

Abstract

This paper investigates systemic risk in multilateral netting payments systems. A four-period model is constructed to investigate the effects of random liquidity shocks. There are three different types of agents in this model: banks, the payments system operator, and the central bank. Banks pay one another via the payments system. The payments system operator sets the rules for participation. These include total asset requirements, collateral requirements, and net debit caps. The central bank serves as a source of liquidity during a financial crisis.

The model is constructed along the lines of Diamond and Dybvig (1983). In period 0, banks optimize their holdings of non-interest earning central bank reserves to meet their payment obligations and any additional liquidity obligation. Their alternative is to invest in a non-liquid asset that earns a rate of return R . In period 1, a number of banks are unable to make their payments. The number of banks defaulting is random and realized after banks decide their optimal reserve holdings. In period 2, the remaining banks must cover the net payments of defaulting banks minus the defaulting banks' collateral holdings. Each remaining bank has three options to meet its liquidity event: deliver reserves it holds at the time of the shock, borrow funds in the interbank market from banks in net credit positions, or default since it cannot meet its additional obligations. In period 3, final wealth of each due to bank is calculated. All banks want to maximize final wealth in period 3.

The model provides the following results. The model calculates the threshold point where the payments system collapses. An interbank funds market increases the efficiency of the payments system. Implementation of policy options such as total asset requirements, collateral requirements, and net debit caps decrease systemic risk. The central bank's role as provider of liquidity to the financial system is investigated in the context of the model.

Analysis of Systemic Risk in the Payments System

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1 Introduction

In any financial transaction, there are risks involved with the transfer of monetary value. Settlement risk is almost eliminated when currency is used as the payment medium, because at the time of the transaction, good funds are exchanged for the good or service.¹ Settlement risk is the risk that the receiver of a payment instrument is unable to convert it to good funds.² However, the use of currency for large-value transactions are cumbersome and pose safety concerns regarding its transportation. Checks, on the other hand, do not have these disadvantages, but checks need to be converted to good funds before the funds can be used for other transactions. The receiver of the check bears the risk that he will be unable to convert the check into good funds.

Currency outranks other payment media in terms of the frequency of use, but only account for a small fraction of the total value of payments in developed countries. In the United States, large-value electronic payments between banks comprise the bulk of the value of all financial transactions. This paper focuses on these types of funds transfer systems.

Large-value payments systems are at the foundation of any nation's financial system. Horii and Summers (1994, p74) explain the importance of safe and efficient large-value payments systems:

Large-value transfer systems supporting the interbank markets are the main arteries of a nation's payment system. The safe and efficient operation of the money and capital markets hinges upon the smooth functioning of these systems. ... The safe and efficient operation of large-value transfer systems

¹Good funds are funds that are always accepted as final payment and can be immediately used by the receiver to meet other monetary obligations without further conversion. However, the risk of the currency being counterfeit always exists, but I assume this risk to be negligible.

²In a broader sense, settlement risk is the risk that one party does not deliver his side of the transaction, but has received from the counterparty the other side of the transaction. Timing differences between delivery of one asset for another increases settlement risk. For example, in a foreign exchange transaction, one party may receive currency from the counterparty before he delivers funds in a different currency. In 1974, Bankhaus Herstatt received deutsche mark payments and was closed by the German banking supervisory authorities before it could make dollar payments in New York. Counterparties that were expecting dollar payments from Herstatt incurred losses. This type of settlement risk in foreign exchange markets is known as Herstatt risk.

has a bearing not only on the markets they directly serve but on a nation's whole financial system.

Both safety and efficiency of the payments system are important characteristics that payments system operators should be concerned with. In this paper, I study the tradeoff between the increase in efficiency of multilateral netting and exposure to systemic risk of the participants. In this paper, efficiency refers to technical efficiency of the payments system.³

Payments systems that net transactions multilaterally allow banks to settle their positions at the end of the period. During the period, banks receive and send payment messages to other banks. Banks that owe funds are called due to banks, and banks that receive funds are called due from banks. There is a clearinghouse that acts as an intermediary and collects good funds from due to banks and releases good funds to due from banks. Final settlement occurs when the clearinghouse has successfully completed this process. In large-value payments systems, good funds are reserves held at the central bank.

In payments systems that net transactions, banks often interpret payment messages as good funds and release the funds to their customers. Consider the behavior of the Clearing House for Interbank Payments System (CHIPS) participants.⁴ The Federal Reserve Bank of New York (FRBNY) (1991, p.17) explains the behavior of CHIPS participants: "competitive pressures have, however, resulted in a common practice of permitting receivers access to funds immediately." The time difference between when funds are released and when they are settled increases settlement risk.

The risk that banks face in a net payments system where payment messages are interpreted as good funds is the inability of other banks to deliver their net due tos at settlement time. It is possible for banks that never dealt directly with the defaulting bank to suffer. For example, if bank A sends a payment message to bank B and bank B uses it to make a payment to bank C, the default of bank A can affect bank C.

³The issue of economic efficiency should not be ignored by policymakers when establishing the operating rules of a payment system. However, it is not addressed in this paper.

⁴CHIPS is the multilateral netting payments systems used to clear and settle cross border dollar transactions and the dollar leg of foreign exchange transactions.

Settlement risk can lead to systemic risk. Systemic risk is the risk that one bank is unable to settle its obligations resulting from another bank not settling its obligations.⁵ Systemic crises occur when settlement cannot be met by normal means and alternate measures must be taken. Systemic crises result in systemic collapse when alternate measures cannot prevent a complete unwinding of payments.

The risk of systemic collapse must be weighed against the gains in efficiency. The primary reason that payments systems settle net payment positions at the end of the day is to reduce the amount of reserves needed for the settlement process. If banks had to settle payments individually, they would on average need to hold more reserves.⁶ Banks face higher costs by holding a greater level of reserves, because reserves held at the central bank are non-interest bearing assets.⁷ A funds transfer system that uses less reserves for settling the same value of transactions in a given amount of time is defined to be more technically efficient.

Humphrey (1986) conducted one of the first studies of systemic risk in net settlement systems. By using actual daily records of net positions of banks at settlement, Humphrey considered what would happen if one bank were to default its net debit position. He chose one bank and deleted all its incoming and outgoing payment messages during the day. Then, he recalculated each remaining bank's settlement positions; a process known as partial unwinding. He found that the default of one bank could lead to the default of other banks.

A few years ago, CHIPS adopted measures to reduce systemic risk with the intent of avoiding an unwinding of payments. Two of these risk-reducing measures are: a collateral pool and additional settlement obligations (ASOs). According to FRBNY (1991), CHIPS' collateral pool was slightly over \$3 billion by late 1990. In a system that

⁵This definition is used in to describe systemic risk in the payments system context. In the broader banking literature, systemic risk is defined as the failure of one or more financial institutions, with adverse consequences to both the financial system and the economy as a whole. Systemic risk in the payments system context may lead to systemic risk in the broader banking literature.

