A Factor-Proportions Theory of Endogenous Firm Heterogeneity

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Abstract

In the model where the choices of technology by firms endogenously determine productivity differences, we investigate the link between factor endowment and the productivity both in the firm and industry levels. We find among others that firms in capital-abundant countries tend to adopt new advanced technologies more in their production processes, and that opening to international trade will not equalize factor prices across countries if their up-to-date technology levels are different, which in turn depends on their factor endowments.

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1 Introduction

Productivity varies across firms even within the same industry. Some firms are more efficient than others since they hire more-productive workers or they adopt more-advanced production technology. Indeed, Bartel and Lichtenberg (1987) show the evidences that educated workers have comparative advantage in implementing new technology. Doms et al. (1997) also find that the plants that adopt new automation technology tend to have more skilled workers. Even if new advanced production technology is in the public domain, some firms choose not to adopt it perhaps due to the lack of human capital that is essential to the technology adoption.

This linkage between the difference in the intrinsic productivity of firms, due to the difference in the amount of human capital, and the adoption rate of new technology explains the coexistence of capital-intensive advanced technology and labor-intensive old technology within the same industry. Anecdotal evidence suggests that even the same firm often uses different technology depending on the plant location. Firms tend to adopt capital-intensive advanced technology in capital-abundant developed countries while they tend to use labor-intensive old technology in labor-abundant less-developed countries. Firms in less-developed countries use less-advanced technology on average, not just because of its lack of capacity to absorb new technology but perhaps because that less-developed countries tend to be labor-abundant so firms that operate there deliberately adopt old technology to save capital.[1]

In this paper, we formalize the idea that factor endowments affect firm-level productivity heterogeneity through firms' technology adoption. For this purpose, we extend the framework of monopolistic competition and heterogeneous firms in Melitz (2003) by introducing firms' appropriate technology choices à la Zeira (1998, 2006). Melitz (2003) introduces productivity difference across firms within the same industry and builds a general equilibrium model in which less productive firms sell only domestically while productive firms sell both domestically and internationally. He also investigates how trade liberalization affects firms with different productivity levels differently. In his model, different productivity levels are assigned to *ex ante* symmetric firms exogenously; and once assigned, productivity of a firm remains unchanged. In reality, however, efficiency gap between productive firms and unproductive ones tend to expand with technological improvement.

[1]Examining a sample of 230 U.S. cities over the period from 1980 to 2000, Beaudry et al. (2006) find that firms located in cities endowed with relatively abundant and cheap skilled labor on average adopted personal computers more aggressively than cities with relatively expensive skilled labor.

In our model, firms are allowed to choose either labor-intensive old technology or capital-intensive new technology. However, unlike Zeira (1998, 2006), firms disperse in terms of their intrinsic productivity levels, which leads to the dispersion of technology adoption within the same industry. On the one hand, labor-intensive old technology is standardized so that all firms can utilize it in the same manner. Capital-intensive new technology, on the other hand, requires cognitive skills that vary across firms, so the difference in productivity arises once they adopt new production technologies. Our model specification is rich enough so that efficiency gap between inherently-productive firms and inherently-unproductive firms expands with technological improvement; inherently-productive firms become more efficient as the technology advances while inherently-unproductive firms stop adopting new technologies once the technological frontier reaches a certain level.

In the model where the choices of technology by firms endogenously determine productivity differences, we investigate the link between factor endowment and the productivity both in the firm and industry levels. We find among others that firms in capital-abundant countries tend to adopt new advanced technologies more in their production processes, and that opening to international trade will not equalize factor prices across countries if their up-to-date technology levels are different, which in turn depends on their factor endowments.

Bernard et al. (2007) also investigate the link between factor endowment and the industry-wide productivity distribution across firms, assuming as Melitz (2003) that the productivity of each firm is exogenously given. Ederington and McCalman (2004, 2006) consider models in which ex ante identical firms become heterogenous in terms of productivity levels as a result of their different choices of the date of technology adoption. In their models, firms are indifferent to which timing they would choose since while delaying the date of technology adoption reduces firms' profits, the adoption cost of the new technology gradually falls over time. Thus, the logic of technology adoption and firm-level heterogeneity are very different from ours. Also, they do not explore the effects of factor endowments on technology adoption.

2 The Model

This section lays out the model that contains two sectors, two factors, and a continuum of potentially heterogenous firms. One sector (sector Z) produces a homogenous good that serves as the numeraire in the model. The other sector (sector Y) is characterized by a continuum of differentiated goods à la Melitz (2003). However, firms' heterogeneity stems from each firm' choice of production technologies.

2.1 Preferences

We consider a closed economy populated with the unit mass of identical households. The number of members of each household is L and each member of household supplies one unit of labor. Also, each household is endowed with K units of capital. The preferences of consumers over final goods are represented by the utility function

$$U = \left[\int_{i \in \Omega} y(i)^{\alpha} di \right]^{\gamma/\alpha} z^{1-\gamma}, \quad \gamma \in (0,1), \tag{1}$$

where $y(i)$ denotes consumption of variety i in sector Y, Ω the set of available differentiated goods, and z consumption of the homogeneous good. The varieties of differentiated goods are substitutable and an elasticity of substitution between any two varieties is equal to $\sigma = 1/(1-\alpha) > 1$. As is well known, these preferences yield the following iso–elastic demand function for differentiated good i of

$$y(i) = \frac{\gamma E p_y(i)^{-\sigma}}{P^{1-\sigma}}, \tag{2}$$

where E represents total expenditure, $p(i)$ the price of differentiated good i and P an aggregate price index for sector Y such that

$$P = \left[\int_{i \in \Omega} p_y(i)^{1-\sigma} di \right]^{1/(1-\sigma)}. \tag{3}$$

2.2 Production Technology

We assume that any firms can produces the homogenous good using labor and capital with a constant-returns-to-scale production technology. Since we set $p_z = 1$, perfect competition ensures that

$$c_z(r, w) = 1, \tag{4}$$

where w and r are the returns to labor and capital, respectively, and c_z is the unit production cost of the homogenous good. For analytical concreteness, we specify the functional form of $c_z(r, w)$ such that $c_z(r, w) = a^{-1} r^{m_z} w^{1-m_z}$ where $a > 1$ is a given productivity parameter and $m_z \in [0, 1)$.

Production of differentiated goods requires a continuum of intermediate goods of which the set is the interval $[0, 1]$. Specifically, the following Cobb-Douglas type function is employed for the production of each variety of differentiated goods:

$$y(i) = \exp\left[\int_0^1 \ln x_i(j) dj\right], \tag{5}$$

where $x(j)$ is input of the intermediate good j.

The intermediate goods are produced by labor and capital. There potentially exist two types of technologies for each intermediate good so that firms producing differentiated products choose which technology they adopt for each intermediate good. The first technology is a labor-intensive technology in which the production of intermediate good j involves a constant marginal requirement $l(j)$ units of labor. We assume that this production technology is well known to the economy and always available to any firms.

The second-type technology contrasts with the first-type in several respects. First, the second type is capital-intensive: it saves labor but requires more capital relative to the first-type technology. Second, the extent to which firms can efficiently use the second-type technology is positively correlated to their intrinsic productivity levels that we will discuss shortly. Third, the second-type technology is available only for the intermediate goods below a technology frontier; technological progress is expressed by an expansion of the range of intermediate goods for which the second-type technology is available. Therefore, a newly discovered technology has factor intensity biased toward capital and a skill complement in the sense that firms with higher productivity levels use more efficiently than those with lower productivity levels. As suggested by the literature of skill-biased technological change such as Autor et al. (1998), Doms et al. (1997), and Bresnahan et al. (2002), these specifications are empirically consistent.

More specifically, in order to simplify the algebra, we assume that the first and the second technologies are extreme in terms of factor intensity so that the second technology requires a constant marginal requirement $k(j)/\varphi$ units of capital where $\varphi > 0$ denotes firm-specific productivity levels. Without loss of generality, we order intermediate goods j by the extent to which the second technology replaces labor with capital. Namely, defining the following function $\lambda(\varphi, j)$ that measures the capital required to save labor by one unit

$$\lambda(\varphi, j) = \frac{k(j)}{\varphi l(j)}, \tag{6}$$

we order j such that $\lambda(\varphi, j)$ is non-decreasing in j, that is, an intermediate good with higher j has a higher capital cost of labor reduction. Note that the second technology is more efficient for firms with higher productivity levels in the sense that they would save the same amount of labor with fewer capital than those with lower productivity levels.

We introduce the technological frontier to the model by defining a critical intermediate good $\theta \in [0,1]$ such that capital-intensive technologies are known for all intermediate goods for $j \in [0,\theta]$ while they have not yet discovered for the remaining intermediate goods $j \in (\theta, 1]$. Thus, while firms can choose whether or not to adopt the second-type technology for intermediate $j \in [0,\theta]$, they have to use the first-type technology for the remaining. A rise in θ means that the second-type technology becomes available for more intermediate goods with increasing λ. This specification implies that newly discovered technologies are more capital intensive than those discovered before. Although θ is exogenously given in the base model, we will later discuss an endogenous technological frontier as an extension of the model. Imposing these specifications, we will refer to the first-type technology as the "old" technology and the second-type technology as the "new" technology.

Firms adopt the new technology as long as it reduces the production costs. Supposing that a firm adopts the new technology for intermediate goods in $[0, m]$ where $m \leq \theta$, the unit variable cost is given by

$$c(\varphi, r, w, m) = \exp\left[\int_0^m \ln rk(j)dj + \int_m^1 \ln wl(j)dj - m \ln \varphi dj\right]. \tag{7}$$

The partial derivative of c with respect to m,

$$c_m(\varphi, r, w, m) = c(\varphi, r, w, m)[\ln rk(m) - \ln wl(m) - \ln \varphi], \tag{8}$$

implies that as long as $rk(m)/wl(m) - \varphi \leq 0$ holds, *ceteris paribus*, the firm can reduce the unit variable cost by expanding the adoption of the new technologies if they have been discovered. Using the expression in equation (6), this condition can be rewritten as follows:

$$c_m(\varphi, r, w, m) \lesseqqgtr 0 \Leftrightarrow \lambda(\varphi, m) \lesseqqgtr \frac{w}{r}, \tag{9}$$

which also reveals that normalizing $l(j)$ to one for all j does not alter the analysis in essential manners so that we will henceforth simplify λ such that $\lambda(\varphi, j) = k(j)/\varphi$.

