Online Appendix to "Slow Post-Financial Crisis Recovery and Monetary Policy"

Daisuke Ikeda* Takushi Kurozumi†

This appendix is organized as follows. Section A presents equilibrium conditions and the steady state of the model in terms of stationary variables. Section B derives the second-order approximation to the welfare of households around the steady state. Section C shows the result of financial crisis scenario simulations under the welfare-maximizing monetary policy rule and other selected rules. Section D compares the specification of financial frictions between our paper and Queraltó (2013) and derives its implications for the amplification mechanism of financial shocks.

A Equilibrium Conditions and the Steady State

A.1 Equilibrium conditions

This section begins by presenting equilibrium conditions of the model in terms of stationary variables.

For 34 stationary endogenous variables $y_t = Y_t/A^*_{t-1}$, $gdp_t = GDP_t/A^*_{t-1}$, $c_t = C_t/A^*_{t-1}$, $i_t = I_t/A^*_{t-1}$, $k_t = K_t/A^*_{t-1}$, $w_t = W_t/A^*_{t-1}$, $l_t = L_t/A^*_{t-1}$, $d_t = D_t/A^*_{t-1}$, $b_t = B_t/A^*_{t-1}$, $i_{d,t} = I_{d,t}/A^*_{t-1}$, $s_t = S_t/A^{\theta_a-1}_{t-1}$, $i_{a,t} = I_{a,t}A_{t-1}/A^*_{t-1}$, $v_t = V_tA_{t-1}/A^*_{t-1}$, $j_t = J_tA_{t-1}/A^*_{t-1}$, $a_t = A_t/Z_t$, $\gamma_t = A_t/A_{t-1}$, $\gamma^*_t = A^*_t/A^*_{t-1}$, $\gamma^{tfp}_t = TFP_t/TFP_{t-1}$, λ_t, π_t, $\zeta_{p,t}$, $\upsilon_{p1,t}$, $\upsilon_{p2,t}$, $\upsilon_{w1,t}$, $\upsilon_{w2,t}$, u_t, n_t, r_t, μ_t, r^τ_t, φ_t, φ'_t, $\delta_{k,t}$, and $\delta'_{k,t}$, the system of equilibrium conditions consists of the following 34 equations.

$$\varphi_t = d_t + \kappa_d (d_t - d)^2, \tag{A1}$$

$$\varphi'_t = 1 + 2\kappa_d (d_t - d), \tag{A2}$$

$$\gamma^*_t = \gamma_t^{\frac{\theta_a-1}{1-\alpha}}, \tag{A3}$$

$$\gamma^{tfp}_t = \frac{\zeta_{p,t-1}}{\zeta_{p,t}} \left(\gamma^*_{t-1}\right)^{1-\alpha} \frac{x_t}{x_{t-1}}, \tag{A4}$$

$$\delta_{k,t} = \delta_k + \delta_1 (u_t - 1) + \frac{\delta_2}{2} (u_t - 1)^2, \tag{A5}$$

$$\delta'_{k,t} = \delta_1 + \delta_2 (u_t - 1), \tag{A6}$$

*Bank of Japan. E-mail: daisuke.ikeda@boj.or.jp
†Bank of Japan. E-mail: takushi.kurozumi@boj.or.jp

$$r_t^\tau = 1 + (1-\tau)(r_t - 1), \tag{A7}$$

$$l_t = w_t n_t + i_t + v_t[\gamma_t - (1-\delta_a)], \tag{A8}$$

$$l_t = \xi_t\left(k_t + v_t\gamma_t - \frac{b_t}{r_t}\right), \tag{A9}$$

$$0 = \theta_x s_t \varphi_t' x_t n_t^{1-\alpha}\left(\frac{u_t k_{t-1}}{\gamma_{t-1}^*}\right)^\alpha + \frac{b_t}{r_t^\tau} - w_t n_t - i_t - v_t[\gamma_t - (1-\delta_a)] - \varphi_t - \frac{b_{t-1}}{\gamma_{t-1}^* \pi_t}, \tag{A10}$$

$$\frac{1-\alpha}{\alpha} = \frac{w_t n_t}{\delta_{k,t}' u_t k_{t-1}/\gamma_{t-1}^*}, \tag{A11}$$

$$s_t = \frac{1/\varphi_t' + \mu_t}{x_t}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{\delta_{k,t}'}{\alpha}\right)^\alpha, \tag{A12}$$

$$1 = E_t\left[\frac{\beta c_t}{\gamma_t^* c_{t+1}} \frac{\alpha s_{t+1} x_{t+1} u_{t+1}^\alpha \left(\frac{n_{t+1}}{k_t/\gamma_t^*}\right)^{1-\alpha} + (1-\delta_{k,t+1})\big(1/\varphi_{t+1}' + \mu_{t+1}\big)}{1/\varphi_t' + \mu_t(1-\xi_t)}\right], \tag{A13}$$

$$1 = E_t\left[\frac{\beta c_t}{\gamma_t^* c_{t+1}} \frac{r_t^\tau}{\pi_{t+1}} \frac{\varphi_t'}{\varphi_{t+1}'}\right] + \mu_t \xi_t \varphi_t' \frac{r_t^\tau}{r_t}, \tag{A14}$$

$$v_t = E_t\left[\frac{\beta c_t}{\gamma_t c_{t+1}} \frac{(\theta_x - 1) s_{t+1} x_{t+1} n_{t+1}^{1-\alpha}\left(\frac{u_{t+1} k_t}{\gamma_t^*}\right)^\alpha + (1-\delta_a) v_{t+1}\big(1/\varphi_{t+1}' + \mu_{t+1}\big)}{1/\varphi_t' + \mu_t(1-\xi_t)}\right], \tag{A15}$$

$$y_t = \frac{1}{\zeta_{p,t}} x_t n_t^{1-\alpha}\left(\frac{u_t k_{t-1}}{\gamma_{t-1}^*}\right)^\alpha, \tag{A16}$$

$$1 = (1-\xi_p)\left(\theta_y \frac{\upsilon_{p1,t}}{\upsilon_{p2,t}}\right)^{1-\eta_y} + \xi_p\left(\frac{\pi}{\pi_t}\right)^{1-\eta_y}, \tag{A17}$$

$$\upsilon_{p1,t} = \theta_x s_t \varphi_t' \frac{y_t}{c_t} + \beta\xi_p\, E_t\left[\left(\frac{\pi}{\pi_{t+1}}\right)^{-\eta_y} \upsilon_{p1,t+1}\right], \tag{A18}$$

$$\upsilon_{p2,t} = \frac{y_t}{c_t} + \beta\xi_p\, E_t\left[\left(\frac{\pi}{\pi_{t+1}}\right)^{1-\eta_y} \upsilon_{p2,t+1}\right], \tag{A19}$$

$$\zeta_{p,t} = (1-\xi_p)\left(\theta_y \frac{\upsilon_{p1,t}}{\upsilon_{p2,t}}\right)^{-\eta_y} + \xi_p\left(\frac{\pi}{\pi_t}\right)^{-\eta_y} \zeta_{p,t-1}, \tag{A20}$$

$$k_t = (1-\delta_{k,t})\,\frac{k_{t-1}}{\gamma_{t-1}^*} + i_t, \tag{A21}$$

$$\lambda_t = \lambda_0\, i_{a,t}^\omega, \tag{A22}$$

$$\gamma_t = (1-\delta_a)\left[1 + \lambda_t\left(\frac{1}{a_{t-1}} - 1\right)\right], \tag{A23}$$

$$j_t = -i_{a,t} + (1-\delta_a)\left\{\lambda_t v_t + (1-\lambda_t)\, E_t\left[\frac{\beta c_t}{\gamma_t c_{t+1}} j_{t+1}\right]\right\}, \tag{A24}$$

$$i_{a,t} = \omega\,(1-\delta_a)\,\lambda_t\left(v_t - E_t\left[\frac{\beta c_t}{\gamma_t c_{t+1}} j_{t+1}\right]\right), \tag{A25}$$