⁶Payments systems that settle payments individually are commonly referred to as gross settlement systems. There are gross settlement systems where the central bank grants intra-day credit, e.g. Fedwire. This system uses less reserves for settlement than gross settlement systems where the central bank does not extend intra-day credit.

⁷There are other benefits that banks receive in exchange for interest lost on reserves which are not modeled here.

has gross transactions valued over \$1 trillion per day, the collapse of two major banks could potentially exhaust the collateral pool.⁸

CHIPS implemented loss-sharing agreements in October 1990.⁹ Before the implementation of these measures, if one participant defaulted, a partial or total unwind was undertaken. Under this new arrangement, each participant agrees to pay an ASO, if a participant fails to settle.¹⁰ The sum of ASOs will equal the net net debit balance of the failed participant. The calculation of the remaining participant's ASO is as follows:

the net net debit balance owed by the failed participant multiplied by a fraction, the numerator of which is the highest bilateral credit limit that was granted by the remaining participant to the failed participant during the day for which settlement is to be made and the denominator of which is the sum of all of the highest bilateral credit limits that were granted by all the remaining participants to the failed participant during that day. (NYCHA 1990, p.8)

The formula used in the case of multiple failed participants is such that:

the total ASO of a remaining participant may not be greater than 5 percent of the remaining participant's maximum bilateral credit limit. In addition, the ASO of a remaining participant calculated with respect to any single failed participant may not exceed the highest bilateral credit limit extended by the former to the latter. (NYCHA 1990, p.8)

CHIPS may ask if any other participant is willing to make up the shortages in case sufficient funds were not raised by this rule. If ASOs are not adequate to make up the

⁸This was the opinion of individuals that I spoke with at FRBNY.

⁹Although FRBNY (1991) claims that the share-loss rule may allow for more than one participant failing, based on my conversations with the Payments Systems Studies Staff at FRBNY, this arrangement is meant to handle the failure of one large participant and multiple failure of large participants has not really been addressed. Although I was not given actual figures for the net amounts settled by the settling members, it is estimated to be somewhere in the range of \$7 billion to \$10 billion per day with the large banks incurring net debit positions up to \$2 billion per day. Settling participants are participants that are settling agents for themselves and the remaining participants.

¹⁰In the case of a settling participant failing, there are other options available to CHIPS.

debit position of the defaulted net due to banks, a partial unwinding occurs. If the partial unwinding is not successful, a complete unwinding of payments occurs.

By increasing the supply of funds during a financial panic, the central bank can avert systemic collapse. An example of the central bank willingness to increase funds is the Federal Reserve's decision to provide liquidity in the interbank market through open market purchases the morning after the stock market crash of October 1987. Garber and Weisbrod (1992, p.500) cite the Federal Reserve's commitment in general to provide liquidity to the financial markets, "The Federal Reserve affirms its readiness to serve as a source of liquidity to support the economic and financial system."

In addition to open market operations, the central bank can operate a discount window to directly lend to banks that require additional liquidity. However, it is difficult to recognize in a short time frame when a bank is illiquid and when it is insolvent. A bank is defined to be illiquid if it cannot convert its assets into good funds at the present time, but is able to make payment at a later date. A bank is defined to be insolvent if it can never settle its payment obligations.

The model developed in this paper is along the lines of Diamond and Dybvig (1983). Diamond and Dybvig present a multi-period model in which agents make decisions about the amount of liquid funds that they want to hold. Their model considers a liquidity event which is realized after the investment decision is made. Instead of a general equilibrium model, the model presented in this paper is a partial equilibrium model that focuses on institutional features of payments processing.

Bhattacharya and Gale (1987) add an interbank funds market to a Diamond and Dybvig setting. In their model, banks make portfolio decisions between investing in low-yield short-term assets and high-yield long-term assets. Each bank faces a random liquidity event, but the aggregate liquidity shock to the economy is known. Short-term assets can be used to meet liquidity needs instead of the costly liquidation of long-term assets. But, the first best solution is for each bank to invest an equal share in the short-term asset to meet its share of the certain aggregate liquidity shock. The first best solution cannot be achieved, since each bank has an incentive to invest less. Thus, they propose a second best precommitted investment plan.

The liquidity event in the model presented in this paper is caused by the default of banks at settlement. The number of banks defaulting is exogenous to the model.¹¹ Unlike the Diamond and Dybvig model, deposit insurance does not eliminate bad outcomes from occurring without high costs to the insurer. The crisis is caused by an exogenous random variable and not from the perception of savers that a good bank has gone bad leading to a run on the bank. Although this is a strong assumption, there have been random liquidity shocks in financial markets. For example, on November 21, 1985, the Bank of New York (BONY) had a computer malfunction that led to settlement problems. The Federal Reserve made a \$22.6 billion loan at the discount window. This loan increased the monetary base by more than 10 percent overnight.¹² Such operational problems may not be foreseen by market participants, but still lead to settlement problems and may lead to financial panics.

The model determines what is the maximum number of banks that can default without systemic collapse given certain parameters and optimal decisions made by banks. Banks optimize their reserve holdings based on institutional parameters imposed by the payments system operator, a random liquidity shock, and the default rate in the interbank market. The number of banks defaulting is random. The remaining banks must cover the debit positions of the defaulting banks. The additional obligation is a liquidity shock to each of the remaining banks. This additional obligation can result from a partial unwinding or an explicit additional settlement obligation arrangement, e.g. ASOs in CHIPS. Each bank has three options to meet its obligation: 1) use additional reserves it holds, 2) borrow funds in the interbank market from due from banks, or 3) default since it cannot afford to borrow the necessary funds to meet its obligation.

The role of the central bank is also discussed. Although most central banks do not explicitly guarantee payments over privately-owned netting systems, most bankers and regulators feel that the central bank would bail out these systems. However, such

¹¹This assumption is a starting point for the model. A general equilibrium model would consider the depositors at these institutions such as in Diamond and Dybvig.

¹²See Garber and Weisbrod (1992, p.286-288) for detailed analysis of BONY's computer malfunction and its discount window borrowing.

guarantees, whether explicit or implicit, lead to moral hazard problems.¹³

In the next section, I construct a model to determine the amount of reserves held by due to banks given an expectation of revised settlement positions. The model predicts the threshold point where the payments system operator can no longer collect good funds from the remaining banks to complete the settlement process. In section 3, the central bank's ability to intervene in a financial crisis is considered. The instruments used by the central bank are open market purchases or the provision of loans at the discount window. In section 4, payments system policy options to reduce systemic risk are investigated. Section 5 concludes the paper.

2 The Model

To examine the issue of systemic risk, I construct a four-period model. The goal of this model is to analyze how different institutional constraints affect the optimizing behavior of banks and the risk of systemic collapse.

