

# Inflation as a Global Phenomenon—Some Implications for Inflation Modelling and Forecasting: Model Derivations and Additional Results\*

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## Abstract

Kabukçuoğlu and Martínez-García (Forthcoming) model local inflation dynamics using global inflation and domestic slack motivated by a novel interpretation of the implications of the workhorse open-economy New Keynesian model. Then, Kabukçuoğlu and Martínez-García (Forthcoming) evaluate the pseudo out-of-sample performance of inflation forecasts based on the theoretically-consistent single-equation forecasting specification implied by theory, exploiting the international linkages of inflation. In this paper, we provide a detailed description of the structure of the workhorse open-economy New Keynesian model underlying their analysis derived from first principles. Furthermore, we characterize the solution of the log-linearized model under Wicksellian-style Taylor (1993) monetary policy rules and prove analytically the key equilibrium relationships that Kabukçuoğlu and Martínez-García (Forthcoming) use for inflation forecasting. We make note in particular of the role that the slope of the Phillips curve plays in the equilibrium solution of the workhorse open-economy New Keynesian model and for forecasting purposes. We also include additional results (robustness checks) that support and complement the pseudo out-of-sample forecasting evaluation exercises and the in-sample analysis of the fit of the model reported in Kabukçuoğlu and Martínez-García (Forthcoming).

JEL Classification: C21; C23; C53; F41; F47; F62.

KEY WORDS: Inflation Dynamics; Open-Economy Phillips Curve; Forecasting.

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# 1 The Building Blocks of the Model

The workhorse open-economy New Keynesian model of Kabukçuoğlu and Martínez-García (Forthcoming) incorporates two countries: Home and Foreign.<sup>1</sup> Here we describe the main features of the open-economy New Keynesian framework maintaining the assumption of symmetry in the structure of both countries. Furthermore, we assume that there is an equal mass of one (the entire unit interval) of identical households in each country. The country’s population size is equal to the mass of total varieties that each country produces. We illustrate the two-country model with the first principles from the Home country unless otherwise noted, and use the superscript  $*$  to denote Foreign country variables.

**Households’ Labor Supply and Consumption Behavior.** The lifetime utility of the representative household in the Home country is additively separable in consumption,  $C_t$ , and labor,  $L_t$ , i.e.,

$$\sum_{\tau=0}^{+\infty} \beta^\tau \mathbb{E}_t \left[ \frac{1}{1-\gamma} (C_{t+\tau})^{1-\gamma} - \frac{\chi}{1+\varphi} (L_{t+\tau})^{1+\varphi} \right], \quad (1)$$

where  $0 < \beta < 1$  is the subjective intertemporal discount factor,  $\gamma > 0$  is the inverse of the intertemporal elasticity of substitution, and  $\varphi > 0$  is the inverse of the Frisch elasticity of labor supply. The scaling factor  $\chi > 0$  pins down labor in steady state. The household maximizes its lifetime utility in (1) subject to the following sequence of budget constraints which holds across all states of nature  $\omega_t \in \Omega$ , i.e.,

$$\begin{aligned} P_t C_t + \int_{\omega_{t+1} \in \Omega} Q_t(\omega_{t+1}) B_t^H(\omega_{t+1}) + S_t \int_{\omega_{t+1} \in \Omega} Q_t^*(\omega_{t+1}) B_t^F(\omega_{t+1}) \\ \leq B_{t-1}^H(\omega_t) + S_t B_{t-1}^F(\omega_t) + W_t L_t + Pr_t - T_t, \end{aligned} \quad (2)$$

where  $W_t$  is the nominal wage in the Home country,  $P_t$  is the Home consumer price index (CPI),  $T_t$  is a nominal lump-sum tax (or transfer) imposed by the Home government, and  $Pr_t$  are (per-period) nominal profits from all firms producing the Home varieties. We denote the bilateral nominal exchange rate as  $S_t$  indicating the units of the currency of the Home country that can be obtained per each unit of the Foreign country currency at time  $t$ .<sup>2</sup> Similarly, we define the problem of each household in the Foreign country.

We assume within-country labor mobility—although labor remains immobile across countries—ensuring that wages equalize across firms in a given country but not necessarily across countries. From the household’s first-order conditions we obtain a labor supply equation of the following form,

$$\frac{W_t}{P_t} = \chi U_t (C_t)^\gamma (L_t)^\varphi. \quad (3)$$

With flexible prices, all households are paid the same wage rate  $W_t$  and work the same hours in equilibrium.

The household’s budget constraint also includes a portfolio of one-period Arrow-Debreu securities (contingent bonds) internationally traded, issued in the currencies of both countries and in zero-net supply. That is, the pair  $\{B_t^H(\omega_{t+1}), B_t^F(\omega_{t+1})\}$  refers to the portfolio of contingent bonds issued by both countries and held equally by each household of the Home country. Access to a full set of internationally-traded, one-period

<sup>1</sup>The framework is related to Clarida et al. (2002) and, particularly, to that of Martínez-García and Wynne (2010), Kabukçuoğlu and Martínez-García (2016), and Martínez-García (2017) in their implementation of the standard two-country New Keynesian model.

<sup>2</sup>The nominal exchange rate is fully flexible in this environment.

Arrow-Debreu securities completes the local and international asset markets recursively. The prices of the Home and Foreign contingent bonds expressed in their currencies of denomination are denoted  $Q_t(\omega_{t+1})$  and  $Q_t^*(\omega_{t+1})$ , respectively.<sup>3</sup>

Under complete asset markets, standard no-arbitrage results imply that  $Q_t(\omega_{t+1}) = \frac{S_t}{S_{t+1}(\omega_{t+1})} Q_t^*(\omega_{t+1})$  for every state of nature  $\omega_t \in \Omega$ . Hence, Home and Foreign households can efficiently share risks domestically as well as internationally—this implies that the intertemporal marginal rate of substitution is equalized across countries at each possible state of nature, and accordingly it follows that:

$$\beta \left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} \frac{P_{t-1}}{P_t} = \beta \left( \frac{C_t^*}{C_{t-1}^*} \right)^{-\gamma} \frac{P_{t-1}^* S_{t-1}}{P_t^* S_t}. \quad (4)$$

We define the bilateral real exchange rate as  $RS_t \equiv \frac{S_t P_t^*}{P_t}$ , so by backward recursion the *perfect international risk-sharing condition* in (4) implies that,

$$RS_t = v \left( \frac{C_t^*}{C_t} \right)^{-\gamma}, \quad (5)$$

where  $v \equiv \frac{S_0 P_0^*}{P_0} \left( \frac{C_0^*}{C_0} \right)^\gamma$  is a constant that depends on initial conditions. If the initial conditions correspond to those of the symmetric steady state, then the constant  $v$  is equal to one.

Yields on redundant one-period, non-contingent nominal bonds in the Home country are derived from the price of the contingent Arrow-Debreu securities, which results in the following standard stochastic Euler equation for the Home country:

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right], \quad (6)$$

where  $i_t$  is the riskless Home nominal interest rate. The households' optimization problem also produces the budget constraint of the Home country household given by (2), the initial conditions, and the appropriate (no-Ponzi games) transversality conditions. An analogous labor supply equation, stochastic Euler equation, and household budget constraint (with the corresponding initial conditions and transversality conditions) can be derived for the Foreign country.

$C_t$  is the CES aggregator of both countries' bundles of goods for the Home country household and is defined as,

$$C_t = \left[ (1-\xi)^{\frac{1}{\sigma}} (C_t^H)^{\frac{\sigma-1}{\sigma}} + (\xi)^{\frac{1}{\sigma}} (C_t^F)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (7)$$

where  $\sigma > 0$  is the elasticity of substitution between the consumption bundle of locally-produced goods ( $C_t^H$ ) and the consumption bundle of the foreign-produced goods ( $C_t^F$ ). The share of imported goods from the Foreign country in the consumption basket of the Home country satisfies that  $0 \leq \xi \leq \frac{1}{2}$ , allowing for local-consumption bias whenever  $\xi < \frac{1}{2}$  (since each country produces an equal share of varieties  $\frac{1}{2}$ ). Similarly, the CES aggregator for the Foreign country is defined as:

$$C_t^* = \left[ (\xi)^{\frac{1}{\sigma}} (C_t^{H*})^{\frac{\sigma-1}{\sigma}} + (1-\xi)^{\frac{1}{\sigma}} (C_t^{F*})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (8)$$

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<sup>3</sup>The price of each bond in the currency of the country who did not issue the bond is converted at the prevailing bilateral exchange rate with full exchange rate pass-through under the law of one price (LOOP).

where  $C_t^{F*}$  and  $C_t^{H*}$  are respectively the consumption bundle of foreign-produced goods and of home-produced goods for the Foreign country household, and  $\xi$  identifies the share of imported goods from the Home country in the Foreign consumption basket. The consumption sub-indexes aggregate the consumption of the representative household over the bundle of differentiated varieties produced by each country and are defined as follows:

$$C_t^H = \left[ \int_0^1 C_t(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_t^F = \left[ \int_0^1 C_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \quad (9)$$

$$C_t^{H*} = \left[ \int_0^1 C_t^*(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_t^{F*} = \left[ \int_0^1 C_t^*(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \quad (10)$$

where  $\theta > 1$  is the elasticity of substitution across the differentiated varieties within a country.

The CPIs that correspond to this specification of consumption preferences are,

$$P_t = \left[ (1-\xi)(P_t^H)^{1-\sigma} + \xi(P_t^F)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad P_t^* = \left[ \xi(P_t^{H*})^{1-\sigma} + (1-\xi)(P_t^{F*})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad (11)$$

and,

$$P_t^H = \left[ \int_0^1 P_t(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}, \quad P_t^F = \left[ \int_0^1 P_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}}, \quad (12)$$

$$P_t^{H*} = \left[ \int_0^1 P_t^*(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}, \quad P_t^{F*} = \left[ \int_0^1 P_t^*(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}}, \quad (13)$$

where  $P_t^H$  and  $P_t^{F*}$  are the price sub-indexes corresponding to the bundle of varieties produced locally in the Home and Foreign countries, respectively. The price sub-index  $P_t^F$  represents the Home country price of the bundle of Foreign varieties while  $P_t^{H*}$  is the Foreign country price for the bundle of Home varieties. The price of the variety  $h$  produced in the Home country is expressed as  $P_t(h)$  and  $P_t^*(h)$  in units of the Home and Foreign currency, respectively. Similarly, the price of the variety  $f$  produced in the Foreign country is quoted in both countries as  $P_t(f)$  and  $P_t^*(f)$ , respectively.

Each household decides how much to allocate to the different varieties of goods produced in each country. Given the structure of preferences, the utility maximization problem implies that the household's demand for each variety is given by,

$$C_t(h) = \left( \frac{P_t(h)}{P_t^H} \right)^{-\theta} C_t^H, \quad C_t(f) = \left( \frac{P_t(f)}{P_t^F} \right)^{-\theta} C_t^F, \quad (14)$$

$$C_t^*(h) = \left( \frac{P_t^*(h)}{P_t^{H*}} \right)^{-\theta} C_t^{H*}, \quad C_t^*(f) = \left( \frac{P_t^*(f)}{P_t^{F*}} \right)^{-\theta} C_t^{F*}, \quad (15)$$

while the demand for the bundle of varieties produced by each country is simply equal to,

$$C_t^H = (1-\xi) \left( \frac{P_t^H}{P_t} \right)^{-\sigma} C_t, \quad C_t^F = \xi \left( \frac{P_t^F}{P_t} \right)^{-\sigma} C_t, \quad (16)$$

$$C_t^{H*} = \xi \left( \frac{P_t^{H*}}{P_t^*} \right)^{-\sigma} C_t^*, \quad C_t^{F*} = (1-\xi) \left( \frac{P_t^{F*}}{P_t^*} \right)^{-\sigma} C_t^*. \quad (17)$$

These equations relate the demand for each variety—whether produced domestically or imported—to the aggregate consumption of the country.

**The Firms' Price-Setting Behavior.** Home firms produce their variety of output subject to a linear-in-labor technology, i.e.  $Y_t(h) = A_t L_t(h)$  for all  $h \in [0, 1]$ . Each firm located in either the Home or Foreign country supplies its local market and exports its own differentiated variety operating under monopolistic competition. We assume producer currency pricing (PCP), so firms set prices by invoicing all sales in their local currency. The PCP assumption implies that the law of one price (LOOP) holds at the variety level—i.e., for each variety  $h$  produced in the Home country, it must hold that  $P_t(h) = S_t P_t^*(h)$  (similarly, for each variety  $f$  produced in the Foreign country holds that  $P_t(f) = S_t P_t^*(f)$ ). Hence, it follows naturally that the conforming price sub-indexes in both countries for the same bundle of varieties must satisfy that  $P_t^H = S_t P_t^{H*}$  and  $P_t^F = S_t P_t^{F*}$ .

The bilateral terms of trade  $ToT_t \equiv \frac{P_t^F}{S_t P_t^{H*}}$  define the Home country value of the imported bundle of goods from the Foreign country in Home currency units relative to the Foreign value of the bundle of the Home country's exports (quoted in the currency of the Home country at the prevailing bilateral nominal exchange rate). Under the LOOP, terms of trade can be expressed as,

$$ToT_t \equiv \frac{P_t^F}{S_t P_t^{H*}} = \frac{P_t^F}{P_t^H}. \quad (18)$$

Even though the LOOP holds, the assumption of local-product bias in consumption introduces deviations from purchasing power parity (PPP) at the level of the consumption basket. For this reason,  $P_t \neq S_t P_t^*$  and therefore the bilateral real exchange rate between both countries deviates from one—i.e.,  $RS_t \equiv \frac{S_t P_t^*}{P_t} = \left[ \frac{\xi + (1-\xi)(ToT_t)^{1-\sigma}}{(1-\xi) + \xi(ToT_t)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}} \neq 1$  if  $\xi \neq \frac{1}{2}$ .<sup>4</sup>

Given households' preferences in each country, the demand for any variety  $h \in [0, 1]$  produced in the Home country is given as,

$$\begin{aligned} Y_t(h) &\equiv C_t(h) + C_t^*(h) = (1-\xi) \left( \frac{P_t(h)}{P_t^H} \right)^{-\theta} \left( \frac{P_t^H}{P_t} \right)^{-\sigma} C_t + \xi \left( \frac{P_t(h)}{P_t^H} \right)^{-\theta} \left( \frac{P_t^{H*}}{P_t^*} \right)^{-\sigma} C_t^* \\ &= \left( \frac{P_t(h)}{P_t^H} \right)^{-\theta} \left( \frac{P_t^H}{P_t} \right)^{-\sigma} \left[ (1-\xi) C_t + \xi \left( \frac{1}{RS_t} \right)^{-\sigma} C_t^* \right]. \end{aligned} \quad (19)$$

Similarly, we derive the demand for each variety  $f \in [0, 1]$  produced by the Foreign firms. Firms maximize profits subject to a partial adjustment rule à la Calvo (1983) at the variety level (that is, subject to sticky prices). In each period, every firm receives either a signal to maintain their prices with probability  $0 < \alpha < 1$  or a signal to re-optimize them with probability  $1 - \alpha$ . At time  $t$ , the re-optimizing firm producing variety  $h$  in the Home country chooses a price  $\tilde{P}_t(h)$  optimally to maximize the expected discounted value of its profits, i.e.,

$$\sum_{\tau=0}^{+\infty} \mathbb{E}_t \left\{ (\alpha\beta)^\tau \left( \frac{C_{t+\tau}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+\tau}} \left[ \tilde{Y}_{t,t+\tau}(h) \left( \tilde{P}_t(h) - (1-\phi) M C_{t+\tau} \right) \right] \right\}, \quad (20)$$

subject to the constraint that the aggregate demand given in (19) is always satisfied at the set price  $\tilde{P}_t(h)$

<sup>4</sup>For more in-depth analysis on the role of international price-setting on PPP and the design of optimal monetary policy, see Engel (2011).

for as long as that price remains unchanged.  $\tilde{Y}_{t,t+\tau}(h)$  indicates the demand for consumption of the variety  $h$  produced in the Home country at time  $t + \tau$  ( $\tau > 0$ ) whenever the prevailing prices remain unchanged since time  $t$ —i.e., whenever  $P_{t+s}(h) = \tilde{P}_t(h)$  for all  $0 \leq s \leq \tau$ . An analogous problem describes the optimal price-setting behavior of the re-optimizing firms in the Foreign country.

