

Learning, Expectations Formation, and the Pitfalls of Optimal Control Monetary Policy

Athanasios Orphanides
Central Bank of Cyprus

John C. Williams
Federal Reserve Bank of San Francisco

Prepared for the FRB Dallas Conference
“John Taylor’s Contributions to Monetary Theory and Policy”
October 12–13, 2007

The opinions expressed are those of the authors and do not necessarily reflect the views of the Central Bank of Cyprus or the management of the Federal Reserve Bank of San Francisco.

Optimal Control Monetary Policy

- Optimal control approach to monetary is back in vogue among academics

(Svensson (2002), Svensson and Woodford (2003), Woodford (2003), Giannoni and Woodford (2005))
- Svensson and Tetlow (2006) show it's feasible in practice with FRB/US, the Board's large-scale nonlinear models

Optimal Control Monetary Policy

Optimal control methodology (Sargent, 2007):

1. Apply rational expectations econometrics to historical data to estimate parameters that describe private agents preferences, technology, endowments, and information sets.
2. Posit a timing protocol and an objective function for a government, typically a Pareto criterion.
3. Find a new rational expectations equilibrium that maximizes the governments objective.
4. Proclaim as advice the government policy that implements that rational expectations equilibrium.

Source: Thomas J. Sargent (2007), "Evolution and Intelligent Design."

But what if the reference model is misspecified?

- Economists disagree about model specification
- Ironside and Tetlow (2007) show that key properties of the FRB/US model changed over time reflecting evolution in specification
- Robust control can deal with small deviations from reference model
- But, incorporating “realistic” moderately-sized degree of model uncertainty in optimal control is infeasible with today’s methods and computers

Stress Testing Monetary Policies

- This paper follows the approach of “stress testing” proposed monetary policies across a range of non-nested empirical macroeconomic models (McCallum 1988; Taylor 1993)
- This approach helps identify characteristics of policy rules that perform well across different models, and those that don't
- Past research has identified types of simple monetary policy rules are robust to misspecification of model dynamics (Levin, Wieland, Williams 1999, 2003; Orphanides and Williams 2002, 2007)
- Optimal control policies may not be robust to gross misspecification of dynamics (Levin and Williams 2003)

Expectations Formation

- We assume the central bank has the correct model of aggregate dynamics
- But, agents have imperfect knowledge of the macroeconomy and base their forecasts on estimated models
- Evidence from surveys supports deviations from rational expectations
- We evaluate optimal control policies and simple monetary policy rules optimized under the assumption of rational expectations to various models of learning

Estimated Model

IS Curve (measured in unemployment rate)

$$u_t = 0.5 u_{t+1}^e + 0.5 u_{t-1} + 0.056 (\tilde{r}_t^e - r^*) + v_t, \\ (0.022)$$

$$v_t = 0.513 v_{t-1} + e_{v,t}, \quad \hat{\sigma}_{e_v} = 0.30, \\ (0.085)$$

Philips Curve

$$\pi_t = 0.5 \pi_{t+1}^e + 0.5 \pi_{t-1} - 0.294 (u_t^e - u_t^*) + e_{\pi,t}, \\ (0.087)$$

$$\hat{\sigma}_{e_\pi} = 1.35,$$

Sample : 1968:4 - 2004:2

Optimal Control Monetary Policy

Central Bank Loss:

$$\mathcal{L} = Var(\pi - \pi^*) + \lambda Var(u - u^*) + \nu Var(\Delta(i)),$$

We assume $\lambda = 4$ and $\nu = 1$ (implies equal weights on inflation and output gap)

- Optimal control policy minimizes loss subject to constraints of model equations under rational expectations
- Policy implemented using forecasts from structural model.

Simple Rules

Optimized LWW (2003) rule:

$$i_t = i_{t-1} + 1.05 (\bar{\pi}_{t+3}^e - \pi^*) - 1.39 (u_{t-1} - \hat{u}_t^*).$$

Optimized OW (2007) rule:

