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Risk Measurement Illiquidity Distortions

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by

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Abstract

We examine the effects of smoothed hedge fund returns on standard deviation, skewness, and kurtosis of return and on correlation of returns and cross-sectional volatility and covariance of returns using an MA(2)-GARCH(1,1)-skewed-t representation of returns instead of the traditional MA(2) model employed in the literature. We present evidence that our proposed representation is more consistent with the behavior of hedge fund returns and that the traditional method tends to overstate the degree of smoothing observed in hedge fund returns. We present methods for correcting for the distortive effects of smoothing using our representation.

Keywords: hedge fund, return smoothing, illiquidity, GARCH

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1. Introduction

Hedge fund returns are characterized by certain stylized facts. Among them, hedge fund return distributions are often skewed and leptokurtic, returns may exhibit high positive correlation between funds during periods of financial distress, and monthly returns of individual hedge funds are often positively serially correlated, a phenomenon known to hedge fund analysts as smoothed returns.

Several writers have examined return smoothing. These include Asness, Krail, and Liew (2002); Getmansky, Lo, and Makarov (2004); Aragon (2007); Kosowski, Naik, and Teo (2007); Jagannathan, Malakov, and Novikov (2010); and Titman and Tiu (2010). Getmansky et al argue in their well-known work that hedge fund monthly returns are smoothed because of the effects of various forms of illiquidity. For example, for its month-end books a hedge fund may average broker estimates of an illiquid asset's value. Brokers may estimate the month-end value as a markup at an assumed rate from the prior month's value. This and the practice of averaging the estimates produces smoothed returns.

It is established in the literature that with smoothed returns, standard deviation using reported returns is understated so that the Sharpe ratio and information ratio are overstated. It is also established that measures of skewness and kurtosis are also distorted by smoothing as is correlation of returns between funds. We analyze corrected measures of these statistics. We also examine the possibility that smoothing distorts cross-sectional measures of returns, indicators of hedge fund risk, and we present a method for analyzing this issue.

Section 2 examines computational methods employed in our analysis. Section 3 examines the impact of smoothed hedge fund returns on measured standard deviation, skewness, and kurtosis of individual hedge fund return streams. Section 4 examines the impact of smoothing on correlation of returns between hedge funds, and cross-sectional measures of returns between hedge funds. Section 5 presents an empirical study of key statistics under return smoothing. Section 6 presents conclusions.

2. Computational methods

In the literature an MA(2) model of monthly hedge fund returns is often employed as follows:

(2.1) $R_t^o = \theta_0 R_t + \theta_1 R_{t-1} + \theta_2 R_{t-2}$

$$1 = \theta_0 + \theta_1 + \theta_2$$
$$0 \leq \theta_i \leq 1$$

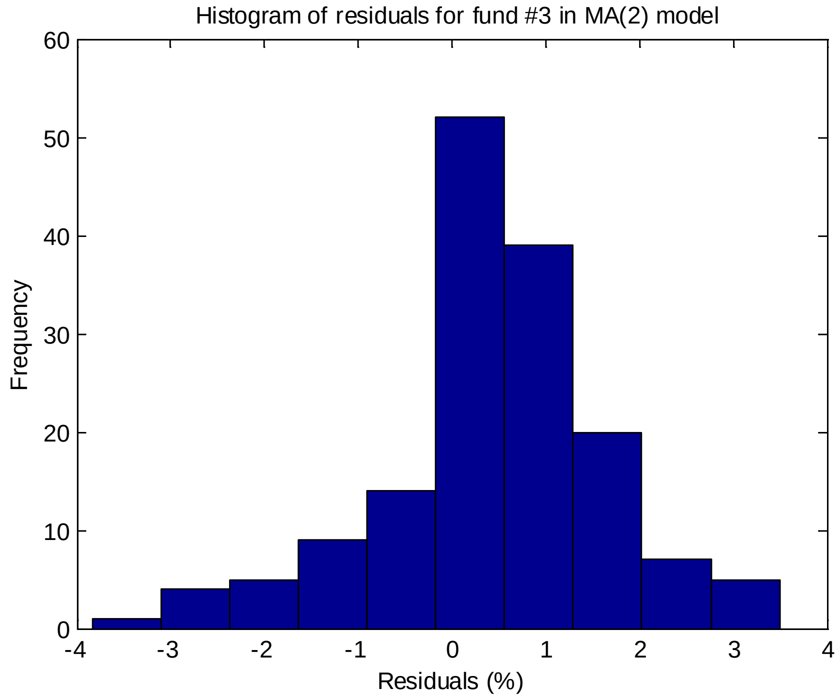
where R_t^o is the observed, or reported, monthly return in period t, R_t is the underlying actual return in t, and the θs are parameters.

Some authors (Getmansky et al 2004) propose to estimate the MA(2) smoothing model using maximum likelihood estimation (MLE) while relaxing the constraint $0 \leq \theta_i \leq 1$. They define the de-meaned observed return $X_t = R_t^o - \mu$. Then:

(2.2) $$X_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$
$$1 = \theta_0 + \theta_1 + \theta_2$$
$$\epsilon_k \sim \text{iid N}(0, \sigma^2)$$

