The Micro-Macro Disconnect of Purchasing Power Parity

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I. Background: Real Exchange Rates

- Define real exchange rate between countries $i$ and $j$:

$$q_{ij,t} = e_{ij,t} + p_{it} - p_{jt} \quad (1)$$

where $e_{ij,t}$ is the log nominal exchange rate, $p_{it}$ and $p_{jt}$ are the log national price indexes.

- As a long-run equilibrium condition, relative purchasing power parity (PPP) says $q_{ij,t}$ reverts to a long-run value.

- Theory is silent regarding adjustment mechanism: arbitrage in goods market or foreign exchange market.

- A large literature documents deviations of $q_{ij,t}$ from PPP are persistent.
A Microeconomic Perspective

- A growing literature studies international relative prices in microeconomic data sets (many papers)
- Define a relative price for an individual good $k$,
  \[ q_{ij,t}^k \equiv e_{ij,t} + p_{it}^k - p_{jt}^k \]  
  where $p_{it}^k$ and $p_{jt}^k$ are the logged price of good $k$.
- As a long-run equilibrium condition, the relative Law of One Price (LOP) states $q_{ij,t}^k$ reverts to long-run value.
A Persistence Puzzle

- Aggregating $q_{ij,t}^k$ over goods produces $q_{ij,t}$, so the dynamics of the two should be related.

- However, Imbs et al. (QJE, 2005) document that $q_{ij,t}^k$ is much less persistent than $q_{ij,t}$: half-life less than a year.

- They offer aggregation bias as an explanation: aggregation over goods with heterogeneous persistence can give upward bias under certain conditions.
Where This Paper Fits In

- We argue that the apparent inconsistency can be reconciled if one conditions on the distinct shocks driving disaggregated and aggregate data. (akin to story for sectoral price stickiness in face of monetary shocks in Boivin et al. 2009)
- We apply time-series tools to a micro data set of international relative prices to study dynamic adjustment.
- Data from the Economist Intelligence Unit (EIU), for 98 traded goods for 20 industrial countries, semi-annual.
- Estimate a series of panel vector error correction models, nesting aggregate and disaggregated data.
- Simple method to identify shocks to foreign exchange market, other macro shocks, industry-specific shocks.
Preview of Paper’s Findings

1) New fact: Dynamic adjustment in disaggregated data not just a faster version of the adjustment in agg. data. While adjustment to PPP occurs in foreign exchange market, adjustment to LOP occurs in goods market.

2) This is due to distinct shocks: macro shocks dominate in aggregate data, but industry shocks in micro data.

3) Conditional on macro shocks, microeconomic prices are just as persistent as aggregate real exchange rate.

4) This challenges theories of real exchange rate based sticky price and aggregation bias (heterogeneity conditional on agg. shock innocuous; omitted variable bias).
Related Literature

- **Crucini and Shintani (2008, JME)**: Also studies dynamics with EIU data.

- **Andrade and Zachariadis (2010)**: Also decompose micro price dynamics by various shocks.

- **Broda and Weinstien (2010, AER forthcoming)**: suggest micro prices adjust faster due to larger micro shocks.

- **Carvalho and Nechio (2010, AER forthcoming)**: demonstrate heterogeneity bias in structural model.


- **This paper distinct**: studies mechanism of adjustment; decomposes shocks in ECM to reconcile agg./disagg. adjustment speeds in international relative price data.
Plan for Rest of Talk

1) **Preliminaries**: test stationarity of real exchange rate; estimate convergence speeds in autoregression (AR).

2) Show new fact by estimating simple panel vector error correction model (VECM) separately for aggregate and disaggregated data.

3) Role of shocks: joint VECM nesting agg. & disagg. data, identify shocks and show impulse responses.

4) Show implications for aggregation bias literature, by augmenting standard AR estimation.
II. Data

- **Source:** Worldwide Cost of Living Survey conducted by the Economist Intelligence Unit

- Survey retail outlets for prices. We have complete data on 98 tradable goods and 37 nontradables:
  - Examples: Coca Cola (1 liter, supermarket), aspirin (100 count), light bulbs (2 count, 60 watt).