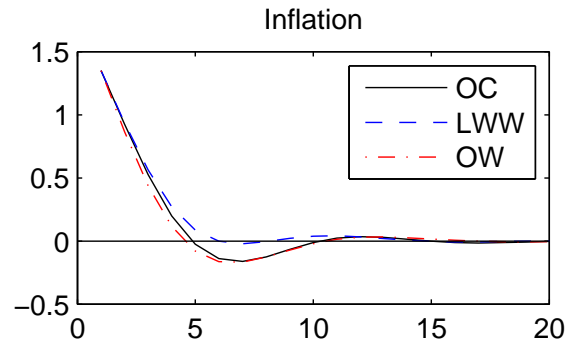
$$i_t = i_{t-1} + 1.74 (\bar{\pi}_{t+3}^e - \pi^*) - 1.19 (u_{t-1} - u_{t-2}).$$

- Parameters chosen to minimize central bank loss under rational expectations
- Inflation expectations formed using same model as private agents

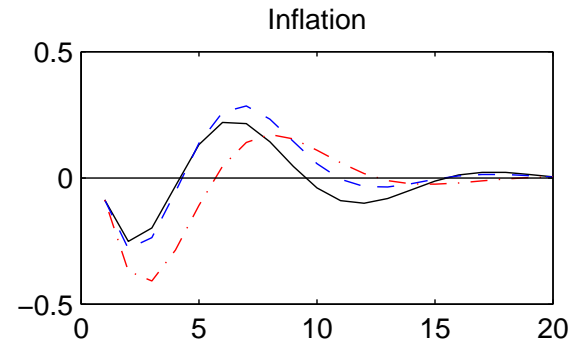
Sources: Levin, Wieland, and Williams (2003), “The Performance of Forecast-Based Policy Rules under Model Uncertainty,” and Orphanides and Williams (2007), “Inflation Targeting Under Imperfect Knowledge.”

Impulse Responses with Rational Expectations

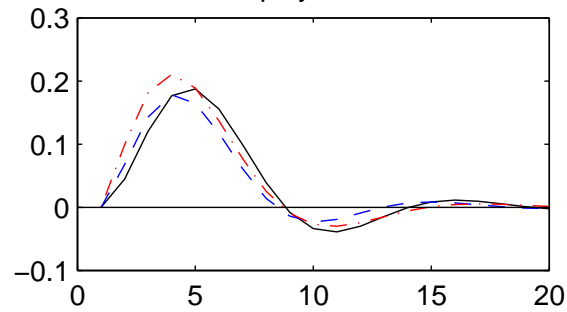
Inflation Shock



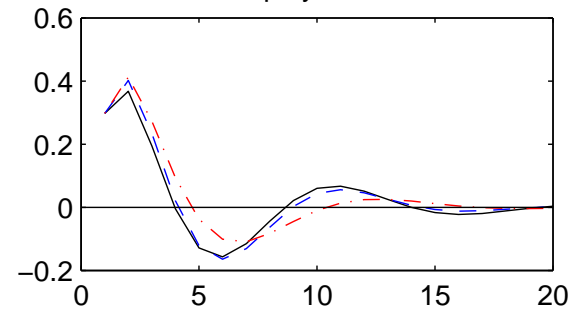
Unemployment Shock



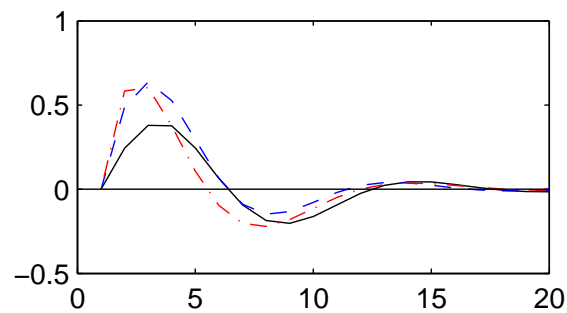
Unemployment Rate



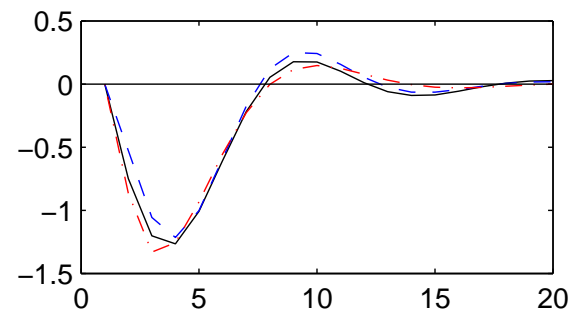
Unemployment Rate



Interest Rate



Interest Rate



Performance with Rational Expectations

Policy	Standard Deviation			Loss
	π	$u - u^*$	Δi	\mathcal{L}
Optimal control	1.83	0.67	1.20	6.59
LWW rule	1.87	0.69	1.23	6.93
OW rule	1.83	0.73	1.39	7.40

