

Time-Varying Risk of Disaster, Time-Varying Risk Premia, and Macroeconomic Dynamics

François Gourio*

February 2009

Preliminary and Incomplete

Abstract

In order to develop a model that fits both business cycles and asset pricing facts, this paper introduces a small, time-varying risk of economic disaster in an otherwise standard real business cycle model. This simple feature can generate large and volatile risk premia. Under some conditions, the risk of disaster does not affect the path of macroeconomic aggregates, but in general it does: there is no “separation theorem” between quantities and prices (unlike Tallarini (2000)). An increase in the perceived probability of disaster can lead to a collapse of investment and a recession, with no current or future change in productivity. This model thus allows to analyze the effect of “time-varying beliefs” or “time-varying risk aversion” on the macroeconomy. Interestingly, this model is (at least qualitatively) consistent with the well-known facts that the stock market, the yield curve, and the short rate predict GDP growth, facts which are difficult to replicate in a standard model.

Keywords: business cycles, equity premium, term premium, return predictability, disasters, rare events, jumps.

JEL code: E32, E44, G12

1 Introduction

The empirical finance literature has provided substantial evidence that risk premia are time-varying (e.g. Campbell and Shiller (1988), Fama and French (1989), Ferson and Harvey (1991), Cochrane (2005)). Yet, standard business cycle models such as the real business cycle model, or the DSGE models used for monetary policy analysis, largely fail to replicate the level and the cyclicity of risk premia. This seems an important neglect, since empirical work suggests a tight connection between risk premia and economic activity. For instance, Philippon (2008) and Gilchrist and Zakrajsek (2007) show that corporate bonds spreads are highly correlated with real physical investment, both in the time series and

*Boston University, Department of Economics, and visiting the Wharton School, U of Pennsylvania. Email: fgourio@bu.edu. Phone: (617) 792 2066. I thank Andrew Abel, Xavier Gabaix, and Joao Gomes for helpful discussions.

in the cross-section. A large research, summarized in Backus, Routledge and Zin (2008), shows that the stock market, the term premium, and (negatively) the short rate all lead the cycle.¹

I introduce time-varying risk premia in a standard business cycle model, through a small, stochastically time-varying risk of a “disaster”, following the work of Rietz (1988), Barro (2006), and Gabaix (2007). Existing work has so far confined itself to endowment economies, and hence does not consider the feedback from time-varying risk premia to macroeconomic activity. I show that an increase in the perceived probability of disaster can create a collapse of investment and a recession, as risk premia rise and increase the cost of capital. These business cycle dynamics occur with no change in total factor productivity. Under some conditions the increase in probability of disaster is observationally equivalent to a preference shock, which is interesting since these shocks appear to be important in accounting for the data (e.g., Smets and Wouters (2003)). The simple model is also, at least qualitatively, consistent with the lead-lag relationships between asset prices and the macroeconomy mentioned above.

This risk of an economic disaster could be a strictly rational expectation, or more generally it could reflect a time-varying belief which may differ from the objective probability (i.e. the “perceived probability” of disaster is what matters). During the recent financial crisis, many commentators have highlighted the possibility that the U.S. economy could fall into another Great Depression. My model studies the macroeconomic effect of such time-varying beliefs. Of course in reality this change in probability of disaster may be an endogenous variable and not an exogenous shock. But it is useful to understand the effect of an increase in aggregate risk premia on the macroeconomy. This simple modeling device captures the idea that aggregate uncertainty is sometimes high, i.e. people sometimes worry about the possibility of a deep recession. It also captures the idea that there are some asset price changes which are not obviously related to current or future TFP, i.e. “bubbles” and “crashes”, and which in turn affect the macroeconomy.

Introducing time-varying risk premia requires solving a model using nonlinear methods, i.e. going beyond the first-order approximation and considering “higher order terms”. Researchers disagree on the importance of these higher order terms, and a fairly common view is that they are irrelevant for macroeconomic quantities (e.g. Tallarini (2000), Campanale et al. (2007)). Lucas (2003) summarizes: “Tallarini uses preferences of the Epstein-Zin type, with an intertemporal substitution elasticity of one, to construct a real business cycle model of the U.S. economy. He finds an astonishing separation of quantity and asset price determination: The behavior of aggregate quantities depends hardly at all on attitudes toward risk, so the coefficient of risk aversion is left free to account for the equity premium perfectly.”² My results show, however, that these higher-order terms can have a significant effect on macroeconomic dynamics, when we consider shocks to the probability of disaster.³

¹Schwert (1989) and Bloom (2008) also show that stock market volatility negatively leads economic activity.

²Note that Tallarini (2000) actually picks the risk aversion coefficient to match the Sharpe ratio of equity, but return volatility is very low in his model since there are no capital adjustment costs.

³Cochrane (2005, p. 296-297) also discusses in detail the Tallarini (2000) result: “Tallarini explores a different possibility, one that I think we should keep in mind; that maybe the divorce between real business cycle macroeconomics and finance isn’t that short-sighted after all (at least leaving out welfare questions, in which case models with identical dynamics can make wildly different predictions). (...) The Epstein-Zin preferences allow him to raise risk aversion while keeping intertemporal substitution constant. As he does so, he is better able to account for the market price of risk (...) but the

The paper is organized as follows: the rest of the introduction reviews the literature. Section 2 studies a simple analytical example in an AK model which can be solved in closed form. Section 3 gives the setup of the full model and presents some analytical results. Section 4 studies the quantitative implications of the model numerically.

Literature Review

This paper is mostly related to three strands of literature. First, a large literature in finance builds and estimates models which attempt to match not only the equity premium and the risk-free rate, but also the predictability of returns and potentially the term structure. Two prominent examples are Bansal and Yaron (2004) and Campbell and Cochrane (1999). However, this literature is limited to endowment economies, and hence is of limited use to analyze the business cycle or to study policy questions.

Second, a smaller literature studies business cycle models (i.e. they endogenize consumption, investment and output), and attempts to match not only business cycle statistics but also asset returns first and second moments. My project is closely related to these papers (A non-exhaustive list would include Jermann (1998), Tallarini (2000), Boldrin, Christiano and Fisher (2001), Lettau and Uhlig (2001), Kaltenbrunner and Lochstoer (2008), Campante et al. (2008), Croce (2005), Gourio (2008c), Papanikolaou (2008), Kuehn (2008), Uhlig (2006), Jaccard (2008), Fernandez-Villaverde et al. (2008)). Most of these papers consider only the implications of productivity shocks, and generally study only the mean and standard deviations of return, and not the predictability of returns. Many of these papers abstract from hours variation. Several of these papers note that quantities dynamics are unaffected by risk aversion,⁴ hence it is sometimes said that matching asset pricing facts need not affect the business cycle implications of the model. Recently some authors have also tried to generate time-varying risk premia in monetary models (e.g. Swanson and Rudebusch (2008a and 2008b)). The long-run target is to have a medium-scale DSGE model (as in Smets and Wouters (2003) or Christiano, Eichenbaum and Evans (2005)) that is roughly consistent with asset prices.

Finally, the paper draws from the recent literature on “disasters” or rare events (Rietz (1988), Barro (2006), Barro and Ursua (2008), Gabaix (2007), Farhi and Gabaix (2008), Martin (2007), Gourio (2008a and 2008b), Julliard and Ghosh (2008), Santa Clara and Yan (2008), Wachter (2008), Weitzmann (2007)). Disasters are a powerful way to generate large risk premia. Moreover, as we will see, disasters are relatively easy to embed into a standard macroeconomic model.

The project will also relate its findings to the empirical finance literature discussed above linking risk premia and the business cycle. There has been much interest lately in the evidence that the stock market leads TFP and GDP, which has motivated introducing “news shocks” (e.g., Beaudry and Portier (2006), Jaimovich and Rebelo (2008)), but my model suggests that this same evidence could also be rationalized by shocks to risk premia (i.e. shocks to the probability of disasters).

Last, the paper has the same flavor as Bloom (2008) in that an increase in aggregate uncertainty creates a recession, but the mechanism and the focus of the paper - asset prices- is different.

quantity dynamics remain almost unchanged. In Tallarini’s world, macroeconomists might well not have noticed the need for large risk aversion.”

⁴Fernandez-Villaverde et al. (2008) use perturbation methods and report that the first three terms are independent of risk aversion (there is, of course, a steady-state adjustment).

2 A simple analytical example in an AK economy

To highlight a key mechanism of the paper, consider a simple economy with a representative consumer who has Epstein-Zin preferences, i.e. his utility V_t satisfies the recursion:

$$V_t = \left((1 - \beta) C_t^{1-\gamma} + \beta E_t (V_{t+1}^{1-\theta})^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}}, \quad (1)$$

where C_t is consumption; note that θ measures risk aversion towards static gambles, γ is the inverse of the intertemporal elasticity of substitution (IES) and β reflects time preference.⁵ This consumer has access to an AK technology:

$$Y_t = A_t K_t,$$

where Y_t is output, K_t is capital, and A_t is a stochastic technology which is assumed to follow a stationary Markov process with transition Q . The resource constraint is:

$$C_t + I_t \leq A_t K_t.$$

The economy is randomly hit by disasters. A disaster destroys a share b_k of the capital stock. This could be due to a war which physically destroys capital, to expropriation of capital holders (e.g. if the capital is taken away and then not used as effectively), or it could be a ‘‘technological revolution’’ that makes a large share of the capital worthless. The law of accumulation for capital is thus:

$$\begin{aligned} K_{t+1} &= (1 - \delta)K_t + I_t, \text{ if } x_{t+1} = 0, \\ &= ((1 - \delta)K_t + I_t)(1 - b_k), \text{ if } x_{t+1} = 1, \end{aligned}$$

where x_{t+1} is a binomial variable which is 1 with probability p_t and 0 with probability $1 - p_t$. The probability of disaster p_t is assumed to vary over time, but to maintain tractability I assume in this section that it is *i.i.d.*: p_t , the probability of a disaster at time $t + 1$, is drawn at time t from a constant cumulative distribution function F . A disaster does not affect productivity A_t .⁶ Finally, I assume that p_{t+1} , A_{t+1} , and x_{t+1} are independent.

This model has one endogenous state K and two exogenous states A and p , and there is one control variable C . The Bellman equation for the representative consumer is:

$$\begin{aligned} V(K, A, p) &= \max_{C, I} \left\{ (1 - \beta) C^{1-\gamma} + \beta \left(E_{p', x', A'} V(K', A', p')^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \right\}^{\frac{1}{1-\gamma}} \\ \text{s.t.} & : \\ C + I &\leq AK, \\ K' &= ((1 - \delta)K + I)(1 - x' b_k). \end{aligned}$$

Define $W(K, A, p) = V(K, A, p)^{1-\gamma}$. Then we can guess and verify that W is of the form $W(K, A, p) = K^{1-\gamma} g(A, p)$, with

$$g(A, p) = \max_i \left\{ \begin{aligned} &(1 - \beta)(A - i)^{1-\gamma} \\ &+ \beta(1 - \delta + i)^{1-\gamma} (1 - p + p(1 - b_k)^{1-\theta})^{\frac{1-\gamma}{1-\theta}} \left(E_{p', A'} g(A', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{aligned} \right\},$$

⁵ As explained in Gourio (2008b) and Wachter (2008), the disaster model with standard power utility has counterfactual implications, which are resolved with Epstein-Zin utility.

⁶ In an AK model, a permanent reduction in productivity would lead to a permanent reduction in the growth rate of the economy, since permanent shocks to A affect the *growth rate* permanently.

where $i = \frac{I}{K}$ is the investment rate.⁷ The first-order condition with respect to i yields, after rearranging:

$$\left(\frac{A - i}{1 - \delta + i} \right)^{-\gamma} = \frac{\beta}{1 - \beta} (1 - p + p(1 - b_k)^{1-\theta})^{\frac{1-\gamma}{1-\theta}} \left(E_{p', A'} g(A', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}}.$$

Given the assumption that p is *i.i.d.*, the expectation of g next period is independent of the current p . Hence, assuming that risk aversion $\theta \geq 1$, i is increasing in p if and only $\gamma > 1$ i.e. the intertemporal elasticity of substitution is less than unity. The intuition for this result is as follows:⁸ if p goes up, the expected risk-adjusted return on capital $(1 - p + p(1 - b_k)^{1-\theta})^{\frac{1-\gamma}{1-\theta}}$ goes down since there is a higher risk of disaster. However, the effect of a change in the expected return on the consumption/savings choice depends on the value of the IES, because of offsetting wealth and substitution effects. If the IES is unity (i.e. utility is log), savings are unchanged and thus the savings or investment rate does not respond to a change in the probability of disaster. But if the IES is larger than unity, the substitution effect dominates, and i is decreasing in p (under the maintained assumption that $\theta \geq 1$). Hence, an increase in the probability of disaster leads initially, in this model, to a decrease in investment and an increase in consumption (since output is unchanged on impact). In the subsequent periods, the decrease in investment leads to a decrease in the capital stock and hence in output. This simple analytical example thus shows that a change in the perceived probability of disaster can lead to a decline in investment and output. While the preceding example is revealing,⁹ a serious examination of the role of beliefs regarding disasters requires a quantitative model.

