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The Taylor Rule and Forecast Intervals for Exchange Rates*

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Abstract

This paper attacks the Meese-Rogoff (exchange rate disconnect) puzzle from a different perspective: out-of-sample interval forecasting. Most studies in the literature focus on point forecasts. In this paper, we apply Robust Semi-parametric (RS) interval forecasting to a group of Taylor rule models. Forecast intervals for twelve OECD exchange rates are generated and modified tests of Giacomini and White (2006) are conducted to compare the performance of Taylor rule models and the random walk. Our contribution is twofold. First, we find that in general, Taylor rule models generate tighter forecast intervals than the random walk, given that their intervals cover out-of-sample exchange rate realizations equally well. This result is more pronounced at longer horizons. Our results suggest a connection between exchange rates and economic fundamentals: economic variables contain information useful in forecasting the distributions of exchange rates. The benchmark Taylor rule model is also found to perform better than the monetary and PPP models. Second, the inference framework proposed in this paper for forecast-interval evaluation can be applied in a broader context, such as inflation forecasting, not just to the models and interval forecasting methods used in this paper.

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1 Introduction

Recent studies explore the role of monetary policy rules, such as Taylor rules, in exchange rate determination. They find empirical support in these models for the linkage between exchange rates and economic fundamentals. Our paper extends this literature from a different perspective: interval forecasting. We find that the Taylor rule models can outperform the random walk, especially at long horizons, in forecasting twelve OECD exchange rates based on relevant out-of-sample interval forecasting criteria. The benchmark Taylor rule model is also found to perform relatively better than the standard monetary model and the purchasing power parity (PPP) model.

In a seminal paper, Meese and Rogoff (1983) find that economic fundamentals - such as the money supply, trade balance and national income - are of little use in forecasting exchange rates. They show that existing models cannot forecast exchange rates better than the random walk in terms of out-of-sample forecasting accuracy. This finding suggests that exchange rates may be determined by something purely random rather than economic fundamentals. Meese and Rogoff's (1983) finding has been named the Meese-Rogoff puzzle in the literature.

In defending fundamental-based exchange rate models, various combinations of economic variables and econometric methods have been used in attempts to overturn Meese and Rogoff's finding. For instance, Mark (1995) finds greater exchange rate predictability at longer horizons.[1] Groen (2000) and Mark and Sul (2001) detect exchange rate predictability by using panel data. Kilian and Taylor (2003) find that exchange rates can be predicted from economic models at horizons of 2 to 3 years, after taking into account the possibility of nonlinear exchange rate dynamics. Faust, Rogers, and Wright (2003) find that the economic models consistently perform better using real-time data than revised data, though they do not perform better than the random walk.

Recently, there is a growing strand of literature that uses Taylor rules to model exchange rate determination. Engel and West (2005) derive the exchange rate as a present-value asset price from a Taylor rule model. They also find a positive correlation between the model-based exchange rate and the actual real exchange rate between the US dollar and the Deutschmark (Engel and West, 2006). Mark (2007) examines the role of Taylor-rule fundamentals for exchange rate determination in a model with learning. In his model, agents use least-square learning rules to acquire information about the numerical values of the model's coefficients. He finds that the model is able to capture six major swings of the real Deutschmark-Dollar exchange rate from 1973 to 2005. Molodtsova and Papell (2009) find significant short-term out-of-sample predictability

[1] Chinn and Meese (1995) and MacDonald and Taylor (1994) find similar results. However, the long-horizon exchange rate predictability in Mark (1995) has been challenged by Kilian (1999) and Berkowitz and Giorgianni (2001) in subsequent studies.

of exchange rates with Taylor-rule fundamentals for 11 out of 12 currencies vis-á-vis the U.S. dollar over the post-Bretton Woods period. Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008a, 2008b) find evidence of out-of-sample predictability for the dollar/mark nominal exchange rate with forecasts based on Taylor rule fundamentals using real-time data, but not revised data. Chinn (forthcoming) also finds that Taylor rule fundamentals do better than other models at the one year horizon. With a present-value asset pricing model as discussed in Engel and West (2005), Chen and Tsang (2009) find that information contained in the cross-country yield curves are useful in predicting exchange rates.

Our paper joins the above literature of Taylor-rule exchange rate models. However, we address the Meese and Rogoff puzzle from a different perspective: interval forecasting. A forecast interval captures a range in which the exchange rate may lie with a certain probability, given a set of predictors available at the time of forecast. Our contribution to the literature is twofold. First, we find that for twelve OECD exchange rates, the Taylor rule models in general generate tighter forecast intervals than the random walk, given that their intervals cover the realized exchange rates (statistically) equally well. This finding suggests an intuitive connection between exchange rates and economic fundamentals beyond point forecasting: the use of economic variables as predictors helps narrow down the range in which future exchange rates may lie, compared to random walk forecast intervals. Second, we propose an inference framework for cross-model comparison of out-of-sample forecast intervals. The proposed framework can be used for forecast-interval evaluation in a broader context, not just for the models and methods used in this paper. For instance, the framework can also be used to evaluate out-of-sample inflation forecasting.

As we will discuss later, we in fact derive forecast intervals from estimates of the distribution of changes in the exchange rate. Hence, in principle, evaluations across models can be done based on distributions instead of forecast intervals. However, focusing on interval forecasting performance allows us to compare models in two dimensions that are more relevant to practitioners: empirical coverage and length.

While the literature on interval forecasting for exchange rates is sparse, several authors have studied out-of-sample exchange rate density (distribution) forecasts, from which interval forecasts can be derived. Diebold, Hahn and Tay (1999) use the RiskMetrics model of JP Morgan (1996) to compute half-hour-ahead density forecasts for Deutschmark/Dollar and Yen/Dollar returns. Christoffersen and Mazzotta (2005) provide option-implied density and interval forecasts for four major exchange rates. Boero and Marrocu (2004) obtain one-day-ahead density forecasts for the Euro nominal effective exchange rate using self-exciting threshold autoregressive (SETAR) models. Sarno and Valente (2005) evaluate the exchange rate density forecasting performance of the Markov-switching vector equilibrium correction model that is developed by Clarida, Sarno, Taylor and Valente (2003). They find that information from the term structure of forward premia help the model to outperform the random walk in forecasting out-of-sample densities of the spot exchange rate. More recently, Hong, Li, and Zhao (2007) construct half-hour-ahead density forecasts for Euro/Dollar and Yen/Dollar exchange rates using a comprehensive set of univariate time series models that capture fat tails, time-varying volatility and regime switches.

There are several common features across the studies listed above, which make them different from our paper. First, the focus of the above studies is not to make connections between the exchange rate and economic fundamentals. These studies use high frequency data, which are not available for most conventional economic fundamentals. For instance, Diebold, Hahn, and Tay (1999) and Hong, Li, and Zhao (2007) use intra-day data. With the exception of Sarno and Valente (2005), all the studies focus only on univariate time series models. Second, these studies do not consider multi-horizon-ahead forecasts, perhaps due to the fact that their models are often highly nonlinear. Iterating nonlinear density models multiple horizons ahead is analytically difficult, if not infeasible. Lastly, the above studies assume that the densities are analytically defined for a given model. The semiparametric method used in this paper does not impose such restrictions.

Our choice of the semiparametric method is motivated by the difficulty of using macroeconomic models in exchange rate interval forecasting: these models typically do not describe the future *distributions* of exchange rates. For instance, the Taylor rule models considered in this paper do not describe any features of the data beyond the conditional means of future exchange rates. We address this difficulty by applying *Robust Semiparametric* forecast intervals (hereon *RS* forecast intervals) of Wu (2009).[2] This method is useful since it does not require the model be correctly specified, or contain parametric assumptions about the future distribution of exchange rates.

We apply RS forecast intervals to a set of Taylor rule models that differ in terms of the assumptions on policy and interest rate smoothing rules. Following Molodtsova and Papell (2009), we include twelve OECD exchange rates (relative to the US dollar) over the post-Bretton Woods period in our dataset. For these twelve exchange rates, the out-of-sample RS forecast intervals at different forecast horizons are generated from the Taylor rule models and then compared with those of the random walk. The empirical coverages and lengths of forecast intervals are used as the evaluation criteria. Our empirical coverage and length tests are modified from Giacomini and White's (2006) predictive accuracy tests in the case of rolling, but fixed-size, estimation samples.

For a given nominal coverage (probability), the empirical coverage of forecast intervals derived from a forecasting model is the probability that the out-of-sample realizations (exchange rates) lie in the intervals. The length of the intervals is a measure of its tightness: the distance between its upper and lower bound.

[2]For brevity, we omit RS and simply say forecast intervals when we believe that it causes no confusion.

In general, the empirical coverage is not the same as its nominal coverage. Significantly missing the nominal coverage indicates poor quality of the model and intervals. One certainly wants the forecast intervals to contain out-of-sample realizations as close as possible to the probability they target. Most evaluation methods in the literature focus on comparing empirical coverages across models, following the seminal work of Christoffersen (1998). Following this literature, we first test whether forecast intervals of the Taylor rule models and the random walk have equally accurate empirical coverages. The model with more accurate coverages is considered the better model. In the cases where equal coverage accuracy cannot be rejected, we further test whether the lengths of forecast intervals are the same. The model with tighter forecast intervals provides more useful information about future values of the data, and hence is considered as a more useful forecasting model.

It is also important to establish what this paper is *not* attempting. First, the inference procedure does not carry the purpose of finding the *correct* model specification. Rather, inference is on how useful models are in generating forecast intervals, measured in terms of empirical coverages and lengths. Second, this paper does not consider the possibility that there might be alternatives to RS forecast intervals for the exchange rate models we consider. Some models might perform better if parametric distribution assumptions (e.g. the forecast errors are conditionally heteroskedastic and $t-$distributed) or other assumptions (e.g. the forecast errors are independent of the predictors) are added. One could presumably estimate the forecast intervals differently based on the same models, and then compare those with the RS forecast intervals, but this is out of the scope of this paper. As we described, we choose the RS method for the robustness and flexibility achieved by the semiparametric approach.

Our benchmark Taylor rule model is from Engel and West (2005) and Engel, Wang, and Wu (2008). For the purpose of comparison, several alternative Taylor rule models are also considered. These setups have been studied by Molodtsova and Papell (2009) and Engel, Mark, and West (2007). In general, we find that the Taylor rule models perform better than the random walk model, especially at long horizons: the models either have more accurate empirical coverages than the random walk, or in cases of equal coverage accuracy, the models have tighter forecast intervals than the random walk. The evidence of exchange rate predictability is much weaker in coverage tests than in length tests. In most cases, the Taylor rule models and the random walk have statistically equally accurate empirical coverages. So, under the conventional coverage test, the random walk model and the Taylor rule models perform equally well. However, the results of length tests suggest that Taylor rule fundamentals are useful in generating tighter forecasts intervals without losing accuracy in empirical coverages.

We also consider two other popular models in the literature: the monetary model and the model of purchasing power parity (PPP). Based on the same criteria, both models are found to perform better than the random walk in interval forecasting. As with the Taylor rule models, most evidence of exchange rate predictability comes from the length test: economic models have tighter forecast intervals than the random walk given statistically equivalent coverage accuracy. The PPP model performs worse than the benchmark Taylor rule model and the monetary model at short horizons. The benchmark Taylor rule model performs slightly better than the monetary model at most horizons.

Our findings suggest that exchange rate movements are linked to economic fundamentals. However, we acknowledge that the Meese-Rogoff puzzle remains difficult to understand. Although Taylor rule models offer statistically significant length reductions over the random walk, the reduction of length is quantitatively small, especially at short horizons. Forecasting exchange rates remains a difficult task in practice. There are some impressive advances in the literature, but most empirical findings remain fragile. As mentioned in Cheung, Chinn, and Pascual (2005), forecasts from economic fundamentals may work well for some currencies during certain sample periods but not for other currencies or sample periods. Engel, Mark, and West (2007) recently show that a relatively robust finding is that exchange rates are more predictable at longer horizons, especially when using panel data. We find greater predictability at longer horizons in our exercise. It would be of interest to investigate connections between our findings and theirs.