$$\frac{1}{a_t} = (1-\delta_a)\frac{1}{\gamma_t a_{t-1}} + \chi_z \frac{i_{d,t}^{\rho}}{\gamma_t a_{t-1}}, \tag{A26}$$

$$1 = \chi_z (1-\delta_a) \frac{1}{a_{t-1} i_{d,t}^{1-\rho}} E_t \left[\frac{\beta c_t}{\gamma_t c_{t+1}} j_{t+1} \right], \tag{A27}$$

$$1 = E_t \left[\frac{\beta c_t}{\gamma_t^* c_{t+1}} \frac{r_t}{\pi_{t+1}} \right], \tag{A28}$$

$$1 = (1-\xi_w) \left(\theta_n \chi_n \frac{\upsilon_{w1,t}}{\upsilon_{w2,t}} \right)^{\frac{1-\eta_n}{1+\eta_n/\nu}} + \xi_w \left(\frac{\pi}{\pi_t} \frac{w_{t-1}}{w_t} \frac{\gamma^*}{\gamma_{t-1}^*} \right)^{1-\eta_n}, \tag{A29}$$

$$\upsilon_{w1,t} = n_t^{1+\frac{1}{\nu}} + \beta \xi_w E_t \left[\left(\frac{\pi}{\pi_{t+1}} \frac{w_t}{w_{t+1}} \frac{\gamma^*}{\gamma_t^*} \right)^{-\eta_n \left(1+\frac{1}{\nu}\right)} \upsilon_{w1,t+1} \right], \tag{A30}$$

$$\upsilon_{w2,t} = \frac{w_t n_t}{c_t} + \beta \xi_w E_t \left[\left(\frac{\pi}{\pi_{t+1}} \frac{w_t}{w_{t+1}} \frac{\gamma^*}{\gamma_t^*} \right)^{1-\eta_n} \upsilon_{w2,t+1} \right], \tag{A31}$$

$$\log \frac{r_t}{r} = \phi_r \log \frac{r_{t-1}}{r} + (1-\phi_r) \left(\phi_\pi \log \frac{\pi_t}{\pi} + \phi_{dy} \log \frac{gdp_t}{gdp_{t-1}} \frac{\gamma_{t-1}^*}{\gamma^*} \right) + \epsilon_{r,t}, \tag{A32}$$

$$gdp_t = c_t + i_t + i_{d,t} + \eta_{g,t} gdp_t, \tag{A33}$$

$$y_t = gdp_t + \kappa_d (d_t - d)^2 + i_{a,t} \left(\frac{1}{a_{t-1}} - 1 \right). \tag{A34}$$

A.2 Steady state

Next, turning to the steady state, the strategy for computing it is to set target values for labor n, the rate of technological change γ^*, and the technology adoption rate λ so as to pin down the values of parameters χ_n, χ_z, and λ_0.

In the steady state with the capital utilization rate of $u = 1$, (A5) and (A6) imply

$$\delta_k = \delta_k, \quad \delta_k' = \delta_1.$$

Equilibrium conditions (A2)–(A4) and (A28) also imply

$$\varphi' = 1, \quad \gamma = (\gamma^*)^{\frac{1-\alpha}{\theta_a - 1}}, \quad \gamma^{tfp} = (\gamma^*)^{1-\alpha}, \quad r = \frac{\gamma^* \pi}{\beta},$$

and then (A7) and (A14) lead to

$$r^\tau = 1 + (1-\tau)(r-1), \quad \mu = \frac{1}{\xi} \left(\frac{r}{r^\tau} - 1 \right).$$

Combining (A17)–(A20) generates

$$s = \frac{1}{\theta_x \theta_y}, \quad \zeta_p = 1.$$

Labor is normalized to unity, i.e., $n = 1$. Equilibrium conditions (A11)–(A13), (A16), and

(A21) then yield

$$k = \gamma^* \left(\frac{1}{\alpha s} \left\{ \frac{\gamma^*[1+\mu(1-\xi)]}{\beta} - (1-\delta_k)(1+\mu) \right\} \right)^{-\frac{1}{1-\alpha}}, \quad \delta'_k = \frac{s\alpha}{1+\mu} \left(\frac{\gamma^*}{k} \right)^{1-\alpha},$$
$$w = \frac{s(1-\alpha)}{1+\mu} \left(\frac{k}{\gamma^*} \right)^{\alpha}, \quad y = \left(\frac{k}{\gamma^*} \right)^{\alpha}, \quad i = \left(1 - \frac{1-\delta_k}{\gamma^*} \right) k.$$

Equilibrium condition (A15) leads to

$$v = \frac{\beta s(\theta_x - 1)}{\gamma[1+\mu(1-\xi)] - \beta(1-\delta_a)(1+\mu)} \left(\frac{k}{\gamma^*} \right)^{\alpha},$$

and then (A8)–(A10) yield

$$l = w + i + v[\gamma - (1-\delta_a)], \quad b = r \left(k + v\gamma - \frac{l}{\xi} \right),$$
$$d = \theta_x s \left(\frac{k}{\gamma^*} \right)^{\alpha} + \left(\frac{1}{r^{\tau}} - \frac{1}{\gamma^* \pi} \right) b - w - i - v[\gamma - (1-\delta_a)].$$

Besides, (A1) generates

$$\varphi = d.$$

Solving (A24) and (A25) for j and i_a leads to

$$j = \frac{\gamma\lambda v(1-\delta_a)(1-\omega)}{\gamma - \beta(1-\delta_a)[1-\lambda(1-\omega)]}, \quad i_a = \frac{\omega\lambda v(1-\delta_a)[\gamma - \beta(1-\delta_a)]}{\gamma - \beta(1-\delta_a)[1-\lambda(1-\omega)]}.$$

Equilibrium conditions (A22) and (A23) yield

$$\lambda_0 = \frac{\lambda}{i_a^{\omega}}, \quad a = \left[1 + \frac{1}{\lambda} \left(\frac{\gamma}{1-\delta_a} - 1 \right) \right]^{-1}.$$

Solving (A26) and (A27) for i_d and χ_z leads to

$$i_d = \frac{\beta(1-\delta_a)[\gamma - (1-\delta_a)]}{\gamma a} j, \quad \chi_z = \frac{\gamma - (1-\delta_a)}{i_d^{\rho}}.$$

Equilibrium conditions (A33) and (A34) yield

$$gdp = y - i_a \left(\frac{1}{a} - 1 \right), \quad c = gdp(1-\eta_g) - i - i_d.$$

Combining (A29)–(A31) generates

$$\upsilon_{w1} = \frac{1}{1-\beta\xi_w}, \quad \upsilon_{w2} = \frac{1}{1-\beta\xi_w} \frac{w}{c}, \quad \chi_n = \frac{w}{\theta_n c},$$

while (A18) and (A19) lead to

$$\upsilon_{p1} = \frac{\theta_x s}{1-\beta\xi_p}\frac{y}{c}, \quad \upsilon_{p2} = \frac{1}{1-\beta\xi_p}\frac{y}{c}.$$

B Second-order approximation to the welfare of households

This section derives a second-order approximation around the steady state to the unconditional expectation of the average utility function over households (56) of the paper. Combining this welfare function and the demand curve for each type of specialized labor (45) of the paper yields

$$SW = (1-\beta)E\left[\sum_{t=0}^{\infty}\beta^t\left(\log C_t - \frac{\chi_n}{1+1/\nu}n_t^{1+1/\nu}\zeta_{w,t}\right)\right],$$

where $\zeta_{w,t}$ denotes wage dispersion given by

$$\zeta_{w,t} = \int_0^1 \left(\frac{W_{f,t}}{W_t}\right)^{-\eta_n\left(1+\frac{1}{\nu}\right)} df.$$