3 Quantitative model: a RBC model with disasters

This section introduces a real business cycle model with time-varying risk of disaster and study its implications, first analytically, and then numerically. This model extends the simple example of the previous section in the following dimensions: (a) the probability of disaster is not *i.i.d.* but can be persistent; (b) the production function is neoclassical; (c) labor is elastically supplied; (d) disasters may affect total factor productivity as well as capital; (e) there are capital adjustment costs.

3.1 Model Setup

The representative consumer has preferences of the Epstein-Zin form, and the utility index incorporates hours worked N_t as well as consumption C_t :

$$V_t = \left((1 - \beta)u(C_t, N_t)^{1-\gamma} + \beta E_t (V_{t+1}^{1-\theta})^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}}, \quad (2)$$

where the per period felicity function $u(C, N)$ is assumed to have the following form:¹⁰

$$u(C, N) = C^v (1 - N)^{1-v}.$$

⁷Note that if $\gamma > 1$ the max needs to be transformed into a min.

⁸This intuition is similar to that spelled out in Weil (1989) in a consumption-savings example with exogenous returns.

⁹This example is related to work by Epaulard and Pommeret (2003), Jones, Manuelli and Siu (2005a, 2005b), and to the earlier work of Obstfeld (1994).

¹⁰Future work will consider different specifications, e.g. with a higher elasticity of labor supply.

There is a representative firm, which produces output using a standard Cobb-Douglas production function:

$$Y_t = K_t^\alpha (z_t N_t)^{1-\alpha},$$

where z_t is total factor productivity (TFP), to be described below. The firm accumulates capital subject to adjustment costs:

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right) K_t, \text{ if } x_{t+1} = 0, \\ &= \left((1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right) K_t\right) (1-b_k), \text{ if } x_{t+1} = 1, \end{aligned}$$

where ϕ is a concave function, which curvature captures adjustment costs, and x_{t+1} is 1 if there is a disaster at time $t+1$ (with probability p_t) and 0 otherwise (probability $1-p_t$). The resource constraint is

$$C_t + I_t \leq Y_t.$$

Finally, we describe the shock processes. Total factor productivity is affected by the “business cycles” normal shocks ε_t as well as jumps (disasters). A disaster reduces TFP by a permanent amount b_z :

$$\begin{aligned} \log z_{t+1} &= \log z_t + \mu + \sigma \varepsilon_{t+1}, \text{ if } x_{t+1} = 0, \\ &= \log z_t + \mu + \sigma \varepsilon_{t+1} + \log(1-b_z), \text{ if } x_{t+1} = 1, \end{aligned}$$

where μ is the drift of TFP, and σ is the standard deviation of small “business cycles” shocks. Moreover, p_t follows a stationary Markov process with transition Q . I assume that p_{t+1} , ε_{t+1} , and x_{t+1} are independent conditional on p_t . This assumption requires that the occurrence of a disaster today does not affect the probability of a disaster tomorrow.¹¹

3.2 Bellman Equation

This model has three states: capital K , technology z and probability of disaster p , and two independent controls: consumption C and hours worked N . Denote $V(K, z, p)$ the value function, and define $W(K, z, p) = V(K, z, p)^{1-\gamma}$. The social planning problem can be formulated as:¹²

$$\begin{aligned} W(K, z, p) &= \max_{C, I, N} \left\{ (1-\beta) (C^v (1-N)^{1-v})^{1-\gamma} + \beta \left(E_{p', z', x'} W(K', z', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \right\}, \quad (3) \\ \text{s.t.} \quad & \\ C + I &\leq z^{1-\alpha} K^\alpha N^{1-\alpha}, \\ K' &= \left((1-\delta)K + \phi\left(\frac{I}{K}\right) K \right) (1-x' b_k), \\ \log z' &= \log z + \mu + \sigma \varepsilon' + x' \log(1-b_z). \end{aligned}$$

¹¹This assumption could be wrong either way: a disaster today may indicate that the economy is entering a phase of low growth or is less resilient than thought; but on the other hand, if a disaster occurred today, and capital or TFP fell by a large amount, it is unlikely that they will fall again by a large amount next year. Rather, historical evidence suggests that the economy is likely to grow above trend for a while (Gourio (2008)). Future work will study the implications of relaxing these independence assumptions.

¹²Here too, if $\gamma > 1$ the max needs to be transformed into a min.

A standard homogeneity argument implies that we can write $W(K, z, p) = z^{v(1-\gamma)}g(k, p)$, where $k = K/z$, and g satisfies the associated Bellman equation:

$$g(k, p) = \max_{c, i, N} \left\{ \begin{array}{l} (1 - \beta)c^{v(1-\gamma)}(1 - N)^{(1-v)(1-\gamma)} \\ + \beta e^{\mu v(1-\gamma)} \left(E_{p', \varepsilon', x'} e^{\sigma \varepsilon' v(1-\theta)} (1 - x' + x'(1 - b_{tfp})^{v(1-\theta)}) g(k', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\} (4)$$

s.t. :

$$c = k^\alpha N^{1-\alpha} - i,$$

$$k' = \frac{(1 - x'b_k) \left((1 - \delta)k + \phi \left(\frac{i}{k} \right) k \right)}{e^{\mu + \sigma \varepsilon'} (1 - x'b_{tfp})}.$$

Here $c = C/z$ and $i = I/z$ are detrended consumption and investment, respectively. This Bellman equation will lead to some analytical results, and can further be studied using standard numerical methods.

3.3 Asset Prices

It is straightforward to compute asset prices in this economy. The stochastic discount factor is given by the formula

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{v(1-\gamma)-1} \left(\frac{1 - N_{t+1}}{1 - N_t} \right)^{(1-v)(1-\gamma)} \left(\frac{V_{t+1}}{E_t (V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}} \right)^{\gamma-\theta}. \quad (5)$$

The price of a purely risk-free asset is

$$P_{rf,t} = E_t (M_{t,t+1}) \stackrel{def}{=} P_{rf}(k, p).$$

Following Barro (2006), I will assume that government bonds are not risk-free but are subject to default risk during disasters.¹³ More precisely, if there is a disaster, then with probability q the bonds will default and the recovery rate will be r . The T-Bill price can then be easily computed as

$$P_{1,t} = E_t (M_{t,t+1} (1 - x_{t+1}q(1 - r))) \stackrel{def}{=} P_1(k, p).$$

Computing the yield curve is conceptually easy using the standard recursion for zero-coupon bonds:¹⁴

$$P_{n,t} = E_t (M_{t,t+1} P_{n-1,t+1} (1 - x_{t+1}q(1 - r))) \stackrel{def}{=} P_n(k, p).$$

The ex-dividend value of the firm F_t is defined through the value recursion:

$$F_t = E_t (M_{t,t+1} (D_{t+1} + F_{t+1})),$$

where $D_t = F(K_t, z_t N_t) - w_t N_t - I_t$ is the payout of the representative firm, and w_t is the wage rate. This value satisfies the q -theoretic relation:

$$F_t = \frac{(1 - pb_k)K_{t+1}}{\phi' \left(\frac{I_t}{K_t} \right)}, \quad (6)$$

¹³Empirically, default often takes the form of high rates of inflation which reduces the real value of nominal government debt.

¹⁴Note the implicit assumption that a disaster simply reduces the face value of the bond (and does not affect its maturity).

so that if we define Tobin's q as $Q_t = \frac{F_t}{(1-pb_k)K_{t+1}}$, we have $Q_t = \phi' \left(\frac{I_t}{K_t} \right)$, and Q_t is one in the limiting case of no adjustment costs. In the standard model, $p = 0$, but here the amount of capital available tomorrow is unknown, since some capital may be destroyed if there is a disaster. Finally, the equity return is obtained as

$$R_{t,t+1} = \frac{D_{t+1} + F_{t+1}}{F_t}.$$

Using equation (6), we can find an equivalent expression for the equity return, the investment return:

$$\begin{aligned} R_{t,t+1} &= \frac{F_{t+1} + D_{t+1}}{F_t} = \frac{\frac{(1-pb_k)K_{t+2}}{\phi' \left(\frac{I_{t+1}}{K_{t+1}} \right)} + D_{t+1}}{\frac{(1-pb_k)K_{t+1}}{\phi' \left(\frac{I_t}{K_t} \right)}} \\ &= \phi' \left(\frac{I_t}{K_t} \right) \left[\frac{1 - \delta + \phi \left(\frac{I_{t+1}}{K_{t+1}} \right)}{\phi' \left(\frac{I_{t+1}}{K_{t+1}} \right)} (1 - x_{t+1}b_k) + \frac{\alpha K_{t+1}^\alpha N_{t+1}^{1-\alpha} - I_{t+1}}{K_{t+1}} \right]. \end{aligned}$$

This expression is similar to that in Jermann (1998) or Kaltenbrunner and Lochstoer (2008), but for the presence of the term $(1 - x_{t+1}b_k)$, which reflects the capital destruction following a disaster. Finally, I will also compute the price of two additional assets, a levered claim to consumption, defined by its payoff C_t^λ , where λ is a leverage parameter, and a levered claim to dividends, defined by its payoff $D_t^{\lambda_2}$.^{15,16}

3.4 Analytical results: sufficient conditions for a separation theorem

Before turning to the quantitative analysis, it is useful to point out two simple analytical results which follow directly from the Bellman equation above:

Proposition 1 *Assume that the probability of disaster p is constant, and that $b_k = b_{tfp}$ i.e. TFP and capital fall by the same amount if there is a disaster. Then, in a sample without disaster, the quantities implied by the model (consumption, investment, hours, output and capital) are the same as those implied by the model with no disasters ($p = 0$), but a different time discount factor $\beta^* = \beta(1 - p + p(1 - b_k)^{v(1-\theta)})^{\frac{1-\gamma}{1-\theta}}$. Moreover, assuming $\theta > 1$, $\beta^* < \beta$ if and only if $\gamma < 1$. Asset prices, however, will be different under the two models.*

Proof. Notice that if $b_k = b_{tfp}$, then $k' = \frac{((1-\delta)k + \phi(\frac{i}{k})k)}{e^{\mu + \sigma\varepsilon'}}$ is independent of the realization of disaster x' . Hence, we can rewrite the Bellman equation as

$$g(k) = \max_{c,i,N} \left\{ \begin{array}{l} (1 - \beta)c^{v(1-\gamma)}(1 - N)^{(1-v)(1-\gamma)} \\ + \beta e^{\mu v(1-\gamma)} \left(E_{x'} (1 - x' + x'(1 - b_{tfp})^{v(1-\theta)}) E_{\varepsilon'} e^{\sigma\varepsilon'v(1-\theta)} g(k')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\},$$

i.e.:

$$g(k) = \max_{c,i,N} \left\{ (1 - \beta)c^{v(1-\gamma)}(1 - N)^{(1-v)(1-\gamma)} + \beta^* e^{\mu v(1-\gamma)} \left(E_{\varepsilon'} e^{\sigma\varepsilon'v(1-\theta)} g(k')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \right\}.$$

¹⁵Note that for this last claim, since net payout may be negative, the leverage parameter λ_2 needs to be an odd integer; moreover, the interpretation in terms of leverage is not fully satisfactory

¹⁶Add financial leverage.

Finally note that the term in $1 - \beta$ is purely a utility normalization, which can be taken out. Hence, we see that this is the same Bellman equation as the one in a standard neoclassical model, with discount rate β^* . As a result, the policy functions $c = C/z$, $i = I/z$, etc. are the same, so the implied quantities are the same, as long as no disaster occurs.¹⁷ Asset prices, on the other hand, are driven by the stochastic discount factor, which has the following expression (see the computational appendix):

$$M(k, k', \varepsilon', x') = \beta \left(\frac{z'}{z} \right)^{(\gamma-\theta)v+v(1-\gamma)-1} \left(\frac{c(k')}{c(k)} \right)^{v(1-\gamma)-1} \left(\frac{1-N(k')}{1-N(k)} \right)^{(1-v)(1-\gamma)} \times \dots$$

$$\left(\frac{g(k')^{\frac{1}{1-\gamma}}}{E_{z',x'} \left(\left(\frac{z'}{z} \right)^{v(1-\theta)} g(k')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1}{1-\theta}}} \right)^{\gamma-\theta},$$

and of course the term z'/z depends on the realization of a disaster x' . ■

Discussion of Result 1: This result is in the spirit of Tallarini (2000): fixing the asset pricing properties of a RBC model may not lead to any change in the quantity dynamics.¹⁸ An economy with a high equity risk premium ($p > 0$) is observationally equivalent to the standard stochastic growth model ($p = 0$), with a different β . The assumption that $b_k = b_{tfp}$ simplifies the analysis: the steady-state of the economy shifts due to a change in z , but the ratio of capital to productivity is unaffected by the disaster, i.e. the economy is in the same position relative to its steady-state after the disaster and before the disaster. As a result, if we start in the steady-state of the economy, a disaster will simply reduce investment, output, and consumption by a factor b , and hours will stay constant.

Without the adjustment of β , the quantity implications are very slightly different. This is illustrated in the top panel of Figure 3 which depicts the impulse response of quantities to a TFP shock in three models: (a) the model with $p = 0$, (b) the model with constant positive p , and (c) the benchmark calibration with time-varying p . The differences can be seen in the scale (y-axis), but they are tiny. For this case, $\beta^* \approx .9893$, which is very close to $\beta = .99$. Of course, asset prices will be different, and in particular the equity premium will be higher, as seen in the bottom panel of Figure 1 - the average returns are very different across the three models. The observational equivalence is broken in a long enough sample since disasters must occur; or if one can trade assets contingent on disasters, since the prices would be different under the two models.

