Demographic Patterns and Household Saving in China∗

Chadwick C. Curtis  Steven Lugauer
University of Notre Dame University of Notre Dame

Nelson C. Mark
University of Notre Dame and NBER

December 2010. This version: May 2012

Abstract

This paper studies how changing demographics can explain much of the evolution of China’s household saving rate from 1955 to 2009. We undertake a quantitative investigation using an overlapping generations model in which agents live for 85 years. Agents begin to exercise decision making when they are 20. From age 20 to 63, they work. From age 20 to 49, they also provide for children. Dependent children’s consumption enters into the parent’s utility, and parents choose the consumption level of the young until they leave the household. Working agents transfer a portion of their labor income to their retired parents and save for their own retirement. Retirees live off of their accumulated assets and support from current workers. We present agents in the parameterized model with the future time-path of the demographics, interest rates and wages as given by the data and observe their saving decisions. The simulated model accounts for nearly all the observed increase in the household saving rate from 1955 to 2009.

Keywords: Saving Rate, Life-Cycle, China, Demographics, Overlapping Generations

JEL: E2, J1

∗We thank Chris Carroll, Marcus Chamon, Joe Kaboski, Laurence Kotlikoff, Hui He and Guonan Ma for comments on an earlier draft. We have also benefited from comments by seminar participants at the Federal Reserve Banks of Kansas City and St. Louis, Notre Dame’s macro research group, the Kellogg Institute of International Studies, Hong Kong Institute for Monetary Research, Columbia-Tsinghua conference, Bundesbank Workshop on China, NBER China working group, NBER IFM group, Western Michigan University, University of North Dakota, and the 2011 Society of Economic Dynamics and Midwest Macro Meetings.
1 Introduction

China’s economy is large. It has recently overtaken Japan as the world’s second largest in terms of aggregate GDP. China’s saving rate is also large and exceeds that of every other large country. The household component of Chinese saving, which in 2009 was 27 percent of income, contributes heavily to the country’s investment led growth and helps to finance large purchases of assets denominated in US dollars. In 2008, the US current account deficit with China was nearly $270 billion or about 2% of US GDP. The emergence and growth of China’s external surplus underlies calls for a ‘rebalancing’ of China’s growth from investment to consumption (which necessarily means lowering the saving rate). We believe having a thorough and accurate understanding of the factors that currently contribute to the high saving rate is necessary for the useful and effective design of rebalancing policies. This paper aims to contribute to this understanding.

China’s household saving rate has not always been so high. From 1955 to 1977 its aggregate household saving rate fluctuated around a low average rate of less than 5 percent per year. Then starting in 1978, the saving rate began to rise so that by 2009 Chinese households were saving an astonishing 27 percent of their income. The timing of this change in saving behavior coincides with the beginnings of an equally dramatic demographic transition. Over the time-span of our study, China’s population went from mostly young to predominantly middle-aged. At the peak of China’s baby-boom in the 1960s and 1970s, nearly half of the population was under 20 years old. Then, largely as a result of the government’s one-child policy, fertility rates plummeted. Today, less than 25 percent of the population is under 20, and the age distribution will continue to skew older for the foreseeable future. The share of China’s population older than 63 could surpass the fraction under 20 before 2035, an event not expected to occur in the US until 2075 according to United Nations projections.

These demographic and saving rate shifts are likely related. This paper studies how the changes in China’s demographic profile have affected its household saving rate over time. We undertake a quantitative investigation to determine the importance of several demographic channels of influence by incorporating support for dependent children and transfers to retirees into a medium-scale overlapping generations (OLG) model. In the model, individuals live for 85 periods or years. From birth through age 19, individuals make no decisions and depend on their parents for consumption. At age 20, individuals begin making their own consumption / saving decisions; from 20 to 49 they work and raise children. Dependent children’s consumption enters into the parent’s utility, and parents choose the consumption level of their young. Individuals continue to work from 50 until retiring at 63, but they no longer have children to support. All working age agents save for their own retirement and transfer a portion of their labor income to current retirees. The income transfer is meant to capture both the formal pension system (which currently has a low participation rate in China) and the informal family network. Retirees live off of their accumulated assets and the support from current workers. A representative firm produces by hiring the workers and renting capital from an intermediary bank.

We conduct the analysis in partial equilibrium. Agents take the evolution over time of the demographic structure (including family size), interest rates, and wages, as given. We present
agents in the parameterized model with the future time-path of the demographics, interest rates and wages as given by the data and observe their saving decisions from 1955 to 2009. In comparing the implied saving rates by agents in the model to the data, we find that the model can account for nearly all of the observed increase in the household saving rate.¹

We emphasize three distinct channels through which the changing age distribution can affect the saving rate. The first channel operates through the decline in the number of dependent children (family size) brought about by the one-child policy which frees up household resources for saving. A household with relatively few children devotes a smaller share of household income to support dependents and therefore has more to save. A second channel operates through the composition effect. Most saving is created from un consumed labor income and it is the working population that earns labor income. The prime working age group (20-63) in China has increased from 46 percent of the population in 1970 to 65 percent today. All else equal, the increased importance of this age group mechanically raises the aggregate saving rate. The third channel operates through the projected future decline in the number of workers per retiree. The amount of pension support retirees receive depends on the relative size of the working age cohort (their children), and current workers have few children. Thus, in the model, the current working age population saves more aggressively because when they retire there will be relatively fewer workers to pay into the pension system.

To separate demographic influences on the saving rate from other aspects of the economy, we simulate the model holding wages and interest rates constant at their 1970 values. This experiment reveals that demographic changes alone (jointly operating through the three channels described above) generate over half of the observed increase in China’s household saving rate between 1955 and 2009. It also matches the timing (in 1978) when the saving rate begins to rise. We run additional experiments that turn off and on various features of the benchmark model to isolate the three demographic sub-channels individually. Here, we find that the decline in the number of children since the 1970s has had a large impact on household saving. Holding the number of dependent children constant at the 1970 level generates a household saving rate in 2009 that is 8 percentage points lower than in our benchmark simulation. The composition effect and variations in expected pension receipts qualitatively impact the saving rate in the expected fashion but their quantitative effects are smaller.

