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**Pricing-to-Market and Optimal Interest Rate Policy\***

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**Abstract**

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I study optimal interest rate policy in a small open economy with consumer search in the product market. When there are search frictions, firms price-to-market, with implications for the design of monetary policy. Country-specific shocks generate deviations from the law of one price for traded goods which monetary policy acts to stabilize by influencing firm markups. However, stabilizing law of one price deviations results in greater fluctuations in output.

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## 1. Introduction

Pricing-to-market by exporting firms, and induced deviations in the law of one price, have important implications for the design of monetary policy in open economies.<sup>1</sup> In this paper, I study optimal interest rate policy with consumer search in the product market and pricing-to-market behavior by firms. I consider consumer search for two reasons. First, the frictions induced by search costs have been shown to help match key statistics on fluctuations in international relative prices.<sup>2</sup> Second, search provides a simple, micro-founded explanation for endogenous markups and equilibrium price dispersion across countries.

I develop a small open economy model where consumers actively search to reduce the price of the goods they consume. This allows for short-run deviations from the law of one price, and because search takes time, its opportunity cost is measured in terms of foregone labor income. Consumption purchases are subject to a cash-in-advance restriction. The opportunity cost of search is therefore adjusted for the cost of holding money. In this case, the path of the nominal interest rate influences the markup charged by the firm, in addition to allocations and international relative prices.<sup>3</sup>

In the open economy, by using the interest rate to influence the terms of trade, a policymaker can raise the level of home consumption, for a given level of output. Absent search frictions, it is optimal to raise (or lower) the nominal interest rate in response to shocks depending on the elasticity of substitution between home and foreign goods - the trade elasticity - and the intertemporal elasticity of substitution in consumption. Because optimal policy can be

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<sup>1</sup>Empirical evidence points to pricing-to-market being strong and persistent. For example, see Atkeson and Burstein (2008) and Gopinath and Itskhoki (2010).

<sup>2</sup>Neither real business cycle models nor sticky price models have been able to account for movements in international relative prices observed in the data. See Backus *et al.* (1995) and Chari *et al.* (2002).

<sup>3</sup>I use a cash-in-advance constraint for simplicity. In general, any transactions technology will mean that the interest rate influences the markup due to the nature of the search frictions I consider.

expressed solely in terms of output - with no additional role for the exchange rate - in a special case, when the two elasticities are equal, the interest rate is invariant to fluctuations in the business cycle.<sup>4</sup>

When there are search frictions in the product market, it is optimal for the interest rate to respond to shocks, even under the special case just described. Moreover, the interest rate can also be written as a function of deviations from the law of one price for the home good. When the home good is relatively expensive in the domestic market - and the home currency is overvalued - interest rates are raised, because a higher interest rate reduces the desired markup, forcing firms to charge a lower price for their product. However, because an overvalued currency is also consistent with a drop in external demand, by focusing exclusively on law of one price deviations, monetary policy generates greater fluctuations in output.

Deviations from the law of one price for the home good are sufficient to explain interest rate policy in the special case because home consumer search for the foreign good is exogenous from the perspective of the policymaker. However, in general, the interest rate reacts to deviations from the law of one price for the foreign good and the mass of shoppers sent out by households to search for (home and foreign) goods. In this case, simple expressions for interest rate policy are unavailable, and I undertake a numerical exercise to determine how law of one price deviations add to fluctuations in the business cycle under optimal policy.

The results I present can be compared to those that study monetary policy in models of pricing-to-market based on sticky-prices.<sup>5</sup> Monacelli (2005) argues that optimal policy under commitment requires the stabilization of law of one price deviations for imported goods using a small open economy model. Engel (2011) studies the role of currency misalignments

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<sup>4</sup>The reasoning behind this result is very standard, and is similar in spirit to the analysis of Benigno and Benigno (2003), albeit with flexible prices.

<sup>5</sup>In models with nominal rigidities, pricing-to-market is the result of fluctuations in the nominal exchange rate and sticky local prices.

(defined as a weighted average of deviations from the law of one price over home and foreign goods) in a two-country setting, and shows why policy should account for exchange rate movements (in addition to inflation and an output gap) under cooperation. In this paper, I find that optimal interest rate policy acts to smooth-out deviations from the law of one price, and hence should also be concerned with currency misalignments.

This paper is also related to research on product search in dynamic general equilibrium models. The rationale for law of one price deviations I adopt is based on the two-country business cycle model of Alessandria (2009). However, in the closed economy, Head *et al.* (2012) embed product search into a search-theoretic model of money with the result that money shocks are neutral but prices adjust sluggishly, and Kaplan and Menzio (2013) develop a model with self-fulfilling fluctuations that result from the interaction of product search and search and matching in labor markets.<sup>6</sup> In the open economy, Liu and Shi (2010) use a monetary search model to study policy coordination when there are deviations from the law of one price driven by a cross-country differential in money growth.

A final point is that open economy RBC models with complete international asset markets and productivity shocks generate procyclical terms of trade.<sup>7</sup> Corsetti *et al.* (2008) have shown how incomplete asset markets coupled with a low trade elasticity can overturn this result, but recent research has pointed to alternative channels, via induced movements in the labor wedge arising from home production (Karabarbounis, 2014), or search frictions in product markets (Bai and Rios-Rull, 2013). In this paper, I do not attempt to explain which friction is more relevant for generating countercyclical terms of trade, rather, I demonstrate

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<sup>6</sup>Arseneau *et al.* (2014) study optimal fiscal and monetary policy with search frictions based on long-term relationships.

