A Canonical Model of the FX Premium*

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Resolving the forward premium puzzle requires a volatile FX risk premium that covaries negatively with the depreciation rate. This paper revisits the forward premium anomaly using a standard open economy macro model. We show that a small open economy model with consumption based external habit formation can only generate such a FX premium if households have very slow-moving habits as well as are subject to very persistent productivity shocks.

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Abstract

Resolving the forward premium puzzle requires a volatile FX risk premium that covaries negatively with the depreciation rate. This paper revisits the forward premium anomaly using a standard open economy macro model. We show that a small open economy model with consumption based external habit formation can only generate such a FX premium if households have very slow-moving habits as well as are subject to very persistent productivity shocks.

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1 Introduction

Would an investment strategy that borrows in a low interest rate currency and invests the proceeds in a high yielding currency be profitable? The naive answer is "yes" since the interest rate differential on the borrowing-lending spread represents a potential profit opportunity. The traditional answer is "no" since any excess return from investing in a high interest rate currency would be offset by an associated depreciation in that particular currency. But, actually, the empirical evidence is that not only this strategy is profitable on average but it yields returns that exceed the interest rate differential. This is because, in practice, high interest rate currencies frequently appreciate over time. And therefore, rather than offsetting any interest rate differentials, the exchange rate movements actually increase the profit of this particular investment strategy (see Cavallo (2006)). This finding is obviously well-known in the academic literature and is known as the forward premium anomaly or the uncovered interest parity (UIP) puzzle (see Fama (1984)).[1]

A traditional open economy model cannot replicate the forward premium anomaly as it typically assumes linear UIP holds. When investors are assumed to be risk-neutral, any cross-country differences in interest rates are exactly offset by exchange rate changes. A large literature have tried previously to account for the forward premium anomaly. One strand of research, first explored in Bekaert (1996), attributes the failure of UIP to the existence of time varying foreign exchange (FX) risk premia. When you allow for risk, risk-adverse investors may require additional compensation to hold riskier assets. A risk-adjusted UIP condition breaks the tight link between changes in the exhange rate and interest rate differentials. But as demonstrated by Fama (1984), the FX risk premium embodied in this UIP condition needs to have certain dynamics properties in order to resolve the forward premium anomaly. The challenge for this strand of the literature has been to come up with an open economy model that generates an FX risk premium with the time series properties that resolves this long-lasting anomaly.[2]

In this paper, we re-examine the forward premium anomaly using a standard open economy macro framework as in Gali and Monacelli (2005) and De Paoli (2004). Having consumption habits in the model is crucial in order to resolve the UIP puzzle. These ensure that in periods where the domestic currency is expected to appreciate, domestic investors are less risk averse than foreign households, which translates into domestic interest rates being higher than foreign interest rates. We are not the first to explore the role of consumption habits in solving the UIP puzzle. Both Verdelhan (2006) and Moore and Roche (2007) have shown how models featuring consumption habits can help rationalise the UIP puzzle. These works assume that investors are subject to permanent shocks and have Campbell and Cochrane (1999) type preferences. The former considers an endowment economy subject to trade costs. The latter presents a monetary model which adds so-called "deep habits" to the Campbell and Cochrane (1999) setting.

Our works differ from the above mentioned in the following way: Our approach has been to

[1] While the original forward premium anomaly is documented by Fama (1982), a number of papers have looked at the robustness of his result. Recent papers include Baillie and Bollerslev (1997), Bansal and Dahlquist (2000), Bansal (1997) and Flood and Rose (1996))

[2] There are other strands of literature that have tried to rationalize the forward premium anomaly using theoretical models. "Peso problem"-type arguments and other explanations related to irrational market participant behavior can be found in the early literature (see Engel (1996) for a survey). More recently, Bacchetta and van Wincoop (2005) and Gourinchas and Tornell (2004) examine the role of imperfect information. Alvarez et al. (2005) look at the properties of the FX risk premium in a model with asset market segmentation.

apply a small open economy model which has no non-standard features (e.g. trade costs, "deep habits" or a non-linear Campbell and Cochrane (1999) type habits) but allows for a slow-moving consumption habit. The structural nature of the model enables us to identify the key features required to address the UIP puzzle. For instance, we can assess the implications of changing parameters that are well-known in the open economy macro literature such as the import share, Frisch labor supply elasticity and the productivity shock persistence.

We find that consumption habits alone are not sufficient to generate a constellation of movements in exchange rates and interest rates that resolve the UIP puzzle. It is crucial that investors expect changes to economic conditions - as captured by their excess consumption level - to persist. We therefore need to assume that investors in our model are subject to very persistent shocks and their consumption habits adjust only gradually to these shocks. These findings are of a similar nature to the results in Bansal and Shaliastovich (2006) who highlight the importance of modelling long term risks.

To understand our result, consider a positive domestic productivity shock which would depreciate the domestic currency. Investors now expect the currency to appreciate back to its pre-shock level. In order to resolve the Fama puzzle, such currency movements would have to be accompanied by a greater rise in domestic relative to foreign interest rates. But the typical *intertemporal substitution* effect of open economy models imply that domestic interest rates fall by more than foreign rates. Nevertheless, in a world where investors are not risk neutral, this shock may lessen agent's incentive to engage in *precautionary savings* and put upward pressure on interest rates. If the precautionary savings effect dominate the intertemporal substitution effect, domestic interest rates will end up higher than foreign interest rates. So precautionary savings needs to be countercyclical and strong in order to solve the forward premium anomaly.

If agents except their consumption level to revert quickly to their pre-shock level or their habits to adjust rapidly, then agents are less inclined to change their precautionary savings. Think about the extreme case where the positive shock only lasts for one period and consumption habits adjust instantly. Here agents expect their consumption level next period to be below their habit level. So agents will actually *increase* their precautionary savings rather than decrease. In the absence of any shock or habit persistence, domestic interest rates fall by even more than they would in a risk-neutral world!

The paper is structured as follows: In Section 2 we present the model. We derive the FX risk premium in Section 3 and show what determines the sign of the FX risk premium. Section 4 analyses the cyclicality of the premium and illustrates how a negative Fama coefficient can be achieved. Finally, the concluding remarks are presented in section 5.

2 Model

In this paper we derive an analytical solution for foreign exchange rate risk premium in a canonical small open economy model similar to the one in Gali and Monacelli (2005) or De Paoli (2004). The next subsections present the goods and asset markets specifications.

2.1 The goods market

Our theoretical framework consists of a two-country general equilibrium model in which real exchange rate fluctuations comes about via the consumption home bias channel. The size of this consumption home bias depends on the degree of openness and the relative size of the economy. This specification allows us to characterize the small open economy by taking the limit of the home size to zero. Prior to applying the limit, we derive the optimal equilibrium conditions for the general two country model. After the limit is taken, the two countries Home and Foreign represent the small open economy and the rest of the world, respectively.

The original Gali and Monacelli (2005) or De Paoli (2004) specifications feature monopolistic competition and a Calvo-type sticky price setting. These assumptions are introduced in the small open economy in order to address issues of monetary policy. However, in the present paper we assume that the monetary authority targets domestic price inflation. Thus, our equilibrium allocation correspond to the flexible price version of these models.

