Interpreting Investment-Specific Technology Shocks*

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Abstract

Investment-specific (IST) technology shocks are often interpreted as multi-factor productivity (MFP) shocks in the investment-producing sector. However, this interpretation is valid only when some stringent conditions are satisfied. We spell out such conditions and, when relaxing some of them, show how the effects of IST shocks in a simple one-sector model differ from those of investment-sector MFP shocks in a two-sector model whose calibration is based on the U.S. Input-Output Tables. Notably, under this calibration, MFP shocks in the investment-producing sector induce a positive short-run correlation between consumption and investment, while the correlation is negative in reaction to IST shocks.

Keywords: DSGE models, Multi-Factor Productivity Shocks, Investment-Specific Technology Shocks

JEL Classification: E13, E32

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1 Introduction

Investment specific technology (IST) shocks have become a standard ingredient of dynamic stochastic general equilibrium (DSGE) models. Following a modeling short cut suggested by Greenwood, Hercowitz, and Krusell (1997) (abbreviated as GHK), these shocks have been introduced into one-sector models to distinguish equipment investment from other final use categories.[1] In particular, when allocated to equipment investment, the single undifferentiated good yields more or less installed capital depending on the level of IST but when allocated to other uses it remains unchanged. This paper is about the interpretation of IST shocks.

GHK point the way to an interpretation. Their one-sector model is a special case of a model with two-sectors, one that produces a good used only for equipment investment and another that produces a good used for both consumption and structures investment. They state conditions under which an IST shock to equipment investment in their one-sector model is equivalent to a multi-factor productivity (MFP) shock to equipment production in the corresponding two-sector model. This result, which we refer to as GHK equivalence, provides a basis for interpreting the IST shock as an MFP shock.

It may come as no surprise that the conditions for GHK equivalence are quite restrictive. As we show, they entail a production structure that differs significantly from the one implied by the U.S. Input-Output (IO) Tables. In addition, capital must be perfectly mobile between sectors. Furthermore, when present, investment adjustment costs must satisfy an unfamiliar constraint.

We investigate the effects of reasonable departures from the conditions for GHK equivalence. We use a model with two production sectors calibrated to the U.S. IO Tables and other sectoral statistics.[2] We find that sectoral MFP increases in this

[1] Throughout this paper we use the term "equipment" investment to refer to refer to what in the NIPA tables is called "Equipment and Software" investment.

[2] One of the first papers to emphasize the importance of the input-output structure for the business cycle is

two-sector model have effects that are qualitatively different from IST increases in a one-sector model even though the models are designed to match many of the same aggregate features. One important difference is that with MFP shocks, consumption is boosted at all horizons while with IST shocks consumption is reduced initially.[3]

Our model can be viewed as a generalization of the GHK model. As in GHK, there are two production sectors and three final goods. However, in our model the outputs of both production sectors are used in "assembling" all three final goods. The final goods are the same as those in GHK, equipment investment, consumption, and structures investment. The two production sectors are the machinery (M) sector and its complement, the non-machinery (N) sector. For example, equipment investment is assembled using machinery from the M sector and distribution services from the N sector. Our model has the same production structure as the GHK model in the limiting case of "complete specialization in assembly" in which M output is used only in the assembly of equipment and N output is used only in the assembly of consumption and structures. In this limiting case, the machinery sector could just as well be referred to as the equipment sector, as it is in GHK, because machinery is used for nothing else.

Our model allows for two types of real rigidities. As has become common, we allow for costs of changing investment. One of our contributions is to show that whether GHK equivalence obtains depends on the specification of these costs.[4] We also allow for costs of moving capital between sectors so that we can consider the case in which capital stocks are predetermined at the sectoral level. Of course, others have considered this case, but none of them have explored the implications

Long and Plosser (1983). More recent contributions include Hornstein and Praschnik (1997) and Edge, Kiley, and Laforte (2008)

[3]In related work, Swanson (2006) showed that MFP shocks at the sectoral level in a multi-sector model can lead to different aggregate implications from those of MFP shocks in a one-sector model.

[4]Investment adjustment costs are no a part of the model developed by Greenwood, Hercowitz, and Krusell (1997), but are common ingredient of models developed subsequently that also incorporate IST shocks.

for the interpretation of IST shocks.

The conditions for GHK equivalence can be divided into two distinct sets. Under one set, *IST shocks and sectoral MFP shocks are equivalent in a two-sector model.* There must be complete specialization in the assembly of final goods. Moreover, there are unfamiliar restrictions on the specification of investment adjustment costs. Under a second set, *a two-sector model can be reduced to a one-sector model for the determination of aggregate variables.* Importantly, there must be no costs of moving capital between production sectors in any period. Additionally, the two sectoral production functions must have the same factor intensities. Furthermore, investment adjustment cost functions must be the same across sectors.[5]

Following the empirical validation for the importance of IST shocks provided by Fisher (2006) and Smets and Wouters (2007), a growing number of papers that attempt to estimate DSGE models have included IST shocks and found them a large driver of business cycle fluctuations. However, these studies struggle with the problem that if IST shocks are prominent, they drive the unconditional correlation between investment and consumption to be counterfactually negative. For example, Justiniano, Primiceri, and Tambalotti (2008) found that IST shocks are the most important drivers of business cycle fluctuations in U.S. output and hours. Within an aggregate context, they show the comovement between investment and consumption to be positive in the data, but negative in the model. MFP shocks in the equipment sector, while sharing many features with IST shocks, have the potential to resolve this incongruence.

A good overview of the literature on comovement is provided by Christiano and Fitzgerald (1998). Recent contributions by Christiano, Ilut, Motto, and Rostagno (2008) and Jaimovich and Rebelo (2009) point respectively to consumption habits in

[5]Two other standard assumptions are required. Production functions must exhibit constant returns to scale, and adjustment costs must be homogeneous of degree zero in current and lagged investment. Also, if the model economy is to have a balanced steady-state growth path, the production functions must be Cobb-Douglas.

combination with investment adjustment costs and departures from utility functions that are additively separable in consumption and leisure as mechanisms to generate comovement, even in the face of IST shocks. Although we allow for investment adjustment costs, our two-sector model does not rely on such costs to generate comovement, nor do we depart at any point from preferences that are consistent with a balanced growth path.

In the final part of the paper we conduct a Monte Carlo experiment to investigate whether taking the our two-sector model as data-generating process, the estimated one-sector model would still imply negative comovement between consumption and investment. The results of the experiment corroborate this hypothesis and confirm that the one- and two-sector models have dramatically different implications for the correlation of consumption and investment even for the small estimation sample size typically used in a macro-econometric context.

2 The model

Our approach to the analysis of productivity changes is a combination of the growth-accounting approach based on industrial breakdowns and the DSGE approach based on final-use breakdowns. We use a closed-economy model with a representative agent that has three central features: multiple industries, multiple final goods, and multiple capital stocks.

2.1 Industries

We analyze productivity developments in two intermediate goods industries which we call the machinery (M) industry and the non-machinery (N) industry. Both industries comprise perfectly competitive firms. Consider the representative firm in industry i (where $i \in \{M, N\}$) in period s. It hires labor (N_{is}) from households at a wage (W_s) that is same for both industries because labor is perfectly mobile between industries. It also rents two types of capital from households: equipment capital (K_{is}^E) and structures capital $\left(K_{is}^S\right)$ at rentals (R_{is}^E and R_{is}^S) that are industry-specific because it is costly to shift capital of either type between industries. The firm minimizes the unit cost of producing a given amount of its industry's output (Y_{is}) subject to an industry-specific Cobb-Douglas production function

$$Y_{is} = (A_{is}N_{is})^{1-\alpha_i^E-\alpha_i^S} \left(K_{is}^E\right)^{\alpha_i^E} \left(K_{is}^S\right)^{\alpha_i^S}. \tag{1}$$

The factor shares for the two types of capital are α_i^E and α_i^S. There is an industry-specific multi-factor productivity (MFP) shock (A_{is}). Since it is competitive and there are constant returns to scale, the firm ends up selling at a price equal to unit cost. Let P_{is} represent the price of output i.[6] Since the N good is the numeraire $P_{Ns} = 1$.

2.2 Final goods

There are three final goods: a consumption good (C_s) and two investment goods, one suited for gross investment in E capital stocks (J_s^E) and one suited for gross investment in S capital stocks (J_s^S). These goods are assembled by perfectly competitive final goods firms using intermediate goods as inputs. The assembly function for C_s is a constant elasticity of substitution (CES) function of consumption inputs of M goods (C_{Ms}) and of N goods (C_{Ns}):

$$C_s = \left[\phi_M^C \left(\frac{C_{Ms}}{\phi_M^C}\right)^{\frac{\sigma^C-1}{\sigma_C^C}} + \phi_N^C \left(\frac{C_{Ns}}{\phi_N^C}\right)^{\frac{\sigma_C-1}{\sigma_C}}\right]^{\frac{\sigma_C}{\sigma_C-1}}, \tag{3}$$

[6]For example, P_N is the mulitiplier in the Lagrangian expression ($\mathcal{N}_N$)used to the minimize costs of producing a given quantity $\bar{Y}_N$:

$$\mathcal{N}_N = WN_N + R_N^E K_N^E + R_N^S K_N^S + P_N \left\{\bar{Y}_N - (A_N N_N)^{1-\alpha_N^E-\alpha_N^S} \left(K_N^E\right)^{\alpha_N^E} \left(K_N^S\right)^{\alpha_N^S}\right\}, \tag{2}$$

where time subscripts have been omitted for simplicity where ϕ_M^C and ϕ_N^C are the weights for M and N goods, and σ^C is the elasticity of substitution between M and N goods in the assembly of C_s.

The assembly functions for J_s^E and J_s^S are CES functions of investment inputs of M goods (I_{Ms}^E, I_{Ms}^S) and of N goods (I_{Ns}^E, I_{Ns}^E):

$$J_s^E = \left[\phi_M^E \left(\frac{I_{Ms}^E}{\phi_M^E} \right)^{\frac{\sigma_E - 1}{\sigma_E}} + \phi_N^E \left(\frac{I_{Ns}^E}{\phi_N^E} \right)^{\frac{\sigma_E - 1}{\sigma_E}} \right]^{\frac{\sigma_E}{\sigma_E - 1}} \tag{4}$$

$$J_s^S = \left[\phi_M^S \left(\frac{I_{Ms}^S}{\phi_M^S} \right)^{\frac{\sigma_S - 1}{\sigma_S}} + \phi_N^S \left(\frac{I_{Ns}^S}{\phi_N^S} \right)^{\frac{\sigma_S - 1}{\sigma_S}} \right]^{\frac{\sigma_S}{\sigma_S - 1}} \tag{5}$$

where $\phi_M^E, \phi_N^E, \phi_M^S$ and ϕ_N^S are the weights given to M and N goods, and σ_S and σ_E are the elasticities of substitution between M and N goods.

