together with measures of operating and entering firms $\{M_t(z), M_{et}, M_t^*(z), M_{et}^*\}$ such that household in each country are maximizing their utility subject to their budget constraints, intermediate goods firms in each country are maximizing within period profits if they operate and are choosing to operate or exit optimally, final goods firms in each country are also maximizing profits, and all of the feasibility constraints are satisfied.

In our model, we have assumed that managers and workers have separate skills and thus have imposed two separate constraints (2.13) and (2.14) on their labor input. We also consider an alternative version of our model in which there is no distinction between the labor of these two types of agents and hence only one wage in each country ($W_t = W_{mt}$) and one feasibility constraint formed from the sum of (2.13) and (2.14).

3. Characterizing Equilibrium

In this section, we characterize the relationship between the endogenous distribution of productivity across firms, the endogenous distribution of firm sizes in the economy, and aggregate output.

We start with an analysis of the static profit maximization problem (2.8) for an operating firm in the home country. If this firm has productivity $\exp(z)^{1/(\rho-1)}$ its optimal prices are given by

$$p_{at}(z) = \frac{\rho}{\rho - 1} \frac{W_t}{\exp(z)^{1/(\rho-1)}}, \text{ and } p_{at}^*(z) = \frac{\rho}{\rho - 1} \frac{DW_t}{\exp(z)^{1/(\rho-1)}}.$$

Likewise, foreign firms' prices are given by

$$p_{bt}(z) = \frac{\rho}{\rho - 1} \frac{DW_t^*}{\exp(z)^{1/(\rho-1)}}, \text{ and } p_{bt}^*(z) = \frac{\rho}{\rho - 1} \frac{W_t^*}{\exp(z)^{1/(\rho-1)}}.$$

The output of the home firms is given by

$$a_t(z) = \exp(z)^{\rho/(\rho-1)} \left(\frac{\rho}{\rho-1} \frac{W_t}{P_t} \right)^{-\rho} Y_t \text{ and } a_t^*(z) = \exp(z)^{\rho/(\rho-1)} \left(\frac{\rho}{\rho-1} \frac{DW_t}{P_t^*} \right)^{-\rho} Y_t^*,$$

and likewise for the foreign firms. Home firms thus have variable profits

$$\exp(z) W_t \left[\frac{1}{\rho-1} \right] \left(\frac{\rho}{\rho-1} \frac{W_t}{P_t} \right)^{-\rho} Y_t \tag{3.1}$$

on their home sales and

$$\exp(z) D^{1-\rho} W_t \left[\frac{1}{\rho-1} \right] \left(\frac{\rho}{\rho-1} \frac{W_t}{P_t^*} \right)^{-\rho} Y_t^* \tag{3.2}$$

on their foreign sales. Hence, for each realization of n_x there is a cutoff firm productivity index $\bar{z}_{xt}$ such that firms with productivity index below $\bar{z}_{xt}$ do not export and those with productivity index above $\bar{z}_{xt}$ do export.

Note that there is a simple relationship between productivity in our model and firm size measured as workers employed in production of output for domestic consumption. This employment is directly proportional to $\exp(z)$. In contrast, there is no relationship between productivity in the model, measured as $\exp(z)^{1/(\rho-1)}$ and standard measures of labor productivity in the firm. This is because the average productivity of workers in the firm (measured as sales per worker) is given by $\rho W_t/(\rho-1)$ and hence is constant across firms. Hence differences in productivity across firms in the model, measured by $\exp(z)^{1/(\rho-1)}$, are manifest in data in measures of firm size and not in measures of labor productivity[1].

It is convenient to define four indices of aggregate productivity across firms. For the home firms these are

$$Z_{at} = \int_{\bar{z}_t}^{\infty} \exp(z) dM_t(z) \text{ and } Z_{a^*t} = \int_{\bar{z}_t}^{\infty} \exp(z) x_t(z) dM_t(z)$$

where the first of these is an index of productivity aggregated across all operating firms and the second is an index of productivity across all firms that export. Likewise, for the foreign firms we have

$$Z_{b^*t}^* = \int_{\bar{z}^*}^{\infty} \exp(z) dM_t^*(z) \text{ and } Z_{bt}^* = \int_{\bar{z}^*}^{\infty} \exp(z) x_t^*(z) dM_t^*(z).$$

[1] Our model gives this stark result because, in our model, intermediate goods producing firms choose a constant markup of price over marginal cost. Note that the presence of fixed mangerial costs, if counted as workers, can only generate differences in value-added per worer for small firms. Bernard, Eaton, Jensen, and Kortum (2003) develop a model in which the markup that firms charge rises with size and hence measured labor productivity rises with firm size.

Aggregates are then determined as the solutions to the following equations. Equations (2.4) and (2.6) imply that

$$1 = \frac{\rho}{\rho - 1}\left[\left(\frac{W_t}{P_t}\right)^{1-\rho} Z_{at} + \left(D\frac{P_t^*}{P_t}\frac{W_t^*}{P_t^*}\right)^{1-\rho} Z_{bt}^*\right]^{1/(1-\rho)},$$

$$1 = \frac{\rho}{\rho - 1}\left[\left(\frac{W_t^*}{P_t^*}\right)^{1-\rho} Z_{b^*t}^* + \left(D\frac{P_t}{P_t^*}\frac{W_t}{P_t}\right)^{1-\rho} Z_{a^*t}\right]^{1/(1-\rho)}.$$

These equations also imply that (2.2) and (2.3) are satisfied. Labor market clearing requires that

$$\left(\frac{\rho}{\rho-1}\right)^{-\rho}\left(\frac{W_t}{P_t}\right)^{-\rho} Y_t\left[Z_{at} + \left(\frac{P_t}{P_t^*}\right)^{1-\rho} D^{1-\rho} Z_{a*t}\frac{P_t^* Y_t^*}{P_t Y_t}\right] = L$$

$$\left(\frac{\rho}{\rho-1}\right)^{-\rho}\left(\frac{W_t^*}{P_t^*}\right)^{-\rho} Y_t^*\left[D^{1-\rho}\left(\frac{P_t^*}{P_t}\right)^{1-\rho} Z_{bt}^*\frac{P_t Y_t}{P_t^* Y_t^*} + Z_{b*t}^*\right] = L^*.$$

Note that with our assumption of log-utility and common intertemporal prices Q_t across countries, the ratio of expenditure across countries $P_t^* Y_t^*/P_t Y_t$ is constant over time with that constant determined by the initial distribution of assets $\bar{W}$ and $\bar{W}^*$. More precisely, we have

$$P_t Y_t = \beta^t \frac{Q_0}{Q_t} P_0 Y_0 \text{ and } P_t^* Y_t^* = \beta^t \frac{Q_0}{Q_t} P_0^* Y_0^*.$$

Hence, given expenditure in period 0, the aggregates $W_t/P_t, W_t^*/P_t^*, Y_t, Y_t^*$, and P_t, P_t^* are the solution to these six equations.

We focus our attention for the remainder of the paper on equilibria that are symmetric in the following sense. First, we assume that the distribution of initial assets is such that expenditure is equal across countries at date 0 and hence in every period. Second, we assume that each country starts with the same distribution of operating firms by productivity and hence, because prices and wages are equal across countries, continue to have the same distribution of operating firms by productivity in each subsequent period. In such a symmetric equilibrium, we have $Z_{at} = Z_{b^*t}^*$, $Z_{a^*t} = Z_{bt}^*$, $Y_t = Y_t^*$, $P_t = P_t^*$, and $W_t/P_t = W_t^*/P_t^*$. With this symmetry, we can solve for real wages and output purely as a static problem. In such a symmetric equilibrium, we have

$$\frac{W_t}{P_t} = \frac{\rho - 1}{\rho}\left[Z_{at} + D^{1-\rho} Z_{a^*t}\right]^{1/(\rho-1)} \tag{3.3}$$

and

$$Y_t = \left[Z_{at} + D^{1-\rho} Z_{a^*t}\right]^{1/(\rho-1)} L. \tag{3.4}$$

Define $l_t(z) = \int l_t(z, n_x) dG_x$. Note that in a symmetric equilibrium, the employment of workers and real profits of operating firms with productivity z in either country are given by

$$l_t(z) = \frac{\exp(z)}{[Z_{at} + D^{1-\rho} Z_{a^*t}]} \left(1 + x_t(z) D^{1-\rho}\right) L \tag{3.5}$$

$$\pi_t(z) \equiv \frac{\Pi_t(z)}{P_t} = \frac{1}{\rho} \frac{l_t(z)}{L} Y_t - (x_t(z) n_x + n_f) \frac{W_{mt}}{P_t} \tag{3.6}$$

With this definition of real profits, we can restate the Bellman equation of a firm in either country deflating by the price level P_t :

$$v_t(z) = \max \left[0, v_t^o(z)\right] \tag{3.7}$$

where

$$\begin{aligned} v_t^o(z) &= \max_{p \in [0,1]} \pi_t(z) - \frac{W_{mt}}{P_t} H(z,p) + \\ & (1-\delta)\beta \frac{Y_t}{Y_{t+1}} \left[p v_{t+1}(z+s) + (1-p) v_{t+1}(z-s)\right] \end{aligned} \tag{3.8}$$

with free entry condition

$$\frac{W_{mt}}{P_t} n_e = (1-\delta)\beta \frac{Y_t}{Y_{t+1}} \int v_{t+1}(z) dG \tag{3.9}$$

Note that $Q_{t+1} P_{t+1} / Q_t P_t = \beta Y_t / Y_{t+1}$.

The decision of a firm to invest managerial time in improving productivity, if interior, must satisfy the first order condition

$$\frac{W_{mt}}{P_t} \frac{\partial}{\partial p} H(z,p) = (1-\delta)\beta \frac{Y_t}{Y_{t+1}} \left[v_{t+1}(z+s) - v_{t+1}(z-s)\right]. \tag{3.10}$$

This first order condition must satisfy the obvious inequality if the optimal choice of $p_t(z)$ is equal to either 0 or 1. We discuss the implications of this first order condition (3.10) for the impact of changes in the costs of trade on innovation in the next several sections.

4. Trade and Innovation: A First Look

In this section, we prove the main analytic result of the paper: the result that in an economy with no fixed costs of international trade, once and for all changes in the marginal costs of trade have no impact at all on the incentives of firms to innovate. This result holds in general equilibrium because, in an economy in which every firm exports, the increased incentive to innovate resulting from the increase in profits that come from a reduction in marginal trade costs are exactly offset by an increase in the cost of managerial time necessary for innovation. As a result, the optimal innovation decision is unchanged.

We state our main result in the following proposition

Proposition 1: Consider two economies with no fixed costs of trade ($n_x = 0$ for all firms) and identical initial distributions of productivity across firms across both countries and economies. Let the marginal cost of trade be $D \geq 1$ in the first economy and $\tilde{D} \neq D$ in the second economy. Then the innovation decisions of firms in a symmetric equilibrium in both economies are identical in that $p_t(z) = \tilde{p}_t(z)$ for all dates t and productivities z.

Proof: We prove this result by using the equilibrium in the first economy to construct an equilibrium in the second economy. Let the equilibrium in the first economy be denoted by variables without tildes and the equilibrium in the second economy be denoted by variables with tildes. In both economies, since $n_x = 0$, all firms export so that $x_t(z) = \tilde{x}_t(z) = 1$ for all t and z and hence $Z_{at} = Z_{a^*t}$ and $\tilde{Z}_{at} = \tilde{Z}_{a^*t}$. With this result, it is straightforward to verify that one can use the equilibrium in the first economy to construct an equilibrium in the second economy as follows. Firm decision rules $\bar{z}_t$, $\bar{z}_t^*$, $x_t(z)$, $x_t^*(z)$, $p_t(z)$, $p_t^*(z)$, and labor $l_t(z)$, $l_t^*(z)$, measures $M_t(z)$, M_{et}, $M_t^*(z)$, M_{et}^*, and asset prices Q_t are symmetric across countries and identical across economies. Output, consumption, and real wages, real domestic prices, Y_t, Y_t^*, C_t, C_t^*, $\tilde{W}_t/\tilde{P}_t$, $\tilde{W}_t^*/\tilde{P}_t^*$, $\tilde{W}_{mt}/\tilde{P}_t$, $\tilde{W}_{mt}^*/\tilde{P}_t^*$, $p_{at}(z)/P_t$, $p_{b*t}(z)/P_t^*$,and real profits and firm value functions $\tilde{\pi}_t(z)$, $\tilde{v}_t(z)$, $\tilde{v}_t^o(z)$ in the second economy can all be constructed from their counterpart in the first economy using the constant factor of proportionality

$$\tilde{Y}_t = \frac{\left[1 + \tilde{D}^{1-\rho}\right]^{1/(\rho-1)}}{\left[1 + D^{1-\rho}\right]^{1/(\rho-1)}} Y_t, \text{ etc.}$$

Likewise, real import prices $p_{a*t}(z)/P_t^*$ and $p_{bt}(z)/P_t$ can be constructed using the constant factor of proportionality

$$\frac{\tilde{p}_{a*t}(z)}{\tilde{P}_t^*} = \frac{\tilde{D}}{D} \frac{\left[1 + \tilde{D}^{1-\rho}\right]^{1/(\rho-1)}}{\left[1 + D^{1-\rho}\right]^{1/(\rho-1)}} \frac{p_{a*t}(z)}{P_t^*}, \text{ etc.,}$$

domestic quantities, $a_t(z)$, $b_t^*(z)$,by the factor of proportionality

$$\left[\frac{1 + \tilde{D}^{1-\rho}}{1 + D^{1-\rho}}\right]^{-1},$$

and imported quantities $a_t^*(z)$, $b_t(z)$ by the factor of proportionality

$$\left(\frac{\tilde{D}}{D}\right)^{-\rho}\left[\frac{1+\tilde{D}^{1-\rho}}{1+D^{1-\rho}}\right]^{-1}.$$

As indicated by the expression for real profits (3.6), a change in trade costs affects profits both directly and also through a change in aggregate output and the real wage (a change in the real wage can also be interpreted as change in foreign competition that affects P). It also affects the wage of managers, and hence the costs to operate and to innovate. The key insight to see that equilibrium innovation is unchanged is that the change in trade costs changes the incentive to innovate given by the term $v_{t+1}(z+s) - v_{t+1}(z-s)$ in the right hand side of (3.10) and the cost of innovation given by the term $\frac{W_{mt}}{P_t}\frac{\partial}{\partial p}H(z,p)$ in the left hand side of (3.10) by the same factor of proportionality

$$\frac{\left[1+\tilde{D}^{1-\rho}\right]^{1/(\rho-1)}}{\left[1+D^{1-\rho}\right]^{1/(\rho-1)}}$$

and hence leaves the optimal innovation decision $p_t(z)$ unchanged.

