Non-Renewable Resources, Extraction Technology and Endogenous Growth

Gregor Schwerhoff and Martin Stuermer

Working Paper 1506

Research Department

https://doi.org/10.24149/wp1506r1

Non-Renewable Resources, Extraction Technology and Endogenous Growth[*]

Gregor Schwerhoff[†] and Martin Stuermer[‡]

December 29, 2015
Revised: August 2019

Abstract

We document that global resource extraction has strongly increased with economic growth, while prices have exhibited stable trends for almost all major non-renewable resources from 1700 to 2018. Why have resources not become scarcer as suggested by standard economic theory? We develop a theory of extraction technology, geology and growth grounded in stylized facts. Rising resource demand incentivises firms to invest in new technology to increase their economically extractable reserves. Prices remain constant because increasing returns from the geological distribution of resources offset diminishing returns in innovation. As a result, the aggregate growth rate depends partly on the geological distribution of resources. For example, a greater average concentration of a resource in the Earth's crust leads to more resource extraction, a lower price and a higher growth rate on the balanced growth path. Our paper provides economic and geologic microfoundations explaining why flat resource prices and increasing production are reasonable assumptions in economic models of climate change.

Keywords: Non-renewable resources, endogenous growth, extraction technology

JEL Codes: O30, O41, Q30, Q43, Q54

[*] The views in this paper are those of the authors and do not reflect the views of the Federal Reserve Bank of Dallas, the Federal Reserve System or the World Bank. We thank Anton Cheremukhin, Thomas Covert, Klaus Desmet, Maik Heinemann, Martin Hellwig, David Hemous, Charles Jones, Dirk Krüger, Lars Kunze, Florian Neukirchen, Pietro Peretto, Gert Pönitzsch, Salim Rashid, Gordon Rausser, Paul Romer, Luc Rouge, Sandro Schmidt, Sjak Smulders, Michael Sposi, Jürgen von Hagen, Kei-Mu Yi, and Friedrich-Wilhelm Wellmer for very helpful comments and suggestions. We also thank participants at the Economic Growth Small Group Meeting at the NBER Summer Institute, AEA Annual Meeting, AERE Summer Meeting, EAERE Summer Conference, SURED Conference, AWEEE Workshop, SEEK Conference, USAEE Annual Meeting, SEA Annual Meeting, University of Chicago, UT Austin, University of Cologne, University of Bonn, MPI Bonn, European Central Bank, and the Federal Reserve Bank of Dallas for their comments. We thank Mike Weiss for editing. Navi Dhaliwal, Achim Goheer, Ines Gorywoda, Sean Howard and Emma Marshall provided excellent research assistance. All errors are our own. An earlier version was published as a Dallas Fed Working Paper in 2015 and as a Max Planck Institute for Collective Goods Working Paper in 2012 with the title “Non-renewable but inexhaustible: Resources in an endogenous growth model."

[†] Gregor Schwerhoff, Mercator Research Institute on Global Commons and Climate Change, gschwerhoff@worldbank.org.

[‡] Martin Stuermer, Federal Reserve Bank of Dallas, Research Department, martin.stuermer@dal.frb.org.

1 Introduction

Economic intuition suggests that non-renewable resources like metals or fossil fuels become scarcer and more expensive over time. However, our new data set for 65 resources from 1700 to 2018 disagrees. Not only has the production of non-renewables increased, but most of their prices have exhibited non-increasing trends. This paper proposes an explanation: innovation in extraction technology exploits a geological law where greater quantities of a resource are found in progressively lower grade deposits. The result is increasing resource production at non-increasing prices to meet growing global demand. Furthermore, it is this interaction between technology and geology that co-determines the rate of long-run aggregate growth.

We document three stylized facts that support the mechanism of our model. First, the Fundamental Law of Geochemistry (Ahrens, 1953) states that resources are log-normally distributed in the Earth's crust. This means greater quantities of a resource are locked in lower grade deposits. Second, non-renewable resources are very abundant in the Earth's crust. However, only a small fraction called reserves is economically recoverable with current extraction technology. Third, firms can increase reserves by investing in new technology but there are diminishing returns in terms of accessing lower grade deposits.

We integrate a more realistic extraction sector into a standard lab-equipment model of endogenous growth (Romer (1987, 1990) and Acemoglu (2002)).[1] Extraction firms observe aggregate resource demand and invest in new extraction technology. This allows them to increase their reserves and to extract the resource from lower grade deposits. They purchase

[1]Besides the extractive sector, the model features a standard intermediate goods sector with goods and sector-specific technology firms. The final good is produced from the intermediate good and the non-renewable resource.

the technology from technology firms.

Technology firms invent new extraction technology because it is rivalrous. Each technology is specific to deposits of particular grades. Most similar to this understanding of innovation is Desmet and Rossi-Hansberg (2014), where non-replicable factors of production like land provide the incentive for innovation despite perfectly competitive markets. Although it becomes progressively harder to develop technologies for lower grade deposits, their resource quantities increase exponentially. Thus the geological distribution of the resource offsets the diminishing returns from technological development. The cost of technology per unit of the resource and its price are constant over the long term.

On the balanced growth path, aggregate output and extraction grow at a constant rate, whereas the resource price is constant. The three variables depend partly on the resource distribution in the Earth's crust. For example, a higher average geological concentration of the resource leads to a higher rate of resource extraction, a lower price level and a higher aggregate growth rate in equilibrium holding other factors constant. The rivalrous nature of extraction technology implies that the extraction sector only exhibits constant returns to scale and is not an engine of growth.

The interaction between resource distribution and extraction technology determines the long-run rate of aggregate output growth along with the usual factors. This contrast with conventional models that include a drag on growth driven by depletion and where this depletion effect can be partially offset by the development of resource-saving technology or substitution (see Nordhaus et al., 1992; Weitzman, 1999; Jones and Vollrath, 2002).

Based on geological and economic micro-foundations our model shows that constant resource prices and increasing extraction are reasonable long assumptions over the long term.

This is relevant to a growing literature studying the effects of fossil fuels on climate change because it suggests that the transition towards clean energy will be more costly (Acemoglu et al., 2012a; Golosov et al., 2014; Hassler and Sinn, 2012; van der Ploeg and Withagen, 2012; Acemoglu et al., 2019). Our model also suggests that demand side policies to curb fossil fuel consumption are effective, because they would slow down innovation in extraction technology. This is in contrast to the so called "Green Paradox" , where an exhaustible stock of fossil fuels incentivices firms to bring forward extraction when faced with demand side policies (see Sinn, 2008; Eichner and Pethig, 2011; Van der Ploeg and Withagen, 2012). At the same time, the availability of critical metals needed for the energy transition may not face constraints. Continued increases in resource consumption might also also not raise the risk of conflicts over resources (see Acemoglu et al., 2012b).

This paper challenges a literature that predicts greater resource scarcity with economic development (see e.g. Stiglitz, 1974; Dasgupta and Heal, 1974; Solow and Wan, 1976; Nordhaus et al., 1992; Aghion and Howitt, 1998; Jones and Vollrath, 2002; Groth, 2007). These models rely on Hotelling's (1931) characterization of optimal depletion: resource extraction declines at a constant rate, while prices rise at the rate of interest. As a result, depletion negatively affects output growth but can potentially be offset by substitution and technological change in resource efficiency. However, the literature also agrees that non-renewable resources have neither become scarcer nor more expensive over time (see Nordhaus et al., 1992; Krautkraemer, 1998; Livernois, 2009). This mismatch between theory and empirical findings presents an open question (see Jones and Vollrath, 2002; Hassler et al., 2016).

Our paper contributes the interaction between geology and endogenous innovation in extraction technology to the literature. We build on a small literature studying innovation in extraction. In Rausser (1974) the non-renewable resource stock can increase due to learning-by-doing, which allows for constant extraction and prices in a partial equilibrium model. Heal (1976) argues that prices and extraction stay constant after a more costly but inexhaustible "backstop technology" is reached. Cynthia-Lin and Wagner (2007) predict increasing resource output and constant prices after adding exogenous technological change and heterogeneous extraction costs to a model with an infinite resource. Tahvonen and Salo (2001) study the transition from a non-renewable to a renewable energy resource with heterogeneous extraction costs based on a growth model with learning-by-doing. Their model implies an inverted U-shaped extraction and a U-shaped resource price path. Acemoglu et al. (2019) study the role of fracking in the transition towards clean energy. In their setup, exogenous technological change augments a constant flow of natural gas leading to a constant price.

The remainder of the paper is organized as follows. In section 2, we present empirical evidence about the long-run trends of global resource extraction and prices based on a new data-set. In section 3, we document stylized facts on geology and extraction technology. Section 4 describes the main mechanism of our model, namely the interaction between geology and technology. Section 5 outlines the micro-economic foundations of the extractive sector and its innovation process. Section 6 presents the growth model, and section 7 derives theoretical results, which are discussed in section 8. Section 9 concludes and discusses policy implications.

2 Long-Run Trends in Non-Renewable Resource Extraction and Prices

We first present a new data-set of inflation adjusted resources prices and global production from 1700 to 2018 for all major non-renewable resources.[2]

2.1 Resource Extraction Has Strongly Increased

The extraction and consumption[3] of non-renewable resources strongly increased over the past three hundred years. Figure 1 shows that global extraction rose from about 3.3 million metric tons in 1700 to 21 billion metric tons in 2018. This is an increase by a factor of more than 6000. About two thirds of the non-renewable resource production is driven by fossil fuels, including crude oil, coal and natural gas, and the other third by metals and non-metals. Global real GDP increased at a factor of about 190 over the same period, while real GDP on a per capita basis multiplied by 15.

In per capita terms global resource extraction increased from roughly 5 to 3,000 kilograms. A closer statistical examination confirms that the mine production of most non-renewable resources exhibits significantly positive growth rates in the long term (see table 2 in the appendix).[4]

[2] See Appendix 1 for data descriptions and sources.

[3] Over the long term, extraction and consumption of resources are about equal, as stockholdings vary over the business-cycle and are generally relatively small compared to consumption. In some cases, where recycling is important, consumption could be higher. Our data is therefore a lower bound estimate for metals and non-metals consumption.

[4] These results also hold by-and-large for per capita production of the respective commodities over the long run. Regressions results are available from the authors upon request.

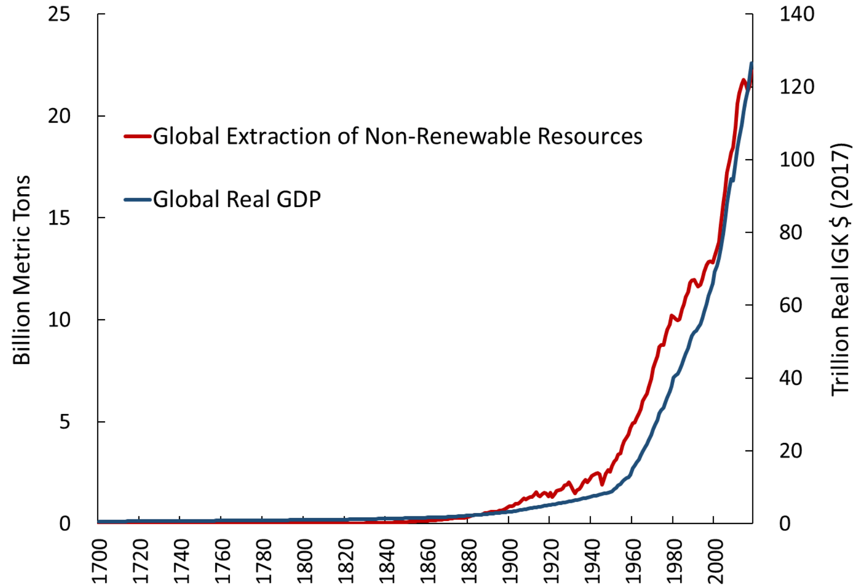

Figure 1: World Extraction of 65 Non-Renewable Resources and World Real GDP, 1700-2018. The total quantity of extracted non-renewable resources increased roughly in line with world real GDP.

2.2 Non-Renewable Resource Prices Exhibit Non-Increasing Trends

Non-renewable resource prices exhibit strong fluctuations but follow mostly non-increasing or even declining trends over the long term. Figure 2 presents an equally weighted and inflation adjusted price index for 65 non-renewable resources, which shows a stable trend over the long term. However, there is a significant uptick in crude oil and natural gas prices since the 1970s, probably due to a structural break related to the changing roles of the Texas Railroad Commission and oligopolistic behavior by OPEC (see Dvir and Rogoff, 2010).

We test the null hypothesis that growth rates of real prices are not significantly different from zero. As the regression results in Table 3 in the appendix show, this null hypothesis cannot be rejected. Real prices are mostly trend-less. Our evidence is in line with the literature, see e.g. Krautkraemer (1998), Von Hagen (1989), Cynthia-Lin and Wagner (2007), Stuermer (2018). The literature is certainly not definitive on price trends (see Pindyck, 1999; Lee et al., 2006; Slade, 1982; Jacks, 2013; Harvey et al., 2010), but we conclude that prices do generally not show increasing trends over the long term.

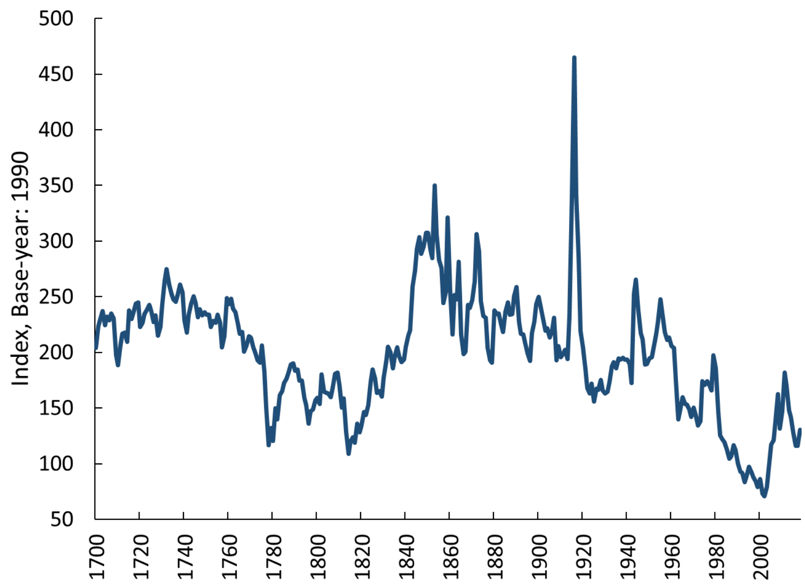

Figure 2: Inflation Adjusted Price Index for 65 Non-Renewable Resources (equally weighted), 1700-2018.

3 Stylized Facts

We lay out stylized facts about geology and extraction technology, which inform the main mechanism of our model.

3.1 Non-Renewable Resources are Abundant in the Earth's Crust

To better understand the interaction between geology and technological change, we first take a closer look at the abundance and distribution of non-renewable resources in the Earth's crust.

We update and extend a data-set by Nordhaus (1974) on the abundance (or estimated total quantity) of mineral non-renewable resources in the Earth's crust. Table 1, second column, shows that the crustal abundance of major non-renewable resources is substantial.[5] The fourth column shows annual mine production, which is several orders of magnitude smaller than the quantities in the Earth's crust. If production stayed constant, resources are basically infinite as current extraction could continue for millions or billions of years depending on the resource (see table 4 in the appendix).

