A Microfounded Model of Chinese Capital Account Liberalisation

Dong He
Hong Kong Monetary Authority and
Hong Kong Institute for Monetary Research

Paul Luk
Hong Kong Institute for Monetary Research

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Abstract

In shaping the evolution of the global financial system in the decade ahead, few events will likely be more significant than capital account liberalisation in China and the internationalisation of the renminbi. This paper provides a theory-based enquiry into the contours of China’s international balance sheets after the renminbi becomes convertible under the capital account. We construct a two-country general equilibrium model with trading in equities and bonds and calibrate the model with US and Chinese data. We interpret Chinese capital account liberalisation as a removal of restrictions that prohibit agents from trading Chinese bonds and US equities. We explore how international risk-sharing can be achieved through portfolio diversification in each of these asset market configurations. We also look at how these holdings would change as China gradually rebalances its production with a higher share of labour income, and as the productivity gap between China and the US narrows. We find that both US and Chinese residents would have incentives to increase their holdings in each other’s equities; and they also have incentives to issue debt in each other’s currency. We interpret the latter observation as the co-existence of the US dollar and the renminbi as major international currencies.

1 Introduction

China has been growing rapidly for the last 30 years and has become the second largest economy in the world. Since China declared renminbi current account convertibility in 1996, the country has transformed itself to a major player in international trade (accounting for 11% of world trade in 2012). Its influence is also clearly visible in global commodity markets. However, China’s participation in international financial markets is small as compared with the size of the economy and the volume of trade. The key reason
is that the renminbi is not freely convertible under the capital account until now and cross-border financial transactions are constrained by capital controls.

In fact, China has endeavoured to attract foreign direct investments since the 1980s, and it has been among the largest recipients of foreign direct investments. In recent years, overseas direct investments by Chinese firms have also been encouraged and risen fast. But restrictions on cross-border portfolio investments have been prevalent. Nevertheless, the Chinese government realises that opening the capital account is important and has put it on the official agenda. The goal of ‘gradually realizing convertibility under the capital account’ was reiterated in the 12th Five-Year Program. In 2012, the People’s Bank of China (PBC) released a report making the case for a faster pace of capital account liberalisation.\footnote{Report by the Financial Survey and Statistics Department of the PBC, 23 February 2012 (in Chinese).} The report proposes a roadmap towards the liberalisation in a span of 10 years. In fact, a number of policies related to liberalisation of portfolio investments has already been rolled out over the last few years. For instance, the Qualified Foreign Institutional Investor (QFII) scheme, under which foreign residents can invest in Chinese equity and bond markets through collective investment schemes, was launched in 2002 and subsequently expanded. An analogous Qualified Domestic Institutional Investor (QDII) scheme, under which Chinese residents can make portfolio investments overseas through collective investment schemes, was set up in 2006. Although these steps are small, piecemeal and subject to quota and restrictions, reflecting China’s gradualist and controllable approach towards financial liberalisation, the intention to move towards a more liberalised capital account is clear. Going forward, we expect the speed of capital account liberalisation will accelerate.

Capital account liberalisation in China is closely related to the process of the internationalisation of the renminbi. Facilitating the international use of the renminbi was put on the policy agenda by the Chinese authorities in the aftermath of the global financial crisis of 2007-2009. Since mid-2009, the Chinese authorities have moved quickly to remove restrictions against the use of renminbi in current account transactions, and have gradually expanded the scope for the use of renminbi in capital account transactions. Market forces have responded quickly to official policies and the use of renminbi in cross-border trade has risen rapidly, and offshore renminbi markets in Hong Kong and other financial centres have also mushroomed. Although a certain degree of international use of the renminbi does not require full capital account convertibility, sufficient provision of renminbi liquidity in global financial markets will ultimately be dependent on a higher degree of capital account convertibility (He, 2012). In particular, the creation of renminbi liquidity in offshore markets requires that non-Chinese residents are willing and able to take on renminbi-denominated liabilities (He and McCauley, 2010).

The relatively low capital account convertibility has important implications for China’s international balance sheet. Figure 1 illustrates China’s gross external assets and liability positions.\footnote{The data before 2007 is obtained from the Lane and Milesi-Ferretti (2007) ‘External Wealth of...} China has a large long position in fixed-income securities and a large short
Figure 1: China’s Gross International Investment Position.

position in foreign direct investment (FDI) and equity portfolios. As the gross financial positions are growing over time, the ‘long debt, short equity’ asymmetry has become more prevalent (Ma and Zhou, 2009). On the asset side, the foreign exchange reserves has been accumulating rapidly since 2000 and constitutes more than two-thirds of the gross assets holding by 2011. The outward FDI assets and portfolio equities combined amounts to only 5.8% of GDP in 2011. On the liability side, the inward FDI and the equity portfolios together account for about 75% of the gross liability position. The large inward FDI is by and large a policy choice by the Chinese authority to facilitate technology transfer.\(^3\) On the other hand, the US external balance sheet is the mirror image of China’s: the US

\(^3\)See Lardy and Douglass (2011) for a summary of these policies.
borrows by issuing treasury bonds and uses the proceeds to make loans abroad, invest overseas as FDI and equity portfolios. For this reason, Kindleberger (1965) and Despres, Kindleberger and Salant (1966) famously call the US ‘the banker of the world’. The evolution of the external balance sheet of the US in recent decades has been described by Gourinchas and Rey (2007) as ‘from world banker to world venture capitalist.’

There is an empirical literature that studies how capital account liberalisation in China would affect the gross financial position in China. These papers typically use cross-country panels for the estimations. Ma and Zhou (2009) find that the gross international investment positions in the OECD countries are positively correlated with country sizes and openness. Based on the OECD experience, they predict that the gross international investment position in China would reach 150% of GDP by 2015, driven by capital account liberalisation and economic growth. He et al. (2012) use data for OECD, Asian and Latin American countries to study the determinants of different types of capital flows. Their results suggest that, after capital account liberalisation, both outward FDI and portfolio investment in China would increase significantly, and at a faster pace than the inward flows, reflecting an intention to diversify risks. Moreover, the Chinese foreign exchange reserves (a large fraction of which is US bonds) will continue to rise till 2019 before falling. However, when expressed as a share of GDP, they predict that the reserves will begin to fall as early as in 2013.

In this paper, we instead take a structural approach to study the impact of the capital account liberalisation to China’s international balance sheet. The key advantage of this approach is that it provides theoretical underpinnings regarding the optimal portfolio choices and the gross international investment positions. It also allows us to take into account country specifics when modelling its residents’ portfolio choices. Moreover, the structural model is immune to the Lucas critique, which allows us to do other counterfactual experiments, analyzing how economic growth and other ongoing transitions in China affect the portfolio holdings.

Specifically, we construct a microfounded two-country general equilibrium model with trading in equities and bonds and calibrate the model with US and Chinese data. We interpret the capital account liberalisation as a removal of restrictions that prohibit agents to trade Chinese bonds and US equities. The optimal steady-state portfolio holdings before and after the liberalisation are computed. We explore how international risk-sharing can be achieved through portfolio diversification in each of these asset market configurations. We also look at how these holdings would change as China gradually rebalances its production with a higher share of labour income, and as the productivity gap between China and the US narrows.

Our results indicate that before capital account liberalisation, China builds up a positive gross position in US bonds and a negative position in FDI and portfolio equities consistent with the observed data. After capital account liberalisation in China, when Chinese and US stocks and bonds are both freely tradable, the optimal portfolio will
change significantly. First, Chinese households will invest a substantial amount in the US firms, and US will likewise invest more in the Chinese firms. This echoes the empirical findings that financial liberalisation facilitates risk-sharing and raise cross-border investments. Second, China’s desired US bond holding will move from a long position to a short position, increasing the gross supply of US-dollar bonds. The US households will hold these bonds to hedge real exchange rate risks. Third, the US will also hold a negative position in Chinese bonds.\(^4\) One way in which this might happen is that the US issues renminbi-denominated bonds, which is then held by the Chinese. We argue that this pattern is consistent with both currencies serving as funding currencies in the international monetary system. Finally, we show that as China grows and rebalances its economy, the Chinese will hold an increasing share of US equity. In addition, the Chinese will reduce its holdings of US bonds as a share of US output.

Our work is related to the theoretical literature on international portfolio choice in general equilibrium models. Much of the theoretical literature focuses on resolving the equity home bias puzzle (French and Poterba, 1991). There are two important insights from the recent literature. First, Engel and Matsumoto (2009), Coeurdacier and Gourinchas (2010), Devereux and Sutherland (2008), Coeurdacier, Kollmann and Martin (2010) and Coeurdacier and Rey (2011) show that having multiple asset classes (e.g. equities and bonds) can lead to very different portfolio allocations from ‘equities-only’ models because the existence of other assets alters the risk-sharing motive of holding equities. Second, models with capital accumulation and differentiated goods (such as Heathcote and Perri (2009), Engel and Matsumoto (2009) and Coeurdacier, Kollmann and Martin (2010)) can give rise to a plausible degree of equity home bias and a low correlation between excess equity return and the real exchange rate, consistent with empirical findings (van Wincoop and Warnock, 2010). We borrow these insights from the literature.

Since most of the literature on portfolio choice is devoted to address theoretical puzzles, these papers study symmetric economies. One notable exception is Devereux and Sutherland (2009), in which there is an advanced economy and an emerging market. That paper considers a configuration in which only the bonds of the advanced economy and the equities of the emerging market are traded, and contrast it with a situation in which equities of both countries together with a real bond are traded. Their model shows that in the former configuration, the advanced economy holds the equity of the emerging market and the emerging market holds the bond of the advanced economy, a result similar to ours. However, their model also implies foreign equity bias in the latter asset market configuration. Our work differs from theirs in three ways. First, we consider a different setup in which each country issues its own bonds, as the bond denomination has important implications for the international monetary system. Second, we assume consumption home bias in each country. Third, our setup generates equity home bias at all times, consistent with empirical findings.