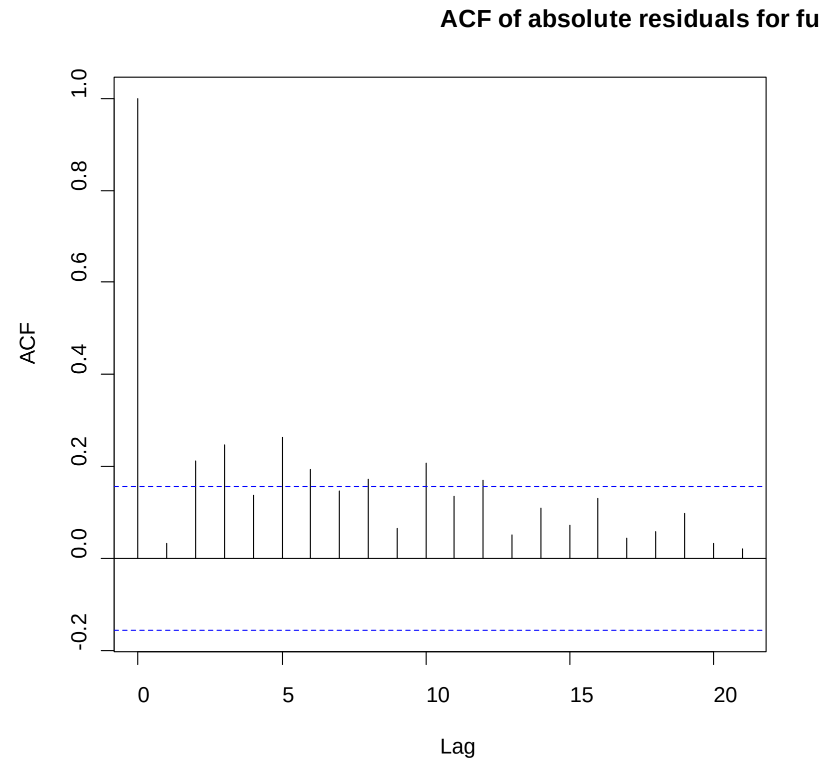
It then becomes a traditional MA time series estimation process and can be estimated using standard statistical software. A key assumption in such an approach is that the underlying true hedge fund return has an iid normal distribution. In reality, the true return rarely has such a distribution. It is not uncommon for the return to show volatility clustering and skewness. To demonstrate this, we employ a dataset of 256 hedge funds from the Hedge Fund Research Inc. database, each with a continuous monthly reporting history for the period from January 1998 to December 2010. We restricted our selection of hedge funds to those with assets under management of at least $100 million and continuous reporting of returns over our sample period. In Figure 1 below, we show the residuals for one of the funds obtained through the MA(2) process with normality assumptions. The distribution of the residuals, which should be iid normal, exhibits fat tails and are highly skewed.

Figure 1. Histogram of the residuals for fund #3 in the MA(2) model



To demonstrate the volatility clustering or heteroscedasticity, we show the autocorrelation function plot of the absolute residuals in the MA(2) model in Figure 2.

Figure 2. ACF of the absolute residuals for Fund #3 in the MA(2) model. The dashed horizontal lines form the 95 percent confidence band for no autocorrelation.



To extract the true return accommodating its skewness, kurtosis and heteroscedasticity properties, we propose using an MA(2)-GARCH(1,1)-skewed-t model:

(2.3)
$$X_t = \theta_0 a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2}$$
$$1 = \theta_0 + \theta_1 + \theta_2$$
$$a_t = \sigma_t \epsilon_t$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

where the error term ϵ_t has a skewed-t distribution defined in Lambert and Laurent (2001) as follows. Suppose x has a Student-t distribution with density $g(.)$ and degrees of freedom υ. Potential skewness is introduced through the parameter ξ such that:

(2.4)
$$E(x \mid \xi, \upsilon) = \frac{\Gamma\,[(\upsilon-1)/2]\,\sqrt{\upsilon-2}}{\sqrt{\pi}\;\Gamma\,(\upsilon/2)}\left(\xi - \frac{1}{\xi}\right) = m$$

$$V(x\,|\,\xi,\upsilon) = \xi^2 + \frac{1}{\xi^2} - 1 - m^2 = s^2$$

$$y = \frac{x-m}{s}$$

$$f(y\,|\,\xi,\upsilon) = \frac{2s}{\xi + 1/\xi}\left\{ g\left[\xi(sy+m)\,|\,\upsilon\right] I_{(-\infty,0)}\left[y + \frac{m}{s}\right] + g\left[\frac{sy+m}{\xi}\,|\,\upsilon\right] I_{[0,\infty)}\left[y + \frac{m}{s}\right]\right\}$$

where $f(y\,|\,\xi,\upsilon)$ is the density.

For our 256 hedge funds, 228 of them have returns different from a normal distribution at the 0.05 significance level based on Jarque and Bera (1987) statistic, which is asymptotically distributed as a chi-square random variable with two degrees of freedom. This is a strong indication that we should incorporate the skewed-t distribution in our estimation process. In our estimation we also relax the constraint of $0 \leq \theta_i \leq 1$ as in the literature. In the following table, we list the estimated θs using our modified model and the traditional MA(2) model together with the Jarque and Bera statistics for the duals ϵ_t. We list only the first 10 funds ranked in increasing order of the estimated smoothing parameter θ_0 from the traditional model to illustrate the difference between the two estimation processes.

Table 1. Comparison of traditional model and skewed-t model across hedge funds.

MA(2)-GARCH(1,1)-skewed-t			MA(2)			
θ_0	θ_1	θ_2	θ_0	θ_1	θ_2	J-B statistic
0.5943	0.2680	0.1377	0.4790	0.3302	0.1908	457.38
0.6004	0.2591	0.1406	0.4821	0.3344	0.1835	418.13
0.6841	0.2626	0.0533	0.4929	0.3945	0.1126	2675.84
0.6496	0.2762	0.0742	0.5162	0.3433	0.1406	525.44
0.6468	0.2388	0.1144	0.5327	0.2830	0.1842	564.30
0.7155	0.2481	0.0364	0.5402	0.2870	0.1729	702.96
0.6541	0.2516	0.0943	0.5463	0.3566	0.0971	1299.21
0.7264	0.2037	0.0700	0.5603	0.3419	0.0978	2967.80
0.6664	0.2316	0.1020	0.5640	0.2688	0.1671	346.54
0.6448	0.2668	0.0884	0.5641	0.2965	0.1394	75.64

From the table we see that there is significant amount of smoothing in hedge fund returns and that the normality assumption is violated as the Jarque and Bera statistic is 5.99 at the 0.05 significance level. When we compare the estimations of θ_0, the traditional process tends to overestimate the smoothing effects in hedge fund returns, and that could exaggerate the distortion of key hedge funds statistics when taking smoothing into consideration.