The extent to which firms adopt the new technology, $m(\varphi, r, w, \theta)$, is given by

$$m(\varphi, r, w, \theta) = \begin{cases} \theta, & \text{if} \quad rk(\theta)/w \leq \varphi \\ k^{-1}(\varphi w/r), & \text{if} \quad rk(0)/w < \varphi < rk(\theta)/w \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

Consequently, the unit variable costs for firms with productivity level φ are

$$c(\varphi, r, w, \theta) = \exp\left[\int_0^{m(\varphi,r,w,\theta)} \ln rk(j)dj + (1 - m(\varphi, r, w, \theta)) \ln w - m(\varphi, r, w, \theta) \ln \varphi\right]. \tag{11}$$

The upper panel of Figure **??** illustrates three examples of λ with different productivity levels for a given wage-rental ratio, $(w/r)'$. λ is nondecreasing in j by construction. Firms with productivity φ_1 have the highest productivity level among the three and $\lambda(\varphi_1, j) < (w/r)'$ for the entire range of intermediate goods, which implies that the unit variable cost is monotonically decreasing in θ as is shown as $c(\varphi_1, r, w, \theta)$ in the lower panel of the figure. Such firms are willing to adopt the new technologies for all intermediate goods if they are available. Thus, the optimal technology adoption equals to the technological frontier θ' in the figure. Firms with φ_2 are in an intermediate case: by adopting the new technologies up to $k^{-1}(\varphi w/r)$, firms can reduce its production cost. However, they do not have incentive to adopt the new technologies for intermediate goods beyond this range since the adoption of the new technologies is too costly for such firms. Hence, the unit variable cost $c(\varphi_2, r, w, \theta)$ is flat for $\theta \geq k^{-1}(\varphi w/r)$. Finally, firms with φ_3 are in the opposite situation to those with φ_1. Adopting new technologies is unprofitable for any intermediate goods so that such firms stick to the old technologies even though the new technologies are available. The corresponding unit variable cost is $c(\varphi_3, r, w, \theta) = w$ in the figure.

As depicted in Figure **??**, firms are categorized into three cohorts according to their intrinsic productivity levels. Adopting the old technology for all intermediate, the least productive firms with $\varphi < rk(0)/w$ are homogenous in the production cost even though their intrinsic productivity levels vary. Firms with mid-range productivity levels of $rk(0)/w < \varphi < rk(\theta)/w$ choose the extent of technological adoption as interior solutions. It follows that technological progress (i.e. a rise in θ) does not directly affect this group's technological adoption. Rather, this group may change the technological adoption through general equilibrium ramifications, such as changes in the rental rate r. In contrast, the most productive firms with $\varphi \geq rk(\theta)/w$ are directly affected by technological progress since their technological choices are corner solutions. Their technological choices are independent from changes in factor prices. For simplifying the notation, we henceforth supress the parameter θ from the arguments of the functions m, c, and others, except in the comparative statics analysis with respect to θ.

Facing the iso-elastic demand function in (2), a firm with productivity level φ sets the price at $p(c(\varphi, r, w)) = \sigma c(\varphi, r, w)/(\sigma - 1)$ and produces

$$x(\varphi, r, w) = \frac{\gamma E}{P} \left[\frac{p(c(\varphi, r, w))}{P} \right]^{-\sigma} \tag{12}$$

units of variety i. The production of differentiated goods requires fixed costs. In order to simplify the analysis, we assume that all firms have a common fixed cost of "hiring" f units of the numeraire good Z. Then, the profit, net of the fixed cost is expressed by

$$\pi(\varphi, r, w) = \frac{\gamma E}{\sigma} \left[\frac{p(c(\varphi, r, w))}{P} \right]^{1-\sigma} - f. \tag{13}$$

The profit $\pi(\varphi, r, w)$ is increasing in φ if φ is high enough to adopt the new technology for at least an intermediate input: i.e., $\varphi > rk(0)/w$. Otherwise, the profit is irrelevant to firms' intrinsic productivity levels sine firms use the old technology for all intermediate inputs.

As is standard in the Dixit-Stiglitz-type monopolistic competition, the ratios of any two firms' outputs $x(\varphi, r, w)$, and revenues $R(\varphi, r, w)$ depend only on the ratio of their unit variable costs: i.e.,

$$\frac{x(\varphi_1, r, w)}{x(\varphi_2, r, w)} = \left[\frac{c(\varphi_1, r, w)}{c(\varphi_2, r, w)} \right]^{-\sigma}, \quad \frac{R(\varphi_1, r, w)}{R(\varphi_2, r, w)} = \left[\frac{c(\varphi_1, r, w)}{c(\varphi_2, r, w)} \right]^{1-\sigma}. \tag{14}$$

2.3 Entry and Exit

Similarly to the production fixed cost, the design of differentiated goods requires a sunk investment, which is f_e units of good Z. After the investment, each firm independently draws its intrinsic productivity level φ from the common probability distribution on $(0, \infty)$ having the cumulative distribution function H with continuous density function h.

Since the the profit $\pi(c(\varphi, r, w))$ is strictly increasing in φ for $\varphi > rk(0)/w$, there exist two cases of the marginal firms earning zero profit. The first case is that the marginal firms' productivity level φ^* is above $rk(0)/w$. Firms with productivity levels below φ^* earn negative profits and immediately exit from the market. Thus, the zero-profit condition for the marginal firms is given by

$$\frac{\gamma E}{\sigma} \left[\frac{p(\varphi^*, r, w)}{P} \right]^{1-\sigma} = f. \tag{15}$$

The second case is that even the least productive firms with $\varphi \leq k(0)/z$ may enter the market, earning zero profit. In such a case, cutoff productivity level φ^* is undetermined.

Although it is not difficult to handle both cases, we concentrate on the first case: φ^* is large enough so that even the marginal firms adopt the new technology with $m(\varphi^*, r, w) > 0$. This is easily ensured, for example, by assuming $k(0) = 0$.

Letting N_e be a mass of new entrant firms and N a mass of surviving firms, the entry-exit dynamics are simply

$$[1 - H(\varphi^*)]N_e = N, \tag{16}$$

where $H(\varphi)$ is a cumulative distribution of productivity parameter φ.

The price index P in equation (3) can be written as follows:

$$P = \left[\int_0^\infty p_y(\varphi, r, w)^{1-\sigma} N d\mu_{\varphi^*}(\varphi)\right]^{1/(1-\sigma)} = N^{1/(1-\sigma)} p_y(\tilde{c}(\varphi^*, r, w)), \tag{17}$$

where $\tilde{c}(\varphi^*, r, w)$ denote the unit variable cost for the average firms such that

$$\tilde{c}(\varphi^*, r, w) = \left[\int_0^\infty c(\varphi, r, w)^{1-\sigma} d\mu_{\varphi^*}(\varphi)\right]^{1/(1-\sigma)}, \tag{18}$$

and $\mu_{\varphi^*}(\varphi)$ is the conditional distribution of $h(\varphi)$ such that

$$\mu_{\varphi^*}(\varphi) = \begin{cases} \dfrac{h(\varphi)}{1 - H(\varphi^*)} & \text{if} \quad \varphi \geq \varphi^* \\ 0 & \text{otherwise.} \end{cases}$$

The price $p_y(\tilde{c}(\varphi^*, r, w))$ is an average price in the sense that the price index P can be replicated by N *homogenous* firms with price $p_y(\tilde{c}(\varphi^*, r, w))$ (See equation (17)). Indeed, as is shown in the Appendix, $p_y(\tilde{c}(\varphi^*, r, w))$ is a weighted average of the prices of differentiated goods. Using equation (17), the zero-profit condition in equation (15) can be expressed by

$$\frac{\gamma E}{\sigma N}\left[\frac{c(\varphi^*, r, w)}{\tilde{c}(\varphi^*, r, w)}\right]^{1-\sigma} = f. \tag{19}$$

The free entry condition ensures that the *ex ante* expected profit equals the entry cost f_e. Noting that the profit of the average firms, net of the fixed cost, is $\tilde{\pi} = (\gamma E)/(\sigma N) - f$, the free entry condition is expressed as

$$[1 - H(\varphi^*)]\left[\frac{\gamma E}{\sigma N} - f\right] = f_e. \tag{20}$$

Using equation (19), the profit earned by the average firm, $\tilde{\pi} = (\gamma E)/(\sigma N) - f$ can be written such that

$$\tilde{\pi}(\varphi^*, r, w) = f\phi(\varphi^*, r, w), \tag{21}$$

where $\phi(\varphi^*, r, w) \equiv [\tilde{c}(\varphi^*, r, w)/c(\varphi^*, r, w)]^{1-\sigma} - 1$. Substituting this expression into the free-entry condition in (20), we obtain

$$\Phi(\varphi^*, r, w) \equiv [1 - H(\varphi^*)]\, \phi(\varphi^*, r, w) = F, \tag{22}$$

where $F \equiv f_e/f$. Equation (22) restates the free-entry condition in terms of the price of capital r and the cutoff productivity level φ^*. It should be noted that unlike Bernard et al. (2007), the free-entry condition in (22) contains terms of factor prices. This is because the current model allows that firms are heterogenous not only in intrinsic productivity levels φ but also in the factor intensity.

2.4 Factor Market

Capital market clearing requires that $K = K_{Yp} + K_{Yf} + K_{Yfe} + K_Z$ where K_{Yp} denotes capital demand by the production in sector Y, K_{Yf} and K_{Yfe} capital hired for covering the fixed cost and the entry investment, respectively, and K_Z capital used in sector Z.

By Shephard's lemma, capital requirements for the unit production of a differentiated product are $c(\varphi, r, w)m(\varphi, r, w)/r$ (see the Appendix). Thus, using equations (12) and (17) and the pricing rule, the total amount of capital used in the production of differentiated goods can be written such that

$$K_{Yp} = \int_0^\infty \frac{c(\varphi, r, w)m(\varphi, r, w)}{r} x(\varphi, r, w) N d\mu_{\varphi^*}(\varphi) = \frac{\alpha\gamma E}{r}\tilde{m}(\varphi^*, r, w), \tag{23}$$

where $\alpha \equiv (\sigma - 1)/\sigma$ represents the share of income that distributed to the intermediate goods and $\tilde{m}(\varphi^*, r, w)$ is the average level of technological adoption such that

$$\tilde{m}(\varphi^*, r, w) = \int_0^\infty m(\varphi, r, w) \left[\frac{c(\varphi, r, w)}{\tilde{c}(\varphi^*, r, w)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi), \tag{24}$$

where the relative revenues of differentiated goods are again used as the weights (see the Appendix). It should be noted that the average level of technological adoption $\tilde{m}$ must be smaller than or equal to θ.

