Inflation Dynamics: The Role of Public Debt and Policy Regimes[*]

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Abstract ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

We investigate the roles of a time-varying inflation target and monetary and fiscal policy stances on the dynamics of inflation in a DSGE model. Under an active monetary and passive fiscal policy regime, inflation closely follows the path of the inflation target and a stronger reaction of monetary policy to inflation decreases the equilibrium response of inflation to non-policy shocks. In sharp contrast, under an active fiscal and passive monetary policy regime, inflation moves in an opposite direction from the inflation target and a stronger reaction of monetary policy to inflation increases the equilibrium response of inflation to non-policy shocks. Moreover, a weaker response of fiscal policy to debt decreases the response of inflation to non-policy shocks. These results are due to variation in the value of public debt that leads to wealth effects on households. Finally, under a passive monetary and passive fiscal policy regime, both monetary and fiscal policy stances affect inflation dynamics, but because of a role for self-fulfilling beliefs due to equilibrium indeterminacy, theory provides no clear answer on the overall behavior of inflation. We characterize these results analytically in a simple model and numerically in a richer quantitative model.

JEL codes: E31, E52, E63

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1 Introduction

Using a micro-founded model, we address two classic questions in monetary economics and policy in this paper. First, can a time-varying inflation target decisively influence the path of actual inflation? In other words, does monetary policy properly control the dynamics and path of inflation? Second, what are the roles of policy stances on inflation dynamics and the effects of changes in policy stances on the equilibrium response of inflation to various shocks impinging on the economy? For example, what happens to the equilibrium behavior of inflation when the monetary policy stance changes to a more aggressive response to inflation? Does the fiscal policy stance with respect to public debt matter for inflation dynamics? If yes, then how does a variation in the fiscal policy stance affect inflation?

These issues, while long of great interest in monetary economics, have received a renewed interest recently in the literature.[1] A prominent illustration is provided by the research that aims to provide an explanation for the low frequency movement of inflation in the U.S., especially, the rise of inflation in the $1970s$ and the subsequent fall in the $1980s$. Proposed explanations typically rely on changes in the dynamics of the inflation target and/or changes in policy stances.[2]

For example, Ireland (2007) and Cogley, Primiceri, and Sargent (2010) propose a rise in a persistent time-varying inflation target as an explanation for the rise of inflation in the $1970s$. Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), and Bhattarai, Lee, and Park (2012a and 2012b) argue that a weak monetary policy stance with respect to inflation, or a passive monetary policy regime, in the pre-Volcker period implied indeterminacy of equilibria, which in turn, led to a rise of inflation due to self-fulfilling beliefs.[3] These papers provide evidence that post-Volcker, inflation stabilization was successful because of an aggressive monetary policy stance with respect to inflation, that is, an active monetary policy regime. Finally, Sims (2011) and Bianchi and Ilut (2012) argue that a weak response of taxes to debt, or an active fiscal policy regime, led to an increase in inflation in the $1970s$ as a response to increases in government spending. These authors argue that after the $1970s$, the fiscal policy stance changed to one that implied a passive policy regime, that is, taxes

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[1]For a recent survey of the literature on monetary and fiscal policy interactions, see Canzoneri, Cumby, and Diba (2011).

[2]There are also some well-known papers that provide a learning-based explanation for the rise and fall of U.S. inflation. See for example, Primiceri (2006) and Sargent, Williams, and Zha (2006). Moreover, some papers have attributed the rise and fall of U.S. inflation mostly to time-varying volatility of shocks. See for example, Sims and Zha (2006).

[3]We use the language of Leeper (1991) in characterizing policies as active and passive. Under an active monetary policy regime, nominal interest rates react strongly to inflation while under an active fiscal policy regime, taxes respond weakly to debt outstanding. What exactly constitutes active and passive monetary and fiscal policy is model-specific. Later, we precisely state the bounds on policy parameters that lead to a particular policy regime combination in our model.

responded strongly to debt. This choked off the possibility of rising inflation in response to fiscal shocks.

Motivated by these theoretical and empirical considerations, in the first part of the paper, we provide a complete and analytical characterization, to the best of our knowledge for the first time in the literature, of these questions in a standard sticky-price dynamic stochastic general equilibrium (DSGE) model that contains an explicit description of both monetary and fiscal policies. For this purpose, we use a relatively simple model that enables us to derive sharp and clear closed-form results. The baseline model that we solve in closed-form is a standard sticky price set-up that features simple monetary and fiscal policy rules, lump-sum taxes, and one-period nominal government bonds. We focus on the correlation between actual inflation and the inflation target, the role of policy stances on inflation dynamics, and conduct comparative static exercises related to the impact on inflation response of changing policy stances. We find that the results of these experiments depend critically on the prevailing monetary and fiscal policy regimes.

In particular, we analyze three different policy regimes. First, an active monetary and passive fiscal policy regime, where a high response of interest rates to inflation is coupled with a high response of taxes to outstanding public debt. This is the most common policy regime considered in the literature where a unique bounded equilibrium exists. In this regime, inflation closely follows the path of the inflation target. In fact, stronger the systematic reaction of monetary policy to inflation, more closely will actual inflation follow the inflation target. Moreover, a stronger reaction of monetary policy to inflation decreases the response of inflation to the various non-policy shocks impinging on the economy and thereby lowers inflation volatility.[4] Finally, as is well-known, in this case, fiscal policy stance plays no role in price level determination.

These results are standard since in this regime, monetary policy controls inflation dynamics. An unanticipated decrease in the inflation target decreases expected inflation in this regime since the systematic response of interest rate to inflation is more than one-for-one.[5] Then, due to the increase in the ex-ante real interest rate, the output gap and thereby, actual inflation, decrease. Moreover, stronger the systematic response of interest rates to inflation, greater is the effect on the ex-ante real interest rate, which decreases the effect on inflation when non-policy shocks hit the economy.

Second, we analyze an active fiscal and passive monetary policy regime, where a low response of interest rates to inflation is coupled with a low response of taxes to outstanding

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[4]Since the model is completely forward looking in this regime, the properties of the initial responses of inflation translate one-to-one to the standard deviation of inflation.

[5]A decrease in the inflation target thus has the same effects as an unanticipated increase in the nominal interest rate, that is, the usual monetary policy shock analyzed in the literature.

public debt.[6] A unique bounded equilibrium exists with this combination of monetary and fiscal policies as well. In this regime, in sharp contrast to the previous regime, inflation moves in an opposite direction from the inflation target on impact.[7] In fact, stronger the systematic reaction of monetary policy to inflation, greater will be the divergence between the inflation target and actual inflation. In addition, and again in sharp contrast to the active monetary and passive fiscal regime, in this regime, a stronger reaction of monetary policy to inflation increases the response of inflation to the various non-policy shocks impinging on the economy.[8] Moreover, now, the fiscal policy stance matters for the dynamics of inflation. In particular, we show that a weaker response of taxes to debt leads to a weaker response of inflation to non-policy shocks.

These results arise because of the wealth effect on households of interest rate and tax changes. Under the previous regime, because the systematic response of interest rates to inflation was greater than one, expected inflation decreases in response to an unanticipated increase in interest rates. In this regime, in contrast, when monetary policy raises interest rates – whether responding to a decrease in the inflation target or to other non-policy shocks – the value of government debt rises to cover the increased interest expense. This leads to a positive wealth effect on those who hold government debt, the households, because they perceive this increase in value of government debt as increasing their wealth since it is not matched by tax increases that are enough to satisfy the intertemporal government budget constraint at prevailing prices.[9] The positive wealth effect then leads to increased spending by households which in turn increases inflation in equilibrium. Thus, inflation will move in the opposite direction from the lowered inflation target. Moreover, greater the systematic response of interest rates to inflation, as long as this response is less than one-for-one, it only serves to make this positive wealth effect stronger. Then, the equilibrium response of inflation to both policy and non-policy shocks will be even higher.

In addition, given the crucial role of government debt dynamics on equilibrium determination in this regime, it is natural that fiscal policy stance now matters for the dynamics of inflation. In particular, when non-policy shocks hit the economy, the resulting changes

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[6]For early treatments of this policy regime in simple models, see among others, Leeper (1991), Sims (1994), and Woodford (1995). Kim (2003) and Canzoneri, Cumby, and Diba (2011) present numerical results on the effects of some shocks in a sticky-price model under this policy regime. Our analytical results in a sticky-price model in this regime are new to the literature.

[7]We analytically characterize the impact responses while computing the entire path of responses numerically. We also compute the standard deviation of inflation numerically and find that our results on initial responses of inflation to shocks also apply for the volatility of inflation.

[8]This result is somewhat similar to that of Loyo (1999), who considered only a flexible price economy and showed that a strong response of interest rates to inflation can lead to a hyperinflationary spiral under a passive monetary and active fiscal policy regime. In contrast, we work with a determinate equilibrium in a sticky-price model.

[9]Government bonds are thus "net wealth" in this regime.

in lump-sum taxes cause a wealth effect on households, again because tax changes do not respond strongly enough to debt.[10] Moreover, weaker the response of taxes to debt, lower is this wealth effect. Thus, spending and thereby inflation, responds by less when non-policy shocks hit the economy.

Third, we explore a passive monetary and passive fiscal policy regime, where a low response of interest rates to inflation is coupled with a high response of taxes to outstanding government debt. In this regime, there is equilibrium indeterminacy and both fundamental and sunspot shocks play a role in price level determination. Importantly, generally, both monetary and fiscal policy stances matter for inflation dynamics.[11] Due to the potential for self-fulfilling beliefs however, theory provides no clear guidance on the overall behavior of inflation, and the equilibrium effects of monetary and fiscal policy on inflation can be very different from the cases where there is equilibrium determinacy.

While at first we provide closed-form solutions for a simple model, in the second part of the paper, we also conduct a quantitative experiment with a richer DSGE model that includes a variety of shocks and frictions. In particular, we use a medium-scale DSGE model along the lines of Smets and Wouters (2007), Del Negro, Schorfheide, Smets, and Wouters (2007), and Justiniano, Primiceri, and Tambalotti (2010). The model features stochastic growth, sticky prices and wages with partial dynamic indexation, habit formation, endogenous capital accumulation, and variable capacity utilization. Moreover, the economy is subject to a wide range of shocks, such as neutral and investment specific technology shocks, preference shock, government spending shock, price and wage markup shocks, and policy shocks. We show that for a wide range of realistic parameter values, our analytical results continue to apply in such a model.[12]

Our results have implications for both the empirical and theoretical literature in monetary economics. For example, consider the recent practice in papers that estimate monetary DSGE models of using a time-varying inflation target process to explain the low frequency movement in actual inflation. In particular, Ireland (2007) and Cogley, Primiceri, and Sargent (2010) show that the estimated inflation target tracks actual inflation remarkably well in post-WWII U.S. data. Moreover, in a recent comprehensive study of various monetary policy reaction functions, Curdia, Ferrero, Ng, and Tambalotti (2011) show that using a time-varying and slow-moving inflation target improves the fit of the model since it helps capture the low

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[10]Changes in lump-sum taxes thus affect consumption in this economy due to the wealth effect. This is one reason this regime is sometimes called a "Non-Ricardian" regime in the literature.

[11]Lubik and Schorfheide (2003) also analytically analyze the solution of a sticky-price model under indeterminacy. They use a model with only monetary policy specification, however. Our characterization with both monetary and fiscal policies is thus a generalization and also contains new results as we explain later.

[12]We also characterize the entire dynamic path and standard deviation of inflation numerically in this case and show that our insights from the simple model go through again.
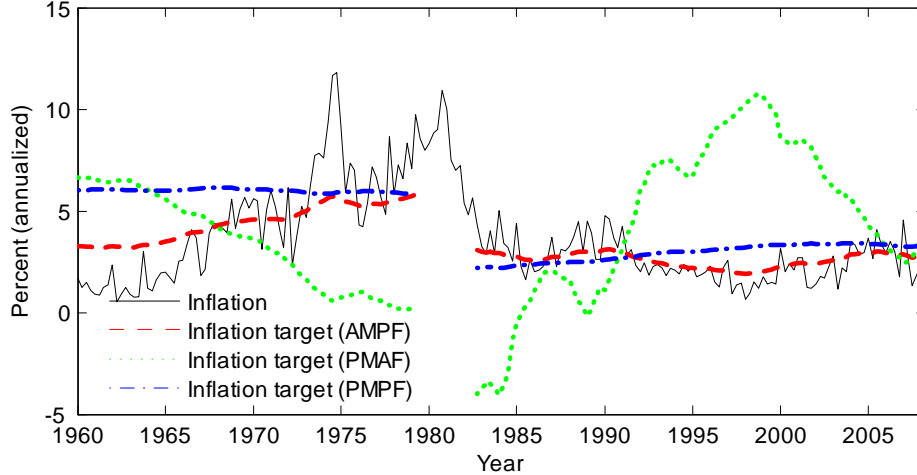
Figure 1: Inflation vs. smoothed inflation target under different policy regimes in an estimated model using U.S. data. Source: Bhattarai, Lee, and Park (2012b).

frequency variation in inflation. Our results show that this strategy works only if one imposes an active monetary and passive fiscal policy regime while estimating the model.[13] Indeed, using an estimated DSGE model and a pre-Volcker and a post-Volcker subsample analysis, in Bhattarai, Lee, and Park (2012b), we show that the correlation between inflation and the smoothed inflation target backed out after estimation varies significantly depending on which policy regime one imposes during estimation.[14] Figure 1, reproduced from that paper, makes the point especially clear. It shows for example that while under an active monetary and passive fiscal policy regime, the long-run correlation between the inflation target and actual inflation is high and positive, under a passive monetary and active fiscal policy regime, it is strongly negative. Moreover, under a passive monetary and passive fiscal policy regime, while theoretically the correlation between the inflation target and actual inflation is not pinned down by theory, the figure shows that empirically, the correlation is close to zero.

Our theoretical results show that under a passive monetary and passive fiscal policy regime, where there is equilibrium indeterminacy, both monetary and fiscal policy stances matter for inflation dynamics. Thus, in bringing a model under indeterminacy to the data, dropping fiscal policy from the model, because fiscal policy is passive, is a source of misspecification.

Finally, we show that the effects of an aggressive monetary policy stance, or a "hawkish" central bank, on inflation depends critically on the joint behavior of monetary and fiscal policy. In particular, we establish that in a passive monetary and active fiscal policy regime,

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[13]Curdia, Ferrero, Ng, and Tambalotti (2011) use U.S. data from $1987:III$ to $2009:III$, a period during which an active monetary and passive fiscal policy regime is certainly a reasonable description of policy.

[14]The estimated model in Bhattarai, Lee, and Park (2012b), while richer than the analytical model we work with in this paper, is relatively small-scale. For example, it does not feature sticky wages and endogenous capital accumulation, features that are present in the quantitative model we use in this paper.

an aggressive reaction to inflation by the central bank actually ends up increasing the response and volatility of inflation to non-policy shocks.[15] Thus, any prescription for monetary policy behavior has to take into account the prevailing fiscal policy regime.[16]

2 Simple Model

We use a standard DSGE model with nominal rigidities that can be solved analytically. We lay out the basic model features below while providing a complete description in the appendix. The main actors and their decision problems are as follows.

2.1 Description

2.1.1 Households

Households, a continuum in the unit interval, face an infinite horizon problem and maximize expected discounted utility over consumption and leisure. The utility function is additively separable over consumption and labor effort.

2.1.2 Firms

Firms, a continuum in the unit interval, produce differentiated goods using labor as input. Firms have some monopoly power over setting prices, which are sticky in nominal terms. Price stickiness is modelled using the Calvo (1983) formulation where every period, firms face a constant probability of not adjusting prices.

2.1.3 Government

The government is subject to a flow budget constraint and conducts monetary and fiscal policies using endogenous feedback rules. For simplicity, we assume that the government issues only one-period nominal debt and levies lump-sum taxes. The government controls the one-period nominal interest rate R_t. Monetary policy is modelled using an interest rate rule that features a systematic response of the nominal interest rate to the deviation of inflation π_t from a time-varying target π_t^*. The feedback parameter on inflation deviation is given by

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[15]Moreover, as is well-known, if the monetary policy stance becomes aggressive enough to imply active monetary policy, then given active fiscal policy, no bounded equilibrium exists. Thus, active fiscal policy generally precludes the possibility of inflation stabilization by relying on an aggressive reaction of interest rates to inflation by the central bank.

[16]Loyo (1999) uses a similar result from a flexible price model to interpret the experience of Brazil in the 1970s and 1980s. Relatedly, Sims (2004) shows in a very different set-up, also a flexible price model, that a central bank might lose control of inflation if it is not adequately backed up by the treasury.

ϕ. Fiscal policy is modelled using a tax rule that features a systematic response of the tax revenues τ_t to the level of outstanding government debt b_{t-1}. The feedback parameter on debt is given by ψ.

2.2 Approximate Model

We first solve the problem of households and firms given the monetary and fiscal policy rules and derive the equilibrium conditions. We then use approximation methods to solve the model: we obtain a first-order approximation to the equilibrium conditions around the non-stochastic steady state.[17] We provide the detailed derivations in the appendix. The resulting model can be summarized by the following linearized equations:

$$\tilde{Y}_t = E_t\tilde{Y}_{t+1} - \left(\hat{R}_t - E_t\hat{\pi}_{t+1}\right) + \hat{r}_t^* \tag{1}$$

$$\hat{\pi}_t = \kappa\tilde{Y}_t + \beta E_t\hat{\pi}_{t+1} \tag{2}$$

$$\hat{R}_t = \phi\left(\hat{\pi}_t - \hat{\pi}_t^*\right) \tag{3}$$

$$\hat{\tau}_t = \psi\hat{b}_{t-1} \tag{4}$$

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\bar{b}\hat{\pi}_t - \beta^{-1}\hat{\tau}_t + \bar{b}\hat{R}_t \tag{5}$$

$$\hat{r}_t^* = \rho_r\hat{r}_{t-1}^* + \varepsilon_{r,t} \tag{6}$$

$$\hat{\pi}_t^* = \rho_\pi\hat{\pi}_{t-1}^* + \varepsilon_{\pi,t}. \tag{7}$$

Here, $\tilde{Y}_t \equiv \hat{Y}_t - \hat{Y}_t^n$ is the output gap. That is, it is the difference between actual output $\hat{Y}_t$ and the natural level of output $\hat{Y}_t^n$, the output that would prevail under flexible prices. Moreover, $\hat{r}_t^*$ is a composite shock that is a linear combination of the structural shocks in the model such as technology and preference shocks. It is often referred to as the natural rate of interest because it is the real interest rate that would prevail under flexible prices.

Equation (1), the dynamic "IS" equation, expresses how the output gap today is determined by the expected output gap tomorrow and the ex ante real interest rate, $\hat{R}_t - E_t\hat{\pi}_{t+1}$. Equation (2), the dynamic "AS" equation, describes how inflation today is determined as a function of discounted expected inflation tomorrow and the output gap today. Here, β is the discount factor of the household and κ, which determines the slope of the AS equation, is a composite parameter of the structural parameters. Equation (3) is the monetary policy rule which governs the response of the nominal interest rate to the deviation of inflation from the

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[17]In the equations below, we use $\hat{X}_t$ to denote the log deviation of a variable X_t from its steady state $\bar{X}$ ($\hat{X}_t = \ln X_t - \ln \bar{X}$), except for two fiscal variables, $\hat{b}_t$ and $\hat{\tau}_t$. Following Woodford (2003), we let them represent respectively the deviation of the maturity value of government debt and of government tax revenues (net of transfers) from their steady-state levels, measured as a precentage of steady-state output: $\hat{b}_t = \frac{b_t - \bar{b}}{\bar{Y}}$ and $\hat{\tau}_t = \frac{\tau_t - \bar{\tau}}{\bar{Y}}$.

Table 1: Monetary/Fiscal Policy Regimes and Equilibrium Properties

	Active Money $(\phi > 1)$	Passive Money $(\phi < 1)$
Active Fiscal $(\psi < 1 - \beta)$	No Equilibrium	Unique Equilibrium
Passive Fiscal $(\psi > 1 - \beta)$	Unique Equilibrium	Multiple Equilibria

inflation target while Equation (4) is the fiscal policy rule which governs the response of taxes to the real maturity value of the outstanding debt. Finally, Equation (5) is the flow budget constraint of the government.

Equation (6) shows that we assume that $\hat{r}_t^*$ follows an exogenous AR (1) process with $0 < \rho_r < 1$. The innovation $\varepsilon_{r,t}$ has zero mean and variance σ_r^2. Equation (7) shows that we assume that the inflation target follows an exogenous AR(1) process with $0 < \rho_\pi < 1$. The innovation $\varepsilon_{\pi,t}$ has zero mean and variance σ_π^2. Ireland (2007) models that the Federal Reserve adjusts the inflation target in response to the economy's supply shocks but finds that the response is not statistically significant. In light of this result, we make the exogeneity assumption on $\hat{\pi}_t^*$. Cogley and Sbordone (2008) and Cogley, Primiceri, and Sargent (2010) also use an exogenous AR process to model the inflation target.[18]

As is well-known, in the approximate model, the existence and uniqueness of equilibrium depends crucially on the prevailing monetary and fiscal policy regime. The equilibrium of the economy will be determinate either if monetary policy is active while fiscal policy is passive (the AMPF regime) or if monetary policy is passive while fiscal policy is active (the PMAF regime). The equilibrium is indeterminate and multiple equilibria exist if both monetary and fiscal policies are passive (the PMPF regime) while no bounded equilibrium exists if both monetary and fiscal policies are active. In our model, monetary policy is active if $\phi > 1$ and fiscal policy is active if $\psi < 1 - \beta$. Table 1 summarizes these policy regime combinations and the associated equilibrium outcomes.

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[18]The assumption that $\hat{\pi}_t^*$ is stationary implies that the monetary authority does not permanently keep the inflation target at the same level but in the long-run drives the inflation target back to the non-stochastic steady state level. We introduce this assumption for two reasons. First, the stationarity assumption allows us to work with a standard framework. To assume that the inflation target has a unit root results in time-varying coefficients of the Phillips curve as in Cogley and Sbordone (2008) or leads to a non-standard monetary policy rule for with which the Taylor principle should be modified. Therefore, to present our point clearly within a familiar framework, we assume that the inflation target is very persistent but stationary. Second, we will generally restrict ρ_π to values close to 1, thereby effectively ensuring that $\hat{\pi}_t^*$ captures the persistent behavior of the inflation target set by the central bank. Cogley, Primiceri, and Sargent (2010) use the same assumption.

2.3   Results

We analytically characterize the solution of the model either when a determinate equilibrium exists or when there are multiple equilibria. We then derive several results regarding the dynamics of inflation. Specifically, we study how the path of inflation depends on the path of the inflation target and how the response of inflation changes when monetary and fiscal policy stances change within a policy regime combination. All the details of the derivations and the proofs of the various propositions are in the appendix.

