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Measuring the World Natural Rate of Interest[*]

Mark A. Wynne
Federal Reserve Bank of Dallas

Ren Zhang
Bowling Green State University

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Abstract

This paper makes the first attempt to estimate the time-varying natural rate jointly with the output gap and trend potential output growth for the world as a whole using a simple unobserved components model broadly following the methodology developed by Laubach and Williams (2003). We find that the world natural rate has been trending down for the past few decades. Nearly half of the variation in the natural rate is accounted for by the trend potential output growth rate. However, the relationship between the world natural interest rate and the world trend growth is modest and not statistically significant.

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[*] Mark A. Wynne, Research Department, Federal Reserve Bank of Dallas, 2200 N. Pearl Street, Dallas, TX 75201. 214-922-5159. mark.a.wynne@dal.frb.org. Ren Zhang, Department of Economics, Bowling Green State University, Bowling Green, OH 43403. renz@bgsu.edu. The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

I. Introduction

The natural rate of interest — also sometimes referred as a neutral or real equilibrium rate of interest — is commonly defined as the real short-term rate of interest consistent with stable inflation and output equal to potential. It's one of the key concepts for interpreting macroeconomic relationships and the effects of monetary policy. For example, it provides a metric for the stance of the monetary policy, which is expansionary (contractionary) when the real interest rate is below (above) the natural rate. Thus, monetary policy makers have a deep interest in estimating the level of the natural interest rate.[1]

Most of the previous work on this topic estimates the natural rate for only a single country or a specific area of the world such as the European Union. However, in light of the increasing degree of global economic integration, measuring the global natural rate is of some interest. This paper tries to make a contribution in this direction. We assume that the world is fully integrated and ask the following questions: How has the world natural rate evolved over the past half century? Does it exhibit a similar pattern to the natural rate in the United States? What are the main contributors to historical fluctuations in the world natural rate? Does it tell us anything about the international interaction between the United States and the rest of the world?

In order to answer the questions above, we broadly apply a commonly used methodology first proposed by Laubach and Williams (2003) to the world, proxied by an aggregate of twenty advanced economies over the period 1961-2015.[2] Laubach and Williams use a state-space model to estimate the unobservable natural rate from observed output, inflation and interest rate data by specifying a couple of simple macroeconomic relationships including a crucial natural rate equation relating the natural rate of interest to the trend rate of potential output growth, an IS curve relating the output gap to the deviation of real interest rate from its natural level and a Phillips curve that links the inflation rate to the deviation of output from potential.

Our specification and estimation deviate from the original Laubach and Williams model in a couple of ways. First, we omit import prices from our Phillips curve specification since we are interested in global aggregates and the world does not trade with anyone. For the same reason, the FRB/US imported oil price in Laubach-Williams' Phillips curve is replaced with the world oil price proxied by the price of West Texas Intermediate crude oil. Second, we apply standard maximum likelihood methods to estimate the trend growth shock instead of the medium unbiased estimator proposed by Stock and Watson (1998). We do so because shocks to world trend growth are bigger than the respective

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[1] See, e.g. Laubach and Williams (2003), Clark and Kozicki (2005), Berger and Kempa (2014), Barsky et al. (2014), Cúrdia et al. (2015), Hamilton et al. (2016), Pescatori and Turunen (2015) and Holston et al. (forthcoming).

[2] The twenty advanced countries include Canada, France, Germany, Italy, Japan, United Kingdom, United States, Australia, Austria, Belgium, Finland, Greece, Ireland, Netherlands, Norway, Portugal, South Korea, Spain, Sweden and Switzerland.

individual country shocks, so that standard maximum likelihood methods do not suffer from the "pile-up problem" (Stock (1994)) as in Laubach and Williams (2003). Third, while implementing the Kalman filter/smoother algorithm, we set the conditional expectation and covariance matrix of the initial states with a diffuse prior instead of the general least squares (GLS hereafter) method proposed by Harvey (1989) since the latter tends to exacerbate the "pile-up problem".

There are several main findings to highlight. First, the world neutral interest rate has been declining for the past half century in a similar pattern as the trend growth rate of potential output. The trend potential output growth can explain nearly half of the forecast error variance of the natural rate at all finite horizons. Nevertheless, consistent with Hamilton et al. (2016), we find that the relationship between the world natural rate and trend potential output growth is modest. The point estimate of the parameter that connects the natural interest rate with the trend growth rate is 0.458, which is less than half of Laubach and Williams' estimate of its U.S. counterpart and is not statistically significant. In addition, our estimates of the output gap pick up the OECD recession turning points quite accurately. The estimation of the IS curve indicates that the world natural rate gap imposes a significant contractionary pressure on the world output gap. Last, the Phillips curve indicates that the world output gap has a significantly positive effect on the global inflation which shows that the short-run output-inflation trade-off exists at the global level.

II. Model Specification

Our benchmark model broadly follows Laubach and Williams (2003). The key motivating equation in Laubach and Williams (2003) is the following version of the relationship between the real rate of interest (r) and the growth rate of consumption (g_c) that falls out of almost any intertemporal household optimization problem:

$$r = \sigma g_c + \theta, \tag{1}$$

where σ is the inverse of intertemporal elasticity of substitution and θ is the pure rate of time preference. Laubach and Williams use this theoretical relationship to motivate a relationship between the unobserved natural rate of interest (r_t^*) and the annualized trend growth rate of potential output (g_t):

$$r_t^* = c g_t + z_t, \tag{2}$$

where z_t captures other determinants of the natural rate of interest such as time preference, fiscal policy and so on.

The dynamics of the output gap are described by a backward-looking IS equation, where the output gap ($\tilde{y}_t$) (defined as the percentage deviation of real output (y_t) from potential output (y_t^*)) is determined by its own lags, the lagged deviation of the real short term interest rate (r_t) from the equilibrium real interest rate (r_t^*) and a serially uncorrelated shock (ϵ_{1t}):

$$\tilde{y}_t = a_1\tilde{y}_{t-1} + a_2\tilde{y}_{t-2} + (a_3/2)\sum_{j=1}^{2}(r_{t-j} - r_{t-j}^*) + \epsilon_{1t}, \tag{3}$$

where the ex-ante real interest rate (r_t) is constructed by subtracting the expected inflation rate ($E_t\pi_{t+1}$) from the nominal interest rate (R_t).

The core consumption price inflation rate (π_t) is assumed to be determined by its own lags, the lagged output gap ($\tilde{y}_{t-1}$) and the crude oil price inflation rate (π_t^O) (as a proxy for global supply shocks) and a serially uncorrelated shock (ϵ_{2t}):

$$\pi_t = B_\pi(L)\pi_{t-1} + b_3\tilde{y}_{t-1} + b_4(\pi_{t-1}^o - \pi_{t-1}) + \epsilon_{2t}. \tag{4}$$

For parsimony, we restrict the coefficients on the lagged inflation terms — not rejected in our sample — to sum to one. This implies that the trade-off between output and inflation exists only in the short run. We also assume that the coefficients on the second through fourth lags are equal to each other, as are the coefficients on the fifth to eighth lags, i.e., $B_\pi(L)\pi_{t-1} = b_1\pi_{t-1} + b_2\sum_{i=2}^{4}\pi_{t-i}/3 + (1-b_1-b_2)\sum_{i=5}^{8}\pi_{t-i}/4$. This specification is similar to the Phillips curve equation in Laubach and Williams (2003), except that we omit the core import price inflation term which they include in their specification, and replace the imported oil price with a measure of the global oil price.

Equations (3) and (4) are the measurement equations of our state space model. Turning to the transition equations, we assume that the variable z_t representing the non-trend-growth determinants of the natural rate in equation (2) follows a random walk:

$$z_t = z_{t-1} + \epsilon_{3t}. \tag{5}$$

The potential output (y_t^*) and annualized trend growth rate of potential output (g_t) are given by[3]:

$$y_t^* = y_{t-1}^* + 0.25g_{t-1} + \epsilon_{4t} \tag{6}$$

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[3] The coefficient before the annualized trend growth rate g_t is 0.25 because our output data are quarterly. This is consistent with the setup in other studies such as Trehan and Wu (2007).

$$g_t = g_{t-1} + \epsilon_{5t}. \tag{7}$$

We assume that ϵ_{1t} through ϵ_{5t} are serially uncorrelated and uncorrelated with one another.

