Chapter 1
Investment and Trade Patterns in a Sticky-Price, Open-Economy Model

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Abstract This paper explores a two-country DSGE model with sticky prices à la Calvo (1983) and local-currency pricing. We analyze the investment decision in the presence of adjustment costs of two types, i.e. capital adjustment costs (CAC) and investment adjustment costs (IAC). We compare the investment and trade patterns with adjustment costs against those of a model without adjustment costs and with (quasi-) flexible prices. We show that having adjustment costs results into more volatile consumption and net exports series, and less volatile investment. We document three important facts on U.S. trade dynamics: (a) the S-shaped cross-correlation between real GDP and the real net exports share, (b) the J-curve between terms of trade and net exports, and (c) the weak and S-shaped cross-correlation between real GDP and terms of trade. We find that adding adjustment costs tends to reduce the model's ability to match these stylized facts. Nominal rigidities cannot account for these features either.

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1.1 Introduction

Adjustment costs on capital accumulation often feature in modern international macro models of the business cycle. The Q theory of investment with adjustment costs (developed among others by Lucas and Prescott, 1971, and Abel, 1983) formalizes the idea that investment becomes more attractive whenever the value of a unit of additional capital is higher relative to its acquisition cost. However, while there is broad agreement on the importance of investment for trade, there is less clarity on the role that adjustment costs play in these models.

In the standard international real business cycle model (IRBC) of Backus, Kehoe and Kydland (BKK) (1995, p. 340), the connection between investment and trade is rather straightforward: "resources are shifted to the more productive location (...). This tendency to 'make hay where the sun shines' means that with uncorrelated productivity shocks, consumption will be positively correlated across countries, while investment, employment, and output will be negatively correlated. With productivity shocks that are positively correlated, (...), all of these correlations rise, but with the benchmark parameter values none change sign."

Heathcote and Perri (2002) elaborate further on this point, explaining that a domestic productivity shock causes domestic investment to increase by much more than the increase in foreign consumption, so the domestic country draws more resources from abroad and the domestic trade deficit widens at the same time as domestic output is raising. Hence, as in the data, the IRBC model implies that the trade balance is countercyclical. Engel and Wang (2007) use a richer model with adjustment costs and durable goods, and find that their IRBC framework can also deliver a countercyclical trade balance.

Raffo (2008, p. 21), however, notes that the IRBC model accounts for this empirical pattern "due to the strong terms of trade effect generated by the change in relative scarcity of goods across countries." This prediction on terms of trade is counterfactual for most countries. Furthermore, consumption volatility in BKK (1992, 1995) and Heathcote and Perri (2002) tends to be noticeably lower than in the data. As our work shows, models that do match the real U.S. GDP volatility generate too much investment volatility, while attaining an excessively smooth consumption series.

The role of the Q theory extension in open economy models requires further consideration. While capital accumulation provides a powerful mechanism to smooth consumption intertemporally that diminishes the benefits of trade, capital adjustment costs are likely to induce smoother investment patterns and a more volatile consumption series. Therefore, costly adjustments on capital could enhance the appeal of trade. The Q extension arises from a long tradition on investment theory, but it definitely has implications for the model's ability to generate incentives to trade as well as empirically-consistent consumption and investment paths.

Another strand of the international macro literature has emphasized the role of deviations of the law of one price (LOOP) that lead to a misallocation of expenditures across countries and, in turn, to sizable effects on trade. The international new neoclassical synthesis (INNS) model is built around the assumptions of monopolistic competition among firms, price stickiness à la Calvo (1983) and local-currency

pricing (LCP) to force a breakdown of the LOOP. An influential paper in this strand of the literature is Chari, Kehoe and McGrattan (CKM) (2002), which also incorporates a form of adjustment costs. Their paper, however, focuses on the behavior of the real exchange rate rather than on trade dynamics.

We believe that the CKM (2002) paper, by its own right a Q theory extension of the INNS model, raises the issue of how adjustment costs interact with deviations of the LOOP to affect the trade patterns implied by the model. The cost function that CKM (2002) use is not necessarily the only one being proposed either. Christiano, Eichenbaum and Evans (CEE) (2005) have popularized an alternative adjustment cost specification, recently advocated by Justiniano and Primiceri (2008) among others, linked to investment growth rates instead of the investment-to-capital ratio.[1]

To our knowledge, the trade predictions of the Q-INNS model with complete international asset markets have not been consistently evaluated against: a) different specifications of the adjustment cost function (including the case without adjustment costs), and b) an approximation of the flexible price environment conventionally assumed in the Q-IRBC literature. In this paper, we develop a two country DSGE model with the distinctive features of the Q-INNS model precisely to help us understand the role of adjustment costs and nominal rigidities on trade. We also examine whether there is any interaction between deviations of the LOOP and adjustment costs that can affect the dynamics of net exports. In other words, this paper aims to provide a broader assessment of whether the Q theory extension of the INNS model can simultaneously be reconciled with the empirical evidence on investment and trade.

We focus our analysis on several important features of the international business cycle data summarized in Table 1.1. First, investment is around three times more volatile than real GDP, while consumption and the net exports share are significantly less volatile. All series tend to be quite persistent. Second, the trade balance is countercyclical. This feature is quite robust across countries, as corroborated by the empirical evidence provided by Engel and Wang (2007). Among 25 OECD countries, they find that the mean correlation between real GDP and the real net exports share is -0.24 and the median is -0.25.

Third, as noted by Ghironi and Melitz (2007) and Engel and Wang (2007), the cross-correlation between real GDP and the real net exports share is S-shaped. Fourth, there is evidence of the J-curve in the cross-correlation between ToT and net exports; a relationship extensively discussed in BKK (1994). Finally, the data shows a weak cross-correlation between real GDP and ToT. This feature is quite robust across countries, as confirmed by the empirical evidence provided by Raffo (2008). For 14 OECD countries plus the EU-15, he finds that the mean correlation between real GDP and ToT is 0.08 and the median is 0.11. We also document that the cross-correlation between real GDP and ToT is S-shaped.

[1] CEE (2005) and Justiniano and Primiceri (2008) are closed economy models. For an application in an open economy model, see e.g. Martínez-García and Søndergaard (2008b).

Table 1.1 Stylized Facts in the U.S. Data

			Cross-correlation of real GDP with						
Variable	Std. Dev.	Autocorr.	x_{t-4}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+4}
GDP	1.54	0.87	0.31	0.70	0.87	1.00	0.87	0.70	0.31
Investment	5.21	0.91	0.29	0.66	0.84	0.94	0.88	0.75	0.37
Consumption	1.24	0.87	0.51	0.79	0.87	0.85	0.69	0.51	0.16
Net Exports	0.38	0.83	−0.46	−0.51	−0.52	−0.48	−0.38	−0.22	0.11
ToT	1.72	0.69	−0.14	−0.05	−0.01	0.07	0.16	0.18	0.20
			Cross-correlation of ToT with						
Net Exports			−0.15	−0.18	−0.14	−0.03	0.14	0.25	0.35

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the dataset in the Appendix. Sample period: 1973q1-2006q4 (except for ToT, which covers only 1983q3-2006q4).

1.2 Equilibrium Conditions

Our baseline is a two-country stochastic general equilibrium model with monopolistic competition, sticky prices and LCP. We posit the existence of a deterministic, zero-inflation steady state (with zero net exports). We log-linearize the equilibrium conditions around this zero-inflation steady state and report them here. We refer the interested reader to Martínez-García and Søndergaard (2008a, 2008b) for a description of the model from its first principles, and for details on the derivation of the steady state and the log-linearization. As a notational convention, any variable identified with lower-case letters and a caret on top will represent a transformation (expressed in log deviations relative to its steady state) of the corresponding variable.

Consumption and Investment Decisions

Aggregate consumption in both countries evolves according to a pair of standard Euler equations,

$$\widehat{c}_t \approx \mathbb{E}_t\left[\widehat{c}_{t+1}\right] - \sigma\left(\widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]\right), \tag{1.1}$$

$$\widehat{c}_t^* \approx \mathbb{E}_t\left[\widehat{c}_{t+1}^*\right] - \sigma\left(\widehat{i}_t^* - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right]\right), \tag{1.2}$$

where $\sigma > 0$ $(\sigma \neq 1)$ is the elasticity of intertemporal substitution, $\widehat{c}_t$ and $\widehat{c}_t^*$ denote consumption, $\widehat{i}_t$ and $\widehat{i}_t^*$ are the nominal short-term interest rates (which are also the instruments of monetary policy), $\widehat{p}_t$ and $\widehat{p}_t^*$ are the consumption-price indexes (CPIs), and $\widehat{\pi}_{t+1} \equiv \widehat{p}_{t+1} - \widehat{p}_t$ and $\widehat{\pi}_{t+1}^* \equiv \widehat{p}_{t+1}^* - \widehat{p}_t^*$ stand for CPI inflation in both

countries.[2] Under complete international asset markets, the *perfect international risk-sharing condition* implies that,

$$\widehat{c}_t - \widehat{c}_t^* \approx \sigma \widehat{rs}_t, \tag{1.3}$$

where the real exchange rate is defined as $\widehat{rs}_t \equiv \widehat{s}_t + \widehat{p}_t^* - \widehat{p}_t$. Consequently, domestic consumption becomes relatively high whenever it is relatively 'cheap' (that is, whenever there is a real depreciation).

