Federal Reserve
Bank of Dallas

Debt Maturity and Commitment to Firm Policies

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April 10, 2023
Revised: February 25, 2026

Abstract

When firms can trade debt only at discrete dates, debt maturity becomes an effective tool to discipline investment and debt policies. In the absence of other frictions, single-period debt restores first-best investment. With market freezes, long-maturity debt amplifies underinvestment and the leverage ratchet effect, while short maturity mitigates these distortions. Calibrating the model to U.S. non-financial firms shows that choosing the optimal debt maturity can reduce the cost of commitment problems and market frictions by up to 4% of firm value. A decomposition of the equilibrium credit spread reveals that the agency component associated with time-inconsistent debt and investment policies is largest when leverage and default risk are low, and is substantially reduced by shorter debt maturities.

JEL Classification: G12, G31, G32, E22.

Keywords: investment, credit risk, debt-equity agency conflicts, leverage ratchet effect, financial contracting, debt maturity.

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1 Introduction

Debt maturity is a central feature of corporate debt. It shapes refinancing costs, credit risk, the value of the interest tax shield, and the conflict between debt and equity over future investment. A large theoretical and empirical literature studies these channels.[1] However, DeMarzo and He (2021) show that when shareholders cannot commit to a debt policy, repeated attempts to dilute existing debt holders eliminate the relevance of future debt financing for firm value. In that case, debt maturity is irrelevant.

This irrelevance result suggests that the role of maturity depends critically on the degree of commitment to future firm policies. While existing work has examined how maturity interacts with the leverage ratchet effect (LRE), its role in addressing commitment problems related to investment and payout policies has received little attention. The few papers that combine the leverage ratchet effect with investment assume no commitment on debt policy, which removes any meaningful role for maturity.[2]

To restore a meaningful role for debt maturity, we introduce limited commitment by assuming that debt trades occur only at discrete dates. This prevents continuous dilution, while still allowing shareholders to revise policies over time. We develop a simple dynamic discrete-time model of a firm subject to productivity shocks. The firm invests in depreciating capital, faces adjustment costs, and finances itself with debt and equity in otherwise frictionless capital markets. Debt is attractive because interest payments are tax deductible. It takes the form of unsecured long-term claims with an average maturity governed by a contractual amortization parameter, ξ. In addition, the firm may face credit market freezes that limit its ability to refinance maturing debt, creating a trade-off between tax benefits and refinancing risk that depends on maturity.

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[1]In capital structure models without investment, Leland (1994) and Leland and Toft (1996) show that longer maturity allows firms to capture a larger interest tax shield. With investment, Myers (1977) emphasizes that debt overhang can make shorter maturity more attractive for firms with valuable growth opportunities, while Diamond and He (2014) show that shorter maturity may also exacerbate debt overhang. Diamond (1991, 1993) highlight the trade-off between benefiting from future improvements in credit quality and higher refinancing risk. He and Xiong (2012) and He and Milbradt (2014) focus on refinancing risk abstracting from investment. Empirically, Barclay and Smith (1995), Guedes and Opler (1996), and Stohs and Mauer (1996) find that firms with stronger growth opportunities tend to choose shorter maturity.

[2]We discuss the related literature at the end of the introduction.

As in the existing literature, shareholders face two commitment problems. First, they cannot commit to a default policy. Upon default, debt holders recover the firm's assets net of bankruptcy costs. Second, shareholders have limited commitment to future debt, investment, and liquidation policies. All decisions are taken at discrete dates to maximize current equity value, and debt is priced at market values that reflect rational expectations about future shareholder behavior. The equilibrium is characterized as a Markov Perfect Equilibrium, in which policies and debt prices depend only on current fundamentals. In this environment, commitment is neither absent nor complete, which allows debt maturity to affect equilibrium outcomes.

Our main result is that, in the absence of credit market freezes, shorter debt maturity acts as a commitment device for investment. In the extreme case of single-period debt, self-interested shareholders choose the firm-value-maximizing level of investment. Debt maturity therefore has real effects. When credit market freezes are possible, however, very short maturity exposes the firm to costly refinancing risk. As a result, firm value is hump-shaped in debt maturity.

To understand the mechanism, note that with limited commitment the interaction between leverage and investment differs from the standard debt overhang logic. On the one hand, because shareholders cannot commit to a default policy, higher leverage reduces the value of the future after-tax cash flows and tax shields generated by investment. This is the familiar debt overhang channel. On the other hand, higher leverage strengthens shareholders' future incentives to dilute existing debt through new issuances. Since investment affects future borrowing capacity and debt prices, the marginal value of investment increases with leverage through this dilution channel. This effect, which is absent in the two polar cases of full commitment and no commitment to debt policy, makes investment relatively insensitive to leverage at moderate leverage levels and partially offsets standard debt overhang.

Debt maturity shapes the strength of this interaction. When maturity is long, the ability to dilute outstanding debt is more persistent, which amplifies the link between underinvestment and future debt issuance. Greater underinvestment then leads to higher leverage relative to a model with fixed investment, reinforcing the leverage ratchet effect. Shorter maturity weakens this mechanism by limiting the horizon over which dilution can operate, thereby reducing underinvestment and mitigating this channel of the leverage ratchet effect.

The joint presence of limited commitment in investment and debt issuance generates richer leverage dynamics than in models with fixed investment. In particular, when maturity is sufficiently long, equilibria may arise in which leverage does not converge but instead depends on initial conditions. Debt maturity is therefore central not only for static trade-offs, but also for the long-run evolution of leverage.

The combined effect of the commitment frictions is not merely a redistribution of value between debt and equity, but a reduction in total firm value. To quantify these forces, we calibrate the model at a quarterly frequency to match empirical moments for non-financial firms in the S&P 500 index, including investment dynamics, financing choices, risk measures, credit spreads, and default frequencies. The calibrated model disciplines the trade-off between the benefits of short maturity, which mitigates commitment distortions, and the benefits of long maturity, which lowers exposure to credit market freezes. In equilibrium, debt financing increases firm value by about 6%, in line with Korteweg (2010) and van Binsbergen et al. (2010). At the same time, the joint presence of commitment problems and market freezes reduces firm value by between 3% and 7%. Hence, optimal maturity can reduce these losses by up to 4%. Consistent with the non-monotonic relation between maturity and firm value, long-run leverage is also hump-shaped, although it is less sensitive to maturity and to the likelihood of market freezes.

We use the calibrated model to study the dynamics of investment, leverage, and payouts. When debt maturity is long and current leverage is above its long-run level, the firm repurchases debt using equity issuance. This reduces distortions from limited commitment in both debt policy and investment. This behavior contrasts sharply with the LRE described in DeMarzo and He (2021), where the commitment problem induced by continuous debt trading prevents such corrective adjustments. In other cases, when long maturity combines with non-stationary leverage, shareholders amplify the LRE by selling assets to finance self-serving payouts.

Simulations of the calibrated economy show that debt maturity has real effects by shaping all three commitment problems. Shorter maturity leads to higher and more persistent investment and Tobin's Q, which respond more strongly to positive productivity shocks and less to negative shocks. By contrast, with long maturity, investment reacts less to positive shocks and underinvests during negative shocks, reflecting the amplified

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commitment distortions. In this way, debt maturity directly influences the stability and responsiveness of investment over time.

The cost of debt increases with debt maturity, reflecting that shorter maturity alleviates both the commitment problem in debt repayment and the commitment problem in future debt issuance and investment. To quantify these effects, we decompose the credit spread into two components: a default spread, capturing shareholders' inability to commit to debt repayment, and an agency spread, arising from time-inconsistent investment and debt policies. This decomposition, which is methodologically novel, shows that the agency component is highest when leverage is low, and that shorter debt maturity can substantially reduce it.

Literature review. Our analysis focusses on the effect of debt maturity on shareholders' commitment on future investment and payout decisions. Several recent contributions analyze the role of debt maturity in the absence of commitment to debt policy. DeMarzo (2019) (and DeMarzo et al. (2023) in the context of sovereign debt models) show that debt maturity affects firm value when debt trades occur only at discrete, possibly random, dates. Benzoni et al. (2022) extend DeMarzo and He (2021) by including debt issuance costs and find that, in equilibrium, the choice of debt maturity involves a trade-off between tax benefits and issuance costs. Malenko and Tsoy (2025), using an alternative equilibrium concept based on a trigger strategy, show that short maturity creates value by committing shareholders to repay debt when the firm is in financial distress after a negative shock and no other credible actions are available. However, all of these contributions abstract from investment.[3] Moreover, different from Benzoni et al. (2022) and Malenko and Tsoy (2025), our solution concept is MPE, which is the equilibrium giving the lowest possible equity value. This choice suites us, because if we show that debt maturity matters in our setting, it will do so also under equilibrium concepts in which the ex ante value of equity is higher.

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[3]There is a large literature extending the baseline *sovereign* debt model without commitment by Eaton and Gersovitz (1981) to long-term debt, facing modeling challenges similar to those of long-term *corporate* debt under no commitment. Within this literature, the contribution most directly related to ours is Chatterjee and Eyigungor (2012). DeMarzo et al. (2023) extend the continuous-time results of DeMarzo and He (2021) to the sovereign debt context. However, endogenous investment is typically absent in sovereign debt models. An exception is Gordon and Guerron-Quintana (2018), who incorporate investment into the baseline sovereign debt model with long-term debt. They show that debt price is increasing and concave in the capital stock, as capital helps reduce the default probability by smoothing consumption. This effect, which operates through the dependence of debt price on the leverage ratio, is present in all LRE models, including ours.

A few papers study the interaction of investment with the LRE under no commitment. In DeMarzo (2019), investment primarily affects firm value through its collateral role, allowing shareholders to exploit tax shields and transfer value from unsecured creditors, which can offset debt overhang at low leverage. By contrast, in our model, limited commitment on debt policy raises the marginal value of investment even when assets are not pledgeable, because more capital increases cash flows which support future debt issuances that benefit shareholders at the expense of current debt holders. DeMarzo and He (2016) show that endogenous investment in a dynamic debt policy model without commitment leads to underinvestment relative to the unlevered case due to debt overhang, a result also found by Crouzet and Tourre (2021). In contrast, in our model with limited commitment, investment always exceeds the unlevered level, and debt overhang is absent at low to moderate leverage due to a positive marginal q component. DeMarzo and He (2021) show that a disinvestment option induces risk-shifting near distress, raising equity value and debt issuance incentives while lowering them when far from distress, amplifying limited-commitment distortions but not creating underinvestment. In our model, underinvestment and asset sales arise endogenously depending on the firm's state, with asset sales increasing payouts rather than triggering debt dilution.

Our main result shows that short-term debt can mitigate commitment problems in investment, and in the extreme case of single-period debt, the (constrained) first-best of firm value maximization is achieved. This is consistent with other findings on the role of short-term debt in reducing frictions, although the mechanism depends on the specific friction. For example, in Abel (2018), short-term debt addresses a commitment problem in debt policy, while Hu and Varas (2025) show that banks unable to commit to retaining loans can resolve the problem by using short-term risky debt. Although the outcomes are similar is spirit, the underlying mechanisms are distinct due to the different frictions.

Finally, the hedging benefits of long-term debt are central in maturity choice models such as Arellano and Ramanarayanan (2012) and Niepelt (2014) for sovereign debt, and Hu et al. (2025) and Jungherr and Schott (2021) for corporate debt. Except for the last, these studies ignore the role of investment. Our quantitative analysis shows that the benefit of long-term debt is too small to outweigh the advantages of shorter maturity as a commitment device against debt dilution and underinvestment.

The paper is organized as follows. Section 2 introduces the model. Section 3 analyzes the MPE and the impact of debt maturity on investment and financing. Section 4 presents the calibrated model and quantitative results on corporate value, leverage, and payouts. Section 5 decomposes the cost of debt into credit risk and agency components. Section 6 concludes. All derivations, proofs, and additional benchmarks are in the Appendix.

2 The model

The model of the firm is in discrete time, and the firm's shareholders and creditors are assumed risk neutral. The firm has capital stock k, with law of motion $k' = I + (1 - \delta)k$, where $\delta > 0$ is the depreciation rate and I is the amount invested, which has unrestricted sign. When the firm invests, the adjustment cost is $\Psi(k, k') = (k'/k - 1 + \delta)^2 \varphi k/2$, with $\varphi > 0$. The after tax cash flow is $(1 - \tau)(y + x)k$, where τ is a tax wedge which captures the effect of corporate and personal taxed, x is an idiosyncratic i.i.d. shock with density $\phi(x)$, support $[\underline{x}, \overline{x}]$ with $\underline{x} < 0 < \overline{x}$ such that $\mu_x = \int_{\underline{x}}^{\overline{x}} x\phi(x)dx = 0$, and $y > 0$ is a persistent shock, with dynamics $\log y' = (1 - \nu)\log \overline{y} + \nu \log y + \sigma\varepsilon$, where $\varepsilon \sim \mathcal{N}(0, 1)$.

We assume the firm can issue long-term debt with face value b and coupon rate $r = 1/\beta - 1$, where β is the risk-free discount factor, which ensures that the debt trades at the face value when it is risk free. Trading takes place in a competitive financial market, where investors make zero profits when buying corporate debt. The use of debt financing by the firm is incentivized by tax-deduction of coupon payments. Each period, the firm commits to repay a fraction ξ of the debt at face value. The parameter ξ can therefore be interpreted as the inverse of the average debt maturity (if the debt remains unchanged and there is no default). The equity holders decide when to default on the debt payments. At default, the absolute priority rule applies and equity holders get nothing, while creditors recover $(1 - \tau)(y + x)k + R(b, k, y)$, conditional on this being positive, where $R(b, k, y) = \min\{(1 - \delta - \alpha)k, \ \tau rb + V(b, k, y)\}$, in which $V(b, k, y)$ is the value of equity defined later, and α is a liquidation cost parameter.[4] For tractability, we will assume the debt is *pari passu* at issuance.

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[4]As will be clarified below, under this specification of $R(b, k, y)$, debt holders' recovery in default never exceeds total debt obligations.

Given the current state (b, k), equity holders choose the new debt level b' *once per period.* This assumption is central in our discrete-time setting. If multiple debt trades were possible within a period, debt maturity would lose its role as a commitment device, as shown by Bizer and DeMarzo (1992). Debt trades at the market price $p(b, k, y)$, which reflects the current state of the firm. All issuances use the same debt contract. Apart from the rollover risk discussed below, there are no frictions in debt or equity trades to raise or distribute capital vis-à-vis the financial market. We focus on three main frictions. First, shareholders cannot commit to service outstanding debt. Second, they have limited commitment with respect to future debt issuances. Third, they cannot commit to future investment policies. Indeed, all three policies aim to maximize current equity value, rather than total firm value.

Our model incorporates debt rollover risk, as in Chatterjee and Eyigungor (2012) and He and Xiong (2012). In each period, the firm is excluded from credit markets with probability π, which limits equity holders' ability to refinance maturing debt and lowers equity value. We denote by V the value of equity under normal credit market conditions, and by V^0 the value of equity when a credit market freeze occurs in the current period, with $V^0 \leq V$. The precise definitions of V and V^0 are provided below. At the end of the period, if $0 < (1 - \tau)\left[(y' + x')k' - rb'\right] - \xi b' + V^0(b', k', y')$, the firm remains solvent even if a market freeze occurs, since $V^0 \leq V$. Conversely, if $(1 - \tau)\left[(y' + x')k' - rb'\right] - \xi b' + V(b', k', y') < 0$, the firm defaults regardless of whether a credit market shock occurs. In both cases, the realization of the market shock is irrelevant. However, if a market freeze occurs and $V^0(b', k', y') \leq \xi b' - (1 - \tau)\left[(y' + x')k' - rb'\right] \leq V(b', k', y')$, equity value equals $V^0(b', k', y')$, and the firm defaults. Overall, debt rollover risk raises the probability of default. Its negative impact is stronger when leverage is higher and when a larger fraction of debt matures, both of which are, directly and indirectly, affected by the debt parameter ξ. The timeline for each period is in Figure 1.

