

Macroeconometric Equivalence, Microeconomic Dissonance, and the Design of Monetary Policy

Andrew T. Levin, J. David López-Salido, Edward Nelson, and Tack Yun

November 2007

Abstract

Recent developments in macroeconomics have focused on the estimation of DSGE models using a system of loglinear expectational difference equations to approximate the equilibrium conditions. In this paper, we use the term *macroeconometric equivalence* to encapsulate the idea that estimates using aggregate data based on first-order approximations of the equilibrium conditions of a DSGE model will not be able to distinguish between alternative underlying preferences and technologies. We then develop the concept of *microeconomic dissonance* in reference to how their underlying microeconomic differences become important when optimal steady-state inflation is analyzed in a nonlinear setting. To illustrate these ideas we use alternative versions of a small, widely estimated, New Keynesian model. We show how identical loglinear approximations to alternative settings of preferences and technologies, including internal *vs* external habits, standard *vs* risk-sensitive preferences, and alternative price-setting specifications, may imply very different optimal steady-state policies.

JEL classification: E22; E30; E52

Keywords: macroeconometric equivalence, alternative microfoundations, Ramsey optimal monetary policy, welfare analysis.

An earlier draft of this paper was presented at the Conference, *John Taylor's Contributions to Monetary Theory and Policy*, Federal Reserve Bank of Dallas, October 12-13, 2007. We thank Mark Gertler, Robert Hall, Jinill Kim, and John Williams for useful comments. The views expressed in this paper are solely those of the authors and do not necessarily reflect the views of the Federal Reserve System, the Federal Reserve Bank of St. Louis, or the Board of Governors. Corresponding author: david.lopez-salido@frb.gov.

1 Introduction

The last fifteen years have witnessed an important shift toward the use of models for monetary policy analysis that feature nominal rigidities but are otherwise recognizable as neoclassical general equilibrium business cycle models of the type advanced by Kydland and Prescott (1982). Taylor (1992) noted that the latter models needed modification because “[f]rom the perspective of formulating monetary policy... the real business cycle model is by definition inadequate; it does not include monetary policy, and it does not explain the strong correlation between price and output fluctuations evident in the data.” The modern monetary policy models bridge the gap by including both monetary policy and nominal rigidity in fully articulated business cycle models. The form of nominal rigidity prevalent in the new-generation models is an inheritance from the pioneering work on staggered contracts by Taylor (e.g., 1980). Several papers integrated the Calvo (1983) staggered price contracts system into dynamic stochastic general equilibrium models with optimizing private agents, while Calvo himself noted that his price-setting scheme was “a close relative of the staggered contracts model... of Taylor (1979, 1980).”¹ This approach attempts to embed realistic effects of monetary policy into a framework that respects the distinction between structural parameters and policy parameters stressed by the Lucas critique.

In this paper, we use the term *macroeconometric equivalence* to encapsulate the idea that macroeconometric estimates based on first-order approximations of the equilibrium conditions will not be able to distinguish, using aggregate data, between alternative underlying preferences and technologies. We then proceed to develop the concept of *microeconomic dissonance*, which refers to how the underlying microeconomic differences bear on the optimal steady-state inflation rate.

We present alternative versions of a compact New Keynesian model exhibiting macroeconometric equivalence but with underlying microeconomic dissonance.² We explore the optimal policy implications of two alternative rationalizations for inflation persistence (i.e., indexation and

¹Calvo (1983, p. 383). For development of staggered contracts models, see King and Wolman (1996), Rotemberg and Woodford (1997), and Yun (1996).

²Because we use the New Keynesian model, our results have clearer relevance for monetary policy analysis than the instances of observational equivalence highlighted by Sargent (1976), Sims (1998), and Barillas, Hansen, and Sargent (2007).

backward-looking price setting), and different time-dependent preferences that lead to output persistence (internal *vs* external habits). We also discuss the implications for optimal monetary policy of two less heavily studied mechanisms that allow for real rigidities and risk-sensitive preferences. All the versions of the New Keynesian model we consider are nested in a loglinear IS equation of the form:

$$y_t = \alpha_b y_{t-1} + \alpha_f E_t y_{t+1} - \rho [r_t - E_t \pi_{t+1}] \quad (1)$$

and in a loglinear hybrid New Keynesian Phillips curve:

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \kappa m c_t. \quad (2)$$

Here y_t is the log-deviation of output from its steady-state growth path, $m c_t$ is the log-deviation of real marginal cost from its steady-state level, and π_t and r_t respectively denote deviations of quarterly inflation and the short-term nominal interest rate from their steady-state values. The model includes as a special case the canonical New Keynesian model.³ That canonical model omits the lagged terms from equations (1) and (2). We consider two versions of the canonical model. First, we consider a baseline sticky-price model that includes different sources of strategic complementarities (real rigidities) in price setting. In this environment, the Phillips curve slope κ can be factorized into two parameters. One captures the degree of nominal rigidities (κ_p), and the parameter γ reflects real rigidities which alter the reaction of prices to marginal cost. Second, we consider a model with risk-sensitive preferences instead of standard expected-utility preferences. We also consider cases where lagged output enters the IS equation ($\alpha_b > 0$), through different forms of time-dependent preferences; and where lagged inflation enters the Phillips curve ($\gamma_b > 0$), through alternative forms of price-setting behavior.⁴

Macroeconometric equivalence is important because these alternative specifications will not be distinguishable by standard macroeconometric procedures, but we show that they deliver different implications for steady-state inflation (i.e., the optimal mean inflation rate).⁵ A straightforward

³See, for instance, Roberts (1995), Rotemberg (1987), Rotemberg and Woodford (1997), Clarida, Galí, and Gertler (1999), King (2000), and Woodford (2003).

⁴Such model features have been proposed as desirable modifications of IS and Phillips curves in a number of studies. See e.g. Fuhrer (2000), Galí and Gertler (1999), Ireland (2001), Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003), Steinsson (2003), and Levin, Onatski, Williams, and Williams (2005).

⁵There are parallels with the discordance issues raised by Browning, Hansen, and Heckman (1999).

implication of the analysis presented in this paper is that a potential remedy for macroeconomic equivalence may be found in econometric procedures capable of estimating versions of the model based on higher-order approximations. But an alternative avenue that we find promising is making use of datasets not consisting purely of macroeconomic time series. Microeconomic and financial data offer themselves as a rich source of information about microeconomic structure. Studies by Bils and Klenow (2004), Nakamura and Steinsson (2007), and the survey by Angeloni *et al* (2006) provide examples of the potential benefits of microeconomic information in understanding inflation dynamics and so optimal policy. And asset price analysis could provide a means of identifying the relevant mechanisms that the existing business cycle models with nominal frictions and money should incorporate.

This paper proceeds as follows. Section 2 describes a prototype New Keynesian model giving the nonlinear environment that we generalize and linearize in subsequent sections. Section 3 focuses on alternative price setting models. We first show how two alternative rationalizations for the presence of lagged inflation in the Phillips curve diverge in their implications for the optimal steady-state inflation rate. Then we consider two real rigidities and their implications for the slope of the Phillips curve and optimal long-run inflation. Section 4 turns the analysis toward the IS equation, considering alternative rationalizations for the output-persistence parameter and studying the corresponding welfare implications. Finally, we study the IS slope parameter, showing the different welfare implications of risk-sensitive preferences compared to the standard expected-utility case. Section 5 concludes.

2 A Prototype New Keynesian Model

Here we describe the New Keynesian model that we use as a baseline to which we add variations in the remainder of the paper.

Representative household: A representative household seeks to maximize intertemporal utility $E_0 \sum_{t=0}^{\infty} \beta^t U_t$, where $U_t = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi_0 \frac{N_t^{1+\chi}}{1+\chi} + \nu_0 \frac{(M_t/P_t)^{1-\nu}}{1-\nu}$, C_t is an aggregate of the different goods consumed, N_t denotes hours worked, and $\frac{M_t}{P_t}$ is household holdings of real money balances.

All parameters are positive, and $\beta \in (0, 1)$ is the discount factor. We allow for money in the utility function so that, as in Khan, King and Wolman (2003), monetary frictions can be among the factors determining the optimal steady-state inflation rate.

Intermediate producers: A continuum of monopolistically competitive firms produces distinct intermediate goods. These goods are then combined as productive inputs to produce a single, final consumption good. The production function for an intermediate-good producing firm j is given by $Y_t(j) = A_t K_t(j)^\alpha N_t(j)^{1-\alpha}$ where A_t is a productivity shock, $K_t(j)$ and $N_t(j)$ are quantities of capital and labor services hired by firm j , and $\alpha \in (0, 1)$. Intermediate firms are assumed to set nominal prices according to the Calvo (1983) scheme. Thus, each period a measure $1 - \xi$ of firms is allowed to reset prices, while a fraction ξ must keep prices unchanged.

Market characteristics and clearing: The labor market is perfectly competitive. Capital and labor are mobile across firms, so all intermediate firms have the same real marginal cost, $MC_t = w_t N_t / (1 - \alpha) Y_t$. Here $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$ is final output produced by a single final goods producer, and $\epsilon > 1$. (We consider an alternative aggregation technology below.) The aggregate capital stock is fixed, so market clearing implies $C_t = Y_t$. Each intermediate firm faces a demand function from the final producer of $\tilde{Y}_t(j) = \tilde{P}_t(j)^{-\epsilon}$, where $\tilde{Y}_t(j) = \frac{Y_t(j)}{Y_t}$, $\tilde{P}_t(j) = \frac{P_t(j)}{P_t}$, and the aggregate price index is given by $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$.

As in Khan, King and Wolman (2003), the relative price dispersion that results from Calvo staggering can be interpreted as an inefficiency that, by misallocating resources across the intermediate goods sector compared to the flexible-price scenario, depresses the equilibrium level of aggregate output. Letting $N_t = \int_0^1 N_t(j) dj$ denote aggregate labor and normalizing aggregate capital at $\bar{K} = 1$, the same misallocation index (Δ_t) as in Khan, King, and Wolman's model is relevant, being related to output as $Y_t = \left(\frac{A_t}{\Delta_t} \right) N_t^{1-\alpha}$, while Calvo contracts imply that this distortion follows a first-order difference equation,

$$\Delta_t = (1 - \xi)(\tilde{P}_t^*)^{-\epsilon} + \xi \Pi_t^\epsilon \Delta_{t-1}. \quad (3)$$

Because all price change in any given period comes from firms given a reset signal, there is a relation

between the aggregate gross inflation rate and an index of the relative reset price:

$$\Pi_t = \left[\frac{1 - (1 - \xi)(\tilde{P}_t^*)^{1-\epsilon}}{\xi} \right]^{\frac{1}{\epsilon-1}} \quad (4)$$

A loglinear approximation of this model yields the two equations given in Section 1. Consumer behavior and market-clearing deliver an IS relation that is a special case of equation (1): one with restrictions $\alpha_b = 0$, $\alpha_f = 1$, and $\rho = \sigma^{-1}$. The supply side of this model corresponds to expression (2), where $\kappa = \kappa_p = \frac{(1-\xi)(1-\xi\beta)}{\xi}$.⁶

3 Inflation Dynamics

3.1 Inflation Persistence

3.1.1 Two Mechanisms: Indexation *vs.* Myopic Price Setters

Galí and Gertler (1999) advance a variation of Calvo contracts, proposing that a fraction of the price-resetting firms uses a backward-looking rule. Galí and Gertler assume that of those able to adjust prices in a given period, only a fraction $1 - \omega$ choose prices optimally, i.e., in terms of the stream of expected marginal costs. A fraction ω uses a simple rule, setting price equal to the average of newly-adjusted prices last period, rescaled by prior inflation. That is, $P_t^b = (P_{t-1}^b)^\omega (P_{t-1}^*)^{1-\omega} \Pi_{t-1}$, where P_t^b is the price set by backward-looking firms.

