Leverage Constraints and the International Transmission of Shocks*

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Abstract

Recent macroeconomic experience has drawn attention to the importance of interdependence among countries through financial markets and institutions, independently of traditional trade linkages. This paper develops a model of the international transmission of shocks due to interdependent portfolio holdings among leverage-constrained financial institutions. In the absence of leverage constraints, international portfolio diversification has no implications for macro-economic co-movements. When leverage constraints bind, however, the presence of diversified portfolios in combination with these constraints introduces a powerful financial transmission channel which results in a high correlation among macroeconomic aggregates during business cycle downturns, quite independent of the size of international trade linkages.

Keywords: Leverage, International Transmission, Portfolios

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1. Introduction

The current international financial crisis has highlighted the critical role of financial markets in the propagation of business cycle shocks, both in transmitting shocks from one country to another and in magnifying the effects of those shocks. One key aspect of this linkage, seen in both the current crisis as well as the Asian and Russion crises a decade ago, is the importance of balance sheet linkages among firms and financial institutions across countries. This implies that asset price collapses in one country are transmitted internationally through deteriorations in the balance sheets of institutions in countries holding portfolios of similar assets.

It is widely agreed that high financial leverage – a high ratio of assets to underlying capital – is a critical factor in the magnifying effects of financial crises. As asset values decline, highly levered financial institutions find their net worth sharply eroded. These institutions are forced to shed assets to avoid unacceptable risks of insolvency. But asset sales drive asset values down further, adversely impairing the balance sheets of other institutions. These institutions in turn are forced to sell assets, creating a vicious cycle of balance sheet deterioration and asset sales. While the financial dynamics of such balance sheet adjustments have been widely discussed elsewhere, it is less well understood how this process affects macroeconomic outcomes, or that this process alone may generate an immediate and powerful international transmission of shocks.

A clear pre-requisite for balance sheet adjustments to have powerful macroeconomic effects is the presence of some type of financial frictions or distortions in credit markets. After all, in a Modigliani-Miller world, leverage is irrelevant. Thus, in order to capture the dynamics of the financial meltdown, financial frictions will be of critical importance.

In the context of the international transmission of business cycles, however, other puzzles arise. Most models of business cycle transmission still rely on international linkages due to trade flows. While global trade has been growing at remarkable rates over the past two decades, it is still the case that the major world regions – the United States, Asia and Europe – are to a large extent 'closed' economies, with the export share from one region to another representing only a small proportion of overall GDP. Kose and Yi (2006) find that using conventional international real business cycle models, it is hard to account for the magnitude of business cycle co-movements among countries. In addition, there is evidence that business cycle co-movement is greater between countries with greater financial integration (Imbs 2004, 2006). Nevertheless, in the standard international business cycle model, enhanced international financial integration actually tends to reduce business cycle co-movement (Heathcote and Perri 2002, 2005). But Krugman (2008) suggests that traditional multi-country business cycle models lack a critical 'international finance multiplier,' by which financial shocks in one country affects investment both in the original country and in other countries through financial or balance sheet linkages.

This paper develops a theoretical model of a balance sheet channel for the international transmission of shocks. The model emphasizes how a process of balance sheet contractions, generated by a downturn in one country, is spread around the globe through inter-connected portfolios. In the presence of leverage constraints, we show that this gives rise to a separate financial transmission mechanism of business cycle shocks that is completely independent of trade linkages.[1] In fact, we work with a highly stripped down 'one world good' model in which, in steady state, there are no trade linkages across countries at all.

[1] In the recent literature, for example Krugman (2008), the adjustment of balance sheets is sometimes referred to as 'de-leveraging'. This term is inaccurate as a description of our model, since, as in Kiyotaki and Moore (1997), the leverage ratio is constant. Nonetheless, the process of satisfying leverage constraints in the wake of asset price

The paper's main contribution is to compare how macro shocks are transmitted under two financial market structures. We do not attempt to provide an integrated explanation of the recent crisis, but instead highlight how the joint process of balance sheet constraints and portfolio interdependence generate an important cross-country propagation effect. We develop a two-country model in which investors borrow from savers in each country, and invest in fixed assets. Investors also diversify their portfolios across countries and hold equity positions in the assets of the other country, as well as their own. Investors cannot commit to repay savers, however, and in order to enforce payment, may face limits on the maximum amount of leverage on their balance sheets. We look at one environment where leverage limits do not bind. In this case the international transmission of shocks is quite limited. Specifically, there is no international transmission due to balance sheet adjustments. A negative productivity shock, which leads to a fall in the value of assets in one country, will cause financial institutions to sell some assets and reduce their debt exposure, but this does not affect other countries. In fact, in other countries, investors increase their borrowing. More broadly, business cycle fluctuations across countries are essentially uncorrelated in the absence of limits on leverage.

When leverage constraints are binding, however, there is a powerful transmission of shocks across countries. A fall in asset values in one country forces an immediate and large process of balance sheet contractions in that country's financial institutions. But the fall in asset values leads to balance sheet deterioration in other countries that have internationally diversified asset portfolios, causing a sell-off in assets and a forced reduction in borrowing around the globe. This, in turn, drives a further sell-off in the first country, establishing a feedback loop. The end

declines does impart a magnification effect on real activity. The endogenization of the leverage ratio represents a separate issue, beyond the scope of this paper. For a recent contribution, see Geanakoplos (2009).

result is a large magnification of the initial shock, a large fall in investment, and highly correlated business cycles across countries during the resulting downturn.

The scale of the propagation linkages depends not just on the presence of leverage constraints, but also on the degree of international portfolio diversification. We show that greater financial integration, which facilitates more diversified portfolios, will increase the degree of common business cycle co-movement of shocks. In this sense there is a trade-off between the benefits of international risk-sharing and the magnified international propagation mechanism.

Finally, the model has implications for gross capital flows. We show that a negative shock that reduces investment in fixed assets in one country causes a scaling back of gross positions, leading to a fall in the holdings of foreign equity and an increase in the holding of home equity. But the magnitude of this gross capital flow contraction is much bigger in the presence of binding leverage constraints.

The model draws heavily on a number of separate literatures. First, and most importantly, we follow Kiyotaki and Moore (1997) in imposing leverage limits on investors. This leads to a wedge between the effective returns faced by investors and savers, and can act as an amplification mechanism for business cycle shocks.[2] Second, we emphasize the linkages among countries through the presence of inter-connected portfolios. Portfolio linkages, in a somewhat different context, have for some time been seen as important in the contagion effects of financial shocks (see Rigobon 2003 and Pavlova and Rigobon 2008, for example). Finally, we introduce

[2] An alternative mechanism where balance sheets play a key role in business cycles is the 'financial accelerator' model of Bernanke et al. (1999). This has been extended to a multi-country setting by Gilchrist (2004).

endogenous portfolio interdependence through the recently developed techniques of Devereux and Sutherland (2009a).[3]

The paper is organized as follows. The next section provides some evidence of the importance of a financial channel in the recent business cycle downturn. We then develop the basic two-country model in which investors and savers interact, but investors may be limited by leverage constraints. In section 4 we explore the effects of a negative productivity shock in one country, and demonstrate the role of balance sheet adjustments in the propagation of business cycle shocks across countries. We then conclude.

2. Empirical evidence

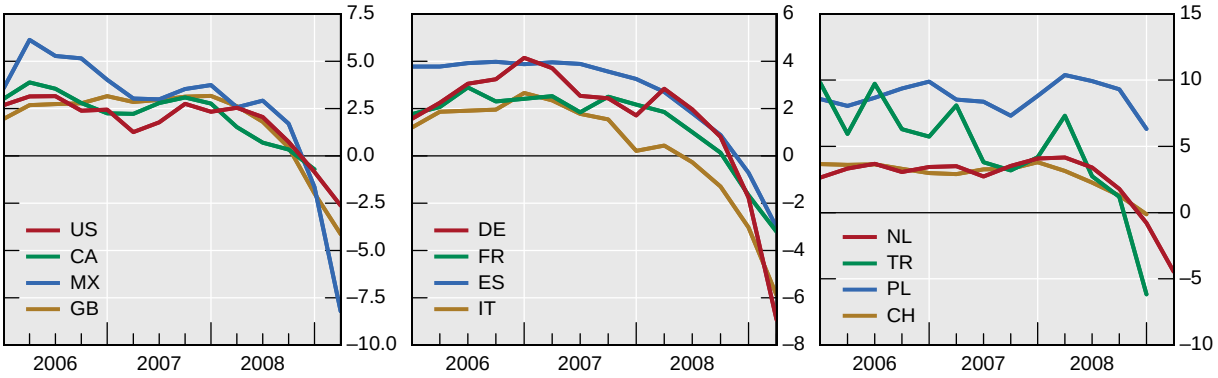
We present some empirical evidence that supports our contention that balance sheet contractions may have been an important propagation mechanism for the current international crisis. First, Figure 1 documents the global nature of the economic crisis, demonstrating a remarkably synchronous collapse in economic growth rates for a sample of OECD countries. It is unlikely that trade linkages alone could account for the simultaneous downturns in all regions. If we take the US economy as the ultimate source of the financial crisis then it would be easy to explain the scale of the downturn in Mexico, for instance. But Figure 1 illustrates dramatic reductions in economic growth in many European economies, only marginally linked to the US through trade flows.

[3] Dedola and Lombardo (2009) develop an interesting model similar to the present paper based on the financial accelerator model, incorporating endogenous portfolios as in the present paper. They emphasise a somewhat different type of transmission effect, unique to the financial accelerator model, coming from the direct connection between risk-premia across countries.

Figure 1

Real GDP growth[1]

In per cent



[1] Year-over-year changes in real GDP. US = United States, CA = Canada, MX = Mexico, GB = United Kingdom, DE = Germany, FR = France, ES = Spain, IT = Italy, NL = Netherlands, TR = Turkey, PL = Poland, CH = Switzerland.

Source: national data.

In addition there is clear evidence that US banks reduced their outstanding claims on the rest of the world. Table 1 contains short-term claims of US banks, for all OECD countries for which data is available. This is the total stock among US reporting banks of all claims on the destination economy with less than one year remaining until maturity. Under normal circumstances new claims are issued and many maturing existing claims are rolled over each quarter. A rapid decline in less than one year, then, implies little new issuance, and few exposures being rolled over. There is a clear pattern overall that the largest OECD economies (by size of claims) have experienced a substantial fall in short-term US bank claims during 2008. In particular France, Germany, Ireland, Italy, Korea and Luxembourg all experienced major withdrawals over 2008. Further, total claims across all countries declined by more than 20 percent, with half of that decline occurring in the final quarter.