- We will study bilateral pairs of 20 industrial countries relative to U.S., for which data are complete.

- Data collection twice yearly, 1990-2007. We negotiated access to historical data at semi-annual frequency to allow use of standard time-series tools.

- Compute a synthetic aggregate real exchange rate as an unweighted average over the goods.
III. Prelimaries: Stationarity Test

- To confirm that real exchange rates and relative prices converge to long-run values, first test for stationarity.

- Apply the cross-sectionally augmented Dickey-Fuller (CADF) test of Pesaran (2007). Estimated separately for each good, $k$, for a cross-section of country pairs, $ij$

\[
\Delta q_{ij,t}^k = a_{ij}^k + b_{ij}^k (q_{ij,t-1}^k) + c_{ij}^k (\bar{q}_t^k) + d_{ij}^k (\Delta q_t^k) + \varepsilon_{ij,t}^k \quad (3)
\]

\[
i j = 1, ..., N, \quad k = 1, ..., K, \quad t = 1, ..., T
\]

($q_{ij}^k$ in logs; $\bar{q}_t^k$ is the cross-section mean of $q_{ij}^k$, included to control for contemporaneously correlated errors)

- Result: reject nonstationarity for large share of traded goods and for traded aggregate; not for nontradeds.

- Hereafter focus on traded goods and their aggregates.
Table 1: Stationarity of Relative Prices

<table>
<thead>
<tr>
<th>Sample</th>
<th>(mean(^1))</th>
<th>(mean(^1))</th>
<th>significance(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(b)</td>
<td>t-stat</td>
<td>1%</td>
</tr>
<tr>
<td><strong>Disaggregated data:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded: (out of 98)</td>
<td>-0.32</td>
<td>-2.43</td>
<td>47</td>
</tr>
<tr>
<td>Nontraded (out of 30)</td>
<td>-0.24</td>
<td>-2.12</td>
<td>8</td>
</tr>
<tr>
<td><strong>Aggregate data:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traded:</td>
<td>-0.28</td>
<td>-2.45</td>
<td>No</td>
</tr>
<tr>
<td>Non-traded</td>
<td>-0.22</td>
<td>-1.87</td>
<td>No</td>
</tr>
</tbody>
</table>

\(^1\)For disaggregated data, table reports \(b\) and t-stat means across the goods.

\(^2\)significance reports the number of goods that reject nonstationarity at the specified significance level.
Preliminaries: Speed of Convergence

- Estimate a second order autoregressive model of real exchange rates with panel data.

- We apply the common correlated regressor (CCE) of Pesaran (2006), pooled with heterogeneous slope coefficient.

- Estimate the following cross-section mean augmented auxiliary equation:

\[ q_{ij,t}^k = c_{ij}^k + \sum_{m=1}^{2} \rho_{ij,m}^k (q_{ij,t-m}^k) + \varepsilon_{ij,t}^k \quad \text{for } k = 1,\ldots, K \quad (4) \]
Table 2. Half-lives in Autoregressions

<table>
<thead>
<tr>
<th>Sample</th>
<th>(Mean) $\rho_1$</th>
<th>(Mean) t-stat</th>
<th>(Mean) $\rho_2$</th>
<th>(Mean) t-stat</th>
<th>(Mean) Half-life$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AR(2):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disaggreg. data</td>
<td>0.72</td>
<td>10.62</td>
<td>0.05</td>
<td>0.70</td>
<td>1.25</td>
</tr>
<tr>
<td>Aggregated data</td>
<td>0.90</td>
<td>13.88</td>
<td>-0.05</td>
<td>-1.20</td>
<td>2.10</td>
</tr>
<tr>
<td><strong>AR(1):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disaggreg. data</td>
<td>0.74</td>
<td>14.25</td>
<td></td>
<td></td>
<td>1.15</td>
</tr>
<tr>
<td>Aggregated data</td>
<td>0.85</td>
<td>20.40</td>
<td></td>
<td></td>
<td>2.13</td>
</tr>
</tbody>
</table>