The inclusion of this backward-looking element in the otherwise forward-looking price-setting environment leads to a hybrid variant of the New Keynesian Phillips curve, represented in expression (2) above. With this specification, the parameters of (2) are given by $\kappa = \frac{(1-\xi)(1-\beta\xi)}{\xi} \frac{\xi(1-\omega)}{\xi+\omega[1-\xi(1-\beta)]}$, $\gamma_f = \beta \frac{\xi}{\xi+\omega[1-\xi(1-\beta)]}$, and $\gamma_b = \frac{\omega}{\xi+\omega[1-\xi(1-\beta)]}$.

A parallel model that leads to an alternative rationalization for the hybrid Phillips curve was proposed by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003). This model introduces dynamic indexation into Calvo contracts. Each firm i faces a constant probability, $1 - \xi$, of being able to reoptimize its price, $P_t(i)$. A firm refused permission to reset prices optimally has its price changed according to some share of lagged inflation, where the parameter v denotes the degree of indexation (i.e., $0 \leq v \leq 1$). So if firm i cannot reoptimize its price in period t ,

⁶The parameter γ is at its baseline value of 1.

it resets price according to the formula $P_t(i) = \Pi_{t-1}^v P_{t-1}(i)$ where Π_t is taken parametrically by the firm. This dynamic indexation yields the loglinear Phillips curve (2), where the parameters are now given by $\gamma_f = \frac{\beta}{1+\beta v}$, $\gamma_b = \frac{v}{1+\beta v}$, and $\kappa = \kappa_p \gamma$, and $\gamma = \frac{1}{1+\beta v}$. γ is a decreasing function of the degree of backward indexation: the greater the indexation to past inflation, the lower the response of aggregate inflation to marginal cost (for a given degree of nominal rigidity). Note that the limiting cases of $\omega \rightarrow 1$ and $v \rightarrow 1$ imply the upper bounds, $\gamma_b \rightarrow \frac{1}{1+\xi\beta}$ and $\frac{1}{1+\beta}$, respectively, and in turn imply an upper bound for γ_b of 0.5.

The foregoing analysis highlights an important difference between these two macroeconomically equivalent characterizations of the inflation process. These two mechanisms impact differently on the connection between marginal cost and inflation, i.e., on the coefficient γ . If the way of putting lagged inflation into the Phillips curve is via backward price setters, then the coefficient $\gamma = \frac{\xi(1-\omega)}{\xi+\omega[1-\xi(1-\beta)]}$ tends to zero as the fraction of backward-looking firms approaches unity. The higher the fraction of backward-looking firms, the weaker the link between inflation and the variable that matters in the forward-looking case (i.e., marginal cost and its expected future values). But if lagged inflation appears in the Phillips curve because of dynamic indexation, then $\gamma = \frac{1}{1+\beta v}$ and the lower bound for this parameter is $\frac{1}{2}$ as $v \rightarrow 1$; that is, the marginal-cost sequence always matters for inflation in the dynamic-indexation case.

We now turn to the use of the nonlinear representations of the models to analyze the implications for the long-run optimal inflation rate.⁷

3.1.2 Implications for Steady-State Inflation

These two models have different nonlinear representations for optimal price setting. In particular, the model with indexation is similar to the baseline case except that now the inflation rate has to be rescaled by the indexation clause in order to characterize the optimal price contract. The presence of myopic price setters implies that the dispersion metric depends upon the relative price at period t charged by rule-of-thumb price-setters, $X_t = \frac{P_t^b}{P_t}$, and the fraction of myopic price setters, ω , as

⁷We do not consider optimal policy in the dynamic stochastic economy; see Steinsson (2003) and Woodford (2003) for the relevant results.

Table 1: Steady-State Inflation and Inflation Persistence

Indexation Model	Myopic Price Setters
$\tilde{P}^* = \left(\frac{1-\xi\Pi^{(1-v)(\epsilon-1)}}{1-\xi}\right)^{\frac{1}{1-\epsilon}}$	$\tilde{P}^* = \left(\frac{1-\xi\Pi^{\epsilon-1}}{1-\xi}\right)^{\frac{1}{1-\epsilon}}$
$\Delta = \left(\frac{1-\xi}{1-\xi\Pi^{\epsilon(1-v)}}\right)(\tilde{P}^*)^{-\epsilon}$	$\Delta = \left(\frac{1-\xi}{1-\xi\Pi^{\epsilon}}\right)(\tilde{P}^*)^{-\epsilon}$
$MC = \left(\frac{\epsilon-1}{\epsilon}\right)\frac{1-\beta\xi\Pi^{(1-v)\epsilon}}{1-\beta\xi\Pi^{(1-v)(\epsilon-1)}}(\tilde{P}^*)$	$MC = \left(\frac{\epsilon-1}{\epsilon}\right)\frac{1-\beta\xi\Pi^{\epsilon}}{1-\beta\xi\Pi^{\epsilon-1}}(\tilde{P}^*)$
$C = \left(\frac{MC}{\chi_0\Delta^\chi}\right)^{1/(\chi+\sigma)}$	$C = \left(\frac{MC}{\chi_0\Delta^\chi}\right)^{1/(\chi+\sigma)}$

follows:

$$\Delta_t = (1 - \xi)[(1 - \omega)(\tilde{P}_t^*)^{-\epsilon} + \omega X_t^{-\epsilon}] + \xi\Pi_t^\epsilon\Delta_{t-1}, \quad (5)$$

The link between inflation and relative prices is given by:

$$1 = (1 - \xi)[(1 - \omega)(\tilde{P}_t^*)^{1-\epsilon} + \omega X_t^{1-\epsilon}] + \xi\Pi_t^{\epsilon-1}, \quad (6)$$

where

$$X_t = X_{t-1}^\omega(\tilde{P}_{t-1}^*)^{1-\omega} \quad (7)$$

Table 1 characterizes the distortion of the markup that steady-state positive inflation introduces. In the Galí-Gertler case, backward-looking reseters use a rule that is identical to the optimal reset price in its steady-state properties. This still leaves the relative price dispersion inefficiency of the standard Calvo case. In the indexation model, complete indexation ($v = 1$) removes this long-run inefficiency because price setters denied a reset signal are able to adjust to recent price movements. In this case, the only long-run distortion is that due to monopolistic competition, which makes $MC > 1$.

Parameterization. We present a quantitative illustration of the foregoing results. We use a discount factor of $\beta = 0.993$, log preferences over consumption ($\sigma = 1$), a Frisch labor elasticity of $(1/\chi) = 1$, and a production function parameter of $\alpha = 0.33$. We assume $\epsilon = 7$ —implying a 16% steady-state markup. We set γ_b equal to 0.1 and 0.4. These choices (in line with Galí, Gertler, and López-Salido, 2001) imply values for the parameters v and ω that range from 0.11 to 0.66 and 0.07

to 0.4, respectively. We calibrate the money demand parameter to $\nu = 11.4$, so as to generate an interest elasticity of money demand of the same magnitude as in Khan, King, and Wolman (2003).

Figure 1 depicts the optimal steady-state inflation rate when the Calvo parameter, ξ , takes two values: 0.6 and 0.75. With the Galí-Gertler source of inflation persistence, the optimal inflation rate is negative, slightly less than -2.5 percent, and this remains unaffected by the presence of backward price setters, since they do not alter the long-run relative price structure. An increase in the degree of price stickiness induces an increase in optimal steady-state inflation, which now moves to -0.7 percent, in line with the results of Khan, King, and Wolman (2003).

By contrast, if the source of inflation persistence is indexation, then persistence produces appreciable departures of optimal steady-state inflation from the Friedman deflation rule. As indexation increases, relative price dispersion dwindles, so the social planner tends to concentrate on imperfect-competition distortions, and so increases steady-state inflation to reduce the steady-state markup. For the parameterization where price changes once a year ($\xi = 0.75$), less than full price indexation generates an optimal positive steady-state inflation.⁸

3.2 Real Rigidities and the Slope of the NKPC

In this section we discuss two types of real rigidity whose effect is to lower firms' incentive to increase prices in the face of a rise in nominal demand (see e.g., Woodford, 2003, Ch. 3). The two real rigidities are isomorphic in what they imply for loglinear dynamics, but differ in their second-order properties and so yield different results for optimal steady-state inflation.

3.2.1 Two Mechanisms: Non-Constant Elasticity of Demand *vs.* Factor Specificity

Quasi-kinked demand curve The first real rigidity we consider is a quasi-kinked demand curve, first applied to Calvo price-setting by Kimball (1995). We follow Dotsey and King's (2005) proposal for aggregating across the intermediate goods $Y_t(j)$. Continuing to let $\tilde{Y}_t(j) = \frac{Y_t(j)}{Y_t}$ denote the share of each intermediate in final output, the Dotsey-King aggregator is:

$$G(\tilde{Y}) = \frac{\phi}{1 + \psi} \left[(1 + \psi)\tilde{Y} - \psi \right]^{\frac{1}{\phi}} - \left[\frac{\phi}{1 + \psi} - 1 \right] \quad (8)$$

⁸This result is in line with the one highlighted by Schmitt-Grohé and Uribe (2004).

Table 2: Price-Setting Behavior

Prototype Model	Quasi-Kinked Demand
$\tilde{P}_t^* = \frac{\epsilon}{\epsilon-1} \frac{Z_{2t}}{Z_{1t}}$	$\tilde{P}_t^* = \phi \frac{Z_{2t}}{Z_{1t}} + \left(\frac{\psi\phi}{\epsilon(1+\psi)} \right) \frac{Z_{3t}}{Z_{1t}} \left(\tilde{P}_t^* \right)^{1+\epsilon(1+\psi)}$
$Z_{1t} = E_t \{ \beta \xi \Pi_{t+1}^{\epsilon-1} Z_{1t+1} \} + C_t \Phi_t$	$Z_{1t} = E_t \{ \beta \xi \Pi_{t+1}^{\epsilon(1+\psi)-1} Z_{1t+1} \} + C_t \Phi_t \lambda_t^{\epsilon(1+\psi)}$
$Z_{2t} = E_t \{ \beta \xi \Pi_{t+1}^\epsilon Z_{2t+1} \} + C_t \Phi_t M C_t$	$Z_{2t} = E_t \{ \beta \xi \Pi_{t+1}^{\epsilon(1+\psi)} Z_{2t+1} \} + C_t \Phi_t \lambda_t^{\epsilon(1+\psi)} M C_t$
$\Phi_t = U_{c,t} = C_t^{-\sigma}$	$Z_{3t} = E_t \{ \beta \xi \Pi_{t+1}^{-1} Z_{3t+1} \} + C_t \Phi_t$

where $\phi = (\epsilon(1+\psi))/(\epsilon(1+\psi)-1)$, and $\epsilon > 1$ is elasticity of demand. The profit-maximizing mix of intermediates is selected, and the aggregator implies $\int_0^1 G(\tilde{Y}_t(j)) dj = 1$.