Table 1

Short term claims of US banks on OECD economies

$US millions

Destination of Funds	2007Q4	2008Q1	2008Q2	2008Q3	2008Q4
Austria	4,179	4,207	4,841	3,574	2,256
Belgium	8,742	13,911	17,453	15,762	15,567
Czech Republic	527	716	798	894	518
Finland	3,191	2,837	2,386	3,024	2,928
France	57,952	69,098	41,790	44,355	55,287
Germany	56,910	65,933	48,407	41,295	39,266
Greece	3,947	4,857	3,005	2,310	2,428
Hungary	894	1,003	900	1,113	491
Ireland	28,317	27,471	28,082	27,767	23,550
Italy	25,180	25,521	26,215	18,617	17,243
Korea	26,254	27,435	28,027	29,873	21,518
Luxembourg	26,050	24,730	22,826	21,650	11,943
Mexico	6,492	7,752	7,497	6,784	7,734
Netherlands	43,132	46,995	52,071	47,617	37,230
Poland	2,356	2,254	2,279	2,308	2,521
Portugal	2,861	2,331	2,054	1,740	1,226
Spain	28,267	28,367	25,370	18,719	18,420
Turkey	7,320	6,916	7,014	6,010	5,107

Source: BIS Consolidated Banking Statistics

Aside from bank balance sheets, we can also find evidence consistent with balance sheet contractions in other instruments. Equities in particular were believed by some policymakers to be a vector of contagion, as the following quote by Rakesh Mohan, Deputy Governor of the Reserve Bank of India, indicates:

"Our problems are mainly due to the sell-off by foreign institutional investors in the domestic equity markets leading to a sharp reduction in net capital inflows and the sharp slowdown in global economic activity and external demand." (Mohan 2009)

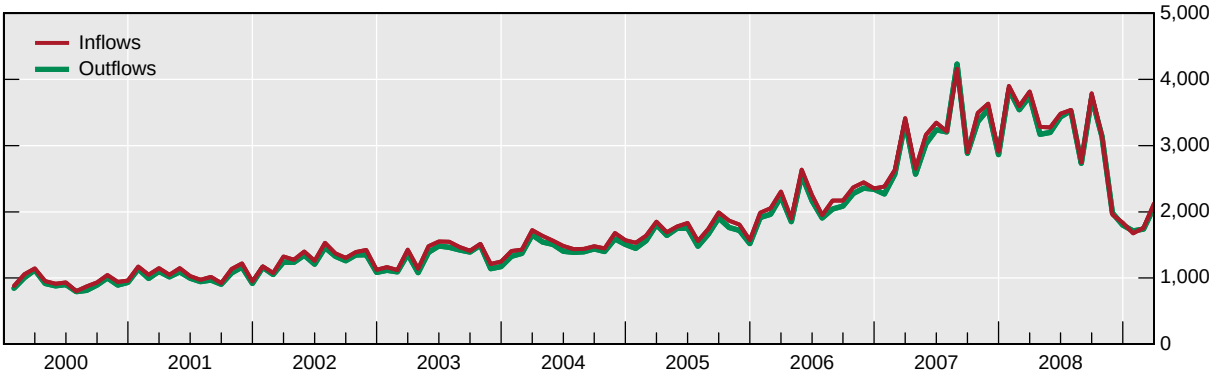
This view is consistent with the data on international capital flows captured by the Treasury International Capital System (Figure 2). The crisis has seen a fall in both inflows and outflows of capital from the US, at the aggregate level. The scale of the fall in flows in early

2009 is unprecedented over the full sample of aggregate TIC data going back to 1980. In the model we will see that the balance sheet contractions implied by this, when combined with binding leverage constraints among financial institutions, can impart an independent international transmission of shocks.

Figure 2

US capital inflows and outflows

In billions of US dollars



Source: US Department of Treasury.

Financial linkages versus trade linkages

The effects of global balance sheet adjustments should be expected to vary by country. Some economies are more dependent on capital inflows than others, and countries with low credit ratings may suffer more from a sudden reduction in flows than higher rated countries, for example. Evidence of the effects of a financial channel should account for the difference in vulnerabilities across countries.

We demonstrate the importance of balance sheet contractions as a propagation mechanism for the crisis using regression analysis. As a rough measure of the international effect of the crisis, we use the change in the growth rate of real GDP between the year ended December 2007 and December 2008. The vulnerability of countries to a sudden outflow of capital is calculated as total capital inflows from the US, as a percent of 2007 GDP, using US Treasury

International Capital data (labeled TIC). Our sample includes all members of the OECD for which TIC data is available. We also include trade linkages, measured using exports to the US in 2007 as a percent of GDP (X). Finally, we interact each of these variables with the sovereign credit rating of the economy (CR), to capture the idea that capital withdrawals are likely to affect lower rated economies more heavily than higher rated ones, due to "flight to quality." Based on the Standards and Poor's sovereign foreign currency credit rating in December 2007, we convert the credit rating to a numerical scale where a value of 0 corresponds to a AAA-rating, 1 to a AA+ rating, and so on, down to 12 for a BB- rating.

Table 2
Explaining the slowdown[1]

	(1)	(2)	(3)	(4)	(5)
X	-0.020 (0.844)	0.003 (0.982)			-0.043 (0.662)
CRX		-0.007 (0.775)			0.039 (0.144)
TIC			**-0.005** **(0.035)**	**-0.006** **(0.006)**	**-0.007** **(0.005)**
CRTIC				**-0.046** **(0.022)**	**-0.078** **(0.009)**
Adj. R^2	-0.036	-0.072	0.133	0.278	0.294
Obs.	29	29	27	27	27

[1] Dependent variable: real GDP growth rate in the year to 2008Q4, less the growth rate in the previous year. P-values are in parentheses; bold indicates significance at the 5% level. X equals exports to the US and TIC is gross capital inflows from the US, each as a percentage of GDP, in 2007. CRX and CRTIC are interactive terms, where CR is S&P sovereign foreign currency credit rating in 2007. CR=0 corresponds to a AAA-rating, 1 for AA+, and so on, to 12 for BB-.

The results are given in Table 2, and provide strong support for our argument that financial flows were a strong causal factor in the propagation of the crisis, while trade channels

appear less important. First the export variables (X and CRX) are never economically or statistically significant, and sometimes enter with the wrong sign. Second our measure of capital flows (TIC) is statistically significant in all cases. Third, when we include an interactive term between the credit rating and the size of capital inflows from the US, this not only enters significantly, consistent with flight-to-quality, but it also further strengthens the statistical support for TIC. Finally, the size of the adjusted R-squared statistics is supportive of capital inflows playing an important role in explaining the downturn, while trade channels are of less importance.

In summary, this evidence suggests the possibility that a financial channel may be important for the international propagation of shocks. Moreover, it is difficult to explain the scale and synchronicity of the global downturn based on trade alone.

3. The model

In this section we describe a basic two-country model with levered borrowers and lenders in each country. The countries are called home and foreign. Within each country there are investors and savers, both of whom use the same fixed asset and have infinite horizons. Investors purchase the fixed asset and rent it to production firms, receiving a risky return in exchange. We may think of this investment as the purchase of an equity claim in the production firm. Investors are more impatient than savers, so they will borrow from savers in order to invest in the fixed asset.[4] Savers also make use of the fixed asset in home production. Savers therefore choose a portfolio in which they hold the debt of investors and the fixed asset. By assumption, savers do not hold domestic or foreign equity. In the current version, we assume that savers do not lend to foreign investors. Because the model is symmetric, this makes little difference for the results.

[4] Because they are more impatient than savers, investors will never accumulate enough resources to cover the cost of investment in any period.

Investors in either country, however, may trade claims with investors in the other country so as to diversify their portfolio of equity holdings. Thus investors in each country hold levered investments, but also have equity portfolios that are inter-connected across countries. Finally, both investors and savers in each country supply labor inelastically to production firms.

Investors

We normalize the population of each country to unity, with a measure n of investors and $1-n$ savers. The representative investor in the home country maximizes:

$$(1) \qquad E_t \sum_{s=t}^{\infty} \theta_s^I U(C_s^I),$$

where C_s^I is consumption of the final good. To keep the analysis solely focused on financial inter-linkages between countries, it is assumed that there is just one world good. Adding an endogenous terms of trade to the analysis would enrich the response, but would not fundamentally alter the cross-country transmission of balance sheet adjustments modeled here, so long as the elasticity of substitution across home and foreign goods is not close to unity.[5]

We define the discount factor for investors such that:

$$\theta_{s+1}^I = \beta^I(\bar{C}_s^I)\theta_s^I, \quad \beta^{I\prime}(\bar{C}_s^I) \leq 0,$$

with $\beta^{I\prime}(\bar{C}_s^I) \leq 0$, where $\bar{C}_s^I$ is the economy-wide average consumption of investors. Thus the investor's time preference is increasing in consumption, but the rate of time preference is taken as given by the individual investor. The assumption of endogenous time preference for investors plays the usual role of ensuring a stationary wealth distribution among groups, both within

[5] As is well known, with an elasticity of substitution equal to unity, there is a high cross-country correlation of consumption purely due to the risk sharing implications of terms-of-trade adjustment. This is seen, for instance, in the paper by Dedola and Lombardo (2009). Aghion et al. (2004) explore the importance of terms-of-trade movements in affecting borrowing constraints in an emerging market economy framework. Extending our model to a setting with endogenous terms-of-trade would affect borrowing constraints through the impact of a terms-of-trade adjustment on net worth in a similar way to the effects of asset price changes in the present version of the model.

countries and across countries. But it also plays a key role in allowing for a comparison of an economy in which financial constraints bind with one where they do not bind, as we discuss below.

Investors receive income from their current holdings of domestic and foreign equity, as well as labor income from working in the domestic production firm. In addition, they must repay their debts owed to domestic savers. They then issue new debt, purchase equity claims on home and foreign investments and consume. The home country investor's budget constraint is written as:

$$C_t^I + q_{1t}k_{1t}^I + q_{2t}k_{2t}^I = W_t^I + (q_{1t} + R_{1Kt})k_{1t-1}^I + (q_{2t} + R_{2Kt})k_{2t-1}^I + B_t^I - R_{t-1}B_{t-1}^I, \quad (2)$$

where q_{1t} and q_{2t} represent the price of the fixed asset (or equity) in the home and foreign country respectively, and k_{1t}^I and k_{2t}^I are the portfolio holdings of the fixed assets in each country held by the home investor. The fixed asset of the home (foreign) country earns a return of R_{1Kt} (R_{2Kt}). W_t^I is wage income for the investor, who supplies one unit of labor. Finally, B_t^I is the debt issued to domestic savers and $R_{t-1}B_{t-1}^I$ is payment on previously incurred debt.

One may question why only investors can purchase the fixed assets, which are then used by final goods firms. As in Bernanke et al. (1999), we could assume that investors (or, in their model, entrepreneurs) have some special capability for transforming a unit of the fixed asset into a usable factor of production that is rented to production firms. Lenders cannot do this, and so may gain only indirectly from the investment, by lending to the investors.

In addition to constraint (2), we assume that investors face a constraint on total leverage due to an inability to commit to repayment, as in Kiyotaki and Moore (1997). Total debt is

assumed to be restricted to be no greater than κ times the market value of equity assets, where $\kappa < 1$. Thus home investors' choices are constrained by:

(3) $$B_t^I \leq \kappa(q_{1t}k_{1t}^I + q_{2t}k_{2t}^I).$$

The full leverage rate (the value of assets to capital) for investors is then $1/(1-\kappa)$ in the case where the leverage constraint (3) is binding. We take κ as a free variable in our analysis. Leverage constraints in the form of (3) have been used quite widely in the literature on asset prices (Aiyagari and Gertler 1999), emerging market crises (Mendoza and Smith 2006), borrowing in a small open economy (Uribe 2006) and monetary policy with credit frictions (Iacoviello, 2005). Kiyotaki and Moore (1997) show that κ may depend on the borrowing rate and expected capital gains on equity under some circumstances.