Hence, the (before-subsidy) nominal marginal cost in the Home country  $MC_t$  can be expressed as,

$$MC_t \equiv \left( \frac{W_t}{A_t} \right), \quad (21)$$

where the Home productivity (TFP) shock is denoted by  $A_t$ . A similar expression holds for the Foreign country's (before-subsidy) nominal marginal cost. Productivity shocks are described with the following bivariate stochastic process,

$$A_t = (A)^{1-\delta_a} (A_{t-1})^{\delta_a} (A_{t-1}^{*,a*})^{\delta_{a,a*}} e^{\varepsilon_t^a}, \quad (22)$$

$$A_t^* = (A)^{1-\delta_a} (A_{t-1})^{\delta_{a,a*}} (A_{t-1}^*)^{\delta_a} e^{\varepsilon_t^{a*}}, \quad (23)$$

$$\begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^{a*} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{a,a*} \sigma_a^2 \\ \rho_{a,a*} \sigma_a^2 & \sigma_a^2 \end{pmatrix} \right), \quad (24)$$

where  $A$  is the unconditional mean of the process,  $\delta_a$  and  $\delta_{a,a*}$  capture the persistence and cross-country spillovers, and  $(\varepsilon_t^a, \varepsilon_t^{a*})^T$  is a vector of Gaussian innovations with a common variance  $\sigma_a^2$  and possibly correlated across both countries  $\rho_{a,a*}$ .

The optimal pricing rule of the re-optimizing firm  $h$  of the Home country at time  $t$  is given by,

$$\tilde{P}_t(h) = \left( \frac{\theta}{\theta - 1} (1 - \phi) \right) \frac{\sum_{\tau=0}^{+\infty} (\alpha\beta)^\tau \mathbb{E}_t \left[ \left( \frac{(C_{t+\tau})^{-\gamma}}{P_{t+\tau}} \right) \tilde{Y}_{t,t+\tau}(h) MC_{t+\tau} \right]}{\sum_{\tau=0}^{+\infty} (\alpha\beta)^\tau \mathbb{E}_t \left[ \left( \frac{(C_{t+\tau})^{-\gamma}}{P_{t+\tau}} \right) \tilde{Y}_{t,t+\tau}(h) \right]}, \quad (25)$$

where  $\phi$  is a time-invariant labor subsidy which is proportional to the nominal marginal cost  $MC_{t+\tau}$ . An analogous expression can be derived for the optimal pricing rule of the re-optimizing firm  $f$  in the Foreign country to pin down  $\tilde{P}_t(f)$ .

Given the inherent symmetry of the Calvo-type pricing scheme, the price sub-indexes in both countries for the bundles of varieties produced locally,  $P_t^H$  and  $P_t^{F*}$ , respectively, evolve according to the following law of motion,

$$(P_t^H)^{1-\theta} = \alpha (P_{t-1}^H)^{1-\theta} + (1 - \alpha) \left( \tilde{P}_t(h) \right)^{1-\theta}, \quad (26)$$

$$(P_t^{F*})^{1-\theta} = \alpha (P_{t-1}^{F*})^{1-\theta} + (1 - \alpha) \left( \tilde{P}_t^*(f) \right)^{1-\theta}, \quad (27)$$

linking the current-period price sub-index to the previous-period price sub-index and to the symmetric pricing decision taken by all the re-optimizing firms during the current period. Then, the LOOP relates these price sub-indexes to  $P_t^{H*}$  and  $P_t^F$  with full pass-through of the bilateral nominal exchange rate  $S_t$ .

In order to characterize the allocation in the counterfactual case where nominal rigidities (monopolistic competition with staggered price-setting à la Calvo (1983) under PCP), we must replace the optimal pricing

rule in (25) with the standard rule under perfect competition and flexible prices, i.e.,

$$\tilde{P}_t(h) = MC_t, \quad (28)$$

for each firm  $h$  in the Home country at time  $t$ . Solving the model under this alternative price-setting rule would define the equilibrium allocation—and, in particular, the output and real interest rates—that would prevail in a frictionless environment subject to otherwise the same shocks. We refer to output and real rates in this frictionless counterfactual as the output potential and natural (rate) rate of the economy, respectively.

**Monetary and Fiscal Policy.** We model monetary policy in the Home country according to a standard Wicksellian implementation of the Taylor (1993)-type rule on the short-term nominal interest rate,  $i_t$ , i.e.,

$$\frac{1+i_t}{1+\bar{i}} = \frac{M_t}{M} \left( \frac{1+\bar{r}_t}{1+\bar{r}} \right) \left( \frac{\bar{\Pi}_t}{\bar{\Pi}} \right) \left[ \left( \frac{\Pi_t}{\bar{\Pi}_t} \right)^{\psi_\pi} \left( \frac{Y_t}{\bar{Y}_t} \right)^{\psi_x} \right], \quad (29)$$

where  $\bar{i}$  and  $\bar{r} \equiv \beta^{-1}$  denote the nominal and real interest rate in steady state,  $\bar{\Pi} = 1$  is the deterministic steady state inflation rate, and  $\psi_\pi > 0$  and  $\psi_x \geq 0$  are the policy parameters that capture the sensitivity of the monetary policy rule to changes in inflation and the output gap, respectively.  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  is the (gross) CPI inflation rate,  $\bar{\Pi}_t$  is the corresponding (time-varying) inflation rate target,  $Y_t$  defines the aggregate output produced in the Home country, and  $\frac{Y_t}{\bar{Y}_t}$  is the output gap in levels. Here,  $\bar{Y}_t$  defines the potential output level of the Home country and  $\bar{r}_t$  is the natural (real) rate of interest—potential output and the natural rate correspond to the output and real rates that would prevail absent all nominal rigidities (but given the same realization of the shocks).

The monetary policy shock in the Home country is defined as  $M_t$ . Monetary shocks are described with the following bivariate stochastic process:

$$M_t = (M)^{1-\delta_m} (M_{t-1})^{\delta_m} e^{\varepsilon_t^m}, \quad (30)$$

$$M_t^* = (M)^{1-\delta_m} (M_{t-1}^*)^{\delta_m} e^{\varepsilon_t^{m*}}, \quad (31)$$

$$\begin{pmatrix} \varepsilon_t^m \\ \varepsilon_t^{m*} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & \rho_{m,m^*} \sigma_m^2 \\ \rho_{m,m^*} \sigma_m^2 & \sigma_m^2 \end{pmatrix} \right), \quad (32)$$

where  $M$  is the unconditional mean of the process,  $\delta_m$  captures the persistence, and  $(\varepsilon_t^m, \varepsilon_t^{m*})^T$  is a vector of Gaussian innovations with a common variance  $\sigma_m^2$  and possibly correlated across both countries  $\rho_{m,m^*}$ .

Monopolistic competition in production and labor introduces a distortive steady-state price mark-up,  $\frac{\theta}{\theta-1}$ , that drives a wedge between prices and marginal costs. This steady-state distortion is a function of the elasticity of substitution across output varieties within a country  $\theta > 1$ . Home and Foreign governments raise lump-sum taxes from local households within their borders in order to subsidize labor employment and eliminate the steady-state price mark-up distortions. An optimal (time-invariant) labor subsidy proportional to the marginal cost set to be  $\phi = \frac{1}{\theta}$  in every country neutralizes the steady-state monopolistic competition mark-up in the pricing rule (equation (25) in steady state).

Monetary non-neutrality arises in the model from monopolistic competition and producer currency pricing under staggered price-setting behavior à la Calvo (1983). Firms set their prices relative to a (time-varying)

stochastic long-run inflation rate that is pin down in equilibrium by the central bank's inflation target—that is, by  $\bar{\Pi}_t$  in the Home country. Inflation target shocks are described with the following bivariate process:

$$\bar{\Pi}_t = (\bar{\Pi})^{1-\delta_\pi} (\bar{\Pi}_{t-1})^{\delta_\pi} e^{\varepsilon_t^\pi}, \quad (33)$$

$$\bar{\Pi}_t^* = (\bar{\Pi})^{1-\delta_\pi} (\bar{\Pi}_{t-1}^*)^{\delta_\pi} e^{\varepsilon_t^{\pi*}}, \quad (34)$$

$$\begin{pmatrix} \varepsilon_t^\pi \\ \varepsilon_t^{\pi*} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\pi^2 & 0 \\ 0 & \sigma_\pi^2 \end{pmatrix} \right), \quad (35)$$

where  $\bar{\Pi}$  is the unconditional mean of the process which we set to be zero,  $\delta_\pi$  capture the persistence which is arbitrarily close to one to approximate the random walk behavior which we assume in the paper, and  $(\varepsilon_t^\pi, \varepsilon_t^{\pi*})^T$  is a vector of Gaussian innovations with a common variance  $\sigma_\pi^2$  and uncorrelated across both countries. Setting  $\sigma_\pi^2$  arbitrarily close to zero for the random walk inflation target approximates the case where the central bank targets inflation deviations relative to the model's deterministic steady state  $\bar{\Pi}$ .

Although the model permits permanent shifts in the inflation target ( $\bar{\Pi}_t$  and  $\bar{\Pi}_t^*$ ), the resulting inflation process remains stationary around the deterministic zero-inflation steady state (which we retain at  $\bar{\Pi} = 0$ ). Hence, this modification of the economic environment does not alter the functional form of the Phillips curve except that inflation is defined in deviations from the long-run stochastic inflation rate determined by the inflation target rather than directly in deviations from the deterministic steady state. This is unlike what happens when the price-setting rule for firms is log-linearized around a non-zero steady state inflation rate (as noted by Ascari (2004) and Sahuc (2006), among others).



## 2 The Log-Linearized Equilibrium Conditions

Here, we report the system of equations derived after log-linearizing the equilibrium conditions of the workhorse open-economy New Keynesian model described from first principles in the previous section.<sup>5</sup> The derivation of this set of equations is extensively discussed in Martínez-García and Wynne (2010), Duncan and Martínez-García (2015), Kabukcuoğlu and Martínez-García (2016), and Martínez-García (2017).

<b>Workhorse Open-Economy New Keynesian Model</b>	
<b>Home Country</b>	
NKPC	$\widehat{\pi}_t - \bar{\pi}_t \approx \beta \mathbb{E}_t (\widehat{\pi}_{t+1} - \bar{\pi}_{t+1}) + \Phi (\varphi + \gamma) [\kappa \widehat{x}_t + (1 - \kappa) \widehat{x}_t^*]$
Dynamic IS	$\gamma (\mathbb{E}_t [\widehat{x}_{t+1}] - \widehat{x}_t) \approx \Omega [\widehat{i}_t - \mathbb{E}_t [\widehat{\pi}_{t+1}] - \widehat{r}_t] + (1 - \Omega) [\widehat{i}_t^* - \mathbb{E}_t [\widehat{\pi}_{t+1}^*] - \widehat{r}_t^*]$
Taylor rule	$\widehat{i}_t \approx \widehat{r}_t + \bar{\pi}_t + \psi_\pi (\widehat{\pi}_t - \bar{\pi}_t) + \psi_x \widehat{x}_t + \widehat{m}_t$
Natural interest rate	$\widehat{r}_t \approx \gamma \left[ \Theta \left( \mathbb{E}_t [\widehat{y}_{t+1}] - \widehat{y}_t \right) + (1 - \Theta) \left( \mathbb{E}_t [\widehat{y}_{t+1}^*] - \widehat{y}_t^* \right) \right]$
Potential output	$\widehat{y}_t \approx \left( \frac{1+\varphi}{\gamma+\varphi} \right) [\Lambda \widehat{a}_t + (1 - \Lambda) \widehat{a}_t^*]$
<b>Foreign Country</b>	
NKPC	$\widehat{\pi}_t^* - \bar{\pi}_t^* \approx \beta \mathbb{E}_t (\widehat{\pi}_{t+1}^* - \bar{\pi}_{t+1}^*) + \Phi (\varphi + \gamma) [(1 - \kappa) \widehat{x}_t + \kappa \widehat{x}_t^*]$
Dynamic IS	$\gamma (\mathbb{E}_t [\widehat{x}_{t+1}^*] - \widehat{x}_t^*) \approx (1 - \Omega) [\widehat{i}_t - \mathbb{E}_t [\widehat{\pi}_{t+1}] - \widehat{r}_t] + \Omega [\widehat{i}_t^* - \mathbb{E}_t [\widehat{\pi}_{t+1}^*] - \widehat{r}_t^*]$
Taylor rule	$\widehat{i}_t^* \approx \widehat{r}_t^* + \bar{\pi}_t^* + \psi_\pi (\widehat{\pi}_t^* - \bar{\pi}_t^*) + \psi_x \widehat{x}_t^* + \widehat{m}_t^*$
Natural interest rate	$\widehat{r}_t^* \approx \gamma \left[ (1 - \Theta) \left( \mathbb{E}_t [\widehat{y}_{t+1}] - \widehat{y}_t \right) + \Theta \left( \mathbb{E}_t [\widehat{y}_{t+1}^*] - \widehat{y}_t^* \right) \right]$
Potential output	$\widehat{y}_t^* \approx \left( \frac{1+\varphi}{\gamma+\varphi} \right) [(1 - \Lambda) \widehat{a}_t + \Lambda \widehat{a}_t^*]$
<b>Composite Parameters</b>	
	$\Phi \equiv \left( \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \right),$
	$\kappa \equiv (1 - \xi) \left[ 1 - (\sigma\gamma - 1) \left( \frac{\gamma}{\varphi + \gamma} \right) \left( \frac{(2\xi)(1-2\xi)}{1 + (\sigma\gamma - 1)(2\xi)(2(1-\xi))} \right) \right],$
	$\Theta \equiv (1 - \xi) \left[ \frac{\sigma\gamma - (\sigma\gamma - 1)(1-2\xi)}{\sigma\gamma - (\sigma\gamma - 1)(1-2\xi)^2} \right] = (1 - \xi) \left[ \frac{1 + (\sigma\gamma - 1)(2\xi)}{1 + (\sigma\gamma - 1)(2\xi)(2(1-\xi))} \right],$
	$\Omega \equiv (1 - \xi) \left( \frac{1-2\xi(1-\sigma\gamma)}{1-2\xi} \right),$
	$\Lambda \equiv 1 + \frac{1}{2} \left[ \frac{\left( \frac{\gamma}{\varphi + \gamma} \right) (\sigma\gamma - 1)(2\xi)(2(1-\xi))}{1 + \left( 1 - \frac{\gamma}{\varphi + \gamma} \right) (\sigma\gamma - 1)(2\xi)(2(1-\xi))} \right].$

TABLE B1

<sup>5</sup>Any variable identified with lower-case letters and a caret on top represents a transformation (expressed in log-deviations relative to its steady state) of the corresponding variable in upper-case letters.