Several recent studies have attacked the puzzle from a different angle: there are reasons that economic fundamentals cannot forecast the exchange rate, even if the exchange rate is determined by these fundamentals. Engel and West (2005) show that existing exchange rate models can be written in a present-value asset-pricing format. In these models, exchange rates are determined not only by current fundamentals but also by expectations of what the fundamentals will be in the future. When the discount factor is large (close to one), current fundamentals receive very little weight in determining the exchange rate. Not surprisingly, the fundamentals are not very useful in forecasting. Nason and Rogers (2008) generalize the Engel-West theorem to a class of open-economy dynamic stochastic general equilibrium (DSGE) models. Other factors such as parameter instability and mis-specification (for instance, Rossi 2005) may also play important roles in understanding the puzzle. It is interesting to investigate conditions under which we can reconcile our findings with these studies.

The remainder of this paper is organized as follows. Section two describes the forecasting models we use, as well as the data. In section three, we illustrate how the RS forecast intervals are constructed from a given model. We also propose loss criteria to evaluate the quality of the forecast intervals and test statistics that are based on Giacomini and White (2006). Section four presents results of out-of-sample forecast evaluation. Finally, section five contains concluding remarks.

2 Models and Data

Seven models are considered in this paper. Let $m = 1, 2, ..., 7$ be the index of these models and the first model be the benchmark model. A general setup of the models takes the form of:

$$s_{t+h} - s_t = \alpha_{m,h} + \beta'_{m,h}\mathbf{X}_{m,t} + \varepsilon_{m,t+h}, \tag{1}$$

where $s_{t+h} - s_t$ is h-period changes of the (log) exchange rate, and $\mathbf{X}_{m,t}$ contains economic variables that are used in model m. Following the literature of long-horizon regressions, both short- and long-horizon forecasts are considered. Models differ in economic variables that are included in matrix $\mathbf{X}_{m,t}$. In the benchmark model,

$$\mathbf{X}_{1,t} \equiv \begin{bmatrix} \pi_t - \pi_t^* & y_t^{gap} - y_t^{gap*} & q_t \end{bmatrix},$$

where π_t (π_t^*) is the inflation rate, and y_t^{gap} (y_t^{gap*}) is the output gap in the home (foreign) country. The real exchange rate q_t is defined as $q_t \equiv s_t + p_t^* - p_t$, where p_t (p_t^*) is the (log) consumer price index in the home (foreign) country. This setup is motivated by the Taylor rule model in Engel and West (2005) and Engel, Wang, and Wu (2008). The next subsection describes this benchmark Taylor rule model in detail.

We also consider the following models that have been studied in the literature:

- Model 2: $\mathbf{X}_{2,t} \equiv \begin{bmatrix} \pi_t - \pi_t^* & y_t^{gap} - y_t^{gap*} \end{bmatrix}$
- Model 3: $\mathbf{X}_{3,t} \equiv \begin{bmatrix} \pi_t - \pi_t^* & y_t^{gap} - y_t^{gap*} & i_{t-1} - i_{t-1}^* \end{bmatrix}$, where i_t (i_t^*) is the short-term interest rate in the home (foreign) country.
- Model 4: $\mathbf{X}_{4,t} \equiv \begin{bmatrix} \pi_t - \pi_t^* & y_t^{gap} - y_t^{gap*} & q_t & i_{t-1} - i_{t-1}^* \end{bmatrix}$
- Model 5: $\mathbf{X}_{5,t} \equiv q_t$
- Model 6: $\mathbf{X}_{6,t} \equiv \begin{bmatrix} s_t - [(m_t - m_t^*) - (y_t - y_t^*)] \end{bmatrix}$, where m_t (m_t^*) is the money supply and y_t (y_t^*) is total output in the home (foreign) country.
- Model 7: $\mathbf{X}_{7,t} \equiv 0$

Models 2-4 are the Taylor rule models studied in Molodtsova and Papell (2009). Model 2 can be considered as the constrained benchmark model in which PPP always holds. Molodtsova and Papell (2009) include interest rate lags in models 3 and 4 to take into account potential interest rate smoothing rules of the central bank.[3] Model 5 is the purchasing power parity (PPP) model and model 6 is the monetary model. Both models have been widely used in the literature. See Molodtsova and Papell (2009) for the PPP model and Mark (1995) for the monetary model. Model 7 is the driftless random walk model ($\alpha_{7,h} \equiv 0$).[4] Given a date τ and horizon h, the objective is to estimate the forecast distribution of $s_{\tau+h} - s_\tau$ conditional on $\mathbf{X}_{m,\tau}$, and subsequently build forecast intervals from the estimated forecast distribution. Before moving to the econometric method, we first describe the Taylor rule model that motivates the setup of our benchmark model.

2.1 Benchmark Taylor Rule Model

Our benchmark model is the Taylor rule model that is derived in Engel and West (2005) and Engel, Wang, and Wu (2008). Following Molodtsova and Papell (2009), we focus on models that depend only on *current* levels of inflation and the output gap.[5] The Taylor rule in the home country takes the form of:

$$\bar{i}_t = \bar{i} + \delta_\pi(\pi_t - \bar{\pi}) + \delta_y y_t^{gap} + u_t, \tag{2}$$

where $\bar{i}_t$ is the central bank's target for the short-term interest rate at time t, $\bar{i}$ is the equilibrium long-term rate, π_t is the inflation rate, $\bar{\pi}$ is the target inflation rate, and y_t^{gap} is the output gap. The foreign country is assumed to follow a similar Taylor rule. In addition, we follow Engel and West (2005) to assume that the foreign country targets the exchange rate in its Taylor rule:

$$\bar{i}_t^* = \bar{i} + \delta_\pi(\pi_t^* - \bar{\pi}) + \delta_y y_t^{gap*} + \delta_s(s_t - \bar{s}_t) + u_t^*, \tag{3}$$

where $\bar{s}_t$ is the targeted exchange rate. Assume that the foreign country targets the PPP level of the exchange rate: $\bar{s}_t = p_t - p_t^*$, where p_t and p_t^* are logarithms of home and foreign aggregate prices. In equation (3), we assume that the policy parameters take the same values in the home and foreign countries. Molodtsova and Papell (2009) denote this case as "homogeneous Taylor rules". Our empirical results also hold in the case of heterogenous Taylor rules. To simplify our presentation, we assume that the home and foreign countries have the same long-term inflation and interest rates. Such restrictions have been relaxed in our econometric model after we include a constant term in estimations.

[3]The coefficients on lagged interest rates in the home and foreign countries can take different values in Molodtsova and Papell (2009).

[4]We also tried the random walk with a drift. It does not change our results.

[5]Clarida, Gali, and Gertler (1998) find empirical support for forward-looking Taylor rules. Forward-looking Taylor rules are ruled out because they require forecasts of predictors, which creates additional complications in out-of-sample forecasting.

We do not consider interest rate smoothing in our benchmark model. That is, the actual interest rate (i_t) is identical to the target rate in the benchmark model:

$$i_t = \bar{i}_t. \tag{4}$$

Molodtsova and Papell (2009) consider the following interest rate smoothing rule:

$$i_t = (1 - \rho)\bar{i}_t + \rho i_{t-1} + \nu_t, \tag{5}$$

where ρ is the interest rate smoothing parameter. We include these setups in models 3 and 4. Note that our estimation methods do not require the monetary policy shock u_t and the interest rate smoothing shock ν_t to satisfy any assumptions, aside from smoothness of their distributions when conditioned on predictors.

Substituting the difference of equations (2) and (3) into Uncovered Interest-rate Parity (UIP), we have:

$$s_t = E_t \left\{ (1 - b) \sum_{j=0}^{\infty} b^j (p_{t+j} - p^*_{t+j}) - b \sum_{j=0}^{\infty} b^j \left[\delta_y (y^{gap}_{t+j} - y^{gap*}_{t+j}) + \delta_\pi (\pi_{t+j} - \pi^*_{t+j}) \right] \right\}, \tag{6}$$

where the discount factor $b = \frac{1}{1+\delta_s}$. Under some conditions, the present value asset pricing format in equation (6) can be written into an error-correction form:[6]

$$s_{t+h} - s_t = \alpha_h + \beta_h z_t + \varepsilon_{t+h}, \tag{7}$$

where the deviation of the exchange rate from its equilibrium level is defined as:

$$z_t = s_t - p_t + p^*_t + \frac{b}{1 - b} \left[\delta_y (y^{gap}_t - y^{gap*}_t) + \delta_\pi (\pi_t - \pi^*_t) \right]. \tag{8}$$

We use equation (7) as our benchmark setup in calculating h-horizon-ahead out-of-sample forecasting intervals. According to equation (8), the matrix $\mathbf{X}_{1,t}$ in equation (1) includes economic variables $q_t \equiv s_t + p^*_t - p_t$, $y^{gap}_t - y^{gap*}_t$, and $\pi_t - \pi^*_t$.

[6]See appendix for more detail. While the long-horizon regression format of the benchmark Taylor model is derived directly from the underlying Taylor rule model, this is not the case for the models with interest rate smoothing (models 3 and 4). Molodtsova and Papell (2009) only consider the short-horizon regression for the Taylor rule models. We include long-horizon regressions of these models only for the purpose of comparison.

2.2 Data

The forecasting models and the corresponding forecast intervals are estimated using monthly data for twelve OECD countries. The United States is treated as the foreign country in all cases. For each country we synchronize the beginning and end dates of the data across all models estimated. The twelve countries and periods considered are: Australia (73:03-06:6), Canada (75:01-06:6), Denmark (73:03-06:6), France (77:12-98:12), Germany (73:03-98:12), Italy (74:12-98:12), Japan (73:03-06:6), Netherlands (73:03-98:12), Portugal (83:01-98:12), Sweden (73:03-04:11), Switzerland (75:09-06:6), and the United Kingdom (73:03-06:4).

The data is taken from Molodtsova and Papell (2009).[7] With the exception of interest rates, the data is transformed by taking natural logs and then multiplying by 100. The nominal exchange rates are end-of-month rates taken from the Federal Reserve Bank of St. Louis database. Output data (y_t) are proxied by Industrial Production (IP) from the International Financial Statistics (IFS) database. IP data for Australia and Switzerland are only available at a quarterly frequency, and hence are transformed from quarterly to monthly observations using the quadratic-match average option in Eviews 4.0 by Molodtsova and Papell (2009). Following Engel and West (2006), the output gap (y_t^{gap}) is calculated by quadratically de-trending the industrial production for each country.

Prices data (p_t) are proxied by Consumer Price Index (CPI) from the IFS database. Again, CPI for Australia is only available at a quarterly frequency and the quadratic-match average is used to impute monthly observations. Inflation rates are calculated by taking the first differences of the logs of CPIs. The money market rate from IFS (or "call money rate") is used as a measure of the short-term interest rate set by the central bank. Finally, M1 is used to measure the money supply for most countries. M0 for the UK and M2 for Italy and Netherlands is used due to the unavailability of M1 data.

3 Econometric Method

For a given model m, the objective is to estimate from equation (1) the distribution of $s_{\tau+h} - s_\tau$ conditional on data $\mathbf{X}_{m,\tau}$ that is observed up to time τ. This is the h-horizon-ahead *forecast distribution* of the exchange rate, from which the corresponding *forecast interval* can be derived. For a given α, the forecast interval of coverage $\alpha \in (0, 1)$ is an interval in which $s_{\tau+h} - s_\tau$ is supposed to lie with a probability of α.

Models $m = 1, ..., 7$ in equation (1) provide only point forecasts of $s_{\tau+h} - s_\tau$. In order to construct forecast intervals for a given model, we apply robust semiparametric (RS) forecast intervals to all models.

[7] We thank the authors for the data, which we downloaded from David Papell's website. For the exact line numbers and sources of the data, see the data appendix of Molodtsova and Papell (2009).

The nominal α-coverage forecast interval of $s_{\tau+h} - s_\tau$ conditional on $\mathbf{X}_{m,\tau}$ can be obtained by the following three-step procedure:

Step 1. Estimate model m by OLS and obtain residuals $\widehat{\varepsilon}_{m,t+h} \equiv s_{t+h} - s_t - \widehat{\alpha}_{m,h} + \widehat{\beta}'_{m,h}\mathbf{X}_{m,t}$, for $t = 1, ..., \tau - h$.

