Investment-Specific Technology and International Business Cycles: An Empirical Assessment*

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Abstract

We first introduce investment-specific technology shocks to an otherwise standard international real business cycles setup, and show that an arbitrary calibration of these technology processes successfully addresses several of the existent puzzles in the literature. In particular, we obtain a negative correlation of relative consumption and the terms of trade (Backus-Smith puzzle), and a volatile real exchange rate. Then we use OECD data for investment relative prices and estimate these processes, showing that they are cointegrated and better represented by a vector error correction (VECM) model. Finally, we demonstrate that such estimated technology process is actually powerless to explain any of the existent puzzles.

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1 Introduction

International real business cycle (IRBC) models fail to account for at least four features of the data. First, empirical cross-country consumption correlations are generally similar or lower to cross-country output correlations, whereas existing models typically produce consumption correlations much higher than output correlations. Second, investment and employment tend to be positively correlated across countries, whereas the models predict a negative correlation. Third, models generate far less volatility in the terms of trade and the real exchange rate than is seen in the data (Heathcote and Perri, 2002). Fourth, the standard setup predicts that the real exchange rate is closely related to the ratio of consumptions across the two economies, while instead the correlation in the data is in general negative (Backus and Smith, 1993).

Even when incomplete markets are considered, risk sharing induces strong positive cross-country consumption correlations in the IRBC framework. The efficient response to a neutral total factor productivity (TFP) shock involves increasing investment and labor supply in the more productive country and reducing them in the less productive country. Thus the cross-country correlations of factor supplies and output in the models are lower than those observed empirically. Domestic households should consume more relative to their foreign counterparts when their consumption basket is relatively cheap (i.e. the real exchange rate, P_t^*/P_t, increases), at odds with the data. Since models produce highly correlated consumptions (because of risk sharing), their ratio shows low volatility, and the real exchange rate is consequently less volatile than in the data.

The literature has been energetically trying to fill the gap between theory and data. One alternative to address the discrepancies between the model and the data consists on focusing on the demand side and introduce taste shocks as Stockman an Tesar (1995) and Heathcote and Perri (2008). In particular, Heathcote and Perri (2008) shows how demand shocks can successfully address the above described puzzles. However, it is difficult to measure taste shocks in the data. Another alternative has been analyzed by Raffo (2009), who instead considers investment-specific technology (IST) shocks, along the guideline specifications in Greenwood et al. (1998) and the empirical work of Fisher (2006). Raffo (2009) extends to the open economy framework the growing closed economy literature analyzing the role of this channel to explain a sizable business cycle variation of real variables. As it was the case with taste shocks, Raffo (2009) shows how these shocks can address the above described puzzles. This type of productivity innovations have two appealing features : (1) they resemble demand shocks given that directly affect the relative price of capital goods and (2) they have a clear link to the data. Raffo (2009) cleverly takes advantage of the first feature but does not consider the second. Instead of using the data to parametrize the law of motion of the IST shocks, he calibrates them without much discipline from the data.

Our paper follows the alternative approach. We do estimate the law of motion characterizing these shocks using OECD data. We proceed as follows. First, we provide evidence that IST processes for the U.S. and a sample of main industrialized trade partners have a unit root and are cointegrated. Motivated by this empirical finding, we estimate a vector error correction model (VECM) for the investment-specific technology processes of the U.S. and a sample of main industrialized trade partners. Third, we add investment-specific technology shocks that follow the estimated VECM process into an otherwise standard two-country, two-good model with TFP also following a VECM process as described in Rabanal et. al. (2009). This model should be considered as an extension of Heathcote and Perri (2002) to consider investment-specific technology shocks (as Raffo, 2009) and cointegrated shocks (Rabanal et. al., 2009). Finally, we simulate the model and analyze the results.

Our results indicate that while a calibration of the IST shocks on the lines of Raffo (2009) would sufficient to successfully address the three puzzles mentioned above, the data indicates the contrary: the estimated process for the IST shocks is powerless to solve these puzzles. In particular, Raffo (2009) assumes that the variance of the IST process is more than the double of the one characterizing the neutral TFP process. So that, IST shocks account for about two thirds of the variation in output. Instead our estimation results indicate that the variance of the former technology process is about half of the variance of the latter. Same results hold even when we consider internal amplification mechanisms like endogenous capital utilization, which facilitate investment demand booms, or GHH preferences, which suppress the wealth effect responsible of dampening the response of the labor supply to productivity innovations, are considered.

The rest of the paper is organized as follows. In Section 2 we present the model with cointegrated productivity shocks (both TFP and IST). In Section 3 we report estimates for the law of motion of these processes of the United States and a "rest of the world" aggregate. In Section 4 we present the main findings from simulating the model, leaving Section 5 for concluding remarks.

2 The Model

In this section, we present a standard two-country, two-good IRBC model similar to the one described in Heathcote and Perri (2002). The main difference with respect to the standard IRBC literature is the inclusion of IST shocks and the definition of cointegrated processes for both TFP and IST shocks. For completeness, we also introduce endogenous capital utilization as in Greenwood et al (1998) and investment quadratic adjustment costs.

In the existent literature, productivity processes (both TFP and IST) are assumed to be stationary or trend stationary in logs, and they are modelled as a VAR in levels.[1] In this paper, we consider instead (log) processes (again for both TFP and IST) that are cointegrated of order C(1,1). This implies that (log) processes are integrated of order one but a linear combination is stationary. According to the Granger representation theorem,[2] our C(1,1) assumption is equivalent to defining a VECM for the law of motion of the log differences of the technology processes. The VECM for both TFP and IST shocks is defined later.

In each country, a single final good is produced by a representative competitive firm that uses intermediate goods in the production process. These intermediate goods are imperfect substitutes for each other and can be purchased from representative competitive producers of intermediate goods in both countries. Intermediate goods producers use local capital and labor in the production process. The final good can only be locally consumed or invested by consumers, all trade between countries occurs at the intermediate goods level. In addition, consumers trade across countries an uncontingent international one-period riskless bond denominated in units of domestic intermediate goods. We thus

[1] Interestingly, Baxter and Crucini (1995) estimate a VECM using TFP processes for the United States and Canada, but they dismiss this evidence when simulating their model.

[2] See Engle and Granger (1987).

impose incomplete markets. In each period of time t, the economy experiences one of many finite events s_t. We denote by $s^t = (s_0, ..., s_t)$ the history of events up through period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$ and s_0 is given.

In the remainder of this section, we describe the households' problem, the intermediate and final goods producers' problems, and the VECM processes. Then, we explain market clearing and equilibrium. Finally, we discuss the conditions for the existence of a balanced growth path and explain how to transform the variables in the model to achieve stationarity.

2.1 Households

In this subsection, we describe the decision problem faced by home-country households. The problem faced by foreign-country households is similar, and hence it is not presented. The representative household of the home country solves

$$\max_{\{C(s^t),L(s^t),X(s^t),K(s^t),D(s^t),u(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi\left(s^t\right) \frac{\left\{C\left(s^t\right)^{\tau}\left[1-L\left(s^t\right)\right]^{1-\tau}\right\}^{1-\sigma}}{1-\sigma} \tag{1}$$

subject to the following budget constraint

$$\begin{gathered} P\left(s^t\right)\left[C\left(s^t\right)+X\left(s^t\right)\right]+P_H\left(s^t\right)\overline{Q}\left(s^t\right)D\left(s^t\right) \leqslant \\ P\left(s^t\right)\left[W\left(s^t\right)L\left(s^t\right)+u(s^t)R\left(s^t\right)K\left(s^{t-1}\right)\right]+P_H\left(s^t\right)\left\{D\left(s^{t-1}\right)-\Phi\left[D\left(s^t\right)\right]\right\} \end{gathered} \tag{2}$$

and the law of motion for capital

$$K\left(s^t\right)=\left(1-\delta(s^t\right)K\left(s^{t-1}\right)+V\left(s^t\right)\left(X\left(s^t\right)-\frac{\phi}{2}X\left(s^{t-1}\right)\left[\frac{X\left(s^t\right)}{X\left(s^{t-1}\right)}-\Lambda_X\right]^2\right) \tag{3}$$

$\beta \in (0,1)$ is the discount factor, $L\left(s^t\right) \in (0,1)$ is the fraction of time allocated to work in the home country, $C\left(s^t\right) \geq 0$ are units of consumption of the final good, $X\left(s^t\right) \geq 0$ are units of investment, $K\left(s^t\right) \geq 0$ is the capital level in the home country at the beginning of period $t+1$. $P\left(s^t\right)$ is the price of the home final good, which will be defined below, $W\left(s^t\right)$ is the hourly wage in the home country, and $R(s^t)$ is the home-country rental rate of capital, where the prices of both factor inputs are measured in units of the final good. $P_H(s^t)$ is the price of the home intermediate good. The depreciation of the stock of capital, δ, is a function of its utilization rate $u(s^t)$ .Following Greenwood et al (1998) we assume that $\delta(s^t) = \bar{\delta} + \frac{b}{1+\varepsilon} u(s^t)^{1+\varepsilon}$, $\varepsilon > 0$. ϕ controls the elasticity of the cost of adjustment in the capital stock . Since the presence of two unit roots makes the model non-stationary, we rescale the adjustment cost to account for the long-run gross rate of growth of investment along the balanced growth path, Λ_X. The stochastic process for the IST is characterized $V(s^t)$ .In a competitive setting, $\frac{1}{V(s^t)}$ may be interpreted as the relative price of capital goods. $D(s^t)$ denotes the holdings of the internationally traded riskless bond that pays one unit of home intermediate good (minus a small cost of holding bonds, $\Phi(\cdot)$) in period $t+1$ regardless of the state of nature, and $\overline{Q}(s^t)$ is its price, measured in units of the home intermediate good. Finally, the function $\Phi(\cdot)$ is the arbitrarily small cost of holding bonds measured in units of the home intermediate good.[3]

We assume, following the existing literature, that $\Phi(\cdot)$ takes the following functional form $\Phi[D(s^t)] = \frac{\psi}{2} Z(s^{t-1}) \left[\frac{D(s^t)}{Z(s^{t-1})}\right]^2$. It is straightforward to confirm that along a balanced growth path, the following variable $\frac{D(s^t)}{Z(s^{t-1})}$ is stationary, where $Z(s^t) = A(s^t)^{\frac{1}{1-\alpha}} V(s^t)^{\frac{\alpha}{1-\alpha}}$. We need to include $Z(s^{t-1})$, in the adjustment cost function, both dividing $D(s^t)$ and multiplying $\left[\frac{D(s^t)}{Z(s^{t-1})}\right]^2$. The reason is that $D(s^t)$ will grow at the rate of growth of $Z(s^{t-1})$ along the balanced growth path, making the ratio $\frac{D(s^t)}{Z(s^{t-1})}$ stationary. Also, since all home real variables (excluding capital, as discussed below) will also grow at the rate of growth of $Z(s^{t-1})$ along the balanced growth path, we need to make the adjustment cost (measured in units of home intermediate good) also grow at the same rate in order to induce stationarity.