2.1 The Agents

There are three agents in this model: the banks, the payments system operator, and the central bank. Banks transfer funds via the payments system. The payments system is a multilateral netting system. Good funds are reserves held by banks at the central bank. The central bank's role is to provide the necessary liquidity to banks during a financial crisis.

There are two types of banks in this model. In period zero, each bank knows its type. Banks either receive funds or send funds at the end of period one. There are a total of $2n$ banks of which n are due to banks and n are due from banks. Each due to bank has to deliver a net amount P and each due from bank receives a net amount P . If a bank knows that it will be in a due to position, it will keep a certain fraction of its assets in central bank reserves. If a bank knows it will be in a due from position, it will not hold any reserves.¹⁴

¹³These issues are not discussed in this paper.

¹⁴The qualitative results of the model do not change if due from banks hold reserves. There would

Figure 1 depicts the timing of events. In period zero, banks have an expectation of their payment flows. In period one, payment flows occur between banks. During period one, banks interpret payment messages as good funds to the extent that they release the funds before settlement. Settlement occurs at the end of period one. At settlement time, one or more due to banks default. In period two, the remaining banks are notified of their revised settlement positions. In the model, the extra settlement obligation caused by the defaulting bank(s) is equally shared by the remaining banks. In period three, the final wealth of each bank is calculated.

The due to bank maximizes its expected wealth in period 3. The due to bank must decide on the fraction of its assets it will hold in non-interest earning reserves in period zero based on its net payment position at the end of period one and its expectation of the random liquidity event in period two. Since liquid funds do not earn interest, the opportunity cost of holding reserves is the foregone interest on the amount of reserves held.

The only random element in the model is the number of banks defaulting, denoted as t , at the end of period 1. In this model, I consider the ratio of defaulting banks to remaining banks, denoted $\frac{t}{2n-t}$, to have an underlying distribution. The model is solved for the general case where $\frac{t}{2n-t}$ can take on any distribution. To obtain a simple analytical result, I use a uniform distribution for the proportion of defaulting banks to remaining banks.

2.2 The Due To Bank's Problem

The due to bank's problem is determining the quantity of liquid funds to hold for its net due to settlement position at the end of period one and its additional settlement obligation in period two.¹⁵ I derive the optimal holdings of reserves for the case without an overnight interbank funds market and for the case with an interbank funds market. With

still exist a threshold point where the system collapses. By holding reserves, there would be a greater supply of reserves. However, due from banks would also optimize their holding of non-interest bearing assets.

¹⁵Additional settlement obligations are calculated differently in this model than the procedure that CHIPS uses. In this model, each remaining bank owes the same amount. The additional obligation for each bank will be discussed below.

the addition of an overnight interbank funds market, due to banks can borrow funds from due from banks for their revised settlement obligations in period two.¹⁶

2.2.1 Case I: No Interbank Funds Market

Let us consider the case where there is no interbank funds market. In this case, the due to bank must hold the necessary funds to meet its net payment and additional obligation. If a bank fails to meet its net payment and additional obligation, its period 3 wealth will be zero. The qualitative results do not change if the bank has some positive terminal wealth in period 3 as a result of not meeting its liquidity obligation. However, the calculations become complicated in terms of restrictions placed on the terminal wealth.

If t banks default, where t is less than n , the amount $t(P - E)$ is the total amount defaulted. P is the net payment due by each due to bank. E is the required collateral holdings for each due to bank. I assume that the payments system operator can only recover the collateral holdings from the defaulting banks.¹⁷ Since this amount is split evenly among all the remaining banks, the individual bank's contribution, δ_i , is:

$$\delta_i = \frac{t(P - E)}{2n - t}. \quad (1)$$

We defined the additional settlement obligations, α , in terms of the proportion of banks defaulting to remaining banks as:

$$\alpha = \frac{t}{2n - t}.$$

Therefore, α has the same distribution and realization as the ratio of defaulting banks to remaining banks. α is a proportion of $P - E$ that each due to bank must meet. The relationship between α and the ratio of defaulting banks to remaining banks need not be equal, but keeps the model mathematically attractive while yielding the same qualitative results.

¹⁶If due to banks could also borrow in period one, their line limits would further be reduced for period two borrowing leading to a systemic crisis sooner.

¹⁷Even if additional funds could be collected from the defaulting banks, these funds would not be available for settlement in period 2. In reality, it may take months or even years before the payments system operator can collect additional funds from the defaulting banks.

The due to bank faces the following constraint:

$$\underbrace{(1 - \gamma) A + E}_{\text{Liquid funds held}} \geq \underbrace{P + \alpha (P - E)}_{\text{Liquidity demand}} . \quad (2)$$

where:

- A – Total assets excluding collateral,
- α – Fraction of payment for additional settlement,
 $\alpha \in [0, 1]$,
- γ – Fraction of assets held in interest bearing assets,
 $\gamma \in [0, 1]$.

The value of A is known at period zero and is exogenous. As we will see in section four, Crisis Prevention, A can be used as a parameter by the payments system operator to reduce systemic risk. The payments system operator may place minimum asset requirements on participating banks to reduce the risk of systemic collapse.

P represents the net payment due in period one and is known to banks in period zero. The value of P is exogenous. The sum of A and E must be greater than the value of P , otherwise the bank is insolvent. The payments system operator can restrict the upper bound of P to reduce systemic risk. We will explore restrictions on P in section four, Crisis Prevention.

The value for E is an exogenous parameter set by the payments system operator and earns no interest for the due to bank.¹⁸ If P is less than or equal to E , the additional settlement obligation is zero, since the collateral of defaulting banks is used to meet the defaulting banks' obligations. We will focus on the case where P is greater than E since there are gains in technical efficiency when this is the case.

γ is an endogenous choice variable for the due to bank. We will solve for γ under two different settings. First, we consider a setting without an interbank market. Second, we add an interbank market to see the differences in γ . We would expect to see higher

¹⁸By paying interest, the qualitative results would not change as long as the return on collateral was less than the return on the interest bearing asset.

values for γ with an interbank market, since due to banks can borrow to make up for their shortages of liquid funds.

Rearranging equation (2) in terms of the random variable, α , yields:

$$\frac{(1 - \gamma) A}{P - E} - 1 \geq \alpha . \quad (3)$$

When equation (2) holds, the due to bank can meet its liquidity demand, otherwise the due to bank is bankrupt. Let $B(\gamma)$ be defined as:

$$B(\gamma) = \frac{(1 - \gamma) A}{P - E} - 1 . \quad (4)$$

$B(\gamma)$ is the maximum value of α that can be realized without the bank going bankrupt.