A more realistic assumption is that extraction continues to grow exponentially at current rates. In this case, production could still be sustained for a couple of hundred to a thousand years if there is continued technological progress, as column 4 in table 1 illustrates. This is still close enough to infinity for all practical economic purposes. Note also that the Earth's crust makes up less than one percent of the Earth's mass. There are hence more non-renewable resources in other layers of the Earth.

Hydrocarbons are quite abundant in the Earth's crust. Even though reserves of conventional oil resources – the highest grade fossil fuel – may be exhausted someday, deposits of unconventional oil, natural gas, and coal, which could substitute for conventional oil in the long run, are plentiful in the Earth's crust. These results are in line with numerous stud-

[5]

ies that conclude that fossil fuels will last far longer than many expect (see Aguilera et al. (2012), Rogner (1997) and Covert et al. (2016)).

	Crustal Abundance (Bil. mt)	**Reserves** (Bil. mt)	**Annual Output** (Bil. mt)	**Crustal Abundance/ Annual Output** (Years)	**Reserves/ Annual Output** (Years)
Aluminum	1,990,000,000[e]	30[b1]	0.06[a]	491	42[1]
Copper	1,510,000[e]	0.8[b]	0.02[b]	483	26
Iron	1,392,000,000[e]	83[b2]	1.2[a]	580	39[2]
Lead	290,000[e]	0.1[b]	0.005[b]	1,099	16
Tin	40,000[e]	0.005[b]	0.0003[b]	1,405	14
Zinc	2,250,000[e]	0.23[b]	0.013[b]	668	14
Gold	70[e]	0.00005[b]	0.000003[b]	925	15
Coal[3]		511[d]	3.9[d]		63[c]
Crude Oil[4]	15,000,000[f]	241[d]	4.4[d]	558	41[c]
Nat. Gas[5]		179[d]	3.3[d]		34[c]

Notes: We have used the following average annual growth rates of production from 1990 to 2010: Aluminum: 2.5%, Iron: 2.3%, Copper: 2%, Lead: 0.7%, Tin: 0.4%, Zinc: 1.6%, Gold: 0.6%, Crude oil: 0.7%, Natural gas: 1.7%, Coal: 1.9%, Hydrocarbons: 1.4%. [1]Data for bauxite, [2]data for iron ore, [3]includes lignite and hard coal, [4]includes conventional and unconventional oil, [5]includes conventional and unconventional gas, [6]all organic carbon in the earth's crust. Sources: [a]U.S. Geological Survey (2016), [b]U.S. Geological Survey (2018), [c]British Petroleum (2017), [d]Federal Institute for Geosciences and Natural Resources (2017), [e]Perman et al. (2003), [f]Littke and Welte (1992).

Table 1: Availability of selected non-renewable resources in years of production left in the reserve and crustal mass assuming an exponentially increasing annual mine production (based on the average growth rate over the last 20 years).

Of course, extraction of most of these resource quantities in the Earth's crust is impossible or extremely costly with current technology. Only a small fraction is proven to be economically extractable with current technology. This fraction is defined as reserves (see U.S. Geological Survey (2018)). The term "economic" implies that firms established profitable extraction under defined investment assumptions with reasonable certainty. Table 1, column three, shows that reserves are relatively small compared to their crustal abundance. They amount to only a couple of decades of current extraction (see column six).

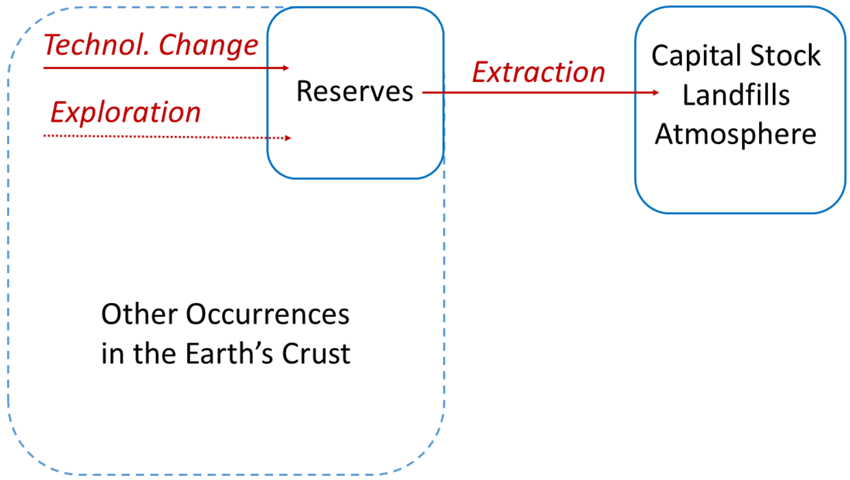

Figure 3: Non-Renewable Resource Flows.

Note: This is a stylized version of the official Resource/Reserve Classification System for Minerals as used by the U.S. Geological Survey (see U.S. Geological Survey (2018)).

The boundary between reserves and other occurrences in the earth's crust is dynamic due to technological change and exploration. Figure 3 shows how resources are classified as either reserves or other occurrences in the Earth's crust.[6] As reserves deplete through extraction, firms explore new deposits and develop new technology to convert other occurrences into reserves. This allows firms to continue extraction. The extracted resource becomes either part of the capital stock, discharges after utilization into landfills or the atmosphere.

3.2 Non-Renewable Resources are Log-Normally Distributed in the Earth's Crust

Non-renewable resources are not uniformly concentrated in the earth's crust. Variations in the geochemical processes have shaped the characteristics of non-renewable resource occur-

[6]Please note that we have left out a major category, the reserve base, to ease understanding. The reserve base encompasses those parts of the resource in the earth's crust that have a reasonable potential for becoming economically available within planning horizons beyond those that assume proven technology and current economics (see U.S. Geological Survey (2018))

rences in the Earth's crust over billions of years. Deposits differ in their geological characteristics along many dimensions, for example, ore grades, thickness and depths. We focus on ore grade, as this is the most important characteristic. Some deposits are highly concentrated with a specific resource (high grade, close to 100 percent ore grade), and other deposits are less so (low grade, close to 0 percent ore grade). The grade distinguishes the difficulty of extraction, where a low grade is very difficult.

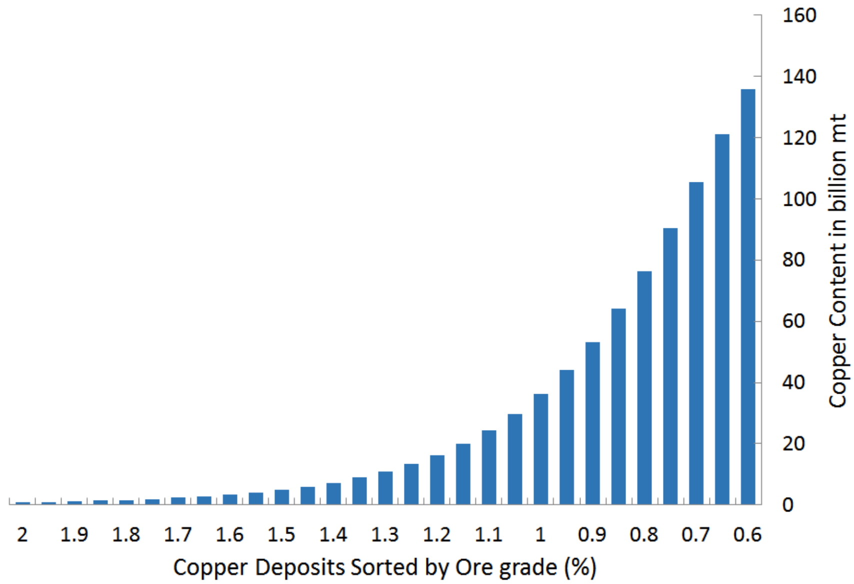

Figure 4: Grade-quantity distribution of copper in the Earth's crust. The total copper content increases, as the ore grades of copper deposits decline. The x-axis has been reversed for illustrative purposes. Source: Gerst (2008).

The fundamental law of geochemistry (Ahrens (1953, 1954)) states that each chemical element exhibits a log-normal grade-quantity distribution in the Earth's crust, postulating a decided positive skewness. Hence, the total resource quantity in low grade deposits is large, while the total resource quantity in high grade deposits is relatively small. The reason for this is that low grade deposits have a far larger volume of rock than high grade deposits. For example, figure 4 shows that the total copper content increases, as the ore grades of copper deposits in the Earth's crust decline.

While a log-normal distribution for the distribution of certain resources is the text-book standard assumption in geochemistry, this literature continues to develop, especially regarding very low concentrations of metals, which might be mined in the distant future. For example, Skinner (1979) and Gordon et al. (2007) propose a discontinuity in the distribution due to the so-called "mineralogical barrier," the approximate point below which metal atoms are trapped by atomic substitution.

Gerst (2008) concludes in his geological study of copper deposits that he can neither confirm nor refute this hypotheses. However, based on worldwide data on copper deposits over the past 200 years, he finds evidence for a log-normal relationship between copper production and ore grades. Mudd (2007) analyzes the historical evolution of extraction and grades of deposits for different base metals in Australia. He also finds that production has increased at a constant rate, while grades have consistently declined.

We conclude that there remains uncertainty about the geological distribution, especially regarding hydrocarbons with their distinct formation processes. However, it is reasonable to assume that non-renewable resources are distributed according to a log-normal relationship between the grade of its deposits and its quantity based on geochemical theory and evidence.

3.3 Diminishing Returns to Innovation in Extraction Technology

Empirical evidence suggests that technological change affects the extractable ore grade with diminishing returns (see Lasserre and Ouellette, 1991; Mudd, 2007; Simpson, 1999; Wellmer, 2008). For example, Radetzki (2009) and Bartos (2002) describe how technological changes in mining equipment, prospecting and metallurgy have gradually enabled the extraction of copper from lower grade deposits. The average ore grades of copper mines have decreased from about twenty percent 5,000 years ago to currently below one percent (Radetzki, 2009). Figure 5 illustrates this development using the example of global copper mines from 1800 to 2000. Mudd (2007) and Scholz and Wellmer (2012) come to similar results for different base metal mines in Australia and for copper mines in the U.S, respectively.

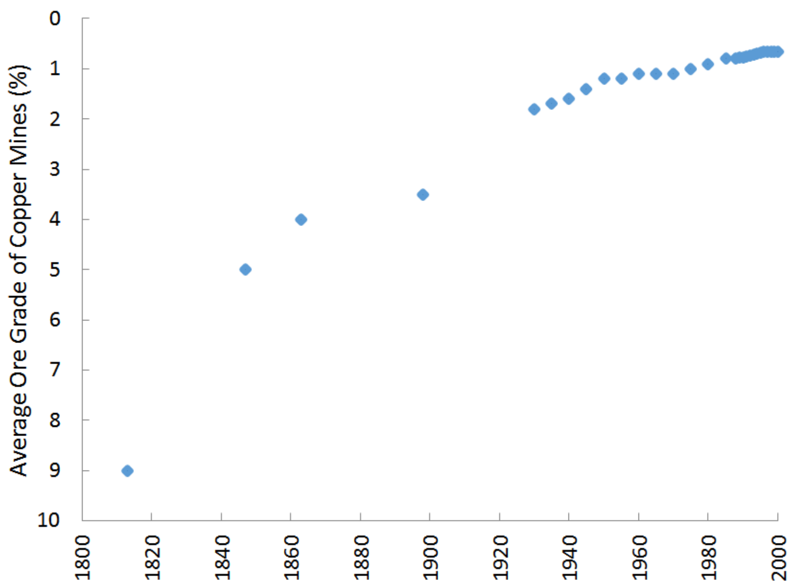

Figure 5: The historical development of average ore grades of copper mines in the world suggests diminishing returns of technological change on extractable ore grades. The y-axis has been reversed for illustrative purposes. Source: Gerst (2008)

However, Figure 5 also shows that decreases in grades have slowed as technological development progressed. Under the reasonable assumption that global real R&D spending in extraction technology and its impact on technological change has stayed constant or increased over the long term, there are decreasing returns to R&D in terms of making mining from deposits of lower grades economically feasible.

We observe similar developments for hydrocarbons. Using the example of the offshore oil industry, Managi et al. (2004) finds that technological change has offset the cost-increasing degradation of resources. Crude oil has been extracted from ever deeper sources in the Gulf of Mexico. Furthermore, technological change and high prices have made it profitable to extract hydrocarbons from unconventional sources, such as tight oil or oil sands (International Energy Agency, 2012).

Overall, we conclude that the long-run data suggests that there are no constant returns from technological change in resource extraction in terms of ore grades. Historical evidence rather suggests diminishing returns to technological development.

4 The Interaction Between Geology and Technology

The stylized facts highlighted the importance for understanding the interaction between geology and technology in the extractive sector. In the following, we describe the key assumptions which we make based on these stylized facts. We point out that there are offsetting effects between geology and technology, which can lead to constant returns from technological development in terms of new reserves.

4.1 Geological Function

We approximate the log-normal distribution of non-renewable resources in the Earth's crust by an increasing relationship between grade and quantity. The geological function (see also

Figure 6) takes the form:

$$R(O) = \frac{\delta}{O}, \quad \delta \in \mathbb{R}_+ \ , \ O^\star \in (0,1) \ . \tag{1}$$

We define the grade O of a deposit as the average concentration of the resource. Parameter δ controls the curvature of the function. If δ is high, the total quantity of the non-renewable resources is large. For example, iron is relatively abundant with an average concentration of 5% in the Earth's crust. A low δ indicates a relatively small quantity of the non-renewable resource in the crustal mass. One example is gold with an average concentration of 0.001%.

The functional form implies that the resource quantity goes to infinity as the grade approaches zero. Although we recognize that non-renewable resources are ultimately finite in supply, we follow Nordhaus (1974) in his assessment that non-renewable resources are so abundant in the earth's crust that "the future will not be limited by sheer availability of important materials" given technological change. Our assumption compares to households maximizing over an infinite horizon.

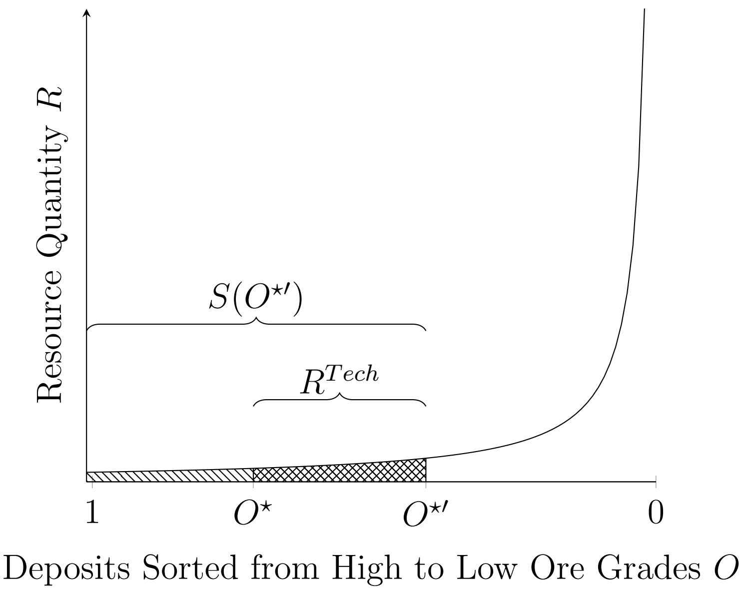

Figure 6: Geological function: Deposits of lower grade O entail a higher resource quantity R. The x-axis has been reserved for illustrative purposes and goes from high grades to low grades. A new technology shifts the extractable grade from $O^{\star}$ to $O^{\star\prime}$. The resulting flow of new reserves is R^{Tech} and indicated by the dark shaded area. The accumulated reserves from the development of all technologies is $S(O^{\star\prime})$ (see light and dark shaded area).