[3] The $\Phi(\cdot)$ cost is introduced to ensure stationarity of the level of $D(s^t)$ in IRBC models with incomplete markets, as discussed by Heathcote and Perri (2002). We choose the cost to be numerically small, so it does not affect the dynamics of the rest of the variables.

2.2 Firms

2.2.1 Final goods producers

The final good in the home country, $Y\left(s^t\right)$, is produced using home intermediate goods, $Y_H\left(s^t\right)$, and foreign intermediate goods, $Y_F\left(s^t\right)$, with the following technology:

$$Y\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} Y_H\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_F\left(s^t\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{4}$$

where ω denotes the fraction of home intermediate goods that are used for the production of the home final good and θ represents the elasticity of substitution between home and foreign intermediate goods. Therefore, the representative final goods producer in the home country solves the following problem:

$$\max_{Y(s^t)\geq 0, Y_H(s^t)\geq 0, Y_F(s^t)\geq 0} P\left(s^t\right) Y\left(s^t\right) - P_H\left(s^t\right) Y_H\left(s^t\right) - P_F\left(s^t\right) Y_F\left(s^t\right)$$

subject to the production function (4).

2.2.2 Intermediate goods producers

The representative intermediate goods producer in the home country uses home labor and capital in order to produce home intermediate goods and sells her product to both the home and foreign final good producers. Taking prices of all goods and factor inputs as given, she maximizes profits. Hence, she solves:

$$\underset{L(s^t)\geq 0, K(s^{t-1})\geq 0}{Max} P_H\left(s^t\right)\left[Y_H\left(s^t\right) + Y_H^*\left(s^t\right)\right] - P\left(s^t\right)\left[W\left(s^t\right) L\left(s^t\right) + u\left(s^t\right) R\left(s^t\right) K\left(s^{t-1}\right)\right]$$

subject to the production function

$$Y_H\left(s^t\right) + Y_H^*\left(s^t\right) = A\left(s^t\right)^{1-\alpha} u\left(s^t\right) K\left(s^{t-1}\right)^{\alpha} L\left(s^t\right)^{1-\alpha} \tag{5}$$

where $Y_H(s^t)$ is the amount of home intermediate goods sold to the home final goods producers, $Y_H^*(s^t)$ is the amount of home intermediate goods sold to the foreign final goods producers.

2.2.3 The VECMs for TFP and IST

As mentioned above, we depart from the standard assumption in the IRBC literature and consider processes for both TFP and IST shocks that are cointegrated of order $C(1,1)$. We specify the following VECM for the law of motion driving the (log) differences of IST processes for both the home and the foreign country $(\log V(s^t), \log V^*(s^t))$:

$$\begin{pmatrix} \Delta \log V(s^t) \\ \Delta \log V^*(s^t) \end{pmatrix} = \begin{pmatrix} c_V \\ c_V^* \end{pmatrix} + \rho_V \begin{pmatrix} \Delta \log V\left(s^{t-1}\right) \\ \Delta \log V^*\left(s^{t-1}\right) \end{pmatrix} \tag{6}$$
$$+ \begin{pmatrix} \kappa_V \\ \kappa_V^* \end{pmatrix} \left[\log V\left(s^{t-1}\right) - \gamma_V \log V^*\left(s^{t-1}\right) - \log \xi_V\right] + \begin{pmatrix} \varepsilon^V(s^t) \\ \varepsilon^{V,*}(s^t) \end{pmatrix}$$

where $\rho_V = \begin{pmatrix} \rho_{V,11} & \rho_{V,12} \\ \rho_{V,21} & \rho_{V,22} \end{pmatrix}$, $(1, -\gamma_V)$ is the cointegrating vector, ξ_V is the constant in the cointegrating relationship. $\varepsilon^V(s^t) \sim N\left(0, \sigma^{\varepsilon,V}\right)$ and $\varepsilon^{V,*}(s^t) \sim N\left(0, \sigma^{\varepsilon,V,*}\right)$, are correlated, and Δ is the first-difference operator. We restrict ourselves to a VECM with one lag. This assumption is motivated by the empirical results to be presented below.

This VECM law of motion implies that deviations of today's log differences of investment-specific innovations with respect to its mean value depend not only on lags of home and foreign log differences of it but also on a function of the ratio of lag home and foreign innovations, $V\left(s^{t-1}\right) / \left[\xi^V V^*\left(s^{t-1}\right)^{\gamma_V}\right]$. The VECM representation implies that $\Delta \log V(s^t)$, $\Delta \log V^*(s^t)$, and $\log V\left(s^{t-1}\right) - \gamma_V \log V^*\left(s^{t-1}\right) - \log \xi^V$ are stationary processes.

For the case of the TFP process we consider the following law of motion driving the (log) differences of TFP processes for both the home and the foreign country $(\log A(s^t), \log A^*(s^t))$ as:

$$\begin{pmatrix} \Delta \log A(s^t) \\ \Delta \log A^*(s^t) \end{pmatrix} = \begin{pmatrix} c_A \\ c_A^* \end{pmatrix} + \rho_A^1 \begin{pmatrix} \Delta \log A(s^{t-1}) \\ \Delta \log A^*(s^{t-1}) \end{pmatrix} + \rho_A^2 \begin{pmatrix} \Delta \log A(s^{t-2}) \\ \Delta \log A^*(s^{t-2}) \end{pmatrix} \tag{7}$$
$$+ \begin{pmatrix} \kappa_A \\ \kappa_A^* \end{pmatrix} \left[\log A\left(s^{t-1}\right) - \gamma_A \log A^*\left(s^{t-1}\right) - \log \xi_A\right] + \begin{pmatrix} \varepsilon^A(s^t) \\ \varepsilon^{A,*}(s^t) \end{pmatrix}$$

where $\rho_A^1 = \begin{pmatrix} \rho_{A,11}^1 & \rho_{A,12}^1 \\ \rho_{A,21}^1 & \rho_{A,22}^1 \end{pmatrix}$, $\rho_A^2 = \begin{pmatrix} \rho_{A,11}^2 & \rho_{A,12}^2 \\ \rho_{A,21}^2 & \rho_{A,22}^2 \end{pmatrix}$, $(1, -\gamma_A)$ is the cointegrating vector, ξ_A is the constant in the cointegrating relationship. Finally, $\varepsilon^A(s^t) \sim N\left(0, \sigma^{\varepsilon,A}\right)$ and $\varepsilon^{A,*}(s^t) \sim N\left(0, \sigma^{\varepsilon,A,*}\right)$, are correlated. Note that in this case, we allow a VECM with two lags. This assumption is also motivated by the empirical results to be presented below.

2.3 Market Clearing

The model is closed with the following market clearing conditions in the final goods markets

$$C\left(s^t\right) + X\left(s^t\right) = Y\left(s^t\right) \text{ and } C^*\left(s^t\right) + X^*\left(s^t\right) = Y^*\left(s^t\right)$$

and the bond markets

$$D\left(s^t\right) + D^*\left(s^t\right) = 0.$$

2.4 Equilibrium

2.4.1 Equilibrium definition

Given our laws of motion for shocks defined in section 2.2.3, an equilibrium for this economy is a set of allocations for home consumers, $C(s^t)$, $L(s^t)$, $X(s^t)$, $K(s^t)$, $u(s^t)$, and $D(s^t)$, and foreign consumers, $C^*(s^t)$, $L^*(s^t)$, $X^*(s^t)$, $K^*(s^t)$, $u^*(s^t)$, and $D^*(s^t)$, allocations for home and foreign intermediate goods producers, $Y_H(s^t)$, $Y_H^*(s^t)$, $Y_F(s^t)$ and $Y_F^*(s^t)$, allocations for home and foreign final goods producers, $Y(s^t)$ and $Y^*(s^t)$, intermediate goods prices $P_H(s^t)$ and $P_F^*(s^t)$, final goods prices $P(s^t)$ and $P^*(s^t)$, rental prices of labor and capital in the home and foreign country, $W(s^t)$, $R(s^t)$, $W^*(s^t)$, and $R^*(s^t)$ and the price of the bond $\overline{Q}(s^t)$ such that (i) given prices, household allocations solve the households' problem; (ii) given prices, intermediate goods producers allocations solve the intermediate goods producers' problem; (iii) given prices, final goods producers allocations solve the final goods producers' problem; (iv) and markets clear.