We can now define the value of the securities issued by the firm. Conditional on the firm being solvent at (b, k, y), the levered equity value is defined recursively as

$$V(b, k, y) = \max_{(b', k')} -\left[k' - (1 - \delta)k\right] + \tau\delta k - \Psi(k, k') + \left[b' - (1 - \xi)b\right]p(b', k', y) + \beta\mathcal{V}(b', k', y),$$

(1)

Figure 1: Timeline

At t, the firm's debt and capital are (b, k), as determined by decisions made at $t - 1$. At t, the shocks (x, y) and the condition of the financial market, ω, where $\omega = 0$ denotes no freeze and $\omega = 1$ denotes a market freeze, are observed. Shareholders then decide whether to default or to continue operations. In the latter case, the value function is $V(b, k, y)$. They subsequently choose (b', k'), which remains in place until $t + 1$, when (x', y') and ω' are observed.
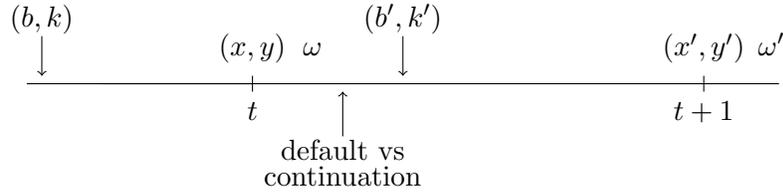
where for convenience we define[5]

$$
\begin{aligned}
\mathcal{V}(b', k', y) = \mathbb{E}_y \Bigg[& \int_{x_d(b',k',y')}^{\overline{x}} \{(1 - \tau)(y' + x')k' - [(1 - \tau)r + \xi] b' + V(b', k', y')\} \phi(x')dx' \\
& - \pi \int_{x_d(b',k',y')}^{x_d^0(b',k',y')} \{(1 - \tau)(y' + x')k' - [(1 - \tau)r + \xi] b' + V(b', k', y')\} \phi(x')dx' \Bigg],
\end{aligned}
$$

in which the second line is the expected loss due to a credit market freeze, and the default threshold is

$$
x_d(b', k', y') = r\frac{b'}{k'} + \frac{\xi b' - V(b', k', y')}{(1 - \tau)k'} - y'. \tag{2}
$$

The threshold $x_d^0(b', k', y')$ has the same expression as (2), except with V^0 in place of V. It is possible to show that if $V^0 \leq V$ then $x_d \leq x_d^0$. The program in (1) reflects the fact that the equity holders cannot credibly commit to future decisions, rather at each date these are made to maximize the current equity value.

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[5]Here and thereafter, the lower limit of integration is actually $x_d(b', k', y') \vee \underline{x}$, which we shorten for the convenience of exposition.

The price of debt, consistent with zero profit for investors, is

$$p(b, k, y) =$$

$$= \beta \mathbb{E}_y \left[\{r + \xi + (1 - \xi)p(b', k', y')\} \int_{x_d(b,k,y')}^{\overline{x}} \phi(x')dx' + \frac{1}{b} \int_{x_f(b,k,y')}^{x_d(b,k,y')} \{(1 - \tau)(y' + x')k + R(b, k, y')\} \phi(x')dx' \right.$$

$$\left. + \pi \left\{ \frac{1}{b} \int_{x_d(b,k,y')}^{x_d^0(b,k,y')} \{(1 - \tau)(y' + x')k + R(b, k, y')\}\phi(x')dx' - \{r + \xi + (1 - \xi)p(b', k', y')\} \int_{x_d(b,k,y')}^{x_d^0(b,k,y')} \phi(x')dx' \right\} \right]$$

$$\tag{3}$$

where $(b', k') = G(b, k, y')$ is the optimal policy of the equity program in (1),

$$x_f(b, k, y') = -\frac{R(b, k, y')}{(1 - \tau)k} - y' \tag{4}$$

is the threshold for x' below which debt holders payoff at default is negative. It can be shown that $x_f(b, k, y') \leq x_d(b, k, y')$. In (3), the second line is the debt service if the firm is solvent plus creditors' recovery in the event of default, considering debt's limited liability, and the third line captures the replacement of the continuation value with the debt recovery in the case of default due to a market freeze, $x_d(b, k, y') < x' < x_d^0(b, k, y')$.

To close the model, we define $V^0(b, k, y)$, representing equity value due to suboptimal debt issuance if a credit market freeze occurs. Let $b^* = b'(k')$ denote the optimal debt in (1) as a function of k'. Then,

$$V^0(b, k, y) = \max_{k'} \; -[k' - (1 - \delta)k] + \tau\delta k - \Psi(k, k')$$

$$+ \left(1 - \frac{\xi b}{k'}\right)\{[b^* - (1 - \xi)b]\, p(b^*, k', y) + \beta\mathcal{V}(b^*, k', y)\}. \tag{5}$$

The impact of a market freeze appears in the second line, where it reduces the value of equity derived from the optimal b^*. We assume that this reduction is greater the larger the amortized debt, ξb, and the smaller the new capital stock, k'. In the numerical solution of the model, we make sure that $\xi b/k' < 1$.

The model in (1)-(5), which is based on state variables (b, k), is non-stationary. Because we must calculate the Markov perfect equilibrium of the model, we need a *stationary* program. Given the linearity of the production function and the assumption

made on capital adjustment cost, the equity price, V, is homogeneous of degree one and the debt price, p, of degree zero in (b, k), and stationarity is achieved by using book leverage as state variable. The following proposition presents a stationary version of the model, which is used in the subsequent analysis. The proof is in Appendix A. With a small abuse of notation, we denote respectively by V and p the equity and debt value for $k = 1$, which solve the stationary program.

Proposition 1. *For given (b, k, y) and defining $\ell = \frac{b}{k}$ and $\kappa = \frac{k'}{k}$, the equilibrium solution for (1)-(5) is found by solving the program*

$$V(\ell, y) = \max_{\kappa} \ -\left[\kappa - 1 + (1 - \tau)\delta\right] - \frac{\varphi}{2}\left(\kappa - 1 + \delta\right)^2 + \kappa \ v\left(\frac{\ell}{\kappa}, y\right), \tag{6}$$

where $\kappa^* = h(\ell, y)$ is the optimal investment policy. In (6), the equity value per unit of capital stock, $v(\cdot, y)$, solves the leverage program,

$$v\left(\frac{\ell}{\kappa}, y\right) = \max_{\ell'} \ \left[\ell' - (1 - \xi)\frac{\ell}{\kappa}\right] p(\ell', y) + \beta \mathcal{V}(\ell', y), \tag{7}$$

where $\frac{\ell}{\kappa} = \frac{b}{k'}$, $\ell' = \frac{b'}{k'}$, and $\ell^* = g(\ell/\kappa, y)$ is the related optimal leverage policy. In (7), the continuation value of equity is

$$\mathcal{V}(\ell', y) = \mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} \left\{(1 - \tau)(y + x' - r\ell') - \xi\ell' + V(\ell', y')\right\} \phi(x')dx' \right.$$
$$\left. -\pi \int_{x_d(\ell', y')}^{x_d^0(\ell', y')} \left\{(1 - \tau)(y + x' - r\ell') - \xi\ell' + V(\ell', y')\right\} \phi(x')dx' \right], \tag{8}$$

where $x_d(\ell, y) = r\ell + \frac{\xi\ell - V(\ell, y)}{1 - \tau} - y$ and $x_d^0(\ell, y) = r\ell + \frac{\xi\ell - V^0(\ell, y)}{1 - \tau} - y$. We define $V^0(\ell, y)$ as in (6), except it is based on $v^0(\ell/\kappa, y) = v(\ell/\kappa, y)(1 - \xi\ell/\kappa)$. The debt price for given leverage ℓ is

$$p(\ell, y) = \beta\mathbb{E}_y \left[\left\{r + \xi + (1 - \xi)p(\ell', y')\right\} \int_{x_d(\ell, y')}^{\overline{x}} \phi(x')dx' + \frac{1}{\ell}\int_{x_f(\ell, y')}^{x_d(\ell, y')}\left\{(1 - \tau)(y' + x') + R(\ell, y')\right\}\phi(x')dx' \right.$$
$$\left. +\pi\left\{\frac{1}{\ell}\int_{x_d(\ell, y')}^{x_d^0(\ell, y')}\left\{(1 - \tau)(y' + x') + R(\ell, y')\right\}\phi(x')dx' - \left\{r + \xi + (1 - \xi)p(\ell', y')\right\}\int_{x_d(\ell, y')}^{x_d^0(\ell, y')}\phi(x')dx'\right\} \right] \tag{9}$$

where $x_f(\ell, y') = -R(\ell, y')/(1 - \tau) - y$, $R(\ell, y') = \min\{1 - \delta - \alpha, \tau r \ell + V(\ell, y')\}$, based on the next-period leverage decision $\ell' = g(\ell/\kappa, y')$, where $\kappa = h(\ell, y')$.

Inspection of expression (8) shows that, for given probability of a credit market shock, the value loss due to a shock is larger the bigger the difference between $x_d^0(\ell', y')$ and $x_d(\ell', y')$. The difference is proportional to the loss due to market freeze, $V(\ell', y') - V^0(\ell', y')$, which is more severe the bigger the contractual reduction in leverage, $\xi\ell$.

3 Markov perfect equilibrium

We focus on an equilibrium concept, Markov perfect equilibrium (MPE), whereby the firm policy depends only on the payoff-relevant state variables, and the policy is time-consistent, in that it is a fixed point in which future policies are the same as this period's policy, and depends only on the state. Specifically, the MPE of the firm model is defined by security prices, $V(b, k, y) = kV(b/k, y)$ and $p(b, k, y) = p(b/k, y)$, and optimal policy $(b', k') = G(b, k, y')$, such that $k' = kh(b/k, y)$ and $b' = k'g(b/k', y)$, where the security prices reflect the expectations regarding the optimal policy, and given the security prices the equity holders will not deviate from said policy. The MPE cannot be found analytically and is calculated by solving equations (7)-(9) simultaneously using a discrete-state value function iteration approach described in Appendix H.

We characterize the MPE and derive the main theoretical results for case without roll over risk ($\pi = 0$). The proof of the following proposition is in Appendix E.

Proposition 2.   *1. For a given y, $\ell = b/k$, and $\kappa = k'/k$, $v(\ell/\kappa, y)$ is convex in ℓ/κ and $V(\ell, y)$ is convex in ℓ.*

   2. For one-period debt, $\xi = 1$, the equilibrium leverage and investment policy in the MPE maximizes the value of the firm (equity plus debt), with full commitment on debt and investment policies.

   3. If $\xi < 1$,

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(a) *assuming $v(\cdot, y)$ is twice differentiable, then $v(\ell/\kappa, y)$ is decreasing in ℓ/κ and $V(\ell, y)$ is decreasing in ℓ; $p(\ell, y)$ is a decreasing function of ℓ, that is $\partial_1 p \leq 0$;*[6]

(b) *if the objective function in (7) is concave with respect to ℓ',[7], there is a unique equilibrium policy function, $\ell' = g(\ell/\kappa, y)$ that satisfies condition*

$$- \left[\ell' - (1-\xi)\frac{\ell}{\kappa} \right] \partial_1 p(\ell', y)$$

$$= \beta \mathbb{E}_y \left[\tau r \int_{x_d(\ell', y')}^{\overline{x}} \phi(x') dx' + \frac{1}{\ell'} \int_{x_f(\ell', y')}^{x_d(\ell', y')} \{(1-\tau)(y'+x') + R(\ell', y')\} \phi(x') dx' \right]. \quad (10)$$

Under the same conditions, $g(\ell/\kappa, y)$ is increasing in ℓ/κ;

(c) *if the objective function in (6) is concave with respect to κ, which is true if and only if proportional adjustment cost, φ, is large enough, the equilibrium investment policy, $\kappa = h(\ell, y)$, satisfies condition*

$$v\left(\frac{\ell}{\kappa}, y\right) + (1-\xi)\frac{\ell}{\kappa}p(\ell', y) = 1 + \varphi\left[\kappa - (1-\delta)\right], \quad (11)$$

where the left-hand side of (11) is marginal q, with $\ell' = g(\ell/\kappa, y)$. Under the same conditions, $h(\ell, y)$ is decreasing in ℓ.

Part 1 shows that, in our model with investment, the value function satisfies the standard properties implied by the optimal leverage policy. Part 2 delivers an important result: debt maturity has real effects, and single-period debt implements the constrained first-best investment. When $\xi = 1$, all debt matures within the period, so the price of new debt fully reflects expected distortions from future leverage choices. Shareholders thus internalize the full cost of these distortions and have no incentive to deviate from the value-maximizing leverage policy. Investment then focuses solely on increasing the value of installed capital. In the absence of other frictions, short debt maturity therefore mitigates commitment problems in investment.

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[6]The notation $\partial_2 f(\hat{x}, \hat{y})$ indicates the first partial derivative of $f(x, y)$ with respect to y, evaluated at $(\hat{x}, \hat{y})$.

[7]In the case of zero recovery at default and no credit market shocks, this condition is satisfied if $p(\ell, y)$ is a concave function of ℓ. However, it is not generally true that the price of debt is concave in ℓ. See e.g. DeMarzo et al. (2023), Section 5.2.2., for a case in which the debt price is convex.

Figure 2: Debt maturity, debt price, and leverage policy

In the left panel we plot the debt price, $p(\ell', y)$, for short and long debt maturity, at a given y against leverage, $\ell = b/k$. The price is calculated $\ell' = g(\ell/\kappa, y)$, where $\kappa = h(\ell, y)$. The right panel shows $\ell' = g(\ell/\kappa, y)$ where $\kappa = h(\ell, y)$, at a given y for short and long debt maturity against $\ell = b/k$. The red dotted line is the leverage policy with long maturity and constant k. The 45 degree (dotted) line determines the long-run leverage, $\hat{\ell}$, as the solution of $\ell = g(\ell/\kappa, y)$.
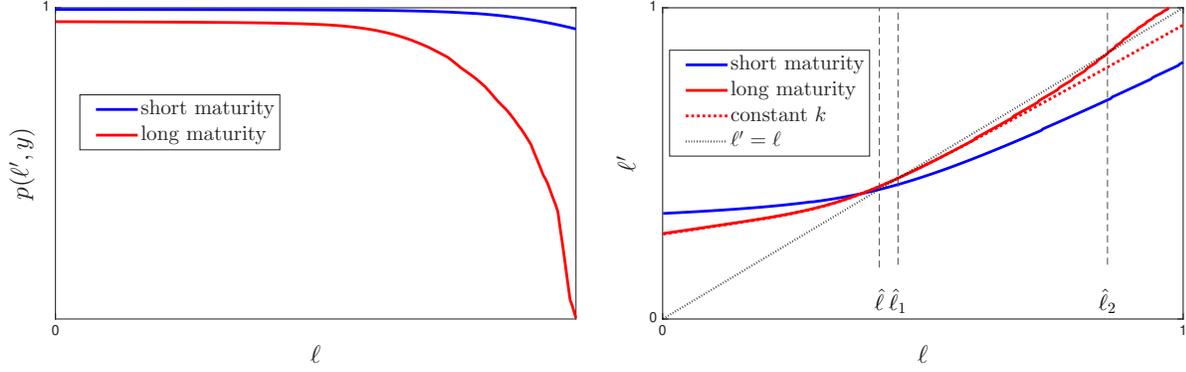
Part 3(a) confirms, under limited commitment on debt policies, properties of the value of securities already shown in similar models with no commitment. Figure 2, left panel, shows that the debt price at the optimal investment and leverage is decreasing also with respect to debt maturity.

While Part 3(b) shows that with limited commitment shareholders have an incentive to dilute debt holders, $\partial_1 g \geq 0$, the right panel of Figure 2 shows that the relationship between leverage policy and debt maturity is non-monotonic. When ℓ is low, new leverage can be higher under short maturity than under long maturity, whereas the opposite holds when ℓ is high. To determine optimal leverage, (10) balances the marginal cost, on the left-hand side, against the marginal benefits of increasing leverage, on the right-hand side. The latter include the present value of future interest tax shields and the debt recovery value at default, which is captured by shareholders through debt issuance.

On the left-hand side of (10) there is the marginal cost of increasing leverage due to the reduction in the debt price for each unit of newly issued debt. Unlike models without commitment to the debt policy, underinvestment plays a central role in our framework. Specifically, a lower κ reduces the marginal cost of leverage, making the optimal ℓ' higher than in a model with fixed investment, thereby exacerbating debt dilution. This effect

can be seen in the right panel of Figure 2, where the equilibrium long-run leverage, denoted by $\hat{\ell}$, is determined as the fixed point of $\ell = g(\ell/\kappa, y)$. For short debt maturity there is a unique $\hat{\ell}$, whereas for long maturity there are two fixed points, $\hat{\ell}_1 < \hat{\ell}_2$, in the interval $[0, 1]$:[8] The second fixed point arises because underinvestment amplifies debt dilution, as discussed above. As we show later, underinvestment becomes more severe at high leverage and with longer debt maturity. The combination of these forces strengthens the debt dilution incentive sufficiently to generate the second fixed point $\hat{\ell}_2$. This qualitative feature is specific to the commitment problem in the investment policy, as illustrated in the right panel of Figure 2. Under long debt maturity, the second fixed point is absent in a model with fixed investment.[9] This shows that models with a constant asset *underestimate* the LRE, since they exclude the underinvestment channel in the debt dilution problem.