Capital accumulation evolves according to the following laws of motion,

$$\widehat{k}_{t+1} \approx (1-\delta)\widehat{k}_t + \delta \widehat{x}_t, \tag{1.4}$$

$$\widehat{k}_{t+1}^* \approx (1-\delta)\widehat{k}_t^* + \delta \widehat{x}_t^*, \tag{1.5}$$

where the parameter $0 < \delta < 1$ denotes the depreciation rate of capital. Investment decisions depend on the technological rate at which aggregate investment goods in either country, $\widehat{x}_t$ and $\widehat{x}_t^*$, can be transformed into new capital, $\widehat{k}_{t+1}$ and $\widehat{k}_{t+1}^*$. The technological constraints on new capital can be summarized with an adjustment cost function, which we normalize to be equal to one in levels and zero in its first derivative whenever evaluated at the steady state.[3]

In a model without adjustment costs (NAC), the rate of transformation of investment into new capital is one-to-one. Hence, the real shadow value of an additional unit of capital (or marginal Q) is equal to one, implying that,

$$\widehat{q}_t = \widehat{q}_t^* \approx 0. \tag{1.6}$$

Naturally, $\widehat{q}_t$ and $\widehat{q}_t^*$ denote the marginal Q in each country in log deviations. Then, the investment decision can be conventionally summarized as,

$$(1-(1-\delta)\beta)\,\mathbb{E}_t\left(\widehat{r}_{t+1}^k\right) \approx \widehat{i}_t - \mathbb{E}_t\left(\widehat{\pi}_{t+1}\right), \tag{1.7}$$

$$(1-(1-\delta)\beta)\,\mathbb{E}_t\left(\widehat{r}_{t+1}^{k*}\right) \approx \widehat{i}_t^* - \mathbb{E}_t\left(\widehat{\pi}_{t+1}^*\right), \tag{1.8}$$

where $\widehat{r}_{t+1}^k$ and $\widehat{r}_{t+1}^{k*}$ denote the real rental rates on capital in both countries. The parameter $0 < \beta < 1$ is the subjective intertemporal discount factor. The real Fisherian interest rates on the right-hand side of $(1.7)-(1.8)$ give us the opportunity cost of investing in capital. The left-hand side, in turn, reflects the real rental rate on capital adjusted to account for capital depreciation over time. In other words, households keep investing in capital until a point where the marginal return of investing in an additional unit of capital equals its marginal cost.

[2] As a matter of notation, the superscript '$*$' distinguishes the foreign country from the domestic country.

[3] Even though the adjustment cost function affects the rate of transformation of investment goods into new capital, this normalization implies that log-linear equations (1.4) and (1.5) are invariant to any such adjustment cost specification.

The Q theory extension of the model means that (1.6) does no longer hold true, and this forces us to revisit our notion of the marginal returns to investment. In this regard, we consider the capital adjustment cost (CAC) function favored by CKM (2002) and the investment adjustment cost (IAC) function preferred by CEE (2005) to make the marginal Q no longer equal to one. Under the CAC specification, we obtain that the marginal Q is,

$$\widehat{q}_t \approx \chi\delta\left(\widehat{x}_t - \widehat{k}_t\right), \tag{1.9}$$

$$\widehat{q}_t^* \approx \chi\delta\left(\widehat{x}_t^* - \widehat{k}_t^*\right), \tag{1.10}$$

which is a function of the contemporaneous investment-to-capital ratio, i.e. $\widehat{x}_t - \widehat{k}_t$ and $\widehat{x}_t^* - \widehat{k}_t^*$. The parameter $\chi \geq 0$ regulates the degree of concavity of the CAC function around the steady state, since $-\frac{\chi}{\delta}$ is the second-order derivative of the function whenever evaluated at the steady state.

Under the IAC specification, the marginal Q is related to investment growth,

$$\widehat{q}_t \approx \kappa\left[(\widehat{x}_t - \widehat{x}_{t-1}) - \beta\mathbb{E}_t\left(\widehat{x}_{t+1} - \widehat{x}_t\right)\right], \tag{1.11}$$

$$\widehat{q}_t^* \approx \kappa\left[\left(\widehat{x}_t^* - \widehat{x}_{t-1}^*\right) - \beta\mathbb{E}_t\left(\widehat{x}_{t+1}^* - \widehat{x}_t^*\right)\right]. \tag{1.12}$$

The parameter $\kappa \geq 0$ regulates the degree of concavity of the IAC function around the steady state, since $-\kappa$ is the second-order derivative of the function whenever evaluated at the steady state. Using the law of motion for capital in (1.4) and (1.5) we re-write equations (1.11) and (1.12) in terms of the investment-to-capital ratio as,

$$\begin{aligned}\widehat{q}_t \approx & -\kappa(1-\delta)\left(\widehat{x}_{t-1} - \widehat{k}_{t-1}\right) + \kappa(1+(1-\delta)\beta)\left(\widehat{x}_t - \widehat{k}_t\right) - \\ & - \kappa\beta\mathbb{E}_t\left(\widehat{x}_{t+1} - \widehat{k}_{t+1}\right),\end{aligned} \tag{1.13}$$

$$\begin{aligned}\widehat{q}_t^* \approx & -\kappa(1-\delta)\left(\widehat{x}_{t-1}^* - \widehat{k}_{t-1}^*\right) + \kappa(1+(1-\delta)\beta)\left(\widehat{x}_t^* - \widehat{k}_t^*\right) - \\ & - \kappa\beta\mathbb{E}_t\left(\widehat{x}_{t+1}^* - \widehat{k}_{t+1}^*\right).\end{aligned} \tag{1.14}$$

Under both adjustment cost functions, we can write the marginal Q as a function of the investment-to-capital ratio. The difference between the two specifications, as can be seen here, is that the CAC case links the marginal Q only to the contemporaneous investment-to-capital ratio while the IAC case introduces a more complex relationship that also depends on the past and the expectations for the future of the investment-to-capital ratio.[4]

We cannot ignore the time-variation of these marginal Q's when computing the marginal returns to investment in capital. The opportunity cost for investment is still given by the Fisherian real interest rate. However, the investment decision under the

[4] In the extreme case where there are no adjustment costs of either type, i.e., either $\chi = 0$ or $\kappa = 0$, then $\widehat{q}_t = \widehat{q}_t^* = 0$ for all t. Then, we are back to the NAC case described in (1.6).

CAC specification implies that,

$$(1-(1-\delta)\beta)\,\mathbb{E}_t\left(\widehat{r}^k_{t+1}\right)-\widehat{q}_t+\beta\mathbb{E}_t\left[\widehat{q}_{t+1}\right]\approx\widehat{i}_t-\mathbb{E}_t\left(\widehat{\pi}_{t+1}\right), \tag{1.15}$$

$$(1-(1-\delta)\beta)\,\mathbb{E}_t\left(\widehat{r}^{k*}_{t+1}\right)-\widehat{q}^*_t+\beta\mathbb{E}_t\left[\widehat{q}^*_{t+1}\right]\approx\widehat{i}^*_t-\mathbb{E}_t\left(\widehat{\pi}^*_{t+1}\right), \tag{1.16}$$

while investment under the IAC specification implies that,

$$(1-(1-\delta)\beta)\,\mathbb{E}_t\left(\widehat{r}^k_{t+1}\right)-\widehat{q}_t+(1-\delta)\beta\mathbb{E}_t\left[\widehat{q}_{t+1}\right]\approx\widehat{i}_t-\mathbb{E}_t\left[\widehat{\pi}_{t+1}\right], \tag{1.17}$$

$$(1-(1-\delta)\beta)\,\mathbb{E}_t\left(\widehat{r}^{k*}_{t+1}\right)-\widehat{q}^*_t+(1-\delta)\beta\mathbb{E}_t\left[\widehat{q}^*_{t+1}\right]\approx\widehat{i}^*_t-\mathbb{E}_t\left[\widehat{\pi}^*_{t+1}\right]. \tag{1.18}$$

Equations $(1.15)-(1.16)$ and $(1.17)-(1.18)$ point out that the marginal benefits of investing in an additional unit of capital should include the properly discounted capital gains between the shadow cost of acquiring capital today, $\widehat{q}_t$ or $\widehat{q}^*_t$, and the shadow value of capital tomorrow, $\widehat{q}_{t+1}$ or $\widehat{q}^*_{t+1}$ (factoring the rate of time preference and the depreciation of capital).

Efficient Factor Use and Market-Clearing Conditions

The factors of production (capital and labor) are homogeneous within a country and factor markets are perfectly competitive, so factor prices equalize within a country.[5] Since the production function is assumed to be homogeneous of degree one (constant returns-to-scale), then all local firms choose the same capital-to-labor ratio. This yields an efficiency condition linking the aggregate capital-to-labor ratios to factor price ratios as,

$$\widehat{k}_t-\widehat{l}_t\approx\widehat{w}_t-\widehat{r}^k_t, \tag{1.19}$$

$$\widehat{k}^*_t-\widehat{l}^*_t\approx\widehat{w}^*_t-\widehat{r}^{k*}_t, \tag{1.20}$$

where $\widehat{w}_t$ and $\widehat{w}^*_t$ denote the real wages, while $\widehat{l}_t$ and $\widehat{l}^*_t$ stand for labor employment in both countries. The evolution of relative factor prices signals the incentives to become more or less capital-intensive in each period. Equations (1.19) and (1.20) establish a link between the real rental rates on capital and the real wages. The market clearing conditions in the labor markets can be fully characterized with the labor supply equations (the intratemporal first-order conditions) from the households' problem,

[5] Production specialization in each country and the immobility of factors accounts for factor price differences across countries even though there are no restrictions to trade. However, it should be noted that while capital is immobile at the aggregate level, the varieties on which it is build are all tradable.

$$\widehat{w}_t \approx \frac{1}{\sigma}\widehat{c}_t + \varphi\widehat{l}_t, \tag{1.21}$$

$$\widehat{w}_t^* \approx \frac{1}{\sigma}\widehat{c}_t^* + \varphi\widehat{l}_t^*. \tag{1.22}$$

The parameter $\varphi > 0$ denotes the inverse of the Frisch elasticity of labor supply. Implicitly, we assume that consumption and labor are additively separable in preferences.

From the supply-side, we can express aggregate output in each country as a function of aggregate labor and aggregate capital,

$$\widehat{y}_t \approx \widehat{a}_t + (1-\psi)\widehat{k}_t + \psi\widehat{l}_t, \tag{1.23}$$

$$\widehat{y}_t^* \approx \widehat{a}_t^* + (1-\psi)\widehat{k}_t^* + \psi\widehat{l}_t^*. \tag{1.24}$$

where the labor share in the production function is captured by the parameter $0 < \psi \leq 1$. The productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$, follow a symmetric $AR(1)$ process of the form,

$$\widehat{a}_t = \rho_a\widehat{a}_{t-1} + \varepsilon_t^a, \tag{1.25}$$

$$\widehat{a}_t^* = \rho_a\widehat{a}_{t-1}^* + \varepsilon_t^{a*}, \tag{1.26}$$

where ε_t^a and ε_t^{a*} are zero mean, possibly correlated, and normally-distributed innovations with a common standard deviation (i.e. $\sigma(\varepsilon_t^a) = \sigma(\varepsilon_t^{a*})$). The persistence of the process is regulated by the parameter $-1 < \rho_a < 1$. From the demand-side, we can derive the following complementary expressions for aggregate output,

$$\widehat{y}_t \approx \eta\widehat{t}_t^W + (1-\gamma_x)\widehat{c}_t^W + \gamma_x\widehat{x}_t^W, \tag{1.27}$$

$$\widehat{y}_t^* \approx -\eta\widehat{t}_t^W + (1-\gamma_x)\widehat{c}_t^{W*} + \gamma_x\widehat{x}_t^{W*}, \tag{1.28}$$

which depend on weighted averages for world consumption, $\widehat{c}_t^W \equiv \phi_H\widehat{c}_t + \phi_F\widehat{c}_t^*$ and $\widehat{c}_t^{W*} \equiv \phi_F\widehat{c}_t + \phi_H\widehat{c}_t^*$, and for world investment, $\widehat{x}_t^W \equiv \phi_H\widehat{x}_t + \phi_F\widehat{x}_t^*$ and $\widehat{x}_t^{W*} \equiv \phi_F\widehat{x}_t + \phi_H\widehat{x}_t^*$. We denote world terms of trade as $\widehat{t}_t^W$, implying that an increase in $\widehat{t}_t^W$ shifts consumption and investment spending away from the foreign goods and into the domestic goods. We discuss the role of $\widehat{t}_t^W$ more extensively in the next section. Equations (1.27) and (1.28) coupled with equations (1.23) – (1.24) give us an aggregate clearing condition for the goods markets.

