

No. 9118

ALLOCATIVE INEFFICIENCY IN EDUCATION

by

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# Research Paper

Federal Reserve Bank of Dallas

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#### Allocative Inefficiency in Education

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#### I. <u>Introduction</u>

Most people consider student achievement to be the primary objective of the United State's multi-billion dollar public school system. A casual examination of the evidence suggests that a good portion of those dollars are wasted - too many students drop out, the students who do graduate are not well-educated, the proportion of administrators to teachers seems too high. In fact, more rigorous analyses support these claims. For example, Hanushek (1986) in his review of the literature on educational production functions found no systematic evidence for a positive correlation between educational expenditures and educational outcomes. Not a surprising conclusion, but one that warrants a deeper examination. In this paper, we provide a methodology by which individual schools can be evaluated on their use of inputs to produce educational outcomes. An examination of the individual scores yields information on the type of school which is more likely to be wasting resources and hopefully suggests a useful reform policy.

We use a distance function to identify shadow prices and to test for allocative inefficiency in Texas school districts. The methodology yields relative measures of resource misallocation for each school district. We then compare the measures of inefficiency across different types of school districts to determine whether school district inefficiency is correlated with characteristics of the student body or school district location.

In the following section, we discuss the contribution to the literature of an analysis of school efficiency using distance functions. We review the distance function methodology and specify the estimating model in section 3. We discuss the data for this analysis in section 4. Section 5 presents the results of our analysis and section 6 concludes.

#### II. The Literature

Most of the literature on educational production focuses on the relationship between a single output (usually measured with achievement tests) and multiple inputs such as the characteristics of schools and school personnel, levels of expenditures and characteristics of students and their families. Much of this work assumes, either implicitly or explicitly, that schools are efficient in their endeavor to educate students.

However, some recent research on public school performance takes a different tack. Researchers have refined their measurement and modeling techniques to allow for multiple schooling outputs and to

examine explicitly questions related to technical, allocative and scale efficiencies.

One group of these studies uses data envelopment analysis (DEA) to analyze the technical efficiency of schools. This technique readily models multiproduct technology and identifies technical inefficiency as deviations from best practice performance using mathematical programming techniques which 'envelop' the data. In this group are Bessent (1980) and Bessent et al. (1982, 1983, 1984) who analyze public schools in Texas. Färe, Grosskopf, and Lovell (1988) use Missouri school district data to illustrate how to implement DEA for a cost-constrained technology. Färe, Grosskopf, and Weber (1989) use the same Missouri school district data to analyze the technical efficiency of these school districts in transforming vectors of inputs and characteristics into a vector of Missouri test scores. A modified version of this technique has also been employed by Ray (forthcoming) for a sample of Connecticut schools and by McCarty and Yaisawarng (1991) for a sample of New Jersey schools. They calculate technical efficiency using standard DEA techniques but include a second stage analysis to purge the technical efficiency measures of effects of home production and factors which schools cannot control using regression techniques.

A second group of studies employs a short run variable cost function approach to analyze education in a multiproduct setting. Included in this group is work by Jimenez (1986), who analyzed primary and secondary education in Bolivia and Paraguay. He finds that these schools are not at optimal scale, and that observed levels of fixed inputs are not consistent with long run minimum cost. Callan and Santerre (1990) use the same general approach applied to school districts in Connecticut. They are particularly interested in allowing for disaggregated types of labor inputs in order to investigate substitution possibilities among teachers and administration, for example. Among other results, they find that support staff are substitutes for instructional staff.

We improve and extend previous research in four ways. First, we construct value-added measures of output that allow us to isolate the contribution of the school to educational achievement by controlling for previous achievement and home inputs to the educational process (see Hanushek and Taylor 1990). Second, we employ an input distance function methodology rather than multiproduct cost functions or DEA to model the production technology. Unlike a cost function, which presumes cost minimizing behavior, a distance

function has no embedded behavioral objective and therefore lends itself well to analyses of the public sector. Estimation of a distance function also has the advantage of examining efficiency in a stochastic framework rather than the non-stochastic framework of the DEA approach. Third, our estimation technique allows us to test for allocative efficiency by calculating district-level shadow prices for inputs that, when compared to observed factor prices, indicate which factors (if any) are earning relative rents. Fourth, we employ bootstrapping techniques to determine the statistical significance of our efficiency measures.

