Common factors in small open economies: Inference and consequences

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Abstract

Inference about a common international stochastic trend is gained using a small open economy model, data from seven developed countries, and Bayesian methods. Shocks to this trend explain up to 14% of the variability of real variables in several economies. Country-specific preference and premium disturbances account for the bulk of the volatility observed in data. There is substantial heterogeneity in the estimated structural parameters as well as stochastic processes for the countries in the sample. This diversity translates into a rich array of impulse responses across countries. The inclusion of a common stochastic factor influences inference of other stochastic processes in the model.

1 Introduction

Understanding the origins of international macroeconomic fluctuations has been a long quest in economics. Over the past decades, research on the topic shows that business cycles across countries are correlated. Indeed, the empirical evidence suggests the existence of a common factor driving fluctuations in the world. Examples of this research agenda include, among others, Stockman (1988), Norrbin and Schlagenhauf (1996), Gregory et al. (1997), Clark and Shin (2000), Kose et al. (2003). Although the relative importance of the common (world) factor varies from study to study, these papers share the feature of being based on reduced form time-series analyses (purely statistical models). As a consequence, the estimated factors lack a structural interpretation besides that of being a world, regional, or country-specific component.

The non-structural nature of these studies is problematic because the economic meaning of the common factor is unclear. Is it a technology related term, an oil shock, or a preference shock that drives the fluctuations? The answer to this question is crucial from a policy point of view. If, for instance, the common factor explains a large fraction of the domestic fluctuations and arises from coordinated monetary/fiscal efforts, then market structure is most likely less relevant for business cycles. Consequently, efforts to mitagate domestic market imperfections may be ineffective. On the other hand, finding that

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the common factor is driven by oil shocks opens the door for policy actions aimed to reduce the country's exposure to such fluctuations.

Providing a structural interpretation to the estimated factors then requires the solution and estimation of a dynamic stochastic general equilibrium (DSGE) model in a multi-country context. The main difficulty behind such an endeavor is that the model's complexity grows with the number of countries. For example, a medium scale DSGE model for 5 countries can easily involve 150 state variables and 200 parameters. The solution of such a model is computationally intensive even in a linear setup. To make things worse, the large dimensionality of the state space makes the evaluation of the likelihood very expensive making the estimation of the model too involved. This paper circumvents these obstacles by proposing and estimating a tractable open economy model that is common for several countries around the world. The estimated model is rich enough to allow for country-specific disturbances as well as a common factor for all economies. Methodologically, the paper shows how to express the likelihood function of an open economy model with a common factor and several idiosyncratic shocks. The proposed approach effectively blends ideas from two strands of the literature: dynamic factor analysis (Stock and Watson, 1993) and DSGE models (Christiano et al., 2005)

Tractability in the model results from three fundamental suppositions. To begin with, a basic premise is that co-movement among countries is due to a single stochastic productivity trend (common factor). This assumption is quite plausible given the significant amount of empirical and theoretical work suggesting that productivity shocks drive international business cycles (Backus et al., 1992 and 1995; Baxter and Crucini, 1995; Rabanal et al., 2009). To deal with the non-stationarity in the data, the model is estimated using the growth rates of output, consumption, and investment. Second, the estimation exercise considers seven developed economies each of which is treated as a small open economy (SOE).[1] The lack of synchronization of business cycles among emerging economies is the main reason to exclude them from the sample (Aguiar and Gopinath, 2007). Finally, based on the empirical findings in Garcia-Cicco et al. (2009), the country-specific factors are identified as preference, risk premium, total factor productivity, and government expenditure disturbances.

The results reveal several interesting features. First, the estimated model can successfully account for the large fluctuations present in Norway and the excess volatility of consumption in Canada and Sweden. Second, as in Kose et al. (2003), there is a common stochastic productivity trend, which is statistically significant. Shocks to this trend are mildly persistent and explain up to 14% of the variability of output, consumption, and investment. Furthermore, when expressed in differences the estimated common factor tracks closely the average growth rate of output in the countries in the sample. In contrast, preference and premium shocks account for a large fraction of the volatility in the sample. For instance, these disturbances together explain 72% and 77% of the volatility of consumption in Australia and Sweden, respectively. Additionally, more than 50% of the dynamics of output in all countries is captured by idiosyncratic TFP shocks.

Third, there is substantial heterogeneity in the estimated structural parameters as well as stochastic processes for the countries in the sample. This diversity translates into a rich array of impulse responses

[1] The countries in the sample are Australia, Belgium, Canada, New Zealand, Norway, Spain, and Sweden.

across countries. Fourth, when the model is estimated separately for each country, the TFP process is less volatile and persistence than in the jointly estimated model. As a consequence, the explanatory power of TFP shocks markedly declines in the model whose parameters were estimated individually.

Finally, several extensions are allowed in the model. For example, when the model is estimated using HP-filtered data, the explanatory power of the common shock rises up to 23%. The addition of a common interest rate shock marginally increases the fraction of the business cycle explained by the productivity shock. Combined the interest rate and productivity disturbances account for 16% of Canada's output volatility. These results suggest that the relatively large importance of the common factor reported in reduced form models (Kose et al., 2003) is probably a convolution of productivity and interest rates shocks and a third source that my model is unable to capture. This missing factor most likely corresponds to an international demand disturbance.

Several strands of the literature relates to this paper. Perhaps the most direct relation comes from the dynamic factor analysis literature as in the seminal contributions of Stock and Watson (1993) and Kose et al. (2003). The structural interpretation of the common international factor is an important improvement of my manuscript over the later paper. Because of its analysis of the implications of structural shocks on open economy models, this paper also relates to the business cycle literature in small open economies (Neumeyer and Perri, 2005; Aguiar and Gopinath, 2007; Garcia-Cicco et al., 2009; Fernandez-Villaverde et al., 2009). The estimation of a structural model with temporal and cross-section data shares some commonalities with the panel data estimation literature (Woodridge, 2002).

This paper is also close in spirit to the contributions of Boivin and Giannoni (2006), Normandin and Powo (2005), and Taylor (1993). Similar to Boivin and Giannoni, this paper estimates a DSGE model with multiple time series. The main difference is that while my study is concerned in extracting a factor common to several countries, they are more interested in understanding the effects of a large database on the estimation of closed economy DSGE models. The second authors study the effects of world and country-specific shocks in a DSGE context. Rather than estimating a full multi-country DSGE model (as is the case in my formulation), the authors use a dynamic factor approach to extract a common factor from Solow residuals for several countries. The estimated factor is then used to simulate a DSGE model. Finally, Taylor (1993) estimates a reduced-form open economy model for the G7 countries. The key discrepancies between our studies are: 1) his approach is non-structural; and 2) unlike my approach, he estimates the model on a country-by-country basis, which rules out a common factor among the countries in his sample.

Garcia-Cicco et al. (2009) argue that inference based on short samples may lead to erroneous claims about the importance of trend shocks on business cycles. Their suggestion is to replace quarterly for yearly data, which is more abundant for both developed and developing small open economies. This paper instead proposes to estimate the common trend using data from multiple countries. This approach effectively increases the sample size and improves inference. Hence, the proposed methodology most likely shields the researcher against vague conclusions regarding stochastic trends in developed SOEs.

The rest of the paper is organized as follows. The next section provides some casual evidence that real variables in developed SOEs have similar trends as well as synchronized cycles. The small open economy model is outlined in Section 3. Sections 4 and 5 discuss the evaluation of the likelihood associated to the model as well as its estimation. The results and some sensitivity checks are reported in Section 6. The final section provides some concluding remarks.

2 Trends and cycles in developed SOE

To motivate the discussion to follow, the top panel in Figure 1 displays the HP-filtered trends for real GDP in several developed economies (see Section 5 for details on the data). For comparison purposes, the trends are normalized to 1 in the year 1990. From a quick look at this figure, it is not difficult to convince ourselves that trends in developed small open economies tend to closely track each other. Indeed, one can hardly distinguish that of New Zealand from that of Australia or, prior to 1990, the trend of Canada from that of Norway. Sweden, however, seems to be a little off track with the rest of countries in the sample. Its trend is below the average prior to 1990 but it has surpassed the trend of all other countries by 1998.

The HP-filtered business cycles (bottom left panel in Figure 1) reveal that output in the small open economies tend to closely co-move over the cycle. The figure also reveals some interesting differences across countries. For example, New Zealand, Norway, and Sweden are more volatile than the other economies in the sample. Furthermore, output in Norway seems countercyclical relative to the other countries, which is especially noticeable in the 1980s. Indeed, while Norway is booming in 1987, the remaining countries are enduring a recession. The roles, though, reverse by the end of the 1980s.

The final panel in Figure 1 portrays the quarterly growth rates of output found in the data. The new plot reinforces our previous finding of substantial co-movement among countries over the past two decades. In sum, the information contained in Figure 1 reveal three important features of the data. First, the developed economies in the sample exhibit very similar trends over the past 25 years. Second, even at the business cycle frequencies real GDP shows substantial correlation across countries. Finally, there is heterogeneity among the small open economies studied in this paper.

Although suggestive, the findings in this section are by no means a formal proof that developed small open economies do indeed share a common trend or their cycles are synchronized. Interestingly, the presence of this common factor has been documented in non-structural setups in Kose et al. (2003) and Rabanal et al. (2009). The later authors, for example, report that TFP processes for the U.S. and some developed economies ("rest of the world") exhibit stochastic trends and are cointegrated. The next sections present a DSGE model and an econometric strategy that help to disentangle the degree of co-movement among the countries in our sample. As will become clear, the chief advantage of the approach is that we can precisely decompose fluctuations in the small open economies in two parts: 1) a common stochastic trend driven by productivity shocks; and 2) structural country specific disturbances, which induce oscillations around the common factor.

3 Model

In the spirit of Gali (2008) and Gali and Monacelli (2005), I assume that there are N small open economies indexed by $j \in [1, N]$. Each of these countries is modeled a la Mendoza (1991) with some modifications to improve the empirical fitting. The small open economy framework is convenient because countries' actions have no impact on the rest of the world. Crucially, it is assumed that all countries share a common stochastic trend (this premise will be relaxed momentarily). There are, though, several shocks that are country specific. These assumptions are intended to capture the salient features reported in the previous section while making the solution and estimation of the model feasible. In what follows, I will describe the government and the problems faced by households and firms in country j. For clarity, variables/parameters not indexed by j are common to all small open economies.

3.1 Firms

A representative firm in country j rents labor, $h_{j,t}$, and capital, $K_{j,t}$, to produce an internationally traded good, $Y_{j,t}$. To that end, the firm solves the following problem

$$\max_{h,K} Y_{j,t} - W_{j,t}h_{j,t} - R^k_{j,t}k_{j,t}$$

subject to

$$Y_{j,t} = a_{j,t}K^{\alpha}_{j,t}\left(X_t h_{j,t}\right)^{1-\alpha}. \tag{1}$$

Here, X_t is a labor-augmenting technology shock that is common to all small economies in the model. The growth rate of this shock is $g_t \equiv X_t/X_{t-1}$. As in the related literature (Backus, Kehoe, and Kydland, 1992), capital can freely move across countries but labor is country specific. The total factor productivity shock $a_{j,t}$ is assumed to be stationary and specific to each country.

The functional form (1) captures the notion that technology progress at home results from a combination of domestic and external components. Several elements support the choice of such a production function. First, it is flexible enough to simultaneously allow for a common trend and country-specific deviations around that trend as suggested by the evidence in Section 2. Next, Glick and Rogoff (1995) assume that domestic productivity consists on a world factor common to all economies and a country-specific term. Their world factor is estimated by first computing the Solow residuals for the countries in their sample and then taking a GNP-weighted average. The country-specific shock is obtained as the difference between the world process and the country's Solow residual. A third motivation comes from the evidence in Baxter and Crucini (1995) and Rabanal et al. (2009). These authors report the presence of cointegrated stochastic trends in several industrialized economies (US, European countries, and Canada). Finally, Backus et al. (1995) fit a VAR to Solow residuals in US and an aggregate of European economies. They find that such residuals are driven by a mixture of idiosyncratic (country-specific) shocks and technology spillovers from abroad, which is consistent with the production function (1).

3.2 Households

Each small economy is populated by a continuum of households. They choose consumption, labor, investment, capital, and purchases of foreign bonds according to the program

$$\max_{C,D,I,h,k} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \upsilon_{j,t} \left[\log \left(C_{j,t} - b^{[j]} \widetilde{C}_{j,t} \right) - \frac{\psi^{[j]}}{2} h_{j,t}^2 \right] \tag{2}$$

subject to

$$\frac{D_{j,t+1}}{1 + r_{j,t}} = D_{j,t} - W_{j,t} h_{j,t} - R_{j,t}^k k_{j,t} + C_{j,t} + I_{j,t} + T_{j,t}.$$

Here, $\widetilde{C}_{j,t}$ corresponds to average consumption, $\upsilon_{j,t}$ is a preference shock, $b^{[j]}$ is the habit formation parameter and $T_{j,t}$ is a lump-sum transfer from the central government in country j.[2] $D_{j,t+1}$ corresponds to foreign indebtedness and $r_{j,t}$ is the interest rate residents of country j have to pay to borrow abroad. Christiano, Eichenbaum, and Evans (2005), Adolfson et al. (2007) and Justiniano and Preston (2008) have shown that a utility specification such as (2) is flexible enough to capture salient features of the data in both closed and open economies.

As in Christiano, Eichenbaum, and Evans (2005) and Fernandez-Villaverde et al. (2009), investment is costly and evolves according to

$$K_{j,t+1} = (1 - \delta) K_{j,t} + \left[1 - \frac{\phi^{[j]}}{2} \left(\frac{I_{j,t}}{I_{j,t-1}} - g \right)^2 \right] I_{j,t}.$$

Again this functional specification is chosen because of being parsimonious enough to describe the dynamics of investment found in the data. In that equation, g corresponds to the steady state growth rate of the common technology shock.

3.3 Government

The government in country j levies lump-sum taxes on households to purchase goods according to $T_{j,t} = \varkappa_{j,t} Y_{j,t}$.

3.4 Interest rates

Following Garcia-Cicco et al. (2009), residents in country j can borrow abroad at the rate

$$r_{j,t} = r^* + \varphi \left[\exp \left(\widetilde{D}_{j,t+1} / X_t - d^{[j]} \right) - 1 \right] + \exp(\mu_{j,t} - 1) - 1.$$

Here, r^* is the international risk-free interest rate. $\widetilde{D}_{j,t+1}$ is the economy-wide foreign asset position. Hence, households take outside their control the adjustment cost in the interest rate process. $d^{[j]}$ corresponds to the de-trended debt in steady state. The literature typically refers to $\mu_{j,t}$ as an exogenous

[2] Heathcote and Perri (2008) introduce a taste shock into a standard international business cycle model to reduce the correlation between real exchange rates and relative consumption.

premium shock. I will respect this denomination although it will become clear that $\mu_{j,t}$ most likely captures domestic nominal shocks.