Step 2. For a range of values of ε (sorted residuals $\{\widehat{\varepsilon}_{m,t+h}\}_{t=1}^{\tau-h}$), estimate the conditional distribution of $\varepsilon_{m,\tau+h}|\mathbf{X}_{m,\tau}$ by:

$$\widehat{P}(\varepsilon_{m,\tau+h} \leq \varepsilon|\mathbf{X}_{m,\tau}) \equiv \frac{\sum_{t=1}^{\tau-h} 1(\widehat{\varepsilon}_{m,t+h} \leq \varepsilon)\mathbf{K}_b(\mathbf{X}_{m,t} - \mathbf{X}_{m,\tau})}{\sum_{t=1}^{\tau-h} \mathbf{K}_b(\mathbf{X}_{m,t} - \mathbf{X}_{m,\tau})}, \tag{9}$$

where $\mathbf{K}_b(\mathbf{X}_{m,t} - \mathbf{X}_{m,\tau}) \equiv b^{-d}\mathbf{K}((\mathbf{X}_{m,t} - \mathbf{X}_{m,\tau})/b)$, $\mathbf{K}(\cdot)$ is a multivariate Gaussian kernel with a dimension the same as that of $\mathbf{X}_{m,t}$, and b is the smoothing parameter or bandwidth.[8]

Step 3. Find the $(1-\alpha)/2$ and $(1+\alpha)/2$ quantiles of the estimated distribution, which are denoted by $\widehat{\varepsilon}_{m,h}^{(1-\alpha)/2}$ and $\widehat{\varepsilon}_{m,h}^{(1+\alpha)/2}$, repectively. The estimate of the α-coverage forecast interval for $s_{\tau+h} - s_\tau$ conditional on $\mathbf{X}_{m,\tau}$ is:

$$\widehat{I}_{m,\tau+h}^{\alpha} \equiv (\widehat{\beta}'_{m,h}\mathbf{X}_{m,\tau} + \widehat{\varepsilon}_{m,h}^{(1-\alpha)/2}, \widehat{\beta}'_{m,h}\mathbf{X}_{m,\tau} + \widehat{\varepsilon}_{m,h}^{(1+\alpha)/2}) \tag{10}$$

For each model m, the above method uses the forecast models in equation (1) to estimate the location of the forecast distribution, while the nonparametric kernel distribution estimate is used to estimate the shape. As a result, the interval obtained from this method is *semiparametric*. Wu (2009) shows that under some weak regularity conditions, this method consistently estimates the forecast distribution,[9] and hence the forecast intervals of $s_{\tau+h} - s_\tau$ conditional on $\mathbf{X}_{m,\tau}$, regardless of the quality of model m. That is, the forecast intervals are *robust*. Stationarity of economic variables is one of those regularity conditions. In our models, exchange rate differences, interest rates and inflation rates are well-known to be stationary, while empirical tests for real exchange rates and output gaps generate mixed results. These results may be driven by the difficulty of distinguishing a stationary, but persistent, variable from a non-stationary one. In this paper, we take the stationarity of these variables as given.

Model 7 is the random walk model. The estimator in equation (9) becomes the Empirical Distribution Function (EDF) of the exchange rate innovations. Under regularity conditions, equation (9) consistently estimates the unconditional distribution of $s_{\tau+h} - s_\tau$, and can be used to form forecast intervals for $s_{\tau+h}$.

[8]We choose b using the method of Hall, Wolff, and Yao (1999).

[9]It is consistent in the sense of convergence in probability as the estimation sample size goes to infinity.

The forecast intervals of economic models and the random walk are compared. Our interest is to test whether RS forecast intervals based on economic models are more accurate than those based on the random walk model. We focus on the empirical coverage and the length of forecast intervals in our tests.

Following Christoffersen (1998) and related work, the first standard we use is the empirical coverage. The empirical coverage should be as close as possible to the nominal coverage (α). Significantly missing the nominal coverage indicates the inadequacy of the model and predictors for the given sample size. For instance, if 90% forecast intervals calculated from a model contain only 50% of out-of-sample observations, the model can hardly be identified as useful for interval forecasting. This case is called under-coverage. In contrast, over-coverage implies that the intervals could be reduced in length (or improved in tightness), but the forecast interval method and model are unable to do that for the given sample size. An economic model is said to outperform the random walk if its empirical coverage is more accurate than that of the random walk.

On the other hand, the empirical coverage of an economic model may be equally accurate as that of the random walk model, but the economic model has tighter forecast intervals than the random walk. We argue that the lengths of forecast intervals signify the informativeness of the intervals given that these intervals have equally accurate empirical coverages. In this case, the economic model is also considered to outperform the random walk in forecasting exchange rates. The empirical coverage and length tests are conducted at both short and long horizons for the six economic models relative to the random walk for each of the twelve OECD exchange rates.

We use tests that are applications of the unconditional predictive accuracy inference framework of Giacomini and White (2006). Unlike the tests of Diebold and Mariano (1995) and West (1996), our forecast evaluation tests do not focus on the asymptotic features of the forecasts. Rather, in the spirit of Giacomini and White (2006), we are comparing the population features of forecasts generated by rolling samples of fixed sample size. This contrasts to the traditional forecast evaluation methods in that although it uses asymptotic approximations to do the testing, the inference is not on the asymptotic properties of forecasts, but on their population *finite sample properties*. We acknowledge that the philosophy of this inference framework remains a point of contention, but it does tackle three important evaluation difficulties in this paper. First, it allows for evaluation of forecast intervals that are not parametrically derived. The density evaluation methods developed in well-known studies such as Diebold, Gunther, Tay (1998), Corradi and Swanson (2006a) and references within Corradi and Swanson (2006b) require that the forecast distributions be parametrically specified. Giacomini and White's (2006) method overcomes this challenge by allowing comparisons among parametric, semiparametric and nonparametric forecasts. As a result, in the cases of semiparametric and nonparametric forecasts, it also allows comparison of models with predictors of different dimensions, as evident in our exercise. Second, by comparing the finite sample properties of RS forecast intervals derived from different models, we avoid rejecting models that are mis-specified,[10] but are nonetheless good approximations useful for forecasting. Finally, we can individually (though not jointly) test whether the forecast intervals differ in terms of empirical coverages and lengths, for the given estimation sample, and are not confined to focus only on empirical coverages or holistic properties of forecast distribution, such as probability integral transform.

3.1 Test of Equal Empirical Coverages

Suppose the sample size available to the researcher is T and all data are collected in a vector $\mathbf{W}_t$. Our inference procedure is based on a rolling estimation scheme, with the size of the rolling window fixed while $T \to \infty$. Let $T = R + N$ and R be the size of the rolling window. For each horizon h and model m, a sequence of $N(h) = N + 1 - h$ α-coverage forecast intervals are generated using rolling data: $\{\mathbf{W}_t\}_{t=1}^{R}$ for forecast for date $R + h$, $\{\mathbf{W}_t\}_{t=2}^{R+1}$ for forecast for date $R + h + 1$, and so on, until forecast for date T is generated using $\{\mathbf{W}_t\}_{t=N(h)}^{R+N(h)-1}$.

Under this fixed-sample-size rolling scheme, for each finite h we have $N(h)$ observations to compare the empirical coverages and lengths across m models ($m = 1, 2, ..., 7$). By fixing R, we allow the finite sample properties of the forecast intervals to be preserved as $T \to \infty$. Thus, the forecast intervals and the associated forecast losses are simply functions of a finite and fixed number of random variables. We are interested in approximating the population moments of these objects by taking $N(h) \to \infty$. A loose analogy would be finding the finite-sample properties of a certain parameter estimator when the sample size is fixed at R, by a bootstrap with an arbitrarily large number of bootstrap replications.

We conduct individual tests for the empirical coverages and lengths. In each test, we define a corresponding forecast loss, propose a test statistic and derive its asymptotic distribution. As defined in equation (10), let $\widehat{I}^{\alpha}_{m,\tau+h}$ be the $h-$horizon ahead RS forecast interval of model m with a nominal coverage of α. For out-of-sample forecast evaluation, we require $\widehat{I}^{\alpha}_{m,\tau+h}$ to be constructed using data from $t = \tau - R + 1$ to $t = \tau$. The *coverage accuracy loss* is defined as:

$$CL^{\alpha}_{m,h} = \left[P(Y_{\tau+h} \in \widehat{I}^{\alpha}_{m,\tau+h}) - \alpha\right]^2. \tag{11}$$

For economic models ($m = 1, ..., 6$), the goal is to compare the coverage accuracy loss of RS forecast intervals

[10] While RS intervals remedy mis-specifications asymptotically, it does not guarantee such corrections in a given finite sample.

of model m with that of the random walk ($m = 7$). The null and alternative hypotheses are:

$$\begin{aligned} H_0 &: \Delta CL^{\alpha}_{m,h} \equiv CL^{\alpha}_{7,h} - CL^{\alpha}_{m,h} = 0 \\ H_A &: \Delta CL^{\alpha}_{m,h} \neq 0. \end{aligned}$$

Define the sample analog of the coverage accuracy loss in equation (11):

$$\widehat{CL}^{\alpha}_{m,h} = \left(N(h)^{-1} \sum_{\tau=R}^{T-h} 1(Y_{\tau+h} \in \widehat{I}^{\alpha}_{m,\tau+h}) - \alpha \right)^2,$$

where $1(Y_{\tau+h} \in \widehat{I}^{\alpha}_{m,\tau+h})$ is an index function that equals one when $Y_{\tau+h} \in \widehat{I}^{\alpha}_{m,\tau+h}$, and equals zero otherwise. Applying the asymptotic test of Giacomini and White (2006) to the sequence $\{1(Y_{\tau+h} \in \widehat{I}^{\alpha}_{m,\tau+h})\}_{\tau=R}^{T-h}$ and applying the Delta method, we can show that

$$\sqrt{N(h)}(\Delta\widehat{CL}^{\alpha}_{m,h} - \Delta CL^{\alpha}_{m,h}) \xrightarrow{d} N(0, \Gamma'_{m,h}\Omega_{m,h}\Gamma_{m,h}), \tag{12}$$

where $\xrightarrow{d}$ denotes convergence in distribution, and $\Omega_{m,h}$ is the long-run covariance matrix between $1(Y_{\tau+h} \in \widehat{I}^{\alpha}_{m,\tau+h})$ and $1(Y_{\tau+h} \in \widehat{I}^{\alpha}_{7,\tau+h})$. The matrix $\Gamma_{m,h}$ is defined as:

$$\Gamma_{m,h} \equiv \left[\begin{array}{cc} 2\left(P\left(Y_{\tau+h} \in \widehat{I}^{\alpha}_{m,\tau+h}\right) - \alpha\right) & 2\left(P\left(Y_{\tau+h} \in \widehat{I}^{\alpha}_{7,\tau+h}\right) - \alpha\right) \end{array} \right]'.$$

$\Gamma_{m,h}$ can be estimated consistently by its sample analog $\widehat{\Gamma}_{m,h}$, while $\Omega_{m,h}$ can be estimated by some HAC estimator $\widehat{\Omega}_{m,h}$, such as Newey and West (1987).[11] The test statistic for coverage test is defined as:

$$Ct^{\alpha}_{m,h} \equiv \frac{\sqrt{N(h)}\Delta\widehat{CL}^{\alpha}_{m,h}}{\sqrt{\widehat{\Gamma}'_{m,h}\widehat{\Omega}_{m,h}\widehat{\Gamma}_{m,h}}} \xrightarrow{d} N(0,1) \tag{13}$$

3.2 Test of Equal Empirical Lengths

Define the *length loss* as:

$$LL^{\alpha}_{m,h} \equiv E\left[leb\left(\widehat{I}^{\alpha}_{m,\tau+h}\right)\right], \tag{14}$$

[11] We use Newey and West (1987) for our empirical work, with a window width of 12.

where $leb(\cdot)$ is the Lesbesgue measure. To compare the length loss of RS forecast intervals of economic models $m = 1, 2, ..., 6$ with that of the random walk ($m = 7$), the null and alternative hypotheses are:

$$\begin{aligned} H_0 &: \Delta LL^{\alpha}_{m,h} \equiv LL^{\alpha}_{7,h} - LL^{\alpha}_{m,h} = 0 \\ H_A &: \Delta LL^{\alpha}_{m,h} \neq 0. \end{aligned}$$

The sample analog of the length loss for model m is defined as:

$$\widehat{LL}^{\alpha}_{m,h} = N(h)^{-1} \sum_{\tau=R}^{T-h} leb(\widehat{I}^{\alpha}_{m,\tau+h}).$$