\(^4\)Our model only implies that the net position is short. But this need not imply on a gross basis the US would not invest in renminbi-denominated bonds.
The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 provides a brief discussion of the solution method used to compute the steady-state portfolios. In section 4, we compute the optimal portfolio holdings before and after the liberalisation of the capital account in China. We explain the risk-sharing motives of the households behind these holdings and discuss the implications for the gross international investment position in China. In section 5, we simulate the optimal portfolios under two likely future scenarios: (i) a rebalancing of the Chinese economy to a less capital-intensive production with a higher share of labour income; and (ii) a narrowing of productivity gap between the US and China. Section 6 concludes.

2 The setup

The model is a two-country infinite-horizon model similar to Coeurdacier, Kollmann and Martin (2010). Homogenous households in each country consume, supply labour and save by buying financial assets. Firms in each country produce country-specific goods using a two-factor production function with capital and labour. Firms accumulate capital to maximise the expected future discounted dividend stream. There are four shocks in the system, namely the productivity shocks and the investment efficiency shocks in each of the two countries. Each country has two assets, equity and bond. We consider two asset market configurations. Without capital account restrictions, households in each of the two countries can freely buy and sell the four assets; whereas in the restricted market, only US bond and Chinese equity are traded. The portfolio choice problem is endogenous. The two countries may be asymmetric. Time is discrete.

2.1 Households

Households are homogenous in each of the two countries, A and B. We aim to study a situation in which the country A is developed and the country B is emerging. We will sometimes refer to country A and B as the US and China respectively. The households consume country A and B’s goods with home bias, trade financial assets and supply labour. The asset market configuration will be discussed in detail below. The utility of the representative household in country i is the following:

\[ U_i = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_{it}^{1-\sigma}}{1-\sigma} - \frac{l_{it}^{1+\omega}}{1+\omega} \right), \quad \text{for } i \in \{A, B\}, \quad (1) \]

where \( C_{it} \) is country i’s aggregate consumption in period t and \( l_{it} \) the labour supply.

Goods are aggregated by a CES aggregator:

\[ C_{it} = \left( a^{\frac{1}{\phi}} (c_{it}^{i})^{\frac{\phi-1}{\phi}} + (1-a)^{\frac{1}{\phi}} (c_{jt}^{j})^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}, \quad \text{for } j \neq i. \quad (2) \]
where $c_{jt}^i$ is the consumption of country $j$’s goods by country $i$’s households in period $t$. $a > 0.5$ is the degree of goods home bias. I assume that the law of one price holds for each type of good. The corresponding aggregate consumer price index is

$$P_{it} = \left( a p_{it}^{1-\phi} + (1-a) p_{jt}^{1-\phi} \right)^{\frac{1}{1-\phi}}, \quad \text{for } j \neq i. \quad (3)$$

where $p_{it}$ is the price of good $i$ in period $t$.

Country $i$’s consumption demand for country $i$ and $j$’s goods are standard:

$$c_{it}^i = a \left( \frac{p_{it}}{P_{it}} \right)^{-\phi} C_{it}, \quad (4)$$

$$c_{jt}^i = (1-a) \left( \frac{p_{jt}}{P_{it}} \right)^{-\phi} C_{it}, \quad \text{for } j \neq i. \quad (5)$$

In the following we describe the configurations of the asset market. We consider two regimes. In the first regime, there is international trade in equities and bonds originated from each of the two countries. Country $i$ firms issue equity which is a claim to its stream of future dividends $\{d_{is}\}_{t=s}$. Country $i$ also issues a bond. The bond entitles the owner one unit of country $i$’s goods in all future periods. Let $S_{jt+1}^i$ denote the number of country $j$’s equity held by country $i$’s households at the end of period $t$, and let $b_{jt+1}^i$ denote the number of country $j$’s bond held by country $i$’s households.\(^5\) Also denote the prices of country $i$’s equity and bond in period $t$ as $p_{it}^S$ and $p_{it}^B$ respectively. Then, the budget constraint for country $i$’s households is

$$P_{it} C_{it} + p_{it}^S S_{it+1}^i + p_{jt}^S S_{jt+1}^i + p_{it}^B b_{it+1}^i + p_{jt}^B b_{jt+1}^i = w_{it} l_{it} + (p_{it}^S + d_{it}) S_{it}^i + (p_{jt}^S + d_{jt}) S_{jt}^i + (p_{it}^B + p_{it}^B) b_{it}^i + (p_{jt}^B + p_{jt}^B) b_{jt}^i, \quad \text{for } j \neq i. \quad (6)$$

where $w_{it}$ is the nominal wage in country $i$. The budget constraint says country $i$’s households use their non-financial income (wage) and financial income (investments in country $A$ and $B$’s assets) for consumption and purchase financial assets.

We normalise the supply of equity in each country to unity. This means that

$$S_{At}^A + S_{At}^B = 1, \quad (7)$$

$$S_{Bt}^A + S_{Bt}^B = 1. \quad (8)$$

Bonds for each of the two countries are in zero net supply, so that

$$b_{At}^A + b_{At}^B = 0, \quad (9)$$

$$b_{Bt}^A + b_{Bt}^B = 0. \quad (10)$$

\(^5\)As will be clear when we describe the equity dividend in the firms, it should be noted that an equity simply means a claim to the firms’ profit. Therefore, equity here can as well be interpreted as FDI or corporate bonds.
We sometimes refer to this regime as a regime after capital account liberalisation in China.

We also study a second configuration in which only US bond and Chinese equity are tradable. These assumptions are reasonable characterizations of the investment options vis-a-vis China until recently. Non-Chinese residents face tight restrictions on the acquisition of Chinese government bonds, and their holding is negligible.\(^6\) The same is true for overseas equity and outward FDI by Chinese residents.\(^7\) We do not attempt to model the institutional arrangements attributed to the growth of the official reserves; but we instead take it as given and study the optimal portfolio allocation between the two countries. If the reserve holding decision is made by a dynamic optimising agent with intertemporal preferences, and if the government is benevolent, this distinction should not make much difference.

Taken together, this asset market configuration implies that \(S^B_{At} = b^A_{Bt} \equiv 0\) for all \(t\). Hence, the budget constraint for the households in country \(A\) is

\[
P_{At} C_{At} + P_{St}^S S^A_{At} + P_{At}^b b^A_{At+1} = w_{At} l_{At} + (P_{St}^b + d_{It}) S^A_{Bt} + (P_{At}^b + P_{At}) b^A_{At} + d_{At}
\]

We refer to this regime as one before capital account liberalisation in China.

In the asset market configuration in which all assets are tradable, households in country \(i\) maximises the utility, Equation (1), subject to the budget constraint (6). The first order conditions are:

\[
1 = \beta E_t \left[ \left( \frac{C_{it+1}}{C_{it}} \right)^{-\sigma} \left( \frac{P_{it}}{P_{it+1}} \right) R^S_{At+1} \right], \quad (12)
\]

\[
1 = \beta E_t \left[ \left( \frac{C_{it+1}}{C_{it}} \right)^{-\sigma} \left( \frac{P_{it}}{P_{it+1}} \right) R^S_{Bt+1} \right], \quad (13)
\]

\[
1 = \beta E_t \left[ \left( \frac{C_{it+1}}{C_{it}} \right)^{-\sigma} \left( \frac{P_{it}}{P_{it+1}} \right) R^b_{At+1} \right], \quad (14)
\]

\[
1 = \beta E_t \left[ \left( \frac{C_{it+1}}{C_{it}} \right)^{-\sigma} \left( \frac{P_{it}}{P_{it+1}} \right) R^b_{Bt+1} \right], \quad (15)
\]

\[
\frac{w_{it}}{P_{it}} = \frac{C_{it}^{\sigma}}{C_{it+1}^{\sigma}}, \quad (16)
\]

\(^6\)There are two ways through which non-residents can acquire Chinese bonds. Under the Qualified Foreign Institutional Investor (QFII) programme launched in 2002, licensed foreign investors can buy and sell in Chinese bond market subject to a quota. Second, the Ministry of Finance issued RMB20 billion in renminbi-denominated bonds in Hong Kong, China on 23 August 2011.

\(^7\)The size of China’s outward FDI is small but growing rapidly. From 2004 to 2011, China’s outward FDI grew from USD$5.5 billion to USD$65 billion a year, and the stock of outward FDI has reached USD$365 billion at the end of 2011 (UnctadStat). Outward portfolio equity must go through the Qualified Domestic Institutional Investor (QDII) scheme, which was introduced in 2006. The total fund raised is less than RMB200 billion, according to Yao and Wang (2012). Taken together the outward FDI and portfolio equity assets constitute less than 6% of Chinese GDP in 2011. The steady state of our model would imply the share of US equities held by the Chinese is roughly 1%, which is negligible.
where
\[
R^S_{it+1} \equiv \frac{p^S_{it+1} + d_{it+1}}{p^S_{it}}, \quad R^b_{it+1} \equiv \frac{p^b_{it+1} + p_{it+1}}{p^b_{it}}, \quad \text{for } i \in \{A, B\}.
\]

The first four equations are the consumption Euler equations for country A and B’s equities and bonds respectively. The last equation is the intratemporal tradeoff between consumption and labour.

In the configuration in which only US bond and Chinese equity are tradable, only the consumption Euler equations for US bond and Chinese equity remain (that is, Equations (13) and (14)). The labour supply conditions are unchanged.