Country-Specific, Exogenous Shocks	
Productivity shock	$\begin{pmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{a}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a^*} \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a^*} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*} \sigma_a^2 \\ \rho_{a,a^*} \sigma_a^2 & \sigma_a^2 \end{pmatrix} \right)$
Monetary shock	$\begin{pmatrix} \widehat{m}_t \\ \widehat{m}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_m & 0 \\ 0 & \delta_m \end{pmatrix} \begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m^*} \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m^*} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & \rho_{m,m^*} \sigma_m^2 \\ \rho_{m,m^*} \sigma_m^2 & \sigma_m^2 \end{pmatrix} \right)$
Inflation target shock	$\begin{pmatrix} \widehat{\varepsilon}_t^\pi \\ \widehat{\varepsilon}_t^{\pi^*} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\pi^2 & 0 \\ 0 & \sigma_\pi^2 \end{pmatrix} \right)$

TABLE B2

## 2.1 Characterization of the Model Solution

We define the world aggregate with output-based weights in this two-country setting as  $\widehat{g}_t^W \equiv \frac{1}{2}\widehat{g}_t + \frac{1}{2}\widehat{g}_t^*$  and label the difference between both countries as  $\widehat{g}_t^R \equiv \widehat{g}_t - \widehat{g}_t^*$ . From here, it follows that any pair of Home and Foreign variables,  $\widehat{g}_t$  and  $\widehat{g}_t^*$  respectively, can be decomposed as,

$$\widehat{g}_t = \widehat{g}_t^W + \frac{1}{2}\widehat{g}_t^R, \quad \widehat{g}_t^* = \widehat{g}_t^W - \frac{1}{2}\widehat{g}_t^R. \quad (36)$$

Hence, if we solve for  $\widehat{g}_t^W$  and  $\widehat{g}_t^R$ , the transformation in (36) suffices to back out the corresponding country variables  $\widehat{g}_t$  and  $\widehat{g}_t^*$ . Then, as shown in Martínez-García (2017), we can orthogonalize the two-country model in Table B1 and Table B2 into one aggregate (or world) economic system for  $\widehat{g}_t^W$  and another differential system for  $\widehat{g}_t^R$  that can be studied separately (and recombined with the help of (36)).

The New Keynesian Phillips curve (NKPC) equations for the world and difference sub-systems can be cast into the following form:

$$\widehat{\pi}_t^s - \overline{\pi}_t^s = \beta \mathbb{E}_t (\widehat{\pi}_{t+1}^s - \overline{\pi}_{t+1}^s) + \Phi(\varphi + \gamma) \kappa^s \widehat{x}_t^s, \quad \text{for } s = W, R, \quad (37)$$

where  $\mathbb{E}_t(\cdot)$  are expectations formed conditional on information up to time  $t$ ,  $\widehat{x}_t^W$  is the global output gap ( $\widehat{x}_t^R$  differential slack),  $\widehat{\pi}_t^W$  is global inflation ( $\widehat{\pi}_t^R$  differential inflation), and  $\overline{\pi}_t^W$  is the global inflation target ( $\overline{\pi}_t^R$  differential inflation target). Furthermore,  $\kappa^W \equiv 1$  is the composite for the slope on global slack and  $\kappa^R \equiv (2\kappa - 1) > 0$  is the slope on differential slack—as defined in Table B1, the composite coefficient  $\kappa \equiv (1 - \xi) \left[ 1 - (\sigma\gamma - 1) \left( \frac{\gamma}{\varphi + \gamma} \right) \left( \frac{(2\xi)(1-2\xi)}{1 + (\sigma\gamma - 1)(2\xi)(2(1-\xi))} \right) \right]$  depends on deep structural parameters but not on the policy parameters.

The dynamic IS equations for the world and difference sub-systems are given by:

$$\widehat{x}_t^s = \mathbb{E}_t [\widehat{x}_{t+1}^s] - \frac{\Omega^s}{\gamma} \left( \widehat{i}_t^s - \mathbb{E}_t [\widehat{\pi}_{t+1}^s] - \widehat{r}_t^s \right), \quad \text{for } s = W, R, \quad (38)$$

where  $\widehat{i}_t^W$  is the world aggregate short-term nominal interest rate ( $\widehat{i}_t^R$  differential nominal interest rate), and

$\widehat{r}_t^W$  is the world natural real rate ( $\widehat{r}_t^R$  differential natural real rate). Furthermore,  $\Omega^W \equiv 1$  is the slope on the world real interest rate gap (i.e., the slope on  $(\widehat{i}_t^W - \mathbb{E}_t[\widehat{\pi}_{t+1}^W]) - \widehat{r}_t^W$ ) and  $\Omega^R \equiv (2\Omega - 1) > 0$  is the slope on the differential real interest rate gap (i.e., the slope on  $(\widehat{i}_t^R - \mathbb{E}_t[\widehat{\pi}_{t+1}^R]) - \widehat{r}_t^R$ )—as defined in Table B1, the composite coefficient  $\Omega \equiv (1 - \xi) \left( \frac{1 - 2\xi(1 - \sigma\gamma)}{1 - 2\xi} \right)$  depends on deep structural parameters but not on the policy parameters.

Finally, we complete the description of the orthogonalized model with the Wicksellian-style Taylor (1993) rules for the world and difference sub-systems which can be written as follows:

$$\widehat{i}_t^s = \widehat{r}_t^s + \bar{\pi}_t^s + \psi_\pi (\widehat{\pi}_t^s - \bar{\pi}_t^s) + \psi_x \widehat{x}_t^s + \widehat{m}_t^s, \text{ for } s = W, R, \quad (39)$$

where  $\bar{\pi}_t^W$  is the world's inflation target ( $\bar{\pi}_t^R$  differential inflation target), and  $\widehat{m}_t^W$  can be interpreted as global innovations on the stance of monetary policy ( $\widehat{m}_t^R$  differential innovations on the stance of monetary policy).

Using the aggregate and differential monetary policy rules in (39) to replace  $\widehat{i}_t^s$  in (37)–(38) for  $s = W, R$ , the sub-system of equations that determines inflation and slack for the aggregates and for the cross-country differentials can be written in the following form:<sup>6</sup>

$$\begin{pmatrix} 1 + \frac{\Omega^s}{\gamma} \psi_x & \frac{\Omega^s}{\gamma} \psi_\pi \\ \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s \psi_x & 1 + \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s \psi_\pi \end{pmatrix} \begin{pmatrix} \widehat{x}_t^s \\ \widehat{\pi}_t^s - \bar{\pi}_t^s \end{pmatrix} = \begin{pmatrix} 1 \\ \Phi(\varphi + \gamma) \kappa^s \end{pmatrix} \begin{pmatrix} \mathbb{E}_t[\widehat{x}_{t+1}^s] \\ \mathbb{E}_t[\widehat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s] \end{pmatrix} - \begin{pmatrix} \frac{\Omega^s}{\gamma} \\ \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s \end{pmatrix} \widehat{m}_t^s, \quad (40)$$

or more compactly as,

$$\begin{pmatrix} \widehat{x}_t^s \\ \widehat{\pi}_t^s - \bar{\pi}_t^s \end{pmatrix} = \Psi^s \begin{pmatrix} 1 & \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \\ \Phi(\varphi + \gamma) \kappa^s & \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s + \beta \left( 1 + \frac{\Omega^s}{\gamma} \psi_x \right) \end{pmatrix} \begin{pmatrix} \mathbb{E}_t[\widehat{x}_{t+1}^s] \\ \mathbb{E}_t[\widehat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s] \end{pmatrix} - \Psi^s \begin{pmatrix} \frac{\Omega^s}{\gamma} \\ \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s \end{pmatrix} \widehat{m}_t^s, \quad (41)$$

where  $\Psi^s \equiv \frac{1}{1 + \frac{\Omega^s}{\gamma} (\psi_x + \Phi(\varphi + \gamma) \kappa^s \psi_\pi)} > 0$ ,  $\Phi(\varphi + \gamma) > 0$ ,  $\psi_\pi > 0$ , and  $\psi_x \geq 0$ . Moreover, we also note that  $\Omega^W = \kappa^W = 1$ ,  $\Omega^R > 0$ , and  $\kappa^R > 0$ .

We can write the aggregate and difference sub-systems of expectational equations in (41) in canonical form as:

$$\widehat{z}_t^s = A^s \mathbb{E}_t(\widehat{z}_{t+1}^s) + a^s \widehat{m}_t^s, \text{ for } s = W, R, \quad (42)$$

where the vector  $\widehat{z}_t^s \equiv (\widehat{x}_t^s, \widehat{\pi}_t^s - \bar{\pi}_t^s)^T$  includes slack ( $\widehat{x}_t^s$ ) and cyclical inflation ( $\widehat{\pi}_t^s - \bar{\pi}_t^s$ ) for  $s = W, R$ , and the driving process is given by the monetary shock  $\widehat{m}_t^s$  for  $s = W, R$  alone. The natural real rate  $\widehat{r}_t^s$  drops out of (42) given the Wicksellian specification of the Taylor (1993) rule that we have incorporated in

<sup>6</sup> When we replace  $\widehat{i}_t^s$  given by (39) into (38) we are introducing the inflation target  $\bar{\pi}_t^s$  as a term while we drop the stochastic natural rate  $\widehat{r}_t^s$  entirely from the resulting system of equations. Here, since we assume that the inflation target follows a random walk we use the property that  $\mathbb{E}_t[\widehat{\pi}_{t+1}^s] = \bar{\pi}_t^s$  to re-write the system accordingly in terms of cyclical inflation alone (i.e., in terms of  $\widehat{\pi}_t^s - \bar{\pi}_t^s$  alone).

(39). The matrices of structural parameters  $A^s \equiv \Psi^s \begin{pmatrix} 1 & \frac{\Omega^s}{\gamma} (1 - \beta\psi_\pi) \\ \Phi(\varphi + \gamma) \kappa^s & \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s + \beta \left(1 + \frac{\Omega^s}{\gamma} \psi_x\right) \end{pmatrix}$

and  $a^s \equiv -\Psi^s \begin{pmatrix} \frac{\Omega^s}{\gamma} \\ \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s \end{pmatrix}$  characterize the dynamics of each sub-system. Hence, we say that cyclical inflation and slack depend on the monetary policy shock ( $\widehat{m}_t^s$ ) while inflation itself depends on the monetary shock ( $\widehat{m}_t^s$ ) through its cyclical component and depends on the inflation target shock ( $\widehat{\pi}_t^s$ ) through its long-run component.

**Existence and Uniqueness of the Solution.** As shown in Martínez-García (2017), under the assumption that  $\widehat{m}_t^s$  is stationary, a system like (42) has a unique nonexplosive solution in which the vector  $\widehat{z}_t^s \equiv (\widehat{x}_t^s, \widehat{\pi}_t^s - \widehat{\pi}_t^s)^T$  is stationary whenever both eigenvalues of the matrix  $A^s$  are inside the unit circle for each sub-system (for  $s = W, R$ ). The eigenvalues corresponding to the matrix  $A^s$  can be written as:

$$\lambda_1^s \equiv \frac{1}{2} \Psi^s \left( \Xi^s - \sqrt{(\Xi^s)^2 - 4 \frac{\beta}{\Psi^s}} \right), \quad \lambda_2^s \equiv \frac{1}{2} \Psi^s \left( \Xi^s + \sqrt{(\Xi^s)^2 - 4 \frac{\beta}{\Psi^s}} \right), \quad (43)$$

where  $\Psi^s \equiv \frac{1}{1 + \frac{\Omega^s}{\gamma} (\psi_x + \Phi(\varphi + \gamma) \kappa^s \psi_\pi)} > 0$  and  $\Xi^s \equiv 1 + \beta + \frac{\Omega^s}{\gamma} (\beta\psi_x + \Phi(\varphi + \gamma) \kappa^s) > 0$  hold given that  $\Phi(\varphi + \gamma) > 0$  and the policy parameters satisfy that  $\psi_\pi > 0$  and  $\psi_x \geq 0$ . Moreover, for any degree of (non-trivial) openness  $0 < \xi < \frac{1}{2}$ , it also holds that  $\Omega^W = \kappa^W = 1$ ,  $\Omega^R > 1$  for all  $\sigma\gamma > \left(\frac{1-2\xi}{1+(1-2\xi)}\right) > 0$ , and  $0 < \kappa^R < 1$  for all  $\sigma\gamma > \max \left\{ 0, 1 - \left( \frac{1}{2\xi + \left(\frac{\gamma}{\varphi + \gamma}\right)(1-2\xi)} \right) \frac{1}{2(1-\xi)} \right\}$ . For standard parameterizations of the model, it naturally follows that  $0 < \lambda_1^s < \lambda_2^s$ . Therefore, both eigenvalues of  $A^s$  lie inside the unit circle if and only if  $\lambda_2^s \equiv \frac{1}{2} \Psi^s \left( \Xi^s + \sqrt{(\Xi^s)^2 - 4 \frac{\beta}{\Psi^s}} \right) < 1$ . This inequality holds, in turn, whenever  $\Psi^s \sqrt{(\Xi^s)^2 - 4 \frac{\beta}{\Psi^s}} < 2 - \Psi^s \Xi^s$ . Taking the square on both sides of the inequality—i.e.,  $(\Psi^s)^2 \left( (\Xi^s)^2 - 4 \frac{\beta}{\Psi^s} \right) < (\Psi^s \Xi^s - 2)^2$ —and then, re-arranging terms, the inequality can be rewritten as:  $\Psi^s (\Xi^s - \beta) < 1$ . From here it follows that  $\lambda_2^s < 1$  if and only if  $\frac{1 + \frac{\Omega^s}{\gamma} (\beta\psi_x + \Phi(\varphi + \gamma) \kappa^s)}{1 + \frac{\Omega^s}{\gamma} (\psi_x + \Phi(\varphi + \gamma) \kappa^s \psi_\pi)} < 1$  or, after further algebraic manipulations, if and only if  $\psi_\pi + \left( \frac{1-\beta}{\Phi(\varphi + \gamma) \kappa^s} \right) \psi_x > 1$ .