The charts below show the comparison of θ_0 estimations using the simple MA(2) model and our MA(2)-GARCH(1,1)-skewed-t model for our 256 funds. The kernel density was calculated using a normal kernel with plug-in optimal bandwidth choice.

Figure 3.1. Histogram comparison of θ_0 estimations

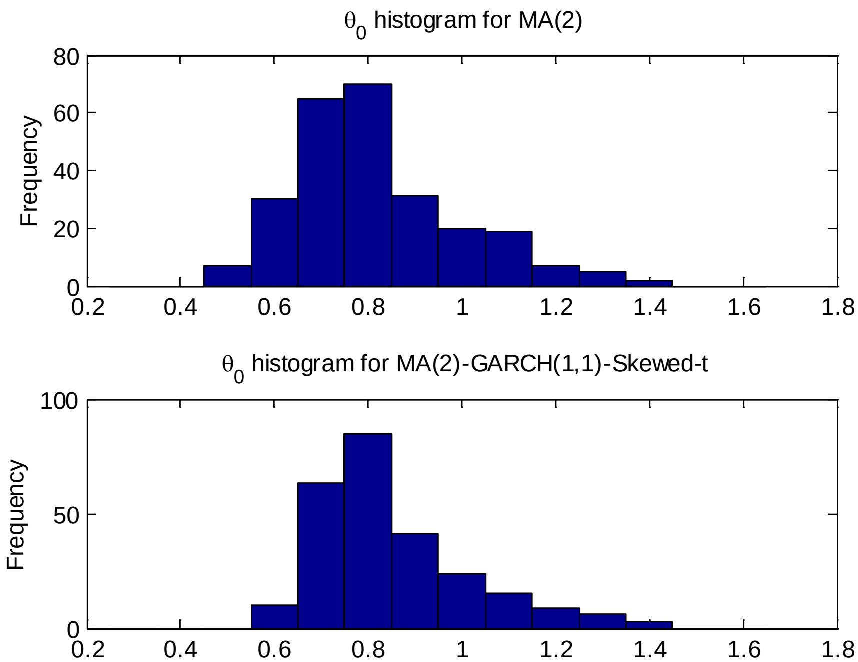


Figure 3.2. Kernel density comparison of θ_0 estimations

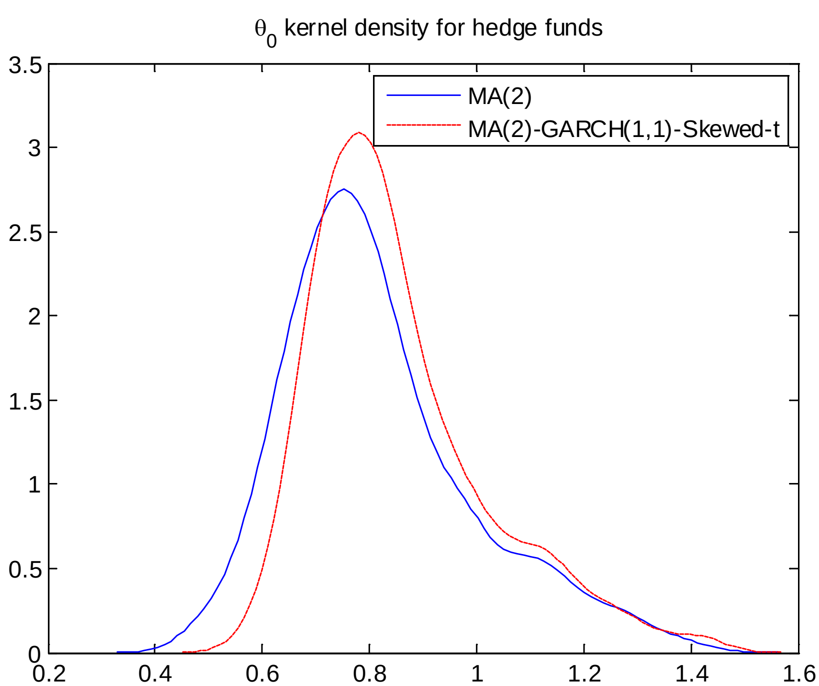
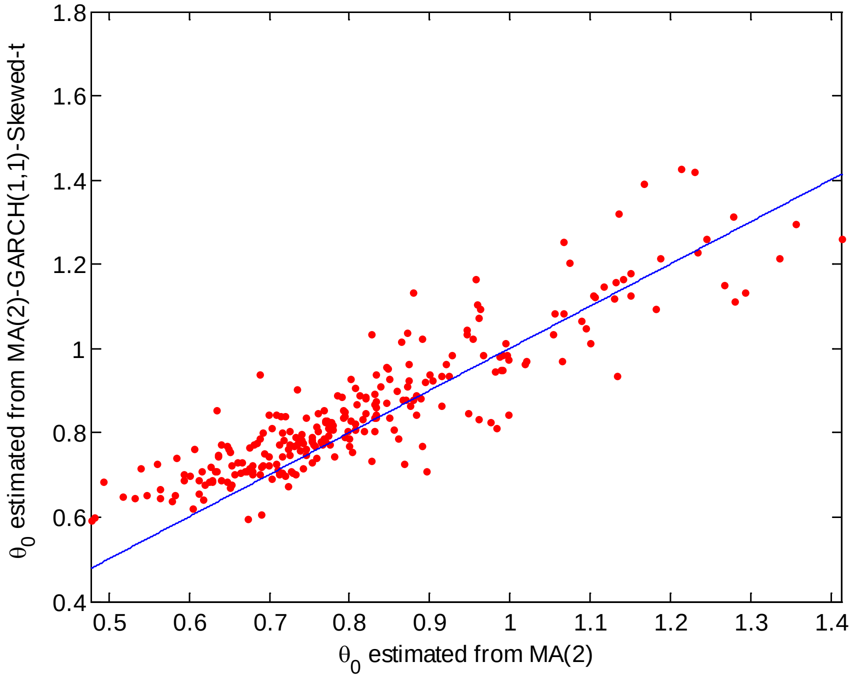


Figure 3.3. Scatter plot comparison of θ_0 estimations



In the chart above, the straight line shows where the two estimates are equal. It is evident from the graphs that the estimates for θ_0 are quite different for these two models. The traditional model tends to overstate the smoothing factor based on our database. Our proposal is closer to reality in that it takes into consideration the skewness, kurtosis, and heteroscedasticity of the return distributions.