The assembly firms minimize the unit cost of producing what they sell and because they are perfectly competitive end up selling final goods at prices that are equal to these costs which are indicated by P_s^C, $P_s^{J^E}$, and $P_s^{J^S}$.[7] We assume that the assembly functions for both C_s and J_s^S are N- intensive relative to the function for J_s^E

2.3 Tastes and the budget constraint

In period t, the representative agent supplies a fixed amount of labor $\bar{N}$ and maximizes the intertemporal utility function

$$\sum_{s=t}^{\infty} \beta^{s-t} \frac{(C_s)^{1-\gamma} - 1}{1-\gamma}. \tag{7}$$

The agent also chooses holdings of a single bond (B) denominated in the N industry good (the numeraire good for the model). In addition, the agent chooses investment

[7]For example, P^C, is the multiplier in the Lagrangian expression ($\mathcal{N}_C$) used to the minimize costs of producing a given quantity $\bar{C}$:

$$\mathcal{N} = P_M C_M + C_l + P^C \left\{ \bar{C} - \left[\phi_M^C \left(\frac{C_M}{\phi_M^C} \right)^{\frac{\sigma^C - 1}{\sigma_C^C}} + \phi_N^C \left(\frac{C_N}{\phi_N^C} \right)^{\frac{\sigma_C - 1}{\sigma_C}} \right]^{\frac{\sigma_C}{\sigma_C - 1}} \right\}, \tag{6}$$

where time subscripts have been omitted for simplicity in each of four capital stocks ($J^E_{Ms}, J^E_{Ns}, J^S_{Ms}$, and J^S_{Ns}) and how to distribute the quantity of capital of a given type between industries ($K^E_{Ms}, K^E_{Ns}, K^S_{Ms}$, and K^S_{Ns}, as well as $D^E_{Ms}, D^E_{Ns}, D^S_{Ms}$, and D^S_{Ns}).

The agent is subject to period budget constraints. In each period, factor income plus income from bonds held in the previous period must be at least enough to cover purchases of final goods, consumption goods and the two types of investment goods as well as bonds:

$$\begin{aligned} &P_{M_s}Y_{Ms} + Y_{Ns} + \rho_{s-1}B_{s-1} = \\ &P_{Cs}C_s + P_{J^E_s}\left(J^E_{Ms} + J^E_{Ns}\right) + P_{J^S_s}\left(J^S_{Ms} + J^S_{Ns}\right) + B_s. \end{aligned} \tag{8}$$

Since we assume perfect competition in the M and N industries, factor income is equal to the value of production. The term ρ_s is the gross return on bonds which are denominated in N goods.

The agent is also subject to technological constraints on accumulating capital. It is costly to move capital of either type between industries, so there are four distinct stocks of capital. Households enter the current period with four inherited capital stocks, two stocks of equipment $D^E_{is}, i = M, N$ and two of structures $D^S_{is}, i = M, N$. Each inherited stock is suitable for only one industry and each is the sum of what is left after depreciation plus investment:

$$\begin{aligned} &D^E_{is} = \left(1 - \delta^E\right) K^E_{is-1} \\ &+Z^E_{is}J^E_{is-1}\left\{1 - \frac{\nu^E_0}{2}\left[\frac{J^E_{is-1}}{J^E_{is-2}}\left(\frac{Z^E_{is-1}}{Z^E_{is-2}}\right)^{\nu^E_1} - 1\right]^2\right\}, \quad i \in \{M, N\}, \end{aligned} \tag{9}$$

$$\begin{aligned} &D^S_{is} = \left(1 - \delta^S\right) K^S_{is-1} + \\ &Z^S_{is}J^S_{is-1}\left\{1 - \frac{\nu^S_0}{2}\left[\frac{J^S_{is-1}}{J^S_{is-2}}\left(\frac{Z^S_{is-1}}{Z^S_{is-2}}\right)^{\nu^S_1} - 1\right]^2\right\}, \quad i \in \{M, N\}, \end{aligned} \tag{10}$$

where it is assumed that depreciation rates (δ^E and δ^S) and investment adjustment-cost parameters (ν^E and ν^S) may differ between type of capital but are the same across industry of use.

Finally, the household is subject to technological constraints on shifting capital of a given type between industries. By incurring an increasing marginal cost, capital of a given type can be shifted between industries before it is used in production. For example, equipment capital in the M industry can be shifted to the N industry. Therefore, the capital of type j actually available for production in period s depends on how much has been shifted between industries:

$$\begin{aligned} K^j_{Ms} + K^j_{Ns} \quad &= D^j_{Ms}\left[1 - \frac{\omega^j}{2}\left(\frac{K^j_{Ms}}{D^j_{Ms}} - 1\right)^2\right] \\ &+ D^j_{Ns}\left[1 - \frac{\omega^j}{2}\left(\frac{K^j_{Ns}}{D^j_{Ns}} - 1\right)^2\right], \qquad j \in \{E, S\}. \end{aligned} \tag{11}$$

The distinction between capital inherited from the previous period, D, and capital allocated to production, K, allows us to nest in the same model the special case of capital predetermined at the *sectoral* level, and the special case of capital predetermined at the *aggregate* level.

2.4 Market clearing

For goods markets, market clearing requires that the supplies of intermediate goods be used up in the assembly of final goods:

$$Y_{Ms} = C_{Ms} + I^E_{Ms} + I^S_{Ms}, \qquad Y_{Ns} = C_{Ns} + I^E_{Ns} + I^S_{Ns},$$

that

$$J^E_s = J^E_{Ms} + J^E_{Ns}, \qquad J^S_s = J^S_{Ms} + J^S_{Ns},$$

and that all the C_s, J^E_s, and J^S_s goods that are assembled are purchased.

Market clearing in factor markets requires that labor supply equal labor demand:

$$N_{Ms} + N_{Ns} = \bar{N}. \tag{12}$$

and that firms demand for $K^E_{Ms}, K^E_{Ns}, K^S_{Ms}$, and K^S_{Ns} exhausts households' supply.

3 Equivalence

We list below general conditions under which the aggregate effects of an MFP shock in the investment-producing sector of the multi-industry model can be reproduced by an IST shock in an aggregate one-industry model. A proof that these conditions are necessary and sufficient for equivalence is given in an appendix. The conditions we identify broaden the examples provided by Greenwood, Hercowitz, and Krusell (1997) and Greenwood, Hercowitz, and Krusell (2000) who confined the analysis to Cobb-Douglas production functions. Furthermore, our proof considers the introduction of investment adjustment costs and spells out the role of complete sectoral specialization in the delivery of the final consumption and investment goods.

This section also presents simulation results illustrating the equivalence between the effects of IST and MFP shocks. The calibration used is such that equivalence holds. We will use results from this simulation as a benchmark against which to compare results from calibrations that do not meet the requirements for equivalence by breaking the conditions spelled out below.

3.1 Conditions for equivalence

IST shocks are typically embedded in one-sector models without attempting to characterize explicitly the investment-producing sector, even though their interpretation is as an MFP shock specific to investment-producing sector.[8]

The first set of assumptions concerns the equivalence in a two-sector model between an MFP shock in the machinery sector (A_{Ms}) and IST shocks that push up the effectiveness of equipment investment in both sectors (Z^E_{Ms} and Z^E_{Ns}).

Assembly of the investment good that gets the IST shock is completely special-

[8]The Appendix A provides a proposition that substantiates the claims made in this section for a static version of our model where only production decisions are considered. The proof for the static version could be extended to a dynamic setting but we resort to numerical simulations instead.

ized. In other words, the output of the industry that experiences the MFP shock in the twp-sector model is used only in the assembly of the investment good that is hit by the IST shock in the one-sector model.[9]

If there are adjustment costs for investment, they apply to changes in effective units, not changes in physical units. Cost of installing computers faster depends not on how many are installed but on how efficient they are.

The second set of assumptions is needed to rewrite a two-sector model as a one-sector model—whose solution is formulated in terms of aggregate variables only, but from which the sectoral details could be retrieved. The assumptions needed to accomplish this task are the following:

1. Production functions in all industries are identical and exhibit CRTS. If capital stocks are predetermined, as is usually assumed, they are perfectly mobile among industries.
2. The depreciation rates of capital stocks and investment adjustment costs are identical across sectors.

Under these conditions, and with the restriction that $Z^j_{Ms} = Z^j_{Ns} = Z^j_s$ with $j \in \{E, S\}$, sectoral capital stocks and capital accumulation equations can be aggregated as:

$$K^j_s = K^j_{Ms} + K^j_{Ns}, \tag{13}$$

$$K^j_s = \left(1 - \delta^j\right) K^j_{s-1} + Z^j_s J^j_{s-1} \left\{ 1 - \frac{\nu^j_0}{2} \left[\frac{J^j_{s-1}}{J^j_{s-2}} \left(\frac{Z^j_{s-1}}{Z^j_{s-2}} \right)^{\nu^j_1} - 1 \right]^2 \right\}. \tag{14}$$

[9]Even though it is standard to assume specialization in assembly in DSGE model, in fact several intermediate goods are often used in assembling a final good. In particular, the final-use equipment investment as it appears in the NIPA is a combination of machinery with transportation and distribution.

3.2 Calibration ensuring equivalence

The parameter choices for the simulation that illustrates equivalence between IST and MFP shocks if the conditions in our proof are summarized in Table 1. To facilitate comparisons with previous work on IST shocks, whenever possible, we adhere to the calibration choices of Greenwood, Hercowitz, and Krusell (1997)[10]. Accordingly, for the baseline calibration, the output share of equipment in both the M and N sectors is 17% and the share of structures is 13%. The parameters governing the assembly functions are set so that there is complete specialization: consumption and structures investment comprise inputs from the N sector only, while equipment investment is produced through inputs from the M sector only.[11] The elasticity of substitution between factor inputs in the production of M and N goods is taken to be 1. Following Greenwood, Hercowitz, and Krusell (1997) the depreciation rates for equipment and structures capital are 5.6% per quarter and 12.4% per quarter respectively. The discount factor is set at 0.99, consistent with an annualized real interest rate of 4%. Finally, the intertemporal substitution elasticity for consumption is taken to 1.

