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How Does Government Spending Stimulate Consumption? *

Daniel P. Murphy
Darden School of Business, University of Virginia

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Abstract

Recent empirical work finds that government spending shocks cause aggregate consumption to increase over the business cycle, contrary to the predictions of Neoclassical and New Keynesian models. This paper proposes a mechanism to account for the consumption increase that builds on the framework of imperfect information in Lucas (1972) and Lorenzoni (2009). In my model, owners of firms targeted by an increase in government spending perceive an increase in their permanent income relative to their future tax liabilities. Owners of firms not targeted remain unaware of the implicit increase in future tax liabilities, causing aggregate consumption to increase. A testable implication of the proposed model is that the value of firms should increase, implying all else equal an increase in aggregate stock returns. This prediction of the model is shown to be consistent with empirical evidence.

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* Daniel P. Murphy, Darden School of Business, Charlottesville, VA 22906-6550. 303-884-5037. murphyd@darden.virginia.edu. I thank Rudi Bachmann, Bob Barsky, Alan Deardorff, Yuriy Gorodnichenko, Chris House, Lutz Kilian, John Laitner, Nathan Seegert, Matthew Shapiro, and Alberto Trejos for helpful comments and discussions. The views in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

1. Introduction

The onset of the economic crisis in 2008 brought some urgency to the ongoing debate over whether fiscal policy can stimulate the economy. While standard neoclassical models imply that the response of output to increases in government spending is close to zero and that the response of consumption is negative, a broad range of empirical evidence suggests that the output multiplier is close to unity and that the consumption multiplier is positive. For example, Perotti (2008) finds that the consumption multiplier is on average positive. Ramey (2011) finds a positive response of consumption of services to defense spending shocks, and Galí, López-Salido, and Vallés (2007) find that the positive consumption multiplier is robust to a number of specifications.[1]

This empirical evidence is puzzling in light of the inability of standard macroeconomic models to generate a positive consumption response. For example, Hall (2009) demonstrates the difficulty of generating a positive consumption multiplier in Neoclassical and New Keynesian models and calls for, "new ideas outside the New Keynesian framework" to reconcile economic theory with the empirical evidence. (p. 228). Below I present one such idea. I show that the positive consumption multiplier can arise from perceptions of increased permanent income in response to government spending shocks. The basic insight is that government spending is focused on a subset of firms, while tax liabilities are spread across all firms. Therefore government expenditure on an individual firm increases firm owners' expectations of their permanent income if they perceive the fraction of government expenditure directed toward their firm to be large relative to aggregate government spending.

To demonstrate the effects of aggregate demand shocks on consumption, I develop a model of imperfect information in which heterogeneous agents observe government expenditure on their firm's output, but they are imperfectly aware of aggregate government expenditure.[2] Imperfect information serves two purposes in the model: First, it prevents agents from perfectly observing aggregate demand. Second, it generates wage rigidity due to workers' inability to

[1] While many studies find a positive response of consumption to government spending shocks, not all do. For example, Barro and Redlick (2011) generally find smaller multipliers than those in the studies mentioned, including a consumption response of nondurables that statistically insignificantly different from zero. The analysis below provides a theoretical foundation for positive consumption multipliers, as well as for consumption responses that are negative but higher than predicted by standard models.

[2] Imprecise information about aggregate government expenditure is consistent with the fact that published data on government spending is subject to large revisions, as discussed in Section 2.6.

coordinate. Wage rigidity arising from coordination failure is itself sufficient to amplify the effects of government spending shocks relative to a standard neoclassical model. The idea that wage rigidities are responsible for deviations of output from its natural rate dates back to Keynes' *General Theory* (1936). In an influential paper, Ball and Romer (1991) discuss the inefficiencies that arise from agents' inability to coordinate and suggest that government intervention may help offset such inefficiencies. This view is consistent with the implications of my baseline DSGE model. The model features imperfect information among heterogeneous workers and shocks to aggregate demand (represented by shocks to agents' marginal utility and shocks to government expenditure). When agents are nearly correct about the value of aggregate government expenditure, the model predicts a consumption multiplier slightly below zero. These results are robust to a range of parameter values when the elasticity of substitution across workers is high.

Under an alternative calibration in which information about macroeconomic aggregates is imprecise, the model predicts a positive response of consumption and an output multiplier above unity. Firm owners perfectly observe idiosyncratic demand from the government for their specific good, but due to a noisy signal of aggregate government spending, they believe that the government expenditure is directed disproportionately toward their own good. This increases profits, firm value, and expectations of permanent income, which in turn causes an increase in desired consumption.[3] I show that such a mechanism is not only consistent with the empirical evidence of high multipliers cited earlier, but also with the response of stock returns to government spending shocks. Finally, I show that the multiplier is higher the less persistent is aggregate government expenditure, consistent with the evidence in Perotti (2008).

One interesting feature of the model is that recessions occur in the form of an overreaction of consumption to a negative marginal utility shock. In this case, government spending unambiguously directs output back toward its natural rate. Thus the model is consistent with the intuition that wages are too high and output too low relative to its potential during a recession.

[3] As discussed in Section 4, the wealth effect is larger when idiosyncratic demand is persistent relative to aggregate government spending shocks because, to the extent that agents perceive an increase in aggregate government spending, they will also perceive an increase in the present value of their tax liability. This tax liability is increasing in the persistence of government spending shocks.

The notion that imperfect information may induce shifts in perceptions that help amplify demand shocks has a long tradition in macroeconomics. The analysis most closely related to my work is Lucas (1973), who demonstrates that a monetary shock has a larger effect on output when prices are stable because agents attribute aggregate price changes to relative price changes. The model below demonstrates that a government spending shock has a larger effect on output when idiosyncratic demand is volatile relative to government spending because agents attribute changes in aggregate demand to changes in idiosyncratic demand.

My modeling approach also builds on Woodford (2003) and Lorenzoni (2009), each of which contains features that extend Lucas (1972) in different dimensions. In my model, like in Woodford (2003), agents set prices to maximize their expected real income, and they satisfy demand at that price. In Woodford (2003) the source of uncertainty is nominal GDP, which follows an exogenous process. In my model agents face uncertainty regarding the exogenous evolution of real government expenditure and average marginal utility, but nominal GDP responds to both demand shocks and to agents' beliefs.

In other related work, Lorenzoni (2009) provides a method for determining the endogenous evolution of agent believes and aggregate states in the economy when nominal GDP is endogenous. While Lorenzoni's focus is the business cycle effects of agents' perceptions of aggregate productivity in a setting with sticky nominal prices, I adapt his method for use in a general equilibrium model with local state variables, constant aggregate productivity, and worker heterogeneity. Agents are separated across islands on which they receive noisy signals of aggregate output, total government spending, the price level, and average marginal utility. They also receive persistent idiosyncratic shocks to the demand for firms' output. Agents in my model can distinguish between private and government demand for local final goods, and they use their signals to infer the values of the aggregate states and the law of motion of the economy.

An additional departure from the model in Lorenzoni (2009) is that I do not incorporate sticky prices following Calvo (1983). Rather, nominal wages are rigid due to coordination failure, which in turn causes nominal price rigidity. This form of wage and price rigidity serves two purposes: first, it facilitates the derivation of analytic results that would be impossible to derive using a Calvo pricing mechanism. Second, under the proposed form of wage rigidity, agents are not concerned that government expenditures will increase future marginal costs, as they would be in a standard New Keynesian model (including models with a Calvo evolution of sticky nominal wages such as in Erceg, Henderson, and Levin (2000)). Rather, agents believe that coordination failure will persist in the present and future period and they incorporate those beliefs into their inflation expectations. Thus agents' inflation expectations do not rise to the extent that they would in a New Keynesian model, which in turn permits a less aggressive nominal interest rate hike by the monetary authority in response to a fiscal stimulus.

Finally, the mechanism in the proposed model is different from the one operating in models that focus on the effects of government demand shocks when the nominal interest rate is at the zero lower bound. In Woodford (2011), for example, government spending increases expected inflation *because* prices are sticky *á la* Calvo (1983), which in turn encourages consumption at the given period's lower prices.[4] The multiplier in my model, in contrast, does not hinge on a zero nominal interest rate, and the mechanism described applies more generally.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 presents the response of output to shocks to aggregate demand (government spending shocks and marginal utility shocks) under the baseline calibration. Section 4 demonstrates an alternative parameterization that produces a multiplier well above unity. Section 5 concludes.

2. Model

The economy consists of a continuum of islands, indexed by $j \in [0,1]$. Each island contains a worker who supplies labor to the market, and a firm that produces a final good using labor from workers on other islands as inputs. The worker on island j owns firm j, so the worker's income consists of the wage earned by selling labor to firms across islands and profits from the sale of good j.

The random assignment of consumption is similar to that in Lorenzoni (2009). In each period $t \in \{0,1,2 \ldots\}$ a random subset $\mathcal{D}_{j,t} \subset [0,1]$ of workers from other islands consumes the good from island j. Symmetrically, each worker in island j consumes a subset $\mathcal{C}_{j,t} \subset [0,1]$ of goods from other islands. The demand for labor inputs is likewise random. Each period a random subset $\mathcal{F}_{j,t} \subset [0,1]$ of firms from other islands requires labor type j as a production input. Symetrically, each firm j purchases labor input from a random subset $\mathrm{F}_{j,t} \subset [0,1]$ of workers on other islands.

[4] See also Eggertson (2010) and Christiano et al. (2011) for the effects of government spending when the nominal interest rate is at the zero lower bound.

Workers and firms set the nominal price of their labor and output in each period, and they fully satisfy all demand at that price. Information is common to a worker and firm within an island but is not shared across islands. Thus each worker faces his own signal extraction problem.

The economy features two aggregate shocks: an average marginal utility shock, and a shock to government expenditure. Technology is constant over time. Therefore additional output of good j in period t requires additional labor input. As in Blanchard and Kiyotaki (1987), labor inputs are imperfect substitutes into the production of final goods. The production technology for final good j in period t is

$$Y_{j,t} = \left(\int_{\mathrm{F}_{j,t}} N_{m,j,t}^{\frac{\rho-1}{\rho}} dm \right)^{\frac{\rho}{\rho-1}},$$

where $N_{m,j,t}$ is the amount of labor sold by worker m to firm j at time t and ρ is the elasticity of substitution across labor inputs. Firm j sells output at price $P_{j,t}$ in period t.

2.1 Consumption

Worker j maximizes

$$\sum_{t=0}^{\infty} \beta^t E_0\left[U\left(C_{j,t}, N_{j,t}\right)\right]$$

where

$$U\left(C_{j,t}, N_{j,t}\right) = X_{j,t} \log C_{j,t} - \frac{1}{1+\xi} N_{j,t}^{1+\xi}, \tag{1}$$

and

$$C_{j,t} = \left(\int_{\mathcal{C}_{j,t}} C_{m,j,t}^{\frac{\gamma-1}{\gamma}} dm \right)^{\frac{\gamma}{\gamma-1}}.$$

$C_{m,j,t}$ is the consumption of final good m by worker j at time t and ξ is the inverse of the Frish elasticity of labor supply. The elasticity of substitution across final goods is $\gamma > 1$ and $X_{j,t}$ represents a shock to worker j's marginal utility at time t.

Nominal income is subject to a time-invariant income tax at rate τ. In steady state, the income tax fully funds all government expenditures. In addition, workers are subject to an idiosyncratic lump-sum tax $T^L_{j,t}$, the dynamics of which are discussed below. Workers can trade nominal one-period bonds but cannot fully insure against idiosyncratic shocks. Thus the budget constraint of worker j is

$$\begin{aligned} Q_t B_{j,t+1} + \int_{\mathcal{C}_{j,t}} P_{m,t} C_{m,j,t} dm + T^L_{j,t} \\ = B_{j,t} + (1-\tau) W_{j,t} N_{j,t} + (1-\tau)\Pi_{j,t}, \end{aligned} \tag{2}$$

where $B_{j,t+1}$ are holdings of nominal bonds that trade at price Q_t, $W_{j,t}$ is the price of labor type j, and $\Pi_{j,t}$ are the profits of firm j. The assumption that workers trade bonds only with each other implies that bonds are in zero net supply.