Our paper is part of an active research program that studies Chinese household saving. Modigliani and Cao (2004) and Horioka and Wan (2007) investigate life-cycle considerations and demographic variation in empirical studies.² These authors show that China’s age structure and saving rate have been related over a 50 year period, but they rely on dependency ratios in reduced form regressions rather than accounting for the entire age distribution as we do.³ Wei and Zhang (2011) hypothesize that the male sex imbalance, resulting from the one-child policy of population control

1The model captures the long-run trend in the household saving rate. The model does less well with the timing of high frequency changes, particularly before 1970 when China was still experiencing the after effects from the Great Leap Forward and the Great Famine.

²See also Horioka (2010) who analyzes the effect of an aging population on China’s household saving rate.

³Horioka and Terada-Hagiwara (2011) find that demographics help explain the evolution of saving rates in a panel of Asian countries, again using forms of the dependency ratio in reduced form regressions.
and the Chinese cultural preference for sons, has raised the saving rate because families with one son compete for a spouse in the marriage market through wealth accumulation, and as the sex ratio has become more imbalanced, the intensity of this wealth competition has increased in recent years. Banerjee et al. (2010) study individual-level survey data and report evidence that saving by the parents of daughters is higher than by the parents of sons. Their explanation is that parents of daughters need to save more so as to provide for themselves in retirement due to the cultural convention that sons (not daughters) provide for parents in old age. The role of the precautionary motive for saving and increasing riskiness of economic life caused by changes in industrial policy (the relative shrinking of the state-owned enterprise sector) and social welfare (shifting responsibility for paying for education, housing, and medical care) has been studied by Meng (2003), Chamon et al. (2010) and Chamon and Prasad (2010).

Recent work that employs the OLG framework to study saving includes Ferrero (2010), who finds an important role for demographics in explaining the long run trend in US saving relative to other G6 countries. Krueger and Ludwig (2007) and Fehr et al. (2007) show the importance of demographic change in multi-country OLG models. Chen et al. (2006) argue that the decline in population growth has had only a small effect on the Japanese saving rate. Our approach differs from these; we enter consumption by children directly into household utility as in the Barro and Becker (1989) model, and we apply the model specifically to China.4

Our paper also makes contact with the broader literature on how demographic changes affect the macro-economy. Shimer (2001) details how the age distribution impacts unemployment rates; Feyrer (2007) relates demographic change to productivity growth; and Jaimovich and Siu (2009) and Lugauer (2010) connect the age distribution to the magnitude of business cycles. Next, we present data on the unprecedented changes to China’s age distribution and household saving rate.

The remainder of the paper is organized as follows. The next section undertakes an examination of the data to motivate connections between Chinese demographics and household saving rates and to inform our modeling choices. Section 3 presents the model used in the quantitative analysis. Section 4 discusses our parameterization of the model. Results of the quantitative analysis are presented in Section 5. Section 6 concludes.

2 Household Saving Rate and Demographic Features of the Chinese Economy

This section draws out potential connections between the demographics and the saving rate in China. The first subsection begins with a discussion of the saving rate data that we want to understand and how its variation over time may be dependent on demographic variation at the macro level. Subsection 2.2 provides motivational evidence from micro-level data on the importance of family size for household saving.

---

4 Song and Yang (2010) also study Chinese household saving using an OLG model. They mainly focus on the effects of the flattening age-earnings profile observed in China.


2.1 Motivation from Macro Data

Figure 1 plots China’s household saving rate (household saving divided by household income) from 1955 to 2009. This is the time-series data that we seek to understand. These data come from Modigliani and Cao (2004) and various issues of the China Statistical Yearbook. We use the available information as provided by these sources without modification.5 The saving rate stayed low until 1978 when it began to increase rather dramatically over time. In contrast, the rising household saving rate did not occur in other large countries over the same time period. For example, the OECD reports China’s household saving as 22.2 percent of GDP in 2007, compared to 4.1 percent in the US and 5.8 percent in Japan. If anything, households in other large economies have been decreasing their saving rates.

The upward trend of China’s saving rate since 1978, however, has not been altogether steady and has been punctuated by sizable fluctuations. In a short 6 year span between 1978 and 1984 the saving rate increased by 15 percentage points. Then, by 1988 household saving fell to 10 percent, after which it resumed a more or less steady upward trend.6

Over this same time period, China experienced considerable demographic transitions. Using historical and projected demographic estimates from the United Nations World Population Prospects, Figure 2 displays the evolution of the age distribution by showing the share of the population in 3 age groups from 1950 to 2050. The lower line (bottom, light colored section) is the fraction of the population under 20 years old, which we take as a coarse measure of the share of dependent children. The top line (middle, dark colored section) is the fraction of the population age 64 and older, an approximation for the number of retirees. The last category is the share of people aged 20 to 63 which we take to measure the working age population.

We hypothesize that the demographic changes depicted in Figure 2 affect household saving through three main channels. First, the relative size of the youngest age group shrank from 49 percent of the population in 1970 to 25 percent today, reflecting the dramatic decline in family size. As family size declines, the saving rate should increase since having fewer children to support (all else constant) frees up household resources, some of which can directed toward saving. As a further indication of this channel at work, Figure 3 shows the positive relation between the ratio of parents (ages 20-50) to dependent children (ages 0-19) and the aggregate household saving rate over a 55 year period.

The speed with which China’s fertility rate declined has been extraordinary. Figure 4 compares fertility rates for China with the US and Japan. Chinese women had a fertility rate of over 6 between 1950 and 1954, but this figure is now even lower than for the US. Much of the decline in China’s fertility rate was due to government policies on family planning. China’s one-child policy began in earnest in 1979 and officially remains in effect although enforcement differs among jurisdictions and rural and urban areas. But even before the one-child policy, other lesser known

---

5See Curtis and Mark (2010) for more about Chinese data and studying China using standard economic models. Ma and Yi (2010) also discuss the data and provide some empirical evidence suggesting that demographic change has helped increase the Chinese household saving rate.

6The rise and fall of savings in the 1980s was driven by rural households. In the mid-1980s, rural household saving rose above 20 percent before falling to 11 percent in 1989, close to that of the urban saving rate (12 percent). From 1990 onward, both urban and rural household saving rates have followed the same upward trajectory.
(and less rigorously enforced) fertility reduction programs were implemented. As can be seen in Figure 4, fertility rates had already begun to decline in response to the 1971 “Later-Longer-Fewer” campaign in which the government suggested that families be limited to two children in urban areas and three in rural areas (Kaufman et al. (1989)).

The second hypothesized channel works through the composition effect, which refers to the change in the relative size of the working age population. Again looking at Figure 2, the relative size of the working age population is seen to have grown considerably, going from less than half of the population in 1970 to over 65% today. Since the largest component of an economy’s saving is un consumed labor income (earned by those who work), we expect the rise in the share of the working age group to coincide with the surge in household saving via this composition effect.