Through the Dynare set of programs we obtain a linearization of the necessary conditions for an equilibrium and a linear approximation to the model's decision rule.

3.3 A Numerical Illustration

Figures 1 and 2 show the effects of two distinct shocks. The solid lines relate to a shock that pushes up the level of both Z^E_{Ms} and Z^E_{Ns} equally and permanently. In this case, we could have cut off the model's sectoral details following Greenwood,

[10]For simplicity, we abstract for trend growth, capital and labor taxes, while Greenwood, Hercowitz, and Krusell (1997) incorporate them in their model.

[11]The substitution elasticities between factor inputs in assembly become irrelevant under complete specialization.

Hercowitz, and Krusell (1997), and have simply obtained the aggregate responses from a canonical one-sector RBC model augmented with IST shocks in the capital accumulation equation, Z_s^E as shown in Equation (14).

The dashed lines relate instead to a permanent MFP shock that pushes up the level of A_{Ms}, the technology for the production of the M sector good. In all the figures presented, the sizes of the shocks are normalized so that aggregate output (in quality-adjusted units at constant prices) increases by 1 percent in the long run.[12]

As implied by the calibration, M sector goods are used to assemble equipment investment only. The baseline calibration also implies that these shocks will produce equal effects on the aggregate variables shown in Figure 1 as the requirements for aggregate equivalence between IST shocks and MFP shock in the machinery sector (discussed above) are satisfied.

The capital accumulation process adds persistence to the effects of the shocks so that output takes a considerable number of quarters to approach its new steady state level. The top two panels in the figure show the output response, but focus on different horizons so as to allow inspection of both the short- and long-run effects.

Both shocks make it possible to produce equipment investment with smaller amounts of factor inputs, regardless of which sector receives the investment. Were it not for investment adjustment costs, the substitution effect associated with the shocks, would be so strong as to immediately build up the equipment and structures capital stock in the M sector. In that case, labor and both kinds of capital inputs would immediately drop in the N sector to contribute to the buildup in the M sector. Without investment adjustment costs, consumption would drop on impact, and then increase as higher production in the M sector could push up the equipment capital stock in the N sector. However, with quadratic adjustment costs

[12] In multi-sector models there are multiple ways of aggregating sectoral outputs depending, for instance, on which good is chosen as the numeraire. We focus on a measure of aggregate output that sums sectoral outputs at constant prices after adjusting for quality. This measure is defined as $Y_{CPs} = C_{Ms} + C_{Ns} + Z_{Ms}^E J_{Ms}^E + Z_{Ns}^E J_{Ns}^E + Z_{Ms}^S J_{Ms}^S + Z_{Ns}^S J_{Ns}^S$. This is an approach commonly adopted in practice by statistical agencies.

in investment, it becomes costly to ramp up the production of equipment investment, slowing down the transfer of factor inputs across sectors. Instead of spiking up, aggregate investment acquires a hump shape. Conversely, consumption declines more gradually.

The consumption share of output takes a long time to recover as shown in Figure 2. According to the baseline calibration, N sector goods are the sole input in the assembly of consumption. First, N sector output goes down, as factor inputs are moved to the sector that received the shock. Then, part of N sector output is devoted to push up the N sector's stock of structures.

Finally, notice that the response of output in the M sector can be used to differentiate the source of the shock for each of the lines shown in Figures 1 and 2. The bottom right panel of 2 depicts M sector output in physical units (*not quality-adjusted*). The IST shock allows lower physical output to yield the same amount of equipment capital.

4 Departures from equivalence

The simulations that follow illustrate that the effects of an IST shock in a one-sector model and MFP shocks in the machinery sector of a two-sector model can differ substantially as the conditions for equivalence summarized in Section 3.1 are relaxed. A key role is shown to be played by incomplete sectoral specialization in the assembly of final goods, reflecting the U.S. input-output tables.

4.1 Comparing MFP shocks in a two-sector model with IST shocks in a one-sector model

Figure 3 compares again the effects of IST shocks in a model use calibration satisfies the conditions for aggregation spelled out in Section 3.1 with the effects of an MFP shock in a model that does not meet those conditions. For ease of comparison, the solid lines in Figure 3 reproduces those shown in Figure 1. The dotted lines, show instead the effects of an IST shock in a model that satisfies all conditions for equivalence, but for the for of the investment adjustment costs. In that case, by setting the parameters ν_1^S, ν_1^E equal to 0, the adjustment cost for investment are specified in physical units, instead of quality-augmented, effective units. Specifying adjustment costs in physical units temporarily lowers the cost of adjustment relative to the specification in quality-adjusted units in reaction to IST shocks. The difference is largest in the first period, as can be seen comparing the solid and dotted lines in Figure 3.

This change alone has no bearing on MFP shocks in the machinery sector (which would still yield the solid lines). Thus, the figure implies that the specification of investment adjustment costs can, by itself, drive a wedge between the effects of aggregate IST shocks and of MFP shocks in the machinery sector of a two-sector model.

We should note that neither Greenwood, Hercowitz, and Krusell (1997) nor Greenwood, Hercowitz, and Krusell (2000) incorporate investment adjustment costs. However, papers that attempt to capture the importance of IST shocks for the business cycle routinely follow Christiano, Eichenbaum, and Evans (2005) (who instead do not consider IST shocks) by incorporating investment adjustment costs expressed in physical units, even within the context of an aggregate one-sector model.[13]

4.2 Alternative calibration

The calibration used for the MFP shock denoted by the dashed lines in Figure 3, while maintaining aggregate shares constant, departs from that used in constructing the IST shock along the three dimensions listed below.

1. *Predetermined capital stocks*

[13]For examples, see Smets and Wouters (2007), or Christiano, Motto, and Rostagno (2007).

By setting $\omega^E = \omega^S = 100$ capital stocks become predetermined in each sector. As such, capital allocations do not vary in the shocks' impact period as is typical of capital in one-sector models.

2. *Sector-specific production functions*

   Following Greenwood, Hercowitz, and Krusell (1997), the baseline calibration implies identical production functions across sectors. However, for the three factor inputs in the model, U.S. data imply different input intensities across the equipment (M sector in the model) and non-equipment sectors (N sector in the model).

   To differentiate the intensities of factor inputs across sectors we used the following restrictions: a) while allowing variation across sectors, the aggregate factor input intensities are the same as in Greenwood, Hercowitz, and Krusell (1997); b) factor payments are equalized across sectors, making the factors' shares of sectoral output proportional to the sectoral stocks of capital; c) factor input intensities are equal regardless of where an industry's output is used.

   We combined data for the net capital stock of private nonresidential fixed assets from the U.S. Bureau of Economic Analysis, with data from the Input-Output Bridge Table for Private Equipment and Software. The first dataset contains data on the size of equipment and non-equipment capital stocks by industry. The second dataset allowed us to peer on the commodity composition of private equipment and software. Finally, we used BEA data to establish an industry's value added output. We focused on the year 2004, the latest available at the time of writing, but similar sector-specific production functions would be implied by older vintages of data.

   Our calculations show that the equipment-producing sector is less intensive in structures and labor than the aggregate economy, but more intensive in equipment capital. For the equipment sector, the share of structures is 11 percent, the labor share 46 percent, and the share of equipment capital the remainder 43 percent (thus, $\alpha_M^S = 0.11$,$\alpha_M^N = 0.46$, $\alpha_M^E = 0.43$). For the non-equipment sector the share of structures is 13 percent, the share of labor 72 percent, and the share of equipment capital 15 percent.

3. *Incomplete specialization*

   The baseline calibration assumes complete specialization in the production of investment and consumption goods. Equipment investment is assembled by using output from the M sector only. Conversely, structures investment and consumption goods are assembled using output from the N sector only. This complete degree of specialization is a simplification of the richer composition of equipment investment and consumption, as captured by the U.S. Input-Output Tables. Specifically, using 2004 data, one can see that a substantial input into the making of both equipment investment, as well as structures investment and consumption are retail and wholesale services, accounting for 15 percent of the total output of private equipment and software. Furthermore, electric and electronic equipment also contribute to the making of consumption, accounting for 4 percent of the total.

   The model captures this co-mingling implied by the Input-Output Tables through assembly functions that combine the output of the M and N sectors to produce consumption, structures investment, and equipment investment. The alternative calibration used to construct Figure 4 sets the parameters in the consumption assembly function to $\phi_M^C = 0.04, \phi_N^C = 0.96$ and for equipment investment $\phi_M^E = 0.85, \phi_N^E = 0.15$.

4.3 The effects of MFP shocks under the alternative calibration

The size of the MFP shock hitting the M sector shown in Figure 3 was chosen again to bring about a permanent 1 percent increase in aggregate output.

Some key differences between the IST and MFP shocks can be captured decomposing the responses of consumption into substitution and wealth effects. The bottom left panels of the figure show the Hicksian decomposition laid out by King (1991) for general equilibrium models. For this decomposition the change in utility ΔU is computed in the following way:

$$\Delta U = E_t \sum_{t=0}^{\infty} \beta^t C^{1-\gamma} \hat{C}_t \tag{15}$$

where a caret symbol denotes a variable in log deviation from its steady state. The wealth effect on consumption is given by the change in consumption that would yield the sane change in utility as that generated by the shock, while the real interest rate is kept constant at its steady state value. Accordingly, the Euler equation for consumption for the model implies that the wealth effect on consumption $\tilde{C}_t$ is constant over time and equal to:

$$\tilde{C}_t = (1-\beta)\frac{\Delta U}{C^{1-\gamma}} \tag{16}$$

The substitution effect is the path of consumption that would induce no change in utility in reaction to the interest rate changes induced by the shock. Accordingly, the substitution effect on consumption, $\tilde{\tilde{C}}_t$, is the path of that solves the system:

$$0 = E_t \sum_{t=0}^{\infty} \beta^t C^{1-\gamma} \tilde{\tilde{C}}_t \tag{17}$$

$$E_t \tilde{\tilde{C}}_{t+1} = \tilde{\tilde{C}}_t + \frac{1}{\gamma}\hat{R}_t \tag{18}$$

where $\hat{R}_t$ is expressed as the difference of the interest rate from its steady state value. Simple algebraic manipulations yield that $\tilde{\tilde{C}}_0 = -\frac{\beta}{\gamma}\sum_{t=0}^{\infty}\beta^t \hat{R}_t$, which allows one to solve for the full path of the substitution effect by combining knowledge of $\tilde{\tilde{C}}_0$ with Equation (18) above.