2.4.2 Equilibrium conditions

It is useful to define the following relative prices: $\widetilde{P}_H(s^t) = \frac{P_H(s^t)}{P(s^t)}$, $\widetilde{P}_F^*(s^t) = \frac{P_F^*(s^t)}{P^*(s^t)}$ and $RER(s^t) = \frac{P^*(s^t)}{P(s^t)}$. Note that $\widetilde{P}_H(s^t)$ is the price of home intermediate goods in terms of home final goods, $\widetilde{P}_F^*(s^t)$ is the price of foreign intermediate goods in terms of foreign final goods, which appears in the foreign country's budget constraint, and $RER(s^t)$ is the real exchange rate between the home and foreign countries. In our model the law of one price holds; hence, we have that $P_H(s^t) = P_H^*(s^t)$ and $P_F(s^t) = P_F^*(s^t)$. In the model the only source of real exchange rate fluctuations is the presence of home bias.

We now determine the equilibrium conditions implied by the first order conditions of households, intermediate and final goods producers in both countries, as well as the relevant laws of motion, production functions, and market clearing conditions. The marginal utility of consumption and the labor supply are given by:

$$U_C\left(s^t\right) = \lambda\left(s^t\right), \tag{8}$$

$$\frac{U_L\left(s^t\right)}{U_C\left(s^t\right)} = W\left(s^t\right), \tag{9}$$

where U_x denotes the partial derivative of the utility function U with respect to variable x. The first order condition with respect to capital and investment delivers:

$$\mu\left(s^t\right) = \beta \sum_{s^{t+1}} \pi\left(s^{t+1}/s^t\right) \left\{u\left(s^{t+1}\right) R\left(s^{t+1}\right) \lambda\left(s^{t+1}\right) + \mu\left(s^{t+1}\right)\left(1-\delta(u\left(s^{t+1}\right))\right)\right\}$$

$$\lambda\left(s^t\right) = \mu\left(s^t\right) V\left(s^t\right)\left(1-\phi\left(\frac{X\left(s^t\right)}{X\left(s^{t-1}\right)}-\Lambda_X\right)\right)+\beta\sum_{s^{t+1}}\pi\left(s^{t+1}/s^t\right)\mu\left(s^{t+1}\right)V\left(s^{t+1}\right)[(*)] \quad (10)$$

$$[(*)] = \left[\phi\left(\frac{X\left(s^{t+1}\right)}{X\left(s^t\right)}-\Lambda_X\right)\frac{X\left(s^{t+1}\right)}{X\left(s^t\right)}-\frac{\phi}{2}\left(\frac{X\left(s^{t+1}\right)}{X\left(s^t\right)}-\Lambda_X\right)^2\right] \quad (11)$$

$$\lambda\left(s^t\right)R\left(s^t\right)=\mu(s^t)\delta'(u\left(s^t\right))=\mu(s^t)bu(s^t)^{\varepsilon} \quad (12)$$

where $\pi\left(s^{t+1}|s^t\right)=\frac{\pi\left(s^{t+1}\right)}{\pi\left(s^t\right)}$ is the conditional probability of s^{t+1} given s^t.

Analogous expressions for the foreign country hold. The optimal choice by households of the home country delivers the following expression for the price of the riskless bond:

$$\overline{Q}\left(s^t\right)=\beta\sum_{s^{t+1}}\pi\left(s^{t+1}|s^t\right)\frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^t\right)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)}-\frac{\Phi'\left[D\left(s^t\right)\right]}{\beta}. \quad (13)$$

The risk-sharing condition is given by the optimal choice of the households of both countries for the riskless bond:

$$\sum_{s^{t+1}}\pi\left(s^{t+1}|s^t\right)\left[\frac{\lambda^*\left(s^{t+1}\right)}{\lambda^*\left(s^t\right)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)}\frac{RER\left(s^t\right)}{RER\left(s^{t+1}\right)}-\frac{\lambda\left(s^{t+1}\right)}{\lambda\left(s^t\right)}\frac{\widetilde{P}_H\left(s^{t+1}\right)}{\widetilde{P}_H\left(s^t\right)}\right]=-\frac{\Phi'\left[D\left(s^t\right)\right]}{\beta}. \quad (14)$$

From the intermediate goods producers' maximization problems, we obtain the result that labor and capital are paid their marginal product, where the rental rate of capital and the real wage are expressed in terms of the final good in each country:

$$W\left(s^t\right)=(1-\alpha)\widetilde{P}_H\left(s^t\right)A\left(s^t\right)^{1-\alpha}u\left(s^t\right)K\left(s^{t-1}\right)^{\alpha}L\left(s^t\right)^{-\alpha}, \quad (15)$$

$$R\left(s^t\right)=\alpha\widetilde{P}_H\left(s^t\right)A\left(s^t\right)^{1-\alpha}u\left(s^t\right)K\left(s^{t-1}\right)^{\alpha-1}L\left(s^t\right)^{1-\alpha}, \quad (16)$$

$$W^*\left(s^t\right)=(1-\alpha)\widetilde{P}_F^*\left(s^t\right)A^*\left(s^t\right)^{1-\alpha}u^*\left(s^t\right)K^*\left(s^{t-1}\right)^{\alpha}L^*\left(s^t\right)^{-\alpha}, \quad (17)$$

and

$$R^*\left(s^t\right) = \alpha \widetilde{P}_F^*\left(s^t\right) A^*\left(s^t\right)^{1-\alpha} u^*\left(s^t\right) K^*\left(s^{t-1}\right)^{\alpha-1} L^*\left(s^t\right)^{1-\alpha}. \tag{18}$$

From the final goods producers' maximization problem, we obtain the demands of intermediate goods, which depend on their relative price:

$$Y_H\left(s^t\right) = \omega \widetilde{P}_H\left(s^t\right)^{-\theta} Y\left(s^t\right), \tag{19}$$

$$Y_F\left(s^t\right) = (1-\omega)\left(\widetilde{P}_F^*\left(s^t\right) RER\left(s^t\right)\right)^{-\theta} Y\left(s^t\right), \tag{20}$$

$$Y_H^*\left(s^t\right) = (1-\omega)\left(\frac{\widetilde{P}_H\left(s^t\right)}{RER\left(s^t\right)}\right)^{-\theta} Y^*\left(s^t\right), \tag{21}$$

and

$$Y_F^*\left(s^t\right) = \omega \widetilde{P}_F^*\left(s^t\right)^{-\theta} Y^*\left(s^t\right). \tag{22}$$

Finally, good, input, and bond markets clear. Thus:

$$C\left(s^t\right) + X\left(s^t\right) = Y\left(s^t\right), \tag{23}$$

$$C^*\left(s^t\right) + X^*\left(s^t\right) = Y^*\left(s^t\right), \tag{24}$$

$$Y\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} Y_H\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_F\left(s^t\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{25}$$

$$Y^*\left(s^t\right) = \left[\omega^{\frac{1}{\theta}} Y_F^*\left(s^t\right)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} Y_H^*\left(s^t\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{26}$$

$$Y_H\left(s^t\right) + Y_H^*\left(s^t\right) = A\left(s^t\right)^{1-\alpha} u\left(s^t\right) K\left(s^{t-1}\right)^{\alpha} L\left(s^t\right)^{1-\alpha}, \tag{27}$$

$$Y_F\left(s^t\right) + Y_F^*\left(s^t\right) = A^*\left(s^t\right)^{1-\alpha} u^*\left(s^t\right) K^*\left(s^{t-1}\right)^{\alpha} L^*\left(s^t\right)^{1-\alpha}, \tag{28}$$

and

$$D\left(s^t\right) + D^*\left(s^t\right) = 0. \tag{29}$$

The law of motion of the level of debt

$$\begin{aligned}\widetilde{P}_H\left(s^t\right)\overline{Q}\left(s^t\right)D\left(s^t\right) &= \widetilde{P}_H\left(s^t\right)Y_H^*\left(s^t\right)-\widetilde{P}_F^*\left(s^t\right)RER\left(s^t\right)Y_F\left(s^t\right) \\ &\quad +\widetilde{P}_H\left(s^t\right)D\left(s^{t-1}\right)-\widetilde{P}_H\left(s^t\right)\Phi\left[D\left(s^t\right)\right]\end{aligned} \tag{30}$$

is obtained using (2) and the fact that intermediate and final goods producers at home make zero profits. Finally, the laws of motion for shocks are as defined in section 2.2.3.

2.5 Balanced Growth and the Restriction on the Cointegrating Vector

Equations (8) to (30) and the VECM processes for characterize the equilibrium in this model. Since we assume that both $\log A\left(s^t\right)$ and $\log A^*\left(s^t\right)$, as well as $\log V\left(s^t\right)$ and $\log V^*\left(s^t\right)$ are integrated processes, we need to normalize the equilibrium conditions in order to obtain a stationary system more amenable to study. Following King, Plosser, and Rebelo (1988) we divide the home-country variables that have a trend by $Z\left(s^{t-1}\right)$, where $Z\left(s^t\right)=A\left(s^t\right)^{\frac{1}{1-\alpha}}V\left(s^t\right)^{\frac{\alpha}{1-\alpha}}$, foreign-country variables are divided by $Z^*\left(s^{t-1}\right)$, where $Z^*\left(s^t\right)=A^*\left(s^t\right)^{\frac{1}{1-\alpha}}V^*\left(s^t\right)^{\frac{\alpha}{1-\alpha}}$. One exception is the capital stocks, which are instead divided by $Z\left(s^{t-1}\right)V\left(s^{t-1}\right)$ and $Z^*\left(s^{t-1}\right)V^*\left(s^{t-1}\right)$ respectively. In the appendix, we detail the full set of normalized equilibrium conditions.