Technological development makes extraction from lower grades possible and converts deposits into reserves. For example, a new technology shifts the extractable deposits from grade $O^{\star}$ down to grade $O^{\star\prime}$. The cut-off grade $O^{\star}$ indicates the lowest grade that firms can extract with the new technology level. This technological change adds resources to the reserves that are equal to: $R^{Tech} = \int_{O^{\star\prime}}^{O^{\star}} R(O^{\star})dO^{\star}, \ \ \delta \in \mathbb{R}_{+} \ , \ O^{\star} \in (0,1)$. The total amount of resources converted to reserves due to technological change over the entire time horizon $[O^{\star\prime}, 1)$ is:

$$S(O^{\star\prime}) = \int_{O^{\star\prime}}^{1} \frac{\delta}{O^{\star}} dO = -\delta ln(O^{\star\prime}), \ \ \delta \in \mathbb{R}_{+} \ , \ O^{\star} \in (0,1) \tag{2}$$

4.2 Diminishing Returns to Technology

We accommodate the diminishing returns of technological change by an extraction technology function, which maps the state of the technology N_R onto the extractable grade $O^\star$ of the deposits (see figure 7):

$$O^\star(N_R) = e^{-\mu N_R},\ \mu \in \mathbb{R}_+ \ \ N_R \in (0, \infty)\ . \tag{3}$$

The grade $O^\star$ is the lowest grade that firms can extract with technology level N_R. Technological change, N_R, expands the range of grades that can be extracted. The extractable grade is a decreasing convex function of technology implying decreasing marginal returns. The curve in Figure 7 starts with deposits of close to a 100 percent ore grade, which represents the state of the world several thousand years ago. For example, humans picked up copper in pure nugget form in Cyprus and beat it to the desired form, given its malleability (see Radetzki, 2009). However, the quantity of copper that is in these high grade deposits is relatively small. With technological change lower grade deposits became available, e.g. today copper is mined from ore that contains below one percent of copper. The quantities of copper contained in these deposits is much larger than in the high grade deposits.

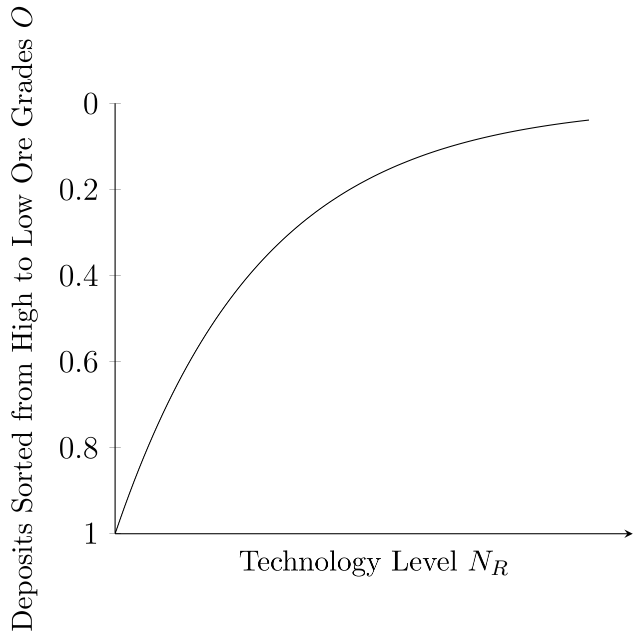

Figure 7: The extraction technology function assumes diminishing returns to technological development in terms of grades. The y-axis has been reserved for illustrative purposes.

The curvature parameter of the extraction technology function is μ. If, for example, μ is high, the average effect of new technology on converting deposits to reserves in terms of grades is relatively high.

4.3 Marginal Effect of Extraction Technology on Reserves

We show that the interaction of the geological and technology function produces a linear relationship between technological development and reserves. Figure 8, Panel A, depicts the technology function. Two equal steps in advancing technology from 0 to N and from N to N', lead to diminishing returns in terms of extractable ore grades $O^\star$ and $O^{\star\prime}$, where $O^{\star\prime} - O^\star \prec O^\star$.

Panel B shows equation 2, which is the integral of the geological function. The figure presents how the different advances in extractable ore grades $O^\star$ and $O^{\star\prime}$ map into equal increases in the accumulated reserve levels S and S', where $S' - S = S$.

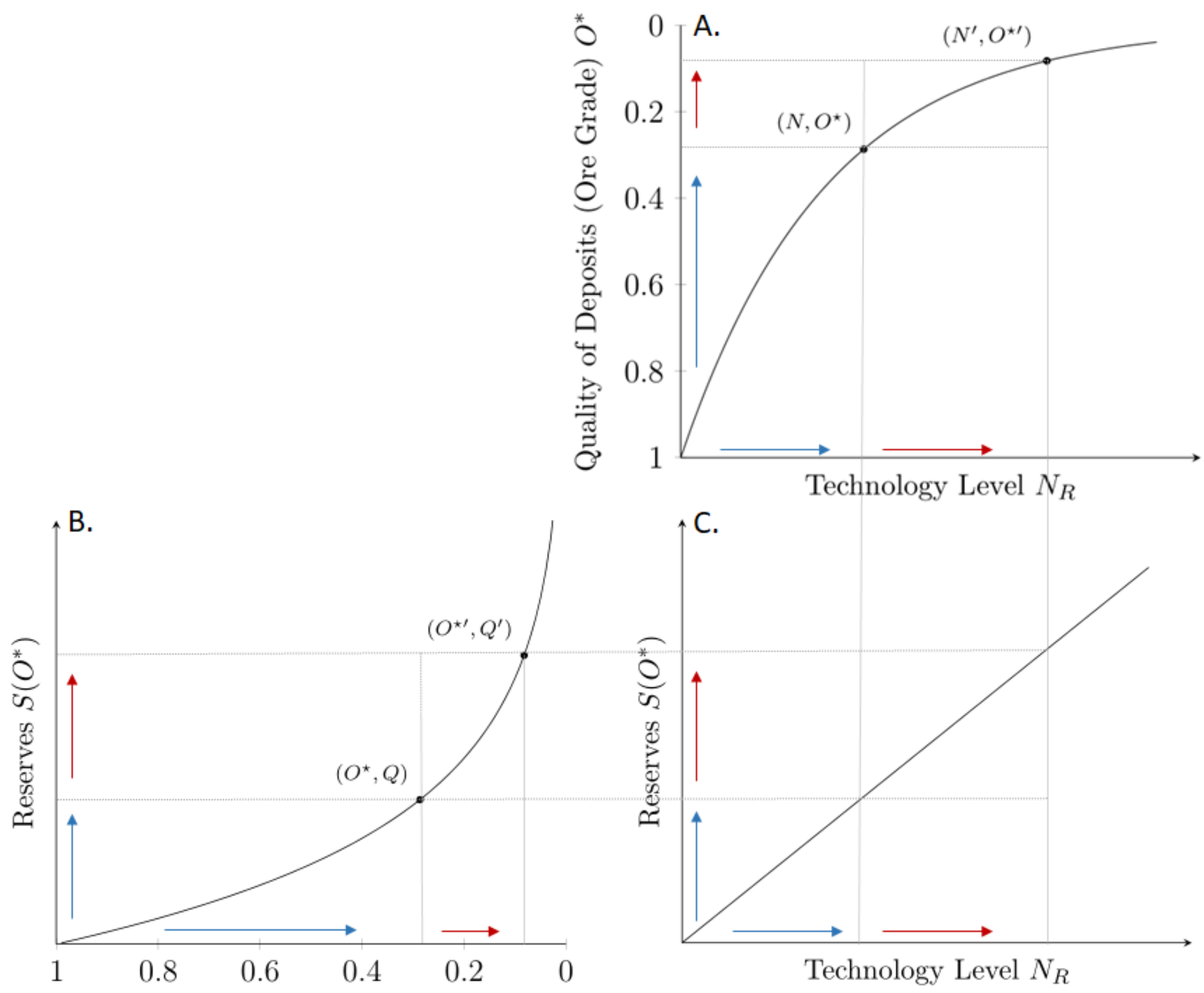

Figure 8: The interaction between the extraction technology function (Panel A) and the accumulated geological function (Panel B) leads to a linear relationship between technology level N_R and reserves S (Panel C). Note that the y-axis in panel A and the x-axis in panel B have been reversed for illustrative purposes.

Finally, Panel C summarizes how the extraction function and the accumulated geological function offset each other and lead to a linear relationship between the technology level and the reserve level.

Proposition 1 *Reserves S increase proportionally to the level of extraction technology N_R:*

$$S(O^{\star}(N_{Rt})) = \delta\mu N_{Rt} \ .$$

The marginal effect of new extraction technology on reserves equals:

$$\frac{dS(O^{\star}(N_{Rt}))}{dN_R} = \delta\mu \ .$$

The intuition is that two offsetting effects cause this result: (i) the resource is geologically distributed such that it implies increasing returns in terms of new reserves as the grade of deposits decline; (ii) new extraction technology exhibits decreasing returns in terms of making lower grade deposits extractable.

As the natural log in the accumulated geological function and the natural exponent in the technology function cancel out, there is a linear relationship between the state of technology N_R and the total quantity of the resource converted into reserves S.

Proof of Proposition 1

$$\begin{aligned} S(O^{\star}(N_{Rt})) &= -\delta \ln(O^{\star}(N_{Rt})) \\ &= -\delta \ln(e^{-\mu N_{Rt}}) \\ &= \mu\delta N_{Rt} \end{aligned}$$

□

The equations in Proposition 1 depend on the shapes of the geological function and the technology function. If the respective parameters δ and μ are high, the marginal return on new extraction technology will also be high.

The constant marginal effect of technology on new reserves is a first approximation and we allow for wide parameter spaces for the functional forms of the underlying functions. If the technology function did not assume decreasing returns in terms of lower ore grades but constant returns, this would result in an increasing marginal effect of technology on new reserves. We discuss other function forms in section 8.

5 The Extractive Sector

We now describe the micro-foundations of the extractive sector and firms' incentives to develop technologies. Our extractive sector includes two types of firms: extraction and technology firms. Extraction firms buy technology from technology firms and extract the resource from deposits of declining grades, while the latter innovate and produce extraction technology.[7] Both types of firms know fully about the geological distribution and the technology function.

5.1 Extraction Firms

We consider a large number of infinitely small extraction firms. They operate in a fully competitive sector where demand for the non-renewable resource, a homogenous good, is given.[8]

[7] To ease comparison, the extractive sector is constructed in analogy to the intermediate goods sector in Acemoglu (2002).

[8] We assume that the firm level production functions exhibit constant returns to scale, so there is no loss of generality in focusing on aggregate production functions. We assume a fully competitive sector, because we model long-run trends. Historically, producer efforts to raise prices were only successful in some non-oil commodity markets in the short run, as longer-run price elasticities proved to be high (see Radetzki, 2008; Herfindahl, 1959; Rausser and Stuermer, 2016). Similarly, a number of academic studies discard OPEC's

Firms can hold reserves S. Reserves are defined as non-renewable resources in underground deposits that can be extracted with grades-specific technology (or machine varieties) at a constant extraction cost $\phi > 0$. The marginal extraction cost for non-reserves is infinitely high, $\phi = \infty$. Firms' reserves $\dot{S}$ evolve according to:

$$\dot{S}_t = -R_t^{Extr} + R_t^{Tech}, \quad S_t \geq 0, R_t^{Tech} \geq 0, R_t^{Extr} \geq 0. \tag{4}$$

Firms can extract the resource from its reserves using grade-specific technology, a flow that we denote as R_t^{Extr}. Machines fully depreciate after use. However, firms can also expand the quantity of their reserves by investing in new grades-specific technology, a flow denoted as R_t^{Tech}.

Extraction firms can purchase the new technologies from sector-specific technology firms at price χ_R. A new grades-specific technology allows firms to claim ownership of all non-renewable resources in the related deposits. Firms declare these deposits their new reserves.

In our setup, reserves are a function of geology and extraction technology. They are comparable to working capital in the spirit of Nordhaus (1974), as they are inventories of resources in the ground that can be used as input to production. To put it differently, the non-renewable resource is not defined as a fixed, primary factor but as a production factor produced by technological change.

Due to the combination of constant returns to technological change in terms of new reserves (Proposition 1) and the assumption of grade-specific technology leads, Firms' new

ability to raise prices over the long term (see Aguilera and Radetzki, 2016, for an overview). This is in line with historical evidence that OPEC has never constrained members' capacity expansions, which would be a precondition for long-lasting price interventions (Aguilera and Radetzki, 2016) reserves are a function of technological change $\dot{N}$, the geological parameter δ and the technological parameter μ:[9]

$$R_t^{Tech} = \delta\mu\dot{N}_R. \tag{5}$$

This production function for reserves exhibits only constant returns to scale, which implies that the social value of an innovation is equal to the private value. As R&D lifts resource scarcity, future innovations are not reduced in profitability. No positive or negative spill-overs occur in our model.

In our setup, extraction firms are basically like car producers, facing a marginal cost curve and producing what is demanded at a given price. Firms only maximize current profits, which are a function of the revenue received from selling the resource, extraction cost and investment in new technologies to expand reserves:

$$\pi_R^E = p_R R^{Extr} - \phi R^{Extr} - \chi_R \delta\mu\dot{N}_R, \tag{6}$$

5.2 Technology Firms in the Extractive Sector

Sector-specific technology firms j invent patents for new varieties of grades-specific extraction technology (or machines). We assume that there is free entry of technology firms into research. Technology firms observe the demand for grades-specific machine varieties by the extraction firms. The innovation possibilities frontier, which determines the creation of new technologies takes the form:[10]

[9] Please see Appendix 1.3 for the derivation of this equation.

[10] We assume in line with Acemoglu (2002) that there is no aggregate uncertainty in the innovation process. There is idiosyncratic uncertainty, but with many different technology firms undertaking research, equation 7 holds deterministically at the aggregate level.

$$\dot{N}_R = \eta_R M_R \ . \tag{7}$$

Each technology firm can spend one unit of the final good for R&D investment M at time t to generate a flow rate $\eta_R > 0$ of new patents, respectively. The cost of inventing a new machine variety is $\frac{1}{\eta}$. Each firm can invent only one new machine variety at a time in line with Acemoglu (2002).

A firm that invents a new extraction machine receives a perpetual patent. The patent grants the firm the right to build the respective machine at a fixed marginal cost $\psi_R > 0$. However, the knowledge about building the machine diffuses to all technology firms and can be used to invent new machine varieties for lower ore grades. The economy starts at the initial technology level $N_R(0) > 0$.

Based on the patent, firms produce a machine, which makes a particular deposits of lower grades O extractable and can only be used for this specific geological formation. The use of a machine by one extraction firm prevents other extraction firms from using it because of this feature. Once these deposits are extracted, new machine varieties must be invented. Technology is hence rivalrous in the context of extracting non-renewable resources.[11]

As each machine variety is specific to deposits of certain grades, only one machine is build and sold per variety. As a consequence, each technology firm stays in the market for only one time period. The value of a technology firm that discovers a new machine depends on instantaneous profits:

[11] This is in contrast to the intermediate goods sector, where technology is non-rivalrous.

$$V_R(j) = \pi_R(j) = \chi_R(j)x_R(j) - \psi_R x_R(j) \ , \tag{8}$$

The present value of a patent is the difference between the machine price $\chi_R(j)$ and the cost to produce a machine ψ_R times the number of produced machines $x_R(j) = 1$.