Note that this same result implies that the steady-state level of capital stock (i.e., if the variance of ε is zero) will be changed, too. If risk aversion θ is greater than unity, then $\beta^* < \beta$ if and only if $\gamma < 1$, i.e. an IES above unity. In this case, the lower risk-adjusted return leads people to save less, which decreases the steady-state capital stock.

While this result is interesting, it is not fully satisfactory however, since the constant probability of disaster implies (nearly) constant risk premia. This motivates extending the result for a time-varying p .

Proposition 2 *Assume still that $b_k = b_{tfp}$, but let now p vary over time. Then, in a sample without disaster, the quantities implied by the model are the same as those implied by a model with no disasters, but with stochastic discounting (i.e. β follows an exogenous stochastic process).*

¹⁷Even after a disaster, the policy functions are the same. However, the destruction of capital in a disaster is not possible in the model with $p = 0$ (the TFP decline is highly unlikely, it is possible).

¹⁸Gabaix (2009) proves some results with a similar flavor.

Proof. This follows from a similar argument: rewrite the Bellman equation as:

$$g(k, p) = \max_{c, i, N} \left\{ \begin{array}{c} (1 - \beta)c^{v(1-\gamma)}(1 - N)^{(1-v)(1-\gamma)} \\ + \beta e^{\mu v(1-\gamma)} \left(E_{x'} (1 - x' + x'(1 - b_{tfp})^{v(1-\theta)}) E_{\varepsilon', p'} e^{\sigma \varepsilon' v(1-\theta)} g(k', p')^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\},$$

then define $\beta(p) = \beta E_{x'} (1 - x' + x'(1 - b_{tfp})^{v(1-\theta)})$, we have:

$$g(k, p) = \max_{c, i, N} \left\{ (1 - \beta)c^{v(1-\gamma)}(1 - N)^{(1-v)(1-\gamma)} + \beta(p)e^{\mu v(1-\gamma)} \left(E_{\varepsilon', p'} e^{\sigma \varepsilon' v(1-\theta)} g(k', p')^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1-\gamma}{1-\theta}} \right\},$$

i.e. the Bellman equation of a model with time-varying p . ■

Discussion of result 2: Result 2 shows that the time-varying risk of disaster has the same implications for quantities as a preference shock. It is well known that these shocks have significant effect on macroeconomic quantities (a point we will quantify later). Hence, this version of the model breaks the “separation theorem” of Tallarini (2000): the source of time-varying risk premia in the model will affect quantity dynamics.

This result is interesting in light of the empirical literature which suggests that “preference shocks” may be important (e.g., Smets and Wouters (2003)). Chari, Kehoe and McGrattan (2008) complain that these shocks lack microfoundations. My model provides an (admittedly simple) microfoundation, which allows to tie these shocks to asset prices. Of course, my model is much “smaller” than the medium-scale models of Smets and Wouters (2003), or Christiano, Eichenbaum and Evans (2005), but I conjecture that this equivalence should hold in larger versions.

3.5 Steady-state Effects when $b_k \neq b_{tfp}$

When $b_k \neq b_{tfp}$, the analytical results above do not hold. It is important to recognize that these disasters have different effect both in terms of the responses following a disaster, and in terms of their effect on steady-states (i.e. when no disaster is realized). The capital destruction acts like a reduction in the return on capital (risk-adjusted) so its effect on capital accumulation depends on the value of the IES. The TFP disaster, on the other hand, has a standard precautionary savings effect.

To understand the effect of disasters when $b_k \neq b_{tfp}$, it is useful to first consider the effect on the steady-state of capital, in a simple special case: expected utility ($\gamma = \theta$), inelastic labor, no adjustment costs, no shock ε , and a constant probability of disaster. In this case, the Bellman equation reads

$$g(k) = \max_{c, i} \left\{ \begin{array}{c} (1 - \beta)(k^\alpha - i)^{1-\sigma} \\ + \beta e^{\mu(1-\gamma)} \left((1 - p) g \left(\frac{(1-\delta)k+i}{e^\mu} \right) + p(1 - b_{tfp})^{1-\gamma} g \left(\frac{(1-\delta)k+i}{e^\mu} \left(\frac{1-b_k}{1-b_{tfp}} \right) \right) \right) \end{array} \right\}.$$

Taking the first-order condition with respect to i yields:

$$(1 - \sigma)(1 - \beta)(k^\alpha - i(k))^{-\sigma} = \beta e^{-\gamma\mu} \left(\begin{array}{c} (1 - p) g' \left(\frac{(1-\delta)k+i(k)}{e^\mu} \right) \\ + p(1 - b_{tfp})^{-\gamma} (1 - b_k) g' \left(\frac{(1-\delta)k+i(k)}{e^\mu} \left(\frac{1-b_k}{1-b_{tfp}} \right) \right) \end{array} \right),$$

where $i(k)$ is the optimal investment given the current k . The envelope condition is, for all $k > 0$:

$$g'(k) = (1 - \beta)(k^\alpha - i(k))^{-\sigma} \alpha k^{\alpha-1} (1 - \sigma) + \beta e^{-\gamma\mu} (1 - \delta) \left(\begin{array}{c} (1 - p) g' \left(\frac{(1-\delta)k + i(k)}{e^\mu} \right) \\ + p(1 - b_{tfp})^{-\gamma} (1 - b_k) g' \left(\frac{(1-\delta)k + i(k)}{e^\mu} \left(\frac{1 - b_k}{1 - b_{tfp}} \right) \right) \end{array} \right).$$

Combining the two equations yields:

$$g'(k) = (1 - \beta)(k^\alpha - i(k))^{-\sigma} (1 - \sigma) (\alpha k^{\alpha-1} + 1 - \delta).$$

Consider a long sample path where no disasters are realized. Then, the level of capital converges to the value \bar{k} , and i to \bar{i} which satisfy:

$$g'(\bar{k}) = (\alpha \bar{k}^{\alpha-1} + 1 - \delta) \beta e^{-\gamma\mu} \left\{ (1 - p) g'(\bar{k}) + p(1 - b_{tfp})^{-\gamma} (1 - b_k) g'(\eta \bar{k}) \right\},$$

or

$$1 = (\alpha \bar{k}^{\alpha-1} + 1 - \delta) \beta e^{-\gamma\mu} \left\{ (1 - p) + p(1 - b_{tfp})^{-\gamma} (1 - b_k) \frac{g'(\eta \bar{k})}{g'(\bar{k})} \right\}, \quad (7)$$

where $\eta = \frac{1 - b_k}{1 - b_{tfp}}$. Hence, if $\eta = 1$ we find a special case of Proposition 1: \bar{k} satisfies the adjusted user-cost rule:

$$1 = \beta^* e^{-\gamma\mu} \left(\alpha \bar{k}^{\alpha-1} + 1 - \delta \right).$$

Equation (7) shows the forces which determine the “steady-state” capital level \bar{k} . On the one hand, a high b_k reduces the return on capital. On the other hand, it increases the marginal value of a unit of capital in the disaster (i.e. $g'((1 - b_k)\bar{k}) > g'(\bar{k})$). Which effect dominates depends on the IES.

This is illustrated by Figures 4 5 which present the average (ergodic) capital stock as a function of b_k and b_{tfp} , for two elasticities of substitution. When the IES is high, an increase in b_k reduces the stock of capital. When the IES is low, changing b_k has essentially no effect.

The effect of a change in b_{tfp} is more standard: an increase in b_{tfp} leads to larger precautionary savings and an increase in the stock of capital. These comparative statics are useful, because when we explore the effect of a change in the probability of disaster, a central effect is that the economy tries to alter its capital stock from one steady-state to another (up or down, depending on the composition of disaster in b_k and in b_{tfp}), which determines the investment and output responses.

4 Quantitative Results

In general of course, the model cannot be solved analytically, so I resort to a numerical approximation. Of course, a nonlinear method is crucial to analyze time-varying risk premia. I use a simple value function iteration (or policy function iteration) algorithm, which is described in detail in an Appendix.

This section first presents the calibration, which is still preliminary. Next, I study the implications of the model for business cycle quantities and for the first and second moments of asset returns. Finally, I discuss the cyclicity of asset returns.

4.1 Calibration

Parameters are listed in Table 1. (This calibration and quantitative results are still preliminary and can likely be improved.) Many parameters are fairly standard (see e.g. Cooley and Prescott (1995)). Risk aversion is 8, but note that this is the risk aversion over the consumption-hours bundle. Since the share of consumption in the utility index is .3, the effective risk aversion is less than 3. Since hours does not vary much when there is a disaster, this utility index is about three times less volatile than consumption. The IES is set equal to 2, and adjustment costs are zero in the baseline model. One crucial element is the probability and size of disaster. As in Barro (2006), I assume that $b_k = b_{tfp} = .43$ and the probability is $p = .017$ per year on average. (Barro actually uses the historical distribution of sizes of disaster. In his model, this distribution is equivalent to a single disaster with size 43%.) The second crucial element is the persistence and volatility of movements in this probability of disaster. For now I simply assume that this change in probability of disaster is volatile and highly persistent: $\pi = .98$ and $\varepsilon = .015$. This is motivated by the results in Gourio (2008b) and Wachter (2008) who show that in an endowment economy, these parameters are necessary to match the stock market volatility. Finally, the leverage parameters λ and λ_2 are both set equal to 3.

On top of this benchmark computation, I will also present results from different calibrations (no disasters, constant probability of disasters, and different b_k and b_{tfp}) to illustrate how the model works.

4.2 Response to shocks

4.2.1 The dynamic effect of a disaster

Figures 1 and 2 present the dynamics of quantities and returns following a disaster, for each of the three main cases: the benchmark model ($b_k = b_{tfp} = .43$); a capital disaster ($b_k = .43$ and $b_{tfp} = 0$); and a TFP disaster ($b_{tfp} = .43$ and $b_k = 0$). Note that to generate a large equity premium, a model must endogenously generate that consumption and stock returns are extremely low during disasters. In the benchmark model, as implied by proposition 2, there are no transitional dynamics following the disaster: if the economy was close to the steady-state before the disaster, it will be close to a new steady-state with lower TFP and lower capital after the disaster. Hence, a disaster will simply reduce investment, output, and consumption by a factor b , and hours will stay constant.

The case of a capital disaster is interesting because it leads endogenously to a recovery. The return on capital is low on impact because of the destruction, but consumption does not fall that much given the anticipated recovery. Adding adjustment costs slows down the recovery, but makes the return on capital not as bad since marginal Q increases after the disaster.

Finally, a TFP disaster without a capital destruction leads to a situation where the economy has too much capital relative to its productivity. Investment falls to zero: the aggregate irreversibility constraint binds. Consumption and output fall over time.¹⁹ In that case, the initial low return on capital is solely due to the binding irreversibility constraint.

¹⁹For this last case, the investment return is not correct, because the FOC for investment does not hold.

4.2.2 The dynamic effect of a TFP shock

As illustrated in figure 3, the dynamics of quantities in response to a TFP shock are similar to those of a standard model without disasters. Consumption, investment and employment are procyclical, and investment is the most volatile series. The T-bill rate is procyclical, as is the equity return and Tobin's q . The equity premium is acyclical.²⁰ These dynamics are very similar for all the calibrations considered here. (The only difference is that large adjustment costs tend to make employment countercyclical, as noted in Boldrin, Christiano and Fisher (2001)).

4.2.3 The dynamic effect of a shock to the probability of disaster

We can now perform the key experiment of a downward “shock” to the probability of disaster, which leads to a decrease in risk premia. Figure 6 plots the (nonlinear) impulse response function to such a shock.²¹ Investment increases, and consumption falls, as in the analytical example of section 2, since the elasticity of substitution is assumed to be greater than unity. Employment increases too, through an intertemporal substitution effect: the risk-adjusted return to savings is high and thus working today is more attractive. (This is in spite of a positive wealth effect.) Hence, output increases because both employment and the capital stock increase, even though there is no change in current or future total factor productivity. This is one of the main result: this shock to the “perceived risk” leads to a boom. After impact, total resources available grow, and so does consumption. These results are robust to changes in parameter values, except of course for the IES which crucially determines the sign of the responses, and the composition of disaster.²² The size of adjustment costs affects the magnitude of the response of investment and hours.²³ The model predicts some negative comovement between C and I , which is reminiscent of Barro and King (1984), but the quantitative significance of this point depends on the labor supply specification.

Regarding asset prices, figure 7 reveals that following the shock, the risk premium on equity decreases, the yield curve becomes downward-sloping, and the short rate increases. Hence, in the model, a reduction in risk premia leads to an economic expansion. Of course, on impact there is a reduction in risk premia, so equity prices move up. The effect is moderate (or even tiny) for an unlevered equity, but can be huge for a levered equity.

²⁰This model generate some positive autocorrelation of consumption growth, hence the dynamics of consumption are qualitatively similar to those which are studied in Bansal and Yaron (2004). This could in principle generate larger risk premia, however, as argued by Kaltenbrunner and Lochstoer (2008), this effect is not quantitatively very important if shocks are permanent and the IES is not small.