Investors in the home country choose investment in the home equity and the foreign equity, as well as borrowing, to maximize their expected utility subject to their budget constraint (2) and leverage constraint (3), giving the conditions:

(4) $$U'(C_t^I) = E_t\beta^I(C_t^I)U'(C_{t+1}^I)\frac{(q_{1t+1} + R_{1Kt+1})}{q_{1t}} + \kappa\mu_t,$$

(5) $$U'(C_t^I) = E_t\beta^I(C_t^I)U'(C_{t+1}^I)\frac{(q_{2t+1} + R_{2Kt+1})}{q_{2t}} + \kappa\mu_t,$$

(6) $$U'(C_t^I) = E_t\beta^I(C_t^I)U'(C_{t+1}^I)R_t + \mu_t,$$

where μ is the multiplier on the leverage constraint, or equivalently the utility benefit of an extra unit of debt to the investor. If this is positive, it means that the investor would like to borrow more, but is constrained by (3). Therefore current marginal utility is less than expected future marginal utility times the return on investing in either the home or foreign country. Thus μ is a

measure of the value of the opportunity to make a levered investment. To show this, put (4), (5) and (6) together to obtain:

$$\mu_t = E_t \beta^I (C_t^I) U'(C_{t+1}^I) \left[\frac{\omega_t r_{1,t+1} + (1-\omega_t) r_{2,t+1} - R_t}{1-\kappa} \right], \tag{7}$$

where $\omega_t = q_{1,t} k_{1,t} / (q_{1,t} k_{1,t} + q_{2,t} k_{2,t})$ is the share in home equity, $r_{1,t+1} = (q_{1,t+1} + R_{1K,t+1}) / q_{1,t}$ is the return on the home equity and $r_{2,t+1} = (q_{2,t+1} + R_{2K,t+1}) / q_{2,t}$ is the return on the foreign equity. Equation (7) shows that, for a given distribution of excess returns and consumption, μ is higher the higher is the leverage rate. It also implies that, when $\mu > 0$, the expected return on the portfolio, up to the first-order, exceeds the cost of borrowing.

Note that the leverage constraint does not directly affect the investors' incentive to diversify equity holdings across countries, since (3) applies equally to borrowing for domestic or foreign equity purchases. Thus we may put (4) and (5) together to get the standard portfolio selection condition:

$$E_t U'(C_{t+1}^I) \left(\frac{(q_{1t+1} + R_{1Kt+1})}{q_{1t}} - \frac{(q_{2t+1} + R_{2Kt+1})}{q_{2t}} \right) = 0 . \tag{8}$$

Given that the portfolio choice may be written in the form (8), we can use the recent methods described in Devereux and Sutherland (2009a) to derive the optimal equity portfolio of each country's investors. This involves using a second-order approximation of condition (8) in conjunction with a linear approximation of the remaining aspects of the model. We discuss the details involved in portfolio choice more fully below.

Savers

Savers have preferences given by:

$$E_t \sum_{s=t}^{\infty} \theta_s^S U(C_s^S) . \tag{9}$$

Again, as for investors, we define the discount factor such that $\theta_{s+1}^S = \beta^S(\bar{C}_s^S)\theta_s^S$, with $\beta^S{}'(\bar{C}_s^I) \leq 0$, where $\bar{C}_s^I$ is the economy-wide aggregate consumption of savers. We make the assumption that savers are inherently more 'patient' than investors in the sense that:

(10) $$\beta^S(x) > \beta^I(x),$$

for all feasible values of x. Assumption (10) ensures that savers will lend to investors, even in a steady state where the leverage constraint (3) is not binding.[6]

Savers purchase the fixed asset, and buy debt from investors. They receive wage income from working in the final goods sector, and returns on their lending to investors. In addition, they have a residual 'home production function' that uses the fixed asset. Thus an individual saver owning $k_{1,t}^S$ of the fixed asset produces $G(k_{1,t}^S)$ in terms of home production, where $G'(k_{1,t}^S) < 0$. For simplicity, we assume that home production is perfectly substitutable with the final good in savers' preferences. With this assumption, we may write the saver's budget constraint as:

(11) $$C_t^S + q_{1t}k_{1t}^S = W_t^S + q_{1t}k_{1t-1}^S + G(k_{1t-1}^S) + B_t^S - R_{t-1}B_{t-1}^S.$$

Note that, by assumption, savers purchase only the domestic fixed asset. They do not have access to the same investment opportunities as investors and therefore only have use for the domestic fixed asset, as it may be utilized in home production. On the other hand, savers' purchases of debt from investors are unconstrained.

The first-order conditions for the optimal choice of k_{1t}^S and B_t^S are simply:

(12) $$U'(C_t^S) = E_t\beta^S(C_t^S)U'(C_{t+1}^S)\frac{(q_{1t+1} + G'(k_{1,t+1}^S)}{q_{1t}},$$

[6] An alternative, but considerably more difficult, approach to achieving an equilibrium with levered investment is to assume that investors are less risk averse than savers. Solving a model with leverage based on risk preferences would be substantially harder than the approach we follow, because we would need to solve the full stochastic model to a higher order of approximation.

(13) $$U'(C_t^S) = E_t \beta^S (C_t^S) U'(C_{t+1}^S) R_t .$$

Production firms

Production firms in each country hire capital and fixed assets in order to produce. Firms are competitive, and maximize profits given the production function:

(14) $$Y_t = A_t F(L_t, K_t),$$

where L_t is effective employment and K_t is the firm's use of the fixed asset. We allow for labor supplied by investors and savers to have different fixed productive content. Thus $L_t = \eta^I L_t^I + \eta^S L_t^S$, where η^I and η^S are fixed effective productivity factors. Profit maximization then implies that:

(14) $$W_t^I = \eta^I A_t F_1(L_t, K_t),$$

(15) $$W_t^S = \eta^S A_t F_1(L_t, K_t),$$

(16) $$R_{1K,t} = A_t F_2(L_t, K_t).$$

Equilibrium

Equilibrium of the two-country world economy entails market clearing for the world market of the fixed asset, as well as each country's debt market. Thus, for the home economy, it must be the case that:

(17) $$nk_{1,t}^I + nk_{1,t}^{*I} + (1-n)k_{1,t}^S = 1,$$

(18) $$nB_t^I + (1-n)B_t^S = 0,$$

where $k_{1,t}^{*I}$ represents foreign country investors' real holdings of the home asset at the beginning of time $t+1$. In addition, the world market clearing condition must be satisfied:

$$n(C_t^I + C_t^{*I}) + (1-n)(C_t^S + C_t^{*S}) = A_t F(\eta^I n + \eta^S (1-n), n(k_{1,t}^I + k_{1,t}^{*I}))$$

(19)

$$+A_t^* F(\eta^{*I} n + \eta^{*S}(1-n), n(k_{2,t}^I + k_{2,t}^{*I})) + (1-n)(G(k_{1,t}^S) + G(k_{2,t}^{*S})).$$

This condition incorporates the fact that the total labor supply of investors and savers is n and $1-n$ respectively, and total use of the fixed factor by final goods firms is equal to total holdings by domestic and foreign investors. The full equilibrium is then described by equations (2)-(6) and (11)-(18) for both the home and foreign country, and the world market clearing condition (19). This gives 27 equations in the 26 variables C_t^I, C_t^S, C_t^{*I}, C_t^{*S}, $k_{1,t}^I$, $k_{2,t}^I$, $k_{1,t}^S$, $k_{1,t}^{*I}$, $k_{2,t}^{*I}$, $k_{2,t}^{*S}$, B_t^I, B_t^S, B_t^{*I}, B_t^{*S}, $q_{1,t}$, $q_{2,t}$, R_t, R_t^*, μ_t, μ_t^*, W_t^I, W_t^S, W_t^{*I}, W_t^{*S}, $R_{1K,t}$ and $R_{2K,t}$, with one equation redundant by Walras' law.

Properties of the steady state

Before examining the dynamics of balance sheet adjustments within the model, we first discuss some properties of the non-stochastic steady state. This is particularly easy in the case of $\mu = \mu^* = 0$, which is when leverage constraints do not bind. Then it follows from a combination of (4) and (6), together with (12) and (13), that the fixed asset is allocated efficiently between the final good sector and home production. That is, for the home economy, we have:

(20) $$G'(k_1^S) = AF_2(L, n\hat{k}_1^I),$$

where $\hat{k}_1^I = k_1^I + k_1^{*I}$ represents the total quantity of the fixed asset used in the final goods production sector. Thus the fixed asset is allocated efficiently in the sense that its marginal product is equalized between home production and final goods production.

In combination with the resource constraint $n\hat{k}_1^I + (1-n)k_1^S = 1$, this uniquely determines the allocation of assets in final goods production. Therefore there is no interdependence across countries in asset allocation in steady state when leverage constraints do not bind. Hence output levels are independent across countries – a permanent increase in productivity A affects home output, but not foreign output.

In fact, we can extend this result further. In the case where leverage constraints never bind, it is easy to see that there is no interaction between asset allocations across countries at all, at least up to a first-order approximation. This can be seen by taking a linear approximation of (4), (6), (12) and (13) to obtain the condition:

$$\gamma_1 \frac{dk_{1,t+1}^S}{k_1^S} = E_t \frac{dA_{t+1}}{A} + \gamma_2 \frac{d\hat{k}_{1,t+1}^I}{\hat{k}_1^I},$$

where γ_1 and γ_2 are constant coefficients. Hence the dynamic paths of asset allocations are independent across countries in the absence of balance sheet constraints.[7] Note that this holds despite the fact that, up to a first-order, expected returns on all assets are equalized both within and across countries.

A more general feature of this environment is that portfolio holdings have no feedback effect on asset allocations. That is, allocation of the fixed asset between home production and investment in the final good is independent of the ownership of equities. This property will not hold in an economy with binding leverage constraints.

When leverage constraints bind, we again use (4), (6), (12) and (13) to obtain the steady state condition:

[7] Of course this is not a robust feature, and would be altered in a model with endogenous labor supply or capital accumulation. But the main point here is to show that the presence of balance sheet constraints introduces substantial additional forces for cross-country correlations that would otherwise be absent.

(21) $$G'(k_1^S) = \frac{\beta^I(1-\beta^S)}{\beta^S(1-\beta^I)-\kappa(\beta^S-\beta^I)} AF_2(L, n\hat{k}_1^I).$$

From condition (3) it must be true that $\beta^I(1-\beta^S)/[\beta^S(1-\beta^I)-\kappa(\beta^S-\beta^I)]<1$ so it follows that, under binding leverage constraints, the final goods sector has an inefficiently low level of the fixed asset. More generally, however, since discount factors are endogenous, the allocation of fixed assets across sectors will no longer be independent across countries. Asset allocation in the home country will depend on the level of productivity in the foreign country. Intuitively this holds because, with free trade in equities across countries, returns to investors must be equal in both countries. Since returns interact with movements in consumption through the endogenous rate of time preference, (21) shows that the division of resources between home production and final goods must be linked across countries as well. It therefore follows that, unlike the case without binding leverage constraints, the cross-country ownership of equity holdings will in general have implications for the allocation of investment in fixed assets.