This formulation allows us to boil down a dynamic optimization problem to a static one. It makes the model solvable and computable. At the same time, the model stays rich enough to derive meaningful theoretical predictions about the relationship between technological change, geology and economic growth.

5.3 Timing

Figure 9 illustrates the timing in our model. At the start of period t, the aggregate production sector demands resources from the extraction firms. The extraction firms request new machine varieties from the technology firms to access deposits of lower grades.

In the early period of t, technology firms observe this demand. They invest into new machines that are specific to the grades of the respective deposits. Firms enter the market until the value of entering, namely profits, equals market entry cost, which is the cost to invent a new technology. Each technology firm obtains a patent for their newly developed machine variety, produces one machine based on the patent and sells it to the extraction firms. The knowledge about the machine directly diffuses to the other firms.

In the mid-period of t, extraction firms convert deposits to reserves based on the new machines. In the later period of t, extraction firms extract the resource and sell it to the final good producers.

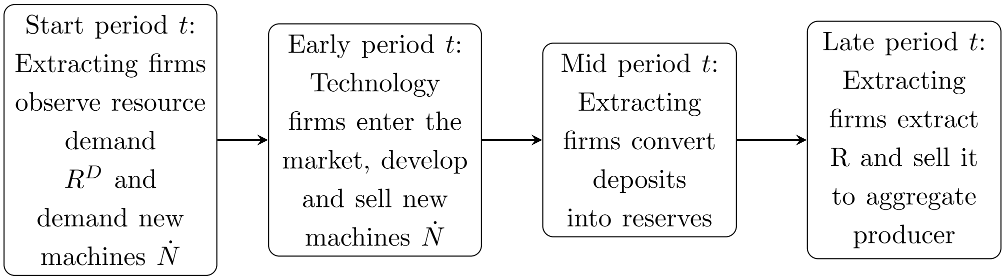

Figure 9: Timing and Firms' Problem

6 The Endogenous Growth Model

To study the aggregate effects of the interaction between geology and extraction technology, we embed the extractive sector in an endogenous growth model by Acemoglu (2002). Endogenizing technological development allows us to show how increases in resource demand affect technological change in extraction technology.

6.1 Setup

We consider a standard setup of an economy with a representative consumer that has constant relative risk aversion preferences:

$$\int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \ .$$

The variable C_t denotes consumption of aggregate output at time t, ρ is the discount rate, and θ is the coefficient of relative risk aversion.

The aggregate production function combines two inputs, namely an intermediate good Z and a non-renewable resource R, with a constant elasticity of substitution:

$$Y = \left[\gamma Z^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) R_{Extr}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad . \tag{9}$$

The distribution parameter $\gamma \in (0,1)$ indicates their respective importance in producing aggregate output Y. The parameter ε is the elasticity of substitution between the non-renewable resource and is $\varepsilon \in (0,\infty)$. Inputs Z_t and R_t are substitutes for $\varepsilon > 1$. In this case, the resource is not essential for aggregate production. For $\varepsilon \leq 1$ the two inputs are complements and the resource is essential for aggregate production. The Cobb-Douglas case is $\varepsilon = 1$ (see Dasgupta and Heal, 1974).

The budget constraint of the representative consumer is: $C + I + M \leq Y$. Aggregate spending on machines is denoted by I and aggregate R&D investment by M, where $M = M_Z + M_R$. The usual no-Ponzi game conditions apply.

The intermediate good sector follows the basic setup of Acemoglu (2002) and consists of a large number of infinitely small firms producing the intermediate good Z and technology firms producing sector-specific technologies. Technological change in the intermediate goods sector expands input varieties, which increases the division of labor and raises the productivity of final good firms (see Romer, 1987, 1990). .[12] Firms in the extractive and intermediate sectors use different types of machines. The representative household owns all firms.

7 Equilibrium

We now solve the model in general equilibrium such that extractive firms determine the rate of change in the extraction technology.

[12]Please find a more detailed description of the sector in appendix Appendix 1.2.

7.1 Non-Renewable Resource Demand

The final good producer demands the non-renewable resource and the intermediate good for aggregate production. Prices and quantities for both are determined in a fully competitive equilibrium. Taking the first order condition with respect to the non-renewable resource in equation (9), the demand for the resource is[13]

$$R^D = \frac{Y(1-\gamma)^{\varepsilon}}{p_R^{\varepsilon}} \; . \tag{10}$$

7.2 Demand for Extraction Technology

To characterize the (unique) equilibrium, we first determine the demand for machine varieties in the extractive sector. Machine prices and the number of machine varieties are determined in a market equilibrium between extractive firms and technology firms. Firms' optimization problem is static since machines depreciate fully after use.

In equilibrium, it is profit maximizing for firms to not keep reserves, $S(j) = 0$.[14] It follows that the production function of extractive firms is

$$R_t^{Extr} = R_t^{Tech} = \delta \mu \dot{N}_{Rt}. \tag{11}$$

Extractive firms face a cost for producing R_t^{Extr} units of resource given by $\Omega(R_t^{Extr}) =$

[13]Please see Appendix 1.4.2 for the respective derivations for the intermediate goods sector in this and the following subsections.

[14]See appendix Appendix 1.4.1 for the derivation of this result. If we assumed stochastic technological change, extractive firms would keep a positive stock of reserves S_t to insure against a series of bad draws in R&D. Reserves would grow over time in line with aggregate growth. The result would, however, remain the same: in the long term, resource extraction equals new reserves.

$R_t^{Extr}\chi_R\frac{1}{\delta\mu}$, where χ_R is the machine price charged by the extraction technology firms. The marginal cost is $\Omega'(R_t^{Extr}) = \chi_R\frac{1}{\delta\mu}$. The inverse supply function of the resource is hence constant and we obtain a market equilibrium at resource price

$$p_R = \chi_R\frac{1}{\delta\mu} \tag{12}$$

and resource demand:

$$R_t^D = \frac{Y(1-\gamma)^\varepsilon}{(\chi_R\frac{1}{\delta\mu})^\varepsilon} \ . \tag{13}$$

Using (11) and (13), we obtain the demand for machines:

$$\dot{N}_R = \frac{1}{\delta\mu}\frac{Y(1-\gamma)^\varepsilon}{(\chi_R\frac{1}{\delta\mu})^\varepsilon} \ . \tag{14}$$

7.3 Extraction Machine Prices

The demand function for extraction machines (14) is isoelastic, but there is perfect competition between the different suppliers of extraction technologies, as machine varieties are perfect substitutes in terms of producing the homogenous resource.[15]

Because extraction technology is grades-specific, only one machine is produced for each machine variety j. The constant rental rate χ_R that the monopolists charge includes the cost of machine production ψ_R and a mark-up that refinances R&D costs. The rental rate

[15] Please see Appendix 1.4.3 for the respective derivations for technology firms in the intermediate good sector.

is the result of a competitive market and derived from (13). It equals:

$$\chi_R(j) = \left(Y/R^{Extr}\right)^{\frac{1}{\varepsilon}} (1-\gamma)\delta\mu. \tag{15}$$

To complete the description of equilibrium on the technology side, we impose the free-entry condition:

$$\pi_{Rt} = \frac{1}{\eta_R} \text{if} M_R \succ 0 \ . \tag{16}$$

Firms enter the market until the value of entering, namely profits, equals market entry cost, which is the cost to develop a new technology. Like in the intermediate sector, markups are used to cover technology expenditure in the extractive sector. Combining equations profit function of extraction technology firms, equation (8), and the machine rental rate, equation (15), we obtain that the net present discounted value of profits of technology firms from developing one new machine variety is:

$$V_R(j) = \pi_R(j) = \chi_R(j) - \psi_R = \left(Y/R^{Extr}\right)^{\frac{1}{\varepsilon}} (1-\gamma)\delta\mu - \psi_R \ . \tag{17}$$

To compute the equilibrium quantity of machines and machine prices in the extractive sector, we first rearrange equation (17) with respect to R and consider the free entry condition. We obtain

$$R_t^{Extr} = \frac{Y(1-\gamma)^{\varepsilon}}{\left(\left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\delta\mu}\right)^{\varepsilon}} \ . \tag{18}$$

Inserting (18) into the rental rate equation (15) we obtain the equilibrium machine price.

$$\chi_R(j) = \frac{1}{\eta_R} + \psi_R \; . \tag{19}$$

7.4 Resource Price

We can now derive the price of the non-renewable resource and the corresponding impacts by its geological distribution and technological change.

The resource price equals marginal production cost due to perfect competition in the resource market. The equilibrium machine price, equation (19), and the equilibrium resource price, equation (12):[16]

Proposition 2 *The resource price depends negatively on the average crustal concentration of the non-renewable resource and the average effect of extraction technology on ore grades:*

$$p_R = \left(\frac{1}{\eta_R} + \psi_R \right) \frac{1}{\delta \mu} \; , \tag{20}$$

where ψ_R reflects the cost of producing the machine and η_R is a markup that serves to compensate technology firms for R&D cost.

The intuition is as follows: If, for example, δ is high, the average crustal concentration of the resource is high (see geological function, equation (1)) and the price is low. If μ is high, the average effect of new extraction technology on converting deposits of lower grades to reserves is high (see technology function, equation 3). This implies a lower resource price. The resource price level also depends negatively on the cost parameter of R&D development

[16]Please see Appendix 1.4.4 for the equilibrium price of the intermediate good.

η_R.

7.5 The Growth Rate on the Balanced Growth Path

We can now study the effects of non-renewable resources and technological change in extraction on the growth rate of aggregate output.

We define the BGP equilibrium as an equilibrium path where consumption grows at the constant rate g^* and the relative price p is constant. From (33) this definition implies that p_{Zt} and p_{Rt} are also constant.

Proposition 3 *There exists a unique BGP equilibrium in which the relative technologies are given by equation* (40) *in the appendix, and consumption and output grow at the rate*[17]

$$g = \theta^{-1}\left(\beta\eta_Z L\left[\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\frac{1}{\eta_R\delta\mu} + \frac{\psi_R}{\delta\mu}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}\frac{1}{\beta}} - \rho\right). \tag{21}$$

The growth rate of the economy is positively influenced by (i) the crustal concentration of the non-renewable resource δ and (ii) the effect of R&D investment in terms of lowering ore grades μ.

Adding the extractive sector to the standard model by Acemoglu (2002), changes the interest part of the Euler equation, $g = \theta^{-1}(r-\rho)$.[18] Instead of two exogenous production

[17] Starting with any $N_R(0) > 0$ and $N_Z(0) > 0$, there exists a unique equilibrium path. If $N_R(0)/N_Z(0) < (N_R/N_Z)^*$ as given by (40), then $M_{Rt} > 0$ and $M_{Zt} = 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. If $N_R(0)/N_Z(0) > (N_R/N_Z)^*$, then $M_{Rt} = 0$ and $M_{Zt} > 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. It can also be verified that there are simple transitional dynamics in this economy whereby starting with technology levels $N_R(0)$ and $N_Z(0)$, there always exists a unique equilibrium path, and it involves the economy monotonically converging to the BGP equilibrium of (21) like in Acemoglu (2002).

[18] There is no capital in this model, but agents delay consumption by investing in R&D as a function of the interest rate.

factors, the interest rate r in our model only includes labor, but adds the resource price, as p_Z depends on p_R according to equation (38).

If $(1-\gamma)^{\varepsilon}(\eta_R \delta\mu)^{1-\varepsilon} < 1$ holds, then the substitution between the intermediate good and the resource is low and R&D investment in extraction technology has a small yield in terms of additional reserves. The effect that economic growth is impossible if the resource cannot be substituted by other production factors is known as the "limits to growth" effect in the literature (see Dasgupta and Heal, 1979, p. 196 for example). When this effect occurs, growth is *limited* in models with a positive initial stock of resources, because the initial resource stock can only be consumed in this case. In our model, growth is *impossible*, because there is no initial stock and the economy is not productive enough to generate the necessary technology. When the inequality does not hold, the economy is on a balanced growth path.

7.6 Resource Intensity of the Economy

Substituting equation (20) into the resource demand equation (10), we obtain the ratio of resource consumption to aggregate output.

Proposition 4 *The resource intensity of the economy is positively affected by the average crustal concentration of the resource and the average effect of extraction technology:*

$$\frac{R^{Extr}}{Y} = (1-\gamma)^{\varepsilon}\left[\left(\frac{1}{\eta_R}+\psi_R\right)\frac{1}{\delta\mu}\right]^{-\varepsilon} .$$

The resource intensity of the economy is negatively affected by the elasticity of substitution if $(1-\gamma)^{\varepsilon}\left[(\frac{1}{\eta_R}+\psi_R)\frac{1}{\delta\mu}\right]^{-\varepsilon} < 1$ *and positively otherwise.*

7.7 Technology Growth

We derive the growth rates of technology in the two sectors from equations (11), (10), and (20). The stock of technology in the intermediate good sector grows at the same rate as the economy.

Proposition 5 *The stock of extraction technology grows proportionally to output according to:*

$$\dot{N}_R = (1-\gamma)^{\varepsilon} Y \left(1/\eta_R + \psi_R\right)^{-\varepsilon} (\delta\mu)^{\varepsilon-1} .$$

In contrast to the intermediate good sector, where firms can make use of the *stock* of technology, firms in the extractive sector can only use the *flow* of new technology to convert deposits of lower grades into new reserves. Previously grade-specific technology cannot be employed because the deposits of that particular grade have already been depleted. Firms in the extractive sector need to invest a larger share of total output to attain the same rate of growth in technology in comparison to firms in the intermediate good sector.

The effects of the parameters δ from the geological function and μ from the extraction technology function on $\dot{N}_R$ depend on the elasticity of substitution ε. Like in Acemoglu (2002), there are two opposing effects at play: the first is a price effect. Technology investments are directed towards the sector of the scarce good. The second is a market size effect, meaning that technology investments are directed to the larger sector.

If the goods of the two sectors are complements ($\varepsilon < 1$), the price effect dominates. An increase in δ or μ lowers the cost of resource production and the resource price, but the technology growth rate in the resource sector decelerates, because R&D investment is directed towards the complementary intermediate good sector. If the resource and the intermediate good are substitutes ($\varepsilon > 1$), the market size effect dominates. An increase in δ or μ makes resources cheaper and causes an acceleration in the technology growth rate in the resource sector, because more of the lower cost resource is demanded.

8 Discussion

Our model can be generalized to different functional forms of the geological function and the extraction technology function. If they have different forms, the effects on resource price, resource intensity of the economy, and growth rate will depend on the resulting changes in proposition 1. In the first case, where increasing returns in the geology function more than offset the decreasing returns in the technology function, the unit extraction cost declines and the resource becomes more abundant. As a result, the resource price is declining, the resource intensity increasing, and the growth rate of the economy also increasing. The condition that resource prices equal marginal resource extraction cost would still extend to this case. Prices cannot be below marginal extraction cost, since firms would make negative profits.

In the second case where the increasing returns in the geology function do not offset the decreasing returns in the technology function, the resource price increases over time. As the unit extraction technology cost goes up, the resource intensity declines and the growth rate of the economy declines as well. There would still be no scarcity rent like in Hotelling (1931)[19], but an additional social cost if extraction firms hold infinite property rights (Heal, 1976). This social cost reflects that present extraction pushes up future unit extraction technology

[19]Note that a scarcity rent has not yet been found empirically (see e.g. Hart and Spiro, 2011)

cost. This would drive a wedge between the resource price and the unit extraction cost.

