

Two Versions of John Taylor's Model of Nominal Price Adjustments

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1. Introduction

John Taylor has many outstanding contributions on topics including techniques for econometric policy analysis in RE models, solution multiplicities in RE models, formulation of models with nominal contracts, procedures for estimation and simulation of RE models (linear and nonlinear), and computational procedures for nonlinear models. Besides these technical contributions, he did work that contributed both substantively and sociologically. First, he specified and estimated various econometric models—large and small, open and closed—during the years 1982-1993, thereby keeping alive the role of quantitative

models with both RE and nominal frictions—thus preparing the way for the upsurge of today's mainstream monetary policy analysis. And of course there is his most famous contribution of all, the 1993 C-R paper that introduced the Taylor Rule. It is my perception that this paper played a huge role sociologically. It was doubly fruitful, as it encouraged academics to think of policy as being implemented by interest rates and simultaneously encouraged CB economists to think in terms of policy rules while also showing that the successful Fed policy over 1987-1992 was quite well-described by his rule:

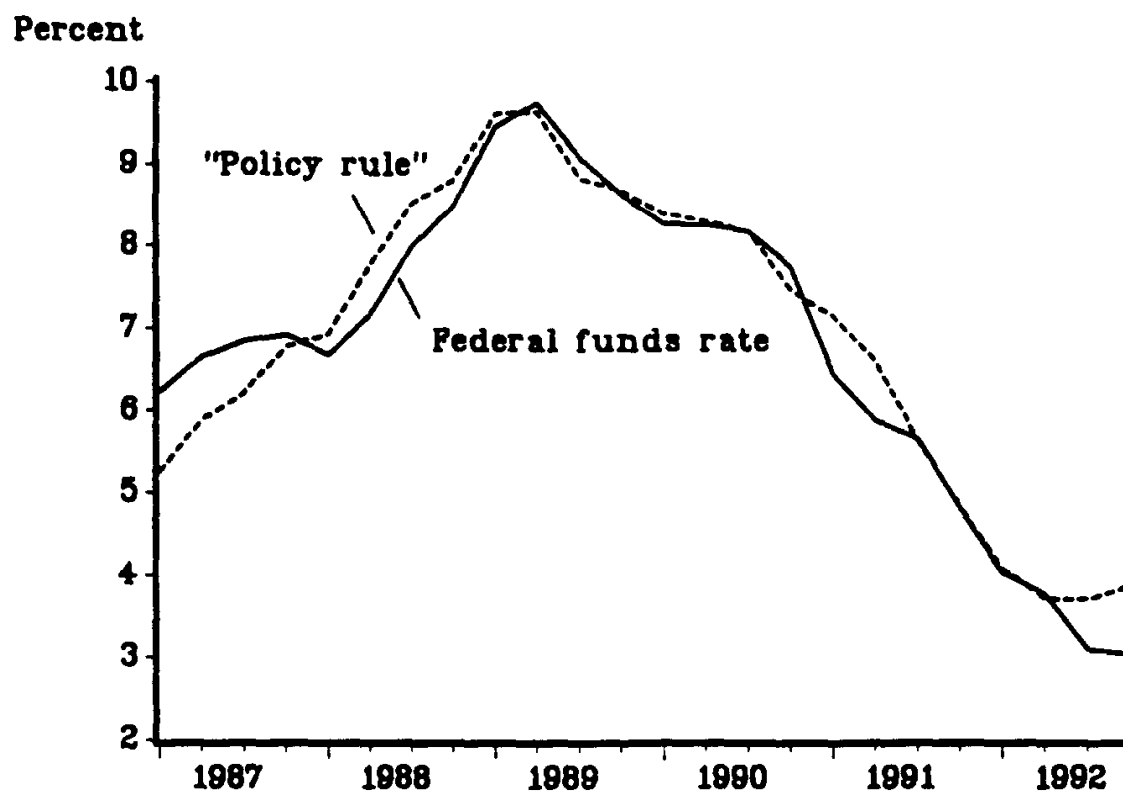


Figure 1. Federal funds rate and example policy rule.

