The Effects of Foreign Shocks when Interest Rates are at Zero*

Martin Bodenstein, Christopher J. Erceg, and Luca Guerrieri**
Federal Reserve Board

August 2009

Abstract

The influence of foreign shocks is minor when the central bank can use interest rates to lean against undesirable economic fluctuations and stabilize the economy. However, in a liquidity trap when nominal policy rates have reached the zero lower bound, economies become more sensitive to additional news and experience heightened volatility. At that point, foreign shocks become an important source of volatility even for large and relatively closed countries.

Keywords: zero lower bound, spillover effects, DSGE models.

JEL Classification: F32, F41

* We thank Roberto Billi, Lawrence Christiano, Martin Eichenbaum, Mark Gertler, Christopher Gust, Michel Juillard, Jinil Kim, Lars Svensson, Linda Tesar, Daniel Waggoner, John Williams, and Tao Zha for insightful discussions and comments. We also benefited from comments at presentations at the Atlanta Fed, SAIS Johns Hopkins, the San Francisco Fed, the Bank of Canada, the Bank of Italy, and the NBER Summer Institute (Impulse and Propagation). The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

** Contact information: Martin Bodenstein: phone (202) 452 3796, Martin.R.Bodenstein@frb.gov; Christopher Erceg: phone (202) 452 2575, Christopher.Erceg@frb.gov; Luca Guerrieri: Telephone (202) 452 2550, E-mail Luca.Guerrieri@frb.gov.

1 Introduction

For large and not very open economies like the United States or the Euro Area, foreign shocks are often perceived as being of minor importance. Thus, researchers, policy-makers, and forecasters tend to abstract from the open economy dimension in analyzing business cycle fluctuations of large economic areas. The correlation between U.S. growth and that of its major trading partners is low and has shown little tendency to rise even as trade ties have grown, as documented by Doyle and Faust (2005). [1]

The wide literature on the transmission of country-specific shocks suggests that even a pronounced slowdown abroad would only cause a modest contraction in economic activity across countries. For instance, Baxter and Crucini (1995), as Backus, Kehoe, and Kydland (1992), resort to imposing an exogenous correlation in the innovations to generate large spillover effects from country-specific technology shocks. Recent papers, such as Lubik and Schorfheide (2005) and Adolfson, Laséen, Lindé, and Villani (2007) also rely on technology shocks that are correlated across countries and find small spillover effects for country-specific shocks.[2] In the presence of nominal rigidities, the influence of foreign shocks is minor when the central bank can use interest rates to lean against undesirable economic fluctuations and stabilize the economy.

However, in a liquidity trap when nominal policy rates have reached the zero lower bound, economies become more sensitive to additional news and experience heightened volatility. At that point, foreign shocks become an important source of

[1] In large scale policy models – such as the FRB/Global and SIGMA – foreign shocks have little impact on the United States. For details see Erceg, Guerrieri, and Gust (2006).

[2] Identifying the sources of comovement across countries is an area of considerable interest and uncertainty, see Kose, Otrok, and Whiteman (2003) or Stock and Watson (2005). Dynamic factor models have become the preferred approach to discriminate between country-specific and global factors. However, it is hard to resolve whether the global factors reflect the spillover effects of country-specific shocks or shocks that are correlated across countries.

volatility even for large and relatively closed economies.

We examine the spillover effects of country-specific shocks using a two country DSGE model that imposes the zero bound constraint on the short term nominal interest rate in each country, our proxies for the policy rates. The model incorporates nominal and real rigidities, such as sticky prices, wages, and consumption habits, that have been found empirically relevant for both the closed and open economy DSGE literature. If the model is calibrated to reflect the United States and an aggregate of its trading partners, a foreign demand shock that reduces foreign output by 1 percent would induce U.S. GDP to fall only around 0.3 percent in normal circumstances in which U.S. interest rates could decline. With the United States in a liquidity trap lasting 10 quarters (our benchmark case), the same foreign shock would cause U.S. output to fall nearly 0.7 percent persistently, close to the effect in the foreign country.

As the foreign shock weakens the demand for domestic goods, factor prices fall and lead to a decline in inflation. Because of the liquidity trap, policy rates can only drop in the future and short term real rates rise immediately. As a result, private domestic demand offsets the contraction in net exports associated with weaker foreign demand only to a smaller extent. Depending on the duration of the liquidity trap, private demand may even fall. This contrasts with the familiar textbook case in which policy rates can freely adjust. Then, both short and long real interest rates fall and fuel an expansion in private domestic demand that cushions the impact on U.S. output.

In the model, country-specific shocks propagate across countries through two trade channels, either by influencing the relative price of imports and exports, or the activity measures that drive trade. Demand shocks can generate some spillover effects as their impacts on the relative price of imports and activity reinforce, rather than offset, each other. By contrast, Hicks-neutral technology shocks, the typical source of fluctuations in open economy models, induce offsetting effects on these two channels and a Taylor rule would call for little change in the policy stance when policy rates are unconstrained.[3] Unsurprisingly then, technology shocks continue to have modest effects even when policy rates are constrained, whereas the spillover effects of foreign demand shocks acquire much greater spillover consequences.[4]

Our benchmark simulations assume that the liquidity trap is generated by an adverse demand shock in the United States, and that agents expect that policy rates will remain at zero for ten quarters prior to an additional negative foreign demand shock. The size of the original shock in the United States influences the length of the liquidity trap and affects the size of the spillover effects. Similarly, shocks that lead to larger foreign contractions can more than proportionately increase the spillover effects.

The magnification of the spillover effects of country-specific shocks when the home economy is in a liquidity trap is robust to changes in monetary policy. However, rules that are more aggressive on either the output gap, or inflation, may extend the duration of the liquidity trap for a given shock. Higher elasticities and increased trade openness imply larger spillovers away from the liquidity trap but are associated with a similar degree of magnification when policy rates are constrained. Finally, when the home country is in a liquidity trap, its GDP response for a given foreign demand shock is little changed whether or not the foreign country is also in a liquidity trap.

In related work, Reifschneider and Williams (2000) argue that there is a significant increase in the volatility of output in a liquidity trap, but their methodology does not allow them to link this higher volatility to structural shocks. Other papers that are related to our analysis but abstract from the open economy dimension are Eggertsson (2006) and Christiano, Eichenbaum, and Rebelo (2009). These authors

[3]As highlighted by Cole and Obstfeld (1991), the terms of trade movement act as an insurance mechanism against country-specific technology shocks.

[4]Appendix B allows comparisons of the benchmark results with those for technology shocks, government spending, and shocks affecting investment demand.

argue that whenever the zero bound on interest rates binds, the multiplier effect of an unanticipated government spending shock is much bigger than when monetary policy leans against the wind by adjusting interest rates. We show that this result applies to a broader array of shocks and extends to the open economy. Coenen and Wieland (2003) investigate the quantitative effects of exchange rate based policies in a model that is partly optimization-based but does not explore the spillover effects of foreign shocks and their dependence on different model parameters.[5]

2 The Model

Apart from the explicit treatment of the zero-lower bound on policy rates, our two-country model is close to Erceg, Guerrieri, and Gust (2006) and Erceg, Guerrieri, and Gust (2008) who themselves build on Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003). We focus on describing the home country as the setup for the foreign country is analogous. The calibration for the home country reflects key features of the United States.

2.1 Firms and Price Setting

Production of Domestic Intermediate Goods. There is a continuum of differentiated intermediate goods (indexed by $i \in [0,1]$) in the home country, each of which is produced by a single monopolistically competitive firm. Firms charge different prices at home and abroad, i.e., they practice pricing to market. In the home market, firm i faces a demand function that varies inversely with its output price $P_{Dt}(i)$ and directly with aggregate demand at home Y_{Dt} :

$$Y_{Dt}(i) = \left[\frac{P_{Dt}(i)}{P_{Dt}}\right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_{Dt}, \tag{1}$$

[5] In an open economy setting, Svensson (2004), and Jeanne and Svensson (2007) show how to use an exchange rate depreciation in order to implement the optimal policy. McCallum (2000) and Orphanides and Wieland (2000) also advocate a depreciation of the exchange rate to escape from the liquidity trap.

where $\theta_p > 0$, and P_{Dt} is an aggregate price index defined below. Similarly, in the foreign market, firm i faces the demand functi:

$$X_t(i) = \left[\frac{P^*_{Mt}(i)}{P^*_{Mt}}\right]^{\frac{-(1+\theta_p)}{\theta_p}} M^*_t, \tag{2}$$

where $X_t(i)$ denotes the foreign quantity demanded of home good i, $P^*_{Mt}(i)$ denotes the price, denominated in foreign currency, that firm i sets in the foreign market, P^*_{Mt} is the foreign import price index, and M^*_t is aggregate foreign imports.

Each producer utilizes capital services $K_t(i)$ and a labor index $L_t(i)$ (defined below) to produce its respective output good. The production function has a constant-elasticity of substitution form:

$$Y_t(i) = \left(\omega_K^{\frac{\rho}{1+\rho}} K_t(i)^{\frac{1}{1+\rho}} + \omega_L{}^{\frac{\rho}{1+\rho}} (z_t L_t(i))^{\frac{1}{1+\rho}}\right)^{1+\rho}, \tag{3}$$

where z_t is a country-specific shock to the level of technology. Firms face perfectly competitive factor markets for hiring capital and labor.

The prices of intermediate goods are determined by Calvo-style staggered contracts, see Calvo (1983). Each period, a firm faces a constant probability, $1-\xi_p$, to reoptimize its price at home $P_{Dt}(i)$ and probability of $1-\xi_{px}$ to reoptimize the price that it sets in the foreign country of $P^*_{Mt}(i)$. These probabilities are independent across firms, time, and countries.

Production of the Domestic Output Index. A representative aggregator combines the differentiated intermediate products into a composite home-produced good Y_{Dt} according to

$$Y_{Dt} = \left[\int_0^1 Y_{Dt}(i)^{\frac{1}{1+\theta_p}} di\right]^{1+\theta_p}. \tag{4}$$

The optimal bundle of goods minimizes the cost of producing Y_{Dt} taking the price of each intermediate good as given. A unit of the sectoral output index sells at the price P_{Dt}:

$$P_{Dt} = \left[\int_0^1 P_{Dt}(i)^{\frac{-1}{\theta_p}} di\right]^{-\theta_p}. \tag{5}$$

Similarly, a representative aggregator in the foreign economy combines the differentiated home products $X_t(i)$ into a single index for foreign imports:

$$M_t^* = \left[\int_0^1 X_t\,(i)^{\frac{1}{1+\theta_p}}\, di \right]^{1+\theta_p}, \tag{6}$$

and sells M_t^* at price P_{Mt}^*:

$$P_{Mt}^* = \left[\int_0^1 P_{Mt}^*\,(i)^{\frac{-1}{\theta_p}}\, di \right]^{-\theta_p}. \tag{7}$$

Production of Consumption and Investment Goods. Assuming equal import content of consumption and investment, there is effectively one final good A_t that is used for consumption or investment, (i.e., $A_t \equiv C_t + I_t$, allowing us to interpret A_t as private absorption). Domestically-produced goods and imported goods are combined to produce final goods A_t according to

$$A_t = \left(\omega_A^{\frac{\rho_A}{1+\rho_A}} A_{Dt}^{\frac{1}{1+\rho_A}} + (1-\omega_A)^{\frac{\rho_A}{1+\rho_A}} M_t^{\frac{1}{1+\rho_A}} \right)^{1+\rho_A}, \tag{8}$$

where A_{Dt} denotes the distributor's demand for the domestically-produced good and M_t denotes the distributor's demand for imports. The quasi-share parameter ω_A determines the degree of home bias in private absorption, and ρ_A determines the elasticity of substitution between home and foreign goods. Each representative distributor chooses a plan for A_{Dt} and M_t to minimize its costs of producing the final good A_t and sells A_t to households at a price P_t. Accordingly, the prices of consumption and investment are equalized.

2.2 Households and Wage Setting

A continuum of monopolistically competitive households (indexed on the unit interval) supplies a differentiated labor service to the intermediate goods-producing sector. A representative labor aggregator combines the households' labor hours in the same proportions as firms would choose. This labor index L_t has the Dixit-Stiglitz form:

$$L_t = \left[\int_0^1 N_t\,(h)^{\frac{1}{1+\theta_w}}\, dh \right]^{1+\theta_w}, \tag{9}$$

where $\theta_w > 0$ and $N_t(h)$ is hours worked by a typical member of household h. The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household's wage rate $W_t(h)$ as given. One unit of the labor index sells at the unit cost W_t:

$$W_t = \left[\int_0^1 W_t(h)^{\frac{-1}{\theta_w}}\, dh\right]^{-\theta_w}. \tag{10}$$

W_t is referred to as the aggregate wage index. The aggregator's demand for the labor services of household h satisfies

$$N_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\frac{1+\theta_w}{\theta_w}} L_t. \tag{11}$$

The utility functional of a representative household h is:

$$\widetilde{\mathbb{E}}_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1-\sigma}\left(C_{t+j}(h) - \varkappa \frac{C_{t+j-1}}{\zeta} - \nu_{ct}\right)^{1-\sigma} \right.$$
$$\left. \frac{\chi_0}{1-\chi}(1 - N_{t+j}(h))^{1-\chi} + V\left(\frac{MB_{t+j+1}(h)}{P_{t+j}}\right)\right\}, \tag{12}$$

where the discount factor β satisfies $0 < \beta < 1$. As in Smets and Wouters (2003), we allow for the possibility of external habits. At date t household h cares about consumption relative to lagged per capita consumption, C_{t-1}. The preference shock ν_{ct} follows an exogenous first order process with a persistence parameter of ρ_ν. The parameter ζ controls for population size. The household's period utility function depends on current leisure $1 - N_t(h)$, the end-of-period real money balances, $\frac{MB_{t+1}(h)}{P_t}$. The liquidity-service function $V(\cdot)$ is increasing in real money balances at a decreasing rate up to a satiation level. Beyond the satiation level, utility from liquidity services is constant. With this specification of the utility function, the demand for real money balances is always positive regardless of the level of the nominal interest rate.[6]

[6] More formally, we follow Jeanne and Svensson (2007) in assuming that $V(MB_{t+1}/P_t) < V_0$, $V'(MB_{t+1}/P_t) > 0$, $V''(MB_{t+1}/P_t) < 0$ for $MB_{t+1} < \bar{m}$, the satiation level of real money. And $V(MB_{t+1}/P_t) = V_0$ for $MB_{t+1} \geq \bar{m}$, and $V'(MB_{t+1}/P_t) \to \infty$ for $MB_{t+1}/P_t \to 0$.