A common feature among changes implied by the alternative calibration is a reduction in the magnitude of the substitution effect on consumption associated with the MFP shock.

With capital predetermined at the sectoral level, because of complementarities more of the factor inputs remain locked in the N sector, reducing the substitution effect associated with the MFP shock. This props up the response of consumption, as its composition is intensive in the output of the N sector. Similarly, structures and equipment capital take longer to shift back and forth across sectors, making the response of aggregate investment more subdued.

Under the alternative calibration, with sector specific production functions, the making of M sector goods used in equipment investment is more intensive in equipment capital relative to the aggregate. This feature contributes to the reduction in the substitution effect on consumption coming from the MFP shock relative to the IST shock. Accordingly, M sector output and investment increase by less at first.

Finally, the incomplete specialization in the assembly of equipment investment not only reduces the magnitude of the substitution effect but also boosts the wealth effect. Relaxing the assumption of complete sectoral specialization implies that the MFP shock in the M sector acquires a direct effect on consumption through the assembly function.

Altogether, the weaker substitution effect and stronger wealth effect lead to a uniform rise in consumption in reaction to the MFP shock (while consumption temporarily falls for the IST shock) and a corresponding reduction in the rise of investment relative to the effects of the IST shock. The cumulative effect of the departures from the baseline calibration is to decouple qualitatively the responses to IST and MFP shocks as seen comparing the dashed and dotted lines in Figure 3. While in the case of an IST shock in the one-sector model, consumption falls initially, the response of the two-sector model to the MFP shock in the equipment investment sector is such that consumption never falls. Similarly, aggregate investment shows protracted differences, with the response to the IST shock in the one-sector model being persistently more pronounced than the response to the MFP shock in the two-sector model.

4.4 Isolating the role of incomplete specialization

While all of the departures from the baseline aggregate calibration are important in overturning the conditional correlation between consumption and investment implied by MFP shocks in the investment sector, a key role is played by incomplete sectoral specialization in the production of final goods.[14] Figure 4 compares again the effects of an IST shock in a one-sector model with an MFP shock in a two-sector model. The solid lines denoting the effects of the IST shock replicate what is also shown in Figure 1. The calibration used in constructing the effects of the MFP shock in the M sector departs from the aggregate calibration summarized in Table 1 only insofar as it accounts for incomplete sectoral specialization in the production of final goods, as described in Section 4.2. With the baseline calibration for investment adjustment costs, this change alone is sufficient to reverse the short-term correlation between investment and consumption.

5 Sensitivity analysis: investment adjustment costs

High adjustment costs for investment and the introduction of consumption habits, by stifling adjustment have the potential to alleviate the negative correlation between consumption and investment following IST shocks. Simulation results presented in Figure 5 show that the change in correlation between consumption and investment implied by the two-sector model does not stem from the calibration of the investment adjustment cost.

The solid line shows again the effects of an aggregates IST shock as in Figure 1. We depart from the calibration described in Table 1 only insofar as we eliminate the investment adjustment costs by setting $\nu_0^E = \nu_0^S = 0$. As investment can now jump

[14]Appendix B isolates the role of capital being predetermined at the sectoral level and of sector-specific production functions in distancing the effects of MFP shocks in the machinery sector from those of IST shocks in a one-sector model.

on impact, the negative correlation between consumption and investment becomes stronger.

The dashed lines show the effects of an MFP shock. The only departure from the calibration used in Figure 3 and summarized in Table 2 is again the elimination of investment adjustment costs. Even without investment adjustment costs, consumption never falls in reaction to an MFP shock in the equipment-producing sector.

The dotted line in the figure reproduces the effects of the MFP shock shown in Figure 4 in that the calibration departs from the one shown in Table 1 by allowing for incomplete specialization. Furthermore, investment adjustment costs are turned off. The dotted line shows that consumption turns negative on impact. While incomplete specialization plays an important quantitative role in reducing the negative correlation between consumption and investment following shocks to the equipment-producing sector, by itself it cannot reverse it.

6 Investigating the Correlation between Consumption and Investment

As illustrated above, in a one-sector model a permanent increase in the level of investment-specific technology would initially boost investment while compressing consumption. Conversely, in a two-sector model, the corresponding increase in MFP specific to the machinery sector under our benchmark calibration would imply positive conditional comovement between consumption and investment. Economy-wide MFP shocks imply positive conditional comovement of consumption and investment in both the one sector and two-sector models. Thus, such MFP shocks, if important enough, could account for the unconditional positive correlation between consumption and investment observed for the United States.

We investigate this question using the empirical evidence provided by Fisher

(2006), who separately identified economy-wide MFP shocks, productivity shocks specific to the machinery sector (equipment in Fisher's terminology), and other shocks. Fisher (2006) augmented the long-run identification scheme of Gali (1999) so as two distinguish between two kinds of productivity shocks. All productivity shocks are identified by the assumption that they are the only ones to affect the level of labor productivity in the long run. Technology shocks that are specific to the investment sector are separated out from other productivity shocks by the assumption that they are the only ones to affect the relative price of investment in the long run. Interestingly, the sector-specific productivity shocks identified by Fisher could be interpreted as capturing either IST shocks in a one-sector model, or MFP shocks specific to machinery sector in our two-sector model, as both models satisfy the identifying assumptions.

We calibrate the relative importance of the two types of productivity shocks by matching the variance decomposition for output at business cycle frequencies estimated by Fisher (2006). Accordingly, overall MFP shocks account for 35 percent of the variation of output, and productivity shocks specific to the investment sector (or IST shocks in the one sector model) the remainder 65 percent. When matching these statistics, the unconditional correlation between consumption and investment at business-cycle frequencies is -0.19 for the one-sector model and 0.16 for the two-sector model. Similar correlations obtain in the model for the unfiltered quarterly growth rates of consumption and investment: -0.17 for the one-sector model and 0.17 for the two-sector model. Using U.S. data, we estimate the level correlation between consumption and investment at 0.68 and the correlation between the growth rates at 0.20. [15]. The two-sector model matches the sign of the level correlation and is strikingly close to the data for the correlation of the growth rates.

While the one-sector model is not able to deliver a positive correlation between

[15] Both correlations reported were constructed with data from Table 1.1.6 of the National Income and Product Accounts for the period between 1947:q1 and 2009:q2.

consumption and investment with plausibly calibrated investment and MFP shocks, it is still possible that the one-sector mispecification bias might imply a different shock decomposition that could lead to reproduce the correlations observed in the data. To investigate this possibility we relied on a Monte Carlo exercise. In the exercise, the two-sector model is the data-generating process. The model was used to generate 100 replications of a sample containing 200 observations, the typical size of an estimation sample using aggregate time-series data for the United States. The data-generating process adhered to Fisher's estimates for the relative importance of the two technology shocks. For each sample we used a maximum likelihood estimator to estimate the standard deviation of the overall MFP shock and the IST shock. Even though the data-generating process excluded consumption shocks, we considered them at the estimation stage as a way for the the one-sector model to reconcile the positive correlation between consumption and investment implied by data-generating process. We performed the estimation for both the one-sector model and the two-sector model keeping all other parameters as set in tables 1 and 2 respectively. The level of consumption and investment were the two observed variables. In the estimation model we also allowed for a consumption preference shock and estimated its standard deviation and AR(1) persistence parameter.

Figure 6 shows the sampling distributions for the correlation between consumption and investment at business cycle frequencies implied by the Monte Carlo experiment. Figure 7 shows the sampling distributions for the correlation between the unfiltered quarterly growth rates of consumption and investment from the Monte Carlo experiment. As is evident from the figures, even allowing for mispecification bias in the importance of the non-neutral technology shocks and for consumption shocks, the one sector model still implies a negative correlation between consumption and investment. The correlations for consumption and investment for the two-sector model serve as a frame of comparison. Interestingly they are tightly clustered around the pseudo-true values (denoted by the vertical lines in the figures) implied by the data-generating process. Such tight clustering confirms that the two-sector model has observationally different implications from the one sector model as far as the correlation between consumption and investment is concerned.

7 Conclusion

The incorporation of investment specific technology shocks is becoming a standard feature of aggregate DSGE models. Not only did Greenwood, Hercowitz, and Krusell (1997) make the case that there is a separate technology trend in the sector producing equipment investment, but Greenwood, Hercowitz, and Krusell (2000) and Fisher (2006) have argued that investment-specific technology shocks play an important role in explaining the behavior of aggregate variables over the cycle.

The convenient shortcut for the treatment of technology shocks in the machinery sector within the context of a one-sector model suggested by Greenwood, Hercowitz, and Krusell (1997) provides an interpretation for investment specific technology shocks as MFP shocks in the machinery producing sector of a two-sector model. However, we have shown that these assumptions contradict some implications that can be drawn from the U.S. Input-Output Tables for the calibration of two-sector models: namely, 1) the machinery sector is more intensive in the use of equipment capital than the rest of the economy, 2) both commodities from the machinery sector and the non-investment sector are comingled to deliver equipment investment to its final use. When allowing for these features extrapolated from the U.S. Input-Output Tables, together with sector-specific capital stocks and investment adjustment costs in physical units, we showed that MFP shocks in the machinery sector of our model (principally responsible for the production of equipment investment) can have very different aggregate implications from those of an IST shock in a one-sector model. In reaction to the latter consumption initially falls, while in reaction to the former consumption rises through time generating aggregate comovement.

Given the differences between sector-specific MFP shocks in a two-sector model and IST shocks in a one-sector model, we conclude that the more detailed two-sector model is better suited to assess alternative sources of economic fluctuations empirically even if the focus is on aggregate time series. IST shocks may be important in their own right, but unless restrictive assumptions hold cannot be interpreted as MFP shocks at the sectoral level.