2.3.1   Active Monetary and Passive Fiscal Policy

Under an active monetary and passive fiscal policy regime, we can express the solution for inflation as:

$$\hat{\pi}_t = \Phi\left(\phi\right)\hat{\pi}_t^* + \Gamma\left(\phi\right)\hat{r}_t^*, \tag{8}$$

where $\Phi\left(\phi\right)$ and $\Gamma\left(\phi\right)$ are functions of the monetary policy response parameter ϕ.[19] Note that in this case, as Equation (8) makes clear, the dynamics of inflation do not depend on the dynamics of government debt and fiscal policy. Therefore, debt $\hat{b}_{t-1}$ is not a state variable that determines inflation and the fiscal policy response parameter ψ does not affect inflation. Moreover, this implies that inflation is solely a function of the two exogenous processes, $\hat{\pi}_t^*$ and $\hat{r}_t^*$, since other than the budget constraint, the rest of the model is completely forward-looking.

We next characterize several properties of the solution. We first start with the response of inflation to changes in the inflation target and the non-policy shock.

Proposition 1 (Direction of inflation response) *When monetary policy is active and fiscal policy is passive (AMPF), inflation moves in the same direction as the inflation target $\hat{\pi}_t^*$ – that is,*

$$\Phi\left(\phi\right) > 0.$$

Inflation responds more (or less) than one-for-one to changes in the inflation target if prices are sufficiently flexible (or sticky):

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\begin{aligned}
1 &< \Phi\left(\phi\right), & for\ \kappa &> \frac{\left(1 - \rho_\pi\right)\left(1 - \beta\rho_\pi\right)}{\rho_\pi}, \\
0 &< \Phi\left(\phi\right) \le 1, & for\ 0 < \kappa &\le \frac{\left(1 - \rho_\pi\right)\left(1 - \beta\rho_\pi\right)}{\rho_\pi}.
\end{aligned}
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[19]Obviously, $\Phi\left(\phi\right)$ and $\Gamma\left(\phi\right)$ in Equation (8) are a function of other structural parameters as well. Here on after, we write the coefficients in a solution for inflation as a function of policy parameters only so as to highlight their role in determining inflation dynamics.

Moreover, inflation moves in the same direction in response to the non-policy shock, $\hat{r}_t^*$ – that is,

$$\Gamma\left(\phi\right) > 0.$$

In this regime, since inflation moves in the same direction as the inflation target, we see clearly that monetary policy controls the dynamics of inflation. Consider a positive shock to $\hat{\pi}_t^*$. From Equation (3), this leads to a decrease in the nominal interest rate $\hat{R}_t$. Since in this regime the central bank systematically changes nominal interest rates more than one-for-one to changes in inflation ($\phi > 1$), a decrease in $\hat{R}_t$ brings expected inflation up. This implies that the ex-ante real interest rate $\hat{R}_t - E_t\hat{\pi}_{t+1}$ goes down. From Equation (1), this leads to an increase in the output gap $\tilde{Y}_t$ and then from Equation (2), it leads to an increase in actual inflation $\hat{\pi}_t$. Thus, $\hat{\pi}_t$ and $\hat{\pi}_t^*$ are positively correlated.

Moreover, as is natural, the response of inflation to the inflation target shock depends on the extent of price stickiness in the economy: greater the degree of price stickiness, smaller is the response of inflation. As a limiting result, one can easily show that inflation does not respond to the inflation target at all when prices are completely sticky (that is, $\lim_{\kappa \to \infty} \Phi\left(\phi\right) = 0$) because the price level would not respond at all to any shocks. In contrast, when prices are sufficiently flexible, inflation responds more than one-for-one to changes in the inflation target. This is possible due to two factors. First, from Equation (3), we see that on impact, a unit increase in $\hat{\pi}_t^*$ decreases $\hat{R}_t$ by more than a unit (since $\phi > 1$).[20] At a given level of $\hat{\pi}_t$, from Equation (1), $\tilde{Y}_t$ increases and in equilibrium, if κ is large enough, which implies that inflation is quite sensitive to changes in the output gap, then Equation (2) shows that the increase in $\hat{\pi}_t$ can be by more than a unit.

Proposition 1 also establishes that when monetary policy is active, inflation moves in the same direction as the natural rate of interest $\hat{r}_t^*$. In particular, we show in the appendix that demand-type shocks such as preference shocks increase $\hat{r}_t^*$, while supply-type shocks such as technology shocks lower $\hat{r}_t^*$. Hence, the proposition tells us that inflation increases in response to favorable demand shocks and decreases in response to favorable supply shocks. This is a conventional result under the AMPF regime. Equation (1) implies that a positive $\hat{r}_t^*$ increases output gap for a given level of expected output gap and the real interest rate. Then from Equation (2) it is clear that this in turn increases inflation and expected inflation in equilibrium.

Next, we consider a comparative static exercise with respect to the policy parameter ϕ, which is the measure of the monetary policy stance.

Proposition 2 (Magnitude of inflation response and monetary policy stance) *When monetary policy is active and fiscal policy is passive (AMPF), the response of inflation to*

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[20]To emphasize, this is in a partial equilibrium sense.

changes in the inflation target is decreasing (or increasing) in ϕ if prices are sufficiently flex-ible (or sticky):

$$\frac{\partial \Phi(\phi)}{\partial \phi} \quad < \quad 0, \qquad for\ \kappa > \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi},$$

$$\frac{\partial \Phi(\phi)}{\partial \phi} \quad \geq \quad 0, \qquad for\ 0 < \kappa \leq \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi}.$$

The equality holds when $\kappa = \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi}$. Moreover, inflation responds less to non-policy shocks as the monetary authority becomes more aggressive—that is, Γ decreases as ϕ increases:

$$\frac{\partial \Gamma(\phi)}{\partial \phi} < 0.$$

In combination with proposition 1, we now have the intuitive result that $\hat{\pi}_t$ will move more closely with $\hat{\pi}_t^*$ as ϕ increases – for all values of κ. Again, in this sense, monetary policy controls inflation successfully in this regime. Thus, if the central bank's objective is to stabilize the "inflation gap," $\hat{\pi}_t - \hat{\pi}_t^*$, it needs to have a large value for ϕ, that is, it needs to respond strongly to the inflation gap. Proposition 2 also shows that greater the systematic response of monetary policy to inflation, lower will be the response of inflation to non-policy shocks. As ϕ increases, under active monetary policy, the response of the ex ante real interest rate increases and inflation will increase by a lower amount in equilibrium.

To make these results even more transparent, we illustrate them using figures. We show in panel (a) of Figure 2 the impulse response of inflation to an exogenous change in the inflation target, varying the degree of monetary policy stance.[21] Panel (a) clearly shows that inflation dynamics closely mimic those of the inflation target. The completely forward-looking nature of the model, with the fiscal variables being redundant, makes the inflation dynamics particularly simple. Note that if the inflation target moves persistently, so does inflation. In addition, we can see that inflation responds more than one-for-one to changes in the inflation target because $\Phi(\phi) > 1$ at our benchmark parameterization.[22] Not surprisingly, however, inflation is closer to the target rate as the monetary authority responds more strongly to the inflation gap. Thus these results are completely aligned with those of propositions 1 and 2.

We show in panel (b) of Figure 2 the response of inflation to non-policy shocks under three different values of ϕ. It clearly illustrates our analytical findings in proposition 1 and 2 that inflation responds positively to a positive innovation to the non-policy shock and that

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[21] For this and the other figures in this simple model section, we assign some standard values to model parameters: $\beta = 0.99$; $\alpha = 0.75$; $\bar{b} = 0.4$; $\rho_r = 0.9$; and $\rho_\pi = 0.995$.

[22] The lower bound of κ for $\Phi(\phi) > 1$, found in Proposition 1, is very small when ρ_π has a vlaue close to one. At the benchmark parameterization, the lower bound is less than 0.0001, which is not restrictive at all.
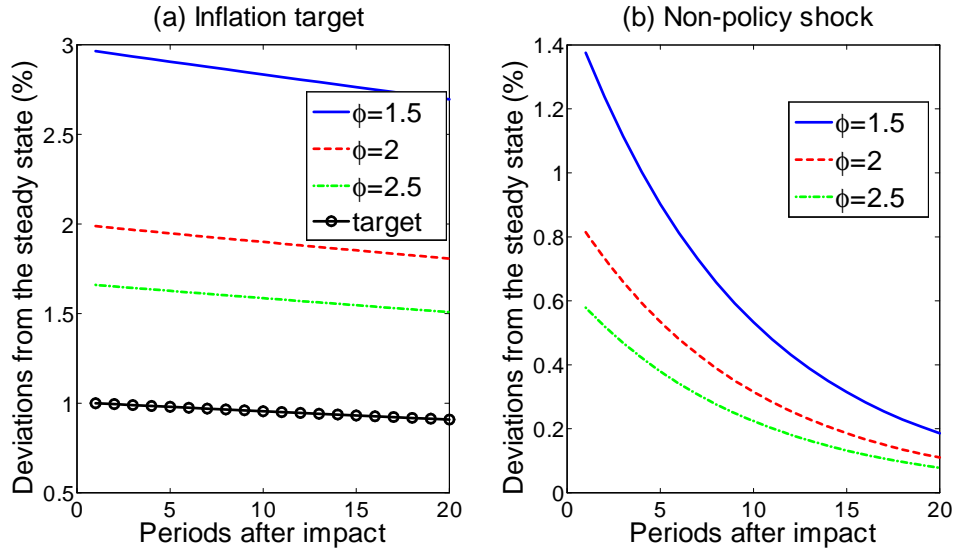
Figure 2: The response of inflation to a one percentage point increase in the inflation target and non-policy shock under the AMPF regime.

this response is lower when ϕ is higher. In addition, we can see that the response of inflation decays at a much faster rate here than in panel (a). The reason for this is straightforward. As is clear from Equation (8), the response of inflation to a shock to each of the exogenous processes – aside from the size of the response – is entirely dictated by the response of the respective exogenous process itself. Therefore, to the extent that the inflation target is more persistent than other exogenous variables ($\rho_\pi > \rho_r$), inflation should return to its steady state level more slowly in response to inflation target shocks. This implies that the inflation target – relative to other shocks – will dominate inflation dynamics, especially in the long run.

Our results in Figure 2 raise an interesting point and provide another interpretation of some recent results in the literature. Empirical studies in the recent DSGE literature such as Cogley, Primiceri, and Sargent (2010) have found that the low-frequency components of the inflation rate are explained almost entirely by a time-varying inflation target. The literature's strategy is to calibrate ρ_π to a large value, close to a random walk, to establish this finding. Our results are consistent with those findings. They at the same time however, also show that this strategy works as long as the inflation target shock is more persistent than the other shocks hitting the economy. If we instead treat all shocks symmetrically (i.e. $\rho_\pi = \rho_r$) rather than fixing ρ_π at a higher value than ρ_r,[23] the model would not distinguish between the two shocks with respect to inflation dynamics.

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[23]As stated earlier, this strategy is often adopted in the literature. It is of course true that empirically this restriction is not very binding since once we fix ρ_π very close to 1, the estimated values of the persistence of other shocks are almost always lower than such calibrated value of ρ_π. Our results thus provide an extra condition that helps interpret the results of the recent estimated monetary DSGE literature.

Finally, because of the forward-looking nature of the model variables, it is straightforward to establish a result regarding the variance of inflation in response to non-policy shocks.

Proposition 3 (Unconditional variance) *When monetary policy is active and fiscal policy is passive (AMPF), the unconditional variance of inflation decreases in ϕ:*

$$\frac{\partial VAR_{NP}(\hat{\pi}_t)}{\partial \phi} < 0,$$

where $VAR_{NP}(\cdot)$ denotes the unconditional (long-run) variance associated with non-policy shocks only (i.e. the influence of policy shifts by the government is shut down: $\varepsilon_{\pi,t} = 0$).

Proposition 3 thus establishes that the long-run variance of inflation in this policy regime decreases when the monetary authority reacts systematically strongly to inflation. Our results in propositions 2 and 3 regarding non-policy shocks have essentially the same implication: as the central bank responds more strongly to the inflation gap, the volatility of inflation decreases.

We now move on to analyzing another policy regime combination that leads to a determinate equilibrium. As we will see, the results related to the correlation between the inflation target and inflation and the comparative statics on monetary policy response parameters will be in stark contrast in this case compared to the active monetary and passive fiscal policy regime analyzed in this section.

2.3.2 Passive Monetary and Active Fiscal Policy

Under a passive monetary and active fiscal policy regime, we can express the solution for inflation as:

$$\hat{\pi}_t = \Omega\left(\phi, \psi\right)\hat{b}_{t-1} - \Phi\left(\phi, \psi\right)\hat{\pi}_t^* + \Gamma\left(\phi, \psi\right)\hat{r}_t^*, \tag{9}$$

where $\Omega\left(\phi, \psi\right)$, $\Phi\left(\phi, \psi\right)$, and $\Gamma\left(\phi, \psi\right)$ are functions of both the monetary policy response parameter ϕ and the fiscal policy response parameter ψ.[24] Note that in this policy regime, as Equation (9) makes clear, the dynamics of inflation also depend on public debt outstanding $\hat{b}_{t-1}$. This implies that there is an endogenous state variable in this case, which in turn imparts endogenous dynamics to the model. These are important differences from the case we analyzed in the previous section where monetary policy was active and fiscal policy was passive. As we explain below in detail, the wealth effect on households due to changes in the value of government debt and taxes, which in turn affects households' spending, is the main mechanism behind our results in this section.

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[24]The complete solution, including the solution for debt, is provided in the appendix.

We next characterize several properties of the solution.[25] We first start with the response of inflation to changes in the inflation target, the non-policy shock, and public debt outstanding.

Proposition 4 (Direction of inflation response) *When monetary policy is passive and fiscal policy is active (PMAF), inflation moves in the opposite direction in response to a change in the inflation target – that is,*

$$\Phi(\phi, \psi) \geq 0 \quad for$$
$$-\infty < \psi < \bar{\psi}^* \ and \ 0 \leq \phi < 1,$$

where $0 < \bar{\psi}^* \leq 1 - \beta$ is a reduced-form parameter. The equality holds when $\phi = 0$. Moreover, inflation moves in the same direction in response to the non-policy shock, $\hat{r}_t^*$ – that is,

$$\Gamma(\phi, \psi) > 0 \quad for$$
$$-\infty < \psi < 1 - \beta \ and \ 0 \leq \phi < 1.$$

Finally, inflation moves in the same direction in response to a change in public debt outstanding – that is,

$$\Omega(\phi, \psi) > 0 \quad for$$
$$-\infty < \psi < 1 - \beta \ and \ 0 \leq \phi < 1.$$

Proposition 4 thus shows that under a PMAF regime, inflation moves in the opposite direction from a change in the inflation target.[26] This result, which is in stark contrast to proposition 1 under the AMPF regime, arises because now changes in the value of government debt influence inflation dynamics. Consider a negative shock to the inflation target. From Equation (3), this increases the nominal interest rate on impact. An increase in the nominal interest rate results in an increase of the value of government debt $\hat{b}_t$, due to an increase in interest expense. In this active fiscal policy regime, since taxes do not adjust by enough to satisfy the intertemporal government budget constraint at prevailing prices, the increase in the value of government debt leads to a positive wealth effect on households who hold government debt. This positive wealth effect then leads to higher spending, which pushes up inflation in equilibrium. Proposition 4 is therefore, the key result behind the negative relationship between inflation and the inflation target under the PMAF regime shown in Figure 1.

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[25]In the propositions below we focus on the empirically realistic range of values for the monetary policy parameter: $0 \leq \phi < 1$.

[26]Note that for the first result of this proposition, there is a condition on ψ such that fiscal policy has to be "sufficiently" active. That is, $\psi < \bar{\psi}^* \leq 1 - \beta$. As we argue in the appendix, this condition is unlikely to be relevant empirically.

Proposition 4 also establishes that under a PMAF regime, inflation moves in the same direction as the non-policy shock. This proposition is the same as proposition 1 under the AMPF policy regime. Thus inflation increases in response to a positive demand shock while it decreases in response to a positive supply shock. This is because even under the PMAF policy regime, the effect of the non-policy shock on the economy is still to increase the output gap given the expectations as implied by Equation (1), and in turn, inflation as implied by Equation (2).

Finally, proposition 4 also shows that under a PMAF regime, inflation is affected positively by changes in public debt outstanding $\hat{b}_{t-1}$. This result is again a direct derivative of the wealth effect on households discussed above that is a crucial mechanism under active fiscal policy. A higher level of public debt outstanding, not matched by sufficient tax increases, is translated into higher wealth for households, which increases spending and thereby inflation.

Next we consider a comparative static exercise with respect to the policy parameter ϕ, which is the measure of monetary policy stance.

Proposition 5 (Magnitude of inflation response and monetary policy stance) *When monetary policy is passive and fiscal policy is active (PMAF), inflation deviates even further from the inflation target when the monetary authority is more aggressive – that is, Φ increases as ϕ increases in the domain of $[0, 1)$:*

$$\frac{\partial \Phi(\phi, \psi)}{\partial \phi} > 0 \qquad for$$

$$-\infty < \psi < \bar{\psi}^{**} \; and \; 0 \leq \phi < 1,$$

where $0 < \bar{\psi}^{**} \leq 1 - \beta$ is a reduced-form parameter. Moreover, inflation responds more to non-policy shocks when the monetary authority is more aggressive—that is, Γ increases as ϕ increases in the domain of $[0, 1)$:

$$\frac{\partial \Gamma(\phi, \psi)}{\partial \phi} > 0 \qquad for$$

$$-\infty < \psi < 1 - \beta \; and \; 0 \leq \phi < 1.$$

Finally, inflation responds more to a change in public debt outstanding when the monetary authority is more aggressive—that is, Ω increases as ϕ increases in the domain of $[0, 1)$:

$$\frac{\partial \Omega(\phi, \psi)}{\partial \phi} > 0 \qquad for$$

$$-\infty < \psi < \bar{\psi}^{***} \; and \; 0 \leq \phi < 1,$$

where $0 < \bar{\psi}^{***} \leq 1 - \beta$ is a reduced-form parameter.

In sharp contrast to our result under the AMPF policy regime, proposition 5 shows that under a PMAF regime, as the reaction of monetary policy to inflation increases, so does the equilibrium impact on inflation of the inflation target shock.[27] The mechanism is as follows. When the reaction of monetary policy to inflation increases, then for a given decrease in the inflation target, the interest rate increase will be higher. This means that the value of government debt $\hat{b}_t$ increases by more, which in turn, increases the size of the wealth effect discussed above. This then implies a greater effect on spending, and thereby, on inflation. Thus, unless the monetary authority decides to respond to inflation by enough to move from a passive to an active regime while at the same time fiscal policy moves from an active to a passive regime, a stronger response of monetary policy to inflation ends up stabilizing inflation by less.

Moreover, proposition 5 establishes that stronger the systematic response of monetary policy to inflation, greater will be the response of inflation to the non-policy shocks in equilibrium. This result is again in contrast to the result under the AMPF regime established in proposition 2. What leads to this result? When a positive $\hat{r}_t^*$ shock hits the economy, it raises inflation. Now with a higher ϕ, interest rates will rise by more in response to this increase in inflation, as given by Equation (3). Under the AMPF policy regime, this increase in interest rates would bring expected inflation down. In this PMAF regime, however, the greater increase in interest rates raises the value of government debt $\hat{b}_t$ by a greater amount. As we have explained before, this leads to a greater wealth effect on the households, which increases inflation by a larger amount.

Finally, proposition 5 shows that a greater systematic response of interest rates to inflation leads to a greater response of inflation to public debt outstanding, $\hat{b}_{t-1}$. Again, this result arises because with a stronger response of interest rates to inflation, the wealth effect gets amplified. As shown in proposition 4, a higher value of $\hat{b}_{t-1}$ increases inflation $\hat{\pi}_t$. While the monetary authority raises the interest rate in response to the increased inflation, the interest rate will rise by more with a higher ϕ, which in turn will deliver a stronger wealth effect.

In this regime, since fiscal policy also matters for inflation dynamics, we next establish a result related to the fiscal policy stance.

Proposition 6 (Magnitude of inflation response and fiscal policy stance) *When monetary policy is passive and fiscal policy is active (PMAF), inflation deviates even further from*

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[27] Again, note that for the first and the third result of this proposition, there is a condition on ψ such that fiscal policy has to be "sufficiently" active. That is, $\psi < \bar{\psi}^{**}, \bar{\psi}^{***} \leq 1 - \beta$. As we argue in the appendix, this condition is unlikely to be relevant empirically.

the inflation target as the fiscal authority becomes more active – that is, Φ increases as ψ decreases in the domain of $(-\infty, 1 - \beta)$:

$$\frac{\partial \Phi(\phi, \psi)}{\partial \psi} < 0 \qquad for$$

$$-\infty < \psi < 1 - \beta \ and \ 0 \leq \phi < 1.$$

Moreover, inflation responds less in response to non-policy shocks as the fiscal authority becomes more active—that is, Γ decreases as ψ decreases in the domain of $(-\infty, 1 - \beta)$:

$$\frac{\partial \Gamma(\phi, \psi)}{\partial \psi} > 0 \qquad for$$

$$-\infty < \psi < 1 - \beta \ and \ 0 \leq \phi < 1.$$

Finally, inflation responds more to a change in public debt outstanding when the fiscal authority becomes more active—that is, Ω increases as ψ decreases in the domain of $(-\infty, 1 - \beta)$:

$$\frac{\partial \Omega(\phi, \psi)}{\partial \psi} < 0 \qquad for$$

$$-\infty < \psi < 1 - \beta \ and \ 0 \leq \phi < 1.$$

In this PMAF regime, proposition 6 shows that as fiscal policy becomes more active, inflation responds more strongly, and in the opposite direction, to changes in the inflation target. This result arises because as ψ decreases, taxes respond less strongly to debt as given by Equation (4). Then the wealth effect due to interest rate changes described above becomes amplified. This increased wealth effect in turn leads to greater spending and thereby a stronger response of inflation.

Proposition 6 also shows that weaker is the response of taxes to debt, lower is the response of inflation to the non-policy shock. When a positive $\hat{r}_t^*$ hits the economy, as we discussed above, it leads to higher inflation. This lowers the value of government debt. From Equation (4) this implies that taxes will decrease. Now, lower is ψ, smaller is the decrease in taxes. Even though taxes are lump-sum in our model, when the regime is PMAF, tax changes lead to a wealth effect on households. With a smaller decrease in taxes, the wealth effect is smaller, which in turn leads to a smaller change in spending and thereby inflation. Finally, the last result in proposition 6 that inflation responds more to a change in public debt outstanding when fiscal authority becomes more active arises because the wealth effect gets magnified when taxes respond less to public debt outstanding.