#### III. The Distance Function

We model technology with a Shephard (1953) input distance function. This function is a mapping from the set of all nonnegative input vectors  $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N)$  and nonnegative output vectors  $\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_M)$  into the real line, i.e.,

$$D(y,x) = \max \{\lambda: (x/(\lambda)) \text{ is an element in } L(y)\}$$
 (1)

where

$$L(y) = \{(x): x \text{ can produce } y\}.$$
 (2)

The distance function is perhaps most easily understood with the aid of a diagram. Consider Figure 1. Observation K employs the input bundle  $(x_i,x_j)$  to produce output level y. The distance function seeks the largest proportional contraction of that input bundle which allows production of the original output level y (which may be a vector). In this example, the value of the distance function for observation K is OK/OK. This illustrates the following characteristic of the distance function, namely

$$D(y,x) \geq 1 <=> x \in L(y). \tag{3}$$

Furthermore, D(y,x) = 1 if and only if the input bundle is an element of the isoquant of L(y).

The distance function satisfies fairly general regularity properties (see Färe and Grosskopf (1990) for

<sup>&</sup>lt;sup>1</sup>This measure is related to Farrell (1957) technical efficiency. In fact, the Farrell input-saving measure of technical efficiency is the reciprocal of the value of the input distance function.

details), including being homogeneous of degree one in inputs, concave in inputs, convex in outputs, and nondecreasing in inputs. The distance function is dual to the cost function; both functions completely describe technology. Note that they have different data requirements; whereas both require data on outputs, the distance function requires data on input quantities rather than input prices.<sup>2</sup> The distance function has the advantage for our purposes of being "agnostic" with respect to the economic motivation of the decision maker, unlike the cost function which presumes cost minimizing behavior. In that sense, the distance function is much less restrictive; empirically, it merely identifies the boundary of the (best practice) technology.

As discussed in Blackorby and Russell (1988) the first derivatives of the input distance function with respect to input quantities yield (cost-deflated) shadow or support prices of those inputs. Since the input distance function is dual to the cost function, this follows from Shephard's (dual) lemma, see, Färe and Grosskopf (1990). We can use these shadow prices to test for cost minimizing behavior. Let  $w = (w_1, w_2, ..., w_N)$ , where w is positive, be the vector of observed input prices. If a municipality is minimizing costs then the following holds:

$$D_i(y,x)/D_i(y,x) = w_i/w_i$$
, for all i,j = 1,2,...N. (4)

 $D_i$  is the first derivative of D(y,x) and is interpreted as the virtual or shadow price of the ith input. Alternatively, we can define a measure  $\kappa_{ij}$  as the degree to which the shadow price ratio agrees with the actual price ratio, where the formulation in (5) follows the nonminimal cost literature,<sup>3</sup>

$$\kappa_{ii} = (D_i(\bullet)/D_i(\bullet)) / w_i/w_p$$
 (5)

see for example Toda (1976) or Atkinson and Halvorsen (1986).

If  $\kappa_{ij} = 1$  for all i,j then the observation is said to be allocatively efficient. When  $\kappa_{ij} \neq 1$  we can have

<sup>&</sup>lt;sup>2</sup> In a cross-section setting where prices do not vary, the distance function provides a good empirical alternative to the cost function.

<sup>&</sup>lt;sup>3</sup> In this literature, firms are assumed to minimize (unobservable) shadow costs given (unobservable) shadow prices. This is achieved by introducing additional parameters into the cost function that essentially allow input prices to "pivot". These parameters are used to construct the  $\kappa_{ij}$  in equation 5. Unlike the distance function methodology, this technique cannot identify firm-specific relative shadow prices.

the following non-optimal situations. If

$$\kappa_{ii} > 1,$$
 (6)

factor i is underutilized relative to j at observed relative prices, and if

$$\kappa_{ii} < 1,$$
 (7)

factor i is overutilized relative to j at observed relative prices. Figure 2 illustrates for  $\kappa_{ij} < 1$ . In this diagram, relative shadow prices are less than the observed relative prices. In other words, the absolute value of the slope of w'w' is less than the absolute value of the slope of ww. Based on observed relative prices, cost minimizing employment occurs at the tangency of the isoquant and w'w' which is parallel to ww. At this point, employment of input i is lower and employment of input j is higher than at the observed employment levels. Another way of interpreting a value of  $\kappa_{ij} < 1$  is that the marginal product per dollar paid for input j exceeds the marginal product per dollar paid for input i at the observed input mix and prices. That is, input i is relatively underpaid and input j is relatively overpaid at the observed input mix and prices.

Thus, the input distance function can be used to model multiple-output, multiple-input technology without information on factor prices. By adding information on observed factor prices, the distance function can be used to test for allocative inefficiency.