**Proposition 1** *An open-economy variant of the Taylor principle which requires that  $\psi_\pi + \left( \frac{1-\beta}{\Phi(\varphi + \gamma) \kappa^s} \right) \psi_x > 1$  for  $s = W, R$  is needed to ensure the uniqueness and existence of the nonexplosive solution for the aggregate and differential sub-systems. The standard Taylor principle ( $\psi_\pi > 1$ ) is sufficient, but not necessary, in order to prove existence and uniqueness of the solution. Moreover, the open-economy Taylor principle reduces to its standard closed-economy variant which requires  $\psi_\pi + \left( \frac{1-\beta}{\Phi(\varphi + \gamma)} \right) \psi_x > 1$  whenever  $\sigma\gamma > \max \left\{ 0, 1 - \left( \frac{1}{2\xi + \left(\frac{\gamma}{\varphi + \gamma}\right)(1-2\xi)} \right) \frac{1}{2(1-\xi)} \right\}$ .*

**Proof.** Analogous to the derivations in Martínez-García (2017). ■

**Solving the Model Using the Method of Undetermined Coefficients.** Based on the method of undetermined coefficients used in Martínez-García (2017), we conjecture that the solution for the endogenous

variables  $(\hat{x}_t^s, \hat{\pi}_t^s, \hat{i}_t^s)$  can be expressed as:

$$\hat{\pi}_t^s - \bar{\pi}_t^s = \chi_1^s (\hat{\pi}_{t-1}^s - \bar{\pi}_{t-1}^s) + \eta_t^s, \quad \eta_t^s \sim N(0, \sigma_s^2), \quad (44)$$

$$\hat{\pi}_t^s - \bar{\pi}_t^s = \chi_0^s \hat{x}_t^s, \quad \text{for all } s = W, R, \quad (45)$$

where the nominal short-term interest rate is given by  $\hat{i}_t^s = \hat{r}_t^s + \bar{\pi}_t^s + \psi_\pi (\hat{\pi}_t^s - \bar{\pi}_t^s) + \psi_x \hat{x}_t^s + \hat{m}_t^s = \hat{r}_t^s + \bar{\pi}_t^s + \left(\psi_\pi + \psi_x \frac{1}{\chi_0^s}\right) (\hat{\pi}_t^s - \bar{\pi}_t^s) + \hat{m}_t^s$  as a function of the natural rate  $\hat{r}_t^s$ , the inflation target  $\bar{\pi}_t^s$ , the cyclical component of inflation  $(\hat{\pi}_t^s - \bar{\pi}_t^s)$ , and the monetary policy shock  $\hat{m}_t^s$  for all  $s = W, R$ .

We can express the model for all  $s = W, R$  as follows:

$$\begin{aligned} \hat{x}_t^s &= \Psi^s \mathbb{E}_t [\hat{x}_{t+1}^s] + \Psi^s \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \mathbb{E}_t [\hat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s] - \Psi^s \frac{\Omega^s}{\gamma} \hat{m}_t^s, \\ \hat{\pi}_t^s - \bar{\pi}_t^s &= \Psi^s \Phi (\varphi + \gamma) \kappa^s \mathbb{E}_t [\hat{x}_{t+1}^s] + \Psi^s \left( \frac{\Omega^s}{\gamma} \Phi (\varphi + \gamma) \kappa^s + \beta \left( 1 + \frac{\Omega^s}{\gamma} \psi_x \right) \right) \mathbb{E}_t [\hat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s] + \dots \\ &\quad - \Psi^s \frac{\Omega^s}{\gamma} \Phi (\varphi + \gamma) \kappa^s \hat{m}_t^s, \end{aligned}$$

where the monetary driving processes for  $s = W, R$  are characterized simply as:

$$\hat{m}_t^s = \delta_m \hat{m}_{t-1}^s + \hat{\varepsilon}_t^{ms}, \quad \hat{\varepsilon}_t^{ms} \sim N(0, \sigma_{ms}^2).$$

The volatility terms for the aggregate and difference monetary policy shocks are given as:

$$\sigma_{mW}^2 \equiv \sigma_m^2 \left( \frac{1 + \rho_{m,m^*}}{2} \right), \quad \sigma_{mR}^2 \equiv 2\sigma_m^2 (1 - \rho_{m,m^*}),$$

which depend solely on the variance-covariance of the underlying monetary shocks.

**Step 1.** We replace the conjecture that states  $\hat{x}_t^s = \frac{1}{\chi_0^s} \hat{\pi}_t^s$  in the corresponding expectational equations of the model so that we can express both of them in terms of inflation alone as,

$$\begin{aligned} \frac{1}{\chi_0^s} (\hat{\pi}_t^s - \bar{\pi}_t^s) &= \Psi^s \mathbb{E}_t \left[ \frac{1}{\chi_0^s} (\hat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s) \right] + \Psi^s \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \mathbb{E}_t [\hat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s] - \Psi^s \frac{\Omega^s}{\gamma} \hat{m}_t^s, \\ \hat{\pi}_t^s - \bar{\pi}_t^s &= \Psi^s \Phi (\varphi + \gamma) \kappa^s \mathbb{E}_t \left[ \frac{1}{\chi_0^s} (\hat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s) \right] + \Psi^s \left( \frac{\Omega^s}{\gamma} \Phi (\varphi + \gamma) \kappa^s + \beta \left( 1 + \frac{\Omega^s}{\gamma} \psi_x \right) \right) \mathbb{E}_t [\hat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s] - \dots \\ &\quad \Psi^s \frac{\Omega^s}{\gamma} \Phi (\varphi + \gamma) \kappa^s \hat{m}_t^s, \end{aligned}$$

or, simply,

$$\begin{aligned} \hat{\pi}_t^s - \bar{\pi}_t^s &= \Psi^s \left[ 1 + \chi_0^s \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \right] \mathbb{E}_t [\hat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s] - \Psi^s \chi_0^s \frac{\Omega^s}{\gamma} \hat{m}_t^s, \\ \hat{\pi}_t^s - \bar{\pi}_t^s &= \Psi^s \left[ \Phi (\varphi + \gamma) \kappa^s \left( \frac{1}{\chi_0^s} + \frac{\Omega^s}{\gamma} \right) + \beta \left( 1 + \frac{\Omega^s}{\gamma} \psi_x \right) \right] \mathbb{E}_t [\hat{\pi}_{t+1}^s - \bar{\pi}_{t+1}^s] - \Psi^s \frac{\Omega^s}{\gamma} \Phi (\varphi + \gamma) \kappa^s \hat{m}_t^s. \end{aligned}$$

**Step 2.** We replace the conjecture that states  $\hat{\pi}_t^s - \bar{\pi}_t^s = \chi_1^s (\hat{\pi}_{t-1}^s - \bar{\pi}_{t-1}^s) + \eta_t^s$  with  $\eta_t^s \sim N(0, \sigma_s^2)$  to

express the cyclical inflation expectations in terms of current cyclical inflation as,

$$\begin{aligned}\widehat{\pi}_t^s - \bar{\pi}_t^s &= \Psi^s \left[ 1 + \chi_0^s \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \right] \chi_1^s (\widehat{\pi}_t^s - \bar{\pi}_t^s) - \Psi^s \chi_0^s \frac{\Omega^s}{\gamma} \widehat{m}_t^s, \\ \widehat{\pi}_t^s - \bar{\pi}_t^s &= \Psi^s \left[ \Phi(\varphi + \gamma) \kappa^s \left( \frac{1}{\chi_0^s} + \frac{\Omega^s}{\gamma} \right) + \beta \left( 1 + \frac{\Omega^s}{\gamma} \psi_x \right) \right] \chi_1^s (\widehat{\pi}_t^s - \bar{\pi}_t^s) - \Psi^s \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s \widehat{m}_t^s,\end{aligned}$$

and then re-write the system of equations as follows,

$$\begin{aligned}\widehat{\pi}_t^s - \bar{\pi}_t^s &= - \left( \frac{\Psi^s \chi_0^s \frac{\Omega^s}{\gamma}}{1 - \Psi^s \left( 1 + \chi_0^s \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \right) \chi_1^s} \right) \widehat{m}_t^s, \\ \widehat{\pi}_t^s - \bar{\pi}_t^s &= - \left( \frac{\Psi^s \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s}{1 - \Psi^s \left( \Phi(\varphi + \gamma) \kappa^s \left( \frac{1}{\chi_0^s} + \frac{\Omega^s}{\gamma} \right) + \beta \left( 1 + \frac{\Omega^s}{\gamma} \psi_x \right) \right) \chi_1^s} \right) \widehat{m}_t^s.\end{aligned}$$

**Step 3.** We replace the solution for cyclical inflation in the monetary shock process as follows,

$$\begin{aligned}\widehat{\pi}_t^s - \bar{\pi}_t^s &= \delta_m (\widehat{\pi}_{t-1}^s - \bar{\pi}_{t-1}^s) - \left( \frac{\Psi^s \chi_0^s \frac{\Omega^s}{\gamma}}{1 - \Psi^s \left( 1 + \chi_0^s \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \right) \chi_1^s} \right) \widehat{\varepsilon}_t^{ms}, \quad \widehat{\varepsilon}_t^{ms} \sim N(0, \sigma_{ms}^2), \\ \widehat{\pi}_t^s - \bar{\pi}_t^s &= \delta_m (\widehat{\pi}_{t-1}^s - \bar{\pi}_{t-1}^s) - \left( \frac{\Psi^s \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s}{1 - \Psi^s \left( \Phi(\varphi + \gamma) \kappa^s \left( \frac{1}{\chi_0^s} + \frac{\Omega^s}{\gamma} \right) + \beta \left( 1 + \frac{\Omega^s}{\gamma} \psi_x \right) \right) \chi_1^s} \right) \widehat{\varepsilon}_t^{ms}, \quad \widehat{\varepsilon}_t^{ms} \sim N(0, \sigma_{ms}^2).\end{aligned}$$

**Step 4.** We apply the method of undetermined coefficients (also known as the method of matching coefficients) to equate this formula for the stochastic dynamics of cyclical inflation with the conjecture above, imposing enough restrictions to ensure that both solutions are identical:

$$\begin{aligned}\chi_1^s &= \delta_m, \\ \left( \frac{\Psi^s \chi_0^s \frac{\Omega^s}{\gamma}}{1 - \Psi^s \left( 1 + \chi_0^s \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \right) \chi_1^s} \right) &= \left( \frac{\Psi^s \frac{\Omega^s}{\gamma} \Phi(\varphi + \gamma) \kappa^s}{1 - \Psi^s \left( \Phi(\varphi + \gamma) \kappa^s \left( \frac{1}{\chi_0^s} + \frac{\Omega^s}{\gamma} \right) + \beta \left( 1 + \frac{\Omega^s}{\gamma} \psi_x \right) \right) \chi_1^s} \right), \\ \eta_t^s &= - \left( \frac{\Psi^s \chi_0^s \frac{\Omega^s}{\gamma}}{1 - \Psi^s \left( 1 + \chi_0^s \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \right) \chi_1^s} \right) \widehat{\varepsilon}_t^{ms}.\end{aligned}$$

The implicit formula for  $\chi_0^s$  comes down to,

$$\chi_0^s = \frac{\Phi(\varphi + \gamma) \kappa^s}{1 - \beta \chi_1^s},$$

given that  $\Psi^s \equiv \frac{1}{1 + \frac{\Omega^s}{\gamma} (\psi_x + \Phi(\varphi + \gamma) \kappa^s \psi_\pi)} > 0$ . Then, it holds that,

$$\left( \frac{\Psi^s \chi_0^s \frac{\Omega^s}{\gamma}}{1 - \Psi^s \left( 1 + \chi_0^s \frac{\Omega^s}{\gamma} (1 - \beta \psi_\pi) \right) \chi_1^s} \right) = \left( \frac{\Phi(\varphi + \gamma) \kappa^s \frac{\Omega^s}{\gamma}}{(1 - \beta \chi_1^s) \left( 1 - \chi_1^s + \frac{\Omega^s}{\gamma} \psi_x \right) + \Phi(\varphi + \gamma) \kappa^s \frac{\Omega^s}{\gamma} (\psi_\pi - \chi_1^s)} \right),$$

and we can summarize the solution as follows,

$$\begin{aligned}\chi_0^s &= \frac{\Phi(\varphi + \gamma)\kappa^s}{1 - \beta\delta_m}, \\ \chi_1^s &= \delta_m, \\ \eta_t^m &= - \left( \frac{\Phi(\varphi + \gamma)\kappa^s \frac{\Omega^s}{\gamma}}{(1 - \beta\delta_m) \left(1 - \delta_m + \frac{\Omega^s}{\gamma} \psi_x\right) + \Phi(\varphi + \gamma)\kappa^s \frac{\Omega^s}{\gamma} (\psi_\pi - \delta_m)} \right) \widehat{\varepsilon}_t^{ms},\end{aligned}$$

where  $\mathbb{E}[\eta_t^s] = 0$  and also that  $\sigma_s^2 \equiv \mathbb{V}[\eta_t^s] = \left( \frac{\Phi(\varphi + \gamma)\kappa^s \frac{\Omega^s}{\gamma}}{(1 - \beta\delta_m) \left(1 - \delta_m + \frac{\Omega^s}{\gamma} \psi_x\right) + \Phi(\varphi + \gamma)\kappa^s \frac{\Omega^s}{\gamma} (\psi_\pi - \delta_m)} \right)^2 \sigma_{ms}^2$ . Notice here that  $\sigma_{mW}^2 \equiv \sigma_m^2 \left( \frac{1 + \rho_{m,m^*}}{2} \right)$  and  $\sigma_{mR}^2 \equiv 2\sigma_m^2 (1 - \rho_{m,m^*})$ .

**Policy Trade-offs and the Slope of the Phillips Curve.** Based on the solution to the model, we argue that an autoregressive specification is a natural benchmark for modelling inflation also under the specification of the workhorse open-economy New Keynesian framework laid out here (as shown by the solution derived above). Openness ( $\xi$ ) and other trade features (specifically the trade elasticity  $\sigma$ ) and the policy parameters ( $\psi_\pi$  and  $\psi_x$ ) matter for inflation volatility but not for its persistence.

Furthermore, inflation (or, to be more precise, cyclical inflation) is related in equilibrium to slack for the world and differential solutions as follows:

$$\widehat{\pi}_t^s - \bar{\pi}_t^s = \left( \frac{\Phi(\varphi + \gamma)\kappa^s}{1 - \beta\delta_m} \right) \widehat{x}_t^s, \text{ for all } s = W, R, \quad (46)$$

where we already established that  $\kappa^W = 1$  determines the NKPC slope on global slack and  $\kappa^R \equiv (2\kappa - 1) > 0$  sets the slope on differential slack. The composite coefficient  $\kappa \equiv (1 - \xi) \left[ 1 - (\sigma\gamma - 1) \left( \frac{\gamma}{\varphi + \gamma} \right) \left( \frac{(2\xi)(1 - 2\xi)}{1 + (\sigma\gamma - 1)(2\xi)(2(1 - \xi))} \right) \right]$ , as defined in Table B1, depends on deep structural parameters but not on the policy parameters. The term  $\Phi(\varphi + \gamma)$  defines the slope of the closed-economy Phillips curve. An adjustment term enters into this expression whenever monetary shocks display some persistence ( $\delta_m \neq 0$ ) given that the intertemporal discount factor satisfies that  $0 < \beta < 1$ .

Therefore, the only composite coefficient that depends explicitly on the strength of the bilateral trade linkages—the share of foreign goods in the local consumption basket,  $0 \leq \xi \leq \frac{1}{2}$ , and the trade elasticity of substitution between Home and Foreign goods,  $\sigma > 0$ —is  $\kappa^R$  on the differential sub-system solution. Figure B1 and Figure B2 aim to illustrate how the composite coefficients  $\kappa$  and  $\kappa^R$  vary with the trade parameters ( $\xi$  and  $\sigma$ ), yet are also influenced by features of the labor market like the inverse of the Frisch elasticity of labor supply  $\varphi > 0$  (via the preference ratio  $\frac{\gamma}{\varphi + \gamma}$ ) and by the sign of  $(\sigma\gamma - 1)$  relating the magnitude of the trade elasticity  $\sigma > 0$  to the value of the inverse of the elasticity of intertemporal substitution  $\gamma > 0$ .

The Open-Economy Phillips Curve Slope on Domestic Slack Relative to the Closed-Economy Phillips Curve Slope ( $\kappa$ ).