Given a probability density function for α , $f(\alpha)$, the bank's expected wealth function is the following:

$$E[wealth] = \int_0^{B(\gamma)} [R \gamma A + (1 - \gamma) A + E - P - \alpha (P - E)] f(\alpha) d\alpha + \int_{B(\gamma)}^1 [0] f(\alpha) d\alpha \quad (5)$$

where:

R – Return on the long – term asset ,

$R > 1$.

The value of R is given exogenously. Due to banks invest in the long-term asset in period 0 and the asset matures in period 3.¹⁹ The due to bank's maximization problem can be stated as:

$$\max_{\gamma \in [0,1]} \int_0^{B(\gamma)} [R \gamma A + (1 - \gamma) A + E - P - \alpha (P - E)] f(\alpha) d\alpha . \quad (6)$$

Differentiating $E[wealth]$ with respect to γ , using Leibnitz's rule gives:

$$\frac{\partial E[wealth]}{\partial \gamma} = - [R \gamma A] \left[\frac{A}{P - E} \right] f[B(\gamma)] + [(R - 1) A] \int_0^{B(\gamma)} f(\alpha) d\alpha . \quad (7)$$

Setting the derivative equal to zero yields:

$$[R \gamma A] \left[\frac{A}{P - E} \right] f[B(\gamma)] = [(R - 1) A] \int_0^{B(\gamma)} f(\alpha) d\alpha . \quad (8)$$

¹⁹I assume that there is no secondary market for the long-term asset in earlier periods.

In the special case where α has a uniform distribution ranging from zero to one, the optimal $\hat{\gamma}$ is:

$$\hat{\gamma} = \frac{R-1}{2R-1} \left(1 + \frac{E-P}{A}\right). \quad (9)$$

Because R and E are both given exogenously, some comparative statics can be done. As R increases, the fraction of assets earning interest, $\hat{\gamma}$, should increase. The model confirms this intuition as observed by:

$$\frac{d\hat{\gamma}}{dR} = \frac{1}{(2R-1)^2} \left(1 + \frac{E-P}{A}\right) \geq 0. \quad (10)$$

Increases in collateral holdings, E , lead to increases in $\hat{\gamma}$ as seen by:

$$\frac{d\hat{\gamma}}{dE} = \frac{R-1}{A(2R-1)} \geq 0. \quad (11)$$

In other words, if the due to bank is forced to hold more collateral, it will invest a greater proportion of A in interest earning assets.

2.2.2 Case II: Interbank Funds Market

This section allows the due to bank to borrow in period two to meet its additional liquidity requirement. The purpose of the interbank market in this model is for due to banks to borrow additional reserves to meet the additional liquidity requirement. As mentioned above, the results of the model would not change qualitatively if due to banks borrow in period one to meet their payment obligation.

There are three possible states for the bank. Case A is where the bank has sufficient funds to meet its liquidity demand as discussed above. Case B is where the bank borrows and can pay back the loan in period 2. Case C is where the bank cannot borrow since it will not be able to pay back the loan.

The bank can only borrow when:

$$R\gamma A - R_{ib} [P + \alpha(P-E) - (1-\gamma)A - E] \geq 0, \quad (12)$$

where:

$$R_{ib} - \text{Return due on interbank loans.}$$

The term in brackets multiplying R_{ib} is greater than zero otherwise, the bank would be in case A.

For the due to bank's optimization problem, I make the assumption that the due to banks' expectation of R_{ib} in period two is greater than the period zero long-term investment return, R . Due to banks form expectations of R_{ib} , in period zero based on all possible states of α . They expect an increase in R_{ib} , at the threshold of systemic collapse. How due to banks weigh each state of α is dependent on their aversion towards risk. Recall that if due to banks cannot access liquidity after a liquidity shock, they become bankrupt. Due to banks will weigh states of α where the system collapses with greater weight than other states of α . If the due to banks are risk averse, the expectation in period zero of R_{ib} in period 2 will be greater than R in period zero.²⁰ However, during normal settlement, the R_{ib} realized in period two will be less than R . Later, we will see that the realized value for R_{ib} is dependent upon the realized α .

The point where the bank can no longer borrow occurs when equation (12) is equal to zero or for α :

$$\alpha = \frac{R\gamma A}{R_{ib}(P - E)} + \frac{(1 - \gamma) A}{P - E} - 1 . \quad (13)$$

Let $C(\gamma)$ be the threshold point where the bank is at the upper bound of being able to borrow:

$$C(\gamma) = \frac{R\gamma A}{R_{ib}(P - E)} + \frac{(1 - \gamma) A}{P - E} - 1 . \quad (14)$$

$C(\gamma)$ is greater than $B(\gamma)$ (See equation 3) because the first term is positive.

The due to bank faces the following maximization problem:

$$\begin{aligned} \max_{\gamma \in [0,1]} & \int_0^{B(\gamma)} [R \gamma A + (1 - \gamma) A + E - P - \alpha (P - E)] f(\alpha) d\alpha + \\ & \int_{B(\gamma)}^{C(\gamma)} [R \gamma A - R_{ib} [P + \alpha (P - E) - (1 - \gamma) A - E]] f(\alpha) d\alpha + \\ & \int_{C(\gamma)}^1 [0] f(\alpha) d\alpha . \end{aligned} \quad (15)$$

When α equals $B(\gamma)$, the term in the brackets next to R_{ib} in equation (12) is zero since

²⁰In reality, sometimes banks hold excess reserves to meet future liquidity needs. This action may represent their reluctance to rely on the interbank funds market for all their liquidity needs since they are willing to give up a higher return to hold their assets in liquid form. In addition, Strongin (1995) states that during the last week of December the Federal funds rate as reached as high as 100% because of the increased demand for liquidity.

$B(\gamma)$ is the point where the bank has just enough reserves to meet the liquidity demand. For this case, period 3 wealth of the bank is $R \gamma A$. Any value greater than $B(\gamma)$, the bank's period 3 wealth would be less than $R \gamma A$. Any point after $C(\gamma)$, the bank's borrowing costs are greater than $R \gamma A$ so the bank can no longer borrow to meet its liquidity need.

Following similar methods used to solve problem (5), the solution for problem (15) is:

$$(R - 1)A \int_0^{B(\gamma)} f(\alpha) d\alpha = (R_{ib} - R)A \int_{B(\gamma)}^{C(\gamma)} f(\alpha) d\alpha. \quad (16)$$

In the special case where α is uniformly distributed, the following optimal γ , $\hat{\gamma}$, results:

$$\hat{\gamma} = \frac{R - 1}{2R - 1 - \frac{R^2}{R_{ib}}} \left(1 + \frac{E - P}{A}\right). \quad (17)$$

Comparing $\hat{\gamma}$ found in equation (17) to equation (9) one finds:

$$\hat{\gamma}_2 = \frac{R - 1}{2R - 1 - \frac{R^2}{R_{ib}}} \left(1 + \frac{E - P}{A}\right) \geq \frac{R - 1}{2R - 1} \left(1 + \frac{E - P}{A}\right) = \hat{\gamma}_1, \quad (18)$$

where:

$$\begin{aligned} \hat{\gamma}_2 & - \text{Optimal } \gamma \text{ given banks can borrow ,} \\ & \text{Equation (17) ,} \\ \hat{\gamma}_1 & - \text{Optimal } \gamma \text{ given banks cannot borrow ,} \\ & \text{Equation (8).} \end{aligned}$$

Given interbank borrowing in period two, banks will invest a greater proportion of A in interest earning assets. The gain in efficiency can be measured by the interest earned as a result of this difference. Let G measure the gain in efficiency then:

$$G = R(\hat{\gamma}_2 - \hat{\gamma}_1)A. \quad (19)$$

An overnight interbank funds market increases the efficiency of the payments system.

