The Elasticity of Trade: Estimates and Evidence

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Big Picture

The elasticity of trade with respect to trade frictions, $\theta$, is critical to any quantitative analysis.

Depending on this elasticity... 

- The “size” of the U.S.-Canada border, Anderson and Van Wincoop (2003)
- Tariff reductions role in growth of world trade, Yi (2003)
- Welfare gains in many trade models, Arkolakis, Costinot, and Rodriguez-Clare (2009)

Eaton and Kortum (2002): innovative approach to recover trade frictions and estimate $\theta$. Their estimate is widely referenced in quantitative applications.
Our Paper...

Our paper:

- Prove the EK (2002) estimate is biased and provide monte carlo evidence that the bias is substantial.

- New method to estimate $\theta$ under same data requirements as EK (2002) that resolves this bias.

- New estimate for 123 countries that comprise 98% of world GDP

- These results $\Rightarrow$ welfare cost of autarky is twice as high with our $\theta$. 

\( N \) countries indexed by \( i \). A continuum of tradable goods \( x(j), j \in [0, 1] \).

- Consumer preferences over goods

\[
U_i = \left[ \int_0^1 x_i(j)^{\frac{1}{\rho-1}} dj \right]^{\frac{\rho}{\rho-1}}.
\]

- Firms from country \( i \) can supply \( x(j) \) to country \( n \) at price

\[
p_{ni}(j) = \tau_{ni} \frac{w_i}{z_i(j)}.
\]

- Productivity, \( z_i(j) \), is drawn from a Fréchet distribution

\[
F_i(z_i) = \exp \left( -T_i z_i^{-\theta} \right),
\]

where \( \theta \) is the key parameter of interest.
Expenditure share of $i$ goods in $n$

$$\frac{X_{ni}}{X_n} = \frac{T_i (\tau_{ni}w_i)^{-\theta}}{\sum_{k=1}^{N} T_k (\tau_{nk}w_k)^{-\theta}}.$$

$\theta$ controls . . .

- dispersion in productivity draws,
- response of trade to changes in trade costs,
- welfare gains from trade.

Expenditure share of $i$ goods in $n$

$$\frac{X_{ni}}{X_n} = \frac{T_i(\tau_{ni} w_i)^{-\theta}}{\sum_{k=1}^{N} T_k(\tau_{nk} w_k)^{-\theta}}.$$


- Across all these models the EK (2002) estimate of $\theta$ is widely referenced in quantitative applications.
They derive the relationship

\[
\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left( \frac{P_{ni} \tau_{ni}}{P_n} \right)^{-\theta}
\]

where:

\[
P_n = \gamma \left[ \sum_{k=1}^{N} T_k (\tau_{nk} w_k)^{-\theta} \right]^{-\frac{1}{\theta}},
\]

\(\gamma\) is constant.
EK (2002) Approach to Estimating $\theta$

Take logs and run the regression...

$$\log \left( \frac{X_{ni}}{X_n} \right) = -\beta \left( \log \tau_{ni} + \log P_i - \log P_n \right)$$

Key issues

- How to approximate $\log \tau_{ni}$?
- Given the approximation, does the expected value of $\beta = \theta$?
Approximating the Trade Friction

Suppose we see prices of good $\ell$ across countries.

If we know that good $\ell$ in country $n$ came from $i$, then

$$\frac{p_n(\ell)}{p_i(\ell)} = \tau_{ni}.$$  

If we do not know where good $\ell$ came from, following inequality must hold:

$$p_n(\ell) \leq \tau_{ni} p_i(\ell).$$

Why? If $p_n(\ell) > \tau_{ni} p_i(\ell)$, then one could have imported $\ell$ at a lower price.
Approximating the Trade Friction

The inequality $p_n(\ell) \leq \tau_{ni} p_i(\ell)$ puts a lower bound on the unknown trade cost

$$\frac{p_n(\ell)}{p_i(\ell)} \leq \tau_{ni}$$

Seeing $L$ goods improves this bound

$$\max_{\ell \in L} \left\{ \frac{p_n(\ell)}{p_i(\ell)} \right\} \leq \tau_{ni}$$
EK (2002) Approach to Estimating $\theta$

Their estimator...

$$\hat{\beta} = -\frac{\sum n \sum_i \log \left( \frac{X_{ni}/X_n}{X_{ii}/X_i} \right)}{\sum n \sum_i \left( \log \hat{r}_{ni} + \log \hat{P}_i - \log \hat{P}_n \right)}$$

Variables in the denominator are...