The parameter ψ governs the curvature of the demand for an intermediate firm's product. It yields the familiar Dixit-Stiglitz (1977) constant-elasticity demand function for $\psi = 0$. We consider instead the less standard case of $\psi < 0$. This implies a quasi-kinked demand curve; consumer demand essentially falls off above a certain satiation quantity, and a reduction in relative price in this region barely stimulates demand. On the other hand, demand is highly price-elastic until the effective upper bound on demand is reached. The relative demand for product j is given by:

$$\tilde{Y}_t(j) = \frac{1}{1+\psi} \left[\tilde{P}_t(j)^{-\epsilon(1+\psi)} \lambda_t^{\epsilon(1+\psi)} + \psi \right] \quad (9)$$

where again $\tilde{P}_t(j)$ is the relative price of intermediate good j . The Lagrange multiplier in (9) is defined as $\lambda_t = \left(\int_0^1 \tilde{P}_t(j)^{1-\epsilon(1+\psi)} dj \right)^{\frac{1}{1-\epsilon(1+\psi)}}$, and so collapses to unity in the Dixit-Stiglitz case of $\psi = 0$. The more general case of $\psi < 0$ implies a variable elasticity of demand for good j , denoted $\eta(\tilde{Y}_j)$, for which the expression is:

$$\eta(\tilde{Y}_j) = \epsilon \left(1 + \psi - \psi \tilde{Y}_j^{-1} \right). \quad (10)$$

so that the demand elasticity is inversely related to relative demand. Note that an intermediate-good producer's desired markup is $\mu(\tilde{Y}_j) \equiv \frac{\eta(\tilde{Y}_j)}{\eta(\tilde{Y}_j)-1}$; this yields the special case of $\mu(1) = \mu = \frac{\epsilon}{\epsilon-1}$ when $\psi = 0$, but otherwise is a function of relative demand.

Table 2 gives optimal price-setting conditions for a firm in this environment, and allows comparison with the baseline case regarding the optimal relative price \tilde{P}_t^* , and the stochastic variables Z_{1t} , Z_{2t} , and Z_{3t} .⁹ The representation of optimal pricing for firms adjusting prices, \tilde{P}_t^* , in Table 2 reflects the fact that, in order to make the marginal present discounted value of profits equal to zero, firms have to take into account changes in the elasticity of demand over those future periods in which prices are fixed (the variable λ_t).

As detailed in many papers, this model implies a loglinear Phillips curve:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \gamma \kappa_p m_{Ct}. \quad (11)$$

This is a standard New Keynesian Phillips curve (with marginal cost the driving process) other than the factorization of the Phillips curve slope into two components. One component governs nominal rigidity; the other, real rigidity (with no real rigidity corresponding to $\gamma = 1$). κ_p is a function of the frequency of price adjustment ξ and the discount factor β : $\kappa_p = \frac{(1-\xi)(1-\xi\beta)}{\xi}$. The real-rigidity parameter, γ , can be expressed as $\gamma = \frac{1}{1-\mu\psi}$, where μ is the prototype-model steady-state markup defined above. The demand-curve kink condition $\psi < 0$ implies that γ is below unity, approaching zero as ψ becomes more negative. Therefore, kinked demand for intermediate-firm output diminishes the sensitivity of inflation to marginal cost variations, and so this model feature falls within the class of strategic complementarities discussed in Woodford (2003).

Firm-specific factor inputs We now revert to the assumption of a Dixit-Stiglitz demand structure ($\psi = 0$) in order to consider a different source of real rigidity. In our prototype model, the capital stock was fixed in aggregate but not for any individual firm, which could access extra capital services via a rental market. Let us instead consider the case of a certain amount of firm-specific capital which cannot be augmented by recourse to the rental market,¹⁰ as well as firm-specific labor (reflecting firm-specific human capital). Then firm-level real marginal cost can diverge from average real marginal cost, so the ratio $\widetilde{MC}_t(j) = MC_t(j)/MC_t$ can depart from 1.0. Intermediate firm j 's

⁹See Levin, Lopez-Salido and Yun (2007a) for a derivation of the law of motion for the relative-price-dispersion metric in this model environment.

¹⁰See Sbordone (2002), Woodford (2003, 2005), and Altig, Christiano, Eichenbaum, and Linde (2005) for further discussion of firm-specific capital.

production function is now

$$Y_t(j) = A_t \bar{K}^{\alpha_{fk}} K_t(j)^{\alpha_{vk}} \bar{N}^{\alpha_{fl}} N_t(j)^{\alpha_{vl}}. \quad (12)$$

Here $\alpha_{fk} > 0$, $\alpha_{vk} > 0$, $\alpha_{fl} > 0$, $\alpha_{vl} > 0$, and $\alpha_{fk} + \alpha_{vk} + \alpha_{fl} + \alpha_{vl} = 1$. Overbars denote fixed factors. Therefore $\alpha_f = \alpha_{fl} + \alpha_{fk}$ constitutes the share of input factors (capital and labor) fixed at the firm level. Firm-specific inputs mean that equilibrium wages, and therefore marginal cost, differ across firms. Deviations of a firm's marginal cost from the aggregate in turn reflect differences in firm output from average output levels. Formally, $\widetilde{MC}_t(j) = \tilde{Y}_t(j)^{\frac{1-\alpha_f}{\alpha_f}}$. So relatively high production levels reduce a firm's marginal cost compared to the economy-wide average. A firm is deterred from raising its price by the fact that the resulting decrease in its production will trigger higher marginal cost.

As shown in Levin, López-Salido, and Yun (2007a), the profit-maximization condition in the case of firm-specific factors (which replaces the prototype-model expression in Table 2) is:

$$\left(\tilde{P}_t^*\right)^{1+\epsilon\frac{\alpha_f}{1-\alpha_f}} = \frac{\epsilon}{\epsilon-1} \frac{Z_{2t}}{Z_{1t}} \quad (13)$$

The expressions for variables Z_{1t} and Z_{2t} revert to the prototype case because in this model we are using Dixit-Stiglitz standard preferences (i.e., $\psi = 0$). But firm-specific factors raise the optimal price \tilde{P}_t^* to the power $1 + \epsilon\frac{\alpha_f}{1-\alpha_f}$. Thus, the optimal price set by adjusting firms is now a concave function of the presented discounted flow of markups. This concavity is a manifestation of strategic complementarity. With this concavity and the presence of positive inflation, firms setting prices at time t have incentives to adjust prices less than in the baseline model.

Like kinked demand, fixed factors deliver the New Keynesian Phillips curve (11). The expression for the nominal rigidity coefficient κ_p is the same as given above, but the real-rigidity coefficient becomes $\gamma = \frac{1}{1+\epsilon\frac{\alpha_f}{1-\alpha_f}}$. This expression illustrates the strategic-complementarity character of fixed inputs. The higher the share of fixed inputs in production (α_f), the lower the responsiveness of inflation to marginal cost.

3.2.2 Implications for Steady-State Inflation

Parameterization. Our criteria are to generate a Phillips curve slope ($\kappa_p\gamma$) of 0.025, in line with time series estimates of the elasticity of inflation to current real marginal cost. We fix κ_p so as to keep the implied average interval between a firm’s price adjustments to about three quarters, in line with microeconomic evidence (i.e., $\xi = 0.6$, as before). These choices imply $\gamma \cong 0.1$. The model-specific parameter is fixed at a value that delivers this γ . So $\psi = -8$ in the case of quasi-kinked demand; and $\alpha_f = 0.58$ in the case of firm-specific inputs.

Figure 2 plots the relationship between the key parameters (ψ and α_f) governing each strategic complementarity and the corresponding optimal inflation rate. These results refer to the case of only a single strategic complementarity being present at a time, so a zero value of the parameter corresponds to the no-real-rigidity case. With no real rigidity present, the Friedman rule is optimal and 2.5% deflation is desirable. But, irrespective of which strategic complementarity is considered, introducing the complementarity shifts the balance in favor of zero inflation, the more so the greater the parameterization of the rigidity.

4 Output Dynamics

4.1 Output Persistence

In this section we turn to the analysis of how alternative assumptions about preferences, of the type that have been used to understand output persistence, can lead to different implications for optimal policy, even though they exhibit a kind of macroeconometric equivalence.

Specifically, by introducing time dependence in preferences, several authors have claimed that one can improve the match of the model to the hump-shaped and persistent responses of output to nominal and real shocks found empirically. Two different types of time-dependence—internal and external habit formation—exhibit similarities in loglinear dynamics and empirical fit at the aggregate level, but are associated with different optimal monetary policies. In DSGE models used for monetary policy analysis, one can find both types of habit formation. Thus, Smets and Wouters (2003) use external habits, while Fuhrer (2000), Amato and Laubach (2004), and

Christiano, Eichenbaum, and Evans (2005) favor an internal-habit specification.¹¹

The welfare implication is that external habits (“catching up with the Joneses”) puts an externality into consumption patterns (Ljungqvist and Uhlig, 2000). A typical household does not internalize the fact that its own increase in consumption will trigger higher consumption demand by other households. This externality will, however, be taken into account by the social planner and so will have important implications for optimal steady-state inflation.

4.1.1 Two Mechanisms: Internal *vs.* External Habits

We encompass internal and external habit specifications by generalizing utility to:

$$U_t = \frac{\tilde{C}_t^{1-\sigma} - 1}{1-\sigma} - \chi_0 \frac{N_t^{1+\chi}}{1+\chi} + \nu_0 \frac{(\frac{M_t}{P_t})^{1-\nu}}{1-\nu} \quad (14)$$

where $\tilde{C}_t = C_t - \phi \bar{C}_{t-1}$, $0 \leq \phi < 1$. With internal habits, the household considers its own past consumption in deciding how much to consume today; so in this case $\bar{C}_{t-1} = C_{t-1}$. This specification corresponds to the one originally introduced by Constantinides (1990) and the setting of $\sigma = 1$ was used by Christiano, Eichenbaum, and Evans (2005). With external habits, \bar{C}_{t-1} corresponds to the prior aggregate consumption, taken parametrically by the representative household.

Other than their effect on consumption decisions, habits impact the aggregate supply block of the model; see e.g. Amato and Laubach (2004) and Dennis (2005). We concentrate here on implications for the IS equation. Define an indicator function so that $\theta = 1$ implies internal habits, and $\theta = 0$ external habit. Then, with $Y_t = C_t$, the loglinear IS curve is:

$$x_t - (1 + \theta\beta\phi)E_t x_{t+1} + \theta\beta\phi E_t x_{t+2} = -\frac{1-\phi}{\tilde{\sigma}} [r_t - E_t \pi_{t+1}] \quad (15)$$

where $x_t = (y_t - \phi y_{t-1})$ represents quasi-differenced (log) output, and we define the parameter $\tilde{\sigma} = \sigma(1 - \theta\beta\phi)^{-1}$. In the case of external habits ($\theta = 0$), the preceding expression collapses to a second-order expectational difference equation for output, corresponding to the IS expression (1) discussed in the introduction (with parameters $\alpha_b = \frac{\phi}{1+\phi}$, $\alpha_f = \frac{1}{1+\phi}$, and $\rho = \frac{1-\phi}{\sigma}$).

In the case of internal habits ($\theta = 1$), expression (15) can be written as a third-order expectational difference equation in c_t (being a second-order equation in x_t). This involves a term

¹¹Amato and Laubach (2004) and Fuhrer (2000) used a ratio representation—like Abel (1990)—while Christiano, Eichenbaum, and Evans (2005) use an additive specification—as in Constantinides (1990).

affecting the expected, at time t , value of consumption two periods ahead, i.e. $E_t c_{t+2}$ (equivalently, $E_t x_{t+2}$). This implies that the appropriate IS curve specification depends on how habit formation is modeled, with both variants leading to the presence of a c_{t-1} term, but the external-habits parameterization embodying an exclusion restriction on $E_t c_{t+2}$.

Put differently, neither specification introduces new state variables relative to one another, but each implies different restrictions on (values for) implied solution coefficients on the state variables. But as found by Dennis (2005) from an aggregate empirical perspective, it is extremely difficult in practice to distinguish between internal and external habits. The aggregate fit of the two specifications seems to be too close to deliver a clear-cut superiority of either one. In that sense, they are nearly macroeconometrically equivalent.