$^1$For disaggregated data, table reports $b$ and t-stat medians across goods. Half-lives in years, computed from impulse responses.
- Half-life is shorter in disaggregated relative prices than aggregated real exchange rate.
- This was noted previously by Imbs et al. (2005 QJE).
- Their explanation was aggregation bias:
  - Goods are heterogeneous in terms of their convergence speeds;
  - under certain conditions, goods with high persistence get more weight in aggregate index.
- Note: Our aggregate half-life (2.1 years) shorter than most past studies (3 years) due to start date of data (1990). IFS data for 1990-2007 confirms 2.1 half-life.
IV. Results: Error Correction Puzzle

- Which variable responds to close relative price deviations: the nominal exchange rate component or price component?

- For each good, \( k \), est. a panel vector error correction model

\[
\Delta e_{ij,t} = \alpha_{ij,e}^k + \rho_{e,ij}^k (q_{ij,t-1}^k) + \mu_{e,ij}^k (\Delta e_{ij,t-1}^k) + \mu_{e,ij}^k (\Delta p_{ij,t-1}^k) + \zeta_{ij,t}^e \tag{5}
\]

\[
\Delta p_{ij,t} = \alpha_{ij,p}^k + \rho_{p,ij}^k (q_{ij,t-1}^k) + \mu_{p,ij}^k (\Delta e_{ij,t-1}^k) + \mu_{p,ij}^k (\Delta p_{ij,t-1}^k) + \zeta_{ij,t}^p
\]

(Where \( p_{ij,t}^k = p_{i,t}^k - p_{j,t}^k \) is the price ratio in local currencies)

- Regress changes in nominal exch. rate and price ratio on lagged price deviation (with lags and cross-section means).

- \( \rho_{e,ij}^k \) and \( \rho_{p,ij}^k \) measure the speed of adjustment of nominal exchange rate and price ratio.

- Estimate also one VECM for aggregate data.
Table 3: Vector Error Correction Estimates

<table>
<thead>
<tr>
<th></th>
<th>(mean) $\rho$</th>
<th>(mean) t-stat</th>
<th>Heterogeneity (Std.Dev.)$^1$</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1%</td>
</tr>
<tr>
<td><strong>Disaggregated Data (98 traded goods):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exch. rate eqn.</td>
<td>-0.028</td>
<td>-2.26</td>
<td>0.015</td>
<td>35</td>
</tr>
<tr>
<td>Price eqn.</td>
<td>-0.203</td>
<td>-4.07</td>
<td>0.087</td>
<td>75</td>
</tr>
<tr>
<td><strong>Aggregate Data:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exch. rate eqn.</td>
<td>-0.126</td>
<td>-3.52</td>
<td></td>
<td>yes</td>
</tr>
<tr>
<td>Price eqn.</td>
<td>-0.044</td>
<td>-3.38</td>
<td></td>
<td>yes</td>
</tr>
</tbody>
</table>

$^1$Standard dev. of estimates across goods reported as a measure of heterogeneity. For disaggregated data, values reported are means across goods. Significance reports the number of goods with coefficients significantly different from zero at the specified significance level.
Interpreting the Result

- For aggregate data:
  - The exchange rate response is large.
  - That for price is small.
  (as in Fisher-Park (1991) & Cheung et al. (2004))

- The opposite is true for disaggregated data:
  - The exchange rate response is small.
  - The price response is large.

- It should not be surprising that $e$ cannot resolve deviations from the LOP for all goods, since there are often as many goods overpriced as underpriced.