The unit production of the average firm requires $\tilde{c}(\varphi^*, r, w)\tilde{m}(\varphi^*, r, w)/r$ units of capital. It follows from (16) that the capital demand for the fixed cost and the investment cost, $K_{Yf} + K_{Yfe}$, is given by $N[f + f_e/(1 - H(\varphi^*))]m_z/r$. The capital demand in sector Z is simply $m_z(1 - \gamma)E/r$. Therefore, the capital market clearing condition is expressed by

$$\alpha\gamma E\tilde{m}(\varphi^*, r, w) + N\left[f + \frac{f_e}{1 - H(\varphi^*)}\right] m_z + (1 - \gamma)E m_z = rK.$$

Using equation (20), the capital market clearing condition is further simplified as follows:

$$\{\alpha\gamma\tilde{m}(\varphi^*, r, w) + [(1-\alpha)\gamma + 1 - \gamma]m_z\}E = rK. \tag{25}$$

It should be noted that the production function for the differentiated goods is a Cobb-Douglas (equation (5)), the average adoption $\tilde{m}$ exactly represents the income share for capital in sector Y.

Likewise, the labor market clearing condition is given by

$$\{\alpha\gamma(1 - \tilde{m}(\varphi^*, r, w)) + [(1-\alpha)\gamma + 1 - \gamma](1 - m_z)\}E = wL. \tag{26}$$

Therefore, eliminating E from these two factor market clearing conditions in (25) and (26), we obtain a rather familiar form of factor market clearing condition:

$$\frac{w}{r} = \frac{1 - \widehat{m}(\varphi^*, r, w)}{\widehat{m}(\varphi^*, r, w)} \frac{K}{L}, \tag{27}$$

where $\widehat{m}$ is the average capital share of the economy such that $\widehat{m}(\varphi^*, r, w) \equiv \alpha\gamma\tilde{m}(\varphi^*, r, w) + [(1-\alpha)\gamma + 1 - \gamma]m_z$.[2]

The model now has three endogenous variables, φ^*, r, and w. Since the zero-profit condition in sector Z in equation (4), the free-entry condition in equation (22), and the full-employment condition in equation (27) simultaneously solve the three endogenous variables, the model is closed. Once φ^*, r, and w are determined, the other endogenous variables, $E, p_y(i), m(i), P, N$, and N_e are pinned down.

Since the extent to which the average firms adopt the new technology, $\tilde{m}$ (or equivalently $\widehat{m}$), is a summary index of technology adoption in the economy, it is important to understand how $\tilde{m}$ is affected by changes in cutoff productivity levels φ^*, the rental rate r, and the wage rate w. If the cutoff productivity level is above a certain value that will be discussed in the next section, all firms adopt the new technologies for all intermediate goods. As a result, the average productivity level $\tilde{m}$ is pinned down at θ and becomes insensitive to φ^*, r, and w. Otherwise, there are the two types of firms: firms that partially adopt the new technologies and firms that fully adopt them (we will discuss these two cases in more detail in the next section).

As is expected, given that not all firms adopt the new technologies up to the technological frontier θ, the average technological adoption $\tilde{m}$ is increasing in φ^* and w and decreasing

[2]By Walras' law, the condition that income equals spending, $rK + wL = E$ is redundant.

in r, Namely,

$$\frac{\partial \tilde{m}}{\partial \varphi^*} > 0, \quad \frac{\partial \tilde{m}}{\partial r} < 0, \quad \frac{\partial \tilde{m}}{\partial w} > 0. \tag{28}$$

The proof is straightforward but tedious so that we relegate it to the Appendix. Here we simply describe the intuition about the signs of the partial derivatives in (28). Intuitively, an increase in the cutoff productivity level φ^* forces firms with the lowest productivity levels to exit from the market so that the average level of technological adoption rises.

The responses of the average technological adoption to changes in the factor prices are more elusive although they also seem intuitive. Obviously, when the rental rate becomes higher, firms decline to adopt the new technologies, which results in a lower average $\tilde{m}$. A higher wage rate should work in the opposite direction. However, in addition to this substitution effect, changes in the factor prices affect the average adoption level through changing firms' weights in the average technological adoption. Recall that $\tilde{m}$ uses the ratios of each firm's revenue to the average revenue as weights (see equations (14) and (24)). The impact of changes in the rental rate on revenues is greater for firms with higher productivity levels than those with lower productivity levels since firms with higher productivity levels have higher capital-labor ratios than those with lower productively levels. Thus, the weights attached with firms that adopt the new technologies above the average level decrease while those attached with firms that adopt the new technologies below the average level increases. As a result, the average technological adoption level falls.

3 Properties of Equilibrium

This section solves the model and examines several comparative statistics. Then, several comparative statics analysis are conducted.

3.1 Equilibrium

As is shown in (10), if φ^* is greater than $rk(\theta)/w$, then all firms fully adopt the new technologies, which leads to $\tilde{m} = \theta$. From the full-employment condition, the wage-rental ratio becomes

$$\frac{w}{r} = \frac{1 - \widehat{m}_\theta}{\widehat{m}_\theta} \frac{K}{L}, \tag{29}$$

where $\widehat{m}_\theta = \alpha\gamma\theta + (1 - \alpha\gamma)m_z$. The wage-rental ratio in equilibrium is linear in terms of the aggregate capital-labor ratio due to the unit elasticity of substitution between sector Y

and Z. The threshold value of $rk(\theta)/w$ can be written as

$$\varphi_\theta \equiv \frac{\widehat{m}_\theta}{1-\widehat{m}_\theta}\frac{L}{K}k(\theta). \tag{30}$$

Using the zero-profit condition in sector Z (equation (4)), the rental rate and the wage rate in such an equilibrium are given by

$$r_\theta = \left[\frac{\widehat{m}_\theta}{1-\widehat{m}_\theta}\frac{L}{K}\right]^{1-m_z} \quad \text{and} \quad w_\theta = \left[\frac{1-\widehat{m}_\theta}{\widehat{m}_\theta}\frac{K}{L}\right]^{m_z}, \tag{31}$$

respectively.

When all firms fully adopt the new technology, the free-entry condition in equation (22) is reduced to

$$[1-H(\varphi^*)]\left[\int_0^\infty \left[\frac{\varphi}{\varphi^*}\right]^{\theta(\sigma-1)} d\mu_{\varphi^*}(\varphi) - 1\right] = F, \tag{32}$$

which implies that the cutoff productivity level φ^* is solely determined by the free-entry condition with the model's parameters, independent of the factor prices (and factor endowments). We denote the solution of (32) by $\bar{\varphi}$ since this cutoff productivity is the highest among equilibrium cutoff productivity levels, given the model's parameters.

In this type of equilibrium, all firms in sector Y employ the same technologies regardless the diversification of intrinsic productivity levels φ. Thus, this case is in essence the same as the one considered in Bernard et al. (2007).

Then, focusing on the case in which firms may be diversified in technology adoption, we examine properties of the free-entry condition in (22) and the full-employment condition in (27). Noting that these two conditions can be expressed with two endogenous variables, φ^* and r, using equation (4), it is rather straightforward to establish the followings:

Lemma 1. *The free-entry condition is represented by a down-sloping schedule in the* (r, φ^*)*-space if firms are diversified in technological adoption. Otherwise, the free-entry schedule is horizontal and the cutoff productivity* φ^* *is independent of factor prices.*

The full-employment condition is an upward-sloping schedule with the upper-bound rental rate of $r_\theta = \left[\frac{\widehat{m}_\theta}{1-\widehat{m}_\theta}\frac{L}{K}\right]^{1-m_z}$ *where* $\widehat{m}_\theta \equiv \gamma\theta + (1-\gamma)m_z$.

Proof. See the Appendix. □

Relegating the proof to the appendix, we here state intuitions of the lemma. Suppose that the extent to which firms adopt the new technologies are diversified. When the entry to the market becomes more difficult (a higher φ^*), the average profit $\tilde{\pi}$ (this is also *ex-ante* expected profit conditioning on successful entry) must rise in order to make firms incur the initial investment f_e. This is enabled by a lower rental rate that raises the profits earned by inframarginal firms through both reducing the production costs and encouraging the technology adoption. Thus, the free-entry (FE) schedule is down-sloping in the (r, φ^*) space as long as firms are diversified in technological adoption.

However, as r becomes smaller (i.e., the wage-rental ratio w/r becomes greater), the number of firms that fully adopt the new technology increases. This implies that firms becomes less diversified in terms of the factor intensity. In extreme, if r becomes sufficiently low, all firms fully adopt the new technology and the FE schedule becomes flat. The cutoff productivity is determined by the model's parameters and independent of the factor prices. Thus, the FE schedule has a flat part for small rental rates (see the Appendix).

With respect to the full-employment (FM) schedule, a higher rental rate lowers the demand for capital by discouraging the technology adoption and raising the production costs. In order to compensate this demand decline, firms must become more productive on average. This implies that firms should be more selective so that a higher φ^* is needed. As is already discussed, the rental rate has the upper bound at r_θ. Thus, once the rental rate reaches r_θ, the FM schedule becomes vertical in the (r, φ^*) space.

When would firms be diversified in terms of technological adoption? From Lemma 1, it is necessary that the horizontal part of the FE schedule and the vertical part of the FM schedule do not intersect each other. For this, we impose the following condition:

Condition 1. $\dfrac{\bar{\varphi}}{k(\theta)} < \dfrac{\widehat{m}_\theta}{1 - \widehat{m}_\theta} \dfrac{L}{K}$, where $\bar{\varphi}$ is the solution of equation (32).

This condition tends to hold when the technological frontier θ is large and K/L is small. In other words, if the capital endowment is abundant relative to the technological progress (a lower θ), it is likely that all firms fully adopt the new technologies.

We record the equilibrium of the model in the following proposition:

Proposition 1. *There exists a unique equilibrium referenced by the vector $(\varphi^*, r, w,)$. If Condition 1 holds, only the most productive firms fully adopt the new technologies; firms with lower productivity levels adopt the new technologies fewer than firms with higher productivity levels (i.e. $\tilde{m} < \theta$). Otherwise, all firms adopt the new technologies up to θ (i.e. $\tilde{m} = \theta$).*

Proof. See Appendix. □

Figure **??** depicts the determination of equilibrium values of r and φ^*. If Condition 1 does not hold, the FE schedule (the free-entry condition) cuts through the FM schedule (the full-employment condition) at a vertical part of the FM schedule as is shown at point B. Since all firms are homogenous in terms of the factor intensity, this case is essentially the same as the one examined by Bernard et al. (2007). We will fucus on the case in which Condition 1 holds and firms are heterogenous in technological adoption (and factor intensity).

Once φ^* and r are determined, the characteristics of the average surviving firms such as the average unit variable cost $\tilde{c}$ (equation (18)), the average profit $\tilde{\pi}$ (equation (21)), and the average technology adoption $\tilde{m}$ (equation (24)) are identified, which means that the firms' distribution including the mass of surviving firms is also determined. Notice that a change in the size of the economy (i.e. a proportional change in K and L) does not alter both the free-entry (FE) schedule and the full-employment (FM) schedule. This means that the profile of the average firm is independent of the size of the economy: an increase in the country size simply make the distribution of firms thick (a rise in N), preserving the shape of the distribution of firms the same as before. Hence, the size independency, a standard property of the monopolistic competition model with the CES preferences, holds in this model from the perspective of the average firm.