A key assumption for this result is that all types of managerial work are substitutable. Hence, Proposition 1 holds as well in the version of the model in which there is only one feasibility constraint for workers and managers and hence one wage for all agents in the economy. It also holds in a version of the model in which the total supply of managerial labor is elastic, so that the result is not dependent on their being a fixed supply of managerial labor.

Note that Proposition 1 fails to hold if there are positive fixed costs of trade so that only a subset of firms export. Here, a change in the marginal cost of trade has an asymmetric effect on the change of profits, and thus the incentives to innovate, for those firms that do and do not export. Specifically, a reduction in trade costs leads to an increase in market size for exporters that is only partially offset in profits by an increase in the real wage (or, similarly, an increase in foreign competition), whereas non-exporters face a decline in profits and incentives to innovate due to the increase in the wage. With respect to the costs of innovation, the reduction in marginal trade costs increases the wage of managers and hence the costs of innovation for all firms. Hence, changes in the marginal costs of trade have differential effects on the terms in (3.10) for firms that do and do not export.[2] We explore

[2] This implication of the model would also apply in a two-sector model with traded and non-trade goods.

the implications of these effects of changes in the marginal costs of trade on firm dynamics and the distribution of firm sizes as well as the implications of these changes for aggregate output in the sections that follow.

In the Appendix we show that in our model, firm level innovation decisions are also unaffected by changes in tax rates on firm profits, revenues, or factor use. The logic of this result, which holds also in the version of the model with positive fixed costs of exporting, is quite similar to that in Proposition 1.

5. Innovation, trade, and firm size dynamics

In this section, we examine our model's implications for innovation, firm's decisions to export, firm size dynamics, and the distribution of firm size. As we have seen in (3.5), in a symmetric equilibrium the size of any individual firm, as measured by its employment of workers, is determined by its current level of $\exp(z)$ relative to an index of the average of $\exp(z)$ across operating firms as well as the firm's export decision indexed by $x_t(z)$. Hence, firms' innovation decisions, $p_t(z)$, together with the stochastic shocks to firm productivity and firms' decisions to export or exit determine the dynamics of firm size and the long run distribution of firms by size in our economy. We focus our discussion in this section on the implications of our model for the expected growth rate of firms by size and compare those implications to the stylized fact summarized by Gibrat's Law — the observation that, at least for large firms, expected growth rates are independent of firm size.

We further specialize our model in two respects. First, we consider the symmetric steady-state of the model in which the distributions of firms $M_t(z)$ and $M_t^*(z)$ have settled down to constant distributions. Second, we consider a specific form of the innovation cost function $H(z,p) = h\exp(z)c(p)$, where h is a constant and $c(p)$ is an increasing and convex function of the choice of p.

Note that in our model, employment is directly proportional to $\exp(z)$, and, holding fixed the export decision, innovation results in the growth of employment by $\exp(s)$ with probability p or the shrinkage of employment by $\exp(-s)$ with probability $1-p$ and thus an expected growth rate (conditional on survival) of $p\exp(s) + (1-p)\exp(-s)$. Hence, with our cost function, the costs of generating a productivity improvement that will result, in equilibrium, in a certain expected growth of employment in the firm, scale proportionately with current employment (holding fixed the export decision). It is this assumption that means that our model has the potential to generate patterns of firm growth consistent with Gibrat's Law.

In a symmetric steady-state, one can rewrite the Bellman equation of the firm to reduce the problem of solving for equilibrium to a problem in one variable as follows. Divide both sides of (3.7, 3.8, and 3.9) by the managerial wage W_m/P to get a new Bellman equation

$$w(z) = \max \left[0, \int w^o(z, n_x) dG_x \right] \ , \tag{5.1}$$

where

$$\begin{aligned} w^o(z, n_x) \ &= \ \max_{p,\xi \in [0,1]} A \exp(z) \left(1 + \xi D^{1-\rho}\right) - \xi n_x - n_f - h \exp(z) c(p) + \\ &\quad (1-\delta)\beta \left[p w(z+s) + (1-p) w(z-s) \right] \ , \end{aligned} \tag{5.2}$$

with free entry condition

$$n_e = (1-\delta)\beta \int w(z) dG \ . \tag{5.3}$$

Here

$$A = \frac{1}{\rho} \left[Z_a + D^{1-\rho} Z_{a*} \right]^{1/(\rho-1)-1} L \left(\frac{W_m}{P} \right)^{-1} \tag{5.4}$$

Note that the first order condition governing the optimal choice of p is

$$h \exp(z) c'(p) = (1-\delta)\beta \left[w(z+s) - w(z-s) \right] \tag{5.5}$$

This Bellman equation can be used to solve the model as follows. Find the $w(z), w_z^0(z, n_x)$, and A that solve (5.1),(5.2), and (5.3). From the decision rules for exit $\bar{z}$, innovation $p(z)$, and exporting $x(z) = \int \xi(z, n_x) dG_x$, find the steady-state distribution of firms by productivity scaled by the measure of entrants, $M(z)/M_e$, and then find M_e and $M(z)$ separately from (2.14). This calculation gives Z_a and Z_{a*} and hence W_m/P, W/P, and Y from (5.4), (3.3), and (3.4) directly.

Note that in solving (5.1),(5.2), and (5.3) for A and the firm decision rules for exit $\bar{z}$, innovation $p(z)$, and exporting $x(z) = \int \xi(z, n_x) dG_x$ and the corresponding distributions of z across firms $M(z)$, the parameter ρ governing the elasticity of demand faced by firms enters only through its impact on the export intensity of firms that do export as measured by $D^{1-\rho}$. Thus, is we consider the symmetric steady-states of two economies with different elasticities ρ and different trade costs D but identical trade intensities for firms that do export as indexed by $D^{1-\rho}$, then these two economies will have identical decision rules for firms and identical distributions of z across firms. In terms of the data, this implies that these two economies will have identical distributions of employment across firms (since employment is directly proportional to $\exp(z)$). Aggregates such as real wages and output clearly vary across these economies with variation in ρ.

The Bellman equation (5.1) and (5.2) is a standard problem of valuing the profits of the firm together with two options: the option to stop exporting and the option to exit. As a result, as z gets large, the value function that solves this Bellman equation approaches the solution to this Bellman equation with both fixed costs $n_x = n_f = 0$. This is because, as z gets large, the options to stop exporting and to exit lose all value. The solution to the Bellman equation with no fixed costs is a function of the form $w^u \exp(z)$ where w^u is a scalar given by

$$w^u = \frac{A(1 + D^{1-\rho}) - hc(\bar{p})}{1 - (1-\delta)\beta \left[\bar{p}\exp(s) + (1-\bar{p})\exp(-s)\right]/(1+g)} \tag{5.6}$$

where $\bar{p}$ satisfies the first order condition

$$hc'(\bar{p}) = (1-\delta)\beta w^u \left[\exp(s) - \exp(-s)\right]. \tag{5.7}$$

Note that here, in the absence of fixed costs to either operate or export, firms' innovation decisions are to choose $p(z) = \bar{p}$ and hence all firms have constant expected growth of employment each period as in Gibrat's Law with this expected growth rate of employment conditional on survival given by $\bar{p}\exp(s) + (1-\bar{p})\exp(-s)$. Note that if our model is to have a steady-state distribution of employment across firms, we must have $\bar{p}$ not too large. Specifically, we must have the expected growth of employment (taking into account exogenous exit δ) less than one.

More generally, our model reproduces Gibrat's Law with a constant expected growth rate of employment across firms whenever the value function w has the property that $(w(z+s) - w(z-s))\,/\exp(z)$ is constant with respect to z. This result can be derived directly by dividing both sides of (5.5) by $\exp(z)$. We refer to the term $(w(z+s) - w(z-s))\,/\exp(z)$ as the *scaled incentive to innovate* as it is directly proportional to the ratio of the returns to increasing the probability of advancing the productivity index from z to $z+s$ to the cost of doing so for a firm with current productivity index z.

Consider now our model's implications if we chose an alternative scaling assumption for the innovation investment cost function. In particular, assume that the costs of innovation scaled with the productivity of the firm so that $H(z,p) = h\exp(z)^{1/(\rho-1)}c(p)$. This scaling assumption would be natural if one assumed that productivity in the firm ($\exp(z)^{1/(\rho-1)}$) was modelled like a capital stock to be accumulated within the firm. This modelling approach, however, leads to an inconsistency with Gibrat's Law. As we have seen above in (5.6) and (5.7), if the growth rate of large firms is independent of firm size, then the incentive to innovate is directly proportional to firm size, $\exp(z)$. But, with this alternative cost function, the cost of innovation grows more slowly than firm size, $\exp(z)$, since the elasticity of demand $\rho > 1$. Hence, this alternative specification of innovation investment costs cannot be consistent with Gibrat's Law over a wide range of firm sizes.

As we have seen, in the absence of the options to stop exporting and to exit, our model has the property that this scaled incentive to innovate is independent of z. We now illustrate how the presence of the two options — to stop exporting and to exit — affects the scaled incentives to innovate in the quantitative version of our model described in the next section. Figure 1 shows the scaled incentive to innovate (Panel A) and the fraction of exporters $x(z)$ (Panel B), for the set of active firms (the lowest z corresponds to the exit threshold $\bar{z}$). Recall that, in the absence of fixed costs to either operate or to export, the scaled incentive to innovate would be independent of z.

As is evident in Panel A, in our model with fixed costs of operating and exporting, the scaled incentive to innovate is not flat. Instead, we see that it is highest and roughly flat for the firms with the largest z. These firms have a very high probability of continuing to operate and to export for the foreseeable future. Moreover, the fixed costs of operating and exporting are a very small portion of profits, so here the value function for these firms approaches the value function that can be calculated analytically when there are no fixed costs. This feature of the model accounts for the fact that this scaled incentive to innovate does not vary substantially with z for these firms. The fact that these firms export means that they enjoy a larger market for their product than purely domestic firms and it is this feature of the model that accounts for the fact that these firms have the highest scaled incentive to innovate.

We see in Figure 1 that the scaled incentive to innovate declines gradually with z to a second plateau as the probability that the firm exports declines from near 1 to near 0. It is the feature of the model that exporters face a larger market than non-exporters that accounts for the difference in the level of the two flat portions of the figure.[3] The scaled incentives to

[3] From (5.6) and (5.7) we can see that, keeping A constant, $\bar{p}$ is increasing in export intensity $D^{1-\rho}$.

innovate transition smoothly from one level to the other for those values of z for which the probability of exporting transitions from near 1 to near 0 because the option of switching back to non-exporting (and hence stop paying the fixed export cost $\bar{n}_x$) implies that small exporters (those that have z low enough that they might soon switch to non-exporting) have a lower scaled incentive to innovate relative to large exporters.

Consider finally the large decline in the scaled incentive to innovate for firms with the lowest values of z. This arises from the option to exit. Firms that have a low z will exit if their productivity falls below the exit threshold $\bar{z}$. Given that the value of exit is independent of z, the scaled benefit of innovating is smaller for these firms relative to others that have high values of z and hence are far from exiting.

The extent to which variation in this scaled incentive to innovate results in variation in expected growth rates across firms, as given by $p(z)$, is determined by the curvature in the cost function $c(p)$ as given by $c''(p)$. We explore the magnitude of these effects in our numerical calculations below.

6. Quantitative Analysis

We now examine a quantitative version of our model to explore its implications for the impact of trade costs on trade and innovation. We have observed analytically that if there are no fixed costs of exporting, so that all firms export, then changes in the marginal cost of exporting have no impact on the equilibrium innovation decisions of firms. In our quantitative work, we explore the extent to which the consideration of fixed costs of exporting changes these analytical results. As in Melitz (2003), changes in trade costs in our model with fixed costs of exporting have immediate effects on overall output and productivity that arise as a result of the induced reallocation of labor across firms that export and firms that do not export. Our model has implications for the impact of a reduction in the marginal cost of trade on the innovation decisions of firms: those that export increase their investments in innovation and those that do not export decrease their investments in innovation. Thus, over the long term, the innovation decisions of firms amplify patterns of comparative advantage and amplify the impact of the reduction in the trade costs on both trade volumes and aggregate productivity and output.

We present a simple quantitative model which is consistent with many features of the data on firm size dynamics (both in terms of employment and export status) and the distribution of firm size in the U.S. economy. We then use that model to examine the quantitative impact of a change in the marginal costs of trade on output and the volume of trade in both the short and the long run as well as on the innovative activity of individual firms.

