Competitive Viability in Banking:
Looking Beyond the Balance Sheet

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Determining the cost and revenue efficiency of financial intermediaries is important in a world economy characterized by sudden change and rapid innovation. Changing laws and regulations (e.g., such as interstate banking and branching, the expansion of banking powers, and intensified competition from nonbank financial institutions) coupled with improved telecommunications and computer technologies have raised questions concerning the competitive viability of different sizes and types of banks. Further, an often cited rationale for the ongoing consolidation of the U.S. banking industry through mergers and acquisitions, is the need to improve cost and revenue efficiencies by achieving greater scale and product mix economies. As argued in Berger, Hunter, and Timme (1993), increased efficiencies might be expected to result in improved profitability, more intermediated funds, better prices and service quality for consumers. Increased efficiency may also improve safety and soundness if some of the efficiency savings are applied toward improved capital buffers to absorb risk. But, as these authors note, the opposite effects might be expected if the evolution of the industry results in less efficient institutions.

The traditional role of U.S. commercial banks as financial intermediaries (i.e., collecting deposits and purchasing funds to be subsequently intermediated into loans and other assets) is widely viewed as a declining industry. Evidence used to support this assertion includes commercial banks' declining share of total U.S. financial intermediation assets and the massive consolidation of the commercial banking industry. Over the past ten years, commercial banks' share of total U.S. financial intermediation assets has fallen from 35 percent to 24 percent and the number of banks has dropped from approximately 14,500 to less than 10,000 today. These observations would appear to support this view of the industry.

This conventional view has been recently challenged. Those challenging it argue that significant portions of the business of banking have now moved beyond the balance sheet and as a result, industry output is being mismeasured. Boyd and Gertler (1994) and Kaufman and Mote (1994) note that off-balance sheet activities (hereafter OBS) such as loan origination, sales, servicing, and securitization, standby letters of credit, and derivative securities, are expanding rapidly. These activities must be considered in any evaluation of the competitive position of commercial banks. After augmenting aggregate measures of on balance sheet assets with estimates of aggregate OBS activities, these authors conclude that banking is not declining but, more accurately, changing.

Because of the changing nature of banking and the growth of OBS activities, measures that have traditionally been used to assess bank efficiency and competitive viability may no longer be accurate because they also do not adequately capture the rapid growth of OBS activities. Failing to include the expansion of banks into OBS activities has the potential to seriously understate actual bank output. The omission of OBS activities from recent empirical estimates of the relationships between bank size, product mix and efficiency has the potential to seriously bias these estimates.

The purpose of this study is to investigate the impact of OBS banking products and services on the measurement of efficiency and competitive viability in the banking industry. The paper utilizes both the thick frontier and the stochastic econometric frontier methodologies to estimate the parameters of production cost, economic cost and profit functions both with and without measures of OBS activities. The parameter estimates are then used to construct estimates of inefficiency, scale economies and competitive viability for a number of alternative size classifications.

The results of the paper indicate that including a measure of off-balance sheet activity in the cost and profit functions is supported statistically. However, the differences in the derived estimates of inefficiency, scale economies and expansion path subadditivity associated with addition of a measure of off-balance sheet activity are generally of small consequence qualitatively. A much more significant result is the sensitivity of these estimates to the frontier methodology employed to estimate the parameters of the cost and profit functions.

Section 1 of the paper provides an abbreviated literature review. The estimation methodologies, efficiency benchmarks and functional forms to be used are discussed in Sections 2 through 4. Sections 5 and 6 discuss the derivation of the two measures of OBS activity, describe the data to be used and provide variable definitions. Sections 7 through 10 present and discuss the measures of inefficiency, scale economies and expansion path subadditivity. The results are discussed in Section 11. The final Section provides a summary and conclusions.

1. A Brief Review of the Literature

1.1 Production Cost Efficiency

Over the past decade, substantial research has been conducted to determine whether production cost (in)efficiency is associated with the size and/or product mix of banks.[1] In general, this research indicates that the

[1] See Clark (1988), Humphrey (1990), Berger, Hunter and Timme (1993), and Clark (1996) for extensive

average production cost curve associated with traditional, on-balance sheet assets has a relatively flat U-shape, with some evidence of scale efficiencies for the smallest and largest banks. Minimum efficient scale appears to occur at a size of approximately $2 billion in total on-balance sheet assets.

Little consensus appears to exist on the presence of scope or product mix efficiencies. The widely divergent estimates of optimal production cost scope economies provide little support for global scope economies and evidence of joint production among product pairs is not robust. Several recent papers have attributed these results to a number of serious conceptual and empirical problems associated with the measurement of scope economies in banking.[2]

More recently, a number of papers have investigated the combined effects of increasing scale and changing product mix on production costs through the estimation and evaluation of the expansion path subadditivity measure developed by Berger, Hanweck and Humphrey (1987). This literature provides some evidence of limited subadditivity. However, the on-balance sheet asset size at which it is exhausted never extends beyond $10 billion and may be as small as $300 million.[3]

1.2 Revenue, Risk, Profitability and Bank Size

Production costs are an important consideration in any decision to expand scale or product mix. However, resource allocation decisions intended to maximize shareholder wealth should not be based only on production cost efficiencies. These decisions should also consider how performance attributes such as revenue, profitability and risk are related to bank size and product mix.

An extensive literature has developed on the relationship between bank size and profitability. In general this literature reports a positive relationship between the average rate of return on equity to bank assets up to a size of about $300 million.[4] Beyond this size the relationship is still positive, but is generally not statistically significant. In a recent extension of this literature, Berger, Hancock, and Humphrey (1993) use a profit function approach which allows for the

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reviews of this literature.

[2] See papers by Berger, Hunter and Timme (1993), Pulley and Humphrey (1993), Clark and Speaker (1994).

[3] Examples of this literature include Berger, Hanweck and Humphrey (1987), Hunter, Timme and Yang (1990), Clark and Speaker (1994), Clark (1996) and Mitchell and Onvural (1996).

[4] See Clark (1986).

measurement of inefficiencies on the output side as well as on the input side of the bank. Interestingly, they find that most of the profit inefficiencies are attributable to deficient output revenues rather than excessive input costs.

Several recent papers examine the relationship between bank size and risk. Papers by McAllister and McManus (1993) and Boyd and Runkle (1993) find that *ex post* measures of bank risk (e.g., the standard deviation of the rates of return on equity, the loan portfolio, and assets) decline with increases in total assets. Papers by Hughes and Mester (1993, 1994) and Hughes et al. (1994) examine the role of risk in the relationship between production costs and bank size by developing a model of manager utility maximization. These papers report evidence that supports a rejection of both risk neutrality and production cost minimization. Further, these papers generally find that (production cost) scale economies appear to increase with bank size, and that banks with lower quality assets tend to choose lower capitalization rates.

1.3 Economic Cost and Bank Size

Given the evidence of significant relationships between bank size and both profits and financial risk in addition to production cost, Clark (1996) derives a measure of economic cost. This measure incorporates a financial market based estimate of the opportunity cost to provide a basis for evaluating the composite of production and revenue (in)efficiencies as well as the role of systematic risk. Clark reports no evidence of statistically significant scale economies or subaddivity regardless of whether costs are measured by the economic or production cost metric. However the actual estimates indicate scale economies and subadditivity do exhibit different size-efficiency profiles. Clark reports that the use of the production cost metric tends to overstate (understate) scale economies (diseconomies) and subadditivity (superaddivity) in comparison with the economic cost metric.

1.4 X-Efficiency and Bank Size

The relationship between X-efficiency and size has been examined in several recent papers.[5] Many alternative methodologies have been used in evaluating X-efficiencies, including the stochastic econometric frontier approach, the thick frontier approach, data envelopment analysis, and the distribution-free approach. In a recent review of this work,

[5] Leibenstein (1966) coined the term "X-efficiency" to describe the resulting difference between actual and minimum cost. Thus, X-inefficiencies refer to deviations from the efficient (or best-practice) frontier, where the inefficiencies could result from allocative, technical, scale, and scope inefficiencies, and "can be the result of a deficiency of external, or market pressures from the firm's environment." [Gardner and Grace (1993)].

Berger, Hunter, and Timme (1993) report that the choice of measurement method appears to affect the level of the measured inefficiency. However, they do conclude that there is a preponderance of evidence that larger banks are more X-efficient (i.e., closer to the efficient frontier) than smaller banks.

1.5 Off-Balance Sheet Activity and Bank Efficiency

While a significant number of studies of efficiency and competitive viability in banking have been reported in recent years, only Jagtiani, Nathan, and Sick (1995) and Jagtiani and Khanthavit (1996) explicitly incorporate measures of OBS activities.[6] Jagtiani, Nathan and Sick include the nominal values of OBS foreign currency, interest rate and guarantee products, individually and in the aggregate along with deposits, and loans and investments. Their data covers the period 1988-90. They report that the overall volume of nominal OBS activities appears to have little to no impact on bank production costs for this time period. Jagtiani and Khanthavit carry out a similar study but replace the nominal values of OBS activities with the sum of the credit equivalent amounts of loan guarantees, interest rate and foreign currency swaps, options, and futures and forward contracts. They also extend the time period to include the years 1984-91. This extension allows them to include a period prior to the introduction of risk based capital requirements (RBCs). Their results indicate that the implementation of RBCs appeared to have little effect on the production cost efficiency of small banks, while increasing diseconomies of scale at money center and super-regional banks. However, Jagtiani and Khanthavit acknowledge that despite their evidence of diseconomies at the larger banks, these banks continued to expand both OBS activities and overall production. Thus they suggest that other factors, such as revenue efficiency, may be important.

Collectively, this literature suggests that bank resource allocation decisions have important implications for revenues, profits, and risk as well as for production costs. However, the extent of these effects appear to be at least somewhat sensitive to the performance metric and methodology utilized. Further, to the extent that OBS activities also affect costs, revenues, profits, and risk, and vary systematically with the size of the banking organization, failure to account for the volume of these services may distort estimates of production cost efficiency, economic cost efficiency, profit efficiency, and overall competitive viability. The assessment of the competitive viability of financial institutions must, therefore, look beyond the balance sheet when considering the effects of resource allocation decisions on risk and

[6] DeYoung (1994) includes fee-based income in a translog cost function.

return, as well as on explicit production costs.