Directly applying the test of Giacomini and White (2006), we have

$$\sqrt{N(h)}(\Delta\widehat{LL}^{\alpha}_{m,h} - \Delta LL^{\alpha}_{m,h}) \xrightarrow{d} N(0, \Sigma_{m,h}), \tag{15}$$

where $\Sigma_{m,h}$ is the long-run variance of $leb\left(\widehat{I}^{\alpha}_{7,\tau+h}\right) - leb\left(\widehat{I}^{\alpha}_{m,\tau+h}\right)$. Let $\widehat{\Sigma}_{m,h}$ be the HAC estimator of $\Sigma_{m,h}$. The test statistic for empirical length is defined as:

$$Lt^{\alpha}_{m,h} \equiv \frac{\sqrt{N(h)}\Delta\widehat{LL}^{\alpha}_{m,h}}{\sqrt{\widehat{\Sigma}_{m,h}}} \xrightarrow{d} N(0, 1). \tag{16}$$

3.3 Discussion

The coverage accuracy loss function is symmetric in our paper. In practice, an asymmetric loss function may be better when looking for an exchange rate forecast model to help make policy or business decisions. Under-coverage is arguably a more severe problem than over-coverage in practical situations. However, the focus of this paper is the disconnect between economic fundamentals and the exchange rate. Our goal is to investigate which model comes closer to the data: the random walk or fundamental-based models. It is not critical in this case whether coverage inaccuracy comes from the under- or over-coverage. We acknowledge that the use of symmetric coverage loss remains a caveat, especially since we are using the coverage accuracy test as a pre-test for the tests of length. Clearly, there is a tradeoff between the empirical coverage and the length of forecast intervals. Given the same center,[12] intervals with under-coverage have shorter lengths than intervals with over-coverage. In this case, the length test is in favor of models that systematically under-cover the targeted nominal coverage when compared to a model that systematically over-covers. This problem

[12]Center here means the half way point between the upper and lower bound for a given interval.

cannot be detected by the coverage accuracy test with symmetric loss function because over- and under-coverage are treated equally. However, our results in section 4 show that there is no evidence of systematic under-coverage for the economic models considered in this paper. For instance, in Table 1, one-month-ahead ($h = 1$) forecast intervals over-cover the nominal coverage (90%) for eight out of twelve exchange rates.[13] Note that under-coverage does not guarantee shorter intervals either in our paper, because forecast intervals of different models usually have different centers.[14] In addition, we also compare the coverage of economic models and the random walk directly in an exercise not reported in this paper. There is no evidence that the coverage of economic models is systematically smaller than that of the random walk.[15]

As we have mentioned, comparisons across models can also be done at the distribution level. We choose interval forecasts for two reasons. First, interval forecasts have been widely used and reported by the practitioners. For instance, the Bank of England calculates forecast intervals of inflation in its inflation reports. Second, compared to evaluation metrics for density forecasts, the empirical coverage and length loss functions of interval forecasts, and the subsequent interpretations of test rejection/acceptance are more intuitive.

4 Results

We apply RS forecast intervals for each model for a given nominal coverage of $\alpha = 0.9$. There is no particular reason why we chose 0.9 as the nominal coverage. Some auxiliary results show that our qualitative findings do not depend on the choice of α. Due to different sample sizes across countries, we choose different sizes for the rolling window (R) for different countries. Our rule is very simple: for countries with $T \geq 300$, we choose $R = 200$, otherwise we set $R = 150$.[16] Again, from our experience, tampering with R does not change the qualitative results, unless R is chosen to be unusually big or small.

For time horizons $h = 1, 3, 6, 9, 12$ and models $m = 1, ..., 7$, we construct a sequence of $N(h)$ 90% forecast intervals $\{\widehat{I}^{0.9}_{m,\tau+h}\}^{T-h}_{\tau=R}$ for the h-horizon change of the exchange rate $s_{t+h} - s_t$. Then we compare economic models and the random walk by computing empirical coverages, lengths and test statistics $Ct^{0.9}_{m,h}$ and $Lt^{0.9}_{m,h}$ as described in section 3. We first report the results of our benchmark model. After that, results of alternative models are reported and discussed.

[13]These nine exchange rates are the Danish Kroner, the French Franc, the Deutschmark, the Japanese Yen, the Dutch Guilder, the Portuguese Escudo, the Swiss Franc, and the British pound. Similar results hold at other horizons.

[14]When comparing the intervals for $S_{\tau+h} - S_\tau$, the random walk model builds the forecast interval around 0, while economic model m builds it around $\widehat{\beta}'_{m,h}\mathbf{X}_{m,\tau}$.

[15]Results are available upon request.

[16]The only exception is Portugal, where only 192 data points were available. In this case, we choose R = 120.

4.1 Results of Benchmark Model

Table 1 shows results of the benchmark Taylor rule model. For each time horizon h and exchange rate, the first column (Cov.) reports the empirical coverage for the given nominal coverage of 90%. The second column (Leng.) reports the length of forecast intervals (the distance between upper and lower bounds). The length is multiplied by 100 and therefore expressed in terms of the percentage change of the exchange rate. For instance, the empirical coverage and length of the one-month-ahead forecast interval for the Australian dollar are 0.895 and 7.114, respectively. It means that on average, with a chance of 89.5%, the one-month-ahead change of AUD/USD lies in an interval with length 7.114%. We use superscripts a, b, and c to denote that the null hypothesis of equal empirical coverage accuracy/length is rejected in favor of the Taylor rule model at a confidence level of 10%, 5%, and 1% respectively. Superscripts x, y, and z are used for rejections in favor of the random walk analogously.

We summarize our findings in three panels. In the first panel (*(1) Coverage Test*), the row of "Model Better" reports the number of exchange rates that the Taylor rule model has more accurate empirical coverages than the random walk. The row of "RW Better" reports the number of exchange rates for which the random walk outperforms the Taylor rule model under the same criterion. In the second panel (*(2) Length Test Given Equal Coverage Accuracy*), a better model is the one with tighter forecast intervals given equal coverage accuracy. In the last panel (*(1)+(2)*), a better model is the one with either more accurate coverages, or tighter forecast intervals given equal coverage accuracy.

For most exchange rates and time horizons, the Taylor rule model and the random walk model have statistically equally accurate empirical coverages. The null hypothesis of equal coverage accuracy is rejected in only six out of sixty tests (two rejections each at horizons 6, 9, and 12). Five out of six rejections are in favor of the Taylor rule model. That is, the empirical coverage of the Taylor rule model is closer to the nominal coverage than those of the random walk. However, based on the number of rejections (5) in a total of sixty tests, there is no strong evidence that the Taylor rule model can generate more accurate empirical coverages than the random walk.

In cases where the Taylor rule model and the random walk have equally accurate empirical coverages, the Taylor rule model generally has equal or significantly tighter forecast intervals than the random walk. In forty-two out of fifty-four cases, the null hypothesis of equally tight forecast intervals is rejected in favor of the Taylor rule model. In contrast, the null hypothesis is rejected in only three cases in favor of the random walk. The evidence of exchange rate predictability is more pronounced at longer horizons. At horizon twelve ($h = 12$), for all cases where empirical coverage accuracies between the random walk and the Taylor rule model are statistically equivalent, the Taylor rule model has significantly tighter forecast intervals than the random walk.

As for each individual exchange rate, the benchmark Taylor rule model works best for the French Franc, the Deutschmark, the Dutch Guilder, the Swedish Krona, and the British Pound: for all time horizons, the model has tighter forecast intervals than the random walk, while their empirical coverages are statistically equally accurate. The Taylor rule model performs better than the random walk in most horizons for the remaining exchange rates except the Portuguese Escudo, for which the Taylor rule model outperforms the random walk only at long horizons.

4.2 Results of Alternative Models

Five alternative economic models are also compared with the random walk: three alternative Taylor rule models that are studied in Molodtsova and Papell (2009), the PPP model, and the monetary model. Tables 2-6 report results of these alternative models.

In general, results of coverage tests do not show strong evidence that economic models can generate more accurate coverages than the random walk at either short or long horizons. However, after considering length tests, we find that economic models perform better than the random walk, especially at long horizons. Taylor rule model 4 (the benchmark model with interest rate smoothing Table 4) and the PPP model (Table 5) perform the best among alternative models. Results of these two models are very similar to that of the benchmark Taylor rule model. At horizon twelve, both models outperform the random walk for most exchange rates under our out-of-sample forecast interval evaluation criteria. The performance of Taylor rule model 2 (Table 2) and 3 (Table 3) is relatively less impressive than other models, but for more than half of the exchange rates, the economic models outperform the random walk at several horizons in out-of-sample interval forecasts.

Comparing the benchmark Taylor rule model, the PPP model and the monetary model, the performance of the PPP model (Table 5) is worse than the other two models at short horizons. Compared to the Taylor rule and PPP models, the monetary model outperforms the random walk for a smaller number of exchange rates at horizons 6, 9, and 12. Overall, the benchmark Taylor rule model seems to perform slightly better than the monetary and PPP models. Molodtsova and Papell (2009) find similar results in their point forecasts.

Table 7 shows results with heterogeneous Taylor rules.[17] In this model, we relaxed the assumption that the Taylor rule coefficients are the same in the home and foreign countries. We replace $\pi_t - \pi_t^*$ and $y_t^{gap} - y_t^{gap*}$ in matrix $\mathbf{X}_{1,t}$ of the benchmark model with $\hat{\delta}_\pi \pi_t - \hat{\delta}_\pi^* \pi_t^*$ and $\hat{\delta}_y y_t^{gap} - \hat{\delta}_y^* y_t^{gap*}$, where $\hat{\delta}_\pi$, $\hat{\delta}_\pi^*$, $\hat{\delta}_y$, and $\hat{\delta}_y^*$

[17]See Appendix A.3 for details.

are Taylor rule coefficients estimated from the data of home and foreign countries. The main findings in the benchmark model also hold in Table 7.

4.3 Discussion

After Mark (1995) first documents exchange rate predictability at long horizons, long-horizon exchange rate predictability has become a very active area in the literature. With panel data, Engel, Mark, and West (2007) recently show that the long-horizon predictability of the exchange rate is relatively robust in the exchange rate forecasting literature. We find similar results in our interval forecasts. The evidence of long-horizon predictability seems robust across different models and currencies when both empirical coverage and length tests are used. At horizon twelve, all economic models outperform the random walk for six exchange rates: the Australian Dollar, French Franc, Italian Lira, Japanese Yen, Swedish Krona, and the British Pound in the sense that interval lengths of economic models are smaller than those of the random walk, given equivalent coverage accuracy. This is true only for the Danish Kroner and Swiss Franc at horizon one. We also notice that there is no clear evidence of long-horizon predictability based on the tests of empirical coverage accuracy only.

Molodtsova and Papell (2009) find strong out-of-sample exchange rate predictability for Taylor rule models even at the short horizon. In our paper, the evidence for exchange rate predictability at short horizons is not very strong. This finding may be a result of some assumptions we have used to simplify our computation. For instance, an α-coverage forecast interval in our paper is always constructed using the $(1-\alpha)/2$ and $(1+\alpha)/2$ quantiles. Alternatively, we can choose quantiles that minimize the length of intervals, given the nominal coverage.[18] In addition, the development of more powerful testing methods may also be helpful. The evidence of exchange rate predictability in Molodtsova and Papell (2009) is partly driven by the testing method recently developed by Clark and West (2006, 2007). We acknowledge that whether or not short-horizon results can be improved remains an interesting question, but do not pursue this in the current paper. The purpose of this paper is to show the connection between the exchange rate and economic fundamentals from an interval forecasting perspective. Predictability either at short- or long-horizons will serve this purpose.

Though we find that economic fundamentals are helpful for forecasting exchange rates, we acknowledge that exchange rate forecasting in practice is still a difficult task. The forecast intervals from economic models are statistically tighter than those of the random walk, but they remain fairly wide. For instance, the distance between the upper and lower bound of three-month-ahead forecast intervals is usually a 20%

[18]See Wu (2009) for more discussion.

change of the exchange rates. Figures 1-3 show the length of forecast intervals generated by the benchmark Taylor rule model and the random walk for the British Pound, the Deutschmark, and the Japanese Yen at different horizons.[19] At the horizon of 12 months, the length of forecast intervals in the Taylor rule model is usually smaller than that in the random walk. However, at shorter horizons, such as 1 month, the difference is quantitatively small.