Under the staggered wage setting, $\zeta_{w,t}$ can be expressed recursively as

$$\zeta_{w,t} = (1-\xi_w)\left(\theta_n\chi_n\frac{\upsilon_{w1,t}}{\upsilon_{w2,t}}\right)^{-\frac{\eta_n(1+1/\nu)}{1+\eta_n/\nu}} + \xi_w\left(\frac{\pi}{\pi_t}\frac{w_{t-1}}{w_t}\frac{\gamma^*}{\gamma^*_{t-1}}\right)^{-\eta_n(1+1/\nu)}\zeta_{w,t-1}, \tag{B1}$$

and its steady-state value is $\zeta_w = 1$. Using $c_t = C_t/A^*_{t-1}$, the welfare measure SW can be rewritten as

$$SW = (1-\beta)\,E\left[\sum_{t=0}^{\infty}\beta^t\left(\log c_t + \log A^*_{t-1} - \frac{\chi_n}{1+1/\nu}n_t^{1+1/\nu}\zeta_{w,t}\right)\right]. \tag{B2}$$

Note that this measure is non-stationary because the definition of γ^*_t leads to the process $\log A^*_t = \log A^*_{t-1} + \log\gamma^*_t$. Thus, letting $\{\bar{A}^*_t\}_{t=-1}^{\infty}$ be a deterministic trend defined as $\bar{A}^*_{-1} = A^*_{-1}$, $\bar{A}^*_t = \gamma^*\bar{A}^*_{t-1}$, we subtract its average discounted sum $(1-\beta)\sum_{t=0}^{\infty}\beta^t\log\bar{A}^*_{t-1}$ from both sides of (B2). The resulting welfare measure SW^* is given by

$$\begin{aligned} SW^* &= SW - (1-\beta)\sum_{t=0}^{\infty}\beta^t\log\bar{A}^*_{t-1} \\ &= (1-\beta)E\left[\sum_{t=0}^{\infty}\beta^t\left(\log c_t + \log\frac{A^*_{t-1}}{\bar{A}^*_{t-1}} - \frac{\chi_n}{1+1/\nu}n_t^{1+1/\nu}\zeta_{w,t}\right)\right]. \end{aligned} \tag{B3}$$

We now approximate the stationary welfare measure SW^* around the steady state up to the second order. The term related to detrended consumption c_t in (B3) is approximated around the steady state as

$$(1-\beta)E\left[\sum_{t=0}^{\infty}\beta^t \log c_t\right] \approx \log c + \frac{\varepsilon_c}{c} - \frac{Var\left(c_t\right)}{2c^2}, \tag{B4}$$

where $\varepsilon_c = E[c_t] - c$ denotes the bias in detrended consumption c_t and is of the second order. Using the relations $\log A_t^* = \log A_{t-1}^* + \log \gamma_t^*$ and $\log \bar{A}_t^* = \log \bar{A}_{t-1}^* + \log \gamma^*$, the term related to $A_{t-1}^*/\bar{A}_{t-1}^*$ in (B3) is approximated as

$$\begin{aligned}
(1-\beta)E\left[\sum_{t=0}^{\infty}\beta^t \log \frac{A_{t-1}^*}{\bar{A}_{t-1}^*}\right] &= (1-\beta)\sum_{t=0}^{\infty}\beta^t E\left[\log\left(\prod_{h=0}^{t-1}\frac{\gamma_h^*}{\gamma^*}\right)\right] \\
= (1-\beta)\sum_{t=0}^{\infty}\beta^t E\left[\sum_{h=0}^{t-1}\log\frac{\gamma_h^*}{\gamma^*}\right] &= E\left[\log\frac{\gamma_t^*}{\gamma^*}\right](1-\beta)\sum_{t=0}^{\infty}\beta^t t \\
= \frac{\beta}{1-\beta}E\left[\log\frac{\gamma_t^*}{\gamma^*}\right] &\approx \frac{\beta}{1-\beta}\left(\frac{\varepsilon_{\gamma^*}}{\gamma^*} - \frac{Var\left(\gamma_t^*\right)}{2\left(\gamma^*\right)^2}\right),
\end{aligned} \tag{B5}$$

where $\varepsilon_{\gamma^*} = E[\gamma_t^*] - \gamma^*$ denotes the bias in the rate of technological change γ_t^* and is of the second order. The term related to labor n_t in (B3) is approximated as

$$(1-\beta)E\left[\sum_{t=0}^{\infty}\beta^t n_t^{1+1/\nu}\zeta_{w,t}\right] \approx 1 + \left(1+\frac{1}{\nu}\right)\varepsilon_n + \varepsilon_{\zeta_w} + \frac{1+1/\nu}{\nu}\frac{Var\left(n_t\right)}{2}, \tag{B6}$$

where $n = \zeta_w = 1$ is used to derive this approximation and where $\varepsilon_n = E[n_t] - n$ and $\varepsilon_{\zeta_w} = E[\zeta_{w,t}] - \zeta_w$ denote the biases in labor n_t and wage dispersion $\zeta_{w,t}$ and they are of the second order.

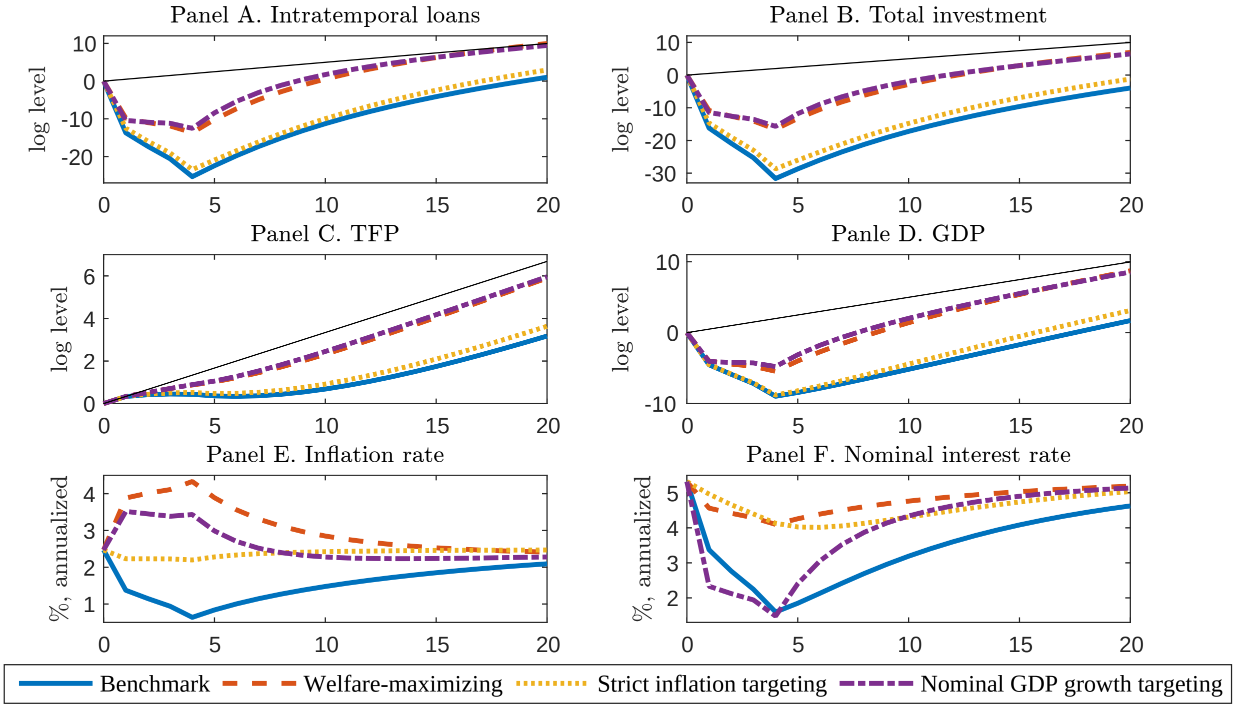
From (B4)–(B6), the second-order approximation to SW^* around the steady state is given by equation (57) of the paper, where terms independent of monetary policy are omitted.

C Financial Crisis Scenario Simulations

This section shows the result of financial crisis scenario simulations under the welfare-maximizing monetary policy rule and other selected rules analyzed in Section III.C of the paper. The section considers the following financial crisis scenario. The economy stays in the steady state in period $t = 0$. During periods $t = 1, 2, 3, 4$, it is hit by the adverse financial shocks that are identified for the U.S. during the Great Recession period 2008Q4–2009Q3 in the estimation conducted in Section II.A of the paper.