2.2 Firms

There are homogenous firms in each country. They produce using a Cobb-Douglas technology with capital and labour inputs as follows:
\[
y_{it} = \theta_{it} k_{it}^{\kappa_i} l_{it}^{1-\kappa_i}, \quad \text{for } i \in \{A, B\}.
\]

(17)

where \(y_{it}\) denotes country \(i\)’s output and \(k_{it}\) country \(i\)’s capital input. \(\theta_{it}\) is the total factor productivity in country \(i\). We assume that productivity in each country follows an exogenous AR(1) process
\[
\log \left( \frac{\theta_{it}}{\bar{\theta}_i} \right) = \rho^\theta \log \left( \frac{\theta_{it-1}}{\bar{\theta}_i} \right) + \epsilon^\theta_{it}, \quad \text{for } i \in \{A, B\}.
\]

(18)

where \(\epsilon^\theta_{At}, \epsilon^\theta_{Bt}\) are exogenous i.i.d disturbances with variances \(\sigma^2_{\theta A}\) and \(\sigma^2_{\theta B}\) respectively. Throughout this paper, \(\bar{X}\) denotes the steady state value of variable \(X\).

Since we want to model asymmetric countries, the capital shares in production \(\kappa_{it}\), steady-state productivity \(\bar{\theta}\), and the volatility of productivity shock, \(\sigma^2_{\theta i}\) in the two countries may not be identical. We will discuss these in detail in the calibration section.

The capital accumulation rule is as follows:
\[
k_{it+1} = (1 - \delta)k_{it} + \chi_{it} I_{it},
\]

(19)

where \(I_{it}\) is aggregate investment and \(\delta\) is the depreciation rate. \(\chi_{it}\) is an investment efficiency shock. \(^8\) The investment efficiency shock, \(\chi_{it}\), follows an AR(1) process:
\[
\log \chi_{it} = \rho^\chi \log \chi_{it-1} + \epsilon^\chi_{it},
\]

(20)

where \(\epsilon^\chi_{At}, \epsilon^\chi_{Bt}\) are exogenous i.i.d disturbances with variances \(\sigma^2_{\chi A}\) and \(\sigma^2_{\chi B}\) respectively.

\(^8\)This specification follows from Coeurdacier, Kollmann and Martin (2010) and Greenwood, Hercowitz and Krusell (1997). Justiniano, Primiceri and Tambalotti (2007) have found empirical evidence supporting that the shock is helpful to account for volatilities in the business cycle.
Investment is, similar to consumption, aggregated by a CES aggregator with home bias as follows:

\[
I_t = \left( a^\phi (i_{it}^i)^{\frac{\phi-1}{\phi}} + (1-a)^\phi (i_{jt}^i)^{\frac{\phi-1}{\phi}} \right)^{\frac{1}{\phi-1}}, \quad \text{for } j \neq i. \tag{21}
\]

Clearly, the demand for country A and B’s investment inputs are given by

\[
i_{it}^i = a \left( \frac{p_t}{P_t} \right)^{-\phi} I_{it}, \tag{22}
\]

\[
i_{jt}^i = (1-a) \left( \frac{p_t}{P_t} \right)^{-\phi} I_{it}, \quad \text{for } j \neq i. \tag{23}
\]

Country i firms maximize the stream of expected discounted future dividends

\[
\max_{l_{it}, I_{it}} E_t \sum_{s=0}^{\infty} Q_{i,t+s} d_{is} \tag{24}
\]

where the dividend for country i’s firms, \(d_{it}\) is given by

\[
d_{it} = p_t y_{it} - w_{it} l_{it} - P_t I_{it} \tag{25}
\]

and \(Q_{i,t+s} \equiv \beta \left( \frac{C_{i,t+s}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{i,t+s}} \right)\) is the pricing kernel for country i’s firms.\(^9\)

The first order conditions are

\[
w_{it} l_{it} = (1 - \kappa_t) p_t y_{it}, \tag{26}
\]

\[
\frac{P_t}{\chi_{it}} = \beta E_t \left( \left( \frac{C_{it+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{it+1}} \right) \left( p_{it+1} \kappa_t y_{it+1} + (1 - \delta) \frac{P_{it+1}}{\chi_{it+1}} \right) \right), \tag{27}
\]

for \(i \in \{A, B\}\). The first equation is the labour demand condition. With a Cobb-Douglas production function, firms pay a constant fraction \((1 - \kappa_t)\) of their revenue to the labour. The second equation is the investment demand, which equates the marginal cost of an additional unit of investment, \(P_t/\chi_{it}\), to the marginal benefits, which is the discounted sum of marginal revenue products of the additional unit of capital, after accounting for depreciation.

\(^9\)Without complete financial market and perfect risk-sharing, the pricing kernel of the firms may depend on the share of equity held by country A and B’s households. Finding the pricing kernel is therefore non-trivial. Fortunately, to find the steady-state portfolio with local approximation, which is what we are interested in, we just need the first-order approximated system. Moreover, in the first-order approximation, the pricing kernels in both countries are identical, so the assumption does not affect the results of the paper.
2.3 Market clearing conditions

Country A’s productions are used to produce country A and B’s aggregate consumption and investment goods. The goods market clearing condition for country A is

\[ c^A_{At} + c^B_{At} + i^A_{At} + i^B_{At} = y_{At}, \]  

(28)

Similarly, the goods market clearing condition for country B is

\[ c^A_{Bt} + c^B_{Bt} + i^A_{Bt} + i^B_{Bt} = y_{Bt}. \]  

(29)

For completeness, the equity and bond market clearing conditions in the system with all four assets tradable are given by Equations (7), (8), (9) and (10). In the system in which only US bond and Chinese equity are tradable, the asset market clearing conditions are Equations (8) and (9).

Finally, the household budget constraint in country A can be re-written as the law of motion of the net foreign assets

\[ NF_{A_{At+1}} = p_{At}y_{At} - P_{At}(C_{At} - I_{At}) + R^b_{At}NF_{A_{At}} + \bar{\rho}_{A_{At}}y_{At} \xi_t, \]  

(30)

where \( \xi_t \) denotes the excess portfolio return. In the configuration in which all four assets are tradable, the net foreign assets and excess portfolio return are defined as

\[ NF_{A_{At+1}} \equiv p^S_{Bt}S^A_{Bt+1} - p^S_{At}S^B_{At+1} + p^b_{At}b^A_{At+1} + p^b_{Bt}b^B_{Bt+1}, \]  

(31)

\[ \bar{\rho}_{A_{At}}y_{At} \xi_t \equiv S^A_{Bt}p^S_{Bt+1}(R^S_{Bt} - R^b_{At}) - S^B_{At}p^S_{At+1}(R^S_{At} - R^b_{At}) + b^A_{At}p^b_{Bt}b^B_{At+1}(R^b_{Bt} - R^b_{At}) \]  

(32)

and in the configuration in which there are trades in US bonds and Chinese equity,

\[ NF_{A_{At+1}} \equiv p^S_{Bt}S^A_{Bt+1} + p^b_{At}b^A_{At+1}, \]  

(33)

\[ \bar{\rho}_{A_{At}}y_{At} \xi_t \equiv S^A_{Bt}p^S_{Bt-1}(R^S_{Bt} - R^b_{At}). \]  

(34)

This completes the description of the model.

3 Model Solution

We are interested in the steady-state optimal portfolio allocations both before and after capital account liberalisation in China. Devereux and Sutherland (2011) and Tille and van Wincoop (2010) recently developed methodologies to compute portfolio allocations under both complete and incomplete asset markets. We follow a two-step procedure developed by Devereux and Sutherland (2011). In the first step, we approximate the non-portfolio system up to the first order around the deterministic steady state. We temporarily treat the deviation of the excess portfolio return, \( \xi_t \), as an exogenous i.i.d process and solve this first-order model in terms of the state of the economy, the exogenous shocks and the excess portfolio return. The second step makes use of the second-order approximation to
the optimal portfolio conditions to compute the portfolio allocations. Below we discuss the two-step procedure in more detail.

In order to approximate the non-portfolio system, we need to first find the steady state. The deterministic non-portfolio steady-state of the model, under either set of asset market configurations, is the solution of the steady-state versions of equations (4), (5), (14), (16), (17), (19), (22), (23), (26), (27), (28), (29) and (30). It is well-known that in open-macro models, the net foreign asset position follows a unit root process (see Schmitt-Grohe and Uribe, 2003). We choose the net foreign asset position in the steady-state to be zero, following the literature on open economy financial macroeconomics. We also normalise the price of country A’s goods to unity i.e. \( \bar{p}_A \equiv 1 \). This set of equations solve for the steady state of the following variables:

\[ \{ NF_A, \bar{y}_A, \bar{y}_B, \bar{k}_A, \bar{k}_B, \bar{l}_A, \bar{l}_B, \bar{w}_A, \bar{w}_B, \bar{\bar{p}}_A, \bar{\bar{p}}_B, \bar{\bar{P}}_A, \bar{\bar{P}}_B, \bar{\bar{I}}_A, \bar{\bar{I}}_B, \bar{\bar{R}} \} , \]

where \( \bar{R} = \bar{R}_A^s = \bar{R}_B^s = \bar{R}_A^k = \bar{R}_B^k \).