The world economy is populated with a continuum of agents of unit mass, where the population in the segment $[0, n)$ belongs to country H (Home) and the population in the segment $(n, 1]$ belongs to country F (Foreign). The utility function of a consumer j in country H is given by:

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j, X_s) - V(y_s(j), \varepsilon_t) \right]. \tag{1}$$

Households obtain utility from consumption $U(C_t, X_t) = \frac{\left(C_t^i - hX_t\right)^{1-\rho} - 1}{1-\rho}$ and contribute to the production of a differentiated good $y(j)$ attaining disutility $V(y_t, \varepsilon_{Y,t}) = \frac{\varepsilon_t^{-\eta} y_t^{1+\eta}}{1+\eta}$.[3] Productivity shocks are denoted by ε_t. We assume that agents have a slow-moving external consumption habits X_ti.e.

$$X_t = (1 - \phi)C_{t-1} + \phi X_{t-1} \tag{2}$$

C_t is a constant elasticity of substitution (CES) aggregate over domestic, C_{Ht}, and foreign produced tradables C_{Ft}:

$$C_t = \left[\upsilon^{\frac{1}{\theta}} \left(C_{Ht}\right)^{\frac{\theta-1}{\theta}} + (1-\upsilon)^{\frac{1}{\theta}} \left(C_{Ft}\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

with the corresponding domestic price index, P_t, defined as:

$$P_t = \left[\upsilon \left(P_{Ht}\right)^{1-\theta} + (1-\upsilon) \left(P_{Ft}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{4}$$

The sub-indices C_H and C_F are Home consumption of the differentiated products produced in countries H and F. These are defined as follows:

$$C_H = \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c\left(z\right)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}} \int_n^1 c\left(z\right)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}} \tag{5}$$

[3]This specification would be equivalent to one in which the labour market is decentralized. These firms employ workers who have disutility of supplying labour and this disutility is separable from the consumption utility.

where $\sigma > 1$ is the elasticity of substitution across the differentiated products. The consumption-based price indices that correspond to the above specifications of preferences are given by

$$P_H = \left[\left(\frac{1}{n} \right) \int_0^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, P_F = \left[\left(\frac{1}{1-n} \right) \int_n^1 p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \tag{6}$$

We define the real exchange rate, Q_t as the number of domestic consumption baskets per foreign consumption baskets.

$$Q_t = \frac{S_t P_t^*}{P_t} \tag{7}$$

Hence, a rise in the real exchange rate, Q_t, is a real depreciation. Assuming that the law of one price holds, we have that $P_{H,t} = S_t P_{H,t}^*$ and $P_{F,t} = S_t P_{F,t}^*$ where S_t is the nominal exchange rate expressed as domestic currency per foreign currency. Obviously with law of one price holding and identical consumer preferences, $\upsilon^* = \upsilon$, purchasing power parity (PPP) holds and the real exchange rate, Q, equals 1. However, as in Sutherland (2002), we assume a particular specification of υ and υ^* which implies that consumers at home and abroad have a consumption home bias. Because of home bias, the real exchange rate, Q, can deviate from its PPP value, $Q_t \neq \frac{S_t P_t^*}{P_t}$.

Specifically, we assume that the parameter governing domestic consumers' preference for foreign goods, $(1-\upsilon)$, is a function of the relative size of the foreign economy, $1-n$, as well as the degree of openness, λ in the domestic economy:

$$(1-\upsilon) = (1-n)\lambda. \tag{8}$$

Hence, a more open domestic economy (higher λ) would - ceterius paribus - imply a large share of foreign goods in the domestic consumption basket. Similarly, the greater the size of the foreign economy relative to the domestic economy (higher $1-n$), the larger the share of foreign goods in the domestic consumption basket.

Agents in the Foreign economy have preferences analogous to (1), (3) and (5). And foreign consumers' preferences for home goods also depend on the relative size of the home economy, n, and the degree of openness of the domestic economy λ, that is

$$\upsilon^* = n\lambda. \tag{9}$$

As demonstrated in De Paoli (2004), one can derive the demand equations, using the relationship between preferences, υ, country size, n, and openness, λ. After invoking our small open economy assumption. (that is, we taking the limit for $n \to 0$) we have

$$Y_t = \left[\frac{P_{H,t}}{P_t} \right]^{-\theta} \left[(1-\lambda)C_t + \lambda \left(\frac{1}{Q_t} \right)^{-\theta} C_t^* \right], \tag{10}$$

$$Y_t^* = C_t^*. \tag{11}$$

Moreover, combining equations (4), (7), (10) and (11) and log-linearising the resulting expression, we can write the following expression

$$y_t = (1-\lambda)c_t + \lambda y_t^* + \gamma q_t \tag{12}$$

which summirises the demand side equilibrium in the small open economy. Note that lower case variables denote log deviations from steady state (i.e. $x = \log(X/\bar{X})$) and $\gamma = \theta\lambda(2-\lambda)/(1-\lambda)$. For the foreign economy the demand condition is simply given by

$$y_t^* = c_t^* \tag{13}$$

Turning now to the supply side of the model, we assume that prices follow a partial adjustment rule à la Calvo (1983). Producers of differentiated goods know the form of their individual demand functions, and maximize profits taking overall market prices and products as given. In each period a fraction, $\alpha \in [0,1)$, of randomly chosen producers is not allowed to change the nominal price of the goods they produce. The remaining fraction of firms, given by $(1-\alpha)$, chooses prices optimally by maximizing the expected discounted value of profits.[4] The optimal choice of producers that can set their price $\tilde{p}_t(j)$ at time T is, therefore:

$$E_t\left\{\sum(\alpha\beta)^{T-t}U_c(C_T)\left(\frac{\tilde{p}_t(j)}{P_{H,T}}\right)^{-\sigma}Y_{H,T}\left[\frac{\tilde{p}_t(j)}{P_{H,T}}\frac{P_{H,T}}{P_T}-\frac{\sigma V_y\left(\tilde{y}_{t,T}(j),\varepsilon_{Y,T}\right)}{(\sigma-1)U_c(C_T)}\right]\right\}=0. \tag{14}$$

Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production, represented by $\frac{\sigma}{(\sigma-1)}$.[5] Given the Calvo-type setup, the price index evolves according to the following law of motion,

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)\left(\tilde{p}_t(h)\right)^{1-\sigma}. \tag{15}$$

If we log-linearise equations (14), (15) and combine with the log-linerised version of equations (4) and (7), we obtain the small open economy's Phillips Curve.

$$\pi_t = k(\rho c_t + \eta y_t + \lambda(1-\lambda)_t^{-1}q_t - \eta\varepsilon_t) + \beta E_t\pi_{t+1}$$

where π_t represents domestic producer price inflation and $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$. Throughout the paper we assume that the central bank targets domestic inflation, that is, the central bank's targeting rule is given by

$$\pi_t = 0, \ \forall t.$$

Note that the flexible price allocation is identical to the one that would prevail were policymakers to target domestic inflation (that is, the case in which $\alpha \to 0$ and, therefore, $k \to \infty$, is equivalent to the case in which $\pi_t = 0$, ∇t). So, under this assumption, our model is essentially a flexible price version of Gali and Monacelli (2005) or De Paoli (2004). In this particular case, the supply side equilibrium can be written as

$$y_t = \varepsilon_t - \eta^{-1}\rho c_t + \eta^{-1}\lambda(1-\lambda)_t^{-1}q_t \tag{16}$$

For the foreign economy the supply condition is simply:

[4] All households within a country (that can modify their prices at a certain time) face the same discounted value of the streams of current and future marginal costs. Thus, they choose to set the same price.

[5] Note that, when demand is infinitely elastic, this wedge is eliminated - this specification characterizes the perfect competition case.

$$y_t^* = \frac{\eta}{\eta+\rho}\varepsilon_t^* \tag{17}$$

Thus, given equation (13) and (17), hereafter we treat the foreign output as an exogenous process.

2.2 The asset market

Following Chri and McGrattan (2002), we assume that financial markets are complete both domestically and internationally. In this environment the intertemporal marginal rate of substitution (in nominal terms) is equalized across countries.

$$\frac{U_C\left(C_{t+1}^*, X_{t+1}^*\right)}{U_C\left(C_t^*, X_t^*\right)}\frac{P_t^*}{P_{t+1}^*} = \frac{U_C\left(C_{t+1}, X_{t+1}\right)}{U_C\left(C_t, X_t\right)}\frac{S_{t+1}P_t}{S_t P_{t+1}}. \tag{18}$$

Equation (18) holds in all states of nature. This specification for the asset market implies that there is perfect risk sharing across borders.

Equivalently we could express the risk sharing condition in terms of the domestic and foreign stochastic discount factors, M_{t+1} and M_{t+1}^*, i.e.

$$M_{t+1} = \frac{U_C\left(C_{t+1}, X_{t+1}\right)}{U_C\left(C_t, X_t\right)} \tag{19}$$

$$M_{t+1}^* = \frac{U_C\left(C_{t+1}^*, X_{t+1}^*\right)}{U_C\left(C_t^*, X_t^*\right)} \tag{20}$$

Thus, the risk sharing condition is:

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{t+1}^*}{M_{t+1}} \tag{21}$$

All households have access to a domestic as well as a foreign bonds. The domestic bond is a one period real bond that pays out in units of the domestic consumption basket. The foreign bond is also a one period real bond but its payout is in units of the foreign consumption basket. The one period riskfree returns on the domestic and the foreign bonds are R_t and R_t^* respectively.