For the model to have balanced growth we require some restrictions on preferences, production functions, and the law of motion of productivity shocks. The restrictions on preferences and technology of King, Plosser, and Rebelo (1988) are sufficient for the existence of balanced growth in a closed economy real business cycle (RBC) model. However, in our two-country model, an additional restriction on the cointegrating vector is needed if the model is to exhibit balanced growth. In particular, we need the ratio $Z\left(s^{t-1}\right)/Z^*\left(s^{t-1}\right)$ to be stationary. Since nominal variables are stationary, if $Z\left(s^{t-1}\right)/Z^*\left(s^{t-1}\right)$ were to be non-stationary, the ratio between $Y_F\left(s^t\right)/Z\left(s^{t-1}\right)$, and $Y_F^*\left(s^t\right)/Z^*\left(s^{t-1}\right)$ would also be non-stationary and consequently the balanced growth path would not exist. A sufficient condition to guarantee the stationarity $Z\left(s^{t-1}\right)/Z^*\left(s^{t-1}\right)$, is to check for the stationarity of both $A\left(s^{t-1}\right)/A^*\left(s^{t-1}\right)$ and $V\left(s^{t-1}\right)/V^*\left(s^{t-1}\right)$. Rabanal et al (2009) indeed show that the first ratio is stationary. We focus the analysis for the IST in what follows.

3 Estimation of the VECM for IST

In this section, we construct our series for the IST shocks using series for the relative price of investment for the U.S. and the "rest of the world" and use them to estimate the parameters of the VECM for IST. We first describe our constructed series for the relative price of investment for the U.S. and the "rest of the world," we show that our assumption that the IST processes are cointegrated of order C(1,1) cannot be rejected in the data. By the Granger representation theorem this implies that our VECM specification is valid. Second, we also show that the restriction imposed by balanced growth, i.e., that the parameter γ_V is equal to one, cannot be rejected in the data either. Finally, we estimate the parameters driving our VECM in order to simulate our model in the next section.

3.1 Data

In order to estimate our VECM we use data for the US and an aggregate for the "rest of the world" (ROW). The rest of the world is comprised by the US most significant trading partners: the 12 countries of the European Union, Canada, Japan, the United Kingdom, Australia and South Korea. Our sample period goes from 1980:1 to 2008:03.

Both for the US and for the ROW, we aim to obtain the real relative price of investment, which will be measured as an investment deflator divided by a consumption deflator. For the US we use the Personal Consumption Expenditure (PCE) deflator as our consumption deflator and the Gross Domestic Investment series as our investment deflator. Both series are derived directly from the National Income and Product Accounts (NIPA) and provided by the Bureau of Economic Analysis (BEA).

For Japan, we employ the Private final consumption expenditure and the Private sector capital formation deflator series obtained from the Cabinet Office and its quarterly GDP estimates. In the case of Canada, we use the Personal expenditure on consumer goods and services and the Business gross fixed capital formation deflator series as our consumption and investment deflators respectively. Both series can be derived from Canada's statistical agency, "Statistics Canada". For the UK, the Final consumption expenditure deflator and the Gross fixed capital formation deflator provided by the UK national statistics agency were employed. The deflators for Australia are derived from the Australian Bureau of Statistics. The particular series used were the Households final Consumption Expenditure and the Gross Fixed Capital formation implicit price deflators. For South Korea we use the Final Consumption Expenditure and Gross Capital Formation deflator series retrieved from the "Navi-Data" database provided by the Korean National Statistical Office. Finally, for the 12-EU countries, we employ the Consumption Deflator and the Gross Investment deflator from the AWM Database constructed by the European Central Bank.

Ideally, one would want to include additional countries that currently represent an important share of trade with the US, such as China, Brazil and Mexico. Unfortunately, long data series for such countries are unavailable.

3.2 Integration and Cointegration Properties

In this section, we present evidence supporting our assumption that the IST processes for the U.S. and the "rest of the world" are cointegrated of order C(1,1). First, we will empirically support the unit root assumption for the univariate processes. Second, we will test for the presence of cointegrating relationships using the Johansen (1991) procedure. Both the trace and the maximum eigenvalue methods support the existence of a cointegrating vector.

Univariate analysis of the (log) IST processes for the U.S. and the "rest of the world" strongly indicates that both series can be characterized by unit root processes with drift. Table 1 presents results for the U.S. process using the following commonly applied unit root tests: augmented Dickey-Fuller (Dickey and Fuller, 1979, and Said and Dickey 1984); the DF-GLS and the optimal point statistic ($P_T GLS$), both of Elliott et al. (1996); and the modified MZ_α, MZ_t, and MSB of Ng and Perron (2001). The lag length is chosen using the modified Akaike Information criterion (MAIC) as Ng and Perron (1995) recommend. In each case a constant and a trend are included in the specification. Table 1 also presents the same unit root test results for the "rest of the world" process. None of the test statistics are even close to rejecting the null hypothesis of unit root at the 5 percent critical value neither for the U.S. nor the "rest of the world". Using the same unit root tests on the first difference of the processes for the U.S. and the "rest of the world" there is also some evidence of them being stationary. For the U.S. all the tests (but ADF) reject the null hypothesis of unit root at the 10 percent critical value. For the "rest of the world" the evidence is weaker and the ADF only rejects at 30 percent while the other tests do even worse.

Table 1: Unit Root tests for IST Processes

	log U.S. IST				log "Rest of the World" IST			
	Level		First Difference		Level		First Difference	
Method	t-statistic	critical value	t-statistic	critical value	t-statistic	critical value	t-statistic	critical value
ADF	-0.76	-3.45	-2.85	-3.45	-0.95	-3.45	-2.42	-3.45
DF-GLS	-1.04	-3.02	-2.81	-3.02	-1.17	-3.02	-1.77	-3.02
P_T-GLS	23.30	5.64*	6.47	5.64*	20.32	5.64*	48.36	5.64*
MZ_α	-3.70	-17.3**	-13.9	-17.3**	-5.48	-17.3**	-1.17	-17.3**
MZ_t	-1.13	-2.91**	-2.61	-2.91**	-1.44	-2.91**	-0.73	-2.91**
MSB	0.30	0.17**	0.18	0.17**	0.26	0.17**	0.63	0.17**

Notes: ADF stands for augmented Dickey-Fuller test. DF-GLS stands for Elliott-Rothenberg-Stock detrended residuals test statistic. P_T-GLS stands for Elliott-Rothenberg-Stock Point-Optimal test statistic. MZ_α, MZ_t, and MSB stand for the class of modified tests analyzed in Ng-Perron (2001). p-values for the ADF test are one-sided p-values as in MacKinnon (1996) at 5 % critical value. p-values for the DF-GLS test are as in Elliott-Rothenberg-Stock (1996, Table 1) at 5 % critical value. * These values do not represent the p-values but the critical values of the test at the 5 percent level as reported in Elliott-Rothenberg-Stock (1996) Table 1. ** These values do not represent the p-values but the asymptotic critical values of the test at the 5 percent level as reported in Ng-Perron (2001) Table 1.

Once we have presented evidence that indicates that the U.S. and the "rest of the world" are well characterized by integrated processes of order one, we now focus on presenting evidence supporting our assumption that the processes are cointegrated. Table 2 presents some statistics calculated from an unrestricted VAR with four lags and a deterministic trend for the two-variables system

$[\log V(s^t), \log V^*(s^t)]$ for the sample period 1980:1 to 2007:4 where the number of lags was chosen using the AIC criterion.

Table 2: Cointegration Statistics I

Eigenvalues Modulus

0.99	0.84	0.84	0.74

Table 2 shows absolute value for the four eigenvalues of the VAR implied by the point estimates. If $\log V(s^t)$ and $\log V^*(s^t)$ share one common stochastic trend (balanced growth), the estimated VAR has to have a single eigenvalue equal to one and all other eigenvalues have to be less than one. As shown in Table 2, point estimates are in accord with this prediction: the highest eigenvalue equals one, while the second highest is less than one. But this is not a formal test of cointegration. Table 3 reports results from the unrestricted cointegration rank test using the trace and the maximum eigenvalue methods as defined by Johansen (1991). One lag is considered, and the cointegration tests are run for the same sample period and we assume no constant in the cointegrating vector. Clearly, the data strongly support a single eigenvalue.

Table 3: Cointegration Statistics II: Johansen's test

Number of Vectors	Eigenvalue	Trace	p-value	Max-Eigenvalue	p-value
0	0.24	37.83	20.26	31.20	15.89
1	0.05	6.64	9.16	6.64	9.16

Note: p-values as reported in MacKinnon-Haug-Michelis (1999)

3.3 The estimated VECM Model

In the last subsection, we presented evidence that $\log V(s^t)$ and $\log V^*(s^t)$ are cointegrated of order C(1,1). In this subsection we show that the null hypothesis of $\gamma_V = 1$ cannot be rejected by the data. This is very important because a cointegrating vector $(1, -1)$ implies that the balanced growth path hypothesis cannot be rejected. In fact, the likelihood ratio test is a Chi-squared with one degree of freedom and takes the value 0.09 which clearly can not reject the joint null hypothesis $\gamma_V = 1$, indicating that there is evidence for balance growth. Conditional on this restrictions and assuming one lags and no constant (as implied by the Johansen, 1991) the VECM estimates are reported in Table 4.