- More surprising is that goods prices are very responsive to LOP deviations, even though at aggregate level prices do not respond to PPP deviations.
Robustness: A similar conclusion is found when estimating ECM using data from Imbs et al. (2005)

Table 4: Vector error correction estimates using data set from Imbs et al. (2005)

<table>
<thead>
<tr>
<th></th>
<th>(mean)</th>
<th>(mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho$</td>
<td>t-stat</td>
</tr>
<tr>
<td><strong>Disaggregated Data:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange rate equation</td>
<td>-0.016</td>
<td>-2.54</td>
</tr>
<tr>
<td>Price ratio equation</td>
<td>-0.036</td>
<td>-3.61</td>
</tr>
<tr>
<td><strong>Aggregated Data:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange rate equation</td>
<td>-0.025</td>
<td>-2.84</td>
</tr>
<tr>
<td>Price ratio equation</td>
<td>-0.016</td>
<td>-2.77</td>
</tr>
</tbody>
</table>
Potential Explanations for the Result

1) **Measurement error** in the disaggregated data.

   Reject: Hausman test rejects measurement error for 1610 of the 1843 country-good series at the 5% level.

2) **Aggregation bias** (analogous to Imbs et al. for AR estimates): due to heterogeneity in the error correction coefficients.

   Unlikely: Since $|\rho_e^k| < |\rho_p^k|$ holds for all 98 goods in our sample, it is hard to imagine a weighting scheme that would reverse this inequality in the aggregate.

3) **Idiosyncratic shocks** …
V. The Role of Idiosyncratic Shocks

- **Main idea**: There are idiosyncratic shocks at the good level (industry-specific) that are distinct from aggregate (macroeconomic) shocks.

- The idiosyncratic shocks are volatile and the responses to them dominate in disaggregated data.

- But the idiosyncratic shocks cancel out upon aggregation, some make goods overpriced while others underpriced.

- So the responses to macro shocks dominate in the aggregate data.
Role of Idiosyncratic Shocks

Estimate a joint Vector Error Correction Model for $e, p^k$ and $p$, with 2 cointegrating vectors $[1 0 1]$ and $[0 1 -1]$. - combines aggregate and disaggregated data series. - and decomposes price deviations into $q^k$-$q$ and $q$.

\[
\Delta e_{ij,t} = \alpha_{ij,e}^k + \rho_{e,ij}^{k1} (q_{ij,t-1}^k - q_{ij,t-1}^k) + \rho_{e,ij}^{k2} (q_{ij,t-1}^k)
+ \mu_{e,ij,1}^k (\Delta e_{ij,t-1}) + \mu_{e,ij,2}^k (\Delta p_{ij,t-1}^k) + \mu_{e,ij,3}^k (\Delta p_{ij,t-1}^k) + \zeta_{e,ij,t}^k
\]

(6)

\[
\Delta p_{ij,t} = \alpha_{p,ij}^k + \rho_{p,ij}^{k1} (q_{ij,t-1}^k - q_{ij,t-1}^k) + \rho_{p,ij}^{k2} (q_{ij,t-1}^k)
+ \mu_{p,ij,1}^k (\Delta e_{ij,t-1}) + \mu_{p,ij,2}^k (\Delta p_{ij,t-1}^k) + \mu_{p,ij,3}^k (\Delta p_{ij,t-1}^k) + \zeta_{p,ij,t}^k
\]

\[
\Delta p_{ij,t}^k = \alpha_{p,ij}^{k1} + \rho_{p,ij}^{k1} (q_{ij,t-1}^k - q_{ij,t-1}^k) + \rho_{p,ij}^{k2} (q_{ij,t-1}^k)
+ \mu_{p,ij,1}^{k1} (\Delta e_{ij,t-1}) + \mu_{p,ij,2}^{k1} (\Delta p_{ij,t-1}^k) + \mu_{p,ij,3}^{k1} (\Delta p_{ij,t-1}^k) + \zeta_{p,ij,t}^{k1}
\]
VECM Parameters

- Parameter estimates extend conclusions from the pair of earlier VECM estimations:
  - \( p_k \) responds to \( q_k - q \) deviations and not to \( q \) deviations.
  - \( e \) responds to aggregate \( q \) deviations but not to \( q_k - q \) deviations.
  - \( p \) responds only to \( q \) deviations.
### 3-Equation VECM Parameter Estimates