The following Euler equations determine how the domestic household would price both types of bonds:

$$1 = E_t\left[\beta M_{t+1} R_t\right] \tag{22}$$

$$1 = E_t\left[\beta M_{t+1} R_t^* \frac{Q_{t+1}}{Q_t}\right] \tag{23}$$

There is a corresponding set of Euler equations that determine how the foreign household would price the two set of bonds.

2.2.1 Time-varying risk aversion

The coefficient of relative risk aversion (CRRA hereafter) for the utility function $U(C_t, X_t)$ can be expressed as:

$$\eta_t = -C_t \frac{U_{cc}(C_t, X_t)}{U_c(C_t, X_t)}$$

If the utility function is defined as in (1), and consumption habits are *external* to households, CRRA is

$$\eta_t = \rho \frac{C_t}{C_t - hX_t},$$

or, if we define the surplus consumption ratio as

$$S_t = \frac{C_t - hX_t}{C_t},$$

the CRRA can be written as

$$\eta_t = \frac{\rho}{S_t},$$

Note that if we assume a utility function with no habits (i.e. the case of $h = 0$), the coefficient of relative risk aversion is constant and equals ρ. But the presence of habits implies that as consumption changes over time, the surplus ratio fluctuates and this leads to variations in the representative consumer's risk aversion. We can also see that the CRRA is countercyclical with respect to consumption levels - i.e. higher consumption implies a decline in risk aversion.

2.3 Equilibrium

The demand and supply equations, (12) and (16), together with the asset pricing conditions (19), (20), (21), (22), (23), and the definition of habits (2) determined the evolution of (Y_t, C_t, Q_t, M_t, M_t^*, R_t^*, R_t, X_t) given the exogenous process for (ε_t, y_t^*) These shocks are assumed to follow an AR(1) process where γ and γ^* are the domestic and foreign autoregressive coefficients respectively.

3 FX Premia

In this section, we introduce the notion of a foreign exchange rate risk premium. We illustrate how the FX premium captures how domestic as well as foreign investors perceive the riskiness of domestic relative to foreign bonds. We then discuss when we would expect the FX premium to be positive or negative.

Suppose the foreign exchange rate risk premium, fxp_t, is defined as the expected return of holding a domestic bond over a foreign bond (where all returns are measured in the same currency) i.e.

$$fxp_t = r_{t+1} - r_{t+1}^* - E_t\left[\Delta q_{t+1}\right]$$

As we show in the Appendix, we can re-express the excess return on domestic bonds as follows:

$$fxp_t = \frac{1}{2} var_t(q_{t+1}) + cov_t(m_{t+1}, q_{t+1}) \tag{24}$$

The above equation shows that the size of the FX risk premium, fxp, depends on two factors: First, the size of the so-called Jensen inequality effect ($\frac{1}{2} var_t(q_{t+1})$) and second on the size of the covariance between the domestic discount factor, m, and the real exchange rate q.[6]

[6]See Obstfeld and Rogoff (1995, pages 586-587) for a discussion on the relationship between the Jensen's inequality term and the forward premium.

Suppose first that the covariance is negative, i.e. $cov_t(m_{t+1}, q_{t+1}) < 0$. In this case, the domestic currency depreciates in real terms (a rise in q_t) when the domestic discount factor, m_t, falls. So the foreign currency appreciates (and, therefore, increases the return on foreign bonds relative to domestic bonds) when consumption is high (and, hence, marginal utility is low). Hence, the foreign bond yields a higher return exactly in those periods where you don't need it (i.e. when marginal utility is low). Therefore the foreign bond is considered riskier than the domestic bond. And relative to the domestic bond it should pay a premium. Therefore the FX premium, defined as the excess return of domestic bonds relative to foreign bonds, is negative.

Now consider the case where $cov_t(m_{t+1}, q_{t+1}) > 0$. Here the foreign currency appreciates in real terms (a rise in q_t) in periods when marginal utility is high (rise in m_t) and domestic consumption is low. So the foreign asset generates high returns in periods where the domestic household most needs it. Hence, the foreign asset is considered less risky than the domestic asset. Therefore it should yield a lower return than the domestic bond. Therefore the FX premium is positive.

Obviously we could tell a similar story from the point of view of the foreign investor. We could show how the size of the excess return on foreign bond would depend on the covariance between the real exchange rate and the foreign discount factor.

In the analysis above, we ignored the term $\frac{1}{2}var_t(q_{t+1})$, which in our model should be endogenously determined. So suppose we combine the expression for the excess return on foreign bonds with the similar expression for the excess return on domestic bonds. As we show in the Appendix, this allows us to re-express the foreign exchange risk premium as the difference between two conditional covariances

$$fxp_t = \frac{1}{2}[cov_t(m^*_{t+1}, q_{t+1}) + cov_t(m_{t+1}, q_{t+1})] \tag{25}$$

The equation above provides important insights into how we should think about the FX premium. A key point here is that the sign as well as size of fxp_t depends not only on how the discount factor in one country - say m_t - correlates with the exchange rate q_t. The equation suggests that we should think of the FX premium as capturing how foreign as well as domestic households perceive the relative riskiness of domestic and foreign bonds. Therefore it is the *sum* of the domestic and the foreign covariances that determine whether the domestic bond yields a positive or a negative excess return relative to the foreign bond (i.e. whether fxp is greater or less than zero).

To illustrate this point, suppose we assume all business cycle fluctuations are generated by domestic shocks (more specifically, assume that $var(\varepsilon^*_t) = 0$). In this case, the domestic discount factor is negatively correlated with the real exchange ($cov_t(m_{t+1}, q_{t+1}) < 0$) while the foreign discount factor is not correlated at all with the real exchange rate ($cov_t(m^*_{t+1}, q_{t+1}) = 0$ - with no foreign shocks and given our small open economy assumption, foreign consumption stays constant and the foreign discount factor, m^*_t, would be completely uncorrelated with the real exchange rate)

In the situation with only domestic disturbances, following positive shocks the foreign currency appreciates (and, therefore, increases the return on foreign bonds relative to domestic bonds) when domestic consumption is high and, hence, domestic marginal utility is low). Thus, the domestic bond would be traded at a *discount* relative to the foreign bond. (i.e. the FX risk premium would be negative).

Consider now the situation with only foreign shocks. In this case, foreign investors would view domestic bonds as more risky than foreign bonds since $cov_t(m^*_{t+1}, q_{t+1}) > 0$. From the foreign investor's point of view, the depreciation of the domestic currency lowers his return on holding domestic bonds. And this occurs in periods where he most needs it i.e. when foreign marginal utility, m_t^*, is high. So the foreign investor would demand a positive compensation to hold domestic bonds relative to foreign bonds. Likewise, the domestic investor would consider domestic bonds riskier than foreign bonds. With just foreign shocks, we have that $cov_t(m_{t+1}, q_{t+1}) > 0$. Thus, in the case of the world economy is only affected by foreign shocks we would expect domestic bonds to be traded at a *premium* relative to foreign bonds (i.e. the FX risk premium would be positive).

4 FX premium and the FAMA puzzle

Having introduced the notion of a FX risk premium in the previous section, this section discusses how a time-varying FX risk premium can potentially resolve the forward premium anomaly (the so-called Fama puzzle). We start off by briefly reviewing the so-called Fama regressions that document the failure of uncovered interest rate parity (UIP). We then go on to discuss under which conditions a time varying FX risk premium could explain the failure of UIP in general. And we end the section by describing how our model could satisfy such conditions.

4.1 The failure of the UIP

Before explaining how a time-varying FX risk premium can resolve the Fama puzzle, let's us first recap what Fama's puzzle is all about. We start off by taking a log-linear (first order) approximation of equations (22) and (23) and obtain the following condition:

$$E_t[\Delta q_{t+1}] \; = \; r_{t+1} - r_{t+1}^* \tag{26}$$

And, given the definition of the real exchange rate (7), we can write the above equation in nominal terms

$$E_t[\Delta s_{t+1}] \; = \; i_{t+1} - i_{t+1}^* \tag{27}$$

where $i_t = r_t + \Delta p_t$ and $i_t^* = r_t^* + \Delta p_t^*$ are domestic and foreign nominal interest rates. Even though the linear UIP condition (27) is widely rejected by the data. there is extensive evidence that the covered interest parity (CIP) condition holds (see Sarno (2005) Sarno and Taylor (2003) for some useful surveys). The CIP is a no-arbitrage condition stating that the interest rate differential is equal to the forward premium

$$f_t - s_t = i_t - i_t^*$$

where f_t is the forward exchange rate. In general, the literature have tested the UIP condition by substituting the interest rate differential in equation (27) by the so called "forward premium" $(f_t - s_t)$, and regressing the change in the exchange rate on this premium. More specifically, the regression used to test the linear UIP condition is:

$$\Delta s_{t+1} = \alpha + \beta(f_t - s_t) + \upsilon_{t+1} \tag{28}$$

where υ_{t+1} is the regression error term. The linear relationship implied by equation (27) suggests that β should be equal to one, reflecting the fact that investors would demand higher interest rates on currencies that are expected to fall in value. However, as documented in Fama (1984) and many other studies (see, for example, Hodrick (1987), Lewis (1995) and Engel (1996)), β is actually found to be negative and close to -1. These findings present strong evidence against equation (27) as a negative β implies that high interest rate currencies tend to appreciate over time. The above results are often referred to as the "forward premium anomaly puzzle" and the regression given by (28) is often called the "FAMA regression".