Perpetual Learning

Agents forecast with a VAR(l), where $l \in [1, 3]$

$$Y_t = (\pi_t, u_t, i_t).$$

$$X_t = (1, \pi_{t-1}, u_{t-1}, i_{t-1}, \dots, \pi_{t-l}, u_{t-l}, i_{t-l}).$$

c_t : $(3 \cdot l + 1) \times 3$ matrix of VAR coefficients

- VAR(3) encompasses rational expectations equilibrium
- VAR(1) and VAR(2) are underparameterized

Perpetual Learning

Agents reestimate forecast model each period:

$$R_t = R_{t-1} + \kappa(X_t X_t' - R_{t-1})$$

$$c_t = c_{t-1} + \kappa R_t^{-1} X_t (Y_t - X_t' c_{t-1})$$

- κ : estimates range from 0.01 and 0.04 and are estimated imprecisely (Orphanides and Williams 2005, Milani 2005, Branch and Evans 2006)
- $\kappa = 0$: No learning; i.e., agents have infinite amounts of data (c set to values implied by RE equilibrium)
- We consider values of κ between 0 and 0.03.
- Model moments computed from stochastic simulations of 44,000 quarters (first 4,000 quarters dropped to eliminate initial conditions)

Forecast Accuracy (RMSE)

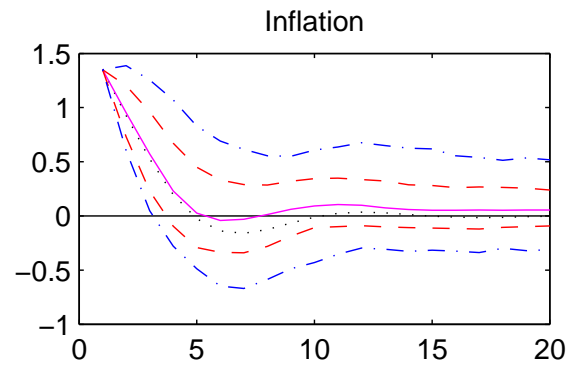
κ	Optimal Control			LWW Rule			OW Rule		
	π	u	i	π	u	i	π	u	i
VAR(3)									
0.00	1.35	0.30	0.00	1.35	0.30	0.00	1.35	0.30	0.00
0.01	1.38	0.31	0.01	1.38	0.31	0.24	1.38	0.31	0.37
0.02	1.43	0.33	0.09	1.41	0.32	0.44	1.41	0.32	0.62
0.03	1.48	0.36	0.16	1.44	0.34	0.57	1.44	0.34	0.87
VAR(2)									
0.00	1.35	0.30	0.23	1.35	0.30	0.00	1.35	0.30	0.00
0.01	1.37	0.31	0.24	1.37	0.31	0.22	1.37	0.31	0.36
0.02	1.39	0.32	0.25	1.38	0.32	0.34	1.39	0.31	0.54
0.03	1.42	0.33	0.26	1.40	0.33	0.49	1.41	0.33	0.74
VAR(1)									
0.00	1.36	0.33	0.44	1.36	0.33	0.11	1.35	0.33	0.36
0.01	1.36	0.33	0.46	1.36	0.33	0.23	1.36	0.33	0.56
0.02	1.37	0.33	0.46	1.37	0.33	0.34	1.37	0.34	0.80
0.03	1.38	0.33	0.47	1.38	0.34	0.45	1.38	0.34	0.99

Forecast Accuracy Horse Race (RMSE) (LWW Rule)

