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A Simple Model of Price Dispersion*

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Abstract

This article considers a simple stock-flow matching model with fully informed market participants. Unlike in the standard matching literature, prices are assumed to be set ex-ante. When sellers pre-commit themselves to sell their products at an advertised price, the unique equilibrium is characterized by price dispersion due to the idiosyncratic match payoffs (in a marketplace with full information). This provides new insights into the price dispersion literature, where price dispersion is commonly assumed to be generated by a costly search of *uninformed* buyers.

JEL codes: C78, D43

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A Simple Model of Price Dispersion

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This article considers a simple stock-flow matching model with fully informed market participants. Unlike in the standard matching literature, prices are assumed to be set ex-ante. When sellers pre-commit themselves to sell their products at an advertised price, the unique equilibrium is characterized by price dispersion due to the idiosyncratic match payoffs (in a marketplace with *full* information). This provides new insights into the price dispersion literature, where price dispersion is commonly assumed to be generated by a costly search of *uninformed* buyers.

Keywords: Price Dispersion, Matching Models.
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1. INTRODUCTION

The standard matching approach originally developed by Diamond (1982), Mortensen (1982), Pissarides (2000) and Kiyotaki and Wright (1991), assumes a purely random search process. Coles (1999) considers the other extreme, a stock-flow matching, where a search is no longer random, but perfectly directed. Both strands of the literature commonly assume that price is determined by ex-post bargaining between buyer and seller after the match has been formed. In most types of markets, however, sellers advertise not only characteristics of products which they want to sell, but they also either fully pre-commit themselves to supply a good for an advertised price or the advertised price represents a signal for subsequent price negotiations.

This article assumes that sellers fully pre-commit themselves to supply a good for a given advertised price. Price distribution in the simplified stock-flow matching model by Coles (1999) with ex-ante price setting is analyzed. It is shown that despite full information on both sides of the market, the unique equilibrium poses price dispersion. Unlike in literature on equilibrium price dispersion, where price dispersion is commonly generated by costly search of imperfectly informed buyers, price dispersion in this model is driven by idiosyncratic match payoffs: each buyer is assumed to either like or dislike the seller's good.

The remainder of the article is organized as follows. Section 2 outlines the general framework. Market equilibrium is characterized in Section 3. Following comparative statics in Section 4, the last section concludes.

2. FRAMEWORK

There is a mass S of sellers (firms), each supplying one type of good for price p. The cost of supplying the good is assumed to be equal to c. The preferences of

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the buyer over goods supplied by sellers are heterogenous. As in Coles (1999), it is assumed that each buyer can either like or dislike the good the seller provides . In case she does, her utility by consuming it is $y - p$; otherwise her utility is zero. Given the mass S of sellers, the probability that a new buyer entering the market likes k of them is assumed to be given by Poisson distribution:

$$P(k \text{ matches}) = \frac{e^{-\lambda S} (\lambda S)^k}{k!} \tag{1}$$

where $\lambda > 0$ is an exogenous preference parameter. If atomistic sellers are assumed instead, the preferences of the buyer would be given by a binomial distribution. With minor modifications, the main findings would continue to hold. There is perfect information in the market - i.e. each buyer knows the location of each seller as well as the characteristic and prices of goods that are offered in the market. Once endowed with a set of valuations over a given set of goods, a buyer's valuation does not change. Each seller has a reservation value denoted by $\overline{v}_s$. A new seller enters the market if the value of being a seller, denoted by V_s, exceeds $\overline{v}_s$. Conversely, if the reservation value exceeds V_s, sellers leave the market. Hence, in the steady-state $V_s = \overline{v}_s$. There is an exogenous inflow $f_b > 0$ of new buyers, each willing to purchase one good. The buyer purchases the good only if its price does not exceed a threshold value y. If the buyer likes more than one good, she will purchase the cheapest. Note that under this set-up, there is a probability $e^{-\lambda S}$ that the new buyer likes none of the existing goods. If the buyer likes no goods, she immediately leaves the marketplace. Hence $\left(1 - e^{-\lambda S}\right) f_b$ represents the flow of buyers who likes at least one good in the market. The flow of new buyers is matched by the stock of sellers. The mass of buyers at any point in time is zero. The model is set in continuous time and the individual discount rate is denoted by $r > 0$. This type of marketplace represents a modified and simplified version of Coles' (1999) stock-flow matching model.