We show that the quantitative magnitudes of these effects depend in large part on two factors. The first is the curvature of the innovation cost function $c(p)$. The second is the opportunity cost of managerial time required for innovation.

There is not much evidence on the curvature of the innovation cost function $c(p)$. In our model, the curvature of this function determines how firm growth rates in steady-state deviate from Gibrat's Law because it determines the extent to which investments in innovation at the firm level respond to changes in the incentives to innovate with firm size and in the value of the two options to exit and export with firm size. We use these observations to discipline our choices of the curvature of this cost function $c(p)$. There is also little evidence on the elasticity of supply of labor time useful for investing in innovation at the firm level. We explore two alternative specifications of our model: our benchmark in which the alternative use of this labor is managing firms and an alternative in which this labor is perfectly substitutable with all forms of labor. We find in both of these specifications that the quantitative impact of changes in trade costs on output are relatively small largely because increased investment in innovation results in reductions in the availability of managers to run firms. We show that trade has a substantially larger impact on innovation and output in the absence of this resource constraint on managerial time.

6.1. Calibration

We now discuss how the model is parameterized to reproduce a number of salient features from US data on firms dynamics, the firm size distribution, and international trade. We choose time periods equal to two months so there are six time periods per year.[4] We parameterize the innovation cost function $H(z,p) = h\exp(z)c(p) = h\exp(z)\exp(bp)$ where h and b are scalars. Note that the parameters h and b together control the average level of investment in innovation and the elasticity of innovation to variation in the scaled incentive to innovate. We parameterize the distribution G of productivity draws of entrants to be normal so that for entering firms, $z \sim N(0, \sigma_e)$. We parameterize the distribution of fixed

[4] As we reduce the period length, we keep the entry period of new firms at one year. If we didn't, then the allocations would change significantly as the cost of waiting for a new draw (and hence the entry threshold $\bar{z}$) would decline.

costs of exporting so that $\log(n_x) \sim N(\log(\bar{n}_x) - \sigma_{nx}^2/2, \sigma_{nx})$ - the mean of n_x is $\bar{n}_x$. As we discussed above, the distribution of employment across firms in a symmetric steady-state depends on the elasticity parameter ρ only through the trade intensity for firms that do export given by $D^{1-\rho}/(1 + D^{1-\rho})$. Since we choose this trade intensity $D^{1-\rho}/(1 + D^{1-\rho})$ as a parameter, our calibration is invariant to the choice of ρ.

The parameters of the model that we must choose then are the steady-state interest rate given by $1/\beta$, the ratio of workers to managers L, the export intensity of exporting firms $D^{1-\rho}/(1 + D^{1-\rho})$, the parameters of the distributions of initial productivity draws for firms, σ_e, and of fixed costs of exporting, $\bar{n}_x$ and σ_{nx}, the parameters of the innovation cost function h and b, the fixed costs of operation n_f and entry n_e, and the parameters governing the variance of employment growth for surviving firms s and the exogenous exit rate of firms δ.

Our calibration strategy is as follows. Our model does not generate dispersion in *measured* productivity (i.e.: value-added per worker) across firms. So, instead of directly focusing on data on firm level productivity, we choose to concentrate on the data on size dynamics and the firm employment-based size distribution. We look to reproduce some features of the data regarding the size dynamics of large firms in the US, as well as keeping their dynamics invariant in the initial steady state as we modify the curvature of the innovation cost function $c(p)$. In the model, we define large firms to be those whose productivity is sufficiently larger than $\exp(\bar{z}_x)^{1/(\rho-1)}$, where $\bar{z}_x = \min\{z \mid x(z) = 1\}$, so that they are expected to remain exporters conditional on survival.

We consider several calibrations. In each calibration, we start with a fixed value of the parameter b governing the curvature of the innovation cost function $c(p)$. In our calibration with a large value of b, the innovation cost function $c(p)$ is highly curved and hence investment in innovation becomes invariant to changes in the incentives to innovate. In this case, our model predicts that all firms choose the same value of $p(z) = \bar{p}$ and our model is quite similar to Luttmer (2006) in that we effectively assume that z follows a binomial approximation to a Brownian motion with constant drift. In our calibrations with lower values of b, firms investment in p varies with z endogenously because the incentives to innovate vary with z. Small firms (those with low z) tend to choose lower values of $p(z)$ because they face a higher risk of endogenous exit. Large firms that export (those with large z) tend to choose higher values of $p(z)$. Thus, our model, with low values of b, implies that there are systematic deviations from Gibrat's Law for small and large firms. We examine the extent of these deviations from Gibrat's Law as implied by the model to assess what is a reasonable range for this parameter.

Also note that, with fixed costs of exporting and the export intensity $D^{1-\rho}/(1+D^{1-\rho})$ for those firms that do export, our model has implications for the identity of firms that export and the ratio of exports to total output. In the case in which $\sigma_{nx} = 0$ so the the fixed costs of exporting are constant for each firm, it is only the large firms that export. With $\sigma_{nx} > 0$, so that the fixed costs of exporting are random, firms switch status from exporting to not exporting and vice-versa more frequently the larger is $\sigma_{nx.}$ We use data on the fraction of employment in firms that switch status from exporting to not exporting and vice versa in our calibration.

In each calibration of the model that we consider, we choose the parameters of the model to reproduce, among other facts, the main features of the distribution of employment across firms in the United States. As discussed in detail in Luttmer (2006), the right tail of this distribution resembles a Pareto distribution. Luttmer shows in a closed economy model similar to ours in which $p(z) = \bar{p}$ is set exogenously, that the steady-state distribution of employment across firms resembles a Pareto distribution in its right tail. Our model also has this property even with endogenous innovation since large firms in our model follow Gibrat's Law. Recall from (5.6) and (5.7) that large firms in our model all choose the same value of $p(z) = \bar{p}$. The actual choice of $\bar{p}$ that emerges in the equilibrium of our model is a function of parameters, but the choice of $\bar{p}$ required to match US data on the distribution of employment across large firms can be backed out directly from the data, as described shortly.

We now describe how all parameters are chosen. Our steady-state interest rate (annualized) is 5%. We normalize $L = 1$. Now consider the parameters shaping the law of motion of firm productivity z (h, n_f, n_e, s, $\sigma_e, D^{1-\rho}$, $\bar{n}_x, \sigma_{nx}$and δ). We choose s so that the standard deviation of the growth rate of employment of large firms in the model is 25% on an annualized basis. ($s = 0.25\sqrt{1/6}$). This figure is in the range of those for US publicly-traded firms reported in Davis et. al. (2006).[5] We choose the exogenous death rate δ so that the model's annual employment-weighted death rate of large firms is consistent with the corresponding one for large firms in the US data.[6] Note that in our model, over the course of one year,

[5] We abstract from the trend in employment growth rate volatility discussed in Davis et. al. (2006) and pick a number that roughly matches the average for the period 1980-2001.

[6] This number is the 1997-2002 average employment-based failure rate of US firms larger than 500 employees, computed using data reported by the Statistics of U.S. Businesses and Nonemployer Statistics.

large firms do not choose to exit endogenously because they have productivity far away from the threshold productivity for exit. Hence δ determines the annual exit rate of these firms directly. We normalize $n_e = 1$ and set $n_f = 0.1$.[7]

Corresponding to each value of b, we choose the remaining parameters $\sigma_e, \bar{n}_x, \sigma_{nx}$, $D^{1-\rho}$, and h as follows. We choose the four parameters $\sigma_e, \bar{n}_x, \sigma_{nx}$, $D^{1-\rho}$ to match the following facts. First, the fraction of employment by entering firms accounted for by entering firms of size under 500 is 90%.[8] Note that firm sizes in terms of number of employees in the model are are normalization. We choose this normalization so that the median firm in the employment-based size distribution is of size 500. In other words, 50% of total employment in the model is accounted for by firms of size under 500, consistent with US data.[9] Second, the fraction of total employment accounted for by exporting firms of 40%, and third, the a fraction of exports in gross output of 7.5% reported in US data[10]. We abstract from variation in trade costs across sectors.[11] Fourth, we match patterns of firms switching from exporters to non-exporters, and vice-versa, over time. We simulate the steady state of the model for seven years to compare the model's predictions seven year predictions with those in the US between 1993 and 2000 reported in Bernard et. al. (2005). The employment-based switching rate is defined as the average of the year seven employment of non-exporters that become exporters, and the year-one employment of exporters that become non-exporters, as a fraction of total exporter's employment. Using the data in Bernard et. al. (2005) we compute this seven year switching rate in US data to be 12%.

Finally, we describe our choice of h . Consider representing the right tail of the distribution of employment across firms in the U.S. data with a function that maps the logarithm

[7] The statistics that we report are invariant to proportional changes in all three fixed costs and h.

[8] This fraction is the 1999-2003 average calculated using US firms birth data, computed using data reported by the Statistics of U.S. Businesses and Nonemployer Statistics.

[9] The size of the median firm corresponds to that in the US firm employment-based size distribution in the period 1999-2003. This information is reported by the Statistics of U.S. Businesses and Nonemployer Statistics.

[10] Bernard et. al. (2005) report that the fraction of total US empoyment (excluding a few sectors such as agriculture, education, and public services) accounted for by exporters is 36.3% in 1993 and 39.4% in 2000. The average of exports and imports to gross output for the comparable set of sectors is roughly 7.5% in 2000. The steady state of our model abstracts from trends in trade costs that would lead to changes in trade volumes over time.

[11] We also explored a two-sector version of the model. In the services-producing sector we assume a trade intensity equal to zero, and in the goods-producing sector all firms participate in the export market ($n_x = 0$). We choose the size of each sector and the trade intensity in the good-producing sector to target the same two measures of aggregate volumes of trade. The results from this alternative model are very similar to those in the benchmark model.

of the number of employees $\log(l)$ into the logarithm of the fraction of total employment employed in firms with this employment or larger. We plot this function in Figure 2, for firms of size 5 to 10000. If the distribution of employment across firms actually was Pareto, this functions would be a straight line. As is evident in this figure, this function is close to linear, with the slope becoming steeper for larger firms. In calibrating the model with exogenous innovation (fixed p for all firms) we choose h (given the other parameters) so that the model matches the slope coefficient for firms within a certain size range. Concretely, we target a slope coefficient of -0.2 for firms ranging between 1000 and 5000 employees, as in Figure 2. The calibrated model then implies a value of $\bar{p}$ for large firms from (5.6) and (5.7). As we change b, we adjust the model parameters to keep $\bar{p}$ constant and thus keep the dynamics of large firms unchanged.[12]

Given that we calibrate the model to data on firm size, we do not take a stand on the value of ρ for our initial steady-state calculations. In our benchmark parametrization, we set $\rho = 5$, but we also perform sensitivity analysis when we examine the impact of changes in trade costs on output and productivity (the latter is affected by the value of ρ).

7. Steady state

We now discuss some steady state implications of our calibrated model. We start with the parametrization that assumes curvature in the innovation cost function, b, that is sufficiently big to imply that all firms choose essentially the same p irrespective of size or export status. This corresponds to the model with exogenous productivity process and constant p. We then discuss how some of these additional implications of the model change as we decrease the curvature parameter b on the innovation cost function. The results are reported in Table 1, as well as in Figures 3-6.

7.1. Exogenous innovation

7.1.1. Size distribution

The benchmark calibration with high b ($b = 3000$) gives $p(z) = \bar{p}$ constant for all z (see Panel A in Figure 2). The parameter choices and the match between the model and the data due to the calibration is reported in Table 1. The match between model and data reported in

[12] Our implied $\bar{p}$ produces a slope coefficient of -0.25 for very large firms (those that choose $p = \bar{p}$ according to (5.6) and (5.7).

rows 8 and 9 of that table come directly from the calibration of s and δ. The match between the model and data in rows 10, 11, 12, 13, and 14 is found from adjusting the five parameters $h, \sigma_e, n_x, \sigma_{nx}$ and $D^{1-\rho}$.

Panel B in Figure 3 shows that the employment-based size distribution implied by the model resembles that in the US for firms ranging between 5 and 10000 employees (recall that our calibration procedure targets the slope in the data for firms between 1000 and 5000 employees).

In calibrating the model, we focused on the right-tail of the distribution of employment across firms. We now consider our model's implications for the size-distribution of firms. If the size distribution were Pareto, then the slope of the right tail distribution based on the number of firms (this function maps $\log(l)$ into the logarithm of the fraction of firms with this employment or larger) would be equal to the slope of the employment-based right tail coefficient plus one.[13] Both the data and the model show a size distribution that departs from a Pareto distribution in the following way. Panel A in Figure 3 plots the firm-based right tail distribution and the employment-based right tail distribution from US data, where the latter is adjusted so that, theoretically, both lines should have the same slope.[14] Panel B displays the same data implied by the model. The model is successful in generating a discrepancy in the slope of both lines. Namely, the firm-based size distribution is flatter than the employment-based size distribution. The distribution of entrants G plays the key role in shaping this departure from a Pareto distribution.

7.1.2. Conditional growth rates

In Panel C of Figure 3, we observe that for small firms, the model generates growth rates for continuing firms that are decreasing in size. This is consistent with Gibrat's law: small surviving firms grow faster than large firms, and among larger firms growth is unrelated to size.[15] This is driven by the selection process: in order to survive, firms that are close to the exit threshold must draw relatively good productivity draws. Note, however, that selection

[13] If l is distributed Pareto with slope parameter $1+k$, then the truncated expectation of l is also distributed Pareto with slope parameter k.