So far, we have provided analytical results in the PMAF regime for the initial response
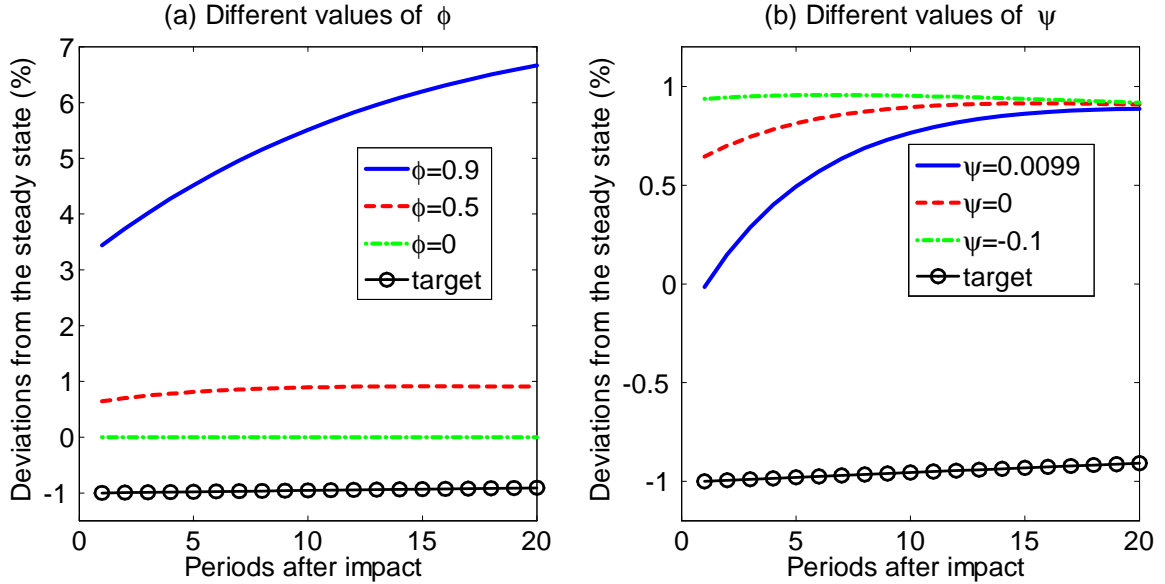
Figure 3: The response of inflation to a one percentage decrease in the inflation target under the PMAF regime.

of inflation. Because of the role of an endogenous state variable, it is cumbersome to derive closed-form results for the full dynamic response of inflation. We therefore, resort to numerical solutions to show that our results are general indications of the overall dynamic responses as well.

We show in Figure 3 the response of inflation to a one percent decrease in the inflation target under varying degrees of monetary and fiscal policy stances. The figure highlights our theoretical results above in propositions 4, 5, and 6. In addition, it shows that although our theoretical findings are focused on the impact response of inflation, the same economic intuition can be extended to longer horizons. Indeed, it clearly shows that the deviation of inflation from the target continues to be greater in periods following the shock, as monetary and fiscal policies become more active.[28] The reason is that when ϕ is higher (and/or ψ is lower), the interest rate will be persistently higher after a negative shock to the inflation target to the extent that the inflation target is persistent. This in turn leads to persistently higher value of public debt. This then leads to a persistently positive wealth effect, which in turn, leads to a persistently higher inflation. What is more, when monetary and/or fiscal policy is more active, inflation depends more strongly on government indebtedness – that is, as shown above in propositions 5 and 6, $\Omega(\phi, \psi)$ is increasing in ϕ and decreasing in ψ. This property

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[28]Kim (2003) and Canzoneri, Cumby, and Diba (2011) present impulse response of inflation to a monetary shock under PMAF and show that a positive interest rate shock leads to an increase in inflation for several periods. Since a positive interest rate shock and negative inflation target shock behave similarly, our results are consistent with theirs. They do not however, consider detailed comparative statics with respect to monetary and fiscal policy parameters.
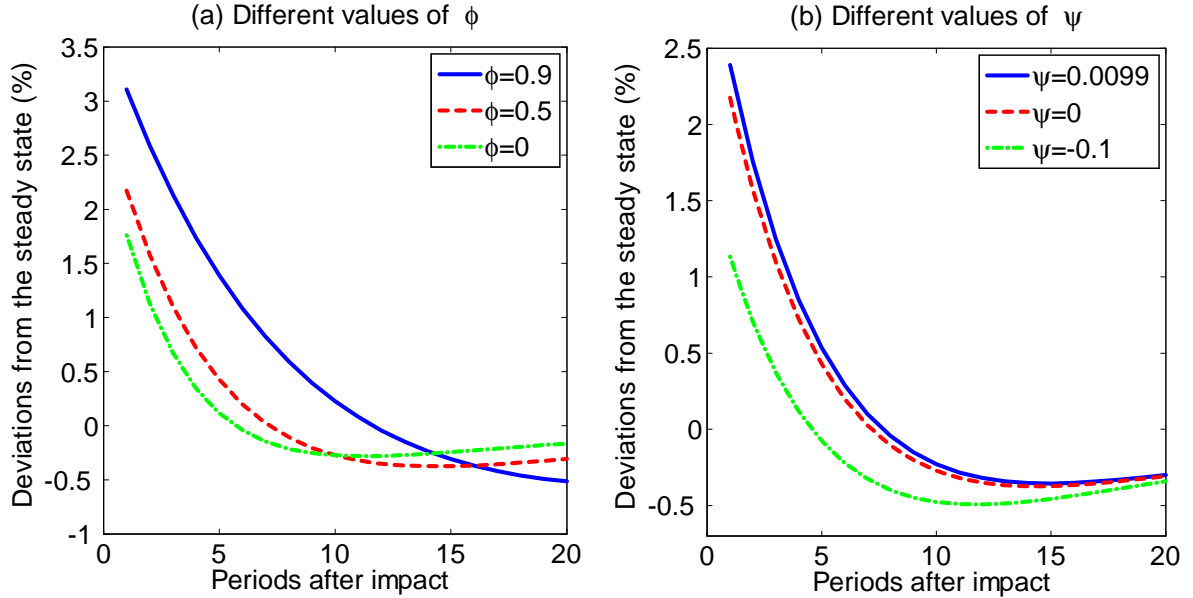
Figure 4: The response of inflation to a one percentage increase in the non-policy shock under the PMAF regime.

obviously magnifies the mechanism through which higher debt influences the dynamics of inflation.

Figure 4 illustrates our results on inflation response to non-policy shocks under varying degrees of monetary and fiscal policy reactions to inflation and debt respectively. The figure highlights our analytical results above on the impact response of inflation. Higher ϕ or a higher ψ leads to a greater initial impact of inflation. In addition to the initial impact, the dynamic responses of inflation reveal an interesting pattern that is different from the case under AMPF. While the initial response of inflation is positive in response to the shock $\hat{r}_t^*$ and it remains positive for a number of periods, after some time, inflation goes below steady state.[29] The intuition for this result is again related to the dynamics of government debt. Initially, the increase in inflation lowers the value of government debt. In this regime, this decrease in the value of government debt leads to a negative wealth effect on households. This negative wealth effect leads to a decrease in spending by households, which in turn, eventually leads to inflation decreasing and going below steady state. Moreover, note that while analyzing the dynamic response of inflation under different values of ϕ, one sees that the paths intersect after a certain number of periods. This feature arises because when inflation goes below steady state, it leads to a decrease in nominal interest rates, as given by Equation (3). This decrease in interest rate leads to a negative wealth effect. Higher the value of ϕ,

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[29]Kim (2003) also present impulse response of inflation to "aggregate demand" and "aggregate supply" shocks under PMAF and notes this behavior of "inflation reversal." He does not however, consider detailed comparative statics with respect to monetary and fiscal policy parameters.
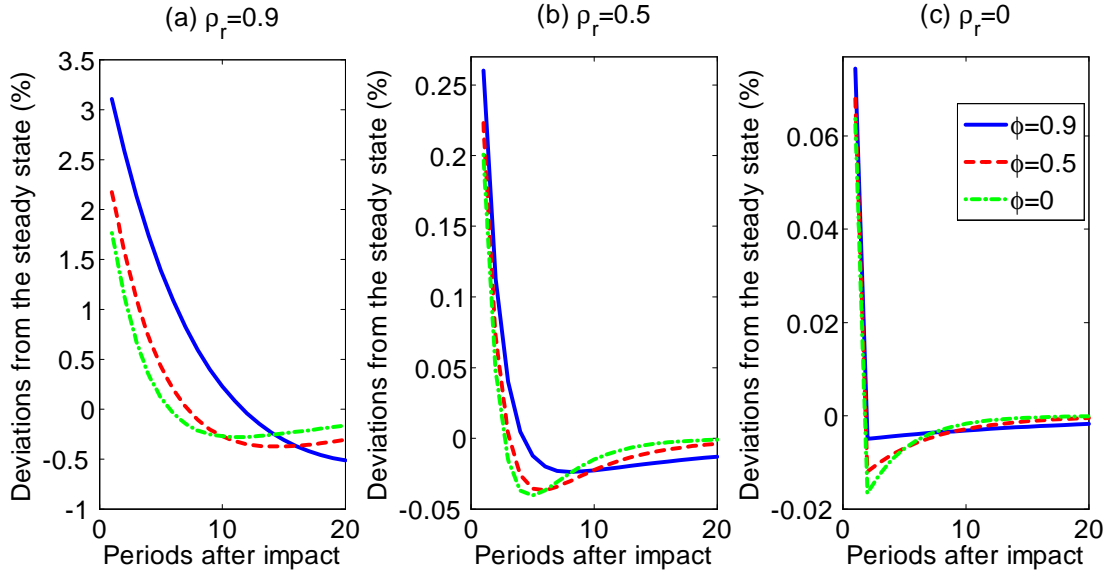
Figure 5: The response of inflation to a one percentage point increase in the non-policy shock with different degrees of persistence under the PMAF regime.

greater is this negative wealth effect. Thus, once inflation goes below steady state, due to the negative wealth effect that depresses spending, there is a tendency for inflation to continue below steady for a while. This effect is more pronounced when ϕ is higher, which in turn, implies that the paths for different levels of ϕ will cross.

We next delve further into this issue. Figure 5 shows results for the inflation response to non-policy shocks under varying degrees of monetary policy reaction to inflation and for different levels of persistence of the non-policy shocks. As is to be expected, the greater the persistence of the shock, the more persistent will be the response of inflation. Moreover, the pattern of inflation initially remaining above steady state and then eventually going below steady state is robust to various levels of persistence of the shock.

The results on the dynamic responses of inflation have an important implication for the relationship between the monetary policy stance and the volatility of inflation – a primary policy objective of central banks. Due to complicated dynamics under this policy regime, we do not provide a counterpart to Proposition 3 under the AMPF regime but resort to numerical illustrations. As Figure 4 illustrates, under PMAF, while the response of inflation deviation is not greater for every time period when ϕ is higher, it is certainly the case for most periods – especially the initial period. To the extent that initial responses of inflation to shocks dominate in the second moment of inflation dynamics, the volatility of inflation will be larger when monetary policy reaction is stronger.[30] Figure 6 illustrates this result. Under AMPF,

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[30]Initial responses are disproportionately important for the variance of the inflation rate because the *squared* size of the initial response to a shock is substantially bigger than those of the responses in the following periods
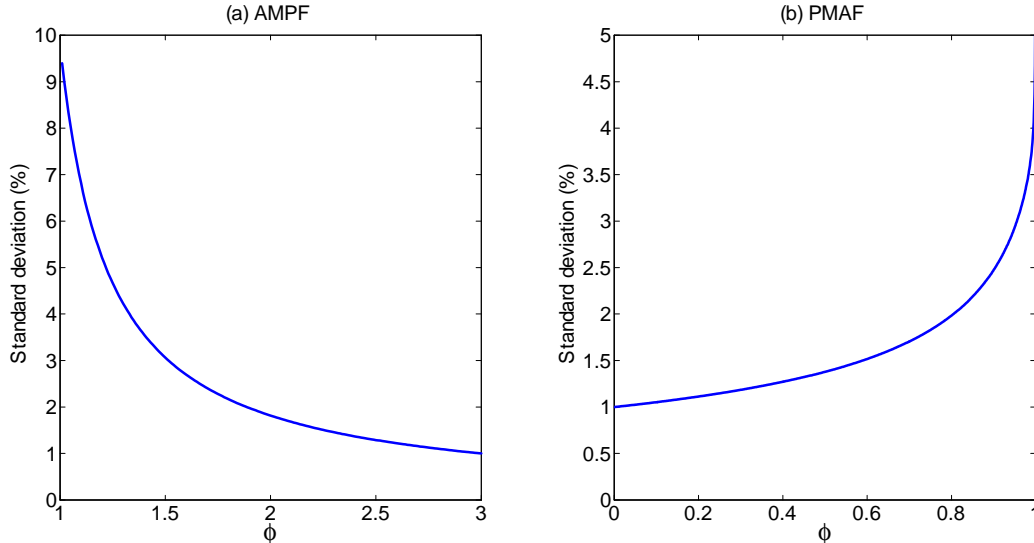
Figure 6: Standard deviation of inflation (in percent) across different values of ϕ conditional on the non-policy shocks. The standard deviation is normalized to unity corresponding to the parameterization with $\phi = 3$ under AMPF and $\phi = 0$ under PMAF.

a more hawkish monetary policy leads to a smaller standard deviation of inflation as proved earlier. Under PMAF, however, a stronger monetary policy reaction to inflation instead leads to a higher volatility of inflation.

Finally, we consider a regime where there is equilibrium indeterminacy as both monetary and fiscal policies are passive.

2.3.3 Passive Monetary and Passive Fiscal Policy

Under a passive monetary and passive fiscal policy regime (PMPF), multiple equilibria exist. We can express the solution for inflation as:

$$\hat{\pi}_t = E_{t-1}\hat{\pi}_t - \Phi(\phi, \psi)\,\hat{\pi}_t^* - \Gamma(\phi, \psi)\,\hat{r}_t^* + \Lambda(\phi, \psi)\,(M_\pi \hat{\pi}_t^* + M_r \hat{r}_t^* + \zeta_t^*),\qquad(10)$$

where ζ_t^* is a sunspot shock that is independent of the fundamental shocks to the economy. Moreover, M_π and M_r are new parameters introduced due to indeterminacy that are not determined uniquely by the structural parameters of the economy.[31] These parameters, multiplied to a function of the structural parameters $\Lambda(\phi, \psi)$, capture self-fulfilling beliefs of agents regarding the initial impact of fundamental shocks.[32] Finally, $\Phi(\phi, \psi)$ and $\Gamma(\phi, \psi)$ capture the

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as can be seen clearly in Figure 5. This argument is reminiscent of the difference in outcomes when monetary policy is analyzed under commitment and under discretion, also known as the *stabilization bias*.

[31]For the complete description of the notation, see the appendix. Our solution methodology follows that of Lubik and Schorfheide (2003).

[32]Note that $\Lambda(\phi, \psi) = 0$ under determinacy.

part of the solution that is determined uniquely by the structural parameters of the economy.

In this regime, we establish the following result.[33]

Proposition 7 *When monetary and fiscal policy is passive (PMPF)*

$$\Phi\left(\phi,\psi\right) \geq 0 \ and \ \Gamma\left(\phi,\psi\right) > 0 \quad for$$
$$\psi > 1 - \beta \ and \ 0 \leq \phi < 1.$$

The equality holds when $\phi = 0$. Moreover, $E_{t-1}\hat{\pi}_t$ is not a function of $\hat{b}_{t-1}$.

Proposition 7 thus shows that under PMPF, for the part of the solution that is determined by the structural parameters of the model, the initial response of inflation to $\hat{\pi}_t^*$ and $\hat{r}_t^*$ depends on both monetary and fiscal policy parameters. In this respect, the solution is similar to that under the PMAF regime and unlike that under the AMPF regime. Thus, under indeterminacy, not explicitly specifying fiscal policy in the model, even though it is passive, is a source of misspecification.

Moreover, for the part of the solution that is determined by the structural parameters of the model, inflation moves in the opposite direction from a change in the inflation target.[34] This is again similar to the case under the PMAF regime. Unlike the case under PMAF (and AMPF) however, the initial effect of the shock $\hat{r}_t^*$ on inflation is negative. Finally, Proposition 7 also establishes that since $E_{t-1}\hat{\pi}_t$ is not a function of $\hat{b}_{t-1}$, $\hat{b}_{t-1}$ does not directly affect inflation dynamics under the PMPF regime. This is a critical difference from the PMAF regime. Thus, while indeterminacy changes the propagation mechanism of fundamental shocks compared to AMPF and PMAF, unlike PMAF, it does not introduce public debt as an endogenous state variable.

Importantly, note that while we prove results above regarding a part of the solution under indeterminacy, it is easy to see from Equation (10) that the *overall* relationship between $\hat{\pi}_t$ and $\hat{\pi}_t^*$ and the *total* initial effect of $\hat{r}_t^*$ on $\hat{\pi}_t$ is ambiguous, depending on the values taken by M_π and M_r. For example, $\hat{\pi}_t$ and $\hat{\pi}_t^*$ may be positively or negatively related. If agents form self-fulfilling expectations that a shock to $\hat{\pi}_t^*$ increases inflation significantly ($M_\pi >> 0$), then the self-fulfilling expectations can dominate the effect from $\Phi\left(\phi,\psi\right)$ and inflation can respond positively to a shock to $\hat{\pi}_t^*$. Therefore, in general, the question of how $\hat{\pi}_t$ is related

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[33]For analytical tractability, we focus on a case where the shocks are iid.

[34]Lubik and Schorfheide (2003) show in a three-equation model without fiscal policy and variables that for the part of the solution determined by structural parameters of the model, the initial effect of a positive monetary shock on inflation is positive. Our results are thus consistent with theirs, since a positive monetary shock and a negative inflation target shock behave similarly, and moreover, provide a generalization since we consider a model with fiscal policy. In addition, as we note above, not including fiscal policy explicitly under indeterminacy is a source of misspecification.

to $\hat{\pi}_t^*$ (and how $\hat{\pi}_t$ responds to $\hat{r}_t^*$) can be answered only empirically. The answer will depend on the numerical values of both non-structural and structural parameters. For example, in our estimated model in Bhattarai, Lee, and Park (2012b), we find that the inflation target is not a significant driving force of inflation dynamics under the PMPF regime, as depicted in Figure 1.

3   Quantitative Model

In this section, we assess whether the results found analytically with the simple model in the previous sections also hold in a quantitative model. We use a standard medium-scale DSGE model that features a rich set of frictions and shocks along the lines of Smets and Wouters (2007), Del Negro, Schorfheide, Smets, and Wouters (2007), and Justiniano, Primiceri, and Tambalotti (2010). We lay out the basic model features below while providing a complete description in the appendix. The main actors and their decision problems are as follows.

3.1   Description

3.1.1   Households

Households, a continuum in the unit interval, face an infinite horizon problem and maximize expected discounted utility over consumption and leisure. The utility function is additively separable over consumption and labor effort. There is time-varying external habit formation in consumption and a discount factor shock. Households own capital that they rent to firms. They choose optimally the variable capacity utilization rate and make the capital accumulation decision taking into account capital adjustment costs. There is an investment shock in the capital accumulation equation that leads to a variation in the efficiency with which the consumption good is converted into capital.

Each household is a monopolistic supplier of differentiated labor. The elasticity of substitution over the differentiated labor varieties is time-varying. A large number of competitive employment agencies combine the differentiated labor services into a homogeneous labor input that is sold to firms. Each household enjoys some monopoly power over setting wages, which are sticky in nominal terms. Wage stickiness is modelled following Calvo (1983). There is a constant probability of not adjusting wages every period, with wages that do not adjust partially indexed to past inflation.

3.1.2 Firms

Firms, a continuum in the unit interval, produce differentiated goods using the homogenous labor input and capital. The elasticity of substitution over the differentiated goods varieties is time-varying. There is a fixed cost in production, which ensures zero profits in steady-state. The production function, which takes a Cobb-Douglas form, is subject to an aggregate neutral technology shock. Each firm enjoys some monopoly power over setting prices, which are sticky in nominal terms. Price stickiness is modelled following Calvo (1983). There is a constant probability of not adjusting prices every period, with prices that do not adjust partially indexed to past inflation.

3.1.3 Government

The government is subject to a flow budget constraint and conducts monetary and fiscal policies using endogenous feedback rules. For simplicity, we assume that the government issues only one-period nominal debt and levies lump-sum taxes. The government controls the one-period nominal interest rate. Monetary policy is modelled using an interest rate rule that features interest rate smoothing and a systematic response of the nominal interest rate to the deviation of inflation from a time-varying target and the deviation of output from the natural level of output.[35] Monetary policy shock is the non-systematic component of this policy rule. Fiscal policy is modelled using a tax rule that features tax smoothing and a systematic response of the tax revenues to the level of outstanding government debt. Government spending-to-output ratio evolves exogenously as a time-varying fraction of output.

3.2   Approximate Model

We first solve the problem of households and firms given the monetary and fiscal policy rules and derive the equilibrium conditions. We then use approximation methods to solve the model. First, the model features a stochastic balanced growth path since the neutral technology shock contains a unit root. Therefore, we de-trend variables on the balanced growth path by the level of the technology shock and write down all the equilibrium conditions of the transformed model. Second, we compute the non-stochastic steady state of this transformed model. Third, we obtain a first-order approximation of the equilibrium conditions around this steady state. We then solve the approximated model using standard methods. The approximated equations are provided in the appendix.

As in the simple model, the existence and uniqueness of equilibrium depends crucially

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[35]The natural level of output is the output that would prevail under flexible wages and prices and in the absence of time-variation in the elasticity of substitution over the different varieties of labor and goods.

on the prevailing monetary and fiscal policy regime. In this richer model, we are unable to analytically characterize the exact parameter boundaries that lead to active and passive policies. We therefore determine the boundaries numerically.

3.3 Results

The results from this quantitative model are consistent with our analytical results, as we discuss below in detail.[36] We will present results with respect to the inflation target shock and six non-policy shocks: neutral technology shock, government spending, investment specific technology shock, price markup shock, wage markup shock, and a preference shock.

3.3.1 Parameter Values

Our model, other than a slightly different specification of monetary policy rule and an inclusion of a fiscal block, is the same as in Del Negro, Schorfheide, Smets, and Wouters (2007), and Justiniano, Primiceri, and Tambalotti (2010).[37] For our numerical exercises, we use the posterior median estimates of Justiniano, Primiceri, and Tambalotti (2010) for all the parameters related to preferences and technology. For all three policy regimes, we also use the same value as their posterior median estimates for the monetary policy feedback parameter on output gap. For the tax smoothing parameter in the fiscal policy rule we use the posterior estimate of Bhattarai, Lee, and Park (2012b), while for the steady state level of the maturity value of debt-to-output, we use the sample average from U.S. data. We then conduct several comparative static exercises with respect to the policy feedback parameters on inflation and debt outstanding to show that the numerical results from this model are consistent with our analytical results from the simple model. All the parameter values that we use are provided in the appendix.