We approximate the set of equations described above up to the first order around the deterministic steady state. Appendix A reports the log-linearised system. This set of equations, together with the shock processes and the initial conditions of the predetermined variables, determines the first-order transition path of the non-portfolio part of the model, taking the behaviour of the excess portfolio return, \( \hat{\xi}_t \), as exogenous. Specifically, denote \( s_t, c_t, \epsilon_t \) as the vector of state variables, non-state variables and the exogenous shock processes respectively, that is

\[
\begin{align*}
    s_t &= \begin{bmatrix} NY_A, \bar{k}_A, \bar{k}_B, \bar{l}_A, \bar{l}_B, \bar{w}_A, \bar{w}_B, \bar{\bar{p}}_A, \bar{\bar{p}}_B, \bar{\bar{P}}_A, \bar{\bar{P}}_B, \bar{\bar{I}}_A, \bar{\bar{I}}_B, \bar{\bar{R}} \end{bmatrix}^t, \\
    c_t &= \begin{bmatrix} \bar{y}_A, \bar{y}_B, \bar{\bar{C}}_A, \bar{\bar{C}}_B, \bar{\bar{I}}_A, \bar{\bar{I}}_B, \bar{\bar{q}}_t \end{bmatrix}^t, \\
    \epsilon_t &= \begin{bmatrix} \epsilon_A, \epsilon_B \end{bmatrix}^t.
\end{align*}
\]

where \( \hat{X}_t \equiv (X_t - \bar{X})/\bar{X} \) is the percentage deviation of the variable \( X_t \) from its steady state \( \bar{X} \) and \( NF_A \equiv \frac{Y_A - \bar{Y}_A}{\bar{p}_A \bar{y}_A} \). Then the transition path of the non-portfolio part of the model is

\[
\begin{align*}
    s_{t+1} &= F_1 s_t + F_2 \epsilon_t + F_3 \hat{\xi}_t, \\
    c_t &= G_1 s_t + G_2 \epsilon_t + G_3 \hat{\xi}_t.
\end{align*}
\]

where \( F_1, F_2, F_3, G_1, G_2, G_3 \) are unknown conformable matrix which can be solved using standard methods.

---

10 The meaning of the second-order approximation is the following: Assets have equal returns in the steady state, and they are only distinguishable in terms of their risk characteristics. First-order approximation is unhelpful due to certainty equivalence. The second-order components contain the covariance terms which capture the risk characteristics of the assets.

11 Choosing a negative net foreign asset will imply that the US runs a trade surplus in the steady state, which is unreasonable. This is because the excess portfolio return is zero in the steady state.
The second step of the procedure involves a second-order approximation to country A and B’s optimal portfolio conditions to capture the covariance relations. For instance, country A’s portfolio choice condition is

$$0 = E_t \left( \beta \left( \frac{C_{A_{t+1}}}{C_{A_t}} \right)^{-\sigma} \left( \frac{P_{At}}{P_{At+1}} \right) R_{xt+1} \right)$$

(35)

and similarly for country B. $R_{xt}$ denotes the excess return. Before capital account liberalisation, since cross-border asset holdings are restricted, the excess return is

$$R_{xt} \equiv R_{SB_{t}} - R_{B_{At}}.$$ 

After capital account liberalisation, the vector of excess return becomes

$$R_{xt} \equiv \left[ R_{SB_{t}} - R_{B_{At}}, R_{B_{B_{t}}} - R_{B_{At}}, R_{SB_{t}} - R_{B_{At}} \right]'.$$ 

The second-order approximation of the country A and B’s portfolio choice conditions results in the following orthogonality condition:

$$0 = E_t \left( \sigma \left( \hat{C}_{A_{t+1}} - \hat{C}_{B_{Bt+1}} \right) + (\hat{a}_A + \hat{a}_B - 1)\hat{q}_{t+1} \right) \hat{R}_{xt+1},$$

(36)

where $\hat{q}_t$ is the terms of trade, defined as

$$\hat{q}_t \equiv \hat{p}_{At} - \hat{p}_{Bt},$$

(37)

so that a fall in $\hat{q}_t$ is a worsening in the terms of trade in country A, or the US. $\hat{R}_{xt}$ is defined as $\hat{R}_{SB_{t}} - \hat{R}_{B_{At}}$ before capital account liberalisation, and $\left[ \hat{R}_{SB_{t}} - \hat{R}_{B_{At}}, \hat{R}_{B_{B_{t}}} - \hat{R}_{B_{At}}, \hat{R}_{SB_{t}} - \hat{R}_{B_{At}} \right]'$ after liberalisation. Finally, the parameters $\hat{a}_A$ and $\hat{a}_B$ are functions of the home bias $a$ and the steady-state prices, defined in Appendix A, and $0 < \hat{a}_A, \hat{a}_B < 1$. Equation (36) is a risk-sharing condition. On the right hand side is the covariance between the relative marginal rate of substitution and the excess portfolio return, both in their first-order approximated forms. From the first step of the procedure, we know the first-order approximated behaviour of each of these two terms conditional on the steady-state portfolio allocation through $\hat{\xi}_{t+1}$. Hence, this equation can be used to back out the unique steady-state portfolio such that the correlation is zero, using a method developed in Devereux and Sutherland (2011).

4 Model Property

4.1 Calibration

Given the complexity of the model, we solve the model numerically. We need to choose the parameter values. We choose the discount factor $\beta = 0.96$ so that the steady-state interest rate is around 4.2%. We set the degree of home consumption and investment

12See Appendix B for details.
bias at $a = 0.75$, a common value for home consumption bias in the macroeconomic literature. We set the coefficient of relative risk aversion to $\sigma = 2.5$, again a standard value in the literature. The inverse of Frisch elasticity of labour supply is set to $\omega = 1$, consistent with the estimate by Kimball and Shapiro (2008). The depreciation, $\delta$, is assumed to be 0.1. There is a wide range of estimates for the Armington elasticity of substitution between home and foreign goods. Feenstra et al. (2012) estimate that the macro elasticity is not significantly different from unity but the micro elasticity is much larger. In the portfolio choice literature, common values used are slightly above unity. Backus, Kehoe and Kydland (1994) and Engel and Matsumoto (2009) use a value of 1.5, Heathcote and Perri (2002) use 1.2. We use $\phi = 1.1$ for our calibration. All the above calibrated values are well within the range of values used in the macroeconomic literature.

We allow the capital shares in production and the levels of productivity to be different for the US and China. We use $\kappa_A = 0.35$ for the capital share in the US production function. For the counterpart in China, Brandt, Hsieh and Zhu (2008) estimate that the labour share in production for agricultural and non-agricultural sectors are both in the vicinity of 0.5. Hence, we set $\kappa_B = 0.5$. For the level of productivity, we normalise the $\bar{\theta}_A = 1$ in the US. Then, according to Zhu (2012), the productivity in China is around 13% of that in the US. We set $\bar{\theta}_B$ to be 0.13.

We calibrate the parameters related to the shocks. Coeurdacier, Kollmann and Martin (2010) estimate VARs for the total factor productivity across G7 countries. We follow their estimates to use $\rho_\theta = 0.75$, $\sigma_{\theta A} = 0.012$ and $\text{Corr}(\epsilon_{\theta A}, \epsilon_{\theta B}) = 0.45$. Garcia-Cicco et al. (2010) find higher volatility of productivity shocks in emerging markets. This assumption is also made in the portfolio choice problem in Devereux and Sutherland (2009). Shi, Wu and Xu (2012) construct a business cycle model of China and estimate that the $\sigma_{\theta B} = 0.017$, which we follow. For the investment efficiency shocks, there is no empirical estimation in China that we know of. We use the following values estimated by Coeurdacier, Kollmann and Martin (2010) for G7 countries: $\rho_\chi = 0.79, \sigma_{\chi A} = \sigma_{\chi B} = 0.0173, \text{Corr}(\epsilon_{\chi A}, \epsilon_{\chi B}) = 0.19$, but also experiment with other values for the volatility in China.

The calibration of the model parameters and shocks are summarized in Table 1.

### 4.2 Steady State Portfolios in the Benchmark Model

#### 4.2.1 Before capital account liberalisation

In this section, we consider the optimal portfolio in the system in which only US bond and Chinese equity are tradable. Given the insight of the optimal portfolio choice equation (36), we consider the relative marginal rate of substitution and the excess portfolio return when the system is hit by each of the four shocks, assuming that there is no portfolio diversification. This assumption of zero portfolio holding is important because in this case ($\hat{\xi}_t = 0$), the solution to the first-order approximation is known.
Table 1: Model Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reasoning/ Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.96</td>
<td>Steady interest rate = 4.2%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
<td>Standard value</td>
</tr>
<tr>
<td>$a$</td>
<td>0.75</td>
<td>G7 mean import to GDP ratio = 25%</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.5</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\omega$</td>
<td>1</td>
<td>Kimball and Shappiro (2008)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.1</td>
<td>Average value in literature.</td>
</tr>
<tr>
<td>$\kappa_A$</td>
<td>0.35</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\kappa_B$</td>
<td>0.5</td>
<td>Brandt, Hsieh and Zhu (2008)</td>
</tr>
<tr>
<td>$\theta_A$</td>
<td>0.13</td>
<td>Zhu (2012)</td>
</tr>
<tr>
<td>$\theta_B$</td>
<td>0.75</td>
<td>Coeurdacier, Kollmann and Martin (2010)</td>
</tr>
<tr>
<td>$\sigma_{\theta A}$</td>
<td>1.2%</td>
<td>Coeurdacier, Kollmann and Martin (2010)</td>
</tr>
<tr>
<td>$\sigma_{\theta B}$</td>
<td>1.7%</td>
<td>Shi, Wu and Xu (2012)</td>
</tr>
<tr>
<td>$Corr(\theta_A, \theta_B)$</td>
<td>0.45</td>
<td>Coeurdacier, Kollmann and Martin (2010)</td>
</tr>
<tr>
<td>$\rho_\chi$</td>
<td>0.79</td>
<td>Coeurdacier, Kollmann and Martin (2010)</td>
</tr>
<tr>
<td>$\sigma_{\chi A}$</td>
<td>1.73%</td>
<td>Coeurdacier, Kollmann and Martin (2010)</td>
</tr>
<tr>
<td>$\sigma_{\chi B}$</td>
<td>1.73%</td>
<td>Coeurdacier, Kollmann and Martin (2010)</td>
</tr>
<tr>
<td>$Corr(\chi_A, \chi_B)$</td>
<td>0.19</td>
<td>Coeurdacier, Kollmann and Martin (2010)</td>
</tr>
</tbody>
</table>

Suppose US productivity ($\theta_{At}$) rises. Both consumption and output in the US rise and the real exchange rate depreciates. In China, consumption also rises, but not by as much as in the US. As a result, relative consumption in the US increases. Due to the real exchange rate effect, Chinese equity now pays a higher dividend relative to US bond. This means that the relative return of the Chinese equity is positively correlated with the relative consumption in the US, and therefore buying US bond and selling Chinese equity can hedge the productivity shock in the US. On the other hand, when Chinese productivity ($\theta_{Bt}$) rises, relative consumption in the US falls. Holding US bond is good for two reasons. First, as Chinese exchange rate depreciates, US prices are higher. Second, Chinese firms increase their investment and hence reduce the dividend payout. Again, US households have an incentive to hold their own bond for risk-sharing purposes.