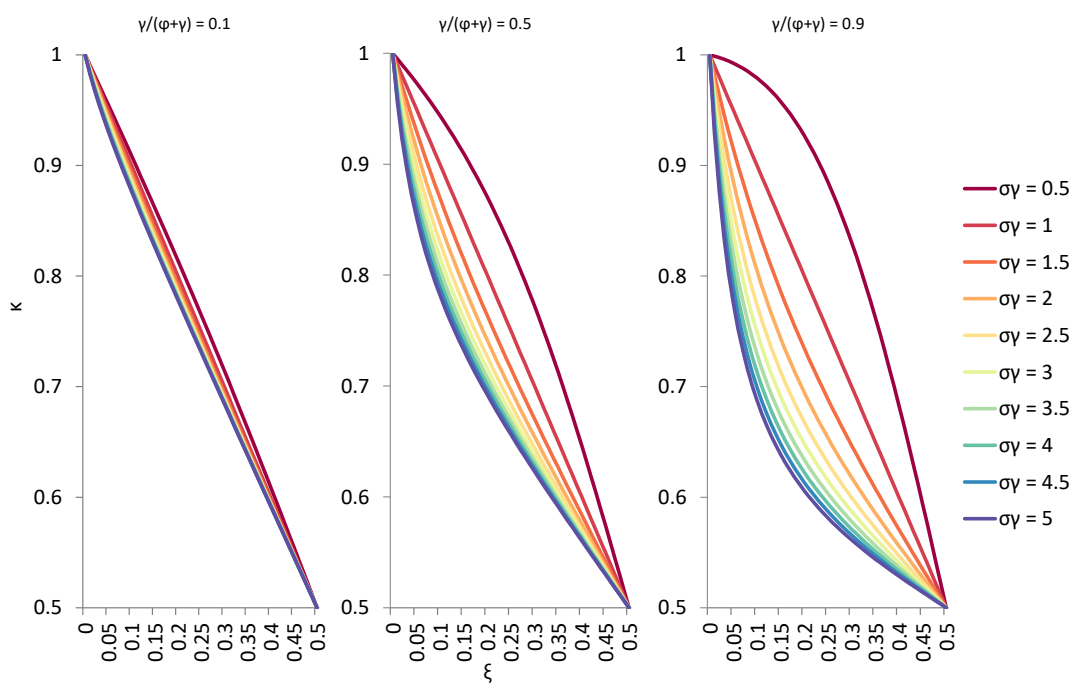


FIGURE B1



The Differential Open-Economy Phillips Curve Slope on Slack Relative to the Closed-Economy Phillips Curve Slope ( $\kappa^R$ ).

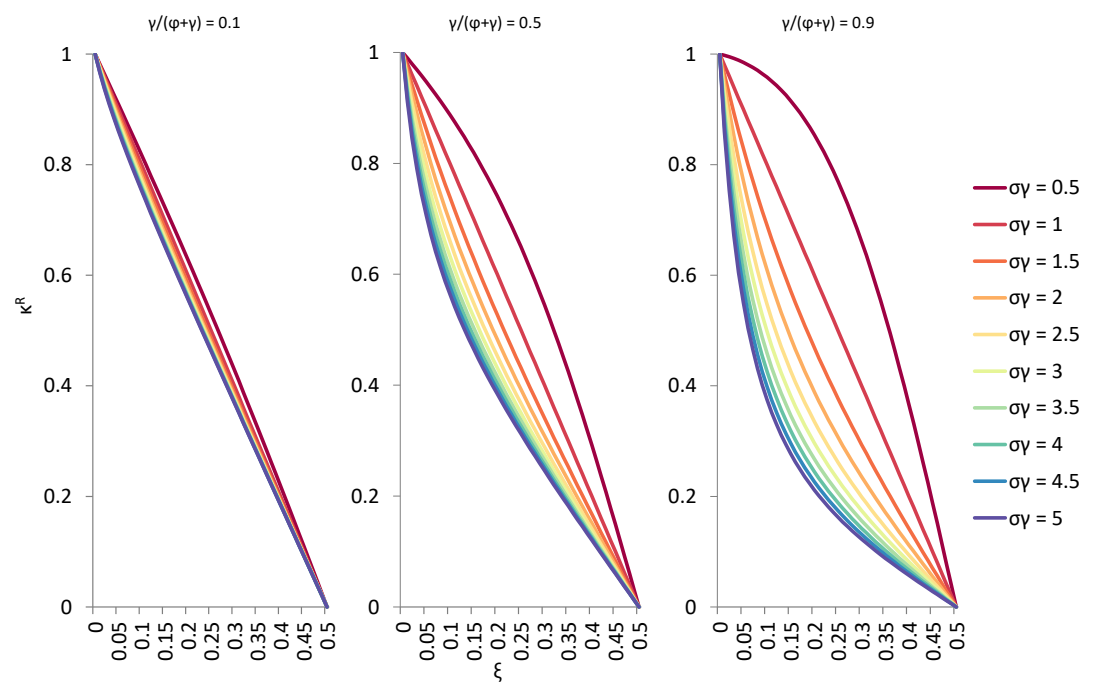


FIGURE B2

### 3 Data Description

This section gives details for the data used in the remainder of the paper.

#### *Abbreviations*

BLS = U.S. Bureau of Labor Statistics; BEA = Bureau of Economic Analysis; DGEI = Database of Global Economic Indicators (Federal Reserve Bank of Dallas); IMF = International Monetary Fund; SA = Seasonally adjusted. All series are quarterly unless indicated otherwise.

#### 1 Inflation measures

We use series starting in 1984:Q1 and ending in 2015:Q1 (SA, 2010=100). CPI (all items) is available from the Bureau of Labor Statistics (BLS) for the U.S. going back to 1947:Q1, while core CPI (all items ex. food and energy) is available from the BLS going back to 1957:Q1. We use headline and core inflation series comparable with those of the U.S. for all 14 advanced economies in our sample—obtained from the database of global economic indicators (DGEI) of the Federal Reserve Bank of Dallas (see the details in Grossman et al. (2014)).

#### 2 Global slack and global inflation measures

The series for the individual countries needed to construct global slack and rest of the world inflation measures are obtained from the Federal Reserve Bank of Dallas' DGEI (see the details in Grossman et al. (2014)). Weighted averages of filtered quarterly Industrial Production and real GDP series (using either first-differencing in logs expressed in percentages or a 1-sided Hodrick-Prescott-filter also in logs and expressed in percentages) for the period 1984:Q1-2015:Q1 are used as proxy measures for unobservable global slack.<sup>7</sup> Annualized log differences of quarterly headline CPI and core CPI series in percentages are used in constructing the rest of the world inflation measures. Country coverage varies with data availability. The list of countries used in each sample is given below.

Table B3 Panel (c): Australia, Austria, Belgium, Canada, China, Colombia, France, Germany, Indonesia, Italy, Japan, Korea, Mexico, Netherlands, Philippines, Portugal, South Africa, Spain, Sweden, Switzerland, Taiwan, United States, United Kingdom.

Table B3 Panel (d): Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Mexico, Netherlands, Spain, Sweden, Switzerland, United States, United Kingdom.

Table B5 Panels (a) and (e): Australia, Austria, Belgium, Canada, Chile, France, Germany, Greece, Hungary, India, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Portugal, South Africa, Spain, Sweden, Switzerland, Taiwan, United States, United Kingdom.

Table B5 Panels (b) and (f): Australia, Austria, Belgium, Canada, Chile, France, Germany, Italy, Japan, Korea, Mexico, Spain, Sweden, Switzerland, Taiwan, United States, United Kingdom.

Table B5 Panels (c) and (g): Australia, Austria, Belgium, Canada, China, Colombia, France, Germany, Indonesia, Italy, Japan, Korea, Mexico, Netherlands, Philippines, Portugal, South Africa, Spain, Sweden, Switzerland, Taiwan, United States, United Kingdom.

Table B5 Panels (d) and (h): Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Mexico, Netherlands, Spain, Sweden, Switzerland, United States, United Kingdom.

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<sup>7</sup>The H-P filter is applied as described in Stock and Watson (1999). This is a one-sided HP filter.

Table B7 Panels (c) and (e): Australia, Austria, Belgium, Canada, China, Colombia, France, Germany, Indonesia, Italy, Japan, Korea, Mexico, Netherlands, Philippines, Portugal, South Africa, Spain, Sweden, Switzerland, Taiwan, United States, United Kingdom.

Table B7 Panels (d) and (f): Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Mexico, Netherlands, Spain, Sweden, Switzerland, United States, United Kingdom.

The same pool of countries (switching the U.S.) is employed for all other 13 advanced countries in our sample for which we perform our analysis.

### 3 Kilian (2009)'s index of global economic conditions

Kilian (2009)'s index of global economic conditions is based on monthly series of dry cargo single voyage ocean freight rates. The series covers the period 1968:M1 till 2015:M1 and can be accessed at: <http://www-personal.umich.edu/~lkilian/reaupdate.txt>. The quarterly series that we use is averaged across the three months of each quarter.

### 4 Oil prices

West Texas Intermediate Crude Oil 40 Deg. Beginning of Month (\$/BBL), quarterly series obtained by averaging monthly series available for the period 1947:Q1-2015:Q1 obtained from the FRED database (FRED codes: MCOILWTICO and OILPRICE) (SA, 2005=100).

### 5 Country weights

The weights for any country  $i$  out of the  $N$  for which we conduct our empirical analysis which corresponds to a sample of 14 advanced economies (Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, and United States) are defined as  $w_{ij}^y$ , for all  $j = 1, \dots, M$ , where  $M$  corresponds to a sample of up to 29 countries for which we can draw data.<sup>8</sup> The weights for rest-of-the-world inflation are consistent for all entries except for the home country (intra-national weights are netted out).<sup>9</sup> Weighted aggregates for inflation have the home country weight set to 0, by construction. In other words, for country  $i$ , we consider rest-of-the-world inflation weights  $w_{ij}^\pi$  which set a country's own weight equal to zero (i.e.,  $w_{ii}^\pi = 0$ ) while other weights are re-scaled accordingly so they still sum up to 1 (i.e.,  $w_{ij}^\pi = \frac{w_{ij}^y}{1 - w_{ii}^y}$  for any  $j \neq i$ ). We use 5 measures of country weights in order to compute our global slack and rest-of-the-world inflation measures:

W1 Equal weights for country  $i$  (for any  $i = 1, \dots, N$ ): The weights are given by  $w_{ij}^y = \frac{1}{M}$ , for all  $j = 1, \dots, M$ , where  $M$  is the number of countries in the sample including the domestic economy.

W2 Contiguity weights for country  $i$  (for any  $i = 1, \dots, N$ ): The weights  $w_{ij}^y$  equal  $\frac{1}{Z}$  if the home country  $i$  and country  $j$  share a border and 0 otherwise, for all  $j = 1, \dots, M$ . Here,  $Z$  is given as the total number of countries that share a border with the home country.

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<sup>8</sup>This set of  $M = 29$  countries used to construct our global measures naturally includes the 14 advanced countries that we investigate in the paper.

<sup>9</sup>A country's own weight is non-zero in all weighting schemes except for the contiguity measure since, by definition, a country does not have a border with itself.

- W3 Distance weights for country  $i$  (for any  $i = 1, \dots, N$ ): These weights are based on geodesic distances that are calculated following the great circle formula, which uses latitudes and longitudes of the most important cities/agglomerations (adjusted by population size). The  $dist$  variable is obtained from the GeoDist dataset.<sup>10</sup> In particular, we use the inverse of the square of the bilateral distances between the home country  $i$  and country  $j$ ,  $\frac{1}{dist_{ij}^2}$ , and construct the weights to be normalized to sum up to 1 as follows  $w_{ij}^y \equiv \frac{\frac{1}{dist_{ij}^2}}{\sum_{j=1}^M \frac{1}{dist_{ij}^2}}$  for all  $j = 1, \dots, M$ .
- W4 Population-adjusted distance weights for country  $i$  (for any  $i = 1, \dots, N$ ): These weights are constructed using the  $distwces$  measure from the GeoDist dataset, based on city-level data to obtain the geographic distribution of population (in 2004) inside each country. The bilateral distances between the biggest cities of the two countries are calculated and the inter-city distances are weighted by the share of the city in the overall country's population. As with the distance-based weights proposed before, we use the inverse of the square of the population-adjusted distance between the home country  $i$  and country  $j$ ,  $\frac{1}{distwces_{ij}^2}$ , and construct the weights to be normalized to sum up to 1 as follows  $w_{ij}^y \equiv \frac{\frac{1}{distwces_{ij}^2}}{\sum_{j=1}^M \frac{1}{distwces_{ij}^2}}$  for all  $j = 1, \dots, M$ .
- W5 Trade weights for country  $i$  (for any  $i = 1, \dots, N$ ): To construct the trade weights we use annual IMF Direction of Trade (DOT) data for every country  $j = 1, \dots, M$  on their merchandise nominal imports from the world,  $imp_j$ , and their merchandise nominal exports to the world,  $exp_j$ , obtained through the Federal Reserve Bank of Dallas' DGEI (see the details in Grossman et al. (2014)).<sup>11</sup> With those two series, we construct trade weights for any home country  $i$  as follows:  $w_{ij}^y \equiv \frac{imp_j + exp_j}{\sum_{i=1}^M imp_j + exp_j}$  for all  $j = 1, \dots, M$ . These weights are based only on each country's share in world trade and do not reflect the actual bilateral trade linkages between country  $i$  and  $j$ —hence, these weights only account for how open each country is relative to the rest of the world through trade. The weights obtained with this formula are the same for any country  $i$  (i.e.,  $w_{ij}^y$ ). The annual IMF DOT data is available for the entire 1980 – 2014 period. We use here trade weights constructed with the average of the full 1984 – 2014 period.

Figure B3 and Figure B4 plot the country and global series for inflation and slack used in U.S. inflation forecasts.

<sup>10</sup>For a detailed explanation of the GeoDist data, see Mayer and Zignago (2011). The GeoDist database can be accessed at [http://www.cepii.fr/CEPII/en/bdd\\_modele/presentation.asp?id=6](http://www.cepii.fr/CEPII/en/bdd_modele/presentation.asp?id=6)

<sup>11</sup>The series for imports and exports are expressed in U.S. dollars for all countries.

