Federal Reserve Bank of Dallas

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**Working Paper 2026** **August 2020**

Research Department

https://doi.org/10.24149/wp2026

# The Business Cycle Mechanics of Search and Matching Models*

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August 18, 2020

**Abstract**

This paper estimates a real business cycle model with unemployment driven by shocks to labor productivity and the job separation rate. We make two contributions. First, we develop a new identification scheme based on the matching elasticity that allows the model to perfectly match a range of labor market moments, including the volatilities of unemployment and vacancies. Second, we use our model to revisit the importance of shocks to the job separation rate and highlight how their correlation with labor productivity affects their transmission mechanism.

**Keywords:** Unemployment; Vacancies; Separation Rate; Real Business Cycles; Estimation

**JEL Classifications**: C13; E24; E32; E37; J63

* This work was supported by computational resources provided by the Big-Tex High Performance Computing Group at the Federal Reserve Bank of Dallas. The views expressed in this paper are our own and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

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## 1 INTRODUCTION

Short-run fluctuations in unemployment are a key component of modern business cycles. However, even in a model without investment or risk aversion, Shimer (2005) shows labor market volatility remains low under common parameterizations. In response to this puzzle, recent work has used simple frameworks to shed light on mechanisms that generate empirically consistent labor market fluctuations (Hagedorn and Manovskii, 2008; Hall and Milgrom, 2008; Ljungqvist and Sargent, 2017; Pissarides, 2009). Using these observations as a point of departure, we provide new qualitative and quantitative insights into the business cycle mechanics of search and matching models.

This paper estimates a real business cycle model with equilibrium unemployment à la Diamond-Mortensen-Pissarides. The model is driven by estimated stochastic processes for labor productivity and the job separation rate, and is consistent with a wide range of business cycle moments for the U.S. economy. In particular, the model simultaneously matches the volatilities of aggregate consumption and investment, unemployment, and vacancies. This result sharply contrasts with earlier quantitative analyses of unemployment and the macroeconomy (Andolfatto, 1996; Merz, 1995).

More specifically, we make two contributions to the literature. First, we develop an identification scheme that allows the estimated model to exactly match a range of labor market moments, including the volatilities of both unemployment and vacancies. Our identification scheme highlights how model parameters are connected to the data in ways that calibration exercises often obscure. For example, we show how the elasticity of matches with respect to unemployment determines the volatility of vacancies relative to the volatility of unemployment. Intuitively, when the elasticity is higher, a given increase in matches requires a smaller increase in unemployment. Therefore, as matches fluctuate over the cycle, unemployment fluctuates less relative to vacancies.

To generate empirically consistent volatilities of unemployment and vacancies, we combine our insights related to the matching elasticity with existing points about the role of the "fundamental surplus," the difference between the marginal product of labor and a value that is not allocated to vacancy creation (Ljungqvist and Sargent, 2017). The fundamental surplus sets the level of labor market volatility, while the parameter governing the matching elasticity determines how the volatility is split between vacancies and unemployment. We demonstrate the power of our new approach by contrasting the results with a model in which the matching elasticity is implied by targeting average labor market tightness (e.g., Hagedorn and Manovskii, 2008). Under this approach, the model produces substantial labor market volatility, but the split between unemployment and vacancies is far from the data. The implied matching elasticity is also outside of the plausible range (Mortensen and Nagypal, 2007; Petrongolo and Pissarides, 2001), in sharp contrast with our estimated model.

Our second contribution uses our model to revisit the transmission of shocks to the job separation rate. We begin by offering a structural interpretation of an open empirical question in the literature: how much unemployment volatility is explained by variation in the job separation rate? While recent empirical work has found that job separation rate shocks drive at most $25\%$ of the variation in unemployment, we reach a different conclusion. Variation in the job separation rate accounts for $60\%$ of short-term unemployment volatility and around $50\%$ of longer-run volatility.

We emphasize that the contribution of job separation rate shocks is consistent with the reduced-form empirical evidence that favors the job finding rate. Our structural decomposition acknowledges that the finding rate is itself an endogenous function of variation in the separation rate and labor productivity. We find that separation rate shocks account for around $30\%$ of variation in the job finding rate. Once we account for this, the contribution of separation rate shocks naturally increases and is potent at short-run horizons. This finding is consistent with Elsby et al. (2009), who argue that job separations are crucial for unemployment dynamics at the start of most recessions.

We then turn to the transmission of job separation rate shocks, distinguishing between shocks that affect only the job separation rate and shocks that also affect labor productivity according to the strength of their empirical correlation. Accounting for this correlation is crucial. In the absence of an associated decline in labor productivity, an increase in job separations causes unemployment and vacancies to increase, which contrasts with the negatively sloped Beveridge curve observed in the data. Allowing for a correlated decline in labor productivity in our model prevents this counterfactual outcome and strengthens the macroeconomic responses to a job separation rate shock.

**Related Literature** The literature on search and matching in a business cycle setting is extensive. Our analysis stays close to the quantitative tradition of the early literature (Andolfatto, 1996; Den Haan et al., 2000; Merz, 1995), while incorporating the insights of the recent literature that abstracts from capital and curvature in utility (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017; Mortensen and Nagypal, 2007; Shimer, 2005). Relative to these influential papers, we show that an estimated model can simultaneously match a range of moments in goods and labor markets. In estimating our model, we provide a new way of identifying a key model parameter that drives relative labor market volatility and complements the insights of the recent literature. We also provide a structural analysis of the importance of job separation rate shocks and their transmission.

We adopt a data-driven approach to modeling job separation rate shocks that contrasts with analyses that model job separations as purely endogenous (Den Haan et al., 2000; Fujita and Ramey, 2012). In these models, job separations and labor productivity are essentially perfectly negatively correlated. We measure this correlation in the data and find that it is negative but far from perfect, indicating there is considerable variation in the job separation rate that is uncorrelated with changes in labor productivity. To capture this fact parsimoniously, we assume variation in the job separation rate is exogenous but correlated with labor productivity, and discipline this correlation using the data. In this sense, our approach is similar to Coles and Kelishomi (2018), who study how vacancy adjustment frictions affect the labor market in a search and matching model.

Our estimation-based approach is related to recent work by Christiano et al. (2016), who estimate a New Keynesian model with labor market frictions to match identified impulse responses of macroeconomic variables to monetary and technology shocks. Relative to their analysis, we offer a complementary approach to the identification of key model parameters. Furthermore, we pay special attention to the role of job separation rate shocks in driving volatility in the macroeconomy.

Finally, our focus on job separation rate shocks speaks to the empirical literature on cyclical gross job flows (Elsby et al., 2009; Shimer, 2012). We complement this literature with a structural model-based decomposition of unemployment volatility into its underlying driving forces. Our decomposition controls for the endogeneity of the job finding rate and highlights a larger role for job separation rate shocks in driving unemployment dynamics than the empirical literature finds.

The paper proceeds as follows. Section 2 introduces our model, while Section 3 details our data and estimation strategy. Section 4 presents our main quantitative results. Section 5 concludes.

## 2 MODEL

We situate our analysis in a real business cycle model, augmented to include a frictional labor market similar to Merz (1995) and Andolfatto (1996). Relative to Shimer (2005), these frameworks include capital and curvature in utility. Time is discrete and the population is normalized to unity.

**Aggregate Shocks** There are two sources of aggregate fluctuations: shocks to labor productivity $\varepsilon_{a,t}$ and the job separation rate $\varepsilon_{s,t}$. The shocks are independent standard normal random variables.

**Search and Matching** At the beginning of period $t$, the employment rate is $n_{t-1}$. A fraction $s_t$ of employed workers are then separated from their jobs. The exogenous job separation rate follows

$$\ln s_{t+1} = (1-\rho_s)\ln \bar{s} + \rho_s \ln s_t + \rho_{as}\sigma_a\varepsilon_{a,t+1} + \sigma_s\varepsilon_{s,t+1}, \tag{1}$$

where $\rho_{as}$ determines the cross-correlation between the job separation rate and labor productivity.

We assume newly separated workers are able to search for new jobs within the same period as their job loss. However, it is natural that these workers will have less time to search for new jobs than those who became unemployed in a previous period. Therefore, let $\chi \in [0,1]$ denote the fraction of a period that newly unemployed workers spend searching for work within the same period as their job loss. Then the number of unemployed people searching for work is given by

$$u_t^s = u_{t-1} + \chi s_t n_{t-1}. \tag{2}$$

Shimer (2005) makes this point when constructing a measure of the monthly job finding rate in the data. To deal with this "time aggregation bias", he sets $\chi = 0.5$, while we estimate the value of $\chi$.

Following Den Haan et al. (2000), if a firm posts $v_t$ vacancies the number of matches is given by

$$m_t = u_t^s v_t/((u_t^s)^\iota + (v_t)^\iota)^{1/\iota}, \tag{3}$$

where $\iota > 0$ controls the elasticity of matches with respect to unemployed searching. Define $\theta_t \equiv v_t/u_t^s$ as labor market tightness. Then the job finding rate $f_t$ and job filling rate $q_t$ are given by

$$f_t = m_t/u_t^s = 1/(1+\theta_t^{-\iota})^{1/\iota}, \tag{4}$$

$$q_t = m_t/v_t = 1/(1+\theta_t^{\iota})^{1/\iota}. \tag{5}$$

Following Blanchard and Galí (2010), we assume newly matched workers begin employment in the same period they are matched with a firm, so aggregate employment evolves according to

$$n_t = (1-s_t)n_{t-1} + m_t. \tag{6}$$

The unemployment rate, $u_t$, includes anyone who is not employed in period $t$, so it satisfies

$$u_t \equiv u_t^s - m_t = 1 - n_t. \tag{7}$$

**Households** Following Merz (1995) and Andolfatto (1996), employed and unemployed workers pool their incomes in a representative family. A family head chooses optimal paths for consumption and capital investment, taking the paths for aggregate employment and unemployment as given. Investment is subject to capital adjustment costs, so the capital stock evolves according to

$$k_t = (1-\delta)k_{t-1} + \left(a_1 + \frac{a_2}{1-1/\nu}\left(\frac{i_t}{k_{t-1}}\right)^{1-1/\nu}\right)k_{t-1}, \tag{8}$$

where $0 < \delta < 1$ is the capital depreciation rate, $\nu > 0$ determines the size of the capital adjustment cost, and $a_1 = \delta/(1-\nu)$ and $a_2 = \delta^{1/\nu}$ are chosen so there are no adjustment costs in steady state.