²¹The figure plots the path implied by the model, starting in steady-state, if, at $t = 6$, the economy shifts from the low probability of disaster to the high probability of disaster. For clarity, there are no further shocks to the probability of disaster, no realized disaster, and no “normal shocks” ε .

²²If the disaster only affects TFP, then an increase in the probability of disaster leads to more savings and hence a boom - the opposite sign of what is shown here; and this appears to be true regardless of the IES. This is consistent with the comparative statics for the average level of capital discussed in section 2.5.

²³The model predicts some negative comovement between C and I , which is reminiscent of Barro and King (1984), but the quantitative significance of this point depends on the labor supply specification.

4.3 First and second moments of asset returns and quantities

Table 2 reports the standard business cycle moments obtained from model simulations for a sample without disasters. (Table 3 presents the same statistics in a full sample, i.e. a sample with disasters.) Overall, the standard business cycle moments are largely preserved in this model - consumption is less volatile than output, and investment is more volatile than output. The volatility of hours is low, a standard defect of the basic RBC model given this labor supply specification.

Introducing a constant probability of disaster does not change the moments significantly (consistent with the IRF shown in the previous section). However, the presence of the new shock - a change in the probability of disaster - leads to additional dynamics. Specifically, the correlation of consumption with output is reduced. Consumption and employment become somewhat more volatile. When the disaster only affect TFP, these effects are very strong.

Turning to returns, table 4 and table 5 show that the benchmark model (row 3) can generate a large equity premium: 60 basis points per quarter for unlevered equity; and 160 basis points (over 6% per year) for a levered equity. Note that these risk premium are obtained with a risk aversion over consumption which is less than 3. Moreover, these risk premia are computed over short-term government bonds, which are not riskless in the model. Of course, without disasters, the model generates very small equity premia. Finally, whether these risk premia are calculated in a sample with disasters or without disasters does not matter much quantitatively - the risk premia are reduced by 15-20 basis point per quarter (see table 5).

The model generates a slightly negative term premium, consistent with the evidence for indexed bonds in the US and UK. In this model, the yield curve is similar to that of a Vacijek model, i.e. long-term bond prices are more stable than short-term bond prices, and are mostly affected by shocks to the probability of disaster (a mean-reverting state variable). However, the model does not generate enough volatility in the term premium.

The model also does not generate enough volatility in unlevered equity returns (only 9 basis points in a sample without disasters). The intuition is that shocks to the probability of disaster affect the risk-free rate and the equity premium in roughly similar ways, so that equity prices and average returns are not much affected by it. The volatility is significantly higher in a sample with disasters (2.12% per quarter). These results are quite similar to Gourio (2008b) and Wachter (2008). Adding some financial or operating leverage, and possibly some wage rigidities may help here. Rather than incorporating in detail all these mechanisms, I consider the implications of the model for a claim to levered equity or levered consumption. In this case, we find that the volatility is of the right order of magnitude: 5.01% and 3.67% per quarter. Overall, I conclude that the model does a good job at fitting the first two moments of asset returns, if one allows for some leverage.

4.4 Asset prices lead the business cycle

While most of the research has focused on the equity premium and the stock market volatility puzzle, the cyclical nature of asset prices is also intriguing. Three facts stand out. First, the stock market return (or excess return) leads the business cycle. Second, the term spread leads the business cycle. Third,

the short rate negatively leads the business cycle. Figure 8 illustrates these facts by plotting the cross-correlogram of the growth rate of industrial production, with the short rate, the yield spread, the stock market return, and the stock market excess return. (Data sources are in appendix; the black lines show the two standard error bands.²⁴ Using employment, consumption or personal income as measures of economic activity rather than industrial production yields a similar picture.) These facts have been documented for a long time.²⁵ Indeed, the cyclical nature of interest rates is often taken to mean that monetary policy is creating, or reacting to, the business cycle.

Standard business cycle models are unable to replicate these facts however. I illustrate this by showing the correlogram implied by a model with no disasters (keeping the same calibration as in the benchmark model). Figure 9 shows that the model misses essentially all of these correlations, because risk premia are nearly constant.

An interesting feature of the model is its ability to improve along this dimension. Figure 10 shows the cross-correlogram implied by the benchmark model. For clarity, this is computed when there are no TFP shocks, so the only impulse here is the probability of disaster shock. (The true correlogram is, naturally, a mix of the correlogram for TFP shocks and for shocks to the probability of disaster.) The model replicates roughly the fact that the term spread, the stock market return and excess return, and the short rate lead output. The intuition is as follows: a shift from high to low probability of disaster leads to an immediate reduction in risk premia, but output is affected in part with a lag. Following the shock, GDP *growth* falls, while the term spread is inverted, and the stock market excess return is lower. The risk-free rate is higher due to lower precautionary savings. Hence, the model fits the facts because it generates some dynamic response of output to a shock to the probability of disaster.

5 Conclusions and Future Work

This work shows how introducing disasters into a standard RBC model both improves its fit of asset return data, and creates some interesting new macroeconomic dynamics. Clearly the model still fails quantitatively in some dimensions (e.g. the volatility of the term premium), which may be overcome by a richer calibration or allowing for some additional frictions. However, the results are attractive given the parsimony of the model.

There are several possible interesting extensions. First, it would be interesting to consider the effect of a time-varying risk of disaster in a richer business cycle model, e.g. one with collateral effects or choice of financial leverage, or a standard New Keynesian model. Second, one could also consider alternative modeling of the dynamics of disasters (e.g. persistence in low growth regimes, recoveries following disasters, and learning about the disaster state or about the disaster probability). Third, a change in the aggregate risk affects macroeconomic aggregates also by affecting the willingness to take on risk. This seems an interesting mechanism to explore: faced with an increase in the probability of an economic disaster, investors shift resources to technologies and projects which are less exposed to

²⁴These standard errors are computed using the Newey-West GMM formulas and the Delta method. See Cochrane (2004), p. 217.

²⁵See Backus, Routledge and Zin (2008), and the references therein.

disasters. In doing so, they move the economy alongside a risk/return frontier, and pick projects which are less risky but also have lower expected returns. As a result, the expected output of the economy falls, and so does productivity.

References

- [1] Angeletos, Mario, Philippe Aghion, Abhijit Banerjee and Kalina Manova, 2007. "Volatility and Growth: Credit Constraints and the Composition of Investment", mimeo MIT.
- [2] Backus, David, Bryan Routledge and Stanley Zin, 2008. "The cyclical component in U.S. asset returns", mimeo NYU.
- [3] Bansal, Ravi, and Amir Yaron, 2004. "Risks for the long run: A potential explanation of asset pricing puzzles." *Journal of Finance* 59, 1481–1509.
- [4] Barro, Robert, 2006. "Rare disasters and asset markets in the twentieth century." *Quarterly Journal of Economics* 121, 823–866.
- [5] Barro, Robert and Robert G. King, 1984. "Time-Separable Preferences and Intertemporal-Substitution Models of Business Cycles", *Quarterly Journal of Economics*, 99(4):817-839
- [6] Barro, Robert, and Jose Ursua, 2008. "Macroeconomic Crisis since 1870." *Brookings Paper on Economic Activity*.
- [7] Beaudry, Paul and Franck Portier, 2006. "Stock Prices, News, and Economic Fluctuations," *American Economic Review*, 96(4):1293-1307.
- [8] Bloom, Nicholas, 2008. "The effect of uncertainty shocks." *Forthcoming, Econometrica*.
- [9] Boldrin, Michele, Lawrence J. Christiano, and Jonas Fisher, 2001. "Habit Persistence, Asset Returns, and the Business Cycle." *American Economic Review*, 91(1), pp. 149-166.
- [10] Campanale, Claudio, Rui Castro, and Gian Luca Clementi. 2007. "Asset Pricing in a Production Economy with Chew-Dekel Preferences". *Mimeo, NYU*.
- [11] Campbell, John Y. "Asset Prices, Consumption, and the Business Cycle." *Handbook of Macroeconomics*, 1999, Volume 1, Chapter 19, pp. 1231-1303.
- [12] Campbell, John, and John Cochrane, 1999. *By force of habit: A consumption-based explanation of aggregate stock market behavior.* *Journal of Political Economy* 107, 205–251.
- [13] Campbell, John Y., and Robert Shiller. "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors." *Review of Financial Studies*, 1988, 1, pp. 195-227.
- [14] Campbell, John Y. and Motohiro Yogo, 2006. "Efficient Tests of Stock Return Predictability", *Journal of Financial Economics*, 81(1):27–60.
- [15] Chari, V. V. , Patrick J. Kehoe and Ellen R. McGrattan, 2009. "New Keynesian models: not yet useful for policy analysis," *forthcoming, AEJ: Macroeconomics*.
- [16] Christiano, Lawrence, Martin Eichenbaum, and Charles L. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, 113(1):1-45.

- [17] Cochrane, John, 2004. "Asset Pricing." Princeton University Press.
- [18] Cochrane, John, 2005. "Financial Markets and the Real Economy." *Foundations and Trends in Finance*, 1(1):1-101.
- [19] Cooley, Thomas and Edward Prescott, 1995. "Frontiers of Business Cycle Research". Princeton University Press.
- [20] Croce, Max. "Long-Run Productivity Risk: A New Hope For Production-Based Asset Pricing." Mimeo, UNC.
- [21] Epaulard, Anne, and Aude Pommeret, 2003. "Recursive utility, endogenous growth, and the welfare cost of volatility". *Review of Economic Dynamics*, 6(3):672-684.
- [22] Epstein, Larry G. and Zin, Stanley E. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica*, 1989, 57(4), pp. 937-969.
- [23] Fama, Eugene, and Kenneth French, 1989. "Business conditions and expected returns on stocks and bonds." *Journal of Financial Economics* 25:23-49.
- [24] Farhi, Emmanuel and Xavier Gabaix, 2008. "Rare disasters and exchange rates". Mimeo, Harvard.
- [25] Fernandez-Villaverde, Jesus, Ralph Koijen, Juan F. Rubio-Ramirez and Jules H. van Binsbergen, 2008. "Working with Epstein-Zin Preferences: Computation and Likelihood Estimation of DSGE Models with Recursive Preferences", Mimeo.
- [26] Ferson, Wayne E., and Campbell R. Harvey, 1991. "The Variation of Economic Risk Premiums", *Journal of Political Economy*, 99(2):385-415.
- [27] Gabaix, Xavier, 2007. "Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance." Mimeo, NYU.
- [28] Gabaix, Xavier, 2008. "Fixing the Asset Pricing Properties of RBC Models with Variable Rare Disasters." Mimeo, NYU.
- [29] Gilchrist, Simon, and Egon Zakrajsek, 2007. "Investment and the Cost of Capital: New Evidence from the Corporate Bond Market". Mimeo, BU.
- [30] Gourio, Francois. 2008. "Disasters and Recoveries." *American Economic Review - Papers and Proceedings*, 98(2): 68-73.
- [31] Gourio, Francois, 2008b. "Time Series Predictability in the Disaster Model." *Finance Research Letters*, 5(4): 191-203.
- [32] Gourio, Francois, 2008c. "Putty-clay technology and stock market volatility". Mimeo, BU.
- [33] Gourio, Francois, 2009. "Time-varying risk premia and Reallocation". Work in Progress.

- [34] Jaccard, Ivan, 2008. "Asset Pricing, Habit Memory, and the Labor Market " Mimeo, ECB.
- [35] Jermann, Urban J., 1998. "Asset Pricing in Production Economies." *Journal of Monetary Economics*, 41(2), pp. 257-275.
- [36] Julliard, Christian and Anita Gosh, 2008. "Can Rare Events Explain the Equity Premium Puzzle?", Mimeo LSE.
- [37] Kaltenbrunner, Georg, and Lars Lochstoer, 2007. "Long-Run Risk through Consumption Smoothing." Mimeo, Columbia.
- [38] Kehoe, Tim, and Edward Prescott, 2007. "Great Depressions of the Twentieth Century", Federal Reserve Bank of Minneapolis.
- [39] Kuehn, Lars, 2008. "Asset Pricing with Real Investment Commitment " Mimeo, CMU.
- [40] Jones, Larry, Rodolfo Manuelli, and Henry Siu, 2005a. "Fluctuations in convex models of endogenous growth, I: Growth effects", *Review of Economic Dynamics*, 8(4):780-804.
- [41] Jones, Larry, Rodolfo Manuelli, and Henry Siu, 2005b. "Fluctuations in convex models of endogenous growth, II: Business cycle properties", *Review of Economic Dynamics*, 8(4):805:828.
- [42] Judd, Kenneth, 1998. "Numerical methods in economics". MIT Press.
- [43] Lettau, Martin, and Harald Uhlig, 2000, "Can Habit Formation Be Reconciled with Business Cycle Facts?", *Review of Economic Dynamics*, 3(1):79-99.
- [44] Lettau, Martin and Sydney Ludvigson, 2002, "Time-varying risk premia and the cost of capital: An alternative implication of the Q theory of investment", *Journal of Monetary Economics*, 49(1):31-66.
- [45] Liu, Laura Xiaolei , Toni Whited and Lu Zhang, 2008. "Investment-based expected stock returns" Mimeo, U of Michigan.
- [46] Longstaff, Francis, and Monika Piazzesi, 2004: "Corporate earnings and the equity premium", *Journal of Financial Economics*, 74(3):401-421.
- [47] Lucas, Robert E., Jr, 2003. "Macroeconomic Priorities", *American Economic Review*, 93(1):1-14.
- [48] Papanikolaou, Dimitris, 2008. "Investment-Specific Shocks and Asset Prices". Mimeo, Kellogg.
- [49] Philippon, Thomas, 2008. "The Bond Market's Q". Forthcoming, *Quarterly Journal of Economics*.
- [50] Martin, Ian, 2007. "Consumption-Based Asset Pricing with Higher Cumulants". Mimeo, Stanford.
- [51] Ramey, Garey and Valery Ramey, 1995. "Cross-Country Evidence on the Link Between Volatility and Growth Cross-Country Evidence on the Link Between Volatility and Growth", *American Economic Review*, 85(5):1138-1151.
- [52] Rietz, Thomas, 1988. "The equity premium: A solution." *Journal of Monetary Economics* 22, 117-131.