2. *Estimation Methodology*

Beginning with Berger and Humphrey (1991), the bank efficiency literature provides substantial empirical evidence of significant production cost inefficiencies. Further, there is some evidence that the degree of inefficiency may be systematically related to bank size. As a result, the potential exists for traditional measures of efficiency to be biased by the presence of inefficiency in the data.

Given the evidence of a potential relationship between cost inefficiency and size, it is important to utilize an estimation methodology which is capable of controlling for the effects of any inefficiency that may be present in the data. Because there is no consensus concerning the appropriate methodology for this purpose and because previous research indicates some tendency for the estimates to be sensitive to the estimation methodology, both the stochastic econometric and thick frontier methodologies are utilized.[7]

2.1 *Stochastic Econometric Frontier*

Under the stochastic econometric frontier methodology the cost frontier is obtained by estimating a cost function with a composite error terms that is the sum of two separate component parts. The first component is a two-sided error term, v_i , representing random fluctuations or noise unrelated to inefficiencies. The second component is a one-sided error term, u_i , which represents inefficiency. Letting *ln* denote natural log, C_i denote the observed cost of bank i, y_i, w_i, and B_i represent vectors of output levels, input prices and cost function parameters for bank i, the stochastic frontier can be written generally as:

$$\ln C_i = \ln C(y_i, w_i, B_i) + u_i + v_i .^8$$

Assuming that v_i is normally distributed with a mean of zero and a variance of s_v^2 and u_i is distributed as a half-normal with a mean of zero and a variance of s_u^2, the log-likelihood function of the model can be written as:

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[7] See Berger and Humphrey (1997) for a recent review of the international literature addressing frontier efficiency at financial institutions.

[8] When a stochastic econometric profit function is to be estimated, the one sided error term carriers a negative sign indicating that profit inefficiency causes profits to fall below the frontier.

$$\ln L = \frac{N}{2}\ln\frac{2}{p} - N\ln s - \frac{1}{2s^2}\sum_{i=1}^{N} e_i^{\,2} + \sum_{i=1}^{N}\ln\left[\Phi\left(\frac{e_i\, l}{s}\right)\right].$$

N denotes the number of banks, $e_i = u_i + v_i$, $s^2 = s_u^2 + s_v^2$, $l = s_u / s_v$, and $\Phi(\cdot)$ denotes the standard normal cumulative distribution. The parameters of the cost function can then be estimated using maximum likelihood techniques.

2.2 Thick Frontier Methodology

The thick frontier methodology segments the sample into cost efficiency quartiles. Banks which fall in the lowest average cost quartile are assumed to be of highest average efficiency and form a thick frontier rather than an edge. Banks falling in the highest average cost quartile are regarded as being below average efficiency. Separate cost functions are estimated for both groups. Differences between the highest and lowest average cost quartiles are assumed to reflect inefficiencies and any exogenous differences in average outputs and input prices for each of the respective size categories. Differences in error terms within the respective cost quartiles are assumed to represent random error.

$$C_{Q_1 i} = C_1(y_{Q_1 i}, w_{Q_1 i}, B_{Q_1}) + v_{Q_1 i},$$

$$C_{Q_4 i} = C_4(y_{Q_4 i}, w_{Q_4 i}, B_{Q_4}) + v_{Q_4 i},$$

$$\% INEFF = [\hat{C}_4(y_{Q_4}, w_{Q_4}, \hat{B}_{Q_4}) - \hat{C}_1(y_{Q_1}, w_{Q_1}, \hat{B}_{Q_1})] / \hat{C}_1(y_{Q_1}, w_{Q_1}, \hat{B}_{Q_1}).$$

3. Efficiency Benchmarks

As recently discussed in Berger and Humphrey (1997), studies of bank efficiency issues should, ideally, rely on engineering information pertaining to technology used along the “best-practice frontier.” As the authors point out, engineering information on the technology used by financial institutions is largely unavailable. As a consequence, most empirical studies of bank efficiency focus on accounting measures of cost such as operating expenses, interest expenses, revenues and net income that are easily obtained from financial statement data. In this paper net income and production costs are used to investigate efficiency issues, consistent with much of the existing literature. In addition, an empirical measure of economic costs is derived and utilized.

Under classical economic theory, a firm is optimally or efficiently allocating its resources when there is no alternative reallocation capable of increasing shareholder wealth. Thus as Clark (1996) points out, the failure to minimize production costs given the selected levels of outputs, and the failure to minimize the opportunity cost of capital by failing to choose the optimal levels of outputs reduce both economic rent and thus shareholder wealth. Therefore, production cost efficiency is necessary but not sufficient to achieve economic cost efficiency. Thus, given the importance of both production cost efficiency and economic cost efficiency to shareholder wealth maximization, both cost metrics are derived and utilized as dependent variables.

In addition to the two cost metrics, a profit metric is utilized. As discussed by Berger, Hancock and Humphrey (1993) and Humphrey and Pulley (1996), the profit function approach to measuring efficiency captures both input and output inefficiencies. Inefficient input and output usage will be reflected in production costs and revenues and, therefore, in profits. However, the profit metric does have the shortcoming of not explicitly incorporating risk.

The profit metric employed here is defined as bank net income for the year, t, ending December 31st, the last day of the regulatory reporting period. This accounting measure of profitability is net of taxes, provision for loan loss and extraordinary items, and has been used in most previous profit efficiency studies.

In economic theory, costs, both explicit production costs and economic costs, are dependent upon input prices and the level and composition of output. Following convention, the explicit production costs are found by summing actual expenditures made on the inputs purchased in producing the output levels selected by the bank:

$$PC_{bt} = \sum_j w_{jbt} * q_{jbt},$$

where w_{jbt} and q_{jbt} are the prices and quantities, respectively, of the j inputs used by bank b at time t. Consistent with the empirical banking literature, explicit production costs are defined here as the sum of the bank's operating and interest expenses.

3.1 Measuring Economic Costs

Following Clark (1996), economic costs are defined as the sum of the explicit costs of production and the opportunity cost of capital:

$$EC = \sum_t \sum_i \frac{(PC_{it} + OC_{it})}{(1+k_i)^t} = \sum_t \frac{(PC + EC)_t}{(1+k)^t},$$

where $(PC + OC)_t$ is the sum of production and opportunity costs for all i projects at time t, and k is the risk-free interest rate. Thus, determining economic costs is complicated by the need to measure and include the opportunity cost of capital.

A bank's opportunity cost of capital at time t can be approximated, in percentage terms, using the capital asset pricing model:

$$r_{bt} = r_{ft} + b_{bm}(r_{mt} - r_{ft}) + b_{bi}(r_{it} - r_{ft}),$$

where r_b denotes the opportunity cost of capital for bank b, r_f is the risk-free interest rate, b_{bm} is the sensitivity of the bank's stock returns to overall stock market movements, $\hat{a}_{bi}$ measures the sensitivity of the bank's stock returns to overall interest rate movements. The variable r_i is the rate of return on constant maturity, ten-year Treasury bond index, while the variable r_m is the rate of return on the market portfolio, both for the selected time period *t*.[9] Using this formulation, the expected return necessary to compensate an investor for the risk of holding a given security is the risk-free rate of interest plus risk premiums for the market risks associated with the security. In dollar terms, a bank's opportunity cost of capital is approximated by multiplying the bank's opportunity cost of capital, r_{bt}, by the market value of the bank's equity at the end of the previous time period, MVE_{bt-1}:

$$OC_{bt} = r_{bt} * MVE_{bt-1} = [\, r_{ft} + b_{bm}(r_{mt} - r_{ft}) + b_{bi}(r_{it} - r_{ft})\,] \cdot MVE_{bt-1}.$$

Total economic costs are then approximated by summing the explicit production costs and the opportunity cost of capital over the given time period for each bank as follows:

$$EC_{bt} = PC_{bt} + OC_{bt} = (\sum_j w_{jbt} * q_{jbt}) + ([\, r_{ft} + b_{bm}(r_{mt} - r_{ft}) + b_{bi}(r_{bi} - r_{ft})\,] \cdot MVE_{bt-1}).$$

As Clark (1996) shows, this cost metric is the more broad and relevant cost concept for measuring efficiency and competitive viability because it includes an empirical valuation of opportunity costs associated with both optimal input and optimal output levels.

4. Functional Forms

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[9] A bank's beta (β_t) is defined as the ratio of the covariance of the returns on the bank's security with the returns on the market portfolio, to the variance of the returns on the market portfolio, σ_{im}/σ_m. Estimates of $\hat{a}_{bm}$ and $\hat{a}_{bi}$ are obtained from the estimation of a two factor market model in which daily return data for the bank's stock is regressed on the daily returns on the CRSP stock market index and the daily returns on a constant maturity, ten-year, Treasury bond index.

The estimation of economic and production cost, and profit efficiencies first requires the estimation of the respective efficiency frontiers. This is accomplished by using both the stochastic econometric and thick frontier econometric approaches discussed above. In implementing these estimation methodologies it necessary to specify the functional form of the cost and profit functions.

4.1 General Composite Functional Form

In implementing the thick frontier methodology, general composite statistical cost and profit functions of the form developed by Pulley and Braunstein (1992), Pulley and Humphrey (1993) and Humphrey and Pulley (1997) are utilized. The cost functions have the following form:

$$TC^q = [[B_0 + \sum_{i=1}^{n} B_i y_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} y_i y_j + \sum_{i=1}^{n}\sum_{k=1}^{m} D_{ik} y_i \ln w_k] * \exp[\sum_{k=1}^{m} E_k \ln w_k + \frac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m} F_{kl} \ln w_k \ln w_l]]^q + e$$

where ln denotes the natural logarithm; y_i denotes the output of the *i*th product; w_k denotes the *k*th input price; B_0, B_i, C_{ij}, D_{ik}, E_k, and F_{kl} are all parameters to be estimated; e represents the random

error term; and q denotes the Box-Cox transformation defined by:

$$z^q = \frac{(z^q - 1)}{q} \quad for\ q \neq 0 \quad .$$
$$z^q = \ln z \quad for\ q = 0.$$

Unlike the papers by Jagtiani and Khanthavit (1996) and Jagtiani, Nathan, and Sick (1995), this general composite functional form is used because several papers, including McAllister and McManus (1993), Pulley and Humphrey (1993), and Clark and Speaker (1994), have shown that the translog functional form used in much of the early literature may provide a less accurate approximation when applied to bank sizes and product mixes that are not near their respective means.[10]

Linear homogeneity of input prices requires that $S_k E_k = 1$, $S_k F_{kl} = 0$, and $S_k D_{ik} = 0$, while symmetry requires that $C_{ij} = C_{ji}$ and $F_{kl} = F_{lk}$.[11] The cost function is estimated jointly with n factor share equations and φ using nonlinear-

[10] Berger and Humphrey (1997) also note that there is some evidence that the use of the translog functional form forces the average cost curve to have a U-shape in logs.