3.2 Comparative Statics

Before moving to the analysis in open economy settings, we will check properties of the model's equilibrium by examining a series of comparative statics.

Capital Endowment Consider an increase in the capital endowment K (i.e., an increase in the capital endowment per capita K/L). K shows up only in the full-employment condition. Thus, while the free-entry (FE) schedule is intact, the full-employment (FM) schedule shifts leftward. As a result, the rental rate r declines (and the wage rate w increases) and the cutoff productivity level φ^* rises: firms with the lowest productivity levels will exit from the market as the costs of labor, which they use intensively increases. The key mechanism here is the complementarity between capital and intrinsic productivity of firms: the capital expansion is more beneficial to firms with higher productivity levels than those with lower productivity levels. As a result, some least productive firms have to exit from the market due to their lack of competitiveness. Although not all firms cannot benefit from the capital expansion, the aggregate productivity and the average technological adoption level increase.

Fixed Cost Consider a decrease in the fixed cost f. An example of this exercise is a relaxed regulation of entry of start-up firms. Djankov et al. (2002) find that the regulation of entry of start-up firms significantly vary across countries. For example, they document that while the cost of necessary process for starting operation in New Zealand is virtually nothing (three steps procedures that take three days), that in France is about 14 per cent of GDP per capita (15 steps that takes 53 days).[3]

In the model, a decrease in f (i.e., an increase in F) shifts the free-entry (FE) schedule downward since all operating firms become more profitable for any given factor prices and the cutoff productivity level, hence, must decline. As a result, both the cutoff productivity level φ^* and the rental rate r fall. The model's interpretation is quite intuitive. Lowering the entry cost, for example, invites less productive firms' entry (a lower φ^*). However, since the tail of firm distribution is stretched, the average technological adoption decreases, which leads to a lower rental rate.[4] These results are recorded as follows.

Proposition 2. *An increase in the capital endowment facilitates the adoption of the new technologies and the least productive firms exit from the market (a rise in the cutoff productivity level φ^*). As a result, the aggregate productivity rises.*

A decrease in the fixed cost f raises profitability of operating firms and encourages more firms' entry into the market. However, the average technological adoption level falls, leading to a lower rental rate.

Proof. See the Appendix. □

Technological Progress An expansion of the technological frontier (i.e., an increase in θ) brings about more ambiguous results. First, although the new technology becomes available in more intermediated goods, only firms with $\varphi > rk(\theta)/w$ (i.e., firms choosing the full adoption of the new technologies) adopt the new technologies. Thus, in such a situation, the entry to the market becomes difficult, which leads to an upward shift of the free-entry (FE) schedule. The facts that the most productive firms uses the new technologies more and that the least productivity firms will retire from the market raise the average technological adoption, which leads to a rise in the capital price.

[3] Djankov et al. (2002) study 85 countries and find that the entry cost to per capita GDP ratio uniformly decreases with GDP per capita.

[4] Although each operating firm adopts the new technologies more, the increase in the number of low productivity firms lowers the average technological adoption. The intensive margin of the technological adoption is dampened by the (negative) extensive margin of the technological adoption.

The expansion of the technological frontier also affects the full-employment condition. Since the group of the most productive firms increases the demand for capital, the rental rate goes up (for any cutoff productivity levels) and the full-employment schedule shifts rightward. This change in the factor market raises the rental rate further and dampens the favorable effect of the technological progress on the most productive firms (recall the complementarity between skill and capital). Therefore, it is not clear whether or not the cutoff productivity rises in a new equilibrium (see Figure **??**).

Since r increases in new equilibrium, the wage rate will necessarily decline (equation (4)). Thus, the wage-rental rate will go down. From the full-employment condition, we can immediately observe that the average technological adoption $\tilde{m}$ will rise.

Productivity in the Other Sector Consider now a rise in a, namely, an increase in the productivity of the homogenous good sector. Facing a decline in the relative price of capital (r/w), firms with partially adopting the new technologies (i.e. productivity level below $rk(\theta)/w < \varphi$) adopt the new technologies more. The unit variable costs of firms with lower productivity levels increase more than the unit variable cost of the average firms since low productivity firms have high labor intensity. Thus, the average profit $\tilde{\pi}$ increases, which leads to an upward shift of the free-entry (FE) schedule. At the same time, as in an expansion of the technological frontier, the capital market becomes tight and the full-employment (FM) schedule shifts rightward. Thus, in total, while the rental rate rises, it is not clear if the cutoff productivity rises. The impact of the technological progress in the other sector on r and φ^* is similar to a rise in θ. Also, in both cases, the average technological adoption rises. However, benefitted firms are opposite since while the most productive firms expand technological adoption in the case of a higher θ, the mid-range and low productivity firms expand technological adoption in the case of a higher a. This observation is recorded in the following proportion:

Proposition 3. *An expansion of the technological frontier (a rise in z) increases the rental rate but the effect on the cutoff productivity level is ambiguous. An increase in the productivity in the homogenous good sector has the similar effects on the rental rate and the cutoff productivity level.*

However, an expansion of θ benefits the most productive firms while a productivity improvement in the homogeneous good sector may benefit less productive firms.

Proof. See the Appendix. □

4 Two Country Model

This section extends the basic model to a two country model in which two countries, Home and Foreign, differ in their factor proportions and technology frontiers. Before we investigate the impact of international trade on the factor prices and resulting technology choices, we first argue that the capital abundant country tends to have a higher technology frontier when the technology frontier is endogenously determined. Then we show that factor prices will not be equalized even in free trade; the capital abundant country with a higher technology frontier will be faced with a lower wage-rental ratio than the labor abundant country in free trade. This contrasts with the autarky equilibrium in which capital abundant country is faced with a higher wage-rental ratio.

We have shown in section 3.2 that an increase in the capital endowent will increase the wage-rental ratio as expected. The capital abundant country will be faced with a higher wage-rental ratio than the labor abundant country, and firms there have more incentive to adopt labor-saving new technologies. Having more firms that adopt the up-to-date technology, the capital abundant country is expected to have a higher technological progress at each point in time. So the technology frontier of the capital abundant country is higher than that of the labor abundant country when they open to trade.

Now, we show that in free trade, the factor prices are equalized if and only if the technology frontiers are the same between Home and Foreign, i.e., $\theta_H = \theta_F$. Suppose to the contrary that factor prices are the same between the countries when $\theta_H = \theta_F$. Then the expected profits for a firm in Home, $\tilde{\pi}_H$, is higher than that in Foreign, $\tilde{\pi}_F$. But this means that $\varphi_H^* > \varphi_F^*$ in order for each country to have the good Y industry since the equation (**??**), which can be written as $[1 - H(\varphi_i^*)]\tilde{\pi}_i = f_e$ for $i = H, F$, must be satisfied for both countries. Now consider a potential firm in Home with a φ that is smaller than φ_H^* but higher than φ_F^*. This firm does not operate by the definition of φ_F^* despite that it would earn a higher profit if it operated in the world market as a less-efficient firm in Foreign with φ_F^* earns zero profits. This is contradiction.

Proposition 4. *Factor prices are equalized in free trade if and only if the technology frontiers are the same between the two countries.*

Proof. See the Appendix. □

The difference in factor prices remain even in free trade when the two countries differ in their technology frontiers. The above argument suggests that the expected profits must be the same in free trade equilibrium if both countries produce good Y. Given that potential firms prefer the situation in which the rental rate is low so that they adopt new technology for a wide range of intermediate goods, the wage-rental rate must be greater in labor-abundant Foreign than capital-abundant Home. The ranking of the wage-rental ratio between the two countries are reversed when they open to trade.

5 Concluding Remarks

We have set up an endogenously-heterogeneous-firm general equilibrium model and showed, among others, that firms in capital-abundant countries tend to adopt new advanced technologies more in their production processes and that opening to international trade increases the average adoption rate of new technologies although firms with intermediate productivity levels adopt less of new technologies than in autarky.

We have also shown that the technological improvement, represented by an increase in the technology frontier beyond which only old technology is available, raises the rental rate so firms with high productivity levels adopt more of new technology than before while those with intermediate productivity levels adopt less. Technological improvement is brought to the firms through costly innovation. The incentive to innovate new technology rises with the capital endowment, for example, since it would raise the wage-rental ratio.

Then we have extended our analysis to a two-country model and derive that factor prices are not equalized and hence firms are different in their technology adoption between the two countries if and only if their technology frontiers are different, which in turn reflects the difference in their factor endowments.

A Appendix

A.1 The Average Price

From $\tilde{p}(\varphi^*, r, w)^{1-\sigma} = \int_0^\infty p(\varphi, r, w)^{1-\sigma} d\mu_{\varphi^*}(\varphi)$,

$$\begin{aligned}
\tilde{p}(\varphi^*, r, w) &= \int_0^\infty p(\varphi, r, w) \left[\frac{p(\varphi, r, w)}{\tilde{p}(\varphi^*, r, w)}\right]^{-\sigma} d\mu_{\varphi^*}(\varphi) \\
&= \int_0^\infty p(\varphi, r, w) \left[\frac{c(\varphi, r, w)}{\tilde{c}(\varphi^*, r, w)}\right]^{-\sigma} d\mu_{\varphi^*}(\varphi) \\
&= \int_0^\infty p(\varphi, r, w) \left[\frac{x(\varphi, r, w)}{\tilde{x}(\varphi^*, r, w)}\right] d\mu_{\varphi^*}(\varphi) \quad \text{(equation (14) is used)}
\end{aligned} \tag{A.1}$$

where $\tilde{x}(\varphi^*, r)(= [E/P]\,[\tilde{p}(\varphi^*, r)/P]^{-\sigma})$ is the output of firms with price $\tilde{p}(\varphi^*, r)$. Hence, $\tilde{p}$ is the average price weighted by the relative outputs of differentiated products.

A.2 The Unit Capital Requirement

Denoting the partial derivative of c with respect to r as c_r, we obtain

$$c_r(\varphi, r, w) = c(\varphi, r, w) \left[\int_0^{m(\varphi,r,w)} r^{-1} dj + m_r(\varphi, r, w) \left[\ln rk(m) - \ln w - \ln \varphi\right]\right], \tag{A.2}$$

where m_r is the partial derivative of $m(\varphi, r, w)$ with respect to r. For firms with $\varphi \geq rk(z)/w$, $m_r = 0$. For firms with $rk(0)/w < \varphi < rk(z)/w$, $\ln rk(m) - \ln w - \ln \varphi = 0$. Thus, the unit capital requirement is given by $c(\varphi, r, w)m(\varphi, r, w)/r$.