The representative family solves

$$J_t^H = \max_{c_t, i_t, k_t} \ln c_t + \beta E_t[J_{t+1}^H] \tag{9}$$

subject to (8) and

$$c_t + i_t = w_t n_t + r_t^k k_{t-1} + d_t, \tag{10}$$

$$n_{t+1} = (1 - s_{t+1}(1-\chi f_{t+1}))n_t + f_{t+1}u_t, \tag{11}$$

$$u_{t+1} = s_{t+1}(1-\chi f_{t+1})n_t + (1-f_{t+1})u_t, \tag{12}$$

where $w_t$ is the wage, $r_t^k$ is the rental return on capital, $d_t$ is firm dividends, and $E_t$ is the mathematical expectation operator conditional on information at time $t$. The optimality conditions imply

$$\frac{1}{a_2}\left(\frac{i_t}{k_{t-1}}\right)^{1/\nu} = E_t\left[x_{t+1}\left(r^k_{t+1} + \frac{1}{a_2}\left(\frac{i_{t+1}}{k_t}\right)^{1/\nu}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{i_{t+1}}{k_t}\right)\right], \tag{13}$$

where $x_t = \beta(c_{t-1}/c_t)$ is the household's stochastic discount factor. Condition (13) says the marginal cost of investing in period $t$ is equal to the marginal benefit in period $t+1$, which includes the return on capital, the undepreciated capital stock, and the foregone capital adjustment cost.

**Firms** A representative firm produces output with a Cobb-Douglas production function given by

$$y_t = k^{\alpha}_{t-1}(a_t n_t)^{1-\alpha}, \tag{14}$$

where $0 < \alpha < 1$ is the income share of capital, and labor productivity $a_t$ evolves according to

$$\ln a_{t+1} = (1-\rho_a)\ln \bar{a} + \rho_a \ln a_t + \rho_{as}\sigma_s\varepsilon_{s,t+1} + \sigma_a\varepsilon_{a,t+1}. \tag{15}$$

To hire workers, the firm posts vacancies $v_t$ that are subject to a per unit cost $\kappa$. In addition, the firm rents capital from the household at rental rate $r^k_t$ and pays its workers a wage $w_t$ determined by a Nash bargaining process described below. Therefore, the firm maximizes profits by solving

$$J^F_t = \max_{k_{t-1},n_t,v_t} y_t - w_t n_t - r^k_t k_{t-1} - \kappa v_t + E_t[x_{t+1}J^F_{t+1}] \tag{16}$$

subject to (14) and

$$n_t = (1-s_t)n_{t-1} + q_t v_t, \tag{17}$$

$$v_t \geq 0. \tag{18}$$

Letting $\lambda_{n,t}$ denote the Lagrange multiplier on (17), the optimality conditions are given by

$$r^k_t = \alpha y_t/k_{t-1}, \tag{19}$$

$$\lambda_{n,t} = (1-\alpha)y_t/n_t - w_t + E_t[x_{t+1}(1-s_{t+1})\lambda_{n,t+1}], \tag{20}$$

$$q_t\lambda_{n,t} = \kappa - \lambda_{0,t}. \tag{21}$$

The first condition (19) sets the marginal product of capital equal to its rental rate, while (20) recursively defines the marginal benefit of hiring an additional worker. Finally, (21) states that the firm's optimal vacancy creation choice sets the expected marginal benefit of a vacancy $q_t\lambda_{n,t}$ equal to its marginal cost: the costs of creating the vacancy today minus the savings from relaxing the non-negativity constraint. Also note that $\lambda_{n,t}$ is the marginal surplus of a new match to the firm.

**Wage Rate** As noted by Hall (2005), there are many ways to determine wages in search and matching models. To keep our analysis transparent, we follow the bulk of the literature and assume wages are determined via Nash Bargaining between an employed worker and the firm. The total surplus of a match is $\Lambda_t = \lambda_{n,t} + J_t^E$, where a worker's surplus from employment is given by[1]

$$J_t^E = w_t - b + E_t[x_{t+1}(1 - f_{t+1} - s_{t+1}(1 - \chi f_{t+1}))J_{t+1}^E]. \tag{22}$$

In this definition, $b$ captures a worker's flow value from unemployment, measured in units of consumption. Crucially, $b$ determines the worker's outside option in the Nash bargaining process.

The equilibrium wage rate maximizes $(J_t^E)^\eta \lambda_{n,t}^{1-\eta}$, where $\eta \in [0, 1]$ is the household's bargaining weight. The optimality condition implies $J_t^E = \eta\Lambda_t$ or, equivalently, $\lambda_{n,t} = (1 - \eta)\Lambda_t$. To derive the equilibrium wage rate, plug (22) in for $J_t^E$. Then plug (20) in for $\lambda_{n,t}$ and combine to obtain

$$w_t = \eta((1 - \alpha)y_t/n_t + E_t[x_{t+1}(1 - \chi s_{t+1})f_{t+1}\lambda_{n,t+1}]) + (1 - \eta)b. \tag{23}$$

The wage rate in period $t$ is a weighted average of the firm's value of a new match and the worker's outside option. The firm's value of a new worker includes the additional output produced plus the discounted expected value of the foregone vacancy cost net of separations that occur in period $t+1$.

We are deliberately agnostic about the sources of the worker's outside option $b$. In reality, it could consist of many components from both the worker's and firm's microeconomic environments. For example, the worker may receive unemployment benefits or attain a utility payoff from leisure time. Alternatively, if the firm faces fixed costs of hiring such as training costs or layoff taxes (Petrosky-Nadeau et al., 2018; Pissarides, 2009), then the worker may be able to bargain over how much of these costs are reflected in wages, effectively improving the threat point. Furthermore, other wage bargaining protocols could alter the worker's outside option. For example, in the alternative-offer bargaining protocol of Hall and Milgrom (2008), the worker's bargaining position is stronger when firms incur higher costs from delaying the period when the wage is agreed upon.

We estimate $b$ to match the volatilities of unemployment and vacancies in the data. Following Ljungqvist and Sargent (2017), what matters for this estimation is the size of $b$ relative to the marginal product of labor. The decomposition of $b$ into its underlying components is irrelevant. In addition, it is misleading to only model a subset of the possible components of $b$. For example, in a model in which $b$ reflects mainly the value of the leisure component, Chodorow-Reich and Karabarbounis (2016) show the resulting pro-cyclicality of the worker's outside option dampens

[1] To derive (22), note that $J_t^E \equiv W_t - U_t$, where

$$W_t = w_t + E_t[x_{t+1}((1 - s_{t+1}(1 - \chi f_{t+1}))W_{t+1} + s_{t+1}(1 - \chi f_{t+1})U_{t+1})]$$
$$U_t = b + E_t[x_{t+1}(f_{t+1}W_{t+1} + (1 - f_{t+1})U_{t+1})]$$

are the values of employment and unemployment to an individual worker.

labor market volatility even if its average level is high. Therefore, there must be another component of $b$ that is strongly countercyclical to offset the dampening effect. We make $b$ an estimated parameter, since it is beyond the scope of this paper to decipher the exact microeconomic nature of $b$.[2]

**Equilibrium** The aggregate resource constraint is given by

$$c_t + i_t + \kappa v_t = y_t. \tag{24}$$

A competitive equilibrium includes sequences of quantities $\{c_t, i_t, n_t, k_t, y_t, u_t, u_t^s, v_t, q_t, \lambda_{0,t}\}_{t=0}^{\infty}$, prices $\{w_t, r_t^k\}_{t=0}^{\infty}$, and exogenous variables $\{a_t, s_t\}_{t=1}^{\infty}$ that satisfy (1), (2), (5)-(8), (13)-(15), (19)-(21), (23), and (24), given the initial conditions $\{k_{-1}, n_{-1}, a_{-1}, s_{-1}\}$ and shocks, $\{\varepsilon_{a,t}, \varepsilon_{s,t}\}_{t=0}^{\infty}$.

## 3 Data and Estimation Procedure

This section begins by describing our data and the empirical targets in our estimation. It then outlines our new identification scheme and provides a detailed account of our estimation methodology.

### 3.1 Empirical Targets

The model is disciplined using a balanced sample from 1955Q1-2019Q4. Appendix A provides a description of our data sources and how they were transformed.