4.1.2 Implications for Steady-State Inflation

The case of time-dependent preferences, either from internal or external habits, does not change the impact of inflation on the average markup and on relative price dispersion. These two distortions only depend upon the real interest rate, the probability of changing prices, and the elasticity of demand. This implies that the expressions for \tilde{P}^* , Δ , and MC remain those displayed in the second column of Table 1.

Habits matter for the aggregate level of consumption and therefore welfare. In particular, with nonzero steady-state inflation, the social-planning consumption level is given by:

$$C = \left[\frac{MC(1 - \theta\phi\beta)}{\chi_0 \Delta^x (1 - \phi)^\sigma} \right]^{\frac{1}{\sigma+x}} \quad (16)$$

The preceding expression can be decomposed as:

$$C = \left[\frac{MC}{\chi_0 \Delta^x} \right]^{\frac{1}{\sigma+x}} \left[\frac{1 - \theta\phi\beta}{(1 - \phi)^\sigma} \right]^{\frac{1}{\sigma+x}} = C^{nh} \Delta^h$$

where C^{nh} corresponds to the consumption level implied by the baseline model—see e.g. Table 1—and Δ^h is an extra term implied by time-dependent preferences. In the case of internal habits, we have $\Delta^h = \left[\frac{1 - \phi\beta}{(1 - \phi)^\sigma} \right]^{\frac{1}{\sigma+x}}$ which, as $\beta \rightarrow 1$, can be written as $\Delta^h = (1 - \phi)^{\frac{1 - \sigma}{\sigma+x}}$. External habits produce a welfare externality of a higher level of steady-state consumption: i.e., $\Delta^h =$

$(1 - \phi)^{-\frac{\sigma}{\sigma+\chi}} > 1$. For a given labor supply elasticity, χ , this overconsumption depends positively on the habit parameter, ϕ , as well as the degree of risk aversion, σ .

In Figure 3 we display optimal steady-state inflation with internal or external habits, for different values of the habit persistence parameter ϕ . Allowing for internal habits changes the nature of the stochastic discount factor but does not introduce any new distortion to the social planning problem. Therefore, optimal steady-state inflation depends only on the monetary and price frictions. As can be seen from Figure 3, external habits substantially alter steady-state optimal policy. With strong external habit formation, optimal steady-state inflation becomes close to zero and even slightly positive, notwithstanding monetary frictions and low price stickiness, reflecting the planner’s desire to hold down the tendency to excessive consumption and output levels.¹²

4.2 Risk-Sensitive Preferences

The Euler equation for consumption is the basis for the IS curve in New-Keynesian models. But, as Sargent (2007, p. 50) observes, “A long list of empirical failures called puzzles come from applying... that Euler equation. Until we succeed in getting a consumption-based asset pricing model that works well, the New Keynesian IS curve is built on sand.” That is, this IS function cannot account for important financial market regularities. The standard case (hereafter called “expected utility”) cannot explain the large premium priced into risky assets and cannot explain the high volatility of returns on long-term assets.

Recognizing the vulnerability of the expected utility specification, in this section we examine the implications for monetary policy of Epstein-Zin (1989) (risk-sensitive) preferences; in so doing, we demonstrate another case of macroeconomic equivalence and microeconomic dissonance. To our knowledge, this section provides the first attempt to integrate the Epstein-Zin framework into an otherwise standard sticky-price New Keynesian setup.

The notable feature of Epstein-Zin preferences that they admit a distinction between the coefficient of relative risk aversion and the intertemporal elasticity of substitution in consumption.

¹²Chugh (2004) discusses the possibility of the suboptimality of Friedman deflation in the presence of catching-up-with-the-Joneses preferences. But he also reports that the optimality of the Friedman deflation rule prevails in the presence of internal habit formation.

They therefore offer the attraction of being able to match both securities market facts—i.e., low risk-free real interest rates—and equity market facts—i.e., the equity premium puzzle (see e.g. Tallarini, 2000; Brevik, 2005).

Households Bringing real balances into the single period utility function, we use the specification of preferences in Tallarini (2000), defining the representative household’s preferences *recursively* as:

$$U_t = V_t + \frac{\beta}{\sigma} \log(E_t[\exp(\sigma U_{t+1})]), V_t = \log C_t + \varphi_0 \log(1 - N_t) + \nu_0 \log\left(\frac{M_t}{P_t}\right), \quad (17)$$

where $\sigma = \frac{(1-\beta)(1-\varphi)}{1+\varphi_0}$, while φ captures relative risk aversion attitudes. This specification assumes that the elasticity of intertemporal substitution is 1, while φ governs preference for resolution of uncertainty. In the limit of $\varphi \rightarrow 1$, preferences coincide with the expected utility case. When $\varphi > 1$, the household is more risk averse relative to the expected utility case (i.e., the concavity of $\exp(\sigma U_{t+1})$ increases risk aversion without affecting intertemporal substitution). In this case, agents prefer early resolution of uncertainty, and thus will require compensation for long-run risk. The opposite holds for $\varphi < 1$.

We follow Uhlig (2006) in characterizing the optimal plan for the representative household. The plan consists of maximization from period 0 of intertemporal utility, subject to the evolution of preferences, expression (17), and the flow-budget constraint for period t :

$$C_t + E_t\left[Q_{t,t+1} \frac{B_{t+1}}{P_{t+1}}\right] + M_{t+1} = \frac{B_t + M_t}{P_t} + \frac{W_t}{P_t} N_t + D_t - T_t, \quad (18)$$

where B_{t+1} denotes a portfolio of nominal state contingent claims in the complete contingent claims market, $Q_{t,t+1}$ is the stochastic discount factor for computing the real value at period t of one unit of consumption at period $t + 1$, W_t is the nominal wage rate, T_t is the real lump-sum tax, and D_t is real dividend income. The first-order conditions with respect to U_t , C_t , and B_t lead to:

$$\Omega_0 = 1 \quad (19)$$

$$\Omega_t = \Omega_{t-1} X_{U_t}, \text{ for } t \geq 1 \quad (20)$$

$$\Lambda_t = \Omega_t \Phi_t \quad (21)$$

$$E_t[Q_{t,t+1} R_t \frac{P_t}{P_{t+1}}] = E_t[\beta \frac{\Lambda_{t+1}}{\Lambda_t} R_t \frac{P_t}{P_{t+1}}] = 1, \quad (22)$$

where $\Phi_t = \frac{\partial V_t}{\partial C_t}$, and we have defined X_{Ut} as the ratio:

$$X_{Ut} = \frac{\exp(\sigma U_t)}{E_{t-1}[\exp(\sigma U_t)]}, \quad (23)$$

where Ω_t and Λ_t are Lagrange multipliers on constraints (17) and (18), respectively. Expression (22) gives the optimality condition for bond holdings, where the stochastic discount factor is $Q_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}$. With R_t the (gross) risk-free nominal short-term rate of interest, combining (20)-(22) yields:

$$E_t \left\{ \beta \frac{\Omega_{t+1}}{\Omega_t} \frac{\Phi_{t+1}}{\Phi_t} R_t \frac{P_t}{P_{t+1}} \right\} = 1 \quad (24)$$

which generalizes the standard expected-utility version of the Euler equation. The Lagrange multiplier Ω_t incorporates the impact of risk aversion on the marginal intertemporal rate of substitution of consumption. To see this more clearly, note that with specification (17) the consumption optimality condition (24) can be written as:

$$\beta E_t \left\{ \left[\frac{\exp(\sigma U_{t+1})}{E_t[\exp(\sigma U_{t+1})]} \right] \left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{R_t}{\Pi_{t+1}} \right\} = 1. \quad (25)$$

The model also implies standard equations for labor supply and money demand,

$$\varphi_0 \frac{(1 - N_t)^{-1}}{\Phi_t} = \frac{W_t}{P_t} \quad (26)$$

$$\nu_0 \left(\frac{M_t}{P_t} \right)^{-1} = (1 - R_t^{-1}) \Phi_t. \quad (27)$$

Intermediate Firms As in the prototypical model, optimal price contracts, \tilde{P}_t^* , can be written as $\tilde{P}_t^* = \frac{\epsilon}{\epsilon-1} \frac{\tilde{Z}_{2t}}{\tilde{Z}_{1t}}$, where the expressions for \tilde{Z}_{1t} and \tilde{Z}_{2t} are given by $\tilde{Z}_{1t} = \beta \xi E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t+1}^{\epsilon-1} \tilde{Z}_{1t+1} \right\} + C_t$, and $\tilde{Z}_{2t} = \beta \xi E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_{t+1}^\epsilon \tilde{Z}_{2t+1} \right\} + C_t M C_t$. Using expressions (20), (21), and (23), and proceeding as before, it is possible to rewrite the preceding expressions for \tilde{Z}_{1t} and \tilde{Z}_{2t} as: $Z_{1t} = \beta \xi E_t \left\{ X_{Ut+1} \Pi_{t+1}^{\epsilon-1} Z_{1t+1} \right\} + C_t \Phi_t$ and $Z_{2t} = \beta \xi E_t \left\{ X_{Ut+1} \Pi_{t+1}^\epsilon Z_{2t+1} \right\} + C_t M C_t \Phi_t$; where $Z_{1t} = \Phi_t \tilde{Z}_{1t}$, and $Z_{2t} = \Phi_t \tilde{Z}_{2t}$.

From these expressions, it follows that Epstein-Zin preferences alter price setting in the economy by changing the stochastic discount factor; this now incorporates the extra compensation for long-run cash-flow risks associated with the limited scope to change prices. This is captured by the term $X_{U_{t+1}} = \frac{\exp(\sigma U_{t+1})}{E_t[\exp(\sigma U_{t+1})]}$.

Loglinear Approximation Under non-expected utility, a loglinear approximation around the steady state delivers the prototypical New Keynesian equations. To see this, note that profit maximization leads to the following loglinear conditions: $\hat{z}_{1t} = \xi\beta E_t[(\epsilon - 1)\hat{\Pi}_{t+1} + \hat{z}_{1t+1}]$; $\hat{z}_{2t} = \xi\beta E_t[\epsilon\hat{\Pi}_{t+1} + \hat{z}_{2t+1}] + (1 - \beta\xi)\hat{M}C_t$; and $\hat{P}_t^* = \hat{z}_{2t} - \hat{z}_{1t} = \frac{\xi}{1-\xi}\hat{\Pi}_t$. As a result, we arrive at a standard New Keynesian Phillips curve:

$$\pi_t = \kappa(\hat{C}_t - \hat{A}_t) + \beta E_t[\pi_{t+1}]$$

where $\kappa = \frac{(1-\xi)(1-\xi\beta)}{\xi}(\frac{\chi_0 + MC}{\chi_0})$. With $Y_t = C_t$, the loglinear IS equation is given by:

$$\hat{y}_t = E_t[\hat{y}_{t+1}] - (\hat{R}_t - E_t[\hat{\pi}_{t+1}])$$

which is the same as that arising from the logarithmic-preferences, expected-utility case (i.e., it corresponds to expression (1) in Section 1 with $\rho = 1$ and $\alpha_b = 0$).

4.2.1 Optimal Policy

In this section we characterize optimal monetary policy under commitment. Before characterizing the problem, it is convenient to rewrite the relevant restrictions into a more parsimonious form. First, we assume that the production function is linear in hours, which implies that total hours, N_t , can be written as $N_t = \frac{\Delta_t C_t}{A_t}$, where we have used the aggregate resource constraint $Y_t = C_t$. Second, using expression (27), we can write real balances as $\frac{M_t}{P_t} = (\frac{\nu_0 C_t}{1 - R_{i,t}})$, where $R_{i,t} \equiv R_t^{-1}$ is the inverse of the (gross) nominal interest rate. Third, real marginal cost is given by $MC_t = MC_t(i) = \frac{W_t}{A_t P_t}$ and, using labor supply expression (26), it follows that $MC_t = \frac{\varphi_0 C_t}{(A_t - \Delta_t C_t)}$.