3.5 Stochastic processes

There is a common trend, X_t, affecting all small open economies. The growth rate of this trend, g_t, follows

$$\log g_t = \left(1-\rho_g\right)\log g + \rho_g \log g_{t-1} + \sigma_g \varepsilon_{g,t} \tag{3}$$

For future reference, denote $g_{[\%]} \equiv 100\log g$ as the average growth rate in percentage points. The stochastic processes driving the four idiosyncratic shocks are:

$$\begin{aligned}
\log a_{j,t} &= \rho_a^{[j]} \log a_{j,t} + \sigma_a^{[j]} \varepsilon_{a,t}^{[j]}, \\
\log \mu_{j,t} &= \rho_\mu^{[j]} \log \mu_{j,t} + \sigma_\mu^{[j]} \varepsilon_{\mu,t}^{[j]}, \\
\log \vartheta_{j,t} &= \rho_\vartheta^{[j]} \log \vartheta_{j,t} + \sigma_\vartheta^{[j]} \varepsilon_{\vartheta,t}^{[j]}, \\
\log \varkappa_{j,t} &= \left(1-\rho_\varkappa^{[j]}\right) \log \varkappa^{[j]} + \rho_\varkappa^{j} \log \varkappa_{j,t} + \sigma_\varkappa^{[j]} \varepsilon_{\varkappa,t}^{[j]}.
\end{aligned} \tag{4}$$

The shocks $\varepsilon_{\cdot,t}^{j}$ are assumed to be independent normal distributed with mean 0 and variance 1.

3.6 First Order Conditions

For clarity, the index j is omitted when there is no risk of confusion. Let $\widetilde{\lambda}_t$ and $\widetilde{\zeta}_t$ denote the multipliers associated with the budget constraint and the investment process, respectively. Then the optimality conditions for the small open economy model are

$$\begin{gathered}
\frac{\vartheta_t}{C_t - bC_{t-1}} = \widetilde{\lambda}_t,\ \psi\vartheta_t h_t = \widetilde{\lambda}_t W_t,\ \frac{\widetilde{\lambda}_t}{1+r_t} = \beta \mathbb{E}_t \widetilde{\lambda}_{t+1}, \\
\widetilde{\lambda}_t + \widetilde{\zeta}_t \left[-1 + \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - g\right)^2 + \phi \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - g\right)\right] = \\
\phi\beta\mathbb{E}_t \widetilde{\zeta}_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - g\right), \\
\widetilde{\zeta}_t = \beta\mathbb{E}_t\left[\left(1-\delta\right)\widetilde{\zeta}_{t+1} + R_{t+1}^K \widetilde{\lambda}_{t+1}\right],\quad W_t = \left(1-\alpha\right)\frac{Y_t}{h_t},\ R_t^K = \alpha\frac{Y_t}{K_t},
\end{gathered} \tag{5}$$

The first order conditions plus the country's budget constraint, the government policy, the interest rate rule, the evolution of investment equation, and the shock processes define the equilibrium in the model.

4 Likelihood with one common factor

The solution to the log-linearized version of the stationary model for country j can be represented as

$$
\begin{aligned}
\overline{S}_{j,t} &= \Pi^{[j]}\overline{S}_{j,t-1} + \eta^{[j]}\overline{\xi}_{j,t}, \\
\mathbb{Y}_{j,t} &= \Phi^{[j]}\overline{S}_{j,t},
\end{aligned}
$$

where $\overline{S}_{j,t} = [S'_{j,t}, \widehat{g}_t]'$, $\mathbb{Y}_{j,t} = [\widehat{y}_{j,t}, \widehat{c}_{j,t}, \widehat{i}_{j,t}, \widehat{r}_{j,t}, \widehat{h}_{j,t}, \widehat{\zeta}_{j,t}]'$, and $\overline{\xi}_{j,t} = [\xi'_{j,t}, \varepsilon_{g,t}]'$ denote the vectors of states, controls, and structural shocks, respectively. Here, $S_{j,t} = [\widehat{y}_{j,t-1}, \widehat{c}_{j,t-1}, \widehat{i}_{j,t-1}, \widehat{k}_{j,t}, \widehat{d}_{j,t}, \widehat{\vartheta}_{j,t}, \widehat{\varkappa}_{j,t}, \widehat{\mu}_{j,t}, \widehat{a}_{j,t}]'$, and $\xi_{j,t} = [\varepsilon^{[j]}_{\vartheta,t}, \varepsilon^{[j]}_{\varkappa,t}, \varepsilon^{[j]}_{\mu,t}, \varepsilon^{[j]}_{a,t}]'$. A lowercase variable corresponds to a de-trended (stationary) variable, $c_t = C_t/X_t$, while a hat indicates log-deviations from steady state. The matrices Π, Φ, and η depend on the structural parameters of each country. The first order condition for consumption is used to eliminate the multiplier, $\widetilde{\lambda}_t$, from the solution; hence, $\widetilde{\lambda}_t$ is neither part of the states nor the controls. The structure behind $\overline{\xi}_{j,t}$ allows us to disentangle the fluctuations in the small open economy j due to the common factor — represented here by the shock $\varepsilon_{g,t}$, which may have a diffuse effect on all country data series — from those due to country-specific conditions represented by the shocks $\varepsilon^{[j]}_{\cdot,t}$.

Define the expanded vector of states as $\mathbb{S}_t = [\mathbb{Y}'_{1,t}, \mathbb{Y}'_{2,t}, \cdots, \mathbb{Y}'_{N,t}, S'_{1,t}, S'_{2,t}, \cdots, S'_{N,t}, \widehat{g}_t]'$, and the expanded vector of structural shocks as $\xi_t = [\xi'_{1,t}, \xi'_{2,t}, \cdots, \xi'_{N,t}, \varepsilon_{g,t}]'$. Here, $\dim(\mathbb{S}_t) = [N \times N_c + N \times (N_s - 1) + 1]$ and $\dim(\xi_t) = [(N_{sh} - 1) \times N + 1]$, where N is the number of countries in the sample, $N_c = \dim(\mathbb{Y}_{j,t})$, $N_s = \dim\left(\overline{S}_{j,t}\right)$, and $N_{sh} = \dim\left(\xi_{1,t}\right)$. Then the state space representation of the model with N countries is

$$
\begin{aligned}
\mathbb{S}_t &= \mathbb{F}\,\mathbb{S}_{t-1} + \eta\xi_t, \\
\mathbb{Y}^{Data}_t &= g_{[\%]} + \mathbb{H}\,\mathbb{S}_t.
\end{aligned} \tag{6}
$$

With this formulation in hand, the Kalman filter can be used to evaluate the likelihood. Since there are three observables per country (growth rates of output, consumption, and investment) and four country-specific shocks, the state space model does not require measurement errors. In the discussion to follow a 0 denotes a conformable matrix of zeros. Let $\sum^{[j]}$ be a diagonal matrix composed of the scalars $\left\{\sigma^{[j]}_{\vartheta}, \sigma^{[j]}_{\varkappa}, \sigma^{[j]}_{\mu}, \sigma^{[j]}_{a}\right\}$ and partition the matrices $\Pi^{[j]}$, $\Phi^{[j]}$, and $\eta^{[j]}$ as follows

$$
\begin{aligned}
\Pi^{[j]} &= \begin{bmatrix} \Pi^{[j]}_s & \Pi^{[j]}_{sg} \\ \Pi^{[j]}_{gs} & \Pi_g \end{bmatrix}, \ \Phi^{[j]} = \begin{bmatrix} \Phi^{[j]}_s & \Phi^{[j]}_g \end{bmatrix}, \\
\eta^{[j]} &= \begin{bmatrix} 0 & 0 \\ \sum^{[j]} & 0 \\ 0 & \sigma_g \end{bmatrix}, \ \sum{}^{[j]} = \begin{bmatrix} \sigma^{[j]}_{\vartheta} & 0 & 0 & 0 \\ 0 & \sigma^{[j]}_{\varkappa} & 0 & 0 \\ 0 & 0 & \sigma^{[j]}_{\mu} & 0 \\ 0 & 0 & 0 & \sigma^{[j]}_{a} \end{bmatrix}.
\end{aligned} \tag{7}
$$

Here, the matrices $\Pi^{[j]}_s$, $\Pi^{[j]}_{sg}$, $\Pi^{[j]}_{gs}$, Π_g, $\Phi^{[j]}_s$, and $\Phi^{[j]}_g$ have dimensions $(N_s - 1) \times (N_s - 1)$, $(N_s - 1) \times 1$, $1 \times (N_s - 1)$, 1×1, $N_c \times (N_s - 1)$, and $N_c \times 1$, respectively. Then for the case of two countries, the matrices $\mathbb{F}$ and η takes the following forms

$$\mathbb{F}=\begin{bmatrix} 0 & 0 & G_s^{[1]} & 0 & G_g^{[1]} \\ 0 & 0 & 0 & G_s^{[2]} & G_g^{[2]} \\ 0 & 0 & \Pi_s^{[1]} & 0 & \Pi_{sg}^{[1]} \\ 0 & 0 & 0 & \Pi_s^{[2]} & \Pi_{sg}^{[2]} \\ 0 & 0 & \Pi_{gs}^{[1]} & \Pi_{gs}^{[2]} & \Pi_g \end{bmatrix}, \ \eta = \begin{bmatrix} \Phi_s^{[1]}\begin{bmatrix} 0 \\ \sum^{[1]} \end{bmatrix} & 0 & \Phi_g^{[1]}\sigma_g \\ 0 & \Phi_s^{[2]}\begin{bmatrix} 0 \\ \sum^{[2]} \end{bmatrix} & \Phi_g^{[2]}\sigma_g \\ \begin{bmatrix} 0 \\ \sum^{[1]} \end{bmatrix} & 0 & 0 \\ 0 & \begin{bmatrix} 0 \\ \sum^{[2]} \end{bmatrix} & 0 \\ 0 & 0 & \sigma_g \end{bmatrix},$$

where, $G_s^{[j]} \equiv \Phi_s^{[j]}\Pi_s^{[j]} + \Phi_g^{[j]}\Pi_{gs}^{[j]}$, $G_g^{[j]} \equiv \Phi_s^{[j]}\Pi_{sg}^{[j]} + \Phi_g^{[j]}\Pi_g$. The element Π_g has not been indexed because it is common for both countries. Finally, $\mathbb{H}$ is a matrix of ones and zeros that selects the appropriate elements of $\mathbb{S}_t$ needed to build the model's equivalent to the observables found in the data.

The curse of dimensionality in the state space representation (6) arises from three sources. First, it is the number of countries, N. Indeed, adding an extra country requires $N_c + N_s$ elements in the expanded state vector. Second, the complexity of the DSGE model under study determines N_c, N_s, and N_{sh}. An additional state, control, or structural shock increases the size of $\mathbb{S}_t$ by N. Finally, the number of observables in the measurement equation, $\dim\left(\mathbb{Y}_t^{Data}\right)$, grows with the number of countries and observables to be explained. All these sources amount to increasing the dimensions of $\mathbb{F}$, $\mathbb{H}$, and η, which is problematic since the Kalman filter requires multiplying and inverting objects that depend on those matrices. The larger those matrices are, the more expensive the evaluation of the likelihood is.

Assuming a small open economy framework effectively controls the dimensionality problem by limiting N_c, N_s and N_{sh}, i.e, containing the model's complexity. However, such an assumption does not preclude the first and third sources of dimensionality from happening. To control for those sources, we restrict our study to explain output, consumption, and investment in seven developed small open economies. Even after these restrictions, the expanded state vector, $\mathbb{S}_t$, has dimension 106; an extra country increases the number of states in $\mathbb{S}_t$ by 15 while an extra disturbance increase its size by 7.

Let Θ^j denote the set of all structural parameters in country j. Operationally, the following steps are used to evaluate the likelihood of the system (6):

1. For country j, solve the DSGE model using Θ^j as the relevant parameters.
2. Using the model's solution build the matrices $\Pi^{[j]}$, $\Phi^{[j]}$, $\eta^{[j]}$, and $\sum^{[j]}$ as shown in equation (7)
3. Repeat steps 1 and 2 for all countries in the sample.
4. Finally, compute the state representation (6) and build its likelihood, $\mathcal{L}$, using the Kalman Filter.

Although this section shows how to evaluate the likelihood with only one common factor, which corresponds to the stochastic trend in Section 6, it can be easily extended to allow multiple common factors (Section 7.3).

5 Estimation

This section describes the data as well as the econometric approach used to estimate the multi country model proposed in the previous sections. For each country in the sample, the parameter space is divided in non-estimated parameters

$$\Theta_1^j = \left[\alpha, \beta, \varphi, \delta, d^{[j]}, \varkappa^{[j]}, \psi^{[j]}\right]$$

and estimated parameters

$$\Theta_2^j = \left[\rho_a^{[j]}, \rho_\mu^{[j]}, \rho_\vartheta^{[j]}, \rho_\varkappa^{[j]}, \sigma_a^{[j]}, \sigma_\mu^{[j]}, \sigma_\vartheta^{[j]}, \sigma_\varkappa^{[j]}, b^{[j]}, \phi^{[j]}, g_{[\%]}, \rho_g, \sigma_g\right].$$

The lack of indexation in the last three parameters reflect our assumption that there is a common shock buffeting all small open economies. The parameters $d^{[j]}$ and $\varkappa^{[j]}$ are set to match the ratio of net exports to output and government expenditures to output observed in the data for each country (see the last two columns in Table 1). Without loss of generality, the steady state of labor is normalized to 1. This assumption in turn pins down the value for $\psi^{[j]}$. The remaining parameters in Θ_1^j are set to $\alpha = 0.32$, $\beta = 0.995$, $\delta = 0.025$, and $\varphi = 0.001$, which are standard choices in the literature (Schmitt-Grohe and Uribe, 2003). In total, there are 70 country-specific parameters (10 per country) plus 3 common parameters to be estimated.

- **Data**

The data, which are taken from Aguiar and Gopinath (2007), consist on quarterly growth rates of real output, consumption, and investment for the following developed economies: Australia, Belgium, Canada, Norway, New Zealand, Spain, and Sweden. The data are quarterly and span from the second quarter of 1987 to the first quarter of 2003. The shorter sample is a consequence of the lack of information for New Zealand prior to 1987. As in the dynamic factor literature (Kose et al., 2003), the use of growth rates helps to control for country size in the estimation.