**Labor Market Moments** The job finding rate, $f_t = 1 - (U_{t+1} - U_{t+1}^s)/U_t$ is based on Shimer (2005), where $U_t$ is total unemployed and $U_t^s$ is the subset who are unemployed $1$ month or less. Following Shimer (2012), the monthly job separation rate is $s_t \equiv 1 - \exp(-\tilde{s}_t)$, where $\tilde{s}_t$ satisfies

$$U_{t+1} = (1 - e^{-\tilde{f}_t - \tilde{s}_t})\tilde{s}_t LF_t/(\tilde{f}_t + \tilde{s}_t) + e^{-\tilde{f}_t - \tilde{s}_t} U_t,$$

$LF_t$ is the labor force, and $\tilde{f}_t \equiv -\log(1 - f_t)$. The unemployment rate is $u_t = U_t/LF_t$. The vacancy rate $v_t$ is based on the series in Barnichon (2010) until 2000, after which it is equal to job openings as a share of the labor force in the Job Openings and Labor Turnover Survey. These series correct for trends in the print and online help-wanted indexes published by the Conference Board. The rates are converted to a quarterly frequency by averaging across the months in each quarter.

Following Shimer (2005), labor productivity is output per job in the non-farm business sector, while the wage rate is the ratio of labor compensation to employment in the non-farm business sector. To remove time trends, we filter the data following Hamilton (2018) (henceforth, Hamilton) by regressing each variable on its most recent four lags after an $8$ quarter window. We use this approach over the more common Hodrick and Prescott (1997) filter because Hodrick (2020) shows the Hamilton filter performs better when time series, such as these, are first difference stationary.

Using these time series, we compute the following estimation targets: the means of the quarterly unemployment, job finding, and job separation rates, the standard deviations of the unem-

[2] Mortensen and Nagypal (2007) combine multiple components of $b$ to generate realistic labor market volatility.

ployment and vacancy rates, the standard deviation and autocorrelation of the job separation rate, the standard deviation and autocorrelation of labor productivity, the cross-correlation between labor productivity and the job separation rate, and the elasticity of wages with respect to productivity (computed as the slope coefficient from regressing log wages on an intercept and log productivity).

**Goods Market Moments** In addition to the 11 labor market moments, we target the standard deviations and autocorrelations of consumption and investment growth. Consumption includes expenditures on services and nondurables. Investment is composed of durable consumption and private fixed investment. The growth rates are computed as quarter-over-quarter log differences.

3.2 IDENTIFICATION Before estimating the model, we first describe the mapping between the model parameters and moments that are measurable in the data. We estimate 12 model parameters: $b, \iota, \eta, \kappa, \chi, \bar{s}, \rho_s, \sigma_s, \rho_a, \sigma_a, \sigma_{a,s}, \nu$. While these parameters are jointly estimated, we can heuristically describe how each parameter is identified from specific moments that we compute in the data.

| Parameters | Identifying Moments |
|---|---|
| $b, \iota$ | $SD(u), SD(v)$ |
| $\eta$ | $Cov(w,a)/Var(a)$ |
| $\kappa, \chi$ | $E(u), E(f)$ |
| $\bar{s}, \rho_s, \sigma_s, \rho_a, \sigma_a, \sigma_{a,s}$ | $E(s), AC(s), SD(s), AC(a), SD(a), Corr(a,s)$ |
| $\nu$ | $SD(c), SD(i), AC(c), AC(i)$ |

Table 1: Identification heuristic. $E$, $SD$, $Var$, $AC$, $Corr$, $Cov$ denote the mean, standard deviation, variance, autocorrelation, cross-correlation, and covariance over our balanced sample from 1955Q1-2019Q4.

Table 1 summarizes the identification scheme. The outside option $b$ governs the economy's "fundamental surplus fraction" (Ljungqvist and Sargent, 2017), defined as the upper bound on the fraction of a worker's output that can be allocated to vacancy creation. It is now well understood that a small fundamental surplus fraction is crucial to deliver realistically large volatilities of unemployment and vacancies (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017). To see this, consider the steady-state vacancy creation condition in a model without capital ($\alpha = 0$),

$$\frac{\kappa}{\bar{q}} = \frac{(1-\eta)(\bar{a}-b)}{1-\beta(1-\bar{s})+\eta\beta(1-\chi\bar{s})\bar{f}}, \tag{25}$$

where bars denote steady states. The elasticity of tightness with respect to productivity is given by,

$$\bar{\epsilon}_{\theta,a} = \frac{\bar{a}}{\bar{a}-b} \times \frac{1-\beta(1-\bar{s})+\eta\beta(1-\chi\bar{s})\bar{f}}{\bar{\epsilon}_{m,u^s}(1-\beta(1-\bar{s}))+\eta\beta(1-\chi\bar{s})\bar{f}} \tag{26}$$

where $(\bar{a}-b)/\bar{a}$ is the fundamental surplus fraction and $\bar{\epsilon}_{m,u^s}$ is the steady-state elasticity of matches with respect to the mass of unemployed searching. The second term in this expression is near unity since $\beta(1-\bar{s}) \approx 1$ at a monthly frequency. Therefore, to generate a large response of tightness (and hence unemployment and vacancies) to changes in productivity, the fundamental surplus fraction must be small, which requires that $b$ is close to the marginal product of labor $\bar{a}$. A small fundamental surplus fraction makes the fundamental surplus very sensitive to changes in productivity, which causes volatile changes in the resources allocated to vacancy creation. Hence, we estimate $b$ by targeting the standard deviations of unemployment and vacancies in the data.

While $b$ affects the overall level of labor market volatility, we now show that $\iota$ affects the relative volatilities of vacancies and unemployment. First note that our matching function specification implies that $\bar{\epsilon}_{m,u^s} = \bar{f}^{\iota}$, where $\bar{f}$ is the steady-state job finding rate. Therefore, given an average job finding rate (that we target using other parameters), $\iota$ pins down the elasticity of matches with respect to unemployed searching.[3] To see the role that $\bar{\epsilon}_{m,u^s}$ plays, we compute the elasticities of unemployment and vacancies with respect to tightness in the simplified model, which are given by[4]

$$\bar{\epsilon}_{u,\theta} = -(1-\bar{u})(1-\bar{\epsilon}_{m,u^s})/(1-\chi\bar{f}), \tag{27}$$

$$\bar{\epsilon}_{v,\theta} = 1-(1-\bar{u})(1-\bar{\epsilon}_{m,u^s})/(1-\chi\bar{f}). \tag{28}$$

As $\bar{\epsilon}_{m,u^s}$ increases, the responsiveness of vacancies to changes in tightness grows relative to the responsiveness of unemployment. Intuitively, when the elasticity is higher, a given increase in matches requires a smaller increase in unemployed searching, and hence in unemployment. Therefore, when matches fluctuate, unemployment fluctuates less relative to vacancies. Hence, we estimate $\iota$ by targeting the *relative* standard deviations of unemployment and vacancies in the data.

Recall from (23) that $\eta$ governs the responsiveness of wages to changes in the marginal product of labor, which is driven by labor productivity. Hence, we follow Hagedorn and Manovskii (2008) and estimate $\eta$ by targeting the empirical elasticity of wages with respect to labor productivity.

The last two labor market parameters $\kappa$ and $\chi$ are estimated by targeting the average unemployment rate and job finding rate. To see this mapping, consider the following steady-state conditions

$$\bar{f} = \bar{\theta}/(1+\bar{\theta}^{\iota})^{1/\iota}, \tag{29}$$

$$(\kappa/\bar{q})(1-\beta(1-\bar{s})+\eta\beta(1-\chi\bar{s})\bar{f}) = (1-\eta)(\bar{a}-b), \tag{30}$$

$$\bar{u} = \bar{s}(1-\chi\bar{f})/(\bar{s}(1-\chi\bar{f})+\bar{f}). \tag{31}$$

Given $\iota$, targeting the average job finding rate identifies the average tightness $\bar{\theta}$ from (29), and hence $\bar{q} = \bar{f}/\bar{\theta}$. Combining with a target for average unemployment, we can then solve (30) and (31) for $\kappa$ and $\chi$, given all of the other parameters. The parameters governing the exogenous processes $\bar{s}, \rho_s, \sigma_s, \rho_a, \sigma_a, \rho_{a,s}$ have empirical counterparts in the data, so we estimate these parameters

[3] Our argument also applies to the Cobb-Douglas matching function $m_t = \mu(u_t^s)^{\alpha} v_t^{1-\alpha}$. In this case, $\bar{\epsilon}_{m,u^s} = \alpha$.

[4] These come from differentiating the steady-state conditions $\bar{u} = \bar{s}(1-\chi\bar{f})/(\bar{s}(1-\chi\bar{f})+\bar{f})$ and $\bar{v} = \bar{\theta}\bar{u}^s$.

by directly targeting these moments. Finally, we estimate the capital adjustment cost parameter $\nu$ by targeting the standard deviations and autocorrelations of investment and consumption growth.

**3.3 ESTIMATION PROCEDURE** First, we externally set three parameters in line with the business cycle literature. The time discount factor, $\beta$, is set to $0.9983$, which implies an annual real interest rate of $2\%$. The capital depreciation rate, $\delta = 0.0077$, matches the annual average rate on private fixed assets and consumer durable goods converted to a monthly rate. The income share of capital, $\alpha = 0.3845$, equals the complement of the quarterly labor share in the non-farm business sector.

The $15$ target moments are stored in $\hat{\Psi}_T^D$ and estimated using a two-step Generalized Method of Moments (GMM) estimator with a balanced sample of $T = 260$ quarters. Given the GMM estimates, we estimate our model with Simulated Method of Moments (SMM) to account for potential short-sample bias. For parameterization $\vartheta$ and shocks $\mathcal{E} = \{a, s\}$, we solve the log-linear model using Sims (2002) `gensys` algorithm and simulate it $R = 1{,}000$ times for $T$ periods. The model analogues of the target moments are the median moments across the $R$ simulations, $\bar{\Psi}_{R,T}^M(\theta, \mathcal{E})$.