As detailed in Appendix A, the model can be expressed in the form of a state-space model:

$$Y_t = HS_t + AX_t + u_t \tag{8}$$

and

$$S_t = FS_{t-1} + v_t, \tag{9}$$

where $Y_t = \left(\begin{array}{cc} y_t, & \pi_t \end{array} \right)'$, $S_t = \left(\begin{array}{ccccccc} y_t^*, & y_{t-1}^*, & y_{t-2}^* & g_{t-1}, & g_{t-2}, & z_{t-1}, & z_{t-2} \end{array} \right)'$, and $X_t = \left(\begin{array}{ccccccccc} y_t, & y_{t-1}, & y_{t-2} & r_{t-1}, & r_{t-2}, & \pi_{t-1}, & \pi_{t-2,4} & \pi_{t-5,8} & \pi_{t-1}^o - \pi_{t-1} \end{array} \right)'$.

In applying the Kalman filter to the model, standard maximum likelihood estimation of σ_3, the standard deviation of the shock to z_t, is biased towards zero because of the so-called "pile-up problem" which usually arises when the shock to the random walk process is of small size.[4] Accordingly, our estimation proceeds in two steps. In the first step, we use the median unbiased estimator proposed by Stock and Watson (1998) to estimate the noise to signal ratio $\lambda_z = a_3(\sigma_3/\sigma_1)$. In the second step, we impose the estimated value of λ_z obtained in the previous step and estimate the remaining model parameters with standard maximum likelihood methods.[5]

The above estimation procedure deviates from the three-step method designed by Laubach and Williams (2003). Laubach and Williams (2003) find that the U.S. trend growth shock is small, so that the standard maximum likelihood estimates of the standard deviation of the trend growth shock (σ_5) suffers from the "pile-up problem". Accordingly, they include an extra step to estimate the ratio $\lambda_g = \sigma_5/\sigma_4$ with Stock and Watson's median unbiased estimation method and impose that ratio in latter steps. However, as we will show later, the world trend growth shock exhibits more volatility than the U.S. trend growth shock estimated by Laubach and Williams (2003), and is immune to the "pile-up problem". Thus, we skip the extra step and estimate the standard error of the

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[4] For more detailed discussion on the "pile-up problem", see Stock (1994), Stock and Watson (1998) among others

[5] To proceed the Kalman filter/smoother procedure, we need to set the the conditional expectation and covariance matrix of initial states. In both steps, different from Laubach and Williams (2003), the conditional expectation and covariance matrix of initial states are set by diffuse prior instead of the GLS method introduced in Harvey (1989). There are two reasons for the deviation. First, as is mentioned in Laubach (2002) the GLS method tends to exacerbate the "pile-up problem". Second, as in Laubach and Williams (2003), the GLS method fails in the last step because of singularity problems. Thus, it's more consistent to use diffuse prior in both steps rather than using GLS method in the first step while using a diffuse prior in the second step.

trend growth shock together with other parameters simultaneously using the standard maximum likelihood method instead.

III. Estimation Results

A. Data

The model is estimated using quarterly data for the world from 1961Q1 to 2015Q4. Due to data availability, we proxy the world by an aggregate of twenty advanced economies: Canada, France, Germany, Italy, Japan, United Kingdom, United States, Australia, Austria, Belgium, Finland, Greece, Ireland, Netherlands, Norway, Portugal, South Korea, Spain, Sweden and Switzerland. These twenty countries account for a substantial fraction of global economic activity. Moreover, the set of countries are all market economies and exhibit a high degree of economic and financial integration with each other, which justifies the assumption underlying our model. Nevertheless, we recognize that emerging market economies especially the BRIC (Brazil, Russia, India and China) countries play an increasingly important role in recent decades. Including those countries into the model is left to future research.

The aggregated GDP data are constructed by adding up the GDP series (measured in constant purchasing power parity (PPP) dollars) of each individual country. The nominal world interest rate and inflation rate are derived by taking weighted averages of the corresponding indicators for individual countries with the time-varying PPP-adjusted GDP shares displayed in Figure 1 as the weights. The GDP shares are calculated by the ratios of the real GDP of the individual countries to the aggregated GDP of the twenty countries included in our sample. We compute the expectation of world inflation rate four quarters ahead from a univariate AR(3) model of inflation estimated over the 80 quarters prior to the date at which expectations are being formed.[6] Then, we construct the ex-ante real interest rate by subtracting the world expected inflation from the nominal world interest rate. We use the West Texas Intermediate oil price as a measure of the global oil price. The sources and construction of the data are detailed in Appendix B.

[Figure 1 about here]

B. Parameter Estimation

The first column of Table 1 reports the estimates of parameters. To facilitate comparison with previous studies of U.S. natural rate, we also update the estimation of the

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[6] Due to data availability, before 1981 we use a fixed window of the data from 1959 to 1981 to estimate the coefficients of the AR model. After 1981 the AR coefficients are estimated using a rolling window with the sample size fixed at 80 quarters

model in Laubach and Williams (2003) to 2015Q4 with the results of parameter estimates listed in the second column of Table 1.[7]

[Table 1 about here]

Similar to other individual country studies, we find the world output gap to be a fairly persistent process. The summation of the autoregressive parameters in the IS equation, a_1 and a_2, is as high as 0.922. The coefficient relating the output gap to the real rate gap (a_3) is negative and statistically significant, which indicates that a positive world real interest rate gap is indeed contractionary. In terms of the evidence on inflation, we find that the slope of the Phillips curve (b_3) is significantly positive as is predicted by standard economic theory. Our estimated value of b_3 is four times the size of its U.S. counterpart. One possible reason for this is that the Phillips curve equation estimated using individual country data insufficiently captures the effect of foreign demand on domestic inflation. Lastly, for the natural rate equation, the link between the world natural rate and the world trend growth is weak. The point estimate of the parameter c is 0.458 which is only one third of its U.S. counterpart. By contrast to the U.S. estimate, the parameter c is insignificantly different from zero which indicates that the relationship between the natural rate and the trend growth rate is modest. This finding is consistent with the findings of Hamilton et al. (2016), who draw a similar conclusion by studying the simple cross-country correlation between the average GDP growth rate and the average real interest rate.

For the estimates of the standard errors, the shock to trend growth rate (ϵ_5) is more volatile than its U.S. equivalent. The standard deviation of the trend growth shock (σ_5) equals 0.143, which is more than three times its U.S. counterpart. The large size of trend growth shock makes it possible to avoid the “pile-up problem” in estimating σ_5 which usually arises in single country studies. On the other hand, the standard deviation of the other determinants of the natural rate (σ_3) is 0.076 which is less than one third of the U.S. estimate. The estimates of the standard errors shed some light on the driving forces underlying the natural rate. By combining equations (2), (5) and (7), the natural rate of interest (r_t^*) follows a random walk:

$$r_t^* = r_{t-1}^* + \epsilon_{rt}, \tag{10}$$

where the shock to the natural rate equation $\epsilon_{rt} = c\epsilon_{5t} + \epsilon_{3t}$. Given the estimates above, the standard deviation of the world natural rate shock $\sigma_r = \sqrt{c^2\sigma_5^2 + \sigma_3^2}$ is 0.115 while

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[7] The U.S. natural rate estimated by Laubach-Williams model is also updated in real time on the website of the San Francisco Fed. Our replication matches with their results closely. The slight difference might arise as a result of the different observation vintage. Our data are observed in June 2016 as our world natural rate estimates while the data used by Laubach and Williams for the same sample period are observed in March 2016.

the standard deviation of the corresponding U.S. shock is 0.254. Thus, our estimation suggests that the shock to the world natural rate is of smaller size than the shock that drives the U.S. natural rate. Furthermore, the forecast error variance of the natural rate contributed by the trend growth at all finite horizons, measured by $c^2\sigma_5^2/\sigma_r^2$, is 42.6 percent which is much greater than the respective U.S. ratio of 4.8 percent. Thus, a substantial amount of the variation in the world natural rate is contributed by the world trend potential output growth.

C. Output gap, trend growth and natural rate

Figure 2 plots our two-sided estimates of the world output gap, where the shaded areas indicate recession periods as defined by the OECD.[8] It turns out that the estimated output gap picks up the business cycle turning points quite accurately. The output gap decreases significantly in each of the OECD recessions. In particular, the world output gap decreases most sharply during the global oil crisis of 1973M5-1975M5 and 1979M9-1982M12 as well as the recent 2007M12-2009M5 global financial crisis.