As previously argued, the assortment of countries in our sample seems to share a common trend. Furthermore, they are geographically located in different continents and export different goods. For example, Australia and Norway are big exporters of commodities while Spain relies on tourism and financial services. Ultimately, one expects that these countries be buffeted by shocks that are not necessarily correlated across them. Altogether, these features should provide enough information to identify the stochastic trend in the model, g_t, as well as the country-specific terms. Australia, Canada, and New Zealand have been studied elsewhere (Lubik and Schorfheide, 2007; Justianiano and Preston, 2008) so their inclusion facilitates comparison with the related literature.

A fair question to ask is why we do not include additional countries in the exercise. One reason is computational costs. As argued above, adding a country increases the size of the state space by 15%. More important, most of the remaining developed small open economies are located in Europe. Therefore, by including them in the estimation we risk recovering a regional shock common to all countries in the European area. The correlation of this disturbance with the remaining countries in the sample is most likely weak, which may lead us to conclude that the factor is unimportant to, say, Canada and Australia.

But this is problematic because we are interested in recovering a factor that is meaningful for countries in different geographical areas. Alternatively, we could incorporate emerging economies into the analysis. Aguiar and Gopinath (2007) forcefully argue that these economies display markedly different business cycles, which makes their inclusion unfitted for our purposes.

- **Bayesian Inference**

Following the recent literature, e.g. Schorfheide (2000) and Smets and Wouters (2003), the log-linearized version of the model is estimated using Bayesian methods. Let $p(\Theta_2)$ denotes the prior distribution on the parameters of interest and $\mathcal{L}$ the likelihood of the model. The Bayes theorem in turn implies that the posterior of the structural parameters, $p\left(\Theta_2|\mathbb{Y}^{Data}\right)$, is proportional to $p(\Theta_2)\,\mathcal{L}$, where the likelihood can be evaluated following the algorithm outlined in the previous section. Then $p\left(\Theta_2|\mathbb{Y}^{Data}\right)$ is characterized using the random walk Metropolis-Hasting (MH) procedure (for details, see the excellent survey of An and Schorfheide, 2007). The results are based on 1.5 million draws from the posterior simulator after an initial burn-in phase of $500,000$ iterations. The acceptance rate for the MH algorithm was set to approximately 0.23 as suggested by Casella and Roberts (2004).

- **Priors**

The priors imposed during estimation are reported in Table 1. The prior mean $g_{[\%]}$ is set to the average quarterly growth rate in percentage points across all countries and observables in the sample. The priors for the persistence parameters reflect the view that the structural shocks display some autocorrelation. The relatively large standard deviation helps to account simultaneously for processes with low and high persistence. Following Justiniano and Preston (2008) and Garcia-Cicco et al. (2009), it is assumed that there is some habit formation present in the data. Since there is little information about the adjustment cost of investment in open economies, I choose a very wide uniform prior. Finally, the priors for the volatilities allow for a wide range of values with a 95 percentile credible set given by $[0.35, 8.5]$.

6 Results

Given the considerable attention that productivity has received in the small open economy literature, the second panel in Table 1 presents the posterior distributions for the common productivity process, g_t, and the country specific productivity processes (the columns $\rho_.$ and $\sigma_.$ correspond to the estimates for ρ_a and σ_a, respectively). For completeness, the estimates for habit formation and the cost of adjusting investment are displayed as well (all estimated parameters are in Table 3). The estimation exercise reveals that the growth rate of the common factor displays some mild persistence and its volatility is bounded away from zero. Furthermore, the average growth rate median is 0.68%, which is close to the average quarterly growth rate across all countries and observables in the sample. This result is expected as the prior was centered at the average growth rate across countries and observables (output, consumption, and investment). Interestingly, the estimates are consistent with those reported in Aguiar and Gopinath (2007) for Canada. Indeed, these authors find the following values: $\rho_g = 0.29$ and $\sigma_g = 0.47$. The less volatile productivity process in Table 1 most likely results from a combination of the different econometric procedures (they rely on GMM), the shorter sample in the present paper and, more importantly, the richer array of countries and structural shocks in my formulation.

In terms of the country-specific productivities, two outcomes deserve some discussion. First, these productivities are at least twice as volatile as the common productivity process. Norway has the largest productivity shock, which, as explained in the next sections, is needed to account for the large fluctuations present in that country. Second, the idiosyncratic productivities display substantial persistence (close to one in some cases). This finding raises the question of whether the persistence of the stationary productivity shock is too large. The answer is no. Aguiar and Gopinath (2007), for example, use a partial information approach to estimate a production function similar to the one used here. They find that the persistence of productivity to be 0.97 for Canada. Justiniano and Preston (2008) and Garcia-Cicco et al. (2009) report very persistent productivity process for Canada and Argentina, respectively. Finally, Glick and Rogoff (1995) find that productivity in the G7 countries has a persistence close to one (Section 6.3 provides further discussion about why our approach delivers such a persistent productivity process).

The results also indicate that habit formation and costly investment are present in the countries, albeit in different degrees. For example, while Belgium has the largest habit formation (0.47), the smallest cost of adjusting investment corresponds to Norway (0.04). As the next sections reveal, this heterogeneity in the estimates affects the model's predictions in terms of variance decomposition and impulse responses.

What can we learn from the common factor? To answer this question, Figure 2 displays the filtered common factor, $\widehat{g}_t$, against several time series (the posterior medians are used to compute the implied factor). The upper left panel plots the factor and the unweighted average of the output growth rates in the sample. To facilitate comparison, the variables are de-meaned and normalized so that their highest value is 1. It is quite clear that the factor does a good job in tracking the major movements in the average growth rate, e.g., the contraction in the early nineties, the boom in the middle and last part of the nineties, and finally the recession in 2001. Indeed, the correlation between $\widehat{g}_t$ and the average growth is 0.70, which is a strong confirmation of the relation between those two variables.

The second upper panel shows the factor against the growth rate of real GDP in US. We observe some co-movement but in a lesser degree than with the average growth. Yet the correlation is still relatively high 0.52, which suggests that GDP in US may be an important driving force behind the common factor. Consequently, its omission for the estimation may well be biasing our findings. This possibility is explored in more detail in the next sections. The dynamics of the common factor and the US T-bill, R_t^{US}, rate are portrayed in the left lower panel. Eyeball econometrics indicates that the T-bill rate seems to lead the factor over the business cycle. For example, the interest rate spike in 1989 is followed by a sharp decline in the estimated factor around 1991. This observation is readily confirmed by the correlation between R_{t-8}^{US} and $\widehat{g}_t$, which is -0.44.[3] As before, this result suggests that US interest rates affect the common factor and consequently one should include such a variable for estimation purposes (more on this in the next sections).

[3] The contemporaneous correlation between those variables is -0.08.

The final panel in Figure 2 shows the common factor and the end-of-quarter price of oil. There is some co-movement especially during the last part of the sample. The boom of the late 1990's is associated with a decline in oil prices. The co-movement seems to be weaker than in the previous cases, which is corroborated by the correlation between those variables: -0.31.

6.1 Second Moments and Variance Decomposition

The results in this and the next sections are based on the posterior medians of the parameters. The first panel in Table 2 presents the second moments predicted by the model, and those found in the data (numbers in square brackets). The first three rows correspond to the standard deviation of output and the volatilities of consumption and investment relative to that of output. The empirical moments display the usual patterns found in small developed economies: output is more volatile than consumption but substantially less volatile than investment. There are, though, two features worth stressing. First, output in Norway is on average three times as volatile as output in the other countries. More interesting, consumption is more volatile than output in Canada and Sweden. This finding entirely results from the short sample under consideration.[4]

The theoretical moments from the model expose some interesting features. To begin with, the model closely replicates the volatility of output in all countries but Norway and Sweden. For these economies, our model overpredicts the variability of output by factors of 5 and 2, respectively. Based on the estimates in Table 1, this excess volatility results from the relatively large economy-wide productivity processes, σ_a. Note, for example, that the size of this shock for Norway is about three times larger than those for the other countries in the sample. Second, except for Spain, the model generates very volatile investment series. Spain's relatively smooth investment profile is in part due to its relatively large adjustment cost (see Table 1). If one sets this cost to 0.1 (one fourth of the original estimate), the volatility of investment relative to that of output jumps to 37.

Third, the proposed model is capable of replicating the substantial variability of consumption found in Canada and Sweden. This is a remarkable feature of the model given that it can simultaneously account for the less volatile pattern of consumption in the remaining countries. Aguiar and Gopinath (2007) attribute the excessive consumption volatility to the growth productivity shocks. Here, however, the stochastic trend is common to all countries so it cannot alone account for the large volatility of consumption in only a couple of countries. The combination of low habit formation and volatile risk premium shocks, $\mu_{j,t}$, in Canada and Sweden induces the excess volatility.[5] To sustain this argument, I recomputed the second moments for Canada but setting its habit formation to 0.5 and its premium volatility, σ_μ, to zero. Under this counterfactual scenario, the volatility of output drops to 0.51 while the ratios σ_c/σ_y and σ_i/σ_y are now 0.47 and 6.02, respectively. Clearly, the model predicts smoother profiles for consumption and investment. Notably, the volatility of the later variable declines drastically, which highlights the importance of the premium shock in driving investment in the model.

[4] Aguiar and Gopinath (2007), using the same data but with a longer series, report that consumption is smoother than output in Canada and Sweden.

[5] Neumeyer and Perri (2005), Fernandez-Villaverde et al. (2009), and Garcia-Cicco et al. (2009) provide a comprehensive discussion of the role of premium shocks in generating volatile consumption series in emerging economies.

The second panel in Table 2 displays the fraction of the volatilities attributed to each structural shock. For example, the preference shock in Australia, $\vartheta_{j,t}$, explains 21%, 52%, and 0% of the variability of output, consumption, and investment, respectively. The row labeled g reports the contribution of the common shock to the fluctuations in our model. For Australia, note the common shock only explains 7%, 8%, and 4% of output, consumption, and investment, respectively. Rather than discussing individual results I find more illustrative to highlight common patterns resulting from the variance decomposition exercise. To begin with, the common productivity shock, g_t, accounts for less than 15% of the variability of the variables of interest. Furthermore, the shock contributes more to the fluctuations of consumption than to those of output and investment (the effect on these last two variables is remarkably small in Sweden and New Zealand). Since the productivity shock is permanent, households anticipate output to increase permanently in the future. This anticipation generates a wealth effect that makes consumption contemporaneously rise by more than the initial response of output (Garcia-Cicco et al., 2009). It is precisely this initial spike in consumption what drives its larger volatility. In addition, the long lasting feature of the productivity shock implies that the return to capital will be high in the future. Households feel no urgency to boost capital today and, as a consequence, investment is not very volatile in response to the shock.

Next, let us consider the effects of the TFP shock, $a_{j,t}$. The numbers in Table 2 indicate this shock explains at least 50% of the volatility of output in all countries in our sample. In fact, it captures 75% of the output dynamics in Norway and New Zealand. Furthermore, no other shock contributes so much to the fluctuations in output. The TFP disturbance also accounts for an important part of the movements in investment and in a lesser degree in consumption. The intuition behind these results is well understood in the literature: Households perceive the shock as a temporal affair and hence they prefer to save rather than consume. Labor supply increases in the short term to take advantage of the temporary high productivity. Ultimately, output is very responsive to that disturbance.

When we turn to the premium shock, note that this disturbance is the main driving force behind movements in investment. For example, it explains 82% of the variability of that variable in Canada. This disturbance accounts for at least 18% of the volatility in consumption. To understand the effect on investment, let us momentarily assume there are no adjustment costs: $\phi = 0$. Under this premise, a no arbitrage opportunity argument implies that interest rates and the return on capital are related by

$$1 + r_t = \mathbb{E}_t \left[1 - \delta + R^K_{t+1} \right]. \tag{8}$$

Clearly, more volatile interest rates (larger premium shocks) imply very volatile expected returns. Given that R^K_{t+1} drives households' desire to accumulate capital, it is not surprising that investment becomes more volatile when premium shocks buffet the economy.

The interest rate shock $\mu_{j,t}$ is most likely a convolution of three underlying sources: domestic interest rate, risk premium, and foreign interest rate shocks. In Section 7.2, it is shown that accounting for the effects of foreign interest shocks does not change significantly the explanatory power of $\mu_{j,t}$. Furthermore, the countries in the sample are typically viewed as less riskier than developing economies, which implies that the influence of risk premium on r_t should be small. Given that inflation has been relatively stable in the SOEs in my sample, $\mu_{j,t}$ is probably capturing shocks to the domestic nominal interest rates. Hence, the results in Table 2 suggest that nominal shocks at home may play an important role in explaining consumption and investment.

Table 2 shows that consumption is very responsive to the preference shock, $\vartheta_{j,t}$, which is consistent with the results from the closed economy literature (Fernandez-Villaverde and Rubio-Ramirez, 2008). The first order condition for consumption (equation 5) clearly shows those shocks distorts the marginal utility of consumption, which accounts for the large explanatory power of the preference disturbance. Furthermore, this shock has essentially no implications for investment but it captures 14% or more of the variability of output predicted by our model. Finally, given the preponderant role of the productivity, interest rate, and preference shocks in explaining the dynamics of the model, it is hardly surprising to find that government shocks account for less than 1% of output, consumption and investment in any of the countries under consideration. For this reason, they are not reported in Table 2 and the ones to come.

The variance decomposition exercise reveals a general pattern: country-specific disturbances explain a substantially larger fraction of the countries' volatilities than the common productivity shock does. For Canada, the common disturbance explains only 5% of the variability of ouput. Kose et al. (2003) report that the world (common) factor roughly captures one third and one half of the volatility of output in Canada and Belgium, respectively. By contrasting their results with mine, we can argue that one-sixth of that 33% explanatory power for Canada can be attributed to shocks to the stochastic trend.

The small explanatory power of the trend shocks is consistent with the results in Norrbin and Schlagenhauf (1996) and Aguiar and Gopinath (2007). Using a dynamic factor model, Norrbin and Schlagenhauf report that 40% and 11% of the variance of the forecast error in Canada can be attributed to country-specific and international (common) factors, respectively. The later authors find that such trend shocks account for a negligible fraction of the fluctuations in developed economies.