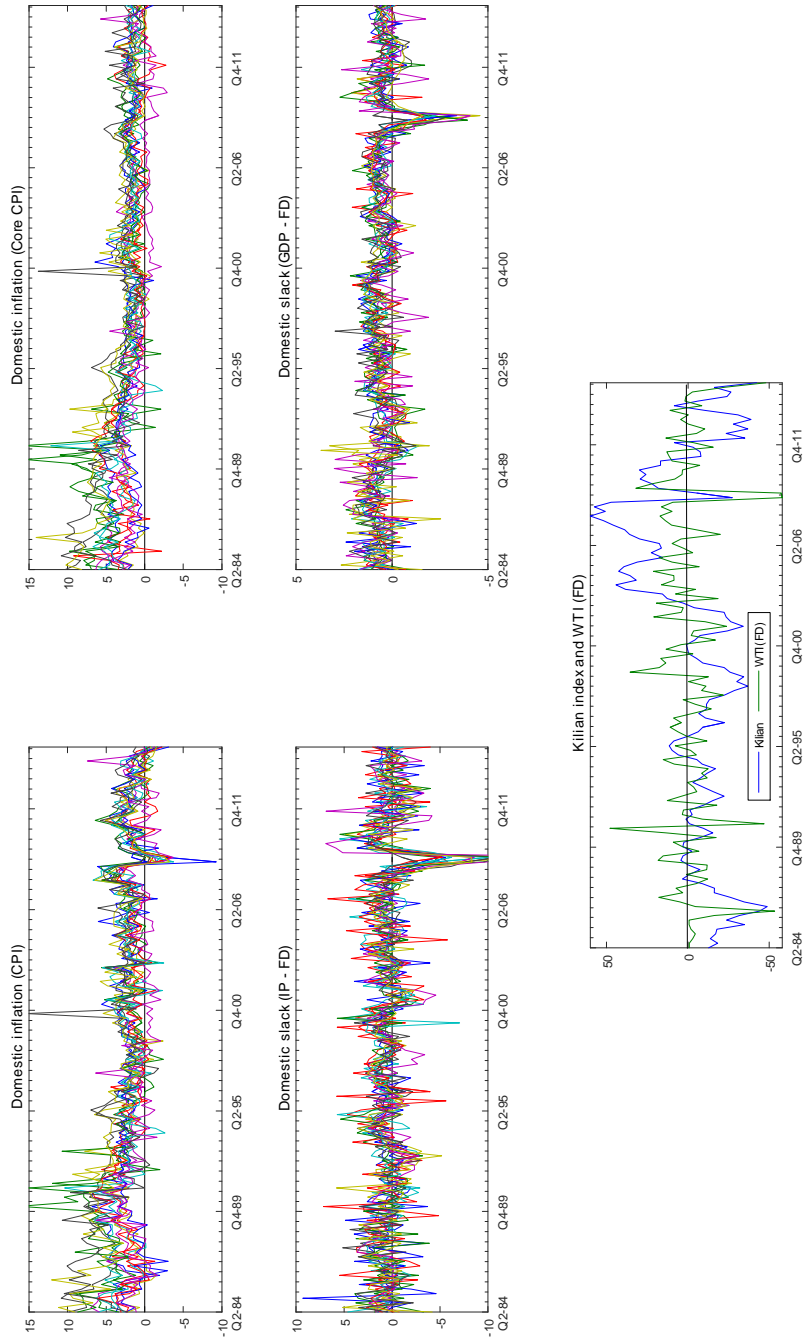


FIGURE B3

Note: This figure illustrates our domestic inflation and domestic slack measures, plus the WTI oil price series and the Kilian index. All series are in logged first differences except the Kilian index. We plot the series for the following countries: Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, and United States.

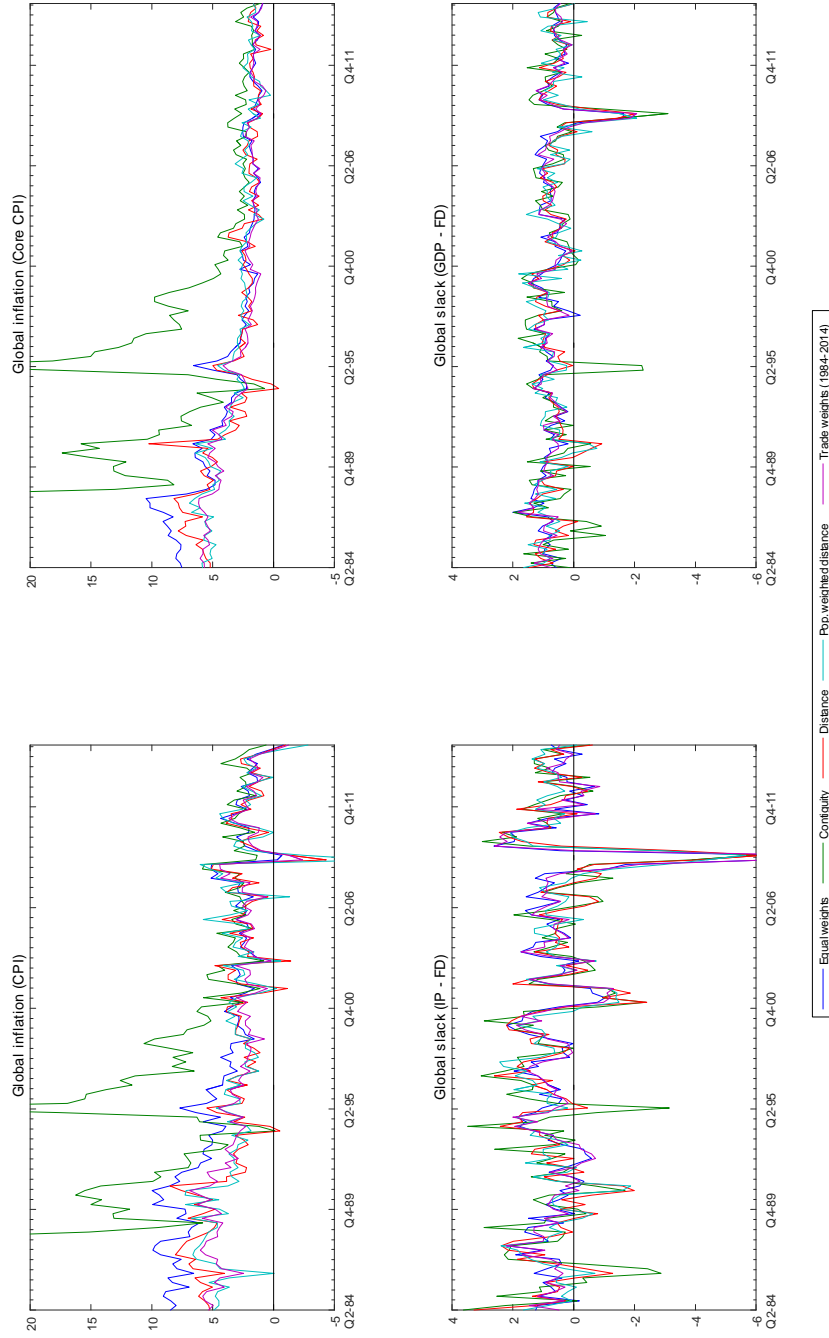


FIGURE B4

Note: This figure illustrates our global inflation and global slack measures, based on 5 different weighting schemes, using data from the following countries: Australia, Austria, Belgium, Canada, Chile, France, Germany, Greece, Hungary, India, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Portugal, South Africa, Spain, Sweden, Switzerland, Taiwan, United Kingdom, and United States. Notice that the weighting schemes used in these calculations are computed only for the United States as the Home country. All series are reported in logged first differences.

## 4 Empirical Findings: Additional Robustness Checks

We explore additional robustness checks that complement the pseudo out-of-sample forecasting evaluation and the in-sample analysis of model fit reported in Kabukcuoğlu and Martínez-García (Forthcoming).

### 4.1 Alternative Pseudo Out-of-Sample Specifications

We consider the empirical specifications proposed by Kabukcuoğlu and Martínez-García (Forthcoming) for our forecast evaluation exercise:

1. A benchmark autoregressive (AR) model to predict inflation (with no international linkages or economic predictors), i.e.,

$$\pi_{i,t+h|t}^1 = c_i^1 + \sum_{s=0}^p \gamma_{i,s}^1 \pi_{i,t-s} + \epsilon_{i,t+h}^1, \text{ for country } i \text{ and horizon } h, \quad (\text{Model 1})$$

which forecasts future inflation of country  $i$  solely with the distributed lag of earlier inflation rates  $\pi_{i,t}$ . The optimal number of lags  $p$  is selected based on the SIC. To keep the model parsimonious and since we work with quarterly series, the maximum possible lags allowed is set at four. We use the lag length selected under this benchmark with all other models to keep them nested in our forecasting exercises.

2. The standard open-economy Phillips curve specification based on global slack (which incorporates global output linkages) for forecasting domestic inflation, i.e.,

$$\pi_{i,t+h|t}^2 = c_i^2 + \sum_{s=0}^p \gamma_{i,s}^2 \pi_{i,t-s} + \sum_{s=0}^q \psi_{i,s}^2 y_{i,t-s}^* + \epsilon_{i,t+h}^2, \text{ for country } i \text{ and horizon } h. \quad (\text{Model 2})$$

In this case, for each country  $i$ , we forecast  $h$ -quarters ahead inflation with the distributed lag of earlier inflation rates,  $\pi_{i,t}$ , and the distributed lag of the explanatory variable,  $y_{i,t}^*$ , where we define  $y_{i,t}^*$  as global slack—the weighted average of domestic slack measures given by  $\sum_{j=1}^M w_{ij}^y y_{j,t}$ . We use the SIC to select the optimal number of lags  $q$  for the explanatory economic variable, with the maximum possible lags allowed set at four. The optimal lag length for local inflation dynamics  $p$  is the same as for [Model 1](#). The global slack weights for country  $i$  are  $w_{ij}^y$ , for all  $j = 1, \dots, M$  where  $M$  corresponds to the sample of up to 29 countries for which we can draw data to construct our aggregates.

3. An alternative specification of the open-economy Phillips curve that takes into account its spatio-temporal dimensions for forecasting inflation in each country  $i = 1, \dots, N$  with the distributed lag of earlier inflation rates in country  $i$ ,  $\pi_{i,t}$ , of rest-of-the-world (weighted) inflation,  $\pi_{i,t}^*$ , and of domestic slack,  $y_{i,t}$ , i.e.,

$$\begin{aligned} \pi_{i,t+h|t}^3 = c_i^3 + \sum_{s=0}^p \gamma_{i,s}^3 \pi_{i,t-s} + \sum_{s=0}^z \lambda_{i,s}^3 \pi_{i,t-s}^* + \dots \\ \sum_{s=1}^z \psi_{i,s}^3 y_{i,t-s} + \epsilon_{i,t+h}^3, \text{ for country } i \text{ and horizon } h. \end{aligned} \quad (\text{Model 3})$$

The right-hand side of [Model 3](#) augments that of [Model 1](#) with the introduction of an additional pair of regressors,  $\pi_{i,t}^*$  and  $y_{i,t}$ , with coefficients  $\lambda_{i,s}^3$  and  $\psi_{i,s}^3$  respectively with up to  $z$  lags. We calculate the rest-of-the-world inflation as  $\pi_{i,t}^* = \sum_{j=1}^M w_{ij}^\pi \pi_{j,t}$ . We consider rest-of-the-world inflation weights

$w_{ij}^\pi$  were a country’s own weight is set equal to zero (i.e.,  $w_{ii}^\pi = 0$ ) in order to compute our measures of rest-of-the-world inflation,  $\pi_{i,t}^*$ . Also, all other country weights other than the own-country weight are re-scaled to maintain the principle that they should sum up to 1 (i.e.,  $w_{ij}^\pi = \frac{w_{ij}^y}{1-w_{ii}^y}$  for any country  $j \neq i$ ). We use the SIC to select the optimal number of lags  $z$  for the economic variables in the specification with the maximum possible lags allowed set at four, taking as given the optimal  $p$  determined based on the SIC applied to [Model 1](#).

With these empirical models at hand and using the same forecasting procedure laid out in Kabukçuoğlu and Martínez-García (Forthcoming), we consider a number of additional forecasting exercises for robustness:

- Using [Model 2](#), we evaluate the performance of the Kilian (2009) index, as an alternative proxy for the global output gap. As seen in Table B3 and Table B4, the Kilian (2009) index produces weak forecasts relative to the benchmark autoregressive in [Model 1](#) for the U.S. and in general across the 14 advanced countries in our sample. In other words, we find this alternative measure of global slack to have only limited value for forecasting within the framework of model [Model 2](#).
- We evaluate inflation forecasts augmenting [Model 3](#) with a broader set of regressors than those suggested by theory to capture unmodelled features and to evaluate their predictive accuracy accordingly. We consider models with: (i) global inflation and the Kilian (2009) index (Table B3 and Table B4), and (ii) global inflation and global slack—where global slack is based on either industrial production (IP) or real GDP measures, 1-sided HP-filtered and first-differenced in logs (Table B5 and Table B6). All these results can be summarized as follows: While the Kilian (2009) index together with global inflation performs well only occasionally in our experiments for the U.S. and the majority of other advanced countries, the model with global inflation and global slack measures yields competitive results compared to our [Model 3](#). In any event, the evidence suggests that [Model 3](#) provides a theoretically-grounded and empirically successful forecasting specification which cannot be significantly and systematically improved if we replace domestic slack with a measure of global slack.

In Table B7 and Table B8, we then evaluate inflation forecasts based on: (i) WTI oil prices under the form of [Model 2](#) (i.e., we redefine  $y_{i,t}^*$  to capture the WTI oil price series instead of global slack); and (ii) WTI oil prices and global inflation under the form of [Model 3](#) (i.e., where  $y_{i,t}$  represents oil prices instead of domestic slack).

- In the forecasts under the proposed reinterpretation of [Model 2](#), the 1-sided HP-filtered WTI oil price series does not help yield more accurate forecasts of headline CPI inflation while log-first-differenced WTI oil prices appear to help forecast headline inflation at long horizons more accurately than the benchmark ([Model 1](#)). Core CPI inflation, which excludes food and energy, cannot be forecasted more accurately (relative to [Model 1](#)) with any of the filtered WTI oil price measures that we consider here.
- Under the reinterpretation of [Model 3](#), the most accurate forecasts of headline CPI inflation are those we obtain with a combination of global inflation and WTI oil prices (particularly after log-first-differencing the series). These results appear to be true for the majority of the advanced countries studied in the sample. Once again, our findings support the inflation forecasting model favored in Kabukçuoğlu and Martínez-García (Forthcoming) based on the open-economy Phillips curve with global inflation and domestic slack.



**Forecasts of U.S. Inflation - Relative MSFEs (1997:Q1-2015:Q1)**

Horizon	CPI						Core CPI (CPI ex. Food & Energy)					
	1	4	6	8	10	12	1	4	6	8	10	12
<b>Model 2 against Model 1</b>												
<b>Global slack (Kilian)</b>												
Model 3 against Model 1	1.031	1.098	1.112	1.102	1.114	1.081	(b)	1.029	1.096	1.233	1.411	1.542
<b>Global inflation &amp; Kilian</b>												
Equal weights	0.856***	1.001	1.079	1.045	1.009	0.950	(d)	0.966*	0.892**	0.891*	0.978	0.935
Contiguity	1.003	1.037	1.021	0.969	0.897	0.755**		1.010	0.961	0.945	1.082	1.079
Distance	0.749***	0.889**	0.886**	0.850**	0.794**	0.692***		0.996*	0.948*	0.903*	0.987	1.009
Pop. weighted distance	0.973**	1.004	0.988	0.934	0.857*	0.713**		1.006	0.952	0.934	1.062	1.058
Trade weights (1984-2014)	0.771***	0.918*	1.024	1.058	1.089	1.048		0.947**	0.866**	0.883*	0.963	0.907

TABLE B3

Note: This table reports forecasting performance with an estimation sample covering 1984:Q1-1996:Q4 and a pseudo out-of-sample forecasting sample over 1997:Q1-2015:Q1. The entries in this table display the MSFEs of the forecasts under alternative specifications of Model 2 and Model 3 relative to the MSFEs of the benchmark model (Model 1). Asterisks denote that the MSFE of the corresponding variant of Model 2 or Model 3 is statistically different and more accurate than the MSFE of the benchmark model (Model 1) at 1 (\*\*\*) , 5 (\*\*), and 10 (\*) percent significance levels.