Even in the case of constant time preference, however, the presence of leverage constraints would still imply a dynamic interaction between output levels across countries that is absent without these constraints, because productivity shocks to one country will affect the tightness of leverage constraints across all financial markets. We explore this in detail below.

Portfolio choice

We have already solved for the overall allocation of the fixed asset in each country in steady state, but not the ownership structure of equities. Thus, while $\hat{k}_1^I$ is determined by (20) or (21) with non-binding or binding leverage constraints respectively, k_1^I and k_1^{*I} are not yet determined. Clearly, in order to analyze the dynamic response to productivity shocks in one country, it is necessary to understand the structure of equity holdings. To do this, we follow

Devereux and Sutherland (2009a) in using a second-order expansion of (8) to obtain an approximation of optimal portfolio holdings.

Since only investors have access to equity markets by assumption, it is sufficient to look at the portfolio decisions of home and foreign investors. To illustrate the application of Devereux and Sutherland (2009a) to the present model, take the budget constraint for home country investors (1). This may be rewritten as:

$$(22) \qquad C_t^I + NFA_t = W_t^I + R_{1Kt}\hat{k}_{1t-1}^I - q_{1t}(\hat{k}_{1t}^I - \hat{k}_{1t-1}^I) + r_{2t}NFA_{t-1} + r_{xt}\left[q_{1t-1}(k_{1t-1}^I - \hat{k}_{1t-1}^I)\right] + B_t - R_{t-1}B_{t-1},$$

where NFA_t denotes net foreign assets, defined as $NFA_t = q_{2t}k_{2t}^I - q_{1t}(\hat{k}_{1t}^I - k_{1t}^I)$, and r_{xt} is the *excess* return on the portfolio:

$$r_{xt} = r_{1t} - r_{2t} \equiv \left(\frac{q_{1t} + R_{1K,t}}{q_{1t-1}}\right) - \left(\frac{q_{2t} + R_{2K,t}}{q_{2t-1}}\right).$$

For given NFA_t, the portfolio choice may be described as the choice of $\alpha_t = q_{1t-1}(k_{1t-1}^I - \hat{k}_{1t-1}^I)$, which is the net holding of home country equity by home agents. If $\alpha_t < 0$, the investors diversify in the sense that less than 100 percent of all home equity is owned by home investors. Devereux and Sutherland (2009a) show that, when the model is analyzed up to a first-order approximation, α_t is a constant and is determined by a combination of a second-order approximation of (8), together with a first-order approximation of the rest of the model.

In the solution below, following Tille and van Wincoop (2007), we extend (8) to allow for transactions costs of international financial trade that effectively limit international portfolio diversification. This represents a brute-force technique for generating an equilibrium with home equity bias. In particular, we assume that an 'iceberg' cost factor given by $\exp(-\tau) \leq 1$ reduces the returns that home investors receive from foreign investment so that condition (8) becomes:

(8') $$E_t U'(C^I_{t+1})\left(\frac{(q_{1t+1}+R_{1Kt+1})}{q_{1t}}-\frac{(q_{2t+1}+R_{2Kt+1})}{q_{2t}}\exp(-\tau)\right)=0\,.$$

In addition, we follow Tille and Van Wincoop in assuming that τ is a small, second-order term. This means that while it does affect the solution for the equilibrium portfolio, which is evaluated using a second-order approximation of (8'), it does not impact on the first-order dynamics of the model, except insofar as it affects the choice of the portfolio itself.

Note that given the revised definition of net foreign assets, the leverage constraint for home country investors becomes:

(23) $$B_t \le \kappa(NFA_t + q_{1t}\hat{k}^I_{1t})\,.$$

Thus, holding home asset prices constant, an increase in net foreign assets generated by either a current account surplus or a capital gain on the external portfolio will loosen the leverage constraint. But since $NFA_t + NFA^*_t = 0$, this will simultaneously tighten the leverage constraint facing foreign investors. Thus the degree to which leverage linkages govern the transmission of shocks across countries depends on the dynamics of net foreign assets, and these in turn are linked to the portfolio choices made by home and foreign investors.

Calibration

Because the model is such a stripped-down representation of a full-scale DSGE framework, lacking capital accumulation and dynamics in the labor supply and containing only a single world good, there are many dimensions in which the model's predictions will depart from reality. The aim of the exercise is solely to explore the way in which financial leverage constraints affect the cross-country dynamics of asset prices, asset allocations and levered investments, and to investigate the international transmission of balance sheet contractions. To do this, however, we need to choose parameter values for preferences, production technologies

and the leverage constraint itself. Table 3 gives the set of parameter values used in the baseline model.

We assume that the measure of investors and savers is equal, so that $n = 0.5$. In the leverage constrained economy, this accords with the estimates of Campbell and Mankiw (1990) regarding the share of households that are subject to credit constraints in the US economy.

Table 3
Calibration

Parameter	Value	Parameter	Value
n	0.5	ε	0.5
η	0.01	ω	0.36
ζ	Discount factors 0.96 and 0.94	σ	5
κ	0.5, 0.8.	ρ	0.9

We assume a discount factor defined as:

$$\beta(C) = \zeta C^{-\eta}.$$

We set $\eta = 0.01$, and choose ζ so that, in a steady state with binding leverage constraints, lenders and borrowers have discount factors of 0.96 and 0.94 respectively. The parameter κ directly determines the total value of new assets that investors can borrow. Since the model is calibrated in a symmetric way, net foreign assets are zero in steady state so that investors' net worth, measured as total assets less debt, equals $q\hat{k}^I(1-\kappa)$.

Total leverage (investment relative to capital) is equal to $1/(1-\kappa)$. This leverage ratio has a significant affect on the model's dynamics quantitatively. We examine two alternatives. First we choose a relative low ratio of 2 ($\kappa = 0.5$), as in Bernanke and Gertler (1999). In response to

the discussion of the high rates of leverage seen in the financial system in recent years we also explore the implications of a higher value of $\kappa = 0.8$, corresponding to total leverage of 5. From a qualitative point of view, we will see that the results are very similar for both leverage ratios.

We assume a Cobb-Douglas final goods production technology, and let $F(L,K) = L^{\varepsilon} K^{1-\varepsilon}$. In order to have substantial propagation effects from leverage constraints, Kiyotaki and Moore (1997) require that production in the borrowing sector is linear in the fixed asset. Kocherlakota (2000) shows that, with a more conventional calibration allowing for decreasing returns, credit constraints have much less impact. We set $\varepsilon = 0.5$, implying substantial decreasing returns, yet find substantial effects of leverage constraints, as we will see. Our choice of ε implies that fixed assets are slightly more important than conventional measures of capital's share in calibrations of the US economy.[8] In addition we assume that effective labor productivity of savers and lenders is initially equal, so that $\eta^I = \eta^S = 1$.

Regarding the home production sector, we assume that $G = Z(k_1^S)^{\omega}$ and $\omega = 0.36$, implying that the fixed asset is less important in this sector, consistent with convention. We set $A = Z = 1$ in steady state. The combination of these assumptions implies that, in steady state, 80 percent of the fixed asset is employed in final goods production. We follow the asset pricing literature (see, for example, Bansal and Yaron 2004) in setting a relatively high degree of risk aversion with $\sigma = 5$ in $U(C) = C^{1-\sigma}/(1-\sigma)$. Lower values of σ reduce the volatility of asset prices, but have little qualitative effect on the results otherwise.

We focus on shocks to the productivity of final goods in each country. The stochastic process for final goods productivity is modeled as:

$$\log(A_t) = \rho \log(A_{t-1}) + \upsilon_t, \tag{24}$$

[8] For many emerging market economies, however, estimates of capital share equal to 50 percent are quite common.

where $\rho = 0.9$, $E_{t-1}\upsilon_t = 0$ and $\sigma_\upsilon^2 = 0.02^2$. We assume that foreign productivity is driven by the same process, and foreign and domestic productivity shocks are uncorrelated.

4. Effects of productivity shocks

In this section we look at how the joint process of balance sheet constraints with portfolio interdependence affects the model's response to a productivity shocks in one country. The focus on a productivity shock is not of central importance. The main elements of the propagation mechanism hold also for other shocks. The key aspect of the model is to show that the two features of portfolio interdependence and balance sheet constraints introduce a substantial process of macroeconomic co-movement that is absent when these features are not present.