There are two relevant price indices on each island. The first is the price firm j pays for its labor inputs,

$$\bar{P}^I_{j,t} = \left(\int_{\mathrm{F}_{j,t}} W_{m,t}^{1-\rho} dm \right)^{\frac{1}{1-\rho}},$$

where $W_{m,t}$ is the price of labor type m. Firm j's producer price index will in general differ from the economy-wide producer price index, defined as

$$P^I_t = \left(\int_0^1 W_{j,t}^{1-\rho} dj \right)^{\frac{1}{1-\rho}}.$$

The second relevant price index on island j is the consumer price index, which aggregates over the prices of final goods in worker j's consumption basket $\{P_{m,t}\}_{m \in \mathcal{C}_{j,t}}$:

$$\bar{P}_{j,t} = \left(\int_{\mathcal{C}_{j,t}} P_{m,t}^{1-\gamma} dm \right)^{\frac{1}{1-\gamma}}.$$

Island j's consumer price index will differ in general from the aggregate price level,

$$P_t = \left(\int_0^1 P_{j,t}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}},$$

which is the price of a unit of the aggregate consumption good

$$C_t = \left(\int_0^1 C_{j,t}^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}}.$$

Worker j's demand for good m follows from utility maximization subject to the budget constraint (2):

$$C_{m,j,t} = \left(\frac{P_{m,t}}{\bar{P}_{j,t}}\right)^{-\gamma} C_{j,t}.$$

Aggregating the demand for good j from all visitors in $\mathcal{D}_{j,t}$ yields total private demand for the output of firm j:

$$Y_{j,t}^{P} = P_{j,t}^{-\gamma} \int_{\mathcal{D}_{j,t}} \bar{P}_{m,t}^{\gamma} C_{m,t} dm. \tag{3}$$

Total demand for final good j, $Y_{j,t}^{D}$ consists of demand from the private sector, $Y_{j,t}^{P}$, and demand from the public sector, $Y_{j,t}^{G}$, the nature of which is discussed below. Given demand $Y_{j,t}^{D}$, firm j's cost minimization problem dictates the quantity of each labor type it uses to produce good j. Demand from firm j for labor input m is:

$$N_{m,j,t} = \left(\frac{W_{m,t}}{\bar{P}_{j,t}^{I}}\right)^{-\rho} Y_{j,t}^{D}.$$

Aggregating the demand for labor input j from all firms in $\mathcal{F}_{j,t}$ yields total demand for the labor input of worker j:

$$N_{j,t}^{d} = W_{j,t}^{-\rho} \int_{\mathcal{F}_{j,t}} \left(\bar{P}_{m,t}^{I}\right)^{\rho} Y_{m,t}^{D} dm. \tag{4}$$

2.2 Government

The government balances its budget each period by imposing a time-invariant income tax and by collecting lump-sum taxes. Its budget constraint is

$$\int_0^1 P_{j,t} Y_{j,t}^{G}\, dj = \tau \int_0^1 \left(W_{j,t} N_{j,t} + \Pi_{j,t}\right) dj + \int_0^1 T_{j,t}^{L}\, dj, \tag{5}$$

where $Y_{j,t}^{G}$ is the government's demand for final good j. I assume that government demand for the output good of firm j is a function of the good's price and the aggregate price level:

$$Y_{j,t}^{G} = \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} G_t Z_{j,t}^{G}, \tag{6}$$

where G_t is total government consumption and $Z_{j,t}^G$ is the price-insensitive fraction of total government demand directed toward island j. Let $Y_{j,t}^D \equiv Y_{j,t}^P + Y_{j,t}^G$ be total demand for an island's output and note that profits of firm j can be written as $\Pi_{j,t} = Y_{j,t}^D\left(P_{j,t} - \bar{P}_{j,t}^I\right)$. Then integrating (5) across islands yields

$$P_t G_t = \tau P_t Y_t + T_t^L, \tag{7}$$

where $Y_t \equiv C_t + G_t$ is real GDP and $T_t^L \equiv \int_0^1 T_{j,t}^L$ is total collection of lump-sum taxes. In steady state, T_t^L is zero, although any given household's lump-sum tax will be positive or negative. The total lump-sum tax, T_t^L, adjusts to maintain a balanced budget in response to deviations in government spending.

2.3 Price Setting

Final Goods. We can rewrite total demand for good j as the sum of demand from the private sector and demand from the government:

$$Y_{j,t}^D = P_{j,t}^{-\gamma}\left[\int_{\mathcal{D}_{j,t}} \bar{P}_{m,t}^{\gamma} C_{m,t} dm + P_t^{\gamma} G_t Z_{j,t}^G\right] \tag{8}$$

Firm j chooses $P_{j,t}$ to maximize profits. The resulting optimal price for final good j is a constant markup over the marginal cost $\bar{P}_{j,t}^I$:

$$P_{j,t} = \frac{\gamma}{\gamma - 1}\bar{P}_{j,t}^I. \tag{9}$$

Firm profits, expressed as a function of the output price, are

$$\Pi_{j,t} = \frac{1}{\gamma} Y_{j,t}^D P_{j,t}. \tag{10}$$

Wages. Each worker chooses the price of his labor input in order to maximize (1) subject to his budget constraint (2) and demand for his labor (4). Given his price choice, worker j supplies labor to satisfy all demand at price $W_{j,t}$.

2.4 Signals

Each agents observes a public signal of government expenditure,

$$s_t = g_t + \epsilon_t, \tag{11}$$

where ϵ_t is i.i.d over time with distribution $N(0, \sigma_\epsilon^2)$ and g_t is the log deviation of government spending from its steady state level.[5] In the baseline calibrations below the signal represents real-time government expenditure data, while the errors represent the difference between the initially reported value and most recent vintage of revised data. In addition to the common public signal, agents also receive idiosyncratic signals, which they use to infer values of the aggregate state variables.

Marginal Utility. In each period the marginal utility for a worker on island j is a noisy signal of the average marginal utility across islands:

$$x_{j,t} = x_t + \eta_{j,t}, \qquad (12)$$

where $\eta_{j,t}$ is i.i.d across islands with distribution $N(0, \sigma_\eta^2)$. Average marginal utility x_t is white noise across time with distribution $N(0, \sigma_x^2)$.

Prices. I assume that the random selection of consumers in $\mathcal{D}_{j,t}$ is such that worker j's consumer price index is, in log-linear deviations from steady state,

$$\bar{p}_{j,t} = p_t + \zeta_{j,t}^{CPI}, \qquad (13)$$

where $\zeta_{j,t}^{CPI}$ is Normally distributed with mean zero and variance σ_{CPI}^2, is i.i.d. across islands, and satisfies $\int_0^1 \zeta_{j,t}^{CPI} dj = 0$. The random selection of labor inputs in $\mathrm{F}_{j,t}$ is such that the producer price index for firm j is

$$\bar{p}_{j,t}^I = p_t^I + \zeta_{j,t}^{PPI},$$

where $\zeta_{j,t}^{PPI}$ is Normally distributed with mean zero and variance σ_{PPI}^2, is i.i.d. across islands, and satisfies $\int_0^1 \zeta_{j,t}^{PPI} dj = 0$. Substituting in the log-linearized version of (9) yields

$$\bar{p}_{j,t}^I = p_t + \zeta_{j,t}^{PPI}. \qquad (14)$$

Demand for Final Goods. The log-linearized version of private demand for output from firm j (equation 3) is

$$y_{j,t}^P = -\gamma p_{j,t} + \int_{\mathcal{D}_{j,t}} (\gamma \bar{p}_{m,t} + c_{m,t}) dm$$

[5] In the equations that follow, lower-case endogenous variables represent the log deviations of the corresponding upper-case variable unless otherwise specified.

I assume that the random selection of private customers for final good j is such that the above equation takes the form

$$y_{j,t}^{P} = c_t - \gamma(p_{j,t} - p_t) + \zeta_{j,t}^{P} + \zeta_{j,t}^{1}. \quad (15)$$

The linearized fraction of total demand for final good j consists of a white noise component, $\zeta_{j,t}^{1}$, which has distribution $N(0, \sigma_{\zeta,1}^2)$ and satisfies $\int_0^1 \zeta_{j,t}^{1} dj = 0$, and a persistent component $\zeta_{j,t}^{P}$. The latter component is a local state variable that follows the AR(1) process

$$\zeta_{j,t}^{P} = \rho_\zeta \zeta_{j,t-1}^{P} + \mu_{j,t}^{P},$$

where $\mu_{j,t}^{P}$ is i.i.d., Normally distributed with mean zero and variance $\sigma_{\mu,P}^2$, and integrates to zero across islands.[6]

Government spending on worker j's output evolves according to

$$y_{j,t}^{G} = g_t - \gamma(p_{j,t} - p_t) + \zeta_{j,t}^{G} + \zeta_{j,t}^{2}, \quad (16)$$

which is the log-linearized version of equation (6). Equation (16) states that demand for good j from the government is a function of aggregate government consumption, g_t, and the idiosyncratic fraction of government demand directed toward final good j, $(\zeta_{j,t}^{G} + \zeta_{j,t}^{2})$. The white noise shock $\zeta_{j,t}^{2}$ has distribution $N(0, \sigma_{\zeta,2}^2)$ and satisfies $\int_0^1 \zeta_{j,t}^{2} dl = 0$. The persistent component of government demand, $\zeta_{j,t}^{G}$, follows

$$\zeta_{j,t}^{G} = \rho_\zeta \zeta_{j,t-1}^{G} + \mu_{j,t}^{G},$$

where $\mu_{j,t}^{G}$, is i.i.d., Normally distributed with mean zero and variance $\sigma_{\mu,P}^2$, and integrates to zero across islands.

I assume that agents observe demand for their product from the private sector and from the government sector, and that they can distinguish between demand from the private and public sectors. We can rewrite equations (15) and (16) in terms of the price firms choose and the remaining components of demand, which I will refer to as the demand signals from the private sector and public sector, $d_{j,t}^{P}$ and $d_{j,t}^{G}$, respectively:

[6] An alternative specification is to assume that the idiosyncratic demand shifter has only a temporary component, as in Lucas (1972), Woodford (2003), and Lorenzoni (2009). I choose to allow for a persistent component because persistent idiosyncratic demand is a feature of the data, as demonstrated in section 2.6. In addition, persistent idiosyncratic demand generates the wealth effect responsible for an output multiplier above unity under the alternative parameterization in Section 4.

$$y_{j,t}^P = d_{j,t}^P - \gamma p_{j,t} \qquad\qquad y_{j,t}^G = d_{j,t}^G - \gamma p_{j,t}. \tag{17}$$

The demand signals can be expressed as functions of unobservables:

$$d_{j,t}^P = c_t + \gamma p_t + \zeta_{j,t}^P + \zeta_{j,t}^1 \qquad\qquad d_{j,t}^G = g_t + \gamma p_t + \zeta_{j,t}^G + \zeta_{j,t}^2. \tag{18}$$

Total demand for final good j is the sum of private and public demand, $y_{j,t}^d = \theta_C y_{j,t}^P + \theta_G y_{j,t}^G$, where θ_C is the steady state fraction of consumption in total output and $\theta_G \equiv 1 - \theta_C$ is defined analogously. Define the total demand signal to be $d_{j,t} \equiv \theta_C d_{j,t}^P + \theta_G d_{j,t}^G$. Then demand for final good j, in terms of the total demand signal and the worker's output price, is

$$y_{j,t}^d = d_{j,t} - \gamma p_{j,t}. \tag{19}$$

Demand for Labor. The log-linearized version of demand for labor input j (equation 4) is

$$n_{j,t}^d = -\rho w_{j,t} + \int_{\mathcal{D}_{j,t}} \left(\rho \bar{p}_{m,t}^I + y_{m,t}\right) dm.$$