4.2 Risk Premium Explanation of UIP failure

In order to obtain the linear UIP condition (26) one needs to assume that investors are both rational and risk neutral. Therefore, relaxing these assumption could help explain the failure of equation (26) to match economic data. In the current work, we relax the second assumption and consider that investors are risk averse. In particular, we consider the case in which investor's risk aversion changes over time and so does the FX risk premium. But what properties does the risk premium need to inherit in order to resolve the forward premium puzzle? As already mentioned, the FAMA regression results suggest that economies with higher interest rates tend to see an expected appreciation. That is :

$$E_t[\Delta q_{t+1}] \downarrow \text{ when } (r_{t+1} - r^*_{t+1}) \uparrow \tag{29}$$

So if we consider that agents investors not risk neutral and allow for the presence of a foreign exchange rate risk premium (defined as $fxp_t = r_{t+1} - r^*_{t+1} - E_t[\Delta q_{t+1}]$), we have:

$$r_{t+1} - r^*_{t+1} = E_t[\Delta q_{t+1}] + fxp_t. \tag{30}$$

Therefore, in order to obtain the conditions stated in (29) we need an fxp_t that: (1) covaries negatively with $E_t[\Delta q_{t+1}]$ and (2) is more volatile than $E_t[\Delta q_{t+1}]$. This two conditions imply that movements in the fxp_t more than offset movements in $E_t[\Delta q_{t+1}]$ in equation (30). In the next section we demonstrate under which conditions our model can generate an FX risk premium with such characteristics.

4.3 Time varying risk aversion and FX risk premium

As we have just seen, in order to resolve the Fama puzzle, we would want our model to imply a FX risk premium that co-varies negatively with the expected depreciation rate of the real exchange rate. So the FX risk premium - the excess return on domestic assets - should increase in periods when the domestic currency depreciate (and therefore is expected to appreciate back to its steady-state).

The cyclical nature of the exchange rate in our model is standard and well understood. A positive domestic productivity shock depreciates the domestic currency today and implies that the currency is expected to appreciate back to its initial steady state. But would an increase in domestic productivity necessarily be associated with higher domestic interest rates and hence a higher excess return on domestic assets (i.e. an increase in the FX premium)?

An increase in domestic productivity is typically associated with a fall in domestic interest rates. This is the so-called *inter-temporal* substitution effect. Households enjoy higher consumption today and increase their savings in order to smooth consumption over time. Hence, domestic interest rates fall.

But suppose now that households become less risk-adverse in good times which would lower the amount of precautionary savings that households engage in. This *precautionary savings* effect would put upward pressure on domestic interest rates.

Whether a positive domestic productivity shock is associated with lower or higher domestic interest rates then depend on the strength of the precautionary savings effect relative to the standard inter-temporal substitution effect. As we will demonstrate below, a model calibration which implies a very strong precautionary savings effect will imply that domestic interest rates rise in response to a positive domestic productivity shock. This translates into an increase in the excess return on domestic assets and hence a rise in the FX premium. And lo and behold the FX premium co-varies negatively with the expected depreciation rate of the domestic currency.

To illustrate this argument we can write the FX premium as the difference between the conditional variance of the stochastic discount factor (SDF) abroad and at home (see Appendix for derivation). Thus,

$$fxp_t = \frac{1}{2}var_t(m^*_{t+1}) - \frac{1}{2}var_t(m_{t+1}). \tag{31}$$

The model would therefore generate a negative covariance between fxp_t and $E_t[\Delta q_{t+1}]$ if a shock that generates an expected appreciation (as positive domestic shocks or negative foreign shocks do) also generates an decrease in precautionary savings by domestic households relative to foreign households. But is it the case that precautionary savings at home fall relative to foreign precautionary saving abroad after positive domestic shocks or negative foreign shocks? One could imagine that countercyclical risk aversion could generate such pattern. But as explain bellow, this is only the case under certain conditions.

Let's us now consider how consumption habits implies both a time varying risk aversion and a time varying FX premium. As shown in the appendix, the foreign exchange rate risk premium can also be written as

$$fxp_t = -\frac{1}{2}\left[\lambda_t cov_t\left(c_{t+1}, q_{t+1}\right) + \lambda^*_t cov_t\left(c^*_{t+1}, q_{t+1}\right)\right] \tag{32}$$

where

$$\lambda_t = E_t(\rho/S_{t+1}) = E_t(\eta_{t+1}) \text{ and } \lambda^*_t = E_t(\rho/S^*_{t+1}) = E_t(\eta^*_{t+1})$$

As illustrated in Section 2.2.1, agent's level of risk aversion is time varying and countercyclical - when consumption is above its habit level, agents are less risk averse. However, the variable determining the cyclicality of agents attitude towards risk - λ_t -depends on *expected* risk aversion.[7] And, as shown in De Paoli and Zabczyk. (2007) - and replicated in the Appendix - the business cycle properties of λ_t are a function of the persistence of the exogenous stochastic process (i.e. the parameter γ in the present paper) and the degree of persistence in the consumption habits process (i.e. the value of ϕ). The authors show that λ_t is countercyclical only when there is sufficient

[7]Note that $cov_t\left(c_{t+1}, q_{t+1}\right)$ can also vary over time even if exogenous shocks are homoskedastic. This would be the case if the reduce form for consumption is a non-linear function of the past and present values of the stochastic shock. Therefore, movements in $cov_t\left(c_{t+1}, q_{t+1}\right)$ could also affect the business cycle properties of fxp_t. But these are less direct effects and are found to be of second order in our numerical simulations. Thus, we focus on movements of λ_t following shocks and their implications for the fxp_t.

persistence in these processes.[8] That is, even though η_t always varies countercyclically, $E_t(\eta_{t+1})$ might not have the same property. Consider for example the case of a negative productivity shock that reduces consumption. If after the bad shock agents expect consumption to return to their previous level, agents attitude towards risk may not move. A similar reasoning can be made if agents expect their habit level to adjust quickly after the shock.

Therefore, the structural parameters γ and ϕ determine the cyclical properties of λ_t. But what determines the overall behaviour of the FX risk premium? As shown in section 3 the risk premium can be positive or negative depending on the importance of different shocks hitting the economy. And it follows that the sign of the FX premium also influences its cyclical properties.

Consider, for example, the case in which there are only domestic shocks and γ and ϕ are high enough to ensure that λ_t is countercyclical. In this case $cov_t\left(c_{t+1}, q_{t+1}\right) > 0$ and $cov_t\left(c^*_{t+1}, q_{t+1}\right) = 0$ and the risk premium is negative (i.e. domestic bonds are insurance). It follows that under this specification, the risk premium will vary procyclically - this is because $\partial\lambda_t/\partial\varepsilon_t < 0$ and $cov_t\left(c_{t+1}, q_{t+1}\right) > 0$.[9] Suppose the economy is hit by a bad shock. This would make agents more risk averse and, thus, increase demand for insurance and this would make the risk premium even more negative.

In the case of foreign shocks $cov_t\left(c_{t+1}, q_{t+1}\right) < 0$ and $cov_t\left(c^*_{t+1}, q^*_{t+1}\right) > 0$ (or $cov_t\left(c^*_{t+1}, q_{t+1}\right) < 0$) and, thus, the risk premium is positive (domestic bonds are risky relative to foreign bonds). In this case, the FX premium varies countercyclically: After bad shocks, agents require a higher premium to hold domestic bonds and the fxp increases.