The parameter estimates, $\hat{\vartheta}$, are obtained by minimizing the following loss function:

$$J(\vartheta, \mathcal{E}) = [\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\vartheta, \mathcal{E})]'[\hat{\Sigma}_T^D(1 + 1/R)]^{-1}[\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\vartheta, \mathcal{E})],$$

where $\hat{\Sigma}_T^D$ is the diagonal of the GMM estimate of the variance-covariance matrix. We use Monte Carlo methods to calculate the standard errors on the parameter estimates. For different sequences of shocks, we re-estimate the structural model $N_h = 100$ times and report the mean and $(5, 95)$ percentiles of the parameter estimates.[5] Appendix B provides additional details on the methodology.

# 4 QUANTITATIVE ANALYSIS

In this section, we first report the estimated parameters and demonstrate the quantitative fit of our model. We then use the estimated model as a lens to study the transmission of job separation rate shocks and explore how inflows and outflows from unemployment affect business cycle dynamics.

**4.1 PARAMETER ESTIMATES AND MODEL FIT** Table 2 reports the targeted empirical and simulated moments across a range of specifications. To gauge the success of our identification scheme for the labor market parameters, we first compare the data to the model when we only target labor market moments. As shown in the "Baseline-Labor" column, the model is able to perfectly match the data for each targeted moment, indicating the strength of our proposed identification scheme.

The "Baseline-Labor" column of Table 3 reports the corresponding estimated parameters. The mean estimates are well within conventional ranges and the standard errors are small. Relative

[5] Ruge-Murcia (2012) applies SMM to several nonlinear business cycle models and finds that asymptotic standard errors tend to overstate the variability of the estimates. This underscores the importance of using Monte Carlo methods.

| | | Labor | Labor & Goods | |
|---|---|---|---|---|
| Moment | Data | Baseline | Baseline | $\theta = 0.634$ |
| $E(u)$ | 5.89 | 5.89<br>(0.00) | 5.92<br>(0.13) | 5.95<br>(0.24) |
| $E(f)$ | 42.14 | 42.14<br>(0.00) | 42.02<br>(−0.11) | 43.36<br>(1.08) |
| $E(s)$ | 3.27 | 3.27<br>(0.00) | 3.27<br>(−0.01) | 3.27<br>(−0.01) |
| $SD(u)$ | 22.25 | 22.25<br>(0.00) | 22.70<br>(0.24) | 30.97<br>(4.79) |
| $SD(v)$ | 22.99 | 22.99<br>(0.00) | 22.95<br>(−0.02) | 16.16<br>(−3.57) |
| $SD(s)$ | 8.66 | 8.66<br>(0.00) | 8.67<br>(0.01) | 8.67<br>(0.01) |
| $SD(a)$ | 2.63 | 2.63<br>(0.00) | 2.68<br>(0.27) | 2.68<br>(0.27) |
| $SD(\Delta \log c)$ | 0.51 | 0.16*<br>(−8.36) | 0.50<br>(−0.22) | 0.56<br>(1.10) |
| $SD(\Delta \log i)$ | 2.12 | 2.88*<br>(4.22) | 2.06<br>(−0.34) | 2.23<br>(0.61) |
| $AC(s)$ | 0.79 | 0.79<br>(0.00) | 0.79<br>(0.09) | 0.79<br>(0.09) |
| $AC(a)$ | 0.90 | 0.90<br>(0.00) | 0.91<br>(0.34) | 0.91<br>(0.34) |
| $AC(\Delta \log c)$ | 0.29 | 0.45*<br>(1.93) | 0.23<br>(−0.76) | 0.23<br>(−0.71) |
| $AC(\Delta \log i)$ | 0.44 | 0.21*<br>(−3.17) | 0.21<br>(−3.10) | 0.22<br>(−3.05) |
| $Corr(s, a)$ | −0.47 | −0.47<br>(0.00) | −0.47<br>(−0.01) | −0.47<br>(−0.01) |
| $Cov(w, a)/Var(a)$ | 0.47 | 0.47<br>(0.00) | 0.45<br>(−0.30) | 0.41<br>(−0.94) |
| $J$ | | 0.00 | 10.72 | 49.41 |

Table 2: Model fit for targeted moments. The t-statistic for the null hypothesis that a model-implied moment equals its empirical counterpart is shown in parentheses. The first column reports the mean GMM estimates of the empirical targets. The second column sets $\nu \to \infty$ and only targets the labor market moments (asterisks indicate non-targeted moments). The third column includes the goods market moments and estimates $\nu$. The final column sets $\iota$ to target average labor market tightness, $\bar{\theta} = 0.634$, following Hagedorn and Manovskii (2008) and sets $\kappa$ to retain the mean finding rate in the data. All monthly time series are averaged to a quarterly frequency and the data is detrended using a Hamilton (2018) filter with an 8 quarter window.

to Hagedorn and Manovskii (2008), we estimate a considerably larger bargaining weight $\eta = 0.46$ (cf., $\eta = 0.052$). The higher estimate of $\eta$ is due to the presence of capital in our model, which weakens the response of the marginal product of labor to changes in labor productivity. The estimate of $\kappa$ implies that vacancy creation costs account for less than $1\%$ of output on average. Our estimate of $b = 0.96$ is consistent with the small fundamental surplus required to generate realistic labor market volatility (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017).

| | Labor | Labor & Goods | |
|---|---|---|---|
| Parameter | Baseline | Baseline | $\theta = 0.634$ |
| $\bar{s}$ | $0.0325$ $(0.0325, 0.0326)$ | $0.0325$ $(0.0325, 0.0326)$ | $0.0325$ $(0.0325, 0.0326)$ |
| $\rho_a$ | $0.9533$ $(0.9524, 0.9542)$ | $0.9577$ $(0.9569, 0.9587)$ | $0.9577$ $(0.9569, 0.9587)$ |
| $\sigma_a$ | $0.0073$ $(0.0073, 0.0074)$ | $0.0071$ $(0.0070, 0.0073)$ | $0.0071$ $(0.0070, 0.0073)$ |
| $\iota$ | $0.5955$ $(0.5908, 0.6007)$ | $0.6046$ $(0.5964, 0.6115)$ | $1.1419$ $(1.1419, 1.1419)$ |
| $\eta$ | $0.4621$ $(0.4445, 0.4836)$ | $0.4787$ $(0.4155, 0.5457)$ | $0.4787$ $(0.4155, 0.5457)$ |
| $b$ | $0.9599$ $(0.9596, 0.9603)$ | $0.9622$ $(0.9617, 0.9626)$ | $0.9622$ $(0.9617, 0.9626)$ |
| $\kappa$ | $0.0214$ $(0.0196, 0.0229)$ | $0.0202$ $(0.0157, 0.0255)$ | $0.0572$ $(0.0434, 0.0721)$ |
| $\chi$ | $0.5334$ $(0.5278, 0.5388)$ | $0.5337$ $(0.5271, 0.5404)$ | $0.5337$ $(0.5271, 0.5404)$ |
| $\rho_s$ | $0.8940$ $(0.8927, 0.8952)$ | $0.8961$ $(0.8937, 0.8982)$ | $0.8961$ $(0.8937, 0.8982)$ |
| $\sigma_s$ | $0.0414$ $(0.0412, 0.0416)$ | $0.0410$ $(0.0406, 0.0415)$ | $0.0410$ $(0.0406, 0.0415)$ |
| $\rho_{as}$ | $-0.0999$ $(-0.1016, -0.0983)$ | $-0.1002$ $(-0.1030, -0.0966)$ | $-0.1002$ $(-0.1030, -0.0966)$ |
| $\nu$ | – | $4.9659$ $(4.9201, 5.0096)$ | $4.9659$ $(4.9201, 5.0096)$ |

Table 3: Mean estimates of the model parameters. The $(5, 95)$ percentiles are shown in parentheses. Each column corresponds to a different estimation specification. The first column sets $\nu \to \infty$ and only targets the labor market moments. The second column includes the goods market moments and estimates $\nu$. The final column sets $\iota$ to target average labor market tightness, $\bar{\theta} = 0.634$, following Hagedorn and Manovskii (2008) and sets $\kappa$ to retain the mean finding rate in the data (the remaining parameters are not re-estimated).

Our identification strategy yields an estimate for $\chi = 0.53$. This value is remarkably close to the assumption of $\chi = 0.5$ that Shimer (2005) makes when computing his empirical measure of the job separation rate series. Following his intuition, $\chi = 0.53$ implies newly separated workers have around two weeks on average to find another job before the next measurement of unemployment.[6]

Having established the success of our identification scheme, we now examine the first column of the "Labor & Goods" moments section of Table 2. In this specification, we estimate all parameters including $\nu$, using the full list of moments in Table 1. The first takeaway is that the model's performance in the labor market dimension remains strong when we also target the goods market moments. The t-statistics for the null hypothesis that a model-implied moment equals its empirical counterpart remain close to zero. The estimated model parameters are also essentially unchanged.

Second, targeting good market moments and estimating $\nu$ improves the model's ability to match

[6]We compute our own series for the empirical separation rate using the continuous time methodology in Shimer (2012). Therefore, our finding that $\chi$ is close to 0.5 is not simply a result of our construction of the separation rate.

the volatilities and autocorrelations of consumption and investment growth. When only targeting the labor market moments, the estimated model fails to reproduce the standard deviations and autocorrelations of consumption and investment growth in the data. In the fully estimated model, only the autocorrelation of investment growth remains significantly different from the data.[7] Given this success, we conduct the rest of our analyses using the model that targets all 15 data moments.