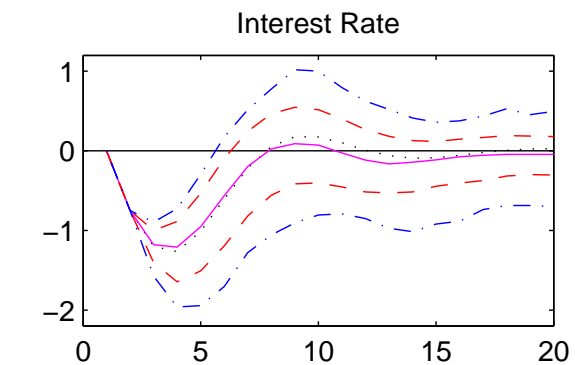
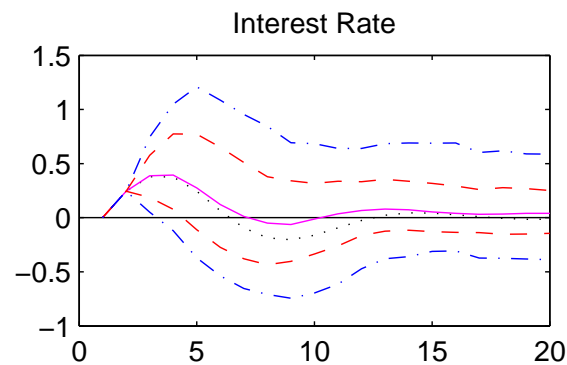
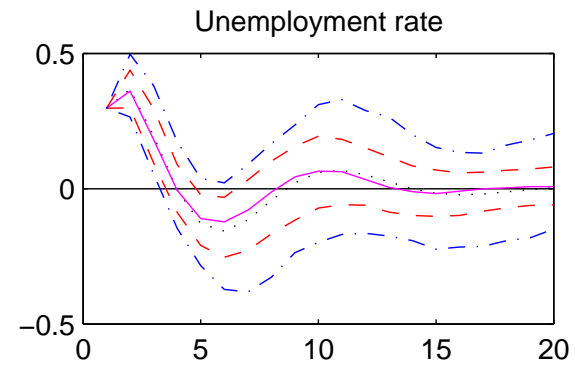
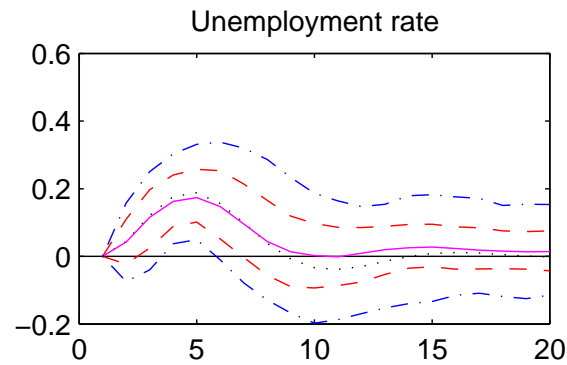
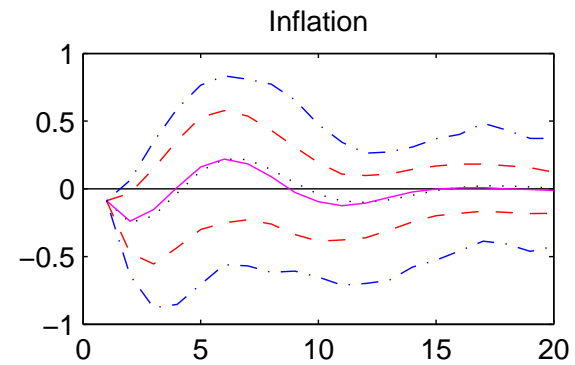
κ	Forecast Model								
	VAR(3)			VAR(2)			VAR(1)		
	π	u	i	π	u	i	π	u	i
Forecasting model used in simulation: VAR(3)									
0.00	1.35	0.30	0.00	1.35	0.30	0.00	1.36	0.36	0.11
0.01	1.38	0.31	0.24	1.37	0.32	0.26	1.39	0.38	0.37
0.02	1.41	0.32	0.44	1.40	0.34	0.43	1.41	0.39	0.54
0.03	1.44	0.34	0.57	1.43	0.36	0.57	1.44	0.42	0.70
Forecasting model used in simulation: VAR(2)									
0.00	1.35	0.30	0.00	1.35	0.30	0.00	1.36	0.36	0.11
0.01	1.38	0.30	0.18	1.37	0.31	0.22	1.38	0.38	0.29
0.02	1.41	0.32	0.31	1.38	0.32	0.34	1.39	0.40	0.42
0.03	1.45	0.33	0.46	1.40	0.33	0.49	1.42	0.42	0.58
Forecasting model used in simulation: VAR(1)									
0.00	1.36	0.32	0.18	1.36	0.34	0.19	1.36	0.33	0.11
0.01	1.38	0.30	0.12	1.37	0.30	0.13	1.36	0.33	0.23
0.02	1.41	0.31	0.20	1.39	0.31	0.21	1.37	0.33	0.34
0.03	1.45	0.32	0.30	1.41	0.32	0.30	1.38	0.34	0.45