<sup>7</sup>The opposite appear to hold in the data. The raw correlation between the terms of trade and output is reported in Heathcote and Perri (2002) as being  $-0.24$ . Similar values are reported in Alessandria (2009) and Bai and Rios-Rull (2013). Evidence based on vector autoregressions is presented in Enders and Müller (2009).

why consumer search in the product market is important for the conduct of monetary policy in the open economy.

The remainder of the paper is organized as follows. In section 2, I develop a small open economy monetary model with consumer search in the product market. I consider optimal stabilization policy in section 3. Section 4 concludes.

## 2. Model

I consider a two-country economy. The home country is populated by a continuum of households of total measure 1 and the foreign country is populated by a continuum of households of total measure  $\omega$ . Thus,  $\omega$  is the population size of the home country relative to the foreign country. In both countries households maximize lifetime utility, where period utility is a function of consumption and the mass of workers and shoppers. In each country, firms supply an identical good produced using labor.

In what follows, I focus the exposition of the model on the home country, with the understanding that analogous expressions hold for the foreign country. Consumption, output, and the nominal price of the home/foreign output are denoted with  $h/f$ -subscripts. Asterisks denote foreign country variables.

### 2.1. Households

Households have the following intertemporal utility function,

$$U = E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \kappa_t(l_t + s_t)] \quad ; \quad c_t = \left[ (1 - \chi)^{1/\xi} c_{h,t}^{(\xi-1)/\xi} + \chi^{1/\xi} c_{f,t}^{(\xi-1)/\xi} \right]^{\xi/(\xi-1)} \quad (1)$$

The variable  $c_t$  is a consumption aggregate over the home ( $c_{h,t}$ ) and foreign ( $c_{f,t}$ ) good,  $l_t$  is total labor supplied for production, and  $s_t \equiv s_{h,t} + s_{f,t}$  is the measure of shoppers (of which there are  $s_{h,t}/s_{f,t}$ ) searching for the home/foreign. The parameter  $\xi > 0$  is the elasticity of

substitution between home and foreign goods (the trade elasticity) and  $\chi = \alpha\omega / (1 + \omega)$  is a weight placed on the foreign good in consumption, where  $\alpha$  is a measure of trade openness.

The disutility of shopping and labor are assumed to be linear and the variable  $\kappa_t$  is a shock to the marginal disutility of labor as in Hall (1997). With search frictions this shock generates countercyclical movement in the terms of trade. Since without search the shock mimics shocks to productivity, it provides a simple benchmark from which to assess the importance of search-based pricing-to-market for optimal monetary policy.<sup>8</sup>

Households start period  $t$  with nominal wealth  $\mathcal{W}_t$ , receive monetary transfers  $T_t$ , and choose money holdings  $M_t$ , and nominal bond holdings  $B_t$ , that pay  $R_t B_t$  one period later.<sup>9</sup> The gross nominal interest rate at date  $t$  is  $R_t$ . Households also buy  $A_{t+1}$  units of home currency state-contingent nominal securities. Each security pays one unit of money at the beginning of period  $t + 1$  in a particular state where  $Q_{t,t+1}$  is the beginning of period  $t$  price of these securities normalized by the probability of the occurrence of the state. In the asset market at the beginning of period  $t$  households face the constraint,

$$M_t + B_t + E_t Q_{t,t+1} A_{t+1} - T_t \leq \mathcal{W}_t \quad (2)$$

After leaving the asset market households enter the goods and labor markets. Consumers search to reduce the price of home and foreign goods. With probability  $z$  each shopper gets a price quote from only one (1) firm and with probability  $(1 - z)$  they get price quotes from two (2) firms. The cumulative distribution of the lowest price drawn by a shopper is,

$$J(p_i) = zF(p_i) + (1 - z) [1 - (1 - F(p_i))^2] \quad (3)$$

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<sup>8</sup>For a quantitative analysis of this type of shock, without search frictions, see Holland and Scott (1998). Nakajima (2005) presents a microeconomic foundation for the shock, which, more generally, should also be viewed as part of the labor wedge, as stressed in Chari *et al.* (2007).

<sup>9</sup>Each period is divided into two sub-periods, with the asset market operating in the first sub-period and the goods market in the second.

where  $p_i \in [\underline{p}_i, \bar{p}_i]$  and  $F(p_i)$  is the cumulative density function of price quotes for goods  $i = \{h, f\}$  in the economy. With a reservation-price  $\tilde{p}_i \geq p_i$ , a shopper buys the good, with probability  $J(p_i)$ , at the expected price of  $P(\tilde{p}_i)$ . Thus, in period  $t$ , the amount the shopper expects to buy is  $P(\tilde{p}_{i,t}) J(\tilde{p}_{i,t}) = \int_0^{\tilde{p}_{i,t}} p_i dJ(p_i)$ . The household sends  $s_{i,t}$  agents to shop, and since the total amount of good purchased is equal to the number of shopping trips, consumption is  $c_{i,t} = s_{i,t} J(\tilde{p}_{i,t})$ .