[Figure 2 about here]

Figure 3 displays our two-sided estimates of the growth rate of potential output in blue dashed lines along with trend growth in black solid lines. The world potential output growth rate fluctuates around the trend growth rate as expected. It becomes less volatile between the mid-1980s and 2007 which corresponds to the so-called Great Moderation period in the U.S. The potential output growth rate reaches its trough at a historically low value of -1.2 in 2009Q1 during the global financial crisis, which was the worst recession since World War II. The world trend growth rate captures the low-frequency movement in the potential output growth which has been declining since the mid-1960s until the recent global financial crisis. The annualized trend growth rate drops from 4.8 percent in 1966Q1 to 0.8 percent in 2009Q1 and then recovers slowly to 1.2 percent in 2015Q4 at the end of our sample. Based on the discussion above, our estimates of the output gap, potential output and trend growth are consistent with global economic history, which provides some support to our estimates of the natural rate.

[Figure 3 about here]

Figure 4 depicts the two-sided estimates of the natural rate of interest in black solid lines together with the historical realization of the ex-ante real interest rate in blue

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[8] We use the OECD business cycle chronology for two reasons. First, our sample of countries are all OECD members and the aggregate of the twenty countries we select makes up dominant share of the total GDP of OECD countries. Second, it's the only public source we are aware which dates the turning points of global economic activity back to the 1960s. Martínez-García et al. (2015) provide a global business cycle chronology for a broader group of countries but their chronology only begins in 1980.

dashed lines. Similar to the single country estimates of Laubach and Williams (2003), the world natural rate of interest has been trending down during the past half century. The declining pattern is also consistent with the world real yields on 10-year government bonds estimated by King and Low (2014) which is plotted in red in Figure 4. Based on our estimates, the real interest rate lies below the natural rate for most of the period prior to 1980 which has expansionary effects on output. This loose monetary policy helped raise the global inflation rate in the 1970s as documented in Ciccarelli and Mojon (2011). In the late 1970s, the central banks in major advanced economies raised policy rates to fight inflation. The real interest rate exceeded the natural rate starting in 1980Q2 and the real interest rate gap reached almost 4.6 percentage points in 1982Q3. This positive real interest rate gap, signifying the contractionary stance of world monetary policy, persisted until 2001Q4. The natural rate started to decline more significantly in the late 1990s and kept falling even as the real interest rate rose from 2004Q2 to 2007Q3. The divergent movement in the real interest rate and the natural rate created a big 2.1 percentage point real interest rate gap in 2007Q3 which was followed by the global financial crisis and the Great Recession. The natural rate drops to a historically low level of 0.2 percent in 2009Q3 and then recovers slowly until the end of our sample in 2015Q4. During and after the Great Recession, major central banks lowered their policy rates and launched Quantitative Easing (QE) programs to support economic activity. In light of the low natural rate, global monetary policy was not overly aggressive but necessary to help the world economy recover from the Great Recession.

[Figure 4 about here]

The natural rate equation above shows that the world natural rate is determined by two factors: the world trend growth rate g_t and the other determinant z_t. Figure 5 displays the natural rate along with the contribution of each of the underlying determinants. Most of the fluctuation in the world natural rate is determined by the trend growth rate while the other determinant (z_t) plays a rather limited role. This is quite different from the previous estimates of the U.S. natural rate, where the other determinant (z_t) acts as a significant contributor to the natural rate, especially in recent years as is shown in Figure 6. A possible explanation to account for such a difference is that much of the z_t for the U.S. natural rate is contributed by the trend growth of the rest of the world. Nevertheless, to further verify this possibility requires a two-country model where the natural rate in the U.S. is determined by both home country trend growth and the foreign country trend growth as explored in Wynne and Zhang (forthcoming), which is beyond the discussion of this paper.

[Figure 5 and Figure 6 about here]

D. Robustness Analysis

In the section above, we have shown that the natural rate and trend growth have been declining in the baseline model. One assumption implicitly underlying the baseline model is that the population growth is stable across the sample period. However, as shown in Figure 7, world population growth declined significantly from an annual rate of 1.2 percent in 1961 to 0.4 percent in 2015. In order to examine whether this considerable shift in demographic factors contributed to the decline in the trend growth and thus the natural rate, we implement a robustness check where we redefine y_t in the baseline model as the output per capita.[9]

[Figure 7 about here]

The third column of table 1 displays the parameter estimates for the robustness check. Most of the parameters are very close to the baseline case. Nevertheless, the parameter c that connects the natural rate with the trend growth is 0.6, which is 31 percent bigger than the baseline estimate but still insignificantly different from zero. The standard deviation of the shock to trend growth (σ_5) equals 0.168 which is moderately larger than the respective estimate of 0.143 in the baseline model. Lastly, the standard deviation of the shock to the other determinant of the natural rate (σ_3) rises from 0.076 in the benchmark model to 0.116 in the model with per capita output. All of these features contribute to a larger size of the natural rate shock (ϵ_{rt}) than the baseline model. The standard deviation of the natural rate shock (σ_r) rises from the baseline estimate of 0.115 to 0.154 in the model with per capita output.

Figures 8 to 10 plot the two-sided estimates of the output gap, the trend growth rate and the natural rate of interest for the per capita model. To facilitate comparison, the baseline estimates are plotted as red dotted lines. The output gap per capita matches very closely to the output gap estimated by the baseline model. Moreover, the trend growth rate of the potential output per capita is lower than the baseline potential output trend growth rate as expected where the gap diminishes in recent years as the population grows more slowly. Finally, Figure 10 shows that the per capita estimates of the natural rate tracks the baseline estimates very closely for most of history. However, with a larger natural rate shock, the natural rate in the per capita model exhibits moderately richer dynamics than the baseline estimates. The two estimates diverge most substantially in 2009Q1 when the natural rate in per capita model reaches its trough at 0 percent compared to 0.2 percent in the baseline model. Nevertheless, the baseline estimates of the world natural rate is by and large robust to the historical demographic shifts.

[Figure 8 to 10 about here]

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[9] Here we assume that this demographic shift is exogenous.

IV. Conclusion

A growing number of literature utilizes unobserved components models to estimate equilibrium rate of interest by means of multivariate trend-cycle decompositions. However, most of such models focus on either an individual country or a specific area like the European Union. In this paper, we contribute to the literature by jointly estimating the world natural interest rate, potential output and the trend growth rate of output using an unobserved components model broadly following Laubach and Williams (2003). We find that both the world natural interest rate and the trend potential output growth rate have been declining significantly in the past fifty years. The trend growth rate contributes substantially to the variation in the natural interest rate. Nevertheless, our estimation shows that the relationship between the world natural rate and the world trend growth rate is modest. The estimates of the natural interest rate are robust even while controlling for demographic shifts.

Our project also inspires a reflection on the previous estimation on the U.S. natural rate of interest. By comparing the determinants of the natural rate between the U.S. and the world, we find that the other determinants of the natural rate in Laubach and Williams (2003) might be mostly contributed by the trend growth in the rest of the world. However, formally testing this inference requires a two-country model which is beyond the discussion of this paper and is left for future research.

Appendix A: the state-space representation of the model

The model in the text can be written in state space form:

$$Y_t = HS_t + AX_t + u_t \tag{A1}$$

$$S_t = FS_{t-1} + v_t. \tag{A2}$$

Here, Y_t and X_t are respectively vectors of contemporaneous endogenous, and of exogenous and predetermined variables. S_t is the vector of unobserved states. The vectors of stochastic disturbance u_t and v_t are assumed to be Gaussian and mutually uncorrelated with mean zero and covariance matrices R and Q, respectively.