Optimal Control Policy: Impulse Responses with Learning ($\kappa = 0.02$)

Inflation Shock



Unemployment Shock

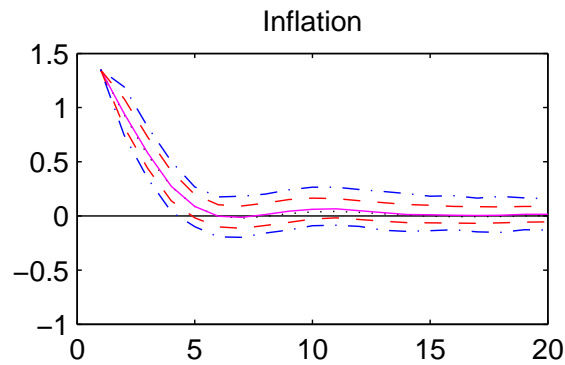


Performance of Optimal Control Policy with Learning

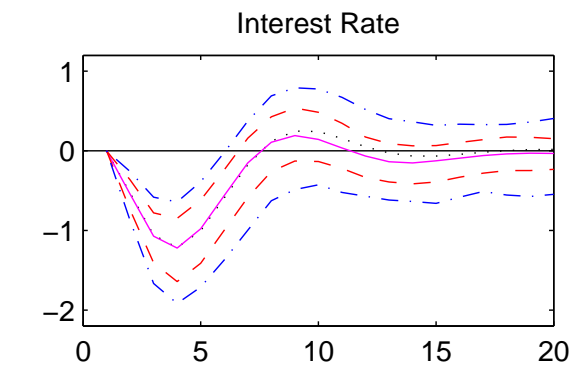
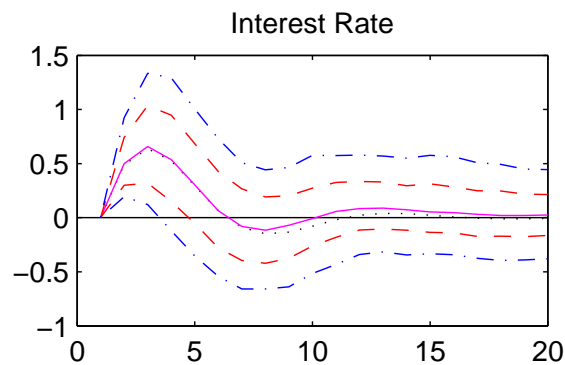
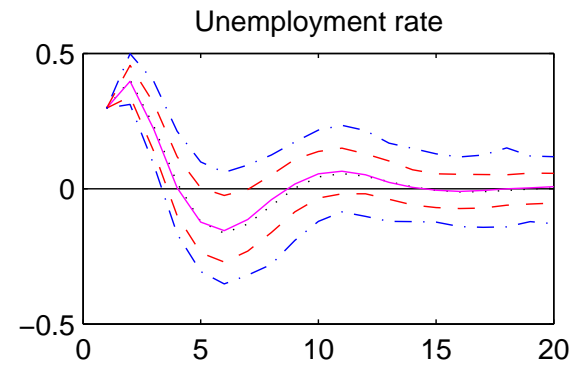
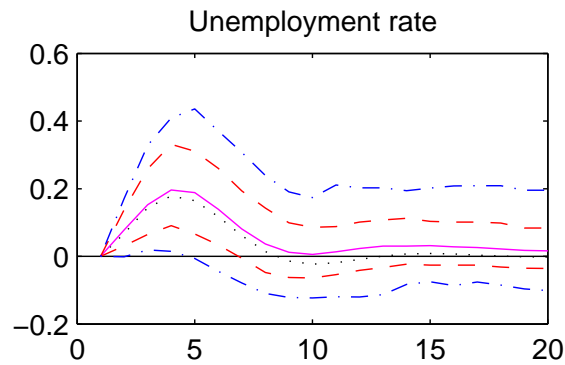
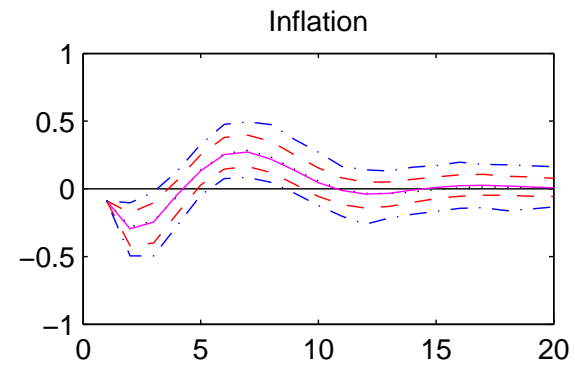
κ	Standard Deviation			Loss
	π	$u - u^*$	Δi	\mathcal{L}
VAR(3)				
0.00	1.84	0.68	1.20	6.65
0.01	2.14	0.76	1.32	8.63
0.02	2.75	0.92	1.57	13.39
0.03	3.17	1.05	1.81	17.76
VAR(2)				
0.00	1.83	0.68	1.22	6.71
0.01	2.06	0.74	1.29	8.14
0.02	2.42	0.86	1.47	10.93
0.03	2.76	0.97	1.66	14.12
VAR(1)				
0.00	1.94	0.78	1.43	8.27
0.01	2.15	0.75	1.36	8.75
0.02	2.46	0.84	1.48	11.06
0.03	2.70	0.92	1.59	13.21