#### IV. The Data

The Texas Research League provides data for the 1988-89 school year on the number of teachers, administrators, staff and teacher aides employed in each Texas school district, the average salaries paid to each type of employee and other school characteristics. Such labor data represent roughly 80 percent of operating expenditures and probably capture all of the short-run decision parameters for school districts.

Although capital stocks are practically fixed in the short run, school district decisions regarding the use of labor clearly depend on the (exogenous) stock of non-labor inputs. Unfortunately, there are no direct measures of the quantities or prices of non-labor inputs. Therefore, because expenditures on maintenance and operations should be a function of the size of the capital stock, we use data on school district expenditures on maintenance and operations as our proxy for the quantity of non-labor inputs

(M&OINPUT). By construction, the observed price per unit of these non-labor inputs is one.4

The Texas Education Agency uses the Texas Educational Assessment of Minimum Skills (TEAMS) tests to collect data on student achievement. However, as shown by Hanushek and Taylor (1990), raw test scores are a poor proxy for the actual output of the school system. We use the residual analysis techniques in Hanushek and Taylor (1990) to estimate the value added by Texas school districts. This approach also yields an estimate of predicted achievement. We consider the estimates of predicted achievement as exogenous fixed effects representing home production or the student input to the production process.

Specifically, we estimate four measures of school district output, using TEAMS scores in mathematics, reading and writing, and demographic data on the racial and socio-economic composition of the student body (Texas Education Agency, 1987, 1989). For each of four grade levels — 3rd, 5th, 9th and 11th — we estimate the value added by the school district according to equation (8).

TEAMS89<sub>i,g</sub> = 
$$\alpha_g + \delta_{i,g}$$
 NONWHITE<sub>i</sub> +  $\delta_{2,g}$  SES<sub>i</sub> +  $\sum_{i=3}^{5} \delta_{j,g}$  TEAMS87<sub>ij,g-2</sub> +  $\epsilon_{i,g}$  (8)

where TEAMS89<sub>ig</sub> is the average total TEAMS scores for school district i for grade level g in 1989, TEAMS87<sub>i,ig2</sub> is the average TEAMS score in subject j (reading, writing and mathematics) for the same cohort two years previously, NONWHITE<sub>i</sub> is the fraction of the student body of school district i that is a member of a minority group, SES<sub>i</sub> is the fraction of the student body of school district i that is receiving free or reduced-price lunches (the best available proxy for socio-economic status), and the estimated residual,  $\epsilon_{i,g}$ , represents the average value added in school district i. Because the four value-added equations share common regressors (NONWHITE<sub>i</sub> and SES<sub>i</sub>), we suspected a cross-equations correlation between the error terms, and therefore among our output measures. We found that the correlations between error terms were surprisingly low (in the neighborhood of 0.20), but significant and therefore estimate the output measures simultaneously using the standard SAS package for seemingly unrelated regression (SUR). We present these equation estimates in Table 1.

Estimating school outputs as equation residuals generates output measures that represent deviations

<sup>&</sup>lt;sup>4</sup>Callan and Santerre (1990) use a similar proxy for capital stock.

from the state average. School districts that add less value than the state average have negative output measures. Since the distance function methodology cannot handle negative outputs, we transform the value-added residuals into tractable output measures by adding the estimated value of the intercept from each equation to the value-added residual for that equation. Therefore,

$$OUTPUT_{ig} = \hat{\alpha}_{g} + \epsilon_{ig}$$
 (9)

and

STUINPUT<sub>i,g</sub> = 
$$\hat{\delta}_{1,g}$$
 NONWHITE<sub>i</sub> +  $\hat{\delta}_{2,g}$  SES<sub>i</sub> +  $\sum_{i=3}^{5} \hat{\delta}_{j,g}$  TEAMS87<sub>ij,g-2</sub> . (10)

Thus STUINPUT is our measure of the contribution of home production, which we treat as a quasi-fixed input, i.e., an input over which the school district has no control.<sup>5</sup> OUTPUT is our proxy of the value-added of the school district. It is the net gain in achievement purged of the effect of home production and earlier achievement test gains.<sup>6</sup>

#### V. Estimation

To estimate the distance function a functional form for D(.) must be chosen. In this analysis we use a translog form for the distance function. To avoid numerous econometric issues in the estimation of the

<sup>&</sup>lt;sup>5</sup>We also specified a model in which the variables included in the right hand side of (10) were included directly in the distance function as quasi-fixed inputs. Our qualitative and quantitative results were very similar.