- [53] Santa Clara and Yan, 2008. "Crashes, Volatility, and the Equity Premium: Lessons from S&P 500 Options". *Review of Economics and Statistics*, forthcoming.
- [54] Schwert, G. William, 1989, "Why Does Stock Market Volatility Change Over Time?", *Journal of Finance*, 44(5):1115-1153.
- [55] Schwert, G. William, 1990. "Stock Returns and Real Activity: A Century of Evidence Stock Returns and Real Activity: A Century of Evidence", *Journal of Finance*, 45(4):1237-1257.
- [56] Smets, Frank, and Ralf Wouters, 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area". *Journal of the European Economic Association*, 1(5):1123-1175.
- [57] Swanson, Eric and Glenn Rudebusch, 2008a. "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks", Mimeo.
- [58] Swanson, Eric and Glenn Rudebusch, 2008b. "Examining the Bond Premium Puzzle with a DSGE Model" *Journal of Monetary Economics*, forthcoming.
- [59] Tallarini, Thomas D. Jr., 2000. "Risk-Sensitive Real Business Cycles." *Journal of Monetary Economics*, 45(3), pp. 507-532.
- [60] Uhlig, Harald, 2006. "Macroeconomics and Asset Markets: Some Mutual Implications." University of Chicago Mimeo.
- [61] Wachter, Jessica, 2008. "Can time-varying risk of rare disasters explain aggregate stock market volatility?" Mimeo, Wharton.
- [62] Weil, Philippe, 1989. "Unexpected Utility in Macroeconomics Unexpected Utility in Macroeconomics", *Quarterly Journal of Economics*, 105(1):29-42.

6 Appendix

6.1 Computational Method

This method is presented for the case of a Cobb-Douglas production function, and a Cobb-Douglas utility function, but it can be used for arbitrary homogeneous of degree one production function and utility function. The Bellman equation for the “rescaled” problem is:

$$g(k, p) = \max_{c, i, N} \left\{ \begin{array}{l} (1 - \beta)c^{v(1-\gamma)}(1 - N)^{(1-v)(1-\gamma)} \\ + \beta e^{\mu v(1-\gamma)} \left(E_{p', \varepsilon', x'} e^{\sigma \varepsilon' v(1-\theta)} (1 - x' + x'(1 - b_{tffp})^{v(1-\theta)}) g(k', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\},$$

s.t. :

$$\begin{aligned} c &= k^\alpha N^{1-\alpha} - i, \\ k' &= \frac{(1 - x'b_k) \left((1 - \delta)k + \phi \left(\frac{i}{k} \right) k \right)}{e^{\mu + \sigma \varepsilon'} (1 - x'b_{tffp})}. \end{aligned}$$

Here too, the max needs to be transformed in a min if $\gamma > 1$. To approximate numerically the solution of this problem, I proceed as follows:

(1) Pick a grid for k , and a grid for i , and approximate the process for p with a Markov chain with transition matrix Q . Discretize the normal shock ε , with probabilities $\pi(\varepsilon)$. I used 120 points for the grid for k , 1200 points for the grid for i , 2 points for the grid for p , and 5 points for the grid for ε .

(2) Compute for any k, i in the grid the value $N(k, i)$ which solves

$$V(k, i) = \max_N (k^\alpha N^{1-\alpha} - i)^v (1 - N)^{(1-v)}.$$

Next define $R(k, i) = (1 - \beta)V(k, i)^{1-\gamma}$.

(3) The state space and action space are now discrete, so this is a standard dynamic discrete programming problem, which can be rewritten as follows, with one endogenous state, one exogenous state, and two additional shocks: a binomial variable x equal to one if a disaster occurs (probability p) and the normal shock ε :

$$g(k, p) = \max_i \left\{ \begin{array}{l} R(k, i) + \beta e^{\mu v(1-\gamma)} \times \dots \\ \left(\sum_{p', \varepsilon', x' \in \{0,1\}} \pi(\varepsilon') Q(p, p') e^{\sigma \varepsilon' v(1-\theta)} pr(x', p) (1 - x' + x'(1 - b_{tffp})^{v(1-\theta)}) g(k', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\},$$

s.t. :

$$k' = \frac{(1 - x'b_k) \left((1 - \delta)k + \phi \left(\frac{i}{k} \right) k \right)}{e^{\mu + \sigma \varepsilon'} (1 - x'b_{tffp})},$$

where $pr(x', p) = p1_{x'=1} + (1 - p)1_{x'=0} = px' + (1 - p)(1 - x')$. I solve this Bellman equation using modified policy iteration (Judd (1998), p. 416), starting with a guess value close to zero.²⁶ Recursive utility implies that the Blackwell sufficient conditions do not hold here, hence it is not obvious that the Bellman operator is a contraction. Note that to compute the expectation, we need the value function outside the grid points. I use linear interpolation in the early steps of the iteration, then switch to spline

²⁶This turns out to be significantly faster than value iteration for this application.

interpolation. The motivation is that linear interpolation is more robust, hence it is easier to make the iterations converge; but spline interpolation is more precise.

(4) Given g , we have $V(K, z, p) = z^v g(k, p)^{\frac{1}{1-\gamma}}$. We also obtain the policy functions $C = zc(k, p)$, $I = zi(k, p)$, $N = N(k, p)$, and the output policy function $Y = zk^\alpha N(k, p)^{1-\alpha}$.

(5) To compute asset prices, we need the stochastic discount factor, which is given by the standard formula:

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{v(1-\gamma)-1} \left(\frac{1 - N_{t+1}}{1 - N_t} \right)^{(1-v)(1-\gamma)} \left(\frac{V_{t+1}}{E_t (V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}} \right)^{\gamma-\theta}.$$

Using homogeneity, the SDF between two states $s = (k, p)$ and $s' = (k', p')$ is:

$$\begin{aligned} M(s, s', \varepsilon' x') &= \beta \left(\frac{z' c(k', p')}{zc(k, p)} \right)^{v(1-\gamma)-1} \left(\frac{1 - N(k', p')}{1 - N(k, p)} \right)^{(1-v)(1-\gamma)} \left(\frac{z'^v g(k', p')^{\frac{1}{1-\gamma}}}{E_{z', p', x'} (z'^v (1-\theta) g(k', p')^{\frac{1-\theta}{1-\gamma}})^{\frac{1}{1-\theta}}} \right)^{\gamma-\theta} \\ &= \beta \left(\frac{z'}{z} \right)^{(\gamma-\theta)v+v(1-\gamma)-1} \left(\frac{c(k', p')}{c(k, p)} \right)^{v(1-\gamma)-1} \times \dots \\ &\quad \dots \left(\frac{1 - N(k', p')}{1 - N(k, p)} \right)^{(1-v)(1-\gamma)} \left(\frac{g(k', p')^{\frac{1}{1-\gamma}}}{E_{z', p', x'} \left(\left(\frac{z'}{z} \right)^{v(1-\theta)} g(k', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1}{1-\theta}}} \right)^{\gamma-\theta}. \end{aligned}$$

Note that we first need to compute the conditional expectation which appears on the denominator of the last term. Denote $k' = j(k, p, \varepsilon', x')$ the detrended capital next period, which depends on the detrended investment $i(k, p)$ and on the realization of the shocks next period ε' and x' (but not p'). The conditional expectation is obtained as:

$$\begin{aligned} E_{z', p', x'} \left(\left(\frac{z'}{z} \right)^{v(1-\theta)} g(k', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1}{1-\theta}} &= \\ = \sum_{p', \varepsilon'} Q(p, p') \Pr(\varepsilon') e^{v(1-\theta)\mu + v(1-\theta)\sigma\varepsilon'} \left(p(1 - b_{tfp})^{v(1-\theta)} g(j(k, p, \varepsilon', 1), p')^{\frac{1-\theta}{1-\gamma}} + (1-p)g(j(k, p, \varepsilon', 0), p')^{\frac{1-\theta}{1-\gamma}} \right). \end{aligned}$$

(6) We can now obtain the price of a one-period asset, with payoff $d(k', z', p', x', \varepsilon')$. e.g. a pure risk-free asset $d = 1$, or a short-term government bond: $d = 1 - b(1-r)x'$, as $P(k, p) = E_{x', p', \varepsilon'} M(s, s', x', \varepsilon') d(k', z', p', x', \varepsilon')$. For instance, for a pure risk-free asset, the formula is:

$$E_t M_{t,t+1} = \frac{\beta \sum_{p'} \sum_{\varepsilon'} Q(p, p') \Pr(\varepsilon') e^{((\gamma-\theta)v+v(1-\gamma)-1)(\mu+\sigma\varepsilon')} \times \dots \left(\begin{aligned} &p(1 - b_{tfp})^{(\gamma-\theta)v+v(1-\gamma)-1} c(j(k, p, \varepsilon', 1), p')^{v(1-\gamma)-1} \times \\ &(1 - N(j(k, p, \varepsilon', 1), p'))^{(1-v)(1-\gamma)} g(j(k, p, \varepsilon', 1), p')^{\frac{\gamma-\theta}{1-\gamma}} \\ &+ (1-p)c(j(k, p, \varepsilon', 0), p')^{v(1-\gamma)-1} \times \\ &(1 - N(j(k, p, \varepsilon', 0), p'))^{(1-v)(1-\gamma)} g(j(k, p, \varepsilon', 0), p')^{\frac{\gamma-\theta}{1-\gamma}} \end{aligned} \right)}{c(k, p)^{v(1-\gamma)-1} (1 - N(k, p))^{(1-v)(1-\gamma)} E_{z', p', x'} \left(\left(\frac{z'}{z} \right)^{v(1-\theta)} g(k', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{\gamma-\theta}{1-\theta}}}.$$

(7) Next, we can obtain the term structure of interest rates on government bonds, using the recursion:

$$P_n(k, p) = E(M(s, s', x', \varepsilon') ((1 - x'b(1-r)) P_{n-1}(k', p'))),$$

where $b(1-r)$ is the expected loss rate of government bonds during disasters. It is assumed that government bonds are risk-free if there is no disaster.

(8), Finally, we can compute the price of equity claims. I consider three types of “equities”. Define the representative firm’s dividend as earnings less investment:

$$D_t = Y_t - w_t N_t - I_t.$$

(i) The first equity is simply a claim to the stream $\{D_t\}$. Let F_t denote its price, which satisfies the standard recursion:

$$F_t = E_t (M_{t,t+1} (F_{t+1} + D_{t+1})).$$

Note that D_t can be written as $D_t = z_t d(k_t, p_t)$, where $d(k, p) = \alpha k^\alpha N^{1-\alpha} - i(k, p)$. Hence, we can rewrite the firm value recursion as:

$$z f(k, p) = E_{s'|s} (M(s, s') \times (z' d(k', p') + z' f(k', p'))), \quad (8)$$

where $F_t = z_t f(k_t, p_t)$. The equity return is then

$$\begin{aligned} R_{t+1}^e &= \frac{F_{t+1} + D_{t+1}}{F_t} \\ &= \frac{z_{t+1} f(k_{t+1}, p_{t+1}) + d(k_{t+1}, p_{t+1})}{z_t f(k_t, p_t)}. \end{aligned} \quad (9)$$

To solve the recursion 8 in practice, I iterate starting with an initial guess $f(k, p) = 0$. The recursion can be rewritten as:

$$\begin{aligned} f(k, p) &= E_{s'|s} \left(M(s, s') \frac{z'}{z} (d(k', p') + f(k', p')) \right) \\ &= \beta E_{p', \varepsilon', x'} \left[\left(\frac{z'}{z} \right)^{(\gamma-\theta)v+v(1-\gamma)} \left(\frac{c(k', p')}{c(k, p)} \right)^{v(1-\gamma)-1} \left(\frac{1-N(k', p')}{1-N(k, p)} \right)^{(1-v)(1-\gamma)} \times \dots \right. \\ &\quad \left. \left(\frac{g(k', p')^{\frac{1}{1-\gamma}}}{E_{z', p', x'} \left(\left(\frac{z'}{z} \right)^{v(1-\theta)} g(k', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1}{1-\theta}}} \right) (d(k', p') + f(k', p')) \right]. \end{aligned}$$