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Table 1: Model Calibration for Baseline Experiment

Parameter	Determines	Parameter	Determines
	Utility Function		
$\gamma = 1$	Intertemporal consumption elast. $= 1/\gamma$	$\beta = 0.99$	Discount factor
	Depreciation Rates		
$\delta^E = 0.124$	Equipment capital	$\delta^S = 0.056$	Structures capital
	Adjustment Costs		
$\nu_0^E = 0.5$	M, N Equipment Investment	$\nu_0^S = 0.5$	M, N Structures Investment
$\nu_1^E = 1$	M, N Equipment Investment	$\nu_1^S = 1$	M, N Structures Investment
$\omega^E = 0$	M, N Equipment Capital	$\omega^S = 0$	M,N Structures Capital
	M Goods Production		
$\alpha_M^N = 0.7$	Labor share	$\alpha_M^E = 0.17$	Equipment share
$\alpha_M^S = .13$	Structures share		
$\sigma_M = 1$	Substitution elast. for factor inputs		
	N Goods Production		
$\alpha_N^N = 0.7$	Labor share	$\alpha_N^E = 0.17$	Equipment share
$\alpha_N^S = 0.13$	Structures share		
$\sigma_N = 1$	Substitution elast. for factor inputs		
	Consumption Assembly		
$\phi_M^C = 0$	M goods intensity	$\phi_N^C = 1$	N goods intensity
$\sigma_C = 0.5$	Substitution elast. for M and N goods		
	Assembly of Equipment Investment		
$\phi_M^E = 1$	M goods intensity	$\phi_N^E = 0$	N goods intensity
$\sigma_E = 0.5$	Substitution elast. for M and N goods		
	Assembly of Structures Investment		
$\phi_M^S = 0$	M goods intensity	$\phi_N^S = 1$	N goods intensity
$\sigma_S = 0.5$	Substitution elast. for M and N goods		

Table 2: Alternative Calibration: changes relative to baseline*

Parameter	Determines	Parameter	Determines
	Adjustment Costs		
$\omega^E = 100$	M, N Equipment Capital	$\omega^S = 100$	M,N Structures Capital
	M Goods Production		
$\alpha_M^N = 0.46$	Labor share	$\alpha_M^E = 0.43$	Equipment share
$\alpha_M^S = .11$	Structures share		
	N Goods Production		
$\alpha_N^N = 0.72$	Labor share	$\alpha_N^E = 0.15$	Equipment share
$\alpha_N^S = 0.13$	Structures share		
	Consumption Assembly		
$\phi_M^C = 0.04$	M goods intensity	$\phi_N^C = .96$	N goods intensity
	Assembly of Equipment Investment		
$\phi_M^E = .85$	M goods intensity	$\phi_N^E = .15$	N goods intensity
	Assembly of Structures Investment		
$\phi_M^S = 0.04$	M goods intensity	$\phi_N^S = .96$	N goods intensity

* For ease of comparison with Table 1, this table only reports the parameters that vary from the baseline calibration.

Figure 1: Equivalent IST and MFP shocks under baseline calibration

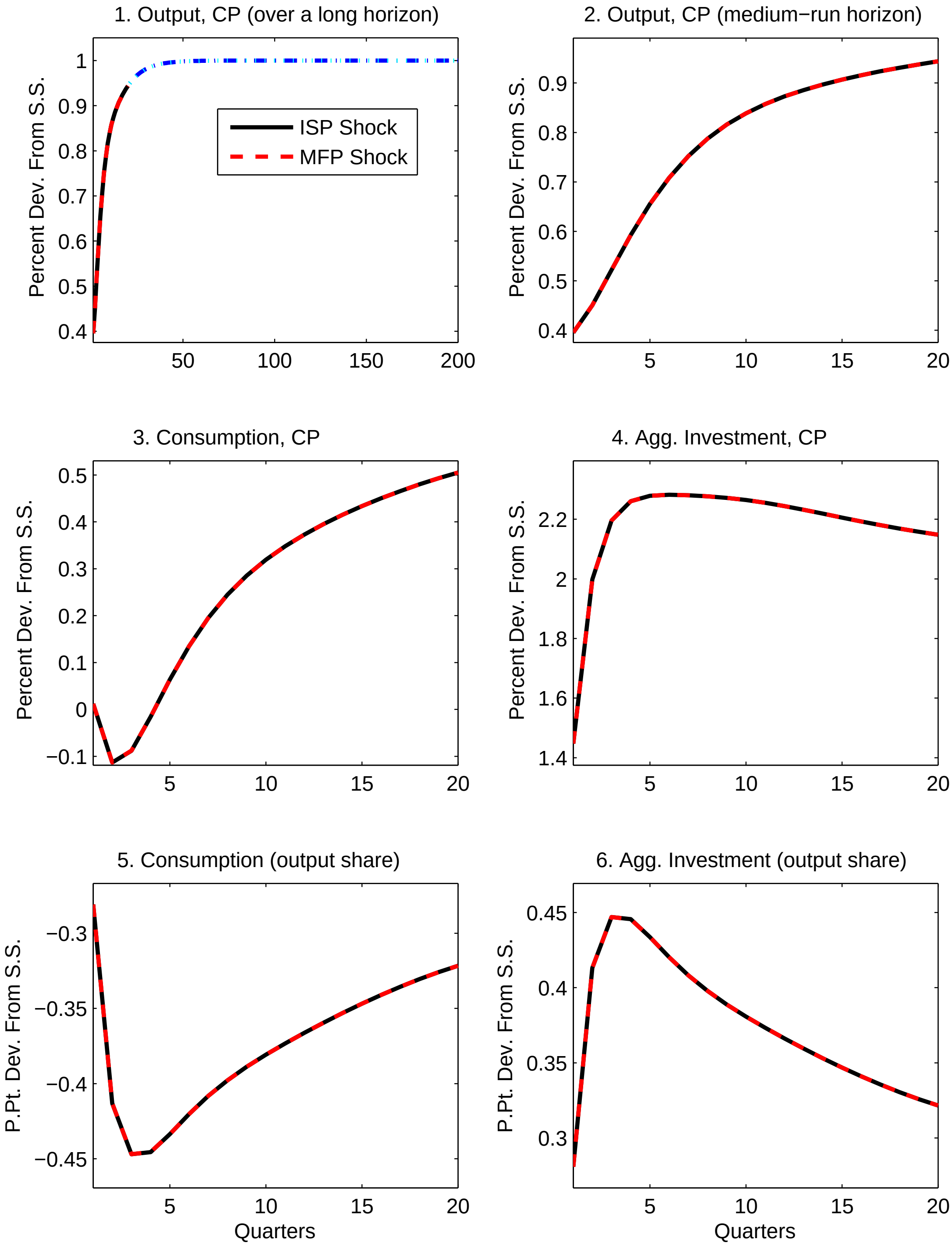


Figure 2: Equivalent IST and MFP shocks under baseline calibration (sectoral details)