<sup>&</sup>lt;sup>6</sup>We note that this general technique was also employed by Callan and Santerre (1990) to arrive at a measure of educational quality. However, Callan and Santerre did not have access to pretest information and therefore were unable to derive a value-added quality measure.

distance function,<sup>7</sup> we proceed by estimating only the share equations which are derived from D(y,x) using Shephard's Dual Lemma (see Färe and Grosskopf (1990)). Suppressing the observational subscript, the system we estimate is:

$$S_{j} = \beta_{j} + \sum_{\ell} \beta_{j\ell} \ln x_{\ell} + \sum_{m} \rho_{mn} \ln y_{m} + \sum_{n} \gamma_{nj} \ln z_{n} + \xi \qquad j = 1,...,3$$

$$(11)$$

where  $x_t$  is the quantity for endogenous inputs (teachers, administrators, staff and aides),  $z_n$  is the quantity for exogenous inputs (STUINPUT<sub>g</sub> for g=3,5,9, and 11 and M&OINPUT), and  $y_m$  are the output quantities (OUTPUT<sub>g</sub> for g=3,5,9 and 11). We impose homogeneity in the inputs:  $\Sigma \beta_n = 1$ ,  $\Sigma \beta_{mn} = 0$ ,  $\Sigma \rho_{mn} = 0$ , as required by the definition of the input distance function.

Dividing the predicted values from the share equations by the observed input quantities yields input shadow prices that when compared to data on the observed factor prices generate point estimates of the  $\kappa_{ij}$  for each school district. However, we would also like to be able to indicate whether or not those  $\kappa_{ij}$  are significantly different from one, i.e., whether or not allocative efficiency is significantly violated. Therefore, we performed a bootstrap based on the residuals of the SUR estimation of (11) to produce confidence intervals for our point estimates. The bootstrapping procedure involved estimating the system 100 times using randomly-selected residuals which were added to the predicted values of the dependent variables to create new "pseudo" dependent variables. We maintained the original covariance structure by using the residuals from the same (randomly-selected) observation for each of the four dependent variables (see Freedman and Peters (1984)). For each observation, we calculated every  $\kappa_{ij}$  100 times.

Briefly, the bootstrap generates a new distribution for each of the observations in the sample. We can then construct the empirical probability density function and cumulative distribution for each observation (see Efron 1982). If 95 percent of the estimated  $\kappa_{ij}$ s for an observation are greater 1, then we consider the mean

<sup>&</sup>lt;sup>7</sup> Estimation of a system of equations including the share equations and the distance function raises several difficult econometric problems. First, the left-hand side of the distance equation is unobservable. Second, an intercept cannot be estimated. While estimation of the distance function by setting the left hand side equal to unity is feasible (see Grosskopf and Hayes (1991)), we felt that the required instrumental variables approach could introduce additional problems and lead to erroneous predictions from the share equations.

value of that  $\kappa_{ij}$  to be statistically greater than 1. Similarly, if 95 percent of the estimated  $\kappa_{ij}$ s for an observation are less than 1, then we consider the mean value of that  $\kappa_{ij}$  to be statistically less than 1.

#### VI. Results

Mean  $\kappa_{ij}$ s that are statistically different from one signal allocative inefficiency.<sup>8</sup> For example, when the marginal productivity of teachers per dollar paid is significantly greater than the marginal productivity of administrators per dollar paid( $\kappa_{teachers, administrators} > 1$ ), then the school district is using relatively too few teachers and relatively too many administrators ceteris paribus. Similarly, when the marginal productivity of teachers per dollar paid is significantly less than the marginal productivity of support staff per dollar paid ( $\kappa_{teachers, staff} < 1$ ), then the school district is over-utilizing teachers relative to support staff. Put differently, at the observed personnel mix,  $\kappa_{ij} > 1$  implies that factor i is relatively underpaid and factor j is relatively overpaid and factor j is relatively underpaid given their relative productivities. By the same token, if  $\kappa_{ij} < 1$  then factor i is relatively overpaid and factor j is relatively underpaid given their relative productivities.

Given four types of labor input (teachers, administrators, staff and teacher aides), there are six dimensions in which a school district could be allocatively inefficient and therefore six relevant  $\kappa$ ijs ( $\kappa_{\text{teachers,administrators}}$ ,  $\kappa_{\text{teachers,staff}}$ ,  $\kappa_{\text{teachers,aides}}$ ,  $\kappa_{\text{administrators,staff}}$ ,  $\kappa_{\text{administrators,aides}}$ ,  $\kappa_{\text{staff,aides}}$ ). We report the distributions of the mean  $\kappa_{ii}$ s in Tables 3 and 4.