Figure A1 plots the developments of intratemporal loans, total investment (i.e., the sum of capital investment, technology adoption investment, and R&D investment), TFP, GDP, the inflation rate, and the nominal interest rate in the financial crisis scenario simulations

Figure A1: Financial crisis scenario simulations



under the benchmark monetary policy rule (the solid line), the welfare-maximizing rule (the dashed line), the strict inflation targeting rule (the dotted line), and the nominal GDP growth targeting rule (the dot-dashed line). Note that the coefficients values of the strict inflation targeting rule and the nominal GDP growth targeting rule are set equal to the welfare-maximizing ones in the case of all three shocks reported in Table 2 of the paper. In this figure, three findings are detected.

First, in response to the estimated financial shocks, intratemporal loans drop sharply under the benchmark rule, whereas the decline in the loans is subdued under the welfare-maximizing rule. As a consequence, slowdowns in growth of total investment and TFP are much less pronounced under the welfare-maximizing rule. Moreover, GDP approaches the pre-shock steady-state growth path under the welfare-maximizing rule, while it does not under the benchmark rule, implying that the welfare gain from the welfare-maximizing rule relative to the benchmark rule is huge, as shown in Section III.C of the paper. The inflation rate then drops sharply under the benchmark rule, whereas it rises under the welfare-maximizing rule. This rise happens because the decline in intratemporal loans of intermediate-good firms is smaller than that in the value of their collateral (i.e., net assets held by the firms), which tightens the borrowing constraint (7) of the paper and increases the associated Lagrange multiplier μ_t (i.e., real marginal cost of funds), thereby raising wholesalers' real marginal cost and hence inflation. The welfare-maximizing rule thus aims to stabilize output even at the cost of stability of inflation in the short run.

Second, under the strict inflation targeting rule, its strong policy response to inflation stabilizes inflation much more than under the welfare-maximizing rule, which has a weak response to inflation. Yet the strict inflation targeting rule cannot directly mitigate a slowdown in TFP growth induced by the estimated severe financial shocks, because it includes no response to output. Consequently, GDP and total investment recover to the pre-shock steady-state growth path much more slowly than under the welfare-maximizing rule, implying that the welfare loss from the strict inflation targeting rule relative to the welfare-maximizing rule is sizable, as stressed in Section III.C of the paper.

Third, under the nominal GDP growth targeting rule, the achieved levels of GDP and total investment are almost the same as those under the welfare-maximizing rule, implying that the welfare gain from the welfare-maximizing rule relative to the nominal GDP growth targeting rule is small, as indicated in Section III.C of the paper. The inflation rate then rises for the same reason as that under the welfare-maximizing rule mentioned above. Yet the nominal interest rate drops sharply, causing relatively high interest-rate volatility, as noticed in Section III.C of the paper. This is mainly because the nominal GDP growth targeting rule has a stronger response to inflation and lower policy rate smoothing than the welfare-maximizing rule.

Before proceeding, it is worth noting that the simulations presented here take no account of the zero lower bound on the nominal interest rate. If the rate in period 0 were around 2 percent annually as in 2008Q3, instead of the steady-state rate of above 5 percent annually, the adverse financial shocks added in the simulations would lead the nominal rate to hit the zero lower bound under the benchmark rule and the nominal GDP growth targeting rule, according to panel F of Figure A1. The binding lower bound would cause a more severe recession than those displayed in the figure.[1] The welfare-maximizing rule, however, could help avoid the zero lower bound. This is because that rule yields a much milder drop in the nominal interest rate through a higher degree of policy rate smoothing and a stronger response to output, which increase inflation. Thus, in the presence of the zero lower bound, the lower interest-rate volatility is an important advantage of the welfare-maximizing rule over the nominal GDP growth targeting rule. Because the welfare-maximizing rule could attain higher welfare than the strict inflation targeting rule, we argue that the welfare-maximizing rule would still achieve higher welfare than other rules considered, even when taking into account the zero lower bound.

D Comparison of Financial Friction Specifications

This section compares the specification of financial frictions between our paper and Queraltó (2013) in terms of the amplification mechanism of financial shocks. Specifically, instead of those of Jermann and Quadrini (2012), the financial friction and shock of the sort proposed by Gertler and Kiyotaki (2011) are embedded in our model along the lines of Queraltó. The resulting model is called the "Q-type" model. This model is presented in Section D.1. The

[1] Moran and Queraltó (2017) show that the zero lower bound on the nominal interest rate during the period 2009–15 led the levels of TFP and GDP in the U.S. to be permanently lower by 1.75% and 2.5%, respectively, under a standard Taylor-type rule, using a DSGE model with endogenous TFP growth and nominal rigidities (but no financial friction).

notations used in our model are also employed in the Q-type model, unless otherwise noted. Section D.2 compares impulse responses to financial shocks between the Q-type model and ours to derive implications for the amplification mechanism of financial shocks.

D.1 The Q-type model

The Q-type model, along the lines of Queraltó (2013), replaces the financial friction and shock of Jermann and Quadrini (2012) with those of the sort proposed by Gertler and Kiyotaki (2011) in our model. One point to be emphasized here is that intermediate-good firms—the production sector—face the financial friction in our model, while technology adopters do so in Queraltó.

Final-Good and Intermediate-Good Firms.—The problem of final-good firms is the same as in our model, while that of intermediate-good firms differs because they no longer face financial frictions, as opposed to our model.

As in our model, capital K_t and adopted ideas A_t are accumulated according to equations (4) and (5) of the paper (without the intermediate-good firm index h),

$$K_t = (1 - \delta_{k,t}) K_{t-1} + I_t, \tag{D1}$$

$$A_t = (1 - \delta_a) A_{t-1} + \triangle_{a,t}. \tag{D2}$$

Because of no financial friction, combining the first-order conditions for capital K_t, labor n_t, and the utilization rate u_t leads to

$$1 = E_t\left[m_{t,t+1}\left(\alpha S_{t+1}\frac{u_{t+1}^{\alpha}n_{t+1}^{1-\alpha}}{K_t^{1-\alpha}} + 1 - \delta_{k,t+1}\right)\right], \tag{D3}$$

$$\frac{1-\alpha}{\alpha} = \frac{W_t n_t}{\delta'_{k,t}u_t K_{t-1}}, \tag{D4}$$

$$S_t = \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{\delta'_{k,t}}{\alpha}\right)^{\alpha}, \tag{D5}$$

and the value of adopted technologies (ideas) V_t is given by

$$V_t = E_t\left[m_{t,t+1}\left\{(\theta_x - 1)S_{t+1}\frac{n_{t+1}^{1-\alpha}(u_{t+1}K_t)^{\alpha}}{A_t} + (1-\delta_a)V_{t+1}\right\}\right], \tag{D6}$$

which is basically consistent with equations (B.14) and (B.15) of Queraltó (2013). Equations (D3)–(D6) are the financial frictionless version of equations (10), (12), (13), and (23) of the paper (without the TFP shock, i.e., $x_t = 1$).

Retailers and Wholesalers.—The problems of retailers and wholesalers are the same as in our model. Then, the marginal cost MC_t and aggregate output Y_t are given by

$$MC_t = P_t\theta_x S_t A_t^{1-\theta_a}, \tag{D7}$$

$$Y_t = \frac{(A^*_{t-1})^{1-\alpha}}{\zeta_{p,t}} n_t^{1-\alpha}(u_t K_{t-1})^\alpha, \tag{D8}$$

where

$$A^*_t = A_t^{\frac{\theta_a - 1}{1-\alpha}}. \tag{D9}$$

Then, TFP is given by

$$TFP_t = \frac{(A^*_{t-1})^{1-\alpha}}{\zeta_{p,t}}. \tag{D10}$$

Equation (D7) is the financial frictionless version of equation (21) of the paper, while (D8)–(D10) are the same as equations (24), (26), and (27) of the paper (without the TFP shock, i.e., $x_t = 1$).