This had the effect of bringing CB and academic research much closer together, to the point that now one cannot distinguish between them

2. Taylor-Style Contracts

Simplest two-period version for prices: each period half of sellers choose new price x_t to prevail for two periods t and $t+1$. So average (log) price in t is

$$(1) \quad p_t = 0.5(x_t + x_{t-1}).$$

In Taylor (1980) and (1999, p. 1028) the “reset” price x_t is

$$(2) \quad x_t = 0.5E_{t-1}(p_t + \gamma ed_t) + 0.5E_{t-1}(p_{t+1} + \gamma ed_{t+1}), \quad \gamma > 0,$$

where ed_t is “excess demand,” often approximated as output deviations from trend. Taylor (1999, p. 1034) suggests that ed_t might be related to real marginal cost in a manner that would make (2) represent optimal

pricing, subject to fixity for $N = 2$ periods. One possibility is

$$(3) \quad x_t = 0.5\bar{p}_t + 0.5E_t\bar{p}_{t+1}$$

where \bar{p}_t is optimal under price flexibility.

3. Flexible-Price Relations

Consider household with utility at $t = 1$ of form

$$(4) \quad E_1[U_1 + \beta U_2 + \beta^2 U_3 + \dots], \quad \text{with} \quad U_t = \frac{V_t C_t^{1-\sigma}}{1-\sigma} - \Psi \frac{N_t^{1+\psi}}{1+\psi}$$

where V_t is preference shock process, AR(1). Prod fn is $Y_t = A_t k^{1-\alpha} N d_t^\alpha$

and demand for its product is $Y_t^A (P_t / P_t^A)^{-\theta}$, where A denotes aggregate:

$$(5) \quad W_t(N_t - Nd_t) + Y_t^A (P_t / P_t^A)^{1-\theta} - \frac{B_{t+1}}{1+r_t} + B_t - Tx_t - C_t = 0$$

$$(6) \quad A_t k^{1-\alpha} Nd_t^\alpha - Y_t^A (P_t / P_t^A)^{-\theta} = 0.$$

$$(7) \quad V_t C_t^{-\sigma} - \lambda_t = 0$$

$$(8) \quad -\Psi N_t^\psi + \lambda_t W_t = 0$$

$$(9) \quad -\lambda_t W_t + \xi_t \alpha A_t k^{1-\alpha} Nd_t^{\alpha-1} = 0$$

$$(10) \quad \frac{\lambda_t}{1+r_t} + \beta E_t \lambda_{t+1} = 0$$

$$(11) \quad \lambda_t (1-\theta) Y_t^A \frac{P_t^{-\theta}}{(P_t^A)^{1-\theta}} - \xi_t Y_t^A (-\theta) \frac{P_t^{-\theta-1}}{(P_t^A)^{-\theta}} = 0 \quad \Rightarrow \quad \lambda_t P_t = \xi_t P_t^A \frac{\theta}{\theta-1}.$$

In symmetric monop comp equil, $P_t = P_t^A$ and $N_t = Nd_t$. Government determines G_t and Tx_t subject to

$$(12) \quad G_t - Tx_t = \frac{B_{t+1}}{1 + r_t} - B_t.$$

We complete the model with

$$(13) \quad R_t = r_t + E_t \pi_{t+1},$$

$$(14) \quad \pi_{t+1} = p_{t+1} - p_t$$

with $p_t = \log(P_t^A)$, and specify that R_t is set by the CB acc to rule such as

$$(15) \quad R_t = \mu_0 + \mu_1(\pi_t).$$

Then (5)-(15) governs C_t , N_t , B_{t+1} , P_t , λ_t , ξ_t , W_t , Y_t , r_t , R_t , and π_t , given exogenous G_t , Tx_t , V_t , and A_t . In symm flex-price equil, $\lambda_t/\xi_t = \theta/(\theta-1)$ and we can derive that log output is given as

$$(20) \quad \bar{y}_t = a_t + \frac{\alpha}{[(\sigma\alpha/\omega) + \psi + (1-\alpha)]} [v_t + (1 - (\sigma/\omega))a_t + \sigma((1-\omega)/\omega)g_t]$$

where $a_t = \log A_t$, $v_t = \log V_t$, and ω is consumption share.

Note from (9) that λ_t/ξ_t equals real marginal cost.