A fraction  $\frac{1}{v_t}$  of the households consumption purchases must be made with money such that the following cash-in-advance constraint is satisfied,

$$\frac{\Omega_t}{v_t} \leq M_t \quad (4)$$

where  $\Omega_t \equiv s_{h,t} \int_0^{\tilde{p}_{h,t}} p_{h,t} dJ(p_{h,t}) + s_{f,t} \int_0^{\tilde{p}_{f,t}} p_{f,t} dJ(p_{f,t})$ . Households also receive labour income,  $W_t l_t$ , where  $W_t$  is the nominal wage rate, and dividends from all firms, denoted  $\Phi_t$ . The nominal wealth at the beginning of period  $t + 1$  is,

$$W_{t+1} = M_t + R_t B_t + A_{t+1} - \Omega_t + W_t l_t + \Phi_t \quad (5)$$

The households problem is to choose  $\{B_t, A_{t+1}, M_t, l_t, s_{i,t}, \tilde{p}_{i,t}\}_{t=0}^{\infty}$  to maximize, (1), subject to (2)-(5). For securities, we get standard expressions, albeit evaluated at the reservation price.

$$\frac{u'(c_t)}{\tilde{p}_t} = R'_t E_t \left[ \frac{\beta u'(c_{t+1})}{\tilde{p}_{t+1}} \frac{R_{t+1}}{R'_{t+1}} \right] \quad \text{and} \quad Q_{t-1,t} = \beta \frac{(u'(c_{t+1}) / \tilde{p}_{t+1}) (R_{t+1} / R'_{t+1})}{(u'(c_t) / \tilde{p}_t) (R_t / R'_t)} \quad (6)$$

where  $R'_t = 1 + (R_t - 1) / v_t$  and  $E_t Q_{t,t+1} = 1 / R_t$ . When  $v_t = 1$ , then  $R'_t = R_t$ , and equation (4) coincides with the standard cash-in-advance constraint. As  $v_t \rightarrow \infty$ , the cash constraint becomes irrelevant,  $R'_t = 1$ , and the model reduces to a real model. The choice of labor and search effort generates the following conditions.

$$\frac{\kappa_t}{u'(c_t)} = \frac{W_t}{\tilde{p}_t R'_t} \quad \text{and} \quad \frac{W_t}{R'_t} = [\tilde{p}_{i,t} - P(\tilde{p}_{i,t})] J(\tilde{p}_{i,t}) \quad (7)$$

The first condition captures the labor-leisure trade-off where the intratemporal marginal rate of substitution between leisure and consumption equals the real wage adjusted for the cost of holding money. The second condition captures the trade-off between sending out  $(1/J(\tilde{p}_{i,t}))$  more shoppers, with the expectation of paying  $P(\tilde{p}_{i,t})$  for a unit, and receiving a lower price. Note that the opportunity cost of search is measured in terms of foregone labor income, and is also adjusted for the cost of holding money.

Finally, the choice over home and foreign goods is characterized by,

$$\frac{c_{f,t}}{c_{h,t}} = \left( \frac{\chi}{1-\chi} \right) \left( \frac{\tilde{p}_{h,t}}{\tilde{p}_{f,t}} \right)^\xi \quad \text{and} \quad \tilde{p}_t^{1-\xi} = (1-\chi)\tilde{p}_{h,t}^{1-\xi} + \chi\tilde{p}_{f,t}^{1-\xi} \quad (8)$$

We can also generate the demand functions for home and foreign goods in terms of overall consumption,  $c_{h,t} = (1-\chi)(\tilde{p}_t/\tilde{p}_{h,t})^\xi c_t$  and  $c_{f,t} = \chi(\tilde{p}_t/\tilde{p}_{f,t})^\xi c_t$ , respectively. As  $\xi \rightarrow 0$  we recover the Cobb-Douglas case with unit-elastic demand functions. It is worth restating that conditions (6)-(8) are all in terms of reservation prices, as these determine marginal decisions.

## 2.2. Firms

In each country, there are many firms producing a country-specific good. Each firm has technology,  $y_{h,t} + y_{h,t}^* = l_{h,t} + l_{h,t}^*$ . Home (foreign) households send  $s_{h,t}$  ( $s_{h,t}^*$ ) shoppers to search for the home good. The total profit for a firm serving both markets is denoted  $\phi_t$ , and I write this as,

$$\phi_{h,t} + \phi_{h,t}^* = (p_{h,t} - W_t) d_{h,t} + (e_t p_{h,t}^* - W_t) d_{h,t}^* \quad (9)$$

where  $d_{h,t}/q = 1 + \varsigma [1 - F(p_{h,t})]$  and  $d_{h,t}^*/q = 1 + \varsigma [1 - F(p_{h,t}^*)]$  are the demand curves per-shopper faced by the firm (in the home and export market) and  $\varsigma \equiv 2(1-q)/q$ .<sup>10</sup> The

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<sup>10</sup>A fraction  $q/[q+2(1-q)]$  of consumers get one (1) quote, and a fraction  $2(1-q)/[q+2(1-q)]$  get two (2) quotes. Combining these expressions, the probability that a shopper purchases from a firm charging  $p_t$  is,  $q(p_t) \equiv \{q+2(1-q)[1-F(p_t)]\}/(2-q)$  for  $p_t \leq \tilde{p}_t$  and  $q(p_t) = 0$  if  $p_t > \tilde{p}_t$ . The demand curve per shopper is then,  $d(p_t) \equiv q(p_t) \times (2-q)$ .

firm maximizes profit per-shopper, choosing  $p_{h,t}$  and  $p_{h,t}^*$ , with reservation prices,  $\tilde{p}_{h,t}$  and  $\tilde{p}_{h,t}^*$ , and input costs  $W_t$  given, such that,  $y_{h,t} = d_{h,t}$  and  $y_{h,t} = d_{h,t}^*$ . The maximum price a firm can charge in the domestic (export) market is the consumers local-currency reservation price so the solutions to the firms problems imply the following price distributions,

$$F(p_{h,t}) = 1 - \frac{1}{\varsigma} \left( \frac{\tilde{p}_{h,t} - p_{h,t}}{p_{h,t} - W_t} \right) \quad \text{and} \quad F(p_{h,t}^*) = 1 - \frac{1}{\varsigma} \left( \frac{\tilde{p}_{h,t}^* - p_{h,t}^*}{p_{h,t}^* - W_t/e_t} \right) \quad (10)$$

where  $F(\underline{p}_{h,t}) = 0$  and  $F(\underline{p}_{h,t}^*) = 0$  determine the lower point on the price distributions. Notice that firms are indifferent between charging any price on  $p_i \in [\underline{p}_i, \bar{p}_i]$  and because firms can be viewed as randomizing, the both price distributions,  $F(p_{i,t})$ , are continuous.