5 Conclusion

There is a growing strand of literature that uses Taylor rules to model exchange rate movements. Our paper contributes to the literature by showing that Taylor rule fundamentals are useful in forecasting the distribution of exchange rates. We apply Robust Semiparametric forecast intervals of Wu (2009) to a group of Taylor rule models for twelve OECD exchange rates. The forecast intervals generated by the Taylor rule models are in general tighter than those of the random walk, given that these intervals cover the realized exchange rates equally well. The evidence of exchange rate predictability is more pronounced at longer horizons, a result that echoes previous long-horizon studies such as Mark (1995). The benchmark Taylor rule model is also found to perform better than the monetary and PPP models based on out-of-sample interval forecasts.

Though we find some empirical support for the connection between the exchange rate and economic fundamentals, we acknowledge that the detected connection is weak. The reductions of the lengths of forecast intervals are quantitatively small, though they are statistically significant. Forecasting exchange rates remains a difficult task in practice. Engel and West (2005) argue that as the discount factor gets closer to one, present value asset pricing models place greater weight on future fundamentals. Consequently, current fundamentals have very weak forecasting power and exchange rates appear to follow approximately a random walk. Under standard assumptions in Engel and West (2005), the Engel-West theorem does not imply that exchange rates are more predictable at longer horizons or that economic models can outperform the random walk in forecasting exchange rates based on out-of-sample interval forecasts. However, modifications to these assumptions may be able to reconcile the Engel-West explanation with empirical findings of exchange rate predictability. For instance, Engel, Wang, and Wu (2008) find that when there exist stationary, but persistent, unobservable fundamentals, for example risk premium, the Engel-West explanation predicts long-horizon exchange rate predictability in *point forecasts*, though the exchange rate still approximately follows a random walk at short horizons. It would also be of interest to study conditions under which our findings

[19] Figures in other countries show similar patterns. Results are available upon request.

in *interval forecasts* can be reconciled with the Engel-West theorem.

We believe other issues, such as parameter instability (Rossi, 2005), nonlinearity (Kilian and Taylor, 2003), real time data (Faust, Rogers, and Wright, 2003, Molodtsova, Nikolsko-Rzhevskyy, and Papell, 2008a, 2008b), and model selection (Sarno and Valente, forthcoming) are all contributing to the Meese-Rogoff puzzle. Panel data are also found helpful in detecting exchange rate predictability, especially at long horizons. For instance, see Mark and Sul (2001), Engel, Mark, and West (2007), and Rogoff and Stavrakeva (2008). It would be interesting to incorporate these studies into interval forecasting. We leave these extensions for future research.

Table 1: Results of Benchmark Taylor Rule Model

	$h=1$		$h=3$		$h=6$		$h=9$		$h=12$	
	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.
Australian Dollar	0.895	7.114	0.888	14.209^c	0.959	21.140	0.942	26.613	0.963	29.175^c
Canadian Dollar	0.814	3.480	0.794	6.440^c	0.738	8.483^c	0.675	8.669^c	0.596^x	9.707^c
Danish Kroner	0.920	8.676^c	0.939	17.415^c	0.954	26.198	0.922	28.712^c	0.968	37.123^c
French Franc	0.912	8.921^c	0.860	15.728^c	0.928^c	26.007^c	0.957	29.924^c	0.934	36.883^c
Deutschmark	0.927	8.327^c	0.879	18.634^c	0.894	27.923^c	0.960^a	33.734^c	0.969	39.618^c
Italian Lira	0.899	8.291^c	0.875	18.305	0.910	26.788^c	0.846	32.545^c	0.874	37.151^c
Japanese Yen	0.915	9.633^z	0.909	19.762	0.892	28.451^c	0.932	33.793^c	0.883	37.728^c
Dutch Guilder	0.917	8.726^c	0.907	18.615^c	0.933	27.458^c	0.941^b	30.902^c	0.959^a	40.177^c
Portuguese Escudo	0.901	8.580^z	0.928	18.758^z	0.894^c	23.552^b	0.825	27.086	0.867	32.092^c
Swedish Krona	0.839	7.360^c	0.860	15.413^c	0.874	23.930^c	0.820	28.090^c	0.834	37.432^c
Swiss Franc	0.947	9.358^c	0.916	19.655	0.963	26.553^c	0.963	30.780^c	0.899	35.008^c
British Pound	0.919	8.413^a	0.923	16.592^c	0.912	23.317^c	0.900	26.942^c	0.855	25.905^c
(1) Coverage Test†										
Model Better	0		0		2		2		1	
RW Better	0		0		0		0		1	
(2) Length Test Given Equal Coverage Accuracy‡										
Model Better	8		8		8		8		10	
RW Better	2		1		0		0		0	
(1)+(2)§										
Model Better	8		8		10		10		11	
RW Better	2		1		0		0		1	

Note:

–h denotes forecast horizons for monthly data.

–For each horizon (h), the first column (Cov.) reports empirical coverages given a nominal coverage of 90%. The second column (Leng.) reports the length of forecast intervals in terms of percentage change of the exchange rate. Empirical coverages and lengths are averages across $N(h)$ out-of-sample trials.

–Superscripts a, b, c in the column of Cov. (Leng.) denote rejections of equal coverage accuracy (equal length) in favor of the economic model at a 10%, 5% and 1% confidence level respectively. Superscripts x, y, z are defined analogously for rejections in favor of the random walk.

†–In this panel, a better model is the one with more accurate empirical coverages. RW is the abbreviation of Random Walk.

‡–In this panel, a better model is the one with tighter forecast intervals given equal coverage accuracy.

§–In this panel, a better model is the one with either more accurate coverages or tighter forecast intervals given equal coverage accuracy.

Table 2: Results of Taylor Rule Model Two

	$h=1$		$h=3$		$h=6$		$h=9$		$h=12$	
	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.
Australian Dollar	0.884	7.146^y	0.899	15.086^c	0.928	21.327	0.901	27.329	0.872	30.815^b
Canadian Dollar	0.825	3.442^c	0.783	6.321^c	0.814	8.490^c	0.858	10.034^c	0.825	11.921^c
Danish Kroner	0.915	8.753^a	0.939	17.764^c	0.954	27.479^z	0.953	33.426^y	0.942	40.717
French Franc	0.951	9.042^c	0.930	18.783	0.949^c	29.161^c	0.936	34.994^c	0.868	42.081^c
Deutschmark	0.917	9.090	0.869	19.217	0.952	29.746	0.941^a	39.093^z	0.980	44.571^z
Italian Lira	0.928	9.196	0.875	18.322	0.895	26.926^c	0.869	35.883^c	0.898^a	41.235^z
Japanese Yen	0.915	9.568^x	0.914	19.734	0.912	29.344^c	0.937	36.834	0.942	44.385^c
Dutch Guilder	0.908	8.586^c	0.888	18.782^c	0.962	29.777	0.990	39.507^z	0.990	47.514^z
Portuguese Escudo	0.916	8.005	0.957	17.924^z	0.909^c	24.270^b	0.889	28.533^z	0.883^a	35.338^c
Swedish Krona	0.867	7.624	0.860	16.132	0.857	24.500^c	0.837	32.825	0.811	37.772^c
Swiss Franc	0.941	9.953^b	0.928	20.105	0.982	29.758^c	0.994	38.267^z	0.962	45.965^z
British Pound	0.919	8.627^z	0.933	17.334^c	0.922	26.227^c	0.937	31.044^c	0.957	36.397^c
(1) Coverage Test†										
Model Better	0		0		2		1		2	
RW Better	0		0		0		0		0	
(2) Length Test Given Equal Coverage Accuracy‡										
Model Better	5		5		6		4		6	
RW Better	3		1		1		4		3	
(1)+(2)§										
Model Better	5		5		8		5		8	
RW Better	3		1		1		4		3	

Note:

–h denotes forecast horizons for monthly data.

–For each horizon (h), the first column (Cov.) reports empirical coverages given a nominal coverage of 90%. The second column (Leng.) reports the length of forecast intervals in terms of percentage change of the exchange rate. Empirical coverages and lengths are averages across $N(h)$ out-of-sample trials.

–Superscripts a, b, c in the column of Cov. (Leng.) denote rejections of equal coverage accuracy (equal length) in favor of the economic model at a 10%, 5% and 1% confidence level respectively. Superscripts x, y, z are defined analogously for rejections in favor of the random walk.

†–In this panel, a better model is the one with more accurate empirical coverages. RW is the abbreviation of Random Walk.

‡–In this panel, a better model is the one with tighter forecast intervals given equal coverage accuracy.

§–In this panel, a better model is the one with either more accurate coverages or tighter forecast intervals given equal coverage accuracy.

Table 3: Results of Taylor Rule Model Three

	$h = 1$		$h = 3$		$h = 6$		$h = 9$		$h = 12$	
	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.
Australian Dollar	0.884	7.229^z	0.899	15.055^c	0.881	21.055	0.885	26.359^c	0.867	30.234^c
Canadian Dollar	0.831	3.453^b	0.789	6.408^c	0.814	8.629^c	0.864	10.220^c	0.819	11.971^c
Danish Kroner	0.920	8.753^b	0.934	17.649^c	0.949	27.523^z	0.948	33.307	0.936	40.070
French Franc	0.951	9.171	0.740	14.488^c	0.722	20.313^c	0.915	35.562^a	0.813	41.350^c
Deutschmark	0.908	9.020	0.897	19.303	0.914	29.676	0.901^c	37.291^a	0.878	44.761^z
Italian Lira	0.928	8.900^a	0.875	17.206^c	0.872	26.674^c	0.839	34.819^c	0.787	39.569^c
Japanese Yen	0.905	9.179^c	0.878	18.907^c	0.892	25.883^c	0.927	31.259^c	0.894	37.049^c
Dutch Guilder	0.927	8.910	0.907	19.204^a	0.952	29.426^a	0.951^a	36.896^c	0.959	46.321^z
Portuguese Escudo	0.930	7.961	0.942	16.883^c	0.955^a	23.786	0.905	26.620^c	0.850	33.745^c
Swedish Krona	0.867	7.316^c	0.848	15.017^c	0.840	23.241^c	0.791	29.265^c	0.757	33.751^c
Swiss Franc	0.929	9.761^b	0.922	19.517^c	0.939	28.437^b	0.926^c	37.519	0.911	45.619
British Pound	0.929	8.239^a	0.939	16.213^c	0.927	23.951^c	0.905	28.720^c	0.952	34.900^c
(1) Coverage Test†										
Model Better	0		0		1		3		0	
RW Better	0		0		0		0		0	
(2) Length Test Given Equal Coverage Accuracy‡										
Model Better	7		11		8		8		8	
RW Better	1		0		1		0		2	
(1)+(2)§										
Model Better	7		11		9		11		8	
RW Better	1		0		1		0		2	

Note:

–h denotes forecast horizons for monthly data.

–For each horizon (h), the first column (Cov.) reports empirical coverages given a nominal coverage of 90%. The second column (Leng.) reports the length of forecast intervals in terms of percentage change of the exchange rate. Empirical coverages and lengths are averages across $N(h)$ out-of-sample trials.

–Superscripts a, b, c in the column of Cov. (Leng.) denote rejections of equal coverage accuracy (equal length) in favor of the economic model at a 10%, 5% and 1% confidence level respectively. Superscripts x, y, z are defined analogously for rejections in favor of the random walk.

†–In this panel, a better model is the one with more accurate empirical coverages. RW is the abbreviation of Random Walk.

‡–In this panel, a better model is the one with tighter forecast intervals given equal coverage accuracy.

§–In this panel, a better model is the one with either more accurate coverages or tighter forecast intervals given equal coverage accuracy.