3. Smoothed returns and statistical moments of individual hedge funds

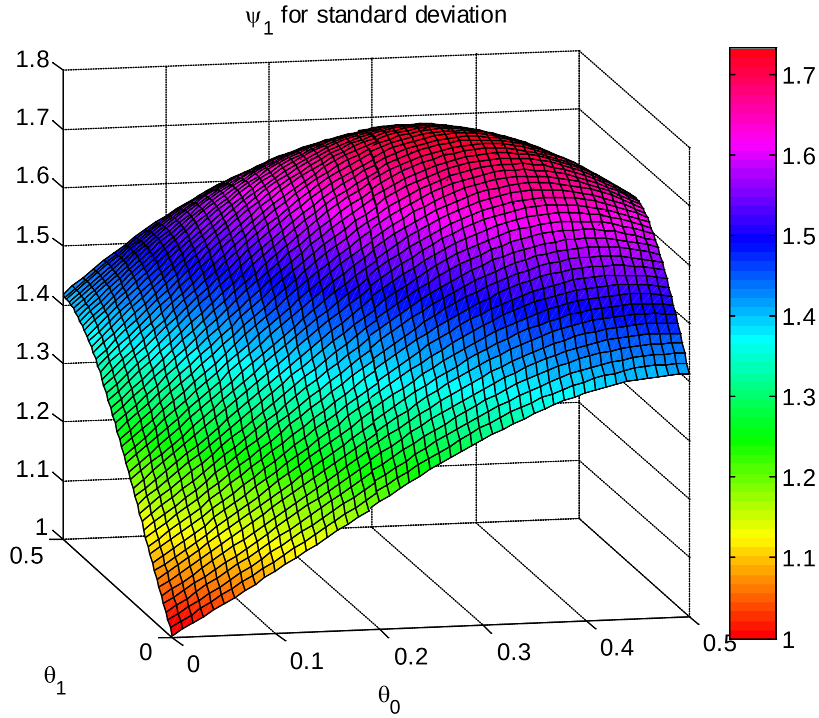
Return smoothing and standard deviation of return

Geltner (1991) shows that for (2.1) the standard deviation SD^{o} of the reported returns R_t^o and the standard deviation SD of the actual returns R_t are related as follows:

(3.1) $\mathrm{SD} = \psi_1\, \mathrm{SD}^{\mathrm{o}}$
$$\psi_1 = 1/(\sum \theta_i^2)^{1/2}$$

The term ψ_1 is greater than 1 where $0 \le \theta_i < 1$; i=0,1,2, i.e., where returns are smoothed, so that the standard deviation of reported returns is less than the standard deviation of the actual underlying returns. The chart below shows ψ_1 plotted as functions of θ_0 and θ_1 ($\theta_2 = 1 - \theta_0 - \theta_1$, of course).

Figure 4. Distortion factor ψ_1 under return smoothing



The distortive effect of return smoothing, i.e., the value of ψ_1, is greatest where $\theta_0 = \theta_1 = \theta_2 = 1/3$. The understatement of standard deviation means that the Sharpe ratio and the information ratio computed from reported returns are higher than they would be using actual returns. For example, where $\theta_0 = \theta_1 = \theta_2 = 1/3$, $\psi_1 = 1.732$ so that the standard deviation of the reported return is 0.5774 (= 1/1.732) that of the actual return, and the reported Sharpe ratio and the information ratio are each 1.732 higher than their respective actual measures. Getmansky et al find that distortions of standard deviation are sometimes substantial in empirical hedge fund data.

With smoothed returns, reported standard deviation is less than actual standard deviation, and the reported Sharpe ratio and information ratio are higher than their respective actual measures, and the degree of distortion is sensitive to the method of estimation.

Return smoothing and higher statistical moments

Higher moments are also distorted by smoothing (Cavenaile et al, 2011). Let SK, SK^o, K, and K^o denote, respectively, skewness of actual returns, skewness of observed returns, excess kurtosis of actual returns, and excess kurtosis of observed returns, respectively. Then:

(3.2) $SK = \psi_2 \, SK^o$

$\psi_2 = (\sum \theta_i^2)^{3/2} / (\sum \theta_i^3)$

(3.3) $K = \psi_3 \, K^o$

$\psi_3 = (\sum \theta_i^2)^2 / (\sum \theta_i^4)$

Of course, in the absence of return smoothing, $\psi_2 = \psi_3 = 1$, but these distortion factors are elevated where returns are smoothed and may be well above 1. The charts below show ψ_2 and ψ_3 plotted as functions of θ_0 and θ_1.