### 2.3. Equilibrium

Solving the model involves finding the reservation price using the search conditions in (7) along with (10).<sup>11</sup> A key feature of the model is that (7) contains the nominal interest rate. Although this is standard feature of a cash-in-advance model without search, with consumer search, this means that the interest rate also affects firms behavior. To see why, consider the following conditions, that describe the distribution of prices.

$$\tilde{p}_{h,t} = \left[ 1 + \left( \frac{1}{1-z} \right) \frac{1}{R_t'} \right] W_t \quad \text{and} \quad \underline{p}_{h,t} = \left[ \frac{2(1-z)}{2-z} \right] W_t + \left( \frac{z}{2-z} \right) \tilde{p}_{h,t} \quad (11)$$

with average price  $P(\tilde{p}_{h,t}) = \tilde{p}_{h,t} - W_t/R_t'$  and where  $J(\tilde{p}_{h,t}) = 1$ . The square brackets of the first expression in (11) is the markup charged by a firm over the marginal cost of production,  $W_t$ . The markup is decreasing in the nominal interest rate. An analogous condition holds for the foreign good, albeit adjusted for the exchange rate,  $e_t$ , which enters via firm profits. The export price is therefore given by,  $e_t \tilde{p}_{h,t}^* = \left[ 1 + \left( \frac{1}{1-z} \right) \left( \frac{e_t W_t^*}{W_t} \right) \frac{1}{R_t^{*'}} \right] W_t$ . Now we see that the law of one price need not hold. I define additional variables that track deviations from the law of one price,  $\psi_{h,t} \equiv e_t \tilde{p}_{h,t}^* / \tilde{p}_{h,t}$  and  $\psi_{f,t} \equiv e_t \tilde{p}_{f,t}^* / \tilde{p}_{f,t}$ , for the home and foreign good, respectively.

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<sup>11</sup>The details of this are relegated to *Appendix A.1*.

Now consider the efficiency condition for bonds' holdings by foreign consumers, which can be written as,  $Q_{t-1,t} u'(c_{t-1}^*) / e_{t-1} \tilde{p}_{t-1}^* = \beta u'(c_{t-1}) / e_t \tilde{p}_t^*$ . Combining this with the equivalent condition for home consumers, the risk-sharing condition is,

$$\tilde{q}_t = u'(c_t^*) / u'(c_t) \quad (12)$$

where  $\tilde{q}_t \equiv e_t \tilde{p}_t^* / \tilde{p}_t$  is the real exchange rate, expressed in terms of reservation prices. Finally, I also derive resource (output and shopping) constraints. Output is equal to home consumption plus exports,  $c_{h,t} + \omega c_{h,t}^*$ , and using the demand conditions above,

$$y_t = (1 - \chi) \left( \frac{\tilde{p}_t}{\tilde{p}_{h,t}} \right)^\xi c_t + \chi^* \omega \left( \frac{\tilde{p}_t^*}{\tilde{p}_{h,t}^*} \right)^\xi c_t^* \quad (13)$$

where  $\chi = \alpha\omega / (1 + \omega)$ ,  $\chi^* = \alpha / (1 + \omega)$ , and  $\alpha = \alpha^*$ . This condition is standard except insofar as deviations from the law of one price for home goods affect export demand. The shopping constraint is given by,  $s_t = s_{h,t} + s_{f,t}$ , and so,

$$s_t = (1 - \chi) \left( \frac{\tilde{p}_t}{\tilde{p}_{h,t}} \right)^\xi c_t + \chi \left( \frac{\tilde{p}_t}{\tilde{p}_{f,t}} \right)^\xi c_t \quad (14)$$

where  $y_{i,t} = s_{i,t}$ . Thus, shopping depends on home consumption only. From here on, I assume the home economy is small in the sense that  $\omega \rightarrow \infty$ .<sup>12</sup> In this case, we can write the foreign economy resource constraints as,  $y_t^* = c_t^* = s_t^*$ , which is exogenous from the perspective of the home economy.

### 3. Optimal Monetary Policy

In the following sections I characterize optimal monetary policy under commitment. Each monetary authority sets its interest rate and injects money in the economy through lump-

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<sup>12</sup>In this case, there is home-bias in consumption in the sense that the home country is small and the limit of the ratio of expenditure share on home goods to population share is not equal to one. For example, see Faia and Monacelli (2008).

sum transfers.<sup>13</sup> I assume throughout that the foreign economy is at a steady-state with a zero nominal interest rate. In the absence of foreign shocks, such a policy is optimal under commitment, and this allows me to focus on the policy problem for the home economy.