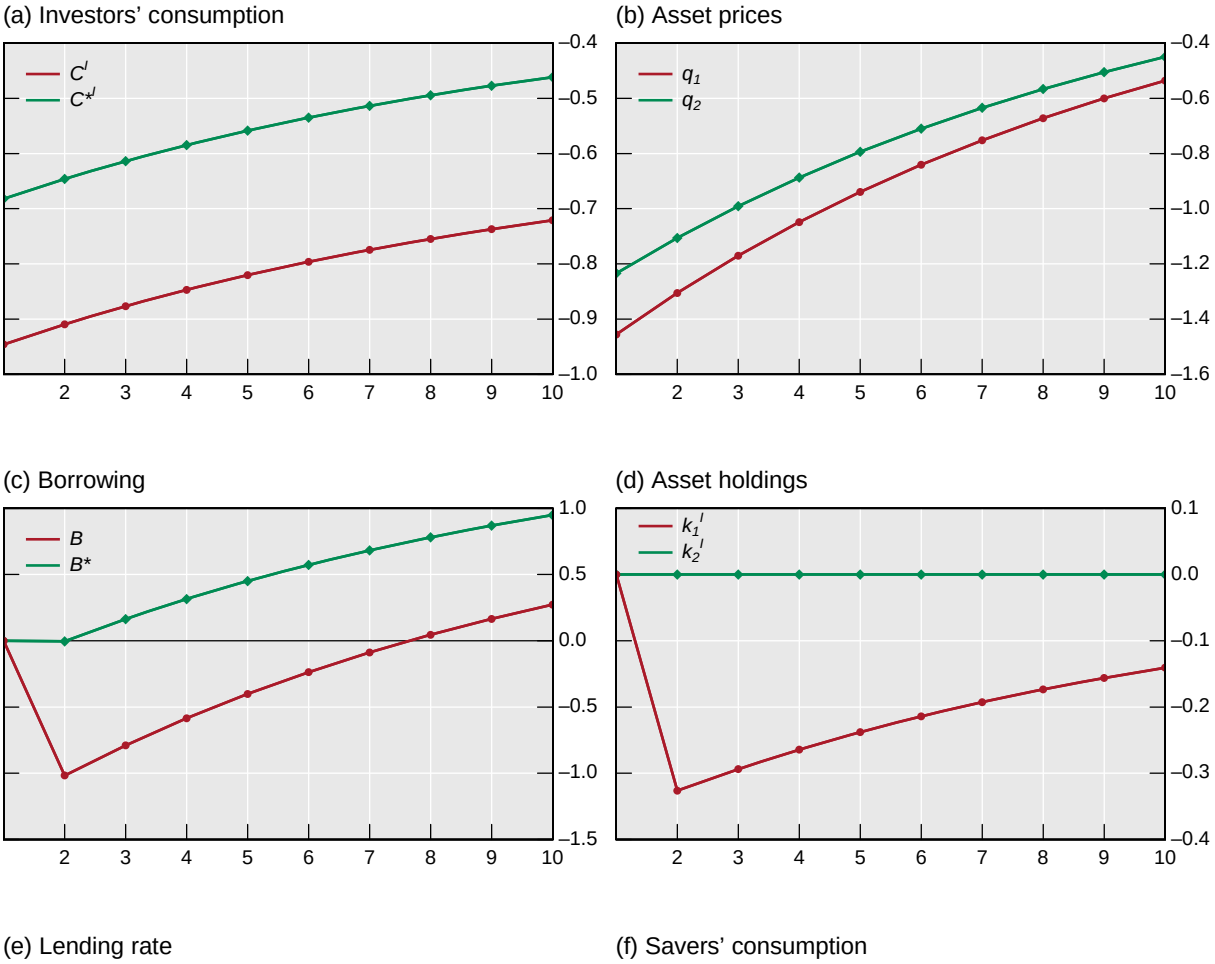
No leverage constraints

We first examine the impact of a 1 percent negative productivity shock in the home country, in the environment without leverage constraints. Figures 3 and 4 describe the impact of the shock on consumption of investors, asset prices, lending by savers, asset allocation, the internal lending rate and the consumption of savers. Figure 3 represents the case where portfolio diversification is restricted by second-order transactions costs as described above, while Figure 4 describes the case of unrestricted portfolios. In the unrestricted case, investors in the home country choose values for k_1^I and k_2^I to satisfy (8'), evaluated up to second-order, with $\tau = 0$. This involves home investors having a bias *against* home equities. Investors are exposed to non-diversifiable risk from wage income, which is positively correlated with the return on home equity. With an unrestricted portfolio, they will hedge this risk by taking a larger position in foreign equity than home equity, as discussed in Baxter and Jehrmann (1997). Given the calibration of the model, in an unrestricted equilibrium $k_1^I = .2\hat{k}_1^I$. That is, home investors would

hold only 20 percent of total home equity (i.e. 20 percent of the fixed assets which are invested in the home final goods technology), with foreign investors holding the remaining 80 percent.

Figure 3

No leverage constraints, partial diversification



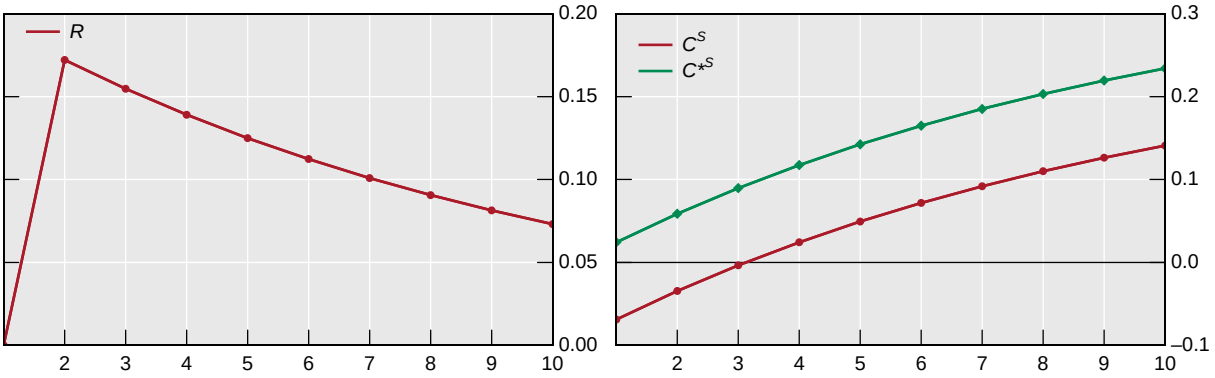
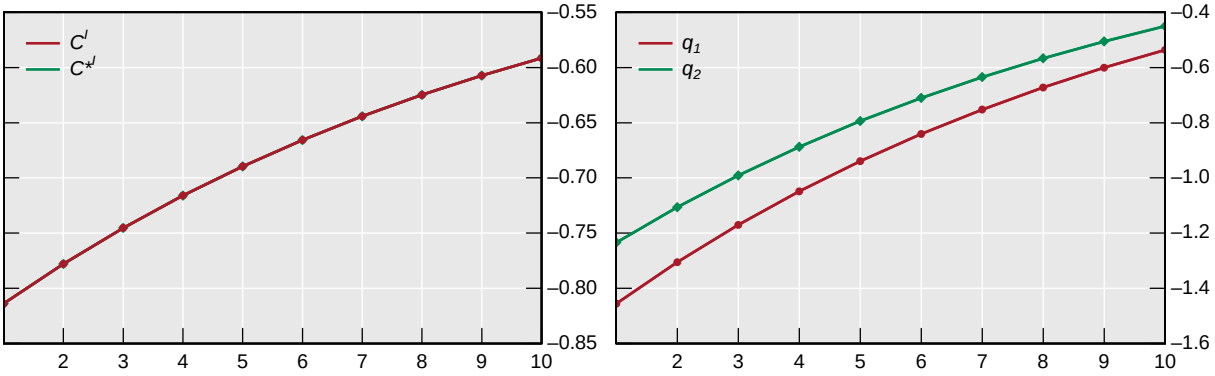


Figure 4

No leverage constraints, full diversification

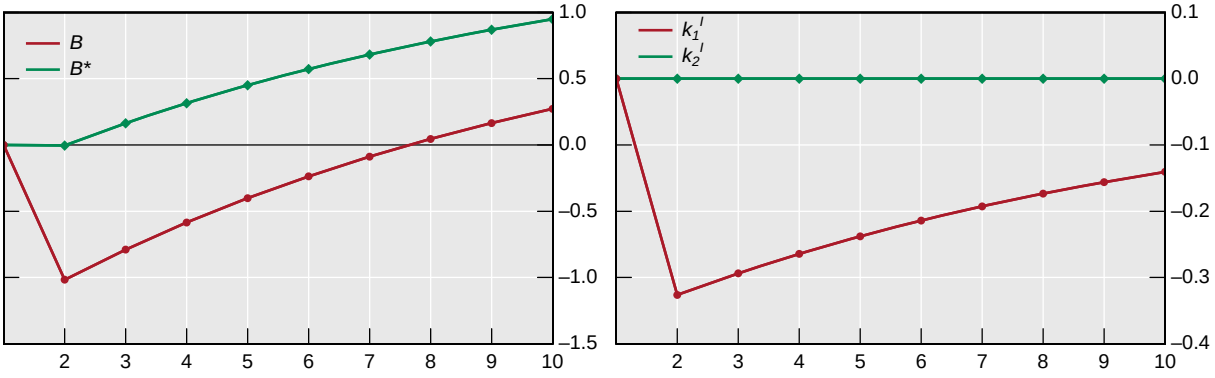
(a) Investors' consumption

(b) Asset prices



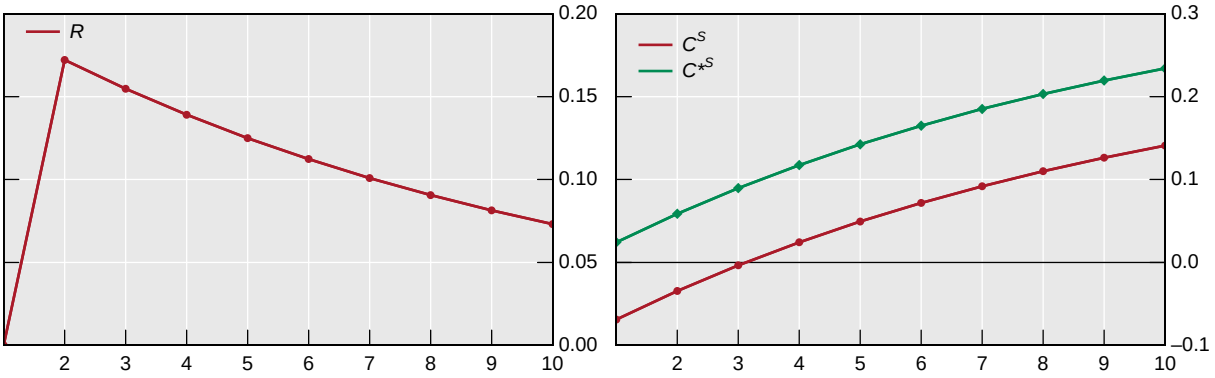
(c) Borrowing

(d) Asset holdings



(e) Lending rate

(f) Savers' consumption



Since this is clearly counterfactual, we use the iceberg cost variable τ as a crude mechanism to match the optimal portfolios more closely with observed home bias in equity holdings. In Figure 3 τ has been chosen so that $k_1^I = .75\hat{k}_1^I$, implying that home investors hold 75 percent of home equity. Figure 4, by contrast, illustrates the counterfactual case where there is full consumption risk sharing for investors due to unrestricted diversification.

The responses to a productivity shock are quite similar in each of the figures. Without leverage constraints, the impact of a fall in home country productivity is to reduce consumption of investors in both countries, by identical amounts in the case of unrestricted diversification. The shock represents a temporary fall in consumption. Since consumption is expected to increase in the future, real interest rates must rise. The combination of a lower return on the home asset and rising real interest rates means that the home asset price must immediately fall.

Without leverage constraints, all returns are equalized, at least up to a first-order approximation, for investors to be willing to hold all assets in their portfolios. Thus the price of foreign assets must also fall. That is, arbitrage implies that the rate of return to lenders rises by the same amount in both countries, even though lenders do not directly engage in international borrowing or lending. But the pattern of lending moves in different directions in the two countries, as do lenders' portfolios. In the home country, there is a fall in investment in the fixed

asset in the final goods sector simply because this sector has suffered a persistent negative technology shock. This leads to an increase in the holdings of the fixed asset by lenders. They shift the composition of their portfolios from debt towards increased holdings of the fixed asset. Thus lending falls in the home country. In the foreign country, by contrast, there is no change at all in the allocation of the fixed asset. But lending in the foreign country actually rises, as investors borrow more from lenders in order to cushion against the temporary fall in their investment income.

A different way to see this is that in the foreign country, lenders are offered a higher rate of return on their lending, and are willing to purchase more debt from foreign investors. Either way we look at it, lending rises in the foreign country, while it falls in the home country. In this sense, there is no international transmission of balance sheet contractions.

The impact of the shock on lenders' consumption in the two countries also moves in opposite directions. Lenders in the home country lose, since they suffer a direct fall in their wage income. Lenders in the foreign country gain, since they lend more at higher interest rates, and their wage income and holdings of the fixed asset are unaffected. Clearly lenders cannot achieve full consumption risk-sharing, since they cannot directly hold a claim on the equity of the other country.