The budget constraint of each household is given by:

$$P_tC_t\left(h\right)+P_tI_t\left(h\right)+MB_{t+1}\left(h\right)-MB_t(h)+\tfrac{e_tP^*_{Bt}B_{Ft+1}(h)}{\phi_{bt}}-e_tB_{Ft}(h)$$
$$=W_t\left(h\right)N_t\left(h\right)+\Gamma_t\left(h\right)-T_t\left(h\right)+R_{Kt}(1-\tau_{Kt})K_t(h)-P_{Dt}\phi_{It}(h). \tag{13}$$

Final consumption and investment goods are purchased at a price P_t. Investment in physical capital augments the per capita capital stock $K_{t+1}(h)$ according to a linear transition law of the form:

$$K_{t+1}\left(h\right)=(1-\delta)K_t(h)+I_t(h), \tag{14}$$

where δ is the depreciation rate of capital. The term $R_{Kt}(1-\tau_{Kt})K_t(h)$ in the budget constraint represents the proceeds to the household from renting capital to firms net of capital taxes.

Financial asset accumulation consists of increases in nominal money holdings $MB_{t+1}\left(h\right)-MB_t\left(h\right)$ and the net acquisition of international bonds. Trade in international assets is restricted to a non-state contingent nominal bond. $B_{Ft+1}(h)$ represents the quantity of the international bond purchased by household h at time t that pays one unit of foreign currency in the subsequent period. P^*_{Bt} is the foreign currency price of the bond, and e_t is the nominal exchange rate expressed in units of home currency per unit of foreign currency. Following Turnovsky (1985) households pay an intermediation fee ϕ_{bt}.[7] The intermediation fee depends on the ratio of economy-wide holdings of net foreign assets to nominal output according to:

$$\phi_{bt}=\exp\left(-\phi_b\left(\frac{e_tB_{Ft+1}}{P_{Dt}Y_t}\right)\right). \tag{15}$$

If the home economy has an overall net lender position, a household will earn a lower return on any holdings of foreign bonds. By contrast, if the economy has a net debtor position, a household will pay a higher return on any foreign debt.

[7]The assumption of an intermediation fee ensures that given our solution technique the evolution of net foreign assets is stationary. See Schmitt-Grohe and Uribe (2003) and Bodenstein (2006) for a discussion. The intermediation cost is asymmetric, as foreign households do not face these costs. Rather, they collect profits on the monopoly rents associated with these intermediation costs.

Households earn labor income, $W_t(h) N_t(h)$, lease capital to firms at the rental rate R_{Kt}, and receive an aliquot share $\Gamma_t(h)$ of the profits of all firms. Furthermore, they pay a lump-sum tax $T_t(h)$. We follow Christiano, Eichenbaum, and Evans (2005) in assuming that households bear a cost of changing the level of gross investment from the previous period, so that the acceleration in the capital stock is penalized:

$$\phi_{It}(h) = \frac{1}{2}\phi_I \frac{(I_t(h) - I_{t-1}(h))^2}{I_{t-1}(h)}. \tag{16}$$

Households maximizes the utility functional (12) with respect to consumption, investment, (end-of-period) capital stock, money balances, and holdings of foreign bonds, subject to the labor demand function (11), budget constraint (13), and transition equation for capital (14). They also set nominal wages in staggered contracts that are analogous to the price contracts described above. In particular, with probability $1 - \xi_w$, each member of a household is allowed to reoptimize its wage contract.

2.3 Monetary and Fiscal Policy

Monetary policy follows an interest rate reaction function as suggested by Taylor (1993). However, when policy rates reach zero we assume that no further actions are taken by the central bank. The notional rate that is dictated by the interest rate reaction function is denoted by i_t^{not}, whereas the effective policy rate that is implemented is denoted by i_t. The two differ only if the notional rate turns negative:

$$i_t^{not} = \bar{i} + \gamma_i(i_{t-1}^{not} - \bar{i}) + (1 - \gamma_i)(\gamma_\pi(\pi_t - \bar{\pi}) + \frac{\gamma_y}{4} y_t^{gap}), \tag{17}$$

and the actual (short-term) policy interest rate satisfies

$$i_t = max(0, i_t^{not}). \tag{18}$$

The terms $\bar{i}$ and $\bar{\pi}$ are the steady-state values for the nominal interest rate and inflation, respectively. The inflation rate π_t is expressed as the logarithmic percentage change of the price level, $\pi_t = log(P_{Dt}/P_{Dt-1})$. The term y_t^{gap} denotes the output gap, given by the log difference between actual and potential output, where the latter is the level of output that would prevail in the absence of nominal rigidities. Notice that the coefficient γ_y is divided by four as the rule is expressed in terms of quarterly inflation and interest rates. The parameter γ_i allows for interest rate smoothing.[8]

Government purchases are a constant fraction of output $\bar{g}$ and they fall exclusively on the domestically-produced good. These purchases make no direct contribution to household utility. To finance its purchases, the government imposes a lump-sum tax on households that is adjusted so that the government's budget is balanced every period.

2.4 Resource Constraints

The home economy's aggregate resource constraint satisfies:

$$Y_{Dt} = C_{Dt} + I_{Dt} + G_t + \phi_{It}. \tag{19}$$

The composite domestically-produced good Y_{Dt}, net of investment adjustment costs ϕ_{It}, is used to produce final consumption and investment goods ($A_{Dt} = C_{Dt} + I_{Dt}$), or directly to satisfy government demand. Moreover, since each individual intermediate goods producer can sell its output either at home or abroad, there are also a continuum of resource constraints that apply at the firm level.

2.5 Calibration of Parameters

The model is calibrated at a quarterly frequency. The values of key parameters are presented in Table 1 and reflect standard calibration choices. We choose $\omega_A = 0.15$

[8] Jung, Teranishi, and Watanabe (2005), Eggertsson and Woodford (2003), Adam and Billi (2006), and Adam and Billi (2007) derive the optimal policy under the zero bound constraint in a closed economy. In the face of contractionary shocks, optimal monetary policy calls for keeping interest rates lower for an extended period in a liquidity trap relative to normal times, a feature captured by interest rate smoothing in our model.

to be consistent with an import share of output of 15%. The domestic and foreign population levels, respectively ζ and ζ^*, are set so that the home country constitutes 25 percent of world output. Balanced trade in steady state implies an import (or export) share of output of the foreign country of 5 percent. Because the foreign country is assumed to be identical to the home country except in its size, $\omega_A^* = 0.05$. We set $\rho_A = 10$, so that the price elasticity of import demand is 1.1.

Nominal rigidities in prices and wages have an average duration of four quarters, determined by the parameters $\xi_p = 0.75$ and $\xi_w = 0.75$. Export price rigidities have a shorter duration of 2 quarters, as implied by the parameter $\xi_{px} = 0.5$. Monetary policy follows a simple Taylor-type rule with interest rate smoothing. The smoothing parameter γ_i is set at 0.7. The parameter γ_π governing the rule's response to inflation deviation from the target rate is 1.5. The parameter γ_y that governs the rule's response to the output gap is 0.5. The steady state real interest rate is set to 2% per year ($\beta = 0.995$). Given steady state inflation $\bar{\pi}$ equal to zero, the implied steady state nominal interest rate is two percent. The values of remaining parameters are fairly standard in the literature, and are summarized in Table 1.

3 Solution Method

All equilibrium conditions except the non-linear policy rule are linearized around the model's non-stochastic steady state. We solve the model using a shooting algorithm first proposed by Laffargue (1990) and extended by Boucekkine (1995) and Juillard (1996), which in turn builds on earlier work by Fair and Taylor (1983). This algorithm stacks all equations through time, which is equivalent to collapsing the Type I and II iterations in the Fair-Taylor shooting algorithm into one step. The size of the first-derivative used to implement a Newton-type recursion is kept manageable by exploiting the sparsity of the stacked system.

The end point of the shooting algorithm imposes that the economy will eventually exit from the liquidity trap. Following Anderson (1999), instead of using the steady state values as end point, we use a mid-way point from the linear solution computed from standard algorithms.[9] As shown by Anderson (1999), this alternative procedure leads to a shorter length of the simulation horizon needed to achieve any desired level of accuracy for those values that are at the beginning of the simulation.

An appropriately long simulation horizon makes the solution from our algorithm numerically equivalent to that obtained following the method described by Eggertsson and Woodford (2003) and Jung, Teranishi, and Watanabe (2005). The solution proposed by these authors recognizes that the model is piecewise-linear. All model equations are linear when the zero bound constraint binds, and they are also linear, yet modified, when the zero bound constraint does not bind. However, the time period for which the economy is at the zero bound is a non-linear function of the exogenous disturbances.[10]

Relative to Eggertsson and Woodford (2003) and Jung, Teranishi, and Watanabe (2005), our method deals easily with shocks whose effects build up over time and only eventually lead to zero short-term interest rates. Moreover, our algorithm extends naturally to deal with the case when both countries are constrained by the zero lower bound on nominal interest rates.

4 Initial Baseline Path

Our principal goal is to compare the impact of foreign shocks on the home country when it faces a liquidity trap with the effects that occur when policy rates can be freely adjusted. In the former case, the impact of a foreign shock depends on the

[9] For example, see Anderson and Moore (1985).

[10] Christiano (2004) suggests an alternative shooting algorithm that also exploits piecewise linearity. Linde and Svensson implement the max-operator through a sequence of anticipated monetary policy innovations, which can be shown to be equivalent to the method of Jung, Teranishi, and Watanabe (2005).

economic conditions that precipitated the liquidity trap. Intuitively, the effects of an adverse foreign shock against the backdrop of a recession-induced liquidity trap in the home country should depend on the expected severity of the recession, and the perceived duration of the liquidity trap. In a shallow recession in which interest rates are only constrained for a short period, the effects of the foreign shock would not differ substantially from the usual case in which rates could be cut immediately.[11] By contrast, the effects of the foreign shock on the home country might be amplified substantially if it occurred against the backdrop of a steep recession in which policy rates were expected to be constrained from falling for a protracted period.

We use the term "initial baseline path" to describe the evolution of the economy that would prevail in the absence of the foreign shock. Given agents' full knowledge of the model, the initial baseline path depends on the underlying shocks that push the economy into a liquidity trap, including their magnitude and persistence, as these features play an important role in determining agents' perceptions about the duration of the liquidity trap.

Our analysis focuses on the effects of foreign shocks against the backdrop of an initial baseline path that is intended to capture a severe recession in the home country. This "severe recession" baseline is depicted in Figure 1 by the solid lines. It is generated by a preference shock ν_{ct} that follows an autoregressive process with persistence parameter equal to 0.75. The shock reduces the home country's marginal utility of consumption. As the shock occurs exclusively in the home country, the foreign economy has latitude to offset much of the contractionary impact of the shock by reducing its policy rate.[12]

As shown in Figure 1 policy rates immediately fall to 0, (2 percentage points below their steady state value at annualized rates) and remain frozen at this level for ten quarters.[13] Given that the shock drives inflation persistently below its steady

[11] In the case of a linear model, the effects of a shock are unrelated to the initial conditions.

[12] We investigate the sensitivity of our results to the initial baseline path in Section 5.1.

[13] In Figure 1, real variables are plotted in deviation from their steady-state values, while nominal variables are

state value and that nominal interest rates are constrained from falling by the zero bound, real rates increase substantially in the near term. This increase in real interest rates accounts in part for the substantial output decline, which peaks in magnitude at about 9 percent below its steady state value. Real interest rates decline in the longer term, helping the economy recover.[14] This longer term decline also causes the home currency to depreciate in real terms, and the ensuing expansion of real net exports mitigates the effects of the shock on domestic output. However, the improvement in real net exports is delayed due to the zero bound constraint, since higher real interest rates limit the size of the depreciation of the home currency in the near-term.

For purposes of comparison, the figure also shows the effects of the same shocks in the case in which the home country's policy rates can be adjusted, i.e., ignoring the zero bound constraint. The home nominal interest rate falls more sharply and dips negative inducing a decline in real interest rates in the near-term. Hence, the fall in home output is smaller than in the benchmark framework in which the zero bound constraint is binding. The home output contraction is also mitigated by more substantial improvement in real net exports. Given that real interest rates fall very quickly, the real depreciation is considerably larger and more front-loaded, contributing to a more rapid improvement in real net exports.

---

in *levels* to highlight the zero bound constraint. The policy rate, real interest rate and inflation are annualized.

[14] A flatter Phillips curve due to firm-specific capital as in Woodford (2003) or a higher degree of inflation inertia due to lagged indexation as in Christiano, Eichenbaum, and Evans (2005) imply a smaller reaction of inflation on impact. However, the inflation response becomes more persistent. The real interest rate responds less on impact but remains elevated relative to the case shown in Figure 1 as time progresses. Accordingly, the behavior of output and the output gap turns out to be more or less unchanged as do the spillover effects as shown in the Appendix B.