3.3.2 Inflation Target Shock

Panels (a)-(d) of Figure 7 show for the three policy regimes the impulse response of inflation to an exogenous change in the inflation target, varying the degree of monetary policy stance. They clearly illustrate one of the main results of our paper: under AMPF, inflation moves in the same direction as the inflation target and that higher the systematic response of monetary policy to inflation, lower is the gap between inflation and the inflation target, while under PMAF, in sharp contrast, inflation moves in an opposite direction from the inflation target

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[36]There are two cases of differences with respect to the intial impact on inflation, for which we provide detailed explanations.

[37]The main differences in the monetary policy rule specification is that we include a time-varying inflation target while excluding the growth rate of the output gap.

and higher is the systematic response of monetary policy to inflation, higher is the gap between inflation and the inflation target. Moreover, under PMPF, depending on the value of the parameter M_π, which governs how self-fulfilling beliefs are formed under indeterminacy, inflation could either move in the same direction as the inflation target or in an opposite direction. Finally, under PMAF, panel (e) of Figure 7 shows that lower is the response of taxes to debt, greater is the gap between inflation and the inflation target.

3.3.3 Non-policy Shocks

Figure 8 shows under AMPF the impulse response of inflation to six non-policy shocks, varying the degree of monetary policy stance. It is clear that inflation responds less on impact to a non-policy shock when the systematic response of monetary policy to inflation is higher for all cases, except for the investment specific technology shock. Even for this shock however, after 5 periods or so, the response is lower for a greater ϕ_π. In our simple model, where we abstract from investment, this shock is not present. In this quantitative model, the initial response of inflation is higher for a greater ϕ_π because this shock directly and significantly affects the capital rental cost for firms. Thus, when ϕ_π is greater, it can be the case that the rise in marginal cost due to a positive investment specific shock outweighs the usual inflation stabilization effect, thereby leading to a greater response of inflation. This result however, depends on all the other parameters of the model, in particular, the extent of wage stickiness in the economy. This is because wage stickiness determines the dynamics of wages, an important component of marginal cost. In fact in the appendix, in an alternate parameterization, we show a case where inflation responds less on impact to this shock with a greater ϕ_π, which makes it completely consistent with our analytical results. In this alternate parameterization, we decrease the extent of wage stickiness compared to the baseline case presented here, which magnifies the inflation stabilization effect of monetary policy on wage costs and, thereby, damps down the increase in inflation following an investment specific shock.

Figure 9 shows under PMAF the impulse response of inflation to six non-policy shocks, varying the degree of monetary policy stance. It is clear that for all cases and in sharp contrast to AMPF, inflation responds more on impact to a non-policy shock when the systematic response of monetary policy to inflation is higher.

Figure 10 shows under PMAF the impulse response of inflation to six non-policy shocks, varying the degree of fiscal policy stance It is clear that inflation responds less on impact to a non-policy shock when the systematic response of fiscal policy to debt is lower for all cases, except for the neutral technology shock. This result is different from our analytical results in the simple model. The reason is that this quantitative model features stochastic growth and the technology shock is therefore a shock to the growth rate as opposed to a shock to the

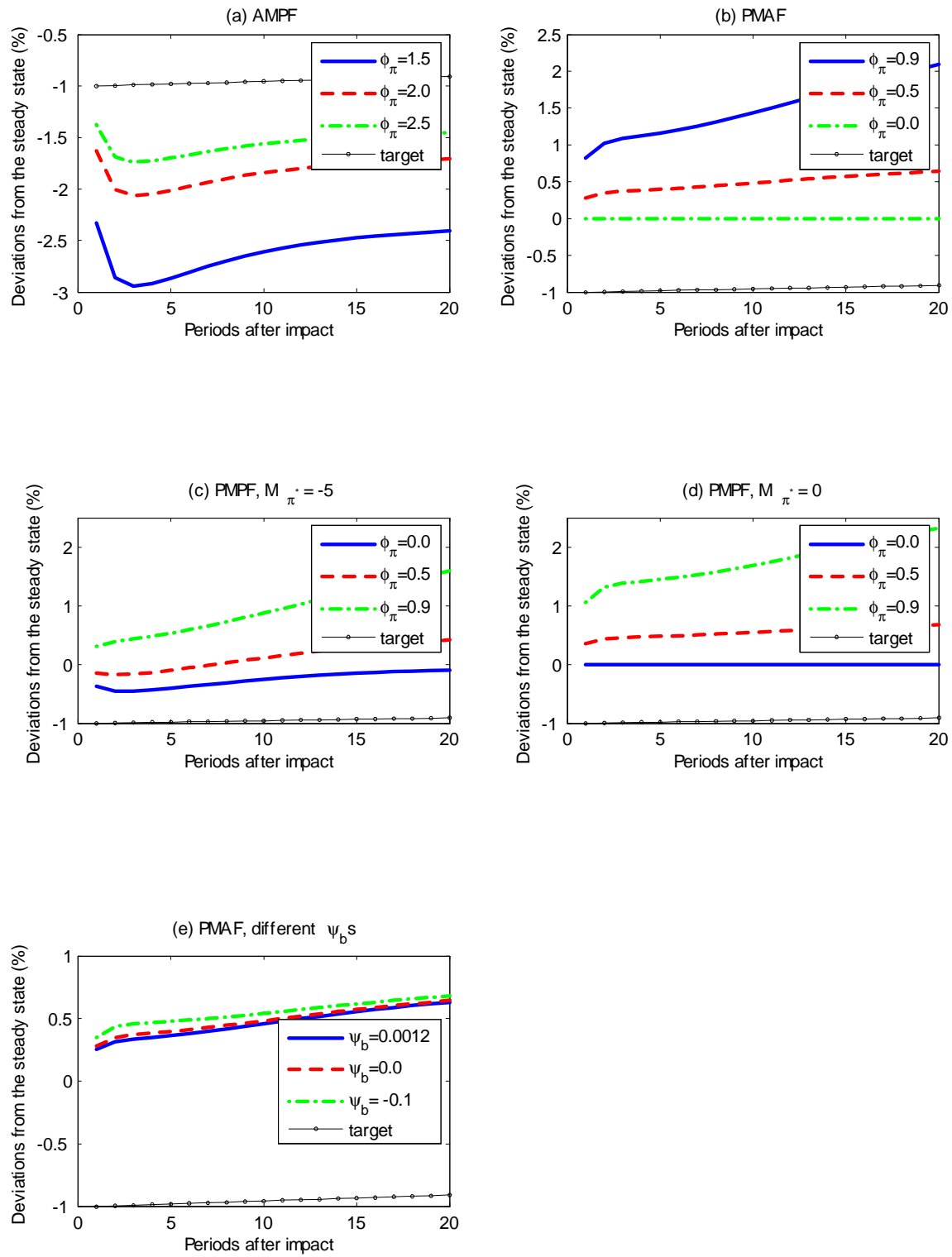Figure 7: The response of inflation to a one percentage decrease in the inflation target.
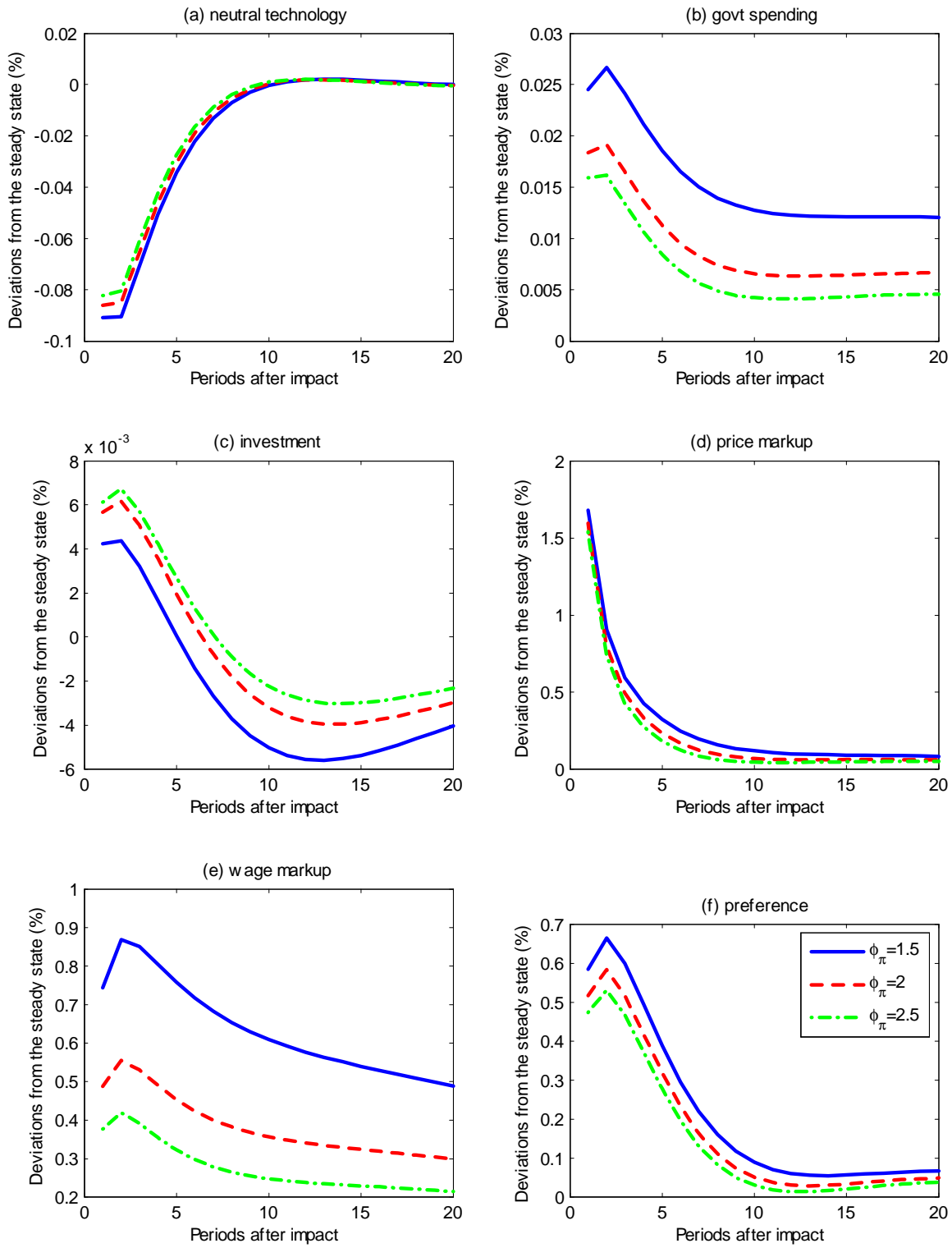
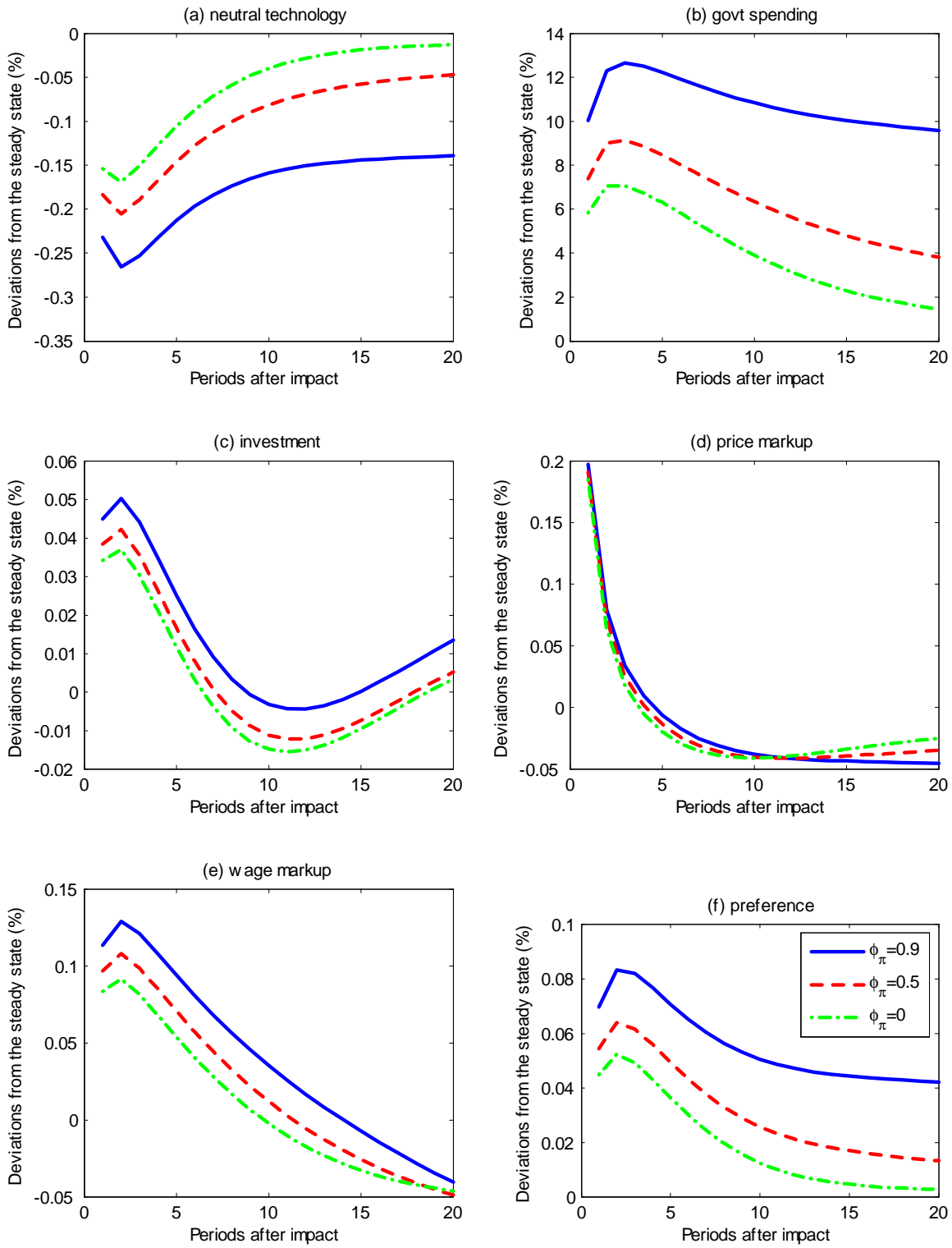Figure 8: The response of inflation to a one standard deviation increase in the non-policy shock under AMPF.

Figure 9: The response of inflation to a one standard deviation increase in the non-policy shock under PMAF.
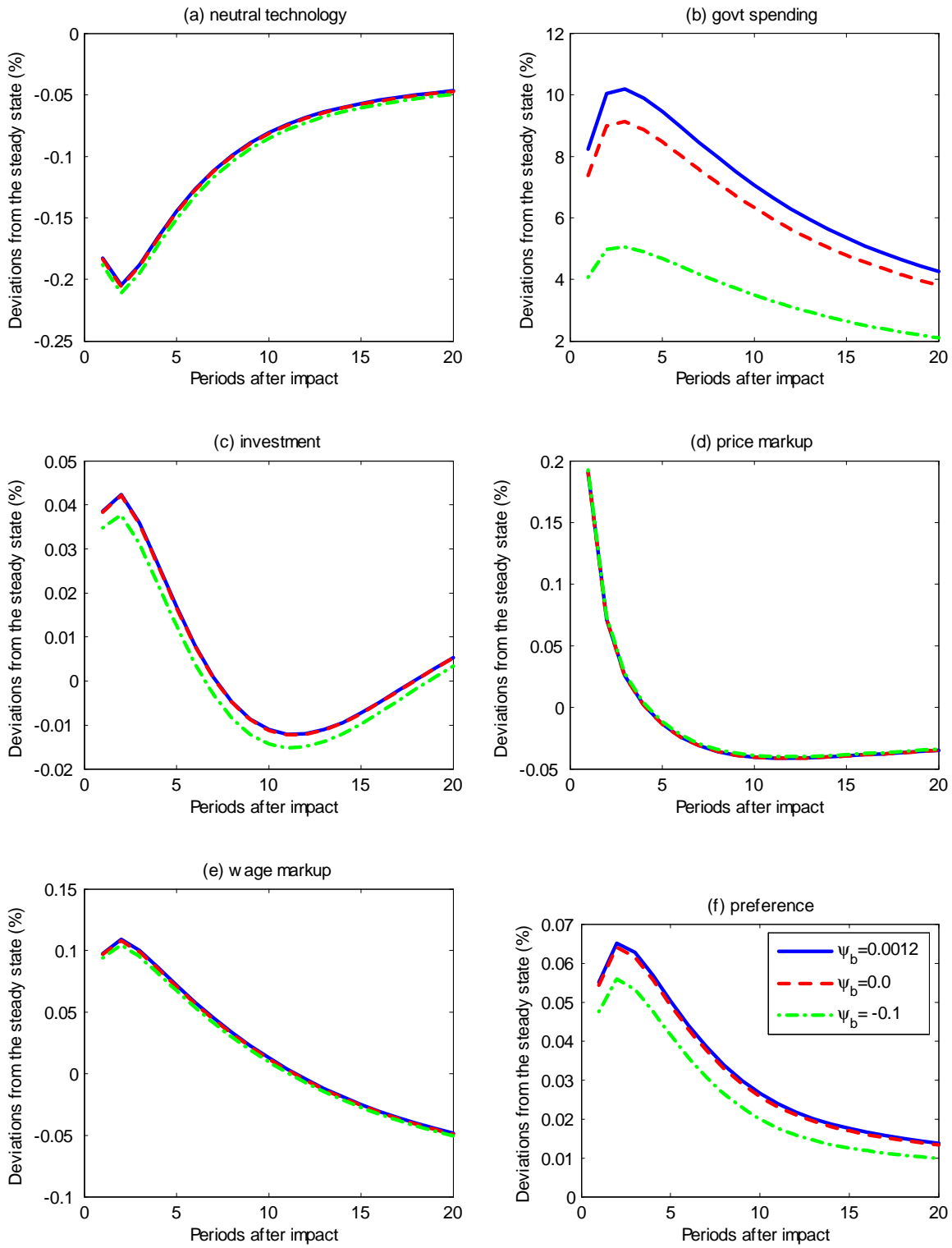
Figure 10: The response of inflation to a one standard deviation increase in the non-policy shock under PMAF.
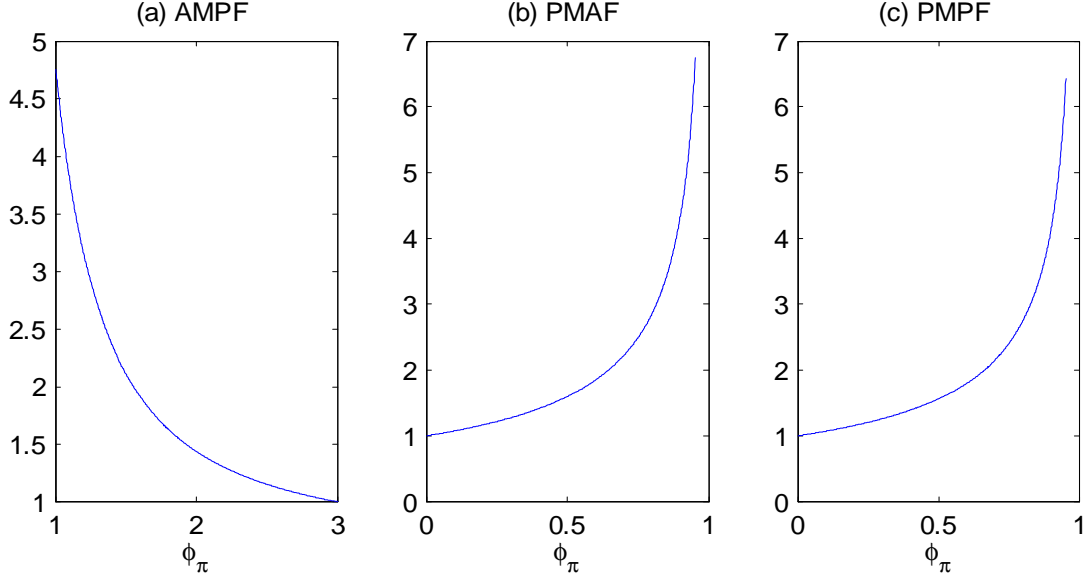
Figure 11: Standard deviation of inflation across different values of ϕ_π conditional on the non-policy shocks. The standard deviation is normalized to unity corresponding to the parameterization with $\phi = 3$ under AMPF and $\phi = 0$ under PMAF and PMPF.

level of technology, which was the case in the simple model. Thus, due to this, the shock can significantly affect the dynamics of inflation as it plays a prominent role in the government budget constraint. To preserve space, we do not present impulses responses under PMPF as the results clearly depend on the calibration of M_π.

We now move on to presenting results on the volatility of inflation. This is especially pertinent because arguably, focusing on the volatility of inflation is a more sensible metric for inflation dynamics in this quantitative model which features various adjustment costs and internal propagation mechanisms. Figure 11 shows under the three policy regimes the standard deviation of inflation, varying the degree of monetary policy stance. Here we present the standard deviation of inflation when all six non-policy shocks hit the economy. Panels (a) and (b) clearly depict one of the main results of our paper: in response to non-policy shocks under AMPF, inflation volatility decreases as monetary policy responds strongly to inflation, while in sharp contrast, under PMAF, inflation volatility increases. In this particular parameterization, panel (c) shows that under PMPF, inflation volatility increases when monetary policy responds strongly to inflation.

4   Conclusion

In this paper we characterize the dynamics of inflation under different monetary and fiscal regime combinations in a standard DSGE model. First, using a simple set-up that allows for

closed-form solutions, we show that answers to some classic questions on inflation dynamics depend crucially on the prevailing policy regime. Second, we show that our insights continue to hold in a richer quantitative model.