In the economy without leverage constraints, then, the international transmission of shocks is limited, and clearly counterfactual, relative to the discussion of the empirical evidence of financial spillovers in section 2. A negative productivity shock in the home country leads to balance sheet contractions domestically, as investors reduce both their borrowing and holdings of fixed assets. But there are no foreign balance sheet contractions. Investment in fixed assets is completely unaffected in the foreign country, and foreign investors actually increase their

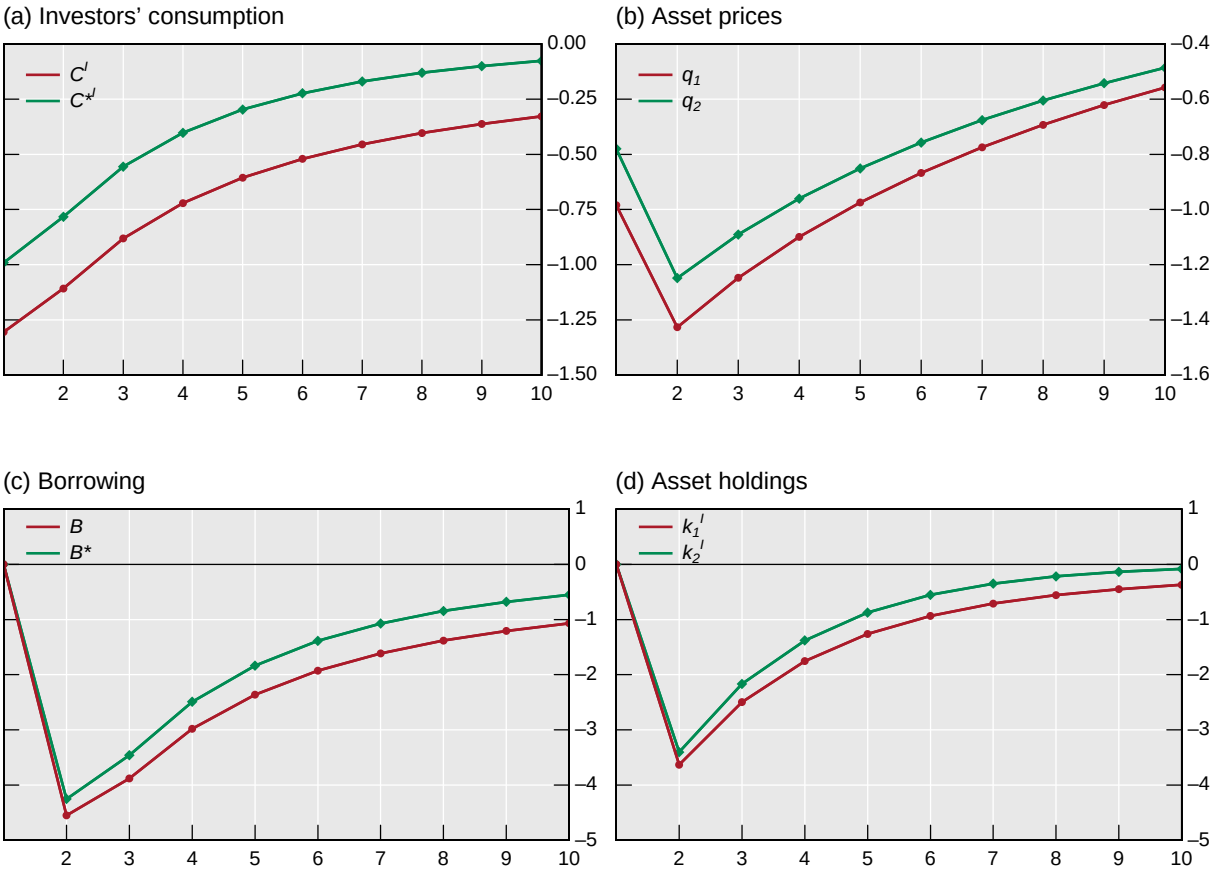
borrowing. More critically, there is no international transmission of the shock to GDP at all. Since the foreign asset allocation is unaffected by the domestic shock, foreign output is unchanged. Thus, in the absence of credit market imperfections, the possibility for the international transmission of shocks through balance sheet contractions is limited.

Leverage constraints and international transmission

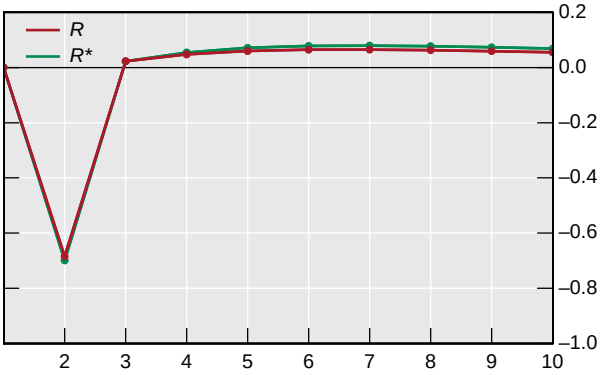
Figures 5-8 show the impact of a negative productivity shock in the home country in the model when leverage constraints bind in both countries. Figures 5 and 6 illustrate the case where the leverage ratio is 5, the former when investors' portfolios are only partly diversified due to the presence of transactions costs and the latter with unrestricted portfolio diversification.

Figure 5

High leverage constraints, partial diversification



(e) Lending rate



(f) Savers' consumption

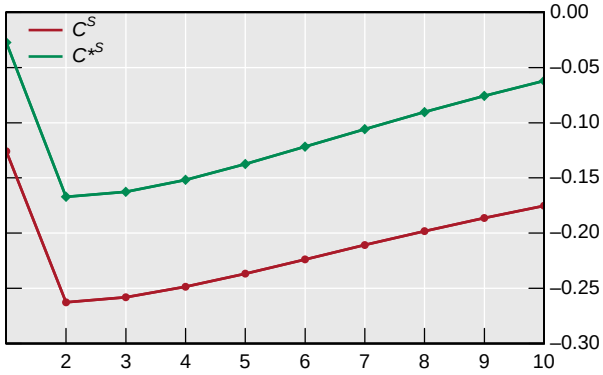
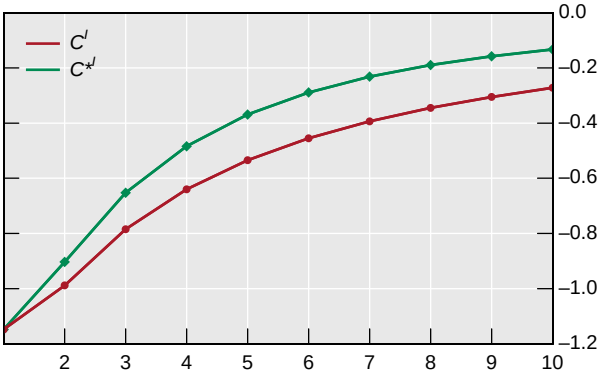


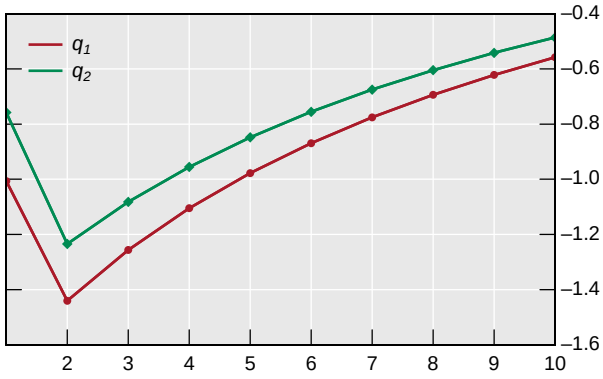
Figure 6

High leverage constraints, full diversification

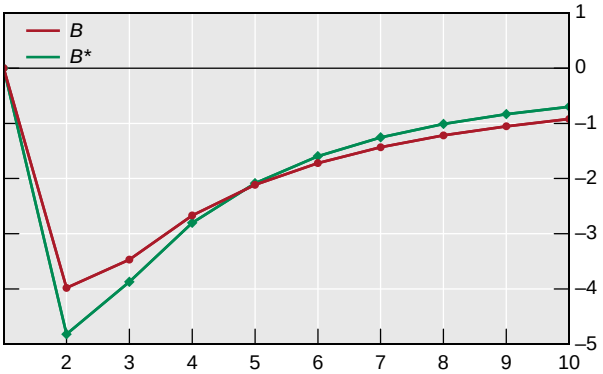
(a) Investors' consumption



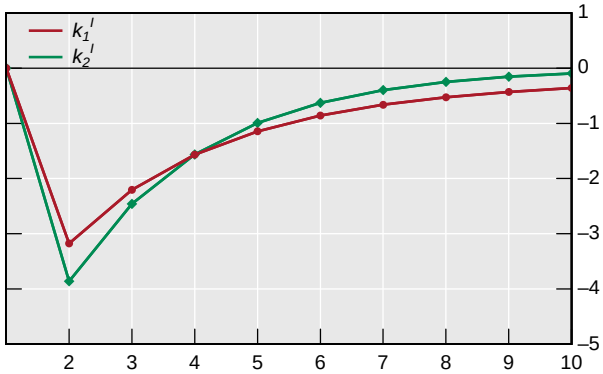
(b) Asset prices



(c) Borrowing

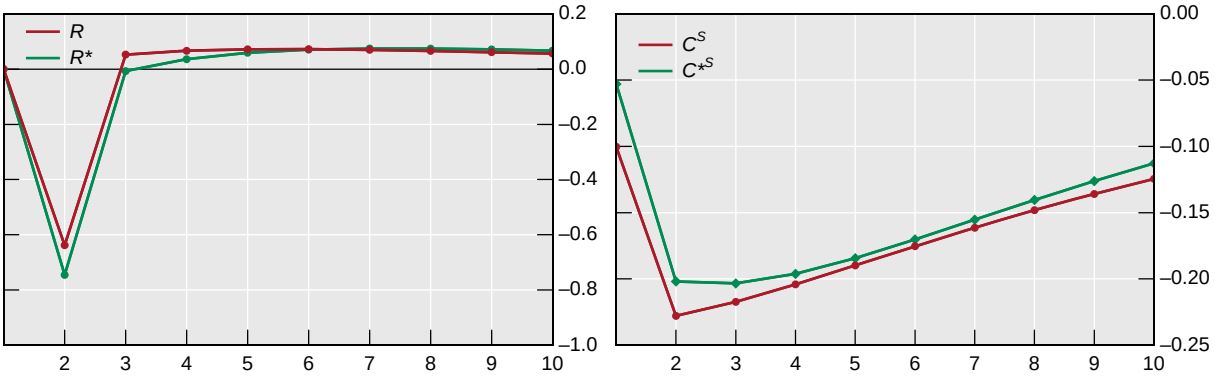


(d) Asset holdings



(e) Lending rate

(f) Savers' consumption



Figures 7 and 8 illustrate the case of a lower leverage ratio of 2, in the case of partial and full portfolio diversification respectively. In all cases there is a clear pattern of global balance sheet contractions in response to the shock.