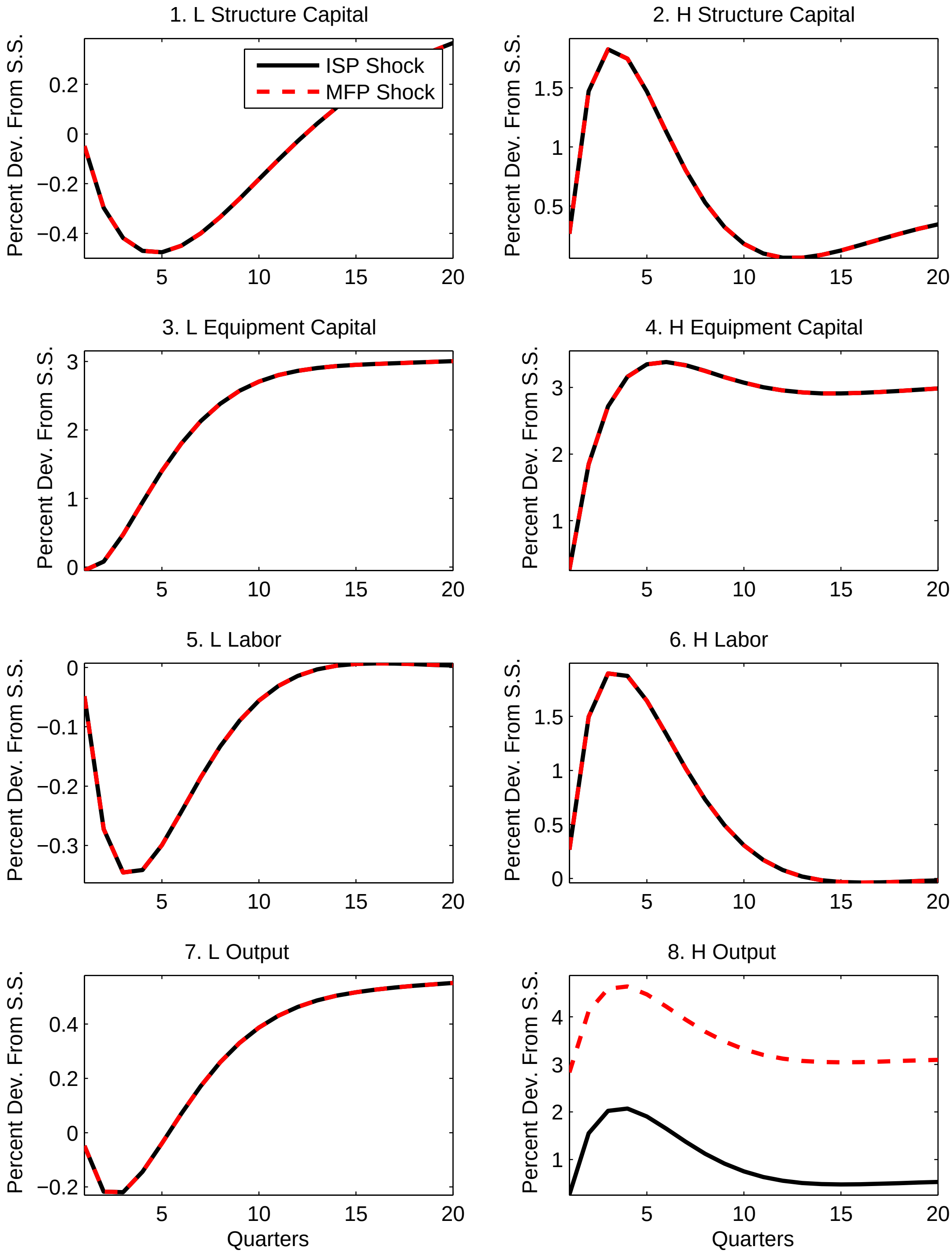


Figure 3: Cumulative effects of departures from baseline calibration

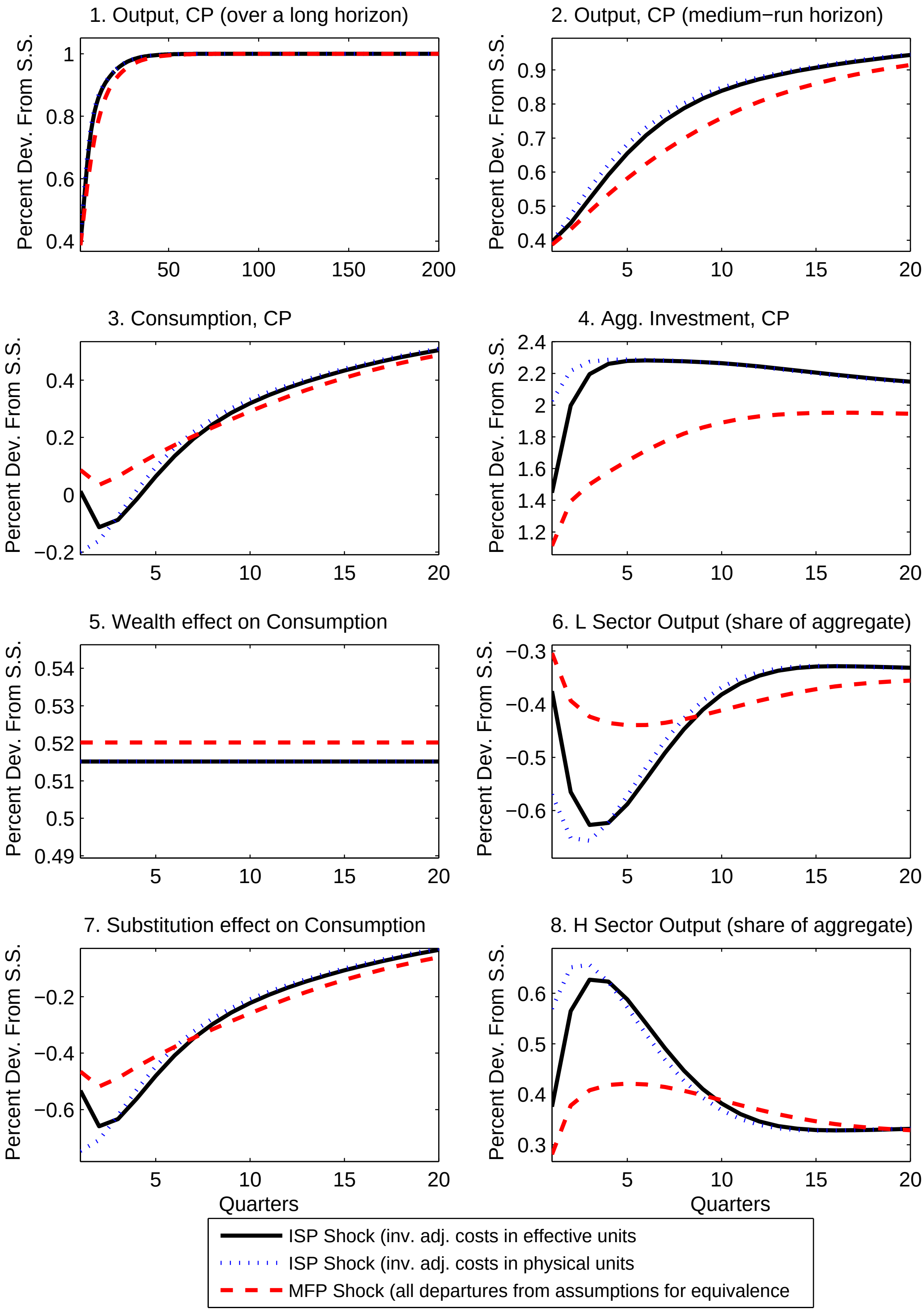


Figure 4: IST under baseline calibration and MFP shocks with incomplete specialization in assembly

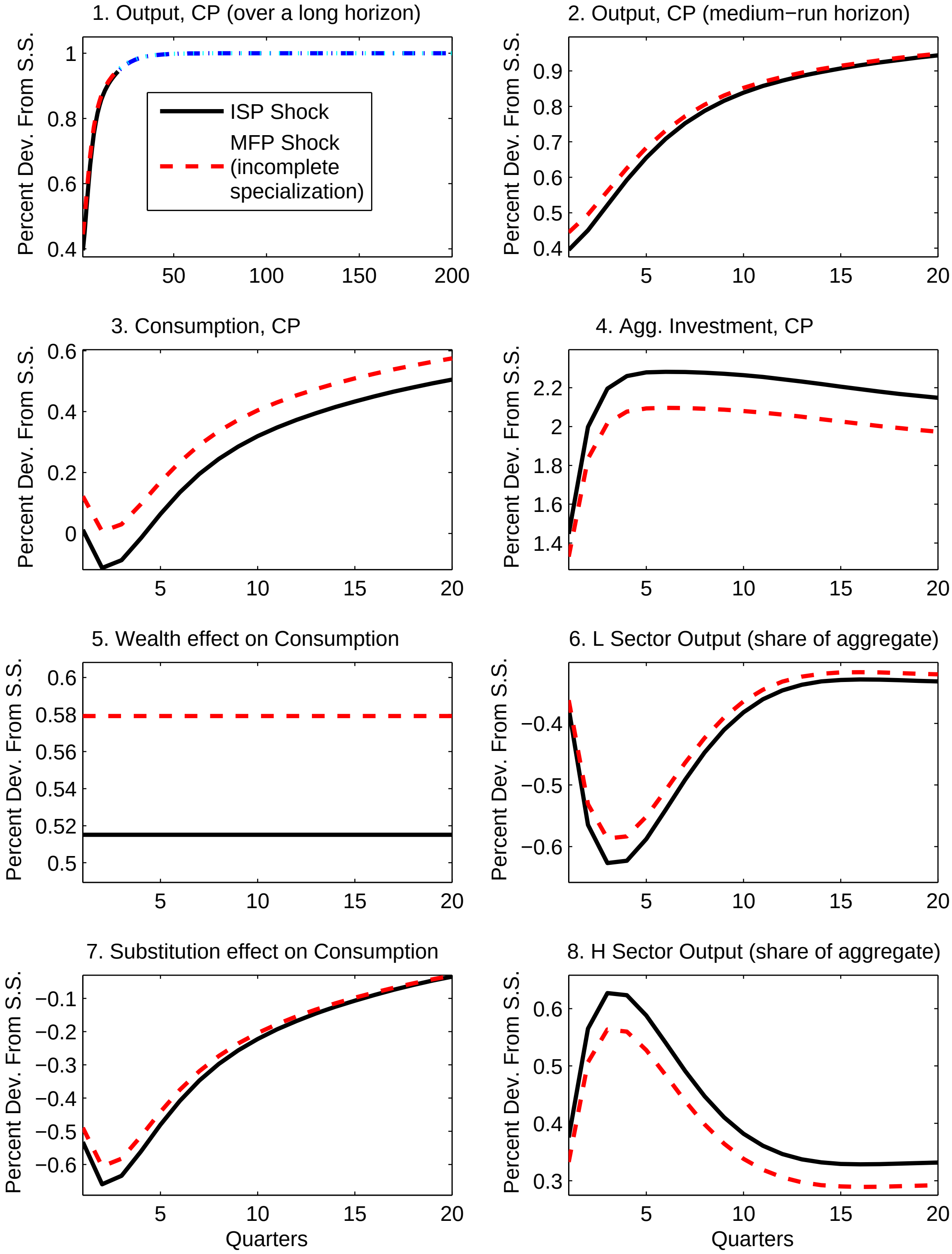


Figure 5: Sensitivity analysis: no investment adjustment costs

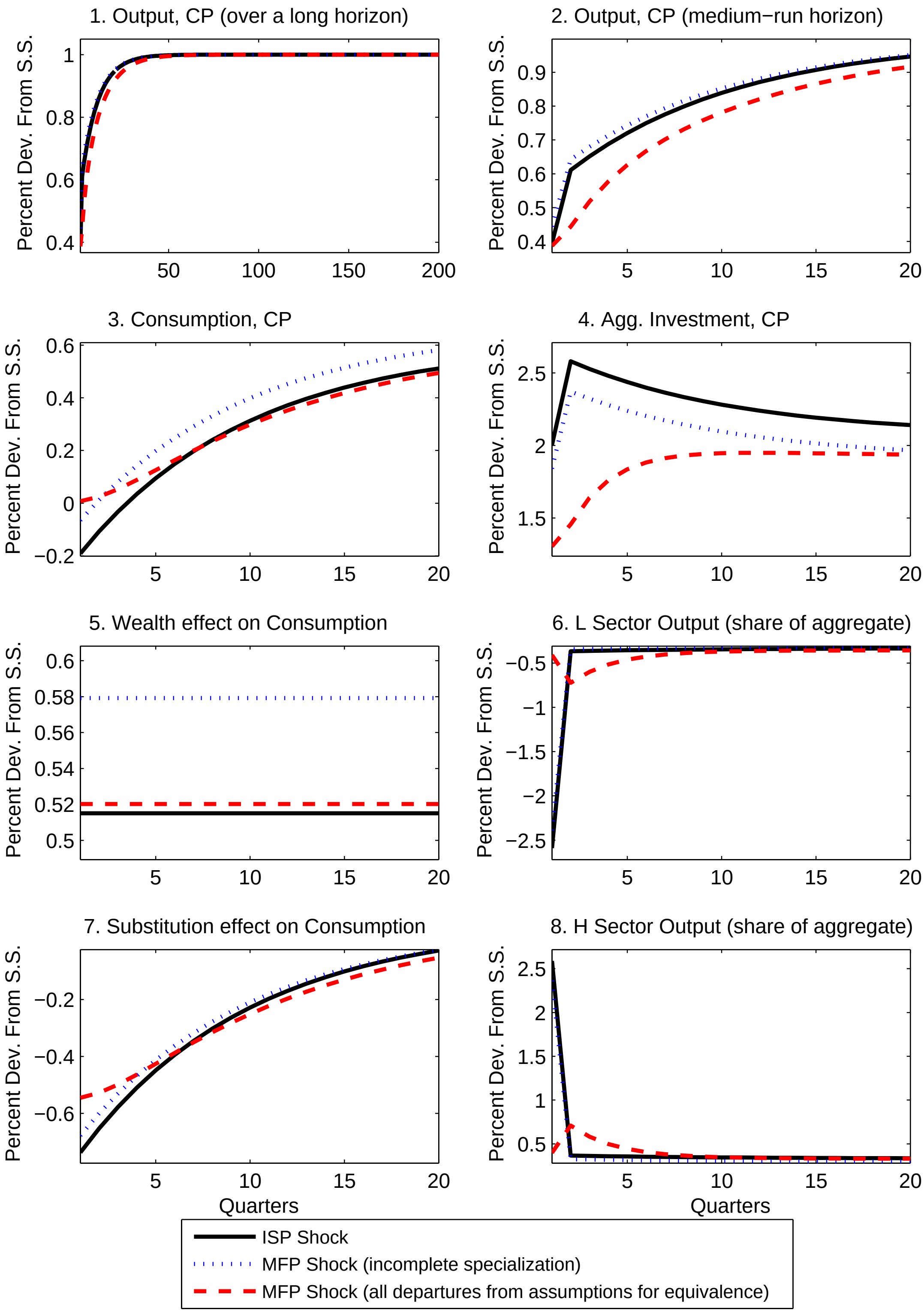


Figure 6: Probability Density Functions: Correlation Between Consumption and Investment at Business Cycle Frequencies