### 3.1. International Relative Prices

I first make some definitions which help explain the role of international relative prices in determining policy decisions. I begin by denoting  $\rho_t \equiv \tilde{p}_{f,t}/\tilde{p}_{h,t}$  and  $\rho_t^* \equiv \tilde{p}_{f,t}^*/\tilde{p}_{h,t}^*$  for relative prices within each economy. It is worth stressing that these variables are not the terms of trade. The terms of the trade for the home economy, for example, are given by  $\tilde{p}_{f,t}/e_t\tilde{p}_{h,t}^*$ , which is the relative price of foreign to home output. The variable  $\rho_t$  only moves one-for-one with the terms of trade when the law of one price holds and  $\psi_{h,t} = \psi_{f,t} = 1$ .

The real exchange rate,  $\tilde{q}_t$ , and the ratio of the producer price to consumer price index,  $g_t \equiv \tilde{p}_t/\tilde{p}_{h,t}$ , are functions of these newly defined variables.

$$g_t(\rho_t) = \left[ (1 - \alpha) + \alpha (\rho_t)^{1-\xi} \right]^{1/(1-\xi)} \quad \text{and} \quad \tilde{q}_t(\rho_t) = \psi_{f,t} \left[ (1 - \alpha) (\rho_t)^{\xi-1} + \alpha \right]^{1/(\xi-1)} \quad (15)$$

where  $g'_t(\rho_t) > 0$ ,  $\tilde{q}'_t(\rho_t) > 0$ , and the term  $\psi_{f,t}$  - the law of one price gap for imported goods - also affects the real exchange rate (and deviations from purchasing power parity). Note that we can re-express the real exchange rate as,  $\tilde{q}_t = \psi_{h,t}/\rho_t^* g_t(\rho_t)$ , where  $g_t^* \approx 1$ , due to the small open economy restriction. When the law of one price holds, and  $\psi_{h,t} = 1$ , we further find,  $\rho_t = 1/\rho_t^*$ .

The final international price I define is the terms of labor, denoted  $\omega_t \equiv e_t W_t^*/W_t$ . Given risk-sharing, the labor-leisure conditions imply  $\omega_t = 1/(\kappa_t R'_t)$ , where the home law of one price gap is  $\psi_{h,t} = \left[ 1 + \left( \frac{1}{1-z} \right) \omega_t \right] / \left[ 1 + \left( \frac{1}{1-z} \right) / R'_t \right]$ . We now see that, for a given home

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<sup>13</sup>Setting an exogenous path for the interest rate does not uniquely determine the path of prices. However, given lump-sum taxes, i.e,  $T_t = M_t - M_{t-1}$ , this indeterminacy does not affect the real allocations or the relative prices, which is the focus of the analysis.

nominal interest rate, both shocks to the marginal utility of leisure and money demand generate deviations from the law of one price. For example, a low realization of  $\kappa_t$  results in a fall in the relative wage (and  $\omega_t$  rises). Home consumers then face a higher opportunity cost of search, which means  $p_{h,t} > e_t p_{h,t}^*$ . In this case, and as in Engel (2011), I say that the home currency is over-valued.

### 3.2. Optimal Monetary Policy with Walrasian Product Markets

In this section I focus on the case without search and where  $v_t \rightarrow 1$ . In this case, the terms of labor and terms of trade coincide ( $\rho_t = \omega_t$ ) and both a higher interest rate and marginal utility of labor lead to an improvement in the terms of trade (a lower  $\omega_t$ ). Optimal policy is determined by a monetary authority that maximizes the expected discounted sum of utilities of all agents, given by  $U = E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \kappa_t l_t]$ .

The constraints faced by the policymaker are,

$$l_t = [g(\omega_t)]^\xi \left\{ (1 - \alpha) c_t + \alpha [q(\omega_t)]^\xi \right\} \quad \text{and} \quad q(\omega_t) = 1/u'(c_t) \quad (16)$$

The first of these conditions is simply the resource constraint, given by equation (13), using the definitions of relative prices presented in *section 3.1*. The second condition is risk-sharing, given by equation (12), where  $u'(c_t^*) = 1$ , which is consistent with the rest of the world being at the Friedman rule. Given the constraints in (16), the policy problem consists in choosing consumption ( $c_t$ ), labor ( $l_t$ ), and the terms of trade/labor ( $\omega_t$ ).

Making the further assumption,  $u(c_t) = c_t^{1-\sigma} / (1 - \sigma)$ , optimal interest rate policy is given by,

$$R_t = 1 + \frac{\alpha}{1 - \alpha} \left[ (1 - \alpha) (\xi\sigma - 1) q_t^{1-\xi} + \sigma \xi q_t^{\xi-1/\sigma} \right] \quad (17)$$

Equation (17) has some very clear implications. The special case  $\xi\sigma = 1$  implies  $R_t = 1/(1 - \alpha)$  such that policy is invariant to fluctuations in the business cycle. Moreover, since

we can write the real exchange rate in terms of output, optimal policy implies,  $q_t = y_t^{1-\alpha} = [(1-\alpha)/\kappa_t]^{1-\alpha}$ , both of which fall with  $\alpha$ . As  $\alpha \rightarrow 1$ , then  $q_t \rightarrow 1$ , and purchasing power parity holds. But because also means  $c_t = c^*$ , and  $l_t = 1/R_t$ , it is optimal for policy to set  $l_t = 0$ , which is an extreme version of a beggar-thy-neighbor strategy. In general, in the open economy, interest rates need not be constant, nor set at the Friedman rule on average (i.e.,  $R = 1$ ), because the policy maker has an incentive to the use interest rate to manipulate the terms of trade.<sup>14</sup>

To evaluate the response of the economy to shocks I linearize equation (17) around its long-run outcome. This generates,

$$r_t = [\chi(q)](\xi\sigma - 1)\hat{q}_t \quad (18)$$

where  $r_t$  is the deviation of  $R_t$  from its long-run level and  $\chi(q) > 0$ .<sup>15</sup> Consider what happens when there is a low realization of  $\kappa_t$ . Output rises and the terms of trade deteriorate (a rise in  $\omega_t$ ). In the open economy, it is possible for consumption to rise, given output, when the terms of trade improve (see the first condition in (16)). Thus, by raising the nominal interest rate, the policymaker can restrict the rising value of  $\omega_t$ . It is optimal to do this when  $\xi\sigma > 1$ .