Table 4: VECM model for IST

κ_V	κ_V^*	$\rho_{V,11}$	$\rho_{V,12}$	$\rho_{V,21}$	$\rho_{V,22}$
-0.0141	-0.0194	0.3801	-0.1607	-0.0826	-0.0287
(-3.41)	(-5.23)	(4.30)	(-0.79)	(-0.36)	(-1.72)

t-statistics in parenthesis.

Whenever the ratio $V\left(s^{t-1}\right)/V^*\left(s^{t-1}\right)$ is larger than the its long-run value ξ_V, $\kappa_V < 0$ and $\kappa_V^* < 0$ imply that both $\Delta \log V\left(s^t\right)$, and $\Delta \log V^*\left(s^t\right)$ will tend to fall. The coefficient on $\rho_{V,11}$ implies a significant and positive autocorrelation for the IST process in the US. This will imply that, on impact, $V^*\left(s^t\right)$ will tend to fall whenever a positive shock hits $V\left(s^t\right)$. However, the negative autocorrelation, $\rho_{V,22}$, in $V^*\left(s^{t-1}\right)$ will guarantee slow convergence towards the balance growth path over time. All these findings are important to explain the results below. Finally, we estimate the standard deviation of the innovations $\sigma^{V,\varepsilon}$ and $\sigma^{V,\varepsilon,*}$ to be around 0.0045 and 0.0040. When simulating our model, we calibrate the stochastic process using the point estimates reported in Table 5, including those for σ^V and $\sigma^{V,*}$. In the simulations, we will also assume that $\varepsilon^V\left(s^t\right)$ and $\varepsilon^{V,*}\left(s^t\right)$ are uncorrelated, since this null hypothesis could not be rejected in the data. In the counterfactual arbitrary scenarios, we modify the magnitude of these last two parameters to assess the importance of the volatility of these shocks in the quantitative experiments.

3.4 Estimation of the VECM for TFP

In this paper we do not estimate the VECM model for TFP. Instead we borrow from Rabanal et al (2009). Rabanal et al (2009) describe the process to constructed TFP series for the U.S. and the "rest of the world," and perform three exercises. First, they show that our assumption that the TFP processes are cointegrated of order C(1,1) cannot be rejected in the data. Second, they show that the restriction imposed by balanced growth, i.e., that the parameter γ_A is equal to one, cannot be rejected in the data either.

In the IRBC literature, it is typically assumed that the coefficients driving TFP processes are symmetric across countries. Rabanal et al (2009) present evidence supporting simmetry across the estimated parameters in (7). In particular, they show that (1) the coefficients related to the speed of adjustment in the cointegrating vector are equal and of opposite sign, i.e., $\kappa_A = -\kappa_A^*$, (2) the coefficients of the constant terms are the same, i.e., $c_A = c_A^*$, and (3) they also show symmetry in the coefficients of the VAR, i.e. they show that it is the case that $\rho^1_{A,11} = \rho^1_{A,22}$, $\rho^2_{A,11} = \rho^2_{A,22}$, $\rho^1_{A,12} = \rho^1_{A,21}$, and $\rho^2_{A,12} = \rho^2_{A,21}$. Finally, their reported point estimates are

Table 5: VECM model for TFP

c_A	κ^A	$\rho^1_{A,11}$	$\rho^2_{A,11}$	$\rho^1_{A,12}$	$\rho^2_{A,12}$
0.0071	-0.0045	0.2041	0.1026	0.1035	-0.1497
(5.83)	(-2.65)	(2.97)	(1.54)	(1.55)	(-2.40)

t-statistics in parenthesis.

They estimate the standard deviation of the innovations $\sigma^{A,\varepsilon}$ and $\sigma^{A,\varepsilon,*}$ is around 0.0082.

4 Results

4.1 Parameterization

Our baseline parameterization follows that in Heathcote and Perri (2002) closely. The discount factor β is set equal to 0.99, which implies an annual rate of return on capital of 4 percent. We set the consumption share, τ, equal to 0.34 and the coefficient of risk aversion, σ, equal to 2. Backus, Kehoe, and Kydland (1992) assume the same value for the latter parameter. We assume a cost of bond holdings, ϕ, of 1 basis points (0.01). Parameters on technology are fairly standard in the literature. Thus, the depreciation rate, δ, is set to a quarterly value of 0.025, the capital share of output is set to $\alpha = 0.36$, and home bias for domestic intermediate goods is set to $\omega = 0.9$, which implies the observed import/output ratio in steady state for the US. As in Raffo (2009) we assume a relatively low value for the elasticity of substitution $\theta = 0.62$. This is the same value used by Corsetti et al (2008). The technology processes are calibrated as described in Tables 4 and 5.

4.2 Solving puzzles with an arbitrary calibration

We will start by showing how the baseline IRBC framework of Heathcote and Perri (2002) can not solve the mentioned puzzles. Then we will add IST, endogenous capital utilization, and GHH utility. This three features (as long as we calibrate IST shocks as in Raffo, 2009) will solve the puzzles.

Let us start by analyzing a model with only stationary TFP shocks (with neither IST, GHH preferences, investment adjusment costs, nor endogenous capital utilization). For this case, we set the parameters of the TFP shocks as in Heathcote and Perri (2002).[4] The first two rows of table 6(a), 6(b) and 6(c) show HP-filtered moments from the data and those from the baseline model. When comparing them, the well-known "puzzles" that characterize the framework are evident. First, the baseline model tend to predict relatively high cross-country consumption correlation, whereas the data indicate that consumption correlation tend to be lower than output correlations. Second, the standard model delivers a standard deviation of the terms of trade and the real exchange rate that is much lower than in the data. Backus et al (1995) refers to these two anomalies as the "quantity puzzle" and "price puzzle" respectively. The "international comovement puzzle" (Baxter, 1995) addresses the cross-country correlations of factor inputs: investment and employment are positively correlated across countries, whereas model tend to imply negative correlations. Finally, the "Backus-Smith" puzzle refers to the fact that while models' strong risk-sharing conditions predict a positive (and close to one) correlation between the real exchange rate and the ratio of consumption between countries[5], the data indicates that such correlation is instead negative.

If TFP shocks are stationary, changes in permanent income following asymmetric shocks are small, implying little need for insurance markets. A single international asset allow households to obtain allocations similar to those when markets are complete. As discussed, in Heathcote and Perri (2002),

[4] In particular, Heathcote and Perri (2002) use the following VAR(1) process:

$$\begin{aligned} A_t &= \rho_A A_{t-1} + \rho_A^* A_{t-1}^* + \varepsilon_t^A \\ A_t^* &= \rho_A A_{t-1}^* + \rho_A^* A_{t-1} + \varepsilon_t^{A,*} \end{aligned}$$

where $\rho_A = 0.97$, $\rho_A^* = 0.025$, $Var(\varepsilon_t^A) = Var(\varepsilon_t^{A,*}) = 0.0073^2$, and $corr(\varepsilon_t^A, \varepsilon_t^{A,*}) = 0.29$.

[5] Optimal risk-sharing condition mimicks the behavior of a benevolent social planner who allocates consumption such that the marginal rate of transformation $\frac{P_t^*}{P_t} = RER_t$ equates the marginal rate of substitution $\frac{U^*_{c^*,t}}{U_{c,t.}}$ (directly linked to relative consumption).

imposing a stationary technology process does not seem to constitute an important feature to judge the quantitative relevance of the model. In principle, if TFP shocks were near unit root with no spill-overs to the other country, we could therefore expect large differences between the behavior of the models with incomplete asset markets as a result of significant changes in relative wealth (and thus relative consumption) following the shock. However, the elasticity of substitution is an important additional determinant of the extent to which productivity shocks affect relative wealth. That is, an increase in aggregate productivity in one country (due to a TFP shock), leads to an increase in the relative world supply of the good such country produces. This will imply an increase in the terms of trade of the other country, since the good its produces becomes relatively scarcer. Standard trade elasticity values used within this IRBC framework imply that movements in the terms of trade almost exactly offset changes in relative productivity.

The inclusion of IST shocks will break this logic, as the terms of trade will not necessarily reflect the relative scarcity of production, but instead the relative demand for capital goods. We first proceed by activating the stochastic process for the IST and the investment adjustment costs (still neither GHH preferences nor endogenous capital utilization). Let us first assume that is process is near-unit root and present no spillovers across countries, i.e. $V(s^t) = \rho_V V(s^t) + \varepsilon^V(s^t)$, $V^*(s^t) = \rho_V V^*(s^t) + \varepsilon^{V^*}(s^t)$, and $\rho_V = 0.999$. Mimicking Raffo (2009), we adjust the investment adjustment costs to match the relative standard deviation of investment and assume that the variance of the IST shock is about three times as big as the one characterizing the TFP shocks, so that IST shocks explains most of the fluctuation of the model's endogenous variables. As seen in the in the third row of tables 6(a), 6(b) and 6(c), this shock appears to be the "silver bullet" needed to successfully address the four puzzles in the literature: consumption across countries is less correlated than output, the volatility of the real exchange rate increases significantly, and both the "international comovement" and "Backus-Smith" puzzle are effectively solved. The intuition for this result is provided in Figure 1 (refer to the thick solid line). As an IST shock hits the home country, the local investment demand significantly increases. Given the aggregate resource constraint, domestic consumption demand instead decreases. Since home investment goods are produced using foreign intermediate goods, the price of intermediate goods produced in the foreign country increases, making the real exchange rate depreciate. Foreign households feel richer because of the improvement in the terms of trade and opposite to their domestic neighbors consume more. This demand effect on the terms of trade (and real exchange rate) is enhanced by a supply effect. That is, over time, the investment boom increases the stock of capital available in the home economy. This more capital-intensive technology results in increasing availability of domestic output which becomes relative abundant and further improves the terms of trade for the foreign economy. Foreign households take advantage of this sizable price effect and increase their labor supply and investment, magnifying over time their joint comovement with their home counterparts.