The vector of observables Y_t is given by:

$$Y_t = \left(\begin{array}{cc} y_t, & \pi_t \end{array} \right)', \tag{A3}$$

where y_t denotes $100 \times log$ real GDP and π_t denotes inflation. The predetermined and exogenous variables are:

$$X_t = \left(\begin{array}{ccccccccc} y_t, & y_{t-1}, & y_{t-2} & r_{t-1}, & r_{t-2}, & \pi_{t-1}, & \pi_{t-2,4} & \pi_{t-5,8} & \pi^o_{t-1} - \pi_{t-1} \end{array} \right)', \tag{A4}$$

where r_t is the real interest rate, $\pi_{t-j,k}$ is shorthand for the moving average of inflation between dates $t-k$ and $t-j$ and π^o_t is oil price inflation. The state vector is:

$$S_t = \left(\begin{array}{ccccccc} y^*_t, & y^*_{t-1}, & y^*_{t-2} & g_{t-1}, & g_{t-2}, & z_{t-1}, & z_{t-2} \end{array} \right)', \tag{A5}$$

where y^*_t is $100 \times log$ potential GDP, g_t denotes the trend growth and z_t represents other determinants of the natural rate. The coefficient matrices are:

$$A = \begin{bmatrix} a_1 & a_2 & a_3/2 & a_3/2 & 0 & 0 & 0 & 0 \\ b_3 & 0 & 0 & 0 & b_1 & b_2 & 1-b_1-b_2 & b_4 \end{bmatrix} \tag{A6}$$

$$H = \begin{bmatrix} 1 & -a_1 & -a_2 & -ca_3/2 & -ca_3/2 & -a_3/2 & -a_3/2 \\ 0 & -b_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{A7}$$

$$R = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \tag{A8}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{A9}$$

$$Q = \begin{bmatrix} \sigma_5^2 + \sigma_4^2 & 0 & 0 & \sigma_5^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_5^2 & 0 & 0 & \sigma_5^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & (\lambda_z \sigma_1 / a_3)^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A10}$$

The signal-to-noise ratio λ_z is estimated with the median unbiased method introduced in Stock and Watson (1998). Given λ_z, the vector of parameters to be estimated by maximum likelihood is $\Theta = \left(\begin{array}{cccccccccccc} a_1, & a_2, & a_3 & b_1, & b_2, & b_3, & b_4, & c, & \sigma_1, & \sigma_2, & \sigma_4. & \sigma_5 \end{array} \right)$.

Appendix B: data sources

This appendix describes the data used in this project. The data are constructed by aggregating quarterly data from 1960Q1 to 2015Q4 for twenty advanced countries: Canada, France, Germany, Italy, Japan, United Kingdom, United States, Australia, Austria, Belgium, Finland, Greece, Ireland, Netherlands, Norway, Portugal, South Korea, Spain, Sweden and Switzerland.

The variable y refers to the log of aggregated PPP-adjusted real GDP (seasonally adjusted at annual rate) measured in millions of 2011 U.S. dollars. The aggregated data are obtained by taking the sum of the real GDP from each of the individual countries. Except for South Korea, the PPP-adjusted real GDP data are available from the OECD Quarterly National Accounts dataset (OECDNAQ) in Haver Analytics. For South Korea, the PPP-adjusted real GDP data from OECDNAQ only goes back to 1970. Nevertheless, the real GDP data in local currency from 1960 to 1970 are available from the Emerging Market dataset (EMERGEPR). We combine the two series by adjusting the observations of earlier periods with the formula: $y_t^{EMERGEPR} * (y_{1970Q1}^{OECDNAQ} / y_{1970Q1}^{EMERGEPR})$ for t from 1960Q1 to 1969Q4.

The aggregated nominal interest rate is the weighted average of the quarterly average annualized short term interest rate in each individual country using GDP share as the weight.[10] We use the central bank policy rate for most of the countries.[11] For the rest of the countries, we use money-market rates instead due to the lack of availability of the central bank policy rate. For the Eurozone countries, we splice their old interest rates with the Main Refinancing Rate in 1999Q1 when the European Central Bank is formed. The only exception is Greece which joined the Eurozone in 2001 so that we stack the earlier Bank of Greece Bank Rate with the European Main Refinancing Rate in 2001Q1.

The aggregated core inflation rate is created by taking a weighted average of the annualized quarterly growth rate of each country's seasonally adjusted core consumer price index using the GDP share as weights. For some countries, the core consumer price index is unavailable back to the 1960s. As a result, we proxy the core CPI inflation rates with the CPI inflation rate when the former rates are missing.

To construct the ex-ante real interest rate, we compute the expectation of average aggregate inflation over the four quarters ahead from a univariate AR(3) of inflation estimated over the 80 quarters prior to the date at which expectations are being formed. In practice, because of the limited sample, for the first 20 years we use the data from 1959-1981 to estimate the coefficients of the AR model. After 1981, the AR model is estimated using a rolling window with the size fixed at 80 quarters. Finally, the oil price

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[10] The GDP share is time-varying as is depicted in Figure 1. It's the ratio between the PPP-adjusted real GDP and the aggregated real GDP of the twenty countries.

[11] Specifically, Canada, Italy, United Kingdom, United States, Australia, Austria, Finland, Greece, Netherlands, Norway, Portugal, South Korea, Spain and Switzerland.

is the West Texas Intermediate spot oil price.

All the data, except for the early Consumer Price Index of Ireland,[12] are from Haver Analytics. To facilitate replication of our results, we list the Haver mnemonics in the following:

GDP: Canada: B156GDPC@OECDNAQ; France: B132GDPC@OECDNAQ;
Germany: B134DPC@OECDNAQ; Italy: B136GDPC@OECDNAQ;
Japan: B158GDPC@OECDNAQ; United Kingdom: B112GDPC@OECDNAQ;
United States: B111GDPC@OECDNAQ; Australia: B193GDPC@OECDNAQ;
Austria: B122GDPC@OECDNAQ; Belgium: B124GDPC@OECDNAQ;
Finland: B172GDPC@OECDNAQ; Greece: B174GDPC@OECDNAQ;
Ireland: B178GDPC@OECDNAQ; Netherlands: B138GDPC@OECDNAQ;
Norway: B142GDPC@OECDNAQ; Portugal: B182GDPC@OECDNAQ;
South Korea: S542NGPC@EMERGEPR(prior 70Q1), B542GDPC@OECDNAQ(post 70Q1);
Spain: B184GDPC@OECDNAQ; Sweden: B144GDPC@OECDNAQ;
Switzerland: B146GDPC@OECDNAQ.

Interest Rate: Canada: Central Bank Rate, C156FROS@OECDMEI; France: Overnight Interbank Rate, C132FRUO@OECDMEI; Germany: Overnight Interbank Rate C134IM@IFS; Italy: Discount Rate, C136IC@IFS; Japan: Tokyo Overnight Call Rate, C158IM@IFS; United Kingdom: Official Bank Rate, N112RTAR@G10; United States: Federal Funds Rate, B111GDPC@DAILY; Australia: Official Cash Rate, N193RTAR@G10; Austria: Discount Rate, C122IC@IFS; Belgium: three-month Interbank Rate, C124IM@IFS; Finland: Discount Rate, C172IFC@IFS; Greece: Central Bank Rate, C174IC@IFS; Ireland: short term facility rate, C178IC@IFS; Netherlands: Discount Rate (prior 93Q4) C138IC@IFS, Inter Bank Offer Rate(94Q1-98Q4) C138FRIO@IFS; Norway: Discount Rate, C142IC@IFS; Portugal: Discount Rate, C182IC@IFS; South Korea: Discount Rate, C542IFC@IFS; Spain: Central Bank Rate, C184IC@IFS; Sweden: Overnight Money Rate, C144FRUO@OECDMEI; Switzerland: Discount Rate, B146IC@IFS; Eurozone (post 99Q1): Main refinancing Rate, N023RTAR@G10.

Price Index:[13]
Canada: CPI(prior 61Q1), C156CZN@OECDMEI, Core CPI, C156CZCN@OECDMEI;
France: CPI(prior 70Q1), C132CZN@OECDMEI, Core CPI, C132CZCN@OECDMEI;
Germany: CPI(prior 62Q1), C134CZN@OECDMEI, Core CPI, C134CZCN@OECDMEI;

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[12] The Consumer Price Index of Ireland before 1975Q4 are acquired from the Central Statistics Office of Ireland.

[13] Except for the United States, we import the Non-Seasonal-Adjusted price series from the Haver since they have longer samples. Then we make the seasonal adjustment to the data using Haver built in function. The early Ireland CPI data are seasonally adjusted by Tramo-Seats.