It is commonly assumed in the literature that in the marketplace with decentralized trade prices are formed by bargaining between the buyer and seller after the match had been formed. In many types of markets, however, when sellers advertise their products, they advertise prices as well. For simplicity, we assume that sellers set prices ex-ante and no negotiations are possible thereafter. The same results would be obtained if sellers could affect the expected price from bargaining arbitrarily by sending ex-ante price signals. The assumption of ex-ante price setting has several interesting implications on the distribution of prices in the market.

3. EQUILIBRIUM

Before proceeding further, we formally define equilibrium in this simple market.

Definition 1 (Equilibrium). Equilibrium is a distribution of prices, denoted by $F^*(p)$, and a mass of sellers S^* satisfying:

$$\forall p \in R^+ : V_s(p) = \overline{v}_s \text{ for } p \in \text{supp}F^*(p) \text{ and } V_s(p) \leq \overline{v}_s \text{for } p \in \left\{R^+ - \text{supp}F^*(p)\right\} \tag{2}$$

It is easy to see that there are no sellers willing to charge price $p > y$ in equilibrium, since even the buyer who likes the good would consider it too expensive. Also, no seller charges price $p \leq 0$, since her reservation value $\overline{v}_s$ is strictly positive. Another interesting observation follows.

LEMMA 1. *In equilibrium, any price $p \in (0, y]$ can not be charged by a positive mass of sellers.*

The proof is in the appendix.

The essential idea is that if a positive mass S_a of sellers charge price p_a, then an arbitrarily small decrease in the charged price p_a will discontinuously increase profits. This is because there is a positive flow of buyers who likes at least two goods for the price p_a. The direct implication of Lemma 1 is that if equilibrium exists, it is characterized by price dispersion. This is despite the fact there is perfect information. Note that, if a finite number of atomistic sellers is assumed instead, the same price would not be charged by two or more sellers in equilibrium. We can now turn to studying the equilibrium properties.

THEOREM 1 (Characterisation of Equilibrium). *The highest and the lowest price $(\overline{p}, \underline{p})$ and the distribution of prices $F(p)$ charged in equilibrium satisfy:*

$$\overline{p} = y \tag{3}$$

$$\underline{p} = c + \frac{\overline{v}_s r}{f_b \lambda} \tag{4}$$

$$(p - c) f_b \lambda e^{-\lambda S F(p)} = \overline{v}_s r \tag{5}$$

Proof. The highest price, $\overline{p}$, charged by a seller cannot exceed y, otherwise no one would willingly purchase her or his good. Zero turnover implies a lower value for the seller than the strictly positive reservation value $\overline{v}_s$. Note also that the seller's highest price cannot be strictly lower than y, otherwise an increase in $\overline{p}$ would not alter demand, hence profits would increase. Now let us focus on the equilibrium price distribution $F(p)$. Let $f(p)$ denote the density of $F(p)$. The value of being seller $V_s(p)$ can be written as:

$$\begin{gathered} rV_s(p) = (p - c)\, \tfrac{f_b}{f(p) S} \cdot \\ \cdot \left[e^{-\lambda S} \lambda S f(p) + e^{-\lambda S} \tfrac{(\lambda S)^2}{2!} 2 f(p) (1 - F(p)) + e^{-\lambda S} \tfrac{(\lambda S)^3}{3!} 3 f(p) (1 - F(p))^2 + ... \right] \\ = (p - c) f_b e^{-\lambda S} \lambda \sum_{i=0}^{\infty} \frac{[\lambda S (1 - F(p))]^i}{i!} \\ = (p - c) f_b \lambda e^{-\lambda S F(p)} \end{gathered} \tag{6}$$

The value of seller $V_s(p)$ must equal the reservation value $\overline{v}_s$. Equating $V_s(p)$ to $\overline{v}_s$ in (6) establishes (5). The lowest price charged in equilibrium can be computed from (5) by noting that $F(\underline{p}) = 0$. ∎

Now, we focus our attention to the existence of equilibria.