The right panel of Figure 2 shows that when commitment increases, that is, when maturity is shorter, reversion toward $\hat{\ell}$ from below is faster, meaning that the slope of $g(\ell/\kappa, y)$ is lower. This occurs because the marginal value of new debt issuance is higher.[10] The figure also shows that the effect of the investment commitment problem on debt dilution is *asymmetric*. With long maturity, for low ℓ, the speed of *upward* adjustment toward the long-run leverage is the same under fixed k and endogenous k. Hence, it is unaffected by the lack of commitment in investment. In contrast, for high ℓ, underinvestment accelerates debt issuance and reduces the speed of *downward* adjustment.[11]

Depending on whether there is one, more than one, or no fixed point, and on the current level of leverage, the leverage dynamics of the firm can take three forms. (i) If $\ell < \hat{\ell}$, leverage adjusts upward toward $\hat{\ell}$. (ii) With short debt maturity, if $\ell > \hat{\ell}$, leverage

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[8]For very long maturity, additional fixed points may arise above 1. We restrict attention to economically plausible levels of leverage.

[9]The benchmark case with constant k is described in Appendix C.

[10]While Dangl and Zechner (2021) also highlight the effect of maturity on downward debt adjustments, the upward effect in our setting arises solely from the role of maturity as a commitment device against time-inconsistent debt policies and is absent in their model. Dangl and Zechner (2021) rule out debt time inconsistency by requiring the firm to repurchase all outstanding debt before increasing leverage.

[11]Chaderina et al. (2022) show that debt maturity affects equity returns through its effect on debt dynamics in the presence of systematic risk. With longer maturity, debt adjustment is slower and exposure to economic downturns is more persistent, which in equilibrium requires a higher risk premium. Although we do not study asset pricing implications here, we complement Chaderina et al. (2022) by showing that debt maturity affects not only leverage dynamics but also investment dynamics, providing an additional channel through which systematic shocks can influence equity returns.

adjusts downward toward $\hat{\ell}$. (iii) With long debt maturity, if $\ell < \hat{\ell}_2$, leverage adjusts downward toward $\hat{\ell}_1$, whereas if $\ell > \hat{\ell}_2$, leverage diverges to 100%.[12] The behavior in (i) and (ii) is also described in DeMarzo et al. (2023), where the fixed point is always attracting, so that long-run leverage converges to it. In contrast, case (iii), in which $\hat{\ell}_2$ is not an attracting point, is specific to a model with limited commitment to the debt policy and no commitment to the investment policy.

In a model with limited commitment to the debt policy, the effect of leverage on investment goes beyond standard debt overhang. To see this, we start from the first-order condition for investment in (11), with a positive marginal q on the left-hand side and the cost of newly installed capital on the right-hand side. Since $v(\ell/\kappa, y) + (1-\xi)\frac{\ell}{\kappa}p(\ell', y) = \beta\mathcal{V}(\ell', y) + \ell'p(\ell', y)$, marginal q is given by

$$
\mathbb{E}_y\left[\int_{x_f(\ell',y')}^{\overline{x}}(1-\tau)(y'+x')\phi(x')dx' + \{\tau r\ell' + V(\ell',y')\}\int_{x_d(\ell',y')}^{\overline{x}}\phi(x')dx' + R(\ell',y')\int_{x_f(\ell',y')}^{x_d(\ell',y')}\phi(x')dx'\right.
$$
$$
\left. +(1-\xi)\ell'p(\ell'',y')\int_{x_d(\ell',y')}^{\overline{x}}\phi(x')dx'\right], \quad (12)
$$

where $\ell'' = g(\ell'/\kappa', y')$. This expression highlights the two main components of marginal q in our model. For given ℓ', the first component, corresponding to the first line of (12), is the marginal value of capital that arises from future after-tax cash flows, interest tax shields when the firm is solvent, and the recovery value of defaulted debt, which is captured by current shareholders through debt issuance. This component of q, shown in the left panel of Figure 3 with dashed lines, is decreasing in ℓ, reflecting the standard debt overhang problem.[13]

Marginal q has a second positive component, corresponding to the second line of (12), which arises from limited commitment to the debt policy and reflects the value captured by shareholders through future debt dilution. Intuitively, an additional unit of physical capital today increases cash flows, allowing shareholders, in future solvent states, to issue

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[12]In unreported results, for very high debt maturity, in the calibration used in Section 4, when $1/\xi > 40$, the equilibrium leverage policy satisfies $g(\ell/\kappa, y) > \ell$ for all $\ell \in [0,1]$, so that no fixed point exists. In this case as well, leverage diverges to 100%.

[13]This can be understood as follows: because g is strictly increasing in its first argument, higher current leverage implies a higher ℓ'. Together with the fact that $V(\ell', y')$ decreases in ℓ', this raises the default threshold $x_d(\ell', y')$. If $V(\ell', y')$ decreases in ℓ' faster than $-\tau r\ell'$, the recovery value $R(\ell', y')$ is lower and the limited liability threshold for debt, $x_f(\ell', y')$, is higher.

Figure 3: Debt maturity, marginal q, and investment

The left panel plots, with solid lines, marginal $q = v\left(\frac{\ell}{\kappa}, y\right) + (1-\xi)\frac{\ell}{\kappa}p\left(\ell', y\right)$, with $\ell' = g(\ell/\kappa, y)$, for given κ and y, against $\ell = k/b$, under short and long maturity. With the same color, the dashed lines represent the first component of (12), and the dotted lines represent the second component of (12). The right panel plots $\kappa = h(\ell, y)$, for given y, under short and long debt maturity, against ℓ. We also plot the total firm value (TFV) maximizing case (solid black line) and the case with permanently no debt (dashed black line). Investment is positive when $\kappa > 1 - \delta$.
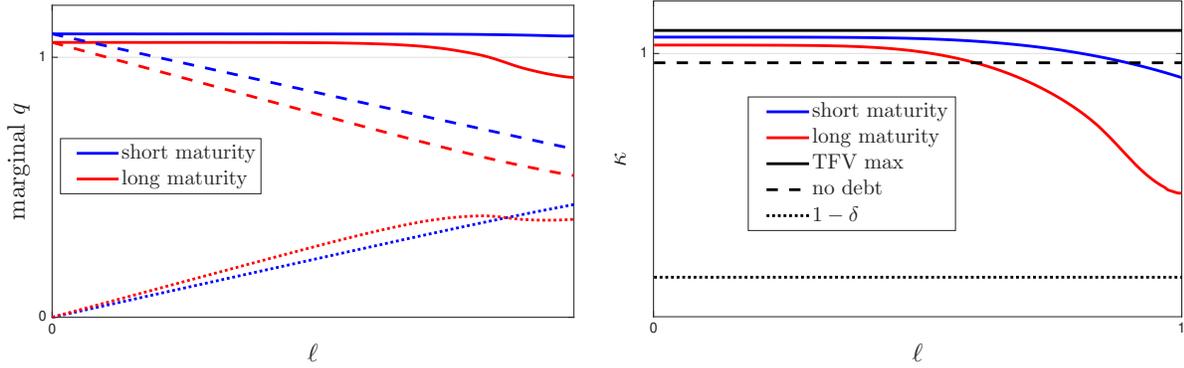
more debt at price $p(\ell'', y')$. This effect is scaled by the residual leverage after investment, $(1 - \xi)\ell'$. Unlike the first component, this second component increases with current leverage ℓ, although its marginal effect weakens at high leverage because p decreases as ℓ' rises, as shown with dotted lines in the left panel of Figure 3. Moreover, this second component is inversely related to commitment in the debt policy: it is stronger for longer maturity.[14] Figure 3 shows that for low leverage the second component partly offsets the debt overhang effect embedded in the first component, thereby mitigating underinvestment. For high leverage, however, underinvestment dominates, as stated in Part 3(c) of Proposition 2.

Overall, for given ℓ, investment is higher under shorter debt maturity, everything else equal, as shown in the right panel of Figure 3, this interaction increases investment when maturity is shorter. The effect of ξ on investment arises from limited commitment to debt policy. When $\xi < 1$, three channels are at work. The first is the positive

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[14]This component arises solely from limited commitment to the debt policy. In DeMarzo and He (2016) and Crouzet and Tourre (2021), under no commitment, the first-order condition for investment is independent of the debt price and maturity, and there is only underinvestment due to debt overhang.

effect of shorter maturity on the first component of marginal q, given by the first line of (12). The second operates through the term $(1 - \xi)$, which decreases with ξ. The third operates through $p(\ell')$, which increases with ξ, that is, decreases with maturity, as shown in Figure 2 left panel. The first and third channels dominate. As a result, shorter debt maturity increases marginal q and therefore investment.

In the right panel of Figure 3, we also plot investment for two benchmark models: the full-commitment case with the total firm value maximizing policy, and the case in which the firm remains permanently unlevered.[15] The figure shows that, for a given ℓ, investment declines more when debt maturity is longer, reflecting weaker commitment to the debt policy. Limited commitment leads to underinvestment relative to the full-commitment benchmark. When current leverage is low, debt financing—even under limited commitment—supports higher investment compared to the permanently unlevered firm. When leverage is high, however, limited commitment depresses investment to the point that the firm would be better off without debt.[16]

4   Quantitative analysis

In this section, we examine the quantitative impact of the commitment problems introduced in the model on firm value and policy dynamics. We also provide a quantitative analysis of how debt maturity helps address these frictions.

We calibrate the model at a quarterly frequency using the baseline parameters in Table 1. The discount factor is $\beta = 0.987$, consistent with similar discrete-time settings (e.g., Cooley and Quadrini (2001)), implying a risk-free rate $r = 1/\beta - 1 = 1.25\%$. The depreciation rate is $\delta = 0.025$, as in Gomes et al. (2016), Jungherr and Schott (2021), and Xiang (2024). The tax rate is $\tau = 32\%$, between the values used in Gomes et al. (2016), Jungherr and Schott (2021), Xiang (2024), and Benzoni et al. (2022). Given δ, τ, and r, the capital adjustment cost parameter φ and average profitability $\overline{y}$ are jointly chosen to satisfy condition (26), which ensures that the unlevered model has a

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[15]These benchmark cases are described in Appendices D and B.

[16]This differs from DeMarzo and He (2016) and Crouzet and Tourre (2021), where, under no commitment to the debt policy, investment is lower even relative to the permanently unlevered benchmark.

Table 1: Base case parameters

This table reports the baseline parameter values. The parameters $\overline{y}$, σ, τ, and α are calibrated to match the observed leverage, credit spread, and default probability. The remaining parameters are taken from the literature. The model is calibrated so that one period corresponds to a quarter in calendar time.

symbol	description	value
β	risk-free discount factor	0.987
$[\underline{x}, \overline{x}]$	support iid profitability	$[-1.15, 1.15]$
$(\overline{y}, \nu, \sigma)$	persistent profitability	$(4.35\%, 0.9, 0.07)$
τ	tax wedge	32%
φ	capital adjustment cost	2.4
δ	depreciation rate	0.025
α	bankruptcy costs	0.23
ξ	debt amortization rate	0.05
π	probability of credit market freeze	0.1

solution.[17] We set $\varphi = 2.4$, and choose the profitability parameters ν and σ, in line with Xiang (2024). The debt maturity parameter is $\xi = 1/20$, corresponding to a 5-year average maturity, consistent with Gomes et al. (2016), Jungherr and Schott (2021), and Xiang (2024). The idiosyncratic profitability component x follows the continuous distribution described in Appendix H, with support $[-1.15, 1.15]$, as in Xiang (2024). Proportional bankruptcy costs are $\alpha = 0.23$, within the range considered by Jungherr and Schott (2021) and Gomes et al. (2016), and consistent with Xiang (2024). The parameter π shapes the tradeoff between mitigating debt dilution with shorter maturity and increasing exposure to freezes. Our baseline assumes a freeze every 10 quarters. Since this parameter is difficult to discipline empirically, we provide sensitivity analysis, including the case without freezes. Finally, we jointly choose $\overline{y}$, σ, τ, and α to match average leverage, credit spreads, and default probabilities in the data.

Our calibration delivers a realistic quantitative fit, as shown in Table 2, which compares average annualized moments from the simulated economy to those of non-financial firms in the S&P 500. Appendices H and I describe the numerical procedure and the data. The model closely matches leverage in both book and market terms. As is typical

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[17]As discussed in Appendices H and B, existence of an MPE requires that the investment problem in (6) admits a solution, which is guaranteed if the unlevered firm program has a solution.

Table 2: Moments

This table compares sample averages of the moments of the simulated economy from the model, obtained following the procedure described in Appendix H, using the baseline parameters in Table 1, to the corresponding moments for the sample of S&P 500 non-financial firms described in Appendix I. Credit spreads in the data are for 1-year CDS and 5-year CDS, respectively. All moments, from the data and from the simulated economy, are annualized for ease of comparison. The data sample spans the years from 2001 to 2014.

|	model	data
debt/asset	0.283	0.281
market leverage	0.259	0.274
market-to-book	1.040	1.202
investment/asset	0.071	0.071
cash flow/asset	0.121	0.149
debt maturity (years)	5	[2.92 7.46]
credit spread (bps)	119	[110 190]
default probability (pct)	0.172	0.200

in this class of models, matching average Q is more difficult because we do not model a stochastic discount factor. Nonetheless, the model matches average investment and profitability well. It implies an average debt maturity of 5 years, between the 2.92-year average term-loan maturity and the 7.46-year average bond maturity in the data. Given a bond share of 0.762, the implied average maturity in the data is 6.54 years. The model also generates realistic average credit spread and default probability, broadly consistent with BBB-rated US corporate debt. Although the observed CDS bracket [110, 169] for 1- and 5-year maturities reflects market risk premia absent from the model, the average credit spread implied by the model remains in line with the data.

4.1 Debt maturity, firm value, and long-run leverage

DeMarzo and He (2021) show that in continuous time with no commitment—where it is only optimal to follow a smooth leverage policy—the MPE value of levered equity is independent of future debt policy. It equals the equity value for a firm with the same current debt that is subsequently excluded from the debt market. Since debt policy does not affect the ex-ante value of equity, debt maturity is also irrelevant ex ante. As

Figure 4: Debt maturity, firm value, and long-run leverage

The left panel shows the value of unlevered equity (black lines, left axis), $V(0, \overline{y}; \xi)$, which coincides with total firm value, as a function of debt maturity, $1/\xi$. We benchmark against the value of a firm with permanently no debt, $V^u(\overline{y})$ (blue line). We show the firm value shortfall (red dotted line, right axis) relative the case with $1/\xi = 1$, as $1 - V(0, \overline{y}; 1)/V(0, \overline{y}; \xi)$ for $\pi = 0.1$. The right panel shows the long-run target leverage, $\hat{\ell}(\xi)$, for a firm starting at zero leverage, as a function of debt maturity. In both panels, we also show the effect of variations on π around the baseline value (solid red line). All remaining parameters are as in Table 1.
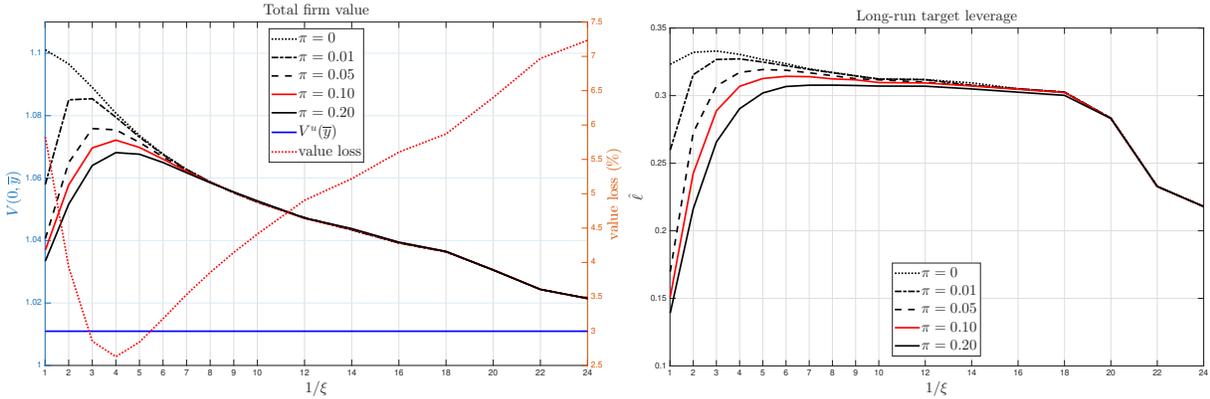
DeMarzo and He (2021) point out, this debt irrelevance result is a consequence of the continuous-time setting.