However, extraction firms typically do not hold property rights for the resources. They mostly lease extraction rights from private owners or the government for a definite period of time. These leases typically require the firm to start production at some time or the lease is terminated early. In addition, there is a substantial risk of ex-appropriation for extractive firms in many countries (see e.g. Stroebel and Van Benthem, 2013). If there is no exclusive property right of extraction firms in the resource, and there is free entry and exit like in our model, firms will increase their production until the resource price equals the unit extraction cost (Heal, 1976).

Finally, if any of the two functions is discontinuous with an unanticipated break, at which the respective parameters change to either $\delta' \in \mathbb{R}_+$ or $\mu' \in \mathbb{R}_+$, there will be two balanced growth paths: one for the period before, and one for the period after the break. Both paths would behave according to the model's predictions.

9 Conclusion

Implementing the interaction between geology and innovation in extraction technology into a standard endogenous growth model predicts stable non-renewable resource prices and exponentially increasing extraction. Increased resource demand due to aggregate output growth incentivises firms to invest in new extraction technologies to convert lower grade deposits into reserves. Firms invest in R&D despite perfect competition in resource markets due to the deposit-specific and hence rivalrous nature of technology. Resource prices remain constant because increasing return from the geological resource distribution offset diminishing returns in innovation.

In contrast to traditional growth models with non-renewable resources, there is no depletion effect that drags down the rate of aggregate growth. Rather it is the concentration of resources in the Earth's crust that co-determines the aggregate output growth rate. Furthermore, the extraction sector is also not an engine of growth because it only exhibits constant returns to scale in the aggregate. This is due to the rivalrous nature of technology and the depletion of higher grade deposits.

The fundamental mechanism of our model builds on Ahrens' fundamental law of geochemistry concerning the geological distribution of resources and the economic history of innovation in the mining sector. The model predicts price and output trends, which are in line with stylized facts from a new data-set that encompasses data for all major non-renewable resources from 1700 to 2018.

If historical trends continue, technological innovation may supply a growing and price-stable flow of fossil fuels into the future. This has important implications for climate change, because it would make a transition towards renewable energy more difficult. At the same time, our model refutes the so called "Green Paradox", which argues that demand-side policies such as a carbon tax are ineffective in reducing greenhouse gas emissions (Sinn, 2008; Eichner and Pethig, 2011; Van der Ploeg and Withagen, 2012). In these models firms manage their finite stock of fossil fuels to maximize returns over time. Knowing a carbon tax would reduce future demand, firms respond by selling their stock of fossil fuels sooner rather than later. Lower prices due to excess supply encourage fossil fuel consumption and inadvertently accelerate climate change. Our model framework suggests otherwise: A demand-side intervention would discourage firm from developing new extraction technology, lowering production and greenhouse gases going forward.[20]

This paper points to a number of different directions for future research on the economics of non-renewable resources and extraction technology. It would be desirable to introduce a more complex cost curve for firms and to study more closely the trade-offs that firms face between R&D investment and higher production cost. This could also include an examination of the role of patents and property rights in the extractive sector. More empirical work in this direction based on micro-data would be valuable. We also observe positive reserve holdings by firms. A model with stochastic R&D could generate this phenomenon and study its implications.

The stylized facts raise questions about the economic mechanisms at work that led to transitions in resource intensity. There was a transition from low intensity in 1700 to a peak in the mid of the 20th century. Following the first transition, there has been a decoupling in intensity between fossil fuels and metals. Fossil fuels have exhibited declining trends while metals have followed trends. This suggests some of the many important factors that we omitted, such as recycling, energy as an input, environmental externalities, technological change in resource efficiency and environmental policies could account for these dynamics. We hope our simple theory proves to be a useful building block for further work in this area.

[20]See also the blog on our paper by Romer (2016).

10 Authors' affiliations

Martin Stuermer is with the Research Department of the Federal Reserve Bank of Dallas.

Gregor Schwerhoff is with the Mercator Research Institute on Global Commons and Climate Change, Berlin.

References

Acemoglu, D. (2002). Directed technical change. *The Review of Economic Studies*, 69(4):781–809. https://doi.org/10.1111/1467-937x.00226.

Acemoglu, D., Aghion, P., Barrage, L., and Hemous, D. (2019). Climate change, directed innovation, and energy transition: The long-run consequences of the shale gas revolution. Technical report, Manuscript.

Acemoglu, D., Aghion, P., Bursztyn, L., and Hemous, D. (2012a). The environment and directed technical change. *American Economic Review*, 102(1):131–66. https://doi.org/10.1257/aer.102.1.131.

Acemoglu, D., Golosov, M., Tsyvinski, A., and Yared, P. (2012b). A dynamic theory of resource wars. *The Quarterly Journal of Economics*, 127(1):283–331. https://doi.org/10.1093/qje/qjr048.

Aghion, P. and Howitt, P. (1998). *Endogenous growth theory*. MIT Press, London.

Aguilera, R., Eggert, R., Lagos C.C., G., and Tilton, J. (2012). Is depletion likely to create significant scarcities of future petroleum resources? In Sinding-Larsen, R. and Wellmer, F., editors, *Non-renewable resource issues*, pages 45–82. Springer Netherlands, Dordrecht.

Aguilera, R. and Radetzki, M. (2016). *The Price of Oil*. Cambridge University Press. https://doi.org/10.1017/CBO9781316272527.

Ahrens, L. (1953). A fundamental law of geochemistry. *Nature*, 172:1148. https://doi.org/10.1038/1721148a0.

Ahrens, L. (1954). The lognormal distribution of the elements (a fundamental law of geochemistry and its subsidiary). *Geochimica et Cosmochimica Acta*, 5(2):49–73.

Bartos, P. (2002). SX-EW copper and the technology cycle. *Resources Policy*, 28(3-4):85–94. https://doi.org/10.1016/s0301-4207(03)00025-4.

British Petroleum (2017). Statistical review of world energy.

Covert, T., Greenstone, M., and Knittel, C. R. (2016). Will we ever stop using fossil fuels? *Journal of Economic Perspectives*, 30(1):117–38. https://doi.org/10.1257/jep.30.1.117.

Cynthia-Lin, C. and Wagner, G. (2007). Steady-state growth in a Hotelling model of resource extraction. *Journal of Environmental Economics and Management*, 54(1):68–83. https://doi.org/10.1016/j.jeem.2006.12.001.

Dasgupta, P. and Heal, G. (1974). The optimal depletion of exhaustible resources. *The Review of Economic Studies*, 41:3–28. https://doi.org/10.2307/2296369.

Dasgupta, P. and Heal, G. (1979). Economic theory and exhaustible resources. *Cambridge Economic Handbooks (EUA).* https://doi.org/10.1017/CBO9780511628375.

Desmet, K. and Rossi-Hansberg, E. (2014). Innovation in space. *American Economic Review*, 102(3):447–452. https://doi.org/10.1257/aer.102.3.447.

Dvir, E. and Rogoff, K. (2010). The three epochs of oil. *mimeo*.

Eichner, T. and Pethig, R. (2011). Carbon leakage, the green paradox, and perfect future markets. *International Economic Review*, 52(3):767–805. https://doi.org/10.1111/j.1468-2354.2011.00649.x.

Federal Institute for Geosciences and Natural Resources (2017). *BGR Energy Survey*. Federal Institute for Geosciences and Natural Resources, Hanover, Germany.

Gerst, M. (2008). Revisiting the cumulative grade-tonnage relationship for major copper ore types. *Economic Geology*, 103(3):615. https://doi.org/10.2113/gsecongeo.103.3.615.

Golosov, M., Hassler, J., Krusell, P., and Tsyvinski, A. (2014). Optimal taxes on fossil fuel in general equilibrium. *Econometrica*, 82(1):41–88. https://doi.org/10.3982/ecta10217.

Gordon, R., Bertram, M., and Graedel, T. (2007). On the sustainability of metal supplies: a response to Tilton and Lagos. *Resources Policy*, 32(1-2):24–28. https://doi.org/10.1016/j.resourpol.2007.04.002.

Groth, C. (2007). A new growth perspective on non-renewable resources. In Bretschger, L. and Smulders, S., editors, *Sustainable Resource Use and Economic Dynamics*, chapter 7, pages 127–163. Springer Netherlands, Dordrecht.

Hart, R. and Spiro, D. (2011). The elephant in Hotelling's room. *Energy Policy*, 39(12):7834–7838. https://doi.org/10.1016/j.enpol.2011.09.029.

Harvey, D. I., Kellard, N. M., Madsen, J. B., and Wohar, M. E. (2010). The Prebisch-Singer hypothesis: four centuries of evidence. *The Review of Economics and Statistics*, 92(2):367–377. https://doi.org/10.1162/rest.2010.12184.

Hassler, J., Krusell, P., and Olovsson, C. (2016). Directed technical change as a response to natural-resource scarcity. Technical report, working paper.

Hassler, J. and Sinn, H.-W. (2012). The fossil episode. Technical report, CESifo Working Paper: Energy and Climate Economics.

Heal, G. (1976). The relationship between price and extraction cost for a resource with a backstop technology. *The Bell Journal of Economics*, 7(2):371–378. https://doi.org/10.2307/3003630.

Herfindahl, O. (1959). *Copper costs and prices: 1870-1957.* Published for Resources for the Future by Johns Hopkins Press, Baltimore.

Hotelling, H. (1931). The economics of exhaustible resources. *Journal of Political Economy*, 39(2):137–175. https://doi.org/10.1086/254195.

International Energy Agency (2012). *World energy outlook 2012.* International Energy Agency, Paris. https://doi.org/10.1787/weo-2012-en.

Jacks, D. S. (2013). From boom to bust: A typology of real commodity prices in the long run. Technical report, National Bureau of Economic Research. https://doi.org/10.3386/w18874.

Jones, C. I. and Vollrath, D. (2002). *Introduction to Economic Growth.* Norton & Company Inc. NY.

Krautkraemer, J. (1998). Nonrenewable resource scarcity. *Journal of Economic Literature*, 36(4):2065–2107.

Lasserre, P. and Ouellette, P. (1991). The measurement of productivity and scarcity rents: the case of asbestos in canada. *Journal of Econometrics*, 48(3):287–312. https://doi.org/10.1016/0304-4076(91)90065-l.

Lee, J., List, J., and Strazicich, M. (2006). Non-renewable resource prices: deterministic or stochastic trends? *Journal of Environmental Economics and Management*, 51(3):354–370. https://doi.org/10.1016/j.jeem.2005.09.005.

Littke, R. and Welte, D. (1992). *Hydrocarbon Source Rocks.* Cambridge University Press, Cambridge, U.K.

Livernois, J. (2009). On the empirical significance of the Hotelling rule. *Review of Environmental Economics and Policy*, 3(1):22–41. https://doi.org/10.1093/reep/ren017.

Managi, S., Opaluch, J., Jin, D., and Grigalunas, T. (2004). Technological change and depletion in offshore oil and gas. *Journal of Environmental Economics and Management*, 47(2):388–409. https://doi.org/10.1016/s0095-0696(03)00093-7.

Mudd, G. (2007). An analysis of historic production trends in Australian base metal mining. *Ore Geology Reviews*, 32(1):227–261. https://doi.org/10.1016/j.oregeorev.2006.05.005.

Nordhaus, W. (1974). Resources as a constraint on growth. *American Economic Review*, 64(2):22–26.

Nordhaus, W. D., Stavins, R. N., and Weitzman, M. L. (1992). Lethal model 2: the limits to growth revisited. *Brookings papers on economic activity*, 1992(2):1–59. https://doi.org/10.2307/2534581.

Perman, R., Yue, M., McGilvray, J., and Common, M. (2003). *Natural resource and environmental economics*. Pearson Education, Edinburgh.

Pindyck, R. (1999). The long-run evolution of energy prices. *The Energy Journal*, 20(2):1–28. https://doi.org/10.5547/issn0195-6574-ej-vol20-no2-1.

Radetzki, M. (2008). *A handbook of primary commodities in the global economy*. Cambridge Univ. Press, Cambridge, U.K. https://doi.org/10.1017/CBO9780511493584.

Radetzki, M. (2009). Seven thousand years in the service of humanity: the history of copper, the red metal. *Resources Policy*, 34(4):176–184. https://doi.org/10.1016/j.resourpol.2009.03.003.

Rausser, G. and Stuermer, M. (2016). Collusion in the copper commodity market: A long-run perspectivel. *Manuscripty*.

Rausser, G. C. (1974). Technological change, production, and investment in natural resource industries. *The American Economic Review*, 64(6):1049–1059.

Rogner, H. (1997). An assessment of world hydrocarbon resources. *Annual Review of Energy and the Environment*, 22(1):217–262. https://doi.org/10.1146/annurev.energy.22.1.217.

Romer, P. (2016). Conditional optimism about progress and climate. https://paulromer.net/conditional-optimism-about-progress-and-climate/index.html (accessed March 21, 2019).

Romer, P. M. (1987). Growth based on increasing returns due to specialization. *The American Economic Review*, 77(2):56–62.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, 98(5, Part 2):S71–S102. https://doi.org/10.1086/261725.

Scholz, R. and Wellmer, F. (2012). Approaching a dynamic view on the availability of mineral resources: what we may learn from the case of phosphorus? *Global Environmental Change*, 23(1):11–27. https://doi.org/10.1016/j.gloenvcha.2012.10.013.

Simpson, R., editor (1999). *Productivity in natural resource industries: improvement through innovation*. RFF Press, Washington, D.C.

Sinn, H. (2008). Public policies against global warming: a supply side approach. *International Tax and Public Finance*, 15(4):360–394. https://doi.org/10.1007/s10797-008-9082-z.

Skinner, B. (1979). A second Iron Age ahead? *Studies in Environmental Science*, 3:559–575. https://doi.org/10.1016/s0166-1116(08)71071-9.

Slade, M. (1982). Trends in natural-resource commodity prices: an analysis of the time domain. *Journal of Environmental Economics and Management*, 9(2):122–137. https://doi.org/10.1016/0095-0696(82)90017-1.

Solow, R. M. and Wan, F. Y. (1976). Extraction costs in the theory of exhaustible resources. *The Bell Journal of Economics*, pages 359–370. https://doi.org/10.2307/3003261.

Stiglitz, J. (1974). Growth with exhaustible natural resources: efficient and optimal growth paths. *The Review of Economic Studies*, 41:123–137. https://doi.org/10.2307/2296377.

Stroebel, J. and Van Benthem, A. (2013). Resource extraction contracts under threat of expropriation: Theory and evidence. *Review of Economics and Statistics*, 95(5):1622–1639.

Stuermer, M. (2018). 150 years of boom and bust: what drives mineral commodity prices? *Macroeconomic Dynamics*, 22(3):702–717. https://doi.org/10.1017/s136510051600050x.

Tahvonen, O. and Salo, S. (2001). Economic growth and transitions between renewable and nonrenewable energy resources. *European Economic Review*, 45(8):1379–1398. https://doi.org/10.1016/S0014-2921(00)00062-3.

U.S. Geological Survey (2016). *Minerals Yearbook*. U.S. Geological Survey.

U.S. Geological Survey (2018). *Mineral Commodity Summaries*. U.S. Geological Survey, Reston, VA.