The staggered price setting à la Calvo (1983) and Yun (1996) leads to equations (28)–(32) of the paper,

$$\frac{\tilde{P}_{h,t}}{P_t} = \theta_y \frac{v_{p1,t}}{v_{p2,t}}, \tag{D11}$$

$$v_{p1,t} = \frac{MC_t}{P_t}\frac{Y_t}{C_t} + \beta\xi_p E_t\left[\left(\frac{\pi}{\pi_{t+1}}\right)^{-\eta_y} v_{p1,t+1}\right], \tag{D12}$$

$$v_{p2,t} = \frac{Y_t}{C_t} + \beta\xi_p E_t\left[\left(\frac{\pi}{\pi_{t+1}}\right)^{1-\eta_y} v_{p2,t+1}\right], \tag{D13}$$

$$1 = (1-\xi_p)\left(\frac{\tilde{P}_{h,t}}{P_t}\right)^{1-\eta_y} + \xi_p\left(\frac{\pi}{\pi_t}\right)^{1-\eta_y}, \tag{D14}$$

$$\zeta_{p,t} = (1-\xi_p)\left(\frac{\tilde{P}_{h,t}}{P_t}\right)^{-\eta_y} + \xi_p\left(\frac{\pi}{\pi_t}\right)^{-\eta_y}\zeta_{p,t-1}. \tag{D15}$$

Banks and Technology Adopters.—Following Queraltó (2013), banks are introduced in the problem of technology adopters. In our model, technology adopters face no financial friction, so that each adopter purchases a developed but not yet adopted technology at the price J_t and invest $I_{a,t}$ to adopt the technology at the adoption rate $\lambda(I_{a,t})$. Yet in the model of Queraltó, technology adopters—who are called entrepreneurs there—borrow J_t to purchase a developed but not yet adopted technology from banks by issuing securities that offer to pay the full value of the technology in the contingency that it is adopted. Another important difference is that the model of Queraltó abstracts from technology adoption investment so that the adoption rate is constant at λ, which is assumed in the Q-type model—the role of exogenous/endogenous technology adoption will be discussed later.

In period t, banks in the aggregate purchase claims X_t on adoption firms at the price J_t. The balance sheet of banks is given by

$$J_t X_t = \phi_t NW_t, \tag{D16}$$

where ϕ_t is the leverage ratio and NW_t is the aggregate net worth of banks. The financial friction à la Gertler and Kiyotaki (2011), in which banks can divert a fraction θ_t of bank assets, determines the leverage ratio as

$$\phi_t = \frac{\nu_t}{\theta_t - \mu_t}, \tag{D17}$$

where

$$\nu_t = E_t\left[m_{t,t+1}\omega_{t+1}\frac{r_t}{\pi_{t+1}}\right], \tag{D18}$$

$$\mu_t = E_t\left[m_{t,t+1}\omega_{t+1}\left(r_{z,t+1} - \frac{r_t}{\pi_t}\right)\right], \tag{D19}$$

$$\omega_t = 1 - \sigma + \sigma(\nu_t + \phi_t\mu_t), \tag{D20}$$

$$r_{z,t} = (1-\delta_a)\frac{\lambda V_t + (1-\lambda)J_t}{J_{t-1}}, \tag{D21}$$

$$\log\frac{\theta_t}{\theta} = \rho_\theta \log\frac{\theta_{t-1}}{\theta} + \epsilon_{\theta,t}. \tag{D22}$$

As in Queraltó (2013), $\epsilon_{\theta,t} \sim$ i.i.d. $N(0, \sigma_\theta^2)$ represents a financial shock in the Q-type model. A small difference from the model of Queraltó is that the real interest rate is state-contingent in the Q-type model because banks borrow and lend in nominal terms. The bank net worth NW_t evolves according to

$$NW_t = \sigma\left[\left(r_{z,t} - \frac{r_{t-1}}{\pi_t}\right)J_{t-1}X_{t-1} + \frac{r_{t-1}}{\pi_t}NW_{t-1}\right] + (1-\sigma)\psi J_{t-1}X_{t-1}. \tag{D23}$$

The number of securities purchased by banks X_t has to be equal to the number of technology adopters in period t,

$$X_t = Z_{t-1} - A_{t-1}. \tag{D24}$$

Moreover, the number of newly adopted technologies $\triangle_{a,t}$ is given by

$$\triangle_{a,t} = (1-\delta_a)\lambda(Z_{t-1} - A_{t-1}), \tag{D25}$$

which is the same as equation (34) of the paper (with the constant adoption rate λ).

Technology Innovators.—The problem of technology innovators is the same as in our model. The technology frontier Z_t follows equation (38) of the paper,

$$Z_t = (1-\delta_a)Z_{t-1} + \Phi_t I_{d,t}, \tag{D26}$$

where

$$\Phi_t = \chi_z \frac{Z_{t-1}}{(A_{t-1}^*)^\rho I_{d,t}^{1-\rho}}, \tag{D27}$$

which is the same as equation (39) of the paper. The zero profit condition for technology innovators leads to equation (40) of the paper,

$$1 = \Phi_t(1-\delta_a)E_t[m_{t,t+1}J_{t+1}]\,, \tag{D28}$$

which implies that the number of newly developed technologies $Z_{N,t}$ ($\equiv \Phi_t I_{d,t}$) is given by

$$Z_{N,t} = \chi_z \frac{Z_{t-1}}{(A^*_{t-1})^\rho} I^\rho_{d,t} = \left[\chi_z \frac{Z_{t-1}}{(A^*_{t-1})^\rho}\right]^{\frac{1}{1-\rho}} \left\{(1-\delta_a)E_t\left[m_{t,t+1}J_{t+1}\right]\right\}^{\frac{\rho}{1-\rho}}.$$

This equation suggests that the specifications of technology innovation in our model and Queraltó (2013) are quite similar. In particular, the number of newly developed technologies $Z_{N,t}$ is increasing in the value of developed but not yet adopted technologies, which is given by $E_t[m_{t,t+1}J_{t+1}]$ in the Q-type model due to a one period lag innovation, while it is given by J_t in the model of Queraltó.

Households and Employment Agencies.—The problems of households and employment agencies are the same as in our model. The consumption Euler equation is the same as equation (44) of the paper,

$$1 = E_t\left[\beta \frac{C_t}{C_{t+1}} \frac{r_t}{\pi_{t+1}}\right]. \tag{D29}$$

The staggered wage setting à la Erceg, Henderson, and Levin (2000) leads to equations (47)–(50) of the paper,

$$\left(\frac{P_t\tilde{W}_{f,t}}{P_tW_t}\right)^{1+\frac{\eta_n}{\nu}} = \theta_n\chi_n \frac{v_{w1,1}}{v_{w2,t}}, \tag{D30}$$

$$v_{w1,t} = n_t^{1+\frac{1}{\nu}} + \beta\xi_w E_t\left[\left(\frac{\pi}{\pi_{t+1}}\frac{\gamma^* W_t}{W_{t+1}}\right)^{-\eta_n\left(1+\frac{1}{\nu}\right)} v_{w1,t+1}\right], \tag{D31}$$

$$v_{w2,t} = \frac{W_t n_t}{C_t} + \beta\xi_w E_t\left[\left(\frac{\pi}{\pi_{t+1}}\frac{\gamma^* W_t}{W_{t+1}}\right)^{1-\eta_n} v_{w2,t+1}\right], \tag{D32}$$

$$1 = (1-\xi_w)\left(\frac{P_t\tilde{W}_{f,t}}{P_tW_t}\right)^{1-\eta_n} + \xi_w\left(\frac{\pi}{\pi_t}\frac{\gamma^* W_{t-1}}{W_t}\right)^{1-\eta_n}. \tag{D33}$$

A Central Bank.—The central bank follows the same Taylor (1993)-type rule as equation (51) of the paper,

$$\log r_t = \phi_r \log r_{t-1} + (1-\phi_r)\left[\log r + \phi_\pi(\log\pi_t - \log\pi) + \phi_{dgdp}\left(\log\frac{GDP_t}{GDP_{t-1}} - \log\gamma^*\right)\right]. \tag{D34}$$