[11] These restrictions are necessary to obtain convergence.

least squares regression methods described by Carroll and Ruppert (1988) and applied by Pulley and Braunstein (1992), with both production and economic cost metrics utilized as dependent variables.

The general composite functional form is also applied to the profit function. Following Humphrey and Pulley (1997), an indirect profit function is adopted:

$$NI^q = [[b_0 + z_i\, MKTBK + \sum_{i=1}^{n} b_i\, y_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, y_i\, y_j + \sum_{i=1}^{n}\sum_{k=1}^{m} d_{ik}\, y_i \ln w_k\,] * \exp[\sum_{k=1}^{m} e_k \ln w_k + \frac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m} e_{kl} \ln w_k \ln w_l\,]\,]^q + e$$

Again, ln denotes the natural logarithm; y_i denotes the output of the *i*th product; w_k denotes the *k*th input price; b_0, b_i, c_{ij}, d_{ik}, e_k, and f_{kl} and z are all parameters to be estimated; ***e*** represents the random error term; and ***q*** denotes the Box-Cox transformation. The variable MKTBK represents the ratio of the market value of bank equity to its book value.[12]

It is important to note that this profit functional form is 'non-standard.' In the standard or neoclassical profit function, profits are assumed to be a function of both output and input prices. Thus in essence firms are treated as price takers in both the output and input markets. As has been argued by Humphrey and Pulley (1997) and DeYoung, Spong and Sullivan (1997), this framework is problematic, since previous empirical evidence suggests that banks have price setting ability in many output markets.[13] Further, financial statement data do not provide sufficient detail to allow the construction of price proxies for the kinds of fee generating activity that generally comprise OBS activities.

The non-standard profit function assumes that banks purchase inputs in competitive markets but have some ability to engage in price setting in output markets. Thus profit is functionally related to input prices and output quantities. An immediate consequence of this treatment is that the standard profit function restrictions of input and output price homogeneity of degree one no longer hold.

4.2 Translog Functional Form

The general composite functional form is employed in the implementation of the thick frontier methodology.

[12] This variable is included to capture the effects of other factors such market structure and regulatory differences.

[13] See Berger and Udell (1995).

An initial attempt was made to use this same functional form in the stochastic econometric frontier methodology. Repeated attempts to estimate the parameters of this functional form using this methodology were unsuccessful. As a result, the translog functional form used in previous implementations of the stochastic econometric frontier is adopted for both the cost function:

$$\ln C = B_0 + \sum_{i=1}^{n} B_i \ln y_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} \ln y_i \ln y_j + \sum_{i=1}^{n}\sum_{k=1}^{m} D_{ik} \ln y_i \ln w_k + \sum_{k=1}^{m} E_k \ln w_k + \frac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m} F_{kl} \ln w_k \ln w_l + e,$$

and the profit function:

$$\ln NI = b_0 + z_i \ln MKTBK + \sum_{i=1}^{n} b_i \ln y_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \ln y_i \ln y_j + \sum_{i=1}^{n}\sum_{k=1}^{m} d_{ik} \ln y_i \ln w_k + \sum_{k=1}^{m} e_k \ln w_k + \frac{1}{2}\sum_{k=1}^{m}\sum_{l=1}^{m} e_{kl} \ln w_k \ln w_l + e.$$

Consistent with the general composite cost function discussed above and with convention, the standard linear homogeneity and symmetry restrictions are employed. The profit function is again non-standard such that the restrictions implied by input and output price homogeneity are not imposed.

5. *Measuring OBS Activities*

The problem of measuring bank output has plagued banking scholars for decades. Service industries in general have serious conceptual and practical problems in measuring output because of the intangible nature of their output. For the banking industry, this problem has intensified in recent years as many banks have moved more and more financial activities off the balance sheet. As a result, traditional measures of on-balance sheet outputs that have been used to evaluate banking performance and efficiency fail to capture the new business of banking that is occurring off the balance sheet.

Boyd and Gertler (1994), Kaufman and Mote (1994), and Edwards and Mishkin (1995) all argue that traditional measures used to evaluate banking are erroneous because they do not reflect the growth of newer, fee-based financial services. As noted above, when industry asset figures are adjusted to incorporate a measure of the OBS activities, these authors find evidence to reject the perception that banking has declined.

Two alternative measures of a bank's aggregate off-balance sheet activities are constructed. The first measure, *OBSCEA*, is the total credit equivalent amount of OBS transactions. This measure is constructed following the guidelines set out by the Basle Committee. The second measure, *BGEST*, is an aggregate measure of adjusted assets which utilizes the rate of return on balance sheet assets to capitalize noninterest income from OBS activities developed by Boyd and Gertler (1994).

5.1 Credit Equivalent Assets

A by-product of the Basle Committee's work on implementing risk based capital requirements is a simple, categorical method for constructing credit equivalent or on-balance sheet proxies for OBS activities. The aggregate *OBSCEA* measure includes all OBS activities from standby letters of credit to derivative contracts. It is comprehensive in that all categories of OBS activities, including recent innovations, are converted to credit risk equivalents by multiplying the nominal principal amounts by a credit conversion factor.[14]

All banks are required to file Call Reports with their respective regulatory agencies. In the Call Reports OBS activities must be allocated into four broad risk categories which carry the following conversion factors:

100% Conversion Factor: Activities which substitute for loans (e.g., general guarantees of indebtedness, bank acceptance guarantees, and standby letters of credit serving as financial guarantees for loans and securities), sale and repurchase agreements and assets sold with recourse (if not already included on the balance sheet), and interest-rate and exchange-rate related derivatives (e.g., swaps, options, futures, and forwards).

50% Conversion Factor: Certain transaction-related contingencies (e.g., performance bonds, bid bonds, warrants and standby letters of credit relating to particular transactions), unused commitments with an original maturity exceeding one year, and revolving underwriting facilities (RUFs) and note issuance facilities (NIFs).

20% Conversion Factor: Short-term, self-liquidating, trade-related contingent liabilities arising from the movement of goods (e.g., documentary credits collateralized by the underlying shipments).

0% Conversion Factor: Unused commitments with an original maturity of one year or less or which are unconditionally cancelable at any time.

The aggregate of the credit equivalent amounts in each category provides a proxy for the on-balance sheet equivalent of a banks' OBS activities.

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[14] As Saunders (1997) notes, the conversion factor for each OBS activity could, potentially, be calculated from a contingent asset valuation or options model. However, this was not the approach taken by the Basle Committee.

The Basle credit equivalent measure may seriously understate the level of OBS activities. The intent of the measure is to approximate the amount of on-balance sheet assets that would result in comparable relative risk exposures to the bank, not as a quantitative measure of their volume or ability to generate income or costs. Further, Boyd and Gertler (1994) suggest two additional reasons for the tendency of the credit equivalent measures to understate the levels of these OBS activities. First, the construction of the OBSCEA measure excludes any activities which the Basle Committee felt did not result in significant risk exposure. Significant revenue producing activities such as loan sales without recourse, loan servicing, consulting and trust services are assigned the zero conversion factor and would, therefore, not be incorporated into the OBSCEA measure. Second, the classification scheme arbitrarily assigns very similar activities to different conversion factor categories.[15]

5.2 Noninterest Income Capitalization Credit Equivalent Measure

Given the potential understatement of OBS activities inherent in the Basle credit equivalent measure, Boyd and Gertler (1994) propose an alternative. Their alternative measure emphasizes the ability of OBS activities to contribute to profitability. Thus it implicitly incorporates both the revenue and cost attributes of these activities.

Boyd and Gertler's objective in deriving their measure is to compute the on-balance sheet assets that would be required to generate a bank's level of non-interest income. As they note, the method "boils down to using the rate of return on on-balance sheet assets to capitalize non-interest income."[16]

The algorithm proposed by Boyd and Gertler begins by defining accounting profits to be equal to:

$$p = I - E - LP - NE + NII,$$

where NII is total noninterest income, I is total interest income, E is total interest expense, NE is total non-interest expense, and LP is loan loss provision. They then make several critical, simplifying assumptions. First, they assume that non-interest income, NII, is generated by hypothetical off-balance sheet assets A_0. Second, they assume that hypothetical assets A_0 are identical to on balance sheet assets, including the mix of liabilities and capital used to fund

[15] Boyd and Gertler provide the example of loan commitments with maturities of less than one year being assigned to the 0% conversion factor category and thus arbitrarily excluded. In contrast, loan commitments with maturities of one year or more receive non-zero conversion factors and are, therefore reflected in the credit equivalent measure.

[16] See Boyd and Gertler (1994), p. 7.

them. Finally, they assume that OBS assets, A_0, and on-balance sheet assets, A_b, are equally profitable. Given the assumed symmetry between on and off-balance sheet assets, Boyd and Gertler show that a estimate of A_0 denoted here as BGEST, can be derived as follows:

$$BGEST = A_0 = A_b[NII / (I - E - LP)] .$$

This measure can be viewed as the hypothetical asset holdings that would be needed to generate the noninterest income stream and has the advantage that the values for A_b, *NII, I, E,* and *LP* necessary to compute it are readily available from Call Report data.[17]

6. The Data

The data used to construct the appropriate efficiency measures and to estimate the cost function and profit function parameters are assembled from three sources. *Bank Compustat* provides annual income and balance sheet data, as well as some market data, for a number of publicly-traded, U.S. commercial banking organizations. The *Y9 Report of Condition and Income* data are collected by the regulatory authorities and contain annual income and balance sheet data as well as OBS transactions. Bank data from *Bank Compustat* and the *Y9 Report of Condition and Income* were matched with stock return data maintained on the *Center for Research in Security Prices* (CRSP) database. Daily return data were collected from 1992-94, with the matching process yielding complete data for 213 banks in 1992, 227 banks in 1993, and 260 banks in 1994, producing a total sample of 603 observations.[18]

6.1 Variable Descriptions

Following Clark (1996), the choice of bank "products" to be included in the cost function is based upon the concept of value added. Those banking functions requiring significant expenditures on non-monetary inputs such as labor and physical capital to produce non-interest banking services are identified as outputs. Using this approach, the

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[17] One obvious shortcoming of this measure is the underlying assumption of symmetry between on and off-balance sheet assets. However, it is important to recall that microeconomic theory suggests that a firm is selling output capacity. As such, the firm should add products to its product mix as long as the marginal revenue from the product is greater than or equal to the marginal costs associated with the use of the productive capacity necessary to produce the product. Since empirical evidence indicates that average production costs are fairly constant over a large range of output and assuming reasonably competitive output markets, profit margins should not differ significantly across product lines. Thus an assumption of equal rates of return for on and off-balance sheet activities may not be unreasonable.