We define the steady state investment as $\gamma_x \equiv \frac{(1-\psi)\delta}{\left(\frac{\theta}{\theta-1}\right)\left(\beta^{-1}-(1-\delta)\right)}$ and the consumption shares as $\gamma_c \equiv 1-\gamma_x$. The parameter $\eta > 0$ is the elasticity of intratemporal substitution between the home and foreign bundles of varieties, while $\theta > 1$ defines the elasticity of substitution across varieties produced within the same country.[6] The share of the home goods in the domestic aggregator for consumption and

[6] The mark-up charged by any monopolistically competitive firm, $\frac{\theta}{\theta-1}$, is a function of the elasticity of substitution across varieties.

investment is ϕ_H, while the share of foreign goods is $\phi_F = 1 - \phi_H$. We define the shares in the foreign aggregator symmetrically (see, e.g., Warnock, 2003).

Inflation Dynamics

Firms supply the home and foreign markets and set their prices under LCP. Furthermore, firms enjoy monopolistic power in their own variety. Frictions in the goods market are modelled with nominal price stickiness à la Calvo (1983). In this environment, the inflation dynamics can be partly summarized with the following pair of Phillip curves,

$$\begin{aligned}\widehat{\pi}_t \approx{}& \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}\right) + \\ &+ \Phi \begin{bmatrix} \left(\sigma^{-1} + (1-\gamma_x)\varphi\omega\right)\left[\phi_H \widehat{c}_t^W + \phi_F \widehat{c}_t^{W*}\right] \\ +\gamma_x \varphi\omega \left[\phi_H \widehat{x}_t^W + \phi_F \widehat{x}_t^{W*}\right] - \left(\frac{(1-\psi)(1+\varphi)}{\psi}\right)\widehat{k}_t^W + \\ +2\phi_H\phi_F \widehat{rs}_t + (\phi_H - \phi_F)\eta\varphi\omega \widehat{t}_t^W - \\ -\left(\frac{1+\varphi}{\psi}\right)\left[\phi_H \widehat{a}_t + \phi_F \widehat{a}_t^*\right] \end{bmatrix},\end{aligned} \tag{1.29}$$

$$\begin{aligned}\widehat{\pi}_t^* \approx{}& \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^*\right) + \\ &+ \Phi \begin{bmatrix} \left(\sigma^{-1} + (1-\gamma_x)\varphi\omega\right)\left[\phi_F \widehat{c}_t^W + \phi_H \widehat{c}_t^{W*}\right] + \\ +\gamma_x \varphi\omega \left[\phi_F \widehat{x}_t^W + \phi_H \widehat{x}_t^{W*}\right] - \left(\frac{(1-\psi)(1+\varphi)}{\psi}\right)\widehat{k}_t^{W*} + \\ -2\phi_F\phi_H \widehat{rs}_t - (\phi_H - \phi_F)\eta\varphi\omega \widehat{t}_t^W - \\ -\left(\frac{1+\varphi}{\psi}\right)\left[\phi_F \widehat{a}_t + \phi_H \widehat{a}_t^*\right] \end{bmatrix},\end{aligned} \tag{1.30}$$

where $\omega \equiv \left(\frac{\varphi\psi^2 + (1-\psi)(1+\varphi)^2}{\varphi\psi + (1-\psi)\psi\varphi^2}\right)$ and $\Phi \equiv \left(\frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\right)$ are two composite parameters, while the weighted averages for world capital are defined as $\widehat{k}_t^W \equiv \phi_H \widehat{k}_t + \phi_F \widehat{k}_t^*$ and $\widehat{k}_t^{W*} \equiv \phi_F \widehat{k}_t + \phi_H \widehat{k}_t^*$. The Calvo parameter $0 < \alpha < 1$ denotes the probability with which a firm is forced to maintain its previous period prices under the Calvo randomization assumption. Under home bias (i.e., if $\phi_H > \phi_F$), an additional equation is required to describe the dynamics of relative CPI inflation, $\widehat{\pi}_t^R \equiv \widehat{\pi}_t - \widehat{\pi}_t^*$,

$$\widehat{\pi}_t^R \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^R\right) + \left[\Phi \widehat{rs}_t + \left(\frac{\phi_H - \phi_F}{\phi_H \phi_F}\right)\left(\beta \mathbb{E}_t \left(\widehat{t}_{t+1}^W\right) - \left(\frac{1+\beta\alpha^2}{\alpha}\right)\widehat{t}_t^W + \widehat{t}_{t-1}^W\right)\right]. \tag{1.31}$$

where the relative CPI inflation is defined as $\widehat{\pi}_t^R \equiv \widehat{\pi}_t - \widehat{\pi}_t^*$. Equations (1.29) and (1.30) show that relative price adjustments through world terms of trade, $\widehat{t}_t^W$, and real exchange rates, $\widehat{rs}_t$, have a direct impact on inflation. Interestingly, equation (1.31) reveals that differences in CPI inflation across countries are explained by relative price effects only.

Monetary Policy Rules

We assume a cashless limit economy as in Woodford (2003). Monetary policy has an impact on inflation by regulating short-term nominal interest rates, and it has real effects because it interacts with the nominal rigidities. Since the Taylor (1993) rule has become the trademark of modern monetary policy, we assume that the monetary authorities set short-term nominal interest rates accordingly, i.e.

$$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1-\rho_i)\left[\psi_\pi \widehat{\pi}_t + \psi_y \widehat{y}_t\right], \tag{1.32}$$

$$\widehat{i}^*_t = \rho_i \widehat{i}^*_{t-1} + (1-\rho_i)\left[\psi_\pi \widehat{\pi}^*_t + \psi_y \widehat{y}^*_t\right]. \tag{1.33}$$

These symmetric policy rules target deviations of output and inflation from their long-run trends. The weights assigned to deviations of output and inflation are $\psi_y > 0$ and $\psi_\pi > 0$, respectively. In keeping with much of the literature, we augment the rule proposed by Taylor (1993) with an interest rate smoothing term regulated by the inertia parameter $0 < \rho_i < 1$, but we do not add discretionary monetary shocks.[7]

1.3 Investment, Trade and ToT

International Relative Prices

Domestic terms of trade, ToT_t, represents the value of the imported good (quoted in the domestic market) relative to the value of the domestic good exported to the foreign market, but expressed in units of the domestic currency. Similarly for the foreign terms of trade, ToT^*_t. This conventional definition of ToT measures the 'foreign market' cost of replacing one unit of imports with one unit of exports of the locally-produced good, and can be formally expressed as,

$$ToT_t \equiv \frac{P^F_t}{S_t P^{H*}_t} = D_t \frac{P^F_t}{P^H_t}, \tag{1.34}$$

$$ToT^*_t \equiv \frac{S_t P^{H*}_t}{P^F_t} = D^*_t \frac{P^{H*}_t}{P^{F*}_t} = \frac{1}{ToT_t}, \tag{1.35}$$

where D_t and D^*_t capture deviations of the LOOP across countries, i.e.

$$D_t \equiv \frac{P^H_t}{S_t P^{H*}_t},\ D^*_t \equiv \frac{S_t P^{F*}_t}{P^F_t}.$$

We also define a pair of international relative prices, T_t and T^*_t, as,

[7] The original Taylor (1993) rule can be seen as a special case of equations (1.32) and (1.33) where $\rho_i = 0$, $\psi_y = 0.5$ and $\psi_\pi = 1.5$.

$$T_t \equiv \frac{P_t^F}{P_t^H}, \tag{1.36}$$

$$T_t^* \equiv \frac{P_t^{H*}}{P_t^{F*}} = \frac{1}{D_t D_t^* T_t}, \tag{1.37}$$

The relative price T_t represents the value of the imported good (quoted in the domestic market) relative to the value of the domestic good sold in the domestic market. Similarly for the foreign relative price, T_t^*. The ratios T_t and T_t^* are the 'local market' cost of replacing one unit of imports with one unit of the locally-produced good (not exported). The joint assumption of nominal rigidities and LCP implies that the LOOP fails, i.e. $D_t \neq 1$ and $D_t^* \neq 1$. Therefore, the distinction between ToT and other international relative prices becomes relevant for our understanding of the patterns of trade in a Q-INNS model.

After log-linearizing the definitions in $(1.34)-(1.35)$ and $(1.36)-(1.37)$, we get that,

$$\begin{aligned} \widehat{tot}_t &= \widehat{d}_t + \widehat{t}_t, \\ \widehat{tot}_t^* &= -\widehat{tot}_t = \widehat{d}_t^* + \widehat{t}_t^*, \end{aligned}$$

and,

$$\begin{aligned} \widehat{t}_t &= \widehat{p}_t^F - \widehat{p}_t^H, \\ \widehat{t}_t^* &= -\left(\widehat{p}_t^{F*} - \widehat{p}_t^{H*}\right) = \widehat{p}_t^{H*} - \widehat{p}_t^{F*}, \end{aligned}$$

where $\widehat{d}_t \equiv \left(\widehat{p}_t^H - \widehat{s}_t - \widehat{p}_t^{H*}\right)$ and $\widehat{d}_t^* \equiv \left(\widehat{s}_t + \widehat{p}_t^{F*} - \widehat{p}_t^F\right)$ are the deviations of the LOOP. With this log-linear equalities, we define the world terms of trade as $\widehat{t}_t^W \equiv \widehat{p}_t^{F,W*} - \widehat{p}_t^{W*}$, where $\widehat{p}_t^{F,W*} \equiv \phi_F \widehat{p}_t^F + \phi_H \widehat{p}_t^{F*}$ and $\widehat{p}_t^{W*} \equiv \phi_F \widehat{p}_t + \phi_H \widehat{p}_t^*$. After some algebra, we find that $\widehat{t}_t^W$ is proportional to the difference between the two international relative prices, $\widehat{t}_t$ and $\widehat{t}_t^*$, i.e.

$$\widehat{t}_t^W \approx (1-\phi_F)\,\phi_F \left[\widehat{t}_t - \widehat{t}_t^*\right]. \tag{1.38}$$

We assume that CES aggregators are used to bundle up consumption and investment. Under standard results on functional separability, the corresponding CPIs can be approximated as,

$$\widehat{p}_t \approx \phi_H \widehat{p}_t^H + \phi_F \widehat{p}_t^F, \tag{1.39}$$

$$\widehat{p}_t^* \approx \phi_F \widehat{p}_t^{H*} + \phi_H \widehat{p}_t^{F*}. \tag{1.40}$$

The transformation of world terms of trade in (1.38) is based on this log-linearization of the CPIs.