Our results show that under an active monetary and passive fiscal policy regime, inflation closely follows the path of the inflation target and a stronger reaction of monetary policy to inflation decreases the response of inflation to shocks. This is the usual case studied in the literature and the results are not surprising since in this regime, monetary policy has control over inflation. In sharp contrast, under an active fiscal and passive monetary policy regime, inflation moves in an opposite direction from the inflation target and a stronger reaction of monetary policy to inflation increases the response of inflation to shocks. These effects arise crucially because of the prevalence of a wealth effect in response to interest rate movements that change the value of government debt. In particular, an increase in interest rate, because of a positive wealth effect, increases spending, and thereby inflation. Moreover, in this case, a weaker response of fiscal policy to debt decreases the response of inflation to shocks. Finally, under a passive monetary and passive fiscal policy regime, because of equilibrium indeterminacy and role for self-fulfilling beliefs, theory provides no clear answer on the behavior of inflation in response to shocks, which can only be ascertained by bringing the model to the data. Even in this regime however, we show that generally, the dynamics of inflation depends on both monetary and fiscal policy stances, even though fiscal policy is passive.

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Appendix

A Simple Model

A.1 Households

Identical households choose sequences of $\{C_t, B_t, N_t, D_{t+1}\}$ to solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t d_t \left[\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

subject to

$$P_t C_t + B_t + E_t\left[Q_{t,t+1} D_{t+1}\right] = R_{t-1} B_{t-1} + D_t + W_t N_t + \Pi_t - P_t \tau_t,$$

where C_t is consumption, N_t is labor hours, P_t is the price level, B_t is the amount of one-period risk-less nominal government bond, R_t is the gross nominal interest rate, W_t is the nominal wage rate, Π_t is profits of intermediate firms, and τ_t is government taxes net of transfers. The parameter, $\varphi \geq 0$, denotes the inverse of the Frisch elasticity of labor supply, while d_t represents an intertemporal preference shock. In addition to the government bond, households trade at time t one-period state-contingent nominal securities D_{t+1} at price $Q_{t,t+1}$.

A.2 Firms

Perfectly competitive firms produce the final good, Y_t, by assembling intermediate goods, $Y_t(i)$, through a Dixit and Stiglitz (1977) technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ denotes the elasticity of substitution between intermediate goods. The corresponding price index for the final consumption good is $P_t = \left(\int_0^1 P_t(i)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$, where $P_t(i)$ is the price of the intermediate good i. The optimal demand for $Y_t(i)$ is given by $Y_t(i) = (P_t(i)/P_t)^{-\theta} Y_t$.

Monopolistically competitive firms produce intermediate goods using the production function, $Y_t(i) = a_t N_t(i)$, where $N_t(i)$ denotes the labor hours employed by firm i and a_t represents exogenous economy-wide productivity. Prices are sticky as in Calvo. A firm adjusts its price, $P_t(i)$, with probability $1 - \alpha$ each period, to maximize the present discounted value of future profits:

$$E_t \sum_{k=0}^{\infty} \alpha^k Q_{t,t+k} \left[P_t(i) - \frac{W_{t+k}}{A_{t+k}} \right] Y_{t+k}(i).$$

A.3 Government

Each period, the government collects lump-sum tax revenues τ_t and issues one-period nominal bonds B_t to finance its consumption G_t, and interest payments. Accordingly, the flow budget constraint is given by:

$$\frac{B_t}{P_t} = R_{t-1} \frac{B_{t-1}}{P_t} + G_t - \tau_t.$$

For simplicity, we assume $G_t = 0$, which is inconsequential for our theoretical results. The flow budget constraint can be rewritten as:

$$R_t^{-1} b_t = b_{t-1} \frac{1}{\pi_t} - \tau_t,$$

where $b_t \equiv R_t \frac{B_t}{P_t}$ denotes the real maturity value of government debt.

The monetary and fiscal policies are described by simple rules. The monetary authority responds to deviations of the inflation rate from its time-varying target rate, π_t^*, by setting the nominal interest rate according to:

$$\frac{R_t}{\bar{R}} = \left(\frac{\pi_t}{\pi_t^*} \right)^{\phi},$$

where $\bar{R}$ is the steady-state value of R_t. Similarly, the fiscal authority sets the tax revenues according to:

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{b_{t-1}}{\bar{b}} \right)^{\psi},$$

where $\bar{\tau}$ and $\bar{b}$ are respectively the steady state value of τ_t and b_t.

A.4   Approximate Model

We log-linearize the equilibrium conditions around non-stochastic steady state values: $\{\bar{\pi}, \ \bar{Y}, \ \bar{R}, \ \bar{b}, \ \bar{\tau}\}$. Since the log-linearized model is completely standard, we omit a detailed derivation. The approximate model is characterized by the following equations:

$$
\begin{aligned}
\hat{Y}_t &= E_t \hat{Y}_{t+1} - \left(\hat{R}_t - E_t \hat{\pi}_{t+1} \right) - E_t[\Delta \hat{d}_{t+1}], \\
\hat{\pi}_t &= \kappa \left(\hat{Y}_t - \hat{Y}_t^n \right) + \beta E_t \hat{\pi}_{t+1}, \\
\hat{R}_t &= \phi \left(\hat{\pi}_t - \hat{\pi}_t^* \right), \\
\hat{\tau}_t &= \psi \hat{b}_{t-1}, \\
\hat{b}_t &= \beta^{-1} \hat{b}_{t-1} - \beta^{-1} \bar{b} \hat{\pi}_t - \beta^{-1} \hat{\tau}_t + \bar{b} \hat{R}_t.
\end{aligned}
$$

In the equations above, we use $\hat{X}_t$ to denote the log deviation of a variable X_t from its steady state $\bar{X}$ ($\hat{X}_t = \ln X_t - \ln \bar{X}$), except for two fiscal variables, $\hat{b}_t$ and $\hat{\tau}_t$. Following Woodford (2003), we let them represent respectively the deviation of the maturity value of government debt and of government tax revenues (net of transfers) from their steady-state levels, measured as a percentage of steady-state output: $\hat{b}_t = \frac{b_t - \bar{b}}{\bar{Y}}$ and $\hat{\tau}_t = \frac{\tau_t - \bar{\tau}}{\bar{Y}}$. In our simple model, (the log-deviation of) the natural level of output and the slope of the Phillips curve are respectively given as $\hat{Y}_t^n = \hat{a}_t$ and $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$.

The model can be reduced to a dynamic system of $\left\{ \hat{\pi}_t, \hat{b}_t, \tilde{Y}_t \right\}$:

$$
\begin{aligned}
\tilde{Y}_t &= E_t \tilde{Y}_{t+1} - \phi \left(\hat{\pi}_t - \hat{\pi}_t^* \right) + E_t \hat{\pi}_{t+1} + \hat{r}_t^*, \\
\hat{\pi}_t &= \kappa \tilde{Y}_t + \beta E_t \hat{\pi}_{t+1}, \\
\hat{b}_t &= \beta^{-1}(1-\psi)\hat{b}_{t-1} - \bar{b} \left(\beta^{-1} - \phi \right) \hat{\pi}_t - \bar{b}\phi\hat{\pi}_t^*,
\end{aligned}
\tag{11}
$$

where $\tilde{Y}_t \equiv \hat{Y}_t - \hat{Y}_t^n$ represents the output gap and $\hat{r}_t^*$ is a linear combination of all non-policy shocks (of both supply and demand types). It is often referred to as the natural rate of interest because it is the real interest

rate that would prevail under flexible prices. In our simple model, it is specifically given as:

$$\hat{r}_t^* = E_t\left[\Delta\hat{a}_{t+1}\right] - E_t[\Delta\hat{d}_{t+1}].$$

Note that demand-type shocks raise $\hat{r}_t^*$, while supply-type shocks lower $\hat{r}_t^*$.

B Solution of the Simple Model

In this section, we solve for the equilibrium time paths of $\left\{\hat{\pi}_t, \hat{b}_t, \tilde{Y}_t\right\}$ given exogenous variables summarized by the policy and non-policy shocks, $\{\hat{\pi}_t^*, \hat{r}_t^*\}$. To this end, we assume the exogenous random variables follow AR(1) processes:

$$
\begin{aligned}
\hat{\pi}_t^* &= \rho_\pi \hat{\pi}_{t-1}^* + \varepsilon_{\pi,t}, \\
\hat{r}_t^* &= \rho_r \hat{r}_{t-1}^* + \varepsilon_{r,t}.
\end{aligned}
$$

We first write (11) in state space form:

$$
\begin{pmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} E_t \begin{pmatrix} \tilde{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = \begin{pmatrix} 1 & \phi & 0 \\ -\kappa & 1 & 0 \\ 0 & -\bar{b}\left(\beta^{-1} - \phi\right) & \beta^{-1}(1-\psi) \end{pmatrix} \begin{pmatrix} \tilde{Y}_t \\ \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix}
$$

$$
+ \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ r_t^* \end{pmatrix} \tag{12}
$$

We then pre-multiply $\begin{pmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1}$ to both sides of the equation (12):

$$
E_t \begin{pmatrix} \tilde{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = \underbrace{\begin{pmatrix} \kappa\beta^{-1} + 1 & \phi - \beta^{-1} & 0 \\ -\kappa\beta^{-1} & \beta^{-1} & 0 \\ 0 & -\bar{b}\left(\beta^{-1} - \phi\right) & \beta^{-1}(1-\psi) \end{pmatrix}}_{=G} \begin{pmatrix} \tilde{Y}_t \\ \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix}
$$

The coefficient matrix, G, can be decomposed as $G = VDV^{-1}$, where D is a diagonal matrix whose elements are the eigenvalues of G. The system then can be written as:

$$
E_t \begin{pmatrix} \tilde{Y}_{t+1} \\ \hat{\pi}_{t+1} \\ \hat{b}_t \end{pmatrix} = V \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} V^{-1} \begin{pmatrix} \tilde{Y}_t \\ \hat{\pi}_t \\ \hat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix},
$$

where

$$e_1 = \frac{1}{2\beta}\left(\beta + \kappa + 1 + \sqrt{(\beta + \kappa + 1)^2 - 4\beta(1 + \kappa\phi)}\right)$$

$$e_2 = \beta^{-1}(1 - \psi)$$

$$e_3 = \frac{1}{2\beta}\left(\beta + \kappa + 1 - \sqrt{(\beta + \kappa + 1)^2 - 4\beta(1 + \kappa\phi)}\right)$$

$$V = \begin{pmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ 1 & 1 & 1 \end{pmatrix} \qquad \text{and} \qquad V^{-1} = \begin{pmatrix} q_{11} & q_{12} & 0 \\ q_{21} & q_{22} & 1 \\ q_{31} & q_{32} & 0 \end{pmatrix}.$$

The elements of V and V^{-1} are nonlinear functions of the model parameters. For later use, we note that:

$$v_{23} = \frac{2(1 - \psi) - \left(\beta + \kappa + 1 - \sqrt{(\beta + \kappa + 1)^2 - 4\beta(1 + \kappa\phi_\pi)}\right)}{2\bar{b}(1 - \beta\phi)}.$$

Finally, letting $X_t \equiv \begin{pmatrix} x_{1,t} & x_{2,t} & x_{3,t} \end{pmatrix}^T \equiv V^{-1}\begin{pmatrix} \tilde{Y}_t & \hat{\pi}_t & \hat{b}_{t-1} \end{pmatrix}^T$, we rewrite the system as:

$$E_t X_{t+1} = \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} X_t + \begin{pmatrix} -\phi q_{11} & -q_{11} \\ -\phi(q_{21} + \bar{b}) & -q_{21} \\ -\phi q_{31} & -q_{31} \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix}. \tag{13}$$

Each element of X_t is given by:

$$x_{1,t} = q_{11}\tilde{Y}_t + q_{12}\hat{\pi}_t,$$

$$x_{2,t} = q_{21}\tilde{Y}_t + q_{22}\hat{\pi}_t + \hat{b}_{t-1},$$

$$x_{3,t} = q_{31}\tilde{Y}_t + q_{32}\hat{\pi}_t.$$

B.1 Active Monetary and Passive Fiscal Policy

Under AMPF, e_1 and e_3 are outside the unit circle, while e_2 is inside the circle. We thus use the first and third rows of the system (13) to draw linear restrictions between model variables. Substituting out the future values of $x_{1,t}$ and $x_{3,t}$ recursively, we obtain:

$$x_{1,t} = \frac{1}{e_1}\sum_{k=0}^{\infty}\left(\frac{1}{e_1}\right)^k E_t z_{1,t+k}^*, \tag{14}$$

$$x_{3,t} = \frac{1}{e_3}\sum_{k=0}^{\infty}\left(\frac{1}{e_3}\right)^k E_t z_{3,t+k}^*, \tag{15}$$

where

$$z_{1,t}^* = \phi q_{11}\hat{\pi}_t^* + q_{11}\hat{r}_t^*,$$

$$z_{3,t}^* = \phi q_{31}\hat{\pi}_t^* + q_{31}\hat{r}_t^*.$$

These equations imply:

$$E_t z_{1,t+k}^* = \phi q_{11} \rho_\pi^k \hat{\pi}_t^* + q_{11} \rho_r^k \hat{r}_t^*$$
$$E_t z_{3,t+k}^* = \phi q_{31} \rho_\pi^k \hat{\pi}_t^* + q_{31} \rho_r^k \hat{r}_t^*$$

Plugging these equations into (14) and (15), we obtain:

$$x_{1,t} = \frac{1}{e_1} \sum_{k=0}^{\infty} \left(\frac{1}{e_1}\right)^k E_t z_{1,t+k}^* = \phi q_{11} \frac{1}{e_1 - \rho_\pi} \hat{\pi}_t^* + q_{11} \frac{1}{e_1 - \rho_r} \hat{r}_t^*,$$

$$x_{3,t} = \frac{1}{e_3} \sum_{k=0}^{\infty} \left(\frac{1}{e_3}\right)^k E_t z_{3,t+k}^* = \phi q_{31} \frac{1}{e_3 - \rho_\pi} \hat{\pi}_t^* + q_{31} \frac{1}{e_3 - \rho_r} \hat{r}_t^*,$$

which leads to:

$$\tilde{Y}_t = -\frac{q_{12}}{q_{11}} \hat{\pi}_t + \phi \frac{1}{e_1 - \rho_\pi} \hat{\pi}_t^* + \frac{1}{e_1 - \rho_r} \hat{r}_t^*$$

$$\tilde{Y}_t = -\frac{q_{32}}{q_{31}} \hat{\pi}_t + \phi \frac{1}{e_3 - \rho_\pi} \hat{\pi}_t^* + \frac{1}{e_3 - \rho_r} \hat{r}_t^*$$

Since the above system has two equations and two endogenous variables, we can easily solve for $\hat{\pi}_t$ and $\tilde{Y}_t$. Moreover, $\hat{\pi}_t$ and $\tilde{Y}_t$ do not depend on $\hat{b}_{t-1}$. We use the method of undetermined coefficients and obtain:

$$\hat{\pi}_t = \Phi(\phi) \hat{\pi}_t^* + \Gamma(\phi) \hat{r}_t^*$$
$$\tilde{Y}_t = \Phi^Y(\phi) \hat{\pi}_t^* + \Gamma^Y(\phi) \hat{r}_t^*$$

where

$$\Phi(\phi) \equiv \frac{\kappa \phi}{\kappa(\phi - \rho_\pi) + (1 - \rho_\pi)(1 - \beta \rho_\pi)}$$

$$\Gamma(\phi) \equiv \frac{\kappa}{\kappa(\phi - \rho_r) + (1 - \rho_r)(1 - \beta \rho_r)}$$

$$\Phi^Y(\phi) \equiv \frac{\phi(1 - \beta \rho_\pi)}{\kappa(\phi - \rho_\pi) + (1 - \rho_\pi)(1 - \beta \rho_\pi)}$$

$$\Gamma^Y(\phi) \equiv \frac{(1 - \beta \rho_r)}{\kappa(\phi - \rho_r) + (1 - \rho_r)(1 - \beta \rho_r)}.$$

B.2 Passive Monetary and Active Fiscal Policy

We consider the case in which $\phi_\pi \in [0,1)$ and $\psi \in (-\infty, \bar{\psi})$ where $\bar{\psi} \equiv 1 - \beta$ is the upper bound for active fiscal policy. We then can show that $e_1 > 1$, $e_2 > 1$ and $e_3 \in (0,1)$ in that parameter space. Consequently the first two rows in (13) provide linear restrictions. From the rows, we obtain:

$$x_{1,t} = \frac{1}{e_1} \sum_{k=0}^{\infty} \left(\frac{1}{e_1}\right)^k E_t z_{1,t+k}^*, \tag{16}$$

$$x_{2,t} = \frac{1}{e_2} \sum_{k=0}^{\infty} \left(\frac{1}{e_2}\right)^k E_t z_{2,t+k}^*, \tag{17}$$

where

$$z_{1,t}^* = \phi q_{11} \hat{\pi}_t^* + q_{11} \hat{r}_t^*,$$
$$z_{2,t}^* = \phi \left(q_{21} + \bar{b} \right) \hat{\pi}_t^* + q_{21} \hat{r}_t^*.$$

The equations above imply:

$$E_t z_{1,t+k}^* = \phi q_{11} \rho_\pi^k \hat{\pi}_t^* + q_{11} \rho_r^k \hat{r}_t^*$$
$$E_t z_{2,t+k}^* = \phi \left(q_{21} + \bar{b} \right) \rho_\pi^k \hat{\pi}_t^* + q_{21} \rho_r^k \hat{r}_t^*$$

Plugging these equations into (14) and (15), we obtain:

$$x_{1,t} = \frac{1}{e_1} \sum_{k=0}^{\infty} \left(\frac{1}{e_1} \right)^k E_t z_{1,t+k}^* = \phi q_{11} \frac{1}{e_1 - \rho_\pi} \hat{\pi}_t^* + q_{11} \frac{1}{e_1 - \rho_r} \hat{r}_t^* \tag{18}$$

$$x_{2,t} = \frac{1}{e_2} \sum_{k=0}^{\infty} \left(\frac{1}{e_2} \right)^k E_t z_{2,t+k}^* = \phi \left(q_{21} + \bar{b} \right) \frac{1}{e_2 - \rho_\pi} \hat{\pi}_t^* + q_{21} \frac{1}{e_2 - \rho_r} \hat{r}_t^* \tag{19}$$

Equation (18) implies:

$$\tilde{Y}_t = -\frac{q_{12}}{q_{11}} \hat{\pi}_t + \phi \frac{1}{e_1 - \rho_\pi} \hat{\pi}_t^* + \frac{1}{e_1 - \rho_r} \hat{r}_t^* \tag{20}$$

We plug (20) into (19) to get:

$$q_{21} \left[-\frac{q_{12}}{q_{11}} \hat{\pi}_t + \phi \frac{1}{e_1 - \rho_\pi} \hat{\pi}_t^* + \frac{1}{e_1 - \rho_r} r_t^* \right] + q_{22} \hat{\pi}_t + \hat{b}_{t-1} = \phi \left(q_{21} + \bar{b} \right) \frac{1}{e_2 - \rho_\pi} \hat{\pi}_t^* + q_{21} \frac{1}{e_2 - \rho_r} \hat{r}_t^*$$

Solving for $\hat{\pi}_t$, we obtain $\hat{\pi}_t$ as a function of state variables, $\left\{ \hat{b}_{t-1}, \hat{\pi}_t^*, r_t^* \right\}$:

$$\hat{\pi}_t = \Omega \hat{b}_{t-1} - \Omega \phi \left[\left(q_{21} + \bar{b} \right) \frac{1}{e_2 - \rho_\pi} - q_{21} \frac{1}{e_1 - \rho_\pi} \right] \hat{\pi}_t^* + \Omega q_{21} \left[\frac{1}{e_1 - \rho_r} - \frac{1}{e_2 - \rho_r} \right] r_t^* \tag{21}$$

where

$$\Omega = \frac{q_{11}}{q_{12} q_{21} - q_{11} q_{22}}$$
$$q_{21} = \frac{\bar{b} \kappa \left(1 - \beta \phi \right)}{\psi^2 + \left(\beta + \kappa - 1 \right) \psi - \kappa \left(1 - \beta \phi \right)}.$$

For further analysis, it is useful to express the coefficients on the state variables in terms of model parameters. To this end, we use the results in the following lemmas.

Lemma 1: $\Omega = \frac{\beta(e_2 - e_3)}{\bar{b}(1 - \beta \phi)} > 0.$

Proof of Lemma 1: Note that

$$\frac{q_{13} q_{21} - q_{11} q_{23}}{\det \left(V^{-1} \right)} = v_{23},$$
$$\frac{q_{11} q_{22} - q_{12} q_{21}}{\det \left(V^{-1} \right)} = v_{33}.$$

Therefore,

$$0 \times q_{21} - q_{11} = -q_{11} = \det\left(V^{-1}\right) \times v_{23},$$

$$q_{11}q_{22} - q_{12}q_{21} = \det\left(V^{-1}\right) \times v_{33} = \det\left(V^{-1}\right) \times 1.$$

It follows that

$$\Omega = \frac{q_{11}}{q_{12}q_{21} - q_{11}q_{22}} = v_{23}$$

$$= \frac{2\left(1 - \psi\right) - \left(\beta + \kappa + 1 - \sqrt{\left(\beta + \kappa + 1\right)^2 - 4\beta\left(1 + \kappa\phi\right)}\right)}{2\bar{b}\left(1 - \beta\phi\right)}$$

$$= \frac{2\left(1 - \psi\right) - 2\beta e_3}{2\bar{b}\left(1 - \beta\phi\right)} = \frac{\beta\left(e_2 - e_3\right)}{\bar{b}\left(1 - \beta\phi\right)} > 0.$$

Lemma 2: $\Omega q_{21}\left(e_1 - e_2\right) = -\kappa\beta^{-1}$.