With high leverage and unrestricted portfolios (Figure 6), the home investor wishes to hold only 47 percent of total home equity. While there is still some 'foreign equity bias' here, it is far less than in the economy without leverage constraints. That is, in equilibrium without portfolio transactions costs, investors wish to hold more of their own equity when there are leverage constraints than when there are not. The reason is that the transmission of balance sheet contractions across countries with binding leverage constraints will make equity returns more positively correlated, as we see below. As a result, the gains from equity diversification are lessened. In Figure 5 we calibrate τ so that investors hold 75 percent of domestic equity, as in Figure 3.

Without leverage constraints, the fall in home country productivity leads to a fall in asset prices in both countries, and a fall in investors' consumption. But now the fall in asset prices leads to a tightening of the leverage constraint, both in the home and foreign countries. The result is a reduction in borrowing by investors in both countries, and a consequent reduction in investment in fixed assets. Note that, for the foreign country, there is no direct fall in the

productivity of the domestic final goods sector. The fall in investment takes place purely through balance sheet linkages.

In addition, the price of foreign equity falls. But in spite of there being no *direct* shock to $R_{2K,t+1}$, and a fall in the price of the asset, there is still a fall in demand for the asset by investors in both countries. This is the essence of the 'inverted demand curve' for assets that characterizes episodes of balance sheet contractions, emphasized by Aiyagari and Gertler (1999). Here it is taking place as a spillover from one levered investor to another, as emphasized by Krugman (2008). That is, a fall in the price of the asset held by one investor leads to a tightening of leverage constraints and a fall in demand for both the original asset and other assets held in the portfolio.

Even in the case where portfolios are only partly diversified, there is a very high correlation across countries in borrowing and investment. The balance sheet contraction is so great that the internal lending rate in each country immediately falls. Again, note that this is in response to a temporary shock so that future consumption of investors is expected to increase. But because investors are subject to leverage constraints, the path of their consumption is de-linked from the path of interest rates. To see this more clearly, note from (6) that there is a conflict between the Fisherian determinants of real interest rates and the effect of binding leverage constraints. Since consumption falls for both home and foreign investors, but is expected to rise in the future, real interest rates should rise. But this effect is more than offset by the increase in the shadow price of borrowing due to the leverage constraint. The fall in asset prices leads to such a large balance sheet contraction in both countries, and correspondingly a large rise in the shadow price of borrowing, that the real interest rate offered by lenders falls rather than rises.

The fall in asset prices is of a similar order of magnitude in the leverage-constrained economy as in the unconstrained economy. Asset prices display a V-shaped response, however, falling by less immediately than in the second period. This is due to the fall in lending rates. Since lenders are unconstrained, the fall in returns on lending must be accompanied by a fall in the expected returns on the lenders holding fixed assets for home production. Hence, immediately following the shock, asset prices are expected to fall further.

Note that there is a distinct difference between the constrained and unconstrained economy, not just in the direction of the international transmission of shocks, but also in the scale. In the unconstrained economy, a one percent decline in final goods' sector productivity leads to an approximately one percent fall in borrowing from lenders, but only a 0.3 percent reduction in fixed asset investment. Not only is there an absence of international transmission via balance sheet contractions, but the domestic impact of the shock is also relatively mild. By contrast, the response of the constrained economy is larger by orders of magnitude. Borrowing falls by almost 5 percent in the home economy and investment in fixed assets by almost 4 percent. Even in the foreign economy, the multiplied effect of the shock is very large – investment falls by over 3 percent and borrowing by 4 percent. In both countries, the response to the shock is proportionally much larger than the shock itself, due to the interaction of asset price declines and binding leverage constraints.

Consumption of home and foreign investors also falls by more in the constrained economy than in the unconstrained economy, although the decline is less persistent with leverage constraints. Also, in contrast to the economy without constraints, consumption falls for all categories of households, both for investors and lenders, in both countries. The fall in interest

rates on lending in the foreign country eliminates the positive wealth effect we saw foreign lenders receive in the unconstrained economy.

When portfolio diversification is unrestricted, Figure 6 shows that the international transmission of the shocks is heightened even more. In fact, we get the surprising prediction that the size of the balance sheet contraction and disinvestment is greater in the foreign country than in the home country! Borrowing falls by 5 percent in the foreign economy, and investment 4 percent, while the equivalent reductions in the home economy are 4 percent and 3.5 percent respectively.

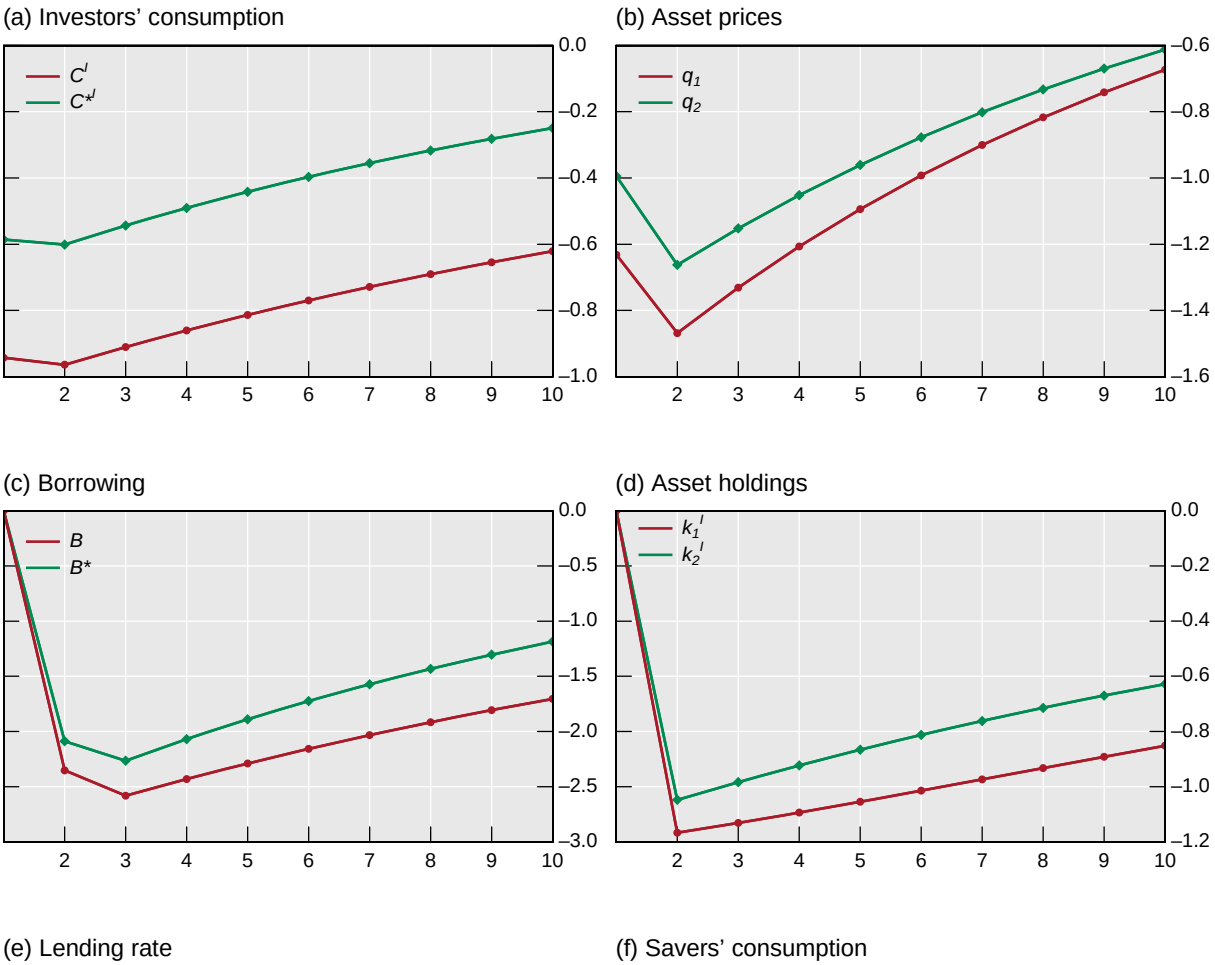
Why does greater portfolio diversification magnify the international propagation effects of the shock? The reason is clear from (3) or (23). Increased diversification leads to a greater sensitivity of foreign balance sheets to the domestic asset price, and therefore a larger balance sheet contraction in response to a negative shock to home productivity. In the completely unrestricted portfolio equilibrium (when $\tau = 0$), the foreign country is over-weighted in home equity, as part of an optimal risk sharing arrangement. So the macroeconomic reverberations of the shock are *greater* in the foreign country than the domestic country, despite the fact that the actual shock takes place only in the home economy. Thus the ownership pattern of equity holdings, and its consequent implication for balance sheet constraints in this case, are more important for the business cycle response than is the geographical source of macro shocks.

A second important feature of Figure 6 is that full portfolio diversification cannot ensure full consumption risk sharing in the leverage-constrained economy, unlike the case of an economy without leverage constraints. This is because the evolution of domestic interest rates, and hence the domestic debt burden facing investors, is no longer identical across the two countries. Since equity market diversification cannot diversify risk associated with country-

specific interest rate movements, unrestricted equity market integration (i.e. when $\tau = 0$) no longer achieves full consumption risk sharing in response to productivity shocks. Figure 6 shows that home and foreign investors' consumption are initially equalized, but foreign consumption rises above home consumption because the home lending rate is higher during the transition to a steady state.

Figure 7

Low leverage constraints, partial diversification



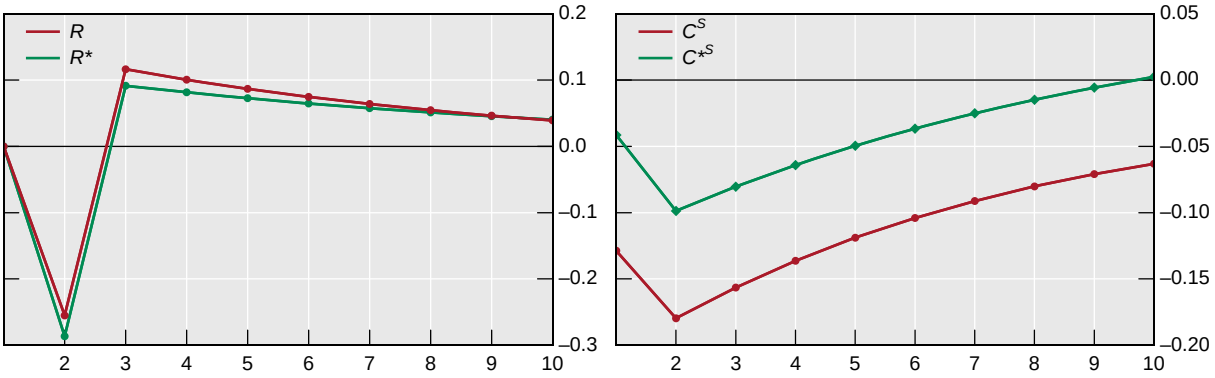
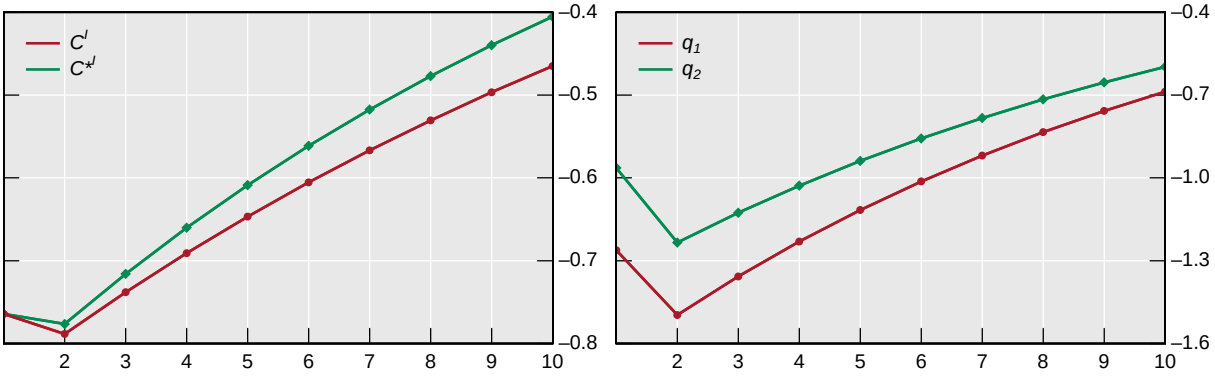