We find that only 2 of the 604 school districts in the sample are allocatively efficient, i.e.  $\kappa_{ij} = 1$  in all six dimensions. On the other hand, no school district violated  $\kappa_{ij} = 1$  in all six dimensions. When comparing one type of personnel to any one other type of personnel, Table 3 indicates that roughly 20 percent of the school districts choose a cost-minimizing mix of any two types of the labor inputs.

Interestingly, some patterns emerge from the information on school districts that misallocate their resources. For example, if a school district is not using teachers efficiently relative to administrators, the school district is 20 percent more likely to be relatively underpaying its administrators than it is likely to be

<sup>&</sup>lt;sup>8</sup>Technically, allocative efficiency obtains if all double ratios equal unity. Once any double ratio does not equal unity, however, it is not necessarily true that other ratios becoming closer to unity implies an improvement in efficiency or that fewer violations are better. That is, we are in the world of the second best.

relatively underpaying its teachers ( $\kappa_{\text{teachers,administrators}}$  is greater than one 36 percent of the time and is less than one 44 percent of the time). If a school district is not using teachers efficiently relative to aides, it is almost 70 percent more likely to be relatively underpaying its aides than it is likely to be relatively underpaying its teachers. In fact, school districts that use aides inefficiently relative to any other type of personnel are more likely to be relatively underpaying their aides. That is, these school districts could reduce costs by substituting aides for the other employment category, ceteris paribus.

We would like to know why some school districts are not cost minimizers and why particular resources are relatively overpaid or used relatively excessively. We suspected that rural school districts might be able to exploit more monopoly power in the delivery of school services than urban school districts, and thus that rural school districts might be subject to less consumer pressure for efficiency than urban school districts. Therefore, we divided the sample based on the census definition which is based on the county in which the school district is located and tested for differences in the distributions of the  $\kappa_{ii}$ s.

From Table 3, it is clear that when school districts misallocate their resources between teachers and administrators or between teachers and staff, rural school districts are more likely to overutilize teachers ( $\kappa_{\text{teachers,j}} < 1$ ) while urban school districts are more likely to underutilize teachers( $\kappa_{\text{teachers,j}} > 1$ ). Further, when school districts misallocate their resources between teachers and aides, both rural and urban school districts tend to underutilize aides ( $\kappa_{\text{teachers,aides}} < 1$ ) but the tendency is particularly strong in rural school districts. An analysis of variance on ranks of the  $\kappa_{ij}$  variables in the urban and rural sub-samples using the NPAR1WAY procedure in SAS support these conclusions.

These results suggested that those rural school districts that are overutilizing teachers and those urban school districts that are overutilizing staff would be fruitful. Specifically, we were curious about whether or not the same school districts that overutilized teachers relative to administrators also overutilized teachers relative to staff. Further, we wanted to determine the number of school districts that overutilized staff relative to teachers as well as staff relative to administrators. From Table 4, it is clear that rural school districts are much more likely than urban districts to overutilize teachers relative to other types of personnel.

Urban school districts have a higher probability of wasting staff relative to other types of personnel.9

We also investigated differences between rural and urban school districts by examining the Pearson correlations between the double ratios ( $\kappa_{ij}$ s) and various revenue and demographic variables (the amount of state aid per pupil, the total enrollment, and the racial and socio-economic composition of the student body). We found two interesting relationships that apply to both rural and urban school districts and one relationship that was significant only for urban school districts.

First, we found evidence that smaller school districts have larger relative productivity wage gaps than do larger school districts. For both urban and rural school districts that misallocate their resources between teachers and administrators, the size of the productivity wage gap appears to decline as the size of the school district increases. For school districts that overutilize teachers relative to administrators ( $\kappa_{\text{teachers,administrators}} < 1$ ), we found a positive correlation between enrollment and  $\kappa_{\text{teachers,administrators}} > 1$ ) we found a negative correlation between enrollment and  $\kappa_{\text{teachers,administrators}} > 1$ ) we found a negative correlation between enrollment and  $\kappa_{\text{teachers,administrators}}$ . We found similar patterns in the correlations between enrollment and the  $\kappa_{ij}$ s for school districts that misallocate their resources between teachers and staff and for school districts that misallocate their resources between administrators and staff. The tendency for smaller school districts to make greater allocative errors than larger school districts may reflect either an indivisibility problem at smaller schools or the relatively greater ability of urban school districts to exert market power over input prices.