When the US receives a positive investment efficiency shock ($\chi_{At}$), the relative consumption in the US falls because it is optimal to switch consumption for investment. Since US firms use more of its own goods for investment, the real exchange rate appreciates. As a result, Chinese firms reduce their investment and increase their dividend payout. In this case, holding Chinese equity enhances risk-sharing for US households. On the other hand, suppose China receives a favourable investment efficiency shock ($\chi_{Bt}$). In order to make room for the rise in investment, Chinese firms cut dividend and households cut consumption relative to the US households. Again, for the US households, Chinese equity provides a better hedge to consumption volatilities.
Hence, productivity shocks and investment efficiency shocks induce households to hold different mixes of assets. Since the financial market is incomplete, risk-sharing is imperfect. The optimal portfolio depends on the relative volatilities of the two types of shocks. In particular, the more volatile the productivity shocks are the more asset home bias in equilibrium.

As explained in the calibration section, we assume that the volatility of the investment efficiency shocks to be larger than that of the productivity shocks in a way that is consistent with empirical data.\textsuperscript{13} As a result, the steady-state asset portfolio is one in which US households short its own bond and purchase Chinese equity. We find that $\bar{S}^B_B = 92\%$ and $\bar{b}^A_A = -0.013$. In terms of gross portfolio holdings, the US holding of Chinese equity is 27% of Chinese nominal income, whereas Chinese holding of US bond as a ratio of US nominal income is about 21%. The numerical values are within reasonable range compared with the actual data. In particular, the gross financial asset position derived here also appears to be consistent with the observed data reported in Figure 1.

### 4.2.2 After capital account liberalisation

After capital account liberalisation in China, equity and bond in both of the countries are tradable. Coeurdacier, Kollmann and Martin (2010) and Coeurdacier and Rey (2011) have analysed similar set-ups. Our model is a generalisation of theirs as we allow for asymmetric country sizes and different capital shares in the production functions. But we can still borrow insights from these models.

There are two sources of risks in this model: the real exchange rate risk and the non-tradable income risk. The real exchange rate risk refers to a situation in which the real exchange rate depreciates so that foreign goods become more costly. The non-tradable income risk refers to the fact that households earn their wage income in their respective countries, and hence the fluctuation of their wage income is non-diversifiable. After the liberalisation of the capital account, since there are as many assets as the number of shocks, and the asset returns are not linearly dependent, the financial market is complete. The optimal combination of the asset portfolio can achieve perfect risk-sharing.\textsuperscript{14}

What combination of the assets can achieve perfect risk-sharing? First, notice that the relative payoff of the US bond is just the terms of trade, $\hat{q}_t$, which is perfectly correlated with the real exchange rate. This means that any real exchange rate risks can be hedged against with the bonds. This leaves to the equity the job of hedging the non-tradable

\textsuperscript{13}Since there is not much empirical guidance regarding the volatility of the investment efficiency shock in China, we have experimented with other values for robustness checks. As $\sigma_B^V$ is increased from 1.5% to 2.5%, the US holding of Chinese equity as a fraction of Chinese nominal GDP rises from 23% to 38%. The main qualitative results are not affected.

\textsuperscript{14}This implies that the portfolios after capital account liberalisation do not depend on the volatilities, correlations and the persistence of shocks.
income risk. We can compute the dividend payments of the equity as follows:

\[
\hat{d}_{At} - \hat{d}_{Bt} = \frac{1 - (1 - \delta)\beta}{1 - \beta} (\hat{y}_{At} - \hat{y}_{Bt}) - \frac{\delta \beta}{1 - \beta} (\hat{I}_{At} - \hat{I}_{Bt}) \\
+ \frac{1}{1 - \beta} ((1 - (1 - \delta)\beta) - \delta \beta (\tilde{a}_A + \tilde{a}_B - 1)) \hat{q}_t
\]  

(38)

What is important is that the relative dividend payment depends on output and investment, which is correlated with the wage income conditional on the real exchange rate. (See the labour demand equation (26).) Hence, suppose a combination of exogenous shocks raises the relative investment in the US but leaves the terms of trade unchanged. Then due to the investment home bias, the relative output rises in the US, raising the labour income of the US households. On the other hand, US firms reduce their dividend payout relative to the Chinese firms in order to increase their investment. This means that the wage income and the relative dividend are negatively correlated, which makes holding US equity helpful to risk-sharing. For other realisations of shocks that move the terms of trade as well, the correlation between non-financial income and relative dividend conditional on the terms of trade will still be negative, and holding local equity will still be desirable. This is because, as discussed, the ‘residual’ real exchange rate risks have been hedged against through bond holdings.

Next, we discuss the holding of bonds. Suppose we ignore the non-tradable income risk, then there is a home bias in bond. A positive productivity shock in the US raises the relative consumption and depreciates the real exchange rate; a positive shock to investment efficiency reduces relative US consumption and appreciates the real exchange rate. In both cases, relative consumption is negatively correlated with the real exchange rate.

But the wage income and equity income are also dependent of the terms of trade, and when households choose their equity portfolio to insure against non-tradable income risks as described previously, the terms of trade components of these income may have already provided some degree of risk-sharing. Taking into account these contingent income, Coeurdacier, Kollmann and Martin (2010) show that theoretically the optimal steady-state portfolio may entail positive or negative holding of the local bond, depending on the preference parameters and the elasticity of substitution between home and foreign goods, \(\phi\). When the elasticity is high (above unity), terms of trade movements are small in response to shocks, and the wage and equity income taken together have only partly offset the need to hedge against real exchange rate risks, and in this case, there is a home bias in bond holdings.

In our calibration, guided by the data before capital account liberalisation, we use a value of \(\phi\) above unity, and the steady-state holding of bond exhibits home bias. We find that \(\bar{b}_A = 0.095\) and \(\bar{b}_B = 0.029\). Moreover, each country takes a positive position in the equity of the other country. We find that \(\bar{S}_A = 81%\) and \(\bar{S}_B = 54%\). The optimal

\[15\text{This can be verified by substituting the goods market clearing conditions into the relative dividend equation.}\]
Table 2: Steady-state Portfolio

<table>
<thead>
<tr>
<th>steady-state Portfolio</th>
<th>$S_A^A$</th>
<th>$S_B^B$</th>
<th>$\tilde{b}_A^A$</th>
<th>$\tilde{b}_B^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Chinese capital account liberalisation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Values</td>
<td>1*</td>
<td>0.92</td>
<td>-0.013</td>
<td>0*</td>
</tr>
<tr>
<td>After Chinese capital account liberalisation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Values</td>
<td>0.81</td>
<td>0.54</td>
<td>0.095</td>
<td>0.029</td>
</tr>
</tbody>
</table>

* Model restriction.

Table 3: Steady-state Gross Financial Asset Positions

<table>
<thead>
<tr>
<th></th>
<th>Before Liberalisation</th>
<th>After Liberalisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>US holding of Chinese equity to Chinese nominal income</td>
<td>27%</td>
<td>163%</td>
</tr>
<tr>
<td>Chinese holding of US bond to US nominal income</td>
<td>21%</td>
<td>-152%</td>
</tr>
<tr>
<td>US holding of Chinese bond to Chinese nominal income</td>
<td>0*</td>
<td>-295%</td>
</tr>
<tr>
<td>Chinese holding of US equity to US nominal income</td>
<td>0*</td>
<td>47%</td>
</tr>
</tbody>
</table>

* Model restriction.
steady-state portfolios are reported in Table 2. The implied gross financial asset positions are reported in Table 3.

One important advantage of our model over Devereux and Sutherland (2009) is that the optimal steady-state portfolio holding computed in our model exhibits equity home bias, that is

\[ \bar{S}_A > \frac{\bar{p}_A y_A}{\bar{p}_A y_A + \bar{p}_B y_B}, \quad \bar{S}_B > \frac{\bar{p}_B y_B}{\bar{p}_A y_A + \bar{p}_B y_B}. \]

Equity home bias is a robust empirical finding by French and Poterba (1991). Our model can generate this feature because shocks generate a redistribution of firm revenue between labour income and dividends, which in turn relies on the introduction of differentiated goods across countries and endogenous capital accumulation. Without differentiated goods, the steady-state portfolios in Devereux and Sutherland’s (2009) model with complete market are ones with a large foreign equity bias. In their model, as well as Baxter and Jermann (1997), home labour income and home equity returns are positively correlated.

We compare the portfolio holdings before and after capital account liberalisation in China. First, consistent with other research in the literature and empirical evidence (such as Sorensen et al., 2007), financial liberalisation allows better risk-sharing and leads to a reduction in equity home bias. For China, this means that both outward FDI and portfolio investment, as well as inward FDI and portfolio investment will increase. This result echoes with the empirical prediction in He et al. (2012).