**Matching Function Identification** We estimate the matching function curvature parameter $\iota$ so the model produces empirically consistent relative volatilities of unemployment and vacancies. This approach contrasts with the literature, which often calibrates $\iota$ to hit a steady-state target. To demonstrate the advantages of our approach, we compare our results to a model calibrated in the style of Hagedorn and Manovskii (2008). Holding all other parameters fixed at their estimated values, we set $\iota$ to target an average tightness of $\bar{\theta} = 0.634$, and set $\kappa$ to retain an average finding rate in the data. The final columns of Table 2 and Table 3 shows the estimated moments and parameters.

Under this calibration, labor market volatility remains substantially elevated as a result of the small fundamental surplus fraction. However, the relative volatilities of unemployment and vacancies are now far from the data. The model over-predicts unemployment volatility and under-predicts vacancy volatility. This discrepancy is explained by the matching elasticities implied by the baseline estimation ($\bar{\epsilon}_{m,u^s} = 0.59$) and alternate calibration ($\bar{\epsilon}_{m,u^s} = 0.37$). The lower elasticity implies that unemployment volatility must increase relative to vacancy volatility. As a result, the model is unable to match these key targets, even though it still performs well in other dimensions.[8] Furthermore, our estimate of $\bar{\epsilon}_{m,u^s}$ is in the middle of the plausible range of elasticities (0.5-0.7) highlighted by Mortensen and Nagypal (2007), while the alternate elasticity is far below the range.

Finally, we stress that our approach is *not* inconsistent with also targeting an average value for labor market tightness or, equivalently, long run values for the job finding and job filling rates. To achieve this, we could augment the matching function with an efficiency parameter $\mu$ so that $m_t = \mu u_t^s v_t/((u_t^s)^\iota + (v_t)^\iota)^{1/\iota}$. We could then estimate $\mu$ to target an average value for $\theta$ using the steady-state relationship $\bar{f} = \mu\bar{\theta}/(1+\bar{\theta}^\iota)^{1/\iota}$, while still using $\iota$ to attain a matching elasticity $\bar{\epsilon}_{m,u^s} = \bar{f}^\iota/\mu^\iota$ that implies the best fit of the unemployment and vacancy volatilities in the data.[9]

**Non-targeted moments** To further validate our estimated framework, we report a range of non-targeted moments. Table 4 shows several key labor market moments. We report the results when only the labor market moments in Table 1 are targeted, but we focus on the case that targets all 15 moments. The model produces a range of non-targeted labor market moments that are close to

[7]Our model is deliberately parsimonious to highlight our main points related to the labor market. Other features such as home production, habits in preferences, and variable capital utilization could further improve the model's fit.

[8]Hagedorn and Manovskii (2008) calibrate their model at a weekly frequency. Their calibration implies a matching elasticity of $\bar{\epsilon}_{m,u^s} = 0.45$ that results in counterfactually high vacancy volatility relative to unemployment volatility.

[9]Using a Cobb-Douglas matching function $m_t = \mu u_t^\alpha v_t^{1-\alpha}$, Shimer (2005) exploits the same feature to argue that average tightness is irrelevant since $\mu$ can always target it. We adopt his logic by normalizing $\mu$ to unity in our model.

| | | Labor | Labor & Goods | |
|---|---|---|---|---|
| Moment | Data | Baseline | Baseline | $\theta = 0.634$ |
| $E(z)$ | 2.55 | 2.55<br>$(-0.01)$ | 2.55<br>$(0.02)$ | 2.54<br>$(-0.08)$ |
| $SD(f)$ | 15.87 | 15.46<br>$(-0.20)$ | 15.82<br>$(-0.03)$ | 22.94<br>$(3.41)$ |
| $SD(z)$ | 12.21 | 10.98<br>$(-1.26)$ | 11.08<br>$(-1.16)$ | 12.64<br>$(0.44)$ |
| $AC(f)$ | 0.90 | 0.93<br>$(0.88)$ | 0.93<br>$(0.83)$ | 0.93<br>$(0.78)$ |
| $AC(u)$ | 0.92 | 0.93<br>$(0.35)$ | 0.93<br>$(0.35)$ | 0.94<br>$(0.56)$ |
| $AC(v)$ | 0.93 | 0.87<br>$(-2.09)$ | 0.87<br>$(-2.27)$ | 0.75<br>$(-6.15)$ |
| $AC(z)$ | 0.87 | 0.82<br>$(-1.37)$ | 0.83<br>$(-1.26)$ | 0.84<br>$(-0.77)$ |
| $Cov(\frac{\bar{s}}{\bar{s}+f}, u)/Var(u)$ | 72.90 | 69.15<br>$(-0.27)$ | 69.16<br>$(-0.27)$ | 73.66<br>$(0.06)$ |

Table 4: Model fit for non-targeted moments. The t-statistic for the null hypothesis that a model-implied moment equals its empirical counterpart is shown in parentheses. The first column reports the mean GMM estimates of the non-targeted data moments. The second column sets $\nu \to \infty$ and only targets the labor market moments in Table 1. The third column includes the goods market moments in Table 1 and estimates $\nu$. The final column sets $\iota$ to target average labor market tightness, $\bar{\theta} = 0.634$, following Hagedorn and Manovskii (2008) and sets $\kappa$ to retain the mean finding rate in the data. All monthly time series are averaged to a quarterly frequency and the data is detrended using a Hamilton (2018) filter with an 8 quarter window.

their empirical counterparts. In particular, the model almost exactly matches the volatility of the job finding rate and closely matches the volatility of the net unemployment inflow rate $z_t = s_t(1-\chi f_t)$. In addition, the model generates realistic persistence in unemployment, vacancies, and the job finding and inflow rates, as indicated by their high autocorrelations, which are all close to the data.

These findings further emphasize the advantage of how we estimate $\iota$. Comparing results to the alternate calibration that sets $\bar{\theta} = 0.634$, the job finding rate becomes far too volatile due to the excess volatility of unemployment, while vacancies are no longer persistent enough. The additional vacancy persistence in our baseline model is generated by the higher matching elasticity, which strengthens the propagation mechanism from the underlying persistent shocks. The baseline model also generates a realistic decomposition of the fluctuations of unemployment into inflows and outflows. Following Shimer (2012), we compute the share of unemployment volatility explained by outflows (i.e., the job finding rate) by regressing $\bar{s}/(\bar{s} + f_t)$ on the unemployment rate. This yields an outflow share of $69\%$ in the model, which accords well with the $73\%$ reported in the data.

We now turn to Table 5, which reports the correlations analyzed by Shimer (2005). The baseline model closely matches most of the cross-correlations in the data. For example, it matches the Beveridge curve (i.e., the correlation between vacancies and unemployment). It is also successful

| | | Labor | Labor & Goods | |
|---|---|---|---|---|
| Moment | Data | Baseline | Baseline | $\theta = 0.634$ |
| $Corr(u, v)$ | $-0.77$ | $-0.78$ $(-0.19)$ | $-0.78$ $(-0.23)$ | $-0.68$ $(1.88)$ |
| $Corr(u, f)$ | $-0.85$ | $-0.94$ $(-3.31)$ | $-0.95$ $(-3.38)$ | $-0.97$ $(-4.14)$ |
| $Corr(u, s)$ | $0.44$ | $0.59$ $(1.69)$ | $0.60$ $(1.72)$ | $0.52$ $(0.94)$ |
| $Corr(u, a)$ | $-0.28$ | $-0.90$ $(-5.66)$ | $-0.92$ $(-5.85)$ | $-0.92$ $(-5.84)$ |
| $Corr(v, f)$ | $0.82$ | $0.94$ $(3.87)$ | $0.94$ $(3.83)$ | $0.84$ $(0.79)$ |
| $Corr(v, s)$ | $-0.39$ | $-0.07$ $(3.59)$ | $-0.08$ $(3.45)$ | $0.05$ $(4.91)$ |
| $Corr(v, a)$ | $0.12$ | $0.82$ $(6.45)$ | $0.84$ $(6.65)$ | $0.73$ $(5.57)$ |
| $Corr(f, s)$ | $-0.25$ | $-0.34$ $(-0.72)$ | $-0.35$ $(-0.82)$ | $-0.34$ $(-0.77)$ |
| $Corr(f, a)$ | $0.20$ | $0.91$ $(6.09)$ | $0.93$ $(6.29)$ | $0.92$ $(6.17)$ |

Table 5: Model fit for non-targeted correlations. The t-statistic for the null hypothesis that a model-implied moment equals its empirical counterpart is shown in parentheses. The first column reports the mean GMM estimates of the non-targeted data moments. The second column sets $\nu \to \infty$ and only targets the labor market moments in Table 1. The third column includes the goods market moments in Table 1 and estimates $\nu$. The final column sets $\iota$ to target average labor market tightness, $\bar{\theta} = 0.634$, following Hagedorn and Manovskii (2008) and sets $\kappa$ to retain the mean finding rate in the data. All monthly time series are averaged to a quarterly frequency and the data is detrended using a Hamilton (2018) filter with an $8$ quarter window.

at producing reasonable cross-correlations with vacancies, though the correlation with the job separation rate is quite small relative to the data. Importantly, our identification scheme produces a closer fit of the data than the alternate calibration along these key dimensions. For example, targeting $\bar{\theta} = 0.634$ results in a much flatter Beveridge curve since unemployment fluctuates more than vacancies. Furthermore, vacancies become positively correlated with the job separation rate.[10]

### 4.2 THE ROLE OF JOB SEPARATION RATE SHOCKS

Our baseline model includes a realistic stochastic process for job separation rate shocks, based on their careful measurement from the underlying employment flows data. Given this, we ask what role job separation rate shocks play in driving and propagating the business cycle. We begin by establishing that variation in the job separation rate is responsible for a significant fraction of business cycle variation in the economy. To show this, Figure 1 reports the normalized forecast error variance decomposition for output, the job finding rate, unemployment, and vacancies. In each plot, we decompose the volatility into

[10]Similar to Shimer (2005) and Hagedorn and Manovskii (2008), the baseline model overstates the correlation between unemployment and labor productivity in the data. However, as Barnichon (2012) shows, the empirical correlation switched sign from negative to positive in the 1980s, making it difficult to draw direct comparisons to the data.