A final implication of (17) is that optimal policy can be written an instrument rule in output alone so that the policy maker need only target the change in output resulting from the shock. Consider equation (16), which can be re-expressed as,

$$y_t = \omega_t^{1/\sigma} \left\{ (1-\alpha) + \alpha [q(\omega_t)]^{\xi-1/\sigma} \right\} [g(\omega_t)]^{\xi-1/\sigma} \equiv y(q_t) \quad (19)$$

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<sup>14</sup>Without shocks, we find that  $R$  is increasing in  $\xi$  and  $\alpha$ . As  $\xi$  rises any change in  $\omega$  exerts a greater effect on  $q$ .

<sup>15</sup>I find,  $\chi(q) \equiv \left[ \left( \frac{\alpha}{1-\alpha} \right) \xi q^{\xi-1/\sigma} - (\xi-1) \alpha q^{1-\xi} \right] / R$ , where  $R$  is determined by (17), and  $\chi'(q) > 0$  for  $\sigma\xi > 1$ . Note that  $\xi\sigma$  alone determines the response of the policy instrument to movements in the real exchange rate generated by the shock I consider.

where  $y'(q_t) > 0$ . For the case  $\xi\sigma < 1$ , the interest rate falls when  $q_t$  rises, and monetary policy is procyclical. However, from an empirical viewpoint, since  $\xi > 1$  and  $\sigma > 1$ , countercyclical policy is a much more likely outcome. Finally, higher values for  $\xi$  make the real exchange rate and output less volatile to shocks. These final points are important when I allow consumer search frictions to influence the policy problem.

### 3.3. Optimal Monetary Policy with Consumer Search

With consumer search, the objective for the policy maker is amended to include resources used in search activities, and reads,  $U_0 = E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - \kappa_t(l_t + s_t)]$ . The constraint on search is given by equation (14) of *section 2.3*.<sup>16</sup>

Optimal interest rate policy is now,

$$R_t = 1 - \left(\frac{z}{1-z}\right) + \alpha \left[ \left(\frac{\xi\sigma}{1-\alpha}\right) \left(\frac{1}{\psi_{h,t}}\right)^{\xi+1} \tilde{q}_t^{\xi-1/\sigma} + (\xi\sigma - 1) \left(\frac{\tilde{q}_t}{\psi_{f,t}}\right)^{1-\xi} \right] + (\xi\sigma - 1) \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{\phi_t}{\psi_{h,t} - 1}\right)^{\xi} \left(\frac{\tilde{q}_t^{1-1/\sigma} s_t}{c_t^* \psi_{f,t}} - 1\right) \quad (20)$$

This is the main equation of the paper. There are important differences between this expression, and equation (17), which describes optimal interest rate policy with Walrasian product markets. Recall, without shocks, the law of one price holds, and  $\psi_{h,t} = \psi_{f,t} = 1$ . Thus, consumer search has two potential implications for long-run policy. First, the term  $z/(1-z) > 0$  appears. This is the monopolistic markup (over average prices) and unambiguously reduces the optimal interest rate.<sup>17</sup> Second, and depending on the specification of preferences, the total mass of shoppers sent out by households affects the policy decision.

<sup>16</sup>More details on the specification of the policy problem are presented in *Appendix A.2*.

<sup>17</sup>It is worth noting the difference between this and the markup based on product differentiation and Dixit-Stiglitz preferences. In that case, the markup interacts with trade openness, whereas in this case, the term is additive.

To isolate the implications of consumer search, I begin by considering the special case  $\xi\sigma = 1$ , where  $s_{f,t}$  - the measure of home consumers searching for the foreign good, is exogenous from the perspective of the policymaker. Since the home policymaker only accounts for shopping activity over home goods, deviations in the law of one for home goods are sufficient to determine optimal interest rate policy (this is immediate from (20)). As before, suppose there is a low realization of  $\kappa_t$ , which leads to a rise in the terms of labor. Because the opportunity cost of search is now relatively low for home consumers, there is a fall in the domestic price of the home good, relative to the price paid in the rest of the world (i.e.,  $p_{h,t}$  falls relative to  $e_t p_{h,t}^*$ , or,  $\psi_{h,t} > 1$ , such that the domestic currency is undervalued). The optimal response to this shock is to lower the interest rate away from its long-run level.