This analysis shows that fxp is varies procyclically (countercyclically) in a world dominated by domestic (foreign) shocks. At the same time, the exchange rate depreciates (appreciate) - creating an expected depreciation (appreciation) - following domestic (foreign) shocks. Therefore, the FX risk premium covaries negatively with the expected depreciation regardless of the source of the shock. But this is only the case under the assumption that λ_t is countercyclical. The next section presents some numerical simulations that illustrate these results.

4.4 Model simulations results

We now turn to our simulation results. Since our model is essentially the flexible price version of the Gali and Monacelli (2005) model, our model calibration follows closely their parameterisation. Given the simplicity of the model, it is unlikely that it will come close to replicating the observed moments of the main macroeconomic variables such as output, consumption, interest rate and the real exchange rate. But our aim is not to match actual empirical moments. Rather, we want to use a familiar open economy model to shed light on how various degrees of consumption habit formation affects the general level of the FX risk premia as well as its business cycle properties. The next section describes our calibration strategy.

[8] De Paoli and Zabczyk. (2007) find that these conditions help generate a countercyclical risk premium.

[9] Again, we should note that $cov_t\left(c_{t+1}, q_{t+1}\right)$ can also vary over time, but we these effects are shown to be of second order in our numerical analysis.

4.4.1 Model calibration

As mentioned earlier, the starting point of our model calibration is the benchmark case in Gali and Monacelli (2005). Column 3 of table 1 summarizes their chosen values.

[Insert Table 1 about here]

The discount factor is set at $\beta = 0.99$ which implies a steady state interest rate of 4% p.a. in a quarterly model. The import share, λ, equals 0.4 which corresponds roughly to the Canadian import/ GDP ratio. The labour supply parameter, η, equals 3, which implies a Frisch labour supply elasticity of $\frac{1}{3}$. The elasticity of substitution across traded goods, θ, and the intertemporal elasticity of substitution, ρ, are both assumed to have unitary values.

Gali and Monacelli (2005) estimate AR(1) processes for log of Canadian labour productivity (their proxy for domestic productivity) and log of US GDP (proxy for world output) using HP-filtered data for the period 1963:1 to 2002:4. We use their estimates and set the domestic AR(1) coefficient, γ, equal to 0.66 while the foreign AR(1) coefficient, γ^*, equals 0.86. Domestic productivity, ε_t, is assumed to have a standard deviation of 0.71 while the foreign productivity shock, ε_t^*, has a standard deviation of 0.78. As in Gali and Monacelli (2003), the two exogenous shocks are assumed to be positive correlated i.e. $corr\left(\varepsilon_t, \varepsilon_t^*\right) = 0.3$. Finally, since Gali and Monacelli (2005) assumed no consumption habits, we set $h = \phi = 0$.

The model's ability to generate significant fluctuations in foreign exchange rate premia is tightly connected to the amount of real exchange rate volatility implied by the model. As we will demonstrate below, the Gali and Monacelli calibration yields a real exchange rate that is considerably less volatile than what we observe in the data. Therefore we also consider an alternative model calibration which does better in terms of matching real exchange rate volatility. Chri and McGrattan (2002) have pointed out that open economy models with complete markets imply a tight link between real exchange rate volatility and consumption volatility. And they claim that you need to have fairly high values for the intertemporal elasticity of substitution, ρ, to get any volatility in the real exchange rate. We follow them and Benigno (2003) and set ρ equal to 5.

Since we are interested in the role that shock persistence plays in determining the cyclical properties of the FX premium, we carry out simulations for different values of the habit parameter, h, and the habit persistence parameter ϕ. Our alternative calibration assumes that h equals 0.85. This is slightly higher than Smets and Wouters (2007) who estimate h to be 0.6 using euro area data. However Juillard et al (2004) and Banerjee and Batini (2003) find $h = 0.8$ using U.S and UK data respectively. Gali and Monacelli (2005) simulates his model using habit persistence parameters, ϕ, ranging from 0.05 to 0.95. But he estimates ϕ to be around 0.42. We simulate our model for various values of ϕ ranging from no habit persistence, $\phi = 0$, to very persistent habits, $\phi = 0.99$.

4.4.2 Simulation Results

We now turn to our simulation results. Using numerical simulations of our model, we intend to illustrate using numerical simulations how the combination of a) very persistent productivity shocks and b) a very slow-moving external consumption habits will generate a 1) sufficiently volatile FX premium that 2) covaries negatively with the expected depreciation rate. In other words, our simulations will illustrate how it is necessary to assume a) and b) in order to resolve the forward premium anomaly.

The Gali and Monacelli Case

How do we proceed? We first discuss a set of simulation results from our model that is calibrated exactly as in the flexible price version of Gali and Monacelli (2003).

[Insert Table 2 about here]

Table 2 reports the volatility of various model variables (all in logs) including domestic output growth, Δy, domestic consumption growth, Δc, real exchange rate growth, Δq and the domestic risk-free rate r. Since one of the requirements to resolve the forward premium anomaly is to have an excess return on domestic bonds (i.e. the FX premium) that varies more than the expected depreciation rate, we report the model implied volatility of the excess return on domestic bonds, σ_{fxp}, as well as the volatility of the expected domestic currency depreciation rate $\sigma_{\Delta q^e}$.

Recall that the second leg of the forward premium anomaly is to have a domestic excess return that covaries negatively with the expected currency depreciation rate. We therefore also report the covariance between these two variables i.e. $cov\left(fxp_t, \Delta q^e_{t+1}\right)$. Finally, we report the model implied Fama (1984) UIP coefficients.

Column 2 in Table 2 contains the simulation results for the Gali and Monacelli calibration. We see that this specific model parameterisation implies consumption growth that is less volatile than output growth. The table also shows that under our benchmark calibration the real exchange rate volatility is far from what we observe in the data. In the data, the standard deviation of real exchange rate growth is about 10%. Our model only generates a standard deviation of 0.56%.

Since this model version does not feature any consumption habits, the degree of risk aversion is constant. Therefore, the FX risk premium is constant i.e. $\sigma_{fxp} = 0$. Hence, with a constant FX risk premia, the risk premia is uncorrelated with the expected depreciation rate i.e. $cov\left(fxp_t, \Delta q^e_{t+1}\right) = 0$. So in the case of no habits, our model generates no FX risk premium and the linear UIP (equation (26)) holds exactly. Our model therefore implies a risk neutral uncovered interest rate parity and therefore the UIP coefficient, α_1, equals 1.

The Gali and Monacelli Case with Consumption Habits

We now add consumption habits to our model by assuming that $h = 0.85$. Column 3 of Table 2 presents the simulation results for the case of consumption habits. Adding consumption habits to the model should imply more volatile stochastic discount factors. Indeed, our model now generates a standard deviation for the risk free rate close to 25%. In an open economy setting with international market completeness, highly volatile riskfree rates would – ceterius paribus – translate into higher real exchange rate volatility. Our simulations suggest that consumption habits have made real exchange rates more volatile. In fact, real exchange rate growth (1.6%) are now more volatile than domestic output growth (1%) but still not as volatile as we observe in the data.

What does consumption habits imply for the cyclical properties of the FX risk premium? As previously explained, adding consumption habits causes risk aversion to vary over time which again implies a time-varying FX risk premium. But as our simulation results illustrate, even with high level of h, the Fama UIP coefficient, α_1, is very close to one. Why is that?

Even with consumption habits , $h = 0.85$, neither conditions (1) or (2) stated in section 4.2 are satisfied. This specification of the model generates very little FX premium volatility. In fact, the standard deviation of the FX risk premium is only 0.5 basis points! Since the volatility of the expected depreciation rate is 0.52%, the FX risk premium is not sufficiently volatile to satisfy the first Fama condition. In addition, the correlation between the FX premium and the expected depreciation is positive. Hence the second Fama condition is not satisfied either.

Adding more shock persistence

Our previous simulation results suggest that consumption habits alone (i.e setting $h = 0.85$) is not sufficient to satisfy either of Fama's two conditions. One problem is that the FX premium, fxp_t, covaries positively with the expected depreciation rate, $E_t[\Delta q_{t+1}]$. We explained in section 4.3 how a countercyclical λ_t was instrumental in terms of producing a negative correlation between fxp_t and $E_t[\Delta q_{t+1}]$. We have also argued that business cycle properties of λ_t are a function of the degree of habit persistence as well as the persistence of the exogenous stochastic process (i.e. the parameter γ in the present paper). Could it be the case that our model calibration is not assuming enough persistence in the shocks to produce a countercyclical λ_t and hence a negative correlation between fxp_t and $E_t[\Delta q_{t+1}]$?