A discrete-time setting reintroduces some commitment, as the borrower cannot default, undertake equity-maximizing investment, or issue new debt before the next trading date. As a result, there is a positive gain from debt trade, as shown by DeMarzo (2019). In our discrete-time setup, we can assess the role that maturity plays on the ex ante value of the firm, before any debt is issued.[18] By setting $y = \overline{y}$, we aim to provide an average valuation. In the left panel of Figure 4, we show that, given the three commitment problems considered in our model, the value of equity (and of the firm) currently at zero leverage, $V(0, \overline{y})$, is hump-shaped with respect to average debt maturity, with the highest value for $1/\xi = 4$. This shape is due to the role of (longer) debt maturity as a hedge against credit market freezes. Indeed, a shorter debt maturity implies that in each period a significant amount of debt matures and needs to be refinanced, exposing the firm to the severity of credit market freezes.

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[18]Since the firm will issue debt in equilibrium, the unlevered state provides the best benchmark to fully capture the value created by debt, and thus the effect of debt maturity. This value also depends on y.

When we exclude credit market freezes, that is $\pi = 0$, the value of equity and of the firm is strictly decreasing in $1/\xi$, as the LRE increases with maturity, without the offsetting effect from potential market freezes. The highest firm value is found for $1/\xi = 1$, the limiting case in which full commitment is restored, as stated in Proposition 2, with the same value as under total firm value maximization. With $\pi = 0$, a corner solution with respect to debt maturity emerges because in our model no other frictions, such as refinancing costs, offset the positive effect of shorter debt maturity.[19] Also, for longer debt maturity the effect of market freeze becomes insignificant, as the value of the firm is independent of π.

To assess the value created by debt policy in the presence of the three commitment problems, the left panel of Figure 4 reports (blue line) the value of a *permanently* unlevered firm, $V^u(\overline{y})$.[20] The figure shows that issuing debt raises firm value by 1% to 6% in the baseline case $\pi = 0.1$, with a maximum at $1/\xi = 4$, consistent with the empirical estimates of 5.5% in Korteweg (2010) and 3.5% in van Binsbergen et al. (2010). Unlike DeMarzo and He (2021), who find two separate MPEs, one permanently unlevered and one with leverage ratcheting, in our model a currently unlevered firm issues debt in equilibrium. As shown in the right panel of Figure 2, leverage increases gradually toward its long-run level, with slower adjustment for higher $1/\xi$. Along this path, shareholders capture tax shields and investment benefits while not internalizing the higher bankruptcy costs imposed on legacy debt. The present value of these gains equals $V(0, \overline{y}) - V^u(\overline{y})$. Figure 4 shows that these benefits decline with maturity, as longer maturity worsens the commitment problem and slows their accrual, but they remain strictly positive.[21]

However, the value of debt financing is lower than it could be due to limited commitment and credit market freezes. Specifically, the impact of these frictions is measured by the shortfall in firm value between the case with full commitment and no market freeze,

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[19] Arellano and Ramanarayanan (2012) argue that longer-maturity debt provides a hedge against negative systematic shocks. To the extent that y captures systematic risk, the fact that, for $\pi = 0$, the firm's value is strictly decreasing in $1/\xi$ (across all calibrations we considered, including higher σ; results are available upon request) suggests that this hedging effect is negligible in our model relative to effect of the commitment problems on investment and debt policies.

[20] This value is derived analytically in Appendix B and differs from $V(0, \overline{y})$, the value of a currently unlevered firm that has access to debt markets and therefore issues debt immediately.

[21] For very long maturities, the gains become small and could be offset by a modest one-time issuance cost, which we do not model. This contrasts with Benzoni et al. (2022), where recurring refinancing costs can make remaining unlevered optimal for high issuance costs and relatively short maturities.

corresponding to $1/\xi = 1$ and $\pi = 0$, and the baseline case. In the left panel of Figure 4 this shortfall (red dotted line) ranges from 3% to 7%, depending on debt maturity.

The right panel of Figure 4 plots long-run target leverage, $\hat{\ell}$, against debt maturity, $1/\xi$, and reveals a hump-shaped relation. When maturity is short, refinancing risk dominates the benefits of debt, keeping leverage low. When maturity is long, credit market freeze risk is negligible, but the value extracted from each unit of debt is limited due to the prevalence of the LRE, again resulting in low leverage. For intermediate maturities, between 6 and 16 quarters in the figure, both credit freeze risk and debt dilution are moderate, so each unit of debt adds more value and the firm chooses higher leverage. Remarkably, leverage varies between 0.14 and 0.34 solely due to changes in debt maturity.

A lower likelihood of credit market freezes affects long-run target leverage only when debt maturity is short. In the limiting case with $\pi = 0$, shorter maturity strengthens the ability of leverage to increase firm value by reducing the LRE. As a result, long-run leverage is higher.

4.2   Firm dynamics

We use the calibrated model to derive implications of debt maturity for the dynamics of the firm policies and security values. In particular, we will focus on the dynamics of the investment and debt policy, and the related dynamics of cash flow and of the payout policy through the flow of funds equation.[22]

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[22]Defining the state at t as (b_t, k_t, y_t), we have $(b_t, k_t, y_t) = k_t(\ell, 1, y)$, and the related optimal policy (κ, ℓ'), where $\kappa = h(\ell, y)$ and $\ell' = g(\ell/\kappa, y)$, the cash flow from investment policy for $k_t = 1$ is $cf^k(\ell, y) = -(\kappa - 1 + \delta) + \tau\delta - \frac{1}{2}\varphi(\kappa - 1 + \delta)^2$ and the cash flow from the debt policy for $k_t = 1$ is $cf^d(\ell, y) = \kappa \left[\ell' - (1 - \xi)\frac{\ell}{\kappa} \right] p(\ell', y)$. From these, given the capital stock at t, k_t, we derive the respective dollar values of the two cash flows at t as $CF_t^k = k_t\, cf^k(\ell, y)$ and $CF_t^d = k_t\, cf^d(\ell, y)$. The (expected) dividend for $k_t = 1$ over the period $[t, t+1]$ is

$$d(\ell, y) = -(\kappa - 1 + \delta) + \tau\delta - \frac{1}{2}\varphi(\kappa - 1 + \delta)^2 + \kappa \left[\ell' - (1 - \xi)\frac{\ell}{\kappa} \right] p(\ell', y)$$

$$+ \kappa\beta\mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\bar{x}} \{(1 - \tau)(y' + x') - ((1 - \tau)r + \xi)\,\ell'\} \phi(x')dx' \right],$$

from which we calculate the dollar dividend $D_t = k_t\, d(\ell, y)$ at t. Finally, the continuation value of the firm at t for $k_t = 1$ is $cv(\ell, y) = \kappa\beta\mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\bar{x}} V(\ell', y')\phi(x')dx' \right]$, from which we determine the dollar continuation value $CV_t = k_t\, cv(\ell, y)$.

Figure 5: Asset and debt dynamics

The figure displays the simulation paths of k_t, b_t, and $\ell_t = b_t/k_t$ (in the left column), and the simulation paths of CF_t^k, CF_t^d, D_t, and CV_t (in the right column). For ease of comparison, the values of CF_t^k and D_t are multiplied by 10. In the simulations, the firm starts with $k_0 = 1$ and initial debt levels $b_0 = 0.2$ (first row), $b_0 = 0.37$ (second row), and $b_0 = 0.45$ (third row). The simulation paths are conditional on the firm being solvent at the beginning of each period. The simulations are conducted assuming that y is non-stochastic and constant at $\overline{y}$, using the parameters in Table 1.
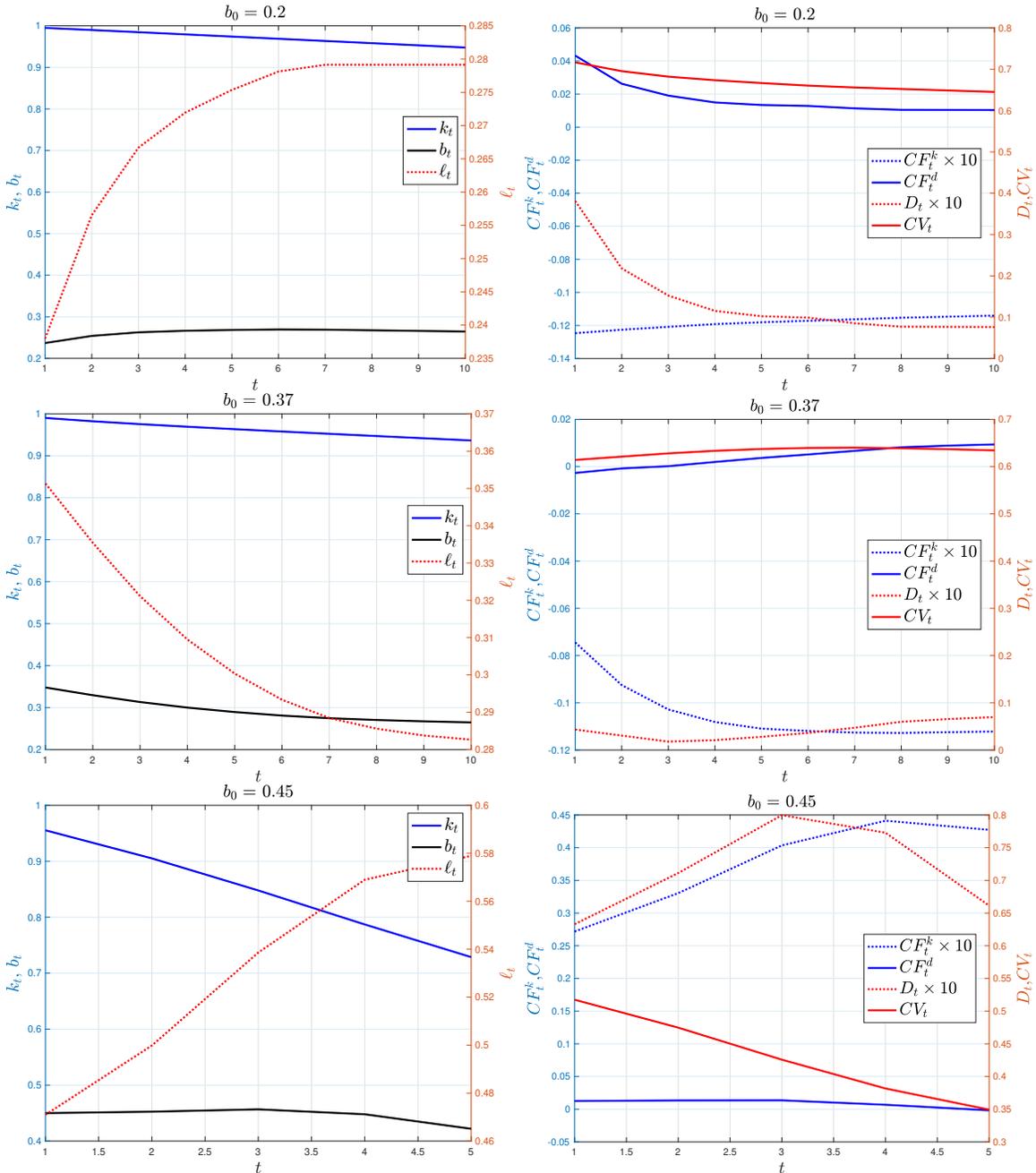
Figure 5, based on the calibration in Table 1, shows the evolution of these variables for different initial levels of b_0. Since the model is homogeneous of degree one in k, without loss of generality we normalize $k_0 = 1$, and so $\ell_0 = b_0$. To isolate the effects of limited commitment on debt policy from the dynamics of y, we fix y at $\overline{y}$. The figure shows that the firm's dynamics are driven by its initial leverage. In the top row of Figure 5, with $b_0 = 0.2$, the firm converges to the long-run leverage level $\hat{\ell} = 0.28$ (or $\hat{\ell}_1$, if there are multiple fixed points in the leverage policy). To do so, capital adjusts downward while debt slightly increases. The right panel of the first row reports the corresponding cash flow and payout dynamics. With low leverage, the commitment problem on debt policy is limited. Hence, the debt price remains relatively high, allowing the firm to raise enough funds through debt issuance to maintain a positive, though declining, dollar dividend and investment. Overall, the firm's (dollar) value, measured by the continuation value, slightly declines over time.

If $b_0 = 0.37$, as shown in the second row of Figure 5, investment is positive and grows over time. Notably, not only is the initially high debt gradually reduced through amortization, but there is also an early phase in which net proceeds from debt issuance are negative. This indicates that shareholders actively reduce debt beyond the scheduled amortization, deviating from the LRE equilibrium. In our model, the LRE amplifies underinvestment, and to restore a positive NPV of capital investment, shareholders reduce debt more aggressively. Later, debt issuance becomes positive again as leverage approaches the long-run level $\hat{\ell} = 0.28$. The initial debt reduction is mainly financed through payout reduction, although dividends remain positive over the observed period. Overall, the firm's value remains relatively stable, even though the capital stock declines. This suggests that, when initial leverage is not too high, debt maturity serves as an effective commitment device to mitigate underinvestment.

The firm's trajectory is quite different when $b_0 = 0.45$, as shown in the third row of Figure 5.[23] Indeed, the firm's path depends on whether the initial leverage ℓ_0 is below or above $\hat{\ell}_2$. If $\ell_0 < \hat{\ell}_2$, leverage converges to $\hat{\ell}_1$, as in the previous cases. However, if

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[23] For illustration, we choose the base case ξ so that equation $\ell = g(\ell, y)$ has multiple solutions for $\ell \in [0, 1]$ and $y = \overline{y}$, as seen in the right panel of Figure 2. Specifically, for the parameters in Table 1 we find $\hat{\ell}_1 = 0.28$ and $\hat{\ell}_2 = 0.42$. In unreported results (available upon request) we show that, for lower $1/\xi$, the dynamic is similar (with the main difference that capital stock is generally increasing over time), but because there is only one fixed point for leverage, we do not observe leverage divergence as in the third row of Figure 5.

$\ell_0 > \hat{\ell}_2$, as in the third row of Figure 5, leverage increases rapidly. This happens because the capital stock k_t shrinks sharply due to asset sales, while debt remains constant or declines only slightly. This outcome fully reflects the effects of limited commitment on debt policy and on investment: shareholders extract value early by paying themselves large dividends funded by asset sales. Meanwhile, the (dollar) value of the firm declines steadily. In other words, the LRE of debt policy leads shareholders to gradually cash out reducing the value of the firm.

Next, we analyze how debt maturity affects the leverage and credit risk dynamics in response to the evolution of the persistent component of productivity, y_t. We first analyze one path of y_t, starting from $y_0 = \bar{y}$. Because of the homogeneity property of the model, it suffices to show the case of a firm starting with unit capital, $k_0 = 1$, as the dynamics would be the same for a different $k_0 \neq 1$. In Figure 6, we illustrate the evolution of capital stock, debt, leverage, and credit spread

$$cs(b', k', y) = cs(\ell', y) = (r + \xi) \left(\frac{1}{p(\ell', y)} - 1 \right), \tag{13}$$

for a firm currently unlevered ($b_0 = 0$), and then implementing the optimal policy from the MPE. We show the evolution for two possible debt maturities: $1/\xi = 10$ and $1/\xi = 20$. Clearly, although y_t follows a stationary process, (k_t, b_t) do not, due to capital accumulation. However, stationarity is recovered when considering ℓ_t as state variable.

By construction, under our assumption of zero adjustment costs for debt and positive adjustment costs for capital, debt policy is more responsive than investment to changes in profitability, as shown in Figure 6. Only during a significant downturn in y_t, such as the one toward the end of the sample period, does the firm's capital stock deviate from its growth path and contract. In contrast, the firm's debt policy reacts immediately to productivity shocks. The speed of this adjustment depends on debt maturity: with shorter maturity, debt adjustment is faster, as seen in Section 3.

A few new facts emerge from Figure 6. First, the response of investment to a negative shock is faster if the debt maturity is longer, as underinvestment is more severe in this case, which a notable real effect of debt maturity. The resulting leverage dynamics shows that with shorter debt maturity the correlation of new leverage and profitability is higher, whereas for longer maturity leverage is more persistent.