2.3 The Supply and Demand of Interbank Funds

The supply of interbank funds is equal to the funds that due to banks owe due from banks.²¹ In this model, the quantity of funds available for lending always exceeds the

²¹Another variation of the model would be to have due from banks hold excess reserves that they could lend.

quantity required for the due to banks to make settlement. However, the due from banks decide on the fraction of incoming funds that they are willing to lend. This lending decision is based on a function ϕ which is a function of α . At α equal to zero, ϕ equals one, because due from banks are willing to lend all of their funds. At α equal to one, ϕ equals zero, because due from banks receive no funds from due from banks to lend since all due to banks are bankrupt.

The only information that due from banks receive in period two to base their lending decisions upon is the amount of the additional settlement obligation, α . As α increases, due from banks are willing to lend a lower proportion of their liquid assets. In other words, as α increases, ϕ decreases. There could be two reasons for this relationship between ϕ and α . First, due from banks may feel that they need to hold a greater proportion of liquid assets as the number of banks defaulting in period 1 increases. Although it is not part of the model, bank defaults may lead to greater liquidity needs for the remaining banks, because other banking related activities may require greater liquidity.

Second, there may be a relationship between the number of banks defaulting in period 1 and the riskiness of interbank lending. In the model, the default of banks in period one is not related to solvent due to banks defaulting on their interbank borrowings. However, due from banks may base their perception of interbank loans becoming more risky as α increases.²² The perception of higher default rates for the remaining due to banks has an impact on the due from banks' lending decisions.

The supply of funds available in the interbank funds can be written as:

$$ss_{lf} = \phi(\alpha) (n - t) [(1 + \alpha) P] . \quad (20)$$

At $\alpha = 0$, due from banks are willing to lend all their incoming funds. Thus, the supply function at $\alpha = 0$ has a value of nP . The supply function at $\alpha = 1$, where all the due to banks default in period 1, is equal to 0. The supply function is assumed to be

²²Although I have not explicitly modeled the risk of interbank lending, this risk can be incorporated in the due from banks' lending decisions. For example, the risk of default in the interbank market could be related to the number of banks defaulting in period 1.

continuous in α . If we place the following restriction on $\frac{d\phi}{d\alpha}$, then the supply function is strictly decreasing in α .

$$-\frac{d\phi}{d\alpha} > \frac{\phi(\alpha)}{1 + \alpha}, \quad (21)$$

In addition, I make the following technical assumption on $\phi(\alpha)$:

$$\frac{d^2\phi}{d\alpha^2} > \frac{-2}{1 + \alpha} \frac{d\phi}{d\alpha}. \quad (22)$$

This condition allows for simpler mathematical analysis in section 4, Crisis Prevention. However, this assumption also gives us insight into the behavior of due from banks. This assumption implies that as α increases, the willingness to lend funds decreases faster.

The demand for interbank funds is inelastic and is dependent upon the realization of α . Due to banks' demand for interbank funds is the difference between their payment obligation including the additional obligation and the sum of their liquid assets and their collateral plus their share of the defaulting banks' collateral. The aggregate demand for funds, dd_{lf} , can be stated as:

$$dd_{lf} = (n - t)[(1 + \alpha)(P - E) - [(1 - \hat{\gamma})A + E]] \quad (23)$$

2.4 The Point of Collapse

In this section, we determine the critical α , α_{cr} , based on the supply and demand for interbank funds for each realization of α . α_{cr} is the realization of α where due to banks can borrow just enough assets to meet their additional settlement. If the realization of α is greater than α_{cr} , the payments system collapses since the payments system operator can no longer clear and settle payments by imposing additional settlement obligations and must unwind all the payments.

In the upper graph of Figure 2, I plot the realizations of α versus quantity of funds supplied and demanded. The demand curve is upward sloping because due to banks demand for interbank funds increases as α increases. The intercept of the α axis occurs at:

$$\alpha = \frac{(1 - \hat{\gamma}) A + E}{P - E} - 1$$

The supply curve's α intercept occurs at 1 and funds intercept occurs at P . The supply curve is downward sloping.

In the upper graph of figure 2, the intersection of the supply and demand occurs at point U . The point U corresponds to the maximum value of α that can be realized without systemic collapse, α_{cr} . The quantity of funds associated with U , labeled F_1 in the graph, is the maximum amount of funds that due to banks can afford to borrow.

In the lower graph of figure 2, I plot the demand and supply curves in terms of R_{ib} and the quantity of funds. In this graph, the demand curve is downward sloping since as R_{ib} increases due to banks can borrow less. The supply curve is flat with a value of $\underline{R_{ib}}$ for realizations of α less than α_{cr} . There are two values for R_{ib} that can be realized. If α is less than α_{cr} , the supply of funds exceeds the demand for funds. In this case, R_{ib} has the value $\underline{R_{ib}}$. The rate of return $\underline{R_{ib}}$ is the lowest rate of return that due from banks are willing to lend funds at.²³ At F_1 , due from banks are willing to lend these funds at a R_{ib} in the open vertical interval seen in lower graph of figure 2. The point R_{ib}^* is determined by the value of R_{ib} at the intersection of the demand curve and the open vertical interval of the supply curve.

3 The Central Bank

Can the central bank prevent a systemic collapse for realizations of α greater than α_{cr} ? The actions of the central bank are not considered in the due to banks' optimal holdings of liquid assets or the due from banks' lending decisions. Banks may alter their behavior if they feel that the central bank will intervene.²⁴ The purpose of this section is to show how the central bank can intervene to avert a systemic collapse. A central bank can provide liquidity in two ways. One method is to conduct open market purchases of securities. These open market purchases are reversed in period 3. The other method is to provide collateralized loans at the discount window. Discount window loans are paid

²³There is no loss in generality if there was a market clearing R_{ib} for each realization of α . The qualitative results are not affected as long as the market clearing R_{ib} is less than R_{ib}^* . In fact, the realized R_{ib} may be less than R for realizations of α less than α_{cr} .

²⁴Including the central bank's response in the due to bank's optimization problem would be an interesting extension of this model and is left for future research.

off in period 3.