[18] Banks with incomplete data or for which poor estimates of the market model parameters produced negative costs of capital were deleted from the final sample.

following four output measures are constructed: commercial and industrial loans (CIL); consumer and real estate loans (CRL); and core deposits (CORE). The balance sheet items included in each output definition are:

CIL = commercial loans, direct leasing, foreign loans, and loans to financial institutions;

CRL = consumer and real estate loans;

CORE = demand deposits, savings deposits and retail time deposits.

Support for these definitions is provided by the Federal Reserve's Functional Cost Analysis program. Based upon these data, Berger and Humphrey (1992) indicate that core deposits absorb approximately 48 percent of bank value added. Real estate loans (4 percent), commercial and industrial loans (14 percent) and installment loans (12 percent) absorbed an additional 30 percent of value added.

There is some disagreement concerning the treatment of core deposits as an output. Much of the disagreement can be attributed to the fact that many bank services are not explicitly priced.[19] Berger and Humphrey (1992) estimate that the implicit revenues for all banks in the U.S. account for approximately 82 percent of total deposit revenues. Approximately two-thirds of these implicit revenues are generated by demand deposits and one-third by time and savings deposits. Thus if all deposit services are explicitly priced, core deposits would produce substantial service output.

Inputs in the production process are assumed to include labor, premises and equipment, and funds. Input price proxies for these inputs are constructed as follows:

w_L = salaries and benefits divided by the number of full-time-equivalent employees;

w_p = total expenses for furniture, equipment and bank premises divided by the book value of bank premises, furniture, equipment and fixtures; and

w_F = weighted average cost of funds.

When the production cost metric is used, the weighted average cost of funds is defined as:

$$w_{Fpc} = S_t (D_t / TF_{pc}) \cdot i_t + S_s (PF_s / TF_{pc}) \cdot i_s$$

[19] This is particularly true for core deposits, which typically do not produce net revenues because of a barter arrangement between the bank and their depositors.

where D_t represents funds of deposit type t, BF_s represents non-deposit, borrowed funds of type s, i_t and i_s denote the respective interest rates paid on each type of deposit and borrowed funds, and $TF_{pc}=S_tD_t+S_sBF_s$. When the economic cost metric is used, the weighted average cost of funds is defined as:

$$w_{Fec}=S_t(D_t\,/TF_{ec})^{\chi}i_t+S_s(PF_s\,/TF_{ec})^{\chi}\,i_s+(E/TF_{ec})^{\chi}r,$$

where r denotes the cost of capital, E denotes the market value of equity and $TF_{ec}=S_tD_t+S_sBF_s+E$.[20]

7. Measuring Inefficiency.

In addition to scale economies and competitive viability, measures of production cost, economic cost, and profit inefficiency are constructed to evaluate X-inefficiency (the relationship between inefficiency and size). Inefficiency is measured using both the stochastic econometric and thick frontier cost and profit functions.

7.1 Measured Inefficiency Using the Stochastic Econometric Frontier

The bank level measure of inefficiency typically used for the case of a normal-half-normal stochastic frontier is the mean of the conditional distribution of u_i given e_i. The mean of this conditional distribution has been shown to be:

$$E(u_i|e_i)=\left(\frac{s_u s_v}{s}\right)\left[\frac{f(\frac{e_i l_c}{s})}{\Phi(\frac{e_i l_c}{s})}+\frac{e_i l_c}{s}\right]$$

and

$$E(u_i|e_i)=\left(\frac{s_u s_v}{s}\right)\left[\frac{f(\frac{e_i l_P}{s})}{\Phi(\frac{e_i l_P}{s})}-\frac{e_i l_P}{s}\right]$$

where C denotes cost and P denotes profit. The difference in sign represents the fact that cost inefficient banks lie above and profit inefficient banks lie below the respective frontiers.

7.2 Measured Inefficiency Using the Thick Frontier

Inefficiency is also measured following the method proposed in Berger and Humphrey (1991). In their approach, *Diff*, the proportionate differences between the estimated costs or profits generated for the frontier (high

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[20]This approach is utilized because it is consistent with a pooled cost of funds approach employed by many banks; it eliminates a source of severe multicollinearity; and it significantly reduces the number of parameters that must be estimated.

efficiency) sub-samples and the costs or profits produced by the least efficient sub-samples measures inefficiency:

$$Diff\,(PC) = (APC^{Q_4} - APC^{Q_1}) \big/ APC^{Q_1} \;,$$

$$Diff\,(EC) = (AEC^{Q_4} - AEC^{Q_1}) \big/ AEC^{Q_1} \quad \text{and}$$

$$Diff\,(PFT) = (ROE^{Q_1} - ROE^{Q_4}) \big/ ROE^{Q_1} \;,$$

where $APC^{\phi} = C^{\phi}(Y^{\phi})/TA^{\phi}$, $AEC^{\phi} = C^{\phi}(Y^{\phi})/TA^{\phi}$ and $ROE^{\phi} = NI^{\phi}(Y^{\phi})/EQ^{\phi}$ are the predicted average production costs, predicted average economic costs and predicted rate of return on equity, repectively, C^{ϕ} , (NI^{ϕ}) is the predicted cost (profit) function using the parameter estimates obtained from the sub-samples when $\phi = Q_4$ and the thick frontier (lowest cost or highest profit quartile) sub-samples when $\phi = Q_1$. Y^{ϕ} indicates the vector of mean outputs and input prices, TA^{ϕ} is the mean total assets and EQ^{ϕ} is the mean book value of equity for the respective $\phi = Q_4$ and $\phi = Q_1$ size classes. Hence, *Diff* is the proportional increase in predicted unit costs or reduction in profit of the inefficient quartile relative to the thick frontier evaluated at the variable means of each of a number of alternative size classifications.

8. Measuring Scale Efficiency

Overall scale efficiency is assessed using the degree of scale economies measure described in Baumol, Panzar and Willig (1982):

$$SCALE = \frac{C(Y;w)}{\sum_i y_i \cdot \frac{\partial C(Y;w)}{\partial y_i}}$$

where *C()* denotes the cost function and ***Y*** and ***w*** are vectors of outputs and input prices, respectively. When *SCALE* > 1 (< 1), total costs rise less (more) proportionately with increases in size and there are overall economies (diseconomies) of scale.

9. Measuring Competitive Viability

To measure the competitive viability of banks in alternative size classifications, the expansion path subadditivity measure is computed as described in Berger, Hanweck, and Humphrey (1987):

$$EPSUB_C(Q_L) = \frac{[C(Q_A) + C(Q_B) - C(Q_L)]}{C(Q_L)}$$

where $C()$ denotes the cost of producing the subscripted output bundle Q_L of large bank L, by small bank A and hypothetical bank B, with output bundles Q_A and $Q_B = Q_L - Q_A$. Expansion path subadditivity provides what appears to be a more reasonable representation of the opportunity of existing banking firms to change their outputs than does the scope economies measure.[21] If $EPSUB_C > 0$ (<0), then output bundle Q_L can be produced more (less) efficiently by large bank L than by smaller banks A and B, and large bank L is competitively viable.

10. Measuring Profit Efficiency

Using the estimated parameters from the profit function, computed measures of scale efficiency and competitive viability are derived in a manner similar to those described in the previous section. However, rather than the scale economies measure, profit elasticity is computed. Profit elasticity measures the percent change in net income that accompanies a one-percent change in all outputs. In addition, a profit EPSUB measure, labeled $EPSUB_P$, is constructed similar to the cost $EPSUB_C$ measure discussed above. The net income associated with the means of the larger quartile is compared to the net income associated with the means of the smaller quartile and a hypothetical bank representing the difference, so that:

$$EPSUB_P(Q_L) \;=\; \frac{[P(Q_S) + P(Q_H) - P(Q_L)]}{P(Q_L)}$$

where $P()$ denotes the profit opportunities of producing the output bundle with the input bundle Q_L of large bank L, by small bank S and hypothetical bank H, with Q_L and $Q_H = Q_L - Q_S$. Thus, when $EPSUB_P < 0$ (>0), Q_L can be produced more (less) efficiently by large bank L, than by smaller banks S and H, and large bank L is competitively viable.

11. Results

Before discussing the derived estimates of inefficiency, scale economies and sub-super-additivity, the statistical importance of including the alternative OBS measures in the production cost, economic cost and the profit functions is examined. The exclusion of an OBS measure from the respective functions is equivalent to restricting to zero the parameter estimates on those terms which include this measure. Thus the restricted (hereafter 3Q) models are nested within the unrestricted (hereafter 4Q) models of the same functional form. Statistical tests of the restrictions implied by

[21] See Berger, Hanweck and Humphrey (1987), Hunter, Timme and Yang (1990), Berger and Humphrey (1991), and Hunter and Timme (1991) for more on expansion path subadditivity.

the 3Q models can then be carried out using a likely ratio test of the following form:

$$2[\log(F_{UR}) - \log(F_R)] \sim c_v^2,$$

where *logL*(F_R) and *logL*(F_{UR}) are the maximum values of the restricted and unrestricted likelihood functions and *v* denotes both the number of restrictions and the number of degrees of freedom.

The logs of the respective likelihood functions, the likelihood ratio test statistics, and their associated p-values are presented in Table 1. Inspection of this Table indicates that in every case the null hypothesis that a measure of OBS activity has no explanatory power in explaining interbank differences in production costs, economic costs or profit, can be rejected at the $a = 0.01$ level of significance. Thus from a statistical standpoint, it appears that a measure of OBS activity, such OBSCEA or BGEST, should be included in cost and profit functions.