I assume that the random demand for labor input j is such that the above equation takes the form

$$n_{j,t}^d = y_t - \rho\left(w_{j,t} - p_t\right) + \zeta_{j,t}^3, \tag{20}$$

where $\zeta_{j,t}^3$ is i.i.d, has distribution $N(0, \sigma_{\zeta,3}^2)$, and satisfies $\int_0^1 \zeta_{j,t}^3 dj = 0$. Note that in equation (20) I use the log-linear approximation of equation (9), $p_t^I = p_t$. Workers choose the price $w_{j,t}$ and observe demand $n_{j,t}^d$ at that price. Therefore their demand signal is $d_{j,t}^N \equiv n_{j,t}^d + \rho w_{j,t}$. Rewriting the signal in terms of unobservables yields

$$d_{j,t}^N = y_t + \rho p_t + \zeta_{j,t}^3, \tag{21}$$

and rewriting (20) in terms of the demand signal yields

$$n_{j,t}^d = d_{j,t}^N - \rho w_{j,t}. \tag{22}$$

Monetary Policy. Although firms are free to fully adjust prices in each period, the actual price level will in general differ from its steady-state value due to rigid wages. Thus there is a role for a monetary authority to set nominal interest rates. As in Lorenzoni (2009), the monetary authority responds to its own noisy signal of inflation:

$$i_t = (1 - \rho_i) i^* + \rho_i i_{t-1} + \varphi \tilde{\pi}_t,$$

where ρ_i and φ are known by all agents, $\tilde{\pi}_t$ is the monetary authority's noisy measure of inflation,

$$\tilde{\pi}_t = (p_t - p_{t-1}) + \omega_t,$$

and ω_t is a Normally distributed shock with zero mean and variance σ_ω^2.

Taxes. Each worker pays a lump-sum tax that is a noisy signal of the total collection of lump-sum taxes in the economy:

$$\tau_{j,t}^L = \tau_t^L + \zeta_{j,t}^\tau, \qquad (23)$$

where $\tau_{j,t}^L \equiv dT_{j,t}^L/Y$ is the ratio of the change in lump sum tax collections to steady state output and $\tau_t^L \equiv \int_0^1 \tau_{j,t}^L dj$. I do not take logs of lump sum taxes because their steady-state value is zero. As with the other idiosyncratic shocks, $\zeta_{j,t}^\tau$ is i.i.d and normally distributed with mean zero and integrates to zero across islands. Its variance is σ_τ^2. While in reality idiosyncratic taxes may have a permanent component, I model the idiosyncratic component as temporary for the sake of simplicity.

2.5 Equilibrium

I assume that the log-linear evolution of government expenditures follows the AR(1) process

$$g_t = \rho_g g_{t-1} + \nu_t,$$

where ν_t is normally distributed over time with mean zero and variance σ_ν^2. The government must satisfy its within-period budget constraint (equation 7), which to a log approximation is

$$\tau_t^L = \theta_G(1-\tau)g_t + (\theta_G - \tau)p_t - \tau\theta_C c_t.$$

The government's budget constraint can be simplified using the steady state result that $\theta_G = \tau$:

$$\tau_t^L = \theta_G\theta_C(g_t - c_t). \qquad (24)$$

The required increase in lump-sum taxes is decreasing in the consumption response to a government spending shock. This is due to the effect of changes in consumption on output and income tax collection.

Given the evolution of government expenditure, workers independently choose their consumption and the price of their output in each period, and the interaction of these choices drives the equilibrium. Worker $j's$ Euler equation is

$$c_{j,t} = x_{j,t} - E_{j,t}[x_{j,t+1}] + E_{j,t}[c_{j,t+1}] - i_t + E_{j,t}[\bar{p}_{j,t+1}] - \bar{p}_{j,t}, \qquad (25)$$

and the budget constraint is

$$\beta b_{j,t+1} = b_{j,t} + (1-\tau)\left(n_{j,t} + w_{j,t} + \frac{1}{\gamma}\left(y_{j,t}^{D} + p_{j,t}\right)\right) - \theta_C c_{j,t} - \theta_C \bar{p}_{j,t} - \tau_{j,t}^{L}, \quad (26)$$

where $b_{j,t} \equiv dB_{j,t}/Y$ is the ratio of the change in nominal bond holdings to steady state output. Nominal income in equation (26) consists of wages derived from sales of labor input j, $n_{j,t} + w_{j,t}$, and profits from firm j, $\frac{1}{\gamma}\left(y_{j,t}^{D} + p_{j,t}\right)$.

The optimal price choice for labor input j is the wage

$$w_{j,t} = c_{j,t} + \bar{p}_{j,t} - x_{j,t} + \xi n_{j,t}^{d}.$$

Since local labor demand $n_{j,t}^{d}$ is a function of the local wage, we can rewrite the above equation by substituting in equation (22) for $n_{j,t}^{d}$:

$$w_{j,t} = \frac{1}{1+\rho\xi}\left(c_{j,t} + \bar{p}_{j,t} - x_{j,t} + \xi d_{j,t}^{N}\right). \quad (27)$$

The worker's best response to an increase in the labor demand signal $d_{j,t}^{N}$ is muted by the factor $1/(1+\rho\xi)$, which is due to the dampening effect of a wage increase on demand for labor type j. While the draw from nature determines the price–insensitive component of demand, total demand for input j is sensitive to its price. Therefore equation (27) captures the worker's internalization of the wage he charges on his required labor effort. The elasticity of idiosyncratic demand with respect to the labor price is determined by the elasticity of substitution across labor inputs ρ. When ρ is high, workers expect demand for their labor to fall sharply in response to a local wage increase. The expectation that idiosyncratic labor demand will fall in response to a wage increase causes real wage rigidity in the sense that nominal wages fail to adjust relative to a perfect information benchmark.

A worker's desired price is also a function of the worker's consumption, which in turn is a function of the demand signal. Therefore to further illustrate the source of real wage rigidity and its effects in the model, it is useful to first complete the description of the model's equilibrium.

Learning and Aggregation. I postulate that $c_{j,t}$ and $w_{j,t}$ follow the linear decision rules

$$c_{j,t} = -\bar{p}_{j,t} + k_x x_{j,t} - k_p \bar{p}_{j,t}^{I} - k_\tau \tau_{j,t}^{L} + k_b b_{j,t} + k_d d_{j,t} + k_n d_{j,t}^{N} + k_z E_{j,t}[\mathbf{z}_{j,t}], \quad (28)$$

$$w_{j,t} = m_x x_{j,t} - m_p \bar{p}_{j,t}^{I} - m_\tau \tau_{j,t}^{L} + m_b b_{j,t} + m_d d_{j,t} + m_n d_{j,t}^{N} + m_z E_{j,t}[\mathbf{z}_{j,t}], \quad (29)$$

where $\boldsymbol{z}_{j,t} = (z_{j,t}, z_{j,t-1}, \dots)$ is an infinite-dimensional vector of state variables with $z_{j,t} = \left(x_t, g_t, c_t, p_t, i_t, \zeta^P_{j,t}, \zeta^G_{j,t}\right)$. Equation (28) represents the optimal consumption decision of worker j, and equation (29) represents the worker's optimal pricing decision. I assume that $\boldsymbol{z}_{j,t}$ evolves according to

$$\boldsymbol{z}_{j,t} = \mathbf{A}\boldsymbol{z}_{j,t-1} + \mathbf{B}\boldsymbol{u}^1_{j,t}, \tag{30}$$

with $\boldsymbol{u}^1_{j,t} \equiv \left(x_t, v_t, \omega_t, \mu^P_{j,t}, \mu^G_{j,t}\right)'$. Agents use the Kalman filter to form expectations of the state variables:

$$E_{j,t}[\boldsymbol{z}_{j,t}] = \mathbf{A}E_{j,t-1}[\boldsymbol{z}_{j,t-1}] + \mathbf{C}\left(\boldsymbol{s}_{j,t} - E_{j,t-1}[\boldsymbol{s}_{j,t}]\right),$$

where $\boldsymbol{s}_{j,t} = \left(s_t\ x_{j,t}\ \bar{p}_{j,t}\ \bar{p}^I_{j,t}\ d^P_{j,t}\ d^G_{j,t}\ d^N_{j,t}\ \tau^L_{j,t}\ \ i_t\right)'$ is the vector of signals received by the agents and $\mathbf{C}$ is a matrix of Kalman gains. There exists a matrix Ξ such that average expectations of the state variables are a linear function of the states themselves:

$$\int_0^1 E_{j,t}[\boldsymbol{z}_{j,t}]dj = \Xi\boldsymbol{z}_{j,t}.$$

A rational expectations equilibrium consists of matrices **A, B, C,** Ξ, and vectors $\{k_x, k_p, k_\tau, k_b, k_n, k_d, k_z\}$ and $\{m_x, m_p, m_\tau, m_b, m_n, m_d, m_z\}$ that are consistent with agents' optimization, Bayesian updating, and with market clearing in the goods, labor, and private bonds markets. The computation method used to solve for the equilibrium is an adaptation of Lorenzoni (2009). Details are in the Appendix.

2.6. Calibration

Table 1: Baseline Calibration

β	ρ	γ	ξ	θ_G	ρ_G	ρ_i	φ	σ_ω	ρ_ζ
0.99	100	7.5	10	0.2	0.5	0.9	1.5	0.0006	0.98
σ_ν	σ_x	σ_η	σ_{CPI}	σ_{PPI}	σ_τ	$\sigma_{\zeta,1}, \sigma_{\zeta,2}$	$\sigma_{\zeta,3}$	$\sigma_{\mu,P}, \sigma_{\mu,G}$	σ_ϵ
0.017	0.002	σ_x				0.166	.016	0.166	0.011

Table 1 shows the baseline calibration. A time period is taken to be a quarter, which implies $\beta = 0.99$. I assume that workers are near-perfect substitutes and set $\rho = 100$, although a multiplier above 0.9 is robust to $\rho > 50$.[7] The monetary policy parameters, as well as the elasticity of substitution across goods, are set according to Lorenzoni (2009): $\rho_i = 0.9,\ \varphi =$

[7] Appendix B discusses the sensitivity of the results to changes in parameter values.

$1.5, \sigma_\omega = 0.0006$, and $\gamma = 7.5$. The Frisch elasticity of labor supply that would prevail in a representative consumer environment, $1/\xi$, has almost no effect on the aggregate labor supply elasticity in the model due to the nature of wage-setting and the high labor supply elasticity that results from workers being off their labor supply curves. To emphasize the independence of aggregate labor supply from the elasticity of the workers' marginal cost curves, the elasticity is set to the low end of estimates based on micro studies: $1/\xi = 0.1$. Results are robust to alternative choices of this parameter.

The steady-state fraction of government consumption in GDP is $\theta_G = 0.2$, and the persistence of government spending is $\rho_G = 0.5$ to match the evolution of contract, grant, and loan funding from the American Recovery and Reinvestment Act of 2009 (ARRA).[8] While deviations of the fraction of government spending in GDP from its steady state value are more persistent, my interest is in the response of output to temporary stimulus spending such as ARRA. The standard deviation of government expenditure, σ_ν, is derived from the residual of the equation $g_t^D = \rho_G g_{t-1}^D + \nu_t^D$, where g_t^D is the demeaned fraction of government expenditure relative to GDP between 1950Q1 and 2011Q1, divided by θ_G.[9] The resulting parameterization is $\sigma_\nu = 0.017$.