If IST are instead transitory, as in Raffo (2009), the investment boom is relatively short lived and home output returns to trend over time. As a result, the real exchange rate does not increase as much, weakening its negative correlation with relative consumption across countries. While the volatility of the real exchange rate decreases, the Backus-Smith puzzle is restored.

The problem is cleverly solved in Raffo (2009) by adding endogenous capital utilization as in Greenwood et al (1998). The parameter ε which represents the elasticity of marginal depreciation with respect to the utilization rate, is set equal to 1, consistent with Baxter and Farr (2001), who rely on estimates provided by Basu and Kimball (1997). The results are again in Figure 1 (refer to the dashed line). Endogenous capital utilization serves as an effective endogenous propagation mechanism that facilitates investment and output expansions in response to shocks. As the demand for foreign intermediates increases due to the home investment boom, on impact, the increase of the exchange rate is larger.

Nonetheless, this mechanism generates instead other counterfactuals. Given the resource constraint, a strong investment boom reduces consumption in the home economy so that the correlation between consumption and output turns out to be very small. Raffo (2009) addresses this new problem using a GHH utility specification, which suppress the wealth effect responsible of dampening the response of the labor supply to positive productivity innovations or change in the terms of trade. Absent this wealth effect in the labor supply, agents in both countries can increase the labor supply (and consumption) in response to shocks.

To conclude, in this section we have shown that if the IST process is calibrated to explain most of the observed macroeconomic fluctuations, in combination with endogenous capital utilization and a GHH utility specification, it provides the sufficient degrees of freedom to successfully address the four mentioned puzzles.

Table 6a: Results

	$SD(Y)$	$SD(C)^+$	$SD(X)^+$	$SD(N)^+$	$SD(RER)^+$	$\rho(RER)$
Data	1.25	0.80	3.40	0.91	4.28	0.84
Baseline IRBC	1.12	0.54	2.51	0.31	1.41	0.75
Model with Near UR Inv process	1.08	0.88	3.40	0.63	2.08	0.65
No Near UR Inv process	1.23	0.90	4.41	0.83	1.52	0.64
No Near UR Inv process+Cap Utiliz.	2.00	0.49	4.02	0.70	1.14	0.62
GHH utility specification	1.23	0.70	2.66	0.12	1.88	0.62

$^+$ denotes relative to output.

Table 6b: Results

	$CORR(Y,N)$	$CORR(Y,C)$	$CORR(Y,X)$	$CORR(RER,C/C^*)$
Data	0.79	0.81	0.91	-0.04
Baseline IRBC model	0.97	0.93	0.97	0.99
Model with Near UR Inv process	0.56	0.51	0.76	-0.20
No Near UR Inv process	0.70	0.12	0.81	0.24
No Near UR Inv process+Cap Utiliz.	0.89	0.09	0.93	-0.01
GHH utility specification	0.99	0.77	0.87	-0.13

Table 6c: Results

	$CORR(Y,Y^*)$	$CORR(C,C^*)$	$CORR(X,X^*)$	$CORR(N,N^*)$
Data	0.52	0.42	0.36	0.51
Baseline IRBC model	0.33	0.81	-0.05	-0.05
Model with Near UR Inv process	0.46	0.30	0.20	0.17
No Near UR Inv process	0.34	0.40	0.13	0.22
No Near UR Inv process+Cap Utiliz.	0.36	0.50	0.20	0.30
GHH utility specification	0.61	0.66	0.18	0.70

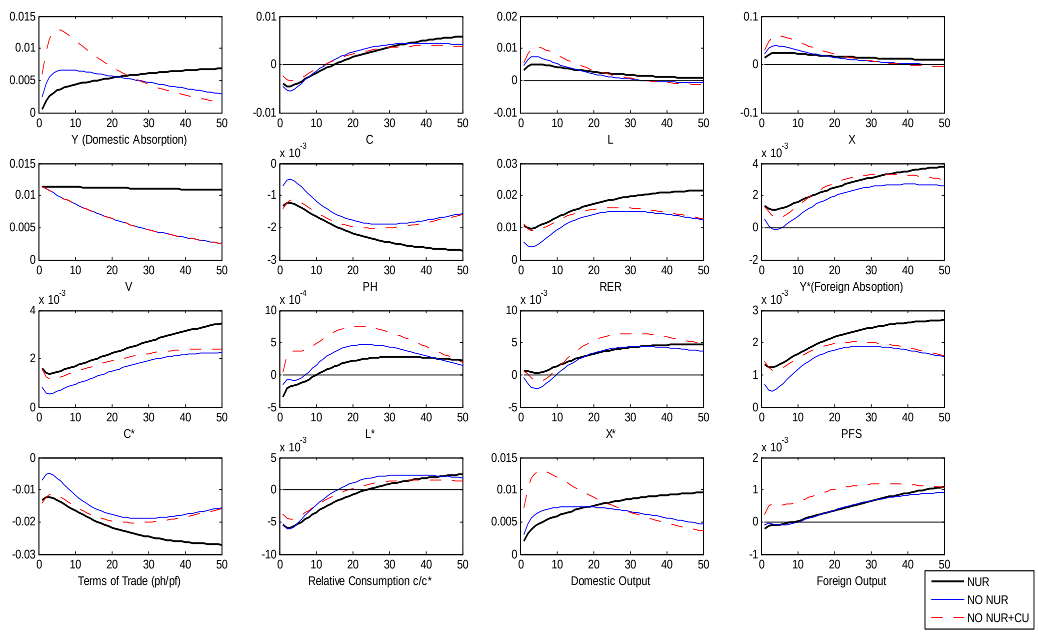

Figure 1-Stationary Case. Impulse response to an IST shock

4.3 IRBC with the estimated IST process

In this section, instead of calibrating the model as Raffo (2009), we are going to simulate the model using the estimates reported in section 3. We will show that when that is the case, the IST shocks can not solve the puzzles. The reason for this failure is that estimated IST shocks are much less volatile than Raffo (2009) assumes.

Since our model with the estimated law of motion is non stationary, we need to rely on simulations to compute the Hodrick-Prescott filtered statistics. We HP-filter the relevant series from the model (output, consumption, investment, employment and the real exchange rate) and compute second moments. To perform the simulation, we solve the model taking a log-linear approximation around the steady state. One might question the use of the HP filter in a model without a stochastic trend. The reason is that we want to replicate patterns studied in the international business cycle literature. The results are in Tables 7a, 7b, and 7c.

The first two row depict the data and the moments obtained from a standard IRBC that only includes TFP shocks as estimated in Rabanal et al (2009). As discussed in that paper the presence of unit root processes in TFP with slow convergence across countries results in very persistent differences in relative productivity that lead to persistent changes in the terms of trade and the RER. As a result, the volatility of this variable increases in the simulations and gets closer in line with the empirical evidence.

In the previous subsection, we showed that an arbitrary near unit root IST process with no spillovers across countries was the "silver bullet" needed to solve the four puzzles. Interestingly, our VECM estimates for the IST process imply similar dynamics: (a) IST processes for the US and the "rest of the world" are well-characterized by unit roots, (b) the estimated very low speed of convergence (κ_V,κ_V^*) of these processes somewhat mimic the scenario with no spillovers in the stationary case. Indeed, the impulse responses (see solid line, figure 2) confirm this intuition, as the macroeconomic dynamics resemble those in Figure 1.

However, the quantitative results at the time of comparing the computed moments with those obtained from the data are disappointing. In the third row, we consider a case with both TFP and

IST and Cobb-Douglas utility[6] In this case, both the "Quantity puzzle" and "Backus-Smith" puzzle remains in place. In addition, both the volatility of the real exchange rate as and the comovement of factor inputs decrease. Results with endogenous capital utilization (fourth row, dashed line in Figure 2 for the dynamics) or GHH utility (fifth row) specification are similarly disappointing.

What is the intuition for these results? In the previous section we arbitrarily fixed the standard deviation of the IST process to be about three times as large as the one characterizing the TFP shock. In the data, the standard deviation of the former is less than half of the magnitude of the variance of the latter. As a counterfactual, we increase the standard deviation of the investment specific shock so that it represents the same relative size with respect to the standard deviation of the neutral process observed in Raffo (2009). The last row of table 7 shows this case. With such manipulation, once again, we succeed in solving all the existent "puzzles."

Table 7a: Results

	$SD(Y)$	$SD(C)^+$	$SD(X)^+$	$SD(N)^+$	$SD(RER)^+$	$\rho(RER)$
Data	1.25	0.80	3.40	0.91	4.28	0.84
Baseline IRBC wih cointegrated TFP	0.70	0.62	2.31	0.28	4.26	0.70
Complete model	0.82	0.72	2.90	0.49	3.68	0.73
Complete model with capital Utiliz.	1.03	0.62	2.82	0.49	2.97	0.70
GHH utility specification	0.79	0.76	2.20	0.16	3.49	0.72
High Variance of the IS Pocess	1.43	0.68	3.85	0.79	3.31	0.72

$^+$ denotes relative to output.

[6] Note that we do not consider investment adjusment costs. The reason is that since our estimated shocks have smaller variance that the ones used by Raffo (2009) we do not need them to match investment volatility.