- $\log \hat{r}_{ni} = \max_{\ell \in L} \{ \log p_n(\ell) - \log p_i(\ell) \}$
- $\hat{P}_i = \frac{1}{L} \sum_{\ell=1}^L p_i(\ell)$
- $p_i(\ell)$ are r.v.'s sampled from the equilibrium distribution of prices.
Proposition 1: EK (2002) Estimator is Biased Upward

If a sample of $L$ prices is available, then

$$
E(\hat{\beta}) = -\theta \frac{\sum_n \sum_i \log \left( \frac{X_{ni}/X_n}{X_{ii}/X_i} \right)}{\sum_n \sum_i (\Psi_{ni}(L; S) - \Omega_{ni})} > \theta
$$

where:

$$
\Psi_{ni}(L; S) = \int_{-\theta \log(\tau_{in})}^{\theta \log(\tau_{ni})} \epsilon_{ni} f_{\max}(\epsilon_{ni}; L, S) d\epsilon_{ni} < \theta \log(\tau_{ni})
$$

$$
\epsilon_{ni} = \theta (\log p_n - \log p_i), \quad f_{\max}(\epsilon_{ni}; L) = \text{pdf of max}\{\epsilon_{ni}\},
$$

$$
\Omega_{ni} = \int_{-\theta \log(\tau_{in})}^{\theta \log(\tau_{ni})} \epsilon_{ni} f(\epsilon_{ni}) d\epsilon_{ni}, \quad S = \{\log(T_1 w_1^{-\theta}), \ldots, \log(T_n w_n^{-\theta})\}.
$$
Why? Because Max Price $\Delta < True Trade Cost$

If the value of the max price $\Delta$ equals the trade cost in expectation, no problem.

$$\int_{-\theta \log(\tau_{in})}^{\theta \log(\tau_{ni})} \epsilon_{ni} f_{\max}(\epsilon_{ni}; L, S) d\epsilon_{ni} = \theta \log(\tau_{ni}) \Rightarrow E(\hat{\beta}) = \theta$$

This can not be the case.

- With positive probability the max price $\Delta <$ the true trade cost.
- With zero probability it can not be larger than the true trade cost.

Thus

$$\int_{-\theta \log(\tau_{in})}^{\theta \log(\tau_{ni})} \epsilon_{ni} f_{\max}(\epsilon_{ni}; L, S) d\epsilon_{ni} < \theta \log(\tau_{ni}) \Rightarrow E(\hat{\beta}) > \theta$$
Simple Monte Carlo exercise:

- Simulate the model with a known $\theta$.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Mean Estimate of $\theta$ (S.D.)</th>
<th>Median Estimate of $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EK (2002) Estimator</td>
<td>12.5 (0.60)</td>
<td>12.5</td>
</tr>
<tr>
<td>Least Squares</td>
<td>12.1 (0.60)</td>
<td>12.1</td>
</tr>
</tbody>
</table>

**Note:** In each simulation there are 19 countries and 100,000 goods. Only 50 realized prices are randomly sampled and used to estimate $\theta$. 100 simulations performed.
Proposition 2: As $L \to \infty$, the Bias is Eliminated

As the sample size $L$ approaches $\infty$,

$$\text{plim}_{L \to \infty} \left( \sum_n \sum_i \log \left( \frac{X_{ni} / X_n}{X_{ii} / X_i} \right) / \sum_n \sum_i (\Psi_{ni}(L; S) - \Omega_{ni}) \right) = \theta$$

In words:

- As the sample size increases, the probability that the max price $\Delta < \tau$ becomes vanishingly small.

- Problem — the sample size needs to be very large.
Another monte carlo exercise:

- Simulate the model with a known $\theta$.
- Vary the sample size and apply EK (2002) estimation approach.

### Monte Carlo Results, **True $\theta = 8.28$**

<table>
<thead>
<tr>
<th>Sample Size of Prices</th>
<th>Mean Estimate of $\theta$ (S.D.)</th>
<th>Median Estimate of $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>12.14 (0.60)</td>
<td>12.15</td>
</tr>
<tr>
<td>500</td>
<td>9.41 (0.22)</td>
<td>9.40</td>
</tr>
<tr>
<td>5,000</td>
<td>8.47 (0.08)</td>
<td>8.46</td>
</tr>
<tr>
<td>50,000</td>
<td>8.29 (0.06)</td>
<td>8.29</td>
</tr>
</tbody>
</table>

**Note:** S.E.M. is the standard error of the mean. In each simulation there are 19 countries and 100,000 goods. 100 simulations performed.
What Does This All Mean? Need A New Estimation Strategy

Proposition 1 ⇒

- EK (2002) estimate is, at best, an upper bound.
  - Not a tight one though . . .

- Welfare gains from trade are underestimated.

Proposition 2 ⇒

- Finding and adding more data is not feasible.

This motivates an alternative estimation strategy.
New Estimation Approach — Overview

Use simulated method of moments . . .

- Moment is the estimate from EK (2002) approach.
  - Proposition 1 ⇒ this is a meaningful moment.

- Our estimator minimizes the distance between the moment on artificial and real data.

Next couple of slides

- Formalize this idea.

- Monte carlo evidence that our approach works.

- Results using new ICP price data for 123 countries.
Simulation Approach Overview

1. Fix $\theta$.

2. Estimate marginal cost parameters ($w_i^{-\theta} T_i$) and $\theta \log(\tau_{ni})$ — up to an unknown scalar $\theta$ — from only trade data, via gravity.