Using the definition of $\widehat{t}_t$ and $\widehat{t}_t^*$ we can alternatively re-write $\widehat{t}_t^W$ as,

$$\widehat{t}_t^W \approx 2\,(1-\phi_F)\,\phi_F \widehat{tot}_t - \widehat{d}_t^W, \tag{1.41}$$

where $\widehat{d}_t^W \equiv (1-\phi_F)\,\phi_F\left[\widehat{d}_t - \widehat{d}_t^*\right]$ is our measure of world deviations of the LOOP. World terms of trade can be thought of as coming from fluctuations in $\widehat{d}_t^W$ or from fluctuations in a conventional measure of domestic ToT (i.e., $\widehat{tot}_t$). In a standard Q-IRBC model with flexible prices, $\widehat{d}_t = \widehat{d}_t^* = \widehat{d}_t^W = 0$ and ToT is proportional to world terms of trade. Otherwise, we must recognize that the relevant international relative price for expenditure-switching effects, $\widehat{t}_t^W$, does not exactly correspond to the data available on ToT.

Another important international relative price is the real exchange rate, which we define as $\widehat{rs}_t \equiv \widehat{s}_t + \widehat{p}_t^* - \widehat{p}_t$. Using the log-linearization of the consumption-price indexes in (1.39) and (1.40), it can be shown that,

$$\begin{aligned}\widehat{rs}_t &\approx \tfrac{1}{\phi_F}\widehat{t}_t^W - \widehat{tot}_t \\ &\approx (1-2\phi_F)\,\widehat{tot}_t - \tfrac{1}{\phi_F}\widehat{d}_t^W .\end{aligned} \tag{1.42}$$

This expression neatly shows that real exchange rate fluctuations arise from two channels: Compositional differences in the basket of goods due to home bias and deviations from the LOOP. In a flexible price model, the real exchange rate is purely proportional to conventional ToT, and that severely restricts the ability of the Q-IRBC framework (when it relies on home bias alone) to match the empirical features of both the real exchange rate and ToT. Equation (1.42) implies that world terms of trade are proportional to the real exchange rate plus the domestic ToT, i.e.

$$\widehat{t}_t^W \approx \phi_F\left(\widehat{tot}_t + \widehat{rs}_t\right). \tag{1.43}$$

In other words, the world terms of trade is equivalent to a linear combination of domestic ToT and the real exchange rate, which are both observable in the data (unlike $\widehat{t}_t^W$ itself). Equation (1.43) suggests that in models with deviations of the LOOP the real exchange rate is really crucial to help us account for the role of deviations of the LOOP on international relative price effects.[8]

Net Exports Share over GDP

The home and foreign consumption bundles of the domestic household, C_t^H and C_t^F, as well as the domestic investment bundles, X_t^H and X_t^F, are aggregated by means of a CES index as,

[8] While the exploration of the dynamics of the real exchange rate goes beyond the scope of this paper, we refer the interested reader to Martínez-García and Søndergaard (2008b) for a deeper exploration of the issue in the Q-INNS model.

$$C_t^H = \left[\int_0^1 C_t(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \; C_t^F = \left[\int_0^1 C_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}, \tag{1.44}$$

$$X_t^H = \left[\int_0^1 X_t(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \; X_t^F = \left[\int_0^1 X_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}, \tag{1.45}$$

while domestic aggregate consumption and investment, C_t and X_t, are defined with another CES index as,

$$C_t = \left[\phi_H^{\frac{1}{\eta}} \left(C_t^H\right)^{\frac{\eta-1}{\eta}} + \phi_F^{\frac{1}{\eta}} \left(C_t^F\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{1.46}$$

$$X_t = \left[\phi_H^{\frac{1}{\eta}} \left(X_t^H\right)^{\frac{\eta-1}{\eta}} + \phi_F^{\frac{1}{\eta}} \left(X_t^F\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}. \tag{1.47}$$

Given these aggregators and their foreign counterparts, we can easily characterize the system of demand equations underlying the model. These aggregators are also consistent with the log-linearization of the CPIs in (1.39) and (1.40). Then, the real exports and imports of domestic goods can be inferred as follows,

$$\begin{aligned} EXP_t &\equiv \int_0^1 \left(C_t^*(h) + X_t^*(h)\right) dh \\ &= \left[\int_0^1 \left(\frac{P_t^*(h)}{P_t^{H*}}\right)^{-\theta} dh\right] \phi_H^* \left(\frac{P_t^{H*}}{P_t^*}\right)^{-\eta} \left[C_t^* + X_t^*\right], \end{aligned} \tag{1.48}$$

$$\begin{aligned} IMP_t &\equiv \int_0^1 \left(C_t(f) + X_t(f)\right) df \\ &= \left[\int_0^1 \left(\frac{P_t(f)}{P_t^F}\right)^{-\theta} df\right] \phi_F \left(\frac{P_t^F}{P_t}\right)^{-\eta} \left[C_t + X_t\right], \end{aligned} \tag{1.49}$$

under the symmetric home bias assumption (i.e., $\phi_H^* = \phi_F$).

In a two-country model, it suffices to determine the net exports share of the domestic country. A simple log-linearization of equations (1.48) and (1.49) allows us to obtain the following pair of equations,

$$\begin{aligned} \widehat{exp}_t &\approx -\eta\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) + (1-\gamma_x)\,\widehat{c}_t^* + \gamma_x \widehat{x}_t^*, \\ \widehat{imp}_t &\approx -\eta\left(\widehat{p}_t^F - \widehat{p}_t\right) + (1-\gamma_x)\,\widehat{c}_t + \gamma_x \widehat{x}_t, \end{aligned}$$

where the relative price distortion at the variety level, captured by the terms within square brackets in (1.48) and (1.49), turns out to be only of second-order importance. The net exports share over GDP is defined as,

$$\begin{aligned} \widehat{tb}_t &\equiv \phi_F\left(\widehat{exp}_t - \widehat{imp}_t\right) \\ &\approx -\eta\left(\phi_F\left[\left(\widehat{p}_t^{H*} - \widehat{p}_t^*\right) - \left(\widehat{p}_t^F - \widehat{p}_t\right)\right]\right) - \\ &\quad - (1-\gamma_x)\,\phi_F\left(\widehat{c}_t - \widehat{c}_t^*\right) - \gamma_x \phi_F\left(\widehat{x}_t - \widehat{x}_t^*\right). \end{aligned} \tag{1.50}$$

In steady state, ϕ_F is the domestic imports share over domestic GDP and, under symmetric home bias, also the foreign imports share over foreign GDP. Given that the steady state is symmetric, i.e. $\overline{Y} = \overline{Y}^*$, the weighted difference between real exports and imports in (1.50) can be reasonably interpreted as the net exports share over GDP.[9]

We define two measures of world price sub-indexes, $\widehat{p}_t^{H,W} \equiv \phi_H \widehat{p}_t^H + \phi_F \widehat{p}_t^{H*}$ and $\widehat{p}_t^{F,W*} \equiv \phi_F \widehat{p}_t^F + \phi_H \widehat{p}_t^{F*}$, and two measures of the relative price sub-indexes, $\widehat{p}_t^{H,R} \equiv \widehat{p}_t^H - \widehat{p}_t^{H*}$ and $\widehat{p}_t^{F,R} \equiv \widehat{p}_t^F - \widehat{p}_t^{F*}$. We already used $\widehat{p}_t^{F,W*}$ and $\widehat{p}_t^{W*}$ to define the world terms of trade before. Here, we use these definitions coupled with the log-linearization of the consumption-price indexes in (1.39) and (1.40) in order to express the relative prices embedded in equation (1.50) in the following terms,

$$\begin{aligned}
\widehat{p}_t^{H*} - \widehat{p}_t^* &= \widehat{p}_t^{H,W} - \widehat{p}_t^W - \phi_H \left(\widehat{p}_t^{H,R} - \widehat{p}_t^R \right), \\
\widehat{p}_t^F - \widehat{p}_t &= \widehat{p}_t^{F,W*} - \widehat{p}_t^{W*} + \phi_H \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R \right),
\end{aligned}$$

where the relative CPI is $\widehat{p}_t^R \equiv \widehat{p}_t - \widehat{p}_t^*$.