5 International Transmission at the Zero Bound

We turn to assessing the impact of a negative foreign consumption preference shock ν_{ct}^* when the home country faces a liquidity trap. The foreign shock is scaled to induce a 1 percent reduction in foreign output relative to the initial baseline when it occurs against the backdrop of the severe recession scenario associated with Figure 1. The size of the foreign shock is small enough that the duration of the liquidity of the liquidity trap in the home country remains at ten quarters.

Figure 2 shows the effects of the foreign shock abroad, while Figure 3 reports the effects on the home country. The solid lines show the responses when the zero bound constraint is imposed on home policy rates, while the dashed lines report the responses to the same shock when the zero lower bound is ignored. To be specific, the responses in Figures 2 and 3 are derived from a simulation that adds both the adverse domestic taste shock from Figure 1 and the foreign taste shock from Figure 3, and then subtracts the impulse response functions associated with the domestic taste shock alone.[15] Thus, all variables are measured as deviations from the baseline path shown in Figure 1.

Figure 2 shows that whether the home country is at the zero lower bound or not has minimal reverberations back on the foreign responses. The solid and dashed lines are virtually indistinguishable reflecting the foreign economy's latitude to adjust interest rates to help offset the shock. In response to the foreign consumption shock, foreign policy rates are cut in both cases. As real rates drop, investment is stimulated. The lower real rates also contribute to a real exchange rate depreciation that boosts foreign exports.

The effects of the foreign demand shock on the home country, shown in Figure 3, are strikingly different whether the zero lower bound is imposed or not. Although the foreign shock has nearly the same effect on foreign output across the two cases,

[15] Because of linearity, the dashed lines in Figures 2 and 3 could also be interpreted as the responses starting from the model's steady state, rather than the severe recession.

the effects on home output are several times larger when the zero bound constraint is imposed. In that case, home real net exports contract for the same reasons as when the zero bound is not imposed. The effect on trade of the decline in foreign activity, is reinforced by the appreciation of the home real exchange rate. However, in a liquidity trap, the decline in home export demand causes a fall in the marginal cost of production and inflation that is not accompanied by lower policy rates. The zero bound constraint keeps nominal rates from declining for ten quarters and real rates rise sharply in the short run. Only after the end of the liquidity trap, with persistently lower policy rates, do real rates abate. Consequently, domestic absorption does not expand as much as when policy rates can be cut immediately. In fact, if the initial recession were more pronounced, private absorption could even fall, as shown below. With net exports falling and with domestic absorption not being able to fill in the gap, output falls by nearly as much in the home country as abroad.

5.1 Alternative Initial Conditions and Monetary Policy

The analysis so far has been based on one particular choice of the size of the underlying baseline shock and the size of the additional foreign shock. These choices are revisited below and results are compared for different monetary policy rules.[16]

Alternative Initial Baseline Paths

In Figure 4, we change the assumptions concerning the initial domestic recession by increasing its persistence. The underlying initial domestic preference shock ν_{ct} is now assumed to follow an autoregressive process of order one with persistence parameter equal to 0.9 instead of 0.75. With this prolonged recession, the liquidity trap is initially expected to last 16 quarters, instead of the 10 quarters considered previously. The figure compares the effects of the same additional foreign consump-

[16] In Appendix B, we show that the magnification of the spillover effects of foreign shocks when the home economy is at the zero loser bound is not particular to the consumption shock discussed so far.

tion shock with the liquidity trap lasting 10 quarters and with the trap lasting 16 quarters. When the duration of the liquidity trap is extended, the rise in short-term real interest rate at home is so large as to generate a drop in absorption thus widening the fall in home output.[17] The analysis that follows traces more systematically how the duration of the liquidity trap affects the spillover of foreign shocks.

In Figure 5, we consider the impact of the same foreign consumption shock ν^*_{ct} under different initial baseline paths and policy rules. For each baseline path, we choose the size of the domestic shock to ensure that the zero lower bound will bind for the number of quarters in the figure's abscissae, thus charting the marginal effects of the shock. We calculate the marginal spillover effects of the foreign shock ν^*_{ct} as the ratio of the shock's effects on home GDP (expressed in deviation from the baseline path) to the effects on foreign GDP (also expressed in deviation from the baseline path). The figure's ordinates show an average of these spillover effects for the first four quarters.[18]

Focusing first on the results for the benchmark Taylor rule, the same rule used for Figures 1 to 3, the spillover effects become larger as the number of periods spent at the zero lower bound increases. Intuitively, the longer the policy rates are constrained from adjusting, the higher is the increase in the home real interest rates stemming from the contractionary foreign demand shock. As real interest rates rise more, they provide greater and greater hindrance for domestic absorption to cushion the contraction in home GDP that is caused by the fall in net exports. When policy rates in the home economy are expected to be constrained for longer than two years, the spillover effects from a small foreign consumption shock more than double relative to the unconstrained case.

[17] Comment on effects of introducing financial frictions in the analysis and how it would increase persistence of rise in the real interest rate.

[18] An implication of the propositions presented in Appendix A is that as long as the size of the foreign consumption shock is chosen to be small enough not to influence the number of periods for which the zero lower bound on policy rates is binding, our measure of spillovers is not affected by the size of the shock.

The figure also shows the same measure of spillover effects under alternative interest rate rules. Both rules leave the basic form of reaction function described in Equation (17) unchanged. However, the rule that is labeled "more aggressive on inflation" doubles the elasticity with respect to inflation γ_π from 1.5 to 3, while the rule that is labeled "more aggressive on output gap" uses an elasticity with respect to the output gap γ_y equal to 4 instead of 0.5. Under both alternative rules, when the baseline conditions imply a higher number of periods spent at the zero lower bound, there is a substantial increase in the home spillover effects of the foreign consumption shock, confirming that our previous results are independent of the specific choice of policy rule.

Alternative Foreign Consumption Shocks

The marginal spillover effects shown in Figure 5 abstract from non-linear dynamics that are associated with changes in the number of periods for which the zero lower bound is expected to bind. As long as the foreign consumption shock does not affect the duration of the liquidity trap, the effects of the shock are linear in the size of its innovation. However, there is a size of the innovation above which the duration of the liquidity trap is extended, thus decoupling the marginal and average effects of shocks. Furthermore, the duration of the liquidity trap is a nonlinear function of the size of the innovations.[19] These effects are instead considered in Figure 6.

In contrast to Figure 5, in Figure 6 we return to the initial baseline path shown in Figure 1 (i.e. the economy is at the zero bound for 10 quarters), but vary the size of the additional foreign shock. The magnitude of the shock is measured by the change induced in foreign GDP relative to the baseline path (on average over the first four quarters). The top panel of Figure 6 shows how the additional foreign shock affects the duration of the liquidity trap. The bottom panel displays the same measure of spillover effects as in the preceding exercise.

We first consider the case of the benchmark Taylor rule (the solid lines). If the

[19] We relegate the formal proofs to Appendix A.

foreign shock is sufficiently small, the number of periods at the zero lower bound does not change relative to the initial baseline and remains at 10 quarters as reported in the upper panel. Then, the spillover effect, shown in the lower panel, is roughly 3/4, the same magnitude shown in Figure 5 for the case of the trap lasting 10 quarters. The spillover effects are linear in the size of the shock and remain 3/4 as long as the additional shock does not vary the duration of the liquidity trap. Hence marginal and average effects of the foreign shock coincide.

Once the foreign shocks are allowed to be large enough, they can affect the duration of the liquidity trap, as shown in the top panel. As negative foreign shocks increase the number of periods at the zero lower bound, the spillover effects become larger. Conversely, larger and larger expansionary shocks abroad can reduce the number of periods for which the zero lower bound constraint binds at home and thus reduce the spillover effects. However, as shown in the bottom right panel, even shocks that are sufficiently large to push the economy out of the liquidity trap cause spillovers that are elevated relative to the case when the zero bound does not bind initially. The reason is that the average effect of the shock differs from the shock's marginal effect. The latter is below the former and the two will only coincide again asymptotically.

We now turn to comparing the effects of the foreign shocks under alternative monetary policy rules. For the given initial baseline shock, the rules that are more aggressive on inflation or the output gap tend to increase the duration of the liquidity trap although they dampen the contractionary effects of macroeconomic shocks. Intuitively, more aggressive rules call for a more sustained fall in the interest rate in reaction to a deflationary shock, and may extend the number of periods spent at the zero lower bound. For the specific rules chosen, the benchmark Taylor rule still delivers larger marginal spillover effects when the foreign shock is too small to affect the number of periods spent at the zero lower bound, as shown in the bottom panel.

The top panel of Figure 6 also shows that different rules imply different threshold sizes for shocks to influence the duration of the liquidity trap. The rule that is more aggressive on inflation requires larger foreign expansionary shocks to reduce the home economy's permanence at the zero lower bound. Because the marginal effect of the foreign shock is an increasing function of the liquidity trap's duration, depending on the shock size, the rule that is more aggressive on inflation can have larger average spillover effects than the benchmark Taylor rule. [20]

5.2 Alternative Trade Elasticities

The value of the import price elasticity of demand is an important determinant of the duration of a liquidity trap and the spillover effects of country-specific shocks. Figure 7 shows how the spillover effects of the foreign consumption shock are affected by choice of a higher elasticity, equal to 1.5 versus 1.1 in our original calibration, or a lower elasticity, equal to 0.75. The figure's bottom right panel, shows that when the policy rule is unconstrained, a higher elasticity increases the spillover effects. The higher elasticity reduces the responsiveness of exchange rates to country-specific shocks. However, the increased sensitivity to movements in relative import prices more than offsets the decreased volatility of exchange rates. Accordingly, with the higher elasticity, home country net exports drop by more in response to a contractionary foreign consumption shock, leading to a larger fall in home GDP.

Away from the zero lower bound, the linearization of the model ensures that spillover effects are unrelated to the size of shocks. The figure's bottom left and top panels consider instead how the spillover effects are influenced by the size of the foreign shock against the backdrop of the same domestic recession considered above. The top panel of Figure 7 shows that the higher the trade elasticity the smaller is

[20] Alternative parameterizations of the interest rate reaction functions considered could change the ordering of the spillover effects even when the additional foreign consumption shocks do not influence the number of periods for which the zero lower bound binds.

the size of foreign shocks that can lift the home economy out of the liquidity trap. The lower panel confirms that the zero lower bound constraint induces the same kind of magnification in the size of the spillover effects regardless of the elasticity chosen.

5.3 Both Countries In a Liquidity Trap

We showed that when one country is in a liquidity trap, the spillover effects of foreign shocks are greatly amplified. We next consider if these spillover effects reverberate back and forth when both countries are mired a liquidity trap, further exacerbating the domestic spillovers of a foreign shock.

Figure 8 illustrates the effects of an additional foreign consumption shock under three distinct initial baseline paths: both countries are at the zero bound for 10 quarters (the dotted line), only the home country is at the zero bound for 10 quarters (the solid line), no country is at the zero bound (the dashed line). In each case, the baseline paths were constructed using different consumption shocks.

The size of the additional foreign consumption shock is unchanged across the three scenarios. This shock is sized to induce a 1% decline of foreign GDP if neither country is at the zero bound. The effects of the foreign consumption shock on foreign GDP are greatly amplified if the foreign country is constrained by the zero bound. The maximum decline of foreign GDP is about 5% relative to baseline if the zero bound binds (dotted line) but only 1% if the policy rate is unconstrained. In the home country the effect of the foreign consumption shock is amplified if the home country is at the zero bound. However, under our benchmark calibration the reverberation effects stemming from the liquidity trap of the shock on home GDP remain small as can be seen from the fact that the dotted and solid lines almost overlap.

In all three cases, the shock to foreign consumption causes an appreciation of the home exchange rate and increases foreign import prices, boosting home imports.

Conversely, with weaker activity in the foreign country, homes exports decline and depress home GDP.

When the foreign economy is at the the zero lower bound monetary policy can no longer boost foreign activity by lowering rates. The greater contraction in the foreign country's aggregate demand (the solid versus dashed lines) is then accompanied by a smaller increase in the foreign import price. As the price movement offsets the movement in foreign activity, home exports and GDP are little varied.

Figure 9 considers changes in the import price elasticity of demand to understand the determinants of the reverberation effects. Each line in the figure is constructed by subtracting the impulse responses to a foreign consumption shock in the case when both countries are at the zero bound from those which obtain when only the home country is at the zero bound. This difference captures the reverberation effects on the home country associated with the liquidity trap in the foreign country.[21]

We consider two cases: our benchmark elasticity equal to 1.1 (the solid lines), and a case in which the elasticity is equal to 0.5. The lower trade elasticity skews the determination of trade flows towards the activity channel, rather than the price channel. When the foreign economy is also at the zero lower bound, lower foreign activity causes a bigger contraction in home exports, which exacerbates the contraction in home GDP relative to the case when only the home economy is at the zero lower bound. [22]

6 Conclusions

When monetary policy is unconstrained by the zero lower bound on nominal interest rates, foreign disturbances have limited spillover effects across countries as monetary

[21] More specifically, the solid line in Figure 9 shows the difference between the dotted and dolid lines of Figure 8.

[22] A high value for the trade elasticity skews the determination of trade flows towards the price channel. In that case, the contraction of home GDP is reduced if the foreign country is also mired in a liquidity trap.

policy can influence domestic demand by adjusting interest rates. By contrast, at the zero lower bound, monetary policy cannot manage domestic demand and the spillover effects of foreign shocks can be amplified greatly.

The amplification of foreign idiosyncratic shocks depends on the duration of the liquidity trap and the size of the foreign shock, to the extent that this shock may change the duration of the trap. The choice of policy rule also affects the spillover effects of foreign shocks. A rule that is more aggressive on inflation, for instance, may increase the spillover effects of foreign shocks by prolonging the duration of the liquidity trap.