Italy: CPI(prior 60Q1), C136CZN@OECDMEI, Core CPI, C136CZCN@OECDMEI; Japan: Core CPI, C134CZCN@OECDMEI; United Kingdom: CPI(prior 70Q1), C112CZN@OECDMEI, Core CPI, C112CZCN@OECDMEI; United States: Core CPI, S111PCXG@G10; Australia: CPI(prior 76Q3), C193CZN@OECDMEI, Core CPI, C193CZCN@OECDMEI; Austria: CPI(prior 66Q1) , C122CZN@OECDMEI, Core CPI, C122CZCN@OECDMEI Belgium: CPI(prior 76Q2), C124CZN@OECDMEI, Core CPI, C124CZCN@OECDMEI; Finland: Core CPI, C172CZCN@OECDMEI; Greece: CPI(prior 70Q1), C174CZN@OECDMEI, Core CPI,
C174CZCN@OECDMEI; Ireland: CPI(prior 75Q4), Central Statistics Office of Ireland, Core CPI, C178CZCN@OECDMEI; Netherlands: CPI(prior 60Q2), C138PC@IFS, Core CPI, C138CZCN@OECDMEI; Norway: CPI(prior 79Q1), C142CZN@OECDMEI, Core CPI, C142CZCN@OECDMEI; Portugal: CPI(prior 88Q1), C182CZN@OECDMEI, Core CPI, C182CZCN@OECDMEI; South Korea (prior 90Q1): C542CZN@OECDMEI, Core CPI, C542CZCN@OECDMEI; Spain: CPI(prior 76Q1), C184CZN@OECDMEI, Core CPI, C184CZCN@OECDMEI; Sweden: CPI(prior 70Q1), C144CZN@OECDMEI, Core CPI, C144CZCN@OECDMEI; Switzerland: Core CPI, C146CZCN@OECDMEI.

Population:
Canada: C156TB@UNPOP; France: C132TB@UNPOP; Germany: C134TB@UNPOP; Italy: C136TB@UNPOP; Japan: C158TB@UNPOP; United Kingdom: C112TB@UNPOP; United States: C111TB@UNPOP; Australia: C193TB@UNPOP; Austria: C122TB@UNPOP; Belgium: C124TB@UNPOP; Finland: C172TB@UNPOP; Greece: C174TB@UNPOP; Ireland: C178TB@UNPOP; Netherlands: C138TB@UNPOP; Norway: C142TB@UNPOP; Portugal: C182TB@UNPOP; South Korea: C542TB@UNPOP; Spain: C184TB@UNPOP; Sweden: C144TB@UNPOP; Switzerland: C146TB@UNPOP.

TABLE 1

Model parameter estimates

Parameters	Baseline	LW	Per Capita
a_1	1.554 (14.56)	1.553 (14.61)	1.569 (15.41)
a_2	-0.632 (5.88)	-0.598 (5.71)	-0.653 (6.24)
a_3	-0.035 (1.93)	-0.058 (3.18)	-0.034(1.94)
b_1	0.782 (10.94)	0.569 (8.52)	0.763 (9.91)
b_2	0.114 (1.35)	0.379 (4.34)	0.123(1.44)
b_3	0.159 (2.45)	0.040(1.36)	0.186(2.25)
b_4	0.002 (1.81)	0.0025(2.18)	0.002 (1.84)
b_5	–	0.036(3.38)	–
c	0.458 (1.14)	1.321(2.22)	0.600 (1.21)
σ_1	0.343	0.360	0.330
σ_2	0.711	0.767	0.700
$\sigma_3 = \lambda_z \sigma_1 / a_3$	0.076	0.248	0.116
σ_4	0.244	0.599	0.243
σ_5	0.143	0.042	0.168
$\sigma_r = \sqrt{c^2 \sigma_5^2 + \sigma_3^2}$	0.115	0.254	0.154
λ_z	0.012	0.040	0.012
λ_g	0.147	0.017	0.174

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Note: MLE estimation results. t-statistics are reported in parenthesis.