11.1 Inefficiency

The derived inefficiency measures for both the thick frontier (hereafter TFM) and stochastic econometric frontier methodologies (hereafter SFM) are presented in Table 2. For each of the two methodologies, inefficiency estimates are evaluated for the production costs, economic costs, and profit metrics, with and without the alternative OBS measures. Given the strong statistical rejection of the null hypothesis that OBS activity does not help to explain interbank differences in costs or profits, it is also important to evaluate the differences in SCALE and EPSUB produced by the restricted and unrestricted models to assess the economic importance of including a measure of OBS activities.

Inspection of the inefficiency tables indicates that probably the most striking aspect is the diversity of the results. Consider first the efficiency results obtained from the SFM (Part 2 of the Table). These results indicate that across the seven size classifications, inefficiency appears to run between ten and thirty percent for both economic and production costs. Inefficiency appears to be slightly higher when the credit equivalent measure of OBS activity, OBSCEA, is used. And, there does not appear to be a systematic relationship between asset size and inefficiency. In contrast, profit inefficiency is substantially higher, generally running between sixty and ninety percent and is generally higher when the income capitalization measure of OBS activity, BGEST, is used. Further, profit efficiency appears to decline with increases in asset size up to approximately five billion dollars before rising again in the largest two size categories.

Inefficiency results obtained from the application of the TFM (Part 1 of Table 1) differ markedly from their

SFM counterparts. While one would expect to observe numerical differences in the estimated values for inefficiency between the two methodologies, many of the relationships appearing in the SFM estimates are reversed in the TFM results. When the TFM is used, economic cost inefficiency appears to be much higher than either production cost or profit inefficiency, running between forty and eighty percent. Profit inefficiency and production cost inefficiency appear to be significantly smaller through approximately $2 billion of assets, generally running between ten and twenty-five percent. However, beyond a size of approximately $5 billion of assets, profit inefficiency drops to less than five percent and substantially below production cost inefficiency, which continues to be near twenty percent. The tendency for inefficiency estimates to be higher when the credit equivalent measure is used to proxy OBS activity identified with SFM, no longer appears to hold with the TFM. However, inefficiency estimates do increase significantly for those institutions larger than $25 billion of assets for all three metrics when BGEST is used while falling in two of the three instances in which OBSCEA is used. Finally there does appear to be a systematic though modest decline in profit inefficiency up to a size of between $5 and $10 billion of assets apparent with both OBS measures and both frontier methodologies. No such systematic relationship between size and inefficiency is apparent for the other performance metrics.

11.2 Scale Economies

Table 3, Parts 1 and 2 provides estimates of production and economic cost scale economies and profit elasticities for seven asset size ranges, both OBS proxies and both frontier methodologies. Inspection of this Table indicates that to the extent that scale economies or diseconomies associated with both cost metrics are found, they appear to be very slight. Using the TFM (Part 1 of the Table), estimates of SCALE>1 can be found for production costs at size levels as high as $25 billion of assets. However these estimates are generally not statistically significant.[22] At asset size levels of $2 billion or less, when the SFM approach (Part 2 of the Table) is utilized, statistically significant scale economies are found for the production cost metric. Statistically significant scale diseconomies appear at approximately $5 billion of assets. When the economic cost metric is utilized, no statistically significant scale economies are found at any size level, while statistically significant scale diseconomies appear at asset sizes as low as $1 billion. In general,

[22] An exception occurs when the credit equivalent measure, OBSCEA, is included in the production cost function. SCALE is found to be statistically significant at the 5 and 10 percent levels for asset sizes between $5 and $25 billion.

once scale diseconomies appear for either cost metric, they increase in size with total assets.

The estimates of SCALE appear to be little affected by the introduction of either OBS proxy up to an asset size of between $2 and $5 billion. Beyond this size, somewhat larger differences can be found when an OBS proxy is included and the TFM is employed. The exclusion of an OBS measure from the cost functions, under the TFM, does appear to result in larger scale economies between $5 and $25 billion of assets, before falling significantly below one for banking organizations above $25 billion. When the SFM is used, SCALE estimates are qualitatively and quantitatively very similar across all size categories.

The estimated profit elasticities provide yet a different perspective. As Part 1: Panels E and F of Table 3 indicate, when the TFM is used profit elasticity is not statistically different from one up to a value of $2 billion of assets. Thus in this broad size range there are neither profit economies nor diseconomies. This result holds regardless of whether a 3Q or 4Q model is estimated and regardless of which OBS proxy is employed in any 4Q model. Differences do, however, appear at asset sizes above $5 billion. When the credit equivalent proxy, OBSCEA, is used, the profit-scale elasticity drops below one and becomes statistically significant from $2 billion through $25 billion of assets. This result would suggest potential profit diseconomies along the thick frontier as bank asset size increases. In contrast, when the income capitalization proxy, BGEST, is used there continue to be no statistically significant profit economies or diseconomies, except in the 4Q model when asset size ranges between $2 and $5 billion.

When the SFM is employed the profit-scale elasticities are less than one and statistically significant at the five percent level up to an asset size of between $2 and $5 billion. Beyond this size, there are no statistically significant profit economies available regardless of whether or which OBS proxy is used.

11.3 Expansion Path Sub-Additivitity

Since the output mix of banks generally changes with increases in asset size, it is important to consider the efficiency impact of these changes as well as changes in scale. As noted in a previous section, the expansion path sub-additivity measure is evaluated for that purpose. Table 4 provides the estimates of $EPSUB_c$ and $EPSUB_p$ for the same seven size classifications, derived using both the TFM and SFM estimation methodologies and all three performance metrics.

Part 1 of Table 4 provides the estimates of $EPSUB_c$ and $EPSUB_p$ obtained using the parameters from the best practice frontier under the TFM. In general, these results provide no strong quantitative or statistical evidence supporting sub-additivity. This result holds fairly consistently across both 3Q and 4Q models, both definitions of OBS activity and all three performance metrics. When OBSCEA is employed, estimates of both $EPSUB_c$ and $EPSUB_p$ generally fall with a range of $\pm$ 0.10 with most estimates falling within the much narrower range of $\pm$ 0.03. When BGEST is employed most estimates of $EPSUB_c$ and $EPSUB_p$ fall within the narrow range of $\pm$ 0.025.

Given the quantitatively small estimates of $EPSUB_c$ and $EPSUB_p$, reported in Part 1 of Table 4, it not surprising that a test of the hypothesis that they are equal to zero is, in most instances, not rejected. A small number of notable exceptions do occur. When the production cost and profit metrics are used along with OBSCEA, estimates of $EPSUB_c$ and $EPSUB_p$ are generally positive though not statistically significant across four of the six adjacent size classifications. However, $EPSUB_c$ and $EPSUB_p$ estimates are positive and statistically significant for expansion from size categories 4 to 5 and 5 to 6 in the case of both metrics.[23] This suggests that increases in size along with the accompanying changes in product mix appear to provide banks that are able expand from $2 through $10 billion of asset size with a production cost and profit advantage. However this conclusion is abrogated, at least in part, by negative and statistically significant estimates of $EPSUB_c$ that are reported for expansion from size classifications 5 to 6 and 6 to 7 that occur when the BGEST proxy is employed along with economic cost metric.

The derived estimates of $EPSUB_c$ and $EPSUB_p$ obtained using cost and profit function parameters from the SFM (Part 2 of Table 4) paint a somewhat different picture. The positive and statistically significant estimates of $EPSUB_c$ obtained using both OBS proxies in conjunction with the production cost metric suggests size and product mix differences may provide a small (one to four percent) cost advantage to expansion through $1 billion and perhaps as far as $2 billion of total assets. The profit function results (using both OBSCEA and BGEST) suggest the presence of a larger, four to ten percent, profit efficiency advantage associated with expanded size and mix that may extend as far as $10 billion in assets since $EPSUB_p$ estimates are found to be positive and statistically significant throughout the first five

[23] In addition, the result indicate the potential for additional production cost advantage from expansion beyond $25 billion in assets.

adjacent size categories. However, the economic cost function results produce estimates of $EPSUB_c$ which are consistently negative (-3.5 to -10%) and statistically significant for expansion across adjacent size classifications 2 through 7. These estimates suggest that from the more comprehensive standpoint of economic costs, changes accompanying expansion beyond $1 billion (OBSCEA) or $2 billion in assets may place larger banking organizations at an economic cost disadvantage.

In general, the estimates of $EPSUB_c$ and $EPSUB_p$ derived from the two frontier methodologies differ both quantitatively and qualitatively more than do estimates of these measures obtained from models which include or exclude an OBS measure. Especially across the three largest adjacent size comparison categories, the TFM appears to suggest the possibility of cost and profit advantages to increase in size and accompanying change in product mix. In contrast, the SFM produces results which suggest that increases in size and the accompanying change in mix may place banks in the three largest adjacent size comparison categories at a cost and profit efficiency disadvantage.

12. Summary and Conclusions

This paper addresses the statistical and economic significance of off-balance sheet (OBS) activities on production cost, economic cost and profit efficiency. Cost and profit functions are estimated using both the thick frontier and stochastic econometric frontier methodologies to control for inefficiency in the data and to allow an evaluation of inefficiency. Two measures are constructed to proxy for on-balance sheet equivalents of off-balance sheet activities. One measure is the Basle Committee's credit equivalent measure. The other measure is an income capitalization approach suggested by Boyd and Gertler.

Tests of the models which exclude a measure of OBS activity against more general models which include OBS activity as an output indicate that the restrictions implied by the exclusion can be strongly rejected statistically. Further, the derived estimates of inefficiency, scale economies and expansion path sub-additivity indicate that the exclusion of a measure of OBS activity does produce small differences in these measures. However, except in the largest and smallest size classification these differences are not substantive. Of much bigger concern are the larger differences associated with the two estimation methodologies employed. The lack of robustness across these two methodologies, especially for the sub-additivity measure, suggests that great care be taken in drawing policy conclusions about efficiency issues when the results are obtained using a single efficiency frontier methodology.

Despite the lack of robustness in the results several tentative conclusions can be advanced. First, the inefficiency results do not provide support for a conclusion of X-inefficiency. Second, there does appear to be evidence of small cost diseconomies of scale but profit economies in the largest size categories. Finally, the results indicate that despite the apparent presence of small cost diseconomies of scale, the changing product mix that accompanies increases in size does appear to provide potential production cost and profit advantages to larger banks. However, these apparent advantages do not appear when the broader economic cost is utilized.