Even if g_t's low explanatory power accords with some previous studies, it is still important to clarify why the model assigns a small role to that shock. This result is likely driven by the interaction of the non-stationary and stationary productivity shocks in the production function and the almost random-walk behavior of the later shock. The large persistence implies that TFP shock's unconditional variance is substantially large. With the two shocks fighting to explain the variability of output (they enter the model only via the production function), the large volatility of the country-specific productivity shock leaves little room for the common trend shock to account for the fluctuations found in the data. The discussion of why the estimated productivity process a_t is so persistent is postponed to Section 6.3.

To further illustrate the interaction between g_t and the other shocks, the third panel in Table 2 presents the contribution of g to the fluctuations in the model under some counterfactual scenarios. If we turn off the TFP shock, g's explanatory power increases for all variables, in particular output. For example, the portion of output explained by the growth rate shocks increases by a factor of 5 for New Zealand. Furthermore, setting the variance of the preference or premium disturbances to zero increases the fraction of the volatility of consumption and investment accounted for by the non-stationary shocks. Finally, eliminating the TFP and preference disturbances substantially improves the performance of the trend shocks.

6.2 Impulse Responses

This section reports the dynamic paths from the estimated model after a positive one-standard deviation shock to the structural disturbances. All responses are expressed as percentage deviations from steady state. Figure (3) presents the impulse responses following a shock to the stochastic trend in the model ($\varepsilon_{g,t}$). Because households anticipate the productivity shock to be permanent they immediately raise their consumption by more than the initial response of output. Furthermore, households feel they will be wealthier in the future so leisure increases contemporaneously at the expense of lowering labor. The responses for all countries in Figure (3) precisely corroborate this intuition. For example, while consumption is initially 0.4% above its steady state in Sweden, its output level only rises 0.1%. In the long term, output, consumption, and investment increase by 0.42%, which is consistent with the permanent effect of a productivity growth shock: $\sigma_g / \left(1 - \rho_g\right)$.

Figure (3) also reveals some important features: 1) there is substantial heterogeneity in the countries' responses even though the estimated stochastic process (equation 3) is the same for all the economies; 2) the dynamic response of output for Norway is different from those for the other countries. The first point is accounted for by the different estimates of habit formation and cost of adjustment in investment reported in Table 1. More to the point, recall from the previous section that the parameters $d^{[j]}$ and $\varkappa^{[j]}$ are calibrated to match relevant data in each country. This asymmetry in some of the structural parameters necessarily changes the propagation mechanism of the trend shocks inside each country, which in turn explains the heterogeneous impulse responses. For example, the large initial response of investment in Norway is likely a consequence of the fairly low estimate for its adjustment cost function.

To understand the initial decline in Norway's production, note that this country has the largest trade balance surplus. As a consequence, domestic residents accumulate claims on foreigners, which combined with a positive productivity growth shock induces a significant wealth effect in the economy. The resulting decline in labor is so pervasive that initially dominates the expansionary effect of productivity in output. Eventually, the second effect takes over the former leading to the permanent increase in output. This intuition also explains the smaller initial response of output in Sweden, which has the second largest net export-to-output ratio in the sample.

Figure (4) displays the effects of a stationary productivity shock ($\varepsilon_{a,t}^{[j]}$ in equation 4). A quick look at the results shows that output is more responsive than consumption in the short run. As explained in the variance decomposition section, the temporal nature of the shock is behind this finding. Households work harder in the short term to take advantage of the transitory larger wages resulting from the temporal increase in productivity. Interestingly, with the exception of Norway, productivity induces a highly persistent response in all variables, in particular consumption. This persistence results from the large estimates for ρ_a recover by our estimation approach. Swedish output and consumption are notably more persistent that the same variables in other countries. The large productivity shock, σ_a, combined with almost unit root productivity shock accounts for this finding. Finally, we find that the impulse responses for Norway substantially depart from those for the other countries in the sample.

When we turn to the effects of a premium shock, $\varepsilon_{\mu,t}^{[j]}$, Figure (5) reveals that the impulse responses follow similar dynamic patterns. There are, though, some important differences, which are worth stressing.

First, investment is more sensitive to premium disturbances in New Zealand and Norway. The low cost of adjusting investment is likely behind this result. Indeed, note that the response of investment in Spain is small, which coincides with Spain having the largest estimated cost of adjustment. Second, the premium shock induces an initial response in consumption that is in absolute value at least 1.5 times larger than output's initial movement. Third, it is somehow surprising that while consumption and investment contract upon the arrival of the shock, output and labor increase. This result can be explained by first recalling that the functional form for preferences (equation 2) allows wealth effects in labor supply following a shock to international interest rates. To see this point, consider the first order necessary conditions for labor supply and asset accumulation

$$\begin{aligned} \psi\vartheta_t h_t &= \widetilde{\lambda}_t W_t, \\ \frac{\widetilde{\lambda}_t}{1+r_t} &= \beta \mathbb{E}_t \widetilde{\lambda}_{t+1}, \end{aligned} \tag{9}$$

respectively. Clearly, a premium disturbance simultaneously raises interest rates today, r_t, and induces a drop in the ratio of marginal utilities, $\widetilde{\lambda}_{t+1}/\widetilde{\lambda}_t$. As a consequence, future consumption is relatively cheaper than current consumption, which induces a drop in the later. Other things being equal, the contemporaneous jump in marginal utility boosts the marginal benefit of working, $\widetilde{\lambda}_{t+1} W_t$, resulting in an increase in labor supply. Given that capital is predetermined, output has no other option than to increase upon the arrival of the premium shock.

Figures (6) and (7) display the responses after preference and government expenditure disturbances, respectively. From equation (5), it is clear that the former shock increase the marginal cost of labor supply, $\psi\vartheta_t h_t$. Ceteris paribus, households opt for increasing leisure, which explains the contraction in labor and the associated decline in output. Furthermore, the preference shock tries to raise the marginal utility of consumption today. Note, however, the asset accumulation equation (9) requires that the ratio of marginal utilities only moves after a premium shock. Hence, consumption must increase contemporaneously to preserve the constancy of multipliers. Finally, the government disturbance generates the usual crowding out effect: government purchases rises at the expense of domestic consumption and investment. It is also clear that the effect of such a shock has very mild consequences on each country. Moreover, the impulse responses in Figure (7) suggest that the contraction in domestic demand is not enough to cover for the increase in public consumption. As a consequence, labor must increase to produce additional goods needed to cover up the gap.

6.3 Individual estimation

In the previous sections, the presence of a common factor effectively translates into a constrained estimation of the country-specific parameters. This is because the stochastic trend must on average match the trend observed in all seven countries in the sample. Ultimately, the restricted estimation can significantly influence the model's predictions for each individual economy. To fully assess the implications of the common trend assumption, Table 3 presents the results from individual estimations for all countries (95 percentile interval in square brackets). For comparison purposes, the outcome when the countries share a common productivity process is also reported (rows labeled "joint"). This exercise reveals that the estimation of the habit formation and the cost of adjusting investment parameters are robust to the use of a common trend. Indeed, look how their medians and credible sets are very close to each other in the two estimation exercises. For example, Canada's habit formation is 0.23 if estimated with a common trend and 0.21 otherwise. Similarly, the estimates for the premium process, μ_t, and the preference shock, ϑ_t, are robust to whether the growth rate process is common to all countries or not.[6] Second, the estimation of the government expenditure disturbance, $\varkappa_t$, shows some sensitivity to the inclusion of the common factor. This influence is particularly noticeable for the variance of that shock. Third, most of the posterior distributions are more tightly estimated under the premise of a common factor. A 95% probability interval for habit formation in Australia covers $[0.16, 0.50]$ in the individual estimation versus $[0.20, 0.46]$ in the joint estimation (a 30% reduction). This result signals efficiency gains from the joint estimation of our small open economy model.

In an individual estimation setup, the non-stationary productivity process, g_t, is tailored to match the time series of each country. Hence, it is hardly surprising the disagreement between the two sets of estimates for g_t. For example, the average growth rate, $g_{[\%]}$, is the smallest for Belgium and Sweden and the largest for Australia and Norway. Relative to the results with a shared trend, the size of the shock, σ_g, is four times larger in the individual estimation for New Zealand and Sweden. Interestingly, there is no noticeable effect on the persistence of the shock, ρ_g.

Since the growth trend shock and the productivity process, a_t, interact via the production function (1), the results from individual estimations for the later shock should differ from those under the assumption of a joint trend. The numbers in Table 3 confirm significant differences between the two sets of estimates. Particularly puzzling is that the TPF shock is substantially less persistent in the individual estimations. To understand this result, let us momentarily concentrate on Sweden. Note that its mean growth rate with a common factor is larger than the actual growth rate (compare 0.68 versus 0.40). Other things equal, the estimated time series with a shared trend would counterfactually lie above the ones found in the data. In the absence of other trends in the model, the likelihood-based approach reconciles this conundrum by inducing an almost random-walk behavior in the total productivity process. Such a persistent shock effectively pushes the predicted series away from the shared growth rate and closer to the data. To strength my point, I re-estimated the model only for Sweden but fixing the growth rate parameters to $g_{[\%]} = 0.68$, $\rho_g = 0.21$, and $\sigma_g = 0.33$, i.e., to the values when the trend is shared. The estimated parameters for a_t are $\rho_a = 0.99$ and $\sigma_a = 0.93$, which essentially coincide with those estimated using a common trend. Clearly, when the growth rates in the model and the data are at odds, our inference technique makes the stationary productivity shock highly persistent.

To conclude this section, Table 4 displays the results from the variance decomposition exercise but this time using the parameters from the individual estimation (values in square brackets correspond to the results based on the common trend estimation). The most important finding is that the non-stationary productivity shocks explain a sizable fraction of the volatility of output, consumption, and investment

[6] The presence of the common trend, however, slightly affects the estimation of habit formation for New Zealand and Sweden, the persistence of the country premium for Belgium and the persistence of the preference shock for New Zealand.

in all countries but Norway. This additional explanatory power comes at the expense of the stationary productivity process. The decline in the importance of a_t is due to its smaller unconditional volatility, which in turn results from its smaller persistence. The relevance of the two productivity shocks in Norway remains the same regardless of whether one uses values from the joint or individual estimation exercise. In terms of the preference and premium shocks, note that the variance decompositions are insensitive to the use of a common trend. Such a result is expected given that the estimated processes for these shocks remain the same under either the joint or individual estimations.

7 Extensions

This section reports the variance decomposition exercise for several alternative specifications to the benchmark model.

7.1 HP-Filtered Data

Recent contributions have emphasized the importance of the data treatment prior to estimation. Indeed, researchers have found that trend misspecification or more precisely the choice of the filter to detrend the data can severely distort the estimated parameters (Harvey and Jaeger, 1993; Cogley, 2001; Baxter and King, 2003; Gorodnichenko and Ng, 2007). Given that trend shocks are at the heart of this study, it is imperative to evaluate how the results change if the estimation is based on HP-filtered data rather than on observables in growth rates. To that end, it is assumed that the labor-augmenting shock X_t follows an AR(1) process: $\log X_t = \rho_g \log X_{t-1} + \sigma_g \varepsilon_{g,t}$.

Table 5 contains the variance decomposition when the data are HP-filtered. For comparison purposes, the values in square brackets replicate the results from Table 4. The common stochastic shock, g_t, accounts for a larger fraction of the variability of output. Indeed, this shock now explains up to 23% of the volatility of output in Belgium or a fivefold increase relative to the findings with growth rates. This extra explanatory power results from the larger unconditional volatility associated with the common shock (compare 1.33% with HP-data versus 0.34% with data in growth rates).[7] Furthermore, the additional importance of this shock comes at the expense of the premium and preference disturbances. Interestingly, the relevance of the TFP shock is almost unaffected. With respect to consumption and investment, there is no significant change relative to the results based on data in growth rates.

Although the common disturbance explains a bit more of the country fluctuations, the bulk of the volatilities is still captured by the country-specific fluctuations. For instance, if we consider New Zealand, Norway, and Sweden, note that the premium, preference, and TFP disturbances account for more than 50% percent of the volatilities in investment, consumption, and output, respectively.

Figure 8 repeats the exercise in Figure 2 but this time using HP-filtered data. The first upper panel shows the estimated factor versus the average output across the countries in the sample. Clearly, the factor replicates very closely the dynamics of the average output such as the contraction in the early 90s or the expansion and subsequent recession of the late 90s and early 2000. As before, there is some

[7] Recall that the unconditional volatility of an AR(1) process is given by $\sigma/\sqrt{1-\rho^2}$.

co-movement between the common factor and output in US. The factor, however, overpredicts the decline in US production between 1992 and 1994.

In summary, we conclude that regardless of the filtering technique fluctuations the economies in our sample are driven in part by a common factor. Its explanatory power, however, is smaller than that of disturbances specific to each country.

7.2 US GDP Growth

The results documented in Section 6, in particular Figure 2, suggest that GDP in US may be an important driving force behind the common stochastic trend. Ideally, one should incorporate US as an additional block to the structural model outlined in Section 3. This approach, however, requires modeling the interactions between US and the other seven economies making the estimation of the resulting model highly intractable. Instead, I indirectly account for the effects of US by preserving the structure in Section 3 and assuming that output growth in this country obeys

$$\begin{aligned} \Delta y_t &= g_{[\%]} + \widehat{g}_t + \varpi_t, \\ \varpi_t &= \rho^{us}\varpi_{t-1} + \sigma^{us}\varepsilon_t^{us}, \end{aligned} \tag{10}$$

where $\varepsilon_t^{us} \sim N(0,1)$. This specification assumes that output growth is driven by the common international factor plus and $AR(1)$ specific to US. Then, the state space representation (6) is modified accordingly to incorporate the dynamics introduced by (10). This way of treating the US economy is related to that used in Justiniano and Preston (2008).

For space considerations, I only report the variance decomposition exercise (Table 6). As before, numbers in square brackets correspond to the variance decomposition from the benchmark model. The new results are comparable to those in Table 2. Indeed, the common factor explains a small fraction of the fluctuations relative to that capture by the country specific components. Furthermore, this fraction is almost identical to that reported for the benchmark specification. In terms of the idiosincratic shocks, there is a redistribution of the explanatory power for Australia. The contributions of those shocks for the remaining countries remain roughly unchanged. In short, adding information from US output just reinforces our finding that the common shock plays a small role in the dynamics of real variables in our sample.