FIGURE 1

GDP Share: 1960Q1-2015Q4

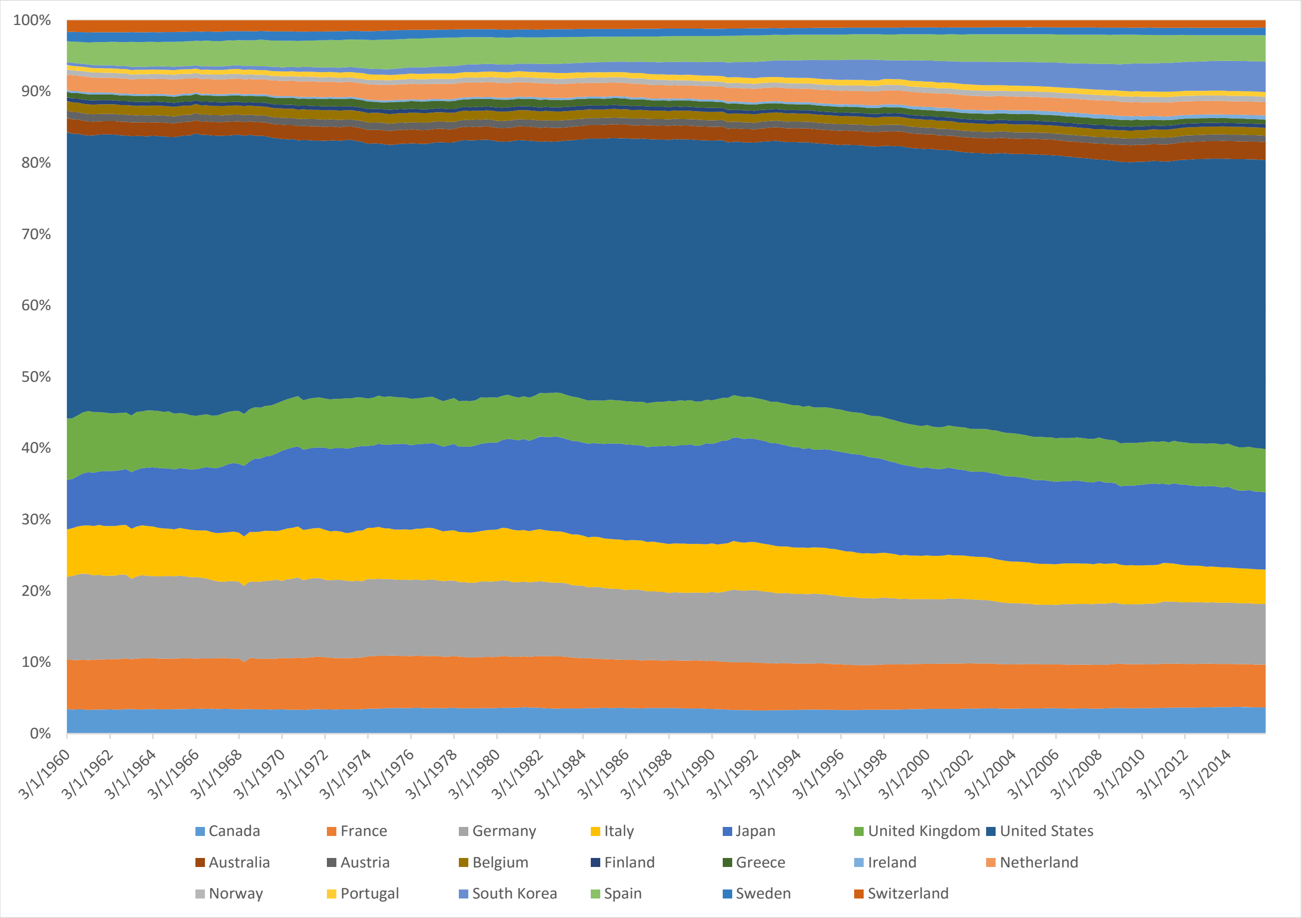


FIGURE 2

The World Output Gap: 1961Q1-2015Q4

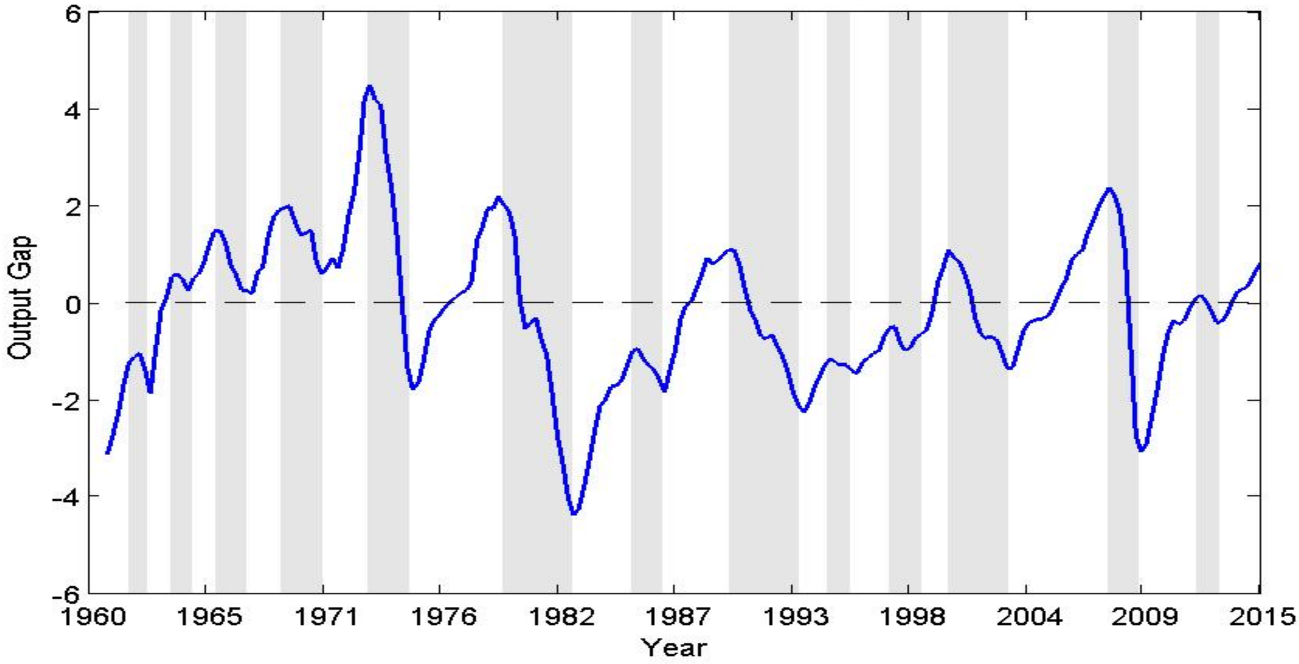


FIGURE 3

The World Potential Output Growth and Its Trend (Annualized)