Firm-level uncertainty. The nature of the uncertainty faced by firms has received increased research interest, although to date there remains a lack of consensus on the exact nature of the evolution of firm-specific demand shocks. Bloom (2009), for example, assumes that firm-level demand and productivity follow a joint random walk, while Cooper and Haltiwanger (2006) estimate an AR(1) process for idiosyncratic shocks to the profit functions of manufacturing firms. To obtain a calibration of the model's idiosyncratic demand process, I estimate an AR(1) process for the logarithm of the ratio of firm-level sales to total sales. The panel of firm level data is identical to that in Bloom (2009, p661) and contains quarterly data over the time period 1981-2000. Let $S_{j,t}$ be the sales of firm j at time t, let S_t represent the total

[8] Total funds awarded are available at www.recovery.gov. I obtain a time series of funds awarded in a given quarter by subtracting total funds awarded at time t from total funds at time $t-1$ for 2009Q3 through 2011Q1. The value $\rho_G = 0.5$ is derived from an AR(1) process for the logarithm of this time series.

[9] The data g_t^D are considered to be deviations of government spending relative to steady state output, $d\left(\frac{G_t}{Y}\right)$. Note that $\frac{dG_t}{Y} = \frac{G}{Y}\frac{dG_t}{G} = \theta_G g_t$. To obtain g_t^D I first divide the data by θ_G.

sales of all firms in the dataset at time t, and define $F_{j,t} \equiv S_{j,t}/S_t$ to be the fraction of firm j's sales relative to total sales. The estimating equation is

$$\log F_{j,t} = \alpha + \rho_\zeta \log F_{j,t-1} + e_{j,t}, \qquad e_{j,t} \sim N(0, \sigma_e^2) \tag{31}$$

which corresponds in the model to the evolution of the price-insensitive fraction of demand for good j:

$$\zeta_{j,t}^P = \rho_\zeta \zeta_{j,t-1}^P + \mu_{j,t}^P.$$

This yields estimates $\hat{\rho}_\zeta = 0.98$ and $\hat{\sigma}_e^2 = 0.055$. Note that the residual $e_{j,t}$ in equation (31) consists of the persistent shock $\mu_{j,t}^P$ and the transitory shock $\zeta_{j,t}^1$, so that its variance can be written $\sigma_e^2 = \sigma_{\mu,P}^2 + \sigma_{\zeta,1}^2$. For the baseline calibration I assume that half of the residual variance is due to permanent shocks, although the results are robust to alternative variance weightings. This yields $\rho_\zeta = 0.98$, $\sigma_\mu^P = 0.166$, and $\sigma_\mu^G = 0.166$. In addition, I impose symmetry on the evolution of idiosyncratic demand from the government and private sectors: $\sigma_{\mu,P}^2 = \sigma_{\mu,G}^2$ and $\sigma_{\zeta,1}^2 = \sigma_{\zeta,2}^2$.[10]

Variance parameters. The primary effect of the remaining variance parameters is to determine agents' expectations of the values of state variables. A common criticism of models of imperfect information is that national statistics are publicly available and that rational, optimizing individuals are aware of the values of aggregate state variables. Woodford (2003) responds to this critique by proposing that individuals have a limited ability to pay attention to all available information. While limited capacity may be a reasonable description of the real world, no data exist on the extent of this inattention. Nonetheless, imperfect information is a result not only of inattention, but also of informational constraints due the fact that reported values of government expenditure in real time are imprecise measures of the true values.[11] Therefore even with fully attentive agents imperfect information arises from imprecision inherent in national statistics. To compute the average revision to government expenditure I use the OECD's real-time database of GDP statistics (available at http://stats.oecd.org). The OECD database contains data release vintages for 1999Q4 through 2009Q2 for government expenditure from the years 1960Q1 through 1999Q2. I treat the most recent data available on government expenditure as

[10] Cooper and Haltiwanger (2006) estimate a persistence parameter of 0.85 for manufacturing firms. The variance of their innovations is 0.09. The results are robust to this alternative parameterization.

[11] The imprecision in real-time data is receiving increasing research attention, especially in forecasting models. See Croushore (2011) for a review. See also Mankiw and Shapiro (1986) and, more recently, Fixler and Grimm (2005) for a discussion of measurement error in national statistics.

the "true" values, while data from 1999Q4 release vintage represent the original release version of the data. The error ϵ_t is assumed to equal the log difference between the original and revised values. This results in a standard deviation $\sigma_\epsilon = 0.011$, which is likely a lower bound on the true value since the statistics for early periods in the 1999Q4 vintage release had already been subject to numerous revisions. Indeed, Fixler and Grimm (2005) use BEA original release data to show that mean absolute revisions to government expenditure are approximately four percentage points.

The remaining parameters are inherently difficult to calibrate. This is less of an issue than it might seem because the results are robust to a range of values for these parameters, as discussed in Appendix B. Variations in the remaining parameter values, especially the variance of marginal utility shocks, primarily affect the extent to which agents believe that idiosyncratic demand has risen in response to aggregate demand shocks. When the variance of average marginal utility is high, for example, agents put more weight on the signal of marginal utility relative to their signal of idiosyncratic demand and thus believe that average marginal utility is high when they perceive high demand and output. For the baseline calibration I set the standard deviation of marginal utility shocks so that the volatility of GDP in the model matches that of HP(1600)-filtered GDP.

The remaining parameters are the variances of the local price indices, lump-sum taxes, marginal utility and labor demand. I assume that σ_{CPI}, σ_{PPI}, σ_τ, and σ_η are equal to σ_x. Finally, I set the standard deviation of idiosyncratic labor demand to be small relative to the other variance parameters. Specifically, I assume that $\sigma_{\zeta,3}$ is a tenth the size of $\sigma_{\zeta,1}$. While this precise parameterization is inconsequential for the model's predictions, I choose a low baseline value for $\sigma_{\zeta,3}$ under the assumption that idiosyncratic demand for labor is far less volatile than that of final goods.

3. Response to Demand Shocks.

The model generates amplification of the response to demand shocks through two primary mechanisms. The first is real wage rigidity, which prevents a price adjustment that would otherwise dampen the change in agents' optimal level of consumption. In this section I begin with some analytic results that help illustrate how imperfect information creates real wage rigidity. I then present the numerical results and discuss the implications for the government multiplier and for amplification of marginal utility shocks.

The second mechanism is a wealth effect due to agents' perceptions of increased permanent income: Agents partially mistake increases (decreases) in aggregate demand for persistent increases (decreases) in idiosyncratic demand, resulting in a wealth effect that increases (decreases) desired consumption. The wealth effect is less important under the calibrated parameter values. Section 4 demonstrates how an alternative parameterization of the model relies on the wealth effect to generate a multiplier above unity.

3.1. Real Wage Rididity

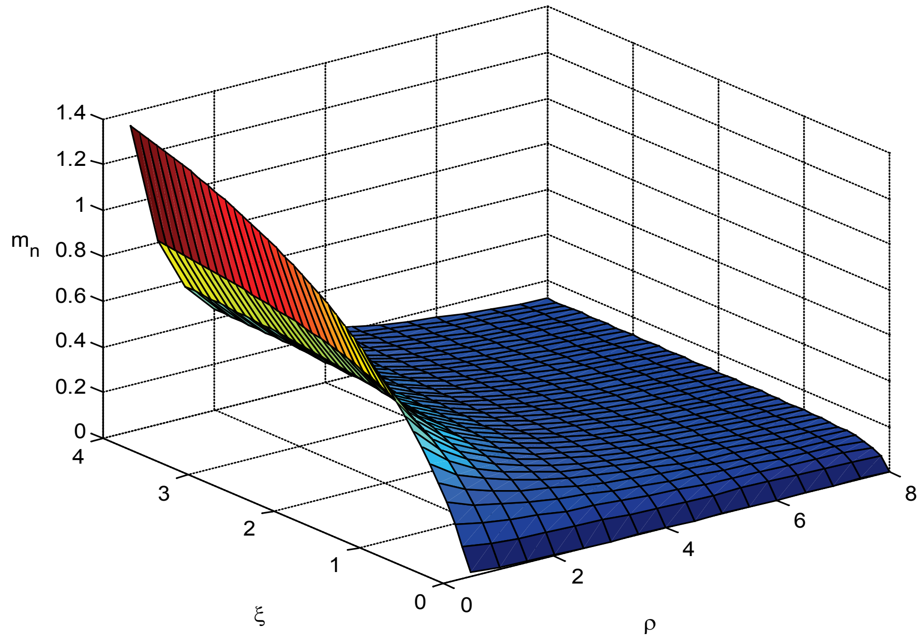
Proposition: Worker j's wage choice in response to the demand signal $d_{j,t}^N$ is

$$m_n = \frac{1}{1+\rho\xi}\left(\xi + \frac{1-\beta}{\rho}\right) \tag{32}$$

Proof: Appendix.

Figure 1 shows m_n as a function of ρ and ξ for the calibrated value of β. The local wage response to an idiosyncratic demand increase is highly sensitive to the Frisch elasticity of labor supply (parameterized by ξ) when labor types are poor substitutes (ρ is low). This is the upper-left region of Figure 1. When labor inputs are strong substitutes (ρ is high), the wage response is muted and relatively insensitive to the inverse of the Frisch elasticity of labor supply, ξ. Therefore even for an inelastic Frisch labor supply elasticity, the nominal wage response to an increase in idiosyncratic demand is muted relative to the response in a model with perfect information. The driving force behind this real wage rigidity is the worker's internalization of the price he charges on demand for his labor, as discussed above in relation to equation (26). Figure 1 demonstrates that this mechanism is strong even allowing for the endogenous response of a worker's consumption decision.

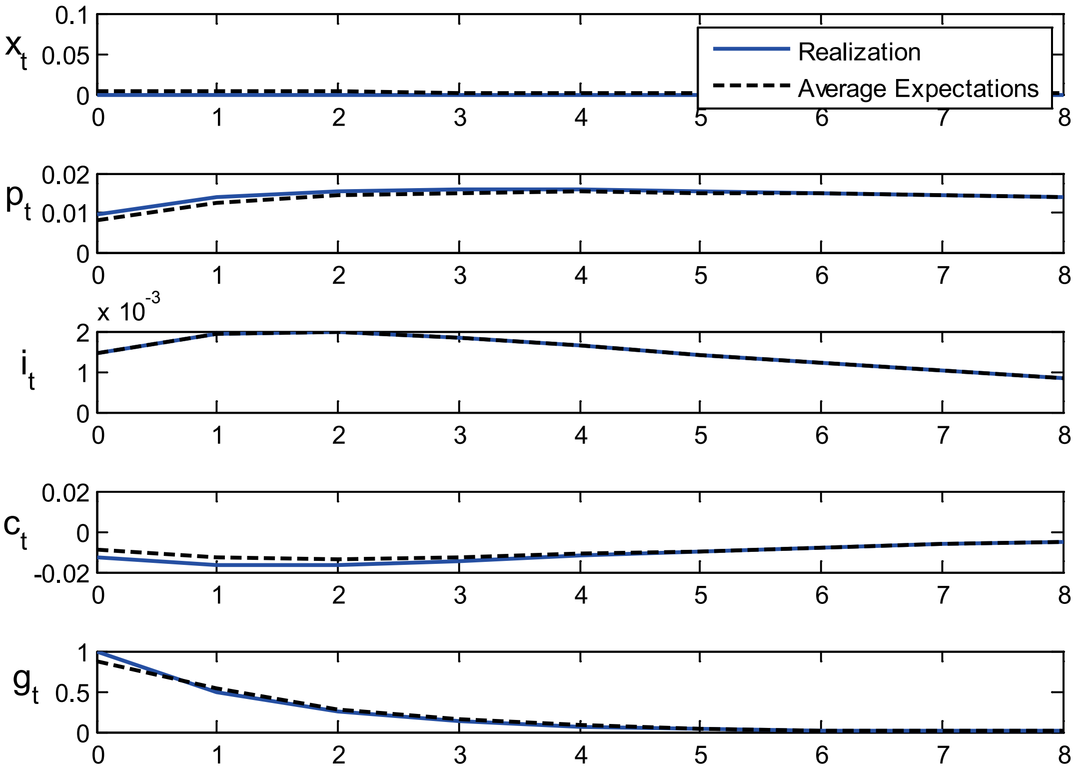
Figure 1: Wage response to an increase in the demand signal.