[14] Specifically, we are plotting $\log(l)$ against the logarithm of the employment of firms with employment l or larger, adjusted by adding the logarithm of l. If l is distributed Pareto with slope coefficient k, then the slope of this adjusted right tail distribution is also equal to k.

[15] For references on Gibrat's law see Sutton (1997) and Caves (1998).

only operates for very small firms.[16]

7.1.3. Exporters

We now assess the benchmark model's implications for the size distribution and growth dynamics of exporters, in relation to the data reported in Bernard et. al. (2005). Panel D in Figure 3 displays the exporters' employment size distribution of the model and the US data in 2000. The size distribution generated by the model is only slightly more concentrated than that in the data. This suggests that the concentration of exporters is not that striking in light of the concentration of firms in the overall economy.[17]

7.2. Endogenous innovation

We now re-calibrate our model using lower values for the innovation cost curvature, b. Specifically, we consider $b = 30$, and $b = 10$. Recall that our calibration strategy involves keeping constant the level of innovation intensity $\bar{p}$ of large firms by appropriately adjusting the parameter h. The results are presented in columns 2 and 3 of Table 1, and in Figures 5 and6.

In Panel A of Figures 5 and 6 we plot the innovation intensity $p(z)$ and the fraction of exporting firms $x(z)$ for active firms z. As we lower b, firms that export (those with $x(z) > 0$) choose a higher investment in innovation than non-exporters (those with $x(z) = 0$). This is consistent with the scaled incentive to innovate displayed in Figure 1.

7.2.1. Conditional growth rates

Panel D in Figures 5 and 6 displays the one-year growth rate of continuing firms as a function of the log of firm size. Recall that in the model with exogenous innovation (or $b = 3000$), consistent with Gibrat's law, the growth rate is positive for small firms due to selection, and is essentially zero - the unconditional growth rate of continuing firms, for larger firms.

As we lower b, the slope of the conditional growth rate schedule is the result of a tension between selection (negative slope) and increasing innovation intensity (positive slope). Panel D in Figures 5 and 6 shows that the second force strengthens as we lower b. Under $b = 10$,

[16] Extending the model to allow for cross-firm variation in fixed costs would generate a smoothly decreasing conditional growth schedule.

[17] Bernard et. al. (2005) report that the exporters' concentration of export values is significantly more concentrated than that of employment. This suggest that large exporters also have a higher trade intensity. Our model abstracts from these considerations.

conditional growth rates become increasing in size, with differences in growth rates across sizes of over 10%. So, very low levels of b deliver a clear violation of Gibrat's law.

7.2.2. Size distribution

Panel B in Figures 5 and 6 compares the employment-based size distribution produced by the model with US data. As we lower b, our calibration procedure of fixing $\bar{p}$ implies that the slope for large firms remains in line with the data. But the model generates a divergence from the data for smaller firms.

In particular, a lower innovation intensity for small firms and non-exporters shows up as a flatter right tail distribution for these firms relative to large firms. We quantify this change in the slope of the firm size distribution (which can be directly observed in Panel B of Figures 3, 5, and 6) as the ratio of the employment-based right tail coefficient for firms of size $100-1000$ to that of firms of size $5000-1000$. This ratio is equal to 0.74 under $b = 3000$, 0.86 under $b = 30$, and 1.38 under $b = 10$. This flattening of the right tail distribution is counterfactual relative to US data (see Panel B).

Overall, we see these implications of the model as useful diagnostics in the search of the parameter that controls the elasticity of innovation intensity to market size. We do not take a definite stand on the value of b, but we simply suggest features of the data one could look at in order to assess its plausibility. In the next section we report our policy experiments under different values of this parameter.

8. Aggregate implications of a decline in marginal trade costs

We now study the aggregate implications of a trade liberalization, defined as a reduction in marginal trade costs D leading to an increase in the trade intensity of firms that export. We choose to lower D so that the export intensity of exporters, $D^{1-\rho}/\left(1+D^{1-\rho}\right)$, increases by 10%. Using this procedure, the resulting steady state changes in aggregate productivity index Z_i and export/GDP (but not aggregate output) are invariant to the elasticity parameter ρ.[18] As a useful benchmark, in a version of the model where all firms export, the percentage change in export/GDP is equal to the percentage change in the export intensity, 10%.

[18] In our calibration, the initial export intensity is 23%. Assuming that $\rho = 5$, our experiment amounts to reducing D by 3%.

8.1. Steady state

Table 2 presents the ratio of new to old steady state values for various aggregate variables, for different levels of b. Table 3 reports sensitivity analysis with respect to key parameters of the model.

We first consider the model with exogenous innovation ($b = 3000$). In Row 1, Column 1 of Table 2, we see that export/GDP increases by 14.5%. This is larger than the percentage change in the export intensity for two reasons. First, exporters are larger than non-exporters, so a change in their export intensity has a more than proportional impact on exports/GDP. Second, the reduction in trade costs induces more firms to pay the fixed cost of exporting.

Now consider the model with less curvature in the innovation cost function ($b = 30$ and $b = 10$). The results are reported in Row 1, Columns 1-2 of Table 2. Now the percentage change in exports/GDP is 25.7% when $b = 30$, and 88.5% when $b = 10$. The model with endogenous innovation amplifies the impact of the reduction in the trade costs on trade volumes because those firms that export increase their investments in innovation and those that do not export decrease their investments in innovation. Initial differences in the productivity between exporters and non-exporters are magnified.

Row 2 in Table 2 reports the steady increase in final output Y resulting from the decline in trade costs. It is 0.02% when $b = 3000$,[19] 0.5% when $b = 30$, and 2% when $b = 10$. Endogenous innovation also magnifies the output increase from a decline in variable trade costs.

In order to understand these output gains, it is useful to go back to the output expression (3.4). Labor is fixed, so output changes as long as the aggregate productivity index $Z_{at} + D^{1-\rho} Z_{a^*t}$ does. The aggregate productivity is the product of the average productivity index $\exp(z)\left(1 + x(z) D^{1-\rho}\right)$ across firms, $\left(Z_{at} + D^{1-\rho} Z_{a^*t}\right)/M_t$, and the total number of firms, $M_t = \int_{\bar{z}}^{\infty} \exp(z) dM_t(z)$. Rows 3 and 4 in Table 2 decomposes the change in the aggregate productivity index into these two components.

A reduction in marginal trade costs has a direct impact on the aggregate productivity index by increasing $D^{1-\rho}$ and thus raising the effective productivity of exporters.[20] As we lower b, there is an additional force that increases the average $\exp(z)$. It results from the

[19] Grossman and Helpman (1984) show that in trade models with monopolistic competition and exogenous innovation, welfare might fall after a trade liberalization.

[20] There is another indirect effect coming from the change in $\bar{z}_X$ that changes the average productivity of exporters. This mechanism is described in Melitz (2003).

increase in innovation investment by large exporters, so Z_{at} and Z_{a^*t} have a larger mass at higher productivity levels.

But increasing trade and innovation is costly, as it takes managers from other productive activities. As more managers are hired to enter the export market, there are fewer managers to run firms, so the number of firms declines. The reduction in M_t reported in Row 4 of Table 2 slightly offsets the gains in average $\exp(z)(1 + x(z) D^{1-\rho})$. This force is even stronger in the model with endogenous innovation, as more managers are used in innovation activities.

This suggests that the opportunity cost of managerial time required for innovation is key in shaping the response of output to a trade liberalization. In our benchmark model the alternative use of this labor is managing firms. We also consider an alternative in which this labor is perfectly substitutable with all forms of labor.[21] Columns 4-6 in Table 3 show that the aggregate consequences of our experiment are roughly equal to those in our benchmark model.

To quantify how the increase in time spent on innovation activities affects the change in aggregate output in our experiment, we examine the a modified version of our model where innovation costs do not take resources from the economy. Suppose that innovation costs $H(z, p)$ are now defined as taxes to firms that are then rebated to the representative household lump sum. In this modified model, each firm optimally chooses the innovation investment as in our benchmark model, but the innovation costs do not show up in the managerial resource constraint (2.14). We now consider the same experiment of increasing the trade intensity of exporters by 10%, which raises innovation investment for exporters and lowers it for non-exporters. The results are reported in Table 3, columns 7-9, for different values of b. The increase in output in the model with exogenous innovation ($b = 3000$) is roughly equal in this model as in our benchmark model. The increase in output is significantly larger in the modified model when we lower b. For example, under $b = 10$, output increases by 2.0% in the benchmark model and 16.1% in the alternative model where innovation does not take up resources.[22]

We conclude that endogenous innovation magnifies the output gains from a reduction in marginal trade costs, but these gains are largely held back by the fact that increasing innovation is costly in that managers are drawn from other alternative productive activities.

[21] Under our calibration procedure, the parameter values are the same under both specifications of the model.

[22] This modified model is related to models of "learning-by-exporting", where the increase in productivity of exporters is modelled as an externality which does not take up resources from the economy.

Columns 10-18 in Table 4 provide some sensitivity analysis with respect to two additional parameters: the elasticity parameter ρ, and the initial steady state right-tail slope of large firms. We report results for $\rho = 2$ and $\rho = 10$, and for a right tail slope coefficient for large firms equal to -0.15 (this is closer to the right tail coefficient that can be inferred using data on the firm-based size distribution). Output gains are larger the lower is ρ (i.e.: from 3.4 a given change in aggregate productivity index Z_i translates into a larger increase in final output), and the closer to zero is the right tail coefficient (i.e.: production becomes more concentrated among the most efficient firms).

8.2. Transition dynamics

Figures $7 - 9$ display the path of export/GDP (Panel A) and output (Panel B) in the transition to the new steady state, for each value of b. While the transition is almost immediate in the economy with exogenous innovation, it takes many periods under endogenous innovation. The change in the innovation intensity of exporters increases the drift in their productivity growth, but this takes time to materialize into higher productivity levels. In the transition, the distribution of firm level productivities $M_t(z)$ slowly shifts rightwards, leading to a gradual increase in final output and export share.

This model with endogenous innovation is capable of producing substantial differences in the short- and long-run response of trade volumes to a trade liberalization. The contemporaneous increase in exports/GDP as a fraction of the overall increase across steady states, reported in Row 1, Columns 1-3 of Table 4, is 99% under $b = 3000$, 56% under $b = 30$, and 18% under $b = 10$. In this transition, the economy takes several periods to reach the new steady state (see Figures 8 and 9, where a period is defined as a year). So, even though we think that this smooth transition is an interesting implication of the model, we do not think that it can fully account for the difference in short- vs long-run elasticities of trade documented in the data by other researchers.[23]

Finally, we report in Row 2 of Table 4 the welfare gains from a reduction in trade costs. Welfare gains are defined as the equivalent variation in consumption that keeps the representative household indifferent between the economy with high and low trade costs (taking into account the transition between steady states). In our benchmark model we find that the welfare gains are almost unchanged as we lower b (they are 0.24% under $b = 3000$ and

[23] See for example, Ruhl (2004).

0.25% under $b = 10$). Columns $4 - 7$ indicates that the welfare gains are larger (0.45% when $b = 30$ and 1.18% when $b = 10$) if the costs of innovation are rebated to the representative household. Note that the very long transition implies that the welfare gains are significantly smaller than the steady state output increase reported in Table 3.

We conclude from these experiments that considerations of endogenous innovation do not substantially alter the welfare implications of a reduction in international trade costs, for the following two reasons. First, investing in innovation is costly in that managers are drawn out of alternative activities such as starting new firms. Second, the productivity gains from this investment come only slowly.

9. Concluding remarks

To be completed

References

[1] Aghion, Phillipe, Richard Blundell, Rachel Griffith, and Peter Howitt (2006) "The Effects of Entry on Incumbent Innovation and Productivity" mimeo

[2] Aghion, Philippe, Nicholas Bloom, Richard Blundell, Rachel Griffith, and Peter Howitt (2005) "Competition and Innovation: An Inverted U Relationship" Quarterly Journal of Economics

[3] Aghion, Philippe, Christopher Harris, Peter Howitt and John Vickers (2003) "Competition, Imitation and Growth with Step-by-Step Innovation" Review of Economic Studies 68: 467-492.

[4] Alessandria, George and Horag Choi (2006) "Do Sunk Costs of Exporting Matter for Net Export Dynamics?" Quarterly Journal of Economics, forthcoming

[5] Atkeson, Andrew and Patrick Kehoe (2005) "Modeling and Measuring Organization Capital", Journal of Political Economy 113: 1026–1053

[6] Bernard, Andrew, Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum (2003) "Plants and Productivity in International Trade" American Economic Review, September, pp. 1268-1290

[7] Bernard, Andrew, J. Bradford Jensen and Peter Schott (2005) "Importers, Exporters and Multinationals: A Portrait of Firms in the U.S. that Trade Goods" in T. Dunne, J.B. Jensen and M.J. Roberts (eds.), Producer Dynamics: New Evidence from Micro Data (University of Chicago Press, forthcoming).

[8] Caves, Richard (1998) "Industrial Organization and New Findings on the Turnover and Mobility of Firms" Journal of Economic Literature 36: 1947:82

[9] Chaney, Thomas (2006) "Distorted gravity: Heterogeneous Firms, Market Structure and the Geography of International Trade" mimeo

[10] Corsetti, Giancarlo, Philippe Martin and Paolo Pesenti (2005) "Productivity spillovers, terms of trade and the 'home market effect" mimeo

[11] Davis, Steven, John Haltiwanger, Ron Jarmin, and Javier Miranda (2006) "Volatility and Dispersion in Business Growth Rates: Publicly Traded versus Privately Held Firms" NBER WP #12354

[12] Eaton, Jonathan and Samuel Kortum (2001) "Technology, Trade, and Growth: A Unified Framework" European Economic Review: Papers and Proceedings, 45(4-6), 742-755.