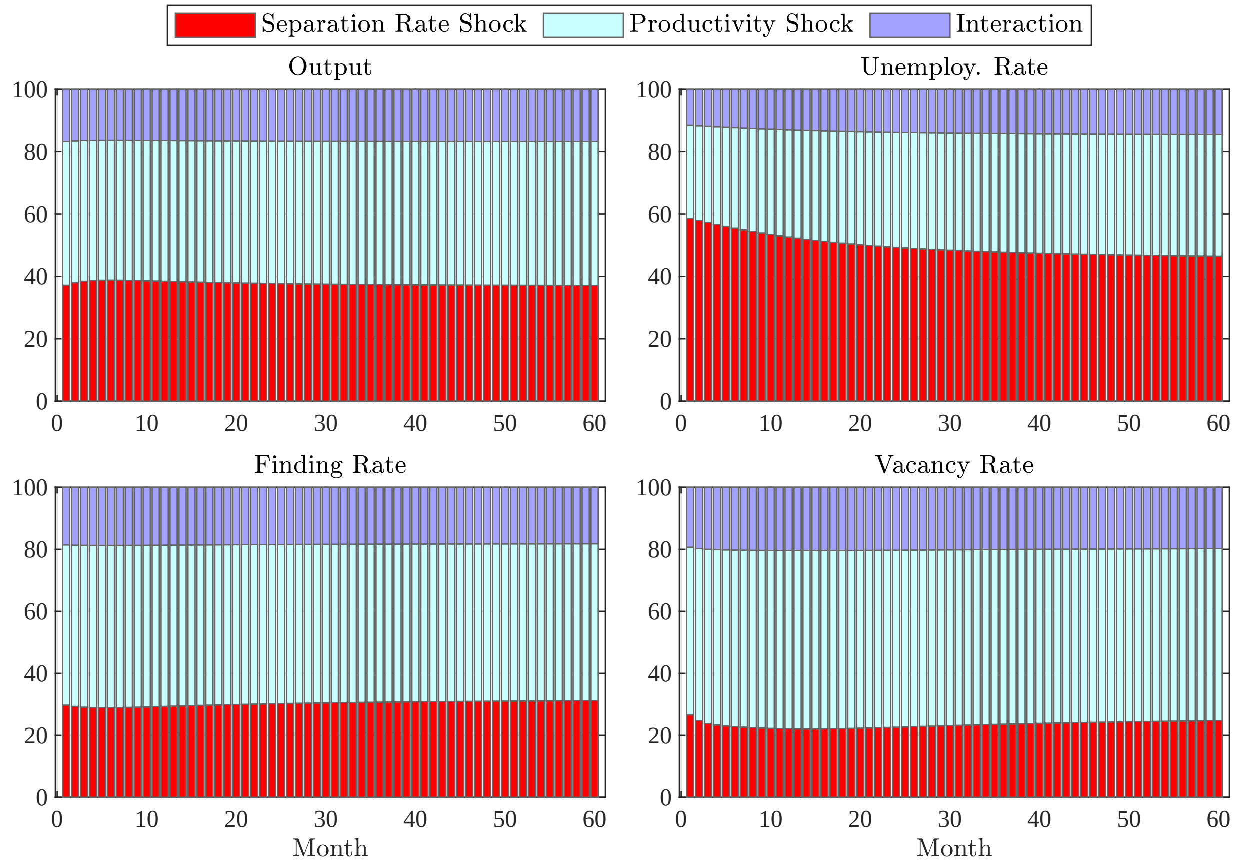


Figure 1: Normalized forecast error variance decomposition. Each plot decomposes the forecast error variance into variation from job separation rate shocks, labor productivity shocks, and their interaction. Values are then normalized by the sum of the components, since the contributions will not necessarily sum to unity.

three components attributable to variation in the job separation rate, labor productivity, and their interaction. The forecast error variance is normalized by the sum of these three components, since the correlation between the shocks implies that the contributions will not necessarily sum to unity.[11]

In all cases, variation in just the job separation rate accounts for at least $20\%$ of the overall volatility. Furthermore, job separation rate shocks account for $60\%$ of short-run unemployment volatility and close to $50\%$ of the volatility at longer horizons. This result contrasts with empirical analyses that conclude that separation rate variation accounts for no more than $25\%$ of unemployment volatility, with most variation driven by volatility in the job finding rate. We emphasize that our model is consistent with this reduced-form result (see Table 4), but also allows us to decompose unemployment into its structural components. This decomposition acknowledges that the finding rate is itself an endogenous function of variation in the job separation rate and labor productivity. As Figure 1 shows, pure separation rate shocks account for around $30\%$ of variation in the job finding rate. Once we account for this, the contribution of separation rate shocks naturally increases,

[11] Isakin and Ngo (2020) use the same approach to normalize the forecast error variance for a fully nonlinear model.

and is particularly potent at short-run horizons. This finding is consistent with Elsby et al. (2009), who argue that job separations are important for unemployment dynamics at the start of recessions.

The left column of Figure 2 plots the impulse responses to two types of job separation rate shocks. In the first (the "interacted" shock), labor productivity responds according to the estimated correlation coefficient $\rho_{as}$. In the second (the "pure" shock), the correlation between labor productivity and job separations is turned off so only the separation rate responds to the shock. In both cases, the shock is such that the job separation rate increases by two standard deviations on impact.

Consider first the more empirically relevant case of the "interacted shock". In response to a $0.27$ percentage point increase in the separation rate and the associated $0.82\%$ decrease in labor productivity, macroeconomic activity declines. Output falls by $0.81\%$ and the unemployment rate increases by $0.52$ percentage points. The response of vacancies reflects two opposing forces. First, the decline in labor productivity lowers the profitability of new hires and causes a drop in vacancy creation. Second, the increase in unemployment raises the job filling rate, lowering the marginal cost of vacancy creation. The drop in marginal costs causes vacancies to quickly rebound before declining again. As a result of the increase in unemployment and decline in vacancy creation, the job finding rate drops by $1.61$ percentage points in response to the positive separation rate shock.

When we artificially shut down the correlation between job separations and labor productivity, the macroeconomic responses are qualitatively different. Without the decline in labor productivity, vacancy creation increases in response to the shock since the job filling rate is higher. As a result, the job finding rate actually increases slightly on impact. Together, these responses mute the increase in unemployment and fall in output stemming from the shock. These results highlight the importance of accounting for the correlation between the job separation rate and labor productivity when analyzing the transmission of separation rate shocks. While pure separation rate shocks produce counterfactual positive co-movements between unemployment and vacancies (Shimer, 2005), allowing for a realistic degree of correlation with labor productivity corrects this behavior (Table 5 shows the unconditional correlation of unemployment and vacancies is close to the data).[12]

## 5 CONCLUSION

This paper shows an estimated real business cycle model with equilibrium unemployment is able to replicate a wide range of empirical business cycle moments. Our identification strategy highlights a new role for the elasticity of matches with respect to unemployment in generating realistic labor market volatility in unemployment and vacancies. We use our model to emphasize the importance

[12] Den Haan et al. (2000) provide a mechanism in which labor productivity shocks drive endogenous movements in the job separation rate. Our analysis suggests that mechanisms with the opposite direction of causality are also relevant. While we are deliberately agnostic about the sources of this correlation, our results suggest that future work could focus on developing micro-foundations for mechanisms that link labor productivity to changes in the rate of job separation.