3.2 Response to a Government Spending Shock

Figure 2 shows the impulse responses to an initial one percent increase in government spending under the baseline calibration. The solid line is the actual realization, while the dotted line is agents' average expectation of the value of the state variable. On average agents are nearly correct about the change in the state variables, but because they do not precisely observe aggregate government expenditures they fail to predict the full extent of movements in the other aggregate states. Rather, they attribute the remaining observed increase in local demand to the idiosyncratic component of the signal $d_{j,t}^{G}$ so that on average each agent believes that the government is purchasing more from their local firm relative to the government's average purchase from other firms. In addition, agents attribute the increase in perceived local demand to a slight increase in average marginal utility.

Figure 2 : Response to a one percent increase in government expenditure.



Misperception of aggregate demand causes workers to believe that other workers' disutility from labor, and hence their desired optimal price, is lower than their own. Each worker j believes that by unilaterally raising w_j, total demand for his labor will fall, the extent to which is increasing in the demand elasticity ρ. Given the severity of the potential fall in income in response to a large wage increase, the worker chooses instead to charge a lower wage and to work harder relative to what he would choose in a model of perfect information. Hence wages and output prices fail to fully adjust relative to a benchmark of perfect information. Workers' real income, and hence their optimal consumption level, falls only slightly due to the small increase in the price level.

The impact multiplier in Figure 2 is 0.95, where I define the multiplier as the absolute change in output divided by the change in government spending. This multiplier is over ten times as high as the impact multiplier in a model of perfect information with identical parameter values.[12] Thus the model obtains a government expenditure multiplier near unity despite preferences that are separable in consumption and leisure.

3.3 Amplification of Marginal Utility Shocks.

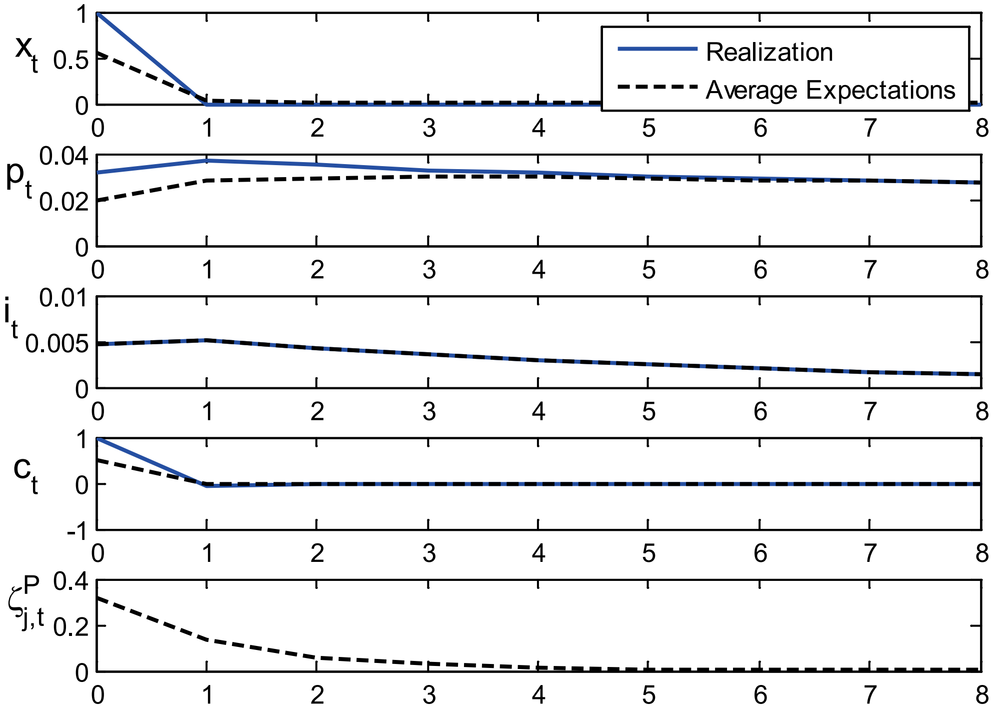
Figure 3 shows the response to a one percent shock to average marginal utility. As with government expenditure shocks, agents are imperfectly aware of the extent of the aggregate shock and thus on average attribute part of the perceived increase in local demand to its idiosyncratic component. The bottom graph of Figure 3 shows the average expectation of persistent idiosyncratic local demand from the private sector, which rises on impact and gradually returns to its steady state level.

Consumption tracks marginal utility nearly one-for one: a one percent increase in average marginal utility corresponds to a 0.96 percent increase in consumption on impact, which is over eight times the consumption response in a standard model with perfect information.[13] As discussed above, the amplification is due to wage rigidity resulting from imperfect knowledge of the position of other workers' labor demand curves. Rigid wages, combined with output prices set as a constant markup over labor costs, prevent a price spike that would crowd out consumption as predicted by a standard neoclassical model. In contrast to New Keynesian models, the amplification in the model here does not rely on countercyclical markups. Indeed, equation (9) holds at all points in time and thus a firm's desired markup does not change over the business cycle.

[12] Under perfect information, equation (27) can be integrated across workers to obtain $\xi y_t = -c_t$. Insert $c_t = (y_t - \theta_G g_t)/\theta_C$ and rearrange to obtain $dY_t/dG_t = \theta_C/(\xi\theta_C + 1)$, where I use $dY_t \approx y_t Y_t$ and $dG_t \approx g_t G_t$. This expression is analogous to equation (1.7) in Woodford (2010). For the parameter values above, the multiplier under perfect information is 0.07.

[13] Under perfect information, the response of consumption to marginal utility is $c_t = x_t/(1 + \xi\theta_C)$. For the calibrated parameter values, this yields $c_t = 0.11x_t$.

Figure 3: Response to a percent shock to average marginal utility.



The consumption responses to positive and negative aggregate demand shocks are symmetric: An adverse demand shock causes an excessive fall in consumption and only a small decline in wages. Thus the model's predictions are consistent with the Keynesian view that downward wage rigidity prevents output from reaching its natural rate during a recession. In the model wages are too high following a negative aggregate demand shock because each worker believes that on average other workers are higher up on their labor supply curves and therefore charging a high price as well. As a result, workers believe that lowering their posted wage will result in excessive demand for their labor and unacceptably high disutility from labor input.

A simultaneous increase in government spending can offset the adverse effects of a recessionary marginal utility shock. For example, an unexpected one percent decrease in marginal utility causes output to fall by 0.77 percent, while the natural level of output, defined as output that would prevail under perfect information, falls by only 0.16 percent. The government can stimulate output to its natural rate with a spending shock equal to 3 percent of steady state government expenditure.

4. The Wealth Effect and a Government Expenditure Multiplier above Unity.

In this section I demonstrate that modifications to the baseline calibration can produce an output multiplier above unity and a consumption multiplier above zero based on perceptions of increased permanent income. This exercise is of special interest in light of recent evidence that consumption increases in response to government spending shocks, causing an output multiplier that may exceed unity.

Table 2 shows the alternative calibration with the changed parameter values in bold. Relative to the baseline calibration, the idiosyncratic noise parameters σ_{CPI}, σ_{PPI}, σ_{τ}, and σ_{η} are multiplied by 10, the standard deviation of the signal government spending multiplied by 100, and the standard deviations of the shocks to the persistent component of idiosyncratic demand multiplied by 10. The effect of these changes is that agents mistake increases in aggregate government expenditure for increases in their firms' share of government expenditure.

Table 2: Alternative Calibration

β	ρ	γ	ξ	θ_G	ρ_G	ρ_i	φ	σ_{ω}	ρ_{ζ}
0.99	100	**1.5**	10	0.2	0.5	0.9	1.5	0.0006	**0.9999**
σ_{ν}	σ_x	σ_{η}	σ_{CPI}	σ_{PPI}	σ_{τ}	$\sigma_{\zeta,1}, \sigma_{\zeta,2}$	$\sigma_{\zeta,3}$	$\sigma_{\mu,P}, \sigma_{\mu,G}$	σ_{ϵ}
0.017	0.002	$\mathbf{10 \times \sigma_x}$				0.166	.016	**1.66**	**1.1**

In addition, the new parameterization features a low elasticity of substitution between final goods and highly persistent idiosyncratic demand. The new parameter values amplify the consumption response to a given expectation of persistent idiosyncratic demand.

It is important to note that while the variance parameter values in Table 2 may be higher than what would be implied by the data (see the baseline calibration), they may capture other real-world phenomena that are not included in the model for the sake of simplicity. For example, the model does not include rational inattention, but rational inattention to aggregate government spending would produce the mechanism emphasized in this paper: agents are inattentive to aggregate government spending but directly observe spending on their individual products (and hence perceive high permanent income relative to tax liabilities in response to government spending).

In other words, even if the variance of idiosyncratic spending from the government is not as high in reality as in Table 2, the high variance parameter causes imperfect knowledge in the model that may match the extent of imperfect information in reality based on a number of factors, including inattention.

Proposition 2: The optimal consumption response to the expected value of the local firm's share of government expenditure, $E_{j,t}\left[\zeta_{j,t}^{G}\right]$, is

$$\boldsymbol{e}_{\zeta^G} k_z' = \frac{\rho_\zeta}{\gamma} \frac{\beta(1-\beta)(1+\rho\xi)\theta_G}{\rho(\xi+1)\left(1-\beta\rho_\zeta\right)}, \tag{33}$$

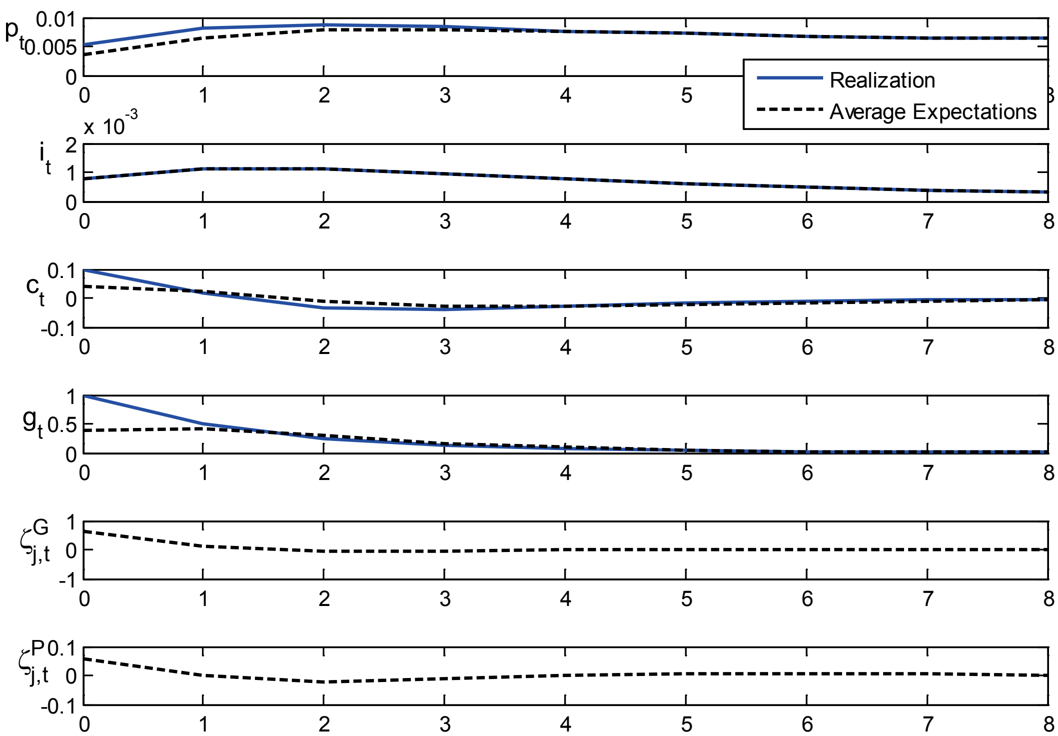
where $\boldsymbol{e}_{\zeta^G}$ selects $\zeta_{j,t}^{G}$ from $z_{j,t}$.