[13] Ericson Richard, and Ariel Pakes (1995) "Markov Perfect Industry Dynamics: A Framework for Empirical Work" Review of Economic Studies 62: 53-82

[14] Ghironi, Fabio and Marc Melitz (2005) "International Trade and Macroeconomic Dynamics with Heterogeneous Firms", Quarterly Journal of Economics, August 2005.

[15] Grossman, Gene and Elhanan Helpman (1991) "Innovation and Growth in the Global Economy" Cambridge: MIT Press

[16] Hopenhayn, Hugo (1992) "Entry, Exit, and Firm Dynamics in Long Run Equilibrium" Econometrica 60: 1127-50

[17] Irarrazabal, Alfonso and Luca David Opromolla (2006) "Hysteresis in Export Market" mimeo

[18] Klette, Jakob and Samuel Kortum (2004) "Innovating Firms and Aggregate Innovation" Journal of Political Economy, 112(5), 986-1018.

[19] Krugman, Paul (1980) "Scale Economies, Product Differentiation, and the Patterns of Trade" American Economic Review 70: 950-59.

[20] Luttmer, Erzo (2006) "Selection, Growth, and the Size Distribution of Firms" Quarterly Journal of Economics, forthcoming

[21] Melitz, Marc (2003) "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity" Econometrica 71: 1695-1721

[22] Ruhl, Kim (2004) "Solving the Elasticity Puzzle in International Economics" mimeo

[23] Sutton, John (1997) "Gibrat's Legacy" Journal of Economic Literature 35: 40-59

10. Appendix: Adding taxes

Here we extend the model to include taxes on firm revenues, profits and factor use. We show that a change in the level of any of these taxes has no effect on firms' innovation decisions, and hence no impact on aggregate productivity and output. The logic of the argument is very similar to that of Proposition 1 in the body of the paper.

We consider three different types of taxes: 1) a tax rate τ_y on firm revenues, 2) a tax rate τ_l on firm wage payments (both workers and managers), and 3) a tax rate τ_c on firm profits. We assume that the profit tax τ_c is also levied on the profits of entering firms. These firms have a negative profit equal to $-(1+\tau_l)\, n_e W_{mt}$, so essentially we are assuming a tax credit on firm's initial investment. Tax revenues are rebated lump-sum to households. We focus on symmetric equilibria with equal tax rates in both countries.

Current profits of a firm with productivity index z are:

$$\begin{aligned}\Pi_t(z) \quad = \quad & \max_{l(n_x),p_a,p_a^*,a,a^*,\xi(n_x)\in[0,1]} (1-\tau_y)\, p_a a + (1-\tau_y) \int \xi(n_x) dG_x p_a^* a^* \\ & -(1+\tau_l)\, W_t \int l(n_x) dG_x - (1+\tau_l)\, W_{mt} \int \xi(n_x) n_x dG_x - (1+\tau_l)\, W_{mt} n_f\end{aligned}$$

subject to the same two constraints as in problem (2.8).

It is straightforward to show that variable real profits on home sales are

$$\exp(z)\left[\frac{1}{\rho-1}\right](1+\tau_l)^{1-\rho}(1-\tau_y)^{\rho}\left(\frac{\rho}{\rho-1}\right)^{-\rho}\left(\frac{W_t}{P_t}\right)^{1-\rho} Y_t$$

and variable real profits on foreign sales are

$$\exp(z)\left[\frac{1}{\rho-1}\right](1+\tau_l)^{1-\rho}(1-\tau_y)^{\rho}D^{1-\rho}\left(\frac{\rho}{\rho-1}\right)^{-\rho}\left(\frac{W_t}{P_t}\right)^{1-\rho}Y_t$$

The Bellman equation of the firm is:

$$V_t(z)=\max\left[0,V_t^o(z)\right]$$

where

$$\begin{aligned} V_t^o(z) &= (1-\tau_c)\max_{p\in[0,1]}\Pi_t(z)-(1-\tau_c)(1+\tau_l)W_{mt}H(z,p)+ \\ &\quad (1-\delta)\frac{Q_{t+1}}{Q_t}\left[pV_{t+1}(z+s)+(1-p)V_{t+1}(z-s)\right]. \end{aligned}$$

Free entry requires that

$$(1-\tau_c)(1+\tau_l)W_{mt}n_e=(1-\delta)\frac{Q_{t+1}}{Q_t}\int V_{t+1}(z)dG$$

Following similar steps as in the benchmark model, we obtain the equilibrium real wage:

$$\frac{W_t}{P_t}=\frac{\rho-1}{\rho}\frac{1-\tau_y}{1+\tau_l}\left[Z_{at}+D^{1-\rho}Z_{a^*t}\right]^{1/(\rho-1)}$$

Aggregate output is given by (3.4) as before.

Using the expression for the real wage, real profits of firm z are:

$$\pi_t(z)=\exp(z)\frac{1}{\rho}(1-\tau_y)\frac{Y_t}{Z_{at}+D^{1-\rho}Z_{a^*t}}\left(1+x_t(z)D^{\rho}\right)-\left(x_t(z)n_x+n_f\right)(1+\tau_l)\frac{W_{mt}}{P_t}$$

We now restate the Bellman equation of a firm deflating by the price level P_t :

$$v_t(z)=\max\left[0,v_t^o(z)\right]$$

where

$$\begin{aligned} v_t^o(z) &= \max_{p\in[0,1]}(1-\tau_c)\pi_t(z)-(1-\tau_c)(1+\tau_l)\frac{W_{mt}}{P_t}H(z,p)+ \\ &\quad (1-\delta)\beta\frac{Y_t}{Y_{t+1}}\left[pv_{t+1}(z+s)+(1-p)v_{t+1}(z-s)\right] \end{aligned}$$

with free entry condition

$$(1-\tau_c)(1+\tau_l)\frac{W_{mt}}{P_t}n_e=(1-\delta)\beta\frac{Y_t}{Y_{t+1}}\int v_{t+1}(z)dG$$

The interior first order condition for investments on innovation is:

$$(1-\tau_c)(1+\tau_l)\frac{W_{mt}}{P_t}\frac{\partial}{\partial p}H(z,p) = (1-\delta)\beta\frac{Y_t}{Y_{t+1}}\left[v_{t+1}(z+s) - v_{t+1}(z-s)\right]. \qquad (10.1)$$

Proposition 2: Consider two economies with identical initial distributions of productivity across firms across both countries and economies. Let taxes be τ_y, τ_l and τ_c in the first economy and τ'_y, τ'_l and τ'_c in the second economy. Then the innovation decisions of firms in a symmetric equilibrium in both economies are identical in that $p_t(z) = \tilde{p}_t(z)$ for all dates t and productivities z.

Proof: Let the equilibrium in the first economy be denoted by variables without tildes and the equilibrium in the second economy be denoted by variables with tildes. It is straightforward to verify that one can use the equilibrium in the first economy to construct an equilibrium in the second economy as follows. Firm decision rules $\bar{z}_t$, $\bar{z}^*_t$, $x_t(z)$, $x^*_t(z)$, $p_t(z)$, $p^*_t(z)$, domestic quantities, $a_t(z)$, $b^*_t(z)$, imported quantities $a^*_t(z)$, $b_t(z)$, real domestic prices $p_{at}(z)/P_t$, $p_{b*t}(z)/P^*_t$, real import prices $p_{a*t}(z)/P^*_t$, $p_{bt}(z)/P_t$, labor $l_t(z)$, $l^*_t(z)$, measures $M_t(z)$, M_{et}, $M^*_t(z)$, M^*_{et}, output Y_t, Y^*_t, consumption C_t, C^*_t and asset prices Q_t are symmetric across countries and identical across economies. Real profits $\tilde{\pi}_t(z)$, $\tilde{\pi}^*_t(z)$ in the second economy can all be constructed from their counterpart in the first economy using the constant factor of proportionality

$$\tilde{\pi}_t(z) = \left(\frac{1-\tilde{\tau}_y}{1-\tau_y}\right)\pi_t(z), \text{ etc.}$$

Likewise, firm real value functions $\tilde{v}^o_t(z)$, $\tilde{v}_t(z)$, $\tilde{v}^{*o}_t(z)$, $\tilde{v}^*_t(z)$ can be constructed using the factor or proportionality:

$$\tilde{v}^o_t(z) = \left(\frac{1-\tilde{\tau}_c}{1-\tau_c}\right)\left(\frac{1-\tilde{\tau}_y}{1-\tau_y}\right)v^0_t(z), \text{ etc.}$$

Workers and manager real wages $\tilde{W}_t/\tilde{P}_t$, $\tilde{W}^*_t/\tilde{P}^*_t$, $\tilde{W}_{mt}/\tilde{P}_t$, $\tilde{W}^*_{mt}/\tilde{P}^*_t$ can be constructed using the constant factor of proportionality

$$\frac{\tilde{W}_{mt}}{\tilde{P}_t} = \left(\frac{1-\tilde{\tau}_y}{1-\tau_y}\right)\left(\frac{1+\tau_l}{1+\tilde{\tau}_l}\right), \text{ etc.}$$

We can then check that condition (10.1) is satisfied at the new equilibrium.