Using the previous simplifications, the constraints on the social planner are:

$$U_t = (1 + \nu_0) \log C_t + \varphi_0 \log(1 - \Delta_t C_t / A_t) + \nu_0 \log\left(\frac{\nu_0}{1 - R_{i,t}}\right) + \frac{\beta}{\sigma} \log(E_t[\exp(\sigma U_{t+1})]), \quad (28)$$

$$\beta E_t \left[\frac{\exp(\sigma U_{t+1})}{E_t[\exp(\sigma U_{t+1})]} C_{t+1}^{-1} \Pi_{t+1}^{-1} \right] = R_{i,t} C_t^{-1}, \quad (29)$$

$$Z_{1t} \tilde{P}_t^* = \frac{\epsilon}{\epsilon - 1} Z_{2t}, \quad (30)$$

$$Z_{1t} = \beta \xi E_t \left[\frac{\exp(\sigma U_{t+1})}{E_t[\exp(\sigma U_{t+1})]} \Pi_{t+1}^{\epsilon-1} Z_{1t+1} \right] + 1, \quad (31)$$

$$Z_{2t} = \beta \xi E_t \left[\frac{\exp(\sigma U_{t+1})}{E_t[\exp(\sigma U_{t+1})]} \Pi_{t+1}^\epsilon Z_{2t+1} \right] + \frac{\varphi_0 C_t}{A_t - \Delta_t C_t}, \quad (32)$$

as well as (3) and (4).

Therefore, the Ramsey problem, along the lines of Uhlig (2006), is characterized by the state-contingent maximization at period 0 by a benevolent social planner of intertemporal household utility subject to (28)-(32), the evolution of the relative price distortion (3), and the inflation equation (4). We give below the first-order conditions of the social planner's problem with respect to U_t , $R_{i,t}$, C_t , Z_{1t} , Z_{2t} , \tilde{P}_t^* , Π_t , and Δ_t :

$$\Omega_0 = 1 \quad (33)$$

$$\Omega_t = \Omega_{t-1} X_{Ut}, \quad \text{for } t \geq 1, \quad (34)$$

$$\Lambda_{It} C_t^{-1} = \left(\frac{\nu_0}{1 - R_{i,t}} \right) \Omega_t, \quad (35)$$

$$C_t^{-2} (\Lambda_{It} R_{i,t} - \Lambda_{It-1} \Pi_t X_{Ut}) = \frac{\Omega_t}{C_t} \left[\frac{\varphi_0 C_t \Delta_t}{A_t - \Delta_t C_t} - (1 + \nu_0) \right] + \frac{\varphi_0 A_t \Lambda_{F_3 t}}{(A_t - \Delta_t C_t)^2}, \quad (36)$$

$$\Lambda_{F_1 t} (\tilde{P}_t^*) + \Lambda_{F_2 t} - \xi \Lambda_{F_2 t-1} \Pi_t^{\epsilon-1} X_{Ut} = 0, \quad (37)$$

$$\frac{\epsilon}{\epsilon - 1} \Lambda_{F_1 t} = \Lambda_{F_3 t} - \xi \Lambda_{F_3 t-1} \Pi_t^\epsilon X_{Ut}, \quad (38)$$

$$\Lambda_{R_1 t} + \left(\frac{\epsilon (P_t^*)^{-1}}{\epsilon - 1} \right) \Lambda_{R_2 t} + \left(\frac{(P_t^*)^\epsilon Z_{1t}}{(1 - \xi)(\epsilon - 1)} \right) \Lambda_{F_1 t} = 0, \quad (39)$$

$$\frac{\Lambda_{It-1}}{C_t} X_{Ut} + \xi(\epsilon - 1) \Pi_t^\epsilon (\Lambda_{F_2 t-1} Z_{1t} X_{Ut} + \Lambda_{R_1 t}) + \epsilon \xi \Pi_t^{\epsilon+1} (\Lambda_{F_3 t-1} Z_{2t} X_{Ut} + \Lambda_{R_2 t} \Delta_{t-1}) = 0, \quad (40)$$

$$\Lambda_{R_2 t} - \beta \xi E_t [\Lambda_{R_2 t+1} \Pi_{t+1}^\epsilon] = \frac{\varphi_0 C_t}{A_t - \Delta_t C_t} \left(\Omega_t + \frac{C_t \Lambda_{F_3 t}}{A_t - \Delta_t C_t} \right), \quad (41)$$

where Ω_t , Λ_{It} , $\Lambda_{F_1 t}$, $\Lambda_{F_2 t}$, $\Lambda_{F_3 t}$, $\Lambda_{R_1 t}$, and $\Lambda_{R_2 t}$ represent the Lagrange multipliers associated with constraints (28)-(32), plus (3) and (4), respectively. Under Ramsey policy, the initial value of the Lagrange multipliers Λ_{I-1} , Λ_{F_2-1} , and Λ_{F_3-1} are set to zero.¹³

¹³Levin, López-Salido and Yun (2007b) show that optimal monetary policies under Epstein-Zin and expected utility

Irrelevance of Recursive Preferences for Steady-State Optimal Inflation Rate

It is straightforward to show that the optimal steady-state inflation rate is the same in models with non-expected and expected utility. Specifically, we assume that $A_t = 1$ for $t = 0, 1, \dots, \infty$. With no shocks in the economy, we have $X_{Ut} = 1$ for $t = 0, 1, \dots, \infty$. This in turn implies that $\Omega_t = 1$ for $t = 0, 1, \dots, \infty$. In this case, the optimality conditions specified above turn out to be identical to those obtained in a model with the corresponding expected-utility preferences.

4.2.2 Optimal Policy in the Stochastic Economy

First-Order Approximation to Optimal Policy Differences with the expected-utility case quickly emerge when we consider loglinear dynamics under optimal policy. That is, while the structural IS equation is equivalent to that under expected utility, the underlying welfare function is not, and so neither are the policymaker first-order conditions that help determine aggregate dynamics.

We loglinearize the optimality conditions of the social planner's problem, assuming that the steady state is distorted by the presence of monopolistic competition, and therefore does not correspond to the efficient allocation.¹⁴ Here, we abstract from the presence of voluntary fiat-money holdings in order to simplify the analysis of the first-order dynamics of the optimal policy.¹⁵ It is worth noting that under this assumption, the Lagrange multiplier for the social planner's optimization turns out to be constant in the case of expected utility. Thus, the difference between expected utility and Epstein-Zin preferences is that $\hat{\Omega}_t = 0$ for $t = 1, \dots, \infty$.

We use several parameter choices of Tallarini (2000), who simulated his model under a risk-aversion parameter set, φ : (1, 10, 25, 100).¹⁶ We set $\beta = 0.9925$, $\varphi = 10$, and $\varphi_0 = 2.97$, as a benchmark parameterization. In addition, the logarithm of aggregate labor productivity follows

preferences are identical in the presence of a fiscal policy that offsets the monopolistic distortion. But this does not mean that the implementation of the optimal allocation in the two economies is identical. In the stochastic economy with an initial relative price distortion, the optimal transition path of the short term nominal interest rate will differ across the two models.

¹⁴See the Appendix for details.

¹⁵This implies that the optimal steady-state inflation rate is zero.

¹⁶Tallarini (2000, Table 5) simulates with two sets of (β, φ_0) : $(\beta = 0.9926, \varphi_0 = 2.9869)$ and $(\beta = 0.9995, \varphi_0 = 3.3050)$.

$\log A_t = 0.95 \log A_{t-1} + \epsilon_t$, where ϵ_t is i.i.d. white noise.

As Tallarini (2000) shows, the coefficient of relative risk aversion in this specification of household preferences is $(\varphi + \varphi_0)/(1 + \varphi_0)$. This means that the coefficient of relative risk-aversion is 1 with expected utility and 3.26 with Epstein-Zin preferences, so the latter implies higher relative risk aversion.

Figure 4 compares optimal policy under Epstein-Zin preferences with that under expected utility in terms of dynamic responses of output and inflation to an exogenous increase in labor productivity. These responses are more volatile with expected utility. The social planner is more risk averse when there are Epstein-Zin preferences than when there is the corresponding expected-utility specification, and so permits fewer fluctuations in output.

We have shown that, up to a first-order approximation, the optimal monetary policy response to transitory technology shocks is affected by the presence of risk-sensitive preferences. But if we had allowed for an employment subsidy that undid the steady-state monopoly distortion, then there would be no differences, up to first order, in optimal monetary policy.

5 Conclusions

In this paper we have shown the consequences for optimal steady-state inflation of models which exhibit macroeconomic equivalence and microeconomic dissonance. We presented alternative versions of the standard New Keynesian model that are isomorphic in their implied linearized macroeconomic dynamics, but whose underlying microeconomic differences return to the surface when optimal policy is analyzed in a fully nonlinear setting. The mechanisms we contemplated were alternative sources of inflation and output persistence. We also considered the implications for optimal monetary policy of some more novel and relatively unexplored mechanisms involving real rigidities and risk-sensitive preferences.

References

- Abel, A.B. (1990). "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review (Papers and Proceedings)* 80, 38-42.
- Altig, D., L.J. Christiano, M. Eichenbaum, and J. Linde (2005). "Firm-Specific Capital, Nominal Rigidities and the Business Cycle." NBER Working Paper No. 11034.
- Amato, J.D., and T. Laubach (2004). "Implications of Habit Formation for Optimal Monetary Policy," *Journal of Monetary Economics* 51, 305-325.
- Angeloni, I., L. Aucremanne, M. Ehrmann, J. Galí, A.T. Levin, and F. Smets (2006). "New Evidence on Inflation Persistence and Price Stickiness in the Euro Area: Implications for Macro Modeling," *Journal of the European Economic Association* 4, 562-574.
- Ball, L.M., and D.H. Romer (1990). "Real Rigidities and the Non-Neutrality of Money," *Review of Economic Studies* 57, 183-203.
- Barillas, F., L.P. Hansen, and T.J. Sargent (2007). "Doubts or Variability?" Manuscript, New York University.
- Bils, M., and P.J. Klenow (2004). "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112(5), 947-985.
- Brevik, F. (2005). "Asset Pricing and Macroeconomic Risk." Manuscript, New York University.
- Browning, M., L.P. Hansen, and J. Heckman (1999). "Micro Data and General Equilibrium Models." In J.B. Taylor and M. Woodford (eds.), *Handbook of Macroeconomics, Vol. 1A*. Amsterdam: Elsevier. 543-633.
- Calvo, G.A. (1983). "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12, 383-398.
- Christiano, L.J., M. Eichenbaum, and C. Evans (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113, 1-45.
- Chugh, S.K. (2004). "Does Monetary Policy Keep Up with the Joneses? Optimal Interest-Rate Smoothing with Consumption Externalities." Federal Reserve Board FEDS Paper No. 812.

Clarida, R., J. Galí, and M. Gertler (1999). “The Science of Monetary Policy: A New Keynesian Perspective,” *Journal of Economic Literature* 37, 1661-1707.

Constantinides, G.M. (1990). “Habit Formation: A Resolution of the Equity Premium Puzzle,” *Journal of Political Economy* 98, 519-543.

Dennis, R. (2005). “Specifying and Estimating New Keynesian Models with Instrument Rules and Optimal Monetary Policies.” Manuscript, Federal Reserve Bank of San Francisco.