Second, the gross financial position in China will rise substantially after the liberalisation. Our numerical results show that the average of gross financial assets and liabilities as a ratio to GDP will rise from 27% before the liberalisation to over 300% after the liberalisation. The qualitative prediction of a rising gross international investment position is shared by Ma and Zhou (2009).

Third, a perhaps striking result is that after liberalisation of the Chinese capital account, the Chinese holding of US bonds will reverse, from a long position to a large short position. The reason is the following. Before capital account liberalisation, when China receives a positive investment efficiency shock, without any portfolio diversification Chinese consumption has to be cut to finance the investment. Holding Chinese equity yields low dividend for the same reason. Hence, Chinese households are better off saving with the other alternative, the US bonds. After capital account liberalisation, the Chinese can choose to hold US equity instead, which is a better hedge against non-financial income.

\[ ^{16} \text{Readers may find the gross position after capital account liberalisation very high. But we stress that this number refers to the steady state position after a full capital account liberalisation in China. We do not speculate on how long the transition will take. Moreover, compared with other countries, the United Kingdom, Netherlands and Switzerland currently has a } (GFA + GFL)/2GDP \text{ ratio of 6, and the Newly Industrialised Asia has a ratio of 3. These ratios are on a rising trend in recent years and are expected to rise further with increasing international financial integration. See Obstfeld (2012)}. \]
risks than US bonds. The large short position in US bonds is then required to reduce exposure in exchange rate risks. US households, in return, will hold a negative position in Chinese bonds.

One way through which the non-Chinese residents can hold a short position in the renminbi-denominated bonds is by issuing the bonds offshore. These bonds are sometimes referred to as ‘panda bonds’. Non-residents in need of renminbi can issue renminbi-denominated bonds to finance their economic activities in China. Currently, the scale of the panda bonds is small and the money raised through such channels can only be used in China (Yu, 2012). The analysis above suggests that once the capital account in China is liberalised, the circulation of renminbi-denominated bonds in the international market will increase substantially. This would be a major boost to renminbi internationalisation.

5 Comparative Statics

In this section, we consider two economic transitions that are likely to happen in China in future and study the implications for the asset portfolios and the gross financial asset holdings between China and the US. We look at two scenarios. In the first, we consider an internal rebalancing in China which reduces the capital share in production. The second scenario considers a sustained rise in productivity in China relative to the US.

5.1 China moves to a more labour-intensive production

China is likely to rebalances its production with a higher share of labour income in future for two reasons. First, China at the moment has a large capital-intensive manufacturing sector, leading to a capital share of about 0.5 (Brandt et al. 2008), which is much higher than the G7 average of about 0.35. Following the experience of the developed economies, a continuing transition is likely to move more workers towards the labour-intensive service sector. This will lower the capital share in the production function. Second, Song et al. (2011) and Brandt et al. (2010) argue that the state sector has a significantly higher capital-to-labour ratio than the non-state sector. As rebalancing takes place and capital market distortions are being removed, the capital-to-output ratio is likely to fall. In terms of the model, the transition is captured by a rise in the labour share in the Chinese production function, or a fall in \( \kappa_B \).

The results are shown in Figure 2 and Figure 3. The red solid lines in Figure 2 report the steady-state portfolios in the economy with restricted capital account for different values of \( \kappa_B \) before capital account liberalisation. The key result is that when China shifts its production towards a labour-intensive service-based one, Chinese households hold more of their own equity and less of the US bond. This reduces the gross financial asset positions, and hence the asymmetry in China’s international balance sheet. The reason is simple: as the steady-state capital-to-output ratio is lower, the investment efficiency shocks become less important in China, \textit{ceteris paribus}, relative to the productivity shocks. When the households care more about productivity shocks, they raise their holding of Chinese
Figure 2: Asset portfolios when China moves to a more labour-intensive production.

equity and reduce the holding of US bond, reducing the gross positions. The red solid line in Figure 3 shows the corresponding gross financial assets. The simulation results suggest that, *ceteris paribus*, when the capital share \( \kappa_B \) falls from 50% to, say, 35%, consistent with the value in major advanced economies, the US holding of Chinese equity as a ratio of Chinese output will fall sharply, from 27% to 10%.

If the rebalancing occurs after capital account liberalisation in China, we obtain a similar qualitative result that the gross financial asset positions will decline, but the quantitative effect is larger. The blue ‘dash-dotted’ lines in Figure 2 and Figure 3 show the optimal portfolios and the gross financial asset positions for different values of \( \kappa_B \) after capital account liberalisation. A rise in labour share in production means that more of the production revenue goes to the wages. Consequently, the non-diversifiable income risks for Chinese households become more severe. To improve risk-sharing, Chinese households switch their portfolio towards the equities (both Chinese and US) in order to hedge against the non-financial income risks. Larger holdings of equities have compensated for some real exchange rate risks, so there is a shift of the portfolio away from the bonds. Quantitatively,
when the capital share $\kappa_B$ falls from 50% to 35%, the US holding of Chinese equity falls from 163% of Chinese output to 85%.

5.2 Productivity growth in China

We also analyse the counterfactual in which the level of productivity in China catches up with that of the US. Zhu (2012) estimates using growth accounting techniques that current productivity in China is very low relative to that of the US. He argues that through gradual and persistent institutional reforms and policy changes that reduce distortions and better align economic incentives, productivity in China is likely to experience a sustained growth in the coming future, relative to the US’s. We therefore do a comparative statics analysis by increasing the steady-state level of productivity in China $\bar{\theta}_B$.

Figure 4 and 5 summarize the results of this exercise. The red solid lines in Figure 4 report the steady-state portfolios for different values of Chinese productivity before capital account liberalisation. The effect of a relative rise to Chinese productivity is similar to
a fall in the share of capital in output: Chinese households hold a larger fraction of local equity. The reason is that a rise in Chinese productivity increases the relative importance of the productivity shock, which can be insured by holding a larger fraction of wealth in domestic equity. The numerical results for the corresponding gross financial positions are displayed in Figure 5. Suppose China replicates its extraordinary growth performance for another two decades, so that by the end of this period the productivity gap will narrow to about 0.4. In this case the US holding of Chinese equity will fall from 27% to 22% of Chinese output. On the other hand, the Chinese holding of US bond as a ratio of US output will fall modestly by less than 1% using our calibration. More generally, this ratio may rise or fall as Chinese productivity rises, depending on the calibration. The reason is that a growing China implies a bigger appetite for assets in general, and this offsets the reduction in relative US bond demand discussed above.

The blue dash-dotted lines in Figure 4 and Figure 5 report the steady-state portfolios and the gross financial asset holdings respectively for different values of Chinese productivity after capital account liberalisation. Chinese households hold a larger share of (both
Chinese and US) equities. This is again because households have an incentive to insure against the increased non-tradable risks associated with the wage income. The Chinese holding of US bonds will reduce further as a share of US output. On the other hand, the US holding of Chinese bond to Chinese output ratio is becoming less negative as the Chinese economy is growing rapidly. To sum up, the effect of a relative rise in Chinese productivity after capital account liberalisation is also similar to that of an internal rebalancing in China. However, the quantitative effect of a sustained productivity growth is smaller, which reflects the fact that the rise in productivity raises the relative income for all factors in China whereas the internal rebalancing redistributes income from the capital owner to the workers.

The bottom line of these comparative statics analysis is that as China rebalances its economy with a higher share of labour income and experiences further economic growth, the asymmetry in China’s international balance sheet will be moderated. In particular, the Chinese will keep increasing its FDI and portfolio equity investment in the US. Moreover, in the long run, as long as the Chinese economy keeps a fast pace of growth and further
integrates with the global economy, we may see growing popularity of the renminbi-denominated bonds in the international bond markets.

6 Conclusions

In this paper, we constructed and calibrated a microfounded model with multiple assets to study the optimal portfolio choices before and after capital account liberalisation in China. The model is able to generate a ‘long debt, short equity’ portfolio in China before the liberalisation, something which is consistent with observed data. We then used the model to predict the portfolio choice after capital account liberalisation and obtained results that broadly agree with the ones in the empirical literature (such as He et al., 2012). Our results suggested the following after capital account liberalisation in China:

1. Portfolio equity and FDI flows, both into and out of China, would increase significantly.
2. China as a whole would reverse its US bond holding, from a long position currently to a short position.
3. The US as a whole would take a short position in Chinese bonds.
4. Continued economic growth in China and a reduction in capital share in production could also rebalance China’s international balance sheet.

This set of results has important implications for the internationalisation of the renminbi. Based on this model, non-Chinese residents will have an incentive to issue renminbi-denominated bonds for risk-sharing purposes. Ultimately, we can envisage an international monetary system in which the private and public sectors use other major currencies such as the renminbi as both investment and funding currencies along with the US dollar.

Our analysis should be interpreted with the following caveats. First, in this paper we focused on the deterministic steady state in the general equilibrium. This means that the equities and bonds in this steady state have equal returns. (Otherwise, the assets with higher return will be in excess demand, which cannot be an equilibrium.) Hence, this model does not generate any equity premium and does not deal with the ‘exorbitant privilege’ of the US, meaning the phenomenon that US bond yields are lower than the yield of other countries’ bonds. Without any excess asset returns in the steady state, our model does not generate both a net foreign liability position and a trade deficit for the US. Second, the analysis assumes either fully open or fully closed capital accounts. This means that it does not deal with transition dynamics, or capital flow management measures that can be adopted in the face of disruptive flows during the process of capital account liberalisation. In addition, barriers to capital flows such as transaction costs, information costs and other non-policy barriers are abstracted in the analysis. We leave these issues for future research.