Figure 6: Firm policies and credit risk dynamics with time-varying profitability

The figure shows one path of $(1-\tau)y_t$ and the corresponding evolution of k_t and b_t, starting from $y_0 = \overline{y}$, $b_0 = 0$ and $k_0 = 1$, and using the optimal policy in the MPE. This produces new leverage, $\ell_t = b_t/k_t$ and credit spread, cs_t. These paths are conditional on the firm being solvent at the beginning of each period. We present with solid lines the case with maturity $1/\xi = 10$ (two and half years), and with dotted lines the case with maturity $1/\xi = 20$ (five years). The equilibrium solution is based on the parameters in Table 1.
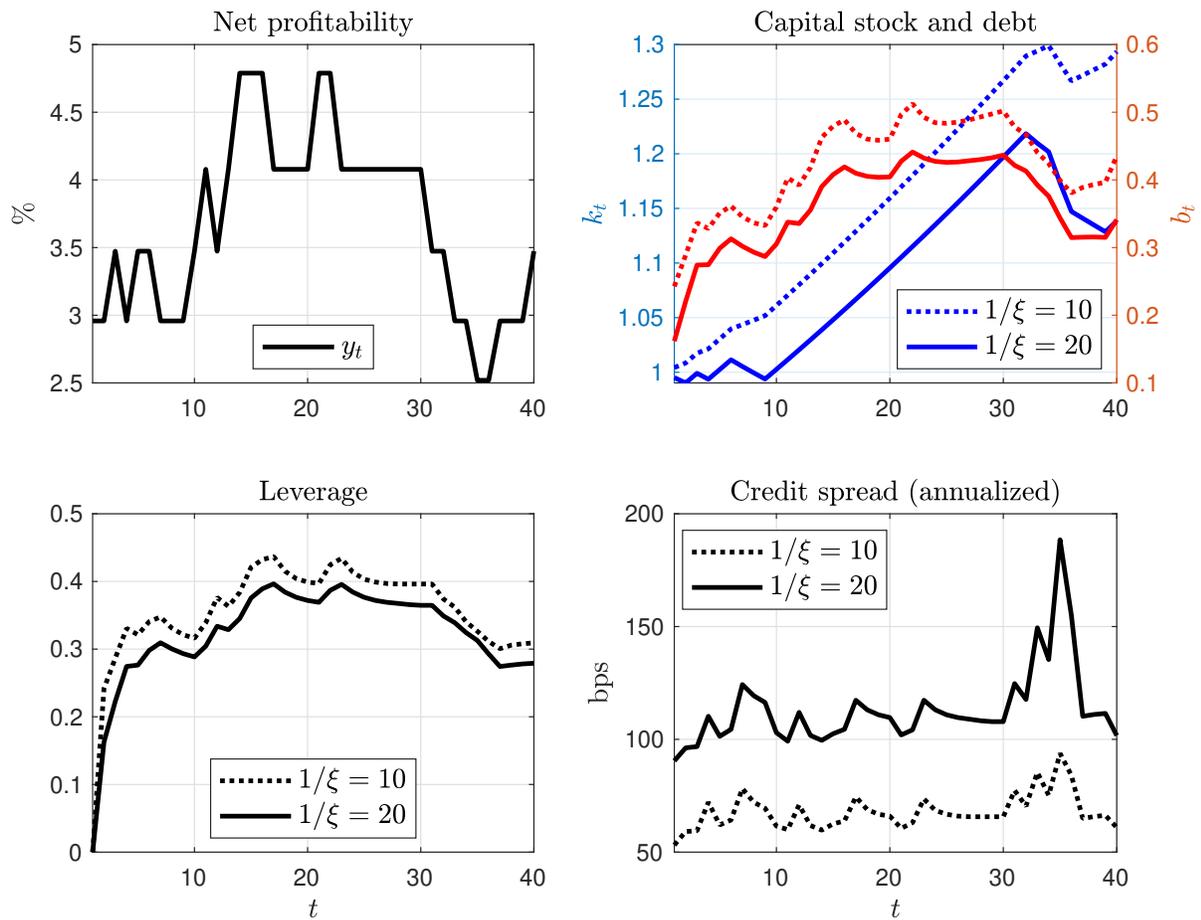
Secondly, Figure 6 shows that the credit spread is affected by the leverage dynamics, and it is lower when the debt has shorter maturity. This is the result of two effects. First, a lower $1/\xi$ reduces the shareholders' commitment problem to debt repayment, as noted also by Dangl and Zechner (2021), and therefore the credit spread for short maturity debt is lower and the spike of the credit spread during the period of economic decline of the firm is contained.[24] The second effect, unique to our setting, is that a lower $1/\xi$ increases the shareholders' commitment to debt and investment policy, and therefore it will reduce the incentive to increase the leverage in those future states in which the firm is solvent. This has a positive ex ante effect on the debt price which reduces the credit spread even further.[25]

Third, capital accumulation in Figure 6 is slower under longer debt maturity, reflecting the fact that for higher $1/\xi$ the commitment problem is more severe. As discussed in Section 3, the more severe the commitment problem, the lower the ex ante value of the tax shield, and therefore the lower marginal q. This reduces investment in each period and, in the long run, lowers the real size of the firm.

A more detailed description of the dynamics of the firm's policies and security prices can be seen in Table 3, in which we report several summary statistics of interesting quantities from a simulated economy under different scenarios for the debt maturity, $1/\xi$. The paths for y and the initial states are the same for all ξ. Within each maturity scenario, given the MPE based on the parameters in Table 1, simulation is done as described in Appendix H. While a firm may default over each quarter, we record the relevant beginning-of-period quantities conditional on a firm being solvent.[26] The statistics presented in Table 3 are calculated as time-series average of cross-sectional averages of the specific statistics.

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[24]In unreported results, we show that 1-period default probability follows a similar path to the credit spread.

[25]In Section 5 we separate the effect of ξ on the credit spread due to the first commitment problem, as analyzed already by Dangl and Zechner (2021), from the one due to the second commitment problem, which is unique to this setting.

[26]In particular, for each (b, k, y) visited with positive probability, given the equilibrium policy (b', k'), where $k' = kh(b/k, y)$ and $b' = k'g(b/k', y)$, we calculate new book leverage as b'/k', new market leverage as $b'p(b', k', y)/F(b, k, y)$, investment as $i = k'/k - (1 - \delta)$, Tobin's Q as $F(b, k, y)/k'$, where $F(b, k, y) = V(b, k, y) + (1 - \xi)bp(b', k', y)$ is total firm value and $V(b, k, y) = kV(\ell, y)$ with $V(\ell, y)$ from (6), and $p(b', k', y) = p(\ell', y)$ from (9).

Table 3: Debt maturity and firm dynamics

The table presents summary statistics of quantities calculated from simulated economies constructed as described in Appendix H, based on the parameters from Table 1, but varying the debt maturity from 1 year (4 quarters) to 5 years (20 quarters). $\mu(\cdot)$ indicates average, $ac(\cdot)$ autocorrelation, $\rho(\cdot, \cdot)$ correlation, bl new book leverage, ml new market leverage, Q Tobin's q, i investment, cs credit spread (in bps), and dp default probability (in %).

$1/\xi$	4	8	12	16	20
$\mu(bl)$	0.31	0.32	0.32	0.31	0.28
$ac(bl)$	0.92	0.95	0.96	0.97	0.97
$\rho(bl, y)$	0.99	0.98	0.96	0.93	0.89
$\mu(ml)$	0.29	0.30	0.29	0.29	0.26
$\mu(Q)$	1.07	1.06	1.05	1.05	1.04
$\mu(i)$	0.11	0.10	0.09	0.08	0.07
$\rho(i, y)$	0.76	0.81	0.88	0.91	0.90
$\mu(cs)$	33.03	59.94	81.36	99.11	119.43
$\mu(dp)$	0.24	0.39	0.49	0.43	0.17

Several remarks from Table 3 are in order. The first and most important is that different debt maturities have real effects because they induce different leverage and investment policies. Tobin's Q is higher (and more persistent, and less sensitive to shocks) when the commitment problem is less severe for shorter debt maturities, which mitigate the commitment problems on debt policy and on investment. Indeed, correlation of investment with y is increasing in debt maturity. This can be explained by considering that, for low $1/\xi$, investment fully reacts to positive shocks on y, but with high $1/\xi$, the correlation of investment with negative productivity shocks increases due to more severe underinvestment, as illustrated in our previous analysis.

The second main point is that longer debt maturity makes leverage more persistent. This aligns with the intuition from Figure 6, which shows that shorter maturity leads to faster adjustment of leverage toward its long-run level after profitability shocks. This is further confirmed by the fact that the correlation between changes in leverage and profitability shocks is nearly perfect for short maturities but declines significantly as maturity increases.

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The final point is that, when there is limited commitment on debt and investment policies, debt maturity significantly affects credit risk and the cost of debt. While default frequency is hump-shaped, in line with average (book and market) leverage, the credit spread increases four-fold when the maturity is 5 years compared to 1 year. We further analyze this point in the next section.

5   Limited commitment and the cost of debt

The previous analysis shows that commitment problems related to debt repayment and future firm policies significantly affect leverage, investment, and ultimately the cost of debt. Both types of commitment issues are influenced by debt maturity, $1/\xi$. On one hand, a higher ξ means that equity holders must repay a larger fraction of the outstanding debt at face value, which increases default risk and the credit spread. On the other hand, in Section 4 we showed that a lower ξ leads to more underinvestment and debt dilution, which lower the price of debt and increase the credit spread. To assess whether (and under what conditions) the commitment problems on debt and investment policie matter more for the cost of debt than the commitment problem on debt repayment—and, relatedly, whether debt maturity is a more effective commitment device for one problem or the other—we decompose the cost of debt financing into two parts: a *default component*, reflecting the commitment problem in debt repayment, and an *agency component*, reflecting underinvestment and debt dilution.

Given the MPE, the credit spread in (13) can be interpreted as the difference between the yield on the firm's debt and the yield on an otherwise identical contract—same maturity and face value—that is not affected by default. Denoting by $p^d(\cdot)$ the price of this default-free contract, evaluated under the equilibrium policy (b', k') in state (b, k, y), equation (13) becomes $(r + \xi) \left(1/p(b', k', y) - 1/p^d(b', k', y)\right)$. To separate the agency and default components of this spread, we would need to isolate the price decline due to commitment problems—on investment and debt issuance—in future states where the firm remains solvent. Let $p^a(\cdot)$ denote the price of a contract identical to the original one, except it is not affected by any commitment problems. Then the agency spread would be $(r + \xi) \left(1/p^d(b', k', y) - 1/p^a(b', k', y)\right)$. However, we have the following proposition.

Proposition 3. *If the equilibrium leverage dynamics is such that the debt is risk free (i.e., it is always $x_d < \underline{x}$), the equilibrium investment and leverage policy in the MPE maximizes the value of the firm (equity plus debt), and there is no agency conflict.*

Because under the MPE eliminating default also removes commitment problems on future debt and investment policies, so that $p^d \equiv p^a \equiv 1$, the default and agency components cannot be separated in (13). Indeed, over an infinite horizon, the accumulation of agency conflicts ultimately leads to the firm's default, meaning the credit spread reflects only the default component.

We propose a workable decomposition of credit spreads into a default and an agency component that is based on the MPE and is conditional on a finite horizon H. The default component is due to the cost of default (according to the equilibrium policy) up to H. The agency component derives from the agency conflicts in *non-default states*, at all dates before H. Because the equilibrium policy is expressed in a dynamic programming setting as one-period decisions, and prices are for infinite-lived securities, we can only define the decomposition in a recursive fashion.[27]

We begin with the one-period horizon case. We define the price at time t of the same debt contract as in the baseline equation (3), but assuming *no default* at $t + 1$, as

$$p_1^d(b, k, y) = \beta \mathbb{E}_y \left[r + \xi + (1 - \xi) \, p(b', k', y') \right], \tag{14}$$

where (b', k') is the equilibrium policy at $t+1$. The one-period *default spread*, capturing the increase in the cost of debt due to the possibility of default at $t + 1$, is defined as

$$ds_1(b', k', y) = (r + \xi) \left(\frac{1}{p(b', k', y)} - \frac{1}{p_1^d(b', k', y)} \right).$$

This spread corresponds to the yield difference between the original debt and an otherwise identical contract without default risk in the first period. It isolates the cost of default occurring specifically at $t + 1$. Next, we define the price at t of the same debt

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[27]In Appendix J, we discuss our decomposition of the credit spread vis-à-vis alternative approaches to decompose the credit spread into a pure default component vs an agency component. Our approach has the key property of being consistent with the MPE, whereas other approaches result in an off-equilibrium decomposition.

contract but without the effects of default and of *underinvestment and debt dilution* on the continuation value at $(t + 1)$ as

$$p_1^a(b, k, y) = \beta \mathbb{E}_y \left[r + \xi + (1 - \xi) \max\{p(b', k', y'), p(b, k, y')\} \right]. \tag{15}$$

The modified continuation value in (15), excludes the *negative effects* on the debt price from changing the state from (b, k) to (b', k') at $t + 1$, i.e., when $p(b', k', y') < p(b, k, y')$, while any positive effects on the debt price from such changes are retained. From $t + 2$ onward, debt holders will be fully exposed to the commitment problems—affecting debt repayment and firm policies—associated with the equilibrium policy, as captured by the equilibrium debt price $p(b', k', y')$. Similarly to what done for the default spread, the one-period *agency spread* is

$$as_1(b', k', y) = (r + \xi) \left(\frac{1}{p_1^d(b', k', y)} - \frac{1}{p_1^a(b', k', y)} \right). \tag{16}$$

Once the default and the agency spreads have been defined, the *credit spread* for one-period exposure to all commitment problems is $cs_1(b', k', y) = ds_1(b', k', y) + as_1(b', k', y)$.

We recursively derive the same decomposition of the credit spread for a predetermined H-period horizon, $H > 1$ and generate the term-structure of the credit spreads and of their two constituents. Specifically, the debt price without the effects of default for n periods, $n = 2, \ldots, H$, is

$$p_n^d(b, k, y) = \beta \mathbb{E}_y \left[r + \xi + (1 - \xi) p_{n-1}^d(b', k', y') \right], \tag{17}$$

where $p_1^d(\cdot)$ is from (14). Hence, the spread for a H-period exposure to default is

$$ds_H(b', k', y) = (r + \xi) \left(\frac{1}{p(b', k', y)} - \frac{1}{p_H^d(b', k', y)} \right), \tag{18}$$

which is the cost of debt due to the chance that the firm defaults in any of the following H periods. In a similar fashion, the price of debt without the effect of commitment problems for n periods, $n = 2, \ldots, H$, is

$$p_n^a(b, k, y) = \beta \mathbb{E}_y \left[r + \xi + (1 - \xi) \max\{p_{n-1}^a(b', k', y'), p_{n-1}^a(b, k, y')\} \right]. \tag{19}$$

where $p_1^a(\cdot)$ is from (15). Then, the H-period agency spread is

$$as_H(b', k', y) = (r + \xi) \left(\frac{1}{p_H^d(b', k', y)} - \frac{1}{p_H^a(b', k', y)} \right), \qquad (20)$$

which is solely due to commitment problems on debt issuance and investment in non-default states in any period from 1 to H. Finally, the credit spread due to exposure to all commitment problems for H periods is $cs_H(b', k', y) = ds_H(b', k', y) + as_H(b', k', y)$.

Proposition 4. *Given the MPE, we have*

1. *for all $H \geq 1$, $ds_H \geq 0$, $as_H \geq 0$, and $cs_H \geq 0$;*

2. *for all $H \geq 1$, $cs_{H-1} \leq cs_H \leq cs$, and for $H \to \infty$, $cs_H \to cs$;*

3. *for $H \to \infty$, $ds_H \to cs$ and $as_H \to 0$.*

The proof is in Appendix G. The intuition is that the longer the horizon, H, over which the creditors are exposed to the two commitment problems, the closer cs_H is to the credit spread in (13), and asymptotically the two are the same. Also, at longer horizons the agency spread becomes negligible and the default spread dominant.

Figure 7 shows how the credit spread at a given horizon H is decomposed into the above defined default and agency components, for debt maturities of 1 and 5 years, current leverage $\ell = 0.1$, and current y at $\bar{y}$. As a benchmark, we report also the credit spread defined in (13), denoted by c^∞. A few remarks are in order. First, c^∞ is higher for higher $1/\xi$. This is intuitive, as ξ controls the commitment of equity holders to future debt and investment policies. A lower ξ (i.e., longer maturity) increases debt dilution and underinvestment incentives, leading to a higher credit spread.