Dixit, A. K., and J. Stiglitz (1977). “Monopolistic Competition and Optimum Product Diversity,” *American Economic Review* 67, 297-308.

Dotsey, M., and R.G. King (2005). “Implications of State Dependent Pricing for Dynamic Macroeconomic Models,” *Journal of Monetary Economics* 52, 213-242.

Epstein, L.G., and S.E. Zin (1989). “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework,” *Econometrica* 57, 937-969.

Fuhrer, J.C. (2000). “Habit Formation in Consumption and Its Implications for Monetary-Policy Models,” *American Economic Review*, 90, 367-390.

Galí, J., and M. Gertler (1999). “Inflation Dynamics: A Structural Econometric Analysis,” *Journal of Monetary Economics* 46, 195-222.

Galí, J., M. Gertler, and J.D. López-Salido (2001). “European Inflation Dynamics,” *European Economic Review* 45, 1237-1270.

Ireland, P.N. (2001). “Sticky-Price Models of the Business Cycle: Specification and Stability,” *Journal of Monetary Economics* 47, 3-18.

Khan, A., R.G. King, and A.L. Wolman (2003). “Optimal Monetary Policy,” *Review of Economic Studies* 60, 825-860.

Kimball, M. S. (1995). “The Quantitative Analytics of the Basic Neomonetarist Model,” *Journal of Money, Credit, and Banking* 27, 1241-1277.

King, R.G. (2000). “The New IS-LM Model: Language, Logic and Limits,” *Federal Reserve Bank of Richmond Economic Quarterly* 86, 45-103.

King, R.G., and A.L. Wolman (1996). "Inflation Targeting in a St. Louis Model of the 21st Century," *Federal Reserve Bank of St. Louis Review* 78, 83-107.

Klenow, P., and J. Willis (2007). "Sticky Information and Sticky Prices," *Journal of Monetary Economics* 54, 79-99.

Kydland, F.E., and E.C. Prescott (1982). "Time to Build and Aggregate Fluctuations," *Econometrica* 50, 1345-1370.

Levin, A.T., J.D. López-Salido, and T. Yun (2007a). "Strategic Complementarities and Optimal Monetary Policy." CEPR Discussion Paper No. 6423.

Levin, A.T., J.D. López-Salido, and T. Yun (2007b). "Risk-Sensitive Monetary Policy." Manuscript, Federal Reserve Board, September.

Levin, A.T., A. Onatski, J.C. Williams, and N. Williams (2005). "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models," *NBER Macroeconomic Annual* 20, 229-287.

Ljungqvist, L., and H. Uhlig (2000). "Tax Policy and Aggregate Demand Management under Catching up with the Joneses," *American Economic Review* 90, 356-366.

Nakamura, E., and J. Steinsson (2007). "Five Facts About Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics*, forthcoming.

Roberts, J. (1995). "New Keynesian Economics and the Phillips Curve," *Journal of Money, Credit, and Banking* 27, 975-984.

Rotemberg, J.J. (1987). "The New Keynesian Microfoundations," *NBER Macroeconomics Annual* 2, 69-104.

Rotemberg, J.J., and M. Woodford (1997). "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," *NBER Macroeconomics Annual* 12, 297-346.

Sargent, T.J. (1976). "The Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics," *Journal of Political Economy* 84, 631-640.

Sargent, T.J. (2007). "Commentary," *Federal Reserve Bank of St. Louis Review* 89, 301-304.

- Sbordone, A.M. (2002). "Prices and Unit Labor Costs: A New Test of Price Stickiness," *Journal of Monetary Economics* 49, 265-292.
- Schmitt-Grohé, S., and M. Uribe (2004). "Optimal Fiscal and Monetary Policy Under Imperfect Competition," *Journal of Macroeconomics*, 26, 183-209.
- Sims, C.A. (1998). "Stickiness," *Carnegie-Rochester Conference Series on Public Policy* 49, 317-356.
- Smets, F., and R. Wouters (2003). "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association* 1, 1123-1175.
- Steinsson, J. (2003). "Optimal Monetary Policy in an Economy with Inflation Persistence," *Journal of Monetary Economics* 50, 1425-1456.
- Tallarini, T. (2000). "Risk-Sensitive Real Business Cycles" *Journal of Monetary Economics* 45, 507-532.
- Taylor, J.B. (1979). "Staggered Wage Setting in a Macro Model," *American Economic Review (Papers and Proceedings)* 69, 108-113.
- Taylor, J.B. (1980). "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy* 88, 1-23.
- Taylor, J.B. (1992). "The Great Inflation, the Great Disinflation, and Policies for Future Price Stability." In A. Blundell-Wignall (ed.), *Inflation, Disinflation and Monetary Policy*. Sydney: Ambassador Press. 9-31.
- Uhlig, H. (2006). "Asset Pricing with Epstein-Zin Preferences." Manuscript, Humboldt University.
- Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.
- Woodford, M. (2005). "Firm-Specific Capital and the New-Keynesian Phillips Curve," *International Journal of Central Banking* 1, 1-46.
- Yun, T. (1996). "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics* 37, 345-370.

Appendix: Characterization of Ramsey Optimal Policy Problem under Alternative Underlying Preferences and Price Settings

1. Indexation and Myopic Price-Setters

1.1. Indexation

Intermediate firms not allowed to change prices here update their prices based on lagged inflation. We let ν denote the degree of indexation, expressed as a fraction. The Ramsey optimal policy problem is the state-contingent maximization from period 0 by a benevolent social planner with the following objective function:

$$\sum_{t=0}^{\infty} E_t \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi_0 \frac{(\Delta_t C_t)^{1+\chi}}{1+\chi} + \nu_0^{1/\nu} \frac{C_t^{\frac{\sigma(1-\nu)}{\nu}} (1 - R_{i,t})^{-\frac{1-\nu}{\nu}} - 1}{1-\nu} \right]. \quad (42)$$

The constraints of the social planner's problem are

$$\beta E_t [C_{t+1}^{-\sigma} \Pi_{t+1}^{-1}] = R_{i,t} C_t^{-\sigma}, \quad (43)$$

$$Z_{1t} \tilde{P}_t^* = \frac{\epsilon}{\epsilon - 1} Z_{2t}, \quad (44)$$

$$Z_{1t} = \beta \xi E_t \left[\left(\frac{\Pi_{t+1}}{\Pi_t^\nu} \right)^{\epsilon-1} Z_{1t+1} \right] + C_t^{1-\sigma}, \quad (45)$$

$$Z_{2t} = \beta \xi E_t \left[\left(\frac{\Pi_{t+1}}{\Pi_t^\nu} \right)^\epsilon Z_{2t+1} \right] + \varphi_0 C_t^{1+\chi} \Delta_t^\chi, \quad (46)$$

$$1 = (1 - \xi) (\tilde{P}_t^*)^{1-\epsilon} + \xi \left(\frac{\Pi_t}{\Pi_{t-1}^\nu} \right)^{\epsilon-1}, \quad (47)$$

$$\Delta_t = (1 - \xi) (\tilde{P}_t^*)^{-\epsilon} + \xi \left(\frac{\Pi_t}{\Pi_{t-1}^\nu} \right)^\epsilon \Delta_{t-1}. \quad (48)$$

From the previous constraints it is straightforward to obtain the expressions presented in Table 1 of the text that characterize the steady state of the model under no monetary distortions.

First-Order Conditions for the Optimal Policy Problem

Given the characterization of the optimal policy problem specified above, we now present its first-order conditions. The first-order conditions of the social planner's problem with respect to $R_{i,t}$, C_t , Z_{1t} , Z_{2t} , \tilde{P}_t^* , Π_t , and Δ_t can be written as follows.

$$\Lambda_{It} = \left(\frac{\nu_0^\nu}{\nu} \right) C_t^{\frac{\sigma(1-\nu)}{\nu}} (1 - R_{i,t})^{\frac{-1}{\nu}}, \quad (49)$$

$$1 - J_t + \left(\frac{\sigma\nu_0^{\frac{1}{\nu}}}{\nu}\right) \frac{C_t^{\sigma-1}}{(1 - R_{i,t})^{\frac{1-\nu}{\nu}}} + \frac{\sigma(\Lambda_{It}R_{i,t} - \frac{\Lambda_{It-1}}{\Pi_t})}{C_t} + (\sigma - 1)\Lambda_{F_2t} = 0, \quad (50)$$

$$\Lambda_{F_1t}(\tilde{P}_t^*) + \Lambda_{F_2t} - \xi\Lambda_{F_2t-1}\left(\frac{\Pi_t}{\Pi_{t-1}^{\nu}}\right)^{\epsilon-1} = 0, \quad (51)$$

$$\frac{\epsilon}{\epsilon - 1}\Lambda_{F_1t} = \Lambda_{F_3t} - \xi\Lambda_{F_3t-1}\left(\frac{\Pi_t}{\Pi_{t-1}^{\nu}}\right)^{\epsilon}, \quad (52)$$

$$(1 - \xi)[(\epsilon - 1)\Lambda_{R_1t} + \epsilon(P_t^*)^{-1}\Lambda_{R_2t}] + (P_t^*)^{\epsilon}Z_{1t}\Lambda_{F_1t} = 0, \quad (53)$$

$$\frac{\Lambda_{It-1}}{C_t^{\sigma}\Pi_t} + \xi(\epsilon - 1)\left(\frac{\Pi_t}{\Pi_{t-1}^{\nu}}\right)^{\epsilon-1}S_{1t} + \epsilon\xi\left(\frac{\Pi_t}{\Pi_{t-1}^{\nu}}\right)^{\epsilon}S_{2t} = S_{3t} + S_{4t}, \quad (54)$$

$$\Delta_t(\Lambda_{R_2t} - \beta\xi E_t[\Lambda_{R_2t+1}\left(\frac{\Pi_{t+1}}{\Pi_t^{\nu}}\right)^{\epsilon}]) = \chi_0\{(\Delta_t C_t)^{1+\chi} + \chi C_t^{1+\chi}\Delta_t^{\chi}\Lambda_{F_3t}\}, \quad (55)$$

where Λ_{It} , Λ_{F_1t} , Λ_{F_2t} , Λ_{F_3t} , Λ_{R_1t} , and Λ_{R_2t} represent the Lagrange multipliers associated with constraints (43)-(48); and where J_t , S_{1t} , S_{2t} , S_{3t} , and S_{4t} are defined as follows:

$$J_t = \chi_0(\Delta_t^{1+\chi}C_t^{\sigma+\chi} + (1 + \chi)C_t^{\sigma+\chi}\Delta_t^{\chi}\Lambda_{F_3t}), \quad (56)$$

$$S_{1t} = \Lambda_{F_2t-1}Z_{1t} + \Lambda_{R_1t}, \quad S_{2t} = \Lambda_{F_3t-1}Z_{2t} + \Lambda_{R_2t}\Delta_{t-1}, \quad (57)$$

$$S_{3t} = \xi\beta\nu(\epsilon - 1)\Lambda_{F_2t}E_t\left[\left(\frac{\Pi_{t+1}}{\Pi_t^{\nu}}\right)^{\epsilon-1}Z_{1t+1}\right], \quad S_{4t} = \xi\beta\nu\epsilon\Lambda_{F_3t}E_t\left[\left(\frac{\Pi_{t+1}}{\Pi_t^{\nu}}\right)^{\epsilon}Z_{2t+1}\right]. \quad (58)$$

1.2. Myopic Price-Setters

In this economy, a fraction of firms, $1-\omega$, behaves optimally when making price decisions, but the remaining fraction of firms, ω , use a simple, backward-looking rule of thumb when setting their prices. The rule-of-thumb price-setters follow the same pricing rule as used in Steinsson (2003). The difference from Steinsson is that our model does not use the yeoman-farmer setup, while Steinsson has assumed that each household specializes in the production of one differentiated good. Our choice means that all cases analyzed in this paper fall into the same general setup. But the difference from Steinsson does not have any impact on the specification of the loglinearized Phillips curve. The optimal policy problem is the state-contingent maximization at period 0 by a benevolent social planner of utility function (42). The constraints of the social planner's problem are (43), (44) and

$$Z_{1t} = \beta\xi E_t[\Pi_{t+1}^{\epsilon-1}Z_{1t+1}] + C_t^{1-\sigma}, \quad (59)$$

$$Z_{2t} = \beta\xi E_t[\Pi_{t+1}^\epsilon Z_{2t+1}] + \chi_0 C_t^{1+\chi} \Delta_t^\chi, \quad (60)$$

$$1 = (1 - \xi)[(1 - \omega)(\tilde{P}_t^*)^{1-\epsilon} + \omega X_t^{1-\epsilon}] + \xi \Pi_t^{\epsilon-1}, \quad (61)$$

$$\Delta_t = (1 - \xi)[(1 - \omega)(\tilde{P}_t^*)^{-\epsilon} + \omega X_t^{-\epsilon}] + \xi \Pi_t^\epsilon \Delta_{t-1}, \quad (62)$$

$$X_t = X_{t-1}^\omega (\tilde{P}_{t-1}^*)^{1-\omega} (C_{t-1}/C_{t-1}^n)^\delta, \quad (63)$$

where X_t is the real price at period t of rule-of-thumb price-setters and ω is the measure of rule-of-thumb price-setters. From the previous constraints it is straightforward to obtain the expressions presented in Table 1 of the text.