PROPOSITION 1. *For $0 < \overline{v}_s < \frac{1}{r}(y - c) \lambda f_b$ unique non-degenerate equilibrium $(S^*, F^*(p))$ with finite nonzero mass S^* of sellers exist:*

$$S^* = \frac{1}{\lambda} \ln \left(\frac{(y - c) \lambda f_b}{r \overline{v}_s} \right) \tag{7}$$

$$F^*(p) = \frac{-1}{\lambda S} \ln \left(\frac{\overline{v}_s r}{(p - c) f_b \lambda} \right) \tag{8}$$

Furthermore, this unique equilibrium is stable.

Both, the proof and the notion of stability considered are in the appendix.

Proposition 1 states that when the reservation value $\overline{v}_s$ of sellers is not very high, non-degenerate equilibrium exists. The density of price distribution $f^*(p) = F^{*\prime}(p) = \frac{1}{\lambda S^*(p-c)}$ is plotted in Figure 1.

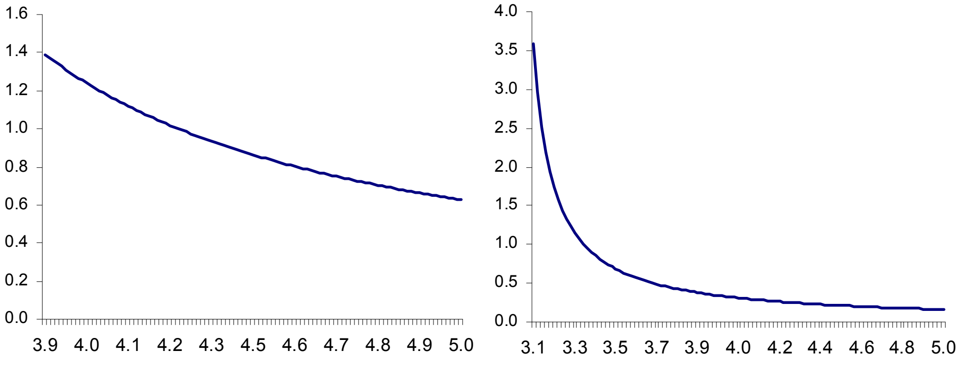

Figure 1: Density of the equilibrium price distribution, $f^*(p)$, for $\overline{v}_s = 0.36, c = 3, y = 5, f_b = 0.4, r = 0.1$, and two choices of $\lambda : 0.1$ (left chart) and 1 (right chart).

4. COMPARATIVE STATICS

There are six exogenous parameters: the preference parameter λ, the seller's reservation value $\overline{v}_s$, the buyer's utility gain y, the flow of new buyers in the market f_b, the cost of production c and the individual discount rate r. We consider the effect of exogenous parameters on the equilibrium stock of sellers S^* and on the equilibrium price distribution $F^*(p)$.

4.1. Equilibrium Mass of Sellers

The equilibrium stock of sellers is given by equation (7). S^* will increase with an increase in the buyer's utility gain y and the flow of new buyers in the market f_b. The mass of sellers will decrease with an increase in the seller's reservation value $\overline{v}_s$, the production cost c and the discount rate r. The effects of λ is unambiguous. For a market with supplied varieties close to each other, i.e. when heterogeneity parameter $\lambda < e\overline{v}_s r / [(y-c) f_b]$, an increase in λ would trigger an increase in the equilibrium stock of sellers. When goods supplied by the market are more heterogenous, i.e. $\lambda > e\overline{v}_s r / [(y-c) f_b]$, an increase in λ would trigger a decrease in S^*.

4.2. Equilibrium Distribution of Prices

A more interesting case is the equilibrium price distribution. We consider the first two moments and the threshold prices $\underline{p}^*, \overline{p}^*$. The highest price $\overline{p}^*$ always equals the buyer's valuation y, and therefore does not depend on the other exogenous parameters. The lowest equilibrium price $\underline{p}^*$ is increasing with the production cost c, the seller's reservation value $\overline{v}_s$ and the discount rate r. An increase in the heterogeneity parameter λ and the flow of new buyers f_b would decrease $\underline{p}^*$, but not below the production cost c.