<table>
<thead>
<tr>
<th>Equation Type</th>
<th>Response to $q_k - q$</th>
<th>Response to $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean $\rho$</td>
<td>Mean t-stat</td>
</tr>
<tr>
<td>Exchange rate equation</td>
<td>-0.002</td>
<td>-0.095</td>
</tr>
<tr>
<td>Aggregated Price equation</td>
<td>0.001</td>
<td>0.006</td>
</tr>
<tr>
<td>Disaggregated Price equation</td>
<td>-0.301</td>
<td>-3.612</td>
</tr>
</tbody>
</table>
Role of Idiosyncratic Shocks

- Next, we identify the disaggregated shocks using Cholesky ordering: $e, p, p_k$.
- which defines an idiosyncratic shock as one that makes $p_k$ move for a particular good, but has no contemporaneous effect on aggregate $p$ (or $e$).
- An aggregate goods market shock makes both $p_k$ and $p$ move (but not $e$): affects goods prices on avg.
- In addition, we can identify a shock to the foreign exchange market, as one that makes $e$ move. This will be a second type of aggregate shock.
- Plot variance decompositions and impulse responses: mean across goods, and bands for 1 standard deviation in the distribution across goods.
Variance Decomposition of $q_k$:

- Most movement in $q_k$ is due to $p_k$ shocks, some portion due to $e$ shocks.
Variance decomposition of $q$:

- Most movement in $q$ is due to $e$ shocks, some portion due to $p$ shocks.
Impulse responses to $p_k$ shock:

- For an idiosyncratic shock, the dynamics of $q_k$ look like $p_k$: $p_k$ does the adjusting conditional on shock. No significant effects on aggregates.
For a foreign exchange market \((e)\) shocks: \(q\) looks like \(e\): \(e\) does the adjusting. (Some effects at disaggregated level, with some adjustment in \(p_k\).)
For an aggregate \((p)\) shock, \(q\) looks like \(e\): \(e\) does the adjusting (no significant effects at disaggregated level)
Summarizing Impulse Responses

- Good’s price ($p_k$) does the adjusting in response to idiosyncratic shocks.
- Nominal exchange rate ($e$) does the adjusting to both foreign exchange market shock and macro shock ($p$).
Formalize Decomposition of Adjustment:


$$\psi_{q_k,n} (t) = \psi_{e,n} (t) + \psi_{p_k,n} (t)$$

- Then $g_{q_k}^n (t) = \Delta \psi_{e,n} (t) / \Delta \psi_{q_k,n} (t)$

measures proportion of adjustment in LOP deviations explained by nominal exch. rate adjustment;

- And $g_{p_k,n}^q (t) = \Delta \psi_{p_k,n} (t) / \Delta \psi_{q_k,n} (t)$

measures the proportion explained by price adjustment, where $g_{e,n}^q (t) + g_{p_k,n}^q (t) = 1$.