Van der Ploeg, F. and Withagen, C. (2012). Is there really a green paradox? *Journal of Environmental Economics and Management*, 64(3):342–363. https://doi.org/10.1016/j.jeem.2012.08.002.

van der Ploeg, F. and Withagen, C. (2012). Too much coal, too little oil. *Journal of Public Economics*, 96(1):62–77. https://doi.org/10.1016/j.jpubeco.2011.08.005.

Von Hagen, J. (1989). Relative commodity prices and cointegration. *Journal of Business & Economic Statistics*, 7(4):497–503. https://doi.org/10.1080/07350015.1989.10509763.

Weitzman, M. L. (1999). Pricing the limits to growth from minerals depletion. *The Quarterly Journal of Economics*, 114(2):691–706. https://doi.org/10.1162/003355399556025.

Wellmer, F. (2008). Reserves and resources of the geosphere, terms so often misunderstood. Is the life index of reserves of natural resources a guide to the future. *Zeitschrift der Deutschen Gesellschaft für Geowissenschaften*, 159(4):575–590. https://doi.org/10.1127/1860-1804/2008/0159-0575.

Appendix 1

Appendix 1.1 Data Description

We include the following 65 non-renewable resources in the data-set: fossil minerals: coal, natural gas, petroleum; metals: aluminum, antimony, arsenic, beryllium, bismuth, boron, cadmium, cesium, chromium, cobalt, copper, gallium, germanium, gold, indium, lead, lithium, magnesium (compounds and metal), manganese, mercury, molybdenum, nickel, niobium, platinum-group metals, pig iron, rare earths, rhenium, salt, selenium, silver, strontium, tantalum, thorium, tin, tungsten, vanadium, zinc, zirconium; non-metals: asbestos, barite, bromine, cement, diatomite, feldspar, fluorspar, garnet, graphite, gypsum, iodine, kyanite, nitrogen, phosphate rock, potash, pumice, silicon, sulfur, talc& pyrophyllite, tellurium, thallium, uranium, vermiculite, wolalstonite.

We currently do not include the following metals: hafnium, cesium; non-metals: natural abrasives, clays, coal combustion products (ashes), diamond (industrial), gemstones, iron oxide pigments, lime, peat, perlite, quartz, sand, soda ash, sodium sulfate, stone, titanium (pigments, metal, mineral concentrates), and helium. These non-renewable resources are excluded for a variety of different reasons, including lack of global historical data, e.g. for stones, no clear separation in the data between natural and synthetic materials like in the case of industrial diamonds, and prevention of double-counting due to different products in the value chain. For example, iron ore is not but pig iron is included. Most of the excluded commodities would not change the results of our analysis, because the extracted quantities and market value are negligible. The only exception is stones, which exhibit relatively large extracted quantities.

The number of resources increases over time, as more they are explored and employed in the manufacturing of goods. In 1700, our data-set includes copper, gold, mercury, pig iron, salt, silver, tin, and coal. These are all non-renewable resources that were in broad use in the global economy at the time with the exception of stones. The number of non-renewable resources increases to 34 in 1900 in our data-set, including petroleum, natural gas and a broad variety of metals and non-metals, and to 65 in 2000.

An online-appendix with further descriptions and sources is in the making.

Appendix 1.2 Description Intermediate Good Sector

The intermediate good sector consists of a large number of infinitely small firms that produce the intermediate good Z, and technology firms that produce sector-specific technologies.[21]

Firms produce an intermediate good Z according to the production function:

$$Z = \frac{1}{1-\beta_Z} \left(\int_0^{N_z} x_z(j)^{1-\beta_Z} dj \right) L_Z^{\beta} \, , \tag{22}$$

where $x_Z(j)$ refers to the number of machines used for each machine variety j in the production of the intermediate good, L is labor, which is in fixed supply, and β_Z is $\in (0,1)$. This implies that machines in the intermediate good sector are partial complements.[22]

All intermediate good machines are supplied by sector-specific technology firms that each

[21] Like in the extractive sector, we assume that the firm level production functions exhibit constant returns to scale, so there is no loss of generality in focusing on aggregate production functions.

[22] While machines of type j in the intermediate sector can be used infinitely often, a machine of variety j in the resource sector is grade-specific and essential to extracting the resource from deposits of certain grades O. A machine of variety j in the extractive sector is therefore only used once, and the range of machines employed to produce resources at time t is $\dot{N}_R$. In contrast, the intermediate good sector can use the full range of machines $[0, N_Z(t)]$ complementing labor.

have one fully enforced perpetual patent on the respective machine variety. As machines are partial complements, technology firms have some degree of market power and can set the price for machines. The price charged by these firms at time t is denoted $\chi_Z(j)$ for $j \in [0, N_Z(t)]$. Once invented, machines can be produced at a fixed marginal cost $\psi_Z > 0$.

The innovation possibilities frontier is assumed to take a similar form like in the extractive sector: $\dot{N}_Z = \eta_R M_Z$. Technology firms can spend one unit of the final good for R&D investment M_Z at time t to generate flow rate $\eta_Z > 0$ of new patents. Each firm hence needs $\frac{1}{\eta_Z}$ units of final output to develop a new machine variety. Technology firms can freely enter the market if they develop a patent for a new machine variety. They can only invent one new variety.

Appendix 1.3 Derivation of Extraction Firms' New Reserves

Equation (5) is derived in the following way: Firms can buy machine varieties j to increase their reserves by:

$$R_t^{Tech} = \delta\mu \lim_{h \to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} x_R(j)^{(1-\beta)} dj \ , \tag{23}$$

where $x_R(j)$ refers to the number of machines used for each machine variety j.

We assume that $\beta = 0$ in the extractive sector, because firms invest into technology to continue resource production. If firms do not invest, extraction cost becomes infinitely high. Firms invest into technology for the next lowest grade deposits. However, firms are ultimately indifferent about the specific deposit from which they extract, because conditioned on new technology the same homogeneous resource can be produced from all deposits. That's why machine varieties are full complements in our setup. This is in contrast to the intermediate goods sector, where machine varieties are partial complements and firms invest into machine varieties to increase the division of labor.

As a machine variety j in the resource sector is grade-specific and essential to extracting the resource from deposits of certain grades, each variety j in the extractive sector is only used once, and the range of machines employed to produce resources at time t is $\dot{N}_R$. In contrast, the intermediate good sector can use machine types infinitely often and hence the full range of machines $[0, N_Z(t)]$ complementing labor. Under the assumption that $x_R(j) = 1$, equation (23) turns into:

$$
\begin{aligned}
R_t^{Tech} &= \delta\mu \lim_{h\to 0} \frac{1}{h} \int_{N_R(t-h)}^{N_R(t)} 1dj \\
&= \delta\mu \dot{N}_R \, .
\end{aligned}
$$

Appendix 1.4 Solving for the Equilibrium

The allocation in the economy is defined by the following objects: time paths of consumption levels, aggregate spending on machines, and aggregate R&D expenditure, $[C_t, I_t, M_t]_{t=0}^{\infty}$; time paths of available machine varieties, $[N_{Rt}, N_{Zt},]_{t=0}^{\infty}$; time paths of prices and quantities of each machine, $[\chi_{Rt}(j), x_{Rt}(j)]_{j\in[0,N_{Rt}]t}^{\infty}$ and $[\chi_{Zt}(j), x_{Zt}(j)]_{j\in[0,N_{Zt}],t}^{\infty}$; the present discounted value of profits V_R and V_Z, and time paths of interest rates and wages, $[r_t, w_t]_{t=0}^{\infty}$.

An equilibrium is an allocation in which all technology firms in the intermediate good sector choose $[\chi_{Zt}(j), x_Zt(j)]_{j\in[0,N_Z(t)],t}^{\infty}$ to maximize profits. Machine prices in the extractive sector $\chi_{Rt}(j)$ result from the market equilibrium, because extraction technology firms are in full competition and technology is grades-specific.

The evolution of $[N_{Rt}, N_{Zt}]_{t=0}^{\infty}$ is determined by free entry; the time paths of factor prices, $[r, w]_{t=0}^{\infty}$, are consistent with market clearing; and the time paths of $[C_t, I_t, M_t]_{t=0}^{\infty}$ are consistent with household maximization.

Appendix 1.4.1 Extraction Firms

To show that it is profit maximizing for extraction firms to not keep any reserves if there is no uncertainty, we first assume that firms have already invested in technology and accessed new reserves R^{Tech}. Firms can either extract the resource for immediate sale R^{Extr} or build reserves S. We obtain the following optimization problem of a firm:

$$\max_{R^{Extr}}(p_R - \phi)R^{Extr} \text{ such that } R^{Tech} \geq R^{Extr}. \tag{24}$$

The maximization problem can be expressed with the following Lagrangian:

$$L = (p_R - \phi)R^{Extr} + \lambda[R^{Tech} - R^{Extr}]. \tag{25}$$

This leads to the following first order conditions:

$$\begin{aligned} (p_R - \phi)R^{Extr} - \lambda &= 0 && (26) \\ \lambda[R^{Tech} - R^{Extr}] &= 0 && (27) \end{aligned}$$

Consider the case that the constraint is not binding. Given (27), we obtain $\lambda = 0$, and from (26) follows $p_R - \phi = 0$. This is a contradiction, since the market entry condition ensures $\pi_R > 0$, which is not in line with $p_R - \phi = 0$. Therefore, the constraint must be binding and $R^{Tech} = R^{Extr}$. In equilibrium, it is thus profit maximizing for firm j to not keep reserves, $S(j) = 0$.

It follows that the production function of the extractive firms is

$$R_t^{Extr} = \delta\mu\dot{N}_{Rt}. \tag{28}$$

Appendix 1.4.2 Intermediate Good Firms

Taking the first order condition with respect to the intermediate good in equation (9), the demand for the intermediate good is

$$Z = \frac{Y(1-\gamma)^{\varepsilon}}{p_Z^{\varepsilon}} ,$$

The maximization problem of the intermediate good firms can be written as

$$\max_{L,\{x_Z(j)\}j\in[0,N_{Zt}]} p_Z Z - wL - \int_0^{N_Z} \chi_Z(j)x_Z(j)dj .$$

The problem is static, as machines depreciate fully.

The FOC with respect to $x_Z(j)$ immediately implies the following isoelastic demand function for machines:

$$x_{Zt}(j) = \left(\frac{p_{Zt}}{\chi_{Zt}(j)}\right)^{1/\beta} L , \tag{29}$$

for all $j \in [0, N_Z(t)]$ and all t,

Appendix 1.4.3 Technology Firms in the Intermediate Good Sector

Substituting (29) into (30), we calculate the FOC with respect to machine prices in the intermediate good sector: $\chi_Z(j)$: $\left(\frac{p_Z}{\chi_Z(j)}\right)^{\frac{1}{\beta}} L - (\chi_Z(j) - \psi_R) p_Z^{\frac{1}{\beta}} \frac{1}{\beta} \chi_Z(j)^{\frac{1}{\beta}-1} L = 0$. Hence, the solution of the maximization problem of any monopolist $j \in [0, N_Z]$ involves setting the same price in every period according to

$$\chi_{Zt}(j) = \frac{\psi_R}{1-\beta} \text{ for all } j \text{ and } t \ .$$

The value of a technology firm in the intermediate good sector that discovers one of the machines is given by the standard formula for the present discounted value of profits:

$$V_Z(j) = \int_t^{\infty} exp\left(-\int_t^s r(s')ds'\right) \pi_Z(j) ds \ .$$

Instantaneous profits are denoted

$$\pi_Z(j) = (\chi_Z(j) - \psi_Z) x_Z(j) \ , \tag{30}$$

where r is the market interest rate, and $x_Z(j)$ and $\chi_Z(j)$ are the profit-maximizing choices for the technology monopolist in the intermediate good sector.

All monopolists in the intermediate good sector charge a constant rental rate equal to a markup over their marginal cost of machine production, ψ_R. We normalize the marginal cost of machine production to $\psi_R \equiv (1-\beta)$ (remember that the elasticity of substitution between machines is $\epsilon \equiv \frac{1}{\beta}$), so that

$$\chi_{Zt}(j) = \chi_Z = 1 \text{ for all } j \text{ and } t\ . \tag{31}$$

In the intermediate good sector, substituting the machine prices (31) into the demand function (29) yields: $x_{Zt}(j) = p_{Zt}^{1/\beta} L$ for all j and all t.

Since the machine quantities do not depend on the identity of the machine, only on the sector that is being served, profits are also independent of machine variety in both sectors. Firms are symmetric.

In particular profits of technology firms in the intermediate good sector are $\pi_{Zt} = \beta p_{Zt}^{1/\beta} L$. This implies that the net present discounted value of monopolists only depends on the sector and can be denoted by V_{Zt}.

Combining the demand for machines (29) with the production function of the intermediate good sector (22) yields the *derived* production function:

$$Z(t) = \frac{1}{1-\beta} p_{Zt}^{\frac{1-\beta}{\beta}} N_{Zt} L, \tag{32}$$

The equivalent equation in the extractive sector is (11), because there is no optimization over the number of machines by the extraction technology firms, as the demand for machines per machine variety is one.

Appendix 1.4.4 Intermediate Good and Resource Prices

Prices of the intermediate good and the non-renewable resource are derived from the marginal product conditions of the final good technology, equation (9), which imply

$$\begin{aligned} p &\equiv \frac{p_R}{p_Z} = \frac{1-\gamma}{\gamma}\left(\frac{R^{Extr}}{Z}\right)^{-\frac{1}{\varepsilon}} \\ &= \frac{1-\gamma}{\gamma}\left(\frac{\delta\mu\dot{N}_R}{\frac{1}{1-\beta}p_L^{\frac{1-\beta}{\beta}}N_Z L}\right)^{-\frac{1}{\varepsilon}} \end{aligned}$$

There is no derived elasticity of substitution in analogy to Acemoglu (2002), because there is only one fixed factor, namely L in the intermediate good sector. In the extractive sector, resources are produced by machines from deposits. The first line of this expression simply defines p as the relative price between the intermediate good and the non-renewable resource, and uses the fact that the ratio of the marginal productivities of the two goods must be equal to this relative price. The second line substitutes from (32) and (11). There are no relative factor prices in this economy like in Acemoglu (2002), because there is only one fixed factor in the economy, namely L in the intermediate good sector.

Appendix 1.4.5 Proof for the Balanced Growth Path

We define the BGP equilibrium as an equilibrium path where consumption grows at the constant rate g^* and the relative price p is constant.

Setting the price of the final good as the numeraire gives:

$$\left[\gamma^{\varepsilon}p_Z^{1-\varepsilon} + (1-\gamma)^{\varepsilon}p_R^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = 1\ , \tag{33}$$

where p_Z is the price index of the intermediate good and p_R is the price index of the non-renewable resource. Intertemporal prices of the intermediate good are given by the interest rate $[r_t]_{T=0}^{\infty}$. This implies that p_{Zt} and p_{Rt} are constant.

Household optimization implies

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}(r_t - \rho),$$

and

$$\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)ds\right)(N_{Zt}V_{Zt} + \dot{N}_{Rt}V_{Rt})\right] = 0,$$

which uses the fact that $N_{Zt}V_{Zt}+\dot{N}_{Rt}V_{Rt}$ is the total value of corporate assets in the economy. In the resource sector, only *new* machine varieties produce profit.