Table 4: Results of Taylor Rule Model Four

	$h=1$		$h=3$		$h=6$		$h=9$		$h=12$	
	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.
Australian Dollar	0.895	7.119	0.888	14.424^{c}	0.928	20.966	0.927	25.304^{c}	0.872	27.492^{c}
Canadian Dollar	0.814	3.425^{c}	0.771	6.366^{c}	0.698	8.019^{c}	0.651	8.631^{c}	0.494^{y}	7.693^{c}
Danish Kroner	0.920	8.703^{c}	0.929	17.536^{c}	0.964	26.025	0.984^{x}	30.891^{c}	0.963	36.545^{c}
French Franc	0.892	8.361^{c}	0.870	16.079^{c}	0.938^{c}	25.950^{c}	0.883	30.016^{c}	0.791	35.755^{c}
Deutschmark	0.927	8.314^{c}	0.879	18.652^{c}	0.894	26.803^{c}	0.931^{c}	33.350^{c}	0.969	36.393^{c}
Italian Lira	0.891	8.663^{c}	0.838	17.575^{c}	0.865	26.387^{c}	0.746	32.270^{c}	0.724	36.422^{c}
Japanese Yen	0.905	9.157^{c}	0.863	18.708^{c}	0.866	24.417^{c}	0.869	28.730^{c}	0.851	31.470^{c}
Dutch Guilder	0.936	8.815	0.897	18.368^{c}	0.914	26.700^{c}	0.931^{c}	30.036^{c}	0.796	29.462^{c}
Portuguese Escudo	0.901	8.525^{z}	0.913^{a}	17.110	0.939^{c}	23.461^{c}	0.889	27.096^{a}	0.917	28.778^{c}
Swedish Krona	0.861	7.289^{c}	0.860	15.321^{c}	0.869	23.340^{c}	0.773	27.198^{c}	0.728	31.843^{c}
Swiss Franc	0.947	9.149^{c}	0.940	19.782^{a}	0.811	22.796^{c}	0.808	26.148^{c}	0.671	26.683^{c}
British Pound	0.919	8.113^{a}	0.913	15.765^{c}	0.875	21.679^{c}	0.825	27.312^{c}	0.839	29.081^{c}
(1) Coverage Test†										
Model Better	0		1		2		2		0	
RW Better	0		0		0		1		1	
(2) Length Test Given Equal Coverage Accuracy‡										
Model Better	9		11		8		9		11	
RW Better	1		0		0		0		0	
(1)+(2)§										
Model Better	9		12		10		11		11	
RW Better	1		0		0		1		1	

Note:

–h denotes forecast horizons for monthly data.

–For each horizon (h), the first column (Cov.) reports empirical coverages given a nominal coverage of 90%. The second column (Leng.) reports the length of forecast intervals in terms of percentage change of the exchange rate. Empirical coverages and lengths are averages across $N(h)$ out-of-sample trials.

–Superscripts a, b, c in the column of Cov. (Leng.) denote rejections of equal coverage accuracy (equal length) in favor of the economic model at a 10%, 5% and 1% confidence level respectively. Superscripts x, y, z are defined analogously for rejections in favor of the random walk.

†–In this panel, a better model is the one with more accurate empirical coverages. RW is the abbreviation of Random Walk.

‡–In this panel, a better model is the one with tighter forecast intervals given equal coverage accuracy.

§–In this panel, a better model is the one with either more accurate coverages or tighter forecast intervals given equal coverage accuracy.

Table 5: Results of Purchasing Power Parity Model

	$h=1$		$h=3$		$h=6$		$h=9$		$h=12$	
	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.
Australian Dollar	0.895	7.114^z	0.883	15.558	0.912	21.311	0.880	26.120^c	0.856	30.316^c
Canadian Dollar	0.819	3.570^z	0.806	6.872	0.767	9.615	0.728	11.078^c	0.615	12.306^c
Danish Kroner	0.925	8.697^c	0.939	18.333	0.938	25.887^c	0.937	31.673^c	0.957	37.447^c
French Franc	0.922	8.904^c	0.940	18.029^c	0.918^c	25.786^c	0.904	29.789^c	0.802	34.209^c
Deutschmark	0.936	9.079	0.935	18.797^c	0.942	27.677^c	1.000	33.585^c	0.990	40.570^c
Italian Lira	0.913	8.780^c	0.868	17.767^c	0.827	25.044^c	0.769	30.190^c	0.772	34.806^c
Japanese Yen	0.920	9.662^z	0.899	19.903	0.912	28.689^c	0.932	33.973^c	0.899	38.568^c
Dutch Guilder	0.936	8.862^y	0.935	18.904^c	0.952	27.928^c	1.000	33.468^c	0.990	41.812^c
Portuguese Escudo	0.916	8.421^y	0.928	19.027^y	0.924^c	23.918	0.857	27.450	0.867	32.467^c
Swedish Krona	0.861	7.541^c	0.876	16.089	0.886	24.345^c	0.855	31.744^b	0.799	37.943^c
Swiss Franc	0.941	9.708^c	0.946	19.694^b	0.976	27.197^c	0.950^b	31.725^c	0.880	36.235^c
British Pound	0.934	8.571^y	0.933	16.954^c	0.932	24.064^c	0.947	28.761^c	0.925^a	31.372^c
(1) Coverage Test†										
Model Better	0		0		2		1		1	
RW Better	0		0		0		0		0	
(2) Length Test Given Equal Coverage Accuracy‡										
Model Better	5		6		8		10		11	
RW Better	6		1		0		0		0	
(1)+(2)§										
Model Better	5		6		10		11		12	
RW Better	6		1		0		0		0	

Note:

–h denotes forecast horizons for monthly data.

–For each horizon (h), the first column (Cov.) reports empirical coverages given a nominal coverage of 90%. The second column (Leng.) reports the length of forecast intervals in terms of percentage change of the exchange rate. Empirical coverages and lengths are averages across $N(h)$ out-of-sample trials.

–Superscripts a, b, c in the column of Cov. (Leng.) denote rejections of equal coverage accuracy (equal length) in favor of the economic model at a 10%, 5% and 1% confidence level respectively. Superscripts x, y, z are defined analogously for rejections in favor of the random walk.

†–In this panel, a better model is the one with more accurate empirical coverages. RW is the abbreviation of Random Walk.

‡–In this panel, a better model is the one with tighter forecast intervals given equal coverage accuracy.

§–In this panel, a better model is the one with either more accurate coverages or tighter forecast intervals given equal coverage accuracy.

Table 6: Results of Monetary Model

	$h=1$		$h=3$		$h=6$		$h=9$		$h=12$	
	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.
Australian Dollar	0.879	7.108	0.848	15.151	0.830	20.090	0.770	24.642^c	0.745	30.099^b
Canadian Dollar	0.842	4.027^x	0.829	7.492	0.744	10.518	0.645	10.689^b	0.675	12.993^c
Danish Kroner	0.905	8.770^b	0.893	17.943	0.897	25.017^c	0.853	28.581^c	0.809	32.504^c
French Franc	0.922	8.791^c	0.910	18.237^c	0.949^b	26.322^c	0.957	31.032^c	0.956	35.971^c
Deutschmark	0.908	8.595	0.841	17.436^c	0.808	24.622^c	0.772	28.052^c	0.704	31.364^c
Italian Lira	0.913	8.858^c	0.882	18.439^b	0.925	26.585^c	0.931	34.857^c	0.913	40.885^c
Japanese Yen	0.930	9.556	0.919	19.374^c	0.887	28.614^c	0.864	33.401^c	0.809	36.520^c
Dutch Guilder	0.917	8.753^a	0.916	19.408	0.962	29.149^b	0.970	38.173	0.898^c	41.716^c
Portuguese Escudo	0.901	8.086	0.986	18.484	0.985	24.744	0.984	27.230	1.000	34.222
Swedish Krona	0.850	7.504^a	0.848	17.097^x	0.811	23.878^c	0.826	31.287	0.805	34.710^c
Swiss Franc	0.905	9.078^c	0.820	17.020^c	0.732	21.212^c	0.609	22.741^c	0.513^x	23.225^c
British Pound	0.909	7.811^c	0.882	14.945^c	0.787	20.788^c	0.677	24.311^c	0.656	26.374^c
(1) Coverage Test†										
Model Better	0		0		1		0		1	
RW Better	0		0		0		0		1	
(2) Length Test Given Equal Coverage Accuracy‡										
Model Better	7		6		8		9		9	
RW Better	1		1		0		0		0	
(1)+(2)§										
Model Better	7		6		9		9		10	
RW Better	1		1		0		0		1	

Note:

–h denotes forecast horizons for monthly data.

–For each horizon (h), the first column (Cov.) reports empirical coverages given a nominal coverage of 90%. The second column (Leng.) reports the length of forecast intervals in terms of percentage change of the exchange rate. Empirical coverages and lengths are averages across $N(h)$ out-of-sample trials.

–Superscripts a, b, c in the column of Cov. (Leng.) denote rejections of equal coverage accuracy (equal length) in favor of the economic model at a 10%, 5% and 1% confidence level respectively. Superscripts x, y, z are defined analogously for rejections in favor of the random walk.

†–In this panel, a better model is the one with more accurate empirical coverages. RW is the abbreviation of Random Walk.

‡–In this panel, a better model is the one with tighter forecast intervals given equal coverage accuracy.

§–In this panel, a better model is the one with either more accurate coverages or tighter forecast intervals given equal coverage accuracy.

Table 7: Results of Heterogenous Taylor Rules

	$h=1$		$h=3$		$h=6$		$h=9$		$h=12$	
	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.	Cov.	Leng.
Australian Dollar	0.915	7.155	0.909	14.690^c	0.959	20.547	0.963^y	26.364	0.947	29.583^c
Canadian Dollar	0.825	3.526	0.794	6.525^c	0.797	9.040^c	0.787	10.370^c	0.693	11.352^c
Danish Kroner	0.915	8.548^c	0.929	17.930^c	0.938	25.328^c	0.890	30.504^c	0.904	36.624^c
French Franc	0.912	8.864^c	0.880	15.970^c	0.845	20.693^c	0.968	30.436^c	0.714	22.789^c
Deutschmark	0.917	8.605^c	0.907	18.356^c	0.894	28.121^c	0.911^c	31.378^c	0.939	33.779^c
Italian Lira	0.913	8.659^c	0.890	18.664	0.887	25.840^c	0.831	32.037^c	0.693	32.236^c
Japanese Yen	0.920	9.637^z	0.888	19.352^b	0.871	28.018^c	0.932	33.388^c	0.878	36.859^c
Dutch Guilder	0.936	8.851	0.916	18.822^c	0.942	27.259^c	0.970	31.410^c	0.990	39.882^c
Portuguese Escudo	0.916	8.881^z	0.870	17.651	0.758	18.730^c	0.746	23.852^c	0.600	20.593^c
Swedish Krona	0.828	7.428^c	0.854	15.658^c	0.903	24.315^c	0.861	29.866^c	0.876	36.235^c
Swiss Franc	0.935	9.731^c	0.940	19.797	0.970	27.227^c	0.969	32.179^c	0.937	36.567^c
British Pound	0.919	8.350	0.908	16.774^c	0.828	20.811^c	0.783	23.105^c	0.720	23.286^c
(1) Coverage Test†										
Model Better	0		0		0		1		0	
RW Better	0		0		0		1		0	
(2) Length Test Given Equal Coverage Accuracy‡										
Model Better	6		9		11		10		12	
RW Better	2		0		0		0		0	
(1)+(2)§										
Model Better	6		9		11		11		12	
RW Better	2		0		0		1		0	

Note:

–h denotes forecast horizons for monthly data.

–For each horizon (h), the first column (Cov.) reports empirical coverages given a nominal coverage of 90%. The second column (Leng.) reports the length of forecast intervals in terms of percentage change of the exchange rate. Empirical coverages and lengths are averages across $N(h)$ out-of-sample trials.

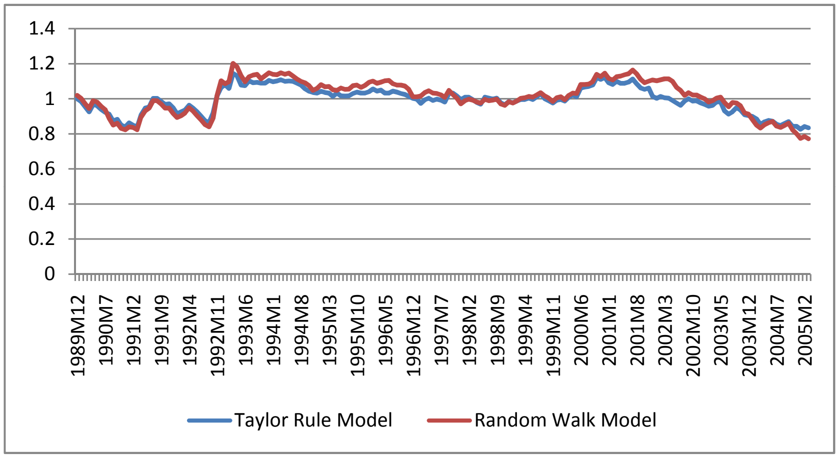
–Superscripts a, b, c in the column of Cov. (Leng.) denote rejections of equal coverage accuracy (equal length) in favor of the economic model at a 10%, 5% and 1% confidence level respectively. Superscripts x, y, z are defined analogously for rejections in favor of the random walk.