The third hypothesized demographic channel concerns the projected aging of China. Looking forward, the current, large, working age cohort will soon enter retirement. In fact, the proportion of the working age population is projected to peak in 2012. As Figure 2 shows, the share of the oldest age group will attain an historic level in the coming decades. The current working age cohort will have relatively few workers available to provide financial support in the future because they have had few children. A 30 year-old today is projected to have a support ratio (number of working age people divided by those 64 and older) of less than 3 upon retiring; whereas, current retirees enjoy a support ratio of 7. Thus, we expect the current working age cohort to save more for their retirement because they expect to receive relatively smaller old-age support due to the projected future age distribution.7

2.2 Motivation from micro data

The above discussion uses inferences about how household (micro-level) decisions may be affected from looking at macro data. Although the quantitative model that we work with below is a macro model, and is meant to capture differences across successive generations (rather than cross-sectional differences) in the number of children and saving, it is still interesting and informative to consult micro-level data for evidence on the relation between family size and the household saving rate.

Here, we run a cross-sectional regression of the household saving rate on the number of children in the household. The data are from the 2007 Urban Household Survey of China, which is part of the Rural-Urban Migration in China and Indonesia survey. The framing of the survey sample is the same as the Urban Household Income and Expenditure Survey used by China’s National Bureau of Statistics. The survey reports household income from all sources as well as total consumption expenditures from which we construct the saving rate as (Income-Consumption)/Income. To compare across similar household types, we follow Banerjee et al. (2010), who use the same data, by restricting the sample to nuclear families (that may or may not contain dependent children). We do this because we do not necessarily want to observe saving behavior associated with multigenerational households since that type of family structure is not part of our model. Additionally, the sample is restricted to families in which the household head is less than 64 years old because the

---

7 See Attanasio and Brugiavini (2003) for evidence on how personal savings substitute for pension wealth in the case of Italy.
Table 1: The Effect of the Number of Children on the Household’s Saving Rate, 2007

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Dependent Variable: Saving Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Number of Children</td>
<td>-0.052 (0.015**)</td>
</tr>
<tr>
<td>log Income</td>
<td>0.241 (0.011**)</td>
</tr>
</tbody>
</table>

Sample Restrictions

<table>
<thead>
<tr>
<th>Age of Children</th>
<th>None</th>
<th>Omit HH with Children Age &gt; 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>3234</td>
<td>2200</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.142</td>
<td>0.127</td>
</tr>
</tbody>
</table>

NOTES: Saving rate is defined as (Income - Consumption)/Consumption. The data is restricted to nuclear families. The regressions include controls for the head of household’s age, age squared, and education level. Standard errors are reported in parentheses. Stars denote significance at the * 5 percent and ** 1 percent level.

households with older members may be at a different stage in the life-cycle. The regressions also include controls for the household heads income (in logs), education, age, and age squared.

Table 1 reports the results. Column (1) shows that having one more child is associated with a reduction of household saving by over 5 percentage points, which is statistically significant at the 1 percent level. Income is positively related to the saving rate. In Column (2), we omit families that have children living at home that are older than 19. Again, having one more child living at home is associated with a reduction of the household saving rate by 5 percentage points which is statistically significant at the 5 percent level. These estimates are in line with others in the literature. Banerjee et al. (2010) estimate that an additional child lowers a household’s saving rate by 7-11%, depending on the exact specification used. Gruber (2012), who uses an alternative data source, also finds a negative relationship between the number of minor children in a household and the saving rate.8

While this cross-sectional regression doesn’t directly map into variation over time, the evidence provides additional motivation for incorporating family size as part of the modeling strategy. Also, the regression evidence is a partial correlation. We are not asserting the direction of cause and effect here. That is the job of the quantitative model.

2.3 Summary

In addition to the demographic changes, the Chinese economy has, since implementation of market reforms in 1978, grown at a torrid pace. Permanent-income / consumption smoothing arguments predict that higher expected labor income growth induces lower saving rates. Figure 5 plots the

8Interestingly, Gruber (2012) finds evidence that having adult children has a positive effect on saving rates. Similarly, in our model, household saving increases once children leave the home.
log wage over time. In the pre-reform period, 1955 to 1978, real wage growth (constructed from the marginal product of labor assuming a Cobb-Douglas production function) averaged 3 percent per year. Then, China began to institute a series of economic reform policies. From 1979 to 2009 wage growth averaged 6.9 percent per year. The low wage growth in the early years and high growth in the later years works against a model based explanation of low saving rates during the pre-reform period and high saving in the post-reform period. These effects need to be more than offset by other factors such as demographic variations.

To summarize, the correlation between the large increase in the saving rate and the dramatic demographic transition represents the main stylized fact that we study. Life cycle considerations predict that household saving should increase in response to exogenously mandated reductions in family size because fewer mouths to feed frees up resources that can be saved. The household saving rate also should depend on the proportion of the working age population simply through changes in the composition (or share) of life-cycle savers. Looking ahead, fewer children today means there will be fewer workers in the future to provide old age support, so the current working age cohort needs to save more for retirement now. Finally, we expect higher wage growth to lower the saving rate. We use these observations to inform the specification of the OLG model, to which we now turn.

3 Overlapping Generations Model

This section presents the model used in the quantitative analysis. It is a partial equilibrium OLG model with 66 generations of decision making agents. A representative firm employs all working age agents and pays them the market wage, which is given by the marginal product of labor. A national financial intermediary bank clears excess supply or demand for capital on an (unmodeled) international market. Perfect foresight agents observe and take current and future observations on the age distribution, wages, and interest rates as given. A distinguishing feature of the model is that dependent children’s consumption enters separately into household utility as in Barro and Becker (1989).

3.1 Consumers

People live 85 periods or years. At any point in time, 85 generations are present but only those aged 20 to 85 make decisions. All agents of the same age are identical. We classify the population into 4 not necessarily disjoint groups: children (age 0 to 19); workers (age 20 to 63); parents of dependent children (age 20 to 49); and retirees (age 64 to 85). Let $N^c_t, N^w_t, N^p_t,$ and $N^r_t$ be the number of people in these respective groups at time $t$. For the first 19 years, people live as children and are dependent upon their parents. They do not save and consume what their parents choose for them. Parental and children’s consumption enter separately into household utility.