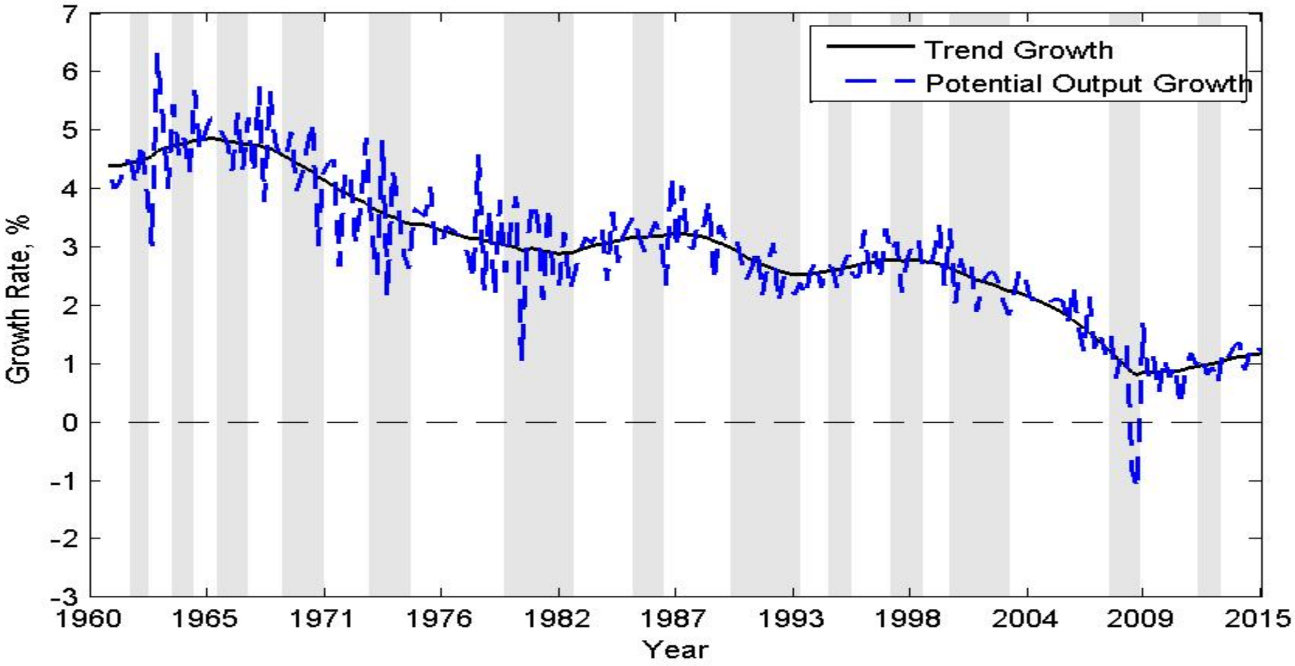


FIGURE 4

The World Real Interest Rate and Natural Rate of Interest

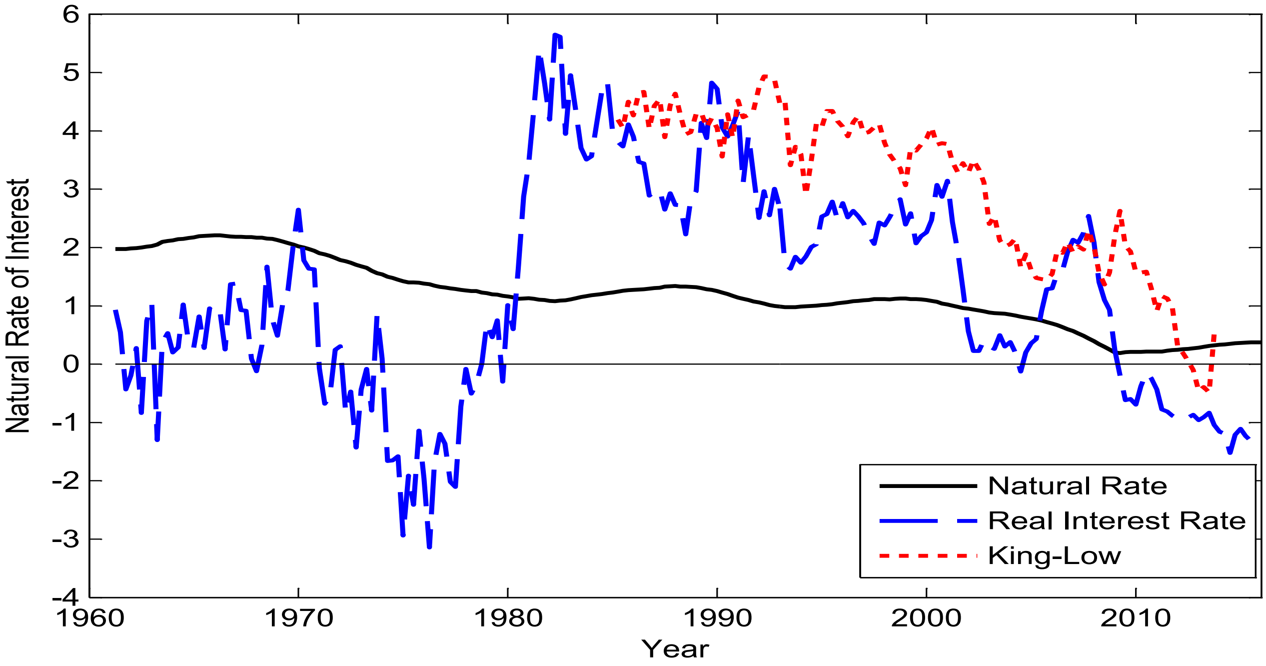


FIGURE 5

The World Natural Rate and Its Decomposition

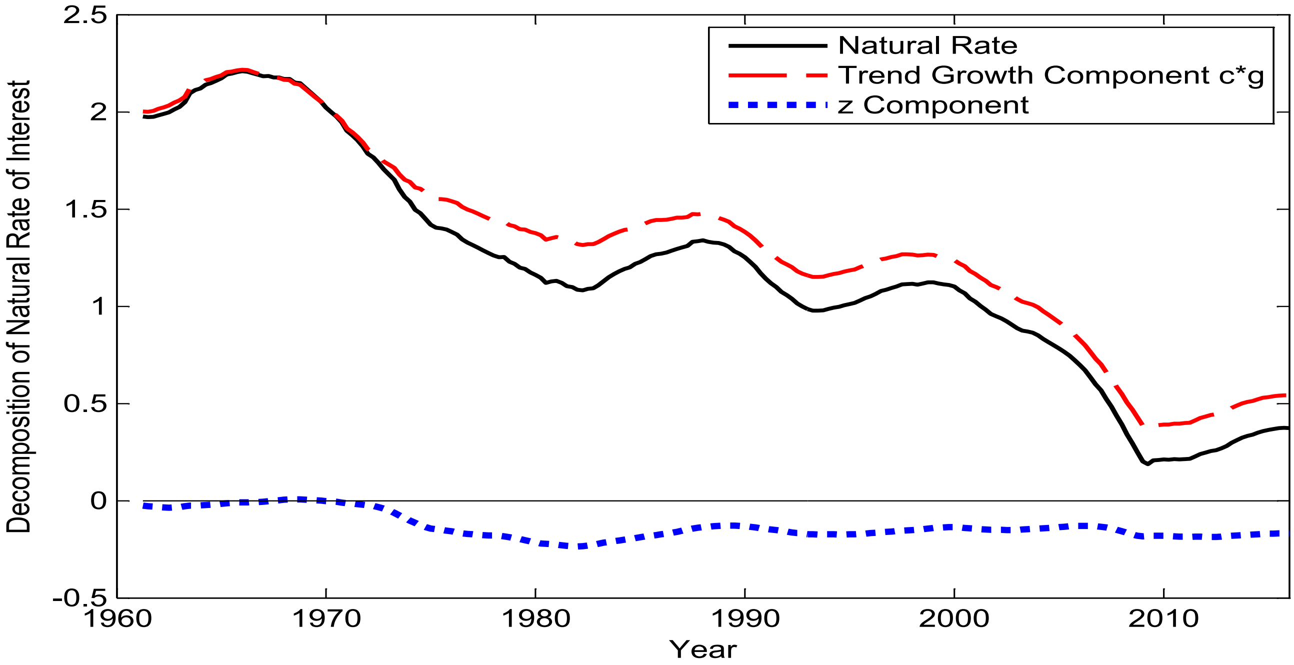


FIGURE 6

The U.S. Natural Rate and Its Decomposition

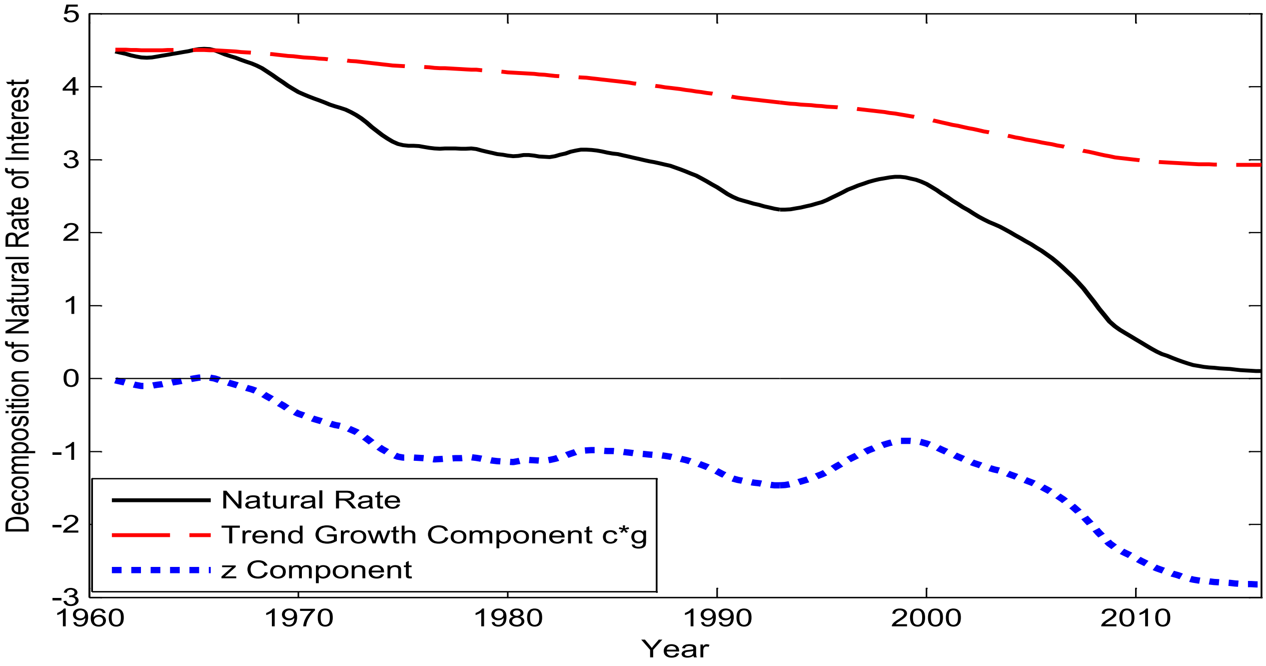


FIGURE 7

Aggregated World Population Growth Rate