In the Q-type model, GDP is defined as

$$GDP_t = C_t + I_t + I_{d,t} + G_t, \tag{D35}$$

where

$$G_t = \eta_g GDP_t. \tag{D36}$$

Because of an exogenous technology adoption as in Queraltó (2013), the right hand side of equation (D35) does not contain technology adoption investment, which is present in our model. In addition, it does not include costs associated with dividend payments, which is also present in our model due to the financial friction of Jermann and Quadrini (2012). Thus, in the Q-type model, the output of retail goods Y_t is equal to GDP,

$$Y_t = GDP_t. \tag{D37}$$

Equilibrium Conditions.—The equilibrium conditions consist of equations (D1)–(D21) and (D23)–(D37), along with the financial shock process (D22). Rewrite these equilibrium conditions in terms of stationary variables and rearranging the resulting equations leads to not only the same equations as (A3)–(A6), (A11), (A16), (A17), (A19)–(A21), and (A26)–(A33) without the TFP, government spending, and monetary policy shocks (i.e., $x_t = 1$, $\eta_{g,t} = \eta_g$, and $\epsilon_{r,t} = 0$) but also the following 14 equations, where there are 32 stationary endogenous variables $y_t = Y_t/A^*_{t-1}$, $gdp_t = GDP_t/A^*_{t-1}$, $c_t = C_t/A^*_{t-1}$, $i_t = I_t/A^*_{t-1}$, $k_t = K_t/A^*_{t-1}$, $w_t = W_t/A^*_{t-1}$, $i_{d,t} = I_{d,t}/A^*_{t-1}$, $s_t = S_t/A^{\theta_a-1}_{t-1}$, $v_t = V_t A_{t-1}/A^*_{t-1}$, $j_t = J_t A_{t-1}/A^*_{t-1}$, $a_t = A_t/Z_t$, $\gamma_t = A_t/A_{t-1}$, $\gamma^*_t = A^*_t/A^*_{t-1}$, $\gamma^{tfp}_t = TFP_t/TFP_{t-1}$, π_t, $\zeta_{p,t}$, $\upsilon_{p1,t}$, $\upsilon_{p2,t}$, $\upsilon_{w1,t}$, $\upsilon_{w2,t}$, u_t, n_t, r_t, μ_t, $\delta_{k,t}$, $\delta'_{k,t}$, x_t, nw_t, ϕ_t, ν_t, ω_t, and $r_{z,t}$.

$$\gamma_t = (1-\delta_a)\left[1 + \lambda\left(\frac{1}{a_{t-1}} - 1\right)\right], \tag{D38}$$

$$y_t = gdp_t, \tag{D39}$$

$$s_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{\delta'_{k,t}}{\alpha}\right)^{\alpha}, \tag{D40}$$

$$1 = E_t\left[\frac{\beta c_t}{\gamma^*_t c_{t+1}}\left\{\alpha s_{t+1} u^{\alpha}_{t+1}\left(\frac{n_{t+1}}{k_t/\gamma^*_t}\right)^{1-\alpha} + 1 - \delta_{k,t+1}\right\}\right], \tag{D41}$$

$$v_t = E_t\left[\frac{\beta c_t}{\gamma_t c_{t+1}}\left\{(\theta_x - 1)s_{t+1} n^{1-\alpha}_{t+1}\left(\frac{u_{t+1}k_t}{\gamma^*_t}\right)^{\alpha} + (1-\delta_a)v_{t+1}\right\}\right], \tag{D42}$$

$$\upsilon_{p1,t} = \theta_x s_t \frac{y_t}{c_t} + \beta\xi_p E_t\left[\left(\frac{\pi}{\pi_{t+1}}\right)^{-\eta_y} \upsilon_{p1,t+1}\right], \tag{D43}$$

$$j_t x_t = \phi_t nw_t, \tag{D44}$$

$$\phi_t = \frac{v_t}{\theta_t - \mu_t}, \tag{D45}$$

$$\nu_t = E_t\left[\frac{\beta c_t}{\gamma^*_t c_{t+1}}\omega_{t+1}\frac{r_t}{\pi_{t+1}}\right], \tag{D46}$$

$$\mu_t = E_t\left[\frac{\beta c_t}{\gamma^*_t c_{t+1}}\omega_t\left(r_{z,t+1} - \frac{r_t}{\pi_{t+1}}\right)\right], \tag{D47}$$

$$\omega_t = 1 - \sigma + \sigma(\nu_t + \phi_t \mu_t), \tag{D48}$$

$$r_{z,t} = (1-\delta_a)\frac{\lambda v_t + (1-\lambda) j_t}{j_{t-1}} \frac{\gamma^*_{t-1}}{\gamma_{t-1}}, \tag{D49}$$

$$nw_t = \sigma \left[\left(r_{z,t} - \frac{r_{t-1}}{\pi_t} \right) \frac{j_{t-1}}{\gamma^*_{t-1}} x_{t-1} + \frac{r_{t-1}}{\pi_t} \frac{nw_{t-1}}{\gamma^*_{t-1}} \right] + (1-\sigma)\psi \frac{j_{t-1}}{\gamma^*_{t-1}} x_{t-1}, \tag{D50}$$

$$x_t = \frac{1}{a_{t-1}} - 1. \tag{D51}$$

Note that equations (D38)–(D43) differ from their counterparts in our model (i.e., (A12), (A13), (A15), (A18), (A23), and (A34)) because of exogenous technology adoption (i.e., no technology adoption investment) and the absence of the financial friction of Jermann and Quadrini (2012) in the Q-type model. Equations (D44)–(D51) pertain to the financial friction à la Gertler and Kiyotaki (2011) used in Queraltó (2013).

Steady State.—The steady state of the Q-type model is computed by setting target values for labor n and the rate of technological change γ^*. In addition, following Queraltó (2013), we set target values for the leverage ratio ϕ and spread $r_z - r/\pi$. Hitting these target values in the steady state allows the Q-type model to pin down the values of parameters χ_n, χ_z, ψ, and θ.

In the steady state with the capital utilization rate of $u = 1$, (A5) and (A6) imply

$$\delta_k = \delta_k, \quad \delta_k' = \delta_1.$$

Equilibrium conditions (A3), (A4), and (A28) also imply

$$\gamma = (\gamma^*)^{\frac{1-\alpha}{\theta_a - 1}}, \quad \gamma^{tfp} = (\gamma^*)^{1-\alpha}, \quad r = \frac{\gamma^*}{\beta}\pi.$$

Combining (A17), (A19), (A20), and (D43) generates

$$s = \frac{1}{\theta_x \theta_y}, \quad \zeta_p = 1.$$

Labor is normalized to unity, i.e., $n = 1$. Equilibrium condition (D41) then implies

$$k = \gamma^* \left[\frac{1}{\alpha s} \left(\frac{\gamma^*}{\beta} - 1 + \delta_k \right) \right]^{-\frac{1}{1-\alpha}},$$

and moreover, (A16) and (A21) lead to

$$y = \left(\frac{k}{\gamma^*} \right)^{\alpha}, \quad i = \left(1 - \frac{1-\delta_k}{\gamma^*} \right) k.$$

Solving (A11) and (D40) for w and δ_k' yields

$$w = s(1-\alpha)\left(\frac{k}{\gamma^*}\right)^{\alpha}, \quad \delta_k' = s\alpha\left(\frac{k}{\gamma}\right)^{\alpha-1}.$$

Equilibrium conditions (D38) and (D42) imply

$$a = \left[1 + \frac{1}{\lambda}\left(\frac{\gamma}{1-\delta_a} - 1\right)\right]^{-1}, \quad v = \frac{\beta s(\theta_x - 1)}{\gamma - \beta(1-\delta_a)}\left(\frac{k}{\gamma^*}\right)^{\alpha}.$$

The target value for $spread = r_z - r/\pi$ yields

$$r_z = \frac{r}{\pi} + spread.$$

Equilibrium conditions (D44), (D49), and (D51) lead to

$$j = \lambda v\left[\frac{r_z}{1-\delta_a}\frac{\gamma}{\gamma^*} - (1-\lambda)\right]^{-1}, \quad x = \frac{1}{a} - 1, \quad nw = \frac{jx}{\phi}.$$