People work and earn labor income from ages 20 to 63. From age 50 to 63 consumers continue to work, but do not have children living at home. During retirement, people live off of their accumulated assets and transfers received from their working adult children (modeled a pay-as-
you-go pension scheme). People die with certainty at age 85. In the last year of life, utility depends only on consumption in that year.\textsuperscript{9}

3.1.1 Budget constraints

Let $c_{t,j}$ be the year $t$ consumption of an individual with decision-making age $j \in [0, 65]$, where decision-making age $j = 0$ corresponds to real-life age 20. When $j \in [0, 29]$, the household includes $n_t = N_t^c/N_t^p$ dependent children, each of whom consume $c_{t,j}^c$.\textsuperscript{10} During the parenting years, agents choose their own consumption $c_{t,j}$, their dependent children’s consumption $c_{t,j}^c$, and assets $a_{t+1,j+1}$ to take into the next period. We require asset holdings to be non-negative (consumers are not allowed to borrow). Agents take the gross return on savings $1 + r_t$ as given. Working agents give a fraction $\tau$ of their labor income $w_t$ to support current retirees. The flow (period-by-period) budget constraints for households with children is

$$c_{t,j} = (1 - \tau) w_t + (1 + r_t) a_{t,j} - a_{t+1,j+1} - n_t c_{t,j}^c, \quad j \in [0, 29].$$

(1)

Children leave the home when parents turn 50 ($j = 30$). Agents continue working until age 63 ($j = 42$). The budget constraints for these ‘empty nester’ working agents are

$$c_{t,j} = (1 - \tau) w_t + (1 + r_t) a_{t,j} - a_{t+1,j+1}, \quad j \in [30, 42].$$

(2)

Retirees consume from their accumulated assets and support from current workers. The per retiree (pension) transfer received in year $t$ equals $P_t = N_t^w/N_t^p \tau w_t$. These agents consume all remaining assets and die at age 85 ($j = 65$). The budget constraints for retirees are

$$c_{t,j} = P_t + (1 + r_t) a_{t,j} - a_{t+1,j+1}, \quad j \in [43, 65].$$

(3)

We can begin to see how the demographics connect to saving through these budget constraints. First, a decline in the number of dependents ($n_t$) frees up resources for saving ($a_{t+1,j+1}$) in Equation (1). Second, a large cohort with ages $j \in [0, 42]$ increases the saving rate because consumers earn income to save during their working years. Finally, a declining support ratio ($N_t^w/N_t^p$) means there will be a relatively small pension ($P_t = N_t^w/N_t^p \tau w_t$) for the current working age cohort when they retire. In Equation (3), the consumers can overcome this shortfall in retirement support by accumulating assets.

\textsuperscript{9}We experimented with variations of the model including either an explicit bequest motive or accidental bequests due to early death. The simulation results were similar to the model without bequests because the size of bequests are quantitatively small compared to workers’ wage income. To maintain simplicity in the model, we proceed without bequests.

\textsuperscript{10}Even though parents support and have children in their utility function for 30 years, children leave the household after age 19, no longer receive consumption from their parents, and support their own children for the next 30 years.
3.1.2 Preferences

For households with dependent children, we use a variation of Barro and Becker (1989) preferences in which consumption of parents and children enter separately into household utility. The per-period utility function for a household head of decision-making age \( j \in [0, 29] \) in year \( t \) is given by

\[
    u_{t,j} = \mu n_t^\eta (c_{t,j}^{c_{t,j}})^{1-\sigma} + \frac{c_{t,j}^{1-\sigma}}{1-\sigma}, \quad j \in [0, 29].
\]

Parameter \( \sigma \) determines the elasticity of intertemporal substitution, and parameters \( \mu < 1 \) and \( \eta < 1 \) determine the weight parents put on their children’s consumption. The number of children \( n_t \) is expressed on a per-person basis; households can be interpreted as single-parent families. Beginning at age 50, individuals no longer support children and have the flow utility function

\[
    u_{t,j} = \frac{c_{t,j}^{1-\sigma}}{1-\sigma}, \quad j \in [30, 65].
\]

A 20 year old agent in year \( t \) chooses a sequence of consumption, consumption for children, and asset holdings subject to the budget constraints given in Equations (1) - (3) in order to maximize lifetime utility,

\[
    U_t = \sum_{j=0}^{29} \beta^j \left( \mu (n_{t+j})^\eta \left( \frac{c_{t+j,j}^{c_{t+j,j}}}{1-\sigma} \right) + \left( \frac{c_{t+j,j}^{1-\sigma}}{1-\sigma} \right) \right) + \sum_{j=30}^{65} \beta^j \left( \frac{c_{t+j,j}^{1-\sigma}}{1-\sigma} \right), \tag{4}
\]

where \( \beta \in (0,1) \) is the subjective discount factor. Agents make decisions taking the time series of interest rates \( r_t \), wages \( w_t \), and pension support \( P_t \) as given.\(^{11}\) Agents also take the parameters (\( \beta, \eta, \mu, \sigma, \tau \)) and the current and future demographic structure and family size \( n_t \) as known and exogenously given. The strong fertility response to the one-child policy provides justification for this assumption.

We are primarily interested in understanding the relationship between the demographic structure and household saving behavior. The following proposition helps to show why family size affects saving through the Barro and Becker preferences.

**PROPOSITION 1 (Effective Discount Rate):** Let \( \bar{c}_{t,j} \) denote total household consumption by parents and their children, \( \bar{c}_{t,j} = n_t c_{t,j}^c + c_{t,j} \). Then, the lifetime utility function for an individual at age 20 can be rewritten as

\[
    U_t = \sum_{j=0}^{29} \hat{\beta}_t^{t+j,j} \frac{\bar{c}_{t+j,j}^{1-\sigma}}{1-\sigma} + \sum_{j=30}^{65} \beta^{t+j,j} \frac{\bar{c}_{t+j,j}^{1-\sigma}}{1-\sigma},
\]

where the effective discount rate

\(^{11}\)These variables are partially determined by equilibrium conditions in conjunction with the production side of the economy described below.
\[ \hat{\beta}_{t,j} = \beta^j \left( 1 + \left[ \mu n^\eta + \eta \right]^{1/\sigma} \right)^\sigma, \] 

is increasing in \( n_t \) if \( \eta + \sigma > 1 \).