This conditional expectation can be written down, as

$$\begin{aligned} f(k, p) &= \frac{\beta \sum_{p'} \sum_{\varepsilon'} Q(p, p') \Pr(\varepsilon') e^{((\gamma-\theta)v+v(1-\gamma))(\mu+\sigma\varepsilon')} \times \dots \\ &\quad \left(p(1-b_{tfp})^{(\gamma-\theta)v+v(1-\gamma)} c(j(k, p, \varepsilon', 1), p')^{v(1-\gamma)-1} (1-N(j(k, p, \varepsilon', 1), p'))^{(1-v)(1-\gamma)} \times \right. \\ &\quad \left. g(j(k, p, \varepsilon', 1), p')^{\frac{\gamma-\theta}{1-\gamma}} (d(j(k, p, \varepsilon', 1), p') + f(j(k, p, \varepsilon', 1), p')) \right. \\ &\quad \left. + (1-p)c(j(k, p, \varepsilon', 0), p')^{v(1-\gamma)-1} (1-N(j(k, p, \varepsilon', 0), p'))^{(1-v)(1-\gamma)} \times \right. \\ &\quad \left. g(j(k, p, \varepsilon', 0), p')^{\frac{\gamma-\theta}{1-\gamma}} (d(j(k, p, \varepsilon', 0), p') + f(j(k, p, \varepsilon', 0), p')) \right) \\ &\quad \left. c(k, p)^{-\gamma} E_{z', p', x'} \left(\left(\frac{z'}{z} \right)^{v(1-\theta)} g(k', p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{\gamma-\theta}{1-\theta}} \right). \end{aligned}$$

$$f(k, p) = .$$

Note that a standard argument homogeneity argument (essentially, Hayashi’s theorem; see Kaltenbrunner and Lochstoer (2008) for instance) implies that the ex-dividend firm value is:

$$F_t = \frac{(1-pb_k)K_{t+1}}{\phi' \left(\frac{I_t}{K_t} \right)},$$

i.e. Tobin's Q is $Q_t = \frac{F_t}{(1-pb_k)K_{t+1}} = \phi' \left(\frac{I_t}{K_t} \right)$. Substituting this in the return equation (9) yields

$$\begin{aligned} R_{t,t+1} &= \frac{F_{t+1} + D_{t+1}}{F_t} = \frac{\frac{(1-pb_k)K_{t+2}}{\phi' \left(\frac{I_{t+1}}{K_{t+1}} \right)} + D_{t+1}}{\frac{(1-pb_k)K_{t+1}}{\phi' \left(\frac{I_t}{K_t} \right)}} \\ &= \phi' \left(\frac{I_t}{K_t} \right) \left[\frac{1 - \delta + \phi \left(\frac{I_{t+1}}{K_{t+1}} \right)}{\phi' \left(\frac{I_{t+1}}{K_{t+1}} \right)} (1 - x_{t+1}b_k) + \frac{\alpha K_{t+1}^\alpha N_{t+1}^{1-\alpha} - I_{t+1}}{K_{t+1}} \right]. \end{aligned}$$

This expression can be calculated using the quantities produced by the model in step (3). In practice it is useful to check that the results obtained using this method are the same as the results using the value F_t .

The second and third types of equity assume respectively that the payoff streams are $\{C_t^\lambda\}$ and $\{D_t^{\lambda_2}\}$. It is easy to adapt the method used in (i) to price the claims to these assets. Finally, I obtain the model statistics by simulating 200 samples of length 500, started at the nonstochastic steady-state, and cutting off the first 300 periods. The Matlab(c) programs will be made available on my web page when the paper is submitted.

6.2 Data Sources

For the cross-correlogram: the stock market return is from CRSP (value-weighted including dividends). The short rate is from Ken French's website. The yield curve is from the Fama-Bliss data (1 to 5 years). The industrial production data is from FRED (series nickname INDPRO). I also used different monthly series to measure economic activity: employment (nickname PAYEMS), disposable income (DSPIC96), consumption (PCE96). All these series are monthly, and the data ranges from January 1959 to December 2007.

For the business cycle and return moments of Tables 2-7: consumption is nondurable + services consumption, investment is fixed investment, and output is GDP, from the NIPA Table 1.1.3, quarterly data 1947q1-2008q4. Hours is nonfarm business hours from the BLS productivity program (through FRED: HOABNS). The return data is from Ken French's webpage, (benchmark factors, aggregated to quarterly frequency, and deflated by the CPI (CPIAUCSL through FRED)).

Parameter	Greek Letter	Value
Capital share	α	.34
Depreciation rate	δ	.02
Adjustment cost curvature	η	0
Trend growth of TFP	μ	.005
Discount factor	β	.99
IES	$1/\gamma$	2
Share of consumption in utility	v	.3
Risk aversion over the C-L bundle	θ	8
Standard deviation of ordinary TFP shock	σ	.01
Size of disaster in TFP	b_{tfp}	.43
Size of disaster for capital	b_k	.43
Probability of disaster in low prob state	p_l	.0005
Probability of disaster in high prob state	p_h	.008
Probability of transition from p_l to p_h	π	.98

Table 1: Parameter values for the benchmark model. The time period is one quarter.

Sample without disasters	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\sigma(\Delta \log Y)$	$\rho_{C,Y}$	$\rho_{I,Y}$
Data (1947q1-2008q4)	0.57	2.68	0.92	0.0098	0.45	0.68
No disaster ($b_k = b_{tfp} = 0$)	0.43	2.88	0.43	0.01	0.97	0.99
Constant p ($b_k = b_{tfp} = .43$)	0.44	2.93	0.43	0.01	0.97	0.99
Benchmark ($b_k = b_{tfp} = .43$)	0.49	3.32	0.53	0.01	0.73	0.94
Capital disasters ($b_k = .43, b_{tfp} = 0$)	0.57	4.02	0.67	0.01	0.43	0.90
Capital disasters, with adj costs	1.09	0.72	0.08	0.01	1.00	0.98
TFP disasters ($b_{tfp} = .43, b_k = 0$)	0.68	4.50	0.89	0.01	-0.01	0.90
TFP disasters, with adj costs	1.08	0.94	0.13	0.01	0.99	0.80

Table 2: Business cycle quantities: second moments implied by the model, for different calibrations. Quarterly data. This is based on a sample without disasters. $\rho(C,Y)$ and $\rho(I,Y)$ are the correlation of the growth rate of C and of Y, and I of Y, respectively. Data sources in appendix.

Sample with disasters	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\sigma(\Delta \log Y)$	$\rho_{C,Y}$	$\rho_{I,Y}$
Data (1947q1-2008q4)	0.57	2.68	0.92	0.0098	0.45	0.68
No disaster ($b_k = b_{tfp} = 0$)	0.44	2.87	0.43	0.01	0.97	0.99
Constant p ($b_k = b_{tfp} = .43$)	0.90	1.40	0.13	0.03	0.98	0.97
Benchmark ($b_k = b_{tfp} = .43$)	0.92	1.46	0.16	0.03	0.88	0.93
Capital disasters ($b_k = .43, b_{tfp} = 0$)	1.66	4.58	1.14	0.01	0.50	0.50
Capital disasters, with adj costs	0.93	1.31	0.10	0.01	1.00	0.98
TFP disasters ($b_{tfp} = .43, b_k = 0$)	0.52	8.27	0.53	0.03	0.47	0.90
TFP disasters, with adj costs	1.07	0.81	0.07	0.02	0.99	0.90

Table 3: Business cycle quantities: second moments implied by the model, for different calibrations. Quarterly data. This is based on a full sample (i.e., including disasters). $\rho(C,Y)$ and $\rho(I,Y)$ are the correlation of the growth rate of C and of Y, and I of Y, respectively. Data sources in appendix.

Sample without disasters	$E(R_f)$	$E(R_b)$	$E(R_e)$	$E(R_{c,lev})$	$E(R_{d,lev})$	$E(R_{ltb} - R_b)$
Data 1947q1-2008q4	—	0.21	1.91	1.91	1.91	n.a.
No disaster ($b_k = b_{tfp} = 0$)	1.22	1.29	1.22	1.28	1.26	0.00
Constant p ($b_k = b_{tfp} = .43$)	0.25	0.66	1.28	2.28	2.25	0.00
Benchmark ($b_k = b_{tfp} = .43$)	0.32	0.70	1.28	2.50	2.41	-0.08
Capital disasters ($b_k = .43, b_{tfp} = 0$)	1.06	1.17	1.34	1.27	1.33	0.00
Capital disasters, with adj costs	1.13	1.23	1.19	1.34	1.25	0.00
TFP disasters ($b_{tfp} = .43, b_k = 0$)	0.88	1.15	0.94	2.15	1.21	0.02
TFP disasters, with adj costs	0.69	0.99	1.40	2.22	2.36	-0.02

Table 4: Mean returns implied by the model for (a) pure risk-free asset, (b) a one-quarter govt bond, (c) a claim to dividends (d) a leveraged claim on consumption (e) a leveraged claim on dividends (f) the difference between the long-term yield and the short-term yield. Quarterly data. Statistics computed in a sample without disasters. Data sources in appendix.

Sample with disasters	$E(R_f)$	$E(R_b)$	$E(R_e)$	$E(R_{c,lev})$	$E(R_{d,lev})$	$E(R_{ltb} - R_b)$
Data	—	0.21	1.91	1.91	1.91	n.a.
No disaster ($b_k = b_{tfp} = 0$)	1.22	1.29	1.22	1.29	1.27	-0.00
Constant p ($b_k = b_{tfp} = .43$)	0.24	0.66	1.11	1.92	1.91	0.00
Benchmark ($b_k = b_{tfp} = .43$)	0.24	0.65	1.10	2.14	2.07	-0.06
Capital disasters ($b_k = .43, b_{tfp} = 0$)	1.18	1.28	1.28	1.29	1.31	-0.00
Capital disasters, with adj costs	1.19	1.29	1.22	1.29	1.27	0.00
TFP disasters ($b_{tfp} = .43, b_k = 0$)	0.75	1.01	0.81	1.70	1.03	0.02
TFP disasters, with adj costs	0.58	0.90	1.13	1.79	1.89	-0.02

Table 5: Mean returns implied by the model for (a) pure risk-free asset, (b) a one-quarter govt bond, (c) a claim to dividends (d) a leveraged claim on consumption (e) a leveraged claim on dividends (f) the difference between the long-term yield and the short-term yield. Quarterly data. This is based on a full sample (i.e., including disasters). Data sources in appendix.

Sample without disasters	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e)$	$\sigma(R_{c,lev})$	$\sigma(R_{e,l})$	$\sigma(R_{ltb} - R_b)$
Data	—	0.81	8.14	8.14	8.14	n.a.
No disaster ($b_k = b_{tfp} = 0$)	0.07	0.09	0.08	2.45	1.68	0.02
Constant p ($b_k = b_{tfp} = .43$)	0.08	0.08	0.08	2.29	0.41	0.00
Benchmark ($b_k = b_{tfp} = .43$)	0.78	0.47	0.09	5.01	3.67	0.16
Capital disasters ($b_k = .43, b_{tfp} = 0$)	0.21	0.14	0.10	2.46	1.71	0.04
Capital disasters, with adj costs	0.06	0.02	0.93	2.56	2.92	0.01
TFP disasters ($b_{tfp} = .43, b_k = 0$)	0.15	0.15	0.32	4.40	1.63	0.05
TFP disasters, with adj costs	0.40	0.18	1.00	4.42	4.71	0.06

Table 6: Standard deviations of returns implied by the model for (a) pure risk-free asset, (b) a one-quarter govt bond, (c) a claim to dividends (d) a leveraged claim on consumption (e) a leveraged claim on dividends (f) the difference between the long-term yield and the short-term yield. Quarterly data. Statistics computed in a sample without disasters. Data sources in appendix.

Sample with disasters	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e)$	$\sigma(R_{c,lev})$	$\sigma(R_{e,l})$	$\sigma(R_{ltb} - R_b)$
Data	—	0.81	8.14	8.14	8.14	n.a.
No disaster ($b_k = b_{tfp} = 0$)	0.07	0.09	0.08	2.44	1.68	0.02
Constant p ($b_k = b_{tfp} = .43$)	0.08	0.08	1.95	4.96	3.86	0.00
Benchmark ($b_k = b_{tfp} = .43$)	0.79	0.48	2.12	7.10	6.12	0.16
Capital disasters ($b_k = .43, b_{tfp} = 0$)	0.32	0.28	1.90	2.91	2.68	0.05
Capital disasters, with adj costs	0.07	0.04	1.02	3.00	2.96	0.01
TFP disasters ($b_{tfp} = .43, b_k = 0$)	0.33	0.29	0.60	6.23	2.13	0.05
TFP disasters, with adj costs	0.44	0.21	2.32	6.31	6.79	0.06

Table 7: Standard deviation of returns implied by the model for (a) pure risk-free asset, (b) a one-quarter govt bond, (c) a claim to dividends (d) a leveraged claim on consumption (e) a leveraged claim on dividends (f) the difference between the long-term yield and the short-term yield. Quarterly data. This is based on a full sample (i.e., including disasters). Data sources in appendix.