We also found some evidence that state aid may encourage school districts to misallocate their resources between teachers and staff and between administrators and staff. For both urban and rural school districts that misallocate their resources between administrators and staff, the size of the productivity wage gap appears to increase as the level of state aid increases. Further, in urban school districts that overutilize staff relative to teachers, we found a positive correlation between  $\kappa_{\text{teachers,staff}}$  and the level of state aid per pupil. Similarly, in rural school districts that overutilize teachers relative to staff, we found a negative correlation between  $\kappa_{\text{teachers,staff}}$  and the level of state aid per pupil. These relationships suggest either that

Again, we note that these are second best situations.

something in the state aid formula encourages inefficiency, or that school districts have fewer incentives to be efficient with the state's money.

We found one interesting relationship that appears to hold only for school districts in urban counties. Whenever an urban school district underutilizes aides relative to some other type of personnel, the productivity wage gap for teachers, administrators and staff (relative to aides) increases as the percentage of minority or low-income students increases.

#### VII. Conclusions

Using an input distance function to model the relationship among the multiple inputs and multiple outputs of Texas school districts, we find evidence of widespread allocative inefficiency. Only 2 of the 604 school districts in our sample efficiently allocated their labor resources among teachers, administrators, staff and aides. In all other school districts, the ratio of relative marginal productivities to relative wages was significantly different from one for at least one combination of labor inputs. This suggests that observed relative input prices do not reflect relative productivity in Texas school districts and could cause bias if used to estimate a cost function, for example. Violation of allocative efficiency also implies that Shephard's lemma is no longer valid; shadow prices should be used instead of observed prices.

We found evidence of a few patterns in the distribution of the ratio of relative shadow to observed input prices. We found that when school districts misallocate resources between administrators and teachers, urban school districts tended to overutilize administrators while rural school districts tended to overutilize teachers. We also found that urban school districts were particularly prone to overutilize staff and that larger school districts had smaller gaps between relative shadow and observed wages than did smaller school districts. Finally we found some evidence that the school financing formula in the state widened the productivity wage gap and that school districts in urban areas with disadvantaged student bodies had systematically higher productivities per dollar paid to teachers aides relative to all other types of labor.

Our work provides an alternative to DEA analysis or the estimation of cost functions in the multiproduct setting typical of the education sector. Further, our results suggest that estimation of a cost function using observed input prices may cause bias and violation of Shephard's lemma.

Table 1
Output Estimation

	3rd Grade	5th Grade	9th Grade	11th Grade
Intercept	1970.60	1800.8	1311.90	859.16
	(80.74)	(75.81)	(96.51)	(41.76)
WD + 1 4000				
TEAMS87 <sub>math,j</sub>	-0.01	0.02	0.20	0.45
	(0.16)	(0.13)	(0.11)	(0.05)
TEAMS87 <sub>writing,j</sub>	0.43	0.52	0.49	0.01
	(0.16)	(0.12)	(0.11)	(0.03)
TEAMS87 <sub>reading,j</sub>	0.25	0.33	0.72	0.46
	(0.17)	(0.14)	(0.18)	(0.07)
NONWHITE	-0.16	0.12	-0.15	-0.18
	(0.21)	(0.18)	(0.17)	(0.07)
SES	-2.03	-1.80	-1.08	-0.49
	(0.28)	(0.25)	(0.23)	(0.10)

System weighted R-square is 0.4169

Table 2
Share Equation Estimates

	Teachers	Administrators	Staff
Intercept	0.83	0.11	-0.16
	(0.01)	(0.01)	(0.01)
L_Teachers	0.01	0.03	-0.04
	(0.003)	(0.003)	(0.003)
L_Administrators	-0.01	0.01	-0.01
	(0.003)	(0.000)	(0.002)
L_Staff	0.02	-0.02	0.003
	(0.001)	(0.001)	(0.000)
L_Aides	-0.02	-0.01	-0.01
	(0.001)	(0.001)	(0.001)
L_OUTPUT <sub>3</sub>	-0.07	0.002	0.05
	(0.02)	(0.01)	(0.01)
L_OUTPUT <sub>5</sub>	-0.02	0.02	-0.01
	(0.02)	(0.009)	(0.01)
L_OUTPUT,	0.07	-0.02	-0.05
	(0.01)	(0.008)	(0.01)
L_OUTPUT <sub>11</sub>	0.02	-0.003	0.001
	(0.02)	(0.01)	(0.01)
L_STUINPUT <sub>3</sub>	0.07	-0.001	-0.04
	(0.01)	(0.005)	(0.01)
L_STUINPUT <sub>5</sub>	0.01	-0.03	0.01
	(0.01)	(0.008)	(0.01)
L_STUINPUT,	0.03	0.01	-0.05
	(0.03)	(0.01)	(0.01)
L_STUINPUT <sub>11</sub>	-0.09	0.02	0.06
	(0.02)	(0.01)	(0.01)
L_M&OINPUT	-0.03	0.01	0.02
	(0.002)	(0.001)	(0.00)