17See Gourinchas et al. (2010) for a model that studies the exorbitant privilege.
References


A Log-linearised non-portfolio system

A.1 The full system

The log-linearised full non-portfolio system of the model is presented as follows:

\[
(1 - \Xi_A)\hat{y}_{At} = \hat{\theta}_{At} + \kappa_A\hat{k}_{At} + \Xi_A(1 - \hat{a}_A)\hat{q}_t - \sigma\Xi_A\hat{C}_{At},
\]
\[
(1 - \Xi_B)\hat{y}_{Bt} = \hat{\theta}_{Bt} + \kappa_B\hat{k}_{Bt} - \Xi_B(1 - \hat{a}_B)\hat{q}_t - \sigma\Xi_B\hat{C}_{Bt},
\]
\[
\hat{k}_{At+1} = (1 - \delta)\hat{k}_{At} + \delta(I_{At} + \hat{\chi}_{At}),
\]
\[
\hat{k}_{Bt+1} = (1 - \delta)\hat{k}_{Bt} + \delta(I_{Bt} + \hat{\chi}_{Bt}),
\]
\[
0 = E_t \left(-\sigma(\hat{C}_{At+1} - \hat{C}_{At}) + (1 - \beta(1 - \delta))(\hat{y}_{At+1} - \hat{C}_{At+1} + (1 - \hat{a}_A)\hat{q}_{t+1})\right)
+ (1 - \beta(1 - \delta)\rho_{\chi})\hat{\chi}_{At},
\]
\[
0 = E_t \left(-\sigma(\hat{C}_{Bt+1} - \hat{C}_{Bt}) + (1 - \beta(1 - \delta))(\hat{y}_{Bt+1} - \hat{C}_{Bt+1} + (1 - \hat{a}_B)\hat{q}_{t+1})\right)
+ (1 - \beta(1 - \delta)\rho_{\chi})\hat{\chi}_{Bt},
\]
\[
0 = E_t \left(\sigma(\hat{C}_{At+1} - \hat{C}_{Bt+1}) + (\hat{a}_A + \hat{a}_B - 1)\hat{q}_{t+1}\right)
- \left(\sigma(\hat{C}_{At} - \hat{C}_{Bt}) + (\hat{a}_A + \hat{a}_B - 1)\hat{q}_t\right),
\]
\[
\mu_A\hat{y}_{At} = \hat{a}_A\mu_A[(1 - \Lambda_A)\hat{C}_{At} + \Lambda_A\hat{I}_{At}] + (1 - \hat{a}_B)\mu_B[(1 - \Lambda_B)\hat{C}_{Bt} + \Lambda_B\hat{I}_{Bt}]
+ [- (1 - \hat{a}_A)\mu_A + (1 - \hat{a}_B)\mu_B]\hat{p}_M
- \phi[\hat{a}_A(1 - \hat{a}_A)\mu_A + \hat{a}_B(1 - \hat{a}_B)\mu_B]\hat{q}_t,
\]
\[
\mu_B\hat{y}_{Bt} = (1 - \hat{a}_A)\mu_A[(1 - \Lambda_A)\hat{C}_{At} + \Lambda_A\hat{I}_{At}] + \hat{a}_B\mu_B[(1 - \Lambda_B)\hat{C}_{Bt} + \Lambda_B\hat{I}_{Bt}]
+ [- (1 - \hat{a}_B)\mu_B + (1 - \hat{a}_A)\mu_A]\hat{p}_B
+ \phi[\hat{a}_A(1 - \hat{a}_A)\mu_A + \hat{a}_B(1 - \hat{a}_B)\mu_B]\hat{q}_t,
\]
\[
\hat{N}FA_{At+1} = \hat{y}_{At} + (1 - \hat{a}_A)\hat{q}_t - (1 - \Lambda_A)\hat{C}_{At} - \Lambda_A\hat{I}_{At} + \frac{1}{\beta}\hat{N}FA_M + \hat{\xi}_t.
\]

where $\hat{X}_t$ denotes the percentage change of the variable $X_t$ from its steady state $\bar{X}$ \textit{i.e.} $\hat{X} \equiv (X_t - \bar{X})/\bar{X}$, $\hat{N}FA_M \equiv \frac{dNFA_M}{p_A y_A}$, and

\[
\hat{q}_t \equiv \hat{p}_M - \hat{p}_B
\]

denotes the terms of trade. The parameters are defined as

\[
\Xi_i \equiv (1 - \kappa_i)/(1 + \omega), \quad \text{for } i \in \{A, B\},
\]
\[
\hat{a}_A \equiv \frac{\alpha p_A^{1-\phi}}{\alpha p_A^{1-\phi} + (1 - a)\tilde{p}_A^{1-\phi}}, \quad \hat{a}_B \equiv \frac{\alpha p_B^{1-\phi}}{\alpha p_B^{1-\phi} + (1 - a)\tilde{p}_A^{1-\phi}}
\]
\[
\mu_A \equiv \frac{\tilde{p}_A y_A}{\tilde{p}_A y_A + \tilde{p}_B y_B}, \quad \mu_B = 1 - \mu_A,
\]
\[
\Lambda_i \equiv \delta\kappa_i \left(\frac{1}{\beta} - (1 - \delta)\right)^{-1}, \quad \text{for } i \in \{A, B\},
\]

\[
30
\]
The excess portfolio returns, $\xi_t = \tilde{\alpha}' \tilde{r}_{xt}$, depends on the asset market configuration.

In the economy after capital account liberalisation in which both equities and bonds are tradable,

$$\tilde{\alpha} \equiv \frac{1}{\beta \bar{P}_A} \left[(1 - \bar{S}_B) \bar{p}_B^S, -\bar{b}_B \bar{p}_B, -(1 - \bar{S}_A) \bar{p}_A^S \right]' ,$$

$$\tilde{R}_{xt} \equiv \left[ \tilde{R}_{Bt}^S - \tilde{R}_{At}^b, \tilde{R}_{Bt}^b - \tilde{R}_{At}^b, \tilde{R}_{At}^S - \tilde{R}_{At}^b \right]' .$$

In the economy with restricted market access, in which only country A bond and country B equity are tradable,

$$\tilde{\alpha} \equiv \frac{1}{\beta \bar{P}_A} (1 - \bar{S}_B) \bar{p}_B^S , \quad \tilde{R}_{xt} \equiv \tilde{R}_{Bt}^S - \tilde{R}_{At}^b .$$

The shock processes are

$$\tilde{\theta}_{At} = \rho \tilde{\theta}_{At-1} + \epsilon_{\theta_{At}} ,$$

$$\tilde{\theta}_{Bt} = \rho \tilde{\theta}_{Bt-1} + \epsilon_{\theta_{Bt}} ,$$

$$\tilde{\chi}_{At} = \rho \tilde{\chi}_{At-1} + \epsilon_{\chi_{At}} ,$$

$$\tilde{\chi}_{Bt} = \rho \tilde{\chi}_{Bt-1} + \epsilon_{\chi_{Bt}} .$$

### A.2 The matrix representation

We can write the system in matrix form:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} s_{t+1} \\ E_t c_{t+1} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} s_t \\ c_t \end{bmatrix} + \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \epsilon_t + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \tilde{\xi}_t ,$$

and $\epsilon_t \sim (0, \Sigma)$. We denote $s_t$ as the vector of predetermined variables and $c_t$ as the vector of jump variables:

$$s_t = [\hat{N} A_t \hat{k}_t \hat{\theta}_t \hat{\chi}_t]' ,$$

$$c_t = [\hat{y}_t \hat{\chi}_t \hat{C}_t \hat{I}_t \hat{q}_t]' .$$

and denote $\epsilon_t$ the vector of shocks

$$\epsilon_t = [\epsilon_{A_t} \epsilon_{B_t} \epsilon_{\chi_t} \epsilon_{\tilde{q}_t}]' .$$

We now write down the matrix representation of the system. The matrix are

$$A_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\delta & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\delta \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} , \quad A_{12} = 0_{7 \times 7} ,$$

31
\[
A_{21} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -(1 - \beta (1 - \delta)) & 0 & 0 & 0 & 0 & 1 - \beta (1 - \delta) \rho_X \\
0 & 0 & -(1 - \beta (1 - \delta)) & 0 & 0 & 0 & 1 - \beta (1 - \delta) \rho_X \\
0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 \\
\end{bmatrix},
\]

\[
A_{22} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma & -\sigma & 0 & 0 & (\bar{a}_A + \bar{a}_B - 1) \\
(1 - \beta (1 - \delta)) & 0 & -\sigma & 0 & 0 & (1 - \bar{a}_A)(1 - \beta (1 - \delta)) \\
0 & (1 - \beta (1 - \delta)) & 0 & -\sigma & 0 & -(1 - \bar{a}_B)(1 - \beta (1 - \delta)) \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
B_{11} = \begin{bmatrix}
\frac{1}{\beta} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (1 - \delta) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (1 - \delta) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_\theta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \rho_\theta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho_X & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \rho_X \\
\end{bmatrix},
\]

\[
B_{12} = \begin{bmatrix}
1 & 0 & -(1 - \Lambda_A) & 0 & -\Lambda_A & 0 & (1 - \bar{a}_A) \\
0 & 0 & 0 & 0 & \delta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \delta & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
B_{21} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_A & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \kappa_B & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
B_{22} = \begin{bmatrix}
\mu_A & 0 & \Delta_1 & \Delta_2 & \Delta_3 & \Delta_4 & \Delta_5 \\
0 & 1 - \mu_A & \Delta_6 & \Delta_7 & \Delta_8 & \Delta_9 & \Delta_{10} \\
0 & 0 & \sigma & -\sigma & 0 & 0 & (\bar{a}_A + \bar{a}_B - 1) \\
0 & 0 & -\sigma & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\sigma & 0 & 0 & 0 \\
-(1 - \Xi_A) & 0 & -\sigma \Xi_A & 0 & 0 & 0 & (1 - \bar{a}_A) \Xi_A \\
0 & -(1 - \Xi_B) & 0 & -\sigma \Xi_B & 0 & 0 & -(1 - \bar{a}_B) \Xi_B \\
\end{bmatrix},
\]