A.3 The Properties of the Average Technological Adoption

Once φ^* passes through $rk(\theta)/w$, all firms fully adopt the new technologies, which leads to $\tilde{m} = \theta$. If this is the case, as shown in the main text, the rental rate becomes

$$r_\theta = \left[\frac{\widehat{m}}{1-\widehat{m}} \frac{L}{K}\right]^{1-m_z}, \tag{A.3}$$

where $\widehat{m} = \alpha\gamma\theta + (1-\alpha\gamma)m_z$. This is the upper bound of the rental rate. Substituting this rental rate into $rk(\theta)/w$, we obtain the threshold value of φ^* as follows:

$$\varphi^*_\theta = \frac{\widehat{m}}{1-\widehat{m}} \frac{L}{K} k(\theta). \tag{A.4}$$

Now suppose that this is not the case. The partial derivative of $\tilde{m}(\varphi^*, r, w)$ with respect to φ^* is given by

$$\begin{aligned}
\frac{\partial \tilde{m}}{\partial \varphi^*}(\varphi^*, r, w) &= \frac{\int_{\varphi^*}^{\infty} \left[c(\varphi^*, \cdot)^{1-\sigma} h(\varphi^*) m(\varphi, \cdot) c(\varphi, \cdot)^{1-\sigma} - m(\varphi^*, \cdot) c(\varphi^*, \cdot)^{1-\sigma} h(\varphi^*) c(\varphi, \cdot)^{1-\sigma}\right] dh(\varphi)}{\left[\int_{\varphi^*}^{\infty} c(\varphi, \cdot)^{1-\sigma} dh(\varphi)\right]^2} \\
&= \frac{c(\varphi^*, \cdot)^{1-\sigma} h(\varphi^*) \int_{\varphi^*}^{\infty} \left[m(\varphi, \cdot) - m(\varphi^*, \cdot)\right] c(\varphi, \cdot)^{1-\sigma} dh(\varphi)}{\left[\int_{\varphi^*}^{\infty} c(\varphi, \cdot)^{1-\sigma} dh(\varphi)\right]^2} \\
&= \frac{h(\varphi^*) \left[\tilde{m}(\varphi^*, r, w) - m(\varphi^*, r, w)\right]}{1 - H(\varphi^*)} \left[\frac{c(\varphi^*, r, w)}{\tilde{c}(\varphi^*, r, w)}\right]^{1-\sigma}.
\end{aligned} \tag{A.5}$$

Since $m(\varphi, \cdot)$ is nondecreasing in φ, it follows that $m(\varphi, \cdot) - m(\varphi^*, \cdot) \geq 0$ on the support of h. Therefore, $\partial \tilde{m} / \partial \varphi^* > 0$ is obtained.

The partial derivative of $\tilde{m}(\varphi^*, r)$ with respect to r is given by

$$\begin{aligned}
\frac{\partial \tilde{m}}{\partial r}(\varphi^*, r, w) &= \int_0^{\infty} \frac{\partial m}{\partial r}(\varphi, r, w) \left[\frac{c(\varphi, r, w)}{\tilde{c}(\varphi^*, r, w)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) \\
&+ \frac{1-\sigma}{r} \left[\int_0^{\infty} \{m(\varphi, r, w)\}^2 \left[\frac{c(\varphi, r, w)}{\tilde{c}(\varphi^*, r, w)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) - \{\tilde{m}(\varphi^*, r, w)\}^2\right].
\end{aligned} \tag{A.6}$$

It is straightforward to see that the first term in the right-hand side (RHS) is non-positive since

$$\frac{\partial m(\varphi, r, w)}{\partial r} = \begin{cases} 0, & \text{if} \quad rk(z)/w \leq \varphi; \\ -\dfrac{k(m(\cdot))}{rk'(m(\cdot))} < 0, & \text{if} \quad rk(0)/w < \varphi < rk(z)/w. \end{cases} \tag{A.7}$$

In order to see that the second term is non-positive, notice that the inside of the square brackets is a weighted variance of $m(\varphi, r, w)$. Using the relative revenues as weights, the variance of $m(\varphi, r, w)$ is

$$Var(m) = \int_0^{\infty} \left[m(\varphi, r, w) - \tilde{m}(\varphi, r, w)\right]^2 \left[\frac{c(\varphi, r, w)}{\tilde{c}(\varphi, r, w)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi).$$

With some algebraic manipulations, we can easily see that

$$\begin{aligned}
&\int_0^{\infty} \left[m(\varphi, r, w) - \tilde{m}(\varphi, r, w)\right]^2 \left[\frac{c(\varphi, r, w)}{\tilde{c}(\varphi, r, w)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) \\
&\qquad = \int_0^{\infty} \{m(\varphi, r, w)\}^2 \left[\frac{c(\varphi, r, w)}{\tilde{c}(\varphi^*, r, w)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) - \{\tilde{m}(\varphi^*, r, w)\}^2 \geq 0,
\end{aligned} \tag{A.8}$$

with equality when all firms are in the same technology adoption level.We now have

$$\frac{\partial \tilde{m}}{\partial r}(\varphi^*, r, w) = \int_0^{\infty} \frac{\partial m}{\partial r}(\varphi, r, w) \left[\frac{c(\varphi, r, w)}{\tilde{c}(\varphi^*, r, w)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) + \frac{1-\sigma}{r} Var(m) < 0. \tag{A.9}$$

The proof of $\partial\tilde{m}/\partial w > 0$ is similar to that of $\partial\tilde{m}/\partial w < 0$. The partial derivative of $\tilde{m}$ with respect to w is given by

$$\frac{\partial\tilde{m}}{\partial w}(\varphi^*, r, w) = \int_0^\infty \frac{\partial m}{\partial w}(\varphi, r, w)\left[\frac{c(\varphi,\cdot)}{\tilde{c}(\varphi^*,\cdot)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi)$$
$$+\frac{1-\sigma}{w}\left[\int_0^\infty m(\varphi,\cdot)\{1-m(\varphi,\cdot)\}\left[\frac{c(\varphi,\cdot)}{\tilde{c}(\varphi^*,\cdot)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) - \tilde{m}(\cdot)\{1-\tilde{m}(\cdot)\}\right]. \quad \text{(A.10)}$$

The first term in the RHS is non-negative since

$$\frac{\partial m}{\partial w}(\varphi, r, w) = \begin{cases} 0, & \text{if } \; rk(z)/w \leq \varphi; \\ \dfrac{\varphi}{rk'(m(\cdot))} > 0, & \text{if } \; rk(0)/w < \varphi < rk(z)/w. \end{cases} \quad \text{(A.11)}$$

The inside of the squire brackets in the second term becomes $-Var(m)$. Therefore,

$$\frac{\partial\tilde{m}}{\partial w}(\varphi^*, r, w) = \int_0^\infty \frac{\partial m}{\partial w}(\varphi, r, w)\left[\frac{c(\varphi,\cdot)}{\tilde{c}(\varphi^*,\cdot)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) + \frac{\sigma-1}{w}Var(m) > 0 \quad \text{(A.12)}$$

A.4 Proof of Lemma 1

From equation (4), the wage rate w is expressed by a decreasing function of the return to capital such that $w(r)$. For later purposes, we define the elasticity of the wage rate in terms of the return to capital: $\eta \equiv -w'(r)r/w \geq 0$. For the specification given in the main text, η is constant at $m_z/(1-m_z)$. Now the partial derivative with respect to r is

$$c_r(\varphi, r) = \frac{c(\varphi, r)}{r}\left[(1+\eta)m(\varphi, r) - \eta\right] \quad \text{(A.13)}$$

instead of $c(\varphi, r, w)m(\varphi, r, w)/r$.

Totally differentiating the free-entry condition in equation (22), the slope of the free-entry condition schedule (FE schedule) is given by

$$\frac{d\varphi^*}{dr} = -\frac{\Phi_r}{\Phi_{\varphi^*}}, \quad \text{(A.14)}$$

where Φ_r and Φ_{φ^*} are the partial derivatives of Φ with respect to r and φ^*, respectively:

$$\Phi_r = [1-H(\varphi^*)]\frac{(1-\sigma)}{r}\left[\frac{\tilde{c}(\varphi^*, r)}{c(\varphi^*, r)}\right]^{1-\sigma}(1+\eta)\left[\int_0^\infty m(\varphi, r)\left[\frac{c(\varphi^*, r)}{\tilde{c}(\varphi^*, r)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) - m(\varphi^*, r)\right] < 0, \quad \text{(A.15)}$$

and

$$\Phi_{\varphi^*} = [1-H(\varphi^*)]\frac{(1-\sigma)}{\varphi^*}\left[\frac{\tilde{c}(\varphi^*, r)}{c(\varphi^*, r)}\right]^{1-\sigma} m(\varphi^*, r) < 0. \quad \text{(A.16)}$$

We here use

$$\Phi_r = [1-H(\varphi^*)][\phi_r + \phi_w w'(r)], \quad \text{(A.17)}$$

where

$$
\begin{aligned}
\phi_w(\varphi^*, r, w) &= (1-\sigma)\left[\frac{\tilde{c}(\varphi^*,\cdot)}{c(\varphi^*,\cdot)}\right]^{-\sigma}\left[\frac{\frac{\partial \tilde{c}(\varphi^*,\cdot)}{\partial w}c(\varphi^*,\cdot) - \frac{\partial c(\varphi^*,\cdot)}{\partial w}\tilde{c}(\varphi^*,\cdot)}{\{c(\varphi^*,\cdot)\}^2}\right] \\
&= \frac{\sigma-1}{w}\left[\frac{\tilde{c}(\varphi^*,\cdot)}{c(\varphi^*,\cdot)}\right]^{1-\sigma}\left[\tilde{m}(\varphi^*,\cdot) - m(\varphi^*,\cdot)\right] > 0.
\end{aligned}
\tag{A.18}
$$

The slope of the free-entry condition schedule (FE schedule) becomes

$$
\begin{aligned}
\frac{d\varphi^*}{dr} &= -\frac{(1+\eta)\varphi^*}{r}\left[\int_0^\infty \frac{m(\varphi, r)}{m(\varphi^*, r)}\left[\frac{c(\varphi, r)}{\tilde{c}(\varphi^*, r)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) - 1\right] \\
&= -\frac{(1+\eta)\varphi^*}{r}\left[\frac{\tilde{m}(\varphi^*, r)}{m(\varphi^*, r)} - 1\right] \le 0,
\end{aligned}
\tag{A.19}
$$

with equality whenever $\tilde{m}(\varphi^*, r) = m(\varphi^*, r)$ where all firms are homogeneous in terms of technological adoption.