Proof of Lemma 2: We have

$$\Omega q_{21}\left(e_1 - e_2\right) = \frac{\beta\left(e_2 - e_3\right)}{\bar{b}\left(1 - \beta\phi\right)} \frac{\bar{b}\kappa\left(1 - \beta\phi\right)}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)}\left(e_1 - e_2\right)$$

$$= \kappa\beta\frac{\left(e_2 - e_3\right)\left(e_1 - e_2\right)}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)} = \kappa\beta\frac{e_1 e_2 - e_2^2 - e_1 e_3 + e_2 e_3}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)}$$

$$= \kappa\beta\frac{e_2\left(e_1 + e_3 - e_2\right) - e_1 e_3}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)} = \kappa\beta\frac{\frac{\left(1 - \psi\right)\left(\beta + \kappa + \psi\right)}{\beta^2} - e_1 e_3}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)}$$

$$= \kappa\beta\frac{\frac{\left(1 - \psi\right)\left(\beta + \kappa + \psi\right)}{\beta^2} - \frac{1 + \kappa\phi}{\beta}}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)} = \kappa\beta\frac{\frac{1}{\beta^2}\left[\left(1 - \psi\right)\left(\beta + \kappa + \psi\right) - \beta - \kappa\beta\phi\right]}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)}$$

$$= \kappa\beta^{-1}\frac{-\left[\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)\right]}{\psi^2 + \left(\beta + \kappa - 1\right)\psi - \kappa\left(1 - \beta\phi\right)} = -\kappa\beta^{-1}.$$

Using the results from these two lemmas, we can simplify (21) as:

$$\hat{\pi}_t = \Omega\left(\phi, \psi\right)\hat{b}_{t-1} - \Phi\left(\phi, \psi\right)\hat{\pi}_t^* + \Gamma\left(\phi, \psi\right)\hat{r}_t^* \tag{22}$$

where

$$\Omega\left(\phi, \psi\right) \equiv \frac{\beta\left(e_2 - e_3\right)}{\bar{b}\left(1 - \beta\phi\right)},$$

$$\Phi\left(\phi, \psi\right) \equiv \phi \times \Theta\left(\phi, \psi\right), \quad \text{where} \quad \Theta\left(\phi, \psi\right) \equiv \frac{\Omega\bar{b}\left(e_1 - \rho_\pi\right) - \kappa\beta^{-1}}{\left(e_1 - \rho_\pi\right)\left(e_2 - \rho_\pi\right)},$$

$$\Gamma\left(\phi, \psi\right) \equiv \frac{\kappa\beta^{-1}}{\left(e_1 - \rho_r\right)\left(e_2 - \rho_r\right)}.$$

It then follows that the law of motion for $\hat{b}_t$ is given as:

$$\hat{b}_t = e_3\hat{b}_{t-1} - \bar{b}\phi\left[1 - \left(\beta^{-1} - \phi\right)\Theta\right]\hat{\pi}_t^* - \bar{b}\left(\beta^{-1} - \phi\right)\Gamma\hat{r}_t^*.$$

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B.3 Passive Monetary and Passive Fiscal Policy

Finally, we consider the case in which $\phi_\pi \in [0,1)$ and $\psi \in (\bar\psi, \infty)$. Then, only one root ($e_1$) is explosive and there will exist multiple solutions to the model, which makes it more difficult to obtain analytical results. For analytical tractability, we focus on a case in which the shocks are i.i.d.

Our solution methodology follows that of Lubik and Schorfheide (2003). We first introduce two new variables, $\xi_t^Y \equiv E_t \tilde Y_{t+1}$ and $\xi_t^\pi \equiv E_t \hat\pi_{t+1}$, and rewrite the model as:

$$
\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde Y_t \\ \hat\pi_t \\ \xi_t^Y \\ \xi_t^\pi \\ \hat b_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \phi & 0 \\ 0 & 0 & -\kappa & 1 & 0 \\ 0 & 0 & 0 & -\bar b(\beta^{-1}-\phi) & \beta^{-1}(1-\psi) \end{pmatrix} \begin{pmatrix} \tilde Y_{t-1} \\ \hat\pi_{t-1} \\ \xi_{t-1}^Y \\ \xi_{t-1}^\pi \\ \hat b_{t-1} \end{pmatrix}
$$

$$
+ \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -\phi & -1 \\ 0 & 0 \\ -\bar b\phi & 0 \end{pmatrix} \begin{pmatrix} \hat\pi_t^* \\ \hat r_t^* \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & \phi \\ -\kappa & 1 \\ 0 & -\bar b(\beta^{-1}-\phi) \end{pmatrix} \begin{pmatrix} \eta_t^Y \\ \eta_t^\pi \end{pmatrix}
$$

Since the system is block-recursive, we solve the lower block first:

$$
\begin{pmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \xi_t^Y \\ \xi_t^\pi \\ \hat b_t \end{pmatrix} = \begin{pmatrix} 1 & \phi & 0 \\ -\kappa & 1 & 0 \\ 0 & -\bar b(\beta^{-1}-\phi) & \beta^{-1}(1-\psi) \end{pmatrix} \begin{pmatrix} \xi_{t-1}^Y \\ \xi_{t-1}^\pi \\ \hat b_{t-1} \end{pmatrix}
$$

$$
+ \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar b\phi & 0 \end{pmatrix} \begin{pmatrix} \hat\pi_t^* \\ \hat r_t^* \end{pmatrix} + \begin{pmatrix} 1 & \phi \\ -\kappa & 1 \\ 0 & -\bar b(\beta^{-1}-\phi) \end{pmatrix} \begin{pmatrix} \eta_t^Y \\ \eta_t^\pi \end{pmatrix}
$$

We pre-multiply $\begin{pmatrix} 1 & 1 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1}$ to both sides of the equation and obtain:

$$
\begin{pmatrix} \xi_t^Y \\ \xi_t^\pi \\ \hat b_t \end{pmatrix} = \underbrace{\begin{pmatrix} \kappa\beta^{-1}+1 & \phi-\beta^{-1} & 0 \\ -\kappa\beta^{-1} & \beta^{-1} & 0 \\ 0 & -\bar b(\beta^{-1}-\phi) & \beta^{-1}(1-\psi) \end{pmatrix}}_{=G} \begin{pmatrix} \xi_{t-1}^Y \\ \xi_{t-1}^\pi \\ \hat b_{t-1} \end{pmatrix} \tag{23}
$$

$$
+ \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar b\phi & 0 \end{pmatrix} \begin{pmatrix} \hat\pi_t^* \\ \hat r_t^* \end{pmatrix} + \begin{pmatrix} \kappa\beta^{-1}+1 & \phi-\beta^{-1} \\ -\kappa\beta^{-1} & \beta^{-1} \\ 0 & -\bar b(\beta^{-1}-\phi) \end{pmatrix} \begin{pmatrix} \eta_t^Y \\ \eta_t^\pi \end{pmatrix}.
$$

As before, the coefficient matrix, G, can be decomposed as $G = VDV^{-1}$. The system then can be written as:

$$\begin{pmatrix} \xi_t^Y \\ \xi_t^\pi \\ \hat{b}_t \end{pmatrix} = V \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} V^{-1} \begin{pmatrix} \xi_{t-1}^Y \\ \xi_{t-1}^\pi \\ \hat{b}_{t-1} \end{pmatrix}$$

$$+ \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix} + \begin{pmatrix} \kappa\beta^{-1}+1 & \phi - \beta^{-1} \\ -\kappa\beta^{-1} & \beta^{-1} \\ 0 & -\bar{b}\left(\beta^{-1}-\phi\right) \end{pmatrix} \begin{pmatrix} \eta_t^Y \\ \eta_t^\pi \end{pmatrix}.$$

Note that G is the same as before, and hence e_1, e_2, e_3, V and V^{-1} are the same as those under AMPF and PMAF. Let $X_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \\ x_{3,t} \end{pmatrix} = V^{-1} \begin{pmatrix} \xi_t^Y \\ \xi_t^\pi \\ \hat{b}_t \end{pmatrix}$, and rewrite the system as:

$$X_t = \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} X_{t-1} + \begin{pmatrix} q_{11} & q_{12} & 0 \\ q_{21} & q_{22} & 1 \\ q_{31} & q_{32} & 0 \end{pmatrix} \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix}$$

$$+ \begin{pmatrix} q_{11} & q_{12} & 0 \\ q_{21} & q_{22} & 1 \\ q_{31} & q_{32} & 0 \end{pmatrix} \begin{pmatrix} \kappa\beta^{-1}+1 & \phi - \beta^{-1} \\ -\kappa\beta^{-1} & \beta^{-1} \\ 0 & -\bar{b}\left(\beta^{-1}-\phi\right) \end{pmatrix} \begin{pmatrix} \eta_t^Y \\ \eta_t^\pi \end{pmatrix}.$$

Note $x_{1,t}$ in particular has an unstable root e_1 and satisfies:

$$x_{1,t} = e_1 x_{1,t-1} + \begin{pmatrix} -\phi q_{11} & -q_{11} \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix} + \begin{pmatrix} \kappa\beta^{-1}\left(q_{11}-q_{12}\right)+q_{11} & -\beta^{-1}\left(q_{11}-q_{12}\right)+\phi q_{11} \end{pmatrix} \begin{pmatrix} \eta_t^Y \\ \eta_t^\pi \end{pmatrix}.$$

Using the singular value decomposition, one can obtain:

$$\begin{pmatrix} \kappa\beta^{-1}\left(q_{11}-q_{12}\right)+q_{11} & -\beta^{-1}\left(q_{11}-q_{12}\right)+\phi q_{11} \end{pmatrix} = \begin{pmatrix} d & 0 \end{pmatrix} \begin{pmatrix} \frac{\kappa\beta^{-1}(q_{11}-q_{12})+q_{11}}{d} & \frac{-\beta^{-1}(q_{11}-q_{12})+\phi q_{11}}{d} \\ \frac{-\beta^{-1}(q_{11}-q_{12})+\phi q_{11}}{d} & -\frac{\kappa\beta^{-1}(q_{11}-q_{12})+q_{11}}{d} \end{pmatrix},$$

where the singular value d is given as:

$$d = \sqrt{\left[\kappa\beta^{-1}\left(q_{11}-q_{12}\right)+q_{11}\right]^2 + \left[-\beta^{-1}\left(q_{11}-q_{12}\right)+\phi q_{11}\right]^2}.$$

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Now apply Proposition 1 of Lubik and Schorfheide (2003; 2004). We then obtain:

$$
\begin{aligned}
\begin{pmatrix} \eta_t^Y \\ \eta_t^\pi \end{pmatrix} &= - \begin{pmatrix} \frac{\kappa\beta^{-1}(q_{11}-q_{12})+q_{11}}{d} \\ \frac{-\beta^{-1}(q_{11}-q_{12})+\phi q_{11}}{d} \end{pmatrix} \times \frac{1}{d} \times 1 \times \begin{pmatrix} -\phi q_{11} & -q_{11} \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix} \\
&\quad + \begin{pmatrix} \frac{-\beta^{-1}(q_{11}-q_{12})+\phi q_{11}}{d} \\ -\frac{\kappa\beta^{-1}(q_{11}-q_{12})+q_{11}}{d} \end{pmatrix} \left\{ M \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix} + \zeta_t^* \right\} \\
&= -\frac{1}{d} \begin{pmatrix} \frac{\kappa\beta^{-1}(q_{11}-q_{12})+q_{11}}{d} \\ \frac{-\beta^{-1}(q_{11}-q_{12})+\phi q_{11}}{d} \end{pmatrix} \begin{pmatrix} -\phi q_{11} & -q_{11} \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix} \\
&\quad + \begin{pmatrix} \frac{-\beta^{-1}(q_{11}-q_{12})+\phi q_{11}}{d} \\ -\frac{\kappa\beta^{-1}(q_{11}-q_{12})+q_{11}}{d} \end{pmatrix} \left\{ M \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix} + \zeta_t^* \right\},
\end{aligned}
$$

where the elements of the matrix, $M = \begin{pmatrix} M_\pi & M_r \end{pmatrix}$, are new parameters introduced due to indeterminacy. Moreover, the dynamics of inflation can be characterized as:

$$
\begin{aligned}
\hat{\pi}_t &= \xi_{t-1}^\pi + \eta_t^\pi = E_{t-1}\hat{\pi}_t + \eta_t^\pi \\
&= E_{t-1}\hat{\pi}_t - \Phi(\phi,\psi)\hat{\pi}_t^* - \Gamma(\phi,\psi)\hat{r}_t^* + \Lambda(\phi,\psi)\left(M_\pi \hat{\pi}_t^* + M_r \hat{r}_t^* + \zeta_t^*\right),
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi(\phi,\psi) &\equiv \phi \frac{q_{11}}{d^2} \left[\beta^{-1}(q_{11}-q_{12}) - \phi q_{11}\right], \\
\Gamma(\phi,\psi) &\equiv \frac{q_{11}}{d^2} \left[\beta^{-1}(q_{11}-q_{12}) - \phi q_{11}\right], \\
\Lambda(\phi,\psi) &= -\frac{\kappa\beta^{-1}(q_{11}-q_{12}) + q_{11}}{d}.
\end{aligned}
$$

C   Proofs

C.1   Proofs under AMPF

Proof of Proposition 1   Showing $\Phi(\phi) \geq 0$ and $\Gamma(\phi) \geq 0$ is straightforward since $\kappa > 0$; $\phi > \rho_\pi$; $(1-\rho_\pi) > 0$; $(1-\beta\rho_\pi) > 0$; $\phi > \rho_r$; $(1-\rho_r) > 0$; and $(1-\beta\rho_r) > 0$.[38] In addition, it is also straightforward to show that $\Phi(\phi)$ is increasing in κ by taking a partial derivative and:

$$
\Phi(\phi) > 1, \qquad \text{for } \kappa > \frac{(1-\rho_\pi)(1-\beta\rho_\pi)}{\rho_\pi}.
$$

Proof of Proposition 2   Take the partial derivative:

$$
\frac{\partial \Phi(\phi)}{\partial \phi} = \frac{-\kappa\left[\kappa\rho_\pi - (1-\rho_\pi)(1-\beta\rho_\pi)\right]}{\left[\kappa(\phi - \rho_\pi) + (1-\rho_\pi)(1-\beta\rho_\pi)\right]^2}.
$$

---

[38]See the previous section for the analytical solution for $\Phi(\phi)$, $\Gamma(\phi)$, and other coefficients.

The denominator is always positive. Therefore, the sign of the numerator will determine the sign of the derivative. We thus have:

$$
\begin{aligned}
\frac{\partial \Phi\left(\phi\right)}{\partial \phi} &> 0, & \text{for } 0 < \kappa < \frac{\left(1-\rho_\pi\right)\left(1-\beta\rho_\pi\right)}{\rho_\pi}, \\
\frac{\partial \Phi\left(\phi\right)}{\partial \phi} &= 0, & \text{for } \kappa = \frac{\left(1-\rho_\pi\right)\left(1-\beta\rho_\pi\right)}{\rho_\pi}, \\
\frac{\partial \Phi\left(\phi\right)}{\partial \phi} &< 0, & \text{for } \kappa > \frac{\left(1-\rho_\pi\right)\left(1-\beta\rho_\pi\right)}{\rho_\pi}.
\end{aligned}
$$

It is trivial to show $\frac{\partial \Gamma\left(\phi\right)}{\partial \phi} < 0$.

Proof of Proposition 3 The result follows directly from Proposition 1 and 2.

C.2   Proofs under PMAF

Proof of Proposition 4

(I) To prove $\Phi\left(\phi,\psi\right) \geq 0$, it suffices to show $\Theta\left(\phi,\psi\right)$ is positive since $0 \leq \phi < 1$. Let us rewrite $\Theta\left(\phi,\psi\right)$ by substituting out Ω:

$$
\begin{aligned}
\Theta &= \frac{\beta\left(e_2-e_3\right)\left(e_1-\rho_\pi\right)-\kappa\beta^{-1}\left(1-\beta\phi\right)}{\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)} \\
&= \frac{\beta\left(e_2-e_3\right)\left(e_1-e_2+e_2-\rho_\pi\right)-\kappa\beta^{-1}\left(1-\beta\phi\right)}{\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)} \\
&= \frac{\beta\left(e_2-e_3\right)\left(e_1-e_2\right)-\kappa\beta^{-1}\left(1-\beta\phi\right)+\beta\left(e_2-e_3\right)\left(e_2-\rho_\pi\right)}{\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)} \\
&= \frac{-\frac{1}{\beta}\left[\psi^2+\left(\beta+\kappa-1\right)\psi-\kappa\left(1-\beta\phi\right)\right]-\frac{\kappa}{\beta}\left(1-\beta\phi\right)+\beta\left(e_2-e_3\right)\left(e_2-\rho_\pi\right)}{\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)} \\
&= \frac{-\frac{1}{\beta}\left[\psi^2+\left(\beta+\kappa-1\right)\psi\right]+\beta\left(e_2-e_3\right)\left(e_2-\rho_\pi\right)}{\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)} \quad\quad (24)
\end{aligned}
$$

Substitute ψ out using $\psi = 1 - \beta e_2$:

$$
\begin{aligned}
\Theta &= \frac{-\frac{1}{\beta}\left[\left(1-\beta e_2\right)^2+\left(\beta+\kappa-1\right)\left(1-\beta e_2\right)\right]+\beta\left(e_2-e_3\right)\left(e_2-\rho_\pi\right)}{\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)} \\
&= \frac{\beta^2\left(e_2-e_3\right)\left(e_2-\rho_\pi\right)-\left(1-\beta e_2\right)^2-\left(\beta+\kappa-1\right)\left(1-\beta e_2\right)}{\beta\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)} \\
&= \frac{\beta^2\left(e_2^2-\left(e_3+\rho_\pi\right)e_2+\rho_\pi e_3\right)-\left(\beta^2 e_2^2-2\beta e_2+1\right)-\left(\beta+\kappa-1\right)+\left(\beta+\kappa-1\right)\beta e_2}{\beta\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)} \\
&= \frac{\beta^2\left(\rho_\pi e_3-\left(e_3+\rho_\pi\right)e_2\right)+2\beta e_2-\left(\beta+\kappa\right)+\left(\beta+\kappa-1\right)\beta e_2}{\beta\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)} \\
&= \frac{\left[\beta^2\left(1-e_3\right)+\beta\left(1-\beta\rho_\pi\right)+\beta\kappa\right]e_2+\beta^2\rho_\pi e_3-\left(\beta+\kappa\right)}{\beta\left(e_1-\rho_\pi\right)\left(e_2-\rho_\pi\right)\left(1-\beta\phi\right)}.
\end{aligned}
$$

The denominator is unambiguously positive for all parameter values under PMAF, and Θ will therefore

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be positive if and only if the numerator is also positive. Note that the numerator is a linear and increasing function of e_2 because the slope, $\left[\beta^2 (1 - e_3) + \beta (1 - \beta\rho_\pi) + \beta\kappa\right]$, is positive. This implies that $\Theta > 0$ for sufficiently large e_2 – or sufficiently small ψ. It is straightforward to show that $\Theta > 0$ if and only if $-\infty < \psi < \bar{\psi}^*$ where:

$$\bar{\psi}^* \equiv 1 - \frac{(\beta + \kappa) - \beta^2 \rho_\pi e_3}{\beta (1 - e_3) + (1 - \beta\rho_\pi) + \kappa} = 1 - \frac{\beta + \kappa - \beta^2 \rho_\pi e_3}{\beta (e_1 - \rho_\pi)}$$

It remains to show that $\bar{\psi}^*$ is positive. Note that the denominator of $\bar{\psi}^*$ is positive. Consider the numerator, $g(\phi) \equiv \beta (e_1 - \rho_\pi) - \left(\beta + \kappa - \beta^2 \rho_\pi e_3\right)$. Given other parameters, $g(\phi)$ has the smallest value at $\phi = 1$ because $g'(\phi) < 0$. Evaluate $g(\phi)$ at $\phi = 1$:

$$g(1) = \beta (e_1 - \rho_\pi) - \left(\beta + \kappa - \beta^2 \rho_\pi e_3\right) = \beta \left(\frac{\kappa + 1}{\beta} - \rho_\pi\right) - \left(\beta + \kappa - \beta^2 \rho_\pi\right)$$

$$= (1 - \beta)(1 - \beta\rho_\pi) > 0,$$

which implies $\bar{\psi}^* > 0$. Finally, redefining $\bar{\psi}^*$ as $\bar{\psi}^* \equiv \min\left\{\bar{\psi}^*, \bar{\psi}\right\}$, we establish that:

$$0 < \bar{\psi}^* \leq \bar{\psi} \equiv 1 - \beta$$

In addition, we can show that $\bar{\psi}^*$ depends crucially on the slope of the Phillips curve and satisfies:

$$\lim_{\kappa \to \infty} \bar{\psi}^* = 0$$
$$\lim_{\kappa \to 0} \bar{\psi}^* = 1 - \beta.$$

Therefore, the interval of ψ that makes Φ positive – i.e. that makes the coefficient on π_t^* negative – *always* contains zero (i.e. always includes the benchmark FTPL case). In addition, the interval covers the AF region $(-\infty < \psi < \bar{\psi})$ *almost* entirely. The interval coincides exactly with the entire AF region when prices are fully sticky (i.e. when $\kappa = 0$). In other cases, the upper bound for positive Φ, $\bar{\psi}^*$, is positive, but may be slightly below the upper bound for AF, $\bar{\psi}$.

(II) It is trivial to show $\Gamma(\phi, \psi) > 0$ because $e_1 > 1 > \rho_r$ and $e_2 > 1 > \rho_r$ under PMAF.