Second, the anticipation by debt holders to be exposed to equity-maximizing debt and investment decisions leads to a sizable agency spread of about 10-12bps at least for horizons of up 10 quarters, when debt maturity is long and current leverage is relatively low. This is because the agency spread reflects the expectation of future transfers of value from debt to equity due to the LRE. The fraction of credit spread that can be attributed solely to agency conflicts related to the leverage policy (right y-axis) is substantial at short horizons. As transfers that are far in the future weigh increasingly less, the agency spread vanishes when longer horizons are considered, and only the default

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Figure 7: Decomposition of credit spread

The figure plots the decomposition of the credit spread into a default spread and agency spread, as defined respectively in equations (18) and (20), for different horizons H. H represents the number of quarters for which debt holders are exposed to the negative effects of equity holders' lack of commitment. The top panel is for maturity $1/\xi = 4$ (1 year) and bottom panel for maturity $1/\xi = 20$ (5 years). The MPE is based on parameters in Table 1, at a current $y = \overline{y}$, and for $\ell = 0.1$.

33
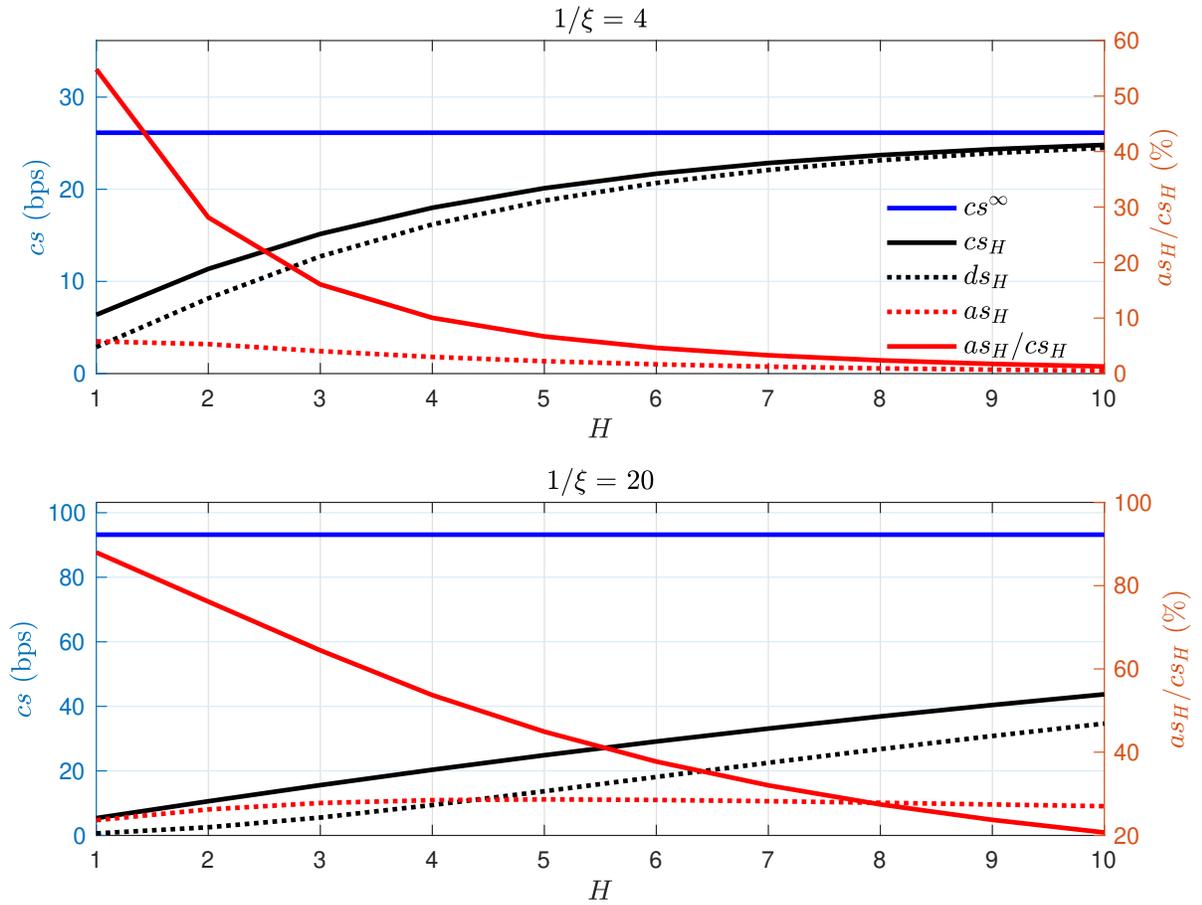
spread remains. In the limit for very long horizons, as seen in Proposition 4, the credit spread only comprises the default component.

Third, the absolute and relative value of the agency spread is strongly influenced by the parameter ξ, being higher at all horizons if debt maturity is longer. As noted by DeMarzo and He (2021), a model based on lack of commitment to future debt issuances, which has negative effect on the investment policy, can generate positive yield spreads for firms even in the presence of small default risk. In the figure, for a case with low current leverage, $\ell = 0.1$, the proportion of credit spread attributable to agency conflicts for $H = 1$ goes from about 50% of the credit spread for 1 year to above 80% for 5 years. These results confirm the intuition provided by Myers (1977) that a longer debt maturity allows for more severe debt dilution and underinvestment. However, in our case the underinvestment issue is exacerbated by lack of commitment on debt policy, which is more severe for longer maturity, as illustrated in Section 3. Overall, a credit risk model based on lack of commitment on future debt policies can succeed where traditional structural models have been unsuccessful (see Huang and Huang (2012)), because it allows for an additional channel of credit risk, an "agency" channel, besides the pure default one, which is sizeable in particular with low leverage firms.

6   Conclusion

Debt maturity is known to help mitigate default risk by committing a firm to repay a fixed amount at a predetermined date. We show that it also plays a key role in reducing the negative effects of shareholders' limited commitment to future debt issuance and investment decisions.

Besides lack of commitment to debt repayment and limited commitment to future debt policies, our model features dynamic asset, with a commitment problem on future investment decisions, to study how the commitment problem on debt policy affects firm's investment, and how the ensuing underinvestment interacts with the incentive to dilute debt holders. To the baseline framework, we also add debt market freezes, which have a more negative effect on firm value the shorter the debt maturity.

Excluding debt market freezes, short debt maturity can effectively eliminate under-investment, as single-period debt can restore value maximizing investment levels. Even with market freezes, short debt maturity mitigates underinvestment and the leverage ratchet effect, although imperfectly.

We calibrate the model and show that debt maturity can be effective at reducing the firm value shortfall created by the three commitment problems that we analyze. Because debt maturity addresses both the commitment problem to debt repayment and the commitment problem to future policies, we decompose the cost of debt into an agency component and a default component. We show that the agency component is more important when the leverage is low and default far, and a lower debt maturity directly affects this part.

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Online Appendix

A   Proof of Proposition 1

Given the linearity of the production function and the assumed capital adjustment cost, it can be proved that $V(b, k, y) = kV(b/k, 1, y)$ (homogeneity of degree one) and $p(b, k, y) = p(b/k, 1, y)$ and $x_d(b', k', y') = x_d(b'/k', 1, y')$ (homogeneity of degree zero). Therefore, we can write the equity program in (1) as

$$
kV\left(\frac{b}{k}, 1, y\right) = k \max_{(b', k')} - \left[\frac{k'}{k} - (1 - \delta)\right] + \tau\delta - \frac{1}{2}\varphi\left[\frac{k'}{k} - (1 - \delta)\right]^2 + \left[\frac{b'}{k} - (1 - \xi)\frac{b}{k}\right] p(b', k', y)
$$

$$
+ \beta\mathbb{E}_y\left[\int_{x_d\left(\frac{b'}{k'}, y'\right)}^{\overline{x}} \left\{(1 - \tau)(y' + x')\frac{k'}{k} - [(1 - \tau)r + \xi]\frac{b'}{k} + \frac{k'}{k}V\left(\frac{b'}{k'}, 1, y'\right)\right\} \phi(x')dx'\right.
$$

$$
\left. - \pi \int_{x_d\left(\frac{b'}{k'}, y'\right)}^{x_d^0\left(\frac{b'}{k'}, y'\right)} \left\{(1 - \tau)(y' + x')\frac{k'}{k} - [(1 - \tau)r + \xi]\frac{b'}{k} + \frac{k'}{k}V\left(\frac{b'}{k'}, 1, y'\right)\right\} \phi(x')dx'\right].
$$

Defining $\kappa = k'/k$, $\ell = b/k$, $\ell' = b'/k'$, and $\eta = b'/k$, simplifying k from both sides, and dropping the capital stock dependence (with a small abuse of notation), $V(\cdot, y) = V(\cdot, 1, y)$ and $p(\cdot, 1, y) = p(\cdot, y)$, from the expressions above we have

$$
V(\ell, y) = \max_{(\eta, \kappa)} - [\kappa - (1 - \delta)] + \tau\delta - \frac{1}{2}\varphi[\kappa - (1 - \delta)]^2 + [\eta - (1 - \xi)\ell]\, p(\ell', y)
$$

$$
+ \beta\mathbb{E}_y\left[\int_{x_d(\ell', y')}^{\overline{x}} \left\{(1 - \tau)(y' + x')\kappa - [(1 - \tau)r + \xi]\eta + \kappa V(\ell', y')\right\} \phi(x')dx'\right.
$$

$$
\left. - \pi \int_{x_d(\ell', y')}^{x_d^0(\ell', y')} \left\{(1 - \tau)(y' + x')\kappa - [(1 - \tau)r + \xi]\eta + \kappa V(\ell', y')\right\} \phi(x')dx'\right],
$$

where $x_d(\ell, y) = r\ell + [\xi\ell - V(\ell, y)]/(1 - \tau) - y$, and $x_d^0(\ell, y)$ has the same expression, but with $V^0(\ell, y)$ in place of $V(\ell, y)$. From this, collecting κ, we have

$$
V(\ell, y) = \max_{(\ell', \kappa)} - [\kappa - (1 - \delta)] + \tau\delta - \frac{1}{2}\varphi[\kappa - (1 - \delta)]^2 + \kappa\left[\frac{\eta}{\kappa} - (1 - \xi)\frac{\ell}{\kappa}\right] p(\ell', y)
$$

$$
+ \kappa\beta\mathbb{E}_y\left[\int_{x_d(\ell', y')}^{\overline{x}} \left\{(1 - \tau)(y' + x') - [(1 - \tau)r + \xi]\frac{\eta}{\kappa} + V(\ell', y')\right\} \phi(x')dx'\right.
$$

$$
\left. - \pi \int_{x_d(\ell', y')}^{x_d^0(\ell', y')} \left\{(1 - \tau)(y' + x') - [(1 - \tau)r + \xi]\frac{\eta}{\kappa} + V(\ell', y')\right\} \phi(x')dx'\right].
$$

We note that $\eta/\kappa = \ell'$. The program above can be decomposed into two parts: the first, for given ℓ and κ (and therefore, the state variable for this program is ℓ/κ) optimizes the last two

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lines with respect to ℓ'; the second part optimizes with respect to κ. More explicitly, defining $\mathcal{V}(\ell', y)$ as in (8) we can write the program above as

$$V(\ell, y) = \max_{\kappa} \ -\left[\kappa - (1 - \delta)\right] + \tau\delta - \frac{1}{2}\varphi\left[\kappa - (1 - \delta)\right]^2$$
$$+ \kappa \max_{\ell'} \left[\ell' - (1 - \xi)\frac{\ell}{\kappa}\right] p(\ell', y) + \beta\mathcal{V}(\ell', y),$$

where it is easy to recognize the first line gives (6) for the outer maximization with respect to κ, and the second line is the inner maximization with respect to ℓ'. From the second line, we define $v(\ell/\kappa, y)$, as in (7).

To determine $V^0(\ell, y)$, we start from (5) and based on the same steps as above, we derive

$$V^0(\ell, y) = \max_{\kappa} \ -\left[\kappa - (1 - \delta)\right] + \tau\delta - \frac{1}{2}\varphi\left[\kappa - (1 - \delta)\right]^2 + \kappa v^0(\ell/\kappa, y)$$

where $v^0(\ell/\kappa, y) = v(\ell/\kappa, y)(1 - \xi\ell/\kappa)$.

As for debt value, we first note that debt recovery can never exceed the asset value at default, $(1 - \tau)(y + x)k + \tau rb + V(b, k, y)$. Consequently, it also cannot surpass the total debt obligation, $(r + \xi)b$. Given $p(b, k, y) = p(\ell, y)$, $R(\ell, y') = \min\{1 - \delta - \alpha, \tau r\ell + V(\ell, y')\}$ and $x_f(\ell, y') = -R(\ell, y')/(1 - \tau) - y'$ we have (9). $\qquad \square$

B   Model of unlevered firm

This is a first benchmark model of a firm with no debt. As the firm is permanently unlevered, the credit market shock is irrelevant. The value of equity (and of the firm) is defined as the fixed point of the recursive program

$$V^u(k, y) = \max_{k'} - \left[k' - (1 - \delta)k\right] + \tau\delta k - \Psi(k, k')$$
$$+ \beta\mathbb{E}_y\left[\int_{x_l(k, y')}^{\bar{x}}\left\{(1 - \tau)(y' + x')k' + V^u(k', y')\right\}\phi(x')dx'\right],$$

where $x_l(k, y') = -\frac{V^u(k, y')}{(1 - \tau)k} - y'$ is the threshold for x' below which the payoff to equity is negative, and therefore excluded by limited liability. The linearity of the production function and the assumed capital adjustment cost ensure that $V^u(k, y) = kV^u(1, y)$ (homogeneity of degree one). We define $\kappa = k'/k$, and (with another small abuse of notation) $V^u(y) = V^u(1, y)$. We have

$$V^u(y) = \max_{\kappa} - (\kappa - 1 + \delta) + \tau\delta - \frac{1}{2}\varphi(\kappa - 1 + \delta)^2 + \kappa\beta f(y), \qquad (21)$$

where

$$f(y) = \mathbb{E}_y\left[\int_{x_l(y')}^{\bar{x}}\left\{(1 - \tau)(y' + x') + V^u(y')\right\}\phi(x')dx'\right] \qquad (22)$$

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and $x_l(y') = -V^u(y')/(1-\tau) - y'$. The solution $V^u(\cdot)$ is found using value iteration together with the optimal κ from first-order condition on (21), $\kappa = \frac{\beta f(y) - 1}{\varphi} + 1 - \delta$, from which

$$V^u(y) = \beta(1-\delta)f(y) + \tau\delta + \frac{1}{2}\frac{(\beta f(y) - 1)^2}{\varphi}. \qquad (23)$$

Under the assumptions that y is non-stochastic and constant at $\overline{y}$ through time, the problem can be solved by replacing V^u from (23) in equation (22), which gives an equation in the unknown f that can be solved using Newton's method.

Under the additional assumption that $x_f(y) \leq \underline{x}$, we have

$$V^u = \max_{\kappa} - [\kappa - (1-\delta)] + \tau\delta - \frac{1}{2}\varphi[\kappa - (1-\delta)]^2 + \kappa\beta[(1-\tau)\overline{y} + V^u]. \qquad (24)$$

The first-order condition is $1 + \varphi[\kappa - (1-\delta)] = \beta[(1-\tau)\overline{y} + V^u]$. Using this equation to eliminate V^u in (24), and after a few manipulations we find the optimal solution of the investment problem:[28]

$$\kappa^* = 1 + r - \sqrt{(r+\delta)^2 + \frac{2}{\varphi}[r - (1-\tau)(\overline{y} - \delta)]}, \qquad (25)$$

where $r = 1/\beta - 1$, as long as

$$\delta + \frac{r}{1-\tau} < \overline{y} < \delta + \frac{r}{1-\tau} + \frac{\varphi}{2}\frac{(r+\delta)^2}{1-\tau}. \qquad (26)$$

By using the optimal investment solution in the first-order condition above, we find the value function:

$$V^u = (1+r)[1 + \varphi(r+\delta)] - (1-\tau)\overline{y} - \varphi(1+r)\sqrt{(r+\delta)^2 + \frac{2}{\varphi}[r - (1-\tau)(\overline{y} - \delta)]}. \qquad (27)$$

If condition (26) is true, the solution in (27) exists and is equal to the solution of (24) found using a value iteration algorithm. This is important, because existence of a solution to the stationary model (6)-(9) hinges upon the existence of a solution to (7)-(9) on the one hand, and on the existence of a solution to (6) on the other. We use condition (26) in the calibration of $\overline{y}$, φ, and δ to make sure that a solution to the investment problem exists.