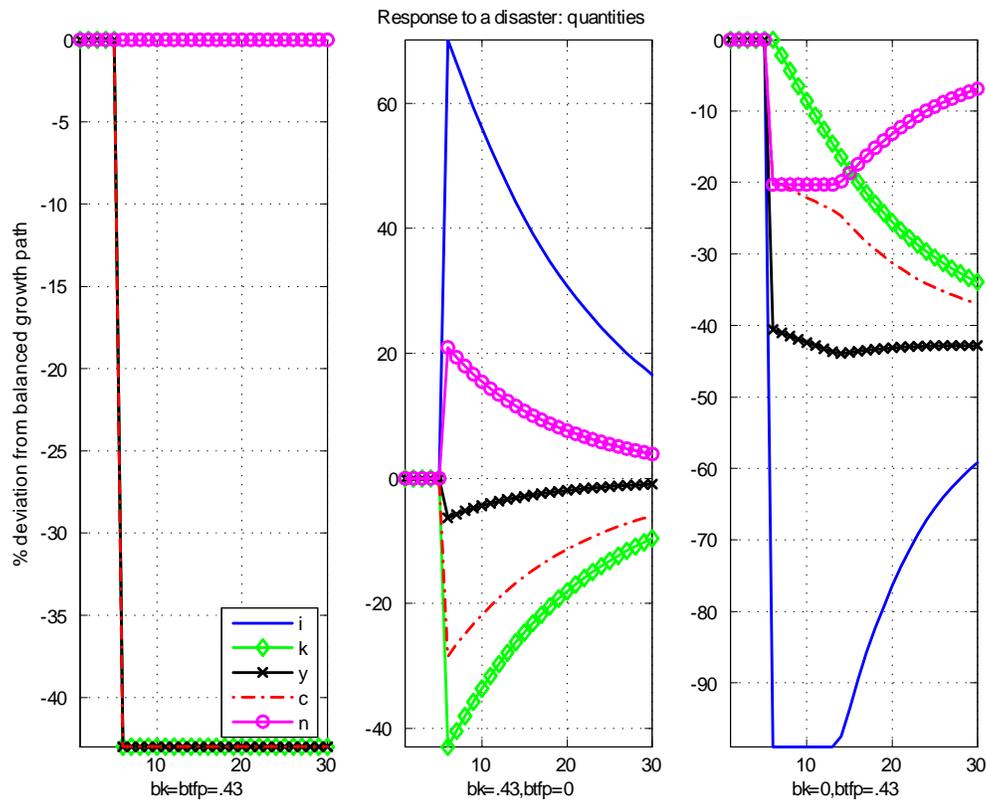


Figure 1: Response of quantities (C,I,K,N,Y) to a disaster at $t = 6$, in % deviation from balanced growth path. Left panel: $b_k = b_{tfp}$; Middle panel: $b_k = .43, b_{tfp} = 0$. Right panel: $b_{tfp} = .43, b_k = 0$.

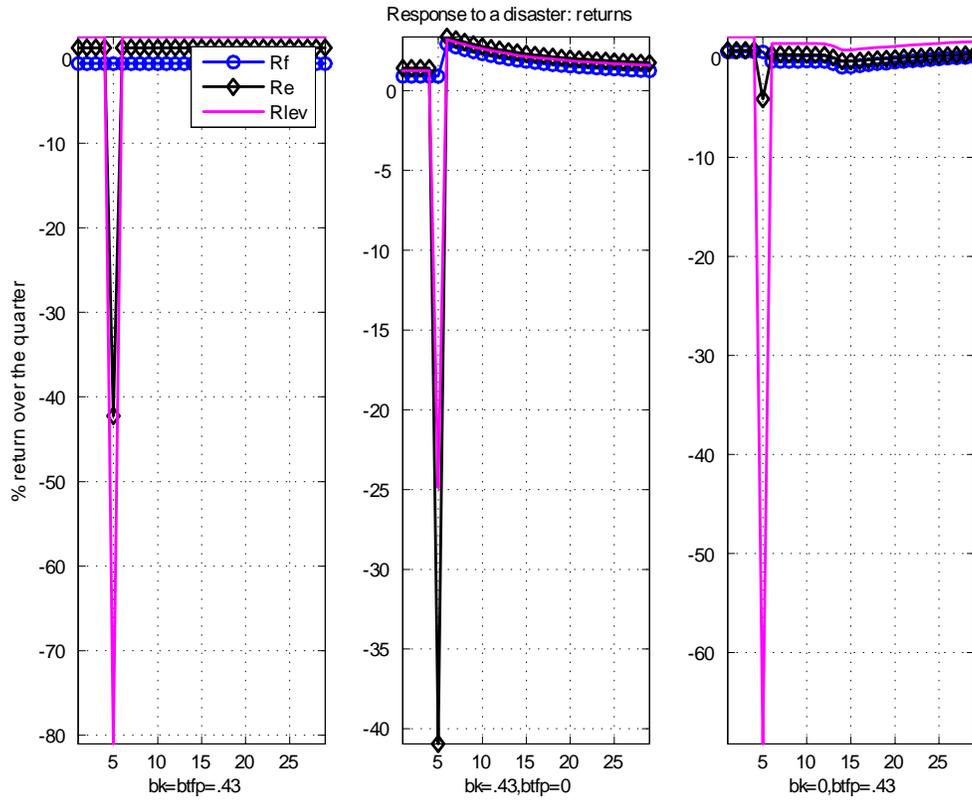


Figure 2: Response of asset returns (risk-free rate, equity return, levered equity) to a disaster at $t = 6$. Left panel: $b_k = b_{tfp}$; Middle panel: $b_k = .43, b_{tfp} = 0$. Right panel: $b_{tfp} = .43, b_k = 0$.

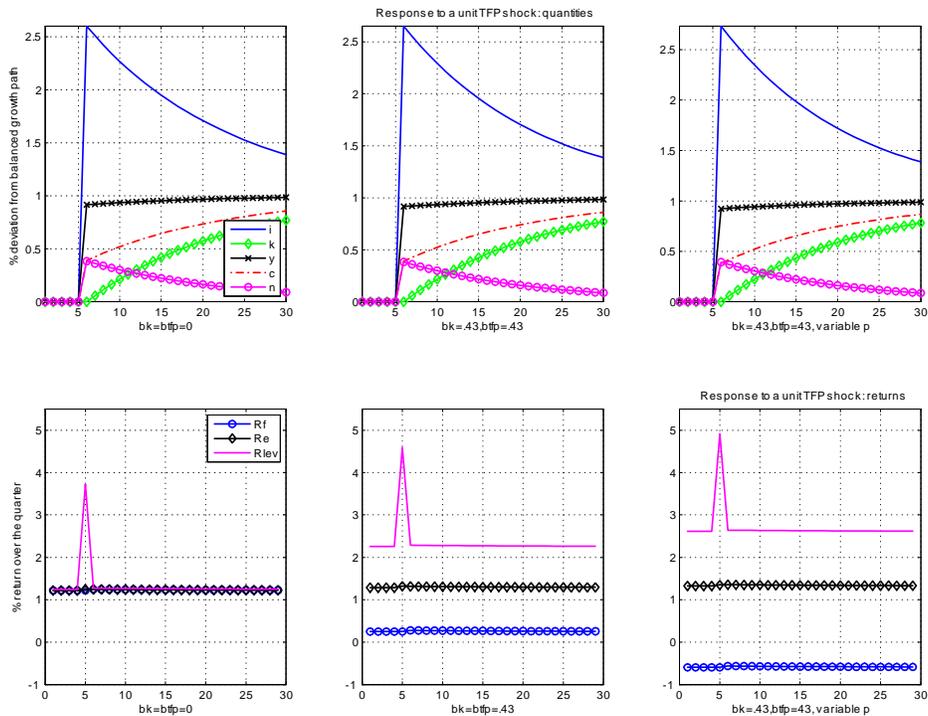


Figure 3: Impulse response function of quantities (C,I,N,K,Y) and returns (risk-free rate, equity return, levered equity) to a permanent TFP shock at $t = 6$. Left panel: model without disasters. Middle panel: model with constant probability of disasters. Right panel: model with time-varying probability of disaster (benchmark).

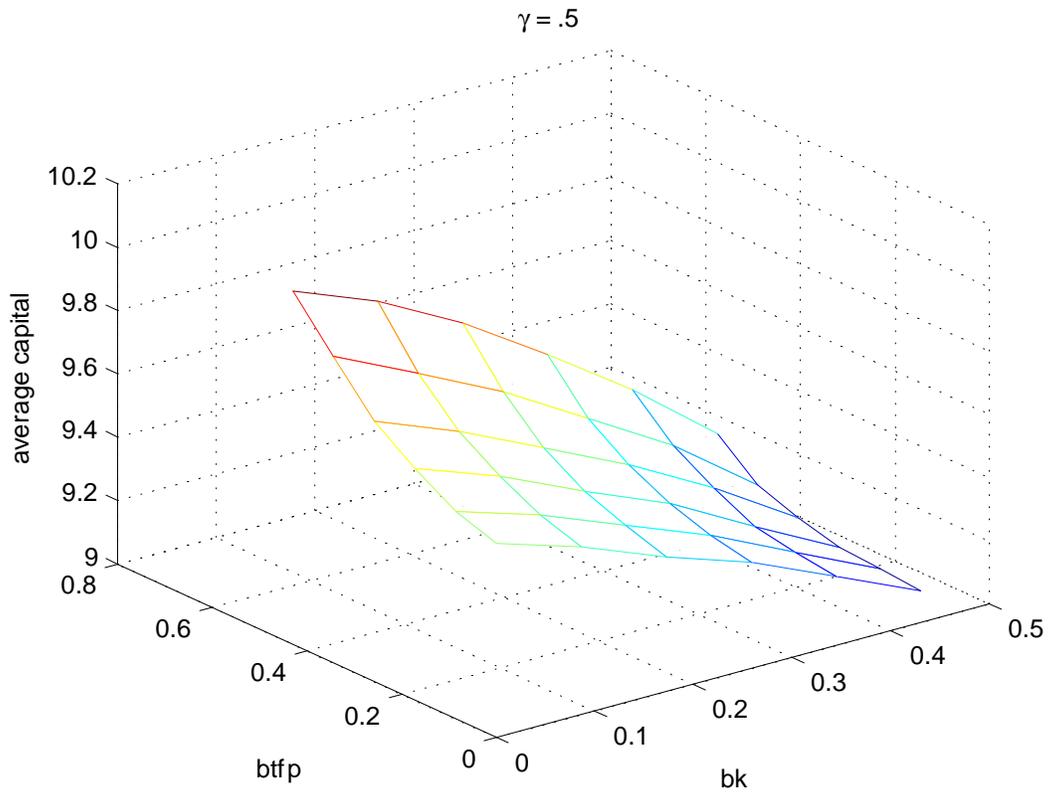


Figure 4: Surface plot of the average capital stock in an economy with $\gamma = .5$, for various values of b_k and b_{tfp} .

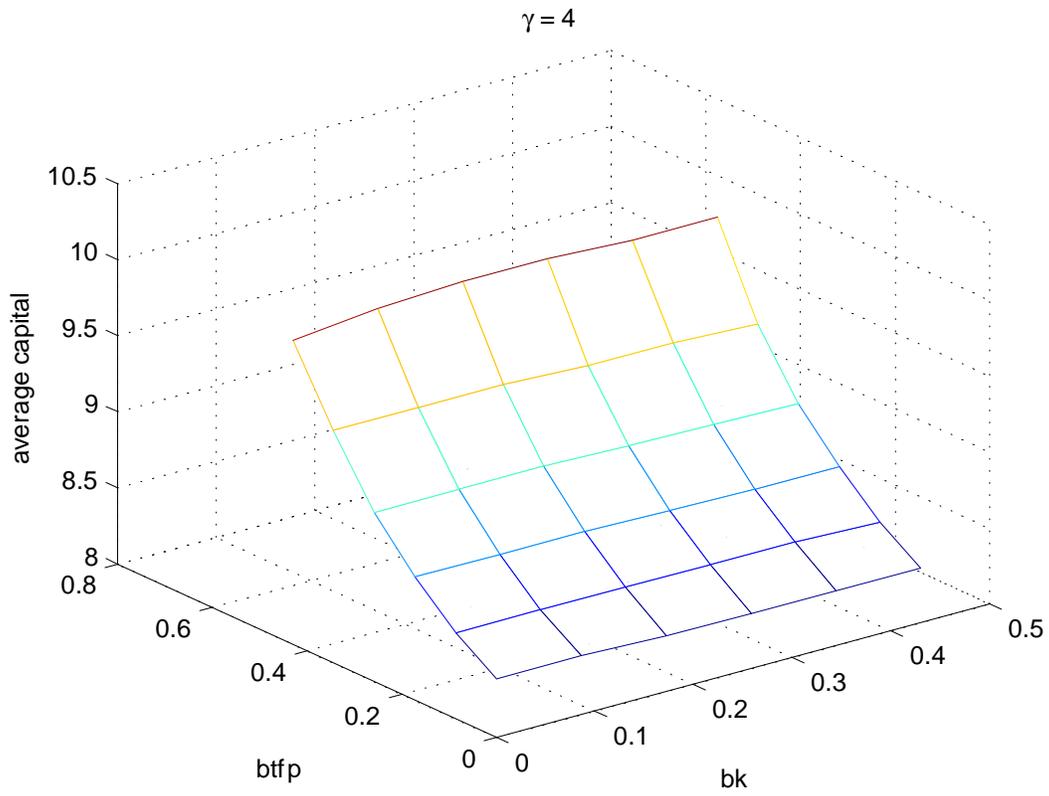


Figure 5: Surface plot of the average capital stock in an economy with $\gamma = 4$, for various values of b_k and b_{tfp} .