(III) It is trivial to show $\Omega(\phi, \psi) = \frac{\beta(e_2 - e_3)}{\bar{b}(1 - \beta\phi)} > 0$ because $e_2 > 1 > e_3$ and $1 > \beta\phi$ under PMAF. (Also see Lemma 1)

Proof of Proposition 5

(I) To show $\frac{\partial\Phi(\phi,\psi)}{\partial\phi} > 0$, let us first consider $\frac{\partial\Theta(\phi,\psi)}{\partial\phi}$:

$$\frac{\partial\Theta(\phi,\psi)}{\partial\phi} = \frac{\left\{\begin{array}{c} \frac{\partial\Omega}{\partial\phi}\bar{b}(e_1 - \rho_\pi)^2(e_2 - \rho_\pi) + \frac{\partial e_1}{\partial\phi}\Omega\bar{b}(e_1 - \rho_\pi)(e_2 - \rho_\pi) \\ - \frac{\partial e_1}{\partial\phi}(e_2 - \rho_\pi)\left[\Omega\bar{b}(e_1 - \rho_\pi) - \kappa\beta^{-1}\right] \end{array}\right\}}{(e_1 - \rho_\pi)^2(e_2 - \rho_\pi)^2}$$

$$= \frac{\frac{\partial\Omega}{\partial\phi}\bar{b}(e_1 - \rho_\pi)^2(e_2 - \rho_\pi) - \frac{\partial e_3}{\partial\phi}(e_2 - \rho_\pi)\kappa\beta^{-1}}{(e_1 - \rho_\pi)^2(e_2 - \rho_\pi)^2} = \frac{\frac{\partial\Omega}{\partial\phi}\bar{b}(e_1 - \rho_\pi)^2 - \frac{\partial e_3}{\partial\phi}\kappa\beta^{-1}}{(e_1 - \rho_\pi)^2(e_2 - \rho_\pi)}$$

Now let us take the partial derivative of $\Phi(\phi, \psi)$ with respect to ϕ:

$$\frac{\partial \Phi(\phi, \psi)}{\partial \phi} \equiv \Theta(\phi, \psi) + \phi \frac{\partial \Theta(\phi, \psi)}{\partial \phi}$$

$$= \frac{\Omega \bar{b}(e_1 - \rho_\pi) - \kappa \beta^{-1}}{(e_1 - \rho_\pi)(e_2 - \rho_\pi)} + \phi \frac{\frac{\partial \Omega}{\partial \phi} \bar{b}(e_1 - \rho_\pi)^2 - \frac{\partial e_3}{\partial \phi} \kappa \beta^{-1}}{(e_1 - \rho_\pi)^2 (e_2 - \rho_\pi)}$$

$$= \frac{\frac{\beta(e_2 - e_3)}{(1 - \beta\phi)}(e_1 - \rho_\pi)^2 - (e_1 - \rho_\pi)\kappa \beta^{-1} + \phi \frac{\partial \Omega}{\partial \phi} \bar{b}(e_1 - \rho_\pi)^2 - \phi \frac{\partial e_3}{\partial \phi} \kappa \beta^{-1}}{(e_1 - \rho_\pi)^2 (e_2 - \rho_\pi)}$$

Since the denominator is positive, $\frac{\partial \Phi(\phi, \psi)}{\partial \phi} > 0$ if and only if the numerator is positive – that is,

$$0 < \frac{\partial \Phi(\phi, \psi)}{\partial \phi} \iff$$

$$0 < \frac{\beta(e_2 - e_3)}{(1 - \beta\phi)}(e_1 - \rho_\pi)^2 - (e_1 - \rho_\pi)\kappa\beta^{-1} + \phi \frac{\partial \Omega}{\partial \phi}\bar{b}(e_1 - \rho_\pi)^2 - \phi \frac{\partial e_3}{\partial \phi}\kappa\beta^{-1}$$

$$\iff \frac{\beta(e_2 - e_3)}{(1 - \beta\phi)} + \phi \frac{\partial \Omega}{\partial \phi}\bar{b} \geq \frac{\phi \frac{\partial e_3}{\partial \phi}\kappa\beta^{-1} + (e_1 - \rho_\pi)\kappa\beta^{-1}}{(e_1 - \rho_\pi)^2}$$

$$\iff \frac{\beta(e_2 - e_3)}{(1 - \beta\phi)} + \phi\beta \frac{\beta e_2 - \beta e_3 - (1 - \beta\phi)\frac{\partial e_3}{\partial \phi}}{(1 - \beta\phi)^2} > \frac{\phi \frac{\partial e_3}{\partial \phi}\kappa\beta^{-1} + (e_1 - \rho_\pi)\kappa\beta^{-1}}{(e_1 - \rho_\pi)^2}$$

$$\iff \beta(e_2 - e_3)\frac{1}{(1 - \beta\phi)^2} > \frac{\phi \frac{\partial e_3}{\partial \phi}\kappa\beta^{-1} + (e_1 - \rho_\pi)\kappa\beta^{-1}}{(e_1 - \rho_\pi)^2} + \frac{\beta\phi}{(1 - \beta\phi)}\frac{\partial e_3}{\partial \phi}$$

$$\iff \beta e_2 > \beta e_3 + (1 - \beta\phi)^2 \left\{ \frac{\phi \frac{\partial e_3}{\partial \phi}\kappa\beta^{-1} + (e_1 - \rho_\pi)\kappa\beta^{-1}}{(e_1 - \rho_\pi)^2} + \frac{\beta\phi}{(1 - \beta\phi)}\frac{\partial e_3}{\partial \phi} \right\}$$

$$\iff \psi < 1 - \left[\beta e_3 + (1 - \beta\phi)^2 \left\{ \frac{\phi \frac{\partial e_3}{\partial \phi}\kappa\beta^{-1} + (e_1 - \rho_\pi)\kappa\beta^{-1}}{(e_1 - \rho_\pi)^2} + \frac{\beta\phi}{(1 - \beta\phi)}\frac{\partial e_3}{\partial \phi} \right\} \right]$$

$$\iff \psi < 1 - \left[\beta e_3 + (1 - \beta\phi)^2 \left\{ \frac{\kappa\beta^{-1}}{(e_1 - \rho_\pi)} + \frac{\phi\kappa\beta^{-1}}{(e_1 - e_3)}\left[\frac{\kappa\beta^{-1}}{(e_1 - \rho_\pi)^2} + \frac{\beta}{(1 - \beta\phi)} \right] \right\} \right]$$

In sum,

$$\frac{\partial \Phi(\phi, \psi)}{\partial \phi} > 0 \iff \psi < \bar{\psi}^{**}$$

where

$$\bar{\psi}^{**} \equiv 1 - \left[\beta e_3 + (1 - \beta\phi)^2 \left\{ \frac{\kappa\beta^{-1}}{(e_1 - \rho_\pi)} + \frac{\phi\kappa\beta^{-1}}{(e_1 - e_3)}\left[\frac{\kappa\beta^{-1}}{(e_1 - \rho_\pi)^2} + \frac{\beta}{(1 - \beta\phi)} \right] \right\} \right]$$

As before, it remains to show that $\bar{\psi}^{**}$ is positive. First, suppose $\bar{\psi}^{**} \geq \bar{\psi}^{*}$ or $\bar{\psi}^{**} \geq \bar{\psi}^{***}$. We have already shown $\bar{\psi}^{*} > 0$ above, and it will be shown $\bar{\psi}^{***} > 0$ below. Thus, it must be that $\bar{\psi}^{**} > 0$. Suppose instead $\bar{\psi}^{**} \leq \bar{\psi}^{*}$ and $\bar{\psi}^{**} \leq \bar{\psi}^{***}$. In this case, if $\psi < \bar{\psi}^{**}$, then $\Theta(\phi, \psi) > 0$ and $\frac{\partial \Omega(\phi, \psi)}{\partial \phi} > 0$. But, we can show from (24) that $\frac{\partial \Theta(\phi, \psi)}{\partial \phi} > 0$ at $\psi = 0$ if $\frac{\partial \Omega(\phi, \psi)}{\partial \phi} > 0$, which implies $\frac{\partial \Phi(\phi, \psi)}{\partial \phi} > 0$ at $\psi = 0$. Since $\psi < \bar{\psi}^{**}$ is the sufficient and necessary condition for $\frac{\partial \Phi(\phi, \psi)}{\partial \phi} > 0$, it should always contain zero. Therefore, $\bar{\psi}^{**} > 0$. Finally, redefining $\bar{\psi}^{**}$ as $\bar{\psi}^{**} \equiv \min\{\bar{\psi}^{**}, \bar{\psi}\}$, we establish that:

$$0 < \bar{\psi}^{**} \leq \bar{\psi} \equiv 1 - \beta$$

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In addition, similar to $\bar{\psi}^*$, it can be shown that:

$$\lim_{\kappa \to \infty} \bar{\psi}^{**} = 0$$

$$\lim_{\kappa \to 0} \bar{\psi}^{**} = 1 - \beta.$$

(II) Showing $\frac{\partial \Gamma(\phi, \psi)}{\partial \phi} > 0$ is trivial because $\frac{\partial e_1}{\partial \phi} < 0$ and $e_2 > 1$ under PMAF.

(III) To show $\frac{\partial \Omega(\phi, \psi)}{\partial \phi} > 0$, take the partial derivative of $\Omega(\phi, \psi)$ with respect to ϕ:

$$\frac{\partial \Omega}{\partial \phi} = \frac{\beta}{\bar{b}} \frac{\beta e_2 - \beta e_3 - (1 - \beta \phi) \frac{\partial e_3}{\partial \phi}}{(1 - \beta \phi)^2} = \frac{\beta}{\bar{b}} \frac{h(\phi)}{(1 - \beta \phi)^2},$$

where

$$h(\phi) \equiv \beta e_2 - \beta e_3 - (1 - \beta \phi) \frac{\partial e_3}{\partial \phi}.$$

Since $\frac{\beta}{\bar{b}} \frac{1}{(1 - \beta \phi)^2}$ is clearly positive, $\frac{\partial \Omega}{\partial \phi}$ and $h(\phi)$ must have the same sign. Note that

$$h(\phi) > 0 \iff \beta e_2 > \beta e_3 + (1 - \beta \phi) \frac{\partial e_3}{\partial \phi}$$

$$\iff \beta e_2 > \beta e_3 + (1 - \beta \phi) \frac{\kappa}{\beta + \kappa + 1 - 2\beta e_3} \quad \left(\because \frac{\partial e_3}{\partial \phi} = \frac{\kappa}{\beta + \kappa + 1 - 2\beta e_3} > 0 \right)$$

$$\iff \beta e_2 > \beta e_3 + (1 - \beta \phi) \frac{\kappa}{\beta (e_1 - e_3)}$$

$$\iff \beta e_2 > \frac{\beta^2 e_1 e_3 - \beta^2 e_3^2 + (1 - \beta \phi) \kappa}{\beta (e_1 - e_3)}$$

$$\iff \beta e_2 > \frac{\beta + \kappa - \beta^2 e_3^2}{\beta (e_1 - e_3)}$$

$$\iff \psi < \underbrace{1 - \frac{\beta + \kappa - \beta^2 e_3^2}{\beta (e_1 - e_3)}}_{\equiv \bar{\psi}^{***}} = \frac{\beta (e_1 - e_3) - (\beta + \kappa - \beta^2 e_3^2)}{\beta (e_1 - e_3)}$$

Once again, it remains to show that $\bar{\psi}^{***}$ is positive. Consider the numerator of $\bar{\psi}^{***}$, $g(\phi) \equiv \beta (e_1 - e_3) - (\beta + \kappa - \beta^2 e_3^2)$. Given other parameters, $g(\phi)$ has the smallest value at $\phi = 1$ because $g'(\phi) < 0$. Evaluate $g(\phi)$ at $\phi = 1$:

$$g(1) = \beta (e_1 - e_3) - (\beta + \kappa - \beta^2 e_3^2) = \beta \left(\frac{\kappa + 1}{\beta} - 1 \right) - (\beta + \kappa - \beta^2)$$

$$= (1 - \beta)^2 > 0.$$

which implies $\bar{\psi}^{***} > 0$. Finally, redefining $\bar{\psi}^{***}$ as $\bar{\psi}^{***} \equiv \min \{\bar{\psi}^{***}, \bar{\psi}\}$, we establish that:

$$0 < \bar{\psi}^{***} \leq \bar{\psi} \equiv 1 - \beta$$

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In addition, similar to $\bar{\psi}^*$ and $\bar{\psi}^{**}$, it can be shown that:

$$\lim_{\kappa \to \infty} \bar{\psi}^{***} = 0$$

$$\lim_{\kappa \to 0} \bar{\psi}^{***} = 1 - \beta.$$

Proof of Proposition 6

(I) To show $\frac{\partial \Phi(\phi, \psi)}{\partial \psi} < 0$ under PMAF, take the partial derivative of $\Theta = \frac{\beta(e_2 - e_3)(e_1 - \rho_\pi) - \kappa \beta^{-1}(1 - \beta\phi)}{(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)}$ with respect to e_2:

$$\frac{\partial \Theta}{\partial e_2} = \frac{\beta(e_1 - \rho_\pi)^2(e_2 - \rho_\pi)(1 - \beta\phi) - (e_1 - \rho_\pi)(1 - \beta\phi)\left[\beta(e_2 - e_3)(e_1 - \rho_\pi) - \kappa \beta^{-1}(1 - \beta\phi)\right]}{\left[(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)\right]^2}$$

$$= \frac{(e_1 - \rho_\pi)(1 - \beta\phi)\left[\beta(e_1 - \rho_\pi)(e_2 - \rho_\pi) - \beta(e_2 - e_3)(e_1 - \rho_\pi) + \kappa \beta^{-1}(1 - \beta\phi)\right]}{\left[(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)\right]^2}$$

$$= \frac{(e_1 - \rho_\pi)(1 - \beta\phi)}{\left[(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)\right]^2} g$$

where

$$g \equiv \beta(e_1 - \rho_\pi)(e_2 - \rho_\pi) - \beta(e_2 - e_3)(e_1 - \rho_\pi) + \kappa \beta^{-1}(1 - \beta\phi).$$

Since $\frac{(e_1 - \rho_\pi)(1 - \beta\phi)}{\left[(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)\right]^2} > 0$, we focus on g, which can be written as:

$$g = \beta\left[\rho_\pi^2 - (e_1 + e_3)\rho_\pi + e_1 e_3\right] + \kappa \beta^{-1}(1 - \beta\phi).$$

Note that

$$e_1 + e_3 = \frac{\beta + \kappa + 1}{\beta}$$

$$e_1 e_3 = \frac{\kappa\phi + 1}{\beta}.$$

Using these, rewrite g and regard g as a function of $\rho_\pi \in (0,1)$ given other parameters:

$$g(\rho_\pi) = \beta\rho_\pi^2 - (\beta + \kappa + 1)\rho_\pi + 1 + \kappa\beta^{-1}.$$

Note that $g(\rho_\pi)$ is a convex and quadratic function of ρ_π, and

$$g(0) = 1 + \kappa\beta^{-1} > g(1) = \kappa\left(\beta^{-1} - 1\right) > 0.$$

Moreover,

$$g'(0) < 0 \text{ and } g'(1) < 0.$$

Therefore, it must be that $g > 0$ for $\rho_\pi \in (0,1)$, $\beta \in (0,1)$ and $\kappa \in (0,\infty)$. Hence,

$$\frac{\partial \Theta}{\partial e_2} = \frac{(e_1 - \rho_\pi)(1 - \beta\phi)}{\left[(e_1 - \rho_\pi)(e_2 - \rho_\pi)(1 - \beta\phi)\right]^2} g > 0.$$

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This implies that

$$\frac{\partial \Theta}{\partial \psi} < 0 \text{ and } \frac{\partial \Phi}{\partial \psi} < 0,$$

because e_2 is decreasing in ψ.

(II) Showing $\frac{\partial \Gamma(\phi,\psi)}{\partial \psi} > 0$ is trivial because $e_1 > 1$ and $\frac{\partial e_2}{\partial \psi} < 0$ under PMAF.

(III) Showing $\frac{\partial \Omega(\phi,\psi)}{\partial \psi}$ is also trivial as $\frac{\partial e_2}{\partial \psi} < 0$.

Discussion on $\bar{\psi}^*$, $\bar{\psi}^{**}$, $\bar{\psi}^{***}$ **and** $\bar{\psi} \equiv 1 - \beta$. Although the upper bounds, $\bar{\psi}^*$, $\bar{\psi}^{**}$ and $\bar{\psi}^{***}$, generally have different values, they converge to the same numbers as the slope of NKPC approaches to infinity and zero. In particular, all of them equal $\bar{\psi} \equiv 1 - \beta$ – the "true" upper bound for AF – when $\kappa = 0$. However, $\bar{\psi}^*$, $\bar{\psi}^{**}$, $\bar{\psi}^{***}$ and $\bar{\psi}$ would be indistinguishable in practice because $\bar{\psi}^*$, $\bar{\psi}^{**}$, $\bar{\psi}^{***} \in (0, \bar{\psi}]$ and $\bar{\psi}$ would have a tiny value.

C.3   Proof under PMPF

Proof of Proposition 7   To prove $\Gamma(\phi,\psi) > 0$ and $\Phi(\phi,\psi) = \phi\Gamma(\phi,\psi) \geq 0$ (the equality holds at $\phi = 0$), we show that

$$\Gamma(\phi,\psi) = \frac{q_{11}}{d^2}\left[\beta^{-1}(q_{11} - q_{12}) - \phi q_{11}\right] \tag{25}$$

is always positive. Note that

$$v_{23} = \frac{-\beta e_3 - (\psi - 1)}{\bar{b}(1 - \beta\phi)},$$

$$v_{13} = \frac{\beta e_3(\kappa - 1 + \psi + \beta) - (\beta + \psi + \beta\kappa\phi - 1)}{\bar{b}\kappa(1 - \beta\phi)},$$

$$q_{11} = \frac{v_{23}}{v_{11}v_{23} - v_{13}v_{21}},$$

$$q_{12} = \frac{-v_{13}}{v_{11}v_{23} - v_{13}v_{21}}.$$

Then, (25) can be written as:

$$\Gamma(\phi,\psi) = \frac{v_{23}\left[\left(\beta^{-1} - \phi\right)v_{23} + \beta^{-1}v_{13}\right]}{d^2(v_{11}v_{23} - v_{13}v_{21})^2}. \tag{26}$$

Note that $v_{13} = v_{23} = 0$ at $\psi = 1 - \beta e_3$, and

$$\frac{\partial v_{23}}{\partial \psi} < 0 \text{ and } \frac{\partial v_{13}}{\partial \psi} < 0,$$

which implies that v_{13} and v_{23} have the same sign for any values of ψ given other parameters. This proves that (26) is always positive since $\beta^{-1} > 1 > \phi$ under PMPF, and consequently $\Phi(\phi,\psi) = \phi\Gamma(\phi,\psi) \geq 0$.

Finally, we can show that b_{t-1} does not directly affect the dynamics of inflation and output expectation,

$\left\{\xi_t^\pi, \xi_t^Y\right\}$ by substituting zero for the unstable part (i.e. the first equation) of the system:

$$X_t = \begin{pmatrix} 0 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} X_{t-1} + \begin{pmatrix} 0 & 0 & 0 \\ q_{21} & q_{22} & 1 \\ q_{31} & q_{32} & 0 \end{pmatrix} \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix} \tag{27}$$

$$+ \begin{pmatrix} 0 & 0 & 0 \\ q_{21} & q_{22} & 1 \\ q_{31} & q_{32} & 0 \end{pmatrix} \begin{pmatrix} \kappa\beta^{-1}+1 & \phi-\beta^{-1} \\ -\kappa\beta^{-1} & \beta^{-1} \\ 0 & -\bar{b}\left(\beta^{-1}-\phi\right) \end{pmatrix} \begin{pmatrix} \eta_t^Y \\ \eta_t^\pi \end{pmatrix}.$$

We then solve for $\left\{\xi_t^\pi, \xi_t^Y\right\}$ by pre-multiplying V back to the system (27):

$$\begin{pmatrix} \xi_t^Y \\ \xi_t^\pi \\ \hat{b}_t \end{pmatrix} = V \begin{pmatrix} 0 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} V^{-1} \begin{pmatrix} \xi_{t-1}^Y \\ \xi_{t-1}^\pi \\ \hat{b}_{t-1} \end{pmatrix}$$

$$+ VV_{-1} \begin{pmatrix} -\phi & -1 \\ 0 & 0 \\ -\bar{b}\phi & 0 \end{pmatrix} \begin{pmatrix} \hat{\pi}_t^* \\ \hat{r}_t^* \end{pmatrix} + VV_{-1} \begin{pmatrix} \kappa\beta^{-1}+1 & \phi-\beta^{-1} \\ -\kappa\beta^{-1} & \beta^{-1} \\ 0 & -\bar{b}\left(\beta^{-1}-\phi\right) \end{pmatrix} \begin{pmatrix} \eta_t^Y \\ \eta_t^\pi \end{pmatrix},$$

where

$$V_{-1} \equiv \begin{pmatrix} 0 & 0 & 0 \\ q_{21} & q_{22} & 1 \\ q_{31} & q_{32} & 0 \end{pmatrix}.$$

It is straightforward to show that both (1,3) and (2,3) elements of the AR coefficient matrix, $V \begin{pmatrix} 0 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{pmatrix} V^{-1}$, are zeros, which implies that the first two equations can be solved separately from the last equation – the government budget constraint.

C.4 A special case with repeated eigenvalues

In the proofs above, we implicitly assume that the matrix G has distinct eigenvalues. There is however *one* special case in which G has repeated eigenvalues and hence is not diagonalizable. Such case arises when $\psi = 1 - \beta e_3$ and only under PMPF.[39] In this case, one should use the Jordan decomposition – instead of the eigenvalue decomposition – as $G = WJW^{-1}$ where

$$J = \begin{pmatrix} e_1 & 0 & 0 \\ 0 & e_3 & 1 \\ 0 & 0 & e_3 \end{pmatrix} \quad \text{and} \quad W = \begin{pmatrix} w_{11} & 0 & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & w_{33} \end{pmatrix}$$

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[39] This special case does not arise under AMPF and PMAF. Under AMPF, $e_3 > 1$, and fiscal policy is active at $\psi = 1 - \beta e_3$. Under PMAF, on the other hand, $0 < e_3 < 1$, and fiscal policy is passive at $\psi = 1 - \beta e_3$.

with

$$e_1 = \frac{1}{2\beta}\left(\beta + \kappa + 1 + \sqrt{(\beta + \kappa + 1)^2 - 4\beta(1 + \kappa\phi)}\right),$$

$$e_3 = \frac{1}{2\beta}\left(\beta + \kappa + 1 - \sqrt{(\beta + \kappa + 1)^2 - 4\beta(1 + \kappa\phi)}\right).$$

Note that the eigenvalues are identical to the two of the three eigenvalues in the previous sections. Note that

$$W^{-1} = \begin{pmatrix} 1 & z_{12} & 0 \\ 0 & -\beta\kappa^{-1} & z_{23} \\ 1 & z_{32} & 0 \end{pmatrix}.$$

In particular,

$$z_{12} = -\frac{\psi}{\kappa} < 0.$$

Then the unstable part of the system is the same as before except that q_{11} and q_{12} are now replaced by 1 and z_{12}, respectively. Therefore

$$
\begin{aligned}
\Gamma(\phi, \psi) &= \frac{1}{d^2}\left[\beta^{-1}(1 - z_{12}) - \phi\right] = \frac{1}{d^2}\left[(\beta^{-1} - \phi) - \beta^{-1}z_{12}\right] > 0, \\
\Phi(\phi, \psi) &= \phi\Gamma(\phi, \psi) \geq 0.
\end{aligned}
$$

In addition, following the same steps as before, one can easily show that b_{t-1} does not directly affect the dynamics of inflation and output expectation.

D Quantitative Model

D.1 Households

There is a continuum of households in the unit interval. Each household specializes in the supply of a particular type of labor. A household that supplies labor of type-j maximizes the utility function:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t\delta_t\left[\log\left(C_t^j - \eta C_{t-1}\right) - \varpi\frac{\left(H_t^j\right)^{1+\varphi}}{1 + \varphi}\right]\right\},$$

where H_t^j denotes the hours of type-j labor services, C_t is aggregate consumption, and C_t^j is consumption of household j. The parameters β, φ, and η are, respectively, the discount factor, the inverse of the (Frisch) elasticity of labor supply, and the degree of external habit formation, while δ_t represents an intertemporal preference shock that follows:

$$\delta_t = \delta_{t-1}^{\rho_\delta}\exp(\varepsilon_{\delta,t}),$$

where $\varepsilon_{\delta,t} \sim$ i.i.d. $N\left(0, \sigma_\delta^2\right)$.

Household j's flow budget constraint is:

$$P_tC_t^j + P_tI_t^j + B_t^j + E_t\left[Q_{t,t+1}V_{t+1}^j\right] = W_t(j)H_t^j + V_t^j + R_{t-1}B_{t-1}^j + R_t^k u_t\bar{K}_{t-1}^j - P_ta(u_t)\bar{K}_{t-1}^j + \Pi_t - T_t,$$

where P_t is the price level, B_t^j is the amount of one-period risk-less nominal government bond held by household j, R_t is the interest rate on the bond, $W_t(j)$ is the nominal wage rate for type-j labor, Π_t denotes profits of intermediate firms, and T_t denotes government taxes.[40] In addition to the government bond, households trade at time t one-period state-contingent nominal securities V_{t+1}^j at price $Q_{t,t+1}$, and hence fully insure against idiosyncratic risk.