\[32\]
where

\[
\begin{align*}
\Delta_1 &= -\tilde{a}_A \mu_A (1 - \Lambda_A), \\
\Delta_2 &= -(1 - \tilde{a}_B) \mu_B (1 - \Lambda_B), \\
\Delta_3 &= -\tilde{a}_A \mu_A \Lambda_A, \\
\Delta_4 &= -(1 - \tilde{a}_B) \mu_B \Lambda_B, \\
\Delta_5 &= \phi [\tilde{a}_A (1 - \tilde{a}_A) \mu_A + \tilde{a}_B (1 - \tilde{a}_B) \mu_B], \\
\Delta_6 &= -(1 - \tilde{a}_A) \mu_A (1 - \Lambda_A), \\
\Delta_7 &= -\tilde{a}_B \mu_B (1 - \Lambda_B), \\
\Delta_8 &= -(1 - \tilde{a}_A) \mu_A \Lambda_A, \\
\Delta_9 &= -\tilde{a}_B \mu_B \Lambda_B, \quad \text{and} \\
\Delta_{10} &= -\phi [\tilde{a}_A (1 - \tilde{a}_A) \mu_A + \tilde{a}_B (1 - \tilde{a}_B) \mu_B] + ((1 - \tilde{a}_A) \mu_A - (1 - \tilde{a}_B) \mu_B).
\end{align*}
\]

And

\[
P_1 = \begin{bmatrix} 0_{3 \times 4} & \end{bmatrix}, \quad P_2 = 0_{7 \times 4}, \quad Q_1 = [1, 0_{1 \times 6}]', \quad Q_2 = 0_{7 \times 1}.
\]

This system can be solved by standard methods such as generalised Schur decomposition (Klein, 2000) or the method of undetermined coefficients.

## B Solution for the Portfolio choice

As shown in the main text, a second-order approximation of the asset portfolio choice equations leads to the following equation:

\[
0 = E_t \left[ \sigma (\hat{C}_{At+1} - \hat{C}_{Bt+1}) + (\tilde{a}_A + \tilde{a}_B - 1) \hat{q}_{t+1} \right] \hat{R}_{xt+1},
\]

where \( \hat{R}_{xt} = \hat{R}^S_{At} - \hat{R}^b_{At} \) before capital account liberalisation, and \( \hat{R}_{xt} = \left[ \hat{R}^S_{At} - \hat{R}^b_{At}, \hat{R}^b_{Bl} - \hat{R}^b_{At}, \hat{R}^S_{At} - \hat{R}^b_{At}, \hat{R}^S_{Bt} - \hat{R}^b_{At}, \hat{R}^S_{At} - \hat{R}^b_{At} \right]' \) after the liberalisation.

We will need to express \( \left[ \sigma (\hat{C}_{At+1} - \hat{C}_{Bt+1}) + (\tilde{a}_A + \tilde{a}_B - 1) \hat{q}_{t+1} \right] \) and the vector \( \hat{R}_{xt+1} \) in terms of \( \{s_{t+1}, \hat{C}_{Bt+1}\} \). The former is easy, and can be done by extracting the relevant rows in \( G_2, G_3 \).

For the latter, we need to express \( \hat{R}_{xt+1} \) in terms of \( \{s_{t+1}, \hat{C}_{Bt+1}\} \). We take the portfolio choice equations and iterate forward to get

\[
\begin{align*}
p^b_{it} &= E_t \sum_{s=1}^\infty Q^i_{t+s} p_{it+s}, \\
p^s_{it} &= E_t \sum_{s=1}^\infty Q^i_{t+s} d_{it+s}, \quad \text{for } i \in \{A, B\}.
\end{align*}
\]
where \( Q_{t,t+1}^i = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \), is the stochastic discount factor.

We log-linearise these four equations. We can difference any two of these equations and use the fact that \( dQ_{t,t+1} - dQ_{t,t+1}^B = 0 \) to obtain:

\[
\begin{align*}
\hat{p}_{At}^b - \hat{p}_{At}^b &= \frac{1 - \beta}{\beta} \sum_{s=1}^{\infty} \beta^s (\hat{p}_{At+s} - \hat{p}_{At+s}^b), \\
\hat{p}_{At}^S - \hat{p}_{At}^S &= \frac{1 - \beta}{\beta} \sum_{s=1}^{\infty} \beta^s (\hat{d}_{At+s} - \hat{d}_{At+s}^S), \\
\hat{p}_{At}^S - \hat{p}_{At}^b &= \frac{1 - \beta}{\beta} \sum_{s=1}^{\infty} \beta^s (\hat{d}_{At+s} - \hat{p}_{At+s}^b).
\end{align*}
\]

Next, we find the relative returns by log-linearising their respective definitions to obtain:

\[
\begin{align*}
\hat{R}_{it+1}^b &= \beta\hat{p}_{it+1}^b + (1 - \beta)\hat{p}_{it+1} - \hat{p}_{it}, \\
\hat{R}_{it+1}^S &= \beta\hat{p}_{it+1}^S + (1 - \beta)\hat{d}_{it+1} - \hat{p}_{it}, \quad \text{for } i \in \{A, B\}.
\end{align*}
\]

Hence the relative returns can be expressed in terms of the expected discounted sum of future payment streams.

\[
\begin{align*}
\hat{R}_{it+1}^S - \hat{R}_{it+1}^b &= \frac{1 - \beta}{\beta} \left( E_{t+1} \sum_{s=1}^{\infty} \beta^s (\hat{d}_{Bs+s} - \hat{p}_{At+s}) - E_t \sum_{s=1}^{\infty} \beta^s (\hat{d}_{Bs+s} - \hat{p}_{At+s}) \right), \\
\hat{R}_{it+1}^b - \hat{R}_{it+1}^b &= \frac{1 - \beta}{\beta} \left( E_{t+1} \sum_{s=1}^{\infty} \beta^s (\hat{p}_{Bs+s} - \hat{p}_{At+s}) - E_t \sum_{s=1}^{\infty} \beta^s (\hat{p}_{Bs+s} - \hat{p}_{At+s}) \right), \\
\hat{R}_{it+1}^S - \hat{R}_{it+1}^b &= \frac{1 - \beta}{\beta} \left( E_{t+1} \sum_{s=1}^{\infty} \beta^s (\hat{d}_{At+s} - \hat{p}_{At+s}) - E_t \sum_{s=1}^{\infty} \beta^s (\hat{d}_{At+s} - \hat{p}_{At+s}) \right).
\end{align*}
\]

Obviously, the relative bond payment is just the terms of trade. For the dividends, we log-linearise equation (25) to obtain:

\[
\hat{d}_{it} = \frac{\kappa_i}{\kappa_i - \Lambda_i} (\hat{g}_{it} + \hat{p}_{it}) - \frac{\Lambda_i}{\kappa_i - \Lambda_i} (\hat{I}_{it} + \hat{P}_{it}), \quad \text{for } i \in \{A, B\}.
\]

This means that

\[
\begin{align*}
\hat{d}_{At} - \hat{p}_{At} &= \frac{1 - (1 - \delta)\beta}{1 - \beta} \hat{y}_{At} - \frac{\delta \beta}{1 - \beta} \hat{I}_{At} - \frac{1}{1 - \beta} ((1 - (1 - \delta)\beta) - \delta \beta \hat{a}_B) \hat{q}_t, \\
\hat{d}_{At} - \hat{p}_{At} &= \frac{1 - (1 - \delta)\beta}{1 - \beta} \hat{y}_{At} - \frac{\delta \beta}{1 - \beta} \hat{I}_{At} + \frac{\delta \beta}{1 - \beta} (1 - \hat{a}_A) \hat{q}_t.
\end{align*}
\]

Finally, it remains to express the expected sums in terms of \( G_2, G_3 \). Denote \( \hat{Z}_t = \{\hat{d}_{At} - \hat{p}_{At}, \hat{p}_{At} - \hat{p}_{At}, \hat{d}_{At} - \hat{p}_{At}\} \), then we can express \( \hat{Z}_{t+1} \) in the following way:

\[
\hat{Z}_{t+1} = z_1 s_{t+1} + z_2 \epsilon_{t+1} + z_3 \xi_{t+1},
\]
where $z_1, z_2, z_3$ are known coefficients. We then use the transition dynamics of the economy, equation (35) and (35), to obtain

$$
\frac{1 - \beta}{\beta} \left( E_{t+1} \sum_{s=1}^{\infty} \beta^s \hat{Z}_{t+s} - E_t \sum_{s=1}^{\infty} \beta^s \hat{Z}_{t+s} \right),
$$

$$
= (1 - \beta) \left( (z_2 + \beta z_1 (I - \beta F_1)^{-1} F_2) \epsilon_{t+1} + (z_3 + \beta z_1 (I - \beta F_1)^{-1} F_3) \hat{\xi}_{t+1} \right).
$$

Hence, the relative interest rates can be extracted from the relevant rows in $G_2, G_3$ accordingly.

Finally, following Devereux and Sutherland (2011), $\bar{\alpha}$ can be found by the following equation:

$$
\bar{\alpha} = \left[ R_2 \Sigma D_2' R_2' - D_1 R_2 \Sigma R_2' \right]^{-1} R_2 \Sigma D_2'.
$$

The steady-state portfolio choice $\bar{S}_j^i$ and $\bar{b}_j^i$ for $i, j \in \{A, B\}$ can be backed out using the asset prices in the steady state.