Solving (A26) and (A27) for i_d and χ_z yields

$$i_d = \frac{\beta(1-\delta_a)[\gamma - (1-\delta_a)]}{\gamma a}j, \quad \chi_z = \frac{\gamma - (1-\delta_a)}{i_d^{\rho}},$$

and then (A33) and (D39) leads to

$$gdp = y, \quad c = gdp(1-\eta_g) - i - i_d.$$

Combining (A29)–(A31) generates

$$\upsilon_{w1} = \frac{1}{1-\beta\xi_w}, \quad \upsilon_{w2} = \frac{1}{1-\beta\xi_w}\frac{w}{c}, \quad \chi_n = \frac{w}{\theta_n c},$$

while (A19) and (D43) lead to

$$\upsilon_{p1} = \frac{\theta_x s}{1-\beta\xi_p}\frac{y}{c}, \quad \upsilon_{p2} = \frac{1}{1-\beta\xi_p}\frac{y}{c}.$$

Equilibrium condition (D50) implies

$$\psi = \frac{nw[\gamma^* - \sigma(r/\pi)] - \sigma jx(r_z - r/\pi)}{jx(1-\sigma)}.$$

Solving (D46)–(D48) for ω, ν, and μ yields

$$\omega = \frac{1-\sigma}{1-\sigma(\beta/\gamma^*)[r/\pi + \phi(r_z - r/\pi)]}, \quad \nu = \frac{\beta}{\gamma^*}\omega\frac{r}{\pi}, \quad \mu = \frac{\beta}{\gamma^*}\omega\left(r_z - \frac{r}{\pi}\right),$$

and then (D45) implies

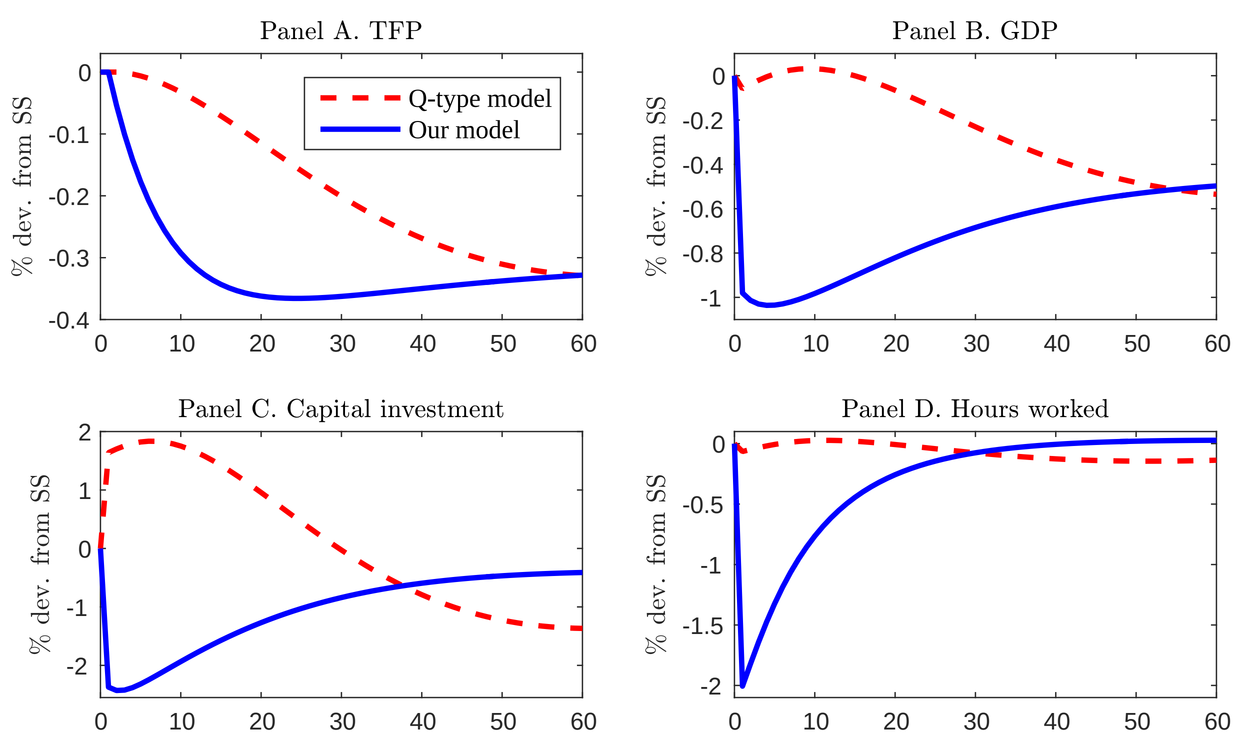
$$\theta = \frac{\nu + \phi\mu}{\phi}.$$

D.2 Comparison between the Q-type model and ours

Parameterization of the quarterly Q-type model.—For the common parameters, the same values as in our model are used. Regarding the parameters pertaining to the financial friction à la Gertler and Kiyotaki (2011), we follow Queraltó (2013) to choose $\sigma = 0.98$ and to set ϕ and θ so as to hit target values for the steady-state leverage ratio ϕ of 4 and the steady-state interest rate spread $(r_z - r/\pi)$ of 1 percent annually. For the persistence of the financial shock, we choose $\rho_\theta = 0.9522$—which is the same value for the persistence ρ_ξ of the financial shock in our model—to make impulse responses to the financial shock comparable to those in our model. As for the standard deviation of the financial shock, we set $\sigma_\theta = 0.29$ so that the permanent decline in TFP induced by the financial shock coincides with that in our model, where the standard deviation of the financial shock is $\sigma_\xi = 0.02$.

Co-movement and Adjustment Speed.—Figure A2 plots impulse responses to an adverse financial shock with the size of a one standard deviation in the Q-type model and ours. By construction, the magnitude of permanent declines in TFP induced by the financial shocks coincides between the two models. The figure reveals two findings.

Figure A2: Impulse responses to an adverse financial shock in the Q-type model and ours



First, the Q-type model fails to generate the co-movement between GDP, capital investment, and hours worked. In response to the adverse financial shock, capital investment increases sharply, while GDP and hours worked are more or less flat in the initial 10 quarters. This contrasts sharply with that in our model, where the three variables co-move.

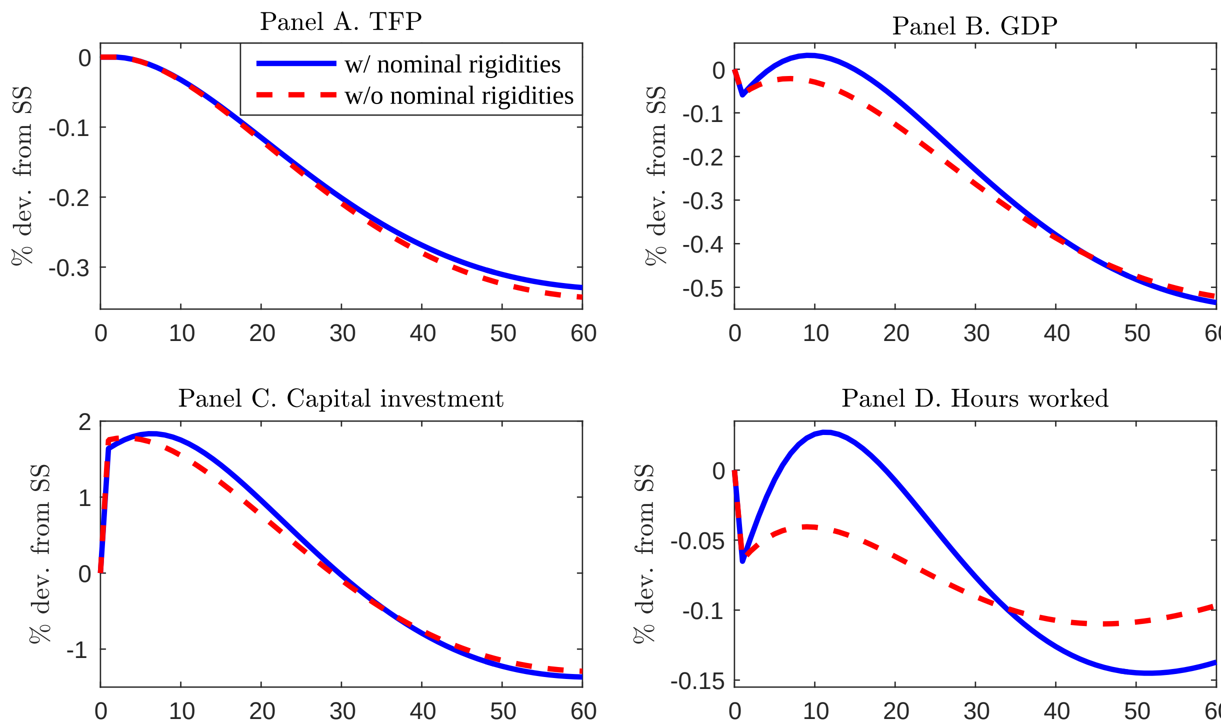
Why does capital investment increase in the Q-type model but decrease in our model in response to the adverse financial shocks? In our model, which embeds the financial friction of Jermann and Quadrini (2012) in the production sector (i.e., intermediate-good firms), the financial shock $\epsilon_{\xi,t}$ affects the firms' borrowing capacity for the expenses of capital investment and hiring. Therefore, a decline in ξ_t directly reduces capital investment and hiring (i.e., hours worked), as shown in panels C and D of Figure A2. Moreover, the decline of real economic activity in the production sector lowers the value of adopted technologies V_t, which in turn decreases technology adoption investment $I_{a,t}$ and the value of developed but not yet adopted technologies J_t, thus leading to a decline in R&D investment $I_{d,t}$. This suggests that, in our model, the financial shock impacts the demand side of the economy—the production sector—and ultimately affects the supply side—the sectors of R&D and technology adoption.