The log-linearization of the CPI in both countries can be re-written as,

$$\begin{aligned}
\phi_H \left[\widehat{p}_t^H - \widehat{p}_t \right] + \phi_F \left[\widehat{p}_t^F - \widehat{p}_t \right] &\approx 0, \\
\phi_F \left[\widehat{p}_t^{H*} - \widehat{p}_t^* \right] + \phi_H \left[\widehat{p}_t^{F*} - \widehat{p}_t^* \right] &\approx 0.
\end{aligned}$$

Based on these relationships, we can infer that,

$$\begin{aligned}
&\phi_F \left[\left(\widehat{p}_t^{H,W} - \widehat{p}_t^W \right) - \phi_H \left(\widehat{p}_t^{H,R} - \widehat{p}_t^R \right) \right] + \\
&\quad + \phi_H \left[\left(\widehat{p}_t^{F,W*} - \widehat{p}_t^{W*} \right) - \phi_F \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R \right) \right] \approx 0.
\end{aligned} \tag{1.51}$$

Using the approximation derived in (1.51) and the definition of the world terms of trade, $\widehat{t}_t^W \equiv \widehat{p}_t^{F,W*} - \widehat{p}_t^{W*}$, we can write the relevant relative prices as follows,

$$\begin{aligned}
\phi_F \left(\widehat{p}_t^{H*} - \widehat{p}_t^* \right) &\approx -\phi_H \left[\widehat{t}_t^W - \phi_F \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R \right) \right], \\
\phi_F \left(\widehat{p}_t^F - \widehat{p}_t \right) &\approx \phi_F \left[\widehat{t}_t^W + \phi_H \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R \right) \right],
\end{aligned}$$

which, after some algebra, implies that,

$$\begin{aligned}
&\phi_F \left[\left(\widehat{p}_t^{H*} - \widehat{p}_t^* \right) - \left(\widehat{p}_t^F - \widehat{p}_t \right) \right] \\
&\quad \approx -\phi_H \left[\widehat{t}_t^W - \phi_F \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R \right) \right] - \phi_F \left[\widehat{t}_t^W + \phi_H \left(\widehat{p}_t^{F,R} - \widehat{p}_t^R \right) \right] \\
&\quad = -\left(\phi_H + \phi_F \right) \widehat{t}_t^W = -\widehat{t}_t^W.
\end{aligned}$$

[9] A simple look at equations (1.41) – (1.42) and equation (1.50) suggests that there is a trade-off between quantities (net exports) and international relative prices which crucially depends on the parameterization of the steady state imports share ϕ_F.

Hence, replacing this expression into equation (1.50) we infer that the net exports share can be calculated as,

$$\widehat{tb}_t \approx \eta \widehat{t}_t^W - (1-\gamma_x)\,\phi_F\,(\widehat{c}_t - \widehat{c}_t^*) - \gamma_x \phi_F\,(\widehat{x}_t - \widehat{x}_t^*)\,. \tag{1.52}$$

This expression for the net exports share illustrates the claim that the world terms of trade, $\widehat{t}_t^W$, is the model-consistent measure of international relative prices that explains the expenditure-switching across countries.

Adjustment in trade comes directly through movements in the world terms of trade, $\widehat{t}_t^W$, or from relative adjustments in consumption and investment across countries. This is the central equation in our analysis of the trade patterns. Our paper revisits the old question of what role does investment play in trade, but we do so with a two-sided strategy. On the one hand, we look at the role of adjustment costs in the accumulation of capital through investment. We recognize that adjustment costs have a role to play in determining the volatility of investment and consumption, and therefore can alter the implied trade dynamics. On the other hand, we recognize that Q-INNS models with deviations of the LOOP could lead to distortions in the allocation of expenditures across countries. We evaluate this additional channel and try to quantify the impact of those distortions on net trade flows.

Our previous discussion on the characterization of an appropriate international relative price measure allows us to re-write equation (1.52) as,

$$\widehat{tb}_t \approx 2\eta\,(1-\phi_F)\,\phi_F \widehat{tot}_t - \eta \widehat{d}_t^W - (1-\gamma_x)\,\phi_F\,(\widehat{c}_t - \widehat{c}_t^*) - \gamma_x \phi_F\,(\widehat{x}_t - \widehat{x}_t^*)\,, \tag{1.53}$$

which mechanically shows the way in which the world relative price distortion, $\widehat{d}_t^W$, operates on the trade balance. In turn, equation (1.43) allows us to express net exports as a function of only observable international relative prices as,

$$\widehat{tb}_t \approx \eta\,\phi_F\left(\widehat{tot}_t + \widehat{rs}_t\right) - (1-\gamma_x)\,\phi_F\,(\widehat{c}_t - \widehat{c}_t^*) - \gamma_x \phi_F\,(\widehat{x}_t - \widehat{x}_t^*)\,. \tag{1.54}$$

This characterization of the net exports share indicates that in a broad class of Q-INNS models the international relative price effects on expenditure-switching can only be accounted if we include domestic ToT and the real exchange rate simultaneously.[10]

The net exports share in equation (1.52) and the domestic ToT implicit in equation (1.43) do not constitute a trade model in themselves. All the other variables on the right- and left-hand side of both equations are endogenous, and their dynamics are determined by the full-blown model described in section 1.2. However, the fact that the relationships in (1.52) and (1.43) hold (up to a first-order approximation) gives us a way to mechanically identify how the propagation of shocks operates.

[10] In fact, under complete international asset markets, equation (1.52) can be re-written more compactly. Using the *perfect international risk-sharing condition* in (1.3) we get that,

$$\widehat{tb}_t \approx \phi_F\left(\eta \widehat{tot}_t + (\eta - (1-\gamma_x)\,\sigma)\,\widehat{rs}_t\right) - \phi_F \gamma_x\,(\widehat{x}_t - \widehat{x}_t^*)\,.$$

Here, we exploit these relationships to focus our attention on the role of investment in trade, and how it is influenced by the presence of adjustment costs and/or large fractions of firms 'unable' to update their prices in every period subject to LCP.

1.4 Quantitative Findings

1.4.1 Model Calibration

Our calibration is summarized in Table 1.2. For comparison purposes, we follow quite closely the parameterization of the Q-INNS model in CKM (2002). We refer the interested reader to their paper for a complete discussion of the calibration. Here, we only comment on those parameters that we calibrate differently. The Calvo price stickiness parameter, α, is assumed to be 0.75. This implies that the average price duration in our model is 4 quarters. Our choice is comparable to CKM (2002) since in their model a quarter of firms re-set prices every period and those prices remain fixed for a total of 4 periods. We also study the implications of the model under (quasi-) flexible prices. We do not simulate an exact solution for a comparable Q-IRBC model. Instead, we approximate that scenario by bringing the Calvo parameter, α, down to 0.00001 in our benchmark Q-INNS model. This implies that 99.999% of the firms are able to re-optimize their prices every period, and only a negligible fraction of them is subject to keeping the previous period prices.[11]

The inverse of the Frisch elasticity of labor supply, φ, is set to 3 instead of 5 as in CKM (2002). This is compatible with the available micro evidence (see, e.g., Browning, Hansen and Heckman, 1999, and Blundell and MaCurdy, 1999), but not consistent with a balanced growth path. This choice is meant to reduce the sensitivity of the CPI inflation to consumption and investment fluctuations (see, e.g., Martínez-García and Søndergaard, 2008b). The parameterization of the monetary policy rule is slightly different than in CKM (2002). The interest rate inertia parameter, ρ_i, equals 0.85, while the weight on the inflation target, ψ_π, equals 2, and the weight on the output target, ψ_y, is 0.5. Our Taylor rule targets current inflation, instead of expected inflation as in CKM (2002). The rule also includes interest rate smoothing and gives more weight to inflation than the one proposed by Taylor (1993).

We adapt the calibration strategy of CKM (2002) and set the parameters of the stochastic real shocks to approximate the features of U.S. real GDP in the data. The aim is to investigate whether it is possible to account for consumption, investment, trade and ToT patterns in a model that replicates key empirical moments of U.S.

[11] The (quasi-) flexible price experiment does not imply that $\widehat{d}_t^W$ is equal to zero. In fact, it will not be. Therefore, we should not view this experiment as if it were equivalent to a standard Q-IRBC model. The (quasi-) flexible price case merely reflects the limiting behavior of the Q-INNS model whenever the share of firms affected by the nominal rigidities becomes marginal (close to zero).

real GDP with only real shocks.[12] We assume the persistence parameter of the real shocks, ρ_a, is set equal to 0.9. We choose the standard deviation of the real innovations to get the exact output volatility in the U.S. data (i.e., 1.54%). In addition, we calibrate the cross-country correlation of the innovations to replicate the observed cross-correlation of U.S. and Euro-zone GDP (i.e., 0.44). This calibration allows us to match exactly the volatility and cross-correlation of U.S. real GDP, and also roughly approximates its persistence.

Table 1.2 Parameters Used in the Benchmark Calibration

		Benchmark	CKM (2002)
Structural Parameters:			
Discount Factor	β	0.99	0.99
Elasticity of Intratemporal Substitution	η	1.5	1.5
Elasticity of Substitution across Varieties	θ	10	10
Elasticity of Intertemporal Substitution	σ	$1/5$	$1/5$
(Inverse) Elasticity of Labor Supply	φ	3	5
Domestic Home Bias Parameter	ϕ_H	0.94	0.94
Foreign Home Bias Parameter	ϕ_F	0.06	0.06
Calvo Price Stickiness Parameter	α	0.75	$N=4$
Depreciation Rate	δ	0.021	0.021
Capital/Investment Adjustment Cost	χ, κ	varies	varies
Labor Share	ψ	$2/3$	$2/3$
Parameters on the Taylor Rule:			
Interest Rate Inertia	ρ_i	0.85	0.79
Weight on Inflation Target	ψ_π	2	2.15
Weight on Output Target	ψ_y	0.5	$0.93/4$
Exogenous Shock Parameters:			
Real Shock Persistence	ρ_a	0.9	0.95
Real Shock Correlation	$corr\left(\varepsilon_t^a, \varepsilon_t^{a*}\right)$	varies	0.25
Monetary Shock Correlation	$corr\left(\varepsilon_t^m, \varepsilon_t^{m*}\right)$	-	varies
Real Shock Volatility	$\sigma\left(\varepsilon_t^a\right) = \sigma\left(\varepsilon_t^{a*}\right)$	varies	0.007
Monetary Shock Volatility	$\sigma\left(\varepsilon_t^m\right) = \sigma\left(\varepsilon_t^{m*}\right)$	-	varies
Composite Parameters:			
Steady State Investment Share	$\gamma_x \equiv \frac{(1-\psi)\delta}{\left(\frac{\theta}{\theta-1}\right)\left(\beta^{-1}-(1-\delta)\right)}$	0.203	(0.203)

This table summarizes our benchmark parameterization. Additional results on the sensitivity of certain parameters can be obtained directly from the authors upon request. The comparison is with CKM's (2002) model specification where monetary policy is represented by a Taylor rule.

CKM (2002) select the adjustment cost parameter to match the empirical ratio of the standard deviation of consumption relative to the standard deviation of output in the data, while Raffo (2008) uses it to reproduce the volatility of investment

[12] CKM (2002) explore a combination of real and monetary shocks. In their calibration, they keep the features of the real shock invariant and match the moments of U.S. real GDP with the help of the monetary shock.

relative to output. We select either the capital adjustment cost parameter, χ, or the investment adjustment cost parameter, κ, to ensure that investment volatility is as volatile as in the data (i.e., 3.38 times as volatile as U.S. real GDP). This is consistent with the goal of adopting a Q theory extension that delivers the best possible fit for investment.