C Model of levered firm with constant capital stock

This is a second benchmark model of a firm that can manage its leverage while the capital stock remains constant, $k' = k = 1$, and therefore investment equals depreciation. For the sake of brevity, we focus on the case without credit market shocks, as extending the analysis

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[28]Of the two roots of the quadratic equation, the solution is the one in the text because of the condition $\kappa < 1 + r$.

to include them is straightforward. We denote by $K = (1-\tau)\delta + \frac{1}{2}\varphi\delta^2$ the burden deriving from replacing depreciated asset. The following model holds for $\xi < 1$, that is debt maturity strictly higher then one period.[29] The value of equity is

$$
V(b,y) = -K + \max_{b'} \ \left[b' - (1-\xi)b\right] p(b',y)
$$

$$
+ \beta\mathbb{E}_y \left[\int_{x_d(b',y')}^{\overline{x}} \{(1-\tau)(y'+x'-rb') - \xi b' + V(b',y')\} \phi(x')dx' \right], \quad (28)
$$

where $x_d(b',y') = rb' + [\xi b' - V(b',y')]/(1-\tau) - y'$. The debt price in the general case with non-zero recovery at default is

$$
p(b,y) = \beta\mathbb{E}_y \left[\{r + \xi + (1-\xi)p(b',y')\} \int_{x_d(b,y')}^{\overline{x}} \phi(x')dx' + \frac{1}{b} \int_{x_f(b,y')}^{x_d(b,y')} \{(1-\tau)(y'+x') + R(b,y')\}\phi(x')dx' \right],
$$
$$(29)$$

where $R(b,y) = \min\{1 - \delta - \alpha, \tau rb + V(b,y)\}$, $x_f(b,y') = -R(b,y')/(1-\tau) - y'$, with $x_f(b,y') \le x_d(b,y')$, and $b' = g(b,y')$ is the optimal policy function from (28). The solution of the model is found using value function iteration based simultaneously on equations (28) and (29).

D   Total firm value maximization

In this section, we derive a third benchmark model in which investment and debt decisions maximize the value of the firm, thus avoiding all agency problems. We focus on the case without credit market shocks ($\pi = 0$), since this benchmark is used to derive several results later on.

Lemma 5. *The default/investment/financing MPE in the case equity holders commit to maximize total firm value is found by solving the dynamic program[30]*

$$
F(k,y) = \max_{(b',k')} \ -\left[k' - (1-\delta)k\right] + \tau\delta k - \Psi(k,k') + \beta\mathbb{E}_y \left[\int_{x_f(b',k',y')}^{\overline{x}} (1-\tau)(y'+x')k'\phi(x')dx' \right.
$$

$$
\left. + R(b',k',y') \int_{x_f(b',k',y')}^{x_d(b',k',y')} \phi(x')dx' + \{\tau rb' + F(k',y')\} \int_{x_d(b',k',y')}^{\overline{x}} \phi(x')dx' \right], \quad (30)
$$

where the default threshold, $x_d(b',k',y')$, is defined in (2), $x_f(b',k',y')$ is defined in (4), with $V(b',k',y') = F(k',y') - (1-\xi)b'p(b'',k'',y')$, and $p(b,k,y)$ is defined in (3), based on the optimal policy resulting from (30).

---

[29]We will show later on a general model for $\xi = 1$, of which the model with no investment is a subcase.

[30]If there is not recovery at default for the debt, then the expression in (30) becomes

$$
F(k,y) = \max_{(b',k')} \ -[k' - (1-\delta)k] + \tau\delta k - \Psi(k,k') + \beta\mathbb{E}_y \left[\int_{x_d(b',k',y')}^{\overline{x}} \{(1-\tau)(y'+x')k' + \tau rb' + F(k',y')\}\phi(x')dx' \right].
$$

Proof. At state (b, k, y) in t, total firm value, F, for an arbitrary (b', k') is

$$V(b, k, y) + (1 - \xi)b \, p(b', k', y) = - \left[k' - (1 - \delta)k\right] + \tau\delta k - \Psi(k, k') + b'p(b', k', y)$$
$$+ \beta\mathbb{E}_y \left[\int_{x_d(b',k',y')}^{\overline{x}} \left\{ (1 - \tau)(y' + x')k' - \left[(1 - \tau)r + \xi\right]b' + V(b', k', y') \right\} \phi(x')dx' \right],$$

the right-hand side of which, by replacing $p(b', k', y)$ from (3) (with $\pi = 0$) and after few simplifications, becomes

$$- \left[k' - (1 - \delta)k\right] + \tau\delta k - \Psi(k, k') + \beta\mathbb{E}_y \left[\int_{x_f(b',k',y')}^{\overline{x}} (1 - \tau)(y' + x')k'\phi(x')dx' \right]$$
$$+ \beta\mathbb{E}_y \left[R(b', k', y') \int_{x_f(b',k',y')}^{x_d(b',k',y')} \phi(x')dx' + \left\{ \tau rb' + V(b', k', y') + (1 - \xi)b'p(b'', k'', y') \right\} \int_{x_d(b',k',y')}^{\overline{x}} \phi(x')dx' \right],$$

where (b'', k'') denotes the choice at $t + 1$. By replacing $V(b', k', y') + (1 - \xi)b'p(b'', k'', y')$, with total firm value at (b', k', y') and choosing (b', k') that maximizes the expression above we define the recursive program in (30), from which we determine F and the related optimal policy. Notably, the program on the right-hand side of (30) does not depend on b, and therefore the optimal choice, $(b', k') = G(k, y)$, and the value function are independent of b. Therefore, a recursive application of (30) gives $F(k, y)$. The debt price is derived from equation (3) (with $\pi = 0$ by assumption) using the optimal policy $(b', k') = G(k, y)$, and the value of equity required to pin down the recovery value, R, and the thresholds x_d and x_f is $V(b', k', y') = F(k', y') - (1 - \xi)b'p(b'', k'', y')$. $\qquad\square$

Lemma 6. *The MPE for the case equity holders maximize total firm value is $F(y) = \beta(1 - \delta)f(y) + \tau\delta + \frac{1}{2}\frac{(\beta f(y) - 1)^2}{\varphi}$ where*[31]

$$f(y) = \max_{\ell'} \ \mathbb{E}_y \left[\int_{x_f(\ell',y')}^{\overline{x}} (1 - \tau)(y' + x')\phi(x')dx' \right.$$
$$\left. + R(\ell', y') \int_{x_f(\ell',y)}^{x_d(\ell',y)} \phi(x')dx' + \left\{ \tau r\ell' + F(y') \right\} \int_{x_d(\ell',y)}^{\overline{x}} \phi(x')dx' \right] \quad (31)$$

with thresholds

$$x_d(\ell', y') = r\ell' + \frac{\xi\ell' - [F(y') - (1 - \xi)\ell'p(y')]}{1 - \tau} - y', \quad x_f(\ell', y') = -\frac{R(\ell', y')}{1 - \tau} - y',$$

---

[31] With no recovery at default, $f(y) = \max_{\ell'} \ \mathbb{E}_y \left[\int_{x_d(\ell',y')}^{\overline{x}} \left\{ (1 - \tau)(y' + x') + \tau r\ell' + F(y') \right\} \phi(x')dx' \right]$. If we consider credit market shocks, the expression is the same as in (31), except that the term $\pi \left\{ \tau r\ell' + F(y') - R(\ell', y') \right\} \int_{x_d(\ell',y')}^{x_d^0(\ell',y')} \phi(x')dx'$ is subracted from it.

recovery at default $R(\ell', y') = \min\{1 - \delta - \alpha, \tau r \ell' + F(y') - (1 - \xi)\ell' p(y')\}$, *and debt price*

$$p(y) = \beta \mathbb{E}_y \left[\{r + \xi + (1 - \xi)p(y')\} \int_{x_d(\ell',y')}^{\overline{x}} \phi(x')dx' + \frac{1}{\ell'} \int_{x_f(\ell',y')}^{x_d(\ell',y')} \{(1 - \tau)(y' + x') + R(\ell', y')\}\phi(x')dx' \right].$$

(32)

Proof. The stationary program to solve (30), following the approach in Appendix A, is

$$F(y) = \max_\kappa - [\kappa - (1 - \delta)] + \tau\delta - \frac{1}{2}\varphi [\kappa - (1 - \delta)]^2$$

$$+ \beta\kappa \max_{\ell'} \mathbb{E}_y \left[\int_{x_f(\ell',y')}^{\overline{x}} (1 - \tau)(y' + x')\phi(x')dx' + R(\ell', y') \int_{x_f(\ell',y')}^{x_d(\ell',y')} \phi(x')dx' + \{\tau r \ell' + F(y')\} \int_{x_d(\ell',y')}^{\overline{x}} \phi(x')dx' \right],$$

where we used $F(k, y) = kF(1, y)$ and then dropped the first argument, with the usual abuse of notation, $F(y) = F(1, y)$. Also, $R(\ell', y') = \min\{1 - \delta - \alpha, \tau r \ell' + V(\ell', y')\}$, $x_d(\ell', y') = r\ell' + (\xi\ell' - V(\ell', y'))/(1 - \tau) - y'$, and $x_f(\ell', y') = -R(\ell', y')/(1 - \tau) - y'$, where $V(\ell', y')$ is the related equity value. From this, we can define the outer investment program $F(y) = \max_\kappa - [\kappa - (1 - \delta)] + \tau\delta - \frac{1}{2}\varphi [\kappa - (1 - \delta)]^2 + \beta\kappa f(y)$, and the inner leverage program in (31). The optimal policy in the leverage program depends only on the current y, that is $\ell' = g(y)$. Therefore, the relevant price of debt at the current date is $p(y) = p(g(y), y)$, which only depends on y and not on current leverage, as shown in (32). Hence, the equity value required to determine the two thresholds and the recovery at default is $V(\ell', y') = F(y') - (1 - \xi)\ell' p(y')$. Once $f(y)$ and the leverage policy $\ell' = g(y)$ are determined, the outer investment program has analytic solution $\kappa^* = \frac{\beta f(y) - 1}{\varphi} + 1 - \delta$, and replacing it in the investment program we have $F(y)$. $\qquad\square$

Under the assumption that y is non-stochastic and constant at $\overline{y}$, the problem can be easily solved as follows: F is replaced in (31), and the optimal leverage, ℓ^*, is found as a function of the unknown f and p. Next, $\ell^*(f, p)$ is replaced on the right-hand side of (31). This gives an expression in f and p. Similarly, equation (32) depends on f and p. Hence, there are two non-linear equations, which can be solved simultaneously in the unknowns f and p using Newton's method.

E   Proof of Proposition 2

1. Convexity of $v(\cdot, y)$ is a consequence of its optimality, as per equation (7). Indeed, for any $\ell' \neq \ell$, shareholders can adjust the leverage to ℓ with proceeds $(\ell - \ell')p$ from the issuance.[32] Given the optimality of v, the shareholders cannot get a higher value than $v(\ell'/\kappa, y)$, that is $v(\ell'/\kappa, y) \geq v(\ell/\kappa, y) - (\ell' - \ell)p/\kappa$, which means v is (weakly) convex. The same argument can be used to show the convexity of $V(\cdot, y)$, given $\kappa = \kappa^*$, the optimal investment policy.

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[32]This is equivalent to adjusting the debt from b' to b, with proceeds $(b - b')p/k$.

2. For $\xi = 1$ and $\pi = 0$, the right-hand side of (1) becomes

$$\max_{(b',k')} -\left[k' - (1-\delta)k\right] + \tau\delta k - \Psi(k,k') + b'p(b',k',y)$$

$$+ \beta\mathbb{E}_y\left[\int_{x_d(b',k',y')}^{\overline{x}}\left\{(1-\tau)(y'+x')k' - \left[1+(1-\tau)r\right]b' + V(b',k',y')\right\}\phi(x')dx'\right],$$

and $p(b',k',y)$ becomes independent of the future debt price in relation to the optimal policy, and only depends of the current state. By replacing in it $p(b',k',y)$ from equation (3) with $\xi = 1$ and simplifying, we find

$$\max_{(b',k')} -\left[k' - (1-\delta)k\right] + \tau\delta k - \Psi(k,k') + \beta\mathbb{E}_y\left[\int_{x_f(b',k',y')}^{\overline{x}}(1-\tau)(y'+x')k'\phi(x')dx'\right.$$

$$\left.+ R(b',k',y')\int_{x_f(b',k',y)}^{x_d(b',k',y)}\phi(x')dx' + \left\{\tau rb' + V(b',k',y')\right\}\int_{x_d(b',k',y')}^{\overline{x}}\phi(x')dx'\right].$$

This program coincides with the one in the right-hand side of (30), except for the value function. However, because current leverage is identically zero for $\xi = 1$, equity and firm value coincide, $V \equiv F$, and the programs yield recursively the same optimal value and policy. Hence, the fixed point for this program is the same as the one for (30) with $\xi = 1$.

To show that also investment is firm-value maximizing, from (30) and following the same steps as in Lemma 6, with $V(y)$ in place of $F(y)$, we have the outer investment program $V(y) = \max_\kappa -\left[\kappa - (1-\delta)\right] + \tau\delta - \frac{1}{2}\varphi\left[\kappa - (1-\delta)\right]^2 + \kappa\beta v(y)$, the inner leverage program

$$v(y) = \max_{\ell'}\mathbb{E}_y\left[\int_{x_f(\ell',y')}^{\overline{x}}(1-\tau)(y'+x')\phi(x')dx'\right.$$

$$\left.+ R(\ell',y')\int_{x_f(\ell',y')}^{x_d(\ell',y')}\phi(x')dx' + \left\{\tau r\ell' + V(y')\right\}\int_{x_d(\ell',y')}^{\overline{x}}\phi(x')dx'\right],$$

and the corresponding debt price

$$p(y) = \beta\mathbb{E}_y\left[(1+r)\int_{x_d(\ell',y')}^{\overline{x}}\phi(x')dx' + \frac{1}{\ell'}\int_{x_f(\ell',y')}^{x_d(\ell',y')}\left\{(1-\tau)(y'+x') + R(\ell',y')\right\}\phi(x')dx'\right],$$

where $\ell' = g(y)$ is the optimal leverage from the leverage program, and $\kappa = h(y)$ the optimal investment from the outer program. Given the independence of $v(y)$ and $V(y)$ from current leverage, $\ell' = g(y)$ and $\kappa = h(y)$ are independent from ℓ.

3. (a) From (6), we have[33]

$$\partial_1 V(\ell, y) = \kappa^* \frac{1}{\kappa^*} \partial_1 v(\ell/\kappa^*, y) = \partial_1 v(\ell/\kappa^*, y) = -(1 - \xi)p(\ell', y), \qquad (33)$$

where κ^* is the optimal solution of the program in (6), and where the second equation follows from (7). Because $p(\ell', y)$ is positive and $\xi < 1$, then both v and V are decreasing in their first argument. To show that $p(\cdot, y)$ is decreasing,

$$\partial_1 p(\ell, y) = \beta \mathbb{E}_y \left[(1 - \xi)\partial_1 p(\ell, y')\partial_1 g(\ell, y') \int_{x_d(\ell, y')}^{\overline{x}} \phi(x')dx' \right.$$
$$\left. - \left[r + \xi + (1 - \xi)p(\ell', y') \right] \phi(x_d(\ell, y'))\partial_1 x_d(\ell, y') \right]. \quad (34)$$

Under the assumption that v is twice differentiable, and denoting for convenience $\tilde{\ell} = \ell/\kappa$, we have that $\partial_1^2 v(\tilde{\ell}, y) = -(1 - \xi)\partial_1 p(\ell', y)\partial_1 g(\tilde{\ell}, y)$ is positive because $v(\cdot, y)$ is convex and $\xi < 1$. Hence, $\partial_1 p\, \partial_1 g \leq 0$ in (34). Also, using (33),

$$\partial_1 x_d(\ell, y) = r + \frac{\xi - \partial_1 V(\ell, y)}{1 - \tau} = r + \frac{\xi + (1 - \xi)p(\ell, y)}{1 - \tau},$$

which is positive. Hence, the right-hand side of (34) is negative.