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TABLE 1: Likelihood Ratio Tests of Restrictions Implied by the Exclusion of Off-Balance Sheet Activities

Part 1: Thick Frontier

Model	Log of the Likelihood Ftn.	LR Test Statistic	p-Value
3Q Model - PC, CE	698.141		
4Q Model - PC, CE	722.742	49.202	0.00000
3Q Model - PC, BG	572.185		
4Q Model - PC, BG	625.808	107.246	0.00000
3Q Model - EC, CE	685.40		
4Q Model - EC, CE	711.916	53.032	0.00000
3Q Model - EC, BG	544.348		
4Q Model - EC, BG	624.198	159.700	0.00000
3Q Model - P, CE	4.9861		
4Q Model - P, CE	16.0662	22.160	0.00049
3Q Model - P, BG	6.1554		
4Q Model - P, BG	35.4482	58.587	0.00000

Table 1: continued

Part 2: Stochastic Econometric Frontier

Model	Log of the Likelihood Ftn.	LR Test Statistic	p-Value
3Q Model - PC	283.772		
4Q Model - PC, CE	309.891	52.238	0.00000
4Q Model - PC, BG	446.008	324.472	0.00000
3Q Model - EC	-14.0904		
4Q Model - EC, CE	-3.2583	21.6642	0.00290
4Q Model - EC, BG	86.8162	201.8132	0.00000
3Q Model - PFT	-213.252		
4Q Model - PFT, CE	-194.225	38.054	0.00001
4Q Model - PFT, BG	-185.105	56.294	0.00000

TABLE 2: Estimated Production Cost, Economic Cost and Profit Inefficiencies

Part 1: Thick Frontier

Panel A: Excluding Off-Balance Sheet Output

Size Category	Asset Size	INEFF (production cost)		INEFF (economic cost)		INEFF (profit)	
		Q_{PC}^{CE}	Q_{PC}^{BG}	Q_{EC}^{CE}	Q_{EC}^{BG}	Q_{P}^{CE}	Q_{P}^{BG}
1	< $500 m	0.6413	0.4579	0.7014	0.3589	0.6815	0.6198
2	$500 m - 1 b.	0.3727	0.7156	0.8036	0.4970	0.6035	0.7016
3	$1 - 2 b.	0.3709	0.4181	0.5966	0.5627	0.5715	0.5925
4	$2 - 5 b.	0.4345	0.5706	0.7182	0.6194	0.6101	0.5307
5	$5 - 10 b.	0.3945	0.2113	0.7355	0.7853	0.4773	0.6176
6	$10 - 25 b.	0.5332	0.4668	0.7287	0.7015	0.5307	0.5307
7	> $ 25 b.	0.4154	0.4676	0.5002	0.4816	0.4518	0.5116

Panel B: Using the Credit Equivalent (CE) Proxy for Off-Balance Sheet Assets

Size Category	Asset Size (TACE)	INEFF (production cost)	INEFF (economic cost)	INEFF (profit)
1	< $500 m	0.34173	0.52787	0.10771
2	$500 m - 1 b.	0.14552	0.65410	0.09851
3	$1 - 2 b.	0.11281	0.57578	0.03367
4	$2 - 5 b.	0.20160	0.58210	0.01378
5	$5 - 10 b.	0.18310	0.50499	-0.09577
6	$10 - 25 b.	0.20437	0.68484	-0.22473
7	> $ 25 b.	0.11690	0.28776	0.05428

Panel C: Using the Boyd-Gertler (BG) Proxy for Off-Balance Sheet Assets

Size Category	Asset Size	INEFF (production cost)	INEFF (economic cost)	INEFF (profit)
1	< $500 m	0.09978	0.57866	0.05371
2	$500 m - 1 b.	0.05888	0.40895	0.15014
3	$1 - 2 b.	0.07227	0.49674	0.10797
4	$2 - 5 b.	0.06689	0.60424	0.00366
5	$5 - 10 b.	0.11463	0.69454	0.00876
6	$10 - 25 b.	0.18938	0.77338	-0.34569
7	> $ 25 b.	0.26428	0.82660	0.02679

Part 2: Stochastic Econometric Frontier

Panel A: Excluding Off-Balance Sheet Output (3Q)

Size Category	Asset Size	INEFF (production cost)		INEFF (economic cost)		INEFF (profit)	
		Q_{PC}^{CE}	Q_{PC}^{BG}	Q_{EC}^{CE}	Q_{EC}^{BG}	Q_{P}^{CE}	Q_{P}^{BG}
1	< $500 m	0.1766	0.1743	0.2288	0.2309	0.9435	0.9418
2	$500 m - 1 b.	0.1756	0.1518	0.2276	0.2103	0.7083	0.7873
3	$1 - 2 b.	0.1520	0.1655	0.2214	0.2235	0.8673	0.7554
4	$2 - 5 b.	0.1662	0.1731	0.2293	0.2341	0.6303	0.7606
5	$5 - 10 b.	0.2139	0.1760	0.2920	0.2489	0.6268	0.5490
6	$10 - 25 b.	0.1766	0.2156	0.2607	0.3027	0.9975	0.8747
7	> $ 25 b.	0.2176	0.2265	0.2354	0.2402	0.6597	0.7189

Table 2: continued

Panel B: Using the Credit Equivalent (CE) Proxy for Off-Balance Sheet Assets

Size Category	Asset Size (TACE)	INEFF (production cost)	INEFF (economic cost)	INEFF (profit)
1	< $500 m	0.16348	0.22241	0.90610
2	$500 m - 1 b.	0.16756	0.22256	0.75394
3	$1 - 2 b.	0.14832	0.21902	0.75617
4	$2 - 5 b.	0.16041	0.22489	0.79396
5	$5 - 10 b.	0.20422	0.28022	0.56076
6	$10 - 25 b.	0.16223	0.25433	0.83169
7	> $ 25 b.	0.16714	0.21284	0.73798

Panel C: Using the Boyd-Gertler (BG) Proxy for Off-Balance Sheet Assets

Size Category	Asset Size	INEFF (production cost)	INEFF (economic cost)	INEFF (profit)
1	< $500 m	0.09582	0.12730	0.92692
2	$500 m - 1 b.	0.08299	0.11730	0.78215
3	$1 - 2 b.	0.09963	0.12692	0.77545
4	$2 - 5 b.	0.09595	0.12364	0.72636
5	$5 - 10 b.	0.09285	0.12901	0.56144
6	$10 - 25 b.	0.08903	0.13148	0.87021
7	> $ 25 b.	0.09983	0.12007	0.79257

TABLE 3: Scale Economies

Part 1: Thick Fontier

Panel A: OBSCEA, Production Costs

Size Category	3Q Model - Q_{PC}^{CE} Frontier Asset Size	SCALE	Standard Error	p-Value			4Q Model - Q_{PC}^{CE} Frontier Asset Size	SCALE	Standard Error	p-Value
1	< $500 m	1.00461	0.025331	0.85590		1	< $500 m	1.00488	0.024438	0.84206
2	$500 m - 1 b.	1.00704	0.010353	0.49782		2	$500 m - 1 b.	1.00313	0.0098973	0.75236
3	$1 - 2 b.	1.00951	0.006728	0.16010		3	$1 - 2 b.	1.00309	0.006035	0.60958
4	$2 - 5 b.	1.01938	1.01938	0.98486		4	$2 - 5 b.	1.00277	0.008643	0.74915
5	$5 - 10 b.	1.04077	0.022801	0.07629		5	$5 - 10 b.	1.02946	0.01662	0.07884
6	$10 - 25 b.	1.01764	0.034736	0.61250		6	$10 - 25 b.	1.09609	0.040978	0.02067
7	> $ 25 b.	0.81387	0.040208	0.00001		7	> $ 25 b.	0.77578	0.037816	0.00000

Panel B: OBSCEA, Economic Costs

Size Category	3Q Model - Q_{EC}^{CE} Frontier Asset Size	SCALE	Standard Error	p-Value			4Q Model - Q_{EC}^{CE} Frontier Asset Size	SCALE	Standard Error	p-Value
1	< $500 m	1.01287	0.044163	0.77123		1	< $500 m	1.00527	0.040137	0.89576
2	$500 m - 1 b.	0.99677	0.01792	0.85726		2	$500 m - 1 b.	0.99126	0.016105	0.58835
3	$1 - 2 b.	0.97271	0.01244	0.03018		3	$1 - 2 b.	0.9637	0.0096385	0.00026
4	$2 - 5 b.	0.94723	0.02165	0.01626		4	$2 - 5 b.	0.92935	0.014689	0.00000
5	$5 - 10 b.	0.87628	0.03565	0.00072		5	$5 - 10 b.	0.9003	0.026932	0.00032
6	$10 - 25 b.	0.85595	0.05332	0.00790		6	$10 - 25 b.	0.92985	0.049288	0.15725
7	> $ 25 b.	0.69411	0.03214	0.00000		7	> $ 25 b.	0.74	0.06222	0.00006

Table 3: Continued

Panel C: BGEST, Production Costs

Size Category	3Q Model - Q_{PC}^{BG} Frontier Asset Size	SCALE	Standard Error	p-Value			4Q Model - Q_{PC}^{BG} Frontier Asset Size	SCALE	Standard Error	p-Value
1	< $500 m	1.00463	0.02585	0.85815		1	< $500 m	1.01098	0.023077	0.63508
2	$500 m - 1 b.	1.0006	0.01518	0.96854		2	$500 m - 1 b.	1.00023	0.012574	0.98544
3	$1 - 2 b.	0.99836	0.00794	0.83671		3	$1 - 2 b.	0.99578	0.006802	0.53617
4	$2 - 5 b.	1.00214	0.00463	0.64477		4	$2 - 5 b.	0.98869	0.007658	0.14233
5	$5 - 10 b.	1.00218	0.01005	0.82864		5	$5 - 10 b.	0.98876	0.011908	0.34712
6	$10 - 25 b.	1.01036	0.01656	0.53276		6	$10 - 25 b.	0.95848	0.02799	0.14059
7	> $ 25 b.	0.85708	0.03745	0.00022		7	> $ 25 b.	0.82618	0.03625	0.00000