1.4.2 Model Exploration

From equation (1.52) we know that the net exports share must be linked to investment, consumption and the world terms of trade. From equation (1.42) we also know that a complex relationship exists between the world terms of trade, domestic ToT and world deviations of the LOOP. Based on the calibration described before, we are able to simulate the log-linearized equilibrium conditions and gain further insight on trade. We are also able to assess the performance of the benchmark model relative to the observable data. The contemporaneous business cycle moments are summarized in Table 1.3.

We find that none of our experiments manages to generate a volatility of consumption above 55% of the observed volatility of U.S. real consumption. Similar patterns can be found in BKK (1992, 1995), Heathcote and Perri (2002) and Raffo (2008). CKM (2002), however, match the consumption volatility, but do so by driving the adjustment cost parameter up at the expense of making investment significantly smoother than in the data. Although consumption is slightly more volatile under investment adjustment costs (IAC) than capital adjustment costs (CAC), this improvement is not sufficient to close the gap.

The trade off between investment and consumption volatility becomes particularly stark when we compare the IAC and CAC specifications against the no adjustment costs (NAC) case. Without adjustment costs, households take full advantage of capital accumulation as a mechanism to smooth consumption intertemporally. The consumption volatility produced by the model with sticky or (quasi-) flexible prices is less than 20% of the empirical volatility, while investment volatility is at least 67% higher. Overall, consumption volatility appears little affected by the choice of the Calvo price stickiness parameter.

The model also has difficulties matching the volatility of net exports. In the (quasi-) flexible price experiments, adding adjustment costs to the model impedes the ability of households to smooth consumption intertemporally. This leads to a higher reliance on trade for risk-sharing and, hence, a more volatile net exports share. The volatility of net exports is quite similar whether prices are (quasi-) flexible or sticky.

Table 1.3 Selected Business Cycle Moments of the Baseline Model

		Sticky Prices				(Quasi-) Flexible Prices		
	U.S. Data	IAC	CAC	NAC	NAC	IAC	CAC	NAC
Std. Dev.								
GDP*	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54
Investment*	5.21	5.21	5.21	7.08	7.09	5.21	5.21	6.62
Consumption	1.24	0.60	0.53	0.22	0.15	0.68	0.51	0.24
Net Exports	0.38	0.17	0.14	0.10	0.07	0.20	0.13	0.04
Autocorrelation								
GDP	0.87	0.91	0.89	0.54	0.71	0.77	0.69	0.70
Investment	0.91	0.94	0.88	0.40	0.67	0.89	0.69	0.69
Consumption	0.87	0.82	0.83	0.75	0.76	0.48	0.70	0.76
Net Exports	0.83	0.84	0.84	−0.12	−0.03	0.45	0.71	0.94
Cross-correlation								
GDP*	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44
Investment	0.33	0.57	0.55	0.37	0.40	0.54	0.56	0.41
Consumption	0.33	0.65	0.63	0.69	0.66	0.68	0.62	0.62
Correlation								
GDP, Net Exp.	−0.47	0.49	0.49	−0.18	−0.11	0.41	0.52	−0.06
GDP, ToT	0.07	0.31	0.21	0.37	0.44	0.47	0.53	0.49
ToT, Net Exp.	−0.03	0.27	0.52	0.42	0.35	0.97	1.00	0.26
Calibration								
$\sigma(\varepsilon_t^a) = \sigma(\varepsilon_t^{a*}) =$		2.07	1.89	1.27	1.785	1.43	1.34	1.15
$corr(\varepsilon_t^a, \varepsilon_t^{a*}) =$		0.4625	0.4475	0.4875	0.44	0.4775	0.465	0.457
$\rho_a =$		0.9	0.9	0.9	0.75	0.9	0.9	0.9
$\chi, \kappa =$		3.35	11.15	–	–	2.12	13.25	–

This table reports the business cycle moments given our benchmark parameterization. All theoretical statistics are computed after H-P filtering (smoothing parameter=1600). NAC denotes the no adjustment cost case, CAC denotes the capital adjustment cost case, and IAC denotes the investment adjustment cost case. Sticky prices implies α = 0.75, while (quasi-) flexible prices implies α = 0.00001. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation.

* We calibrate the volatility and cross-correlation of the real shock innovations to match the observed volatility and cross-country correlation of GDP. Whenever available, we calibrate the adjustment cost parameter to match the observed volatility of U.S. investment.

Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the dataset in the Appendix. Sample period: 1973q1-2006q4 (except for ToT, which covers only 1983q3-2006q4).

Turning to persistence, we observe that output persistence falls below the empirical numbers for U.S. real GDP in the (quasi-) flexible price case with the CAC specification. The same is true for the persistence of consumption, investment and the net exports share. Using the NAC case does not substantially alter this conclusion, which is consistent with the results in BKK (1992, 1995). However, the findings are more mixed when we experiment with adjustment costs of the IAC type. The IAC specification produces higher persistence on output and investment.

At the same time, it also generates counterfactually low first-order autocorrelations for consumption and net exports.

The results are somewhat different in the sticky price case, because adding adjustment costs helps us deliver persistence values for all variables that are roughly in line with the data. The differences between the CAC and IAC specifications are only marginal. The NAC case, however, cannot replicate sufficient persistence. Even when we look at a different calibration of the persistence of the real shock (i.e., $\rho_a = 0.75$) to enhance its odds on output persistence, the model cannot produce sufficient persistence in consumption, investment and net exports. In fact, with sticky prices and no adjustment costs we find a counterfactual, negative first-order autocorrelation for the net exports share. So far, our findings suggest that the Q-INNS model performs better (or certainly not worse) than a competing scenario with (quasi-) flexible prices.

Whether the model relies on sticky prices or (quasi-) flexible prices, the cross-country correlations of consumption and investment are very stable. It should be pointed out that all experiments generate very high cross-correlations of consumption, around twice as much as in the data. This finding is consistent with BKK (1992, 1995) and Heathcote and Perri (2002).[13] The difficulty to match the smaller cross-correlation of consumption relative to the cross-correlation of output found in the data is often known as the "quantity puzzle."

Most notably, we find that only models without adjustment costs can account (qualitatively at least) for the fact that the cross-country correlation of investment is lower than the cross-country correlation of output. Whether prices are (quasi-) flexible or sticky seems to make little difference. BKK (1992, 1995) and Heathcote and Perri (2002) indicate that this stylized fact is not easy to match with a standard calibration of the IRBC model (without adjustment costs). This is, therefore, the first piece of evidence that comes out against the implementation of the Q theory extension by means of either the CAC or the IAC specifications.

On the Contemporaneous Correlations of ToT and Net Exports

The last three correlations reported in Table 1.3 are, however, the litmus test for each one of the experiments that we consider in this paper. The only models that can account qualitatively for the empirical evidence of countercyclical net exports are models without adjustment costs (NAC). BKK (1992, 1995) and Heathcote and Perri (2002) get a similar pattern in standard IRBC models without adjustment costs. Our model shows that it can deliver countercyclical trade patterns with either sticky or (quasi-) flexible prices, but the effects are weaker than in the data. Adding IAC

[13] In a complete asset markets model, this strong consumption cross-correlation has implications for the behavior of the real exchange rate through the *perfect international risk-sharing condition* in (1.3). We refer the interested reader to Martínez-García and Søndergaard (2008b) for additional insight on this issue.

or CAC adjustment costs increases the correlation and alters its sign (i.e., the trade balance is more likely to become procyclical).

Engel and Wang (2007) and Raffo (2008), using different models in the Q-IRBC tradition, are able to replicate the countercyclical trade patterns. The contemporaneous correlation between output and the net exports share is quite sensitive to the calibration of the model and the adjustment cost function. Even minor differences in the structure of the economy or the calibration could explain why they can account for this feature, while our model does not. For example, see BKK (1995, Figure 11.4). Raffo (2008, p. 21) notes that: "Higher substitution between intermediates translates into lower response of the terms of trade. At this value, net exports are already procyclical. In the limiting case of perfect substitute intermediates, this economy resembles a one-good economy and net exports are systematically procyclical."

The elasticity of intratemporal substitution, η, plays an analogous role in our model as suggested by equation (1.52). We leave the exploration of this and other structural parameters for future research. It suffices to say that while including adjustment costs in the model reduces the volatility of investment and increases the volatility of consumption (and net exports), it may also push the contemporaneous correlation between output and net exports up. The effect can be strong enough to make net exports procyclical. This finding suggests that the Q theory extension to an open economy setting has to be undertaken with great care.

Consistent with the results of Raffo (2008), the model produces high and positive contemporaneous correlations between output and ToT. This is true for all variants of the model. However, we find that the model with sticky prices tends to generate lower correlations closer to the data. Adding adjustment costs helps further on this front. Therefore, based on the contemporaneous correlations alone, the Q-INNS model appears to offer a better fit for the data. However, as we shall see shortly, the interpretation becomes more complex when we look at the shape of the cross-correlation function.

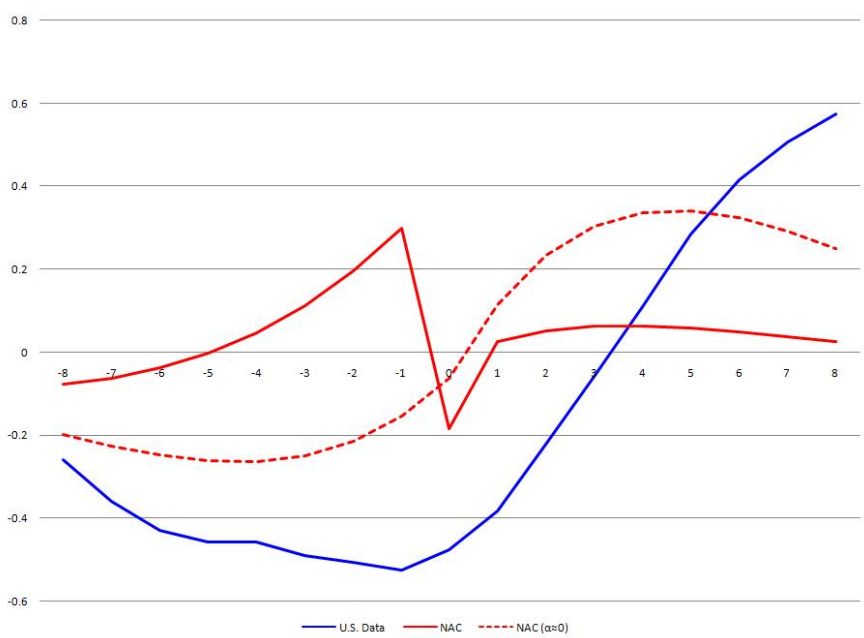
The experiment with (quasi-) flexible prices and no adjustment costs (NAC) generates a contemporaneous correlation of 0.26 between ToT and net exports, which is far away from the value of -0.03 observed in the data. Adding adjustment costs makes matters even worse. In turn, adding adjustment costs in a sticky price scenario helps reduce the correlation. Even though no model does better than the (quasi-) flexible price one without adjustment costs (NAC), the Q-INNS model with IAC adjustment costs also does well. Once again, the interpretation is less straightforward when we look at the entire cross-correlation function.