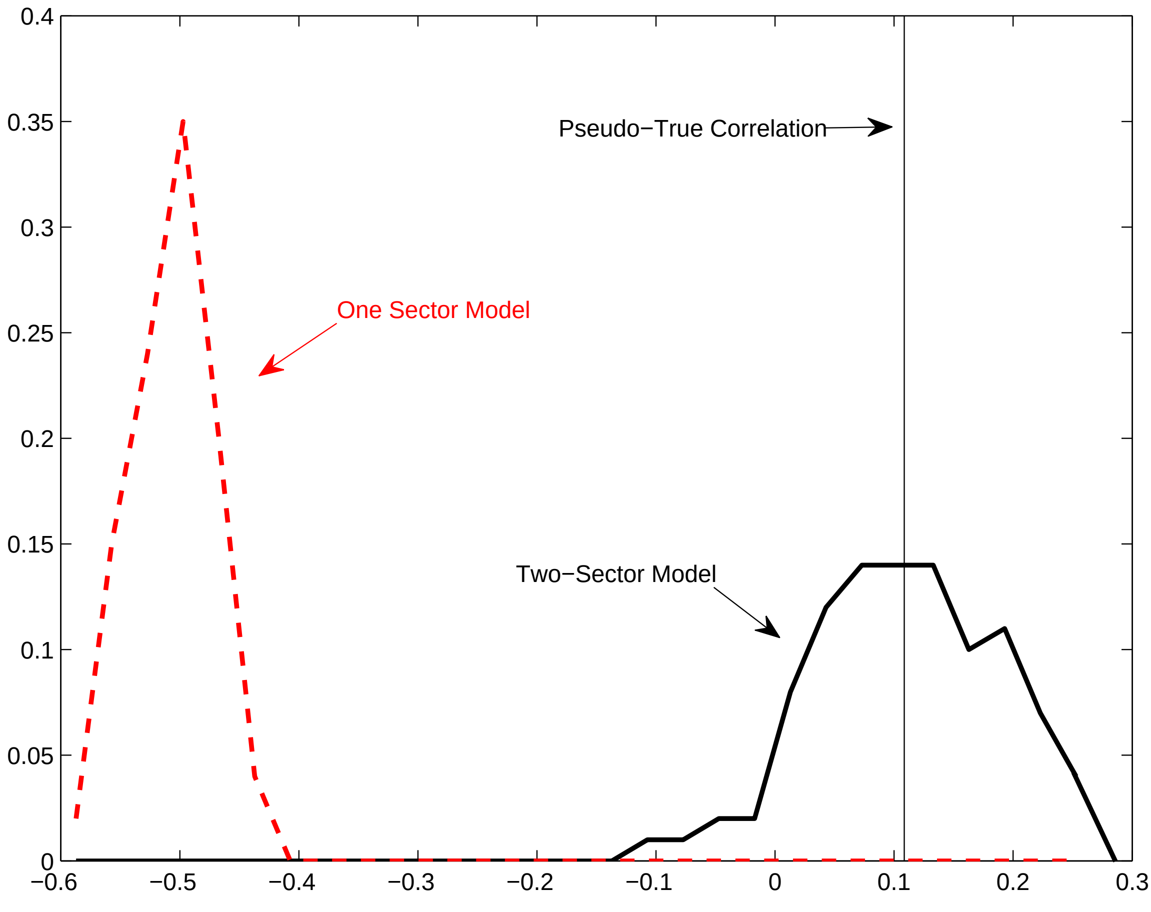
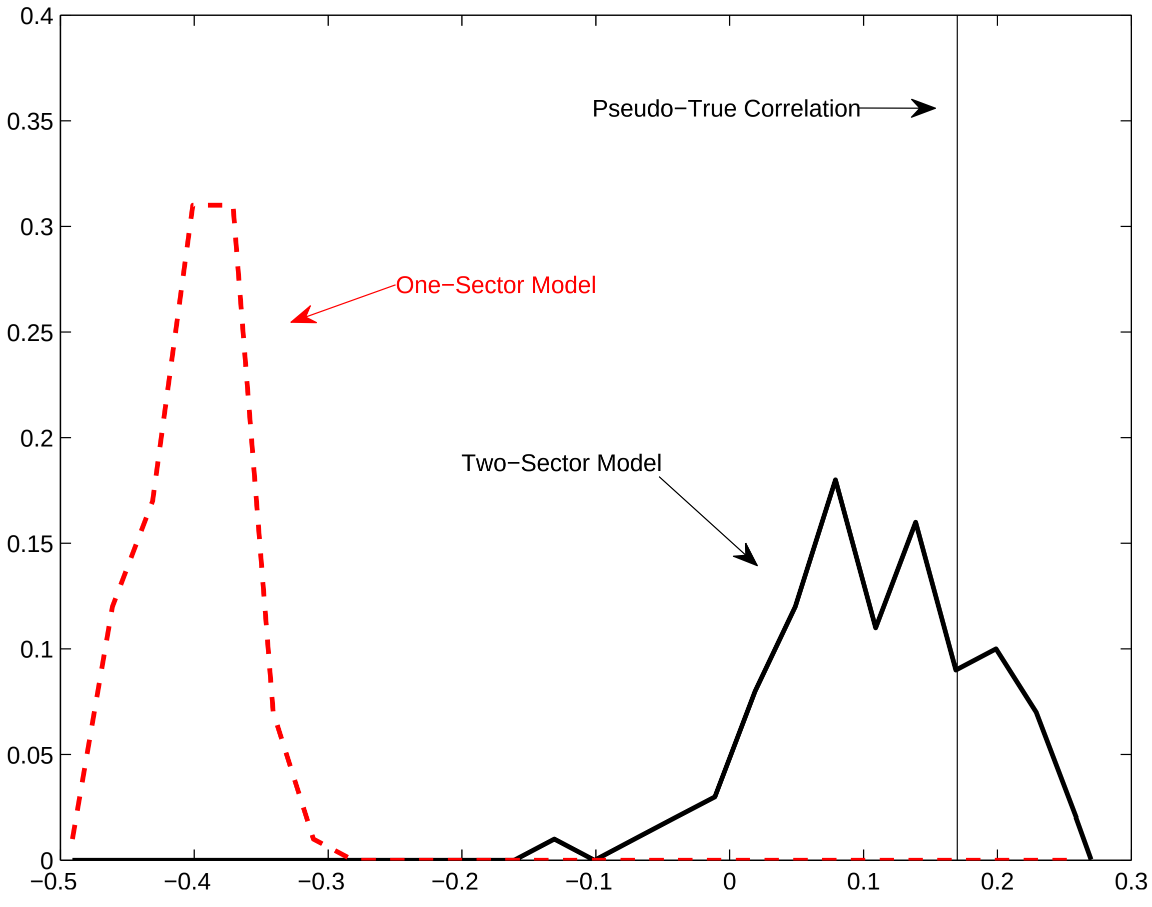


Figure 7: Probability Density Functions: Correlation Between Consumption Growth and Investment Growth



A Equivalence between MFP shocks in the machinery sector of a two-sector model and IST shocks in a one-sector model

This appendix provides a proof of two related propositions. The propositions are stated in the context of the following model:

$$Y_M = A_M F_M(K_M, N_M) \tag{19}$$

$$Y_N = A_N F_N(K_N, N_N) \tag{20}$$

$$K = K_M + K_N \tag{21}$$

$$N = N_M + N_N \tag{22}$$

$$\frac{F_{HK}(K_M, N_M)}{F_{HN}(K_M, N_M)} = \frac{F_{LK}(K_N, N_N)}{F_{LN}(K_N, N_N)} \rightarrow \frac{F_{LK}(K_N, N_N)}{F_{HK}(K_M, N_M)} = \frac{F_{LN}(K_N, N_N)}{F_{HN}(K_M, N_M)} \tag{23}$$

$$A_N F_N(K_N, N_N) = P A_M F_M(K_M, N_M) \rightarrow P = \frac{A_N F_{LK}(K_N, N_N)}{A_M F_{HK}(K_M, N_M)} \tag{24}$$

Sufficiency: The relative price is equal to the ratio of productivity factors if factors are perfectly mobile between sectors and production functions are CRTS and are identical up to a multiplicative productivity factor.

Proof: If the conditions are fulfilled, the model can be written as

$$Y_M = A_M N_M f_N(k_M) \tag{25}$$

$$Y_N = A_N N_N f_N(k_N) \tag{26}$$

$$K = N_M k_M + N_N k_N \tag{27}$$

$$N = N_M + N_N \tag{28}$$

$$\frac{f'_N(k_N)}{f'_N(k_M)} = \frac{f_N(k_N) - f'_N(k_N)\, k_N}{f_N(k_M) - f'_N(k_M)\, k_M} \tag{29}$$

$$P = \frac{A'_N f'_N\left(k_N\right)}{A'_M f'_N\left(k_M\right)} \tag{30}$$

Equation (29) implies $k_M = k_N = k$ and equation (30) implies that $P = \frac{A_N}{A_M}$.

Remark on one-sector models: Under the specified conditions, the two-good model can be written as a one-good model with output measured in terms of the numeraire good:

$$Y = A_N N_N f_N(k) + P A_M N_M f_N(k) = A_N N f_N(k) \tag{31}$$

This measure of output is not affected by changes in A_M which are exactly offset by changes in P. Let the amount of effective gross investment be

$$ZY_M = Z A_M N_M f_M\left(k_M\right) = Z A_M N_M f_N(k) \tag{32}$$

where Z is an investment specific productivity shock. Equal changes in Z and A_M have equivalent effects. They enter expression (32) as a product and do not enter anywhere else in the model.