Figure 1: Scaled incentive to innovate and exporting decision

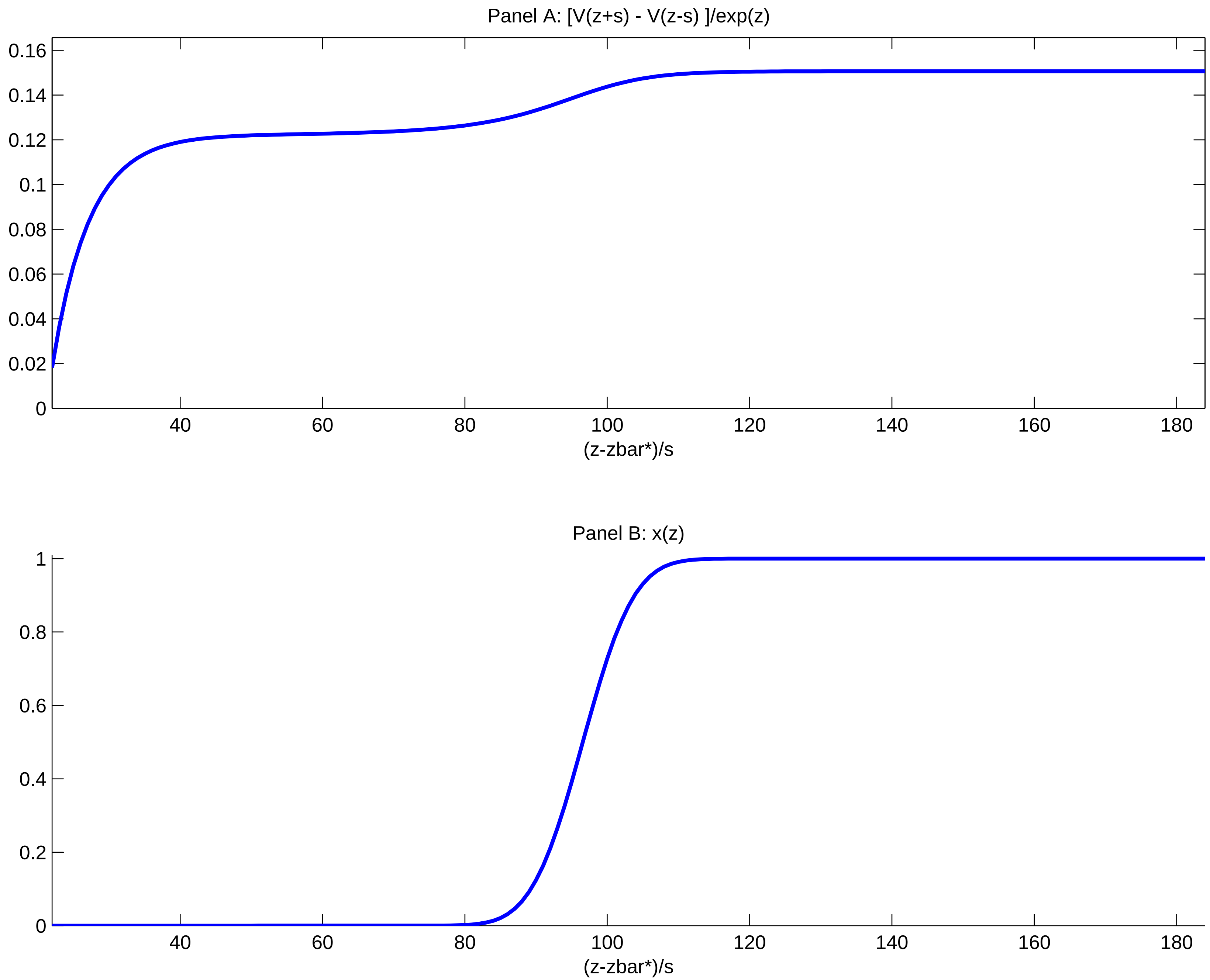


Figure 2: Employment-based size distribution, 2003 US data

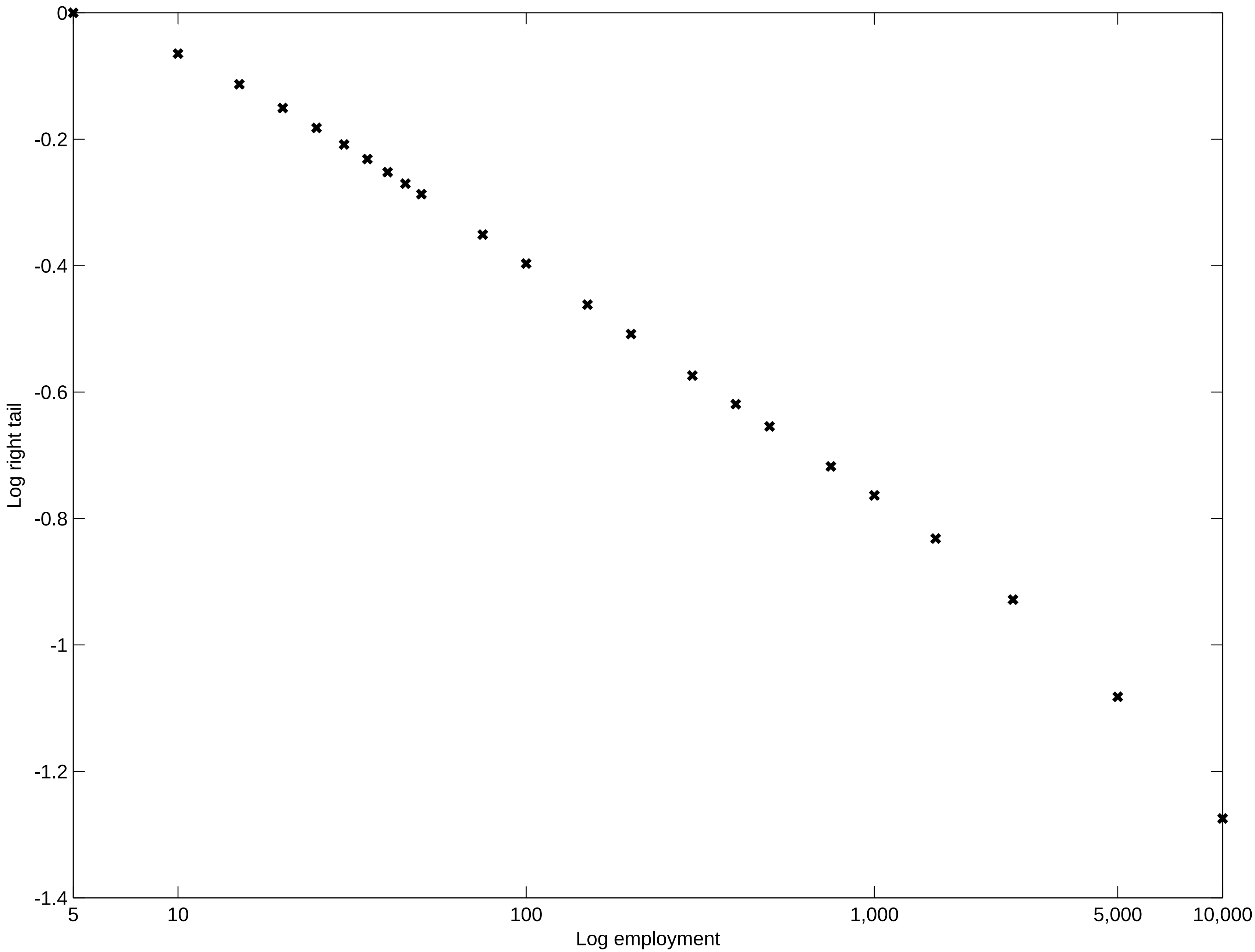


Figure 3: Steady State, b = 3000

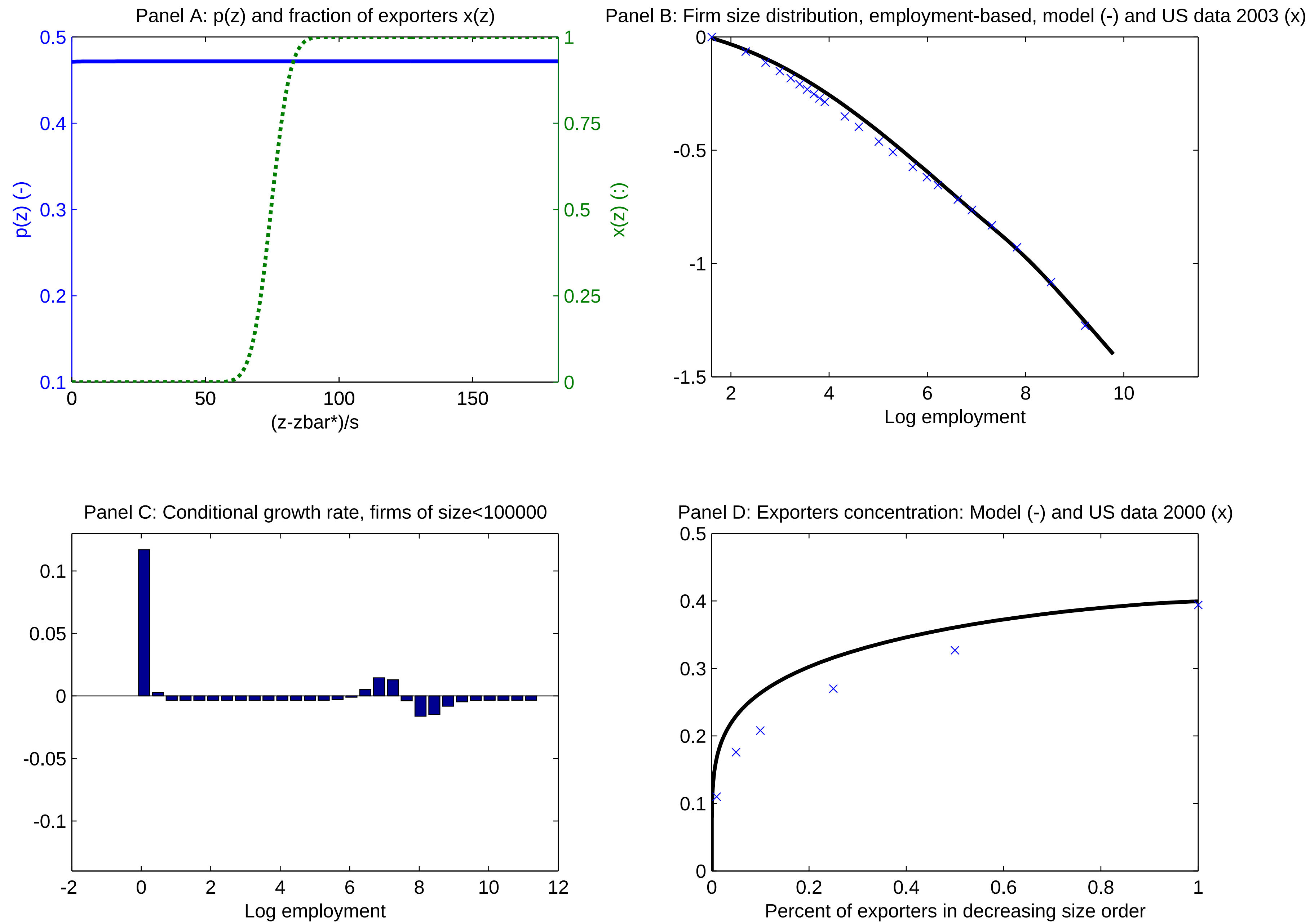
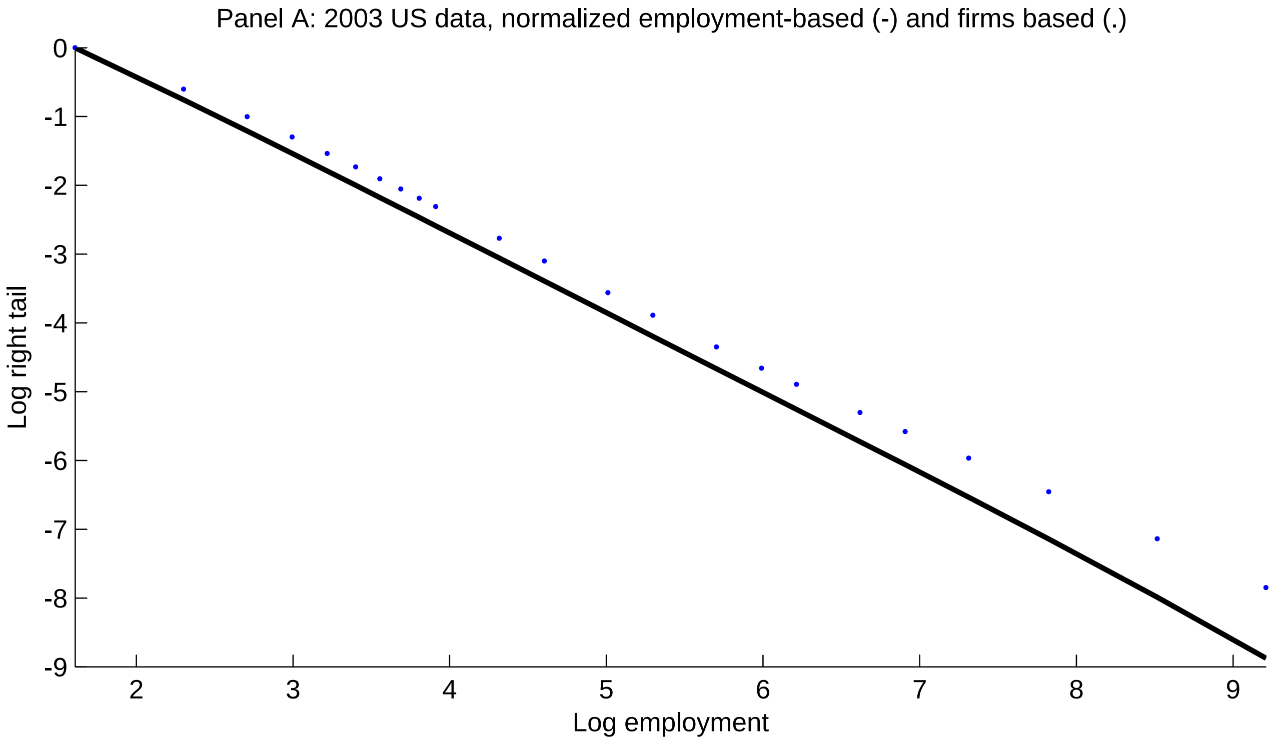