The central bank can increase the supply of reserves through open market purchases. However, it cannot guarantee that illiquid banks will have access to the funds. The allocation of this liquidity is dependent upon the willingness of banks to lend funds to banks in need of liquidity. For the purposes of this paper, I assume that due from banks are willing to lend the excess reserves resulting from the central bank's open market purchase. In figure 3, the supply curve shifts to the right resulting from an open market purchase and the new intersection with the demand curve occurs at V . At V , the value of α , labeled α'_{cr} , is greater than α_{cr} . Looking at the lower graph in figure 3, we see that the demand curve crosses the supply curve at \underline{R}_{ib} . Open market purchases are effective in avoiding systemic collapse if the realized α is below α'_{cr} . By further increasing liquidity via open market purchases, the central bank cannot avert a systemic collapse for values greater than α'_{cr} because due from banks are not willing to lower R_{ib} below \underline{R}_{ib} and due to banks cannot afford to borrow more unless R_{ib} falls for realizations of α greater than α'_{cr} .

The second method that the central bank can use to avert a systemic collapse is providing discount window loans at a lower borrowing rate. Discount window loans are usually collateralized. However, the collateral used for these loans may be fairly risky. Although the central bank prefers relatively safe assets for collateral, it may not have a choice during financial crises. If the proper haircut is not given to the collateral, the central bank could face losses from discount window loans.

The effect of a lower discount rate can be seen in the lower graph of figure 4. The central bank provides a discount window loan at a lower rate of return, R_{ib}^{DW} . Now, due to banks can afford to borrow up to quantity of funds labeled F_3 as seen in the lower graph of figure 4. The value of α associated with the aggregate amount of funds needed for settlement equaling F_3 is the new α_{cr} , α'_{cr} , as seen in the upper graph of figure 4. Depending on the objectives of the central bank, α'_{cr} could be as high as 1. In this case, the central bank completely bails out the due to banks.²⁵

²⁵If the goal of the central bank is to completely bail out banks, the defaults in period 1 would not occur. The discount window operations discussed here are limited to the remaining banks.

4 Crisis Prevention

In this section, we discuss policy options available to the payments system operator that can limit systemic risk. The three parameters that we address are: A , P , and E . Payments system operators often place restrictions on these parameters. Let us consider, the effects of changes in each variable while keeping the others constant.

We would expect, as the total assets, A , of the due to banks increase, α_{cr} would increase. In other words, if total asset holdings of the banks were higher, the system could withstand a greater number of bank failures. Setting the supply and demand functions equal to each other yields:

$$\phi(\alpha)P + \alpha\phi(\alpha)P - \alpha P + \alpha E - P + E + (1 - \hat{\gamma})A = 0 \quad (24)$$

Note the value of α that satisfies this equation is α_{cr} . Implicitly differentiating equation (24) in α_{cr} with respect to A yields:

$$\frac{d\alpha_{cr}}{dA} = \frac{1 - \hat{\gamma}}{-(1 + \alpha_{cr})P \frac{d\phi}{d\alpha_{cr}}(\alpha_{cr}) + (1 - \phi(\alpha_{cr}))P - E} \quad (25)$$

I show that the denominator is positive in the technical appendix. Since the denominator is positive and the numerator is positive, $\frac{d\alpha_{cr}}{dA}$ is positive. By requiring a greater quantity of assets, the payments system operator can reduce systemic risk.

Another policy option is to impose net debit caps during the day as suggested by Humphrey. Net debit caps restrict the maximum value of P .²⁶ Implicitly differentiating equation (24) in α_{cr} with respect to P yields:

$$\frac{d\alpha_{cr}}{dP} = \frac{-(1 - \phi(\alpha_{cr}))(1 + \alpha_{cr})}{-(1 + \alpha_{cr})P \frac{d\phi}{d\alpha_{cr}} + (1 - \phi(\alpha_{cr}))P - E} \quad (26)$$

This derivative is negative since the numerator is negative and the denominator is positive as seen above. By placing net debit caps that are binding, the payments system

²⁶Another policy option to limit the value of P is to impose fees on the level of the net debit. A similar policy was implemented by the Federal Reserve for overdrafts on Fedwire and it was found to reduce daylight overdrafts. See Richards (1995) and Hancock and Wilcox (1995) for effects of debit caps and fees on intra-day overdrafts. Although these policies were implemented on a gross settlement system, similar policies could be implemented on net settlement systems.

operator can reduce systemic risk. However, net debit caps may lead to payments gridlock and increase the cost of participation.

As collateral requirements rise, the difference between P and E decreases. As this difference approaches zero, the additional settlement obligation of remaining banks approaches zero. A 100 percent collateral requirement on the net payment due would eliminate systemic risk, but would also decrease efficiency.²⁷ In this paper, we have focused on payments systems where the collateral requirement is relatively small compared to the net payment. However, we would expect small changes in E also to decrease systemic risk. Implicitly differentiating equation (24) in α_{cr} with respect to E yields:

$$\frac{d\alpha_{cr}}{dE} = \frac{1 + \alpha_{cr}}{-(1 + \alpha_{cr})P \frac{d\phi}{d\alpha_{cr}} + (1 - \phi(\alpha_{cr}))P - E} \quad (27)$$

This derivative is positive since the numerator is positive and we have the same denominator as above which is positive. Thus, increases in collateral requirements would also reduce systemic risk.

In Chakravorti (1995), I discuss differences risk-reducing measures between various payments systems. Some of these risk-reducing measures include collateral requirements and intra-day debit caps. Both these measures would reduce systemic risk as seen by the model presented. In reality, payments system operators differ on types of risk-reducing measures that they use.

5 Conclusion

The model determines the point at which multilateral netting payments systems that use partial unwinds or explicit loss-sharing arrangements collapse. Furthermore, the model provides a framework to evaluate payments system policy. We have seen an interbank funds market reduces the costs incurred by banks to use the payments system. The central bank plays an important role by providing funds to illiquid banks during a financial crisis.

In this paper, we have considered two mechanisms that the central bank can use to provide liquidity. We considered open market operations by the central bank as a means

²⁷I assume that the collateral maintains its value and can be converted to good funds quickly.

to prevent a systemic crisis. In this model, open market operations can be used to avert systemic collapse for a given number of bank defaults. The discount window can be used to help illiquid but solvent banks. However, the central bank faces a cost in providing loans below the market interbank funds rate and may face the risk of default. These additional costs must be weighed against the costs of systemic collapse. An interesting extension of this model would be to explicitly model the central bank's optimization problem.

This paper has provided a framework to compare the tradeoffs between technical efficiency of the payments system and the safety of such systems. Currently, I am working on extending the model in the following ways:

- Consider actions of the central bank in the objective functions of the banks,
- Solve the model for other distributions of α besides the uniform distribution,
- Compare the benefit of netting to the cost of systemic collapse,
- Consider explicit objective functions for the payments system operator and central bank.