7.3 Additional Factors: Productivity, Interest Rates and Demand

There are plenty of disturbances that buffet the countries in our sample besides productivity. For example, one can think of a demand shock originated in a large economy. Such a shock may affect exports from the small developed countries and hence influence their business cycles. Alternatively, Figure 2 suggests that movements in international interest rates as captured by the T-bill rate interact with the estimated common factor. Therefore, assuming a constant international interest rate seems a too strong premise. To relax this assumption, I study a world in which the SOEs share two factors: the productivity disturbance previously analyzed and the international interest rate. In particular, I assume that the domestic interest rates follows

$$
\begin{aligned}
r_{j,t} &= r_t^* + \varphi \left[\exp\left(\widetilde{D}_{j,t+1}/X_t - d^{[j]} \right) - 1 \right] + \exp(\mu_{j,t} - 1) - 1, \\
r_t^* &= \rho^r r_{t-1}^* + \sigma^r \varepsilon_t^r,
\end{aligned}
$$

where $\varepsilon_t^r \sim N(0,1)$. To identify the parameters in the process for r_t^*, a proxy for the real interest rate in US is part of the set of observables. This real rate is constructed by subtracting expected inflation from the T-bill rate.[8] Following Neumeyer and Perri (2005), we compute expected inflation as the average U.S. CPI inflation in the current month and in the eleven preceding months. This assumption is motivated by the observation that inflation in the U.S. is well approximated by a random walk (Atkeson and Ohanian, 2001).

It is tempting to interpret shocks to r_t^* as changes in US monetary policy. The Taylor rule, however, implies that movements in the nominal interest rate, and indirectly in the real interest rate, reflect a combination of the Fed's systematic response to variations in output and inflation and truly unexpected monetary shocks. Consequently, the most we can say about ε_t^r is that captures innovations to the US interest rate not explained by its own lag.

Table 7 reports the variance decompositions. In general, there is a decline in the explanatory power of two country-specific disturbances: TFP and preference. The common productivity disturbance now accounts for a slightly larger fraction of the fluctuations than in the benchmark model. This result indicates some complementarities between the common productivity and the international interest rate, a result already hinted by Figure 2. Shocks to r_t^* explain between 4% and 19% of the variability found in the SOEs, which is larger than what it is accounted for by the trend shocks. Since shocks to the interest rate affect the return on capital via the euler equation (8), it is not surprising that those shocks have a larger impact on investment than in the other variables. Yet we still observe that country-specific shocks capture a big chunk of the volatility in the SOEs.

Given the sizable trade in goods and financial assets between US and Canada, it is perhaps surprising that the the shock ε_t^r explains a fraction of Canada's business cycle that is comparable to that of the remaining countries. To understand this result, recall that when estimating the interest rate shock process r_t^*, its effects on Canada receive the exact same weight as its implications on any other of the countries in the sample.

For completeness, Figure 9 displays the filtered factors versus the average growth rate in the SOEs, output growth in US, the T-bill rate, and the end-of-quarter oil price. Note that the results are not that different from those reported in Figure 2. The common productivity factor tracks closely the average growth in the small open economies. The interest rate shock does a good job in describing the nominal T-bill rate, although there are some periods like the late 1990s and 2003 when the interest rate factor overpredicts and underpredicts the nominal rate, respectively.

As explained above, a possible common shock may arise from demand shocks in, say, US or Germany. A parsimonious way to capture this channel is to include a common shock to each of the countries'

[8] The data correspond to the 3-Month Treasury Bill, secondary market rate taken from the St. Louis Fed database.

consolidated budget constraints

$$\frac{D_{j,t+1}}{1+r_{j,t}} = D_{j,t} - Y_{j,t} + C_{j,t} + I_{j,t} + \varkappa_{j,t}Y_{j,t} + \Upsilon_t,$$

where the demand shock Υ_t follows an $AR(1)$ process with mean 0. Although the estimation of Υ_t delivers statistically significant estimates, its economic impact is negligible in the sense that this shock accounts for less than 1% of the fluctuations in our sample. This result was somehow expected given the small role that the idiosyncratic government shock $\varkappa_{j,t}$ played in the benchmark formulation.

In sum, the two common disturbances (productivity and interest rate) combined explain 16%, 25%, and 17% of the fluctuations in output, consumption, and investment, respectively, in Canada. Interestingly, these numbers are much closer to those reported in Kose et al. (2001). Hence, through the lenses of my very stylized model, one can argue that the common factor uncovered in their reduced-form approach is likely a mixture of a productivity shock, a shock to the international interest rate, and something else (probably a demand shock that my formulation is unable to pick up).

An analysis similar to that for Canada carries over to the remaining countries but New Zeland, Norway, and Sweden. For these economies, we note that the two common shocks account for less than 10% of the fluctuations in output. This lack of explanatory power suggests that our model is too parsimonious to describe the business cycles in those three economies.

8 Concluding Remarks

This paper uncovers the presence of a common factor driving fluctuations in developed small open economies. The main novelty is that this factor is recovered using a DSGE model and Bayesian methods. When the data are expressed in growth rates, the common factor is identified as a stochastic trend in the production function. Shocks to this trend accounts for up to 14% of the cyclical behavior of the countries in the sample. Alternatively, if the data are HP-filtered prior to the estimation, the countries share a common labor-augmenting technology, which is assumed to be stationary. Under this assumption, the common factor explains up to 23% of the fluctuations in the small open economies. In either case, the bulk of the volatility of output, consumption, and investment is accounted for by TFP, preference, and risk premium shocks.

The effects of large developed countries on the estimated factor were partially accounted by incorporating the growth rate of output in US as one of the observables. Such an approach misses the intrinsic rational expectations nature of the model because households do not react to changes in the large economies. As a consequence, the common factor uncovered in this paper may be capturing only a part of a much larger and more important worldwide factor. This is so because large economies drive a substantial fraction of the world output. Hence, shocks buffeting these countries even if they are initially home based, they can easily disseminate around the world. Therefore, the next step in this line of research is to incorporate a richer array of countries. The main hurdle behind this proposal is to overcome the curse of dimensionality arising from 1) modeling the multiple interconnections between countries, e.g., imports and exports and direct foreign investment, and 2) evaluating the likelihood associated with the large world model.

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Table 1: Prior Distributions

	$b^{[j]}$	$\phi^{[j]}$	$\rho_{\cdot}^{[j]}$	$\sigma_{\cdot}^{[j]}$	$g_{[\%]}$	ρ_g	σ_g	$\frac{NX}{Y}$	$\frac{G}{Y}$
	$\mathcal{G}(0.5, 0.2)$	$\mathcal{U}(0, 100)$	$\mathcal{B}(0.5, 0.2)$	$\mathcal{IG}(2, 2)$	$\mathcal{G}(0.65, 0.2)$	$\mathcal{B}(0.5, 0.2)$	$\mathcal{IG}(2, 2)$		
	Posterior Distributions								
Australia	0.34 [0.20,0.46]	0.16 [0.12,0.22]	0.98 [0.97,0.99]	0.57 [0.48,0.68]				-0.2	18.8
Canada	0.21 [0.10,0.32]	0.24 [0.17,0.32]	0.99 [0.98,0.99]	0.55 [0.47,0.64]				2.55	20.8
Norway	0.34 [0.21,0.47]	0.04 [0.03,0.06]	0.80 [0.73,0.87]	1.47 [1.25,1.75]				10	23.3
Spain	0.26 [0.14,0.39]	0.44 [0.30,0.59]	0.96 [0.89,0.99]	0.60 [0.51,0.71]	0.68 [0.59,0.77]	0.21 [0.08,0.37]	0.33 [0.25,0.42]	-1.23	18.6
Sweden	0.17 [0.07,0.28]	0.22 [0.16,0.32]	0.99 [0.98,0.99]	0.94 [0.82,1.13]				4.42	27.7
Belgium	0.47 [0.35,0.57]	0.18 [0.12,0.26]	0.98 [0.97,0.99]	0.62 [0.53,0.74]				2.31	23.4
New Zealand	0.11 [0.04,0.22]	0.09 [0.07,0.12]	0.97 [0.82,0.99]	0.96 [0.83,1.12]				1.48	20.2

For the Gamma, G, and Beta, B, distributions, the values in parenthesis are the mean and standard deviation. For the inverse gamma, IG, the values are the share and scale parameters. For the uniform distribution, U, the values are the lower and upper bounds.

95 percentile probability interval in square brackets.

The last two columns correspond to the ratio of net export to output, NX/Y, and goverment expenditure to output, G/Y, found in the data.

Table 2: Second Moments

	Australia	Canada	Norway	Spain	Sweden	Belgium	New Zeland
σ_y	0.74[0.67]	0.71[0.65]	5.86[1.60]	0.88[0.76]	1.93[0.92]	0.78[0.71]	1.87[1.13]
σ_c/σ_y	0.62[0.82]	1.22[1.07]	0.51[0.67]	0.70[0.94]	1.22[1.33]	0.57[0.69]	0.74[0.93]
σ_i/σ_y	24.2[4.31]	24.6[3.42]	18.3[4.74]	5.99[2.57]	21.2[3.96]	19.8[3.29]	28.8[4.35]
				Variance Decomposition			
ϑ	0.21 0.52 0.00	0.23 0.39 0.00	0.17 0.64 0.15	0.20 0.65 0.16	0.19 0.42 0.03	0.21 0.50 0.07	0.14 0.58 0.03
$\varkappa$	0.00 0.00 0.00	0.00 0.00 0.00	0.00 0.00 0.00	0.00 0.00 0.00	0.00 0.00 0.00	0.00 0.00 0.00	0.00 0.00 0.00
μ	0.13 0.20 0.76	0.23 0.32 0.82	0.07 0.18 0.56	0.09 0.20 0.45	0.22 0.35 0.76	0.14 0.18 0.62	0.08 0.18 0.65
a	0.59 0.20 0.19	0.48 0.18 0.13	0.75 0.03 0.23	0.64 0.10 0.34	0.58 0.17 0.18	0.60 0.22 0.24	0.76 0.17 0.29
g	0.07 0.08 0.04	0.05 0.10 0.05	0.01 0.14 0.07	0.07 0.05 0.05	0.01 0.07 0.03	0.05 0.11 0.07	0.02 0.06 0.03
				Variance Decomposition Counterfactuals			
No a	0.18 0.10 0.05	0.10 0.13 0.06	0.03 0.14 0.09	0.18 0.06 0.08	0.03 0.08 0.04	0.12 0.13 0.10	0.10 0.07 0.04
No ϑ	0.09 0.16 0.04	0.07 0.17 0.05	0.01 0.39 0.08	0.06 0.16 0.06	0.02 0.12 0.03	0.06 0.21 0.08	0.03 0.14 0.03
No μ	0.08 0.10 0.16	0.07 0.15 0.30	0.01 0.17 0.15	0.07 0.07 0.09	0.02 0.10 0.14	0.06 0.13 0.20	0.03 0.07 0.08
No a, ϑ	0.36 0.28 0.05	0.18 0.25 0.06	0.09 0.43 0.11	0.43 0.22 0.10	0.06 0.16 0.04	0.26 0.37 0.11	0.24 0.24 0.04

Table 3: Posterior Distributions Individual Estimation

	$b^{[j]}$	$\phi^{[j]}$	$\rho_\mu^{[j]}$	$\sigma_\mu^{[j]}$	$\rho_\vartheta^{[j]}$	$\sigma_\vartheta^{[j]}$	$\rho_\varkappa^{[j]}$	$\sigma_\varkappa^{[j]}$	$\rho_a^{[j]}$	$\sigma_a^{[j]}$	$g_{[\%]}$	ρ_g	σ_g
Australia	0.34 [0.16,0.50]	0.15 [0.08,0.25]	0.64 [0.09,0.78]	0.19 [0.12,0.31]	0.69 [0.27,0.98]	0.78 [0.56,1.30]	0.60 [0.16,0.90]	2.00 [0.39,20.8]	0.73 [0.24,0.96]	0.33 [0.20,0.51]	0.76 [0.57,0.94]	0.23 [0.07,0.45]	0.71 [0.44,0.99]
(Joint)	0.34 [0.20,0.46]	0.16 [0.12,0.22]	0.67 [0.58,0.75]	0.18 [0.13,0.25]	0.51 [0.09,0.45]	0.72 [0.53,0.80]	0.54 [0.20,0.85]	0.94 [0.38,3.14]	0.98 [0.97,0.99]	0.57 [0.48,0.68]	0.68 [0.59,0.77]	0.21 [0.08,0.37]	0.33 [0.25,0.42]
Belgium	0.45 [0.29,0.59]	0.14 [0.09,0.23]	0.19 [0.04,0.55]	0.26 [0.17,0.40]	0.93 [0.75,0.98]	1.24 [0.81,2.15]	0.51 [0.13,0.88]	1.22 [0.35,6.13]	0.78 [0.64,0.88]	0.62 [0.41,0.79]	0.49 [0.37,0.61]	0.21 [0.05,0.44]	0.40 [0.22,0.77]
(Joint)	0.47 [0.35,0.57]	0.18 [0.12,0.26]	0.57 [0.43,0.69]	0.21 [0.14,0.31]	0.92 [0.85,0.97]	1.21 [0.87,1.85]	0.52 [0.19,0.82]	1.57 [0.49,4.65]	0.99 [0.97,0.99]	0.62 [0.53,0.74]	0.68 [0.59,0.77]	0.21 [0.08,0.37]	0.33 [0.25,0.42]
Canada	0.23 [0.09,0.39]	0.27 [0.17,0.43]	0.76 [0.67,0.84]	0.18 [0.12,0.29]	0.26 [0.05,0.68]	0.29 [0.17,0.53]	0.56 [0.15,0.91]	1.72 [0.39,12.7]	0.87 [0.40,0.99]	0.30 [0.19,0.45]	0.62 [0.45,0.80]	0.29 [0.14,0.43]	0.75 [0.58,0.94]
(Joint)	0.21 [0.10,0.32]	0.24 [0.17,0.32]	0.78 [0.72,0.83]	0.16 [0.12,0.21]	0.24 [0.09,0.45]	0.65 [0.53,0.80]	0.51 [0.18,0.82]	0.99 [0.39,2.91]	0.99 [0.98,0.99]	0.55 [0.47,0.64]	0.68 [0.59,0.77]	0.21 [0.08,0.37]	0.33 [0.25,0.42]
New Zeland	0.22 [0.08,0.37]	0.10 [0.06,0.14]	0.63 [0.46,0.76]	0.24 [0.15,0.40]	0.32 [0.07,0.72]	0.95 [0.68,1.30]	0.54 [0.14,0.92]	1.31 [0.36,7.29]	0.74 [0.28,0.93]	0.52 [0.28,0.80]	0.59 [0.39,0.82]	0.12 [0.03,0.29]	1.30 [0.88,1.74]
(Joint)	0.11 [0.04,0.22]	0.09 [0.07,0.12]	0.57 [0.29,0.71]	0.24 [0.16,0.41]	0.86 [0.66,0.95]	1.26 [0.96,1.76]	0.46 [0.15,0.84]	1.36 [0.45,4.59]	0.97 [0.82,0.99]	0.96 [0.83,1.12]	0.68 [0.59,0.77]	0.21 [0.08,0.37]	0.33 [0.25,0.42]
Norway	0.37 [0.19,0.52]	0.04 [0.03,0.06]	0.14 [0.03,0.40]	0.34 [0.22,0.50]	0.94 [0.24,0.98]	3.58 [0.41,6.81]	0.64 [0.14,0.99]	1.94 [0.39,9.05]	0.72 [0.60,0.82]	1.48 [1.21,1.83]	0.78 [0.65,0.91]	0.21 [0.05,0.50]	0.30 [0.17,0.49]
(Joint)	0.34 [0.21,0.47]	0.04 [0.03,0.06]	0.16 [0.05,0.40]	0.37 [0.25,0.50]	0.93 [0.86,0.97]	3.53 [2.36,5.94]	0.48 [0.17,0.82]	1.04 [0.40,3.06]	0.80 [0.73,0.87]	1.47 [1.25,1.75]	0.68 [0.59,0.77]	0.21 [0.08,0.37]	0.33 [0.25,0.42]
Spain	0.31 [0.16,0.46]	0.40 [0.26,0.59]	0.55 [0.29,0.70]	0.24 [0.15,0.39]	0.89 [0.68,0.98]	1.14 [0.82,2.09]	0.53 [0.14,0.91]	1.15 [0.35,4.30]	0.59 [0.015,0.89]	0.35 [0.21,0.54]	0.61 [0.40,0.82]	0.25 [0.08,0.48]	0.83 [0.56,1.10]
(Joint)	0.26 [0.13,0.39]	0.44 [0.18,0.59]	0.57 [0.37,0.69]	0.23 [0.16,0.34]	0.93 [0.88,0.97]	1.34 [0.99,2.05]	0.50 [0.18,0.88]	1.38 [0.45,5.09]	0.96 [0.89,0.99]	0.60 [0.51,0.71]	0.68 [0.59,0.77]	0.21 [0.08,0.37]	0.33 [0.25,0.42]
Sweden	0.28 [0.14,0.41]	0.25 [0.16,0.40]	0.88 [0.81,0.93]	0.18 [0.13,0.27]	0.83 [0.61,0.95]	1.43 [1.06,2.13]	0.50 [0.13,0.89]	0.98 [0.34,2.36]	0.61 [0.15,0.95]	0.39 [0.22,0.69]	0.53 [0.30,0.81]	0.12 [0.03,0.31]	1.38 [0.94,1.83]
(Joint)	0.17 [0.07,0.28]	0.22 [0.16,0.32]	0.86 [0.81,0.90]	0.16 [0.13,0.22]	0.88 [0.77,0.96]	1.58 [1.232.35]	0.51 [0.19,0.84]	1.24 [0.44,3.39]	0.99 [0.98,0.99]	0.94 [0.82,1.13]	0.68 [0.59,0.77]	0.21 [0.08,0.37]	0.33 [0.25,0.42]