We therefore consider a model calibration that assumes a much higher shock persistence than in the previous two cases. Until now, we have followed Gali and Monacelli (2003) and assumed a fairly modest degree of shock persistence i.e. $\gamma = 0.66$ and $\gamma^* = 0.86$. However, these values are lower than the findings in Smets and Wouters (2007) which would imply $\gamma = \gamma^* = 0.9977$. The Smets and Wouters estimates suggest that real supply shocks are much more persistent than what the Gali and Monacelli (2003) calibration implies.

Column 3 in Table 2 contains simulation results from our model when we increase the shock persistence to $\gamma = \gamma^* = 0.9977$. Increasing the shock persistence does not appear to have any large effect on the volatility of output, consumption and the real exchange rate. In fact, the second moments of those variables appear very similar to the previous case where $\gamma = 0.66$ and $\gamma^* = 0.86$.

However, we see that increasing the shock persistence does generate a negative covariance between the FX premium and the expected depreciation rate. With $\gamma = \gamma^* = 0.9977$ the risk aversion, λ_t, is countercyclical and so is the compensation that agents require to hold risk assets. But having a negative correlation is not sufficient to solve the forward premium anomaly. We also need an FX risk premium that is more volatile than the expected depreciation rate. A higher shock persistence does imply a higher FX premium volatility. The standard deviation of the FX premium is now 26 basis point - considerably higher than the 0.5 basis point we found earlier. But the volatility of the expected depreciation rate generated by our model is even higher (0.46%). So our FX premium is not sufficiently volatile which means that the first Fama condition is not satisfied. And as a result this model calibration actually imply a UIP coefficient that exceeds 1.

Our benchmark calibration - the Gali & Monacelli setup featuring high shock persistence and slow moving consumption habits

Where do we go next? We have shown how persistent productivity shocks as well consumption habits help ensure a negative correlation between the FX premium and the expected depreciation rate. But our model calibration has so far not generated any significant volatility in the premia.

Suppose now that we increase the habit level in our model significantly. A higher degree of consumption habits should increase the volatility of the FX risk premium. So far we have not assumed any long memory in consumption habits i.e. $\phi = 0$. In other words, the consumption habit level adjusts instantly following the shock. Assume now we recalibrate our model such that ϕ equals 0.99. We will call this calibration our benchmark model.

Column 4 in Table 2 presents simulation results from our benchmark model which features very persistent productivity shocks ($\gamma = \gamma^* = 0.9977$) as well as very slow moving consumption habits ($\phi = 0.99$).

What does adding slow moving consumption habits do to the dynamic properties of our model variables? Our model implies consumption growth that is slightly less volatile ($\sigma_{\Delta c} = 0.71\%$). Similarly both output growth ($\sigma_{\Delta y} = 0.95\%$) and real exchange rate growth ($\sigma_{\Delta q} = 1.44\%$) are fluctuating less when we add more slow moving consumption habits.

The combination of very persistent shocks and slow moving consumption habits also appear to imply a very persistent real exchange rate and hence a less volatile expected depreciation rate ($\sigma_{\Delta q^e} = 0.03\%$).

The intuition is straightforward: A positive domestic productivity shock increases domestic consumption and depreciates the real exchange rate. The combination of slow moving consumption habits and a very persistent shock process creates a very persistent consumption series. Or in other words, consumption is only slowly expected to revert back to its steady state. In our open economy model with complete markets, the dynamics of the real exchange rate is closely tied to consumption. Hence, the high persistence in consumption translates into high persistence in real exchange rate.

Adding slow moving consumption habits also produces more volatile FX premia ($\sigma_{fxp} = 0.122\%$). In fact, our FX premia is now more volatile than the expected depreciation rate. And this together with a countercyclical λ_t guarantees that both conditions (1) and (2) of section 4.2 are satisfied. So our benchmark model featuring very persistent productivity shocks and very slow moving consumption habits can actually generate a negative UIP coefficient ($\alpha = -0.082$), albeit it is very close to 0.

4.4.3 Sensitivity analysis

Increasing real exchange rate volatility

We now turn to our sensitivity analysis. Our model has so far failed to replicate a real exchange rate that is as volatile as in the data. Our benchmark model can generate a standard deviation for real exchange rate growth, $\sigma_{\Delta q}$, around 1.5%.

Can we recalibrate our canonical model in order for it to generate more relative price volatility? One option is to assume a lower import share λ. The intuition is obvious since a lower import share implies a smaller expenditure shifting effect in the model. If less of your output is traded, then the real exchange rate has to move by more in order for the goods market to clear.

Column 2 in Table 3 presents simulation results for an alternative calibration, where we have lowered the import share, λ, to 5%. All other parameters remain unchanged relative to our benchmark model. Obviously setting λ equal to 5% implies a lower import share than the average import shares we typically observe in the small to medium-sized G7 economies such as Canada, United Kingdom, Italy and France.

[Insert Table 3 about here]

Relative to our benchmark case, this particular calibration with $\lambda = 0.05$ generates a significant more volatile real exchange rate. In fact, the standard deviation of real exchange rate growth, $\sigma_{\Delta q}$, is around 10%. Hence, our model generates real exchange rates that are just as volatile as in the data[10]. The higher real exchange rate volatility is associated with a more volatile expected depreciation rate ($\sigma_{\Delta q^e} = 0.36\%$) which covaries negatively with the FX premia (i.e $cov\left(fxp_t, \Delta q^e_{t+1}\right) = -0.003\%$). Relative to our benchmark case, the FX premia fluctuates more in our model with $\lambda = 5\%$. In fact, we find that the standard deviation of the FX premia, σ_{fxp}, is 1% and therefore is more volatile than the expected depreciation rate. Thus, our simulation results satisfy both of Fama's two conditions and consequently -as in the benchmark case - our model generates a negative UIP coefficient ($\alpha = -0.18$).

The results in this section suggest that the canonical model's ability to resolve Fama's puzzle is tightly connected to the amount of real exchange rate volatility implied by the model. A highly volatile exchange rate is needed in order to create enough comovements between the stochastic discount factor and the real exchange rate. A volatile exchange rate is also necessary to generate changes in expected depreciation and a sufficient correlation between fxp_t and $E_t[\Delta q_{t+1}]$.

Exogenous labour supply

We now turn to the role played by endogenous labour supply. A number of papers have illustrated how closed economy models with endogenous production imply a lower premia relative to the premia generated by an endowment economy. The assumption of endogenous labour supply allows households to either vary hours worked or consumption in response to shocks. When we assume an endowment economy, however, we prevent households from adjusting their hours worked and hence force households to change their consumption patterns in response to any adverse shock. The combination of an endowment economy and consumption habits means that households dislike variations in their consumption streams even more. And they would require an additional risk premia to hold any asset.

Does the same intuition hold in our open economy model? In other words, would shutting down endogenous labour supply make it easier for us to meet Fama's two conditions? Column 3 in Table 3 contains the simulation results for the case of exogenous labour supply[11]. To make the comparison easier with our previous case, we continue to assume the low import share i.e. $\lambda = 5\%$.

The setup with exogenous labour supply implies a lower real exchange rate volatility ($\sigma_{\Delta q} = 6.6\%$) and less volatile domestic output ($\sigma_{\Delta y} = 0.68\%$) relative to the case with endogenous labour supply. Why is that?

Think of how a positive foreign productivity shock affects the real exchange rate and domestic output in our model. This particular shock would tend to appreciate the domestic real exchange rate and make domestic goods more expensive relative to foreign goods. Given the lower demand for domestic output, domestic households would choose to cut back on hours worked and hence domestic output would fall. The fall in domestic output would trigger an even greater real exchange rate appreciation in order to clear relative world demand and supply.

[10]We don't conclude from these simulation results, that our model has resolved the international pricing puzzle i.e. the inability of open economy model's to generate sufficiently volatile real exchange rates. Our result only comes about because with $\lambda = 5\%$ our open economy model has an implausibly low import share. The international pricing puzzle therefore remains.

[11]By setting the Frisch labour supply elasticity very low ($\eta = 300$), we simulate the case of exogenous labour supply.