Figure 4: Firm size distribution, US Data and Model



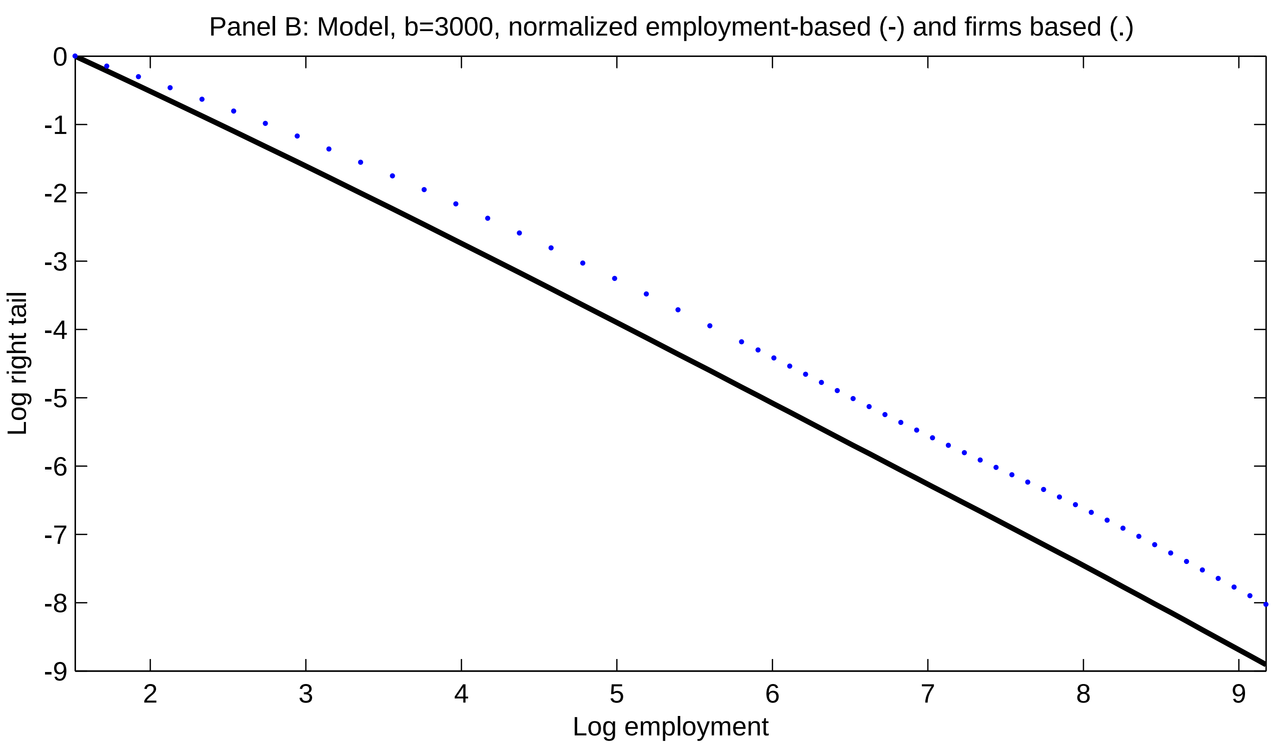


Figure 5: Steady State, b = 30

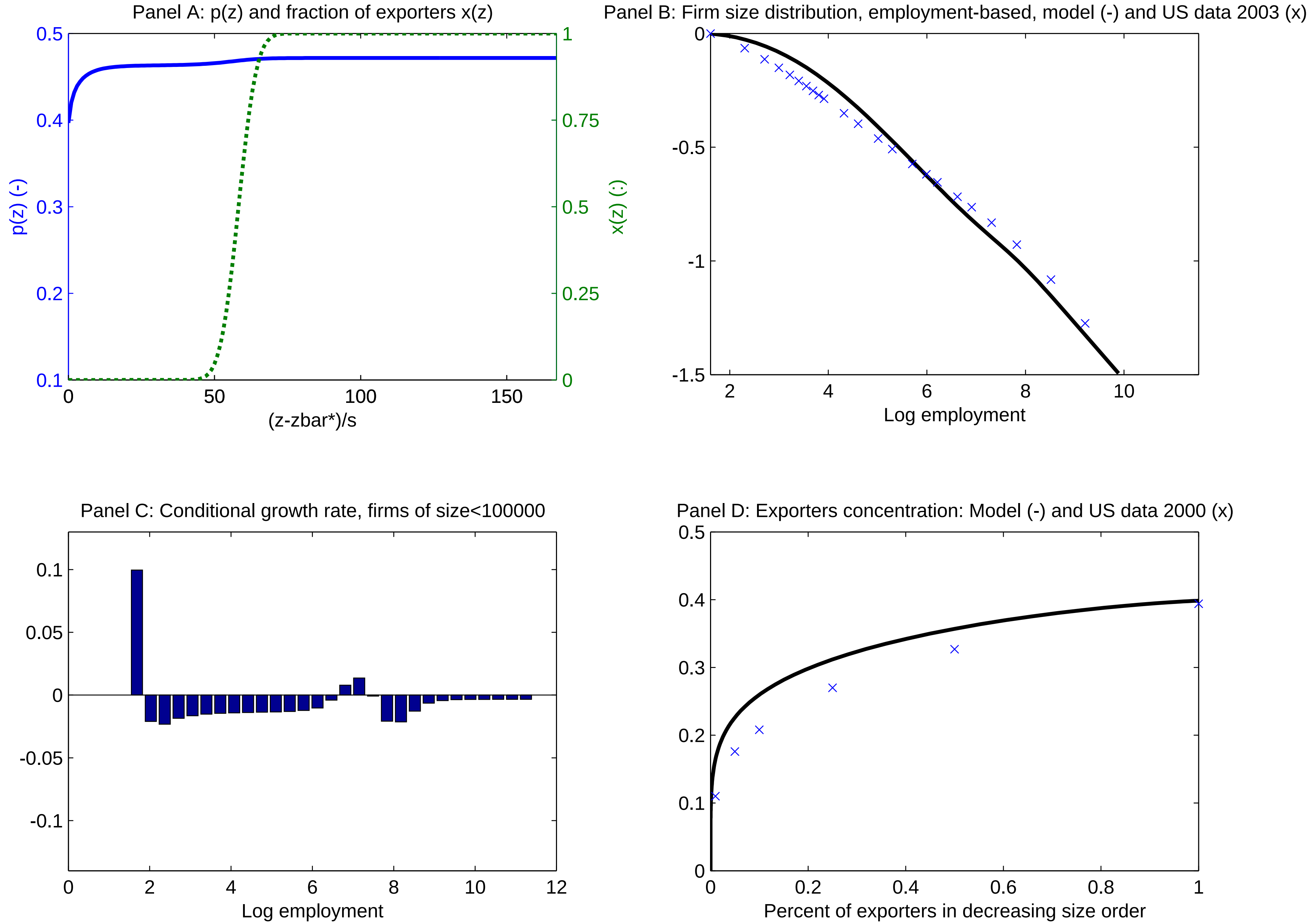


Figure 6: Steady State, b = 10

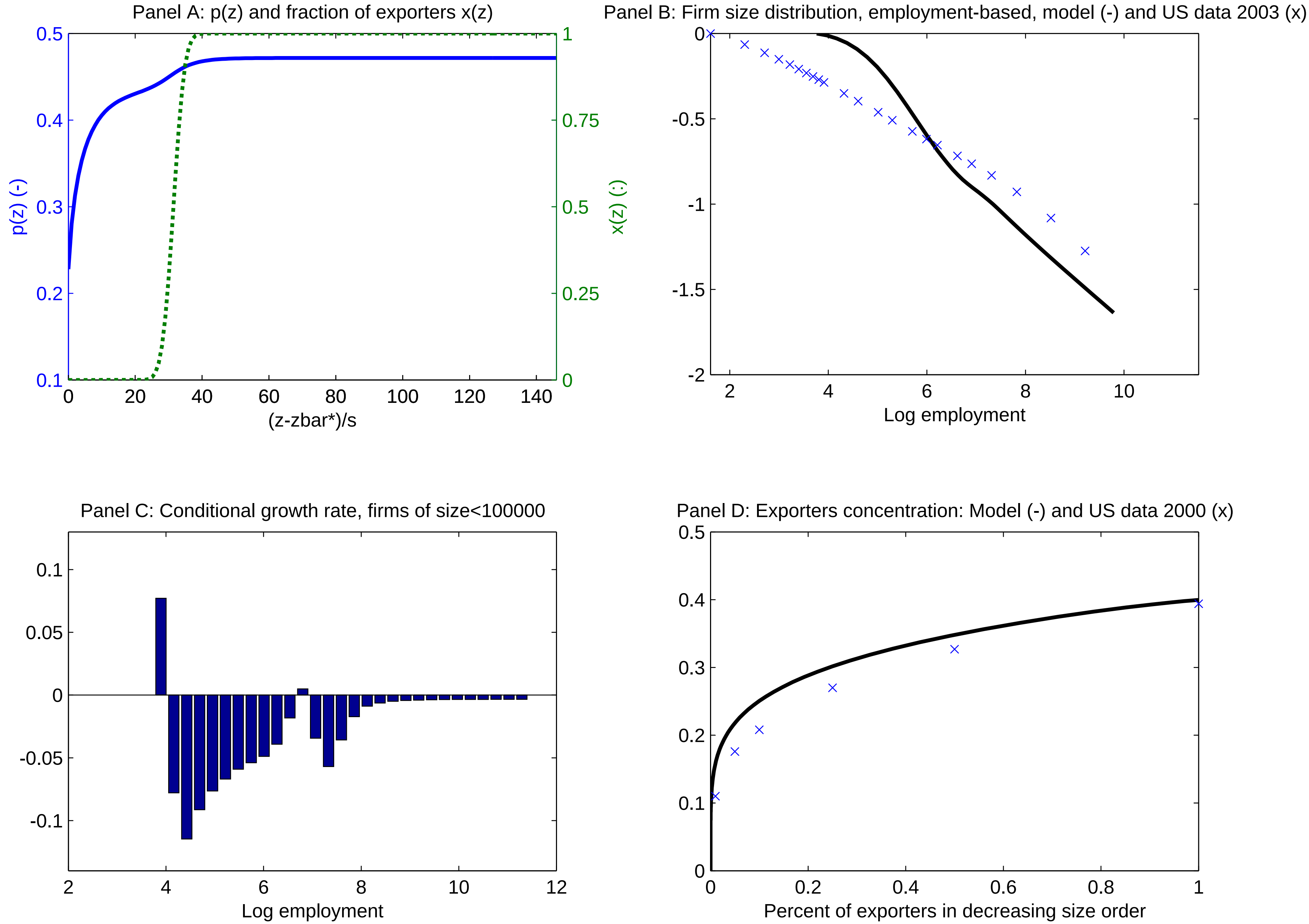
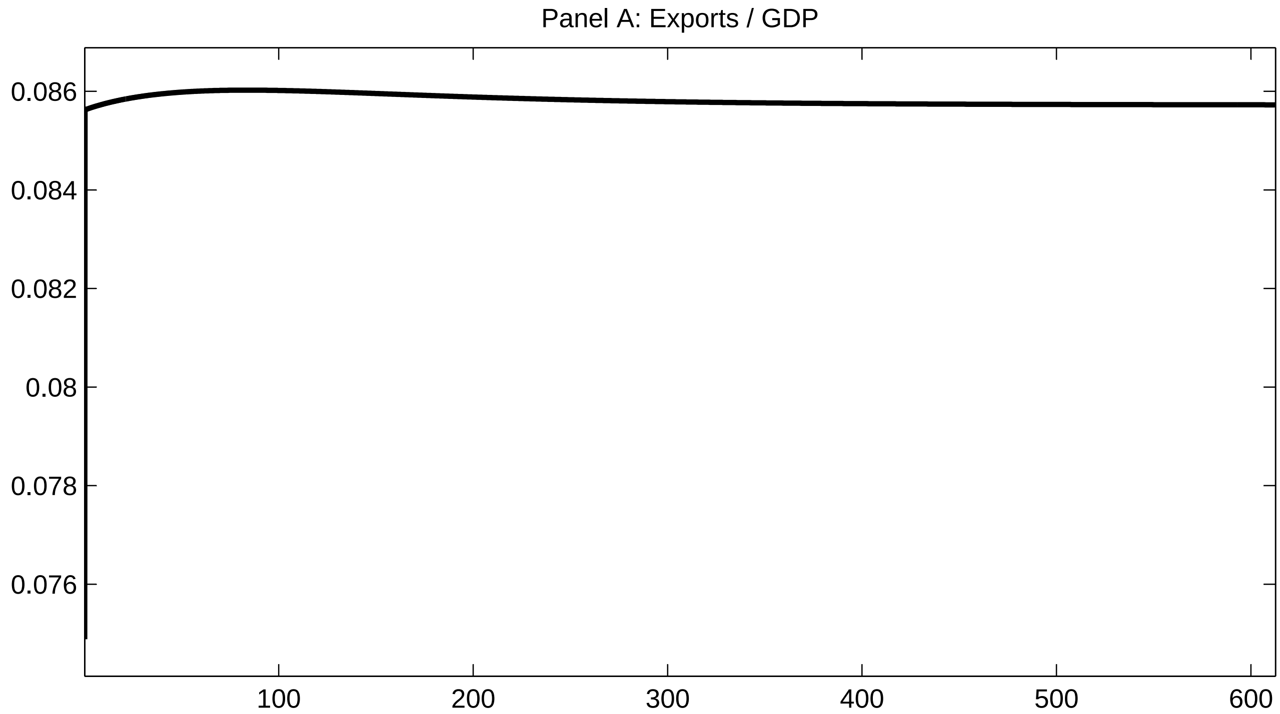


Figure 7: Transition Dynamics, b = 3000



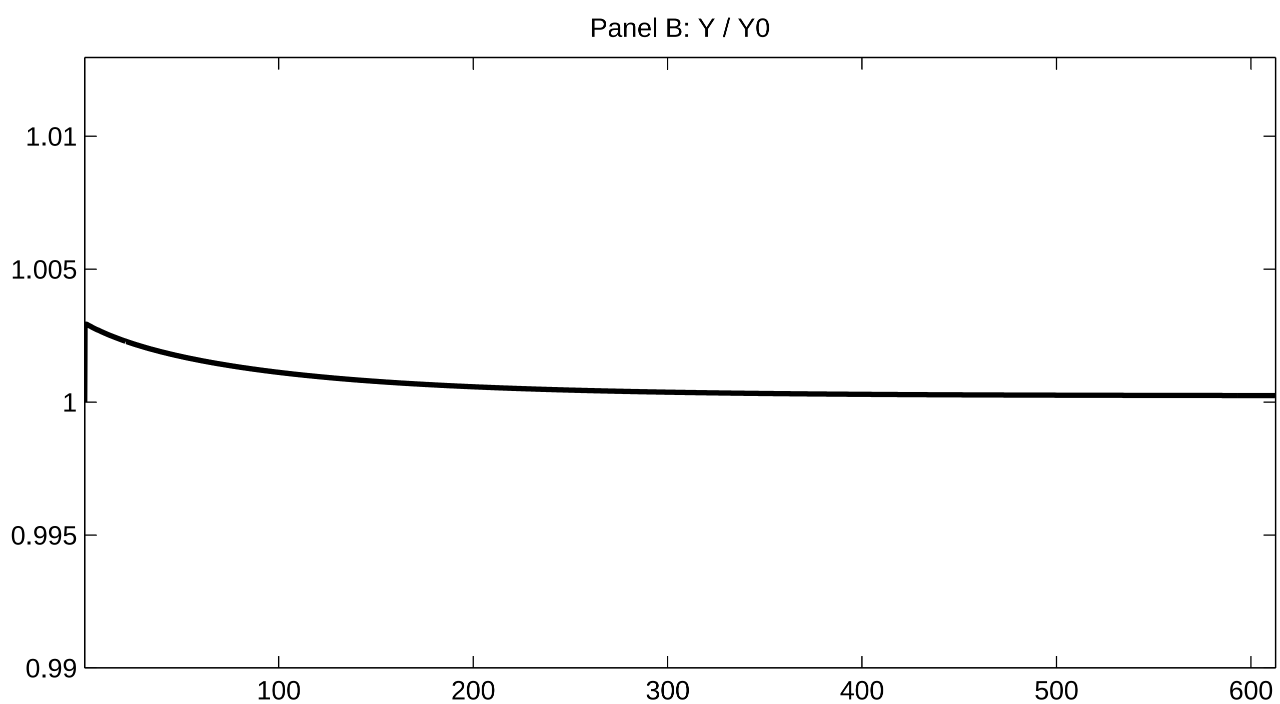
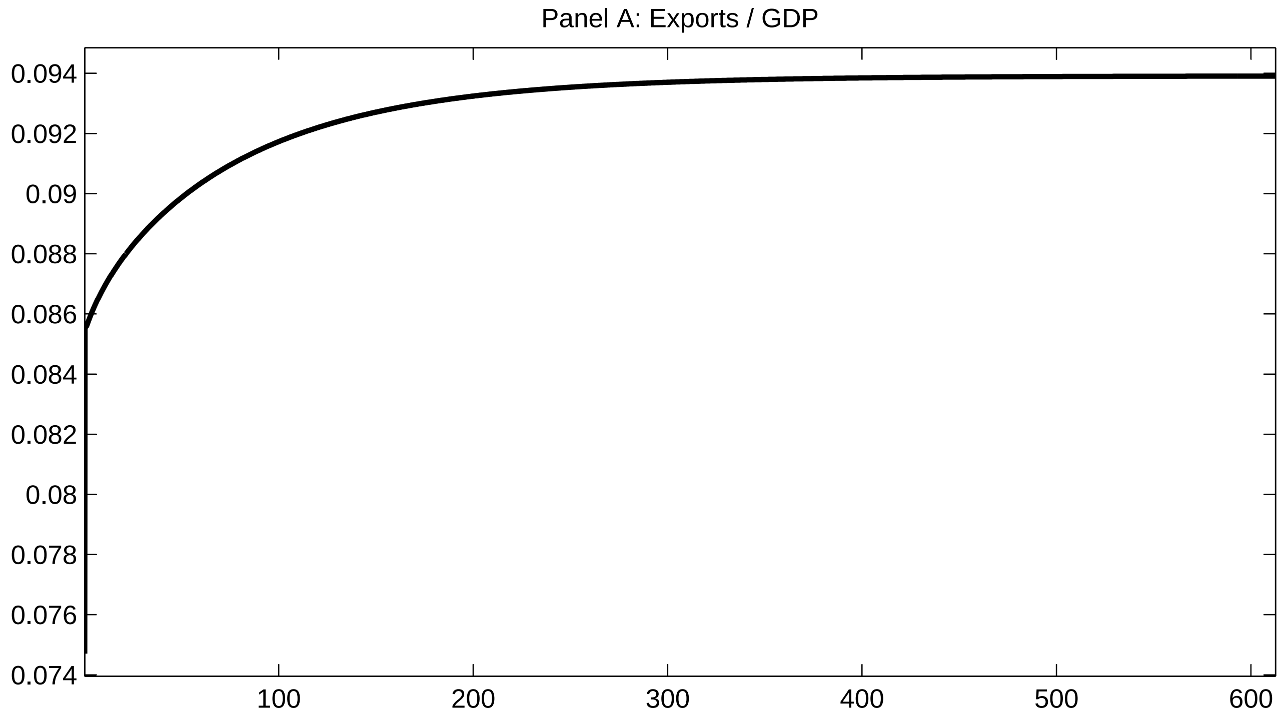