Table 7b: Results

	$CORR(Y,N)$	$CORR(Y,C)$	$CORR(Y,X)$	$CORR(RER,C/C^*)$
Data	0.79	0.81	0.91	-0.04
Baseline IRBC wih cointegrated TFP	0.92	0.93	0.95	0.97
Complete Model	0.67	0.67	0.85	0.65
Model with no Capital Utilization	0.78	0.63	0.90	0.74
GHH utility specification	-0.35	0.82	0.86	0.55
High Variance of the IS Pocess	0.80	0.06	0.90	-0.25

Table 7c: Results

	$CORR(Y,Y^*)$	$CORR(C,C^*)$	$CORR(X,X^*)$	$CORR(N,N^*)$
Data	0.52	0.42	0.36	0.51
Baseline IRBC wih cointegrated TFP	0.38	0.63	0.05	0.16
Complete Model	0.43	0.67	0.18	0.24
Model with no Capital Utilization	0.41	0.70	0.13	0.24
GHH utility specification	0.45	0.42	0.35	-0.21
High Variance of the IS Pocess	0.63	0.42	0.74	0.82

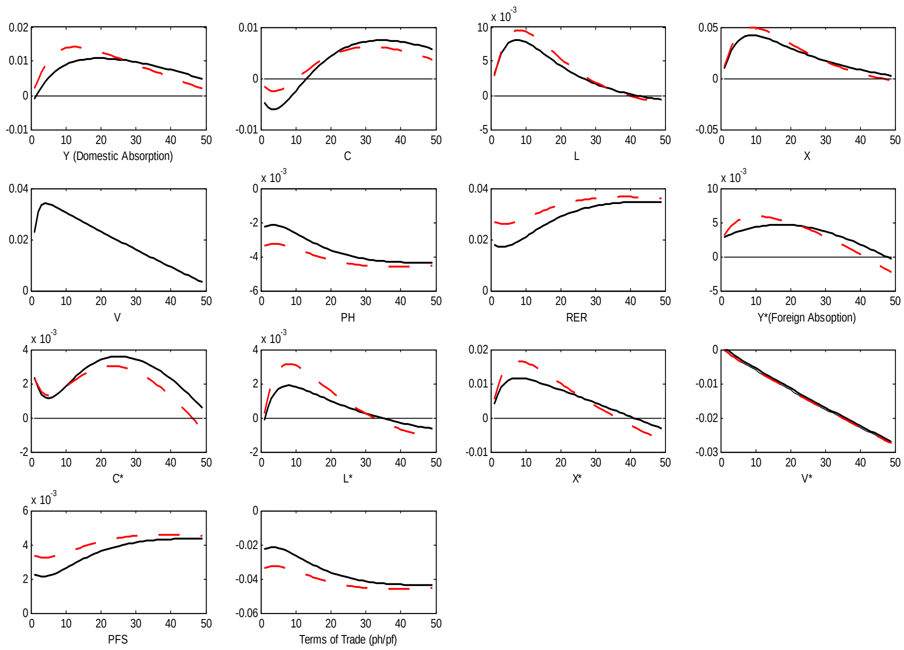

Figure 2-Estimated VECM. Impulse responses to an IST shock

5 Concluding Remarks

Risk sharing induces strong positive cross-country consumption correlations in the standard international real business cycle framework, even when only incomplete markets are considered. It is also the case that the equilibrium real exchange rate is closely related to the ratio of consumptions across the two economies (Backus-Smith Puzzle).

The literature has been trying to fill the gap between theory and data. One alternative to address the discrepancies between the model and the data consists on focusing on the demand side and introduce taste shocks. Raffo (2009) instead considers investment-specific technology shocks, along the guideline specifications in Greenwood et al. (1998) and the empirical work of Fisher (2006). While these productivity innovations resemble demand shocks that directly affect the relative price of capital goods, they also have a clear link to the data. Using an arbitrary calibration, Raffo (2009) shows how these shocks can address the mentioned gap. Instead, in this paper, we use OECD data to parametrize the law of motion of the investment-specific process. The model based on these estimates provide results that do not support the hypothesis in Raffo (2009).

The "Backus-Smith" puzzle could be in part explained by lack of consumption risk sharing across countries. Deeper financial frictions could be an important factor behind this phenomenon. Other mechanisms to address the puzzle could include the inclusion of non-tradable goods, pricing to market, or some form of distribution costs that generate departures from PPP (see Corsetti et al 2008, for instance). All these issues are part of the ongoing research agenda in this the literature.

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A Appendix

A.1 Normalized Equilibrium Conditions

For simplicity in the exposition, redefine $x(s^{t_s})$ as x_{t_s} and $\sum_{s^{t+1}} \pi\left(s^{t+1}/s^t\right)\{\}$ as $E_t\{\}$. Since the presence of two unit roots makes the model non-stationary, we rescale the variables by $\hat{Y}_t = \frac{Y_t}{Z_{t-1}}$, $\hat{C}_t = \frac{C_t}{Z_{t-1}}$, $\hat{X}_t = \frac{X_t}{Z_{t-1}}$, and $\hat{K}_{t-1} = \frac{K_{t-1}}{Z_{t-1}V_{t-1}}$ where $Z_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}$. Similar normalizations will hold for the foreign country.

Market clearing for the intermediate goods:

$$\begin{aligned} \hat{Y}_{H,t} + \hat{Y}^*_{H,t} &= \left(\frac{A_t}{A_{t-1}}\right)\left(\hat{K}^d_{t-1}\right)^{\alpha}(L_t)^{1-\alpha} \\ \hat{Y}_{F,t} + \hat{Y}^*_{F,t} &= \left(\frac{A^*_t}{A^*_{t-1}}\right)\left(\hat{K}^{*,d}_{t-1}\right)^{\alpha}(L^*_t)^{1-\alpha} \end{aligned} \tag{31}$$

Production function of the final good:

$$\hat{Y}_t = \left[\omega^{\frac{1}{\theta}}\hat{Y}_{H,t}^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}}\hat{Y}_{F,t}^{\frac{\theta-1}{\theta}}\left(\frac{Z^*_{t-1}}{Z_{t-1}}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{32}$$

$$\widehat{Y}^*_t = \left[\omega^{\frac{1}{\theta}}\widehat{Y}^{*\,\frac{\theta-1}{\theta}}_{F,t} + (1-\omega)^{\frac{1}{\theta}}\widehat{Y}^{*\,\frac{\theta-1}{\theta}}_{H,t}\left(\frac{Z_{t-1}}{Z^*_{t-1}}\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{33}$$

Market clearing final good:

$$\begin{aligned} \hat{Y}_t &= \hat{C}_t + \hat{X}_t \\ \hat{Y}^*_t &= \hat{C}^*_t + \hat{X}^*_t \end{aligned} \tag{34}$$

Labor supply:

$$\begin{aligned} \frac{1-\gamma}{\gamma}\frac{\hat{C}_t}{(1-L_t)} &= \hat{W}_t \\ \frac{1-\gamma}{\gamma}\frac{\hat{C}_t^*}{(1-L_t^*)} &= \hat{W}_t^* \end{aligned} \tag{35}$$

Marginal utility of consumption:

$$\begin{aligned} \hat{\lambda}_t &= \frac{\gamma}{\hat{C}_t}\left\{\hat{C}_t^{\gamma}\left(1-L_t\right)^{1-\gamma}\right\}^{1-\sigma} \\ \hat{\lambda}_t^* &= \frac{\gamma}{\hat{C}_t^*}\left\{\left(\hat{C}_t^*\right)^{\gamma}\left(1-L_t^*\right)^{1-\gamma}\right\}^{1-\sigma} \end{aligned} \tag{36}$$

where $\hat{\lambda}_t = \lambda_t Z_{t-1}^{1-\gamma(1-\sigma)}$ and $\hat{\lambda}_t^* = \lambda_t^* Z_{t-1}^{*1-\gamma(1-\sigma)}$.

Risk-sharing

$$E_t\left\{\frac{\hat{\lambda}_{t+1}^*}{\hat{\lambda}_t^*}\left[\left(\frac{Z_{t-1}^*}{Z_t^*}\right)^{1-\gamma(1-\sigma)}\right]\frac{\widetilde{P}_{H,t+1}}{\widetilde{P}_{H,t}}\frac{RER_t}{RER_{t+1}}\right\} = E_t\left\{\left[\frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t}\left(\frac{Z_{t-1}}{Z_t}\right)^{1-\gamma(1-\sigma)}\right]\frac{\widetilde{P}_{H,t+1}}{\widetilde{P}_{H,t}}\right\} - \frac{\phi}{\beta}\hat{D}_t \tag{37}$$

where $\hat{D}_t = \left(\frac{D_t - D}{Z_{t-1}}\right)$.