Panel D: BGEST, Economic Costs

Size Category	3Q Model - Q_{EC}^{BG} Frontier Asset Size	SCALE	Standard Error	p-Value			4Q Model - Q_{EC}^{BG} Frontier Asset Size	SCALE	Standard Error	p-Value
1	< $500 m	0.97726	0.0501	0.65073		1	< $500 m	1.00478	0.038234	0.90072
2	$500 m - 1 b.	0.99515	0.024581	0.84392		2	$500 m - 1 b.	1.00288	0.018024	0.87332
3	$1 - 2 b.	1.00519	0.013836	0.70824		3	$1 - 2 b.	1.00349	0.009891	0.72482
4	$2 - 5 b.	1.01332	0.019502	0.49592		4	$2 - 5 b.	0.99106	0.011483	0.43778
5	$5 - 10 b.	1.03573	0.039015	0.36161		5	$5 - 10 b.	0.978	0.02021	0.27852
6	$10 - 25 b.	1.03703	0.073394	0.61481		6	$10 - 25 b.	0.92732	0.032463	0.02701
7	> $ 25 b.	0.80712	0.057299	0.00102		7	> $ 25 b.	0.78864	0.043018	0.00000

Table 3: continued

Panel E: OBSCEA, Profit

Size Category	3Q Model - Q_{PFT}^{CE} Frontier: Asset Size	Elasticity	Standard Error	p-Value			4Q Model - Q_{PFT}^{CE} Frontier: Asset Size	Elasticity	Standard Error	p-Value
1	< $500 m	1.08714	0.06742	0.19867		1	< $500 m	1.0604	0.073995	0.41596
2	$500 m - 1 b.	1.00395	0.03146	0.90029		2	$500 m - 1 b.	0.99204	0.034716	0.81903
3	$1 - 2 b.	0.99014	1.01559	0.99227		3	$1 - 2 b.	0.98711	0.016772	0.44367
4	$2 - 5 b.	0.97205	0.01177	0.01915		4	$2 - 5 b.	0.98804	0.018876	0.52754
5	$5 - 10 b.	0.96504	0.0189	0.06681		5	$5 - 10 b.	0.95364	0.017935	0.01094
6	$10 - 25 b.	0.90573	0.04178	0.02586		6	$10 - 25 b.	0.86229	0.043496	0.00196
7	> $ 25 b.	0.80005	0.17464	0.25452		7	> $ 25 b.	0.84255	0.32885	0.63296

Panel F: BGEST

Size Category	3Q Model - Q_{PFT}^{BG} Frontier: Asset Size	Elasticity	Standard Error	p-Value			4Q Model - Q_{PFT}^{BG} Frontier: Asset Size	Elasticity	Standard Error	p-Value
1	< $500 m	1.05886	0.08778	0.50380		1	< $500 m	0.96041	0.08077	0.62492
2	$500 m - 1 b.	1.01475	0.03569	0.68014		2	$500 m - 1 b.	0.99948	0.03677	0.98874
3	$1 - 2 b.	0.99371	0.01946	0.74709		3	$1 - 2 b.	0.98564	0.02026	0.47983
4	$2 - 5 b.	0.98368	0.02732	0.55139		4	$2 - 5 b.	1.06307	0.02639	0.01841
5	$5 - 10 b.	0.97719	0.04075	0.57669		5	$5 - 10 b.	1.04762	0.03204	0.13983
6	$10 - 25 b.	1.04458	0.06145	0.46958		6	$10 - 25 b.	0.98631	0.0717	0.84890
7	> $ 25 b.	0.92375	0.26325	0.77258		7	> $ 25 b.	1.00965	0.2284	0.96637

Part 2: Stochastic Econometric Frontier

Panel A: OBSCEA, Production Costs

Size Category	Asset Size	SCALE	Standard Error	p-Value			Asset Size	SCALE	Standard Error	p-Value
	3Q Model - Q_{PC}^{CE} Frontier						4Q Model - Q_{PC}^{CE} Frontier			
1	< $500 m	1.06844	0.014297	0.63302		1	< $500 m	1.06635	0.015112	0.00002
2	$500 m - 1 b.	1.03127	0.009559	0.00140		2	$500 m - 1 b.	1.03049	0.010303	0.00372
3	$1 - 2 b.	1.00921	0.006721	0.17314		3	$1 - 2 b.	1.01708	0.007449	0.02359
4	$2 - 5 b.	0.99391	0.005344	0.25672		4	$2 - 5 b.	1.00473	0.006674	0.47987
5	$5 - 10 b.	0.96372	0.008498	0.00004		5	$5 - 10 b.	0.97576	0.010113	0.01808
6	$10 - 25 b.	0.95869	0.010738	0.00019		6	$10 - 25 b.	0.97161	0.013255	0.03423
7	> $ 25 b.	0.94881	0.015054	0.00091		7	> $ 25 b.	0.96232	0.018415	0.04293

Panel B: OBSCEA, Economic Costs

Size Category	Asset Size	SCALE	Standard Error	p-Value			Asset Size	SCALE	Standard Error	p-Value
	3Q Model - Q_{EC}^{CE} Frontier						4Q Model - Q_{EC}^{CE} Frontier			
1	< $500 m	0.99012	0.02639	0.70878		1	< $500 m	0.97178	0.02896	0.33179
2	$500 m - 1 b.	0.96536	0.01706	0.04452		2	$500 m - 1 b.	0.95739	0.01850	0.02299
3	$1 - 2 b.	0.94077	0.01191	0.00000		3	$1 - 2 b.	0.94562	0.01335	0.00008
4	$2 - 5 b.	0.91995	0.01096	0.00000		4	$2 - 5 b.	0.92797	0.01190	0.00000
5	$5 - 10 b.	0.89583	0.01574	0.00000		5	$5 - 10 b.	0.90642	0.01664	0.00000
6	$10 - 25 b.	0.88025	0.02087	0.00000		6	$10 - 25 b.	0.89674	0.02260	0.00001
7	> $ 25 b.	0.85919	0.03032	0.00001		7	> $ 25 b.	0.87192	0.03207	0.00011

Table 3: Continued

Panel C: BGEST, Production Costs

	3Q Model - Q_{PC}^{BG} Frontier						4Q Model - Q_{PC}^{BG} Frontier			
Size Category	Asset Size	SCALE	Standard Error	p-Value			Asset Size	SCALE	Standard Error	p-Value
1	< $500 m	1.06507	0.01459	0.00002		1	< $500 m	1.08287	0.012049	0.00000
2	$500 m - 1 b.	1.03985	0.01034	0.00019		2	$500 m - 1 b.	1.05833	0.009041	0.00000
3	$1 - 2 b.	1.02635	0.00790	0.00113		3	$1 - 2 b.	1.04427	0.007280	0.00000
4	$2 - 5 b.	0.99690	0.00582	0.59526		4	$2 - 5 b.	1.01684	0.005301	0.00189
5	$5 - 10 b.	0.97889	0.00636	0.00120		5	$5 - 10 b.	1.00041	0.006002	0.94565
6	$10 - 25 b.	0.96284	0.00950	0.00015		6	$10 - 25 b.	0.98096	0.008299	0.02351
7	> $ 25 b.	0.94799	0.01463	0.00054		7	> $ 25 b.	0.96491	0.012724	0.00673

Panel D: BGEST, Economic Costs

	3Q Model - Q_{EC}^{BG} Frontier						4Q Model - Q_{EC}^{BG} Frontier			
Size Category	Asset Size	SCALE	Standard Error	p-Value			Asset Size	SCALE	Standard Error	p-Value
1	< $500 m	0.99302	0.02656	0.79315		1	< $500 m	1.00605	0.02376	0.79945
2	$500 m - 1 b.	0.96889	0.01882	0.10094		2	$500 m - 1 b.	0.98682	0.01681	0.43455
3	$1 - 2 b.	0.95444	0.01493	0.00280		3	$1 - 2 b.	0.97055	0.01315	0.02696
4	$2 - 5 b.	0.92756	0.01071	0.00000		4	$2 - 5 b.	0.94590	0.00942	0.00000
5	$5 - 10 b.	0.90456	0.01273	0.00000		5	$5 - 10 b.	0.92639	0.01277	0.00000
6	$10 - 25 b.	0.88686	0.01836	0.00000		6	$10 - 25 b.	0.90360	0.01834	0.00000
7	> $ 25 b.	0.86054	0.02929	0.00001		7	> $ 25 b.	0.87952	0.02887	0.00006

Table 3: continued

Panel E: OBSCEA

Size Category	3Q Model - Q_{PFT}^{CE} Frontier Asset Size	Elasticity	Standard Error	p-Value			4Q Model - Q_{PFT}^{CE} Frontier Asset Size	Elasticity	Standard Error	p-Value
1	< $500 m	0.81048	0.04459	0.00004		1	< $500 m	0.85576	0.049126	0.00398
2	$500 m - 1 b.	0.87739	0.02927	0.00005		2	$500 m - 1 b.	0.87431	0.032570	0.00018
3	$1 - 2 b.	0.91533	0.01841	0.00001		3	$1 - 2 b.	0.90944	0.022127	0.00008
4	$2 - 5 b.	0.93733	0.01681	0.00030		4	$2 - 5 b.	0.93101	0.018285	0.00025
5	$5 - 10 b.	1.00292	0.03214	0.92776		5	$5 - 10 b.	0.94810	0.028405	0.07016
6	$10 - 25 b.	0.99962	0.04279	0.99293		6	$10 - 25 b.	0.94533	0.041473	0.18995
7	> $ 25 b.	1.02132	0.06229	0.73275		7	> $ 25 b.	0.95421	0.064526	0.47931

Panel F: BGEST, Profit

Size Category	3Q Model - Q_{PFT}^{BG} Frontier Asset Size	Elasticity	Standard Error	p-Value			4Q Model - Q_{PFT}^{BG} Frontier Asset Size	Elasticity	Standard Error	p-Value
1	< $500 m	0.82048	0.04640	0.00018		1	< $500 m	0.85262	0.04679	0.00206
2	$500 m - 1 b.	0.84940	0.03151	0.00001		2	$500 m - 1 b.	0.87925	0.03356	0.00047
3	$1 - 2 b.	0.90657	0.02205	0.00004		3	$1 - 2 b.	0.93724	0.02569	0.01602
4	$2 - 5 b.	0.93794	0.01696	0.00038		4	$2 - 5 b.	0.94805	0.01911	0.00753
5	$5 - 10 b.	0.97164	0.02485	0.25604		5	$5 - 10 b.	0.96611	0.02500	0.17777
6	$10 - 25 b.	0.99187	0.03768	0.82954		6	$10 - 25 b.	0.96406	0.03656	0.32756
7	> $ 25 b.	1.01955	0.06048	0.74707		7	> $ 25 b.	0.98686	0.05657	0.81672