With the additional restriction  $\xi = 1$ , the following relationship between the real exchange rate and output holds,  $q_t = \psi_{f,t}^\alpha \{[1 + 1/(1 - z)] [(1 - \alpha) + \alpha/\psi_{h,t}] y_t\}^{1-\alpha}$ . Since optimal policy targets deviations in the home law of one price, this policy cannot be equivalent to targeting output alone, as was the case without search frictions. To see how this works, I linearize the expression for optimal interest rate policy around its long-run level and solve in terms of output and the real exchange rate. This leads to,

$$r_t = \varrho \left[ \hat{y}_t - \left( \frac{1}{1 - \alpha} \right) \left( \hat{q}_t + \alpha \hat{\psi}_{f,t} \right) \right] \quad (21)$$

where  $\varrho \equiv 2(1 - z) / (1 + z(\alpha - 2)) > 0$ ,  $\hat{\psi}_{f,t}$  is exogenous, and the terms in square brackets in equation (21) equals zero without search. Again, a low realization of  $\kappa_t$  (now,  $\hat{\kappa}_t > 0$ ) generates an increase in output. However, the terms of trade (which are equal to  $s_t/\psi_{h,t}$ ) improve as a result of deviations in the law of one price (both  $\hat{\psi}_{h,t} > 0$  and  $\hat{\psi}_{f,t} > 0$  because households substitute into search activities). By lowering interest rates, optimal policy moves the terms of trade in the same direction as the shock, which adds to the movement in output. Put differently, consumer search in the product markets implies that interest rate

policy is procyclical.<sup>18</sup>

### 3.4. The Role of Search in Generating Output Movements

In general, I want to determine how much optimal policy adds to output movements when there is consumer search. For the sake of simplicity, consider a 1% drop in the the marginal disutility of labor.<sup>19</sup> Then consider variations in the trade elasticity,  $\xi > 0$ . In all cases reported below, I fix  $\sigma$  (the intertemporal elasticity of substitution in consumption) at unity, as it is only the value of this parameter, relative to the trade elasticity, that affects interest rate policy. I also set  $\alpha = 0.1$ , which is a measure of home-bias in consumption. In the model with consumer search, I set  $z$  to generate a 10% markup over average prices.

Table 1 presents the change in output from the shock under optimal policy (“Optimal”) - as characterized by equations (17) and (20) - and an interest rate peg at  $R = 1$  (“Peg”), which corresponds to the Friedman rule.

===== **Table 1 Here** =====

As we might expect, output is less volatile with search because, whilst a negative shock to the marginal utility of labor raises output, it also generates a fall in  $\psi_{h,t}$ , which has a subsequent negative impact on output, via the terms of trade. Moreover, under both policies (Optimal and Peg), output is more sensitive to the shock as the trade elasticity rises, consistent with the analytical results presented above. In the case of Walrasian product markets, since  $\xi \geq 1$ , policy is countercyclical as it acts to reduce movements in output, and it is more

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<sup>18</sup>In fact, in this very simple case, we can write output (in levels) as,  $y_t = (1 - \alpha) \left( \frac{1}{\kappa_t R_t + \frac{1}{1-z}} \right) + \alpha \left( \frac{1}{\kappa_t R_t + \frac{1}{1-z}} \right)$  which is falling in  $\kappa_t$  (as is  $\psi_{h,t}$ ). Since a low realization of  $\kappa_t$  requires lower interest rates (since otherwise  $\psi_{h,t}$  would be too high) this clearly acts to stimulate output.

<sup>19</sup>The details of the analysis are presented in *Appendix A.3*.

aggressive as the trade elasticity rises. When there is consumer search, policy is procyclical, even when the trade elasticity is relatively high.

Despite the relative simplicity of the analysis, the overriding point is that the presence of pricing-to-market and deviations from the law of one price have significant effects on policy decisions. The model I develop is one of flexible prices but the results are analogous to those derived within models where nominal price rigidities are present, such as Monacelli (2005) and Engel (2011). One important difference is that I focus on the design of optimal monetary policy following the Ramsey approach which allows for an explicit consideration of all wedges that characterize both the long run and the short-run dynamics of the economy.

## **5. Conclusions**

This paper derives optimal interest rate policy in a small open economy model with consumer search frictions. In response to shocks that generate deviations from the law of one price, optimal policy dictates that the policy stance should tighten (higher interest rates) when the home currency is overvalued. Because policy acts in a direct way to influence deviations from the law of one price this can lead to greater fluctuations in output.

## Appendix

### A.1 Solving for the Reservation Price

Solving the model involves finding the reservation price. Here I supply details on the derivation of equations contained in (11) as reported in the main text. Consider a good with price  $p_t$ . For such a good, the optimal condition for search implies,  $W_t/R'_t = [\tilde{p}_t - P(\tilde{p}_t)] J(\tilde{p}_t)$ , where  $\tilde{p}_t$  is the reservation price and  $P(\tilde{p}_t)$  is the expected (transacted) price. Since  $\int_0^{\tilde{p}_t} dp_t = \tilde{p}_t$ ,  $\int_0^{\tilde{p}_t} dJ(p_t) = J(\tilde{p}_t)$ , and  $P(\tilde{p}_t) J(\tilde{p}_t) = \int_0^{\tilde{p}_t} p_t dJ(p_t)$ , we can use  $\int_0^{\tilde{p}_t} p_t dJ(p_t) = [\tilde{p}_t \times J(\tilde{p}_t)] - \int_0^{\tilde{p}_t} J(p_t) dp_t$  to re-write the right-hand side of the optimal search condition as,  $\int_0^{\tilde{p}_t} J(p_t) dp_t$ . Now we can use the distribution of the lowest price drawn by a shopper and the distribution over price quotes, given by (3) and (10), respectively. The latter can be expressed as,  $F(p_t) = 1 - (\tilde{p}_t - p_t) / \varsigma (p_t - W_t)$ , where  $\varsigma \equiv q/2(1 - q)$ , and this implies,

$$\frac{W_t}{R'_t} = \left(1 + \frac{\varsigma q}{2}\right) \int_{\underline{p}_t}^{\tilde{p}_t} dp_t - \frac{\varsigma q \times (\tilde{p}_t - W_t)^2}{2} \int_{\underline{p}_t}^{\tilde{p}_t} \left(\frac{1}{p_t - W_t}\right)^2 dp_t \quad (22)$$