3. Discretize the continuum. Simulate trade flows and micro-level prices.

4. Define subset of goods common to all countries and collect sample of prices.

5. Estimate $\beta(\theta)$ using artificial data and compare $\beta$ from real data.

6. Update $\theta$ until $\beta(\theta)$ is “close” to $\beta$. 
Moments

Data moment is:

\[
\beta = -\frac{\sum_{n} \sum_{i} \log \left(\frac{X_{ni}/X_{n}}{X_{ii}/X_{i}}\right)}{\sum_{n} \sum_{i} \left(\log \hat{r}_{ni} + \log \hat{P}_i - \log \hat{P}_n\right)}
\]

Model moment \(\beta(\theta, u_s)\) is the analog using artificial data.

- \(u_s\) is vector of random variables specific to simulation \(s\).
A zero function

\[ y(\theta) = \left[ \beta - \frac{1}{S} \sum_{s=1}^{S} \beta(\theta, u_s) \right]. \]

Our estimation is based on the moment condition

\[ E [y(\theta_o)] = 0, \quad \text{where } \theta_o = \text{true value}. \]

Thus our estimator is . . .

\[ \hat{\theta} = \arg \min_{\theta} [y(\theta)'y(\theta)]. \]
Proposition 1 $\Rightarrow \beta$ is biased but linear in $\theta$
Intuition Behind Our Approach...

Estimate of $\beta(\theta)$

Moment Seen in Data

Estimate of $\theta$
Does Our Approach Work? Yes.

<table>
<thead>
<tr>
<th>Estimation Approach</th>
<th>Mean Estimate of $\theta$</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>True $\theta = 8.28$</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMM</td>
<td><strong>8.19</strong></td>
<td>0.41</td>
</tr>
<tr>
<td>EK (2002) Estimator, $\beta$</td>
<td>12.37</td>
<td>0.61</td>
</tr>
</tbody>
</table>

**Note:** In each simulation there are 19 countries and 100,000 goods and 100 simulations performed.
Overview of New 2005 ICP Data

Data:

  - Price data on goods with identical characteristics across retail locations in the participating countries.
  - Analogous to data used in EK (2002), but more countries & goods

- Trade shares from data on bilateral trade and production of manufactures among 123 countries in 2004.
Estimation Results With 2005 ICP Data

<table>
<thead>
<tr>
<th>Estimation Approach</th>
<th>Estimate of $\theta$</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMM</td>
<td>4.22</td>
<td>0.08</td>
</tr>
<tr>
<td>EK (2002) Estimator, $\beta$</td>
<td>7.75</td>
<td>0.03</td>
</tr>
</tbody>
</table>

**Note:** In each simulation there are 123 countries and 100,000 goods. Only 62 realized prices are randomly sampled.
Our estimates range from 3.9 to 4.5, not 7 to 9 as EK (02) approach suggests.

Our estimates are consistent with approaches not using max over price data . . .

- EK (2002) using wage data: 3.6
- BEJK (2003)/EKK (2008) using U.S./French firm level data: 3.6/4.8
- Donaldson (2009) price of salt: 3.8 to 5.2
- Burstein and Vogel (2009) skill intensity in U.S. trade: 4
- Simonovska (2009) average mark-up in OECD: 3.8

Our (implied) estimates of $\rho$ range from 3.4 to 5.2 and are consistent with:

- Romalis (2007): 4-13
- Hummels (2001): 3-8
- Broda and Weinstein (2006): median of 3.1
Why This Matters... Large Welfare Implications

Why care about $\theta$?

Welfare cost of autarky:

$$\frac{1}{\theta} \log \left( \frac{X_{nn}}{X_n} \right).$$

- $\frac{X_{nn}}{X_n}$ = domestic expenditure share.
- $\frac{X_{nn}}{X_n}$ = 1 in autarky.

New $\theta$ of 4 rather than 8 doubles the welfare cost of autarky.
Arkolakis, Costinot, and Rodriguez-Clare (2009):

- New trade models with micro-level heterogeneity and older models without heterogeneity generate previous formula,
- \( \Rightarrow \) new trade models yield no additional welfare gains from trade above older models.

But... 

- Only if these models have the same \( \theta \).
- And micro-level heterogeneity affects the inference about \( \theta \) — our paper demonstrates this.

Moving forward...

- Use our approach within the Melitz framework.
Conclusion

What we did . . .

Proved that EK (2002) estimate is biased upward. Monte carlo evidence supported this.

- Small sample of prices ⇒ poor approximation of trade frictions.

New method to estimate $\theta$ under same data requirements as EK (2002) and applied it to a new data set for 123 countries that comprise 98% of world GDP.

New $\theta$ is 4, rather than 8.

- **Doubles** the welfare cost of autarky.
Robustness

Estimates of $\theta$ are robust to:

- Number of goods — as long as it is large (50,000+)
- Measurement error — as long as it is not too large ($\leq 20\%$ of price)
- Aggregation — used more disaggregate data from EIU
  - Approach yields lower estimates $\approx 2.6$
- Country-specific taxes & distribution costs
  - Model with ad-valorem tax yields identical estimating equation
- Good- and country-specific mark-ups
  - Estimates from high- and low-end stores using EIU data are similar