The consumer earns wages from working in the intermediate good sector and earns interest on investing in technology M_Z. The budget constraint thus is $C = wL+rM_Z$. Maximizing utility in equation (6.1) with respect to consumption and investments yields the first order conditions $C^{-\theta}e^{-\rho t} = \lambda$ and $\dot{\lambda} = -r\lambda$ so that the growth rate of consumption is

$$g_c = \theta^{-1}(r - \rho)\ . \tag{34}$$

This is equal to output growth on the balanced growth path. We can thus solve for the interest rate and obtain $r = \theta g+\rho$. The free entry condition for the technology firms imposes that profits from investing in patents must be zero. Revenue per unit of R&D investment is given by V_Z, cost is equal to $\frac{1}{\eta_Z}$. Consequently, we obtain $\eta_Z V_Z = 1$. Making use of equation

(35), we obtain $\frac{\eta_Z \beta p_Z^{\frac{1}{\beta}} L}{r} = 1$. Solving this for r and substituting it into equation (34) we obtain the following proposition:

$$g = \theta^{-1}(\beta \eta_Z L p_Z^{\frac{1}{\beta}} - \rho) \ .$$

Adding the extractive sector to the standard model by Acemoglu (2002), changes the interest part of the Euler equation, $g = \theta^{-1}(r - \rho)$.[23] Instead of two exogenous production factors, the interest rate r in our model only includes labor, but adds the resource price, as p_Z depends on p_R according to equation (38). Together with (20), this yields the growth rate on the balanced growth path.

Proposition 6 *Suppose that*

$$\beta \left[(1-\gamma)_R^{\varepsilon} (\eta_R R^{Extr})^{\sigma-1} + \gamma_Z^{\varepsilon} (\eta_Z L)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} > \rho, \text{ and}$$
$$(1-\theta)\beta \left[\gamma_R^{\varepsilon} (\eta_R R^{Extr})^{\sigma-1} + \gamma_Z^{\varepsilon} (\eta_Z L)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} < \rho.$$

If $(1-\gamma)^{\varepsilon}(\eta_R \delta \mu)^{1-\varepsilon} < 1$ *the economy cannot produce. Otherwise, there exists a unique BGP equilibrium in which the relative technologies are given by equation* (40)*, and consumption and output grow at the rate in equation* (21)*.*[24]

[23] There is no capital in this model, but agents delay consumption by investing in R&D as a function of the interest rate.

[24] Starting with any $N_R(0) > 0$ and $N_Z(0) > 0$, there exists a unique equilibrium path. If $N_R(0)/N_Z(0) < (N_R/N_Z)^*$ as given by (40), then $M_{Rt} > 0$ and $M_{Zt} = 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. If $N_R(0)/N_Z(0) > (N_R/N_Z)^*$, then $M_{Rt} = 0$ and $M_{Zt} > 0$ until $N_{Rt}/N_{Zt} = (N_R/N_Z)^*$. It can also be verified that there are simple transitional dynamics in this economy whereby starting with technology levels $N_R(0)$ and $N_Z(0)$, there always exists a unique equilibrium path, and it involves the economy monotonically converging to the BGP equilibrium of (21) like in Acemoglu (2002).

Appendix 2 Directed Technological Change

Let V_Z and V_R be the BGP net present discounted values of new innovations in the two sectors. Then the Hamilton-Jacobi-Bellman Equation version of the value function for the intermediate good sector $r_t V_Z(j) - \dot{V}_Z(j) = \pi_Z(j)$ and the free entry condition of extraction technology firms imply that

$$V_Z = \frac{\beta p_Z^{1/\beta} L}{r^*}, \text{ and } V_R = \chi_R(j) - \psi_R \ , \tag{35}$$

where r^* is the BGP interest rate, while p_Z is the BGP price of the intermediate good and $\chi_R(j)$ is the BGP machine price in the extractive sector.

The greater is V_R relative to relative to V_Z, the greater are the incentives to develop machines in the extractive sector rather than developing machines in the intermediate good sector. Taking the ratio of the two equations in (35) and including the equilibrium machine price (19) yields

$$\frac{V_R}{V_Z} = \frac{\chi_R(j) - \psi_R}{\frac{1}{r}\beta p_Z^{\frac{1}{\beta}} L} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta p_Z^{\frac{1}{\beta}} L} \ . \tag{36}$$

This expression highlights the effects on the direction of technological change

1. The price effect manifests itself because V_R/V_Z is decreasing in p_Z. The greater is the intermediate good price, the smaller is V_R/V_Z and thus the greater are the incentives

to invent technology complementing labor. Since goods produced by the relatively scarce factor are relatively more expensive, the price effect favors technologies complementing the scarce factor. The resource price p_R does not affect V_R/V_Z due to perfect competition among extraction technology firms and a flat supply curve.

2. The market size effect is a consequence of the fact that V_R/V_Z is decreasing in L. Consequently an increase in the supply of labor translates into a greater market for the technology complementing labor. The market size effect in the intermediate good sector is defined by the exogenous factor labor. There is no equivalent in the extractive sector.

3. Finally, the cost of developing one new machine variety in terms of final output also influences the direction of technological change. If the parameter η increases, the cost goes down, the relative profitability V_R/V_Z decreases, and therefore the incentive to invent extraction technology declines.

Since the intermediate good price is endogenous, combining (33) with (36) the relative profitability of the technologies becomes

$$\frac{V_R}{V_Z} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta\left(p_R\frac{\gamma}{1-\gamma}\left(\frac{\delta\mu\dot{N}_R}{\frac{1}{1-\beta}p_Z^{\frac{1-\beta}{\beta}}N_Z L}\right)^{\frac{1}{\varepsilon}}\right)^{\frac{1}{\beta}}L} \tag{37}$$

Rearranging equation (33) we obtain

$$p_Z = \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon} p_R^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} . \tag{38}$$

Combining (38) and (20), we can eliminate relative prices, and the relative profitability of technologies becomes:

$$\frac{V_R}{V_Z} = \frac{\frac{1}{\eta_R}}{\frac{1}{r}\beta\left(\left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon}\left(\left(\frac{1}{\eta_R} + \psi_R\right)\frac{1}{\mu\delta}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}\right)^{\frac{1}{\beta}} L} .$$

Using the free-entry conditions and assuming that both of them hold as equalities, we obtain the following BGP *technology market clearing* condition:

$$\eta_Z V_Z = \eta_R V_R. \tag{39}$$

Combining 39 with 37, we obtain the following BGP ratio of relative technologies and solving for $\frac{\dot{N}_R}{N_Z}$ yields:

$$\left(\frac{\dot{N}_R}{N_Z}\right)^* = \left(\left(\frac{r}{\eta_Z \beta L}\right)^{\beta} \frac{1-\gamma}{\gamma p_R}\right)^{\varepsilon} \frac{L p_Z^{\frac{1-\beta}{\beta}}}{(1-\beta)\delta\mu} \tag{40}$$

where the asterisk ($*$) denotes that this expression refers to the BGP value. The relative productivities are determined by both prices and the supply of labor.

Appendix 3 The Case of Multiple Resources

We now extend the model and replace the generic resource with a set of distinct resources. We do so in analogy to a generic capital stock as in many growth models. We define resource extraction R^{Extr}, resource prices p_R and resource investments M_R as aggregates of the respective variables of different resources $i \in [0, G]$,

$$
\begin{aligned}
R^{Extr} &= \left(\sum_i R_i^{Extr\,\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \\
p_R &= \left(\sum_i \frac{R_i^{Extr}}{R^{Extr}} p_{R_i}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{1-\sigma}}, \\
M_R &= \sum_i M_{R_i}, \\
\frac{R^{Extr}}{Y} &= (1-\gamma)^{\epsilon} p_R{}^{-\epsilon}, \\
g &= \theta^{-1} \left(\beta \eta_Z L \left[\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} p_R{}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}\frac{1}{\beta}} - \rho \right),
\end{aligned}
$$

where σ is the elasticity of substitution between the different resources. Note that the aggregate resource price consists of the average of the individual resources weighted by their share in physical production.

This extension can be used to make theoretical predictions. As an example, we focus here on the relative price of two resources, aluminum a and copper c. Using equation (20) and assuming that the cost of producing machines ψ_R and the flow rate of innovations η_R are uniform across resources, we obtain that prices depend solely on geological and technological parameters:

$p_R^c = (\delta^c \mu^c)^{-1}$ and $p_R^a = (\delta^a \mu^a)^{-1}$.

Total resource production equals

$$R^{Extr} = \left(R^{Extr\, c\, \frac{\sigma-1}{\sigma}} + R^{Extr\, a\, \frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad ,$$

From this, we derive the following theoretical predictions:

$\frac{p_R^c}{p_R^a} \ = \frac{(\delta^a \mu^a)}{(\delta^c \mu^c)}$

and $\frac{R^{Extr\, c}}{R^{Extr\, a}} \ = (\frac{(\delta^c \mu^c)}{(\delta^a \mu^a)})^{\sigma}$,

$\frac{p_R^c R^{Extr\, c}}{p_R^a R^{Extr\, a}} \ = (\frac{\delta^a \mu^a}{\delta^c \mu^c})^{\sigma-1}$ and $\frac{\dot{N}_R^c}{\dot{N}_R^a} \ = (\frac{\delta^c \mu^c}{(\delta^a \mu^a)})^{\sigma-1} (\frac{\eta_R^c}{\eta_R^a})^{\sigma}$

We can investigate what happens when a new resource gets used (e.g. aluminum was not used until the end of the XIX^{th}). If we assume that $\sigma > 1$ and that the resource is immediately at its steady-state price, the price of the resource aggregate will immediately decline and the growth rate of the economy will increase: $p_R = ((\delta^c \mu^c)^{\sigma-1} + (\delta^a \mu^a)^{\sigma-1})^{\frac{1}{1-\sigma}}$.

Alternatively, a progressive increase in aluminum technology, $\dot{N}_R^a = \eta_R^a$ min $(N_R^a/\overline{N}, 1)$ M_R^a, would generate an initial decline in the real price (as η_R^a min $(N_R^a/\overline{N}, 1)$ increases) and faster growth in the use of aluminum initially. This is in line with historical evidence from the copper and aluminum markets.

Appendix 4 Regression Results

Table 2: Test for the stylized fact that growth rates of world primary production of non-renewable resources are positive over the long term.

Notes: The table presents results for regressions of non-renewable resource production growth rates (log differences) on a constant and one lagged dependent variable. To check for robustness across time, we run regressions for different sub-samples. As further robustness checks, we run regressions adding a linear trend, and regressions, which regress production in levels on a constant, lags, and a linear trend. These regressions produce similar results. Results are available upon request. Regressions use heteroscedasticity robust standard errors. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

	Aluminum	Antimony	Arsenic	Asbestos	Barite	Beryllium
Range	1855-2018	1867-2018	1892-2018	1880-2018	-	-
Constant	0.104***	0.036	0.011	0.065*	-	-
	(4.007)	(1.618)	(0.493)	(2.156)	-	-
Range	1900-2018	1900-2018	1900-2018	1900-2018	1914-2018	1936-2018
Constant	0.053**	0.024	0.011	0.041	0.041**	0.032
	(2.569)	(1.039)	(0.466)	(1.527)	(2.599)	(0.795)
Range	1875-1975	1875-1975	1892-1975	1880-1975	1914-1975	1936-1975
Constant	0.129***	0.058	0.019	0.117***	0.064***	0.051
	(3.544)	(1.789)	(0.596)	(2.636)	(2.878)	(0.710)

	Bismuth	Boron	Bromine	Cadmium	Cement	Chromium
Range	1826-2018	-	1881-2018	1852-2018	-	1896-2018
Constant	0.062	-	0.063**	0.086	-	0.055***
	(1.719)	-	(2.397)	(1.872)	-	(2.906)
Range	1900-2018	1901-2017	1900-2018	1900-2018	1927-2018	1900-2018
Constant	0.048**	0.047**	0.067*	0.072***	0.028*	0.054***
	(2.376)	(2.402)	(2.254)	(2.877)	(2.026)	(2.808)
Range	1875-1975	1901-1975	1881-1975	1875-1975	1927-1975	1896-1975
Constant	0.067	0.033*	0.089**	0.114*	0.031	0.060*
	(1.173)	(2.229)	(2.370)	(2.190)	(1.385)	(2.233)

	Cobalt	Copper	Diatomite	Feldspar	Fluorspar	Gallium
Range	-	1701-2018	-	-	-	-
Constant	-	0.026***	-	-	-	-
	-	(3.761)	-	-	-	-
Range	-	1800-2018	-	-	-	-
Constant	-	0.033***	-	-	-	-
	-	(3.426)	-	-	-	-
Range	1901-2018	1900-2018	1901-2018	1909-2018	1914-2018	1974-2018
Constant	0.065*	0.029*	0.060	0.057***	0.043	0.088
	(2.109)	(2.008)	(1.666)	(3.600)	(1.926)	(1.839)
Range	1901-1975	1875-1975	1901-1975	1909-1975	1914-1975	-
Constant	0.212	0.088	0.486	-0.100	0.068	-
	(0.414)	(1.749)	(0.737)	(-0.384)	(0.129)	-

	Garnet	Germanium	Gold	Graphite	Gypsum	Indium
Range	-	-	1701-2018	-	-	-
Constant	-	-	0.012***	-	-	-
	-	-	(3.674)	-	-	-
Range	-	-	1800-2018	1897-2018	-	-
Constant	-	-	0.016***	0.020	-	-
	-	-	(3.426)	(1.136)	-	-
Range	1914-2018	1958-2018	1900-2018	1900-2018	1925-2018	1973-2018
Constant	0.057*	0.019	0.012*	0.019	0.028	0.060
	(2.129)	(0.592)	(2.153)	(1.090)	(1.980)	(1.961)
Range	1914-1975	-	1875-1975	1897-1975	1925-1975	-
Constant	0.031	-	0.015*	0.020	0.033	-
	(0.915)	-	(2.115)	(0.857)	(1.557)	-

	Iodine	Kyanite	Lead	Lithium	Magnesium compounds	Magnesium metal
Range	-	-	1701-2018	-	-	-
Constant	-	-	0.012***	-	-	-
	-	-	(3.529)	-	-	-
Range	-	-	1800-2018	-	-	-
Constant	-	-	0.018***	-	-	-
	-	-	(3.590)	-	-	-
Range	1961-2018	1929-2018	1900-2018	1926-2018	1901-2018	1938-2018
Constant	0.032***	0.060*	0.012	0.069	0.060***	0.031
	(2.844)	(2.281)	(1.642)	(1.190)	(2.903)	(0.905)
Range	-	1929-1975	1875-1975	1926-1975	1901-1975	1938-1975
Constant	-	0.100*	0.018*	0.073	0.078**	0.044
	-	(2.086)	(2.084)	(0.679)	(2.461)	(0.653)

	Manganese	Mercury	Mica	Molybdenum	Nickel	Niobium
Range	-	1701-2018	-	-	-	-
Constant	-	0.008	-	-	-	-
	-	(0.834)	-	-	-	-
Range	1881-2018	1800-2018	-	-	1851-2018	-
Constant	0.044	0.004	-	-	0.070***	-
	(1.663)	(0.337)	-	-	(2.860)	-
Range	1900-2018	1900-2018	1901-2018	1901-2018	1900-2018	1965-2018
Constant	0.025	-0.001	-0.001	0.060	0.051**	0.072
	(0.923)	(-0.050)	(-0.031)	(1.811)	(2.480)	(1.782)
Range	1881-1975	1875-1975	1901-1975	1901-1975	1875-1975	-
Constant	0.058	0.011	0.046	0.081	0.086*	-
	(1.517)	(0.697)	(1.629)	(1.573)	(2.276)	-