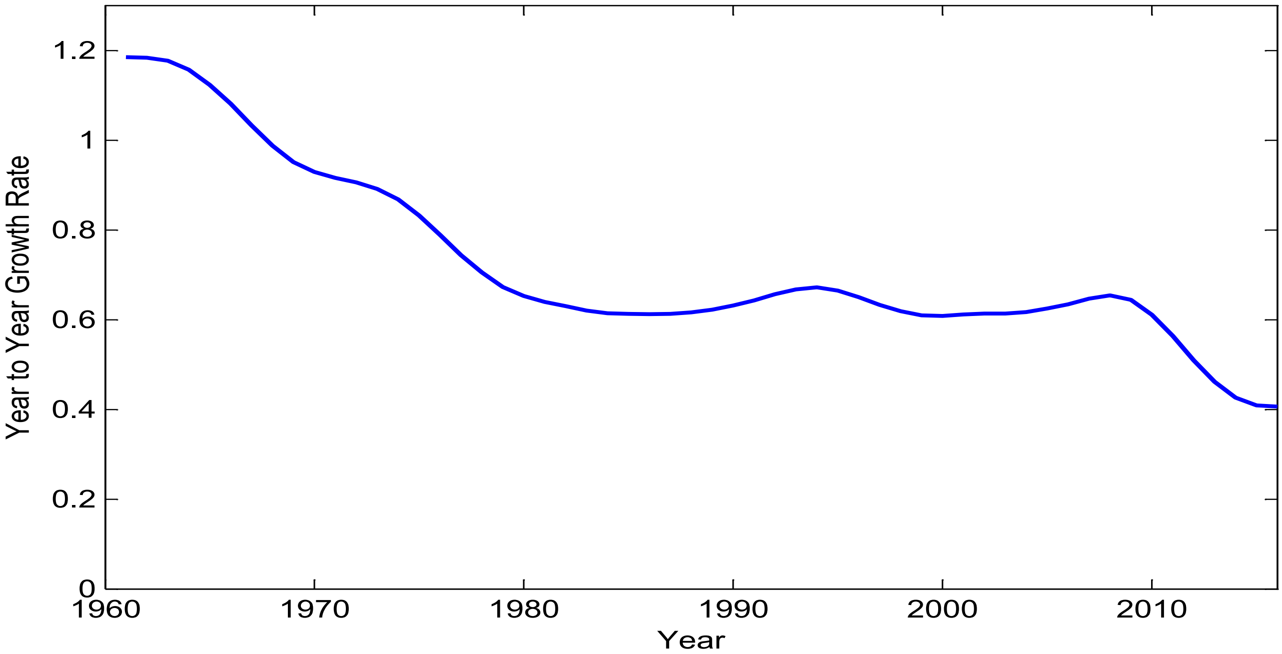


FIGURE 8

The World Output Gap (Per Capita): 1961Q1-2015Q4

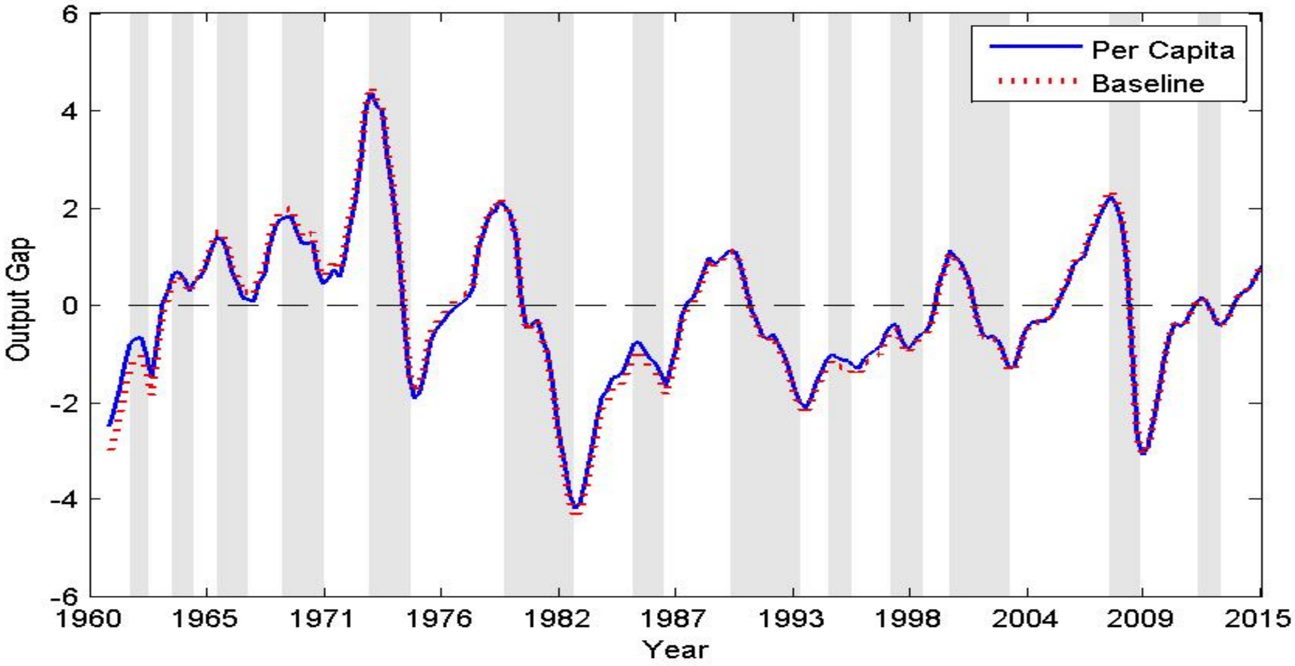


FIGURE 9

The World Potential Output Growth and Its Trend (Per Capita)

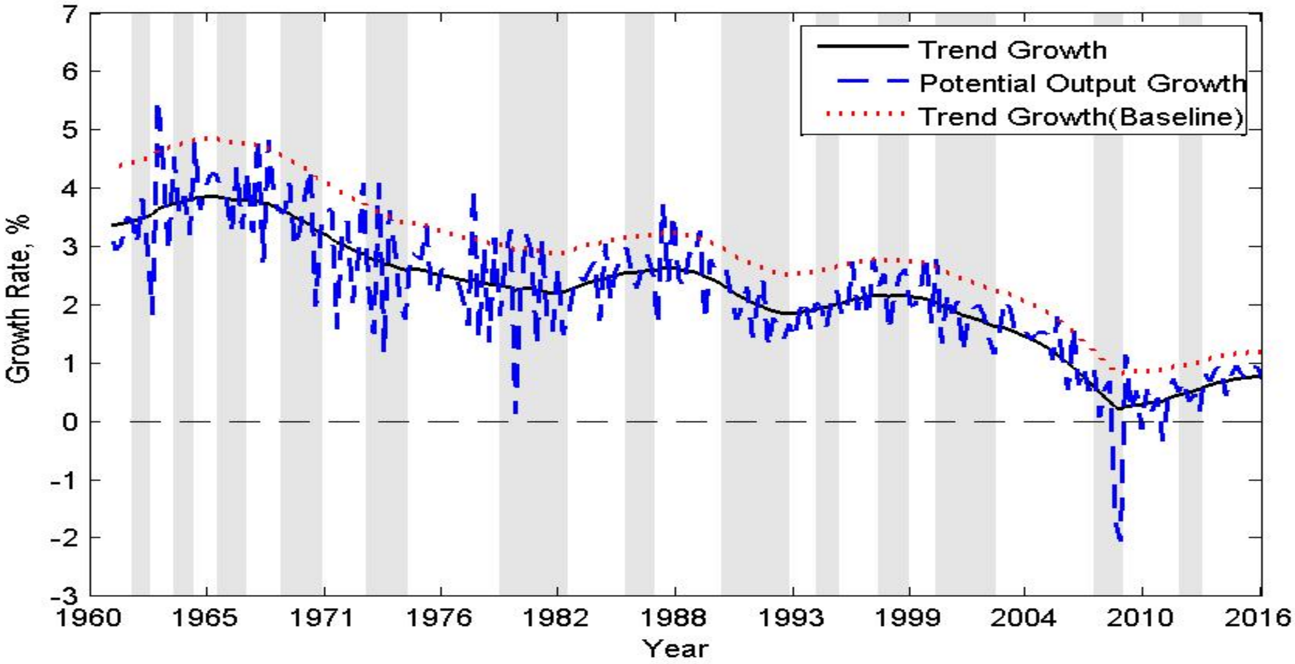
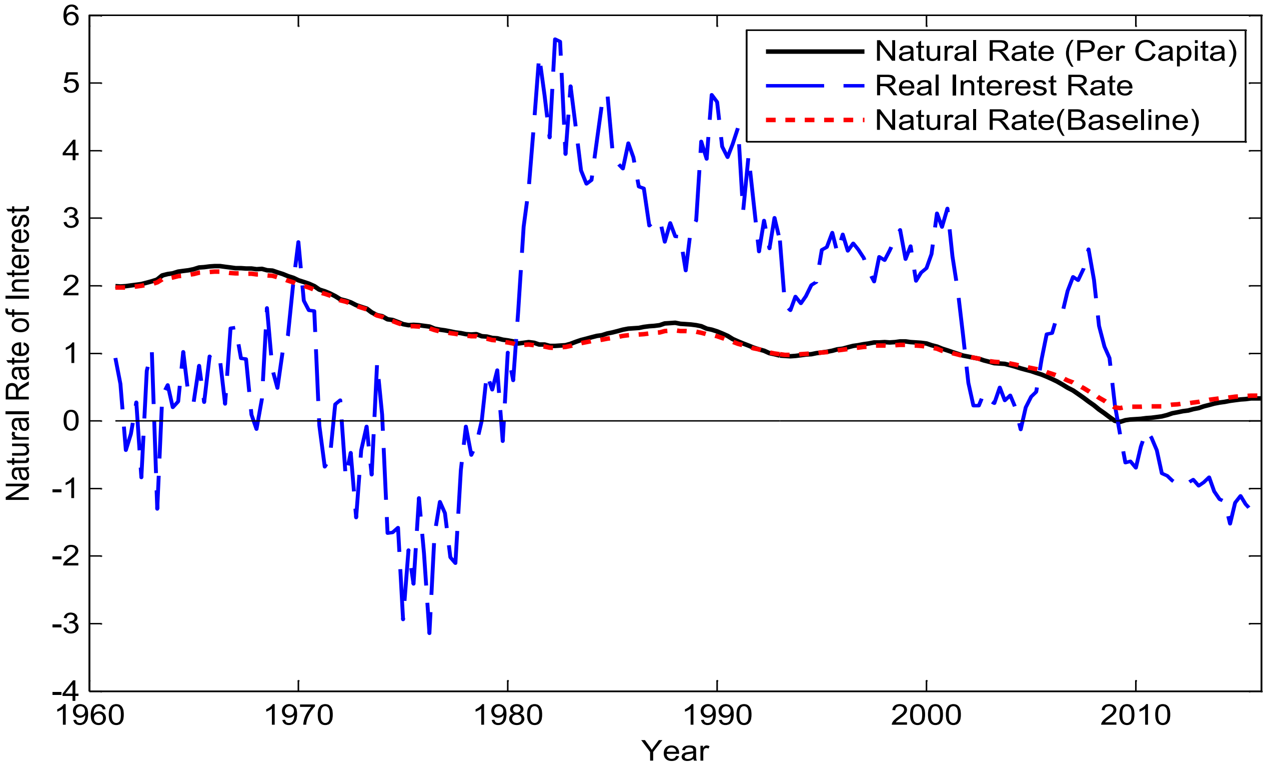


FIGURE 10

The World Real Interest Rate and Natural Rate of Interest (Per Capita)



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