Figure 8: Transition Dynamics, b = 30



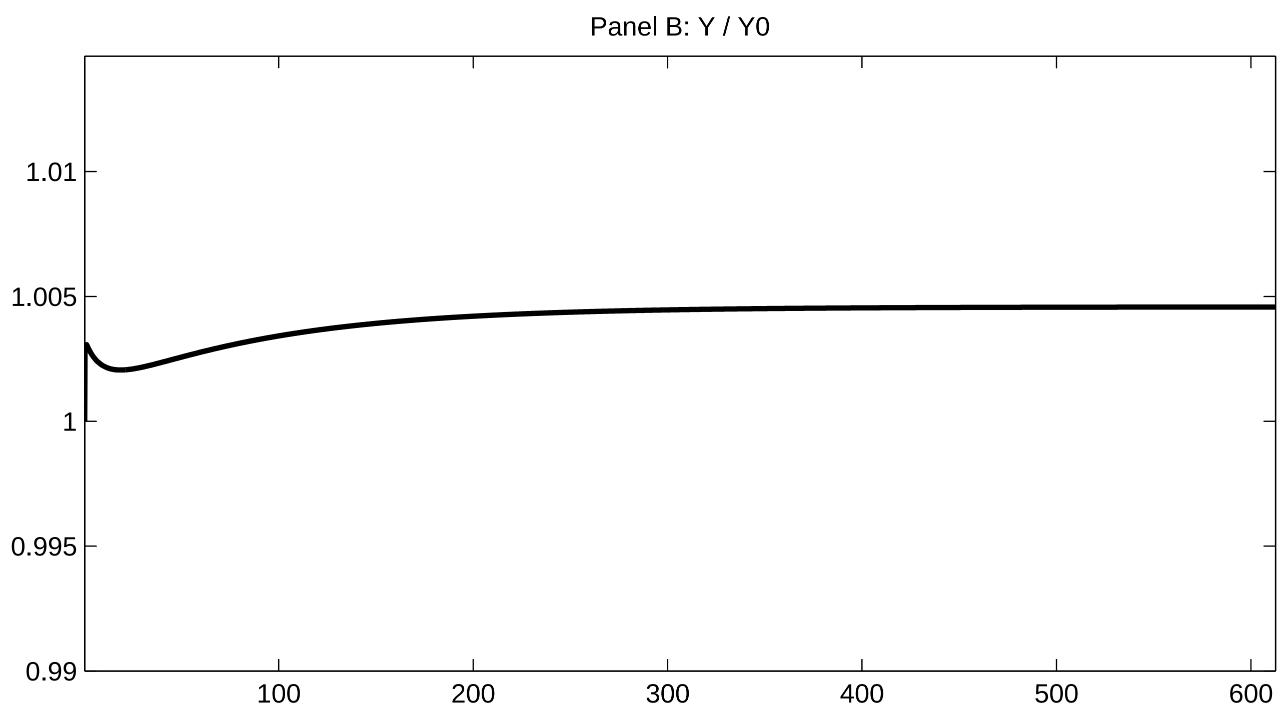


Figure 9: Transition Dynamics, b = 10

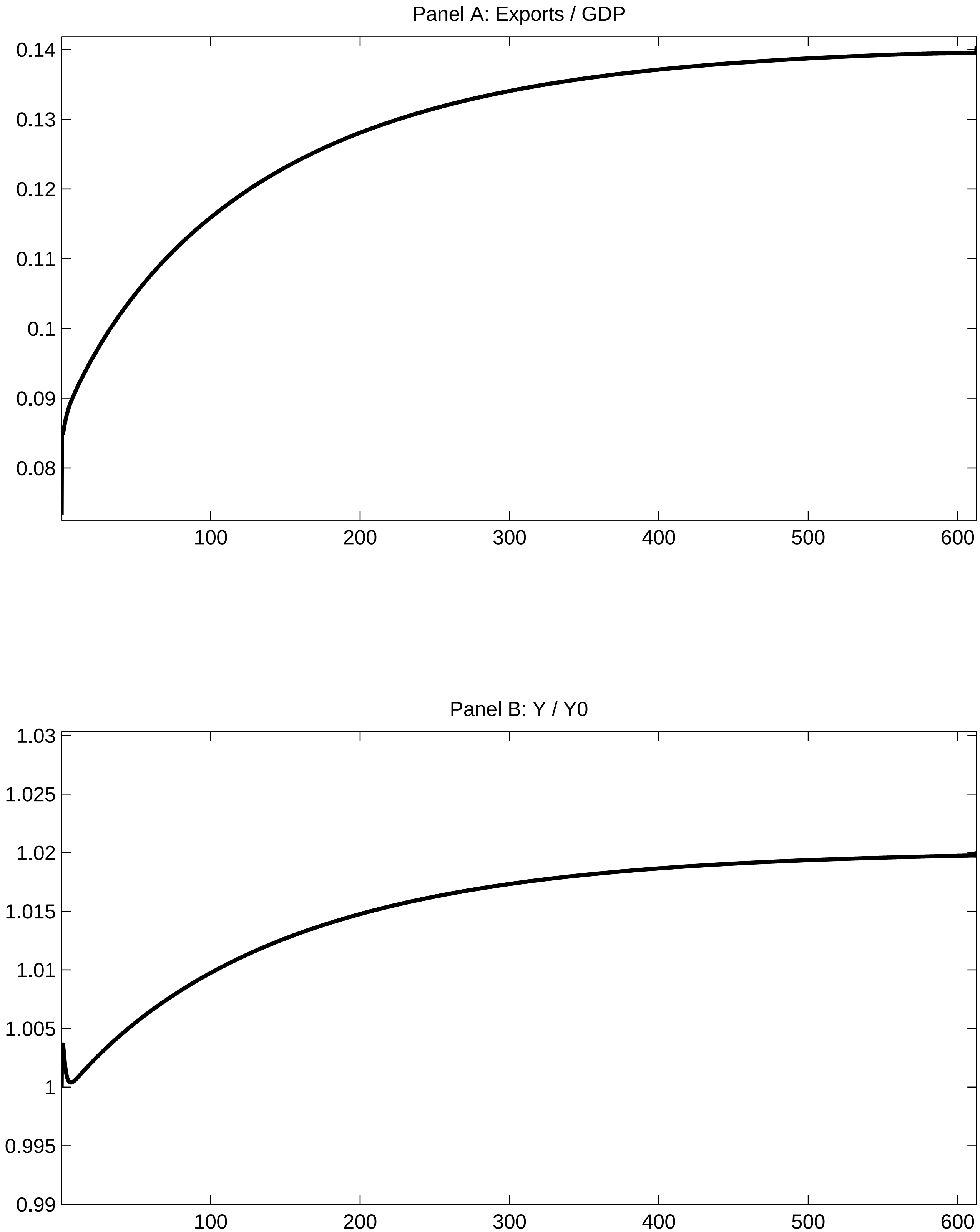


Table 1: Model Calibration

		US Data	1 **b=3000**	2 **b=30**	3 **b=10**
CALIBRATED PARAMETERS					
1	s , annualized		0.25	0.25	0.25
2	δ , annualized		0.0055	0.0055	0.0055
2	h (or employment-based right-tail coefficient of large firms)		4.99E+40 (-0.25)	4.02E-02 (-0.25)	3.58E-01 (-0.25)
4	σ_e , annualized		2.3	1.8	0.7
5	$D^{1-\rho}$		0.231	0.231	0.231
6	$\bar{n}_x$		30	7.9	0.625
7	σ_{nx} , annualized		0.6	0.5	0.3
TARGETS					
8	Employment growth rate of large firms, annual standard deviation	25%	25%	25%	25%
9	Annual employment-based exit rate, firms larger 500 employees	0.55%	0.55%	0.55%	0.55%
10	Employment-based right tail coefficient, firms of size 1000 to 5000	-0.2	-0.20	-0.21	-0.26
11	New firms, % employment of firms smaller than 500 employees	90%	91%	89%	88%
12	Exports / GDP	7.50%	7.5%	7.5%	7.5%
13	Employment share of exporters	40%	40%	40%	40%
14	Average switching rate, seven years	12%	12%	12%	12%

Table 2: Steady State Comparison, 10% increase in exporter's trade intensity

		1	2	3
		Benchmark		
b	Curvature parameter of C(p)	b=3000	b=30	b=10
Ratio of new to old				
1	Exports / GDP	1.145	1.257	1.885
2	Final output	1.0002	1.005	1.020
3	Average productivity index $\exp(z)(1+x(z)D^{1-\rho})$	1.012	1.088	1.872
4	Number of firms	0.989	0.936	0.578

Table 3: Sensitivity Analysis, Steady State Comparison, 10% increase in exporter's trade intensity

	1	2	3	4	5	6	7	8	9
	Benchmark			**Wkers, mgrs perf. substitutes**			**Innovation costs rebated**		
b Curvature parameter of C(p)	b=3000	b=30	b=10	b=3000	b=30	b=10	b=3000	b=30	b=10
Ratio of new to old									
1 Exports / GDP	1.145	1.257	1.885	1.145	1.257	1.885	1.145	1.257	1.885
2 Final output	1.00022	1.005	1.020	1.0005	1.003	1.016	1.00026	1.017	1.161
3 Average productivity index $\exp(z)(1+x(z)D^{1-\rho})$	1.012	1.088	1.872	1.012	1.088	1.872	1.012	1.088	1.872
4 Number of firms	0.989	0.936	0.578	0.990	0.923	0.543	0.989	0.985	0.971

	10	11	12	13	14	15	16	17	18
	$\rho = 10$			$\rho = 2$			**Slope coefficient = -0.15**		
b Curvature parameter of C(p)	b=3000	b=30	b=10	b=3000	b=30	b=10	b=3000	b=30	b=10
Ratio of new to old									
1 Exports / GDP	1.145	1.257	1.885	1.145	1.257	1.885	1.135	1.333	2.752
2 Final output	1.00009	1.002	1.009	1.00095	1.018	1.082	1.000	1.007	1.039
3 Average productivity index $\exp(z)(1+x(z)D^{1-\rho})$	1.012	1.088	1.872	1.012	1.088	1.872	1.012	1.154	65.516
4 Number of firms	0.989	0.936	0.578	0.989	0.936	0.578	0.988	0.889	0.018

Table 4: Transition dynamics, 10% increase in exporter's trade intensity

		1	2	3	4	5	6
		Benchmark			**Innovation costs rebated**		
b	Curvature parameter of C(p)	b=3000	b=30	b=10	b=3000	b=30	b=10
1	Exports / GDP , contemporaneous increase / long run increase	99%	56%	18%	99%	57%	18%
2	Welfare gain (equivalent variation)	0.24%	0.24%	0.25%	0.24%	0.45%	1.18%