(b) The first-order condition of (7) with $\pi = 0$ is

$$p(\ell', y) + \left[\ell' - (1 - \xi)\tilde{\ell} \right] \partial_1 p(\ell', y)$$
$$+ \beta \mathbb{E}_y \left[\int_{x_d(\ell', y')}^{\overline{x}} \left\{ -(1 - \tau)r + \xi + (1 - \xi)\partial_1 V(\ell', y') \right\} \phi(x')dx' \right] = 0,$$

where again $\tilde{\ell} = \ell/\kappa$. Using (33), the first-order condition becomes

$$\left[\ell' - (1 - \xi)\tilde{\ell} \right] \partial_1 p(\ell', y)$$
$$+ \beta \mathbb{E}_y \left[\tau r \int_{x_d(\ell', y')}^{\overline{x}} \phi(x')dx' + \frac{1}{\ell'} \int_{x_f(\ell', y')}^{x_d(\ell', y')} \left\{ (1 - \tau)(y' + x') + R(\ell', y') \right\} \phi(x')dx' \right] = 0.$$

and $\ell' = g(\tilde{\ell}, y)$ is its unique solution under the assumption of concavity of the objective function. Denoting by $\Gamma(\tilde{\ell}, \ell')$ the left-hand side of the equation above, and using the implicit function theorem, the derivative of $g(\tilde{\ell}, y)$ is $\partial_1 g = -\partial_1 \Gamma / \partial_2 \Gamma$. Because for $\xi < 1$ we have $\partial_1 \Gamma = -(1 - \xi)\partial_1 p(\ell', y) > 0$, then the sign of $\partial_1 g$ depends on the sign of $\partial_2 \Gamma$. As the objective function in (7) is concave, the latter is negative, and hence $\partial_1 g \geq 0$.[34]

---

[33] The notation $\partial_1^2 f(\hat{x}, \hat{y})$ denotes the second partial derivative with respect to x, evaluated at $(\hat{x}, \hat{y})$.
[34] If there is zero recovery at default, the partial derivative of Γ with respect to ℓ' becomes

$$\partial_2 \Gamma = \partial_1 p(\ell', y) + \left[\ell' - (1 - \xi)\tilde{\ell} \right] \partial_1^2 p(\ell', y) - \beta \tau r \mathbb{E}_y \left[\partial_1 x_d \phi(x_d) \right],$$

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(c) The derivation of (11) is straightforward using equation (33) and observing that, based on (7), we have

$$v(\ell/\kappa, y) + (1 - \xi)\frac{\ell}{\kappa}p(\ell', y) = \beta\mathcal{V}(\ell', y) + \ell'p(\ell', y).$$

To determine the sign of $\partial_1 h$, we use the implicit function theorem again, although based on the first-order condition for optimal investment in (11), which gives

$$\partial_1 h = \frac{\ell/\kappa^2 \partial_1^2 v\,(\ell/\kappa, y)}{\ell^2/\kappa^3 \partial_1^2 v\,(\ell/\kappa, y) - \varphi}.$$

Under the assumption that the objective function in (6) is concave, the denominator of the ratio is negative. Because v is convex, $\partial_1^2 v \geq 0$, the denominator is negative if and only if φ is large enough. For the same reason, the numerator of the ratio is positive. Hence, $\partial_1 h \leq 0$, that is investment decreases with leverage. □

F   Proof of Proposition 3

If $x_d < \underline{x}$ for all (b, k, y'), then from (3) we have $p(b, k, y) = \beta\mathbb{E}_y\left[r + \xi + (1 - \xi)p(b', k', y')\right]$. Because $r = 1/\beta - 1$, by conjecturing the solution $p(b', k', y') = 1$ on the right-hand side, then the debt price is $p(b, k, y) = 1$, which confirms this is a solution. It is easy to show this is the only solution of the pricing equation. Under the no-default assumption, the right-hand side of (1) becomes

$$\max_{(b',k')} - \left[k' - (1 - \delta)k\right] + \tau\delta k - \Psi(k, k') - (1 - \xi)b$$

$$+ \beta\mathbb{E}_y\left[(1 - \tau)y'k' + \tau rb' + (1 - \xi)b' + V(b', k', y')\right] =$$

$$-(1 - \xi)b + \max_{(b',k')} - \left[k' - (1 - \delta)k\right] + \tau\delta k - \Psi(k, k') + \beta\mathbb{E}_y\left[(1 - \tau)y'k' + \tau rb' + F(b', k', y')\right],$$

where we took out of the objective function the constant term, $-(1 - \xi)b$, which does not affect the optimal policy. Hence, the optimal policy, (b', k'), maximizes the value of the firm (for the no-default case). The equity value resulting from this program, $V(b, k, y)$, plus $(1 - \xi)b$, gives $F(b, k, y)$. Hence, solving the fixed point of the above program gives the optimal firm value and the firm value maximizing investment and debt policies, as shown in Lemma 5. □

G   Proof of Proposition 4

We first derive the following lemma. We focus on the case with no financial market freezes $(\pi = 0)$.

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which is negative if $\partial_1^2 p$, the second partial derivative of p with respect to ℓ, is negative.

Lemma. For all $n \geq 0$, $p_{n+1}^j \geq p_n^j$, where $p_0^j = p$, for $j = d, a$.

Proof. The proof is by induction. We first consider the case $j = d$. For $n = 0$, the statement is $p_1^d(b, k, y) \geq p(b, k, y)$, which is true because in equation (3) in default $(x' < x_d(b, k, y'))$

$$(1 - \tau)(y' + x')k + R(b, k, y') \leq (r + \xi)b.$$

Next, assuming the inequality is true for a given n, that is $p_n^d(b', k', y') \geq p_{n-1}^d(b', k', y')$, because the firm's policy is invariant with respect to n and using equation (17), from this inequality we have $p_{n+1}^d(b, k, y) \geq p_n^d(b, k, y)$, which proves the lemma.

We then consider the case $j = a$. For $n = 0$ the statement is $p_1^a(b, k, y) \geq p(b, k, y)$. We first notice that $p_1^a(b, k, y) \geq p_1^d(b, k, y)$ because using (14) and (19) this is equivalent to $\max\{p(b', k', y'), p(b, k, y')\} \geq p(b', k', y')$, which is obviously true. We have proved above that $p_1^d(b, k, y) \geq p(b, k, y)$, from which we conclude $p_1^a(b, k, y) \geq p(b, k, y)$. Next, assuming the statement is true for given n, that is $p_n^a \geq p_{n-1}^a$, this implies

$$\max\{p_n^a(b', k', y'), p_n^a(b, k, y')\} \geq \max\{p_{n-1}^a(b', k', y'), p_{n-1}^a(b, k, y')\}.$$

Because the firm's policy is invariant with respect to n and using (19), from the above inequality we have $p_{n+1}^a(b, k, y) \geq p_n^a(b, k, y)$, which proves the lemma. $\square$

1. The proof is by induction. As for the non-negativity of the agency spread, for the case $H = 1$ the statement $as_1(b', k', y) \geq 0$ is equivalent to $p_1^a(b', k', y) \geq p_1^d(b', k', y)$. Given the specification of the price of the debt without default in (14) and without default and agency conflicts in (15), $p_1^a(b, k, y) \geq p_1^d(b, k, y)$ is equivalent to $\max\{p(b', k', y'), p(b, k, y)\} \geq p(b', k', y')$, which is obviously true. As for the induction step, assuming the statement $p_{H-1}^a(b', k', y') \geq p_{H-1}^d(b', k', y')$ is true for $H - 1$, for the case $H > 1$, $as_H(b', k', y) \geq 0$ is equivalent to $p_H^a(b', k', y') \geq p_H^d(b', k', y')$. Using the same argument as above, comparing (19) to (17) we have

   $$\max\{p_{H-1}^a(b', k', y'), p_{H-1}^a(b, k, y')\} \geq p_{H-1}^a(b', k', y') \geq p_{H-1}^d(b', k', y'),$$

   which is assumed true. Hence the agency spread is always non-negative.

   To prove that $ds_H \geq 0$, which is equivalent to $p_H^d \geq p$, for all $H \geq 1$ we use the lemma above, and note that $p_H^d \geq p_{H-1}^d \geq \ldots \geq p$.

2. The statement $cs_{H-1} \leq cs_H$ is equivalent to $p_H^a \geq p_{H-1}^a$, which has been proved in the lemma above. The statement $cs \geq cs_H$ is equivalent to $p_H^a \leq 1$, which is true for all $H \geq 1$ as p_H^a is bounded by the risk-free price of \$1. Also, $cs_H \rightarrow cs$ for $H \rightarrow \infty$, because the sequence $\{p_H^a\}$ is increasing and bounded above, hence it has limit and $p_H^a \rightarrow 1$, which concludes the proof.

3. To prove that $ds_H \rightarrow cs$ for $H \rightarrow \infty$, we observe that $ds_H \leq cs_H \leq cs$ for all H. Also, $ds_H \leq cs$ is equivalent to $p_H^d \leq 1$ for all H. As before, the sequence $\{p_H^d\}$ is increasing and bounded above by 1, hence $p_H^d \rightarrow 1$.

Point (1) above states that $as_H \geq 0$, which is equivalent to $p_H^a \geq p_H^d$, for all H. Because $p_H^d \to 1$ and $p_H^a \to 1$, this proves that $as_H \to 0$. $\qquad\square$

H   Numerical algorithm

The algorithm to solve the general model is based on a discretized version of the leverage interval, $[0, \bar{\ell}]$. Although the numerical results are presented for $\ell \in [0, 1]$, we solve the model for $\bar{\ell} > 1$ to avoid the effect of the upper bound on the numerical solution. We discretize the process for y with seven points the compact interval $\left[-3\sigma/\sqrt{1-\nu^2}, +3\sigma/\sqrt{1-\nu^2}\right]$ using Gauss-Hermite quadrature. Therefore, we solve the program (7)-(9) using a value iteration algorithm on a discrete space for (ℓ, y). As for the distribution of x, we set $[\underline{x}, \overline{x}] = [-X, X]$, with $X > 0$, and use the specification $\phi(x) = \frac{3}{4X}\left[1 - \left(\frac{x}{X}\right)^2\right]$. With this specification of ϕ, the integrals involved in the calculations of the iterative algorithm have analytic expressions, which improve efficiency.

To find the MPE of the firm model, at a given step of the iterative procedure, based on $V(\ell, y)$, where $\ell = b/k$, and on $\kappa = k'/k$ from the previous step, we find $v(\ell/\kappa, y)$ and $\ell' = g(\ell/\kappa, y)$ from (7), and we calculate $p(\ell, y)$ in (9). The optimization in (7) with respect to ℓ' is done with an exhaustive search on a grid of 7500 points in $[0, \bar{\ell}]$. The tolerance set for maximum error in two successive iterations of the value function in (7) and of the debt price in (9) is 10^{-5}.[35] Then, in (6) we solve the first-order condition for optimal investment, which requires a numerical approach, as κ is defined implicitly, and because $v(\cdot, y)$ is defined numerically. This is done by using a cubic spline interpolation of $v(\ell/\kappa, y)$, with 100 points, which allows the numerical calculation of its first derivative efficiently. As highlighted in Appendix B, to ensure existence of a solution to the stationary model (7)-(6), on the one hand a solution to (7)-(9) has to exist as discussed above, and on the other a solution to (6) must exist. This requires further restriction of model parameters, as shown in (26). This step finds $\kappa = h(\ell, y)$ and $V(\ell, y)$, from which a new iteration is started. The iterative procedure is halted when the maximum between the improvement of value function and the improvement of debt price over an iteration is lower than a given tolerance.

To obtain a simulated economy, we simulate 1000 firms starting from random initial states, for 200 quarters, and drop the first 50 quartes. We then calculate sample averages, as time series averages of cross-sectional averages in the simulated economy.

I   Data construction

We collect data from multiple sources. Firm-level accounting and financial data are from the merged CRSP-COMPUSTAT database. To match the model structure, we scale variables,

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[35]Notably, Chatterjee and Eyigungor (2012) states that the algorithm fails if convergence is not achieved with tolerance of 10^{-5} in less than 3000 steps. In our numerical experiments, we always achieve convergence in less than 500 steps.

when appropriate, by the sum of the book value of equity and total financial debt (i.e., short-term liabilities plus long-term debt), instead of total assets.

To reduce issues stemming from differences in bond contract terms, tax treatment, and market liquidity, we focus on credit spreads from senior unsecured credit default swap (CDS) contracts. Specifically, we use one and five-year CDS spreads for non-financial firms in the S&P 500 index as our main credit spread measure. Availability of CDS prices restricts the sample to the period of time between 2001 and 2014.

In our model, debt maturity serves as a commitment device because a firm can issue debt only once per period, in our case, once per quarter. When calibrating the model, we therefore focus on firms that issue debt at most once per quarter. To implement this restriction, we merge the dataset defined above with the LSEG Mergent Fixed Income Securities Database, which provides detailed information on bond issuances, and select firms with no more than one issuance of combined bonds per quarter. We have excluded revolvers and credit lines from this calculation. The final dataset includes 458 distinct firms and 14,133 firm-quarter observations.

In our sample of firms there are only 9 default events that can be identified by merging data from Moody's KMV, Bloomberg, Standard & Poor's, and FISD Mergent. These events include Chapter 7 and Chapter 11 filings, as well as missed payments on interest or principal, across both bank loans and publicly issued debt. To avoid the measurement errors implied by basing an estimate on such a particular sample, we use the historical default frequency for BBB-rated firms computed by Moody's. In the most recent default study available (2023), Moody's report a default probability of 0.20% from a very long sample that spans the years between 1920 and 2022.

J Discussion on the proposed approach to decompose the credit spread in the agency vs the default component

The proposed decomposition of the credit spread into a default component and an agency component is motivated by the purpose of separating the affect of the two commitment problems, to repay the debt and to future debt and investment policies, which are both controlled by the same maturity parameter, ξ. Also, the decomposition is for a *given* investment and financing policy in the MPE. We argue that any alternative decomposition approach to the one introduced above would entail an interaction between the policy effects and the price effects of the agency issues, which would cause a deviation from the equilibrium policy and ultimately would violate the assumed equilibrium. Hence, the resulting separation in two components would depend on the extent of such an interaction and therefore would be arbitrary.

This can be better understood by thinking of two (seemingly plausible) decomposition approaches. In the first approach, the effect of agency would be neutralized because the value losses from dilution are 'rebated' to legacy bond holders by shareholders. Alternatively, the 'rebate' may be paid by an external party, who would need to be financed by the shareholders

anyway. Because of this provision, the shareholders' policies would change relative to the equilibrium ones.

A second approach is to assume the shareholders are subject to a 'blanket' protective covenant that precludes them from taking any actions that lowers bond values at the respective horizons. While such a covenant would eliminate the effects of agency on the continuation value on the debt, it would also constrain the shareholders' future decisions, affecting their policy.[36] For both approaches, we would then define the agency spread as the difference between the credit spread obtained in the benchmark model and the one resulting from the approach. Either way, the resulting decomposition into an agency and a default spread would not separate the pure default component from the agency component under the MPE, exactly because this definition entails a deviation from that equilibrium.

Our decomposition of the credit spread naturally follows by applying the same approach used to define the default spread to the case of a debt price reduction due to agency conflicts. Given the equilibrium policy, there is no other way to define the credit spread but $(r + \xi)/p - (r + \xi)/p^d$ (or $(r + \xi)/p - (r + \xi)/p_H^d$ for finite horizons), because the yield $(r + \xi)/p^d$ is for a debt which is identical to the one in the baseline model, except for excluding default. Notably, to define the latter debt we do not need to assume that the debt holders get a 'rebate' or there is a protective covenant (or collateral) that eliminates default risk. In fact, if the debt contract had this provision, we would be calculating the credit spread for a policy different from the one in the MPE.

Finally, the definition in (20) provides a *on-the-equilibrium-path* decomposition of the credit spread into a default component and an agency component exactly because we do not allow the equilibrium policy to be affected. An alternative definition, which entails a deviation from the equilibrium policy, would be off the equilibrium path at any state of the baseline model.

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[36]Also Jungherr and Schott (2021) quantify the cost of corporate debt due to lack of commitment. Differently from our decomposition of the equilibrium credit spread, they calculate this cost in a counterfactual experiment, comparing the credit spread in an economy without commitment to the credit spread in an economy with commitment. This latter case is analogous to the second approach in the text, based on the elimination of the commitment problem. While this may give an idea of the increased cost of capital, in equilibrium the average leverage is lower under commitment, which in part drives the difference of credit spread. Our approach, by using the MPE policies and prices, overcomes this issue.