Figure 5. Distortion factor ψ_2 under return smoothing

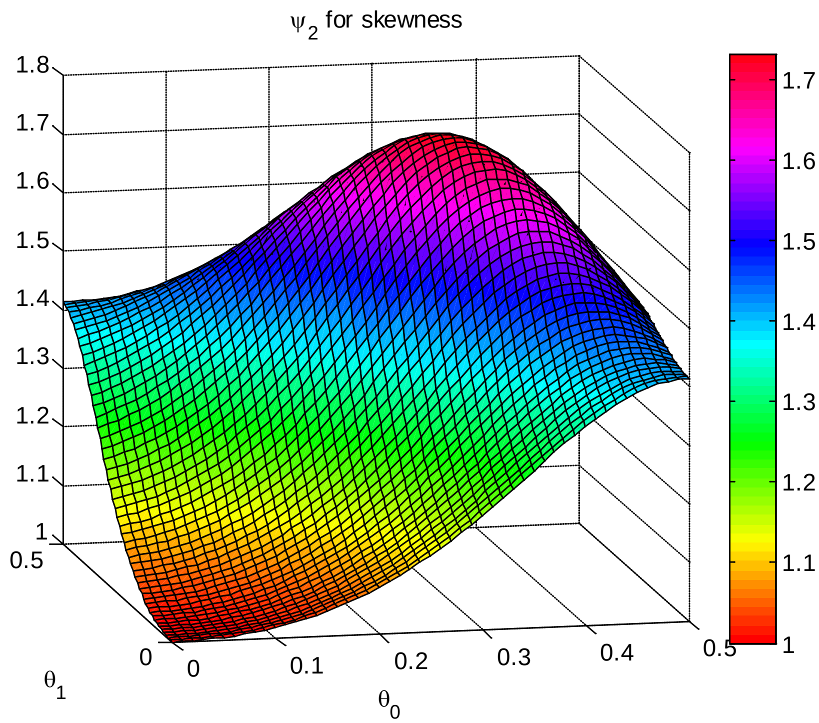
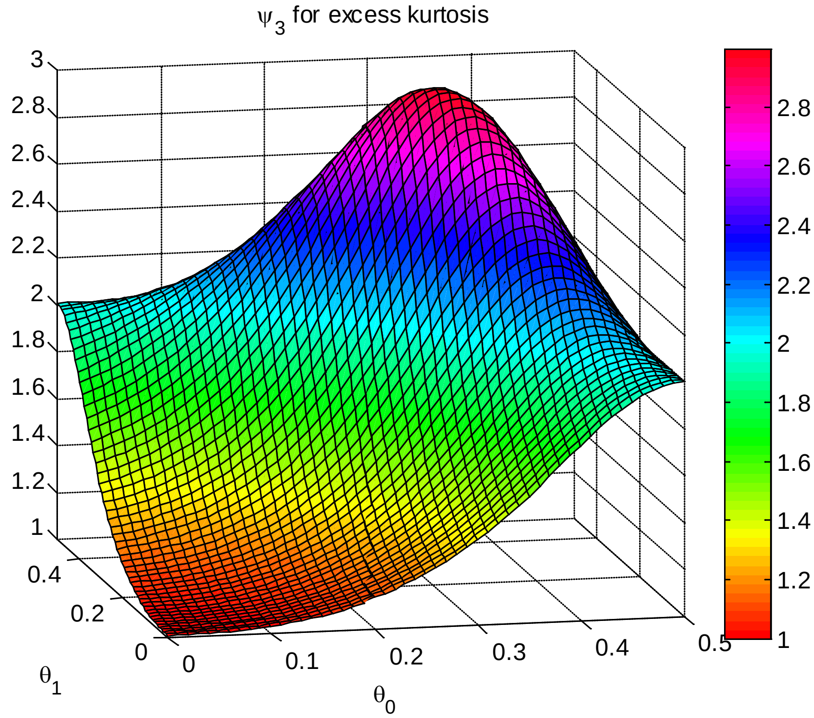


Figure 6. Distortion factor ψ_3 under return smoothing



Again, the distortive effects of return smoothing are greatest where $\theta_0 = \theta_1 = \theta_2 = 1/3$ for both skewness and excess kurtosis.

With smoothed returns, actual skewness and actual excess kurtosis are greater than their respective measures taken from reported returns, and the distortion of kurtosis is greater than the distortion of skewness.

4. Smoothed returns and statistical measurements across hedge funds

Correlation of returns under return smoothing

There is evidence that, during periods of financial distress, hedge fund return streams become positively correlated with each other, and analysts use correlation of hedge fund returns as an indicator of potential financial distress. Unfortunately, the correct computation of correlation of returns may be undermined by return smoothing.

Let x_t^o and y_t^o be the observed returns of two hedge funds, let x_t and y_t be the respective actual returns, and suppose again that the two return streams are subject to MA(2) smoothing so that:

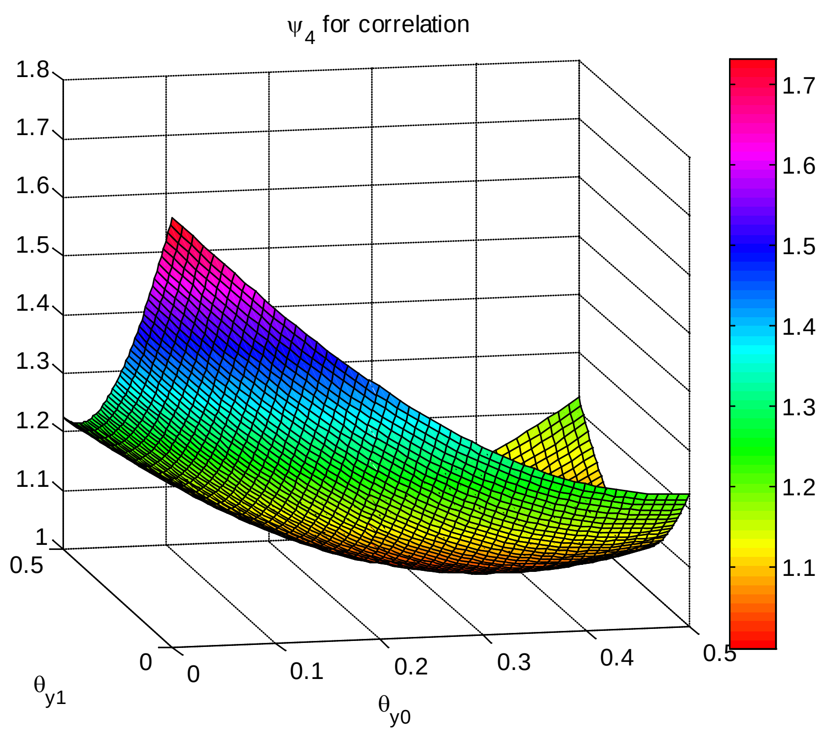
4.1) $x_t^o = \theta_{x0}x_t + \theta_{x1}x_{t-1} + \theta_{x2}x_{t-2}$
$y_t^o = \theta_{y0}y_t + \theta_{y1}y_{t-1} + \theta_{y2}y_{t-2}$
$\sum \theta_{xi} = 1$
$\sum \theta_{yi} = 1$