†–In this panel, a better model is the one with more accurate empirical coverages. RW is the abbreviation of Random Walk.

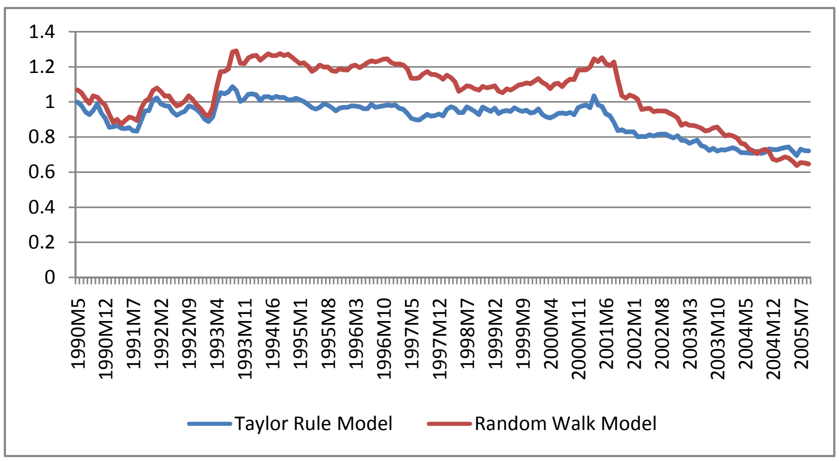
‡–In this panel, a better model is the one with tighter forecast intervals given equal coverage accuracy.

§–In this panel, a better model is the one with either more accurate coverages or tighter forecast intervals given equal coverage accuracy.

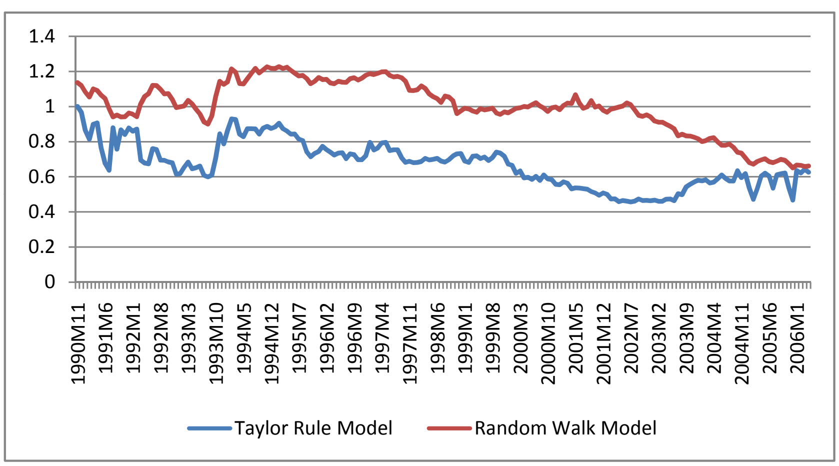
Figure 1: Length of Forecast Intervals for Benchmark Taylor Rule and Random Walk Models (British Pound)



(a) 1-month-ahead forecast



(b) 6-month-ahead forecast

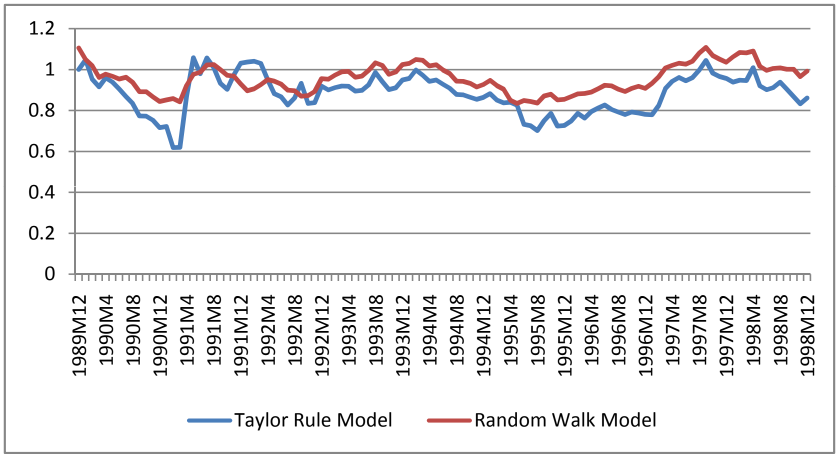


(c) 12-month-ahead forecast

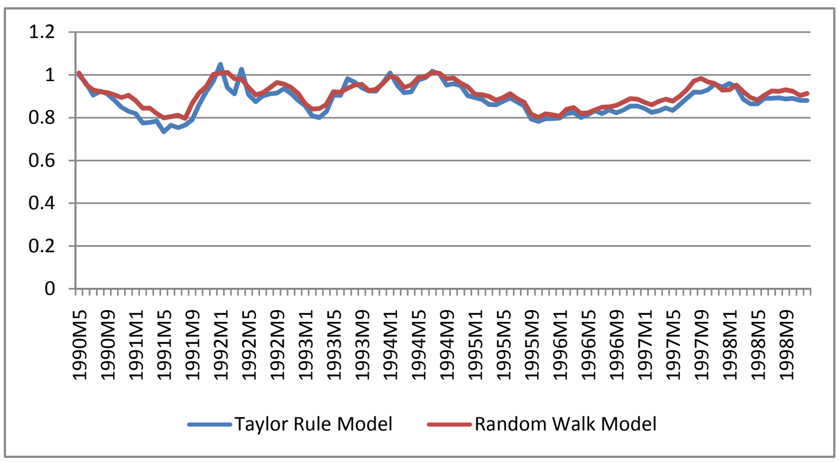
Note:

In each chart, the length of forecast intervals is normalized by the first observation of the benchmark Taylor rule model.

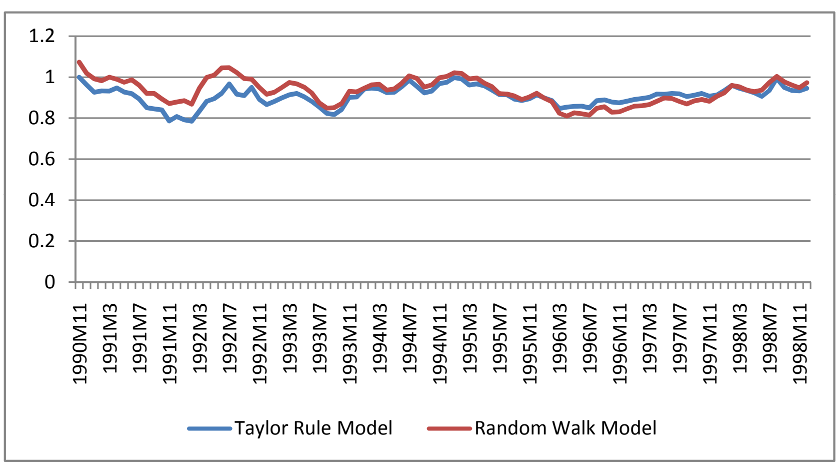
Figure 2: Length of Forecast Intervals for Benchmark Taylor Rule and Random Walk Models (Deutschmark)



(a) 1-month-ahead forecast



(b) 6-month-ahead forecast

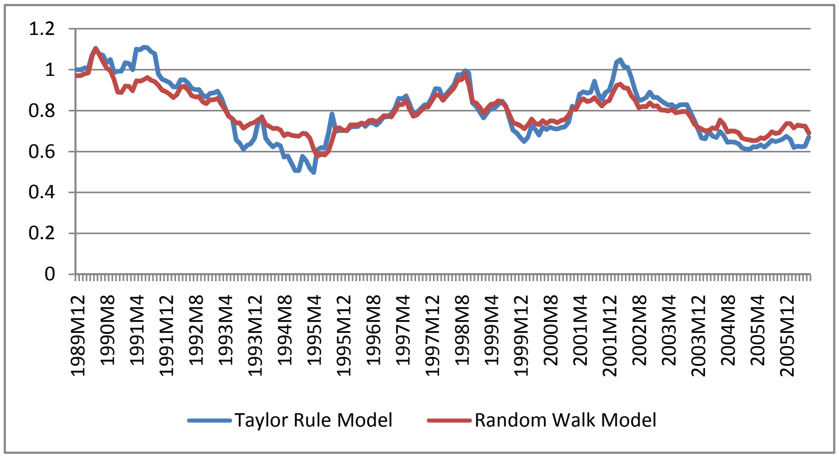


(c) 12-month-ahead forecast

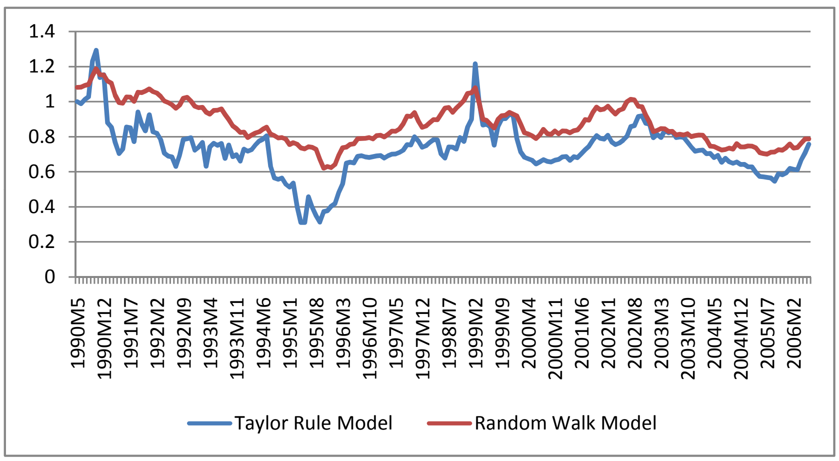
Note:

In each chart, the length of forecast intervals is normalized by the first observation of the benchmark Taylor rule model.

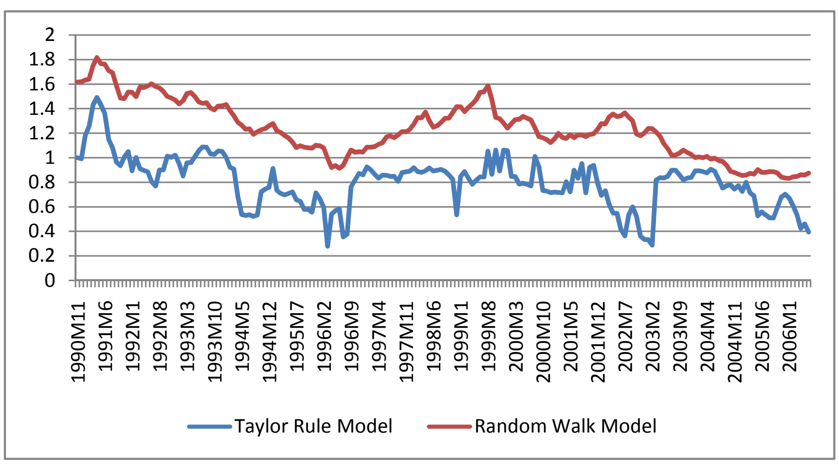
Figure 3: Length of Forecast Intervals for Benchmark Taylor Rule and Random Walk Models (Japanese Yen)



(a) 1-month-ahead forecast



(b) 6-month-ahead forecast



(c) 12-month-ahead forecast

Note:

In each chart, the length of forecast intervals is normalized by the first observation of the benchmark Taylor rule model.

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APPENDIX

A.1 Monetary and Taylor Rule Models

In this section, we describe the monetary and Taylor rule models used in the paper.