LWW Rule: Impulse Responses with Learning ($\kappa = 0.02$)

Inflation Shock

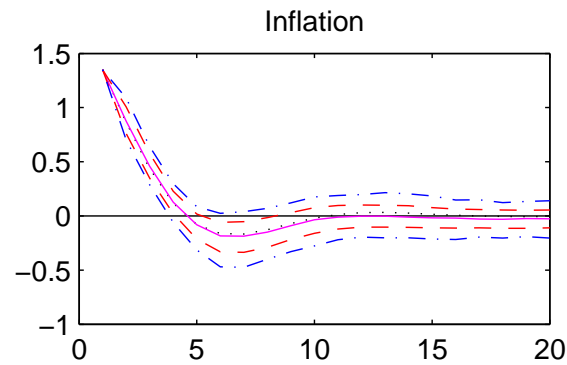


Unemployment Shock

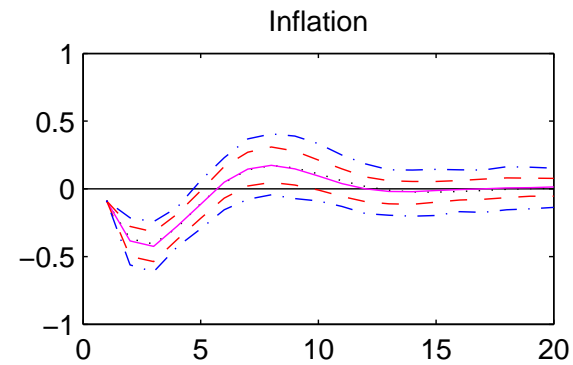


OW Rule: Impulse Responses with Learning ($\kappa = 0.02$)