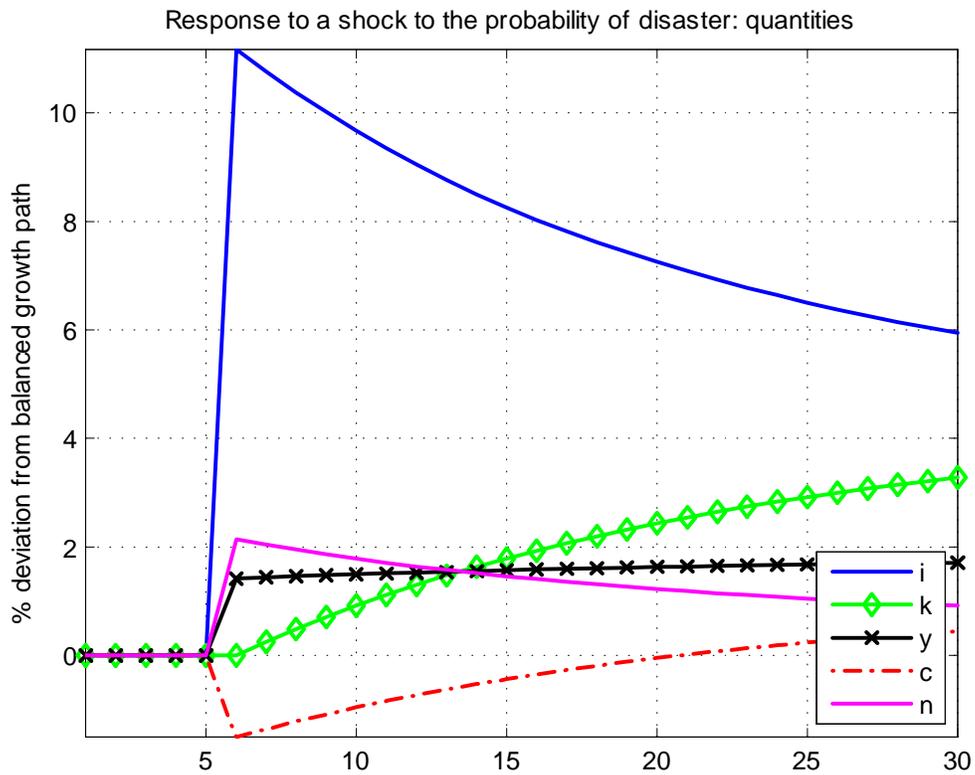


Figure 6: Impulse response of macroeconomic quantities to a shock to the probability of disaster at $t = 5$. The probability of disaster goes from the high state to the low state.

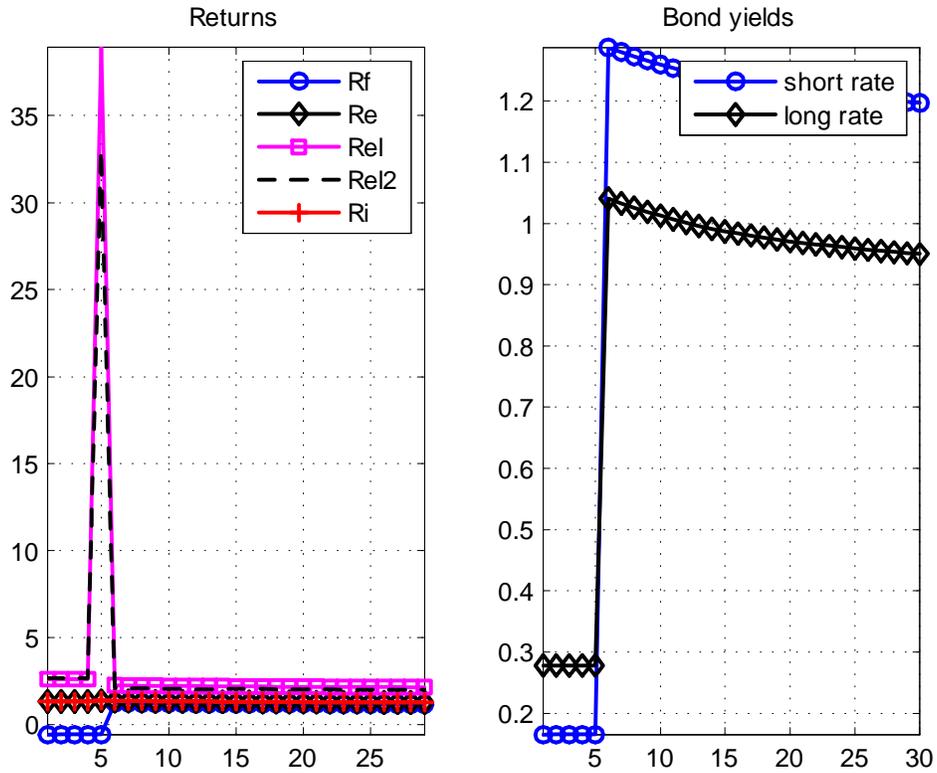


Figure 7: Impulse response of asset returns to a shock to the probability of disaster at $t = 5$. The probability of disaster goes from the high state to the low state. The left panel shows the return on the pure risk-free asset, the equity asset, and a levered claim to consumption. The right panel shows the short (1 quarter) and long (20 quarters) yields.

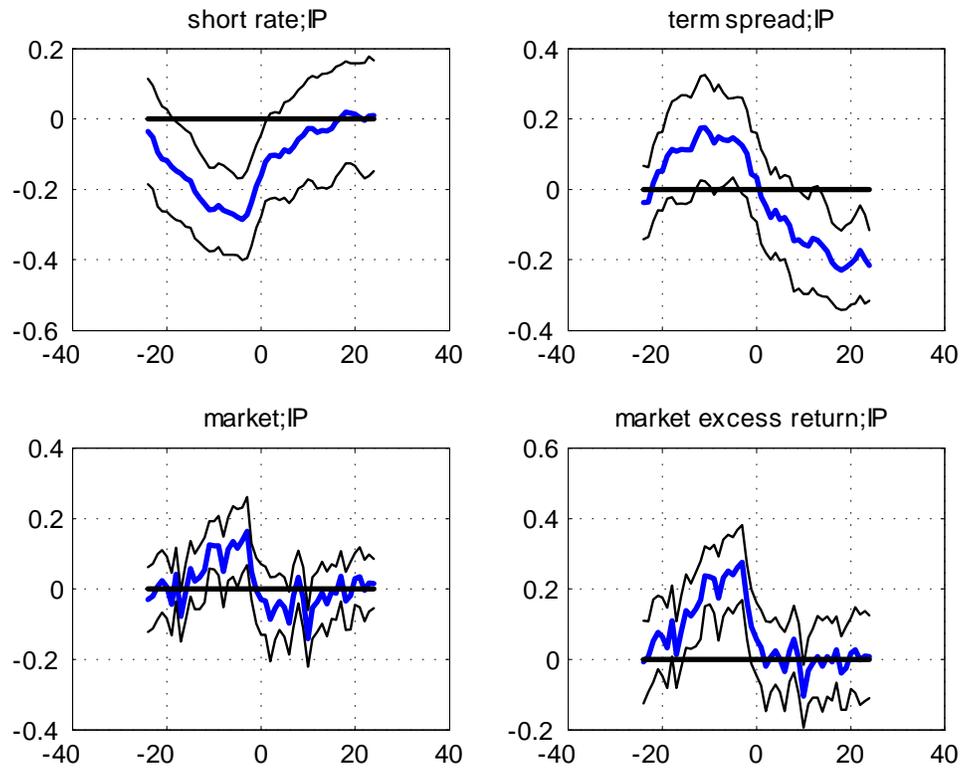


Figure 8: Cross-correlogram of industrial production growth with the short rate, the term spread, the market return, and the market excess return. Monthly data, 1959:1-2007:12. The black line shows the ± 2 S.E. bands for each correlation, based on the GMM formula with Newey-West correction and the Delta method.

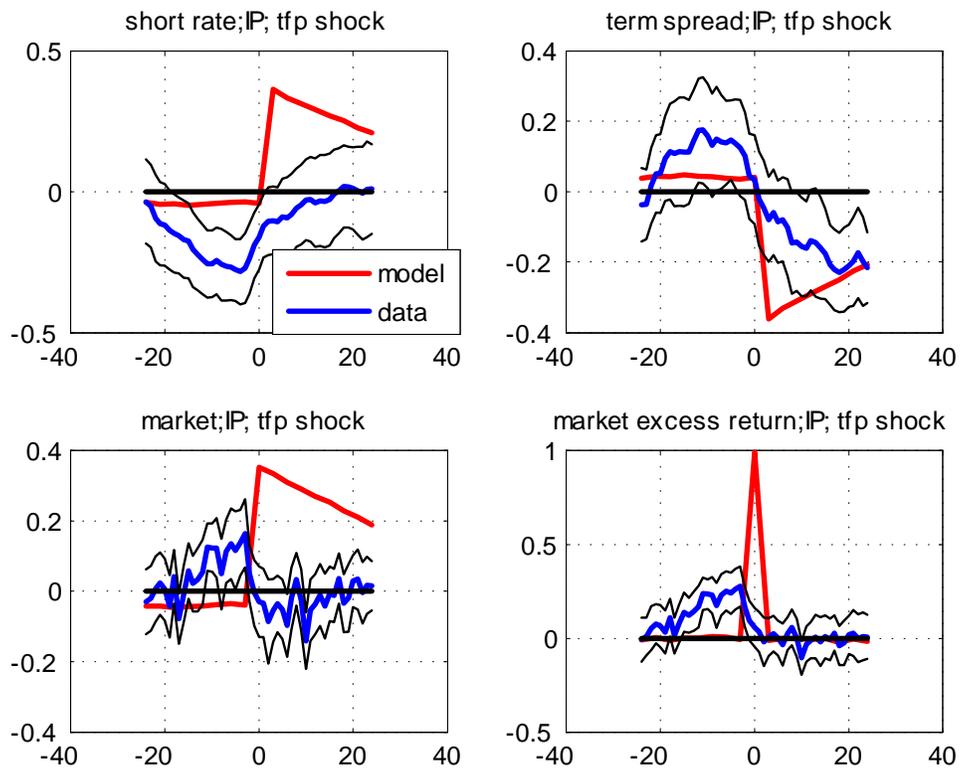


Figure 9: Cross-correlogram of output growth with the short rate, the term spread, the market return, and the market excess return, for the data (blue) and a model with no disasters (red).

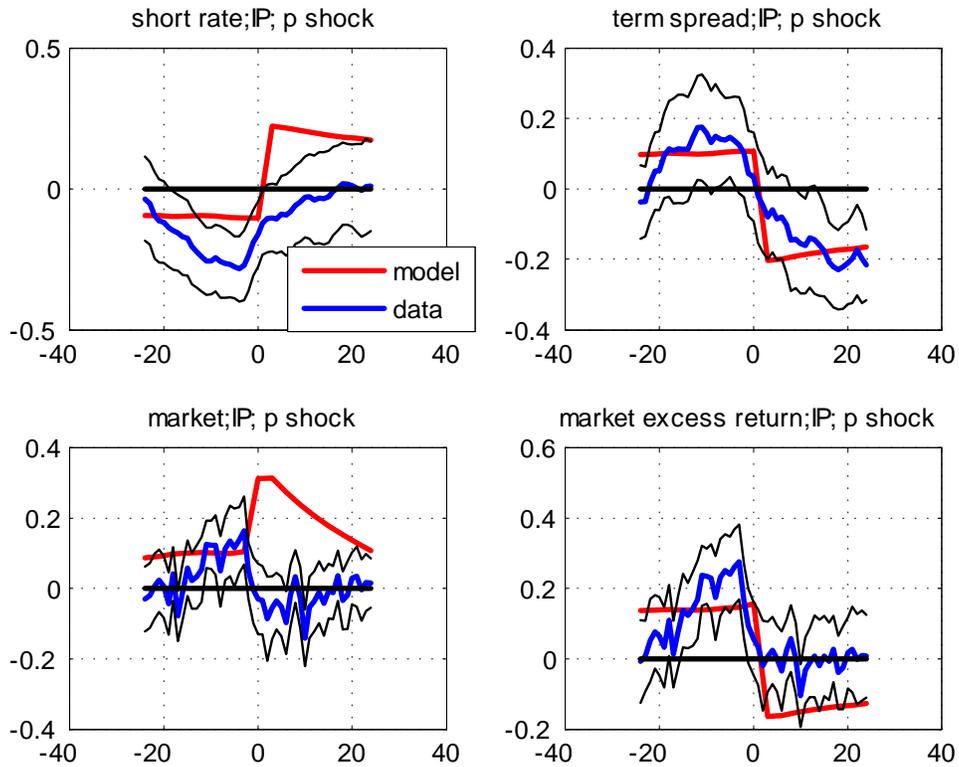


Figure 10: Cross-correlogram of output growth with the short rate, the term spread, the market return, and the market excess return, for the data (blue) and for the benchmark model (red) without TFP shocks. (The only shocks are the shocks to the probability of disaster.)