	Nitrogen	Phosphate rock	Pig Iron	Platinum-group	Potash	Pumice
Range	-	-	1701-2018	-	-	-
Constant	-	-	0.034***	-	-	-
	-	-	(4.293)	-	-	-
Range	-	1897-2018	1800-2018	-	-	-
Constant	-	0.040***	0.042***	-	-	-
	-	(3.431)	(3.816)	-	-	-
Range	1947-2018	1900-2018	1900-2018	1901-2018	1920-2018	1921-2018
Constant	0.040*	0.039***	0.032*	0.036	0.037*	0.066**
	(2.248)	(3.313)	(2.022)	(1.943)	(1.996)	(2.375)
Range	-	1897-1975	1875-1975	1901-1975	1920-1975	1921-1975
Constant	-	0.052***	0.039*	0.046	0.052*	0.123**
	-	(2.997)	(2.072)	(1.615)	(2.026)	(2.445)

	Rare earths	Rhenium	Salt	Selenium	Silicon	Silver
Range	-	-	-	-	-	1701-2018
Constant	-	-	-	-	-	0.010***
	-	-	-	-	-	(2.612)
Range	-	-	1882-2018	-	-	1800-2018
Constant	-	-	0.037***	-	-	0.012*
	-	-	(5.651)	-	-	(2.201)
Range	1901-2018	1974-2018	1900-2018	1939-2018	1965-2018	1900-2018
Constant	0.051	0.059	0.035***	0.036	0.025*	0.011
	(0.848)	(1.858)	(5.160)	(1.810)	(2.198)	(1.381)
Constant	0.049	-	0.047***	0.050	0.072	0.013
	(0.516)	-	(5.148)	(1.364)	(2.032)	(1.333)
Range	1901-1975	-	1882-1975	1939-1975	1965-1975	1875-1975

	Strontium	Sulfur	Talc & pyrophyllite	Tantalum	Tellurium	Thallium
Range	1952-2018	1901-2018	1905-2018	1970-2018	1931-2003	1981-2011
Constant	0.045	0.035	0.048***	0.041	0.054	-0.005
	(1.255)	(1.899)	(3.807)	(1.230)	(0.898)	(-0.524)
Range	-	1901-1975	1905-1975	-	1931-1975	-
Constant	-	0.049	0.076***	-	0.099	-
	-	(1.696)	(4.003)	-	(1.002)	-

	Thorium	Tin	Tungsten	Uranium	Vanadium	Vermiculite
Range	-	1701-2018	-	-	-	-
Constant	-	0.018***	-	-	-	-
	-	(2.772)	-	-	-	-
Range	-	1800-2018	1871-2018	-	-	-
Constant	-	0.018**	0.051	-	-	-
	-	(2.306)	(1.787)	-	-	-
Range	1961-1977	1900-2018	1900-2018	1946-2017	1913-2018	1949-1998
Constant	0.025	0.010	0.033	0.037	0.047	0.013
	(0.438)	(0.887)	(1.254)	(1.076)	(1.118)	(0.871)
Range	-	1875-1975	1875-1975	-	1913-1975	-
Constant	-	0.016	0.078*	-	0.061	-
	-	(1.190)	(2.065)	-	(0.851)	-

	Wolalstonite	Zinc	Zirconium mineral concentrates	Crude Oil	Natural Gas	Coal
Range	-	1801-2018	-	1861-2018	1883-2018	1801-2018
Constant	-	0.029***	-	0.051***	0.018	0.027***
	-	(3.302)	-	(6.584)	(1.939)	(4.107)
Range	1951-2018	1900-2018	1945-2018	1900-2018	1900-2018	1900-2018
Constant	0.040***	0.022	0.070***	0.050***	0.042***	0.020***
	(2.659)	(1.824)	(3.218)	(5.553)	(5.239)	(2.635)
Range	-	1875-1975	1945-1975	1875-1975	1883-1975	1875-1975
Constant	-	0.028	0.131***	0.079***	0.022	0.025***
	-	(1.882)	(3.219)	(6.957)	(1.665)	(2.893)

Table 3: Test for the stylized fact that growth rates of real non-renewable resource prices are zero over the long term.

Notes: The table presents results for regressions of growth rates (log differences) of real non-renewable resource prices on a constant and one lagged dependent variable. To check for robustness across time, we run regressions for different sub-samples. As further robustness checks, we run regressions adding a linear trend, and regressions, which regress production in levels on a constant, lags, and a linear trend. These regressions produce similar results. Results are available upon request. Regressions use heteroscedasticity robust standard errors. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

	Aluminum	Antimony	Arsenic	Asbestos	Barite	Beryllium
Range	1855-2018	1863-2018	1894-2018	1881-2018	-	-
Constant	-0.029*	0.000	-0.008	0.006	-	-
	(-2.058)	(0.019)	(-0.445)	(0.194)	-	-
Range	1900-2018	1900-2018	1900-2018	1900-2018	1901-2018	1936-2018
Constant	-0.016	0.003	-0.011	0.013	-0.000	-0.033
	(-0.983)	(0.098)	(-0.615)	(0.353)	(-0.016)	(-1.164)
Range	1875-1975	1875-1975	1894-1975	1881-1975	1901-1975	1936-1975
Constant	-0.032	0.009	-0.005	-0.003	-0.005	-0.058
	(-1.848)	(0.325)	(-0.218)	(-0.077)	(-0.223)	(-1.921)

	Bismuth	Boron	Bromine	Cadmium	Cement	Chromium
Range	1826-2018	-	1881-2006	1854-2018	-	-
Constant	-0.004	-	-0.014	-0.015	-	-
	(-0.182)	-	(-0.650)	(-0.645)	-	-
Range	1900-2018	1901-2018	1900-2006	1900-2018	1901-2018	1901-2018
Constant	-0.015	-0.018	-0.020	-0.017	-0.001	0.003
	(-0.641)	(-0.737)	(-0.783)	(-0.549)	(-0.112)	(0.155)
Range	1875-1975	1901-1975	1881-1975	1875-1975	1901-1975	1901-1975
Constant	0.004	-0.017	-0.013	-0.007	0.002	0.006
	(0.222)	(-0.443)	(-0.505)	(-0.383)	(0.212)	(0.230)

	Cobalt	Copper	Diatomite	Feldspar	Fluorspar	Gallium
Range	-	1701-2018	-	-	-	-
Constant	-	-0.004	-	-	-	-
	-	(-0.506)	-	-	-	-
Range	-	1800-2018	-	-	-	-
Constant	-	-0.002	-	-	-	-
	-	(-0.194)	-	-	-	-
Range	1901-2018	1900-2018	1901-2018	1901-2018	1901-2018	1944-2018
Constant	-0.008	-0.005	0.001	-0.001	0.003	-0.070***
	(-0.181)	(-0.283)	(0.074)	(-0.081)	(0.260)	(-3.212)
Range	1901-1975	1875-1975	1901-1975	1901-1975	1901-1975	1944-1975
Constant	-0.024	-0.005	0.001	0.003	0.007	-0.064***
	(-0.413)	(-0.308)	(0.087)	(0.174)	(0.704)	(-2.934)

	Garnet	Germanium	Gold	Graphite	Gypsum	Indium
Range	-	-	1701-2018	-	-	-
Constant	-	-	0.001	-	-	-
	-	-	(0.192)	-	-	-
Range	-	-	1800-2018	1897-2018	-	-
Constant	-	-	0.004	0.006	-	-
	-	-	(0.553)	(0.251)	-	-
Range	1901-2018	1946-2018	1900-2018	1900-2018	1901-2018	1937-2018
Constant	-0.020	-0.016	0.003	0.001	-0.014	-0.036
	(-1.655)	(-0.619)	(0.297)	(0.029)	(-1.654)	(-0.908)
Range	1901-1975	-	1875-1975	1897-1975	1901-1975	1937-1975
Constant	-0.011	-	0.001	-0.008	-0.004	-0.072
	(-0.757)	-	(0.136)	(-0.250)	(-0.394)	(-1.779)

	Iodine	Kyanite	Lead	Lithium	Magnesium compounds	Magnesium metal
Range	-	-	1701-2018	-	-	-
Constant	-	-	0.001	-	-	-
	-	-	(0.177)	-	-	-
Range	-	-	1800-2018	-	-	-
Constant	-	-	0.003	-	-	-
	-	-	(0.238)	-	-	-
Range	1929-2018	1935-2018	1900-2018	1937-2018	1901-2015	1916-2018
Constant	-0.012	-0.004	-0.002	-0.015	0.006	-0.029
	(-0.662)	(-0.313)	(-0.102)	(-0.743)	(0.362)	(-1.797)
Range	1929-1975	1935-1975	1875-1975	1937-1975	1901-1975	1916-1975
Constant	-0.023	-0.000	-0.003	-0.054***	0.017	-0.036
	(-1.052)	(-0.022)	(-0.196)	(-3.401)	(0.670)	(-1.888)

	Manganese	Mercury	Mica	Molybdenum	Nickel	Niobium
Constant	-	0.003	-	-	-	-
	-	(0.265)	-	-	-	-
Range	-	1701-2018	-	-	-	-
Constant	0.009	0.005	-	-	-0.008	-
	(0.435)	(0.340)	-	-	(-0.529)	-
Range	1881-2018	1800-2018	-	-	1831-2018	-
Constant	0.014	0.006	-0.020	0.004	-0.003	0.008
	(0.597)	(0.244)	(-0.393)	(0.113)	(-0.162)	(0.210)
Range	1900-2018	1900-2018	1901-2018	1913-2018	1900-2018	1965-2000
Constant	0.009	-0.012	-0.013	0.011	-0.017	-
	(0.357)	(-0.499)	(-0.179)	(0.255)	(-1.229)	-
Range	1875-1975	1875-1975	1901-1975	1913-1975	1875-1975	-

	Nitrogen	Phosphate rock	Pig Iron	Platinum-group	Potash	Pumice
Range	-	-	1701-2018	-	-	-
Constant	-	-	-0.001	-	-	-
	-	-	(-0.152)	-	-	-
Range	-	1881-2018	1800-2018	-	-	-
Constant	-	-0.007	-0.003	-	-	-
	-	(-0.470)	(-0.351)	-	-	-
Range	1951-2018	1900-2018	1900-2018	1901-2018	1901-2018	1903-2018
Constant	-0.019	-0.003	-0.004	0.005	-0.013	-0.012
	(-0.548)	(-0.217)	(-0.295)	(0.230)	(-0.558)	(-0.423)
Range	-	1875-1975	1875-1975	1901-1975	1901-1975	1903-1975
Constant	-	-0.005	0.002	0.001	-0.022	-0.029
	-	(-0.285)	(0.128)	(0.075)	(-0.622)	(-0.728)

	Rare earths	Rhenium	Salt	Selenium	Silicon	Silver
Range	-	-		-	-	1701-2018
Constant	-	-		-	-	-0.004
	-	-	(-1.249)	-	-	(-0.471)
Range	-	-		-	-	1800-2018
Constant	-	-		-	-	-0.002
	-	-	(-1.410)	-	-	(-0.239)
Range	1923-2018	1965-2018	1900-2018	1910-2018	1924-2018	1900-2018
Constant	0.009	-0.028	-0.004	-0.010	0.005	-0.001
	(0.064)	(-0.412)	(-0.547)	(-0.335)	(0.294)	(-0.062)
Range	1923-1975	-	1875-1975	1910-1975	1924-1975	1875-1975
Constant	0.012	-	-0.012	-0.000	0.017	-0.003
	(0.048)	-	(-1.341)	(-0.009)	(0.671)	(-0.229)

	Strontium	Sulfur	Talc & pyrophyllite	Tantalum	Tellurium	Thallium
Range	1918-2018	1901-2018	1901-2018	1965-2018	1918-2018	1943-2018
Constant	0.030	-0.015	-0.002	0.006	-0.002	0.032
	(0.658)	(-0.263)	(-0.180)	(0.096)	(-0.059)	(0.993)
Range	1918-1975	1901-1975	1901-1975	-	1918-1975	1943-1975
Constant	0.059	0.007	-0.021	-	0.002	-0.045
	(0.782)	(0.338)	(-1.474)	-	(0.148)	(-1.841)

	Thorium	Tin	Tungsten	Uranium	Vanadium	Vermiculite
Range	-	1701-2018	-	-	-	-
Constant	-	0.001	-	-	-	-
	-	(0.161)	-	-	-	-
Range	-	1800-2018	1885-2018	-	-	-
Constant	-	0.002	-0.000	-	-	-
	-	(0.181)	(-0.014)	-	-	-
Range	1952-2018	1900-2018	1900-2018	1971-2018	1911-2018	1925-1998
Constant	-0.005	0.000	-0.006	-0.008	0.008	-0.013
	(-0.246)	(0.026)	(-0.196)	(-0.203)	(0.284)	(-0.685)
Range	-	1875-1975	1885-1975	-	1911-1975	1925-1975
Constant	-	0.006	0.002	-	0.004	-0.016
	-	(0.322)	(0.057)	-	(0.200)	(-0.631)

	Wolalstonite	Zinc	Zirconium mineral concentrates	Crude Oil	Natural Gas	Coal
Range	-	1760-2018	-	-	-	1701-2018
Constant	-	-0.005	-	-	-	-0.000
	-	(-0.382)	-	-	-	(-0.035)
Range	-	1800-2018	-	1862-2018	-	1800-2018
Constant	-	-0.004	-	0.005	-	0.000
	-	(-0.314)	-	(0.234)	-	(0.033)
Range	1951-2015	1900-2018	1919-2018	1900-2018	1901-2018	1900-2018
Constant	-0.003	0.001	0.003	0.005	0.005	0.005
	(-0.565)	(0.027)	(0.145)	(0.233)	(0.360)	(0.564)
Range	-	1875-1975	1919-1975	1875-1975	1901-1975	1875-1975
Constant	-	0.003	0.002	0.007	0.003	0.009
	-	(0.160)	(0.063)	(0.295)	(0.250)	(0.838)

	Crustal Abundance (Bil. mt)	**Reserves** (Bil. mt)	**Annual Output** (Bil. mt)	**Crustal Abundance/ Annual Output** (Years)	**Reserves/ Annual Output** (Years)
Aluminum	1,990,000,000[e]	30[b1]	0.06[a]	33,786,078,000	100[1]
Copper	1,510,000[e]	0.8[b]	0.02[b]	76,650,000	40
Iron	1,392,000,000[e]	83[b2]	0.06[a]	1,200,000,000	55[2]
Lead	290,000[e]	0.1[b]	0.005[b]	61,702,000	18
Tin	40,000[e]	0.005[b]	0.0003[b]	137,931,000	16
Zinc	2,250,000[e]	0.23[b]	0.013[b]	170,445,000	17
Gold	70[e]	0.0001[b]	0.000003[b]	22,076,000	17
Coal[3]		510[d]	3.9[d]		131
Crude Oil[4]	15,000,000[6f]	241[d]	4.4[d]	1,297,529	55
Nat. Gas[5]		179[d]	3.3[d]		54

Notes: [1]Data for bauxite, [2]data for iron ore, [3]includes lignite and hard coal, [4]includes conventional and unconventional oil, [5]includes conventional and unconventional gas, [6]all organic carbon in the earth's crust. Sources: [a]U.S. Geological Survey (2016), [b]U.S. Geological Survey (2018), [c]British Petroleum (2017), [d]Federal Institute for Geosciences and Natural Resources (2017), [e]Perman et al. (2003), [f]Littke and Welte (1992).

Table 4: Quantities of selected non-renewable resources in the crustal mass and in reserves, measured in metric tons and in years of production based on current annual mine production.