The price of the bond

$$\overline{Q}_t = \beta E_t\left\{\frac{\hat{\lambda}_{t+1}}{\hat{\lambda}_t}\left(\frac{Z_{t-1}}{Z_t}\right)^{1-\gamma(1-\sigma)}\frac{\widetilde{P}_{H,t+1}}{\widetilde{P}_{H,t}}\right\} - \phi\hat{D}_t \tag{38}$$

Euler Equations:

$$\begin{aligned} \hat{\lambda}_t &= \hat{\mu}_t\frac{V_t}{V_{t-1}}\left(1-\phi\left(\frac{\hat{X}_t}{\hat{X}_{t-1}}\frac{Z_{t-1}}{Z_{t-2}}-\Lambda_X\right)\right) + \beta E_t\hat{\mu}_{t+1}\left(\frac{Z_t}{Z_{t-1}}\right)^{1-\gamma(1-\sigma)}\frac{V_{t+1}}{V_t}[(*)] \\ [(*)] &= \left[\phi\left(\frac{\hat{X}_{t+1}}{\hat{X}_t}\frac{Z_t}{Z_{t-1}}-\Lambda_X\right)\frac{\hat{X}_{t+1}}{\hat{X}_t}\frac{Z_t}{Z_{t-1}} - \frac{\phi}{2}\left(\frac{\hat{X}_{t+1}}{\hat{X}_t}\frac{Z_t}{Z_{t-1}}-\Lambda_X\right)^2\right] \end{aligned} \tag{39}$$

$$\begin{aligned}\hat{\lambda}^*_t &= \hat{\mu}^*_t \frac{V^*_t}{V^*_{t-1}} \left(1 - \phi\left(\frac{\hat{X}^*_t}{\hat{X}^*_{t-1}} \frac{Z^*_{t-1}}{Z^*_{t-2}} - \Lambda^*_X\right)\right) + \beta E_t \hat{\mu}^*_{t+1} \left(\frac{Z^*_t}{Z^*_{t-1}}\right)^{1-\gamma(1-\sigma)} \frac{V^*_{t+1}}{V^*_t}[(*)] \\ [(*)] &= \left[\phi\left(\frac{\hat{X}^*_{t+1}}{\hat{X}^*_t}\frac{Z^*_t}{Z^*_{t-1}} - \Lambda^*_X\right)\frac{\hat{X}^*_{t+1}}{\hat{X}^*_t}\frac{Z^*_t}{Z^*_{t-1}} - \frac{\phi}{2}\left(\frac{\hat{X}^*_{t+1}}{\hat{X}^*_t}\frac{Z^*_t}{Z^*_{t-1}} - \Lambda^*_X\right)^2\right] \qquad (40)\end{aligned}$$

where $\hat{\mu}_t = V_{t-1} Z_{t-1}^{1-\gamma(1-\sigma)} \mu_t$ and $\hat{\mu}^*_t = V^*_{t-1}\left(Z^*_{t-1}\right)^{1-\gamma(1-\sigma)}\mu^*_t$.

$$\begin{aligned}\frac{V_t}{V_{t-1}}\left(\frac{Z_t}{Z_{t-1}}\right)^{1-\gamma(1-\sigma)}\hat{\mu}_t &= \beta E_t\left\{u_{t+1}\hat{R}_{t+1}\hat{\lambda}_{t+1} + \hat{\mu}_{t+1}\left(1 - \delta(u_{t+1})\right)\right\} \qquad (41) \\ \frac{V^*_t}{V^*_{t-1}}\left(\frac{Z^*_t}{Z^*_{t-1}}\right)^{1-\gamma(1-\sigma)}\hat{\mu}^*_t &= \beta E_t\left\{u^*_{t+1}\hat{R}^*_{t+1}\hat{\lambda}^*_{t+1} + \hat{\mu}^*_{t+1}\left(1 - \delta^*(u^*_{t+1})\right)\right\}\end{aligned}$$

where $\hat{R}_t = R_t V_{t-1}$ and $\hat{R}^*_t = R^*_t V^*_{t-1}$

Capital Accumulation ($\hat{K}_{t-1} = \frac{K_{t-1}}{Z_{t-1}V_{t-1}}$)

$$\begin{aligned}\hat{K}_t \frac{Z_t}{Z_{t-1}}\frac{V_t}{V_{t-1}} &= \left(1 - \delta(u_t)\right)\hat{K}_{t-1} + \frac{V_t}{V_{t-1}}\left(\hat{X}_t - \frac{\phi}{2}\frac{Z_{t-2}}{Z_{t-1}}\hat{X}_{t-1}\left[\frac{\hat{X}_t}{\hat{X}_{t-1}}\frac{Z_{t-1}}{Z_{t-2}} - \Lambda_X\right]^2\right) \qquad (42) \\ \hat{K}^*_t \frac{Z^*_t}{Z^*_{t-1}}\frac{V^*_t}{V^*_{t-1}} &= (1 - \delta(u^*_t))\hat{K}^*_{t-1} + \frac{V^*_t}{V^*_{t-1}}\left(\hat{X}^*_t - \frac{\phi}{2}\frac{Z^*_{t-2}}{Z^*_{t-1}}X^*_{t-1}\left[\frac{\hat{X}^*_t}{\hat{X}^*_{t-1}}\frac{Z^*_{t-1}}{Z^*_{t-2}} - \Lambda^*_X\right]^2\right)\end{aligned}$$

Real wages and rental rates of capital:

$$\alpha \hat{W}_t L_t = (1-\alpha)\hat{K}_{t-1}\hat{R}_t \tag{43}$$

$$\alpha \hat{W}_t^* L_t^* = (1-\alpha)\hat{K}_{t-1}^*\hat{R}_t^* \tag{44}$$

$$\hat{W}_t = (1-\alpha)\widetilde{P}_{H,t}(\hat{K}_{t-1}^d)^\alpha \left(\frac{A_t}{A_{t-1}}\right) L_t^{-\alpha} \tag{45}$$

$$\hat{W}_t^* = (1-\alpha)\widetilde{P}_{F,t}^* \left(\hat{K}_{t-1}^{d,*}\right)^\alpha \left(\frac{A_t^*}{A_{t-1}^*}\right) (L_t^*)^{-\alpha} \tag{46}$$

$$\hat{\lambda}_t \hat{R}_t = b\hat{\mu}_t u_t^\varepsilon \tag{47}$$

$$\hat{\lambda}_t^* \hat{R}_t^* = b\hat{\mu}_t^* (u_t^*)^\varepsilon \tag{48}$$

$$\delta_t = \hat{\delta} + \frac{b}{1+\varepsilon} u_t^{1+\varepsilon}. \tag{49}$$

$$\delta_t^* = \hat{\delta} + \frac{b}{1+\varepsilon} (u_t^*)^{1+\varepsilon}. \tag{50}$$

Finally, let us give a look to the demand functions:

$$\hat{Y}_{H,t} = \omega \left(\widetilde{P}_{H,t}\right)^{-\theta} \hat{Y}_t \tag{51}$$

$$\hat{Y}_{F,t} = (1-\omega)\left(\widetilde{P}_{F,t}^* RER_t\right)^{-\theta} \hat{Y}_t \frac{Z_{t-1}}{Z_{t-1}^*} \tag{52}$$

Proceeding in a similar way, we get the demands for the foreign country:

$$\widehat{Y}_{H,t}^* \frac{Z_{t-1}}{Z_{t-1}^*} = (1-\omega)\left(\frac{\widetilde{P}_{H,t}}{RER_t}\right)^{-\theta} \widehat{Y}_t^* \tag{53}$$

$$\widehat{Y}_{F,t}^* = \omega \left(\widetilde{P}_{F,t}^*\right)^{-\theta} \widehat{Y}_t^*$$

Auxiliary variable:

$$nx_t = \frac{\widetilde{P}_{H,t}\hat{Y}_{H,t}^* - \widetilde{P}_{F,t}^* RER_t \hat{Y}_{F,t}\left(\frac{Z_{t-1}^*}{Z_{t-1}}\right)}{\hat{Y}_t} \tag{54}$$

The law of motion of the bond is:

$$\widetilde{P}_{H,t}\overline{Q}_t\hat{D}_t = \widetilde{P}_{H,t}\hat{Y}^*_{H,t} - \widetilde{P}^*_{F,t}RER_t\hat{Y}_{F,t}\left(\frac{Z^*_{t-1}}{Z_{t-1}}\right) + \tilde{P}_{H,t}\hat{D}_{t-1}\frac{Z_{t-2}}{Z_{t-1}} - \tilde{P}_{H,t}\frac{\phi}{2}\left(\hat{D}_t\right)^2 \quad (55)$$

Supplementary equation:

$$\hat{K}^d_{t-1} = u_t\hat{K}_{t-1}, \qquad \hat{K}^{d,*}_{t-1} = u^*_t\hat{K}^*_{t-1}$$

A.2 GHH Utility Specification

Here we follow Raffo (JIE, 2008) and impose a GHH specification. The utility function is:

$$U\left[C\left(s^t\right), 1 - L_t\left(s^t\right)\right] = \frac{\left\{C\left(s^t\right) - \psi L_t\left(s^t\right)^{\nu}\right\}^{1-\sigma}}{1-\sigma}$$

The following FOC's are respectively replaced as follows:

$$\psi\nu L(s^t)^{\nu-1} = W(s^t)$$

$$\left[C(s^t) - \psi L(s^t)^{\nu}\right]^{-\sigma} = \lambda(s^t)$$

As in Raffo (2008) we want to (a) have the same steady state for the labor supply (L_{ss}) than in the Cobb-Douglas case (b) replicate the elasticity of the labor supply pf the Cobb Douglas Specification (*i.e.* $\varepsilon_{CD} = \varepsilon_{GHH}$).

$$\begin{aligned}
\varepsilon_{CD} &= \frac{(1-L_{ss})\left[1-\gamma(1-\sigma)\right]}{\sigma L_{ss}} \\
\varepsilon_{GHH} &= \frac{1}{\nu-1} \Longrightarrow \nu = \frac{1+\varepsilon_{CD}}{\varepsilon_{CD}}
\end{aligned}$$

Finally, we fix ψ to obtain the desired L_{ss}, as follows:

$$W = (1-\alpha)(\frac{K}{L})^{\alpha} = \psi \nu L^{\nu-1} \Longrightarrow \psi = \frac{W}{\nu L^{\nu-1}}$$

Non-Stationary case Notice that for the non-stationary case, we have the following:

$$\hat{\psi} \nu L(s^t)^{\nu-1} = \hat{W}(s^t)$$

$$\left[\hat{C}(s^t) - \hat{\psi} L(s^t)^{\nu}\right]^{-\sigma} = \hat{\lambda}(s^t)$$

Where $\hat{\psi} = \frac{\psi}{z(s^{t-1})}$ and $\hat{\lambda}(s^t) = \lambda(s^t) Z(s^{t-1})^{\sigma}$.