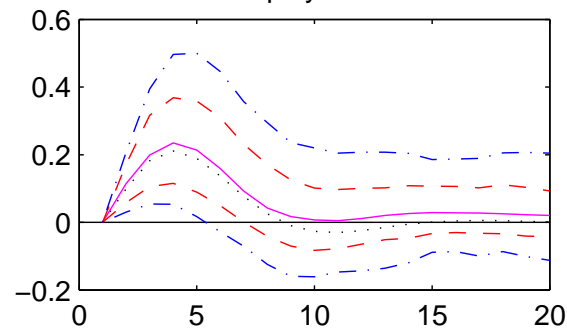
Inflation Shock



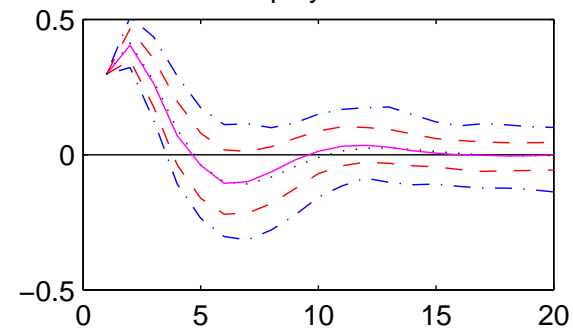
Unemployment Shock



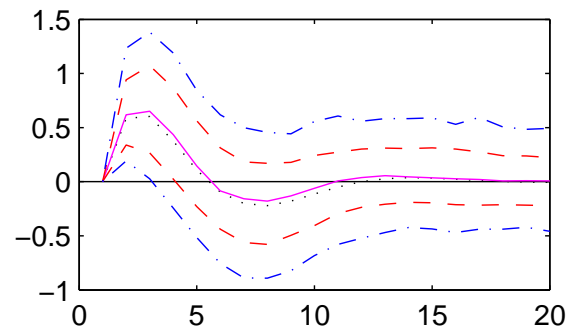
Unemployment rate



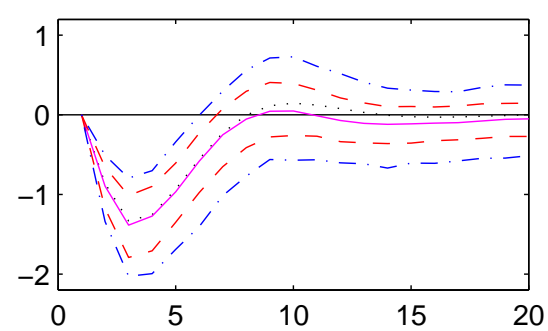
Unemployment rate



Interest Rate



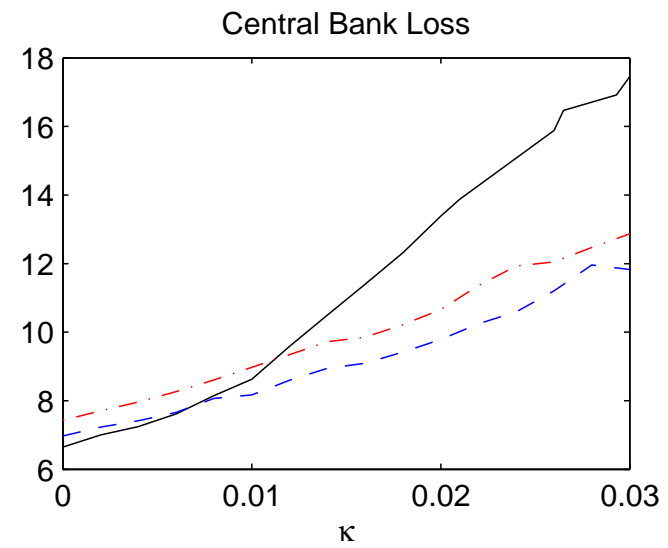
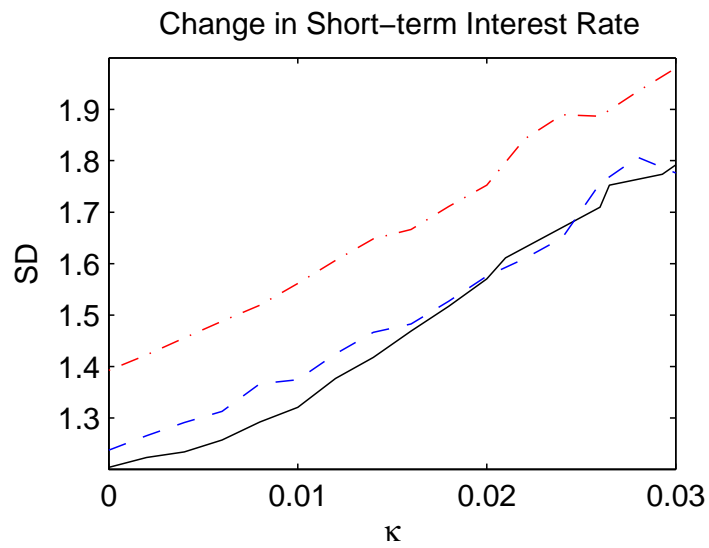
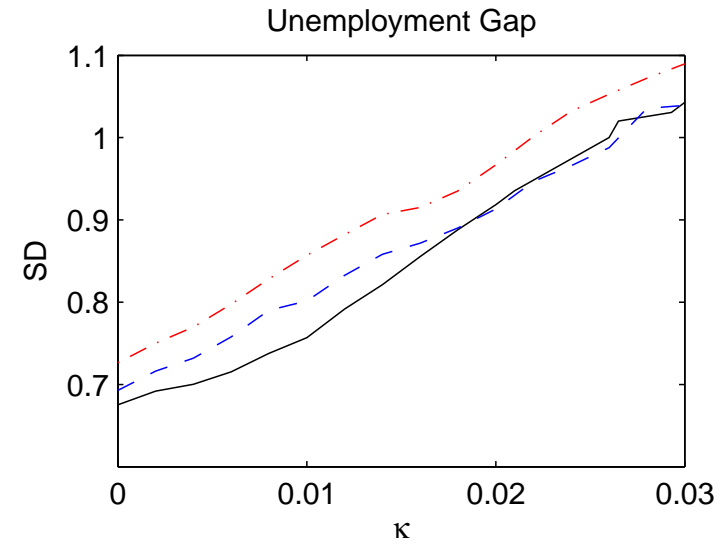
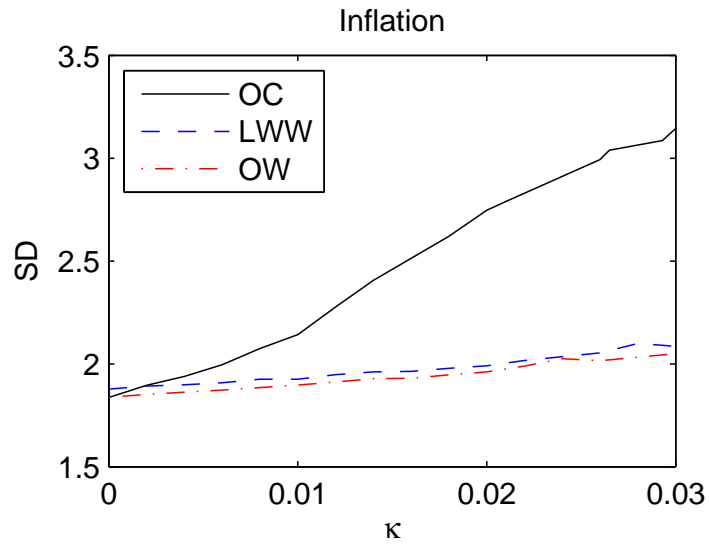
Interest Rate