Proof: Appendix.

Equation (33) is decreasing in γ because profits are decreasing in γ (equation 10) due to a markup that falls as γ increases. When γ is low, the increase in profits is higher for a given increase in output or prices. Workers own the firm on their island and internalize the effect of γ on their income and they therefore choose a consumption response to their signal of local firm demand that is increasing in γ.

The parameter ρ_ζ determines the extent to which higher profits are expected to persist. When agents expect their local firm to continue to receive disproportionate expenditure from the government they perceive an increase in their permanent income, which raises their desired level of consumption. Figure 4 shows the response to a one percent shock to government expenditure under the alternative parameterization. Agents on average expect that government expenditure has increased, but they are unaware of the extent of the increase. Instead, they attribute the remainder of the increase in their local demand signal to its idiosyncratic components. The fifth graph in Figure 4 show that average expectations of the share of government expenditure spent on the local firm rise on impact. The perception of high local demand from the government causes a wealth effect, which increases consumption on impact. As a result of increased private consumption, agents receive a high signal of local private demand (in addition to the high signal of government demand). As shown in the bottom graph in Figure 4, average expectations of the share of private demand captured by the local firm rise on impact, thus rationalizing agents' perceptions of high local demand and their increase in desired consumption.

Figure 4: Response to a percent increase in government expenditure.



Note: This figure is based on the parameter values in Table 2.

The impact output multiplier in Figure 4 is 1.42, and the consumption multiplier is 0.4. As discussed above, the multiplier is increasing in the persistence of idiosyncratic demand shocks ρ_ζ and decreasing in the elasticity of substitution across final goods γ. It is also decreasing in the persistence of government spending ρ_g due to the perception of higher future taxes. Although agents are imperfectly aware of the extent of government spending, on average they are correct that spending has increased. Agents infer their expected future tax liabilities based on their expectations about the current level of government spending and the extent to which spending increases are expected to persist. For a given expectation of the magnitude of a government spending shock, the size of ρ_ζ relative to ρ_g determines whether or not government expenditure is perceived as a net increase in permanent income.

These results are interesting in light of the recent structural vector autoregression-based estimates of the effects of government spending shocks. Perotti (2008) finds a positive consumption response to government spending shocks, while Ramey (2011) finds that the response of consumption of services is significantly above zero. Both of these responses are consistent with the theoretical model in this section. Perotti also demonstrates that the responses of consumption and output are higher the less persistent is the shock to government spending, as predicted by the model.

In another VAR study, Auerbach and Gorodnichenko (2010) demonstrate that, on average, the government expenditure multiplier is higher for defense spending relative to nondefense spending. In the model the dependence of the multiplier on the elasticity of substitution between final goods offers some intuition that may help understand the AG results. When goods are poor substitutes in the model, perceptions of increased demand translate into a wealth effect that increases desired consumption. When final goods are strong substitutes, competition is stiff and higher demand has less impact on profits and perceived wealth.

One way to interpret the AG results relative to the mechanism in the model is to assume that there are few substitutes for government defense spending. Defense contracts are highly profitable, in part due to the paucity of competitors that can provide such a specialized service. In response to an increase in defense spending, contractors perceive an increase in their permanent income and increase their desired consumption. In contrast, when the government purchases a good for which there are strong substitutes and small markups, such as fast food at McDonald's, the impact on profits per dollar spent is smaller, as is the resulting wealth effect.

Testing the Model. The model in this section predicts that current and expectations of future profits should increase in response to a government expenditure shock, which implies that the firm values should also increase. This prediction is consistent with the assumptions that Fisher and Peters (2010) use to identify government spending shocks. While Fisher and Peters identify government spending shocks based on the stock returns of military contractors, the analysis here tests whether government spending shocks cause economy-wide stock price appreciation arising from high government and private spending.

A key identifying assumption below is that government spending shocks coincide with outlays. While this assumption is consistent with the model presented above, a number of papers, including Ramey (2011) and Leeper, Walker, and Yang (2010) highlight the importance of distinguishing between contemporaneous spending shocks and those that are pre-announced. The analysis below abstracts from preannounced shocks to government spending and instead assumes that the exogenous component of government spending coincides with the timing of outlays, as in the model.

I run bivariate structural vector augregressions (VARs) of the form

$$A_0 Y_t = \alpha + \sum_{j=1}^{4} A_j Y_{t-j} + \epsilon_t,$$

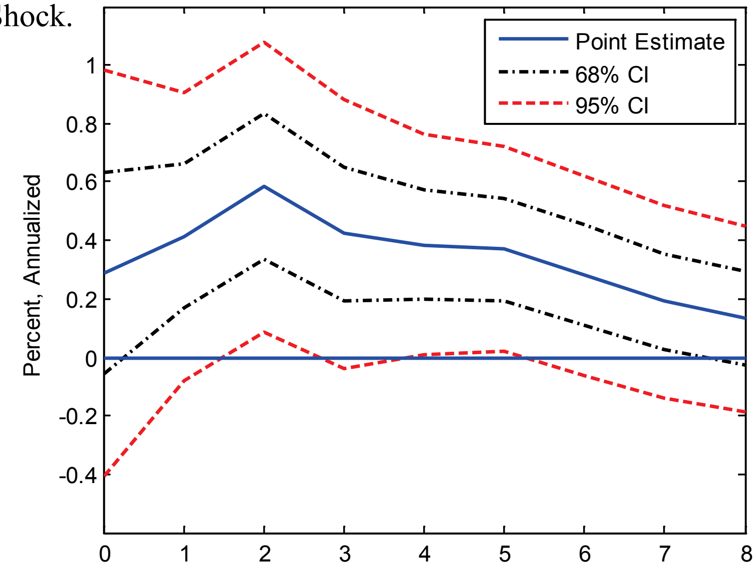
where ϵ_t is a vector of orthogonal structural shocks and $Y_t = [g_t^d \;\; r_t^d]'$ consists of the share of government spending in output, g_t^d, and a measure of real stock returns, r_t^d, from 1950Q1 through 2010Q4. As in Blanchard and Perotti (2002), I assume that the output share of government spending is predetermined with respect to all shocks but its own, which amounts to a zero restriction on the (1,2) element of the impact multiplier matrix A_0, based on the premise that government reacts to other shocks in the economy with a delay. These identifying assumptions are substantively identical with those used in Bachmann and Sims (2011), and Rossi and Zubiary (2011). They are also consistent with the model's assumption that while government spending varies with aggregate output (due to a marginal utility shock, perhaps), its output share only varies in response to a government spending shock (see equation 7). Below I relax this identifying assumption to permit the output share of government spending to respond on impact to all structural shocks.

The stock return series are based on CRSP data from Kenneth French's website. French assigns NYSE, AMEX, and NASDAQ companies to industry portfolios at varying levels of industry aggregation. I convert nominal returns into real returns using the U.S. CPI for all urban consumers.[14]

Figure 5 shows the response of the return of a value-weighted portfolio consisting of companies that produce nondurable goods and services, which is chosen because the model consists only of nondurables. Returns increase on impact, as in the model, and remain positive for over two years. The response is far more persistent than predicted by the model (see the bottom panel of Figure 4), which suggests that the model abstracts from additional frictions, informational or otherwise, that are present in the real world.

[14] The return at quarter t is based on appreciation of the price between date t and date $t+1$. This timing convention is based on the assumption that government expenditure during a quarter will be reflected in earnings reports and stock prices in the following quarter.

Figure 5: Response of Stock Returns for Nondurable Industries to a Government Spending Shock.

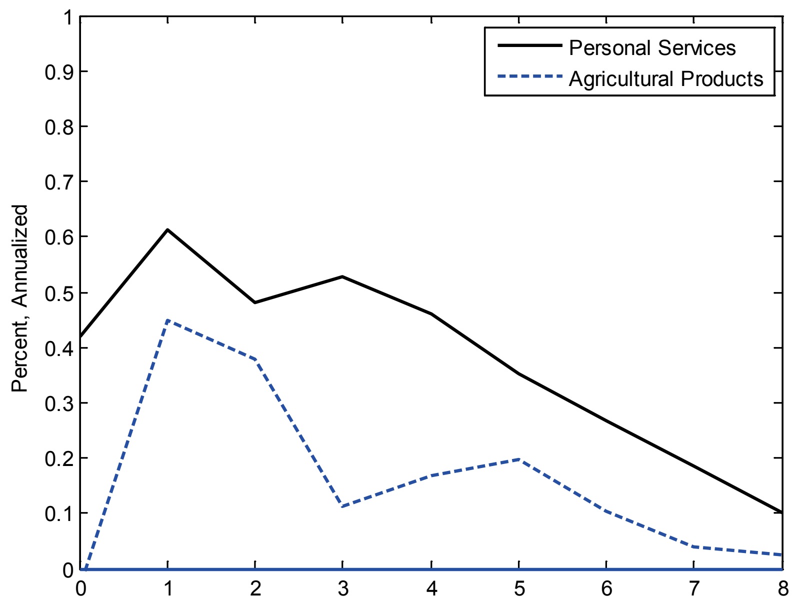


Note: The confidence bands were constructed using the recursive-design wild bootstrap of Goncalves and Kilian (2004).

The model predicts a higher response of firm value as the elasticity of substitution across goods falls due to increasing profit shares of income. This suggests that in the data, firms that produce goods for which there are fewer substitutes should have low earnings relative to stock prices, and should therefore experience a larger appreciation of firm value in response to government spending shocks.

While I am aware of no precise estimates of the within-industry elasticity of substitution for different industries, it is reasonable to assume that agricultural products are strong substitutes for each other. Figure 6 confirms that the response of returns for agricultural firms is higher than that of personal service firms, for which the products are likely to be less substitutable. Similar results hold when comparing the response of agricultural products to a range of other products that are likely to have a lower within-industry elasticity of substitution.

Figure 6: Responses of Portfolio Returns to Government Spending Shock.

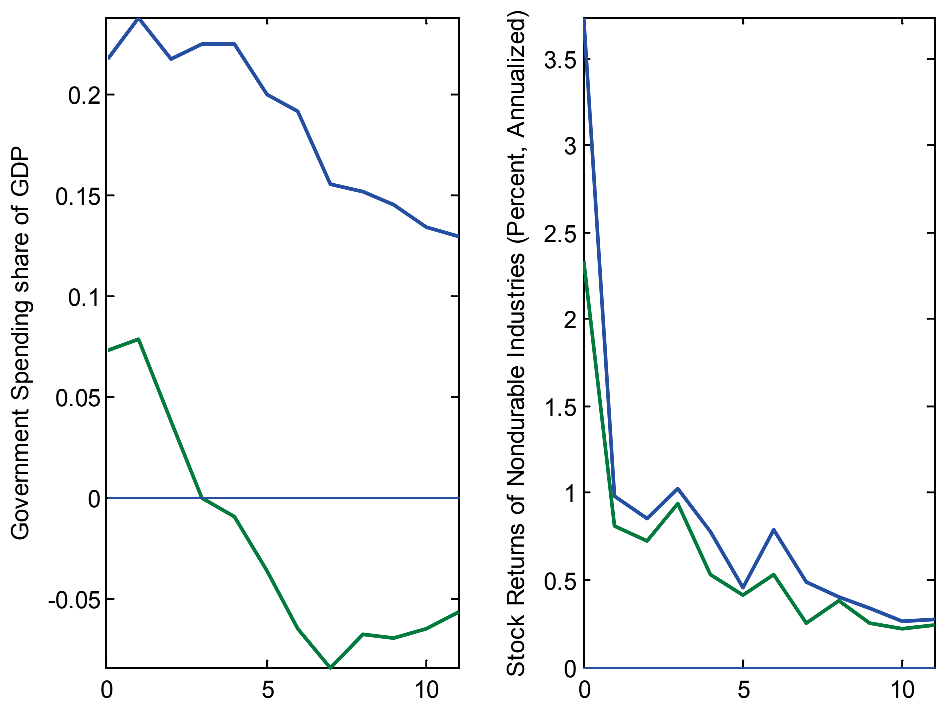


Alternative Identifying Assumptions. The recursively identified VAR relies on the assumption that the share of government spending in GDP is predetermined with respect to other shocks in the economy and that shocks to government spending coincide with government outlays. The assumption is potentially overly restricity, and therefore it is informative to check whether the positive response of stock returns to government spending shocks is robust to alternative identifying assumptions. To do so, I use the model as a guide. The model predicts that a government spending shock causes the output share of government spending to increase on impact and slowly return to its average. Therefore I assume that a government spending shock in the data must cause a similar pattern.