BKK (1994, p. 94) point out that "the contemporaneous correlation between net exports and the terms of trade is weaker, moving from -0.41 in the benchmark case to -0.05" with a higher elasticity of intratemporal substitution between foreign and domestic goods. When discussing the countercyclical nature of net exports, we already quoted a similar argument by Raffo (2008). Indeed, recalling our previous discussion we could say that there are other structural parameters that do matter, as equation (1.52) indicates, but the importance of the adjustment cost parameter cannot be discounted.

On the Cross-Correlations of ToT and Net Exports

Figures 1.1 and 1.2 plot the cross-correlations between real GDP and the real net exports share. The data reveals the same type of S-shaped pattern that Engel and Wang (2007) emphasize in their paper. We show that only models without adjustment costs (NAC) can generate countercyclical trade patterns. We also find that only the (quasi-) flexible price scenario with no adjustment costs (NAC) can qualitatively approximate the S-shaped pattern of the cross-correlation function. The sticky price scenario without adjustment costs (NAC) moves us away from the empirical evidence.

Figure 1.1 Cross-Correlations of Output with Net Exports (without Adjustment Costs)

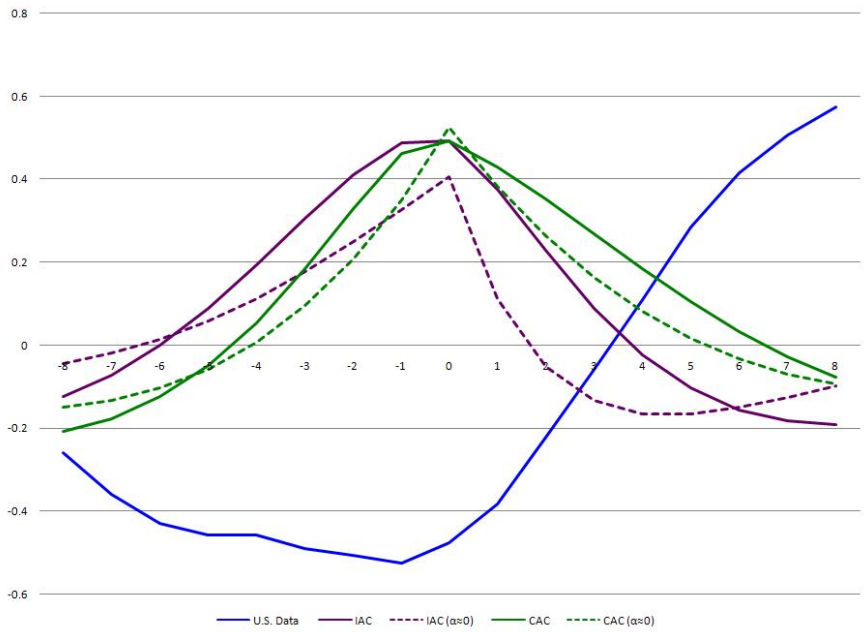


This figure plots the cross-correlation of output at t and net exports at t+s given our parameterization. All theoretical cross-correlations are computed after H-P filtering (smoothing parameter=1600). NAC denotes the no adjustment cost case, while $\alpha \approx 0$ indicates the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation. Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the dataset in the Appendix. Sample period: 1973q1-2006q4.

As Figure 1.2 demonstrates, adding IAC or CAC adjustment costs alters the shape of the cross-correlations in a fundamental way. The cross-correlation function becomes shaped like a tent, with its peak around the contemporaneous correlation. The dominant effect comes from having adjustment costs embedded in the model, but the contribution of sticky prices is also noticeable. Engel and Wang (2007) have a model that also matches qualitatively this cross-correlation function, and they do

so with adjustment costs. Our models are not immediately comparable, but their paper is encouraging. It suggests that there is still room to reconcile the Q theory extension with the empirical evidence.

Figure 1.2 Cross-Correlations of GDP with Net Exports (with Adjustment Costs)



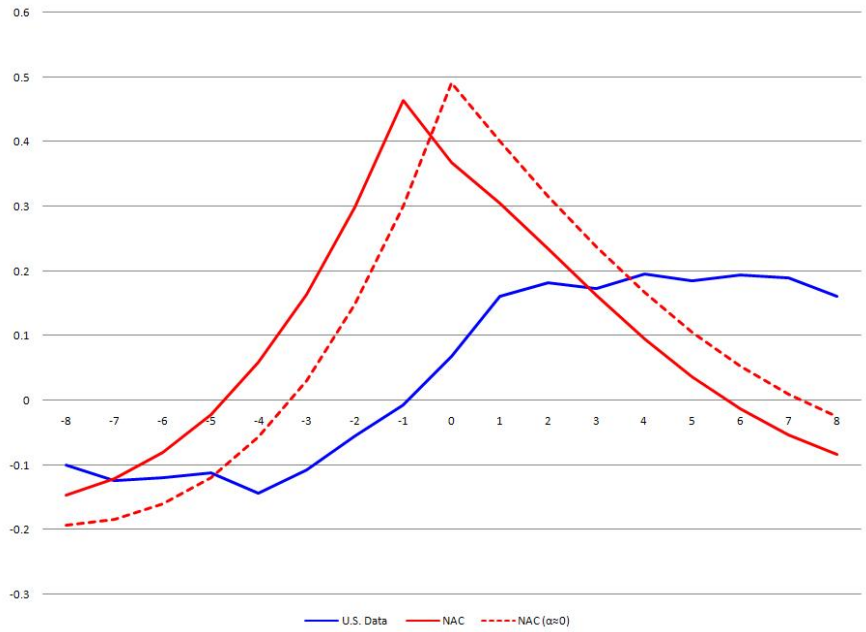
This figure plots the cross-correlation of output at t and net exports at t+s given our parameterization. All theoretical cross-correlations are computed after H-P filtering (smoothing parameter=1600). CAC denotes the capital adjustment cost case, IAC denotes the investment adjustment cost case, while $\alpha \approx 0$ indicates the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation. Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the dataset in the Appendix. Sample period: 1973q1-2006q4.

Our reading of these results is that the (quasi-) flexible price scenario without adjustment costs (NAC) brings back the flavor of the BKK (1992, 1995) model, where investment resources are being shifted across countries in search of (temporarily) higher productivity and higher returns. Adding adjustment costs caps the size of these effects because we set the adjustment cost parameter high enough to ensure that investment flows are not too volatile. The side-effect is that the trade balance becomes procyclical and the cross-correlation function peaks contemporaneously.

Figures 1.3 and 1.4 plot the cross-correlations between real GDP and ToT. Raffo (2008) argues that the IRBC framework delivers a contemporaneous correlation between GDP and ToT that is counterfactually too high. We confirm that the contemporaneous correlation between GDP and ToT is well-above its value in the data (i.e., 0.07). However, we also note that all the experiments display a tent-shaped pattern

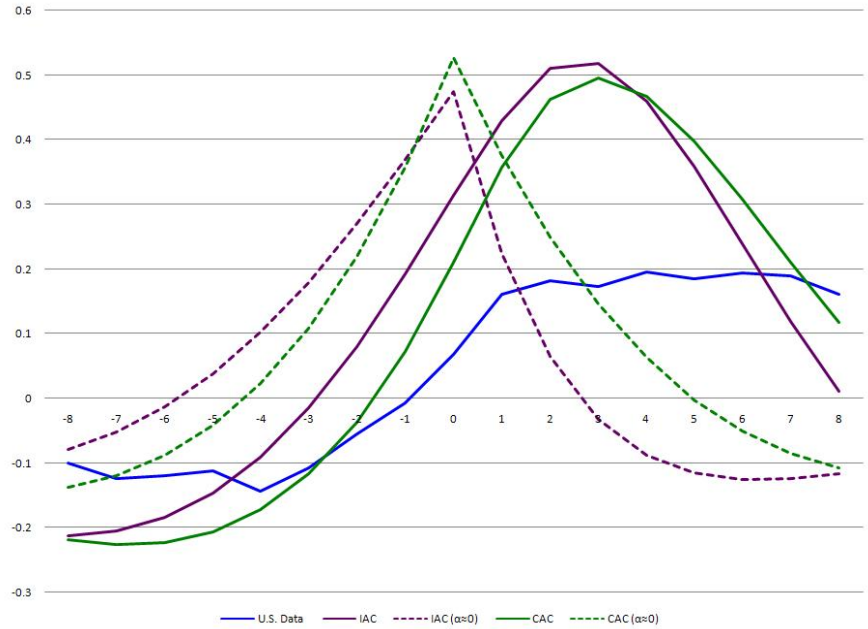
which is inconsistent with the S-shaped empirical cross-correlation function. Combining price stickiness with adjustment costs (preferably of the CAC type) allows us to qualitatively fit the cross-correlations of real GDP with current and lagged ToT, but the leads are significantly different than in the data (specially $3-4$ periods ahead). These features are a challenge for the IRBC literature (see, e.g., Raffo, 2008) as well as for the INNS/Q-INNS model.

Figure 1.3 Cross-Correlations of GDP with ToT (without Adjustment Costs)



This figure plots the cross-correlation of output at t and terms of trade (ToT) at t+s given our parameterization. All theoretical cross-correlations are computed after H-P filtering (smoothing parameter=1600). NAC denotes the no adjustment cost case, while $\alpha \approx 0$ indicates the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation. Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the dataset in Appendix. Sample period: 1983q3-2006q4.