Proposition 1 exploits the Euler equation for the parent / child consumption choice to rewrite the household’s lifetime utility function to look more like the standard model with no children. The difference is that the effective discount rate (\( \hat{\beta} \)) depends on the number of dependent children per worker \( (n) \). Note, if \( n_t = 0 \) or \( \mu = 0 \), then \( \hat{\beta}_{t,j} = \beta^j \). In our benchmark simulation below, however, agents aged 20-49 have and support children with \( \eta + \sigma \) exceeding one. Therefore, according to Proposition 1, the effective discount rate increases with family size which makes the household with more children act as if it is less patient. Consider two household heads, \( A \) and \( B \), of the same age: \( A \) has many children and discounts current utility with \( \hat{\beta}^A \) while \( B \) has fewer children and therefore discounts with \( \hat{\beta}^B < \hat{\beta}^A \). Since \( A \) places more weight on consumption (utility) in the parenting years than \( B \) (\( B \) places relatively more weight on retirement consumption), \( A \) will save less for retirement than \( B \).\(^{12}\) Thus, family size affects saving by altering the households effective discount rate.\(^{13}\) Next, we complete the model by presenting details on the firm and the national bank.

### 3.2 The Firm, an Intermediary Bank, and Equilibrium

We provide details on the firm and the bank here for the sake of completeness. We use a simple model of production to calculate the time series of interest rates \( (r) \), wages \( (w) \), and pension support \( (P) \) faced by households. A representative firm with Cobb-Douglas technology in capital \( K \) and labor \( N \) produces output \( Y \),

\[ Y_t = A_t K_t^\alpha N_t^{1-\alpha}, \] 

where \( A \) is total factor productivity, and \( \alpha \) is the capital share. The firm maximizes profits by hiring workers at wage \( w \) and renting capital at price \( r \) each period. The firm takes prices as given and retains all earnings.

A national intermediary bank finances the capital stock through domestic and foreign borrowing. Let \( F_t \) be the number of internationally traded bonds held by the bank and \( N_{t,j} \) be the number of people of decision-making age \( j \) in year \( t \). The net foreign asset position depends on the difference between deposits (assets supplied by consumers) and loans (capital demanded by

---

\(^{12}\)That the consumer, \( B \), with the lower effective discount rate rate saves more may seem counter-intuitive. Note, though, household head \( B \) puts relatively more weight on future consumption than \( A \) does because both \( A \) and \( B \) have discount rate \( \beta \) upon turning 50.

\(^{13}\)Choi et al. (2008) show that variation in the time discount rate \( (\beta) \) across countries can explain the trending current accounts in Japan and the US. Our model is consistent with the Choi et al. hypothesis, in that different age structures generate different time discount factors.
the firm),
\[ F_t = \sum_{j=0}^{65} (N_{t,j} a_{t,j}) - K_t. \]  
(7)

Changes to the intermediary’s foreign asset position do not correspond to the actual current account because the model only considers household saving.

Equilibrium consists of the firm hiring labor and renting capital to maximize profits and each consumer selecting consumption and assets to maximize utility. Working age consumers \((N^w)\) supply labor inelastically. Therefore, the age distribution exogenously determines the labor supply \(N_t = N^w_t\). The wage \((w)\) equals the marginal product of labor,
\[ w_t = (1 - \alpha) \frac{Y_t}{N^w_t}, \]  
(8)

and the rental rate \((r)\) equals the marginal product of capital less the depreciation rate \((\delta)\),
\[ r_t = \alpha \left( \frac{Y_t}{K_t} \right) - \delta. \]  
(9)

The firm adjusts its capital stock to satisfy Equation (9) to equality. The national demand and supply of assets need not be equal since the bank clears any excess on the international capital market.

4 Parameterization

China is an economy in transition. Our parameterization accounts for the transitional nature of labor’s income share \((1 - \alpha)\). First, the share has declined over time, and second, in recent years it has been comparatively low. Hu and Kahn’s (1997) estimate of labor’s share during the post-reform era (post 1978) is 0.4, substantially lower than the 0.66 share in the US. Hsieh and Klenow (2008) examine Chinese data from 1998 to 2005 and find that the median labor share across all state-owned firms and large (revenues in excess of 5 million yuan) non state-owned firms is 0.3. Table 2 shows our own estimates of the labor share based on selected years of Chinese national accounts data.\(^\text{14}\) Due to the declining labor share, aggregate wage growth has not kept pace with GDP growth in recent years.

Taking into consideration our own calculations and the estimates in the literature, we set \(1 - \alpha = 0.6\) in the pre-reform years (1955-1978). Then, we assume that the share decreases by 0.02 per year between the years of 1979 and 1988 until it reaches 0.4 where it remains from 1989 onwards.\(^\text{15}\)

The formal pension system also has been in transition. In pre-reform years, a government run

\(^{14}\)These estimates include non-wage compensation. Details are available upon request.

\(^{15}\)The choice of \(\alpha\) affects the wages and interest rates faced by households. In several of the simulations, we explicitly pick values for these variables, making the assumptions about \(\alpha\) irrelevant.
Table 2: Declining Labor Share

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor share</td>
<td>0.64</td>
<td>0.57</td>
<td>0.59</td>
<td>0.53</td>
<td>0.48</td>
</tr>
</tbody>
</table>

program covered those working in urban state-owned enterprises, but this was a small portion of the population. Pre-reform China was not an industrialized country and most people lived in rural areas. Even in post-reform China, operation of the formal pension system is tenuous. While a social pay-as-you-go pension system aimed at covering the urban workforce is in place de jure, the system is unfunded and is broken de facto. As a result, participation and pension coverage is estimated to be low, less than 25 percent [Sin (2005) and Dunaway and Arora (2007)]. The majority of people must rely on savings and the Chinese family system whereby children, especially males, are expected to care for elderly parents.16

Our parameterization of the transfer rate $\tau$ is informed by the following. Data on $\tau$ during the pre-reform period is scarce. As noted by Lee and Xiao (1998), in those days, most people lived in rural areas and belonged to collective production units with elderly persons receiving resources directly from the collectives. We view payments from collectives as inter-generational transfers. Lee and Xiao (1998) use a 1992 survey covering children's support for elderly parents to study transfers, and their results imply a value for $\tau$ around 0.08 in urban areas.17 Based on a survey conducted in 1998, Xie and Zhu (2008) find that the (unconditional) fraction of income contributed by urban men to their parents is 0.03.18 Xie and Zhu (2008) also note that a nontrivial proportion of adults in urban areas receive financial support from their elderly parents, making net transfers small. Thus, we set $\tau$ to 0.05, on the low side of the estimates. In the simulations, we experiment with different values of $\tau$ and also investigate the model dynamics with a constant replacement rate social security system.19

In specifying parent’s attitudes towards children, we set the Barro-Becker children in utility parameters to $\mu = 0.65$ and $\eta = 0.76$. Manuelli and Seshadri (2009) choose these values to match US fertility rates in a model with fertility choice. We use the same values under the premise that a typical household in China cares for their children’s consumption the same as a US household. The time discount rate $\beta$ is set equal to 0.97, and the intertemporal elasticity of substitution $(1/\sigma)$ equals 0.67.20 The capital depreciation rate $\delta$ is set to 0.10. Table 3 summarizes the parameter values.