where  $\underline{p}_t$  is determined by  $F(\underline{p}_t) = 0$ , and since, in equilibrium, the highest price is equal to the shoppers reservation price,  $\bar{p}_t = \tilde{p}_t$  (Burdett and Judd, 1983). Using  $\int [1/(p_t - W_t)]^2 dp_t = -1/(p_t - W_t)$  and  $\int dp_t = p_t$  and the condition for the lower support of the distribution,  $F(\underline{p}_t) = 0$ , I generate the following reservation price,  $\tilde{p}_t = W_t + [1/(1 - q)] W_t/R'_t$ . In the text, I also report the transacted price,  $P(\tilde{p}_t)$ . The reservation and transacted price have a very simple relationship. To see why, note,  $P(\tilde{p}_t) = \int_{\underline{p}_t}^{\tilde{p}_t} p_t f^T(p_t) dp_t$ , where  $f^T(p_t) \equiv f(p_t) d(p_t)$  is the transacted price density, and  $d(p_t) = q + 2(1 - q)[1 - F(p_t)]$  is the demand curve per-shopper.

### A.2. Monetary Policy Problem with Product Search

In the home economy, the monetary authority maximizes  $\sum_{t=0}^{\infty} \beta^t [\ln u(c_t) - \kappa(l_t + s_t)]$ . Using the definitions,  $\rho_t = \tilde{p}_{f,t}/\tilde{p}_{h,t}$  and  $\rho_t^* = \tilde{p}_{f,t}^*/\tilde{p}_{h,t}^*$ , resources conditions can be written as,

$$l_t = (1 - \alpha) [g(\rho_t)]^\xi c_t + \alpha (\rho_t^*)^\xi c_t^* ; s_t = \left[ (1 - \alpha) c_t + \alpha \rho_t^{-\xi} \right] [g(\rho_t)]^\xi c_t \quad (23)$$

In both constraints I eliminate home consumption for foreign consumption (which is exogenous) using the risk-sharing condition. The final step involves eliminating the variable  $\rho_t^* = \rho_t (\psi_{f,t}/\psi_{h,t})$ . From the perspective of the home policy maker  $\psi_{f,t}$  is exogenous as it is only a function of the shock and the rest of world nominal interest rate, which is set at unity. Note that the variable  $\rho_t$  and the law of one price gap are not independent choice variables. In particular, I write  $\psi_{h,t} = 1 + \phi_t \rho_t$ , where  $\phi_t \equiv (1 - \kappa_t) / [(1 - z) + \kappa_t]$ . Using this relation, I re-express the output-resource constraint as,

$$l_t = (1 - \alpha) [g(\rho_t)]^\xi c_t + \alpha \left( \frac{\rho_t}{1 + \phi_t \rho_t} \psi_{f,t} \right)^\xi c_t^* \quad (24)$$

Finally, to derive the expression presented in the text, note  $g'(\cdot) = \alpha [g(\rho_t) / \rho_t]^\xi$  and  $q'(\cdot) = \psi_{f,t} (1 - \alpha) [g(\rho_t)]^{\xi-2}$ .

### A.3. Derivations for Calculation of Output Movements

I first linearize the resource equations around  $R > 1$ . For output,

$$\hat{y}_t = (\xi - 1) \hat{g}_t + \frac{(\xi - 1) \alpha}{\alpha + (1 - \alpha) q^{1-\xi}} \hat{q}_t - \frac{\xi \alpha}{\alpha + (1 - \alpha) q^{1-\xi}} \hat{\psi}_{h,t} - \left( \frac{R}{R + \frac{1}{1-z}} \right) \hat{R}_t - \hat{\kappa}_t \quad (25)$$

and shopping,

$$\hat{s}_t = (\xi - 1) \hat{g}_t - \frac{\xi \alpha}{\alpha + (1 - \alpha) \rho^\xi} \hat{\rho}_t - \left( \frac{R}{R + \frac{1}{1-z}} \right) \hat{R}_t - \hat{\kappa}_t \quad (26)$$

which is only relevant for policy decisions when  $\xi \neq 1$ . International relative prices are linked by,  $\hat{g}_t = \alpha (\rho/g)^{1-\xi} \hat{\rho}_t$  and  $\hat{q}_t = \hat{\psi}_{f,t} + (1 - \alpha) (\rho/\tilde{q})^{\xi-1} \hat{\rho}_t$ . When  $R = 1$ , then  $\rho = g = \tilde{q} = 1$ , and we can solve for the the output response to the shock easily as,

$$\hat{y}_t = (\xi - 1) \alpha \left[ (2 - \alpha) \hat{\rho}_t + \hat{\psi}_{f,t} \right] - \xi \alpha \hat{\psi}_{h,t} - \hat{\kappa}_t \quad (27)$$

where  $\rho_t = -\hat{\kappa}_t / (1 + \frac{1}{1-z})$  and  $\psi_{h,t} = \hat{\psi}_{f,t} = (\frac{1}{1-z}) \rho_t$ . I determine  $R > 1$  under optimal policy by solving (25), (26) and (20) jointly as a non-linear system. To determine the response of output to the shock, I linearize (20) around its long-run level.

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**Table 1: Output Movements and Interest Rate Policies**

	Trade Elasticity: $\xi$					
	1		1.2		1.8	
	Peg	Optimal	Peg	Optimal	Peg	Optimal
Product Market						
Walrasian	1	1	1.04	1.02	1.15	1.11
Consumer Search	0.95	1	0.96	1.01	1.01	1.08