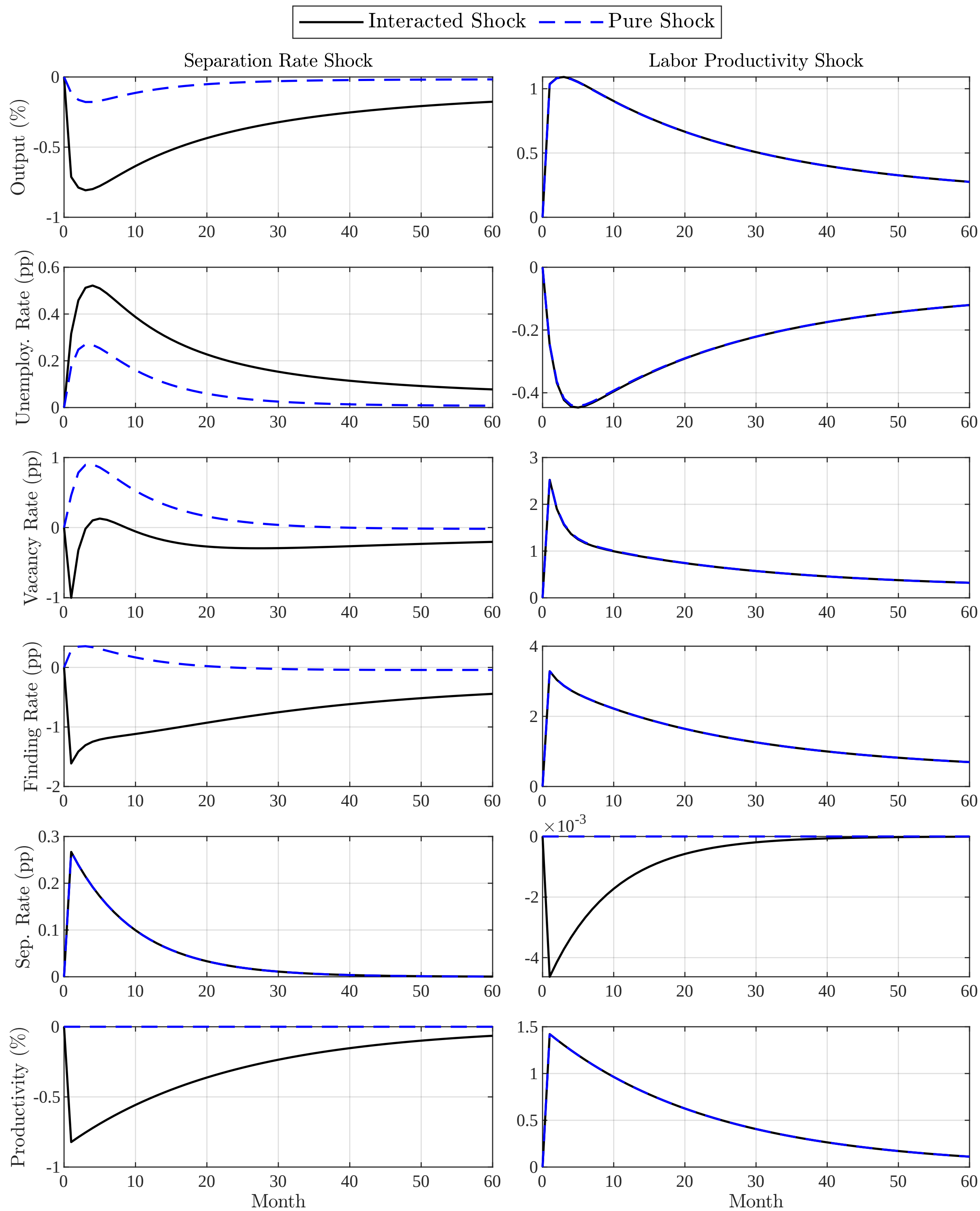


Figure 2: Impulse responses to a +2SD shock in percent (%) or percentage point (pp) deviations from steady state. The "Pure Shock" only affects the exogenous variable being shocked, while the "Interacted Shock" includes the cross-effect caused by the correlation between the job separation rate and labor productivity.

of job separation rate shocks in driving unemployment volatility and also show that accounting for their correlation with labor productivity is crucial to produce a realistic transmission mechanism.

There are several directions one could extend our benchmark model. First, we have abstracted from two margins that seem important for a complete account of business cycle labor market dynamics: on the job search and labor force participation. Second, we have intentionally built our insights on the foundation of the representative household real business cycle model, and as such have abstracted from household heterogeneity in income, consumption, and employment status. Introducing these features into our quantitative framework could be useful goals for future research.

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## A Data Sources and Transformations

We use the following time-series from 1955-2019 provided by Haver Analytics:

1. **Civilian Noninstitutional Population: 16 Years & Over**
   Not Seasonally Adjusted, Quarterly, Thousands (LN16N@USECON)
2. **Gross Domestic Product: Implicit Price Deflator**
   Seasonally Adjusted, Quarterly, 2012=100 (DGDP@USNA)
3. **Gross Domestic Product**
   Seasonally Adjusted, Quarterly, Billions of Dollars (GDP@USECON)
4. **Personal Consumption Expenditures: Nondurable Goods**
   Seasonally Adjusted, Quarterly, Billions of Dollars (CN@USECON)
5. **Personal Consumption Expenditures: Services**
   Seasonally Adjusted, Quarterly, Billions of Dollars (CS@USECON)
6. **Private Fixed Investment**
   Seasonally Adjusted, Quarterly, Billions of Dollars (F@USECON)
7. **Personal Consumption Expenditures: Durable Goods**
   Seasonally Adjusted, Quarterly, Billions of Dollars (CD@USECON)
8. **Output Per Person**, Non-farm Business Sector, All Persons,
   Seasonally Adjusted, Quarterly, 2012=100 (LXNFS@USNA)
9. **Labor Share**, Non-farm Business Sector, All Persons,
   Seasonally Adjusted, Quarterly, Percent (LXNFBL@USNA)
10. **Compensation**, Non-farm Business Sector, All Persons,
    Seasonally Adjusted, Quarterly, 2012=100 (LXNFF@USNA)
11. **Employment**, Non-farm Business Sector, All Persons,
    Seasonally Adjusted, Quarterly, 2012=100 (LXNFM@USNA)
12. **Unemployed, 16 Years & Over**
    Seasonally Adjusted, Monthly, Thousands (LTU@USECON)

13. **Civilian Unemployment Rate: 16 yr & Over**
    Seasonally Adjusted, Monthly, Percent (LR@USECON)
14. **Civilian Labor Force: 16 yr & Over**
    Seasonally Adjusted, Monthly, Thousands (LF@USECON)
15. **Civilians Unemployed for Less Than 5 Weeks**
    Seasonally Adjusted, Monthly, Thousands (LU0@USECON)

We also use the Help Wanted Advertising Index (HWI) from Barnichon (2010), which is in units of the labor force. This series corrects for online advertising and is available on the author's website.

We applied the following transformations to the above data sources:

1. **Per Capita Real Output Growth**:

$$\Delta \log Y_t = 100 \left( \log \left( \frac{GDP_t}{DGDP_t + LN16N_t} \right) - \log \left( \frac{GDP_{t-1}}{DGDP_{t-1} + LN16N_{t-1}} \right) \right).$$

2. **Per Capita Real Consumption Growth**:

$$\Delta \log C_t = 100 \left( \log \left( \frac{CN_t + CS_t}{DGDP_t + LN16N_t} \right) - \log \left( \frac{CN_{t-1} + CS_{t-1}}{DGDP_{t-1} + LN16N_{t-1}} \right) \right).$$

3. **Per Capita Real Investment Growth**:

$$\Delta \log I_t = 100 \left( \log \left( \frac{F_t + CD_t}{DGDP_t + LN16N_t} \right) - \log \left( \frac{F_{t-1} + CD_{t-1}}{DGDP_{t-1} + LN16N_{t-1}} \right) \right).$$

4. **Vacancy Rate**: $HWI$ from 1954M1-2000M12 and $LJJTLA/LF$ from 2001M1-2019M12.
5. **Short-term Unemployed** ($U^s$): The redesign of the Current Population Survey (CPS) in 1994 reduced $u_t^s$. To correct for this bias, we use IMPUMS-CPS data to scale $u_t^s$ by the ratio of $u_t^s/u_t$ for the first and fifth rotations groups to $u_t^s/u_t$ across all rotation groups. In addition to the $9$ mandatory identification variables, we first extract the following: EMPSTAT ("Employment Status"), DURUNEMP ("Continuous weeks unemployed") and MISH ("Month in sample, household level"). Unemployed persons have EMPSTAT of 20, 21, or 22. Short-term unemployed are persons who are unemployed and DURUNEMP is 5 or less. Incoming rotation groups have MISH of 1 or 5. Using the final weights, WTFINL, we then calculate unemployment rates conditional on the appropriate values of MISH and DURUNEMP. We then apply the X-12 seasonal adjustment function in STATA to the time series for the ratio. Finally, we take an average of the seasonally adjusted time series. This process yields an average ratio of $1.1693$, so $U^s$ equals $LU0$ before 1994 and $1.1693 \times LU0$ after 1994.
6. **Job-Finding Rate**: $f_t = 1 - (LTU_t - U_t^s)/LTU_{t-1}$.

7. **Separation Rate**: $s_t = 1 - \exp(-\tilde{s}_t)$, where $\tilde{s}_t$ satisfies

$$LTU_{t+1} = \frac{(1 - \exp(-\tilde{f}_t - \tilde{s}_t))\tilde{s}_t LF_t}{\tilde{f}_t + \tilde{s}_t} + \exp(-\tilde{f}_t - \tilde{s}_t)LTU_t,$$

   and $\tilde{f}_t = -\log(1 - f_t)$.

8. **Net Unemployment Inflow Rate**: $z_t = U_t^s/(LF_{t-1} - LTU_{t-1})$.
9. **Real Wage**: $w_t = 100 \times LXNFF_t/(LXNFM_t \times DGDP_t)$

All monthly time series are averaged to a quarterly frequency. The data is detrended using a Hamilton filter with an $8$ quarter window. All empirical targets are computed using quarterly data.

## B ESTIMATION METHOD

The estimation procedure has two stages. The first stage estimates moments in the data using a 2-step Generalized Method of Moments (GMM) estimator with a Newey and West (1987) weighting matrix with $5$ lags. The second stage is a Simulated Method of Moments (SMM) procedure that searches for a parameter vector that minimizes the distance between the mean GMM estimates in the data and short-sample predictions of the model, weighted by the diagonal of the GMM estimate of the variance-covariance matrix. The second stage is repeated for many different draws of shocks to obtain a sampling distribution for each parameter. The following steps outline the algorithm:

1. Use GMM to estimate the moments, $\hat{\Psi}_T^D$, and the diagonal of the covariance matrix, $\hat{\Sigma}_T^D$.
2. Use SMM to estimate the linear structural model. Given a random seed, $h$, draw a $B + 3T$ period sequence for each shock in the model, where $B$ is a $1{,}000$ period burn-in and $3T$ is the sample size of the monthly time series. Denote the shock matrix by $\mathcal{E}^h = [\varepsilon_s^h, \varepsilon_a^h]_{t=1}^{B+3T})$.