A.1.1 Monetary Model

Assume the money market clearing condition in the home country is:

$$m_t = p_t + \gamma y_t - \alpha i_t + v_t,$$

where m_t is the log of money supply, p_t is the log of aggregate price, i_t is the nominal interest rate, y_t is the log of output, and v_t is the money demand shock. A symmetric condition holds in the foreign country and we use an asterisk in superscript to denote variables in the foreign country. Subtracting the foreign money market clearing condition from the home, we have:

$$i_t - i_t^* = \frac{1}{\alpha}\left[-(m_t - m_t^*) + (p_t - p_t^*) + \gamma(y_t - y_t^*) + (v_t - v_t^*)\right]. \tag{A.1.1}$$

The nominal exchange rate is equal to its purchasing power value plus the real exchange rate:

$$s_t = p_t - p_t^* + q_t. \tag{A.1.2}$$

The uncovered interest rate parity in financial market takes the form:

$$E_t s_{t+1} - s_t = i_t - i_t^* + \rho_t, \tag{A.1.3}$$

where ρ_t is the uncovered interest rate parity shock. Substituting equations (A.1.1) and (A.1.2) into (A.1.3), we have

$$s_t = (1-b)\left[m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (v_t - v_t^*)\right] - b\rho_t + bE_t s_{t+1}, \tag{A.1.4}$$

where $b = \alpha/(1+\alpha)$. Solving s_t recursively and applying the "no-bubbles" condition, we have:

$$s_t = E_t \left\{ (1-b) \sum_{j=0}^{\infty} b^j \left[m_{t+j} - m_{t+1}^* - \gamma(y_{t+j} - y_{t+1}^*) + q_{t+1} - (v_{t+j} - v_{t+j}^*) \right] - b \sum_{j=1}^{\infty} b^j \rho_{t+j} \right\}. \quad \text{(A.1.5)}$$

In the standard monetary model, such as Mark (1995), purchasing power parity ($q_t = 0$) and uncovered interest rate parity hold ($\rho_t = 0$). Furthermore, it is assumed that the money demand shock is zero ($v_t = v_t^* = 0$) and $\gamma = 1$. Equation (A.1.5) reduces to:

$$s_t = E_t \left\{ (1-b) \sum_{j=0}^{\infty} b^j \left(m_{t+j} - m_{t+j}^* - (y_{t+j} - y_{t+j}^*) \right) \right\}.$$

A.1.2 Taylor Rule Model

We follow Engel and West (2005) to assume that both countries follow the Taylor rule and the foreign country targets the exchange rate in its Taylor rule. The interest rate differential is:

$$i_t - i_t^* = \delta_s(s_t - \bar{s}_t^*) + \delta_y(y_t^{gap} - y_t^{gap*}) + \delta_\pi(\pi_t - \pi_t^*) + v_t - v_t^*, \quad \text{(A.1.6)}$$

where $\bar{s}_t^*$ is the targeted exchange rate. Assume that monetary authorities target the PPP level of the exchange rate: $\bar{s}_t^* = p_t - p_t^*$. Substituting this condition and the interest rate differential into the UIP condition, we have:

$$s_t = (1-b)(p_t - p_t^*) - b\left[\delta_y(y_t^{gap} - y_t^{gap*}) + \delta_\pi(\pi_t - \pi_t^*) + v_t - v_t^*\right] - b\rho_t + bE_t s_{t+1}, \quad \text{(A.1.7)}$$

where $b = \frac{1}{1+\delta_s}$. Assuming that uncovered interest rate parity holds ($\rho_t = 0$) and monetary shocks are zero, equation (A.1.7) reduces to the benchmark Taylor rule model in our paper:

$$s_t = E_t \left\{ (1-b) \sum_{j=0}^{\infty} b^j (p_{t+j} - p_{t+j}^*) - b \sum_{j=0}^{\infty} b^j (\delta_y(y_{t+j}^{gap} - y_{t+j}^{gap*}) + \delta_\pi(\pi_{t+j} - \pi_{t+j}^*)) \right\}.$$

A.2 Long-horizon Regressions

In this section, we derive long-horizon regressions for the monetary model and the benchmark Taylor rule model.

A.2.1 Monetary Model

In the monetary model:

$$s_t = E_t \left\{ (1-b) \sum_{j=0}^{\infty} b^j \left(m_{t+j} - m^*_{t+j} - (y_{t+j} - y^*_{t+j}) \right) \right\},$$

where m_t and y_t are logarithms of domestic money stock and output, respectively. The superscript $*$ denotes the foreign country. Money supplies (m_t and m^*_t) and total outputs (y_t and y^*_t) are usually I(1) variables.

The general form considered in Engel, Wang, and Wu(2008) is:

$$\begin{aligned} s_t &= (1-b) \sum_{j=0}^{\infty} b^j E_t \alpha' \mathbf{D}_t \\ (I_n - \Phi(L))\Delta \mathbf{D}_t &= \varepsilon_t \\ E(\varepsilon_{t+j} | \varepsilon_t, \varepsilon_{t-1}, ...) &\equiv E_t(\varepsilon_{t+j}) = 0, \forall j \geq 1, \end{aligned} \tag{A.2.1}$$

where n is the dimension of $\mathbf{D}_t$ and I_n is an $n \times n$ identity matrix. L is the lag operator and $\Phi(L) = \phi_1 L + \phi_2 L^2 + ... + \phi_p L^p$. Assume $\Phi(1)$ is non-diagonal and the covariance matrix of ε_t is given by $\Omega = E_t[\varepsilon_t \varepsilon_t']$. We assume that the change of fundamentals follows a VAR(p) process in our setup. From proposition 1 of Engel, Wang, Wu (2008), we know that for a fixed discount factor b and $p \geq 2$,

$$s_{t+h} - s_t = \beta_h z_t + \delta'_{0,h} \Delta \mathbf{D}_t + ... + \delta'_{p-2,h} \Delta \mathbf{D}_{t-p+2} + \zeta_{t+h}$$

is a correctly specified regression where the regressors and errors do not correlate. In the case of $p = 1$, the long-horizon regressions reduces to

$$s_{t+h} - s_t = \beta_h z_t + \zeta_{t+h}.$$

Following the literature, for instance Mark (1995), we do not include $\Delta \mathbf{D}_t$ and its lags in our long-horizon regressions. The monetary model can be written in the form of (A.2.1) by setting $\mathbf{D}_t = [m_t \quad m^*_t \quad y_t \quad y^*_t]'$, $\alpha = [1 \quad -1 \quad -1 \quad 1]'$. By definition, $z_t = s_t - (m_t - m^*_t) + (y_t - y^*_t)$. This corresponds to $\beta_{m,h} = 1$, $\mathbf{X}_{m,t} = s_t - (m_t - m^*_t) + (y_t - y^*_t)$ in equation (1) of section 3.

A.2.2 Taylor Rule Model

In the Taylor rule model,

$$s_t = E_t \left\{ (1-b) \sum_{j=0}^{\infty} b^j (p_{t+j} - p^*_{t+j}) - b \sum_{j=0}^{\infty} b^j (\delta_y (y^{gap}_{t+j} - y^{gap*}_{t+j}) + \delta_\pi (\pi_{t+j} - \pi^*_{t+j})) \right\},$$

where p_t, y_t^{gap} and π_t are domestic aggregate price, output gap and inflation rate, respectively. δ_y and δ_π are coefficients of the Taylor rule model. The aggregate prices p_t and p_t^* are usually I(1) variables. Inflation and output gap are more likely to be I(0). Engel, Wang, and Wu (2008) consider a setup which includes both stationary and non-stationary variables:

$$\begin{aligned} s_t = (1-b) \sum_{j=0}^{\infty} b^j E_t \left[f_{1t+j} \right] + b \sum_{j=0}^{\infty} b^j E_t \left[f_{2t+j} + u_{2t+j} \right] \\ f_{1t} = \alpha_1' \mathbf{D_t} \sim I(1) \\ f_{2t} = \alpha_2' \mathbf{\Delta D_t} \sim I(0) \\ u_{2t} = \alpha_3' \mathbf{\Delta D_t} \sim I(0) \\ (I_n - \Phi(L)) \Delta \mathbf{X}_t = \varepsilon_t, \end{aligned} \tag{A.2.2}$$

where f_{1t} and f_{2t} (u_{2t}) are observable (unobservable) fundamentals. $\mathbf{\Delta D}_t$ is the first difference of $\mathbf{D}_t$, which contains I(1) economic variables.[20]

From proposition 2 of Engel, Wang, and Wu (2008), we know that for a fixed discount factor b and $h \geq 2$,

$$s_{t+h} - s_t = \tilde{\beta}_h z_t + \sum_{k=0}^{p-1} \tilde{\delta}'_{k,h} \mathbf{\Delta D}_{t-k} + \tilde{\zeta}_{t+h} \tag{A.2.3}$$

is a correctly specified regression, where the regressors and errors do not correlate. In the case of $p = 1$, the long-horizon regressions reduces to:

$$s_{t+h} - s_t = \tilde{\beta}_h z_t + \tilde{\zeta}_{t+h}.$$

[20] To incorporate I(0) economic variables, $\mathbf{D}_t$ contains the levels of I(1) variables and the summation of I(0) variables from negative infinity to time t.

The Taylor rule model can be written into the form of (A.2.2) by setting

$$\mathbf{D}_t = \begin{bmatrix} p_t & p_t^* & \sum_{s=-\infty}^{t} y_s^{gap} & \sum_{s=-\infty}^{t} y_s^{gap*}, \sum_{s=-\infty}^{t} \pi_s & \sum_{s=-\infty}^{t} \pi_s^* \end{bmatrix}'.$$

By definition, $z_t = s_t - p_t + p_t^* + \frac{b}{1-b}(\delta_y(y_t^{gap} - y_t^{gap*}) + \delta_\pi(\pi_t - \pi_t^*))$. This corresponds to $\beta_{m,h} = [1 \quad \frac{b}{1-b}\delta_y \quad \frac{b}{1-b}\delta_\pi]$ and $\mathbf{X}_{m,t} = [q_t \quad y_t^{gap} - y_t^{gap*} \quad \pi_t - \pi_t^*]$, where $q_t = s_t - p_t + p_t^*$ is the real exchange rate. $\beta_{m,h}$ and $\mathbf{X}_{m,t}$ can be defined differently. For instance, $\beta_{m,h} = 1$ and $\mathbf{X}_{m,t} = s_t - p_t + p_t^* + \frac{b}{1-b}(\delta_y(y_t^{gap} - y_t^{gap*}) + \delta_\pi(\pi_t - \pi_t^*))$. Our results do not change qualitatively under this alternative setup.

A.3 Model with Heterogeneous Taylor Rules

In the benchmark model, we assume that the Taylor rule coefficients are the same in the home and foreign countries. In this appendix, we relax the assumption of homogeneous Taylor rules in the benchmark model. It is straightforward to show in this case that the benchmark model changes to:

$$s_{t+h} - s_t = \alpha_h + \beta_h z_t + \varepsilon_{t+h}, \tag{A.3.1}$$

where the deviation of the exchange rate from its equilibrium level is defined as:

$$z_t = s_t - p_t + p_t^* + \frac{b}{1-b}\left[\delta_y y_t^{gap} - \delta_y^* y_t^{gap*} + \delta_\pi \pi_t - \delta_\pi^* \pi_t^*\right]. \tag{A.3.2}$$

According to equation (8), the matrix $\mathbf{X}_{1,t}$ in equation (1) includes economic variables $q_t \equiv s_t + p_t^* - p_t$, $\delta_y y_t^{gap} - \delta_y^* y_t^{gap*}$, and $\delta_\pi \pi_t - \delta_\pi^* \pi_t^*$.[21]

We first estimate the Taylor rules in the home and foreign countries according to equations (2) and (3). Then $q_t \equiv s_t + p_t^* - p_t$, $\hat{\delta}_y y_t^{gap} - \hat{\delta}_y^* y_t^{gap*}$, and $\hat{\delta}_\pi \pi_t - \hat{\delta}_\pi^* \pi_t^*$ are used in the long-horizon regressions and interval forecasts. The results are very similar to the benchmark model and reported in Table 7.

[21] Another option to incorporate heterogenous Taylor rules is to include q_t, y_t^{gap}, y_t^{gap*}, π_t, and π_t^* in $\mathbf{X}_{1,t}$. For instance, see Moldtsova and Papell (forthcoming). However, increasing the number of regressors may cause the "curse of dimensionality" problem for our semiparametric method. To be comparable to our benchmark model, we define $\mathbf{X}_{1,t}$ here such that the number of regressors is the same as in the benchmark model.