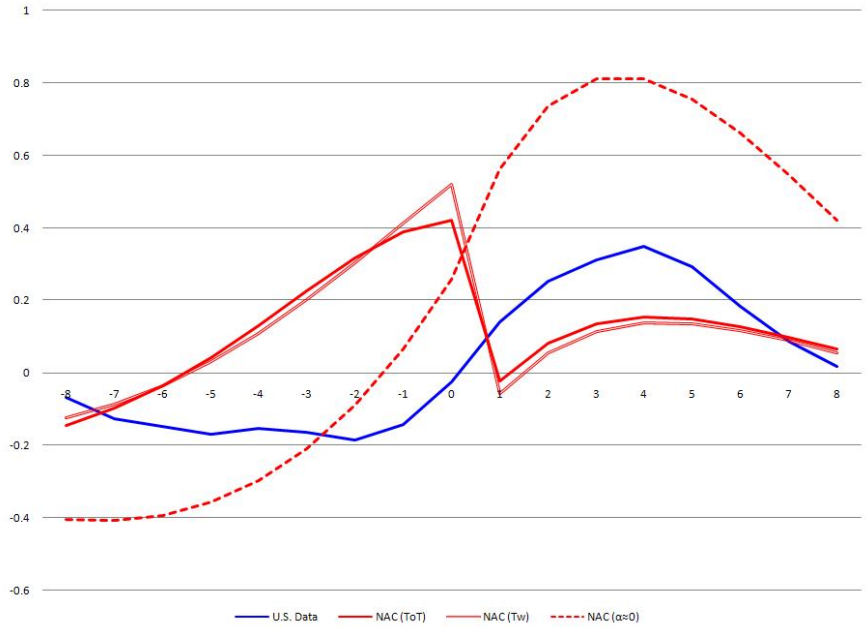
Figure 1.4 Cross-Correlations of GDP with ToT (with Adjustment Costs)



This figure plots the cross-correlation of output at t and terms of trade (ToT) at t+s given our parameterization. All theoretical cross-correlations are computed after H-P filtering (smoothing parameter=1600). CAC denotes the capital adjustment cost case, IAC denotes the investment adjustment cost case, while $\alpha \approx 0$ indicates the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation. Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the dataset in the Appendix. Sample period: 1983q3-2006q4.

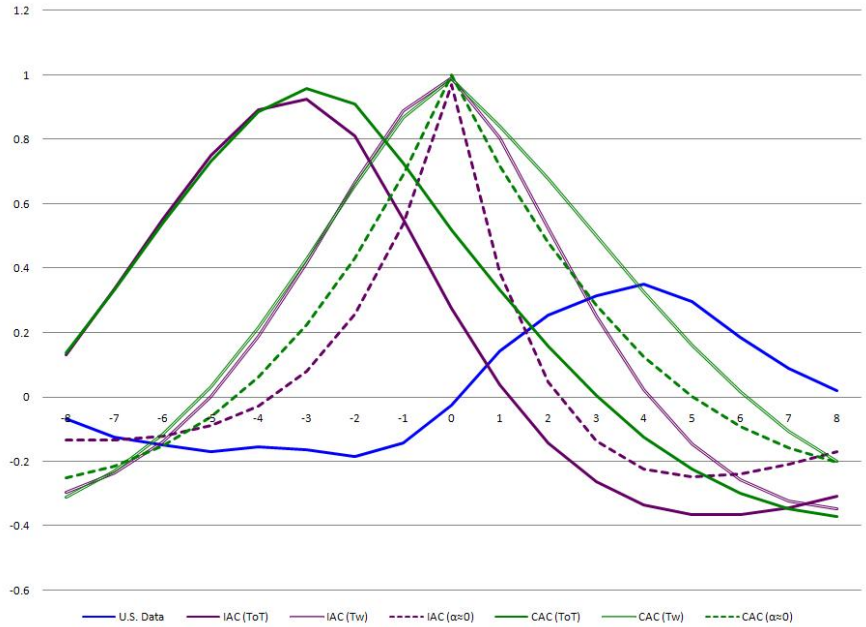
The J-curve has been extensively discussed in the IRBC literature, specially since BKK (1994) showed that the standard IRBC framework was powerful enough to replicate this stylized fact. We still find evidence of a J-curve effect in the data, as reported in Figures 1.5 and 1.6, although the strength of the correlation diminishes beyond a 4 period lead (one year ahead). Our quantitative findings are consistent with the intuition of BKK (1994) given that our best qualitative fit for the cross-correlations between ToT and the net exports share comes from the (quasi-) flexible price scenario without adjustment costs (NAC). Adding adjustment costs and/or sticky prices not only alters the shape of the cross-correlation function, it also shifts its peak from leads to either contemporaneous or lagged cross-correlations.

Figure 1.5 Cross-Correlations of ToT with Net Exports (without Adjustment Costs)



This figure plots the cross-correlation of terms of trade at t and net exports at t+s given our parameterization. We distinguish between conventional terms of trade, ToT, and world terms of trade, Tw. World terms of trade captures the relative price effects in the net exports share. All theoretical cross-correlations are computed after H-P filtering (smoothing parameter=1600). NAC denotes the no adjustment cost case, while $\alpha \approx 0$ indicates the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation. Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the dataset in the Appendix. Sample period: 1983q3-2006q4.

Figure 1.6 Cross-Correlations of ToT with Net Exports (with Adjustment Costs)



This figure plots the cross-correlation of terms of trade at t and net exports at t+s given our parameterization. We distinguish between conventional terms of trade, ToT, and world terms of trade, Tw. World terms of trade captures the relative price effects in the net exports share. All theoretical cross-correlations are computed after H-P filtering (smoothing parameter=1600). CAC denotes the capital adjustment cost case, IAC denotes the investment adjustment cost case, while $\alpha \approx 0$ denotes the experiment with (quasi-) flexible prices. We use Matlab 7.4.0 and Dynare v3.065 for the stochastic simulation. Data Sources: The Bureau of Economic Analysis and the Bureau of Labor Statistics. For more details, see the description of the dataset in the Appendix. Sample period: 1983q3-2006q4.

A consistent message emerges from Figures 1.1 through 1.6. Our experiment with (quasi-) flexible prices and no adjustment costs (NAC) tends to approximate well the good and the bad features of the IRBC model. It qualitatively tracks the J-curve effect and the S-shaped pattern of the cross-correlation between GDP and net exports. It also produces an excessively high correlation between output and ToT, and cannot track the S-shaped pattern of the cross-correlations between these two variables at different leads and lags. Whenever we try to pull the model closer to our Q-INNS benchmark by making price stickiness or adjustment costs a more relevant factor in the dynamics, we end up worsening the trade predictions along some of these dimensions.

1.5 Concluding Remarks

The findings in this paper suggest that a Q theory extension of the standard INNS model has important, although conflicting implications for our ability to replicate observed international business cycle patterns. On the one hand, adding adjustment costs makes investment costlier and, therefore, results in a smoother investment series and a more volatile consumption series. At the same time, the net exports share becomes more volatile. While the model does not perfectly match the properties (on volatility, persistence and cross-country correlations) of consumption, investment and net exports, adding adjustment costs appears to lead us in the right direction overall.

On the other hand, we see that the model with adjustment costs cannot replicate well-known features of the trade data such as the J-curve (see, e.g., BKK, 1994), the S-shaped cross-correlation of GDP and net exports (see, e.g., Engel and Wang, 2007), and the weak and S-shaped cross-correlation between GDP and ToT (see, e.g., Raffo, 2008). Furthermore, our analysis suggests that a full-blown INNS model with sticky prices and LCP does not do any better than an alternative variant with (quasi-) flexible prices. In fact, the (quasi-) flexible price scenario without adjustment costs delivers similar results to those documented in the standard IRBC literature and tracks qualitatively the S-shaped cross-correlation of GDP and net exports and also the J-curve.

An open question is what role monetary policy plays in all of this. In the standard INNS model, with or without the adjustment costs, the size and effect of the relative price distortion resulting from nominal rigidities (price stickiness and LCP) depends on the path of inflation and, by extension, on the choice of monetary policy. We have taken as given a version of the Taylor rule with interest rate inertia and selected a very specific calibration. The predictions of the model for trade are conditional on that calibration of the Taylor rule, and are likely to be different for alternative policy rules or parameterizations. We leave the close examination of the interplay between monetary policy and trade dynamics for future research.

We interpret the findings of the paper mainly as a cautionary tale, and not as a final word on the subject. To sum up: We need to be mindful of the fact that adjustment costs together with nominal rigidities can have unintended consequences for the trade dynamics of the standard Q-INNS model. Therefore, we have to think deeply about how to reconcile the Q-INNS model with the empirical evidence on trade.

1.6 Appendix: Dataset

We collect U.S. quarterly data spanning the post-Bretton Woods period from 1973q1 through 2006q4 (for a total of 136 observations per series). The U.S. dataset includes real output (rgdp), real private consumption (rcons), real private fixed investment (rinv), real exports (rx), the export price index (px), real imports (rm), the import price index (pm), and population size (n). The U.S. import price index and the U.S. export price index cover only the sub-sample between 1983q3 and 2006q4 (for a total of 94 observations). All data is seasonally adjusted.

▷ Real output (rgdp), real private consumption (rcons) and real private fixed investment (rinv): Data at quarterly frequency, transformed to millions of U.S. Dollars, at constant prices, and seasonally adjusted. Source: Bureau of Economic Analysis.

▷ Real exports (rx) and real imports (rm). Data at quarterly frequency, transformed to millions of U.S. Dollars, and seasonally adjusted. Source: Bureau of Economic Analysis.

▷ Import price index (pm) and export price index (px). Data at quarterly frequency, indexed (2000=100), but not seasonally adjusted. Source: Bureau of Labor Statistics. (We compute a conventional measure of terms of trade, tot = pm/px, based on the data for the import and the export price indexes. We seasonally-adjust the resulting series with the multiplicative method X12.)

▷ Working-age Population between 16 and 64 years of age (n): Data at quarterly frequency, expressed in thousands, and seasonally adjusted. Source: Bureau of Labor Statistics. (We compute working-age population as the difference between civilian non-institutional population 16 and over and civilian non-institutional population 65 and over. We also seasonally-adjust the resulting series with the multiplicative method X12.)

The real output (rgdp), real private consumption (rcons), real private fixed investment (rinv), real exports (rx), and real imports (rm) are expressed in per capita terms dividing each one of these series by the population size (n). We compute the terms of trade ratio (tot) and the real net export share over GDP, rnx = ((rx - rm)/rgdp)*100, based on the data for real imports (rm), real exports (rx), the import price index (pm), the export price index (px), and real GDP (rgdp). We express all variables in logs and multiply them by 100, except the real net export share (rnx) which is already expressed in percentages. Finally, all series are Hodrick-Prescott (H-P) filtered to eliminate their underlying trend. We set the H-P smoothing parameter at 1600 for our quarterly dataset.

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