   For shock sequence $h \in \{1, \ldots, N_h\}$, run the following steps:

   (a) Specify a guess, $\hat{\theta}_0$, for the $N_p$ estimated parameters and the covariance matrix, $\Sigma_P^{h,0}$. For all $i \in \{1, \ldots, N_m\}$, apply the following steps:

      i. Draw $\hat{\theta}_i$ from a multivariate normal distribution centered at some mean parameter vector, $\bar{\theta}$, with a diagonal covariance matrix, $\Sigma_0$.

      ii. Solve the linear model with Sims's (2002) `gensys` algorithm given $\hat{\theta}_i$. Repeat the previous step if the solution does not exist or is not unique.

      iii. Given $\mathcal{E}^h(r)$, simulate the monthly model $R$ times for $B + 3T$ periods. We draw initial states from the ergodic distribution by burning off the first $B$ periods. Aggregate variables in levels by summing and rates by averaging to a quarterly frequency. For each repetition $r$, calculate the moments based on $T$ quarters, $\Psi_T^M(\hat{\theta}_i, \mathcal{E}^h(r))$, the same length as the quarterly data.

iv. Calculate the median moments across the $R$ simulations,

$$\bar{\Psi}^M_{R,T}(\hat{\theta}_i, \mathcal{E}^h) = \text{median}\{\Psi^M_T(\hat{\theta}_i, \mathcal{E}^h(r))\}^R_{r=1},$$

and evaluate the loss function:,

$$J^h_i = [\hat{\Psi}^D_T - \bar{\Psi}^M_{R,T}(\hat{\theta}_i, \mathcal{E}^h)]'[\hat{\Sigma}^D_T(1+1/R)]^{-1}[\hat{\Psi}^D_T - \bar{\Psi}^M_{R,T}(\hat{\theta}_i, \mathcal{E}^h)].$$

(b) Find the parameter draw $\hat{\theta}_0$ that corresponds to $\min\{J^h_i\}^{N_d}_{i=1}$, and calculate $\Sigma^{h,0}_P$.

i. Find the $N_{best}$ draws with the lowest $J^h_i$. Stack the remaining draws in a $N_{best} \times N_p$ matrix, $\hat{\Theta}^h$, and define $\tilde{\Theta}^h = \hat{\Theta}^h - \mathbf{1}_{N_{best}\times 1} \sum^{N_d}_{i=N_{best}} \hat{\theta}^h_i/(N_{best})$.

ii. Calculate $\Sigma_{P,0} = (\tilde{\Theta}^h)'\tilde{\Theta}^h/(N_{best})$.

(c) Minimize $J$ with simulated annealing. For $i \in \{0, \ldots, N_d\}$, repeat the following steps:

i. Draw a candidate vector of parameters, $\hat{\theta}^{cand}_i$, where

$$\hat{\theta}^{cand}_i \sim \begin{cases} \hat{\theta}_0 & \text{for } i = 0, \\ \mathbb{N}(\hat{\theta}_{i-1}, c_0\Sigma^{h,0}_P) & \text{for } i > 0. \end{cases}$$

We set $c_0$ to target an average acceptance rate of $50\%$ across seeds.

ii. Under Step 2a, repeats Steps ii-iv.

iii. Accept or reject the candidate draw according to

$$(\hat{\theta}^h_i, J^h_i) = \begin{cases} (\hat{\theta}^{cand}_i, J^{h,cand}_i) & \text{if } i = 0, \\ (\hat{\theta}^{cand}_i, J^{h,cand}_i) & \text{if } \min(1, \exp(J^h_{i-1} - J^{h,cand}_i)/c_1) > \hat{u}, \\ (\hat{\theta}_{i-1}, J^h_{i-1}) & \text{otherwise,} \end{cases}$$

where $c_1$ is the temperature and $\hat{u}$ is a draw from a uniform distribution.

(d) Find the parameter draw $\hat{\theta}^h_{\min}$ that corresponds to $\min\{J^h_i\}^{N_d}_{i=1}$, and update $\Sigma^h_P$.

i. Discard the first $N_d/2$ draws. Stack the remaining draws in a $N_d/2 \times N_p$ matrix, $\hat{\Theta}^h$, and define $\tilde{\Theta}^h = \hat{\Theta}^h - \mathbf{1}_{N_d/2\times 1} \sum^{N_d}_{i=N_d/2} \hat{\theta}^h_i/(N_d/2)$.

ii. Calculate $\Sigma^{h,up}_P = (\tilde{\Theta}^h)'\tilde{\Theta}^h/(N_d/2)$.

(e) Repeat the previous step $N_{SMM}$ times, initializing at draw $\hat{\theta}_0 = \hat{\theta}^h_{\min}$ and covariance matrix $\Sigma_P = \Sigma^{h,up}_P$. Gradually decrease the temperature. Of all the draws, find the lowest $J$ value, denoted $J^h_{guess}$, and the corresponding draws, $\theta^h_{guess}$.

(f) Minimize the same loss function with MATLAB's `fminsearch` starting at $\theta^h_{guess}$. The resulting minimum is $\hat{\theta}^h_{\min}$ with a loss function value of $J^h_{\min}$. Repeat, each time updating the guess, until $J^h_{guess} - J^h_{\min} < 0.001$. The parameter estimates reported in the tables in the main paper, denoted $\hat{\theta}^h$, correspond to the final $J^h_{\min}$.

The set of SMM parameter estimates $\{\hat{\theta}^h\}_{h=1}^{N_h}$ approximate the joint sampling distribution of the parameters. We report its mean, $\bar{\theta} = \sum_{h=1}^{N_h} \hat{\theta}^h / N_h$, and $(5, 95)$ percentiles. For the targeted and non-targeted moments, we report the mean, $\bar{\Psi}^M_T = \sum_{h=1}^{N_h} \bar{\Psi}^M_{R,T}(\hat{\theta}^h, \mathcal{E}^h)/N_h$, and the corresponding t-statistic for moment $m$, $(\bar{\Psi}^M_T(m) - \hat{\Psi}^D_T(m))/(\hat{\Sigma}^D_T(m,m))^{1/2}$.

We set $N_h = 100$, $R = 1{,}001$, and $N_{SMM} = 5$. $N_m$, $N_d$, $N_p$, and $c_1$ are all model-specific. The SMM algorithm is programmed in Fortran 95 with Open MPI and executed on the BigTex cluster.

## C LOG-LINEAR EQUILIBRIUM SYSTEM

We solve the following equilibrium system, where hats denote log deviations from steady state:

$$\hat{n}_t = (1 - \bar{s})\hat{n}_{t-1} + \bar{s}(\hat{q}_t + \hat{v}_t - \hat{s}_t)$$

$$\hat{\theta}_t = \hat{v}_t - \hat{u}^s_t$$

$$\bar{u}^s \hat{u}^s_t = \bar{u}\hat{u}_{t-1} + \chi\bar{s}\bar{n}(\hat{s}_t + \hat{n}_{t-1})$$

$$\bar{u}\hat{u}_t + \bar{n}\hat{n}_t = 0$$

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)(\hat{a}_t + \hat{n}_t)$$

$$\bar{c}\hat{c}_t + \bar{\imath}\hat{\imath}_t + \kappa\bar{v}\hat{v}_t = \bar{y}\hat{y}_t$$

$$\hat{q}_t = -\bar{\theta}^\iota\hat{\theta}_t/(1 + \bar{\theta}^\iota)$$

$$\hat{f}_t = \hat{\theta}_t + \hat{q}_t$$

$$\bar{w}\hat{w}_t = \eta\bar{w}_f\hat{w}_{f,t} + \beta\eta\kappa(1 - \chi\bar{s})\bar{\theta}(E_t\hat{x}_{t+1} + E_t\hat{\theta}_{t+1} - \tfrac{\chi\bar{s}}{1-\chi\bar{s}}E_t\hat{s}_{t+1})$$

$$-(\kappa/\bar{q})\hat{q}_t = \bar{w}_f\hat{w}_{f,t} - \bar{w}\hat{w}_t + \beta(1-\bar{s})(\kappa/\bar{q})(E_t\hat{x}_{t+1} - E_t\hat{q}_{t+1} - \tfrac{\bar{s}}{1-\bar{s}}E_t\hat{s}_{t+1})$$

$$\hat{x}_{t+1} = \hat{c}_t - \hat{c}_{t+1}$$

$$(1/\nu)(\hat{\imath}_t - \hat{k}_{t-1}) = E_t\hat{x}_{t+1} + \beta\bar{r}^k E_t\hat{r}^k_{t+1} + (\beta/\nu)(E_t\hat{\imath}_{t+1} - \hat{k}_t)$$

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{\imath}_t$$

$$\hat{r}^k_t = \hat{y}_t - \hat{k}_{t-1}$$

$$\hat{w}_{f,t} = \hat{y}_t - \hat{n}_t$$

$$\bar{z}\hat{z}_t = \bar{z}\hat{s}_t - \chi\bar{s}\bar{f}\hat{f}_t$$

$$\hat{\theta}_d = \hat{v}_t - \hat{u}_{t-1}$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \rho_{as}\sigma_s\varepsilon_{s,t+1} + \sigma_a\varepsilon_{a,t+1}$$

$$\hat{s}_t = \rho_s\hat{s}_{t-1} + \rho_{as}\sigma_a\varepsilon_{a,t+1} + \sigma_s\varepsilon_{s,t+1}$$