First-Order Conditions for the Optimal Policy Problem

Given the characterization of the optimal policy problem specified above, we now present its first-order conditions. The first-order condition of the social planner's problem with respect to $R_{i,t}$, C_t , Z_{1t} , Z_{2t} , \tilde{P}_t^* , Π_t , and Δ_t can be written as follows.

$$\Lambda_{I_t} = \left(\frac{\nu_0^\nu}{\nu}\right) C_t^{\frac{\sigma(1-\nu)}{\nu}} (1 - R_{i,t})^{\frac{-1}{\nu}}, \quad (64)$$

$$1 - J_t + \frac{(\frac{\sigma\nu_0^\nu}{\nu}) C_t^{\frac{\sigma}{\nu}-1}}{(1 - R_{i,t})^{\frac{1-\nu}{\nu}}} + \frac{\sigma(\Lambda_{I_t} R_{i,t} - \frac{\Lambda_{I_{t-1}}}{\Pi_t})}{C_t} + (\sigma - 1)\Lambda_{F_2t} = \frac{\beta\delta C_t^{\sigma+\delta-1} E_t[\Lambda_{R_3t+1}]}{(C_t^n)^\delta}, \quad (65)$$

$$\Lambda_{F_1t}(\tilde{P}_t^*) + \Lambda_{F_2t} - \xi\Lambda_{F_2t-1}\Pi_t^{\epsilon-1} = 0, \quad (66)$$

$$\frac{\epsilon}{\epsilon - 1}\Lambda_{F_1t} = \Lambda_{F_3t} - \xi\Lambda_{F_3t-1}\Pi_t^\epsilon, \quad (67)$$

$$(\epsilon - 1)\Lambda_{R_1t} + \epsilon(P_t^*)^{-1}\Lambda_{R_2t} + \frac{(P_t^*)^\epsilon Z_{1t}\Lambda_{F_1t}}{(1 - \xi)(1 - \omega)} = \beta E_t\left[\frac{\Lambda_{R_3t+1}(P_t^*)^{\epsilon-\omega} X_t^\omega}{1 - \xi} \left(\frac{C_t}{C_t^n}\right)^\delta\right], \quad (68)$$

$$\frac{C_t^{-\sigma}\Lambda_{I_{t-1}}}{\Pi_t} + \xi(\epsilon - 1)\Pi_t^{\epsilon-1}(\Lambda_{F_2t-1}Z_{1t} + \Lambda_{R_1t}) + \epsilon\xi\Pi_t^\epsilon(\Lambda_{F_3t-1}Z_{2t} + \Lambda_{R_2t}\Delta_{t-1}) = 0, \quad (69)$$

$$\Delta_t(\Lambda_{R_2t} - \beta\xi E_t[\Lambda_{R_2t+1}\Pi_{t+1}^\epsilon]) = \chi_0\{(\Delta_t C_t)^{1+\chi} + \chi C_t^{1+\chi} \Delta_t^\chi \Lambda_{F_3t}\}, \quad (70)$$

$$(\epsilon - 1)\Lambda_{R_1t} + \epsilon X_t^{-1}\Lambda_{R_2t} + \frac{\Lambda_{R_3t} X_t^\epsilon}{(1 - \xi)\omega} = \beta E_t\left[\frac{\Lambda_{R_3t+1}(P_t^*)^{1-\omega} X_t^{\epsilon+\omega-1}}{1 - \xi} \left(\frac{C_t}{C_t^n}\right)^\delta\right], \quad (71)$$

where J_t is defined as

$$J_t = \chi_0(\Delta_t^{1+\chi} C_t^{\sigma+\chi} + (1 + \chi)C_t^{\sigma+\chi} \Delta_t^\chi \Lambda_{F_3t}). \quad (72)$$

2. Time-Dependent Preferences The following formulation of an optimal policy problem takes into account both external and internal habits. The optimal policy problem is the state-contingent maximization at period 0 of a benevolent social planner with the following utility function:

$$\sum_{t=0}^{\infty} E_t \left[\frac{(C_t - \phi C_{t-1})^{1-\sigma} - 1}{1-\sigma} - \chi_0 \frac{(\Delta_t C_t)^{1+\chi}}{1+\chi} + \nu_0 \frac{\nu_0^{\frac{1-\nu}{\nu}} \Phi_t^{-\frac{1-\nu}{\nu}} (1 - R_{i,t})^{-\frac{1-\nu}{\nu}} - 1}{1-\nu} \right]. \quad (73)$$

The constraints of the social planner's problem are

$$\beta E_t[\Phi_{t+1} \Pi_{t+1}^{-1}] = R_{i,t} \Phi_t, \quad (74)$$

$$Z_{1t} \tilde{P}_t^* = \frac{\epsilon}{\epsilon - 1} Z_{2t}, \quad (75)$$

$$Z_{1t} = \beta \xi E_t[\Pi_{t+1}^{\epsilon-1} Z_{1t+1}] + C_t \Phi_t, \quad (76)$$

$$Z_{2t} = \beta \xi E_t[\Pi_{t+1}^{\epsilon} Z_{2t+1}] + \chi_0 C_t^{1+\chi} \Delta_t^{\chi}, \quad (77)$$

$$1 = (1 - \xi)(\tilde{P}_t^*)^{1-\epsilon} + \xi \Pi_t^{\epsilon-1}, \quad (78)$$

$$\Delta_t = (1 - \xi)(\tilde{P}_t^*)^{-\epsilon} + \xi \Pi_t^{\epsilon} \Delta_{t-1}, \quad (79)$$

$$\Phi_t = (C_t - \phi C_{t-1})^{-\sigma} - \theta \phi \beta E_t[(C_{t+1} - \phi C_t)^{-\sigma}]. \quad (80)$$

Φ_t is the shadow value of wealth for the representative household. The difference between external and internal habits is concentrated in the specification of Φ_t , with $\theta = 1$ corresponding to internal habit and $\theta = 0$ external habit. With this formulation of the optimal policy problem, the difference between habit specifications shows up in its constraints. This formulation brings out the fact that the social planner wants to internalize the consumption externality from external habits that arises in decentralized economies.

First-Order Conditions for Optimal Policy

The first-order condition for the social planner's problem with respect to $R_{i,t}$, C_t , Z_{1t} , Z_{2t} , \tilde{P}_t^* , Π_t , Δ_t , and Φ_t can be written as follows.

$$\Lambda_{It} = \left(\frac{\nu_0^{\frac{1}{\nu}}}{\nu} \right) \Phi_t^{-\frac{1-\nu}{\nu}} (1 - R_{i,t})^{-\frac{1}{\nu}}, \quad (81)$$

$$\Phi_{st} + V_{4t} = \chi_0(C_t \Delta_t)^X \Delta_t + (\chi + 1)\chi_0 C_t^X \Delta_t^X \Lambda_{F_3t} + \Phi_t \Lambda_{F_2t}, \quad (82)$$

$$\Lambda_{F_1t}(\tilde{P}_t^*) + \Lambda_{F_2t} - \xi \Lambda_{F_2t-1} \Pi_t^{\epsilon-1} = 0, \quad (83)$$

$$\frac{\epsilon}{\epsilon-1} \Lambda_{F_1t} = \Lambda_{F_3t} - \xi \Lambda_{F_3t-1} \Pi_t^\epsilon, \quad (84)$$

$$\Lambda_{R_1t} + \left(\frac{\epsilon(P_t^*)^{-1}}{\epsilon-1}\right) \Lambda_{R_2t} + \left(\frac{(P_t^*)^\epsilon Z_{1t}}{(1-\xi)(\epsilon-1)}\right) \Lambda_{F_1t} = 0, \quad (85)$$

$$\Phi_t \Lambda_{I_{t-1}} + \xi(\epsilon-1) \Pi_t^\epsilon (\Lambda_{F_2t-1} Z_{1t} + \Lambda_{R_1t}) + \epsilon \xi \Pi_t^{\epsilon+1} (\Lambda_{F_3t-1} Z_{2t} + \Lambda_{R_2t} \Delta_{t-1}) = 0, \quad (86)$$

$$\Lambda_{R_2t} - \beta \xi E_t[\Lambda_{R_2t+1} \Pi_{t+1}^\epsilon] = \chi_0 C_t^{X+1} \Delta_t^{X-1} (\Delta_t + \chi \Lambda_{F_3t}), \quad (87)$$

$$\Lambda_{R_3t} = R_{i,t} \Lambda_{I_t} - \Lambda_{I_{t-1}} \Pi_t^{-1} + C_t \Lambda_{F_2t} + (\nu_0^{\frac{1}{\nu}} / \nu) \Phi_t^{-\frac{1}{\nu}} (1 - R_{i,t})^{-\frac{1-\nu}{\nu}}, \quad (88)$$

where V_{4t} and Φ_{st} are defined as:

$$V_{4t} = \sigma[(C_t - \phi C_{t-1})^{-(\sigma+1)} (\Lambda_{R_3t} (1 + \theta \phi^2 \beta) - \theta \phi \Lambda_{R_3t-1}) - \beta \phi E_t[(C_{t+1} - \phi C_t)^{-(\sigma+1)} \Lambda_{R_3t+1}]]$$

$$\Phi_{st} = (C_t - \phi C_{t-1})^{-\sigma} - \beta \phi E_t[(C_{t+1} - \phi C_t)^{-\sigma}]$$

3. Loglinear approximation of the optimal policy under Epstein-Zin Preferences

We present a loglinear approximation around the steady state in a model without money. This substantially reduces the algebra since, under this assumption, the optimal inflation is equal to zero. We first note that the Lagrange multiplier of the social planner's utility is not constant over time so that its evolution can be described by:

$$\hat{\Omega}_t = \hat{\Omega}_{t-1} + \hat{X}_{Ut} \quad (89)$$

where $\hat{\Omega}_0 = 0$ and \hat{X}_{Ut} is proportional to the utility surprise in period t :

$$\hat{X}_{Ut} = \sigma(\hat{U}_t - E_{t-1}[\hat{U}_t]) \quad (90)$$

$$\hat{U}_t = \frac{1}{U} [(1 - MC)\hat{C}_t + MC\hat{A}_t] + \beta E_t[\hat{U}_{t+1}] \quad (91)$$