The payments system operator and the central bank can eliminate settlement risk, but there is a cost of such a policy. Central bankers must decide their role in payments processing, especially whether the gains in netting outweigh the risk of systemic collapse. Coordination among central bankers in payments issues is vital, since a significant portion of a nation's payments processing is associated with international transactions.

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Technical Appendix

Let $g(\alpha_{cr})$ be defined as:

$$g(\alpha_{cr}) = -(1 + \alpha_{cr})P \frac{d\phi}{d\alpha_{cr}}(\alpha_{cr}) + (1 - \phi(\alpha_{cr}))P - E, \quad (28)$$

where $\alpha_{cr} \in [0, 1]$. We want to show that $g(\alpha_{cr}) > 0$.

If $g(\alpha_{cr})$ is a strictly decreasing function in α_{cr} and the value at $g(1) > 0$, then $g(\alpha_{cr}) > 0$ for any α_{cr} .

$$\frac{dg}{d\alpha_{cr}} = -2P \frac{d\phi}{d\alpha_{cr}} - P(1 + \alpha_{cr}) \frac{d^2\phi}{d\alpha_{cr}^2}. \quad (29)$$

Equality (29) is less than zero, when:

$$\frac{d^2\phi}{d\alpha_{cr}^2} > \frac{-2}{1 + \alpha} \frac{d\phi}{d\alpha_{cr}}. \quad (30)$$

We assumed that this holds in equation (22).

The value of $g(1)$ is:

$$g(1) = -2P \frac{d\phi}{d\alpha_{cr}}(1) + P - E. \quad (31)$$

The first term on the right hand side of equality (31) is positive, since $\frac{d\phi}{d\alpha_{cr}}$ is negative as seen above. We assumed that:

$$\frac{E}{P} < 1,$$

otherwise due to banks will optimize by holding zero of the liquid asset. Therefore, $g(1) > 0$. Thus, $g(\alpha) > 0$.