Necessity: Under homogenous production functions and perfect factor mobility, the relative price is equal to the ratio of productivity factors only if production functions are CRTS and are identical up to a multiplicative productivity factor.

Proof: First, suppose that production functions are CRTS and that $P = \frac{A_N}{A_M}$ so that the model can be written as

$$Y_M = A_M N_M f_M\left(k_M\right) \tag{33}$$

$$Y_N = A_N N_N f_N\left(k_N\right) \tag{34}$$

$$K = N_M k_M + N_N k_N \tag{35}$$

$$N = N_M + N_N \tag{36}$$

$$\frac{f_N\left(k_N\right) - f'_N\left(k_N\right) k_N}{f_M\left(k_M\right) - f'_M\left(k_M\right) k_M} = 1 \tag{37}$$

$$\frac{f'_N\left(k_N\right)}{f'_M\left(k_M\right)} = 1 \tag{38}$$

Totally differentiating equations (37) and (38) yields respectively

$$-f_N''(k_N)\,k_N dk_N + f_M''(k_M)\,k_M dk_M = 0 \tag{39}$$

$$-f_N''(k_N)\,dk_N + f_M''(k_M)\,dk_M = 0 \tag{40}$$

which taken together with equation (35) imply

$$k_N = k_M = k \tag{41}$$

Using this fact and equations (37) and (38) implies

$$f_N(k) = f_M(k) \tag{42}$$

so $P = \frac{A_N}{A_M}$only if the two production functions to be identical.

Now, suppose that production functions are identical and homogeneous of degree n and that $P = \frac{A_N}{A_M}$ so that the model can be written as

$$Y_M = A_M F_M(K_M, N_M) \tag{43}$$

$$Y_N = A_N F_N(K_N, N_N) \tag{44}$$

$$K = N_M k_M + N_N k_N \tag{45}$$

$$N = N_M + N_N \tag{46}$$

$$\frac{F_{LK}(K_N, N_N)}{F_{LK}(K_M, N_M)} = \frac{F_{LN}(K_N, N_N)}{F_{LN}(K_M, N_M)} \tag{47}$$

$$\frac{F_{LK}(K_N, N_N)}{F_{LK}(K_M, N_M)} = 1 \tag{48}$$

Since production functions are identical and homogeneous of degree n, partial derivatives are homogeneous of degree $n-1$ so that, for example,

$$\lambda^{n-1} F_{LK}(K_M, N_M) = F_{LK}(\lambda K_M, \gamma N_M) \tag{49}$$

Suppose $K_N = \lambda K_M$ and $N_N = \gamma N_M$. Then

$$\lambda^{n-1} F_{Li}(K_M, \frac{\gamma}{\lambda} N_M) = F_{Li}(\lambda K_M, \gamma N_M) = F_{Li}(K_N, N_N), \quad i = K, N \tag{50}$$

Thus, equation (47) holds only if $\gamma = \lambda$, that is, only if $k_M = k_N$. Also, since

$$\lambda^{n-1} F_{NK}(K_M, N_M) = F_{NK}(\lambda K_M, \lambda N_M) = F_{NK}(K_N, N_N) \tag{51}$$

equation (48) holds only if $n = 1$ that is only if the production functions are CRTS.

Remark on adjustment costs: Now suppose there are adjustment costs for investment so that the amount of effective gross investment is

$$Z_s A_{Ms} N_{Ms} f_{Ms} \left\{ 1 - \nu \left[\frac{A_{Ms} N_{Ms} f_{Ms}}{A_{Ms-1} N_{Ms-1} f_{Ms-1}} \left(\frac{Z_s}{Z_{s-1}} \right)^{\nu_1} - 1 \right] \right\} \tag{52}$$

where time subscripts have been added for clarity and $f_M(k_s)$ is represnted by f_{Ms} for compactness. The derivatives of $\nu(\cdot)$ obey

$$\nu' \lesseqqgtr 0 \quad as \quad \frac{A_{Ms} N_{Ms} f_{Ms}}{A_{Ms-1} N_{Ms-1} f_{Ms-1}} \left(\frac{Z_s}{Z_{s-1}} \right)^{\nu_1} \lesseqqgtr 1, \qquad \nu'' > 0 \tag{53}$$

and either $\nu_1 = 1$—so that adjustment costs depend on efficiency units—or $\nu_1 = 0$—so that adjusment costs depend on physical units. Changes in A_{Ms} and Z_s have the same effects if and only if the Oultin conditions are met and $\nu_1 = 1$, that is if and only if they enter equation (52) only as a product.

B Additional simulation results

The discussion in the main body of the paper omitted to consider in isolation two departures from our baseline calibration. The effects of relaxing perfect capital mobility across sectors and of varying the factor intensities across sectors are illustrated below.

In Figure 8, the solid lines reproduce the responses to the IST shock from Figure 1. Instead, the dashed lines show the economy's response to an MFP shock in the M sector when relaxing only the assumption of perfect capital mobility across sectors in every period. Perfect capital mobility, as argued before, is necessary to represent our two-sector model as an aggregate one-sector model. To produce the responses shown by the dashed lines, we set the parameters governing the capital adjustment costs ω^E and ω^S both equal to 100. This parametrization implies that sectoral capital allocations only move with a delay of one period. Thus capital stocks are not only predetermined at the aggregate level, but also at the sectoral level.

The size of the MFP shock hitting the M sector was again chosen to bring about a permanent 1 percent increase in aggregate output. While the wealth effect on consumption is identical for the two shocks in Figure 8, the negative substitution effect is reduced in magnitude when the sectoral capital stocks are predetermined.

Figure 9 shows the responses to an IST shock in the aggregate model (replicating, for ease of comparsison, what is also shown in figures 1 and 8), as well as the responses to an MFP shock in the machinery sector of a two-sector model that allows for sector-specific production functions (the only difference relative to the baseline calibration). Again, the magnitude of the MFP shock is chosen to match the 1 percent long-run increase in aggregate output for the IST shock.

The figure shows persistent differences in the responses of consumption and investment. As under the alternative calibration the making of M sector goods used in equipment investment is more intensive in equipment capital relative to the aggregate, the substitution effect on consumption coming from the MFP shock is not as strong initially relative to the IST shock. Accordingly, M sector output increases by less, at first. However, eventually more resources need to be devoted to the M sector to maintain the larger stock of equipment capital implied by the alternative calibration, and the MFP shock in the investment sector leads to a larger long-run increase in equipment investment and a smaller long-run increase in consumption. Consequently, the wealth effect on consumption is smaller for the MFP shock than for the IST shock.

Figure 8: IST under baseline calibration and MFP shock with capital stocks predetermined in each sector

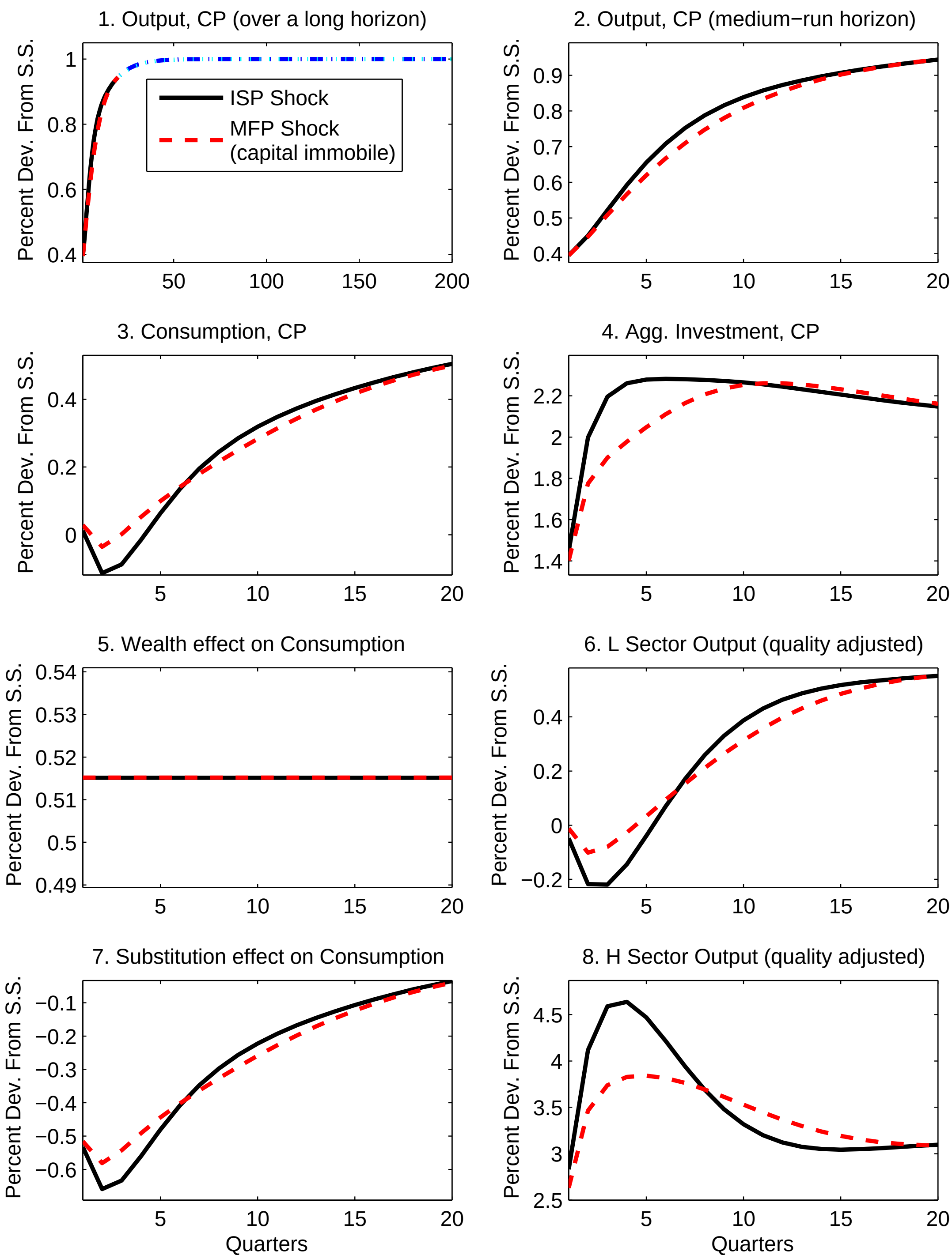


Figure 9: IST under baseline calibration and MFP shocks with sector-specific production functions