Then, the correlation $\mathrm{corr}[x_t, y_t]$ of the actual returns and the correlation $\mathrm{corr}[x_t^o, y_t^o]$ of the observed returns are related as follows (Geltner, 1991):

4.2) $\mathrm{corr}[x_t, y_t] = \psi_4 \,\mathrm{corr}[x_t^o, y_t^o]$
$\psi_4 = (\sum \theta_{xi}^2 \sum \theta_{yi}^2)^{1/2} / (\sum \theta_{xi}\,\theta_{yi})$

When the smoothing parameters are the same for both funds, i.e., where $\theta_{xi} = \theta_{yi}$; i = 0,1,2, we have $\psi_4 = 1$. However, when the smoothing parameters are different, we have $\psi_4 > 1$. Figure 7 shows ψ_4 as a function of θ_{y0} and θ_{y1} where $\theta_{x0} = \theta_{x1} = \theta_{x2} = 1/3$.

Figure 7. Distortion factor ψ_4 under return smoothing



In the case of identical smoothing parameters, actual correlation and observed correlation are equal, but correlation of actual returns is greater in absolute value than correlation using reported returns where the smoothing parameters are different.

Analysts may use correlation of returns as an indicator of the degree of distress in financial markets, and so correcting measured correlation, i.e., computing Ψ_4, may be a consideration in analyzing hedge fund return data.

Cross-sectional volatility, covariance and correlation
of returns under return smoothing

Return smoothing can have effects on other statistical measures, and we round out our analysis with an examination of these effects. Analysts may use cross-sectional measures of returns as indicators of hedge fund risk. Adrian (2007) defines cross-sectional volatility at a point in time as $\sqrt{\frac{1}{N}\sum_{i=1}^{N}(R_i^o)^2}$ where R_i^o is the observed return of fund i and N is the number of hedge funds in the database. We note that there is not a term like ψ above that converts this indicator to $\sqrt{\frac{1}{N}\sum_{i=1}^{N}(R_i)^2}$, i.e., cross-sectional volatility of actual returns. He defines cross-sectional covariance as $\frac{1}{N^2-N}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} R_t^{oi}\, R_t^{oj}$. Using Credit Suisse/ Tremont style indexes, he finds that “increases in … covariances tend to precede elevations in volatility.” Cross-sectional correlation is the ratio of cross-sectional covariance to the square of cross-sectional volatility. Adrian finds “no statistical evidence that increases in … correlations precede rises in … volatility.” He finds that the Long Term Capital Management crisis of August 1998 “was accompanied by a large negative covariance of … returns.” Using the same style indexes, we replicated his results, demonstrating that such measures are useful when working with hedge fund style indexes.

However, problems arise when we deal with individual hedge funds—as we do in this study—which may use very idiosyncratic strategies even within the same style classification. For instance, in our analysis using the definitions above, we are not able to detect significant negative cross-sectional covariance in our database of individual hedge funds during periods of financial crisis.

We also note that, where a large constant is added to all the returns, cross-sectional volatility/covariance is larger while the true underlying performance differences across hedge funds remain the same. Adding a constant to the returns may be thought of as artificially creating a more bullish operating environment for hedge funds. With that in mind, we propose the following modified cross-sectional measures for our study:

Cross-sectional volatility $=\sqrt{\frac{1}{N}\sum_{i=1}^{N}(R_i^o - u)^2}$ $where\ u = \frac{1}{N}\sum_{i=1}^{N} R_i^o$

Cross-sectional covariance $=\frac{1}{N^2-N}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(R_t^{oi} - u) * (R_t^{oj} - u)$

When taking return smoothing into consideration, we will use our approach to filter out the unsmoothed underlying “true” return and estimate the above mentioned cross-sectional measures.

5. Empirical study of key statistics

Using our original dataset of 256 hedge funds, we use both our MA(2)-GARCH(1,1)-skewed-t model and the MA(2) model to examine the distortion effects for standard deviation, skewness, excess kurtosis, and the cross-sectional measures considered above. In this empirical study, we relax the constraint of $0 \leq \theta_i \leq 1$ so that, for instance, standard deviation of reported returns may at times be less than that of actual returns. The following charts show the histogram comparisons of distortions for standard deviation, skewness, excess kurtosis and pair-wise correlations estimated from the two methods. The cross-sectional measures using the observed return and filtered return in our approach are shown from January 1998 to December 2010.