Initial assets equal zero for each 20 year old (decision-age $j = 0$) agent. To solve his or her utility maximization problem, a 20 year old must take into account the next 65 years of wage, interest

---

16Presumably, the role of male children in this regard underlies the preference for boys and the resulting sex imbalance exploited by Wei and Zhang (2011).

17If broken down by gender, the transfer rate in their study is 0.037 for men and 0.16 for women.

18Xie and Zhu (2008) found little difference in the amount contributed by urban women even though women earned substantially less than men. The (unconditional) fraction of women's income contributed was 0.06.

19We define the replacement rate as the percent of final working year ($j=42$) wages received as a pension in the first year of retirement ($j=43$), $\frac{W_{42}+1}{W_{41}}$.

20There exists a large literature attempting to estimate the intertemporal elasticity of substitution, without much agreement. We believe 0.67 is a conservative choice.
rate, and demographic observations. The future demographic data comes from the United Nations projections. Future wages and interest rates are calculated by assuming a gradual transition of output growth to a steady state rate of 1.0 percent with a half-life of adjustment of 3.1 years and with the slowdown beginning in 2012. Since we are focused on explaining the trend of the household saving rate rather than the cyclical component, we smooth the annual wage series with the Hodrick-Prescott filter (smoothing parameter $\lambda = 100$).

## 5 Quantitative Results

In this section, we begin by presenting the model's predictions generated under the benchmark parameterization. The benchmark parameterization is able to explain low saving rates in the pre-reform period and high and rising saving rates in the post-reform period, and constitutes our main results. Additional experiments where we turn off various features of the model to assess the quantitative importance of those model components are then discussed.

### 5.1 Benchmark parameterization results

We simulate the model economy from 1955 to 2009 by exogenously presenting the data on wages, interest rates, and demographics to the model economy agents who, being endowed with perfect foresight, use this information to make saving and consumption decisions over their lifetimes. The demographic data consists of annual observations by single year age groups.

The main results of the paper, presented in Figure 6, come from comparing the predictions of the benchmark model-based aggregate household saving rate with the data. As in the data, people in the model economy generate relatively low aggregate household saving before the mid 1970s. The increase in the model's saving rate leads the data. By 2009, households in the model save 29% of their income, slightly more than the observed 27%. The model also captures the saving boomlet and decline in the 1980s.

---

$^{21}$We think that the inevitable slowing of growth will happen sooner rather than later, as does (apparently) the Chinese leadership. The Wall Street Journal (March 5, 2012) reports that Chinese Premier Wen Jiabao set the 2012 growth target at 7.5 percent, significantly lower than the actual growth rate in 2011 of 9.2 percent.

$^{22}$Again, the run-up starts earlier than in the data, mostly because of the decrease in the number of dependent...
To isolate the quantitative effects of the demographic channels, we next simulate the model economy holding wages and interest rates \( r = 0.04 \) constant at their 1970 values. Figure 7 shows the evolution of the household saving rate when only the demographic composition and family size varies over time. The demographic variation alone is able to explain the low saving rate in the pre-reform period and its rise during the post-reform period. By 2009, the implied saving rate equals 18%, about two-thirds of the observed rate. More than half of the rise in the saving rate generated by the benchmark model is due to the changing age distribution.

Overall, we conclude that the model economy does well in replicating the time series of the Chinese household saving rate and that overall demographic change is a quantitatively important driver of the massive increase in saving over time. As discussed above, we identified three hypothesized demographic influences subsumed in the overall demographic variation. We now present additional experiments to quantify the role of these separate demographic channels.

### 5.2 Three channels of demographic influence

This subsection separately examines the role of the three demographic channels on household saving in the model. We investigate the impact of dependent children (family size), the role of the change in the working age group (composition effect), and support for the retired (pension) on saving.

#### 5.2.1 Dependent Children

To illustrate the total effect of including children in the model, we have simulated the model with \( \mu \) set to zero, which removes explicit valuation of children’s consumption from the utility function. The other parameters equal their values from the benchmark simulation. Chen et al. (2006) and Ferrero (2010) do not consider children’s consumption, so this experiment compares closer to their work. Figure 8 shows the results. Ignoring dependent children causes the saving rate implied by the model to be too high and for most of the sample it vastly overstates saving. The run-up in saving in this exercise begins in the late 1960s, leading the benchmark simulation. Under the benchmark exercise, households wait to increase their saving until the child-depency ratio begins its decline.

Figure 9 shows the saving rates generated by assuming the number of children per household is held constant at 1970 and at 2009 levels. Two-parent households in 1970 had 1.6 more children on average than in 2009. The other determinants of household saving, including pension support, are the same in this experiment as in the benchmark simulation. The economy with larger families have household saving rates that lie below the benchmark, especially after 1990. By 2009, the saving rate for the large family households is 8 percentage points lower than in the benchmark simulation. Fewer children causes households to behave as if they are more patient, just as described by Proposition 1.
For a single cohort (dropping the $j$ subscript), the effective discount factor ($\hat{\beta}$) from Equation (5) increases with family size ($n$) in the benchmark parameterization of preferences because $\sigma + \eta = 2.26 > 1$ (see Proposition 1). Letting total household consumption be $c_t = n_t c_t^f + c_t$, then, we see how China’s declining birth rate has affected household saving decisions by writing the intertemporal marginal rate of substitution as a function of the effective discount factor,

$$
\left( \frac{c_{t+1}}{c_t} \right)^{\sigma} = \frac{\hat{\beta}_{t+1}}{\hat{\beta}_t} (1 + r_{t+1}).
$$

(10)

When the current number of children is high relative to the future ($\hat{\beta}_t > \hat{\beta}_{t+1}$), then the right hand side of Equation (10) is low (and therefore saving is low). Connecting this to the demographic data, Figure 10 traces $\hat{\beta}$ for two cohorts from age 20 to 49 using the benchmark parameter values. The solid line is the effective discount factor for the cohort that turned 20 in 1970, and the dashed line is for the cohort aged 20 in 1990. The 20 year olds in 1970 face a steep slope for $\hat{\beta}$, causing savings for this cohort to be low. In comparison, the relatively flat $\hat{\beta}$ series for 20 year olds in 1990 implies a higher value for $\frac{\hat{\beta}_{t+1}}{\hat{\beta}_t}$, making households in this cohort more apt to save.