Lemma 2. *The first two moments of equilibrium price distribution $F^*(p)$, denoted by M_1 and M_2 are given by:*

$$M_1 = E(p) = \int_{\underline{p}}^{\overline{p}} pF'(p)\,dp = \overline{p} - \underline{p} + c\ln\frac{\overline{p}-c}{\underline{p}-c} \tag{9}$$

$$M_2 = \int_{\underline{p}}^{\overline{p}} p^2 F'(p)\,dp = \frac{\overline{p}^2 - \underline{p}^2}{2} + \left(\overline{p}-\underline{p}\right)(2-c) + c\ln\frac{\overline{p}-c}{\underline{p}-c} \tag{10}$$

An increase in buyer's valuation y increases the expected price, $E(p)$. A higher inflow of new buyers f_b together with higher heterogeneity parameter increases $E(p)$ as well. Note, however, that the expected price does not equal the average purchase price. The latter is lower, since consumers who like more than one commodity purchase the cheaper good. An increase in the seller's reservation value $\overline{v}_s$ and the discount rate r decreases the expected equilibrium price. The impact of changes in the production cost is ambiguous. In the case when $yf_b\lambda/(r\overline{v}_s) > e$, there exists a threshold production cost c^* such that $\partial E(p)/\partial c > 0$ for $c < c^*$ and $\partial E(p)/\partial c > 0$ for $c > c^*$[2]. Otherwise (for $yf_b\lambda/(\overline{v}_s r) < e$) an increase in the production cost c triggers a decrease in $E(p)$.

The variance of the equilibrium price distribution can be computed as $\sigma^2 \equiv var(p) = M_2 - M_1^2$. An increase in the product heterogeneity parameter λ (which implies closer substitutability of goods) always decreases variance σ^2.

5. CONCLUSION

A marketplace where buyers and sellers meet costlessly was considered. Unlike in the standard matching framework, sellers are assumed to advertise not only the characteristics of goods for sale, but also the price. Due to the fact that each buyer likes only some of the supplied goods (if any), the unique equilibrium of this type of market is characterized by price dispersion. Price dispersion is generated due to heterogeneity in consumer' valuations over the goods supplied by sellers in the marketplace with full information.

[2] c^* satisfies: $(y-c^*)\,f_b\lambda/(\overline{v}_s r) = 1 + c^*/(y-c^*)$.

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APPENDIX A: PROOFS

Proof of Lemma 1. (By contradiction). Assume $\beta_1 S$ sellers, where $\beta_1 > 0$, charge price $p_1 \in (0, y]$. Let the value of the seller charging price p be denoted by $V_s(p)$ and let the equilibrium distribution of prices be denoted by $F^*(p)$ (i.e. $F^*(p)$ represents the proportion of sellers charging lower price than p). The value of the seller charging price p_1 can be expressed as:

$$\begin{aligned} rV_s(p_1) = \quad & (p_1 - c)\,\tfrac{f_b}{\beta_1 S}\left(1 - e^{-\lambda\beta_1 S}\right)e^{-\lambda(1-\beta_1)S} + \\ & + (p_1 - c)\,\tfrac{f_b}{\beta_1 S}\left(1 - e^{-\lambda\beta_1 S}\right)\cdot\left(1 - e^{-\lambda F^*(p_1)S}\right)\cdot e^{-\lambda(1-F^*(p_1)-\beta)S}] \end{aligned} \tag{11}$$

With probability $\left(1 - e^{-\beta_1 S}\right)e^{-(1-\beta_1)S}$ a new buyer likes at least one of the goods for price p_1 and dislikes any other good. Hence the first term in the right side of the Bellman equation (11) is the rate at which only some of goods for price p_1 are liked multiplied by the gain $(p_1 - c)$. c represents the cost of production. The second term captures the possibility that some buyers who like at least one good for price p_1 and also some other good(s) for price $p \neq p_1$ might be willing to purchase for price p_1 (provided p_1 is cheaper). Now we will show that every seller charging price p_1 will have incentives to deviate and charge price $p_2 = p_1 - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small, therefore no price can be charged by a positive mass of sellers in equilibrium. The value of being a seller with price $p_2 = p_1 - \varepsilon$ can be written as:

$$\begin{aligned} & rV_s(p_1 - \varepsilon) = \\ & (p_1 - \varepsilon - c)\,\tfrac{f_b}{\beta_1 S}\left[e^{-\lambda\beta_1 S}\lambda\beta_1 S + e^{-\lambda\beta_1 S}\tfrac{(\lambda\beta_1 S)^2}{2!}2 + e^{-\lambda\beta_1 S}\tfrac{(\lambda\beta_1 S)^3}{3!}3 + \ldots\right]e^{-\lambda(1-\beta_1)S} \\ & + (p_1 - \varepsilon - c)\,\tfrac{f_b}{\beta_1 S}\left(1 - e^{-\lambda\beta_1 S}\right)\cdot\left(1 - e^{-\lambda F^*(p_1-\varepsilon)S}\right)\cdot e^{-\lambda(1-F^*(p_1-\varepsilon)-\beta)S} \\ = \quad & (p_1 - \varepsilon - c)\,f_b\lambda e^{-\lambda(1-\beta_1)S} + \\ & + (p_1 - \varepsilon - c)\,\tfrac{f_b}{\beta S}\left(1 - e^{-\lambda\beta_1 S}\right)\cdot\left(1 - e^{-\lambda F^*(p_1-\varepsilon)S}\right)\cdot e^{-\lambda(1-F^*(p_1-\varepsilon)-\beta)S}] \end{aligned} \tag{12}$$

Where $e^{-\lambda\beta_1 S}\frac{(\lambda\beta_1 S)^k}{k!}e^{-\lambda(1-\beta_1)S}$ represents the probability that exactly k goods supplied by $\beta_1 S$ sellers and no other goods are liked. Note that for $k > 1$, the price $p_2 = p_1 - \varepsilon$ attracts higher demand. There is a discontinuous positive jump in the first term of the right side of the seller's Bellman equation, which establishes $V_s(p_1) < V_s(p_1 - \varepsilon)$ for ε being satisfactorily small. ∎

DEFINITION 2. Equilibrium $(S^*, F^*(p))$ is said to be stable if a small increase (decrease) in S^* would trigger a decrease (increase) in V_s for nonzero subset of sellers, i.e.:

$$\begin{gathered}\exists\varepsilon > 0, \forall\delta \in [0,\varepsilon] : \\ i)\ V_s(p, S^* + \delta) \leq V_s(p, S^*)\, \forall p \in F_S \text{ with strict inequality holding for } p \in H_1 \subseteq F_S^* \\ ii)\ V_s(p, S^* - \delta) \geq V_s(p, S^*)\, \forall p \in F_S \text{ with strict inequality holding for } p \in H_2 \subseteq F_S^*\end{gathered} \tag{13}$$

where F_S^* denotes the support of $F^*(p)$ and sets H_1 and H_2 have nonzero measure. Otherwise equilibrium is said to be unstable.

Proof of Proposition 1. Equilibrium mass S^* of sellers can be solved from the value function of the seller charging the highest price $\overline{p}$:

$$rV_s(\overline{p}) = (\overline{p} - c) f_b e^{-\lambda S}\lambda = \overline{v}_s r \tag{14}$$

Noting that $\overline{p} = y$ in equilibrium (Theorem 1), we can solve for S^* in equation (14), which yields (7). It is clear from (7) that a solution exists (note that there cannot be a negative mass of sellers in the market) if and only if $(y - c)\lambda f_b / (r\overline{v}_s) > 1$, i.e.: the reservation value of the seller must be lower than $(y - c)\lambda f_b / r$ for S^* to be positive.

Theorem 1 states that any equilibrium distribution of prices must satisfy equation (5), which can be directly solved for $F(p)$. This establishes (8).

To prove the stability of equilibria, consider equation (6) describing $V_s(p)$. Any increase in equilibrium mass of sellers, would decrease $V_s(p)$ for all $p \in \text{supp}F(p)$. On the other hand, a decrease in S^* would trigger an increase in V_s for every seller in the market. ∎