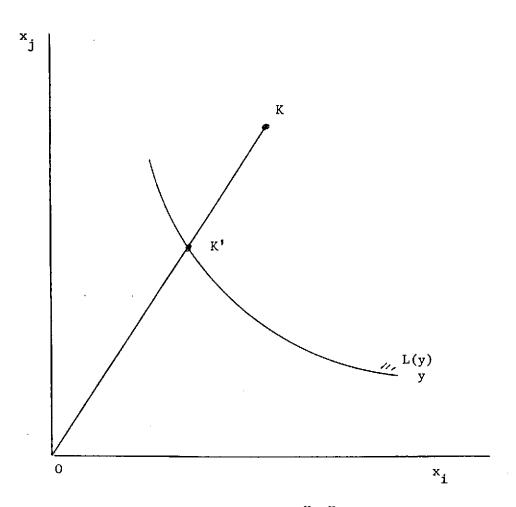
Table 3 Distribution of the  $\kappa_{ij}$ 

	$\kappa_{ij} = 1$	$\kappa_{ij} < 1$	$\kappa_{ij} > 1$
$\kappa_{ ext{teachers},  ext{ administrators}}$			
rural	64 (20%)	165 (50%)	105 (31%)
urban	56 (20%)	100 (37%)	114 (42%)
total	120 (20%)	265 (44%)	219 (36%)
K teachers, staff			
rural	68 (20%)	146 (43%)	120 (36%)
urban	46 (17%)	95 (35%)	129 (48%)
total	114 (19%)	241 (40%)	249 (41%)
$\kappa_{ ext{teachers, aides}}$			
rural	79 (23%)	178 (53%)	77 (23%)
urban	56 (21%)	116 (43%)	98 (36%)
total	135 (22%)	294 (49%)	175 (29%)
$\kappa_{ m administrators, staff}$			
rural	66 (20%)	139 (42%)	129 (39%)
urban	49 (18%)	103 (38%)	118 (44%)
total	115 (19%)	242 (40%)	247 (41%)
K <sub>administrators</sub> , aides			
rural	71 (21%)	151 (45%)	112 (34%)
urban	49 (18%)	124 (46%)	97 (36%)
total	120 (20%)	275 (46%)	209 (35%)
$\kappa_{ m staff, aides}$			
rural	53 (16%)	152 (45%)	129 (39%)
urban	53 (20%)	124 (46%)	93 (34%)
total	106 (18%)	276 (46%)	222 (37%)
	` /	` /	` '

Table 4
Distribution of School Districts
Based on consistent over(under)- utilization
relative to more than one other input

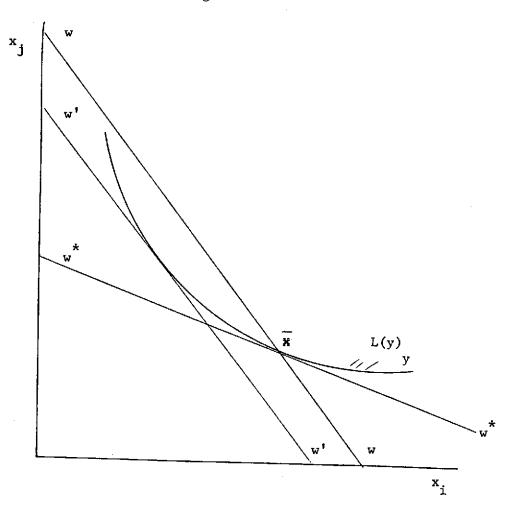
	% of Rural	% of Urban
$\kappa_{ ext{teachers}}$ , administrators $\kappa_{ ext{teachers}}$ , staff		
< 1 > 1	27.5 17.3	17.4 24.0
$\kappa_{ m administrators, teachers}$ $\kappa_{ m administrators, staff}$		
< 1 > 1	14.9 21.5	19.2 20.7
$\kappa_{ ext{staff, teachers}}$ $\kappa_{ ext{staff, administrators}}$		
< 1 > 1	27.5 35.0	37.7 28.1
$\mathcal{K}_{ ext{teachers}}$ , administrators $\mathcal{K}_{ ext{teachers}}$ , staff $\mathcal{K}_{ ext{teachers}}$ , aides		
< 1 > 1	15.9 3.0	7.0 10.3
$\kappa_{ m administrators, teachers}$ $\kappa_{ m administrators, staff}$ $\kappa_{ m administrators, aides}$		
< 1 > 1	11.4 12.5	22.2 16.9
$\kappa_{ ext{staff, teachers}}$ $\kappa_{ ext{staff, administrators}}$ $\kappa_{ ext{staff, aides}}$		
< 1 > 1	25.1 27.5	35.2 25.6

Figure .