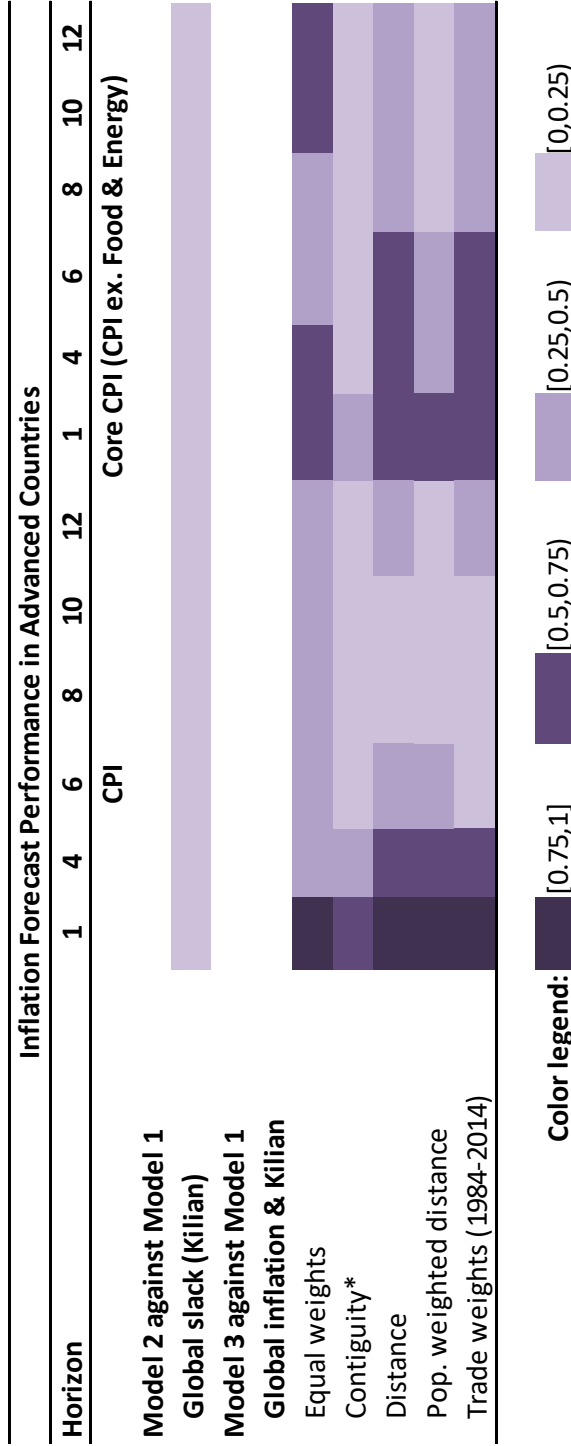


TABLE B4

Note: This table summarizes the forecasting performance for a sample of 14 countries that includes: Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, and United States. The estimation sample covers the period 1984:Q1-1996:Q4 with the pseudo out-of-sample forecasting sample over 1997:Q1-2015:Q1. The entries in this table are the fraction of countries out of the 14 for which we have data where we find that the MSFE of the corresponding forecasting model under consideration (variants of Model 2 and Model 3) is statistically different and more accurate than the MSFE of the country's benchmark model (Model 1) at least at the 10 percent significance level. Specific country results are available from the authors upon request. (\*)The results for the contiguity measure are reported for 10 countries only (Australia, Japan, Sweden, and United Kingdom were omitted).

Forecasts of U.S. Inflation - Relative MSFEs (1997:Q1-2015:Q1)

Horizon	Core CPI (CPI ex. Food & Energy)												
	1	4	6	8	10	12	1	4	6	8	10	12	
<b>Model 3 against Model 1</b>													
<b>Global slack (IP-HP)</b>													
<b>&amp; Global inflation</b>													
Equal weights	2.041	3.783	5.261	6.048	6.555	7.244	(b)	14.840	5.979	15.786	14.713	13.959	14.569
Contiguity	1.584	2.703	3.778	4.505	5.020	5.465	8.317	9.377	9.405	9.005	9.005	8.673	8.849
Distance	0.763***	0.777***	0.740***	0.705***	0.678***	0.618***	0.916**	0.674**	0.588**	0.574**	0.595*	0.604*	0.604*
Pop. weighted distance	0.971**	0.904**	0.848**	0.793**	0.710**	0.962	0.962**	0.781**	0.736**	0.727**	0.717**	0.717**	0.717**
Trade weights (1984-2014)	0.510***	0.766***	0.847***	0.875*	0.856*	0.871*	0.920**	0.612***	0.586***	0.687**	0.805**	1.011	
<b>Global slack (GDP-HP)</b>													
<b>&amp; Global inflation</b>													
Equal weights	2.002	3.870	5.457	6.308	6.734	7.181	(d)	0.907***	0.674***	0.696**	0.805**	0.895	1.041
Contiguity	1.553	2.575	3.555	4.202	4.576	4.845	0.978*	0.832**	0.835**	0.841*	0.829*	0.863*	0.863*
Distance	0.771***	0.780***	0.731***	0.680***	0.639***	0.579***	0.930**	0.745***	0.666***	0.661***	0.699***	0.724**	0.724**
Pop. weighted distance	0.963**	0.884**	0.824**	0.765**	0.682**	0.564**	0.952**	0.730***	0.655***	0.714**	0.810**	0.890	0.890
Trade weights (1984-2014)	0.791***	0.781***	0.850***	0.982*	0.866*	0.825**	0.887***	0.595***	0.597***	0.740**	0.876*	1.060	
<b>Global slack (IP-FD)</b>													
<b>&amp; Global inflation</b>													
Equal weights	1.034	1.039	1.007	1.047	1.105	1.129	(f)	0.953**	0.814**	0.706**	0.677**	0.703**	0.671**
Contiguity	1.009	1.037	1.055	1.053	1.052	1.032	0.999	0.864**	0.789**	0.767**	0.721**	0.658**	0.658**
Distance	1.009	1.030	1.031	1.032	1.246	1.031	1.013	0.818***	0.683***	0.627***	0.619***	0.618***	0.618***
Pop. weighted distance	1.050	1.049	1.057	1.074	1.061	1.066	1.017	0.877**	0.789**	0.751**	0.713**	0.646**	0.646**
Trade weights (1984-2014)	1.096	1.092	1.019	1.070	1.139	1.168	0.962**	0.880**	0.786**	0.772**	0.823*	0.839*	0.839*
<b>Global slack (GDP-FD)</b>													
<b>&amp; Global inflation</b>													
Equal weights	0.842***	0.812**	0.788**	0.834**	0.810**	0.745**	(h)	0.978*	0.843**	0.702***	0.679**	0.707**	0.721**
Contiguity	0.987**	0.928**	0.855**	0.792***	0.703***	0.581***	0.996	0.858**	0.784**	0.762**	0.724**	0.669**	0.669**
Distance	0.773***	0.791***	0.725***	0.667***	0.625***	0.568***	1.057	0.894**	0.767***	0.744***	0.757***	0.820**	0.820**
Pop. weighted distance	0.960**	0.887**	0.796**	0.735**	0.651**	0.532***	0.985	0.853**	0.736**	0.703**	0.691**	0.671**	0.671**
Trade weights (1984-2014)	0.778***	0.800**	0.786**	0.803**	0.791**	0.736**	1.015	0.831**	0.675***	0.703**	0.808**	0.866*	0.866*

TABLE B5

Note: This table reports the forecasting performance with an estimation sample covering 1984:Q1-1996:Q4 and a pseudo out-of-sample forecasting sample over 1997:Q1-2015:Q1. The entries in this table are the MSFEs of the forecasts under different specifications of Model 3 relative to the MSFEs of the benchmark model (Model 1). IP indicates that global slack has been constructed with industrial production data while GDP means that real GDP data was used instead. We distinguish global slack based on whether we use the one-sided Hodrick-Prescott (HP) filter or first-differencing (FD) for filtering the data. Asterisks denote that the MSFE of the corresponding variant of Model 3 is statistically different and more accurate than the MSFE of the benchmark model (Model 1) at 1 (\*\*), 5 (\*), and 10 (\*) percent significance levels.

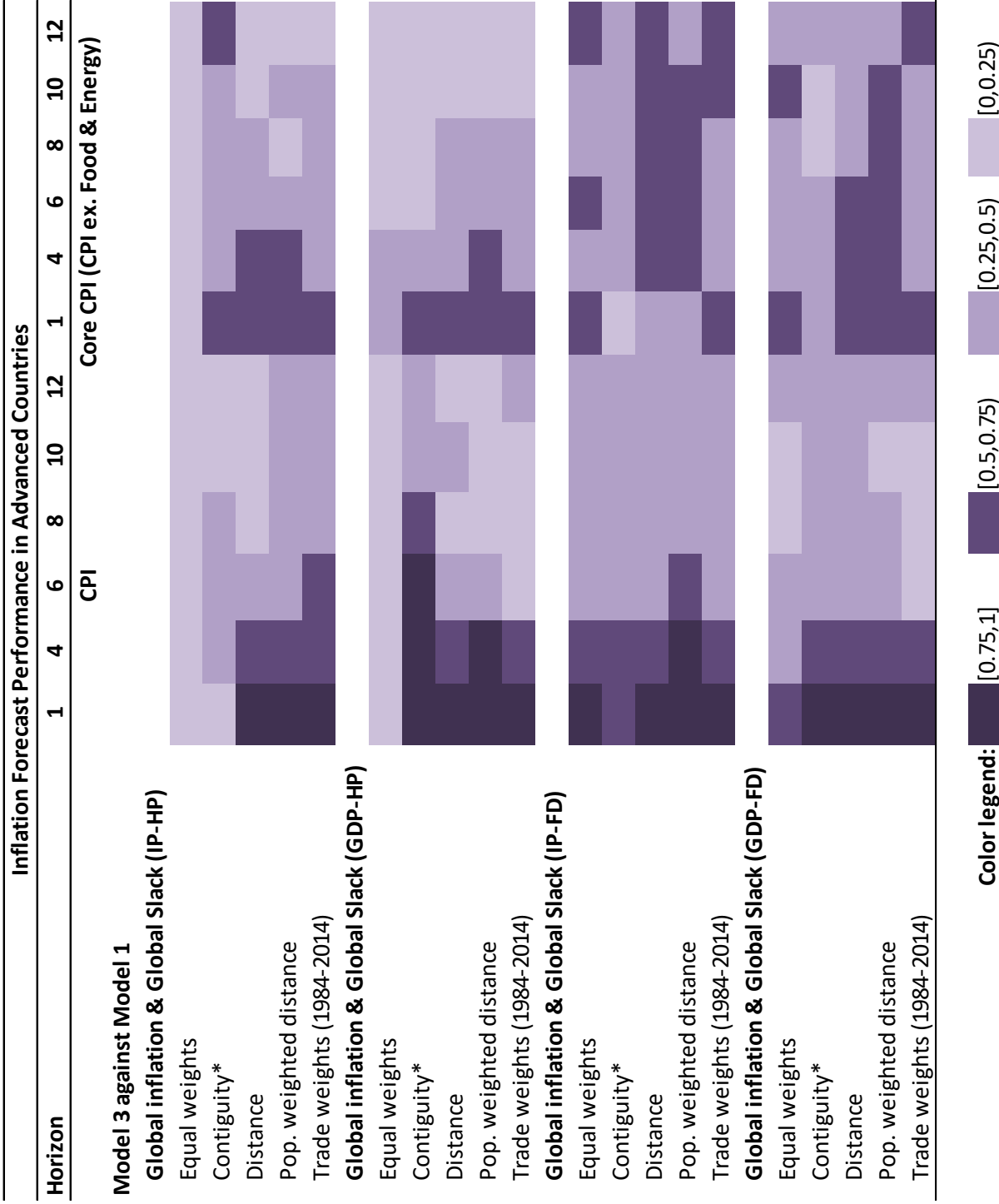


TABLE B6

Note: This table summarizes the forecasting performance for a sample of 14 countries which includes: Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, and United States. The estimation sample covers the period 1984:Q1-1996:Q4 with a pseudo out-of-sample forecasting sample over 1997:Q1-2015:Q1. IP indicates that global slack has been constructed with industrial production data while GDP means that real GDP data was used instead. We distinguish global slack based on whether we use the one-sided Hodrick-Prescott (HP) filter or first-differencing (FD) for filtering the data. The entries in this table are the fraction of countries out of the 14 for which we have data where we find that the MSFE of each forecasting model (some variant of Model 3) is statistically different and more accurate than the MSFE of the country's benchmark model (Model 1) at least at the 10 percent significance level. Specific country results are available from the authors upon request.

(\*)The results for the contiguity measure are reported for 10 countries only (Australia, Japan, Sweden, and United Kingdom were omitted).

Forecasts of U.S. Inflation - Relative MSFEs (1997:Q1-2015:Q1)												
Horizon	1	4	6	8	10	12	1	4	6	8	10	12
	CPI						Core CPI (CPI ex. Food & Energy)					
<b>Model 2 against Model 1</b>												
<b>WTI oil price</b>												
HP-filtered	1.016	2.035	1.354	1.119	0.991	0.905	(b)	0.927**	0.910**	0.967	1.020	1.054
First-differenced	1.010	1.022	1.004	1.014	0.972*	0.921***	1.024	1.016	0.994	0.996	1.010	1.011
<b>Model 3 against Model 1</b>												
<b>WTI oil price (HP) &amp; Global inflation</b>												
Equal weights	0.695***	0.840**	0.879**	0.869**	0.816**	0.740**	(d)	0.702***	0.645***	0.685**	0.699**	0.644**
Contiguity	0.998	0.933*	0.881*	0.871*	0.818**	0.687**	0.992	0.806**	0.749**	0.781**	0.795**	0.751**
Distance	0.783***	0.802***	0.759***	0.747***	0.724***	0.636***	0.957**	0.814***	0.750***	0.773**	0.798**	0.792**
Pop. weighted distance	0.978***	0.903**	0.846**	0.829**	0.771**	0.636**	0.986	0.799**	0.741**	0.771**	0.782**	0.739**
Trade weights (1984-2014)	0.809***	0.790***	0.829***	0.837**	0.797**	0.694**	0.913***	0.647***	0.598***	0.660**	0.683**	0.631**
<b>WTI oil price (FD) &amp; Global inflation</b>												
Equal weights	0.864***	0.892**	0.911*	0.853**	0.755**	0.669**	(f)	0.895*	0.831**	0.808**	0.766**	0.666**
Contiguity	0.993**	0.960*	0.895*	0.858*	0.768**	0.640**	1.045	0.975	0.899*	0.866*	0.832*	0.746**
Distance	0.769***	0.824***	0.776**	0.743***	0.681***	0.586***	1.018	0.965*	0.898**	0.865**	0.842**	0.830**
Pop. weighted distance	0.972**	0.933*	0.868**	0.825**	0.733**	0.602***	1.039	0.964	0.886*	0.851*	0.813*	0.727**
Trade weights (1984-2014)	0.749***	0.812***	0.866**	0.842**	0.787***	0.704***	0.978*	0.872**	0.826**	0.818**	0.775**	0.681**

TABLE B7

Note: This table reports the forecasting performance with an estimation sample covering 1984:Q1-1996:Q4 and a pseudo out-of-sample forecasting sample over 1997:Q1-2015:Q1. We distinguish WTI oil price specifications based on whether we use the one-sided Hodrick-Prescott (HP) filter or first-differencing (FD) for filtering the data. The entries in this table are the MSFEs of the forecasts under different variants of Model 2 and Model 3 relative to the MSFE of the benchmark model (Model 1). Asterisks denote that the MSFE of a given version of Model 2 or Model 3 is statistically different and more accurate than the MSFE of the benchmark model (Model 1) at 1 (\*\*\*) , 5 (\*\*), and 10 (\*) percent significance levels.