It is useful at this point to reexamine the regression results (Table 1) along side the simulation results (Figure 8). In the simulations with ‘no children’ the annual saving rate is 13 percentage points higher than the ‘benchmark’ simulations in 2007. In 2007, a two parent household had 1.6 children. The regression coefficients predict that the saving rate would be 8.2 percentage points higher given a decrease of 1.6 children. Because the regression is from a cross-section and the simulations are dynamic in nature, the two results are not directly comparable. The main points that we wish to underscore is that the number of dependent children has a large effect on household saving behavior and that this effect is apparently robust.

### 5.2.2 Composition Effect

Next, we measure how much the simple composition effect (the growing proportion of life-cycle labor-income savers in the population) contributed to the increase in the saving rate. We do this by decomposing the benchmark simulation’s aggregate saving rate in 2009 ($SR_{2009}$) into contributions by decision-age groups $j \in [0, 65]$. Let $N_{t,j}$ be the number of people who are $j$ years old in year $t$, $\phi_{t,j}$ be the age group’s per-capita income share, and $sr_{t,j}$ be the age specific saving rate. Then, the aggregate saving rate can be decomposed as,

$$
SR_{2009} = \sum_{j=0}^{65} N_{2009,j} (\phi_{2009,j}) (sr_{2009,j}).
$$

(11)

$^{23}$At age 50, the effective discount factor collapses to the subjective discount factor $\beta$, since households no longer have dependent children.
Next, we create a ‘counterfactual’ saving rate by keeping the age-specific saving rates at their 2009 values but setting the age and income distributions to their 1970 values,

\[ SR_{2009} = \sum_{j=0}^{65} N_{1970,j} (\varphi_{1970,j}) (sr_{2009,j}). \]  
(12)

The difference between the benchmark and counterfactual saving rates is \( SR_{2009} - \hat{SR}_{2009} = 0.042 \), meaning that the composition effect accounts for 4.2 saving rate percentage points, or 17 percent of the benchmark simulation increase in the saving rate from 1970-2009. This exercise naively and mechanically calculates the composition effect. The future changes in composition are not taken into account by model agents in deciding their saving rates. The composition effect is not trivial, but it is small relative to the overall increase in the saving rate. Thus, in our model, the change in \( sr \) over time (for all age groups) is responsible for most of the increase in saving.

5.2.3 Retirement support ratio

Here, we consider how the decline in the retirement support ratio (workers per retiree) affects the saving rate. From 1970 to 2009, the support ratio declined from 9 to 7 and it is projected to fall below 3 by 2032. Figure 11 plots the simulation results holding the support ratio constant at the 1970 level. All other values, including family size, are as in the benchmark simulation. The saving rate after 1980 is much lower than in the benchmark. In 2009, the saving rate is only 17%. The current working age generation saves less because there will be many workers to pay into the pension system in the future. In the benchmark simulation households foresee the plummeting retirement support ratio and rely more on personal savings for retirement. The projected low support ratio captures the so-called ‘4-2-1’ problem in China: A married couple who are both only children themselves are expected to provide (pension) for both sets of parents and their only child.

5.3 Experiments that vary non-demographic components

In this subsection, we analyze how our results depend on non-demographic aspects of our model. We quantitatively assess the model’s response to a change in old age support (\( \tau \)), the introduction of a constant replacement rate social security system, a change in the length of parenthood from ages 20-50 in our benchmark model to ages 20-63, variations in wage growth, and relaxation of the perfect foresight assumption. Throughout these exercises, we keep the model’s characteristics the same as the benchmark model except for the dimension being investigated.

5.3.1 Changes in old-age support levels

To further examine the role of expected retirement support, we consider alternative values for the transfer rate. The benchmark parameterization assumed that workers gave a proportion \( \tau = 0.05 \) of their labor income to retirees. Figure 12 shows two alternative scenarios: \( \tau = 0 \) and \( \tau = 0.1 \).\textsuperscript{24}

\textsuperscript{24} Simulations in which the transfer rate changes over time to allow for more generous pre-reform transfers and retirement benefits under the commune system produce saving dynamics that are almost identical to the benchmark.
As expected, without old age support ($\tau = 0$) the saving rate is higher than in the benchmark simulation because households must fully fund their own retirement. When old age support is increased ($\tau = 0.1$), saving is lower than in the benchmark simulation for two reasons. First, workers transfer a large share of their income leaving fewer resources to save. Second, due to rising wages, 10 percent of future wages towards old age support will make for a relatively large transfer. Given the documented evidence on transfers from children to the elderly and a lack of a comprehensive pension system, $\tau = 0.10$ seems unrealistically high. In fact, a 75 year old in 2009 in this scenario would receive almost as much in annual old age support as they earned in their final working year.

### 5.3.2 Constant replacement rate social security system

As an alternative specification of transfers from the working to the retired, we let the economy have a constant replacement rate social security system. Under this system, each retiree earns a constant fraction of their final working year’s wage throughout retirement. To do this, we adjust the transfer rate $\tau$ as required to equate payments from workers and benefits received by retirees. In China in 2004, the replacement rate for retirees in the formal pension system was a generous 75-80 percent of an individual’s final working wage (Trinh (2006)). However, Sin (2005) estimates that only 23.5 percent of the working population participates in the formal pension system. Assuming wages are the same for all workers and an 80 percent replacement rate. This works out to an aggregate replacement rate of 18.4 percent for a fictitious representative retired agent. We also run the model with a constant 30 percent replacement rate. Simulated saving rates under these two cases are plotted in Figure 13. Neither curve deviates much from the benchmark. Overall, the model is not very sensitive to using a constant replacement rate pension system.