Our model results allay fears that a global liquidity trap would further enhance the spillover effects of a given-size country-specific shock, relative to the case in which the trap is limited to one region. When the liquidity trap affects both the United States and the model's foreign block, the effects of foreign demand shocks on relative prices and activity offset each other to produce results in the United States that are very similar to those when the only country in a liquidity trap is the United States.

The increased volatility originating from foreign shocks should play a role in the design of the monetary policy response to a liquidity trap. In fact, episodes that have spurred coordination of policy actions across central banks rarely occur when policy rates are unconstrained, but have become frequent since the United States entered a liquidity trap. The increased interdependence of the global economy in a liquidity trap should reopen the discussion on the benefits of policy coordination.

References

Adam, K. and R. M. Billi (2006). Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates. *Journal of Money, Credit, and Banking 7*, 1877–1905.

Adam, K. and R. M. Billi (2007). Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates. *Journal of Monetary Economics 3*, 728–752.

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2007). Bayesian estimation of an open economy DSGE model with incomplete pass-through. *Journal of International Economics 72*, 482–511.

Anderson, G. (1999). *Analyses in Macroeconomic Modelling*, Chapter 9: Accelerating Non Linear Perfect Foresight Model Solution by Exploiting the Steady State Linearization, pp. 57–85. Springer.

Anderson, G. and G. Moore (1985). A Linear Algebraic Procedure for Solving Linear Perfect Foresight Models. *Economic Letters 17*, 247–52.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992). International Real Business Cycles. *Journal of Political Economy 100*, 745–775.

Baxter, M. and M. Crucini (1995). Business Cycles and the Asset Structure of Foreign Trade. *International Economic Review 36*(4), 821–854.

Bodenstein, M. (2006). Closing Open Economy Models. International Finance

Discussion Paper 867.

Boucekkine, R. (1995). An Alternative Methodology for Solving Nonlinear Forward-Looking Models. *Journal of Economic Dynamics and Control 19*, 711–734.

Calvo, G. A. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics 12*, 383–398.

Christiano, L. (2004). The Zero-Bound, Zero-Inflation Targetting, and Output Collapse.

Christiano, L., M. Eichenbaum, and S. Rebelo (2009). When is the Government Spending Multiplier Large?

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy 113*(1), 1–45.

Coenen, G. and V. Wieland (2003). The zero-interest-rate bound and the role of the exchange rate for monetary policy in Japan. *Journal of Monetary Economics 50*(5), 1071–1101.

Cole, H. and M. Obstfeld (1991). Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter? *Journal of Monetary Economics 28*, 3–24.

Doyle, B. and J. Faust (2005, November). Breaks in the Variability and Co-Movement of G-7 Economic Growth. *Review of Economics and Statis-*

tics 87(4), 721–740.

Eggertsson, G. (2006). Was the New Deal Contractionary? Federal Reserve Bank of New York, Staff Report 264.

Eggertsson, G. B. and M. Woodford (2003). The Zero Bound on Interest Rates and Optimal Monetary Policy. *Brookings Papers on Economic Activity* (1), 139–233.

Erceg, C. J., L. Guerrieri, and C. Gust (2006). SIGMA: A New Open Economy Model for Policy Analysis. *International Journal of Central Banking*, 1–50.

Erceg, C. J., L. Guerrieri, and C. Gust (2008). Trade Adjustment and the Composition of Trade. *Journal of Economic Dynamics and Control*, 2622–2650.

Fair, R. and J. B. Taylor (1983). Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models. *Economectrica*, 1169–1185.

Jeanne, O. and L. E. Svensson (2007). Credible Commitment to Optimal Escape from a Liquidity Trap: The Role of the Balance Sheet of an Independent Central Bank. *American Economic Review 97*, 474–490.

Juillard, M. (1996). DYNARE: A Program for the Resolution and Simulation of Dynamic Models with Forward Variables Through the Use of a Relaxation Algorithm. CEPREMAP Working Paper No. 9602.

Jung, T., Y. Teranishi, and T. Watanabe (2005). Zero Bound on Nominal Interest Rates and Optimal Monetary Policy. *Journal of Money, Credit, and*

Banking 37, 813–836.

Kose, M., C. Otrok, and C. Whiteman (2003). International Business Cycles: World, Region, and Country-Specific Factors. *American Economic Review 93*, 1216–1239.

Laffargue, J. P. (1990). Résolution d'un modèle macroéconomique avec anticipations rationnelles. *Annales d'Economie et Statistique 17*, 97–119.

Lubik, T. A. and F. Schorfheide (2005). A Bayesian Look at New Open Economy Macroeconomics. In M. Gertler (Ed.), *NBER Macroeconomics Annual*. NBER.

McCallum, B. (2000). Theoretical Analysis Regarding a Zero Lower Bound on Nominal Interest Rates. *Journal of Money, Credit, and Banking 32*, 870–904.

Orphanides, A. and V. Wieland (2000). Efficient monetary policy design near price stability. *Journal of the Japanese and International Economies 14*, 327–365.

Reifschneider, D. and J. C. Williams (2000). Three Lessons for Monetary Policy in Low Inflation Era. *Journal of Money, Credit, and Banking 32*(4), 936–966.

Schmitt-Grohe, S. and M. Uribe (2003). Closing Small Open Economy Models. *Journal of International Economics 61*(3), 163–185.

Smets, F. and R. Wouters (2003). An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association 1*, 1124–1175.

Stock, J. and M. Watson (2005). Understanding Changes in International Business Cycle Dynamics. *Journal of the European Economic Association 3*, 968–1006.

Svensson, L. E. (2004). The Magic of the Exchange Rate: Optimal Escape from a Liquidity Trap in Small and Large Open Economies.

Taylor, J. B. (1993). Discretion versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

Turnovsky, S. J. (1985). Domestic and Foreign Disturbances in an Optimizing Model of Exchange-Rate Determination. *Journal of International Money and Finance 4*(1), 151–71.

Woodford, M. (2003). *Interest and Prices.* Princeton: Princeton University Press.

Table 1: Calibration*

Parameter	Determines:	Parameter	Determines:
$\beta = 0.995$	s.s. real interest rate = 2% per year	$\delta = 0.025$	depreciation rate = 10% per year
χ_0	leisure's share of time = 1/2	$\chi = 10$	labor supply elasticity = 1/10
$\sigma = 2$	intertemporal substitution elast. 1/2	$\phi_b = 0.001$	interest elasticity of foreign assets
$\rho = -2$	capital-labor substitution elast. = 1/2	$\rho_A = 10$	long-run import price elasticity = 1.1
$\omega_A = 0.15$	import share of output = 15%	$\omega_A^* = 0.05$	foreign import share of output = 5%
$\zeta = 1$	population size	$\zeta^* = 3$	foreign population size
$\kappa = 0.8$	consumption habits	$\phi_I = 3$	investment adjustment costs
$\theta_w = 0.1$	wage markup = 10%	$\theta_p = 0.1$	domestic/export price markup = 10%
$\xi_p = 0.75$	price contract expected duration = 4 quarters	$\xi_w = 0.75$	wage contract expected duration = 4 quarters
$\xi_{px} = 0.5$	export price contract expected duration = 2 quarters	$\tau_k = 0$	capital tax rate
$\gamma_i = 0.7$	monetary policy's weight on lagged interest rate	$\gamma_\pi = 1.5$	monetary policy's weight on inflation
$\gamma_y = 0.5$	monetary policy's weight on output gap		

* Parameter values for the foreign country are chosen identical to their home country counterparts except for the population size ζ^* and the import share ω_A^*.

Figure 1: Severe Domestic Recession Scenario (Initial Baseline Path)

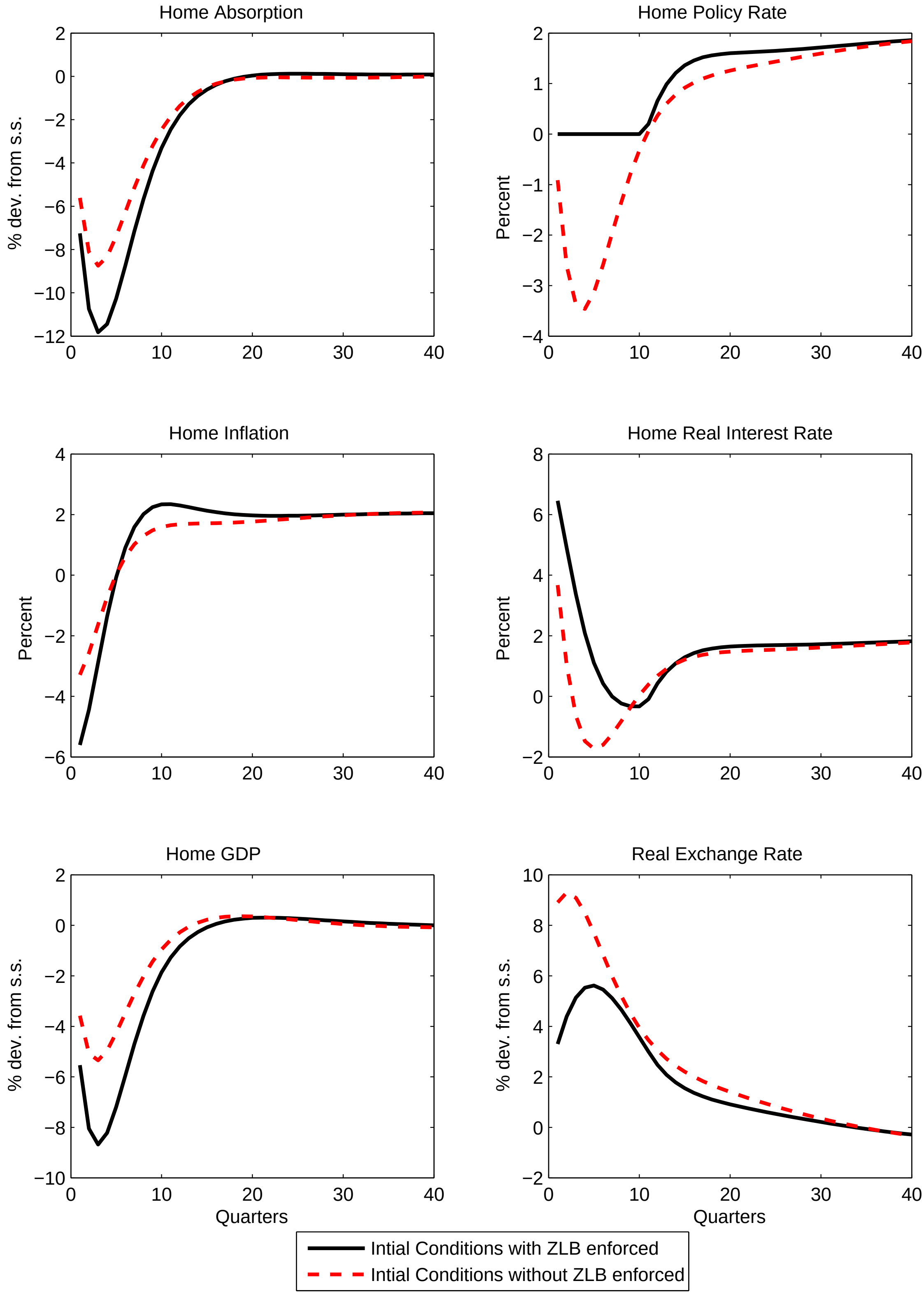


Figure 2: Effects of Foreign Consumption Shock against Backdrop of Domestic Recession

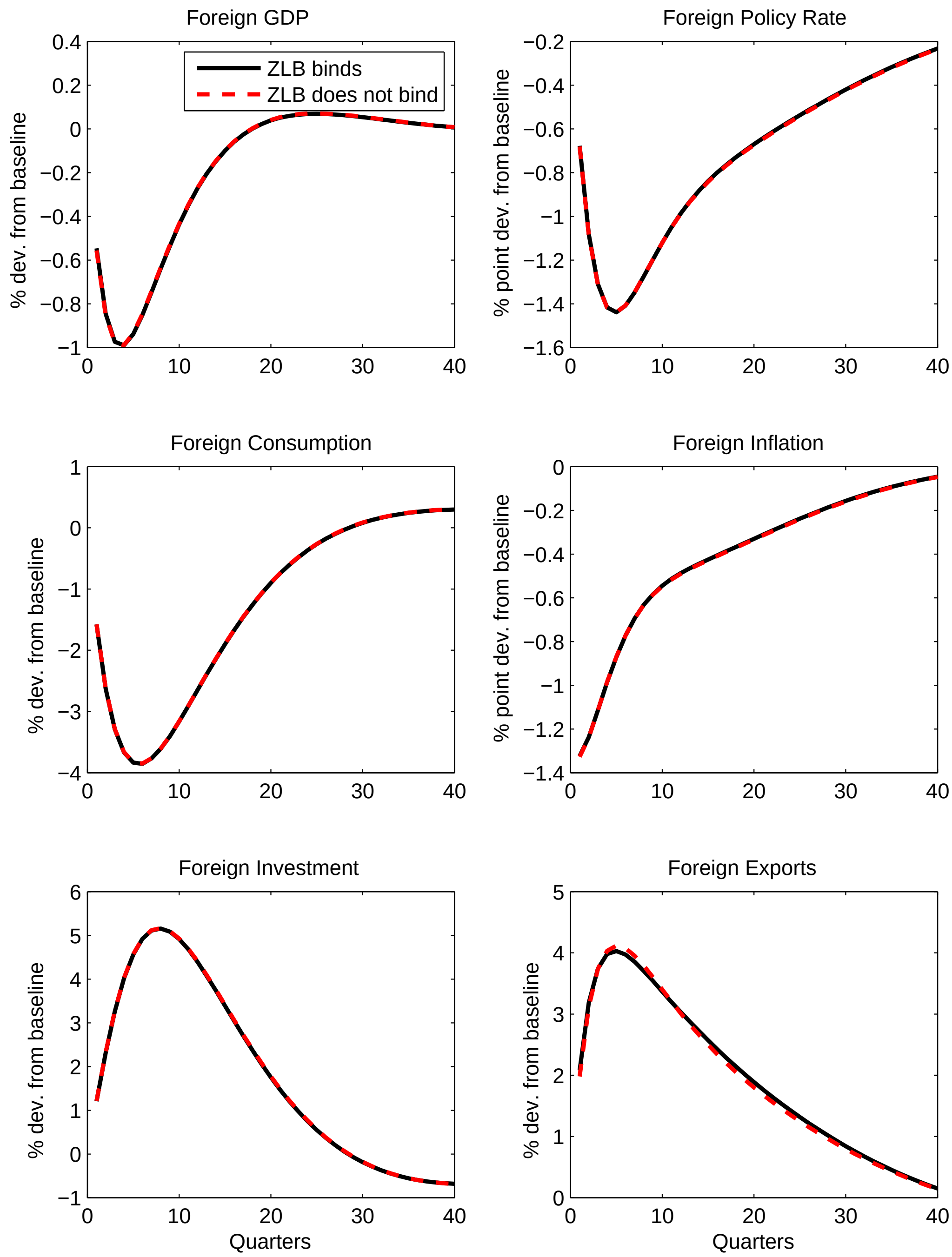


Figure 3: Effects of Foreign Consumption Shock against Backdrop of Domestic Recession