Figure 8

Low leverage constraints, full diversification

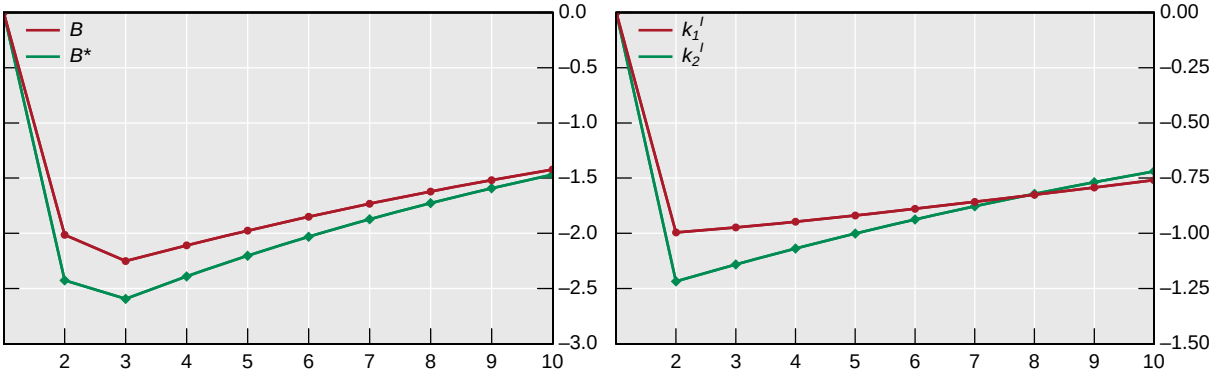
(a) Investors' consumption

(b) Asset prices



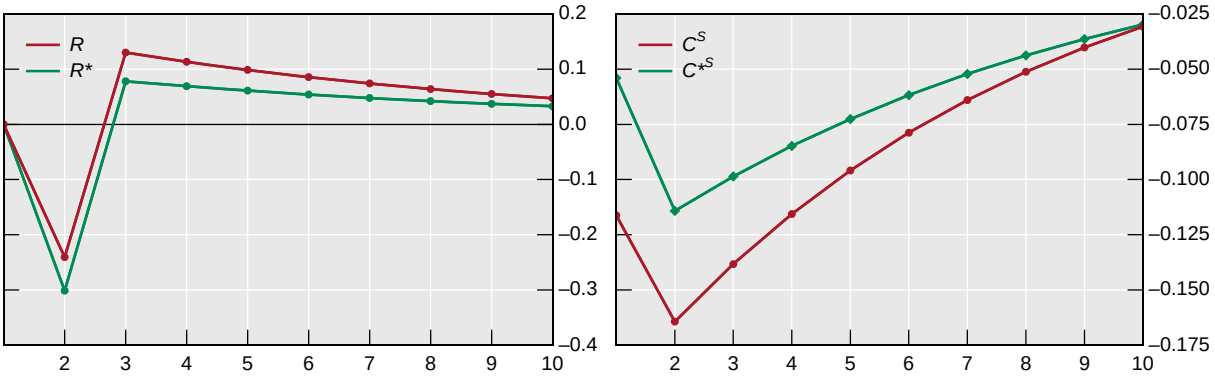
(c) Borrowing

(d) Asset holdings



(e) Lending rate

(f) Savers' consumption



Figures 7 and 8 illustrate the response to a home productivity decline in the case where leverage is lower ($\kappa = 0.5$), when portfolio diversification is partial and full respectively. In Figure 8, full diversification entails the home country investors holding only 25 percent of the home equity, while in Figure 7 home country investors are constrained to hold 75 percent of home equity. The pattern of responses is very similar to that in the high leverage case, but the magnitude of the responses is lower – borrowing and asset disinvestment are both less than in the previous case, although there is still a substantial multiplier effect of the original shock on total leverage.

Unconditional moments

Table 4 reports the unconditional moments of the model under the assumption that productivity shocks in both countries follow identical but uncorrelated distributions, given by (24).

Table 4
Unconditional moments

Leverage constraints	None	None	High	High	Low	Low
Diversification	Partial	Full	Partial	Full	Partial	Full
SDEV(C^I)	2.9	3.0	3.9	4.0	3.1	2.9

SDEV(C^S)	0.4	0.5	0.8	0.9	0.5	0.5
SDEV(q)	4.7	5.2	4.7	4.9	5.1	5.0
SDEV(R)	0.6	0.7	2.0	2.0	0.9	0.9
SDEV($\hat{k}^I$)	0.8	0.9	11.0	11.3	4.2	4.0
CORR(q_1, q_2)	0.98	0.99	0.99	0.99	0.99	0.98
CORR($\hat{k}_1^I, \hat{k}_2^I$)	0.02	0.04	0.99	0.98	0.99	0.98
CORR(R, R^*)	-	-	0.99	0.99	0.99	0.97

As is evident from the figures, the model with binding leverage constraints displays substantially more overall volatility than the model where leverage constraints are absent. Consumption of investors is 50 percent more volatile with high leverage constraints, and consumption of savers is twice as volatile. Asset price volatility is relatively unchanged with and without leverage constraints but investment volatility is vastly higher, as is clear from a comparison of Figures 3 and 5. Investment correlation across countries is essentially zero in the economy without leverage constraints, but almost perfect in the economy with leverage constraints. Table 4 therefore underscores the main message of the paper. In the presence of credit market imperfections, balance sheet linkages across financial institutions can generate a very powerful mechanism for the international transmission of business cycle shocks.

Gross Portfolio Adjustment

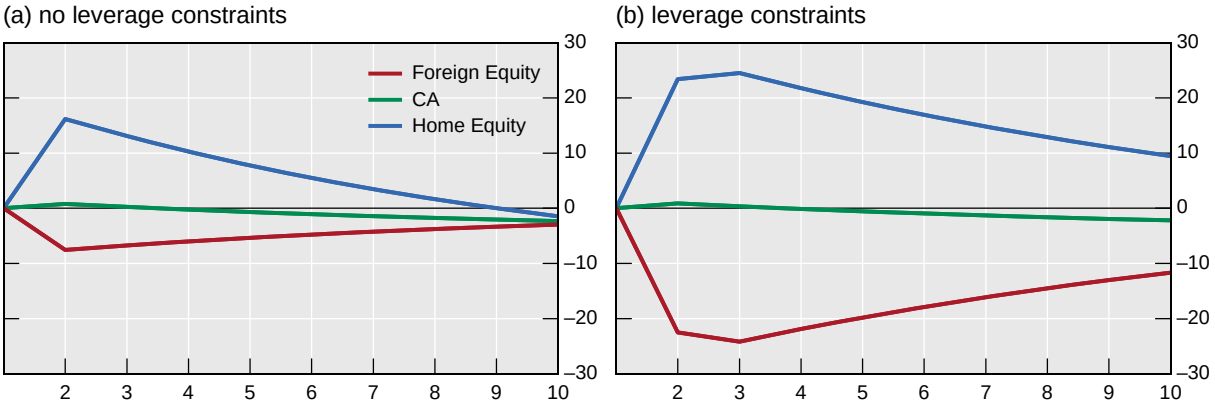
In the first order approximation of the model described in Figures 3-8, the dynamics are computed around a constant portfolio position. Changes in the portfolio affect the evolution of the real economy only at the second-order level. Nevertheless, we may still compute the implied

adjustments in the gross portfolios using the approach of Devereux and Sutherland (2009b).[9] In response to a negative technology shock, investors in both countries will adjust their gross external positions. Initially, the home country investors have a position of $\alpha_t = q_{1t-1}(k^I_{1t-1} - \hat{k}^I_{1t-1})$ <0 , implying that their *net* holdings of domestic equity is negative, while net holdings of foreign equity are positive. In response to a negative home productivity shock, gross asset holdings are adjusted downward. Holding of domestic equity increases (the net position gets less negative), and holding of foreign equity decreases. This occurs in both the economy without leverage constraints and that with leverage constraints, as illustrated in Figure 9. The NFA locus represents the percentage movement of the home economy's net foreign asset position in each case, given that the country begins holding the optimal portfolio. In each case NFA increases, because the country is hedged against the possibility of a negative productivity shock. But the positive movement in NFA is comprised of an increase in the holdings of home equity, and a reduction in the holdings of foreign equity – investors scale down their outstanding positions on each side, selling some of their foreign equity and buying back home equity. Equivalently, home agents reduce their outstanding holdings of foreign equity, and foreign agents reduce their holdings of home equity. Thus there is a reduction in gross capital flows, similar to that seen in the recent data discussed in Section 2. As Figure 9 illustrates, this dynamic occurs in both the unconstrained and constrained economies. But the scale of the drawdown is much bigger in the leverage-constrained economy. In that case, the fall in holdings of foreign equity (and reduction in foreign holdings of home equity) is approximately 3 times as large as in the unconstrained economy. Again, the presence of balance sheet constraints imparts a substantial magnification to the response of gross capital flows to the original shock.

[9] This involves taking a second-order expansion of the dynamic model, in conjunction with a third-order expansion of the portfolio selection equation (8).

Figure 9

Gross capital flows, partial diversification



5. Conclusions

This paper has provided empirical evidence on the importance of balance sheet adjustments in propagating business cycle shocks across countries. Financial interdependence, combined with financial vulnerabilities, can open a channel for the transmission of shocks that may be as important as standard trade linkages. We have used this evidence to construct a simple two-country model in which highly levered financial institutions hold inter-connected portfolios, and may be limited in their investment activity by capital constraints. The combination of portfolio inter-dependence and capital constraints leads a negative shock in one country to precipitate an episode of global balance sheet contractions and disinvestment. In this sense our model may be seen as a formal general equilibrium representation of Krugman (2008), who suggests that inter-connections in financial markets may give rise to an 'international finance multiplier.' In our model we find that, with high initial levels of leverage, the global effects of the shock may be substantially magnified. While the model illustrates the importance of financial

connections, it abstracts away from trade inter-linkages. In a more elaborate model, it would be desirable to quantitatively investigate the relative importance of the two separate channels.

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