4. Two Versions

Use (11) to specify individually optimal prices, with or without price flexibility:

$$(22) \quad \hat{p}_t = p_t^A + \log \frac{\xi_t}{\lambda_t} + \log \frac{\theta}{\theta - 1}.$$

Write the Taylor contract model as specifying that the reset price is

$$(23) \quad x_t = \frac{1}{1 + \beta} \hat{p}_t + \frac{\beta}{1 + \beta} E_t \hat{p}_{t+1}.$$

This is approximately the same as the equal-weights version

$$(23') \quad x_t = \frac{1}{2} \hat{p}_t + \frac{1}{2} E_t \hat{p}_{t+1}.$$

(Sometimes Taylor includes E_{t-1} operators.) Using p_t now as the (log) aggregate price level, (22) implies

$$(24) \quad \hat{p}_t = p_t + mca_t.$$

Thus if mca_t , the log of real marginal cost rel to its flex-price level, were proportional to the output gap, $\tilde{y}_t = y_t - \bar{y}_t$, then could write (23) as

$$(25) \quad x_t = 0.5(p_t + \gamma\tilde{y}_t) + 0.5E_t(p_{t+1} + \gamma\tilde{y}_{t+1}).$$

This would support Taylor's suggestion that "pricing under monop comp gives a more formal underpinning of the staggered price setting model"

(1999, p. 1034). In fact, in model above, we have (ignoring constants)

that mca_t equals $\frac{1 + \psi - \alpha(1 - (\sigma/\omega))}{\alpha} \tilde{y}_t$, except that the mca expression

refers to economy-wide average values, rather than just those adjusting.

Using a correction given by Walsh (2003) and GGL-S (2001), we have

$$(27) \quad \gamma = \frac{\alpha}{\alpha + \theta(1 - \alpha)} \frac{1 + \psi - \alpha(1 - (\sigma/\omega))}{\alpha}.$$

So, (25) represents an optimizing version of a Taylor-contract model.

Let's summarize price level and output determination in a model with Taylor contracts as discussed. With \bar{y}_t as in (20), it is:

$$(28) \quad x_t = 0.5(p_t + \gamma \tilde{y}_t) + 0.5E_t(p_{t+1} + \gamma \tilde{y}_{t+1})$$

$$(29) \quad y_t = E_t y_{t+1} + b(R_t - E_t \Delta p_{t+1}) + v_t - E_t v_{t+1}$$

$$(30) \quad R_t = \mu_0 + \Delta p_t + \mu_1(\Delta p_t - \pi^*) + \mu_2 \tilde{y}_t + e_t$$

$$(31) \quad \tilde{y}_t = y_t - \bar{y}_t$$

$$(32) \quad p_t = 0.5(x_t + x_{t-1})$$

$$(33) \quad \Delta p_t = p_t - p_{t-1}.$$

These determine behavior of the six endog vars x_t , p_t , Δp_t , y_t , \tilde{y}_t , and R_t .

In (28) γ stems from (22) and reflects the link between marg cost and output gap, as in (27). Let's now consider an alternative version, that is

quite different. Instead of relating $\hat{p}_t - p_t$ to marginal cost, recall that with Dixit-Stiglitz aggregation, the demand curve faced by a seller is

$$(34) \quad y_t = y_t^A - \theta(p_t - p_t^A).$$

This pertains to all hypothetical values of y_t . Considering \bar{y}_t , then, and subtracting from (34), we have

$$(36) \quad y_t = \bar{y}_t - \theta(p_t - \bar{p}_t).$$

Using \bar{p}_t from the latter in place of \hat{p}_t in the Taylor two-period relation (23'), then we have

$$(38) \quad x_t = 0.5[p_t + (1/\theta)(y_t - \bar{y}_t)] + 0.5E_t[p_{t+1} + (1/\theta)(y_{t+1} - \bar{y}_{t+1})].$$

The latter represents a different relation governing reset price determination, one of the same form as (28) but with a different constant on the output gap. The model with (28) reflects optimizing behavior, given prices set for two periods, in which marginal cost depends on labor market clearing [(8) and (9)] in each period. The model with (38), by contrast, has output demand-determined in each period. Labor supply and demand relations are in this specification irrelevant to determination of employment and output within each period, although they are essential in determining the reference values \bar{y}_t . This model is more “Keynesian” and less “neoclassical” than the one with (28); but it does

not entirely ignore the supply-side relationships that are crucial in any sensible model.