Table 4: Variance Decomposition

	$\sigma_{\mu}^{[j]}$			$\sigma_{\vartheta}^{[j]}$			$\sigma_{a}^{[j]}$			σ_g		
	y	c	i	y	c	i	y	c	i	y	c	i
Australia	0.13 [0.13]	0.18 [0.20]	0.78 [0.76]	0.20 [0.21]	0.48 [0.52]	0.01 [0.00]	0.34 [0.59]	0.01 [0.20]	0.01 [0.19]	0.33 [0.07]	0.34 [0.08]	0.20 [0.04]
Belgium	0.05 [0.14]	0.12 [0.18]	0.47 [0.62]	0.16 [0.21]	0.68 [0.50]	0.16 [0.07]	0.74 [0.60]	0.02 [0.22]	0.14 [0.24]	0.05 [0.05]	0.18 [0.11]	0.23 [0.07]
Canada	0.28 [0.23]	0.28 [0.32]	0.73 [0.82]	0.23 [0.23]	0.27 [0.39]	0.00 [0.00]	0.24 [0.48]	0.01 [0.18]	0.01 [0.13]	0.25 [0.05]	0.44 [0.10]	0.26 [0.05]
New Zld	0.10 [0.08]	0.13 [0.18]	0.68 [0.65]	0.18 [0.14]	0.39 [0.58]	0.00 [0.03]	0.34 [0.76]	0.00 [0.17]	0.02 [0.29]	0.38 [0.02]	0.47 [0.06]	0.30 [0.03]
Norway	0.06 [0.07]	0.16 [0.18]	0.59 [0.56]	0.16 [0.17]	0.68 [0.64]	0.19 [0.15]	0.78 [0.75]	0.03 [0.03]	0.16 [0.23]	0.01 [0.01]	0.12 [0.14]	0.06 [0.07]
Spain	0.08 [0.09]	0.14 [0.20]	0.49 [0.45]	0.21 [0.20]	0.56 [0.65]	0.11 [0.16]	0.34 [0.64]	0.00 [0.10]	0.00 [0.34]	0.37 [0.07]	0.29 [0.05]	0.39 [0.05]
Sweden	0.34 [0.22]	0.26 [0.35]	0.68 [0.76]	0.21 [0.19]	0.22 [0.42]	0.01 [0.03]	0.22 [0.58]	0.01 [0.17]	0.00 [0.18]	0.23 [0.01]	0.51 [0.07]	0.31 [0.03]

Table 5: Variance Decomposition with HP-Filtered Data

	$\sigma_{\mu}^{[j]}$			$\sigma_{\vartheta}^{[j]}$			$\sigma_{a}^{[j]}$			σ_g		
	y	c	i	y	c	i	y	c	i	y	c	i
Australia	0.01	0.04	0.40	0.09	0.65	0.09	0.69	0.23	0.38	0.21	0.07	0.13
	[0.13]	[0.20]	[0.76]	[0.21]	[0.52]	[0.00]	[0.59]	[0.20]	[0.19]	[0.07]	[0.08]	[0.04]
Belgium	0.02	0.07	0.45	0.15	0.70	0.17	0.60	0.14	0.23	0.23	0.07	0.14
	[0.14]	[0.18]	[0.62]	[0.21]	[0.50]	[0.07]	[0.60]	[0.22]	[0.24]	[0.05]	[0.11]	[0.07]
Canada	0.02	0.07	0.39	0.24	0.78	0.34	0.54	0.10	0.19	0.20	0.04	0.08
	[0.23]	[0.32]	[0.82]	[0.23]	[0.39]	[0.00]	[0.48]	[0.18]	[0.13]	[0.05]	[0.10]	[0.05]
New Zld	0.02	0.07	0.58	0.09	0.66	0.06	0.79	0.22	0.31	0.10	0.04	0.05
	[0.08]	[0.18]	[0.65]	[0.14]	[0.58]	[0.03]	[0.76]	[0.17]	[0.29]	[0.02]	[0.06]	[0.03]
Norway	0.04	0.34	0.73	0.16	0.51	0.13	0.70	0.12	0.08	0.10	0.03	0.06
	[0.07]	[0.18]	[0.56]	[0.17]	[0.64]	[0.15]	[0.75]	[0.03]	[0.23]	[0.01]	[0.14]	[0.07]
Spain	0.01	0.05	0.22	0.13	0.71	0.21	0.71	0.20	0.48	0.15	0.04	0.09
	[0.09]	[0.20]	[0.45]	[0.20]	[0.65]	[0.16]	[0.64]	[0.10]	[0.34]	[0.07]	[0.05]	[0.05]
Sweden	0.03	0.22	0.55	0.25	0.69	0.30	0.60	0.07	0.10	0.12	0.02	0.05
	[0.22]	[0.35]	[0.76]	[0.19]	[0.42]	[0.03]	[0.58]	[0.17]	[0.18]	[0.01]	[0.07]	[0.03]

Table 6: Variance Decomposition with US growth

	$\sigma_{\mu}^{[j]}$			$\sigma_{\vartheta}^{[j]}$			$\sigma_{a}^{[j]}$			σ_g		
	y	c	i	y	c	i	y	c	i	y	c	i
Australia	0.07	0.10	0.44	0.16	0.42	0.09	0.63	0.24	0.36	0.07	0.08	0.08
	[0.13]	[0.20]	[0.76]	[0.21]	[0.52]	[0.00]	[0.59]	[0.20]	[0.19]	[0.07]	[0.08]	[0.04]
Belgium	0.13	0.19	0.63	0.21	0.55	0.09	0.61	0.15	0.21	0.04	0.11	0.07
	[0.14]	[0.18]	[0.62]	[0.21]	[0.50]	[0.07]	[0.60]	[0.22]	[0.24]	[0.05]	[0.11]	[0.07]
Canada	0.22	0.30	0.81	0.24	0.42	0.00	0.48	0.17	0.13	0.05	0.11	0.06
	[0.23]	[0.32]	[0.82]	[0.23]	[0.39]	[0.00]	[0.48]	[0.18]	[0.13]	[0.05]	[0.10]	[0.05]
New Zld	0.09	0.16	0.69	0.15	0.49	0.01	0.73	0.29	0.27	0.02	0.05	0.03
	[0.08]	[0.18]	[0.65]	[0.14]	[0.58]	[0.03]	[0.76]	[0.17]	[0.29]	[0.02]	[0.06]	[0.03]
Norway	0.08	0.21	0.53	0.17	0.63	0.16	0.73	0.04	0.25	0.01	0.13	0.07
	[0.07]	[0.18]	[0.56]	[0.17]	[0.64]	[0.15]	[0.75]	[0.03]	[0.23]	[0.01]	[0.14]	[0.07]
Spain	0.09	0.19	0.43	0.21	0.64	0.18	0.65	0.12	0.35	0.05	0.05	0.05
	[0.09]	[0.20]	[0.45]	[0.20]	[0.65]	[0.16]	[0.64]	[0.10]	[0.34]	[0.07]	[0.05]	[0.05]
Sweden	0.22	0.37	0.76	0.16	0.38	0.05	0.58	0.13	0.17	0.01	0.06	0.03
	[0.22]	[0.35]	[0.76]	[0.19]	[0.42]	[0.03]	[0.58]	[0.17]	[0.18]	[0.01]	[0.07]	[0.03]

Table 7: Variance Decomposition with Two Factors

	$\sigma_\mu^{[j]}$			$\sigma_\vartheta^{[j]}$			$\sigma_a^{[j]}$			σ_r			σ_g		
	y	c	i	y	c	i	y	c	i	y	c	i	y	c	i
Australia	0.14 [0.13]	0.19 [0.20]	0.67 [0.76]	0.22 [0.21]	0.42 [0.52]	0.00 [0.00]	0.49 [0.59]	0.21 [0.20]	0.15 [0.19]	0.06	0.08	0.13	0.09 [0.07]	0.09 [0.08]	0.05 [0.04]
Belgium	0.17 [0.14]	0.22 [0.18]	0.60 [0.62]	0.14 [0.21]	0.21 [0.50]	0.00 [0.07]	0.46 [0.60]	0.15 [0.22]	0.12 [0.24]	0.09	0.12	0.19	0.08 [0.05]	0.17 [0.11]	0.08 [0.07]
Canada	0.23 [0.23]	0.28 [0.32]	0.73 [0.82]	0.22 [0.23]	0.34 [0.39]	0.00 [0.00]	0.39 [0.48]	0.12 [0.18]	0.10 [0.13]	0.09	0.12	0.10	0.07 [0.05]	0.13 [0.10]	0.07 [0.05]
New Zld	0.12 [0.08]	0.19 [0.18]	0.68 [0.65]	0.16 [0.14]	0.43 [0.58]	0.00 [0.03]	0.64 [0.76]	0.26 [0.17]	0.17 [0.29]	0.04	0.06	0.11	0.04 [0.02]	0.07 [0.06]	0.03 [0.03]
Norway	0.19 [0.07]	0.32 [0.18]	0.71 [0.56]	0.20 [0.17]	0.45 [0.64]	0.05 [0.15]	0.56 [0.75]	0.03 [0.03]	0.10 [0.23]	0.04	0.07	0.10	0.01 [0.01]	0.13 [0.14]	0.04 [0.07]
Spain	0.08 [0.09]	0.16 [0.20]	0.31 [0.45]	0.19 [0.20]	0.57 [0.65]	0.18 [0.16]	0.57 [0.64]	0.09 [0.10]	0.29 [0.34]	0.05	0.10	0.14	0.11 [0.07]	0.09 [0.05]	0.08 [0.05]
Sweden	0.27 [0.22]	0.40 [0.35]	0.76 [0.76]	0.12 [0.19]	0.23 [0.42]	0.01 [0.03]	0.51 [0.58]	0.12 [0.17]	0.13 [0.18]	0.05	0.08	0.05	0.02 [0.01]	0.10 [0.07]	0.05 [0.03]