- Analogous for aggregated data.
Decomposition of Adjustment

<table>
<thead>
<tr>
<th>Disagg. $q_k$:</th>
<th>years</th>
<th>under an exchange rate shock</th>
<th>under an aggregate price shock</th>
<th>under a disaggregate price shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$g^{qk}_{e,e}$</td>
<td>$g^{qk}_{p,k,e}$</td>
<td>$g^{qk}_{e,p}$</td>
</tr>
<tr>
<td>1</td>
<td>0.75</td>
<td>0.25</td>
<td>0.66</td>
<td>0.34</td>
</tr>
<tr>
<td>2</td>
<td>0.79</td>
<td>0.21</td>
<td>0.76</td>
<td>0.24</td>
</tr>
<tr>
<td>3</td>
<td>0.79</td>
<td>0.21</td>
<td>0.78</td>
<td>0.22</td>
</tr>
<tr>
<td>5</td>
<td>0.81</td>
<td>0.19</td>
<td>0.82</td>
<td>0.18</td>
</tr>
<tr>
<td>10</td>
<td>0.90</td>
<td>0.10</td>
<td>0.96</td>
<td>0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>aggregate $q$:</th>
<th>years</th>
<th>$g^{q}_{e,e}$</th>
<th>$g^{q}_{p,e}$</th>
<th>$g^{q}_{e,p}$</th>
<th>$g^{q}_{p,p}$</th>
<th>$g^{q}_{e,p,k}$</th>
<th>$g^{q}_{p,k,p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.76</td>
<td>0.24</td>
<td>0.74</td>
<td>0.26</td>
<td>0.88</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.79</td>
<td>0.21</td>
<td>0.77</td>
<td>0.23</td>
<td>0.89</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.78</td>
<td>0.22</td>
<td>0.77</td>
<td>0.23</td>
<td>0.87</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.78</td>
<td>0.22</td>
<td>0.76</td>
<td>0.24</td>
<td>1.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.77</td>
<td>0.23</td>
<td>0.75</td>
<td>0.25</td>
<td>1.07</td>
<td>-0.07</td>
<td></td>
</tr>
</tbody>
</table>
Half-lives conditional on shock, computed from Impulse responses above:

<table>
<thead>
<tr>
<th></th>
<th>$e$ shock</th>
<th>$p$ shock</th>
<th>$p_k$ shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disagg. $q_k$</td>
<td>1.54</td>
<td>1.41</td>
<td>0.87</td>
</tr>
<tr>
<td>Aggregated $q$</td>
<td>1.51</td>
<td>1.66</td>
<td>---</td>
</tr>
</tbody>
</table>

Halflife in years

- Note: conditional on shock, half-lives are very similar for aggregated and disaggregate data. No apparent Imbs aggregation puzzle.
- The main distinction is between agg. v. disag shocks.
Summary of 3-Equation VECM Results:

- Price deviations in aggregate and disaggregated data are driven by different shocks: $q_k$ by idiosyncratic shocks, $q$ by aggregate shocks ($e$ and $p$).
- Dynamic responses differ by shock: $p_k$ responds to $p_k$ shock, $e$ responds to $e$ shocks and $p$ shocks.
- Conditional on shock, the half-lives of aggregated and disaggregate data are very similar.
- So the apparent inconsistency in adjustment speeds and dynamics for aggregated and disaggregated data comes from the distinction in the underlying shocks.
VI. Implication for Aggregation Bias

- First, we provided an alternative explanation for the persistence puzzle to Imbs et al. (2005).
- We show below that some of the heterogeneity observed by Imbs et al. cancels out in aggregation.
- We show below that distinct responses to shocks implies an omitted variable bias in standard estimates of autoregressions not allowing for this.
Implication for Aggregation Bias

- Recall standard AR(1) estimation of persistence:

\[ q_{ij,t}^k = c_{ij}^k + \rho_{ij}^k q_{ij,t-1}^k + \epsilon_{ij,t}^k \quad \text{for} \ k = 1,\ldots,K \quad \text{for disagg. data (7)} \]

\[ q_{ij,t} = c_{ij} + \rho_{ij} q_{ij,t-1} + \epsilon_{ij,t} \quad \text{for aggregates (8)} \]

- Aggregate up (7):

\[ q_{ij,t} = \frac{1}{K} \sum_{k=1}^{K} q_{ij,t}^k = \frac{1}{K} \sum_{k=1}^{K} \left( c_{ij}^k + \rho_{ij}^k q_{ij,t-1}^k + \epsilon_{ij,t}^k \right) \]

- Imbs et al. note that:

\[ \frac{1}{K} \sum_{k=1}^{K} (\rho_{ij}^k q_{ij,t-1}^k) \neq \rho_{ij} q_{ij,t-1} \]

- If \( \rho_{ij}^k \) and \( q_{ij,t-1}^k \) positively correlated, then bias in estimating \( \rho_{ij} \) can be positive.
Implication for Aggregation Bias