Equation (A.19) suggests that if all firms fully adopt the new technologies, the FE schedule becomes flat. If so, the free-entry condition (equation (32)) solely determines the cutoff productivity level. Denoting this cutoff by $\bar{\varphi}$, firms with $\bar{\varphi}$ satisfy

$$
\lambda(\bar{\varphi}, \theta) = \frac{k(\theta)}{\bar{\varphi}} \le \frac{w}{r}. \tag{A.20}
$$

Hence, the minimum rental rate for the full technological adoption for all firms is given by

$$
r = \left[\frac{\bar{\varphi}}{k(\theta)}\right]^{1-m_z}. \tag{A.21}
$$

Then, totally differentiating the full-employment condition in equation (27), we obtain

$$
\frac{d\varphi^*}{dr} = -\frac{E\widehat{m}_r(\varphi^*, r) - \left[1 - \widehat{m}(\varphi^*, r)\right]K + w'(r)L}{E\widehat{m}_{\varphi^*}(\varphi^*, r,)}, \tag{A.22}
$$

where $\widehat{m}_{\varphi^*}(\varphi^*, r)$ and $\widehat{m}_r(\varphi^*, r)$ are the partial derivatives of $\widehat{m}$ with respect to φ^* and r, respectively. It is shown that $\tilde{m}_r(\varphi^*, r, w) < 0$, $\tilde{m}_w(\varphi^*, r, w) > 0$, and $\tilde{m}_{\varphi^*}(\varphi^*, r, w) > 0$ in the previous section. Since $w'(r) < 0$, $\widehat{m}_r(\varphi^*, r) = \alpha\gamma\left[\tilde{m}_r + \tilde{m}_w w'(r)\right] < 0$. Thus, $d\varphi^*/dr > 0$ is immediate.

A.5 Proof of Proposition 1

As is shown in the prof of Lemma 1, the cutoff productivity level monotonically declines from $\bar{\varphi}$ along the FE schedule for $r \ge [\bar{\varphi}/k(\theta)]^{1-m_z}$. Therefore, the FE schedule intersects with the FM schedule at most once.

As long as Condition 1 holds, the FE schedule never cuts through the vertical part of the FM schedule. This is because the FM schedule requires $\tilde{m} = \theta$ along the vertical part. This contradicts with the downs-sloping part of the FE schedule.

A.6 Proof of Proposition 2

In order to obtain the results of comparative statics, totally differentiating the FE schedule in (22) and the FM schedule in (27), we obtain

$$\begin{aligned}\Phi_{\varphi^*} d\varphi^* + \Phi_r dr + \Phi_a da = dF,\\ E\widehat{m}_{\varphi^*} d\varphi^* + [E\widehat{m}_r - (1-\widehat{m})K + w_r \widehat{m} L]dr = r(1-\widehat{m})dK - w\widehat{m}dL - w_a(\widehat{m}L + E\widehat{m}_w)da.\end{aligned} \tag{A.23}$$

where

$$\begin{aligned}
\Phi_{\varphi^*} &= \frac{[1-H(\varphi^*)]\,(1-\sigma)}{\varphi^*}\left[\frac{\tilde{c}(\varphi^*,r)}{c(\varphi^*,r)}\right]^{1-\sigma} m(\varphi^*,r) < 0\\
\Phi_r &= \frac{[1-H(\varphi^*)]\,(1-\sigma)(1+\eta)}{r}\left[\frac{\tilde{c}(\varphi^*,r)}{c(\varphi^*,r)}\right]^{1-\sigma}[\tilde{m}(\varphi^*,r)-m(\varphi^*,r)] < 0\\
\Phi_a &= \frac{[1-H(\varphi^*)]\,(\sigma-1)\eta_a}{a}\left[\frac{\tilde{c}(\varphi^*,r)}{c(\varphi^*,r)}\right]^{1-\sigma}[\tilde{m}(\varphi^*,r)-m(\varphi^*,r)] > 0\\
\widehat{m}_{\varphi^*} &= \gamma\tilde{m}_{\varphi^*} = \frac{\gamma h(\varphi^*)\,[\tilde{m}(\varphi^*,\cdot)-m(\varphi^*,\cdot)]}{1-H(\varphi^*)}\left[\frac{c(\varphi^*,\cdot)}{\tilde{c}(\varphi^*,\cdot)}\right]^{1-\sigma} > 0\\
\widehat{m}_r &= \gamma[\tilde{m}_r + \tilde{m}_w w'(r)] < 0\\
\tilde{m}_r &= \int_0^\infty \frac{\partial m(\varphi,\cdot)}{\partial r}\left[\frac{c(\varphi,\cdot)}{\tilde{c}(\varphi^*,\cdot)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) + \frac{1-\sigma}{r}Var(m) < 0\\
\tilde{m}_w &= \int_0^\infty \frac{\partial m(\varphi,\cdot)}{\partial w}\left[\frac{c(\varphi,\cdot)}{\tilde{c}(\varphi^*,\cdot)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) + \frac{\sigma-1}{w}Var(m) > 0.
\end{aligned} \tag{A.24}$$

Then,

$$\begin{bmatrix} d\varphi^* \\ dr \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} E\widehat{m}_r - (1-\widehat{m})K + w_r\widehat{m}L & -\Phi_r \\ -E\widehat{m}_{\varphi^*} & \Phi_{\varphi^*}\end{bmatrix}\begin{bmatrix} dF - \Phi_a da \\ (1-\widehat{m})rdK - w\widehat{m}dL - w_a(\widehat{m}L + E\widehat{m}_w)da\end{bmatrix}, \tag{A.25}$$

where $\Delta \equiv \Phi_{\varphi^*}[E\widehat{m}_r - (1-\widehat{m})K + w_r\widehat{m}L] - \Phi_r E\widehat{m}_{\varphi^*} > 0$ is immediate. Hence, for the first result (an increase in K), we have

$$\begin{bmatrix} d\varphi^* \\ dr \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} -\Phi_r(1-\widehat{m})rdK \\ \Phi_{\varphi^*}(1-\widehat{m})rdK \end{bmatrix}, \tag{A.26}$$

which follows that $d\varphi^*/dK > 0$ and $dr/dK < 0$.

For the second result (i.e., an increase in $F = f_e/f$), we have

$$\begin{bmatrix} d\varphi^* \\ dr \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} (E\widehat{m}_r - (1-\widehat{m})K + w_r\widehat{m}L)dF \\ -E\widehat{m}_{\varphi^*}dF \end{bmatrix}. \tag{A.27}$$

Therefore, $d\varphi^*/dF < 0$ and $dr/dF < 0$.

Finally, with respect to an increase in a, we have

$$\begin{bmatrix} d\varphi^* \\ dr \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} -[(E\widehat{m}_r - (1-\widehat{m}_r)K + w_r\widehat{m}L) - \Phi_r w_a(\widehat{m}L + E\widehat{m}_w)]da \\ (E\widehat{m}_{\varphi^*}\Phi_a - \Phi_{\varphi^*}w_a(\widehat{m}L + E\widehat{m}_w))da \end{bmatrix}. \tag{A.28}$$

Therefore, while the sign of $d\varphi^*/da$ is unclear, the capita price goes up, $dr/da > 0$.

A.7 Proof of Proposition 3

The average adoption level $\tilde{m}$ can be written as

$$\tilde{m}(\varphi^*, r, \theta) = \int_0^{rk(\theta)/w(r)} m(\varphi, r) \left[\frac{c(\varphi, r)}{c(\varphi^*, r)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) + \theta \int_{rk(\theta)/w(r)}^{\infty} \left[\frac{c(\varphi, r)}{c(\varphi^*, r)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi). \tag{A.29}$$

Thus, the partial derivative with respect to θ, $\tilde{m}_\theta$, is given by

$$\begin{aligned}\tilde{m}_z(\varphi^*, r, \theta) &= rk'(\theta)/w(r) \left[m(rk(\theta)/w(r), r) - \theta\right] \left[\frac{c(rk(\theta)/w(r), r)}{c(\varphi^*, r)}\right]^{1-\sigma} \\ &+ \int_{rk(\theta)/w(r)}^{\infty} \left[1 + \theta(1-\sigma)(\ln rk(\theta)/w(r) - \ln\varphi)\right] \left[\frac{c(\varphi, r)}{c(\varphi^*, r)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi).\end{aligned} \tag{A.30}$$

Noting that $m(rk(\theta)/w(r), r) = \theta$ and $\ln(rk(z)/w(r)) - \ln\varphi < 0$ for the relevant support of φ, $\tilde{m}_\theta$ becomes

$$\tilde{m}_\theta(\varphi^*, r, \theta) = \int_{rk(\theta)/w(r)}^{\infty} \left[1 + \theta(1-\sigma)(\ln rk(\theta)/w(r) - \ln\varphi)\right] \left[\frac{c(\varphi, r)}{c(\varphi^*, r)}\right]^{1-\sigma} d\mu_{\varphi^*}(\varphi) > 0. \tag{A.31}$$

Then, consider the partial derivative of $\Phi(\varphi^*, r, \theta)$ with respect to θ. Denoting it by Φ_θ,

$$\Phi_\theta(\varphi^*, r, \theta) = (1-\sigma) \int_{rk(\theta)/w(r)}^{\infty} \left[\frac{c(\varphi, r)}{c(\varphi^*, r)}\right]^{1-\sigma} [\ln(rk(\theta)/w(r)) - \ln\varphi] dh(\varphi) > 0. \tag{A.32}$$

Thus, the comparative statics with respect to the technology frontier θ is given by

$$\begin{aligned}\begin{bmatrix} d\varphi^* \\ dr \end{bmatrix} &= \frac{1}{\Delta} \begin{bmatrix} E\widehat{m}_r - (1-\widehat{m})K + w_r\widehat{m}L & -\Phi_r \\ -E\widehat{m}_{\varphi^*} & \Phi_{\varphi^*} \end{bmatrix} \begin{bmatrix} -\Phi_\theta d\theta \\ -E\widehat{m}_\theta d\theta \end{bmatrix} \\ &= \frac{1}{\Delta} \begin{bmatrix} (-\Phi_\theta(E\widehat{m}_r - (1-\widehat{m})K + w_r\widehat{m}L) + \Phi_r E\widehat{m}_\theta)d\theta \\ (E\widehat{m}_{\varphi^*}\Phi_\theta - \Phi_{\varphi^*}E\widehat{m}_\theta)d\theta \end{bmatrix},\end{aligned} \tag{A.33}$$

where $\Delta \equiv \Phi_{\varphi^*}\left[E\widehat{m}_r - (1-\widehat{m})K + w_r\widehat{m}L\right] - \Phi_r E\widehat{m}_{\varphi^*} > 0$. Since $\tilde{m}_{\varphi^*} > 0, \Phi_\theta > 0, \Phi_{\varphi^*} < 0$, and $\widehat{m}_\theta > 0$, $E\widehat{m}_{\varphi^*}\Phi_\theta - \Phi_{\varphi^*}E\widehat{m}_\theta > 0$ holds, which implies that $dr/d\theta > 0$. However, $\Phi_r E\widehat{m}_\theta$ is negative while $-\Phi_\theta(E\widehat{m}_r - (1-\widehat{m})K + w_r\widehat{m}L)$ is positive. Thus, in general, the sign of $d\varphi^*/d\theta$ is ambiguous.