Figure 1: Trends and Cycles in Developed Small Open Economies

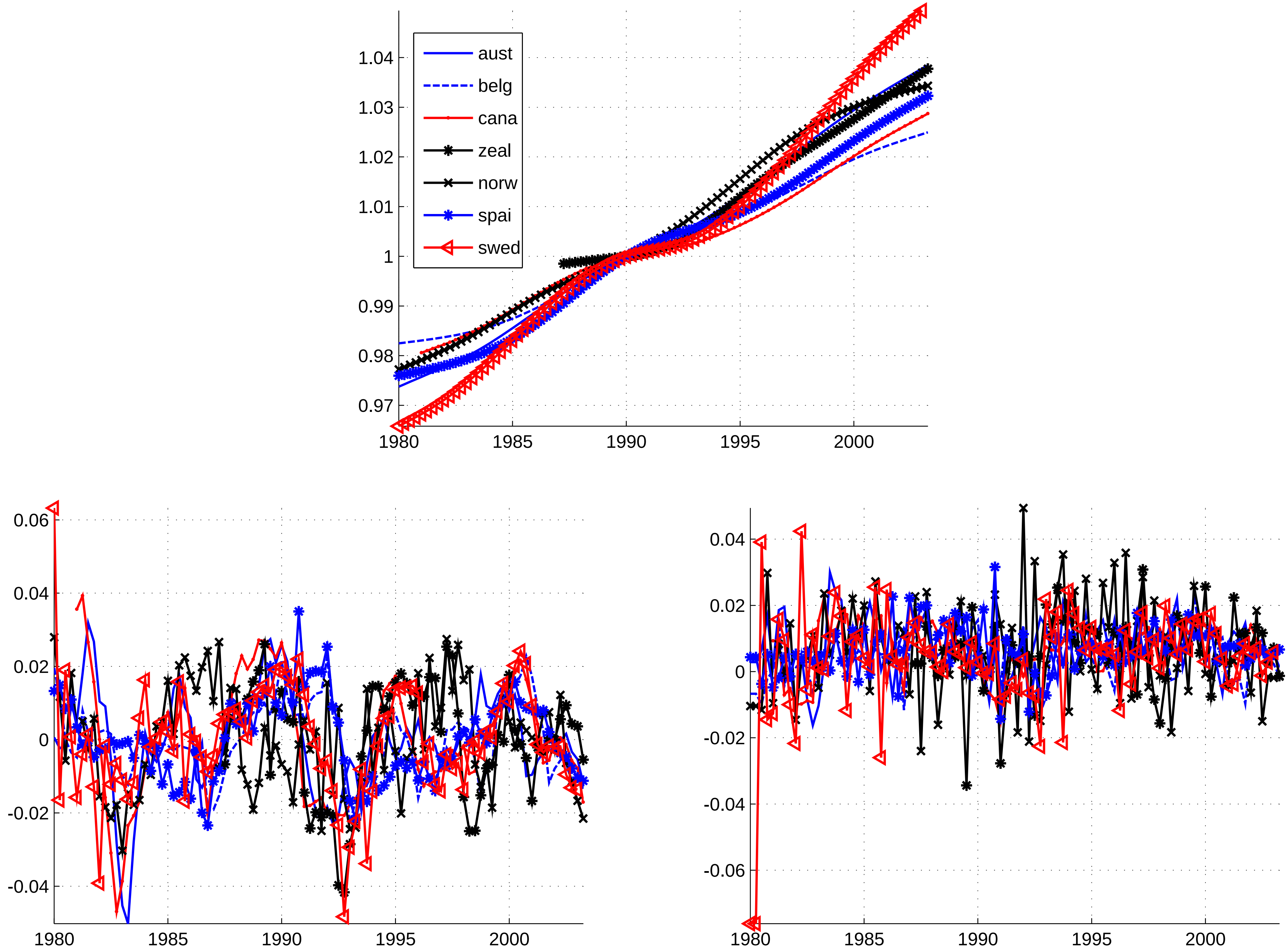

Figure 2: Filtered Common Factor

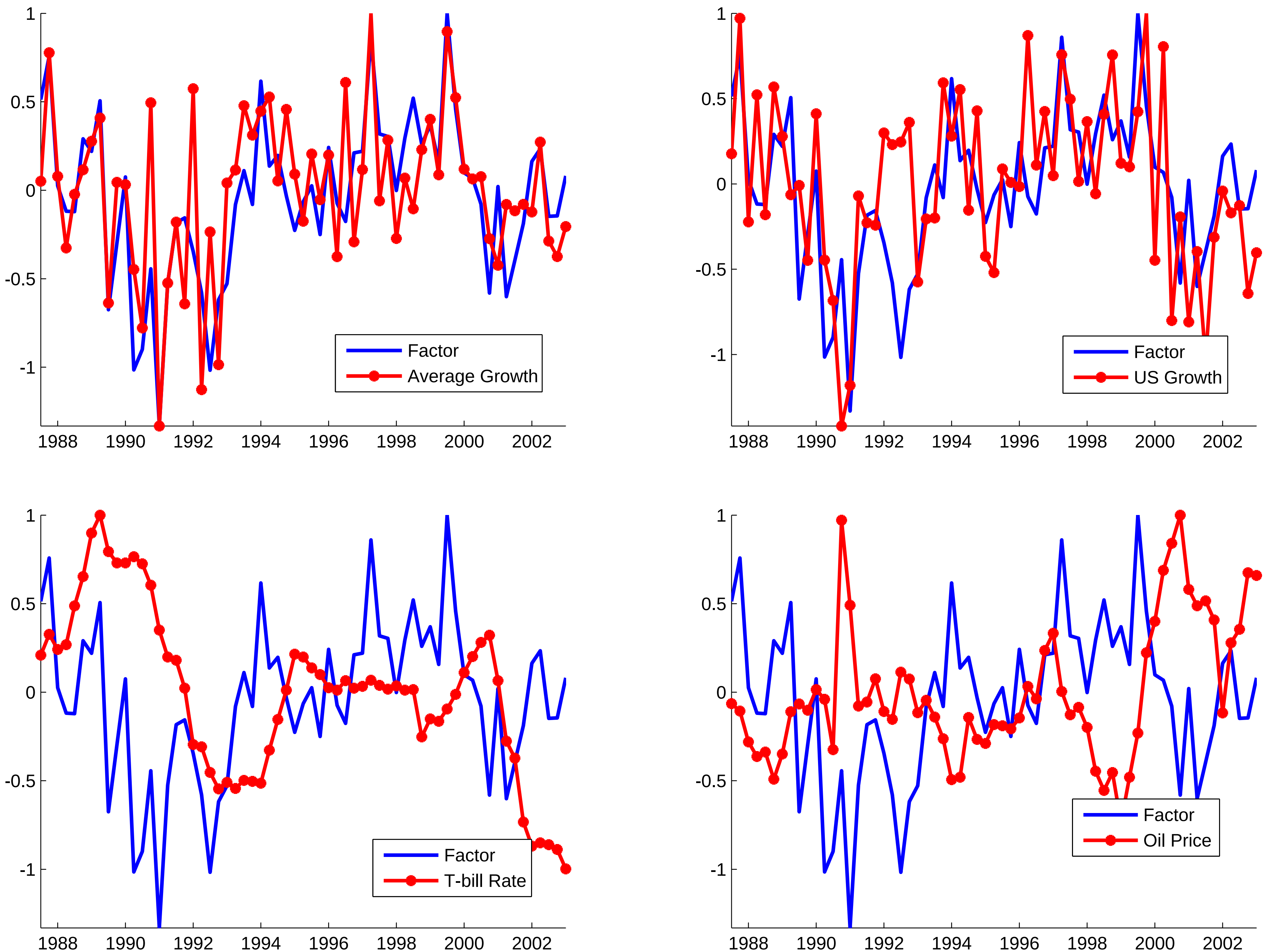

Figure 3: Non-stationary Tech Shock

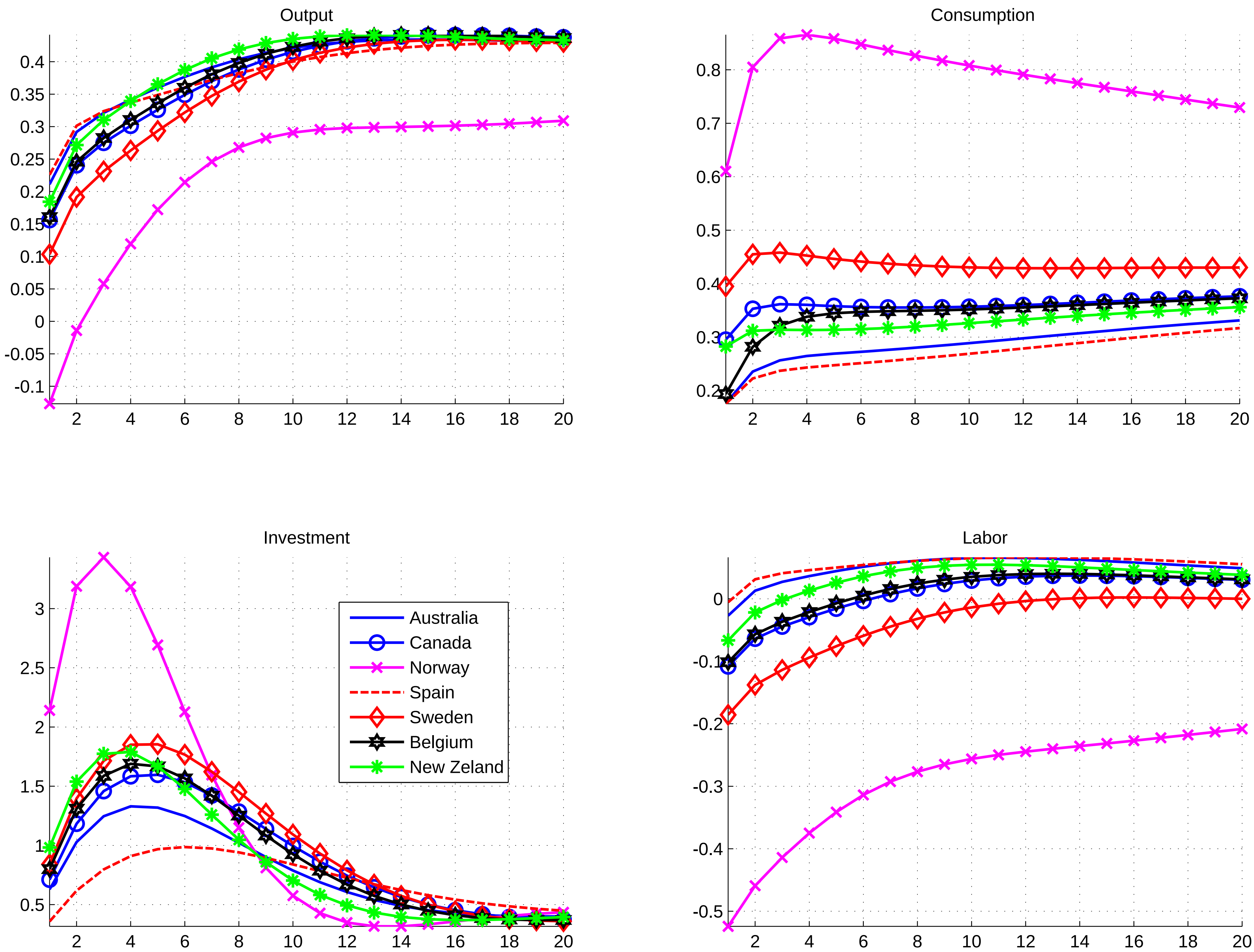

Figure 4: Stationary Tech Shock

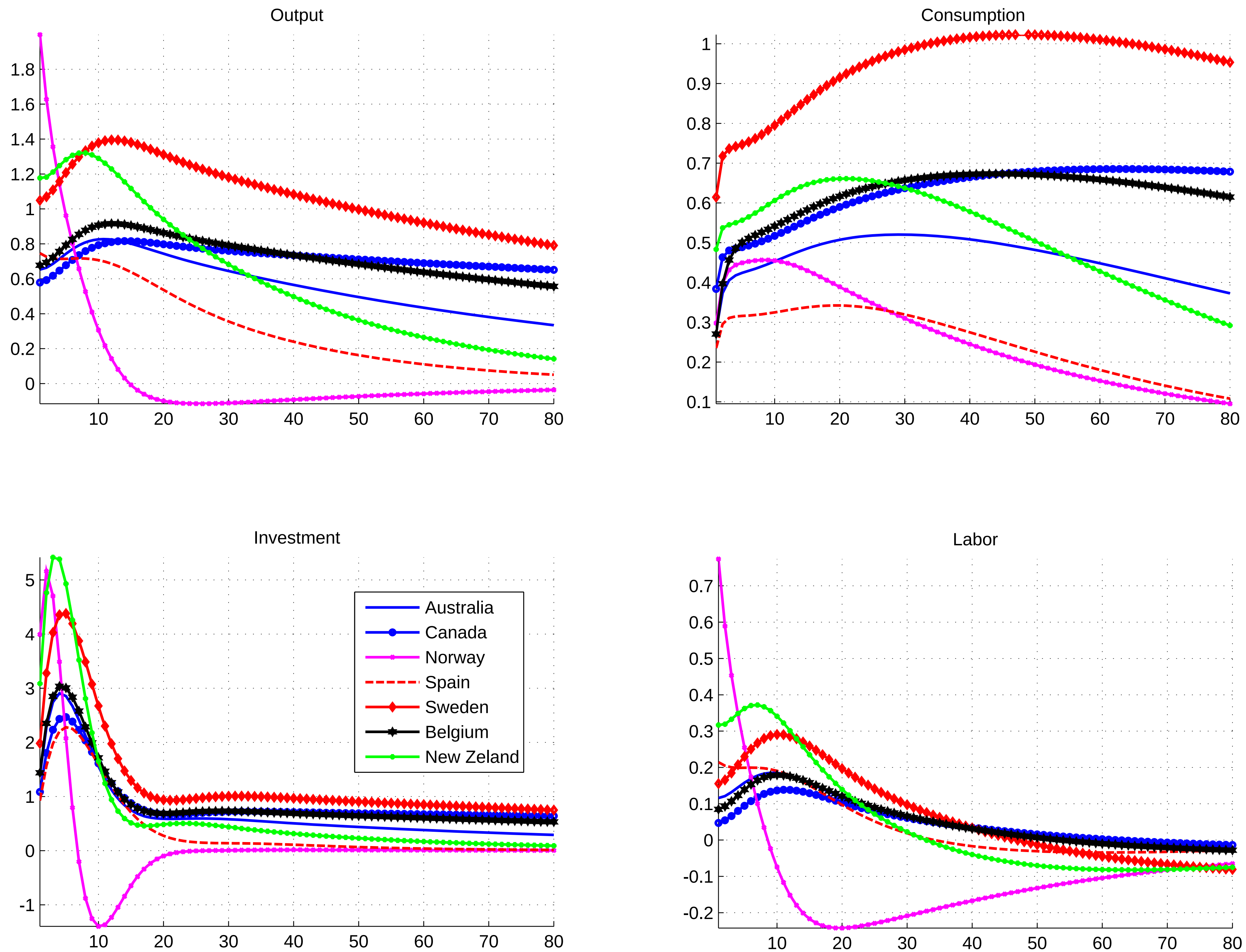