TABLE 4: Expansion Path Subadditivity

Part 1: Thick Fontier

Panel A: OBSCEA, Production Cost

3Q Model - Q_{PC}^{CE} Frontier					4Q Model - Q_{PC}^{CE} Frontier			
Size Category	EPSUB	Standard Error	p-Value		Size Category	EPSUB	Standard Error	p-Value
2-1	0.004151	0.010705	0.69888		2-1	0.0024104	0.10298	0.98136
3-2	0.005219	0.005691	0.36095		3-2	0.0023711	0.0054285	0.66305
4-3	0.00989	0.005047	0.05236		4-3	0.0015422	0.0043325	0.72249
5-4	0.021651	0.010713	0.04551		5-4	0.016348	0.007898	0.04061
6-5	0.016694	0.020764	0.42299		6-5	0.073466	0.02552	0.00473
7-6	-0.054984	0.028536	0.05637		7-6	0.057385	0.028575	0.04687

Panel B: OBSCEA, Economic Costs

3Q Model - Q_{EC}^{CE} Frontier					4Q Model - Q_{EC}^{CE} Frontier			
Size Category	EPSUB	Standard Error	p-Value		Size Category	EPSUB	Standard Error	p-Value
2-1	0.003628	0.01906	0.84936		2-1	-0.000272	0.017193	0.98740
3-2	-0.011132	0.009305	0.23392		3-2	-0.016999	0.008225	0.04091
4-3	-0.026116	0.011476	0.02464		4-3	-0.036909	0.0081102	0.00001
5-4	-0.06644	0.024633	0.00800		5-4	-0.053282	0.017286	0.00255
6-5	-0.059995	0.0429	0.16455		6-5	-0.017883	0.033542	0.59491
7-6	-0.1796	0.054321	0.00125		7-6	0.35498	0.11151	0.00185

Table 4: continued

Panel C: BGEST, Production Cost

3Q Model - Q_{PC}^{BG} Frontier					4Q Model - Q_{PC}^{BG} Frontier			
Size Category	EPSUB	Standard Error	p-Value		Size Category	EPSUB	Standard Error	p-Value
2-1	0.001979	0.0157	0.89990		2-1	-0.00007	0.013179	0.99577
3-2	0.000309	0.0084	0.97072		3-2	0.000224	0.0071735	0.97514
4-3	0.004569	0.00405	0.26151		4-3	0.001461	0.0049342	0.76767
5-4	0.002301	0.00549	0.67587		5-4	0.002072	0.0061502	0.73678
6-5	0.007656	0.00766	0.31957		6-5	0.005031	0.016222	0.75700
7-6	0.025166	0.02731	0.35864		7-6	0.011013	0.034241	0.74829

Panel D: BGEST, Economic Costs

3Q Model - Q_{EC}^{BG} Frontier					4Q Model - Q_{EC}^{BG} Frontier			
Size Category	EPSUB	Standard Error	p-Value		Size Category	EPSUB	Standard Error	p-Value
2-1	-0.008136	0.025797	0.75302		2-1	0.00090347	0.018502	0.96114
3-2	-0.000756	0.013491	0.95541		3-2	0.0018781	0.009766	0.84782
4-3	0.006192	0.009966	0.53557		4-3	-0.0038968	0.006363	0.54142
5-4	0.017458	0.017724	0.32661		5-4	-0.010628	0.010353	0.30669
6-5	0.020954	0.034866	0.54898		6-5	-0.038055	0.019337	0.05138
7-6	-0.11268	0.047888	0.02025		7-6	-0.3075	0.067812	0.00001

Table 4: continued

Panel E: OBSCEA, Profit

3Q Model - Q_{PFT}^{CE} Frontier					4Q Model - Q_{PFT}^{CE} Frontier			
Size Category	PEPSUB	Standard Error	p-Value		Size Category	PEPSUB	Standard Error	p-Value
2-1	-0.00776	0.03183	0.80781		2-1	0.0053677	0.03574	0.88087
3-2	0.00246	0.01531	0.87262		3-2	0.0074886	0.016614	0.65299
4-3	0.01818	0.00942	0.05597		4-3	0.014158	0.011335	0.21408
5-4	0.04174	0.01236	0.00099		5-4	0.082189	0.016185	0.00000
6-5	0.05748	0.02341	0.01551		6-5	0.084995	0.024355	0.00068
7-6	0.11205	0.08652	0.19778		7-6	0.19147	0.20249	0.34626

Panel F: BGEST, Profit

3Q Model - Q_{PFT}^{BG} Frontier					4Q Model - Q_{PFT}^{BG} Frontier			
Size Category	PEPSUB	Standard Error	p-Value		Size Category	PEPSUB	Standard Error	p-Value
2-1	-0.018213	0.03725	0.62578		2-1	0.015029	0.038445	0.69655
3-2	0.00495	0.02075	0.81186		3-2	0.027081	0.022125	0.22335
4-3	0.02229	0.01687	0.18892		4-3	-0.02091	0.015167	0.17057
5-4	0.02071	0.02007	0.30420		5-4	-0.010968	0.015617	0.48384
6-5	-0.00726	0.03467	0.83449		6-5	0.12994	0.0525	0.01472
7-6	0.54668	0.11291	0.00000		7-6	0.057565	0.11114	0.60545

Table 4: continued

Part 2: Stochastic Econometric Frontier

Panel A: OBSCEA, Production Cost

3Q Model - Q_{PC}^{CE} Frontier					4Q Model - Q_{PC}^{CE} Frontier			
Size Category	EPSUB	Standard Error	p-Value		Size Category	EPSUB	Standard Error	p-Value
2-1	0.02626	0.01199	0.03045		2-1	0.02387	0.01263	0.06118
3-2	0.01046	0.01189	0.38076		3-2	0.01472	0.01188	0.21774
4-3	-0.00015	0.01184	0.98991		4-3	0.00473	0.01148	0.68106
5-4	-0.02073	0.01192	0.08458		5-4	-0.01519	0.01181	0.20085
6-5	-0.02606	0.01187	0.03006		6-5	-0.02143	0.01204	0.07762
7-6	-0.03064	0.01332	0.02316		7-6	-0.02658	0.01362	0.05332

Panel B: OBSCEA, Economic Costs

3Q Model - Q_{EC}^{CE} Frontier					4Q Model - Q_{EC}^{CE} Frontier			
Size Category	EPSUB	Standard Error	p-Value		Size Category	EPSUB	Standard Error	p-Value
2-1	-0.01705	0.01948	0.38318		2-1	-0.02697	0.02079	0.19703
3-2	-0.03517	0.01887	0.06479		3-2	-0.03521	0.01981	0.07804
4-3	-0.05149	0.01858	0.00647		4-3	-0.05035	0.01919	0.00983
5-4	-0.06999	0.01867	0.00027		5-4	-0.06716	0.01941	0.00075
6-5	-0.08372	0.01874	0.00002		6-5	-0.07802	0.02005	0.00016
7-6	-0.09658	0.02166	0.00002		7-6	-0.09438	0.02221	0.00004

Table 4: continued

Panel C: BGEST, Production Cost

3Q Model - Q_{PC}^{BG} Frontier					4Q Model - Q_{PC}^{BG} Frontier			
Size Category	EPSUB	Standard Error	p-Value		Size Category	EPSUB	Standard Error	p-Value
2-1	0.032306	0.013144	0.01541		2-1	0.043477	0.010287	0.00005
3-2	0.021705	0.011819	0.06877		3-2	0.032338	0.0091153	0.00056
4-3	0.0029883	0.011479	0.79506		4-3	0.012584	0.0084258	0.13793
5-4	-0.010174	0.011844	0.39205		5-4	0.0019817	0.0087849	0.82191
6-5	-0.02224	0.011564	0.05682		6-5	-0.014653	0.0090729	0.10893
7-6	-0.029459	0.012265	0.01785		7-6	-0.019599	0.0094719	0.04068

Panel D: BGEST, Economic Costs

3Q Model - Q_{EC}^{BG} Frontier					4Q Model - Q_{EC}^{BG} Frontier			
Size Category	EPSUB	Standard Error	p-Value		Size Category	EPSUB	Standard Error	p-Value
2-1	-0.01409	0.02168	0.51699		2-1	-0.00422	0.01541	0.78467
3-2	-0.02529	0.01882	0.18155		3-2	-0.01783	0.01234	0.15109
4-3	-0.04495	0.01792	0.01346		4-3	-0.03803	0.01027	0.00032
5-4	-0.06308	0.01864	0.00097		5-4	-0.05193	0.01115	0.00001
6-5	-0.07813	0.01817	0.00003		6-5	-0.07384	0.01296	0.00000
7-6	-0.08959	0.01974	0.00001		7-6	-0.07801	0.01577	0.00000

Table 4: continued

Panel D: OBSCEA, Profit

3Q Model - Q_{PFT}^{CE} Frontier					4Q Model - Q_{PFT}^{CE} Frontier			
Size Category	PEPSUB	Standard Error	p-Value		Size Category	PEPSUB	Standard Error	p-Value
2-1	0.12334	0.028728	0.00004		2-1	0.098004	0.030606	0.00175
3-2	0.0755	0.017012	0.00002		3-2	0.092893	0.021735	0.00004
4-3	0.04481	0.01236	0.00042		4-3	0.055452	0.013599	0.00008
5-4	0.030176	0.01588	0.05980		5-4	0.042673	0.015831	0.00804
6-5	-0.019055	0.025179	0.45066		6-5	0.035525	0.023732	0.13704
7-6	-0.017958	0.035787	0.61673		7-6	0.015326	0.033948	0.65248

Panel B: BGEST, Profit

3Q Model - Q_{PFT}^{BG} Frontier					4Q Model - Q_{PFT}^{BG} Frontier			
Size Category	PEPSUB	Standard Error	p-Value		Size Category	PEPSUB	Standard Error	p-Value
2-1	0.11959	0.030305	0.00013		2-1	0.10093	0.030403	0.00119
3-2	0.09638	0.02145	0.00002		3-2	0.074547	0.023075	0.00159
4-3	0.054939	0.012973	0.00005		4-3	0.049564	0.013119	0.00025
5-4	0.029199	0.014283	0.04311		5-4	0.031861	0.01499	0.03560
6-5	0.00373	0.020911	0.85873		6-5	0.030447	0.02207	0.17029
7-6	-0.017366	0.032036	0.58877		7-6	0.0026251	0.031216	0.93312