A.8 Proof of Proposition 4

(Case 1) $\theta_H = \theta_F, (K/L)_H > (K/L)_F$

We show that a hypothetical integrated economy can be divided into two distinct economies that are different in terms of factor endowments only.

Consider the free entry condition and the factor market clearing condition in the home country. The free-entry condition is intact:

$$[1 - H(\varphi_H^*)]\, \phi(\varphi_H^*, r, w) = F, \tag{A.34}$$

which implies that $\varphi_H^* = \varphi_F^*$ in an FPE set. The home country and the foreign country have the same firm distribution regardless the difference in the factor endowments. The full-employment conditions are

$$\alpha\gamma n_H E^W \tilde{m} + (1-\alpha)\gamma n_H E^W m_z + (1-\gamma) z_H E^W m_z = rK_H, \tag{A.35}$$

and

$$\alpha\gamma n_H E^W (1-\tilde{m}) + (1-\alpha)\gamma n_H E^W (1-m_z) + (1-\gamma) z_H E^W (1-m_z) = wL_H, \tag{A.36}$$

where $n_H = N_H/N$ is the home share of the differentiated goods and $z_H = Z_H/Z$ is the home share of the homogenous good production. From these, we obtain

$$\omega = \frac{(1-\widehat{m})n_H + (1-\gamma)(1-m_z)z_H}{\widehat{m}n_H + (1-\gamma)m_z z_H} k_H, \tag{A.37}$$

where $\omega = w/r$, $k_H = K_H/L_H$ and $\widehat{m} = \alpha\gamma\tilde{m} + (1-\alpha)\gamma m_z$. We can have an analogous for the foreign country:

$$\omega = \frac{(1-\widehat{m})(1-n_H) + (1-\gamma)(1-m_z)(1-z_H)}{\widehat{m}(1-n_H) + (1-\gamma)m_z(1-z_H)} k_F. \tag{A.38}$$

Thus, the simultaneous equation system for n_H and z_H:

$$[(1-\widehat{m})k_H - \omega\widehat{m}]n_H + (1-\gamma)[(1-m_z)k_H - \omega m_z]z_H = 0 \tag{A.39}$$

$$[(1-\widehat{m})k_F - \omega\widehat{m}]n_H + (1-\gamma)[(1-m_z)k_F - \omega m_z]z_H = [1-\widehat{m}+(1-\gamma)(1-m_z)]k_F - [\widehat{m}+(1-\gamma)m_z]\omega \tag{A.40}$$

Solving these two equations

$$\begin{bmatrix} n_H \\ z_H \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} [-(1-\gamma)[(1-m_z)k_H - \omega m_z][[1-\widehat{m}+(1-\gamma)(1-m_z)]k_F - [\widehat{m}+(1-\gamma)m_z]\omega] \\ [[1-\widehat{m}+(1-\gamma)(1-m_z)]k_F - [\widehat{m}+(1-\gamma)m_z]\omega][(1-\widehat{m})k_H - \omega\widehat{m}] \end{bmatrix}. \tag{A.41}$$

where $\Delta \equiv \omega(1-\gamma)(k_H - k_F)(\widehat{m} - m_z)$ and $\Delta > 0$ since $k_H > k_F$ and $\widehat{m} > m_z$. Thus, for k_H and k_F that satisfy that $n_H \leq 1$ and $z_H \leq 1$, factor price equalization (FPE) holds.

(Case 2) $\theta_H > \theta_F$

When the world market is integrated and goods are traded without any costs, the operating profit earned by a firm with φ is

$$\pi(\varphi, r, w) = \frac{\gamma E^W}{\sigma N^W}\left[\frac{c(\varphi, r, w)}{\tilde{c}^W(\varphi^*, r.w)}\right]^{1-\sigma} - f, \tag{A.42}$$

where $N^W = N_H + N_F$ and $E^W = E_H + E_F$. φ^* is the lowest productivity level with which firms earn zero-profit in equilibrium:

$$\frac{\gamma E^W}{\sigma N^W}\left[\frac{c(\varphi^*, r, w)}{\tilde{c}^W(\varphi^*, r.w)}\right]^{1-\sigma} = f. \tag{A.43}$$

Suppose that factor prices are equalized between the two countries: $r_H = r_F$ and $w_H = w_F$. Since the relative wage rate w/r determines the level of technological adoption level m, firms' technological choice is described as follows: up to $\varphi_F = rk(\theta_F)/w$, both home and foreign firms choose $m = k^{-1}(\varphi w/r)$ (i.e. interior solutions). For productivity level $\varphi \in (\theta_F, \theta_H)$, foreign firms choose $m = \theta_F$ while home firms still choose interior solutions $m = k^{-1}(\varphi w/r) > \theta_F$. After $\varphi \geq \theta_H$, foreign firms' choice is θ_F and home firms' choice is θ_H. Therefore, the average variable cost is lower in the home country than in the foreign country: $\tilde{c}_H(\varphi^*, r, w) < \tilde{c}_F(\varphi^*, r, w)$, which implies that the average firms' profits is greater in the home country than in the foreign country: $\tilde{\pi}_H > \tilde{\pi}_F$.

However, this immediately contradicts with the free-entry condition, which tells that $[1 - H(\varphi^*)]\tilde{\pi} = f_e$ (if Y sector is active in both home and foreign countries). Thus, if Y sector is active in both countries, factor prices are never equalized and $w_F/r_F > w_H/r_H$ must hold since only by discouraging technological adoption in the home country would make $\tilde{\pi}_H = \tilde{\pi}_F$ possible.

References

Autor, David H., Lawrence F. Katz, and Alan B. Kruger, "Computing Inequality: Have Computers Changed the Labor Market?," *Quarterly Journal of Economics*, November 1998, *113* (4), 1169–1213.

Bartel, A.P. and F.R. Lichtenberg, "The Comparative Advantage of Educated Workers in Implementing New Technology," *Review of Economics and Statistics*, 1987, *69* (1), 1–11.

Beaudry, Paul, Mark Doms, and Ethan Lewis, *Endogenous Skill Bias in Technology Adoption: City-Level Evidence from the IT Revolotion*, Vol. 12521 of *NBER Working Paper*, NBER, September 2006.

Bernard, Andrew B., Stephen J. Redding, and Peter K. Schott, "Comparative Advantage and Heterogenous Firms," *Review of Economic Studies*, January 2007, *74* (1), 31–66.

Bresnahan, Timothy F., Erik Brynjolfsson, and Lorin M. Hitt, "Information Technology, Workplace Organization, and The Demand for Skilled Labor: Firm-Level Evidence," *Quarterly Journal of Economics*, February 2002, *117* (1), 339–76.

Djankov, Simeon, Rafael La Porta, Florencio Lopez-Silanes, and Andrei Shleifer, "The Regulation of Entry," *Quarterly Journal of Economics*, February 2002, *117* (1), 1–37.

Doms, Mark, Timothy Dunne, and Kenneth R. Troske, "Workers, Wages, and Technology," *Quarterly Journal of Economics*, February 1997, *112* (1), 253–90.

Ederington, Josh and Phillip McCalman, "Endogenous Firm Heterogeneity and the Dynamics of Trade Liberalization," *mimeo*, August 2004.

_ **and** _ , "Shaking All Over? International Trade and Industrial Dynamics," *mimeo*, February 2006.

Melitz, Marc J., "The Impact of Trade on Intra–Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, November 2003, *71* (6), 1695–1725.

Zeira, Joseph, "Workers, Machines, and Economic Growth," *Quarterly Journal of Economics*, November 1998, *113* (4), 1091–1117.

_ , "Machines as Engines of Growth," *CEPR Discussion Papaer No. 5429*, January 2006.