Moreover, I_t^j is investment, R_t^k is the rental rate of effective capital $u_t \bar{K}_{t-1}^j$ where u_t is the variable capacity utilization rate, and $a(u_t)$ is the cost of capital utilization. In steady-state, $u = 1$ and $a(1) = 0$. Moreover, in the first-order approximation of the model, the only parameter that matters for the dynamic solution of the model is the curvature $\chi \equiv \frac{a''(1)}{a'(1)}$. The capital accumulation equation is then given by:

$$\bar{K}_t^j = (1 - d) \bar{K}_{t-1}^j + \mu_t \left(1 - S \left(\frac{I_t^j}{I_{t-1}^j} \right) \right) I_t^j,$$

where d is the depreciation rate and $S(.)$ is the adjustment cost function. In steady-state, $S = S' = 0$ and $S'' > 0$. μ_t represents an investment shock that follows:

$$\mu_t = \mu_{t-1}^{\rho_\mu} \exp(\varepsilon_{\mu,t}),$$

where $\varepsilon_{\mu,t} \sim$ i.i.d. $N\left(0, \sigma_\mu^2\right)$.

Each household monopolistically provides differentiated labor. There are competitive employment agencies that assemble these differentiated labor into a homogenous labor input that is sold to intermediate goods firms. The assembling technology is a Dixit and Stiglitz (1977) production technology $H_t = \left(\int_0^1 H_t^{j \frac{\theta_{l,t}-1}{\theta_{l,t}}} dj \right)^{\frac{\theta_{l,t}}{\theta_{l,t}-1}}$, where $\theta_{l,t}$ denotes the time-varying elasticity of substitution between differentiated labor. The corresponding wage index for the homogenous labor input is $W_t = \left(\int_0^1 W_t(j)^{1-\theta_{l,t}} dj \right)^{\frac{1}{1-\theta_{l,t}}}$ and the optimal demand for H_t^j is given by $H_t^j = (W_t(j)/W_t)^{-\theta_{l,t}} H_t$. The elasticity of substitution $\theta_{l,t}$ follows:

$$\left(\frac{\theta_{l,t}}{\theta_{l,t}-1} \right) = \left(\frac{\bar{\theta}_l}{\bar{\theta}_l - 1} \right)^{1-\rho_l} \left(\frac{\theta_{l,t-1}}{\theta_{l,t-1}-1} \right)^{\rho_l} \exp(\varepsilon_{l,t} - \upsilon_l \varepsilon_{l,t-1})$$

where $\varepsilon_{l,t} \sim$ i.i.d. $N\left(0, \sigma_l^2\right)$.

As in Calvo (1983), each household resets its nominal wage optimally with probability $1 - \alpha_w$ every period. Households that do not optimize adjust their wages according to the simple partial dynamic indexation rule:

$$W_t(j) = W_{t-1}(j) \left[\pi_{t-1} a_{t-1} \right]^{\gamma_w} \left[\bar{\pi} \bar{a} \right]^{1-\gamma_w},$$

where γ_w measures the extent of indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$. All optimizing households choose a common wage W_t^* to maximize the present discounted value of future utility:

$$E_t \sum_{k=0}^{\infty} \alpha_w^k \beta^k \left[-\delta_{t+k} \frac{\left(H_{t+k}^j \right)^{1+\varphi}}{1+\varphi} + \Lambda_{t+k} W_t^* H_{t+k}^j \right]$$

where Λ_{t+k} is the marginal utility of nominal income.

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[40] The budget constraint reflects our assumptions that each household owns an equal share of all intermediate firms and receives the same amount of net lump-sum transfers from the government.

D.2 Firms

The final good Y_t, which is consumed by the government and households as well as used to invest, is produced by perfectly competitive firms assembling intermediate goods, $Y_t(i)$, with a Dixit and Stiglitz (1977) production technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\theta_{p,t}-1}{\theta_{p,t}}} di \right)^{\frac{\theta_{p,t}}{\theta_{p,t}-1}}$, where $\theta_{p,t}$ denotes the elasticity of substitution between intermediate goods. The corresponding price index for the final consumption good is $P_t = \left(\int_0^1 P_t(i)^{1-\theta_{p,t}} di \right)^{\frac{1}{1-\theta_{p,t}}}$, where $P_t(i)$ is the price of the intermediate good i. The optimal demand for $Y_t(i)$ is given by $Y_t(i) = (P_t(i)/P_t)^{-\theta_{p,t}} Y_t$. The elasticity of substitution $\theta_{p,t}$ follows:

$$\left(\frac{\theta_{p,t}}{\theta_{p,t}-1} \right) = \left(\frac{\bar{\theta}_p}{\bar{\theta}_p-1} \right)^{1-\rho_p} \left(\frac{\theta_{p,t-1}}{\theta_{p,t}-1} \right)^{\rho_p} \exp(\varepsilon_{p,t} - v_p \varepsilon_{p,t-1})$$

where $\varepsilon_{p,t} \sim$ i.i.d. $N\left(0, \sigma_p^2\right)$.

Monopolistically competitive firms produce intermediate goods using the production function:

$$Y_t(i) = \max\{ (A_t H_t(i))^{1-\lambda} K_t(i)^\lambda - A_t F; 0 \},$$

where $H_t(i)$ and $K_t(i)$ denote the homogenous labor and capital employed by firm i and A_t represents exogenous economy-wide technological progress. The gross growth rate of technology $a_t \equiv A_t/A_{t-1}$ follows:

$$a_t = \bar{a}^{1-\rho_a} a_{t-1}^{\rho_a} \exp(\varepsilon_{a,t}),$$

where $\bar{a}$ is the steady-state value of a_t and $\varepsilon_{a,t} \sim$ i.i.d. $N\left(0, \sigma_a^2\right)$. F is a fixed cost of production that ensure that profits are zero in steady state.

As in Calvo (1983), a firm resets its price optimally with probability $1 - \alpha_p$ every period. Firms that do not optimize adjust their price according to the simple partial dynamic indexation rule:

$$P_t(i) = P_{t-1}(i) \pi_{t-1}^{\gamma_p} \bar{\pi}^{1-\gamma_p},$$

where γ_p measures the extent of indexation and $\bar{\pi}$ is the steady-state value of the gross inflation rate $\pi_t \equiv P_t/P_{t-1}$. All optimizing firms choose a common price P_t^* to maximize the present discounted value of future profits:

$$E_t \sum_{k=0}^{\infty} \alpha_p^k Q_{t,t+k} \left[P_t^* X_{t,k} Y_{t+k}(i) - W_{t+k} H_{t+k}(i) - R_{t+k}^k K_{t+k}(i) \right],$$

where

$$X_{t,k} \equiv \begin{cases} (\pi_t \pi_{t+1} \cdots \pi_{t+k-1})^\gamma \bar{\pi}^{(1-\gamma)k}, & k \geq 1 \\ 1, & k = 0 \end{cases}.$$

D.3 Government

D.3.1 Budget Constraint

Each period, the government collects lump-sum tax revenues T_t and issues one-period nominal bonds B_t to finance its consumption G_t, and interest payments. Accordingly, the flow budget constraint is given by:

$$\frac{B_t}{P_t} = R_{t-1} \frac{B_{t-1}}{P_t} + G_t - T_t.$$

The flow budget constraint can be rewritten as:

$$R_t^{-1} b_t = b_{t-1} \frac{1}{\pi_t} \frac{y_{t-1}}{y_y} \frac{A_{t-1}}{A_t} + \tilde{G}_t - \tau_t,$$

where $b_t \equiv R_t \frac{B_t}{P_t Y_t}$ denotes the real maturity value of government debt relative to output, $\tilde{G}_t \equiv \frac{G_t}{Y_t}$, and τ_t $\equiv \frac{T_t}{Y_t}$.

D.3.2 Monetary Policy

The central bank sets the nominal interest rate according to a Taylor-type rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi_t^*} \right)^{\phi_\pi} \left(\frac{X_t}{X_t^*} \right)^{\phi_Y} \right]^{1-\rho_R} \exp\left(\varepsilon_{R,t} \right),$$

which features interest rate smoothing and systematic responses to deviation of GDP from its natural level X_t^* and deviation of inflation from a time-varying target π_t^*.[41] $\bar{R}$ is the steady-state value of R_t and the non-systematic monetary policy shock $\varepsilon_{R,t}$ is assumed to follow i.i.d. $N\left(0, \sigma_R^2\right)$. The inflation target evolves exogenously as:

$$\pi_t^* = \bar{\pi}^{1-\rho_\pi} \left(\pi_{t-1}^* \right)^{\rho_\pi} \exp(\varepsilon_{\pi,t}),$$

where $\varepsilon_{\pi,t} \sim$ i.i.d. $N\left(0, \sigma_\pi^2\right)$.

D.3.3 Fiscal Policy

The fiscal authority sets the tax revenues according to:

$$\frac{\tau_t}{\bar{\tau}} = \left(\frac{\tau_{t-1}}{\bar{\tau}} \right)^{\rho_\tau} \left[\left(\frac{b_{t-1}}{\bar{b}} \right)^{\tilde{\psi}} \right]^{1-\rho_\tau},$$

which features tax smoothing and a systematic response to lagged debt. $\bar{\tau}$ is the steady-state value of τ_t while $\bar{b}$ is the steady-state value of b_t. Government spending follows an exogenous process given by:

$$G_t = \left(1 - \frac{1}{g_t} \right) Y_t$$

where the government spending shock follows:

$$g_t = \bar{g}^{1-\rho_g} g_{t-1}^{\rho_g} \exp(\varepsilon_{g,t}),$$

where $\varepsilon_{g,t} \sim$ i.i.d. $N\left(0, \sigma_g^2\right)$.

D.4 Equilibrium

Equilibrium is characterized by the prices and quantities that satisfy the households' and firms' optimality conditions, the government budget constraint, monetary and fiscal policy rules, and the clearing conditions

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[41] The natural level of output is the output that would prevail under flexible wages and prices and in the absence of time-variation in the elasticity of subsitution over differentiated labor and goods varieties.

for the product, labor, and asset markets:

$$\int_0^1 C_t^j dj + G_t + \int_0^1 I_t^j dj + a(u_t) \int_0^1 \bar{K}_{t-1}^j dj = Y_t,$$

$$\int_0^1 H_t(i) di = H_t$$

$$\int_0^1 V_t^j dj = 0,$$

$$\int_0^1 B_t^j dj = B_t.$$

Note that $C_t^j = C_t$, $I_t^j = I_t$, and $\bar{K}_{t-1}^j = \bar{K}_{t-1}$ due to the complete market assumption and the separability between consumption and leisure. The capital accumulation equation in the aggregate is then given by:

$$\bar{K}_t = (1-d)\bar{K}_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t.$$

and the aggregate resource constraint and the definition of GDP then take the form:

$$C_t + I_t + G_t + a(u_t)\bar{K}_{t-1} = Y_t$$

$$X_t = C_t + I_t + G_t.$$

D.5   Approximate Model

We first solve the problem of households and firms given the monetary and fiscal policy rules and derive the equilibrium conditions. We then use approximation methods to solve the model. First, the model features a stochastic balanced growth path since the neutral technology shock contains a unit root. Therefore, we de-trend variables on the balanced growth path by the level of the technology shock and write down all the equilibrium conditions of the transformed model. Second, we compute the non-stochastic steady state of this transformed model. Third, we obtain a first-order approximation of the equilibrium conditions around this steady state. We then solve the approximated model using standard methods.

For a variable X_t, let $x_t = \frac{X_t}{A_t}$. We denote by $\hat{x}_t$ the log-deviation from steady state of x_t, except for fiscal variables, which are in terms of deviation from steady state.

We also, define some new variables: $\lambda_t = \Lambda_t P_t A_t$, and ϕ_t, which is the lagrange multiplier on the capital accumulation equation for the household's optimization problem (that is, it is the shadow value of installed capital), $\varrho_{p,t} = \frac{\theta_{p,t}}{\theta_{p,t}-1}$, and $\varrho_{l,t} = \frac{\theta_{l,t}}{\theta_{l,t}-1}$. We omit a detailed derivation and the equations characterizing the approximate equilibrium, after some manipulations, are given by:

$$\hat{y}_t = \frac{\bar{y} + F}{\bar{y}} \left[\lambda \hat{k}_t + (1-\lambda)\hat{H}_t\right]$$

$$\chi \hat{u}_t = \hat{w}_t + \hat{L}_t - \hat{k}_t$$

$$\hat{\pi}_t = \frac{\beta}{1 + \beta\gamma_p} E_t \hat{\pi}_{t+1} + \frac{\gamma_p}{1 + \beta\gamma_p} \hat{\pi}_{t-1} + \kappa_p \left[\lambda \left(\chi \hat{u}_t\right) + (1-\lambda)\hat{w}_t\right] + \kappa_p \hat{\varrho}_{p,t}$$

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$$\hat{\lambda}_t = \hat{R}_t + E_t\left(\hat{\lambda}_{t+1} - \hat{a}_{t+1} - \hat{\pi}_{t+1}\right)$$

$$
\begin{aligned}
\hat{\lambda}_t &= \frac{\eta\beta\bar{a}}{(\bar{a}-\eta\beta)(\bar{a}-\eta)}E_t\hat{c}_{t+1} - \frac{\bar{a}^2+\eta^2\beta}{(\bar{a}-\eta\beta)(\bar{a}-\eta)}\hat{c}_t + \frac{\eta\bar{a}}{(\bar{a}-\eta\beta)(\bar{a}-\eta)}\hat{c}_{t-1} \\
&\quad + \frac{\eta\beta\bar{a}\rho_a - \eta\bar{a}}{(\bar{a}-\eta\beta)(\bar{a}-\eta)}\hat{a}_t + \frac{(\bar{a}-\eta\beta\rho_\delta)}{(\bar{a}-\eta\beta)}\hat{\delta}_t
\end{aligned}
$$

$$\hat{\phi}_t = (1-d)\beta\bar{a}^{-1}E_t\left(\hat{\phi}_{t+1} - \hat{a}_{t+1}\right) + \left(1-(1-d)\beta\bar{a}^{-1}\right)E_t\left[\hat{\lambda}_{t+1} - \hat{a}_{t+1} + \chi\hat{u}_{t+1}\right]$$

$$\hat{\lambda}_t = \hat{\phi}_t + \hat{\mu}_t - \bar{a}^2 S''\left(\hat{\imath}_t - \hat{\imath}_{t-1} + \hat{a}_t\right) + \beta\bar{a}^2 S'' E_t\left[\hat{\imath}_{t+1} - \hat{\imath}_t + \hat{a}_{t+1}\right]$$

$$\hat{k}_t = \hat{u}_t + \widehat{\bar{k}}_{t-1} - \hat{a}_t$$

$$\widehat{\bar{k}}_t = (1-d)\bar{a}^{-1}\left(\widehat{\bar{k}}_{t-1} - \hat{a}_t\right) + \left(1-(1-d)\bar{a}^{-1}\right)(\hat{\mu}_t + \hat{u}_t)$$

$$
\begin{aligned}
\hat{w}_t &= \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{w}_{t+1} - \kappa_w\left[\hat{w}_t - \left(\varphi L_t + \hat{\delta}_t - \hat{\lambda}_t\right)\right] \\
&\quad + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1+\beta\gamma_p}{1+\beta}\hat{\pi}_t + \frac{\beta}{1+\beta}E_t\hat{\pi}_{t+1} \\
&\quad + \frac{\gamma_w}{1+\beta}\hat{a}_{t-1} - \frac{1+\beta\gamma_w-\beta\rho_a}{1+\beta}\hat{a}_t + \kappa_w\hat{\varrho}_{l,t}
\end{aligned}
$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\phi_\pi(\pi_t - \pi_t^*) + \phi_y(x_t - x_t^*)\right] + \varepsilon_{R,t}$$

$$\hat{x}_t = \hat{y}_t - \frac{\rho\bar{k}}{\bar{y}}\hat{u}_t$$

$$\frac{1}{\bar{g}}\hat{y}_t = \frac{1}{\bar{g}}\hat{g}_t + \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{\imath}}{\bar{y}}\hat{\imath}_t + \frac{\rho\bar{k}}{\bar{y}}\hat{u}_t$$

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\bar{b}\left[\hat{\pi}_t + \hat{y}_t - \hat{y}_{t-1} + \hat{a}_t\right] - \bar{R}\hat{\tau}_t + \bar{b}\hat{R}_t + \bar{R}\frac{1}{\bar{g}}\hat{g}_t.$$

$$\hat{\tau}_t = \rho_\tau\hat{\tau}_{t-1} + (1-\rho_\tau)\psi\hat{b}_{t-1}$$

where $\rho = \frac{\bar{a}}{\beta} - (1-d)$, $\kappa_p = \frac{(1-\beta\alpha_p)(1-\alpha_p)}{\alpha_p(1+\gamma_p\beta)}$, $\kappa_w = \frac{(1-\beta\alpha_w)(1-\alpha_w)}{\alpha_w(1+\beta)(1+\varphi\theta_l)}$, and $\psi \equiv \frac{\bar{\tau}}{\bar{b}}\tilde{\psi}$.

D.6 Parameterization

Table 2: Benchmark Parameterization

Parameter	Value	Description
λ	0.17	Capital share
d	0.025	Depreciation rate of capital
γ_p	0.24	Price indexation
γ_w	0.11	Wage indexation
$100 \log \bar{a}$	0.48	SS technology growth rate
η	0.78	Consumption habit
$1/\left(1 - \bar{\theta}_p\right)$	0.23	SS price markup
$1/\left(1 - \bar{\theta}_l\right)$	0.15	SS wage markup
$\log \bar{H}$	0.38	SS log-hours
$100(\bar{\pi} - 1)$	0.71	SS quarterly inflation
$100(\beta^{-1} - 1)$	0.13	Discount factor
φ	3.79	Inverse Frisch elasticity
α_p	0.84	Calvo prices
α_w	0.70	Calvo wages
χ	5.30	Elasticity capital utilization costs
S''	2.85	Investment adjustment costs
ρ_R	0.82	Monetary rule smoothing
ρ_τ	0.80	Fiscal rule smoothing
ρ_a	0.23	Neutral technology growth
ρ_g	0.99	Government spending
ρ_μ	0.72	Investment
ρ_p	0.94	Price markup
ρ_l	0.97	Wage markup
ρ_δ	0.67	Intertemporal preference
v_p	0.77	Price markup MA
v_l	0.91	Wage markup MA
$100\sigma_R$	0.22	Monetary policy
$100\sigma_a$	0.88	Neutral technology growth
$100\sigma_g$	0.35	Government spending
$100\sigma_\mu$	6.03	Investment
$100\sigma_p$	0.14	Price markup
$100\sigma_w$	0.20	Wage markup
$100\sigma_\delta$	0.04	Intertemporal preference
$\bar{b}$	0.40	SS public debt
$\bar{g}$	0.22	SS government spending
ϕ_y	0.07	Monetary rule output
ϕ_π	$[0, 3]$	Monetary rule inflation
ψ	$[-0.1, 0.1]$	Fiscal rule public debt

Note: The benchmark values of the parameters other than ρ_τ, $\bar{b}$, ϕ_π and ψ are the estimated median of Justiniano, Primiceri and Tambalotti (2010). The value of ρ_τ is taken from Bhattarai, Lee and Park (2012b) and $\bar{b}$ is the sample average ratio of public debt to output in the U.S. For policy parameters ϕ_π and ψ, we try different values for comparative statics.
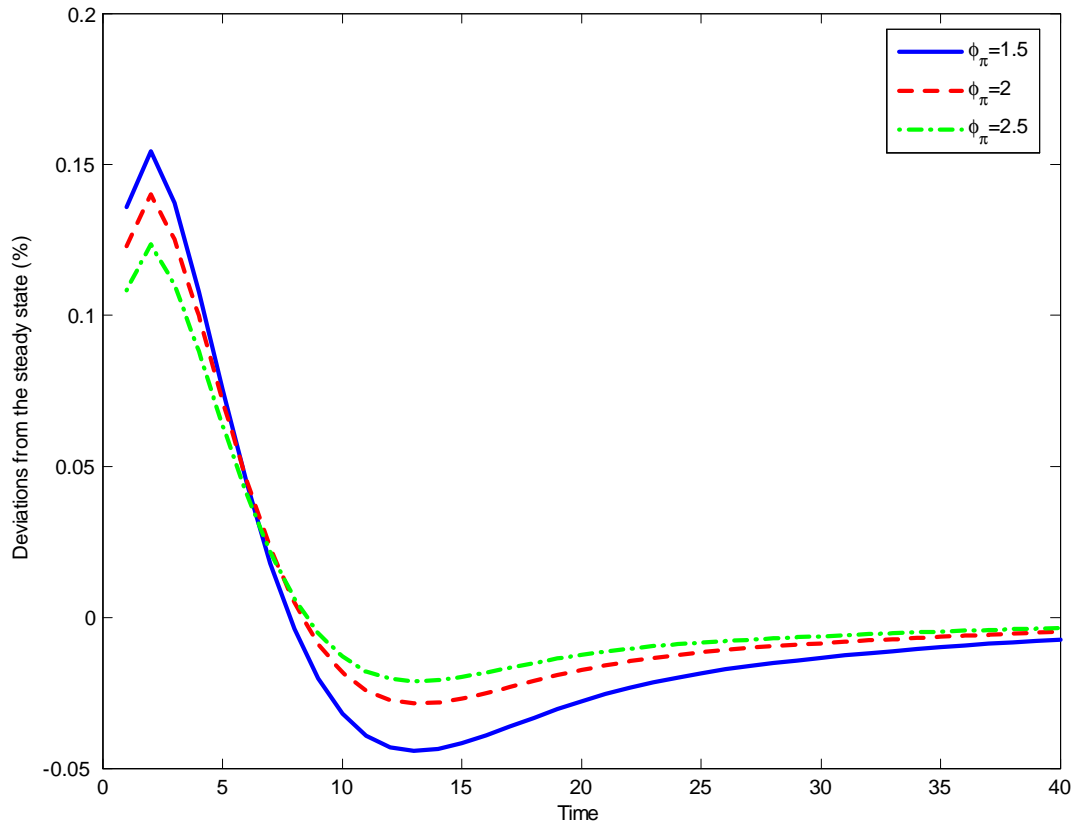
D.7 Dynamics under alternative parameterization

Figure 12: The response of inflation to a one standard deviation increase in the investment specific shock under an alternate parameterization ($\alpha_w = 0.3$).