### 5.3.3 Variations in Wage Growth

Figure 14 shows the simulation results when we slow down the rate of wage growth. Figure 14 shows simulations when the wage grows only by half the historical rate and when it grows only by a quarter of the historical rate. As one might expect from standard income / consumption smoothing arguments, slower future wage growth increases the saving rate. Clearly, wage growth affects saving behavior but the time pattern of low pre-reform growth and high post-reform growth works against explaining the time-path of the saving rate. In light of this, demographic considerations seem to be a critical component in understanding the saving rate.

### 5.3.4 Length of Parenthood

In the benchmark model, household heads aged 20-49 support dependent children. As a check on this modeling choice, we simulate the model assuming consumption by dependent children enters parent’s utility from age 20 to 63, their entire working life. Saving rates in this exercise (graph not shown) are close to the benchmark throughout the sample, although slightly higher after 1990.

---

model, except saving stays near zero until 1972.
The slightly higher saving rate is the result of two countervailing effects. Having fewer dependent children per household (there are more parents than in the benchmark) induces saving; however, parents have dependent children for a longer period of time, reducing saving. In sum, the former effect dominates the latter.

Figure 15 plots the saving rate by age group in 2009 for both the benchmark simulation and the case where households continue to support children until the household head retires. In both, saving displays the typical hump shape over the life-cycle. However, when households are responsible for children their whole working life, they smooth out their saving rather than wait until children leave the house at age 50 as in the benchmark model. As mentioned, even though the age saving profiles differ, the aggregate saving rate is not very sensitive to variations in how long parents support dependent children.

Recent work (Song and Yang (2010) and Chamon and Prasad (2010) among others) have documented a flat (or even U-shaped, rather than hump shaped) saving pattern by age group, in which young workers (age 25-30) in China have had a relatively high saving rate in recent years (though still lower than those 50 and older). Our model does not deliver on this stylized fact but we do not think that matching saving rates by age is a large concern for us, given that our primary interest is in understanding the evolution of the saving rate over time. Additionally, as we just showed, differences in saving rates across age groups have little effect on the trend of the aggregate household saving rate in the model. This same result was also evident in the calculation of the composition effect above. The increase in the household saving rate occurs across all working age groups in our model.

### 5.3.5 Static Expectations

In solving the model, we assumed that households have perfect foresight with respect to the evolution of the interest rate, wage, and demographics (as in the seminal Auerbach and Kotlikoff (1987) model). Within economics and working in a deterministic setting, such a modeling assumption is natural. Outside of economics, this may strike one as a heroic assumption. An alternative assumption that we investigate is that households form static expectations. In our final experiment the household assumes that all future values of the exogenous variables are fixed at those currently observed. The results, displayed in Figure 16 show that assuming static expectations would be a poor modeling choice. The implied saving rate is much too high and has a sudden jump in 1980. The timing of the jump helps to shed some light on why the timing of the increase in saving rate implied by the benchmark parameterization leads the data—people in the benchmark model anticipated the changing demographic changes and factored those changes into their decisions earlier than actual individuals. While perfect foresight may be unrealistic, this assumption is better than...
static expectations. This experiment supports Friedman’s (1966) recommendation to view agents as behaving as if they are perfect foresight optimizers.

6 Conclusion

This paper studies the ability of standard life-cycle considerations combined with demographic changes to explain the evolution of household saving in China from 1955 to 2009. Some of the more conspicuous facets of the saving rate include its relatively low level prior to the economic reforms, its upward trend beginning in the late 1970s, and its current high level. We have shown that it is possible to write down an OLG model that captures these stylized facts. We find that the model generates the low household saving rate in the pre-reform era, the dramatic post-reform increase, and the high current level of household savings. The model is also reasonably successful in replicating the trend break in the saving rate beginning in 1978. We demonstrate that the model mechanisms linking demographics to household saving behavior are quantitatively important for understanding the time path of the saving rate.

Two aspects of the model are central to the analysis. First, consumption by dependent children enters explicitly into household utility, and second, the changing age distribution of the entire economy is represented. In simulations of the model economy holding interest rates and wages constant, the change in the population composition alone generates over half of the observed increase in household saving. Thus, according to our model, the currently observed high saving rate is driven primarily by the reduction in family size resulting from population control policies and the relatively large size of today’s working population.

Projecting forward, the model implies that as the Chinese population ages, and it is aging quite rapidly, the rising household saving rate should be arrested. The large currently observed Chinese external surpluses, to the extent that they are driven by household saving, may also be a temporary phenomenon.
References


Figure 1: Saving as a share of household income, 1955-2009. Source: Modigliani and Cao (2004) and the China Statistical Yearbook (various issues).

Figure 2: Youth (0-19), retirees (64+) and working age (20-63) as a share of population, 1950-2050. Sum of the three groups equals one but figure is truncated above at 0.60. Source: United Nations World Population Prospects 2010.
Figure 3: Household saving rate (left axis) and ratio of parents, age 20-50, to children, age 0-19 (right axis).

Figure 5: Log real wage, 1955-2009. Wage is constructed as the marginal product of labor from Cobb-Douglas production function. Capital stock estimated by perpetual inventory method with depreciation rate of 0.10. Output, labor, and investment data come from the China Statistical Yearbook (various issues).

Figure 6: Household saving rates in the data and benchmark model, 1955-2009.
Figure 7: Demographic changes only. Interest rates ($r = 0.04$) and wages held constant at 1970 levels.

Figure 8: No children. All components from the benchmark exercise remain except parents do not support dependent children (i.e. $\mu = 0$).
Figure 9: Variation in family size. The number of dependent children per household stays constant at their 1970 and 2009 levels, respectively.

Figure 10: Evolution of $\hat{\beta}$ from (actual) age 20-49 for cohorts who were age 20 in 1970 (solid line) and age 20 in 1990 (dashed line).
Figure 11: Age support ratio held constant at 1970 level: 9 workers per retired individual.

Figure 12: Variations in old age support: Alternative transfer shares $\tau = 0$ and $\tau = 0.10$ compared to benchmark simulation $\tau = 0.05$. 
Figure 13: Constant replacement rate retirement plan. Pension income is a fixed percentage of last working year (age 63) wage.

Figure 14: Variations in real wage growth. Wage growth is reduced to one-half and one-quarter of observed wage growth.
Figure 15: Simulated saving rates across cohorts by actual age in 2009. Benchmark parameterization (solid line) parents have dependent children from ages 20-49. Dashed line shows saving rates for parents with dependent children through their entire working life (20-63).

Figure 16: Static expectations. The solid, dotted line depicts the model when agent believe that wages, interest rates, and demographics will be fixed at the currently observed values. In the benchmark model, agents have perfect foresight.