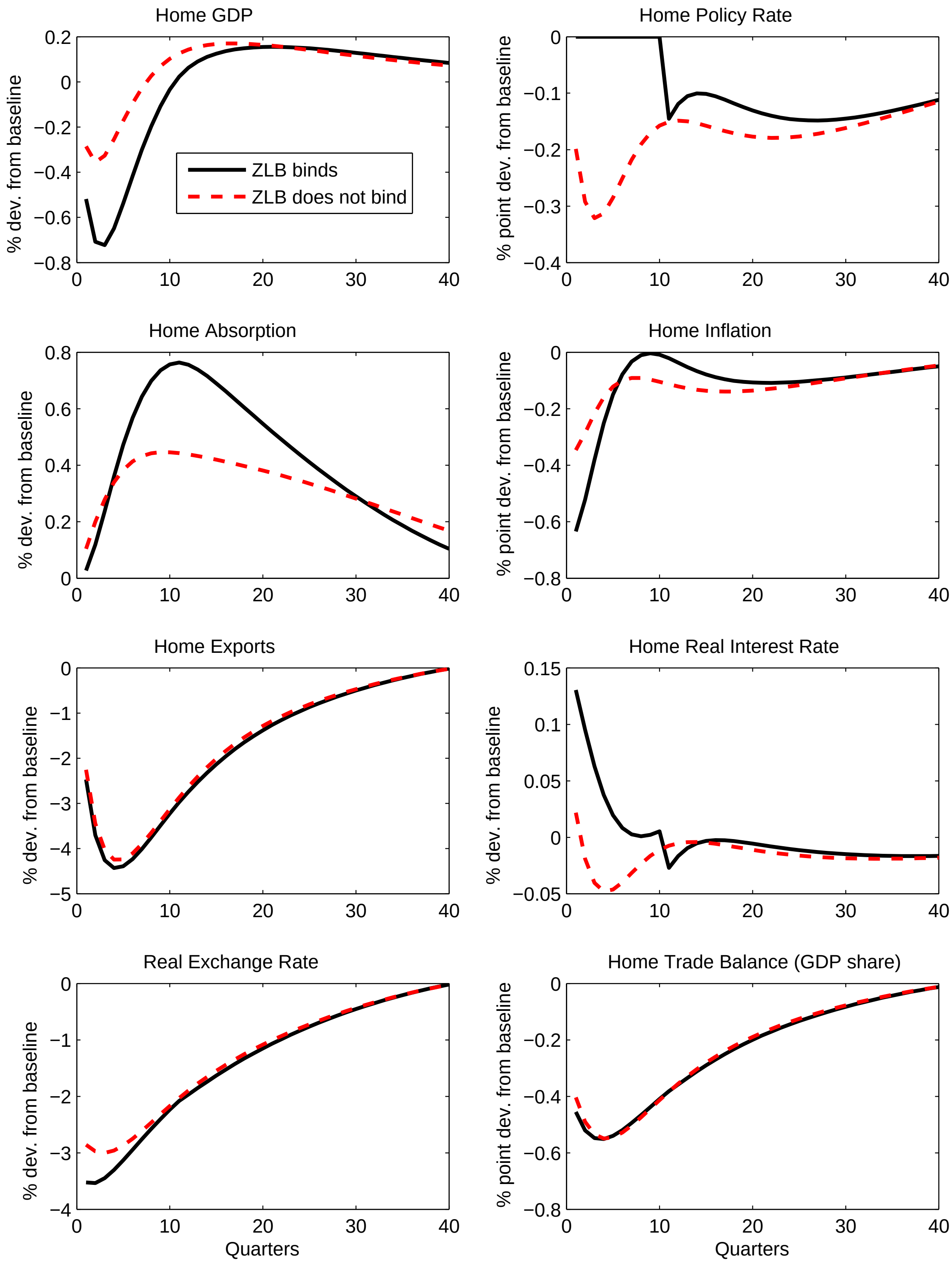


Figure 4: Effects of Foreign Consumption Shock against Backdrop of Deeper Domestic Recession

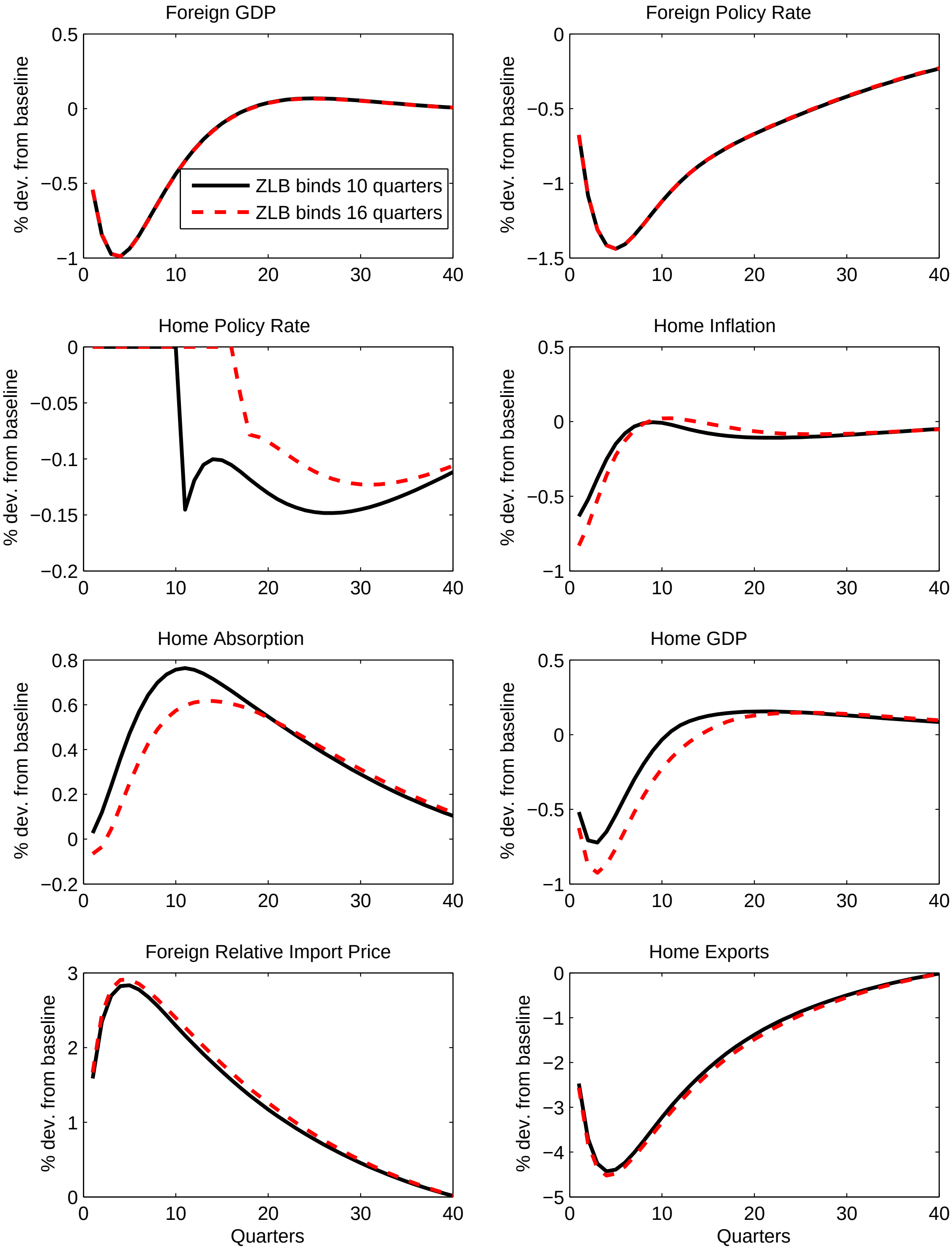
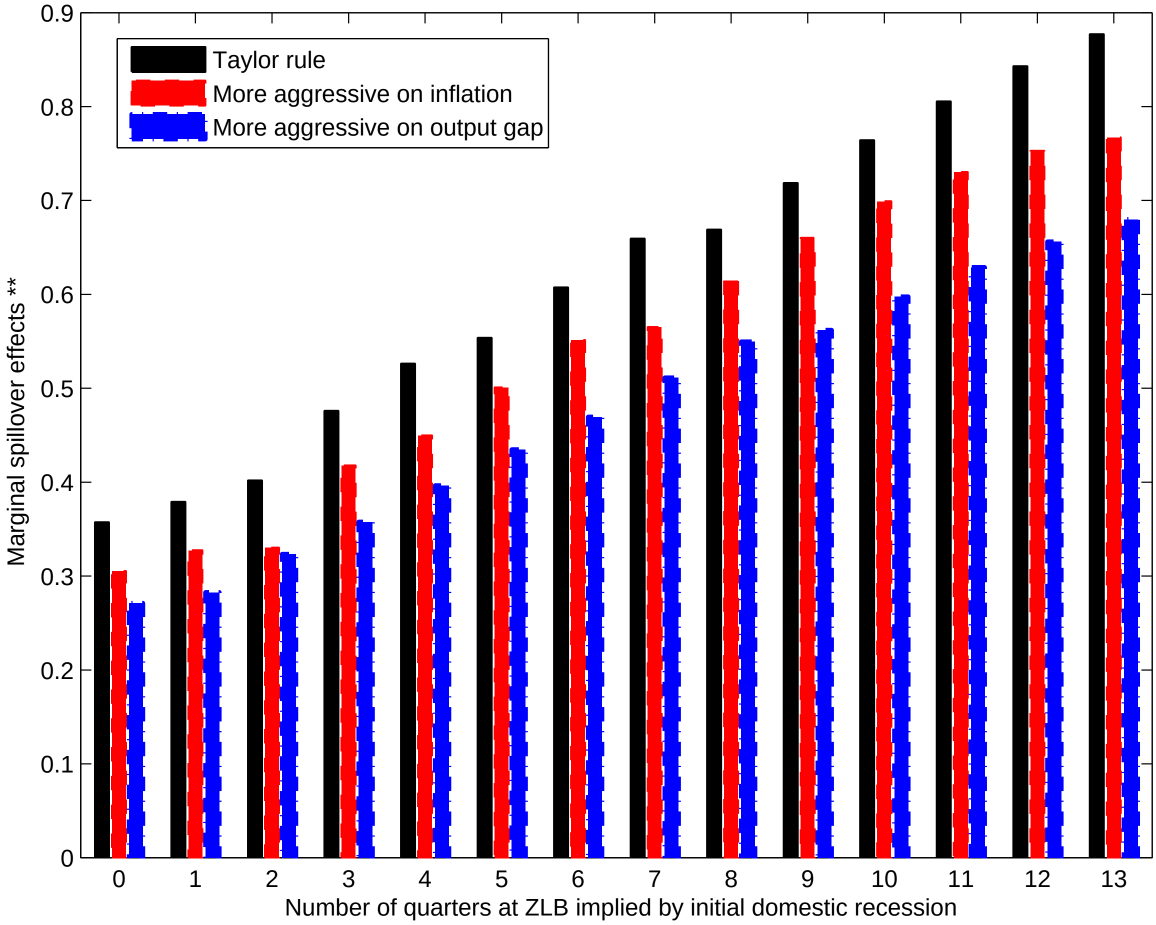


Figure 5: Effects of Foreign Consumption Shock against the Backdrop of Domestic Recession
Alternative Monetary Policy Rules*