Figure 5: Premium Shock

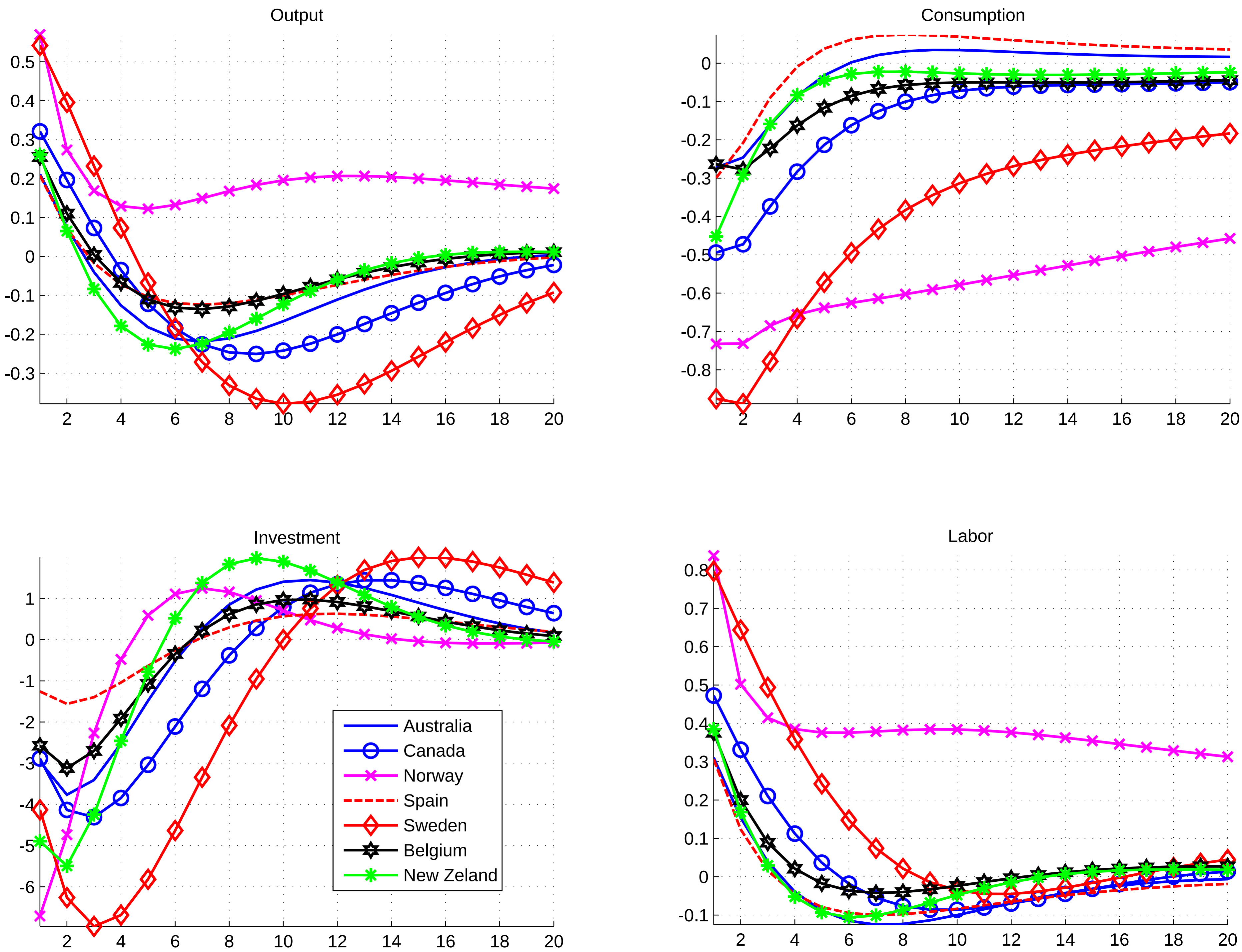

Figure 6: Preference Shock

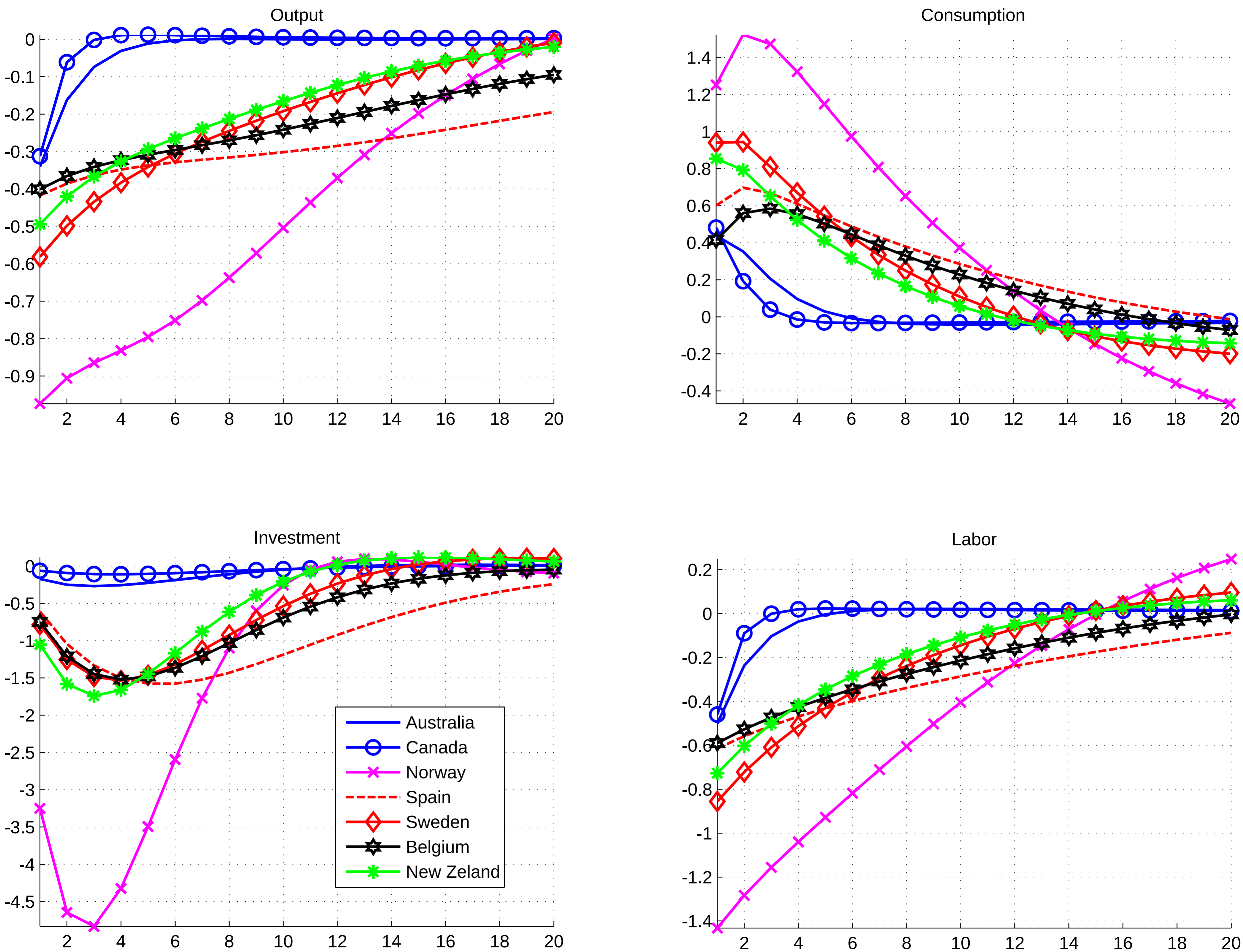

Figure 7: Government Shock

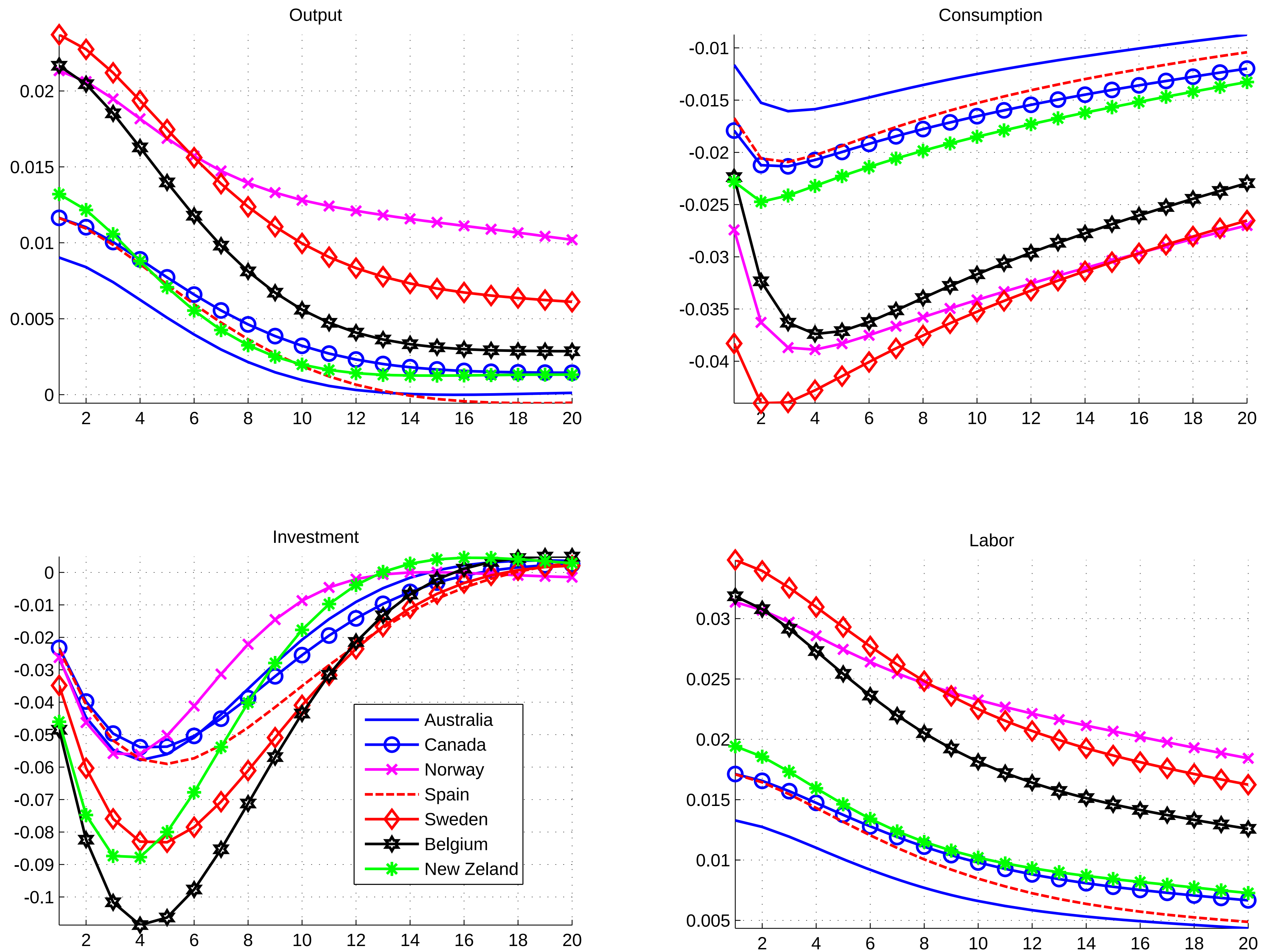

Figure 8: Filtered Common Factor and HP Filtered Data

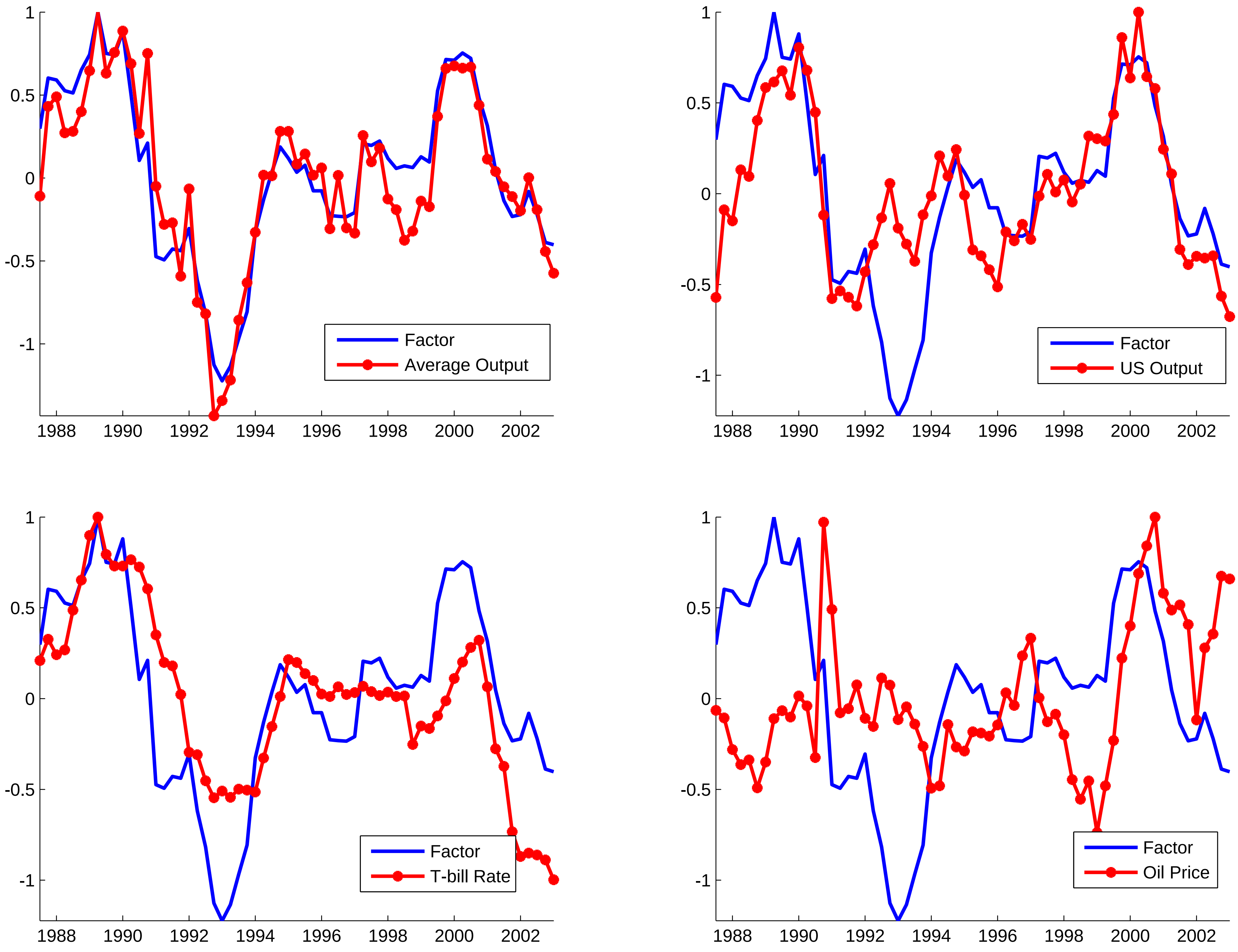

Figure 8: Productivity and Interest Rate Factors