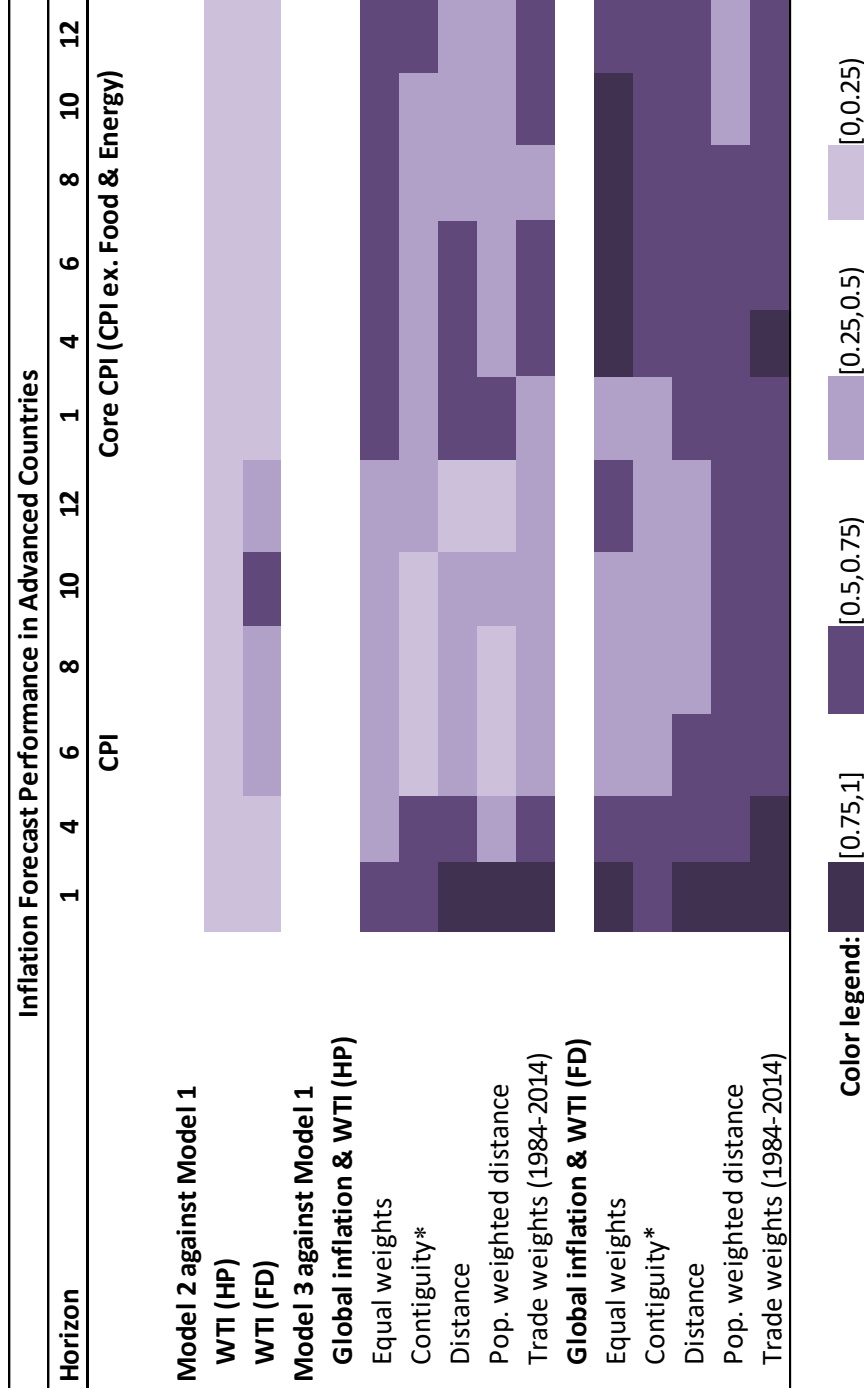


TABLE B8

Note: This table summarizes the forecasting performance for a sample of 14 countries which includes: Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, United Kingdom, and United States. The estimation sample covers the period 1984:Q1-1996:Q4 with a pseudo out-of-sample forecasting sample over 1997:Q1-2015:Q1. We distinguish WTI oil price specifications based on whether we use the one-sided Hodrick-Prescott (HP) filter or first-differencing (FD) for filtering the data. The entries in this table are the fraction of countries out of the 14 for which we have data where we find that the MSFE of each forecasting model (a variant of Model 2 or Model 3) is statistically different and more accurate than the MSFE of the country's benchmark model (Model 1) at least at the 10 percent significance level. Specific country results are available from the authors upon request.

(\*)The results for the contiguity measure are reported for 10 countries only (Australia, Japan, Sweden, and United Kingdom were omitted).

## 4.2 In-sample Model Comparison

Kabukçuoğlu and Martínez-García (Forthcoming) further investigate the in-sample performance of domestic slack in [Model 3](#) as an explanatory variable for domestic inflation. In particular, to understand to what extent adding domestic slack improves inflation predictions over a simpler model with global inflation alone ([Model 4](#)), i.e.,

$$\pi_{i,t+h|t}^4 = c_i^4 + \sum_{s=0}^p \gamma_{i,s}^4 \pi_{i,t-s} + \sum_{s=0}^r \lambda_{i,s}^4 \pi_{i,t-s}^* + \epsilon_{i,t+h}^4, \text{ for country } i \text{ and horizon } h. \quad (\text{Model 4})$$

We define  $\pi_{i,t}^*$  as the weighted rest-of-the-world inflation where  $\pi_{i,t}^* = \sum_{j=1}^M w_{ij}^\pi \pi_{j,t}$  using the weights  $w_{ij}^\pi$ . As noted earlier, we use the SIC to select the optimal number of lags  $r$  of the economic regressor (global inflation), taking as given the lag length  $p$  on the dynamics of inflation determined based on the SIC applied to [Model 1](#). To ensure that [Model 4](#) is nested into [Model 3](#), the lag length of the rest-of-the-world inflation in [Model 4](#) is chosen based on the SIC and this lag length is then used for rest-of-the-world inflation in both [Model 3](#) and [Model 4](#). Then, the lag length of domestic output gap in [Model 3](#) is determined according to the SIC. In both specifications, the lag length for domestic inflation obtained from [Model 1](#) according to the SIC is maintained. The maximum lag length allowed for each variable is four.

To isolate the effect of domestic slack in predicting inflation, Kabukçuoğlu and Martínez-García (Forthcoming) evaluate the in-sample predictive accuracy of these two competing models based on OLS estimates over the full sample period (1984:Q1-2015:Q1), predicting inflation in 14 advanced countries (including the U.S.) and constructing rest-of-the-world inflation measures from the same pool of countries as for their forecasting exercise. As in Kabukçuoğlu and Martínez-García (Forthcoming), our metrics of in-sample fit accuracy for a given model are based on the mean squared error (MSE) and the SIC. A relative MSE, calculated as  $\text{MSE3}/\text{MSE4}$ , that is less than 1 tends to favor [Model 3](#) over [Model 4](#). An SIC difference,  $\text{SIC3}-\text{SIC4}$ , that is less than 0 similarly suggests that [Model 3](#) is preferred by the data over [Model 4](#) even after penalizing for model overfitting. By construction, the SIC imposes a penalty for more complex models (models that are more heavily parameterized). Therefore, the SIC imposes a higher threshold to pass for [Model 3](#) relative to [Model 4](#), as there are a number of additional coefficients in [Model 3](#) introduced with the additional explanatory variable of the model (domestic slack).

We report the results for the in-sample predictive performances of the two models for U.S. CPI and core CPI inflation in Table B9. Except for the two cases where the dependent variable is headline CPI inflation and the output gap measures are based on log-first-differenced series for IP and real GDP, our MSE and SIC findings for the U.S. tend to provide limited support for [Model 3](#) in-sample. In general, our results are mixed and do not show significant differences between [Model 3](#) and [Model 4](#) given how the relative MSE and SIC are very close to 1 and 0, respectively. Kabukçuoğlu and Martínez-García (Forthcoming) summarize the results from the sample of 14 advanced countries showing that the empirical evidence broadly favors [Model 3](#) over [Model 4](#) for the majority of countries under different output gap measures and weighting schemes, based on the relative MSE metric. SIC differences of [Model 3](#) relative [Model 4](#) provide weaker support for some countries, yielding a more nuanced picture. The SIC penalizes overfitting unlike the MSE statistic.

If the coefficient of the domestic output gap under [Model 3](#) is small, then—according to theory—we should expect similar predictive performances under [Model 3](#) and [Model 4](#). We follow Stock and Watson (2003) and Kabukçuoğlu and Martínez-García (Forthcoming) and aggregate the OLS coefficient estimates for

the lags of domestic output gap in [Model 3](#) in order to map them into a measure of persistence. The inverse of one minus this sum of coefficients, referred to as the persistence, is higher if the sum of the estimated coefficient estimates is higher. Table B10 reports the ranges and the median values for domestic output gap persistence calculated in this way, across various model specifications, inflation measures, and countries.

Kabukçuoğlu and Martínez-García (Forthcoming) then go onto illustrate that there is an inverse relationship between the median of the relative MSE (or median SIC difference) of [Model 3](#) versus [Model 4](#) and this measure of domestic output gap persistence. All possible specifications under consideration for each country are summarized with the median. Therefore, the evidence suggests that higher persistence (a higher sum of the OLS-estimated coefficients) is associated with stronger empirical support for [Model 3](#) (both under the MSE and SIC metrics).

As noted in Kabukçuoğlu and Martínez-García (Forthcoming), this finding appears consistent with the theoretical implication of the workhorse open-economy New Keynesian model laid out here. Hence, we can argue that the flattening of the Phillips curve (associated with a lower coefficient on domestic slack) is an important reason explaining the dominant role played by global inflation in explaining the forecasting performance of [Model 3](#).



Relative In-Sample Performance (U.S. Inflation) (1984:Q1-2015:Q1)									
Headline CPI					Core CPI (CPI ex. Food & Energy)				
Model 3 against Model 4	MSE3/MSE4	MSE3	SIC3-SIC4	SIC3	MSE3/MSE4	MSE3	SIC3-SIC4	SIC3	SIC3
<b>M3: Global inflation &amp; Domestic slack (IP-HP)</b>									
<b>M4: Global inflation (a)</b>									
Equal weights	1.005	3.594	0.036	1.403	1.009	2.272	0.039	0.976	
Contiguity	1.006	3.649	0.037	1.418	1.008	2.274	0.039	1.007	
Distance	1.005	3.679	0.035	1.426	1.009	2.250	0.039	0.997	
Pop. weighted distance	1.006	3.648	0.037	1.418	1.008	2.255	0.039	0.999	
Trade weights (1984-2014)	1.005	3.612	0.036	1.408	1.009	2.281	0.039	0.980	
<b>M3: Global inflation &amp; Domestic slack (GDP-HP)</b>									
<b>M4: Global inflation (c)</b>									
Equal weights	1.006	3.598	0.037	1.404	1.008	2.2718	0.039	0.976	
Contiguity	1.008	3.654	0.039	1.419	1.007	2.2722	0.038	1.007	
Distance	1.007	3.688	0.038	1.428	1.009	2.2505	0.039	0.997	
Pop. weighted distance	1.008	3.653	0.038	1.419	1.008	2.2541	0.039	0.999	
Trade weights (1984-2014)	1.006	3.616	0.037	1.409	1.009	2.2807	0.039	0.980	
<b>M3: Global inflation &amp; Domestic slack (IP-FD)</b>									
<b>M4: Global inflation (e)</b>									
Equal weights	0.996	3.560	0.026	1.393	1.007	2.268	0.038	0.974	
Contiguity	0.999	3.623	0.030	1.411	1.005	2.267	0.036	1.005	
Distance	0.995	3.645	0.026	1.417	1.007	2.248	0.038	0.996	
Pop. weighted distance	0.999	3.621	0.030	1.410	1.005	2.249	0.036	0.996	
Trade weights (1984-2014)	0.995	3.576	0.026	1.398	1.007	2.278	0.038	0.978	
<b>M3: Global inflation &amp; Domestic slack (GDP-FD)</b>									
<b>M4: Global inflation (g)</b>									
Equal weights	0.997	3.564	0.028	1.394	1.008	2.271	0.039	0.975	
Contiguity	0.999	3.624	0.030	1.411	1.007	2.271	0.038	1.006	
Distance	0.996	3.646	0.027	1.417	1.008	2.249	0.039	0.996	
Pop. weighted distance	0.999	3.622	0.024	1.410	1.007	2.251	0.037	0.997	
Trade weights (1984-2014)	0.995	3.577	0.026	1.398	1.008	2.280	0.039	0.979	

TABLE B9

Note: This table summarizes the in-sample predictive performance of Model 3 relative to Model 4 for U.S. inflation over the 1984:Q1-2015:Q1 period. We distinguish domestic slack based on whether we use the one-sided Hodrick-Prescott (HP) filter or first-differencing (FD) for filtering the data. We construct alternative measures of domestic slack based on industrial production (IP) and real GDP (GDP) data. We report the performance based on the relative mean squared errors (relative MSEs) and the differences of the Schwarz Information Criteria (SIC), respectively. Hence, a relative MSE less than one or an SIC difference less than zero tends to favor Model 3. The absolute MSE and SIC values for Model 3 are also reported.

Persistence of Domestic Slack (U.S. and Advanced Countries) (1984:Q1-2015:Q1)			
	Headline CPI		Core CPI
	U.S. (a)	Advanced (b)	U.S. (c)      Advanced (d)
<b>M3: Global inflation and domestic slack (IP-HP)</b>			
Max	1.044	1.483	1.006      1.057
Min	1.032	1.117	0.986      1.048
Median	1.040	1.431	0.995      1.051
<b>M3: Global inflation and domestic slack (GDP-HP)</b>			
Max	1.085	2.729	1.011      1.576
Min	1.039	1.165	0.957      1.492
Median	1.070	1.391	0.984      1.531
<b>M3: Global inflation and domestic slack (IP-FD)</b>			
Max	1.230	1.485	0.957      1.124
Min	1.183	1.386	0.932      1.031
Median	1.220	1.441	0.950      1.113
<b>M3: Global inflation and domestic slack (GDP-FD)</b>			
Max	1.568	2.942	0.960      1.683
Min	1.438	2.341	0.896      1.486
Median	1.523	2.816	0.945      1.607

TABLE B10

Note: This table reports the persistence of the domestic output gap defined as the inverse of one minus the sum of coefficients on domestic slack under Model 3 for the U.S. and the 14 advanced countries (including the U.S.) in our sample from OLS estimates over the 1984:Q1-2015:Q1 period. The results for the U.S. are summarized by reporting the maximum, minimum, and median persistence values across 5 different weighting schemes. The results for the advanced countries are summarized by reporting these statistics across the 5 different weighting schemes and across the 14 advanced countries. The world countries used to construct global inflation include: Australia, Austria, Belgium, Canada, Chile, France, Germany, Italy, Mexico, Netherlands, Sweden, Switzerland, Japan, Korea, Spain, Taiwan, United Kingdom, and United States.

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