The Lagrange multiplier for the social planning problem is constant in the case of expected utility. The following equations correspond to the optimality conditions that hold in the case of Epstein-Zin preferences.

$$\hat{\Omega}_t = \kappa_C (\hat{C}_t - \hat{A}_t) + \hat{\Lambda}_{F_3t} \quad (92)$$

$$\hat{\Lambda}_{F_1 t} + \frac{\epsilon \xi}{1 - \xi} \hat{\Pi}_t - \frac{\hat{\Lambda}_{F_2 t}}{1 - \xi} + \frac{\xi}{1 - \xi} (\hat{X}_{U t} + \hat{\Lambda}_{F_2 t - 1}) = 0 \quad (93)$$

$$\hat{\Lambda}_{F_1 t} = \frac{\hat{\Lambda}_{F_3 t}}{1 - \xi} - \frac{\xi}{1 - \xi} (\hat{X}_{U t} + \epsilon \hat{\Pi}_t + \hat{\Lambda}_{F_3 t - 1}) \quad (94)$$

$$\kappa_D \hat{\Pi}_t + \Lambda_{R_1} \hat{\Lambda}_{R_1 t} + \frac{\epsilon \Lambda_{R_2}}{\epsilon - 1} \hat{\Lambda}_{R_2 t} + \frac{z_1 \Lambda_{F_1}}{(1 - \xi)(\epsilon - 1)} (\hat{\Lambda}_{F_1 t} + \hat{z}_{1 t}) = 0 \quad (95)$$

$$\hat{\Lambda}_{R_2 t} - \beta \xi E_t [\epsilon \hat{\Pi}_{t+1} + \hat{\Lambda}_{R_2 t+1}] = \kappa_R (\hat{C}_t - \hat{A}_t) + (1 - \xi \beta) (\eta \hat{\Omega}_t + (1 - \eta) \hat{\Lambda}_{F_3 t}) \quad (96)$$

$$\kappa_U \hat{X}_{U t} + \kappa_{F_2} (\hat{z}_{1 t} + \hat{\Lambda}_{F_2 t - 1}) + \kappa_{R_1} \hat{\Lambda}_{R_1 t} + \kappa_{\Pi} \hat{\Pi}_t + \kappa_{F_3} (\hat{\Lambda}_{F_3 t - 1} + \hat{z}_{2 t}) + \kappa_{R_2} \hat{\Lambda}_{R_2 t} = 0 \quad (97)$$

where $\kappa_U, \kappa_{\Pi}, \kappa_{F_2}, \kappa_{R_1}, \kappa_{F_3}, \kappa_{R_2}$ are defined as

$$\begin{aligned} \kappa_{F_2} &= \Lambda_{F_2} z_1, \kappa_{R_1} = \Lambda_{R_1}, \kappa_{\Pi} = \frac{\epsilon (\Lambda_{F_3} z_2 + \Lambda_{R_2})}{\epsilon - 1}, \kappa_{F_3} = \frac{\epsilon z_2 \Lambda_{F_3}}{\epsilon - 1}, \kappa_{R_2} = \frac{\epsilon \Lambda_{R_2}}{\epsilon - 1}, \kappa_U = \kappa_{F_2} + \kappa_{F_3} \\ \kappa_C &= \frac{\epsilon \varphi_0 + MC(1 + \epsilon)}{\varphi_0}, \kappa_D = \frac{\xi \Lambda_{R_1} + z_1 \Lambda_{F_1}}{1 - \xi}, \eta = \frac{\varphi_0}{\varphi_0 + \Lambda_{F_3} MC} \\ \kappa_R &= \frac{(1 - \beta \xi)(2 - \eta)(\varphi_0 + MC)}{\varphi_0}, \kappa_Z = \frac{(1 - \beta \xi)(\varphi_0 + MC)}{\varphi_0}. \end{aligned}$$

Having presented the loglinear version of optimal conditions, we now move onto their deterministic steady-state settings.

$$\Lambda_{F_1} = \frac{(1 - \xi)(\epsilon - 1)}{\epsilon} \Lambda_{F_3}, \Lambda_{F_2} = -\frac{1}{1 - \xi} \Lambda_{F_1}, \Lambda_{F_3} = \frac{1}{MC} (C(1 + \varphi_0) - 1) \quad (98)$$

$$\Lambda_{R_2} = \frac{MC}{1 - \beta \xi} \left(1 + \Lambda_{F_3} \frac{C}{1 - C}\right), \Lambda_{R_1} = -\frac{\epsilon}{\epsilon - 1} \Lambda_{R_2} - \frac{z_1}{(1 - \xi)(\epsilon - 1)} \Lambda_{F_1} \quad (99)$$

The equilibrium conditions at the deterministic steady state with zero inflation can be written as

$$C = \frac{MC}{\varphi_0 + MC}, N = C, MC = \frac{\epsilon - 1}{\epsilon}, Z_1 = \frac{1}{1 - \xi \beta}, Z_2 = \frac{MC}{1 - \xi \beta} \quad (100)$$

In addition, steady-state welfare is given by

$$U = \frac{1}{1 - \beta} (\log C + \varphi_0 \log(1 - N)). \quad (101)$$

Figure 1: Comparison of Optimal Steady-State Inflation Rates: Observationally Equivalent Mechanisms of Inflation Persistence

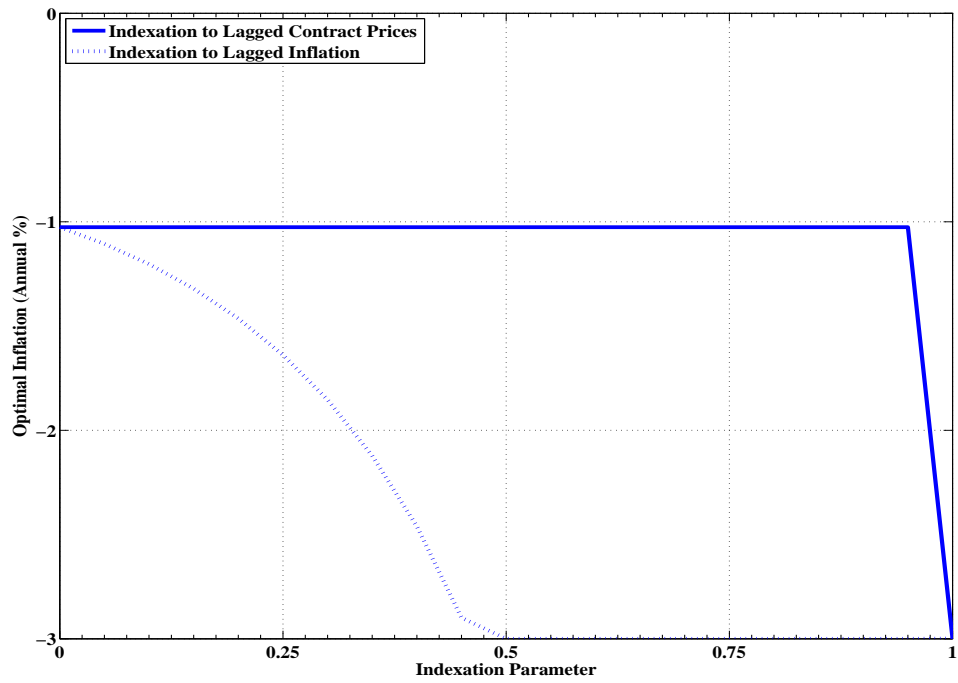


Figure 2: Optimal Steady-State Inflation Rate: Alternative Sources of Real Rigidity

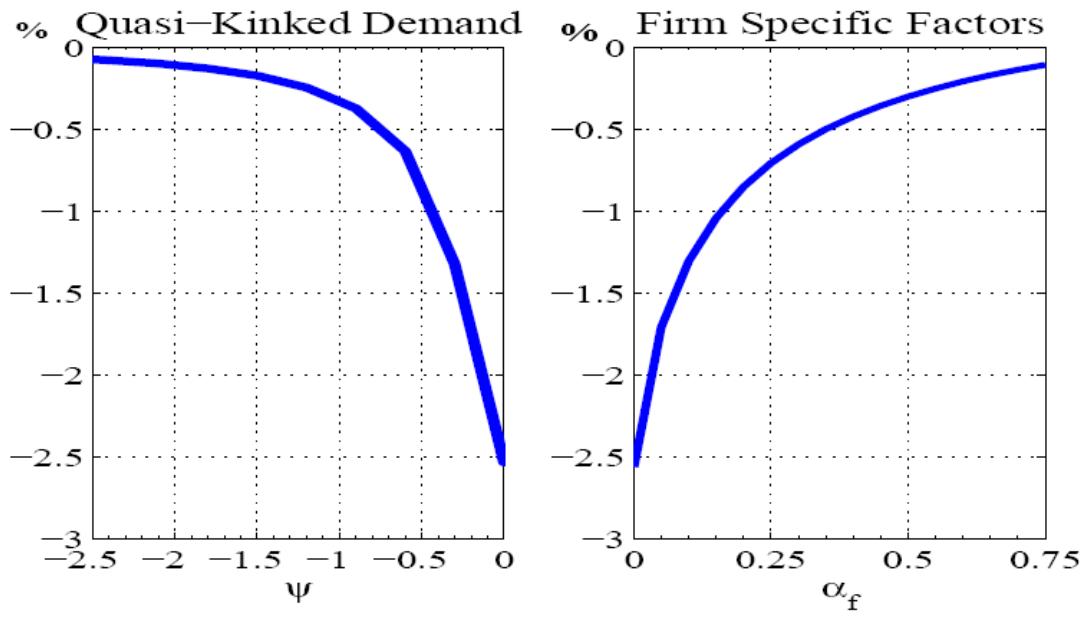


Figure 3: Comparison of Optimal Steady-State Inflation Rates:
External and Internal Habits

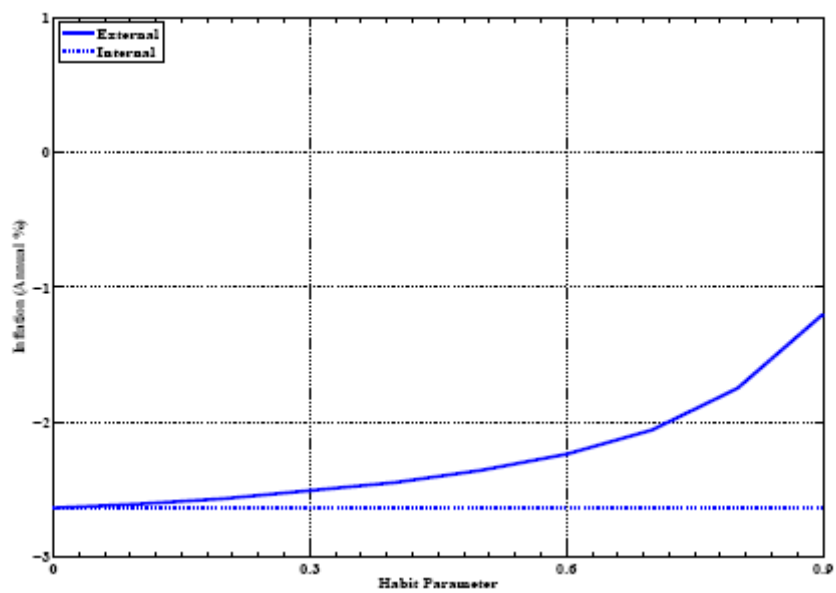


Figure 4: Comparison of Impulse Responses under Optimal Monetary Policies:
Epstein-Zin and Expected Utility Preferences