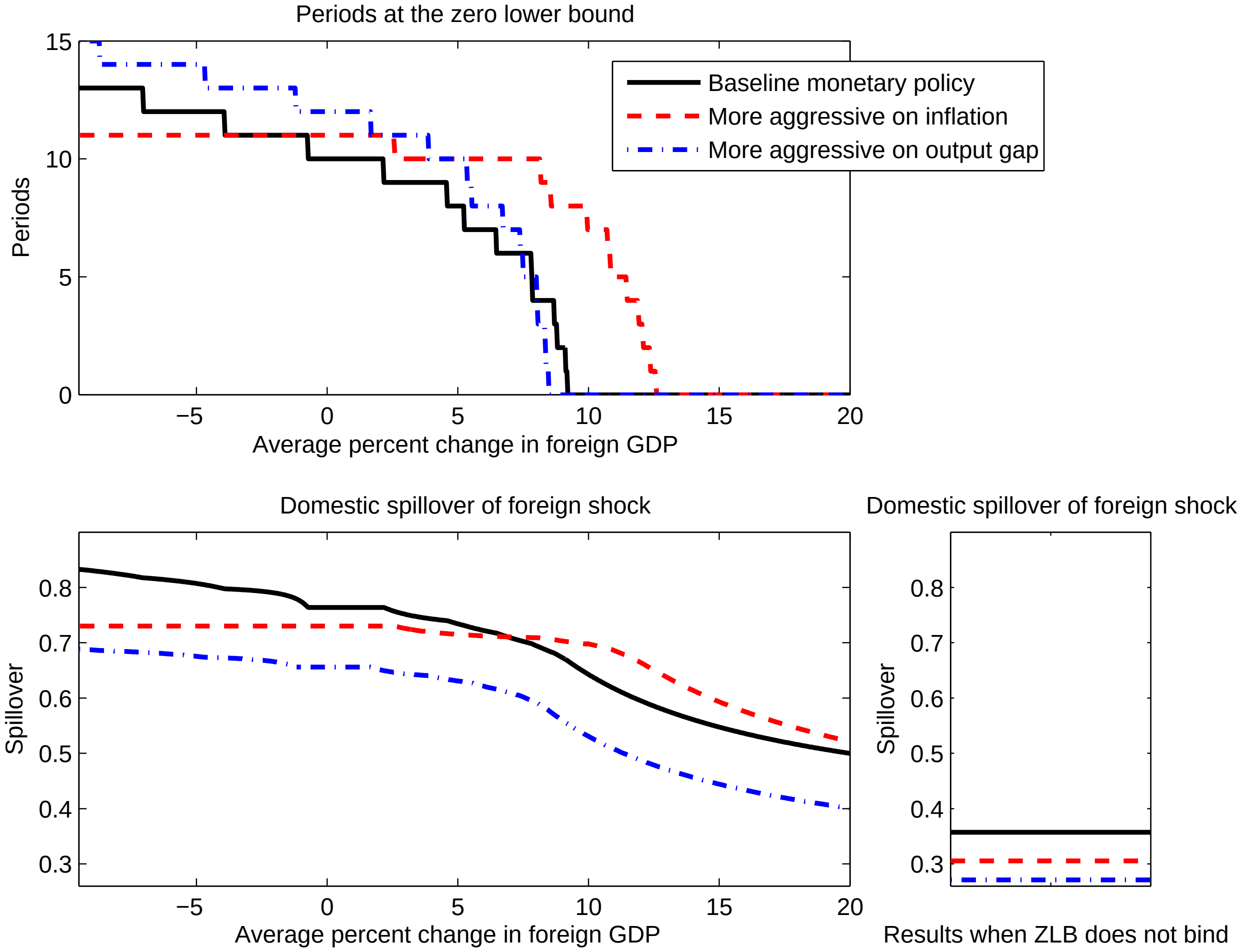


* The parameters for the policy rule described in equation (17) are chosen as: $\gamma_i = 0.7, \gamma_\pi = 1.5, \gamma_y = 0.5$ for the benchmark Taylor rule; the rule more aggressive on inflation takes $\gamma_\pi = 3$ while leaving the other parameters unchanged; and the rule more aggressive on the output gap takes $\gamma_y = 4$ while leaving the other parameters unchanged.

** The spillover effects are defined as the ratio of the response of home GDP (in log deviation from the path implied by the initial baseline recession) to the response of foreign GDP (also in deviation from its initial path). The measure shown is an average of the spillover effects over the first four quarters. The size of the foreign consumption shock was chosen to be small enough not to influence the number of periods for which the zero lower bound on policy rates is binding.

Figure 6: Effects of Foreign Consumption Shock against Backdrop of Domestic Recession

Alternative Monetary Policy Rules*

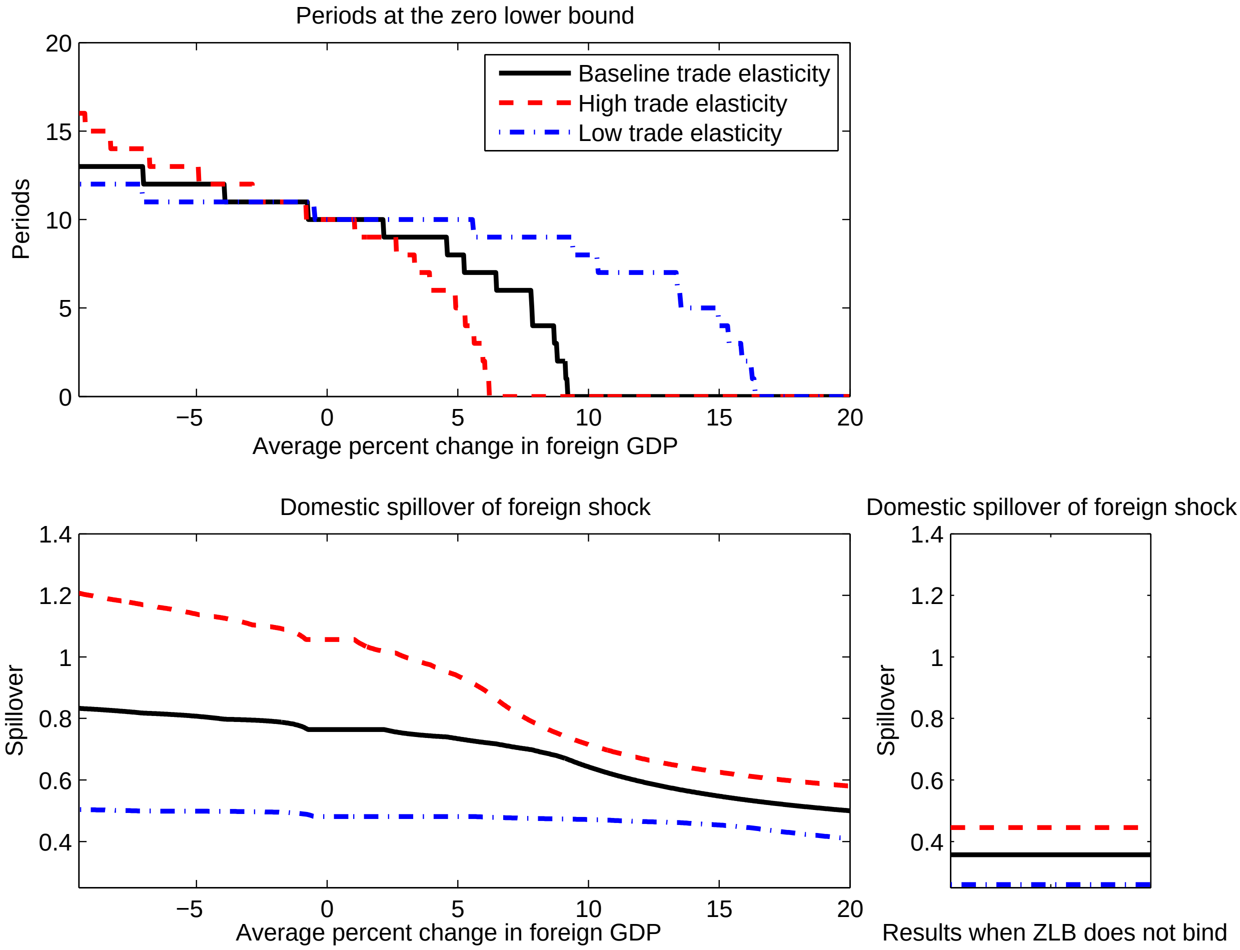


* The parameters for the policy rule described in equation (17) are chosen as: $\gamma_i = 0.7, \gamma_\pi = 1.5, \gamma_y = 0.5$ for the benchmark Taylor rule; the rule more aggressive on inflation takes $\gamma_\pi = 3$ while leaving the other parameters unchanged; and the rule more aggressive on the output gap takes $\gamma_y = 4$ while leaving the other parameters unchanged.

** The spillover effects are defined as the ratio of the response of home GDP (in deviation from the path implied by the initial baseline recession) to the response of foreign GDP (also in deviation from its initial path). The measure shown is an average of the spillover effects over the first four quarters.

Figure 7: Effects of Foreign Consumption Shock against Backdrop of Domestic Recession

Alternative Trade Elasticities*



* The baseline trade elasticity is 1.1; the high trade elasticity is ; the low trade elasticity is 1.5; the low trade elasticity is 0.75.

** The spillover effects are defined as the ratio of the response of home GDP (in deviation from the path implied by the initial baseline recession) to the response of foreign GDP (also in deviation from its initial path). The measure shown is an average of the spillover effects over the first four quarters.

Figure 8: Zero Lower Bound Binds at Home and Abroad

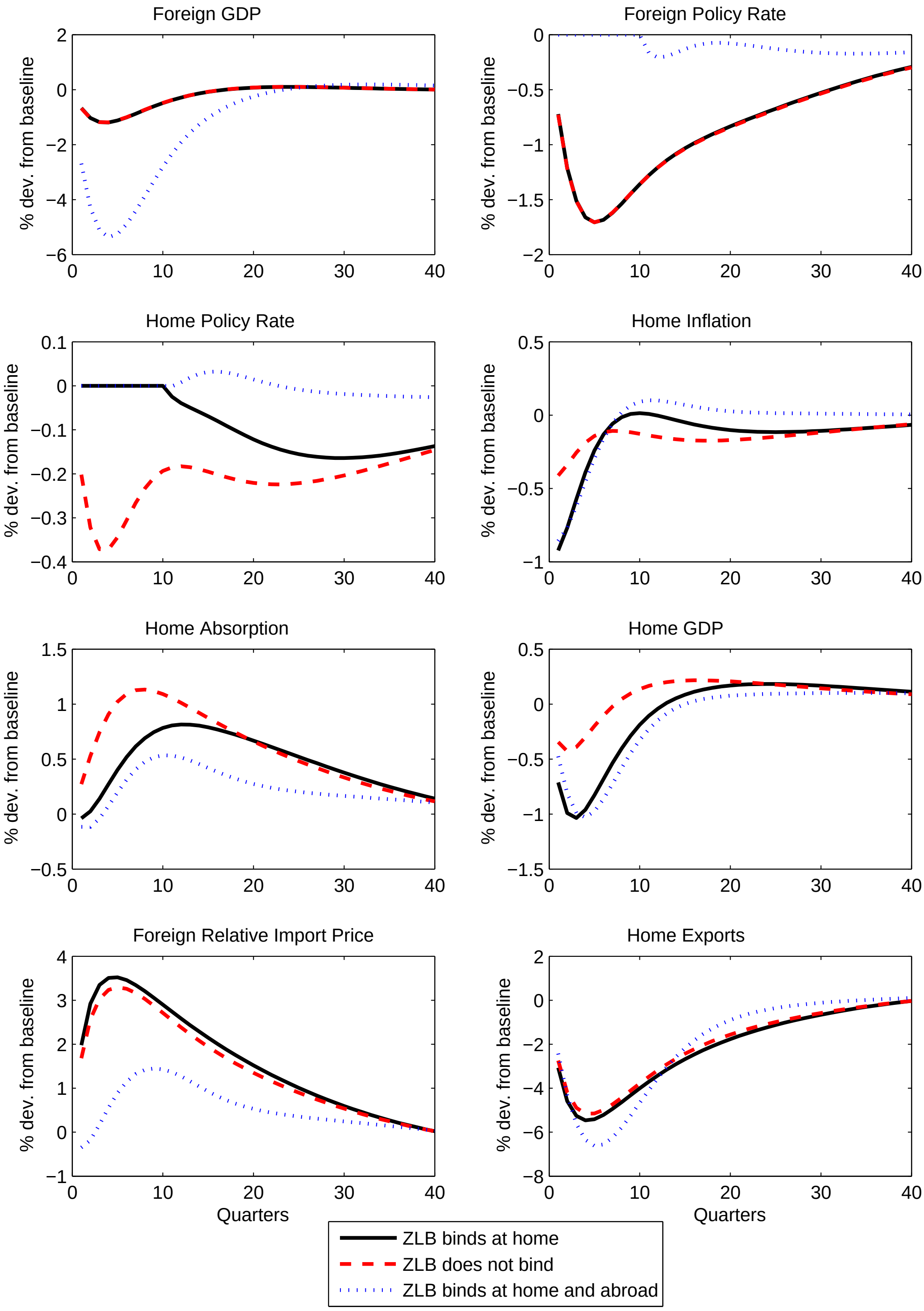
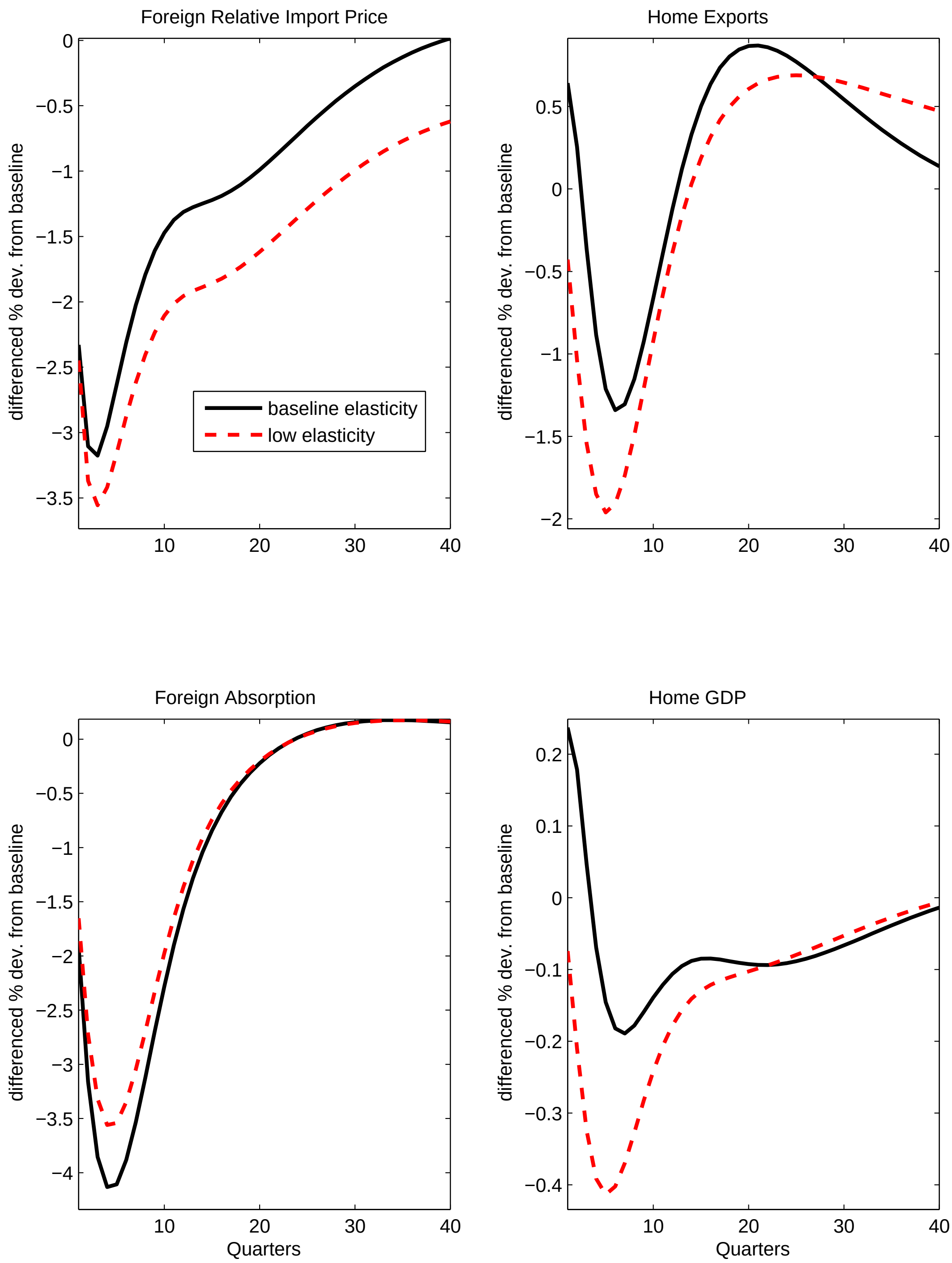