Figure 1: The Timeline

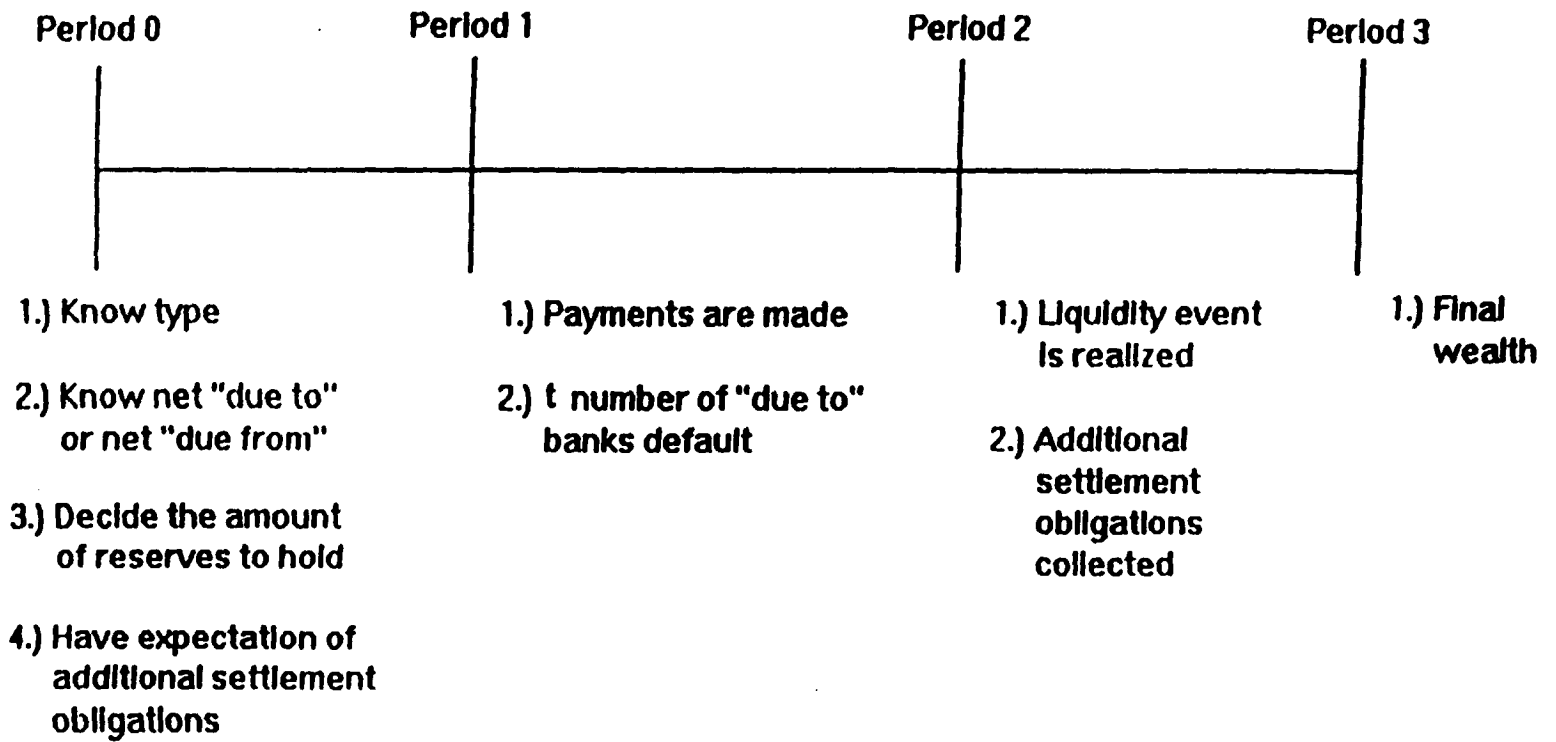


Figure 2: The Point of Collapse

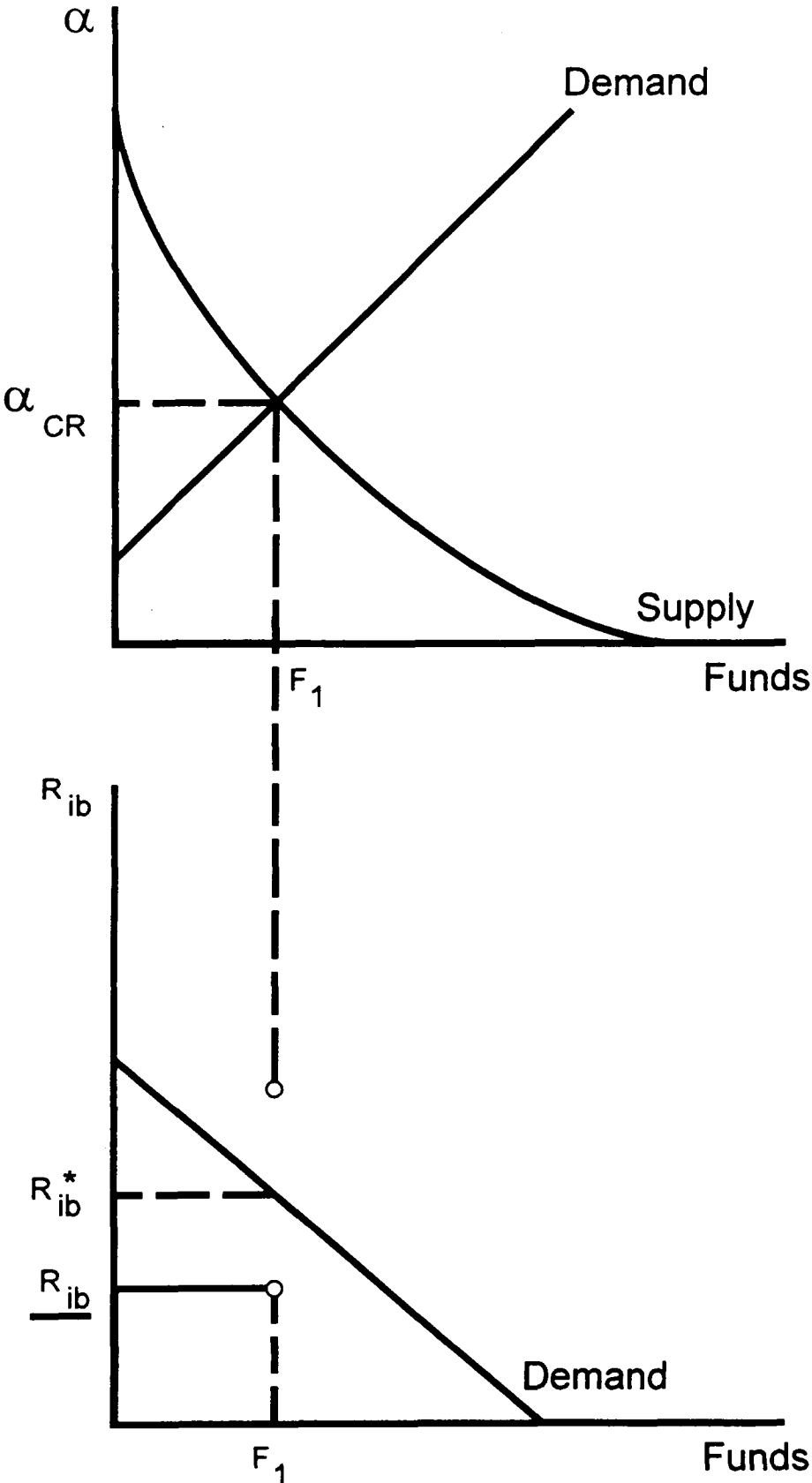


Figure 3: Open Market Operations

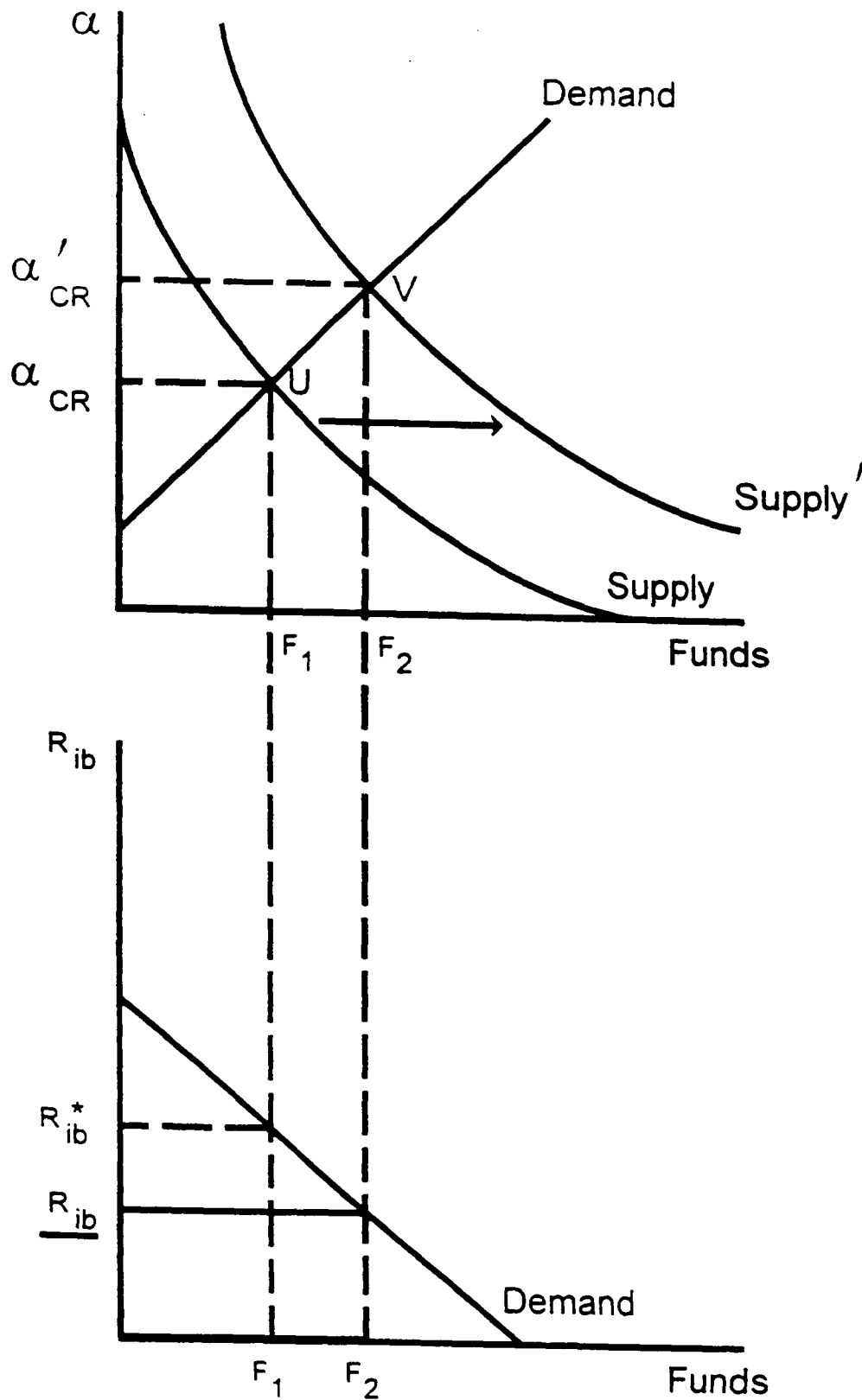


Figure 4: The Discount Window