Figure 8. Histogram of ψ_1

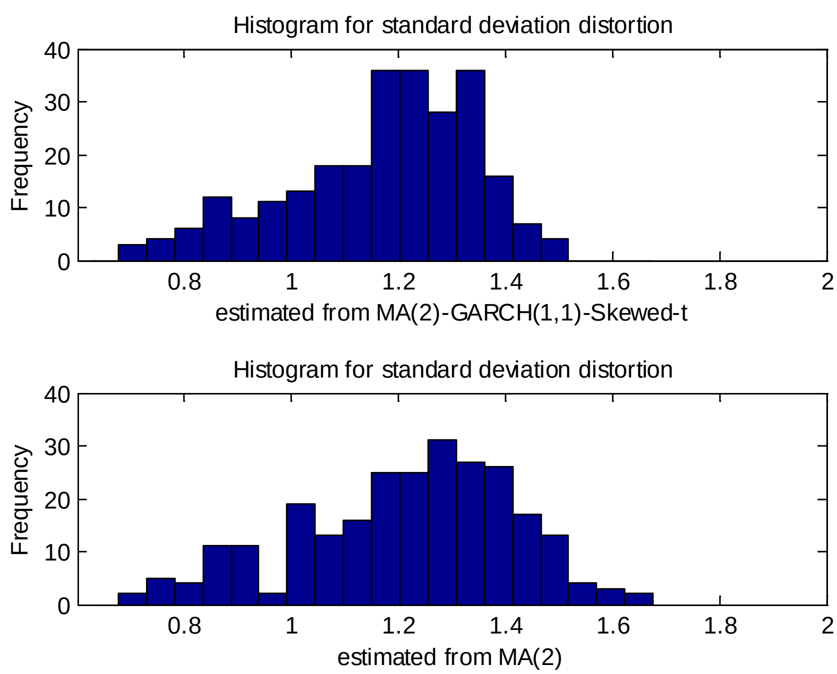


Figure 9. Histogram of ψ_2

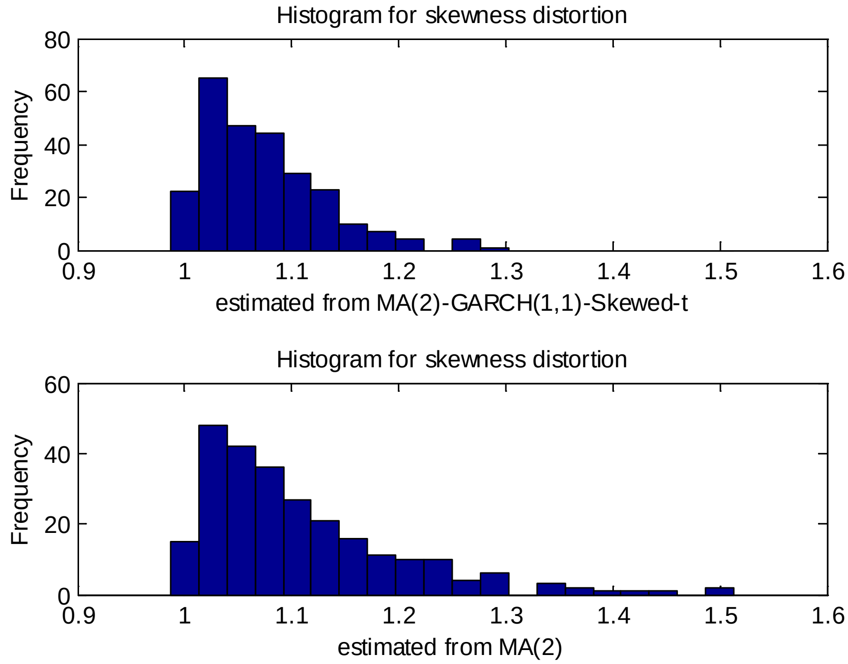


Figure 10. Histogram of ψ_3

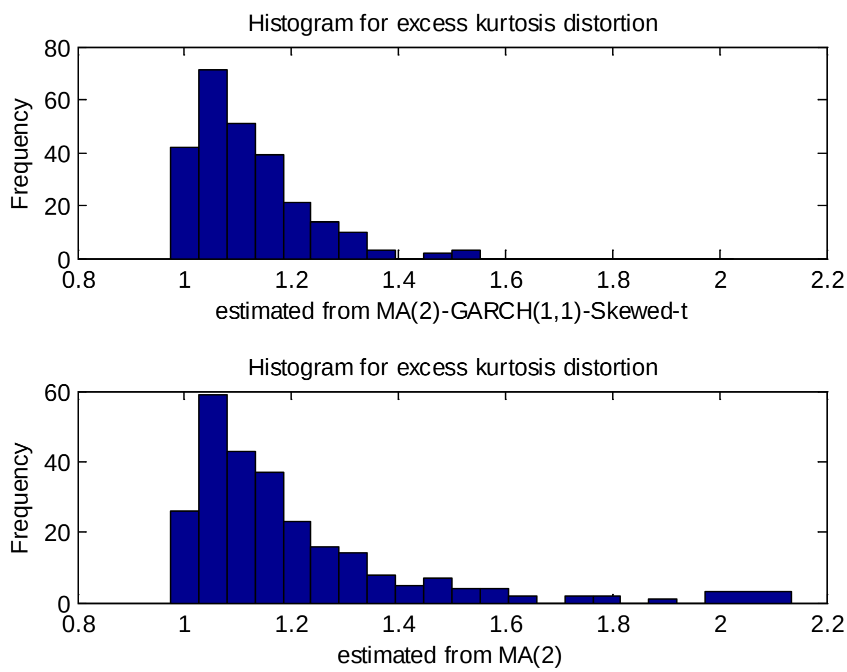


Figure 11. Histogram of ψ_4

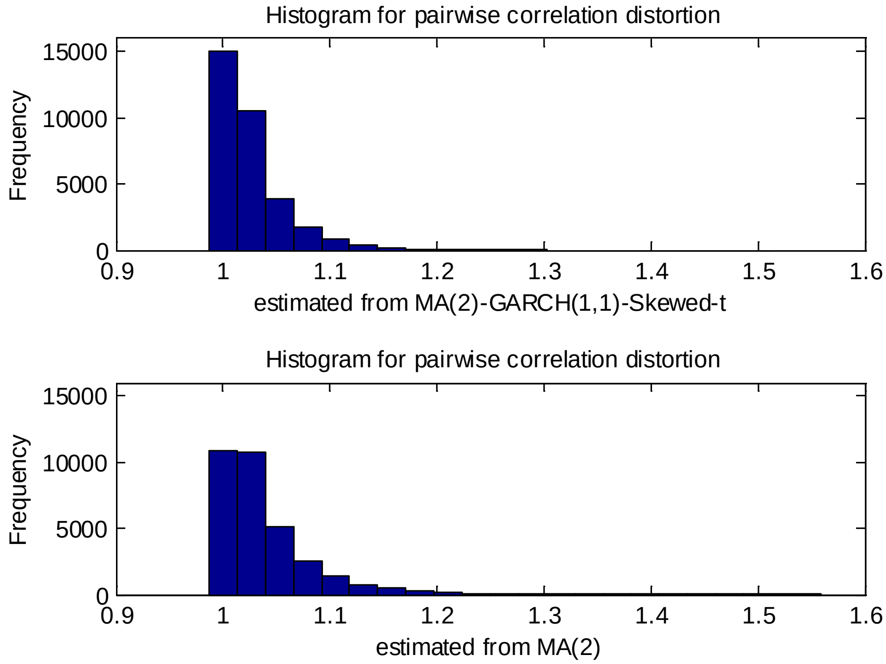


Figure 12. Comparison of cross-sectional volatility

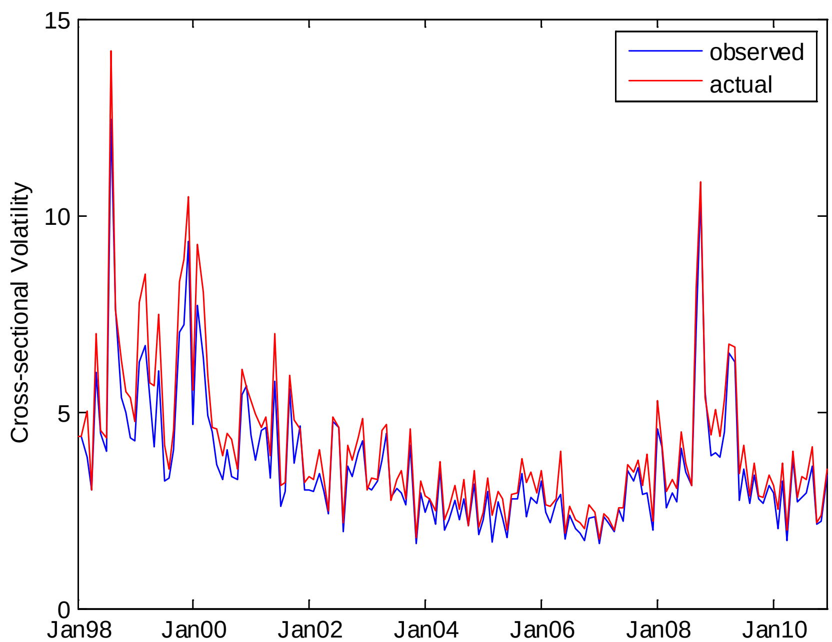
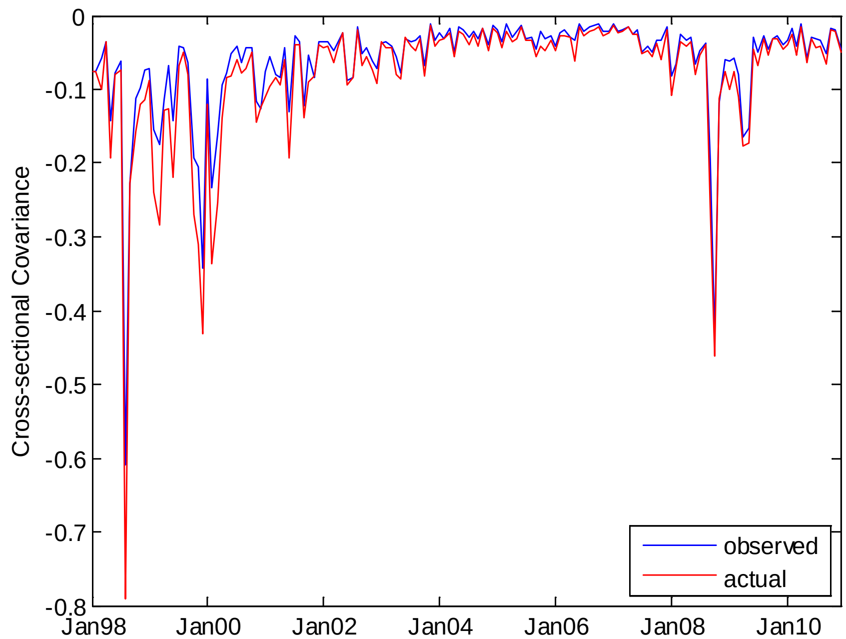


Figure 13. Comparison of cross-sectional covariance



Thus, in general using our method, standard deviation, skewness and excess kurtosis all show some distortions for most of the 256 funds in our study, while the distortions tend to be less than that estimated from the traditional method. The cross-sectional risk measures are affected to a lesser extent, and it would suffice to use only the observed return in this case. Most importantly, there are three big spikes in cross-sectional volatility and three big negative spikes in cross-sectional covariance corresponding to the Long Term Capital Management crisis in 1998, the internet bubble burst in 2000, and the recent financial crisis in 2008.

6. Conclusion

It is established in the literature that hedge fund return smoothing causes distortions in hedge fund statistics. We propose a new MA(2)-GARCH(1,1)-skewed-t model to incorporate the skewness, kurtosis and heteroscedasticity effects often encountered in hedge fund returns. In addition, it has been reported in the literature that return smoothing causes standard deviation of reported returns to understate true volatility and that smoothing distorts correlation of returns. Skewness and excess kurtosis are understated using smoothed returns. In addition, we find that some statistics such as cross-sectional volatility and covariance are not substantially distorted by illiquidity and so they may serve for risk-measurement purposes without correcting for illiquidity effects.

The construction of our indicators of hedge fund illiquidity, i.e., the ψ, is consistent with our analytical framework, and we suggest that such measures may be useful to policy-makers who require correct assessment of risk in hedge fund space.

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