However, in the Q-type model, which embeds the financial frictions à la Gertler and Kiyotaki (2011) in the technology adoption sector, the adverse financial shock $\epsilon_{\theta,t}$—an increase in θ_t—affects that sector without directly affecting the production sector. Such a shock lowers the leverage ratio ϕ_t in equation (D17) and decreases the value of developed but not yet adopted technologies J_t through the bank balance sheet (D16). This in turn decreases R&D investment $I_{d,t}$ through equation (D28). Meanwhile, in the production sector, capital investment becomes relatively attractive than R&D investment and as a consequence, it increases, as displayed in panel C of Figure A2.

The second finding detected in Figure A2 is that TFP in the Q-type model responds much slower than in our model, as can be seen in panel A of the figure. While it takes 20 quarters to reach the bottom in our model, it takes 60 quarters to do so in the Q-type model. This difference reflects the fact that the technology adoption rate is exogenous in the Q-type model: the average duration of technology adoption is fixed at 27 ($\approx 1/(0.15/4)$) quarters. Yet we argue that if technology adoption were endogenized as in our model, the Q-type model augmented with endogenous adoption would feature a weaker amplification mechanism of the financial shock—a weaker response of TFP to the financial shock—than that without it. In the Q-type model, in response to the adverse financial shock, the value of developed but not yet adopted technologies J_t declines, while the value of adopted technologies V_t rises partly due to an increase in capital. As a consequence, $V_t - J_t$ increases. With endogenous technology adoption as in our model, the increase in $V_t - J_t$ would induce a rise in technology adoption investment, which mitigates the negative impact of the financial shock on TFP. Therefore, as long as the financial friction is introduced in the technology adoption sector, endogenous technology adoption would weaken the effect of financial shocks.

Role of Nominal Rigidities—In the Q-type model, nominal rigidities do not play a role as an amplifier of the financial shock. As shown in Figure A3, the impulse responses to the adverse financial shock are quite similar between the models with and without nominal rigidities. This result stems from the fact that in the model the financial shock has no direct effect on the production sector but affects the R&D sector through a change in J_t, which

Figure A3: Role of nominal rigidities in the Q-type model

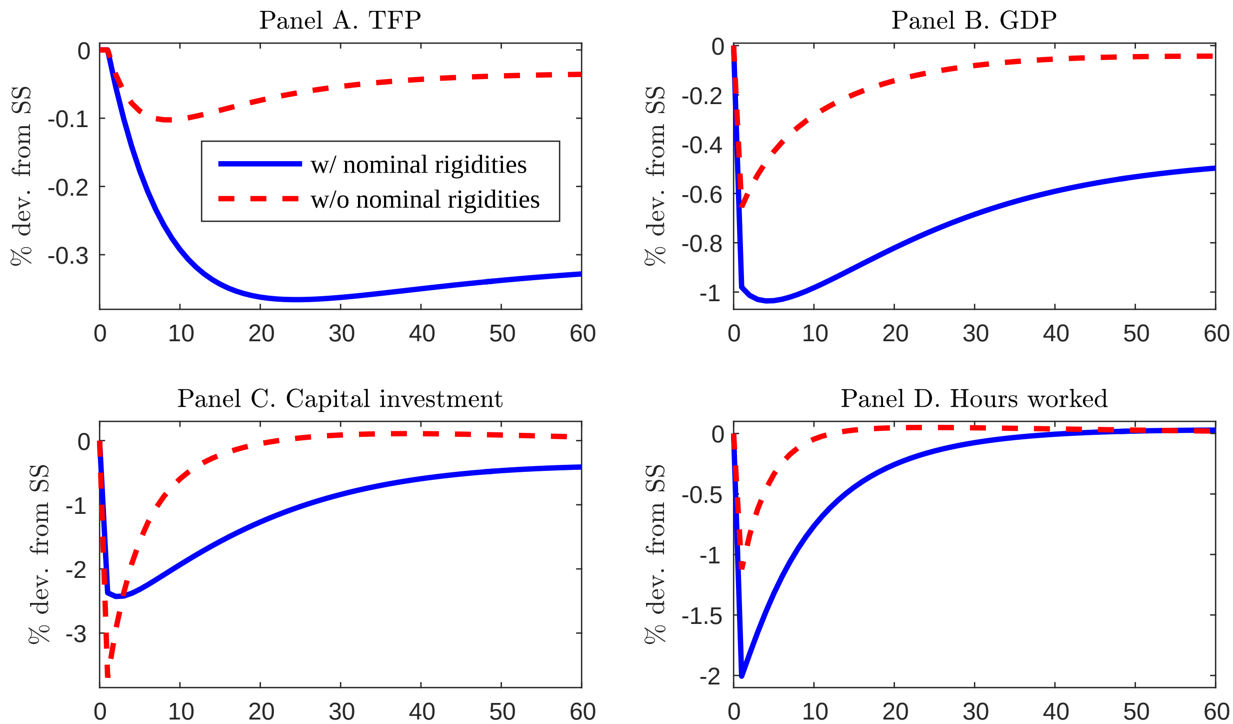


implies that the financial shock is similar to a shock to TFP. It is well known that nominal rigidities do not play a significant role in amplifying TFP shocks.

Unlike in the Q-type model, nominal rigidities greatly amplify the effect of the financial shock in our model. The financial shock has a direct impact on the production sector, which is amplified in the presence of nominal rigidities. The decline of real economic activity in the production sector causes the value of adopted ideas V_t to lower, which decreases $V_t - J_t$. At the same time the decline dampens technology adoption investment and slows down TFP, while a decline in the value of developed but not yet adopted ideas J_t decreases R&D investment and hence potential ideas to be adopted. As shown in Figure A4, the impulse responses of TFP, GDP, capital investment, and hours worked are amplified and more persistent in our model than in that without nominal rigidities. This result supports our argument that, in our model, the financial shock affects the supply side—the R&D sector and TFP—through its effect on the demand side—the production sector, which makes decisions on capital investment and hiring. This is why nominal rigidities greatly amplify the impact of the financial shock in our model.

The arguments above have shown that financial shocks in the Q-type model have much less persistent effects on GDP than those in our model. Besides, in contrast with our model, nominal rigidities do not amplify the effect of financial shocks in the Q-type model. We argue that these results stem from the fact that the Q-type model features the financial friction in the technology adoption sector, as opposed to the production sector in our model.

Figure A4: Role of nominal rigidities in our model



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