- Our results suggest the following AR specification:

\[ q_{ij,t}^k = c_{qk,ij}^k + \rho_{qk,ij}^{k1} \left( q_{ij,t-1}^k - q_{ij,t-1} \right) + \rho_{qk,ij}^{k2} q_{ij,t-1} + \epsilon_{qk,ij,t}^k \] (9)

- Aggregate this up:

\[ q_{ij,t} = \frac{1}{K} \sum_{k=1}^{K} q_{ij,t}^k = \frac{1}{K} \sum_{k=1}^{K} c_{qk,ij}^k + \frac{1}{K} \sum_{k=1}^{K} \rho_{qk,ij}^{k1} \left( q_{ij,t-1}^k - q_{ij,t-1} \right) + q_{ij,t-1} + \frac{1}{K} \sum_{k=1}^{K} \rho_{qk,ij}^{k2} + \frac{1}{K} \sum_{k=1}^{K} \epsilon_{qk,ij,t}^k \] (10)

- Heterogeneity in \( \rho_{qk,ij}^{k1} \) can lead to a heterogeneity bias in Term A as in Imbs et al.

- But heterogeneity in \( \rho_{qk,ij}^{k2} \) has no impact on aggregation of Term B, as the common component passes through the summation and cancels.
Estimating equation (9):

<table>
<thead>
<tr>
<th></th>
<th>Response to $q_{k-q}$</th>
<th>Response to $q$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean $\rho$</td>
<td>Mean t-stat</td>
</tr>
<tr>
<td>Disagg. data</td>
<td>0.68</td>
<td>9.55</td>
</tr>
<tr>
<td>Aggreg. data</td>
<td>-0.00</td>
<td>-1.04</td>
</tr>
</tbody>
</table>

- Conditional on aggregated deviations ($q$), speeds of adjustment very similar for agg and disagg: .79, .80.
- Much of the heterogeneity in disagg. data will cancel in aggregation, since associated with agg. deviations.
- Term A has little impact, since $\rho^{k1}$ uncorrelated with $(q_{ij,t-1} - q_{ij,t-1})$
Omitted Variable Bias

- Lastly, estimating (7) as in Imbs, instead of (9), implies an omitted variable bias in estimate of $\rho_{ij}^k$

$$E\left[\hat{\rho}^{kl}\right] - \rho^{kl} = \left(\sum_{ij=1}^{N} Q_{ij,-1}^k M_w Q_{ij,-1}^k\right)^{-1} \left(\sum_{ij=1}^{N} Q_{ij,-1}^k M_w Q_{ij,-1}^k\right) \left(\rho^{k2} - \rho^{k1}\right)$$

where

$Q_{ij,-1}^k = (q_{ij,1}^k, q_{ij,2}^k, \ldots, q_{ij,T-1}^k)'$

$M_w = I - W( W'W)^{-1} W', W = (W_2', W_3', \ldots, W_T')', W_t = (1, q_t^k, \bar{q}_{t-1}^k)$

- Because $\left(\rho^{k2} - \rho^{k1}\right) > 0$, this bias is positive.
Implication for Sticky Price Models

- We find evidence against a sticky price explanation of real exchange rate volatility:

- Prices are actually very responsive, but selectively to goods-level shocks, not aggregate shocks.

- May suggest a ‘rational inattention model’ instead of standard sticky price model, where firms respond selectively to local industry information.

- Extend closed economy rational inattention model of Mackowiak and Wiederholt (2009 AER), or use sticky information of Crucini, Shintani, Tsuruga (2010 JIE).
Review Paper’s Findings

1) Dynamic adjustment in disaggregated data is not just a faster version of the adjustment in aggregate data. While adjustment to PPP occurs in foreign exchange market, adjustment to LOP occurs in goods market.

2) This is due to distinct shocks: macro shocks dominate in aggregate data, but industry shocks in micro data.

3) Conditional on macro shocks, microeconomic prices are just as persistent as aggregate real exchange rate.

4) Has implications for theories of the real exchange rate based on aggregation bias, and sticky price models.