To implement this assumption, I rotate through the set possible identification matrices based on the reduced-form coefficient estimates and keep each orthoganalization that satisfies the following sign and shape restrictions: Government spending must rise on impact and stay positive for four quarters. Within the second quarter it must be below its impact level. These identifying assumptions are not very restrictive, and they permit a wide range of impulse response functions for which the output share of government spending increases on impact and slowly returns to its steady state. See Appendix C for implementation of the sign and shape restrictions.

Figure 7 shows the upper and lower bounds of the impulse response functions that satisfy the sign and shape restrictions. While the bounds on the government spending impulse responses are fairly large, the range of admissible responses of stock returns for nondurable industries is quite narrow. For any admissible identified structural government spending shock, the initial return on the portfolio of nondurable stocks is over two percent (at an annualized rate). Therefore the alternative identifying assumptions also support the model's predictions with respect to stock returns.

Figure 7: Set of Admissible Impulse Responses to a Government Spending Shock.



Note: The depicted bounds enclose the first 10,000 admissible impulse response functions based on multiple draws from the rotation matrix.

5. Conclusion

Keynes' belief in a multiplier effect of government spending on output was based in part on the perception that wages and prices fail to fully adjust over the business cycle. Since Keynes (1936), many studies have investigated the causes of wage and price rigidity. One contributing factor emphasized by Ball and Romer (1991) is that price rigidity may arise as a consequence of coordination failure among price-setters. My model is in that tradition. The price-setters in my model are imperfectly substitutable workers who charge a wage to final goods firms. Coordination failure arises from workers' imperfect knowledge of the demand for other workers' labor input, and hence from other workers' desired wages. On average workers are on their labor supply curve, but at any point in time they are working more or less than would a representative agent with perfect information.

The real-world counterparts to the model's workers are individual employees who in times of high demand work for a lower wage than they would otherwise. Unable to coordinate with potential replacement workers, these workers choose to accept the current wage rather than to take the chance of leaving the existing firm only to find that market demand for their labor is not sufficient to place them on their labor supply curve. This form of wage rigidity, combined with persistent idiosyncratic demand shocks under imperfect information, generates a positive consumption multiplier when information about aggregate government spending is imperfect, consistent with the empirical evidence on the consumption multiplier, as surveyed by Galí, López-Salido, and Vallés (2007). The key feature of the model that drives this response is persistent idiosyncratic demand. Furthermore, the model's prediction that firm values should appreciate in response to government spending shocks is consistent with the estimated responses of stock prices to government spending shocks.

Model Fit. A key feature of the model presented here is the assumption that agents are imperfectly aware of aggregate government expenditure. A general criticism of models of imperfect information, including Lucas (1972) and Woodford (2003), is that they postulate that agents remain unaware of fluctuations in macro aggregates that are published and available to the public. This assumption may nevertheless be reasonable given the complexity of data on government spending, the delays in the availability of such data, continuous revisions in these data releases, and the view that even rational agents will choose to ignore data that are too costly to process (e.g. Mankiw and Reis 2002, and Sims 2003). Furthermore, the model has two testable implications, both of which are supported by the data.

The calibrated model produces a multiplier consistent with recent empirical estimates without relying on exotic preferences or other strong assumptions about how government expenditure enters utility and production functions. In addition, it features acyclical markups and nominal wages that are only slightly procyclical, consistent with the evidence in Nekarda and Ramey (2010, 2011).[15] Under an alternative calibration agents' signals of macro aggregates are imprecise, consistent with the evidence in Coibion and Gorodnichenko (2011) that informational rigidities are economically large. When combined with persistent shocks to idiosyncratic demand, the output response to government spending shocks causes perceptions of high permanent income. The resulting output multiplier is above unity and the consumption response is positive. In contrast to a neoclassical model, which relies on a negative wealth effect to generate an increase in output and hours, the alternative calibration generates an increase in output and hours through a positive wealth effect.

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[15] Nekarda and Ramey (2011) find that nominal wages do not rise in response to industry-level government spending, and that real product wages fall slightly. In the model above, real product wages are acyclical.

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Appendix A

The matrices **A** and **B** are given by:

$$\mathbf{A}=\begin{bmatrix}\mathbf{0}\\ \rho_g\ \mathbf{0}\\ \boldsymbol{A_C}\\ \boldsymbol{A_P}\\ [0\ 0\ 0-\varphi\ \rho_i\ \ 0\ 0]+\varphi\boldsymbol{A_p}\\ 0\ 0\ 0\ 0\ 0\ \rho_\zeta\ 0\\ 0\ 0\ 0\ 0\ 0\ 0\ \rho_\zeta\\ I\ 0\end{bmatrix}\quad \mathbf{B}=\begin{bmatrix}1\ 0\ 0\ 0\ 0\\ 0\ 1\ 0\ 0\ 0\\ \boldsymbol{B_C}\\ \boldsymbol{B_P}\\ \varphi(\boldsymbol{B_p}+[0\ 0\ 1\ 0\ 0])\\ 0\ 0\ 0\ 1\ 0\\ 0\ 0\ 0\ 0\ 1\\ \mathbf{0}\end{bmatrix},$$

where $\boldsymbol{A_C}, \boldsymbol{B_C}, \boldsymbol{A_P}$, and $\boldsymbol{B_P}$ will be determined in equilibrium.

Optimal Decision Rules

The optimal decision responses to the agents' signals can be expressed analytically as a function of the model's parameters.

Prices. I first solve for the wage decision coefficients in equation (29) as a function of the consumption coefficients by substituting the consumption decision (28) into the optimal wage condition (29) to obtain

$$w_{j,t}=\frac{1}{1+\rho\xi}\left[\bar{p}_{j,t}-x_{j,t}+\xi d_{j,t}^N-\bar{p}_{j,t}+k_x x_{j,t}-k_p\bar{p}_{j,t}^I-k_\tau\tau_{j,t}^L+k_b b_{j,t}+k_n d_{j,t}^N+k_d d_{j,t}+k_z E_{j,t}[\boldsymbol{z}_{j,t}]\right]\quad (A1)$$

Rearranging and matching coefficients with (29) yields

$$m_\tau=\frac{k_\tau}{1+\rho\xi}\quad m_b=\frac{1}{1+\rho\xi}k_b\quad m_n=\frac{1}{1+\rho\xi}(\xi+k_n)\quad m_d=\frac{1}{1+\rho\xi}k_d\quad m_z=\frac{1}{1+\rho\xi}k_z$$

$$m_p=\frac{1}{1+\rho\xi}k_p\quad m_x=\frac{k_x-1}{1+\rho\xi}\quad (A2)$$

Consumption. Substitute for $c_{j,t+1}$ in (28) from the Euler equation (25):

$$c_{j,t}=E_{j,t}\left[-p_{t+1}-k_p\bar{p}_{j,t+1}^I-k_\tau\tau_{j,t+1}^L+k_b b_{j,t+1}+k_n d_{j,t+1}^N+k_d d_{j,t+1}+k_z[z_{jt+1}]\right]-i_t+x_{j,t}-E_{j,t}[x_{j,t+1}]+E_{j,t}[\bar{p}_{j,t+1}]-\bar{p}_{j,t}$$

Substitute for $b_{j,t+1}$ using the budget constraint (26), impose $E_{j,t}[x_{j,t+1}] = 0$, and rearrange terms:

$$(\beta + \theta_C k_b)c_{j,t} = -(\beta + \theta_C k_b)\bar{p}_{j,t} + k_b\left[b_{j,t} - \tau_{j,t}^L + (1-\tau)\left(d_{j,t}^N + \frac{1}{\gamma}d_{j,t} + (1-\rho)w_{j,t} - \frac{\gamma-1}{\gamma}p_{j,t}\right)\right] + \beta\, E_{j,t}\left[-k_\tau \tau_{j,t+1}^L - k_p \bar{p}_{j,t+1}^I + k_n d_{j,t+1}^N + k_d d_{j,t+1} + k_z[z_{jt+1}]\right] - \beta i_t + \beta x_{j,t},$$

Next substitute (A1) for $w_{j,t}$, substitute the period $t+1$ government budget constraint (24) for τ_{t+1}^L, express $E_{j,t}[d_{j,t+1}]$ and all state variables in terms of $\boldsymbol{z}_t$, and collect terms.

$$\begin{aligned}(\beta + \theta_C k_b)c_{j,t} &= -(\beta + \theta_C k_b)\bar{p}_{j,t} + k_b\theta_C\left[\frac{1}{\gamma} - \frac{\rho-1}{1+\rho\xi}k_d\right]d_{j,t} + k_b\theta_C\left[1 - \frac{\rho-1}{1+\rho\xi}(\xi + k_n)\right]d_{j,t}^N \\ &+ k_b\left[1 - \theta_C\frac{\rho-1}{1+\rho\xi}k_b\right]b_{j,t} - k_b\left[1 - \theta_C\frac{\rho-1}{1+\rho\xi}k_\tau\right]\tau_{j,t}^L \\ &+ k_b\theta_C\left[\frac{\rho-1}{1+\rho\xi}k_p - \frac{\gamma-1}{\gamma}\right]\bar{p}_{j,t}^I + \left(k_b(1-\tau)\frac{\rho-1}{1+\rho\xi}(1-k_x) + \beta\right)x_{j,t} \\ &+ \Big(-\beta\boldsymbol{e}_i - k_b(1-\tau)\frac{\rho-1}{1+\rho\xi}k_z \\ &+ \beta\left[k_\tau\theta_G\theta_C(\boldsymbol{e}_c - \boldsymbol{e}_G) + (k_d\gamma + k_I\rho - k_p)\boldsymbol{e}_p + (k_I + k_d)(\theta_C\boldsymbol{e}_c + \theta_G\boldsymbol{e}_G)\right. \\ &\left. + k_d(\theta_C\boldsymbol{e}_{\zeta P} + \theta_G\boldsymbol{e}_{\zeta G}) + k_z\right]\mathbf{A}\Big)E_{j,t}[\boldsymbol{z}_t] \quad (A3)\end{aligned}$$