But in a world with exogenous domestic hours and production, domestic output cannot fall in response to a positive foreign shock. Therefore, the real exchange rate has to appreciate by less in order to clear world demand and domestic output would be less volatile.

The fact that our endowment model generates lower real exchange rate volatility has implications for whether the model can resolve the Fama puzzle. In this case, the volatility of the FX premia is actually lower ($\sigma_{fxp} = 0.48\%$) and our implied UIP coefficient is less negative ($\alpha = -0.01$) than before .

The closed economy literature has often claimed that assuming an endowment economy would make it easier to match the closed economy finance puzzles. But our results suggest that our model's ability to satisfy the two Fama conditions does not hinge on whether labour is exogenous or endogenous.

Introducing a non-linear dynamic process for habit formation

The final part of our sensitivity analysis involves incorporating a non-linear process for habits following Campbell and Cochrane (1999)'s model (C&C hereafter). As previously mentioned, Verdelhan (2007) showed that this specification can help reconcile the forward premium puzzle in an endowment economy with trading costs. In order to introduce this specification, we substitute equation (2) by the following process governing the surplus ratio:

$$s_{t+1} = (1-\delta)\bar{s} + \delta s_t + \lambda(\Delta c_{t+1} - E_t \Delta c_{t+1}) \tag{33}$$

where

$$\lambda(s_t) = \frac{1}{\bar{S}}(1 - s_t - \bar{s})^{1/2} - 1, \text{ if } s < s^{\max} \text{ and } 0 \text{ otherwise,} \tag{34}$$

We denote $\bar{S}$ the steady state level of the surplus ratio and $s^{\max} = \bar{s} + (1 - \bar{S}^2)/2$ is the maximum level of the surplus ratio.[12] We should note that in our framework we do not have the stochastic volatility assumed in the original C&C paper. Moreover, we do not allow for growth.[13]

We find that the introduction of C&C habits does not change the results in any significant way.[14] As shown in column 4 of Table 3, we obtain a value of -0.17 for α which is very similar to the one obtain in our Benchmark case. But even though the UIP coefficient is practically unchanged, the introduction of the non-linear surplus ratio can generate a highly volatile FX premium. And it also helps obtaining a highly negative covariance between the premium and expected depreciation. But this increase in volatility comes at a cost: the volatility of the risk free rate is implausibly high under this specification.

5 Conclusion

In this paper, we examine the properties of the foreign exchange rate risk premium in a canonical general equilibrium small open economy model. We show that in order to resolve the deviations

[12]Our numerical results show that the surplus ratio never exceed its maximum value $s^{\max}$. This is probably a result of our local approximation solution around shocks of low magnitude.

[13]The last assumption imply a small modification in equation (33), which include a term on $E_t\Delta c_{t+1}$ instead of the growth rate of the steady state growth rate of consumption.

[14]The C&C specification is highly non-linear. For this reason we have done some sensitivity analysis of the results by increasing the order of approximation used in our solution method. We have approximated the model up to 5th order and the results were practically unchanged.

from linear UIP observed in the data, investors in our model should be subject to extremely persistent shocks and have a consumption habit process that adjusts slowly to these shocks. We also found that having a model that can generate sufficient exchange rate volatility helps us reconciling the long-standing forward premium puzzle.

Our results are reassuring given that state of the art macro models appear to be far away from matching variability of bond term premia (see for example, Rudebusch and Swanson (2007). One could interpret our findings as evidence of a lower bar for resolving the UIP puzzle. We demonstrate that no additional features, such as deep habits, stochastic volatility, trading costs, or a more exotic Campbell and Cochrane (1999) specification for habits, are required in order to engineer a negative UIP coefficient.

How plausible are our model assumptions? First, the empirical evidence on the persistence of the habit adjustment process is not extensive and, therefore, it constitutes an interesting venue for future research. Second, the assumption of highly persistent shocks appears to be in line with evidences from both the closed and open economy literature. The closed economy framework of Smets and Wouters (2007) estimate an extremely persistent productivity shock in line with what we assume. In an open economy setting, Corsetti et al. (2007) show that highly persistent shocks are important in order to explain the Backus-Smith correlation puzzle. Finally, we do observe high exchange rate volatility in the data. But we have trouble finding models that can generate sufficient volatility as summarised by the international pricing puzzle.

Our model, nevertheless, maintains features that are inconsistent with other stylised facts. As most of the NOEM literature or in the general class of open economy DSGE models, our framework assumes markets are complete internationally. Our model therefore implies a tight link between cross-country consumption differentials and real exchange rates which is strongly rejected by the data. Hence, the Backus Smith anomaly remains unexplained by our model. The Backus-Smith anomaly could be tackled by introducing incomplete marketsas shown by Benigno and Thoenissen (2006). It would therefore be interesting to examine our results in a similar setting.

Another interesting extension could be to introduce capital accumulation to our model. In a closed economy setting, it is well documented that allowing agents to adjust their capital stock tends to increase consumption smoothing and therefore reduce risk premium. A similar result also holds in a closed economy when endogenous labour supply is added to the model. However, as shown in the present work, the latter result does not necessary hold in an open economy. Thus, analysing the implications for the FX risk premium of adding investment in an open economy may also be an interesting venue to follow.

Finally, our results hinges on a strong role for precautionary savings. Without these, our model would imply the traditional linear uncovered interest parity condition. It has been shown that models with credit constraints embodies significant precautionary savings effects (see Aiyagari (1994)). Exploring how credit constraints in an open economy affect the FX risk premium may therefore constitute an interesting venue for future research.

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Appendix: Deriving the FX premium

From the Euler equation (22), we can write that

$$(\beta R_{t+1})^{-1} = E_t\left[M_{t+1}\right] \tag{35}$$

Thus, assuming a log-normal stochastic discount factor, M_{t+1}:

$$r_{t+1} = -\log\beta - E_t\left[m_{t+1}\right] - \frac{1}{2}var_t(m_{t+1}) \tag{36}$$

In the case of foreign bonds, we have:

$$\left(\beta R^*_{t+1}\right)^{-1} = E_t\left[M_{t+1}\frac{Q_{t+1}}{Q_t}\right] \tag{37}$$

And assuming that the stochastic discount factor, M_{t+1}, and the real exchange rate, Q_{t+1}, are jointly log-normal we can write the above pricing equation as follows:

$$r^*_{t+1} = -\log(\beta) - E_t\left[m_{t+1}\right] - E_t\left[\Delta q_{t+1}\right] - \frac{1}{2}var_t(m_{t+1}) - \frac{1}{2}var_t(q_{t+1}) - cov_t(m_{t+1}, q_{t+1}) \tag{38}$$

Therefore, combining equations (36) and (38), we have

$$r_{t+1} - r^*_{t+1} - E_t\left[\Delta q_{t+1}\right] = \frac{1}{2}var_t(q_{t+1}) + cov_t(m_{t+1}, q_{t+1}) \tag{39}$$

or, defining the foreign exchange rate risk premium as the excess return on domestic bonds (that is $fxp_t = r_{t+1} - r^*_{t+1} - E_t\left[\Delta q_{t+1}\right]$)

$$fxp_t = \frac{1}{2}var_t(q_{t+1}) + cov_t(m_{t+1}, q_{t+1}) \tag{40}$$

Moreover, using log-linear relationship given by the risk sharing condition (21), that is,

$$\Delta q_{t+1} = m^*_{t+1} - m_{t+1} \tag{41}$$

we can re-write equation (40) as

$$fxp_t = \frac{1}{2}cov_t(m^*_{t+1} - m_{t+1}, q_{t+1}) + cov_t(m_{t+1}, q_{t+1}) \tag{42}$$

or

$$fxp_t = \frac{1}{2}[cov_t(m^*_{t+1},, q_{t+1}) + cov_t(m_{t+1}, q_{t+1})] \tag{43}$$