Performance of Simple Rules with Learning

κ	OC	LWW Rule			OW Rule				
	Loss	Standard Deviation			Loss	Standard Deviation			Loss
	\mathcal{L}	π	$u - u^*$	Δi	\mathcal{L}	π	$u - u^*$	Δi	\mathcal{L}
VAR(3)									
0.00	6.65	1.88	0.69	1.24	6.97	1.84	0.73	1.39	7.43
0.01	8.63	1.93	0.80	1.37	8.17	1.90	0.86	1.56	8.97
0.02	13.39	1.99	0.91	1.58	9.78	1.96	0.97	1.75	10.66
0.03	17.76	2.09	1.04	1.78	11.83	2.05	1.09	1.98	12.87
VAR(2)									
0.00	6.71	1.88	0.69	1.24	6.97	1.84	0.73	1.39	7.45
0.01	8.14	1.93	0.79	1.35	8.01	1.89	0.82	1.54	8.63
0.02	10.93	1.97	0.89	1.47	9.21	1.94	0.93	1.69	10.08
0.03	14.12	2.04	0.98	1.65	10.75	2.01	1.03	1.88	11.81
VAR(1)									
0.00	8.27	1.89	0.73	1.36	7.56	1.84	0.76	1.41	7.66
0.01	8.75	1.87	0.76	1.28	7.46	1.83	0.80	1.49	8.11
0.02	11.06	1.91	0.85	1.42	8.53	1.89	0.91	1.68	9.72
0.03	13.21	1.96	0.95	1.58	9.95	1.97	1.04	1.87	11.68

Robustness to Learning



Central Bank Loss with Alternative CB Forecast Models

(Private agents forecast with VAR(3))

κ	Optimal	LWW Rule			OW Rule		
	Control	VAR(3)	VAR(2)	VAR(1)	VAR(3)	VAR(2)	VAR(1)
0.00	6.65	6.97	6.97	7.07	7.43	7.43	7.18
0.01	8.63	8.17	7.92	8.96	8.97	8.63	9.38
0.02	13.39	9.78	9.43	11.58	10.66	10.21	12.47
0.03	17.76	11.83	11.11	14.25	12.87	12.27	15.34

Conclusion and Further Research

- Optimal control policies derived under the assumption of rational expectations are not robust to alternative models of expectations formation
- Simple rules that have proven to be robust to other forms of model uncertainty are also robust to alternative models of expectations formation
- Addition of other forms of model misspecification, such as uncertainty about time-varying natural rates, amplifies our results (Orphanides and Williams 2007)