It is my impression that this more Keynesian version of the system is probably closer to the conception of “output determination with sticky prices” that Taylor, Fischer, Gray, and Calvo used in the 1970s and 1980s. Also, I have examined presentation of “The Adjustment Process” in the textbook of Hall and Taylor (1997), and found that it features output determined by aggregate demand with a fully predetermined price level. Relevant pages are 205-226 and 429-446.

5. Digression on the Calvo Model

6. Calibration of the Basic Models

$$\alpha = 0.65, \psi = 1, \sigma = 2, \omega = 0.75, \theta = 10 \text{ or } \gamma = 1.378.$$

shock/AR parameter/innovation SD:

$$a_t/0.95/0.007 \quad g_t/0.98/0.02 \quad v_t/0/0.01 \quad e_t/0/0.002$$

Table 1: U.S Statistics

Cell entries are SDs (per cent p.a.) and AR(1) coeffs for Δp_t , \tilde{y}_t , and R_t

	inflation rate SD/autocorr	output gap SD/autocorr	interest rate SD/autocorr
1954.1- 2005.4	3.00 0.841	1.58 0.848	3.36 0.955
1954.1- 1987.3	3.51 0.863	1.82 0.842	3.74 0.947
1987.4- 2005.4	1.37 0.463	1.00 0.890	2.26 0.970

Table 2: Properties of Model with Taylor Price Adjustment
 Cell entries are SDs (per cent p.a.) and AR(1) coeffs for $\Delta p_t, y_t, \tilde{y}_t,$ and R_t

	two period $1/\theta = 0.1$	four period $1/\theta = 0.1$	two period $\gamma = 1.378$	four period $\gamma = 1.378$
$\mu_1 = 0.5$	1.24	0.97	1.89	1.31
$\mu_2 = 0.5$	1.86	1.77	1.86	1.829
$\mu_3 = 0.0$	0.49	0.55	0.37	0.48
$\mu_4 = 0.0$	2.06	1.74	2.96	2.15
	0.941	0.973	0.630	0.913
	0.789	0.780	0.820	0.789
	-0.047	0.081	-0.279	-0.037
	0.775	0.727	0.581	0.752
$\mu_1 = 0.5$	1.23	0.76	2.38	1.36
$\mu_2 = 0.5$	1.85	1.78	1.84	1.83
$\mu_3 = 0.8$	0.61	0.72	0.47	0.60
$\mu_4 = 0.0$	1.48	1.47	1.53	1.52
	0.837	0.950	0.456	0.801
	0.772	0.754	0.789	0.771
	0.117	0.325	-0.196	0.109
	0.834	0.830	0.830	0.848

$\mu_1 = 0.1$	2.46	1.66	3.95	2.65
$\mu_2 = 0.1$	1.87	1.76	1.85	1.82
$\mu_3 = 0.8$	0.73	0.92	0.54	0.72
$\mu_4 = 0.0$	2.28	2.00	2.48	2.30
	0.882	0.960	0.606	0.858
	0.767	0.743	0.772	0.759
	0.252	0.495	-0.135	0.255
	0.916	0.896	0.923	0.919
$\mu_1 = 0.1$	3.29	2.73	4.66	3.57
$\mu_2 = 0.1$	1.83	1.78	1.83	1.81
$\mu_3 = 0.8$	0.75	0.96	0.54	0.73
$\mu_4 = 1.0$	3.02	2.88	3.28	3.25
	0.913	0.974	0.681	0.895
	0.746	0.735	0.770	0.756
	0.269	0.516	-0.121	0.260
	0.946	0.940	0.945	0.950

Policy parameters are for four cases: orig T rule; add smoothing; reduce coeffs; add RW inflation target. Most realistic is third.

8. Conclusion

For third rule, four-period contracts, case with demand determining, results are not too bad except for persistence (autocorrelation) of output gap. The two different models give results that differ considerably, although not dramatically.