Input Distance Function:  $D(y^K, x^K) = OK/OK'$ 

Figure 2



Overutilization of  $x_i$  at  $\overline{x}$ 

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Appendix Table 1

Descriptive Statistics: Variables in Estimating Equation (11)

Mean
(Standard Deviation)

	Rural	Urban	<u>Total</u>
Expenditure Shares			
Teachers	.76	.76	.76
	(.04)	(.04)	(.04)
Administrators	.11	.11	`.11 <sup>´</sup>
	(.02)	(.02)	(.02)
Support Staff	`.06 <sup>*</sup>	`.08	.07 <sup>°</sup>
	(.03)	(.02)	(.03)
Teacher Aides	.05	,04	.05
	(.02)	(.03)	(.03)
Input Per Student			
Teachers	.06	.06	.06
	(.007)	(.006)	(.01)
Administrators	`.006 <sup>′</sup>	.005	.006
	(.001)	(.002)	(.002)
Support Staff	.005	.006	.005
	(.002)	(.002)	(.002)
Teacher Aides	.01	.009	.01
	(.006)	(.007)	(.007)
Output			
$y_1$	659.17	654.87	657.25
	(26.33)	(25.27)	(25.93)
<b>y</b> <sub>2</sub>	600.28	598.68	599.57
	(23.66)	(21.69)	(22.80)
$y_3$	441.14	436.33	438.99
	(22.06)	(19.34)	(21.008)
<b>y</b> <sub>4</sub>	431,94	429.43	430.82
	(14.44)	(11.93)	(13.43)
Fixed Inputs			
$\mathbf{z_i}$	157,36	160.08	158.57
	(17.70)	(23.71)	(20.63)
$\mathbf{z_2}$	202.48	205.95	204.03
	(16.90)	(24.27)	(20.57)
$z_3$	351.23	351.58	351.38
	(16.77)	(21.41)	(18.97)
$Z_{\mathbf{A}}$	353.69	354.37	353.99
	(15.29)	(20.27)	(17.96)
$z_5$	378.01	369.59	374.25
	(137.84)	(100.71)	(122.62)

# Appendix Table 2

### Descriptive Statistics: Ratios Mean (Standard Deviation)

# Ratios of:

		Ratios of.	Bootstrapped
	Employment	Wages	Mean of Shadow Prices
Teachers vs. Administrators			
Rural	11.68	.61	.60
	(2.72)	(.05)	(.13)
Urban	11.64	.60	.62
	(2.99)	(.05)	(.13)
Total	11.66	.61	`.61 <sup>´</sup>
	(2.84)	(.05)	(.13)
	,	,	,
Teachers vs. Support Staff			
Rural	14.91	.87	.84
	(7.72)	(.07)	(.34)
Urban	12.55	.85	`.89 <sup>´</sup>
	(6.51)	(.06)	(.35)
Total	ì3.85	.86	<b>.</b> .87
	(7.15)	(.07)	(.34)
Teachers vs. Teacher Aides			
Rural	8.28	2.56	2.52
	(12.71)	(.37)	(2.53)
Urban	9.94	2.49	2.58
3.54.2	(10.46)	(.36)	(2.39)
Total	9.02	2.53	2,55
	(11.78)	(.37)	(2.49)
	_		
Administrators vs. Support Sta			
Rural	1.35	1.42	1.41
TT 1	(.82)	(.16)	(.51)
Urban	1.15	1.42	1.46
m + 1	(.71)	(.15)	(.56)
Total	1.26	1.42	1.43
	(.78)	(.16)	(.53)

# Appendix Table 2 (Continued)

	cher Aides		
Rural	.73	4.21	4.39
	(.93)	(.70)	(4.99)
Urban	.93	4.15	4.38
	(1.80)	(.64)	(4.80)
Total	.82	4.18	4.39
	(1.22	(.67)	(4.90)
Support Staff vs. Teach	er Aides		
Support Staff vs. Teach Rural	er Aides .63	2.96	3.55
		2.96 (.47)	3.55 (4.52)
	.63		
Rural	.63 (.93)	(.47)	(4.52)
Rural	.63 (.93) .89	(.47) 2.93	(4.52) 3.19

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