Alternatively, we could combine equations (40) and (41) as follows:

$$fxp_t = \frac{1}{2}var_t(m^*_{t+1} - m_{t+1}) + cov_t(m_{t+1}, m^*_{t+1} - m_{t+1}) \tag{44}$$

or

$$fxp_t = \frac{1}{2}var_t(m^*_{t+1}) + \frac{1}{2}var_t(m_{t+1}) - cov_t(m_{t+1}, m^*_{t+1}) - var_t(m_{t+1}) + cov_t(m_{t+1}, m^*_{t+1}) \tag{45}$$

and, thus,

$$fxp_t = \frac{1}{2}var_t(m^*_{t+1}) - \frac{1}{2}var_t(m_{t+1}) \tag{46}$$

5.1 Time variation in the FX premium

Given the definition of the stochastic discount factor,

$$M_{t+1} = \left(\frac{C_{t+1} - hX_t}{C_t - hX_{t-1}} \right)^{-\rho} \tag{47}$$

and the process for consumption habits,

$$X_t = \phi X_{t-1} + (1 - \phi) C_t \tag{48}$$

we can re-write equation (43) as follows:

$$fxp_t = -\frac{1}{2}\rho(cov_t(q_{t+1}, c^e_{t+1}) + cov_t(q_{t+1}, c^e_{t+1})) \tag{49}$$

where

$$C^e_{t+1} = C_{t+1} - hX_t \tag{50}$$

Defining the surplus ration as

$$S_{t+1} = \frac{C^e_{t+1}}{C_{t+1}} = 1 - \frac{hX_t}{C_{t+1}} \tag{51}$$

we have

$$c^e_{t+1} = s_{t+1} + c_{t+1} \tag{52}$$

So the risk premium can be written as

$$fxp_t = -\frac{1}{2}\rho \left[cov_t \left(c_{t+1}, q_{t+1} \right) + cov_t \left(s_{t+1}, q_{t+1} \right) + cov_t \left(c^*_{t+1}, q_{t+1} \right) + cov_t \left(s^*_{t+1}, q_{t+1} \right) \right] \tag{53}$$

If we employ Stein's lemma (this passage follows Li (2007))

$$fxp_t = -\frac{1}{2} \left[\lambda_t cov_t \left(c_{t+1}, q_{t+1} \right) + \lambda^*_t cov_t \left(c^*_{t+1}, q_{t+1} \right) \right] \tag{54}$$

where

$$\lambda_t = \rho E_t(\partial s_{t+1} / \partial c_{t+1}) + \rho \text{ and } \lambda^*_t = \rho E_t(\partial s^*_{t+1} / \partial c^*_{t+1}) + \rho \tag{55}$$

Finally, from the definition of the surplus rations we have that

$$\lambda_t = E_t(\rho / S_{t+1}) \text{ and } \lambda^*_t = E_t(\rho / S^*_{t+1}) \tag{56}$$

5.2 The cyclicality of λ_t in a closed economy

De Paoli and Zabczyk. (2007) demonstrate how the cyclicality of λ_t depends on γ and ϕ in a closed economy. In this section we replicate their result. Given equation (56), we can write:

$$\lambda_t = \rho E_t (1 - \frac{h(\phi X_{t-1} + (1-\phi)C_t)}{C_{t+1}})^{-1},$$

or

$$\lambda_t = \rho E_t (1 - (C_{t+1})^{-1} h(1-\phi) \sum_{0}^{t} \phi^s C_{t-s})^{-1} \tag{57}$$

And in a closed economy,

$$C_t = Y_t$$

which assumes an endowment process of the following form

$$C_t = C_{t-1}^{\gamma} e^{\varepsilon_t}$$

Hence, if $\gamma = 0$,

$$\partial C_s / \partial \varepsilon_t = C_t \text{ for } s = t \text{ and } \partial C_s / \partial \varepsilon_t = 0 \text{ for } s \neq t$$

To calculate $\partial \lambda_t / \partial \varepsilon_t$ we note that $\partial \lambda_t / \partial \varepsilon_t = (\partial \lambda_t / \partial C_t)(\partial C_t / \partial \varepsilon_t)$ and $\partial E_t(f(x_t))/\partial x_t = E_t(f'(x_t))$. Equation 57 therefore implies

$$\partial \lambda_t / \partial C_t = E_t \left[(C_{t+1})^{-1} h(1-\phi)(1 - (C_{t+1})^{-1} h(1-\phi) \sum_{0}^{t} \phi^s C_{t-s})^{-2} \right]$$

and given that $S_{t+1} = 1 - h(1-\phi)\,(C_{t+1})^{-1} \sum \phi^s C_{t-s}$, we can write

$$\partial \lambda_t / \partial C_t = h(1-\phi) E_t \left[(1/C_{t+1})(1/S_{t+1})^2 \right]$$

and, hence

$$\partial \lambda_t / \partial \varepsilon_t = h(1-\phi) E_t \left[(S_{t+1})^{-2} \left(\frac{C_{t+1}}{C_t} \right)^{-1} \right] > 0$$

The authors also show that if $\gamma = 1$, that is ,

$$\frac{C_t}{C_{t-1}} = e^{\varepsilon_t} \tag{58}$$

$$\lambda_t = E_t \left(1 - h(1-\phi)[\sum_{s=0}^{s=t} \phi^s e^{-\sum_{s=0}^{s=t} \varepsilon_{t-s}}] \right)^{-1} - 1,$$

Hence,

$$\partial \lambda_t / \partial \varepsilon_t = -\phi(1-\phi) h E_t \left[(S_{t+1})^{-2} \sum_{s=0}^{s=t} \phi^s \left(\frac{C_{t+1}}{C_{t-s}} \right)^{-1} \right] < 0$$

Tables

Table 1: Parameters used in the baseline calibration

Parameter	Description	G & M (2003)	Benchmark
β	Discount Factor	0.99	0.99
ρ	Household inverse IES	1	5
h	Habit Parameter	0	0.85
ϕ	Habit Persistence Parameter	0	0.99
λ	Share of imports in domestic economy	0.40	0.40
θ	Import/domestic tradable ES	1	1
η	Inverse of Frisch Elasticity of Labor Supply	3	3
γ	AR(1) Coefficient for Domestic Productivity	0.66	0.9977
γ^*	AR(1) Coefficient for Foreign Output	0.86	0.9977
σ_ε	Standard Deviation of Domestic Productivity	0.71%	0.71%
σ_{ε^*}	Standard Deviation of Foreign Output	0.78%	0.78%
$corr(\varepsilon_t, \varepsilon_t^*)$	Correlation between Domestic and Foreign Shock	0.3	0.3

Notes for Table 1: A full description of the calibration is described in section 4.4.1.

Table 2: Simulation results

Model Variables	$G\&M$	$+habits$	$+shock\ pers.$	$Benchmark$
$\sigma_{\Delta y}$	0.75%	1.04%	1.03%	0.95%
$\sigma_{\Delta c}$	0.61%	0.76%	0.75%	0.71%
$\sigma_{\Delta q}$	0.56%	1.55%	1.53%	1.44%
σ_r	0.21%	24.89%	6.43%	0.73%
Fama risk premia requirements				
$\sigma_{\Delta q^e}$	0.20%	0.52%	0.46%	0.03%
σ_{fxp}	0	0.004%	0.026%	0.122%
$cov\left(fxp_t, \Delta q_{t+1}^e\right)$	0	0.00002%	-0.00009%	-0.00002%
UIP coef α_1	1	0.99%	1.04	-0.082

Notes for Table 2: These results were produced by simulating the model in Dynare ++ freeware under the calibration of table 1. We use a third order approximation of the model presented in section 2 in order to capture the time-variation in risk premium. Note that we use a log-linear version of the demand and supply conditions. This is done in order to facilitate the simulation of our model with highly persistent shocks. For consistency we use this log-linearization throughout our numerical exercises.

Table 3: Sensitivity Analysis

Model Variables	*Lower imp share* λ	*Exogenous labor*	*C&C pref*
$\sigma_{\Delta y}$	0.72%	0.68%	0.74%
$\sigma_{\Delta c}$	0.51%	0.62%	0.49%
$\sigma_{\Delta q}$	10.08%	6.60%	10.09%
σ_{r}	0.58%	0.56%	4.94%
Fama risk premia requirements			
$\sigma_{\Delta q^e}$	0.36%	0.16%	0.47%
σ_{fxp}	1.08%	0.48%	2.96%
$cov\left(fxp_t, \Delta q^e_{t+1}\right)$	-0.003%	-0.0003%	-0.013%
UIP coef α	-0.18	-0.01	-0.17

Notes for Table 3: These results were produced by simulating the model in Dynare ++ freeware under the calibration of table 1. We use a third order approximation of the model presented in section 2 in order to capture the time-variation in risk premium. Note that we use a log-linear version of the demand and supply conditions. This is done in order to facilitate the simulation of our model with highly persistent shocks. For consistency we use this log-linearization throughout our numerical exercises.