Here $\boldsymbol{e}_p$, $\boldsymbol{e}_c$ $\boldsymbol{e}_G$, and $\boldsymbol{e}_i$ select p_t, c_t, g_t and i_t from $\boldsymbol{z}_t$. Let $\Lambda \equiv \beta + \theta_C k_b$. Matching the coefficient for $b_{j,t}$ in (28) yields

$$k_b = k_b\left[1 - \theta_C\frac{\rho-1}{1+\rho\xi}k_b\right]/\Lambda$$

Solving the above expression for k_b yields the consumption response to bonds holdings:

$$k_b = \frac{(1-\beta)(1+\rho\xi)}{\theta_C\rho(\xi+1)} \quad (A4)$$

A similar process of matching coefficients between (A3) and (28), plugging in (A4) for k_b, and rearranging yields expressions for the coefficients $\{k_x, k_p, k_\tau, k_n, k_d, k_z\}$. The expression for k_n is

$$k_n = \frac{1-\beta}{\rho}, \quad (A5)$$

where I use the relationship $\theta_C = (1-\tau)$. I plug (A5) into the expression for m_n in (A2) to obtain the result in Proposition 1. To obtain the result in Proposition 2, note that the expression for k_d is

$$k_d = \frac{(1-\beta)(1+\rho\xi)}{\gamma\rho(\xi+1)}, \quad (A6)$$

and that k_z can be written implicitly as

$$\left(\beta + k_b\theta_C\left(1+\frac{\rho-1}{1+\rho\xi}\right)\right)k_z - \beta k_z A$$
$$= -\beta \boldsymbol{e}_i$$
$$+\beta\big[k_\tau\theta_G\theta_C(\boldsymbol{e}_c - \boldsymbol{e}_G) + (k_d\gamma + k_I\rho - k_p)\boldsymbol{e}_p + (k_I + k_d)(\theta_C\boldsymbol{e}_c + \theta_G\boldsymbol{e}_G)$$
$$+ k_d(\theta_C\boldsymbol{e}_{\zeta P} + \theta_G\boldsymbol{e}_{\zeta G}) + k_z\big]A \quad (A7)$$

by matching coefficients in (A3) and (28). The object of interest is $\boldsymbol{e}_{\zeta G}k_z'$, which is the response of consumption to the expected value of $\zeta_{j,t}^G$. Take the transpose of (A7):

$$\left(\beta + k_b\theta_C\left(1+\frac{\rho-1}{1+\rho\xi}\right)\right)k_z' - \beta(k_z A)'$$
$$= -\beta \boldsymbol{e}_i'$$
$$+\beta A'\left[k_\tau\theta_G\theta_C(\boldsymbol{e}_c' - \boldsymbol{e}_G') + (k_d\gamma + k_I\rho - k_p)\boldsymbol{e}_p' + (k_I + k_d)(\theta_C\boldsymbol{e}_c' + \theta_G\boldsymbol{e}_G')\right.$$
$$\left.+ k_d\left(\theta_C(\boldsymbol{e}_c' + \boldsymbol{e}_\zeta^{P\prime}) + \theta_G\left(\boldsymbol{e}_G' + \boldsymbol{e}_{\zeta G}'\right)\right)\right]$$

Pre-multiply by $\boldsymbol{e}_{\zeta G}$ and note that $\boldsymbol{e}_{\zeta G}\boldsymbol{e}_c' = 0$. Likewise for $\boldsymbol{e}_G'$, $\boldsymbol{e}_i'$, and $\boldsymbol{e}_p'$.

$$\left(\beta + k_b\theta_C\left(1+\frac{\rho-1}{1+\rho\xi}\right)\right)\boldsymbol{e}_{\zeta G}k_z' - \beta\boldsymbol{e}_{\zeta G}(k_z A)' = -\beta\boldsymbol{e}_{\zeta G}\boldsymbol{e}_i' + \beta\boldsymbol{e}_{\zeta G}A'\left[k_d\left(\theta_C\boldsymbol{e}_\zeta^{P\prime} + \theta_G\boldsymbol{e}_{\zeta G}'\right)\right]$$

Rearrange. Note that $\boldsymbol{e}_{\zeta G}A'\left[k_d\left(\theta_C\boldsymbol{e}_\zeta^{P\prime} + \theta_G\boldsymbol{e}_{\zeta G}'\right)\right] = \boldsymbol{e}_{\zeta G}A'\boldsymbol{e}_{\zeta G}' = \rho_\zeta$ and $\boldsymbol{e}_{\zeta G}k_z' = \frac{1}{\rho_\zeta}\boldsymbol{e}_{\zeta G}(k_z A)'$:

$$\left(\beta(1-\rho_\zeta) + k_b\theta_C\frac{\rho(\xi+1)}{1+\rho\xi}\right)\boldsymbol{e}_{\zeta G}k_z' = \beta k_d\theta_G\rho_\zeta. \quad (A8)$$

Substitute the expressions for k_b and k_d and rearrange to obtain the result in Proposition 2.

Individual Inference

Individual inference, aggregation, and the computation of the equilibrium are an adaptation of Lorenzoni (2009). The vector of signals $s_{j,t} = (s_t, x_{j,t}\, \bar{p}_{j,t}, \bar{p}_{j,t}^I, d_{j,t}^P, d_{j,t}^G, d_{j,t}^N, \tau_{j,t}^L, i_t)'$ can be written as

$$s_{j,t} = \mathbf{F}\boldsymbol{z}_{\boldsymbol{j,t}} + \mathbf{G}\boldsymbol{u}^{\mathbf{2}}_{\boldsymbol{j,t}}$$

where $\boldsymbol{u}^{\mathbf{2}}_{\boldsymbol{j,t}} \equiv (\epsilon_t \; \eta_{j,t} \; \zeta^{CPI}_{j,t} \; \zeta^{PPI}_{j,t} \; \zeta^{1}_{j,t} \; \zeta^{2}_{j,t} \; \zeta^{3}_{j,t} \; \zeta^{\tau}_{j,t})'$ and

$$\mathbf{F} \equiv \begin{bmatrix} \boldsymbol{e}_G \\ \boldsymbol{e}_x \\ \boldsymbol{e}_p \\ \boldsymbol{e}_p \\ \boldsymbol{e}_C + \boldsymbol{e}_{\zeta P} + \gamma \boldsymbol{e}_p \\ \boldsymbol{e}_G + \boldsymbol{e}_{\zeta G} + \gamma \boldsymbol{e}_p \\ \theta_C \boldsymbol{e}_C + \theta_G \boldsymbol{e}_G + \rho \boldsymbol{e}_p \\ \theta_G \theta_C (\boldsymbol{e}_G - \boldsymbol{e}_C) \\ \boldsymbol{e}_i \end{bmatrix}, \quad \mathbf{G} \equiv \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0 \end{bmatrix}$$

Bayesian updating for worker j implies that

$$E_{j,t}[\boldsymbol{z}_{\boldsymbol{j,t}}] = E_{j,t-1}[\boldsymbol{z}_{\boldsymbol{j,t}}] + \boldsymbol{C}(\boldsymbol{s}_{\boldsymbol{j,t}} - E_{j,t-1}[\boldsymbol{s}_{\boldsymbol{j,t}}]).$$

Define the variance-covariance matrices

$$\Sigma = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_v^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\omega^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{uP}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{uG}^2 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} \sigma_\epsilon^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_\eta^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{CPI}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{PPI}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\zeta,1}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\zeta,2}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{\zeta,3}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_\tau^2 \end{bmatrix}$$

And let Ω be defined as $\Omega = \mathrm{Var}_{1,t-1}[\boldsymbol{z}_{\boldsymbol{t}}]$.

Then the Kalman gains are given by

$$\mathbf{C} = \Omega \mathbf{F}'(\mathbf{F}\Omega\mathbf{F}' + \mathbf{GVG}')^{-\mathbf{1}}$$

and Ω must satisfy the Riccati equation

$$\Omega = \mathbf{A}(\Omega - \mathbf{CF}\Omega)\mathbf{A}' + \mathbf{B}\Sigma\mathbf{B}'.$$

Fixed Point

The average first-order expectations regarding the state $\boldsymbol{z}_t$ can be expressed as

$$\boldsymbol{z}_{t|t} = \boldsymbol{\Xi}\boldsymbol{z}_t.$$

Aggregating the Bayesian updating equations across workers yields

$$\boldsymbol{z}_{t|t} = (I - \mathbf{CF})\mathbf{A}\boldsymbol{z}_{t-1|t-1} + \mathbf{CF}\boldsymbol{z}_t,$$

which implies that $\boldsymbol{\Xi}$ must satisfy

$$\Xi \boldsymbol{z}_t = (I - \mathbf{CF})\mathbf{A}\Xi \boldsymbol{z}_{t-1} + \mathbf{CF}\boldsymbol{z}_t.$$

Aggregating equations (28) and (29) across workers and rearranging gives

$$c_t = k_x x_t + (\gamma k_d + \rho k_I - 1 - k_p)p_t + k_\tau \theta_G \theta_C (c_t - g_t) + (k_I + k_d)(\theta_C c_t + \theta_G g_t) + k_z \Xi \boldsymbol{z}_{j,t}$$

$$p_t = m_x x_t + (\gamma m_d + \rho m_I - m_p)p_t + m_\tau \theta_G \theta_C (c_t - g_t) + (m_I + m_d)(\theta_C c_t + \theta_G g_t) + m_z \Xi \boldsymbol{z}_{j,t},$$

which is used to update the evolution of the state variables until the impulse responses of c_t and p_t to the shocks in $\boldsymbol{u}_t^1$ converge under the old and updated values of **A** and **B**. In the numerical computation I restrict $\mathbf{A_C}, \mathbf{B_C}, \mathbf{A_P}$, and $\mathbf{A_C}$ so that they do not respond to the local state variables $\zeta_{j,t}^G$ and $\zeta_{j,t}^P$.

Appendix B: Sensitivity Analysis

An output multiplier of 0.95 is robust to an order of magnitude increase or decrease to any of the standard deviation parameters. The results are more sensitive to the persistence and elasticity of substitution parameters. Table 1B shows the multiplier when a single one of these parameters is changed from its value in Table 1 to the value in Table 1B. When ρ is decreased to 20 while keeping the other parameters the same, for example, the impact output multiplier falls to 0.81. The multiplier is less sensitive to the other parameters, including a more reactionary monetary authority, because wage rigidity prevents inflationary pressure.

Table 1B: Sensitivity Analysis

	ρ	ρ	γ	γ	ρ_ζ	ρ_G	ρ_i	φ
New Parameter Value	20	500	2	15	0.5	0.9	0.5	3
Output Multiplier	0.81	0.98	0.99	0.95	0.95	0.89	0.94	0.90

Appendix C: Implementing VAR Sign and Shape Restrictions

Consider the reduced-form VAR model $A(L)y_t = e_t$, where y_t is the N-dimensional vector of variables, $A(L)$ is a finite-order autoregressive lag polynomial, and e_t is the vector of white noise reduced-form innovations with variance-covariance matrix $\Sigma_{\mathrm{e_t}}$. Let ϵ_t denote the corresponding structural VAR model innovations. The construction of structural impulse response functions requires an estimate of the $N \times N$ matrix $\tilde{B}$ in $e_t = \tilde{B}\epsilon_t$. Let $\Sigma_{\mathrm{e_t}} = \mathrm{P\Lambda P}$ and $B = P\Lambda^{0.5}$ such that B satisfies $\Sigma_{\mathrm{e_t}} = \mathrm{BB}'$. Then $\tilde{B} = BD$ also satisfies $\tilde{B}\tilde{B}' = \Sigma_{e_t}$ for any orthonormal $N \times N$

matrix D. One can examine a wide range of possible choices for $\tilde{B}$ by repeatedly drawing at random from the set $\boldsymbol{D}$ of rotation matrices and discarding candidate solutions for $\tilde{B}$ that do not satisfy the set of a priori sign and shape restrictions on the implied impulse response functions. The procedure consists of the following steps:

1) Draw and $N \times N$ matrix K of $NID(0,1)$ random variables. Derive the QR decomposition of K such that $K = Q \cdot R$ and $QQ' = I_N$.
2) Let $D = Q'$. Compute impulse responses using the orthogonalization $\tilde{B} = BD$. If all implied impulse response function satisfy the identifying restrictions, retain D. Otherwise discard D.
3) Repeat the first two steps a large number of times, recording each D that satisfies the restrictions and record the corresponding impulse response functions.

The resulting set $\tilde{\boldsymbol{B}}$ comprises the set of admissible structural VAR models.