Figure 9: Reverberation Effects when Both Countries are in a Liquidity Trap*



* Each line is constructed by subtracting the impulse responses to a foreign consumption shock in the case when both countries are at the zero bound from those which obtain when only the home country is at the zero bound.

A Appendix: Formalizing the Role of the Initial Baseline Forecast

This Appendix provides background notes for implementing the piecewise-linear approach. This approach has proven very helpful in conducting sensitivity analysis. Moreover, we highlight limited relevance of the initial baseline path with regard to the international spillover effects; we also show that the effects of additional shocks are linear provided that the shock does not affect the duration of the liquidity trap.

For simplicity, assume that a shock immediately depresses the policy rate so that the zero lower bound binds from periods 1 to T.[23] If the shock does not also bring down policy rates in the foreign country to the zero lower bound, there are two linear systems that summarize the equilibrium conditions.[24]

Let the linear system that summarizes the equilibrium conditions for $t \geq T+1$ be written as

$$\bar{A}E_t s_{t+1} + \bar{B}s_t + \bar{C}s_{t-1} + \bar{D}\varepsilon_t = 0, \tag{20}$$

where s is a $N \times 1$ vector stacking all the N variables in the model; ε is a $M \times 1$ vector stacking the innovations to the shock processes; and $\bar{A}$, $\bar{B}$, $\bar{C}$, are $N \times N$ matrices and $\bar{D}$ is a $N \times M$ matrix of coefficients. For $1 \leq t \leq T$, the linear equilibrium conditions are denoted by

$$\bar{A}E_t s_{t+1} + \bar{B}^* s_t + \bar{C}s_{t-1} + \bar{D}\varepsilon_t + \bar{d} = 0, \tag{21}$$

where $\bar{B}^*$ is an $N \times N$ matrix and $\bar{d}$ is a $N \times 1$ vector. Furthermore, $\varepsilon_t = 0$ for all $t > 1$.

The matrices $\bar{B}$ and $\bar{B}^*$ differ in one entry only. Without loss in generality, let the Nth row in these two matrices record the relationship between the nominal interest rate r_t and the notional interest rate r_t^{not}, where in the original nonlinear system $r_t = \max(-\bar{r}, r_t^{not})$. Let r_t^{not} be the $n_{r^{not}}$th entry into s_t, and $\bar{B}\left(N, n_{r^{not}}\right)$, $\bar{B}^*\left(N, n_{r^{not}}\right)$ be the entry in row N, column $n_{r^{not}}$ into $\bar{B}$ and $\bar{B}^*$ respectively. Then $\bar{B}\left(N, n_{r^{not}}\right) = -1$ and $\bar{B}^*\left(N, n_{r^{not}}\right) = 0$.[25] The vector $\bar{d}$ contains zeros everywhere except in the Nth row, which equals $\bar{r}$.[26]

[23] The extension to the case in which the interest rate does not reach zero on impact is straightforward, but is omitted for brevity.

[24] There is a proliferation of the number of linear systems for more complex cases in which the ZLB binds in both countries

[25] Notice that r_t is expressed in deviation from its steady state level. Thus, using the notation of equation 17, $r_t = i_t - \bar{i}$ and $r_t^{not} = i_t^{not} - \bar{i}$.

[26] An alternative way to think about the dynamics under the zero lower bound is in terms of monetary policy shocks. Instead of replacing $\bar{B}$ by $\bar{B}^*$ and introducing $\bar{d}$, one can simply add a monetary policy shock in the policy rule of size $\varepsilon_{m,t} = \max\left(-\bar{r} - r_t^{not}, 0\right)$ and $r_t = r_t^n + \varepsilon_{m,t}$.

Dynamics for $t \geq T+1$ The solution of the system (20) above is given by

$$s_t = Ps_{t-1} + Q\varepsilon_t, \tag{22}$$

where P is the matrix that solves the linear rational expectations model in which the zero bound constraint on i_t is ignored.

Dynamics for $t \leq T$ As Eggertsson and Woodford (2003) and Jung, Teranishi, and Watanabe (2005) we derive the solution using backward induction. In the last period in which the economy is at the zero bound, the values of the endogenous variables is computed from (21) and the fact that $s_{T+1} = Ps_T$:

$$\begin{aligned} s_T &= -\left(\bar{A}P + \bar{B}^*\right)^{-1} \bar{C}s_{T-1} - \left(\bar{A}P + \bar{B}^*\right)^{-1} \bar{d} \\ &= G^{(1)}s_{T-1} + h^{(1)} \ . \end{aligned} \tag{23}$$

In all other periods

$$\begin{aligned} s_t &= As_{t+1} + Cs_{t-1} + d, \\ s_1 &= As_2 + Cs_0 + d + D\varepsilon_1, \end{aligned} \tag{24}$$

where $X = -\left(\bar{B}^*\right)^{-1} \bar{X}$.

Combining (23) and (24) we obtain

$$\begin{aligned} s_t &= G^{(T-t+1)}s_{t-1} + h^{(T-t+1)},\ 2 \leq t \leq T \\ s_1 &= G^{(T)}s_0 + h^{(T)} + \left(I - AG^{(T-1)}\right)^{-1} D\varepsilon_1. \end{aligned} \tag{25}$$

G^{T-t} and h^{T-t} are generated recursively with

$$\begin{aligned} G^{(T-t+1)} &= \left(I - AG^{(T-t)}\right)^{-1} C, \\ h^{(T-t+1)} &= \left(I - AG^{(T-t)}\right)^{-1} \left(Ah^{(T-t)} + d\right). \end{aligned} \tag{26}$$

with

$$\begin{aligned} G^{(1)} &= -\left(\bar{A}P + \bar{B}^*\right)^{-1} \bar{C}, \\ h^{(1)} &= -\left(\bar{A}P + \bar{B}^*\right)^{-1} \bar{d}. \end{aligned} \tag{27}$$

We can also express the values of the endogenous variables as a function of the time 1 innovations. If

$$\begin{aligned} s_0 &= \vec{0}, \\ s_1 &= \left(I - AG^{(T-1)}\right)^{-1} D\varepsilon_1 + h^{(T)}, \end{aligned}$$

then for $2 \leq t \leq T$

$$
\begin{aligned}
s_t &= \left(\prod_{i=1}^{t-1} G^{(T-i)} \right) s_1 + \sum_{j=1}^{t-1} \left(\prod_{i=j+1}^{t-1} G^{(T-i)} \right) h^{(T-j)} \\
&= \left(\prod_{i=1}^{t-1} G^{(T-i)} \right) \left(I - AG^{(T-1)} \right)^{-1} D\varepsilon_1 + \sum_{j=0}^{t-1} \left(\prod_{i=j+1}^{t-1} G^{(T-i)} \right) h^{(T-j)}, \qquad (28)
\end{aligned}
$$

where $\prod_{i=j+1}^{t-1} G^{(T-i)} = I$ if $j+1 > t-1$.

From the formulae above, one can see the proof for the following proposition:

Proposition 1 *Linearity at the zero bound: Consider the two shock vectors ε_1 and $\varepsilon_1 + \mu_1$, $\mu_1 \neq 0$. If $T(\varepsilon_1) = T(\varepsilon_1 + \mu_1) = T^*$, then $\left\{ s_t^{(\varepsilon_1, T^*)} \right\}_{t=1}^{\infty} - \left\{ s_t^{(\varepsilon_1+\mu_1, T^*)} \right\}_{t=1}^{\infty} = \left\{ s_t^{(\mu_1, T^*)} \right\}_{t=1}^{\infty}$.*

Two corollaries follow from the proposition above:

Corollary 2 *Consider the four different shock realizations: ε_1, $\varepsilon_1 + \mu_1$, ε_1^*, $\varepsilon_1^* + \mu_1$. Let $T^* = T(\varepsilon_1) = T(\varepsilon_1 + \mu_1) = T^*(\varepsilon_1^*) = T^*(\varepsilon_1^* + \mu_1)$. Then $\left\{ s_t^{(\varepsilon_1, T^*)} \right\}_{t=1}^{\infty} - \left\{ s_t^{(\varepsilon_1+\mu_1, T^*)} \right\}_{t=1}^{\infty} = \left\{ s_t^{\left(\varepsilon_1^*, T^*\right)} \right\}_{t=1}^{\infty} - \left\{ s_t^{\left(\varepsilon_1^*+\mu_1, T^*\right)} \right\}_{t=1}^{\infty}$, i.e. the effect of the μ_1 does not depend on the initial conditions ε_1 or ε_1^*, provided that the duration at the zero bound is unchanged.*

The effect of a positive and a negative shocks are symmetric if the duration at the zero bound is not affected by the shock.

Corollary 3 *Consider the shocks $\varepsilon_1 + \mu_1$ and $\varepsilon_1 - \mu_1$ with $T(\varepsilon_1 + \mu_1) = T(\varepsilon_1 - \mu_1)$. Then $\left\{ s_t^{(\varepsilon_1+\mu_1, T^*)} \right\}_{t=1}^{\infty} = - \left\{ s_t^{(\varepsilon_1-\mu_1, T^*)} \right\}_{t=1}^{\infty}$.*

Finding T Given the guess for T, compute r_T^n and r_{T+1}^n. The current guess for T constitutes is associated with the model's equilibrium solution path if $r_T^n < -i^*$ and $r_{T+1}^n \geq -i^*$. We denote the number of periods for which policy rates are expected to remain at the zero lower bound following a set of innovations ε_1 by $T(\varepsilon_1)$.

Threshold levels Closely related to the question of how to find $T(\varepsilon_1)$, note

that one can define combinations of the innovations such that agents expect the zero lower bound to be binding for any number of periods.

Any shock vector $\bar{\varepsilon}_1$ that is compatible with policy rates at the zero bound for T periods needs to satisfy:

$$e'_{n_{r}not} P \left[\left(\prod_{i=1}^{t-1} G^{(T-i)} \right) \left(I - AG^{(T-1)} \right)^{-1} D\bar{\varepsilon}_1 + \sum_{j=0}^{T-1} \left(\prod_{i=j+1}^{T-1} G^{(T-i)} \right) h^{(T-j)} \right] = -i^*$$

where $e_{n_{r}not}$ is a $N \times 1$ vector with zeros everywhere expect for the $n_{r^{not}}$th position, which has an entry of 1. If $\bar{\varepsilon}_1$ contains only one non-zero elements in its kth position, then

$$\bar{\varepsilon}_{k,1} = \frac{-i^* - e'_{n_{r}not} P \sum_{j=0}^{T-1} \left(\prod_{i=j+1}^{T-1} G^{(T-i)} \right) h^{(T-j)}}{e'_{n_{r}not} P \left(\prod_{i=1}^{t-1} G^{(T-i)} \right) \left(I - AG^{(T-1)} \right)^{-1} De_k}.$$

B Appendix: Additional Sensitivity Analysis

The magnification of foreign spillover effects is not peculiar to foreign preference shocks. Figure 10 shows the impulse responses for the case of a contraction in foreign government spending. The shock follows an AR(1) process with persistence parameter equal to 0.995. The channels for the transmission of the decline in foreign demand are very similar to the ones described for a consumption preference shock. The spillover effects are smaller, because the effects of the government spending shock are less persistent, as consumption habits increase the endogenous persistence of the preference shock. Choosing an AR(2) process for government spending shocks could increase the persistence of the effects of government spending shocks and bring the quantitative responses to this shock much closer to those of the preference shock considered in the paper.