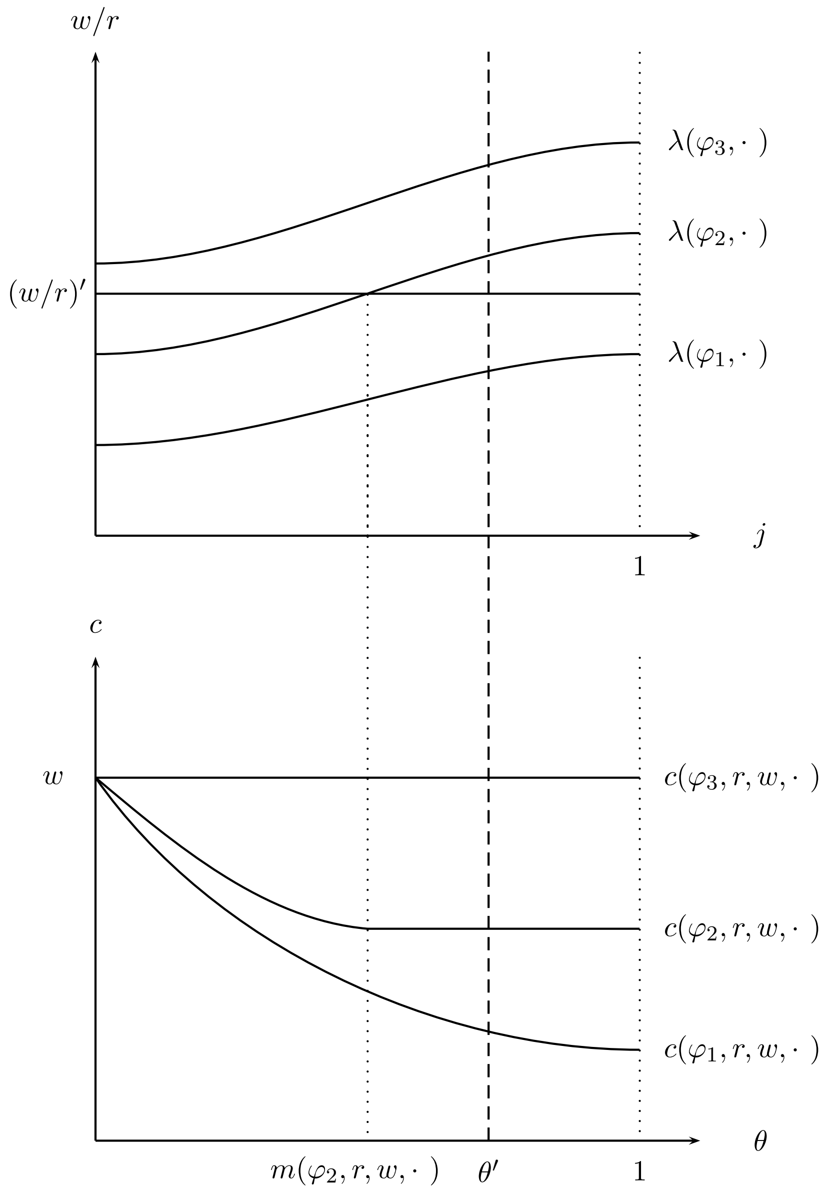

Figure 1: **Technology Adoption and the Unit Variable Costs I**

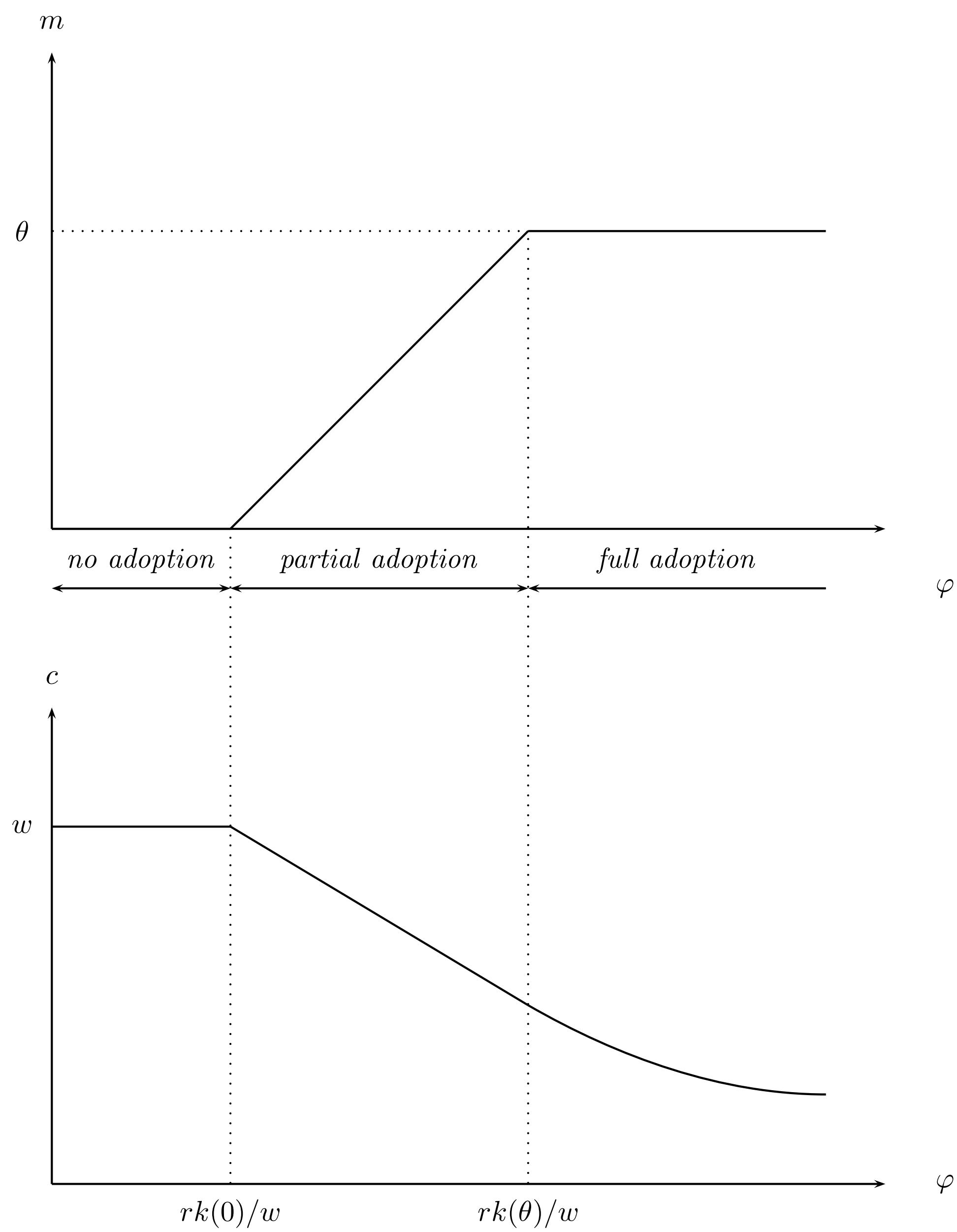

Figure 2: **Technology Adoption and the Unit Variable Costs II**

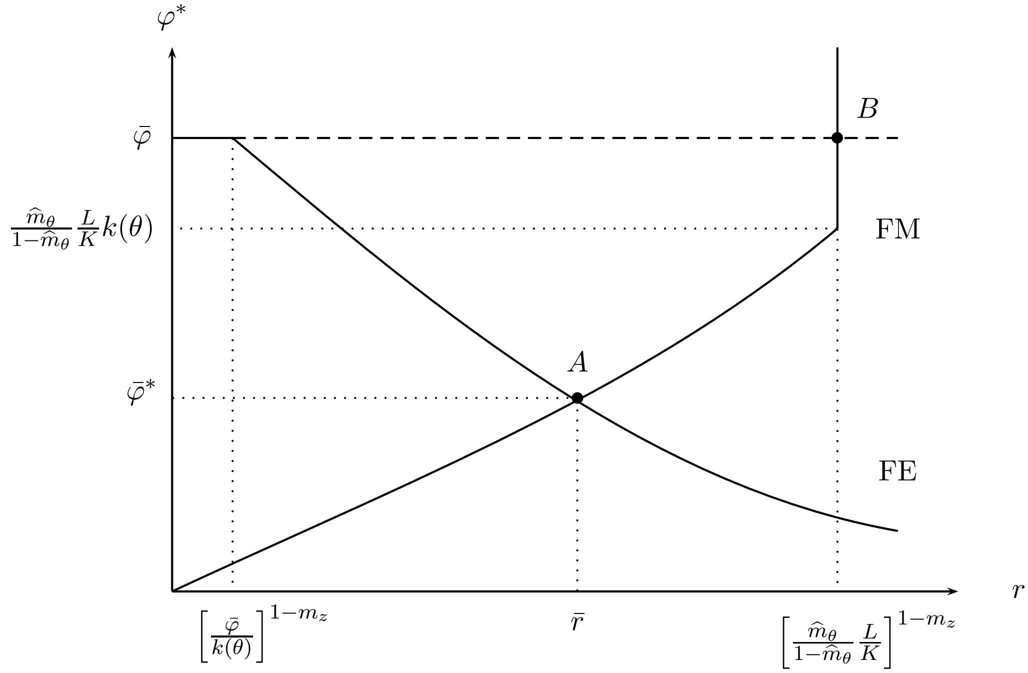

Figure 3: **The Equilibrium Cutoff and Capital Price**

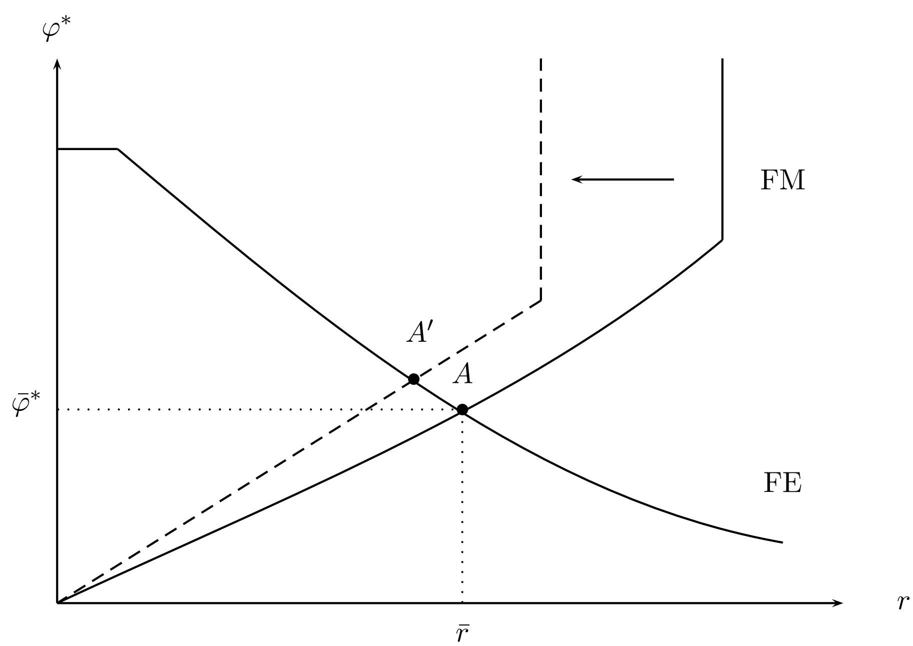

Figure 4: **An Increase in the Capital Endowment**

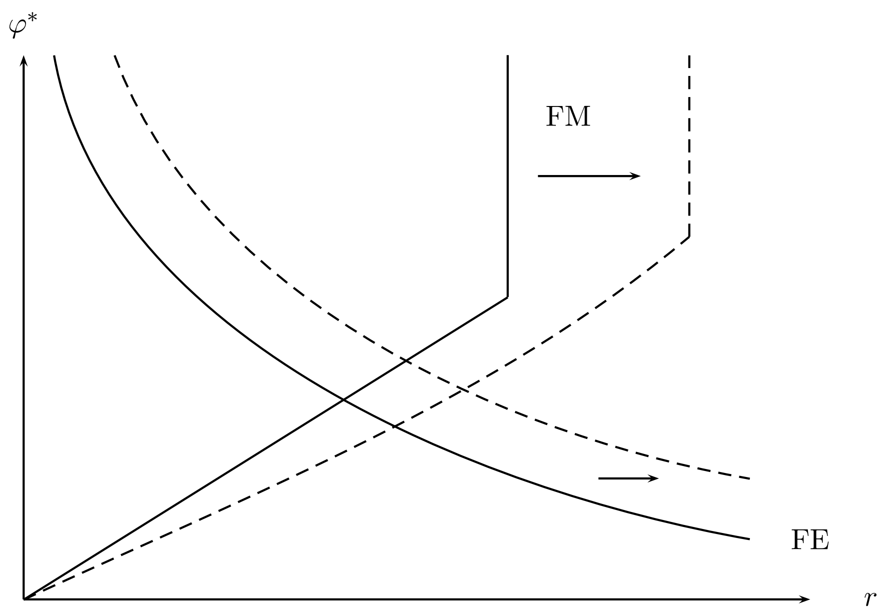

Figure 5: **An Expansion of the Technological Frontier**