Figure 11 shows the effects of a foreign technology shock. The technology shock is assumed to follow an AR(1) process with persistence parameter equal to 0.95. For this shock, the model calibration is changed to eliminate consumption habits and investment adjustment costs. With these real rigidities the spillover effects of a foreign technology shock are even more modest because of J-curve effects. With the real rigidities present, foreign absorption drops sluggishly, which combined with an immediate exchange rate appreciation boosts foreign import demand initially. In that case, based on the strength of its exports, U.S. GDP rises before falling. Without the real rigidities, U.S. exports drop on impact but the spillover effects remain modest because the drop in foreign activity which compresses the demand for U.S. exports, is accompanied by a dollar depreciation whose partial effect is to boost the demand for U.S. exports.

Figure 12 shows the impulse responses for the case of an increase in the foreign capital tax rate. This shock could be interpreted as boosting investment demand. The AR(1) persistence parameter for the shock is set to 0.95. In a liquidity trap, the cross-country spillover effects are magnified at least twofold as measured by the reaction of home GDP relative to the movement in foreign GDP. Increases in the shock persistence would again act to increase the spillover effects.

Finally, Figure 13 offers sensitivity analysis with respect to inflation indexation. The benchmark calibration abstracted from inflation indexation in the setting of domestic prices and wages. The figure considers the effects of a foreign consumption shock when price setting is subject to partial indexation. Those firms that do not receive the Calvo signal are assumed to adjust prices according to a rule of thumb that lets them catch up to 50% of the previous period's aggregate inflation. Whereas the initial fall in inflation is not as dramatic with partial indexation, the persistence of the drop is increased, so that short-term real rates are little changed. This is reflected in a response of domestic private absorption that is little changed across the two columns of figure 13. A flatter New Keynesian Phillips curve, as would obtain with a lower Calvo probability, would result in a similar reduction of the initial inflation response and an increase in the persistence of the inflation movement. There would also be little overall change in the output response relative to the case illustrated for the benchmark calibration.

Figure 10: Foreign Government Spending when Home Country is at Zero Lower Bound

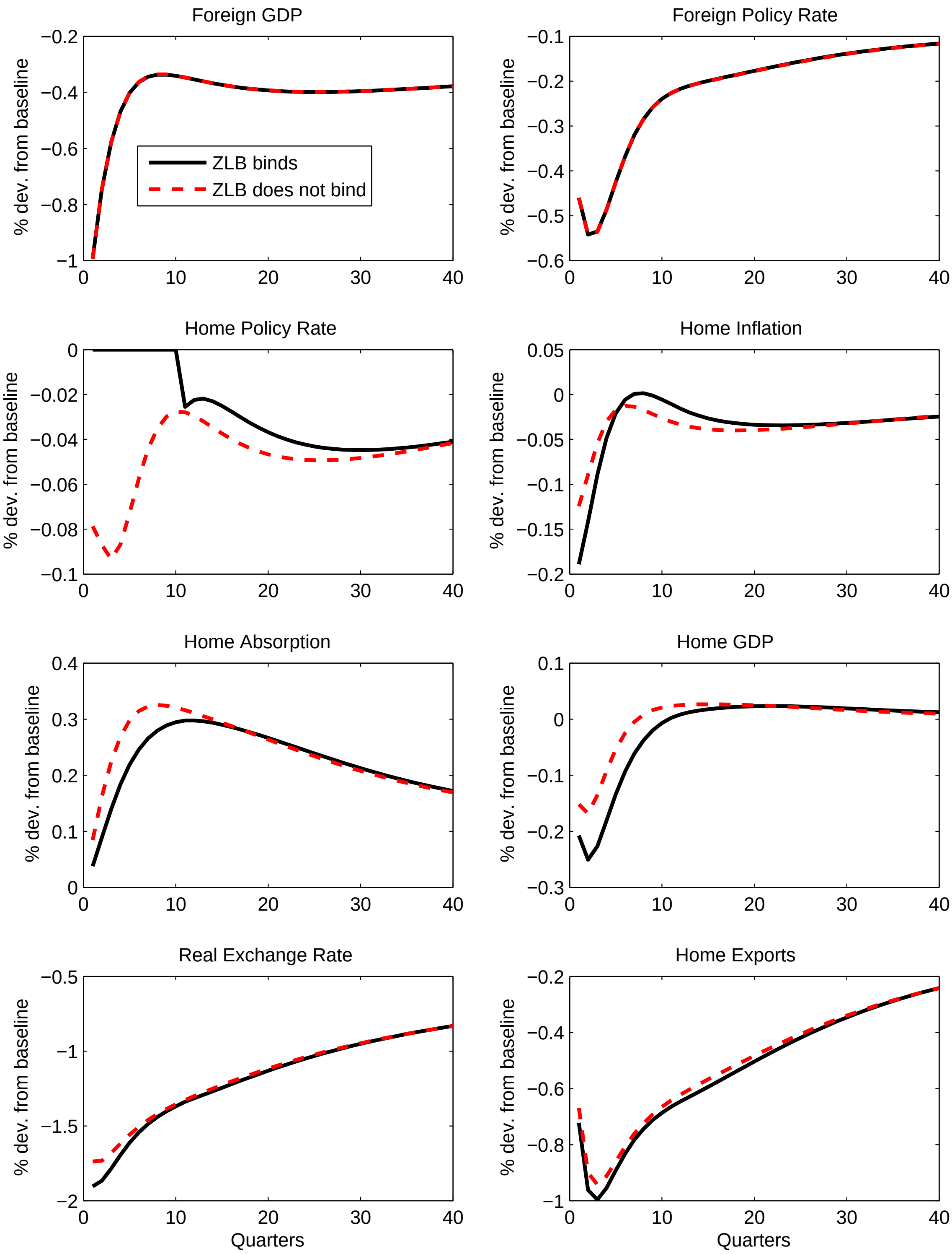


Figure 11: Foreign Technology Shock when Home Country is at Zero Lower Bound

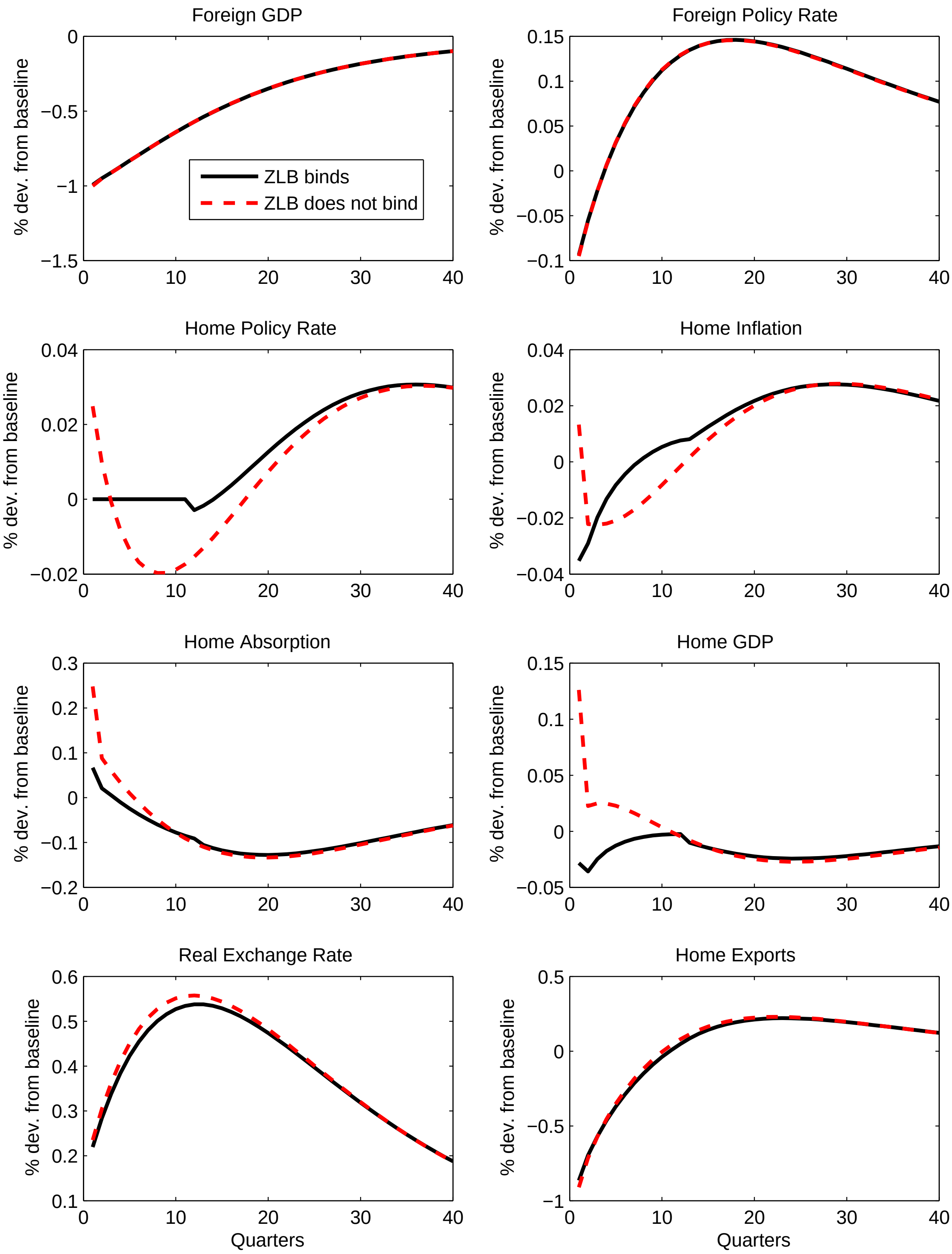


Figure 12: An Increase in the Capital Tax Rate Abroad when Home Country is at Zero Lower Bound

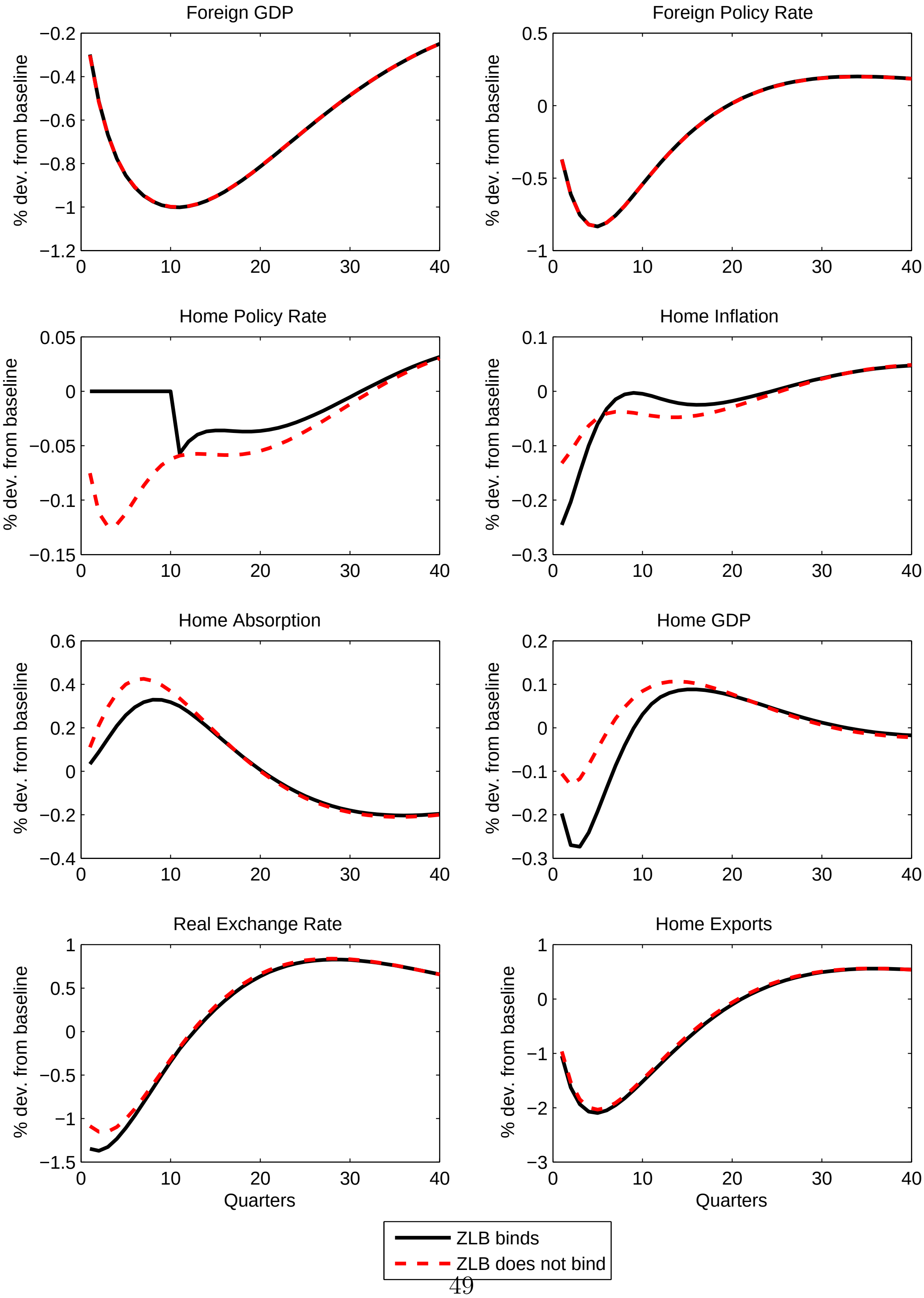


Figure 13: Sensitivity to Degree of Inflation Indexation