

Federal Reserve Bank of Dallas
Globalization and Monetary Policy Institute
Working Paper No. 122

<http://www.dallasfed.org/assets/documents/institute/wpapers/2012/0122.pdf>

The Between Firm Effect with Multiproduct Firms^{*}

Tuan Anh Luong
Shanghai University of Finance and Economics

July 2012

Abstract

This paper studies multi-product firms with two factors of production: unskilled and skilled labor (talent). Creating new products is skill intensive while production is less skill intensive. We show here a new effect: an increase in the skilled labor supply, relatively to unskilled labor, could reduce the number of products but increase the average scale per product. The relative strength of this effect depends on the degree of firm heterogeneity and the extent to which we allow multiple products within the firm. Moreover, the survival cut-off can be higher (or lower) if the fixed costs (or the variable costs) are lower. Economic integration influences this survival cut-off only through the ratio of skilled labor to unskilled labor, but not the market size. This policy is welfare enhancing but the gains might be non uniformly distributed across agents. The paper also sheds light on the pattern of trade with only one industry.

JEL codes: F1, L1

^{*} Tuan Anh Luong, Shanghai University of Finance and Economics, 777 Guoding Road, Shanghai 200433, China. +86-15000047975. tuanluong@gmail.com. The previous title of the paper is "Multiproduct Firms and the Endogeneous Choice of Variety." I would like to thank Marc Melitz, Esteban Rossi-Hansberg, Stephen Reddings, Oleg Itskhoki, Zhihao Yu, Frederic Robert-Nicoud, Larry Qiu, Jonathan Vogel and participants at the seminars of Shanghai University of Finance and Economics, Beijing University of International Business and Economics, the MidWest International Trade Meeting, the North America Econometric Society Meeting, the World Congress for International Economics for helpful comments and suggestions. I am especially grateful for the guidance of my advisors, Gene Grossman and Penny Goldberg. All the remaining errors are my own. The views in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

1. Introduction

The importance of multi-product firms has been well documented in the literature (for instance, Arkolakis and Muendler 2009, Bernard, Redding and Schott 2010). It is therefore not surprising to see a large and growing literature looking at the firm's scope. There are essentially three main approaches to tackle this question. The first one relies on the cannibalization effect: a new variety will cannibalize the existing ones, therefore reduce the incentive for the firm to expand their scope (Dhingra 2011, Feenstra and Ma 2008). The two other approaches assume varieties within a firm can be different. One might assume that each firm has a core product which can be produced with the lowest cost. The efficiency of the firm drops when they decide to expand their scope and therefore depart from their core competence. Therefore the firm will produce the core product first and extend its scope as far as the marginal cost is low enough to make a profit (Eckel and Neary 2010, Mayer, Melitz and Ottaviano 2010). One can also think varieties are different along the demand side. Bernard, Redding and Schott (2010,2011) assume each variety has a consumer taste shock. The firm can expand their scope as long as the taste shocks to their varieties are favorable enough according to their productivity. In particular, a more productive firm can suffer bad taste shocks but a less productive firm will need positive shocks for their survival. In this paper, we revisit the cannibalization effect with a new perspective. To my knowledge, most papers study this effect *within firm*: new varieties introduced reduce the demand of other varieties within the firm. The *between firm* effect is less discussed: new varieties also reduce the demand of the competitors' varieties. This effect is documented to be important (Luong and Chen 2012) and will be the main focus in our paper.

The previous title of the paper is "Multiproduct firms and the endogeneous choice of variety. I would like to thank Marc Melitz, Esteban Rossi-Hansberg, Stephen Reddings, Oleg Itskhoki, Zhihao Yu, Frederic Robert-Nicoud, Larry Qiu, Jonathan Vogel and participants at the seminars of Shanghai University of Finance and Economics, Beijing University of International Business and Economics, the MidWest International Trade Meeting, the North America Econometric Society Meeting, the World Congress for International Economics for helpful comments and suggestions. I am especially grateful for the guidance of my advisors, Gene Grossman and Penny Goldberg. All the remaining errors are my own.

*Corresponding author. Tel.: +86 15000047975; email:tuanluong@gmail.com

¹Present address: Shanghai University of Finance and Economics, 777 Guoding Road Shanghai 200433, China

The paper has a Heckscher-Ohlin flavor. Indeed, similar to the Heckscher-Ohlin set up, there are two factors (skilled and unskilled labor) and two seemingly sectors: the "extensive" sector which is to extend the scope of the firm is skilled labor intensive and the "intensive" sector which is to increase the scale of the firm is unskilled labor intensive. Therefore according to the Heckscher-Ohlin model, one might expect that the country that is skilled labor abundant would produce more scope and become the "scope" exporter. In other words, they will produce more varieties and export them to the other country. The latter country, which is unskilled labor abundant by construction, would be the "scale" producer and exporter. However, the new ingredients of trade theories (namely monopolistic competition and firm heterogeneity) allows me to introduce a new dynamic that might reverse this classical result. Indeed, what is in the Heckscher-Ohlin is the within-firm effect: with more skilled labor, every firms could expand its scope; with more unskilled labor the firm could expand its scale. Then the story will end there. In my paper, this within-firm effect has two implications: On the demand side, new varieties reduce the market shares of the existing varieties. On the supply side, more varieties lead to more demand for unskilled labor, bids up their wages and as a result, increases the production costs. These two implications result in lower profit for the unproductive firms, leading to more of them ceasing production. This between-firm effect is on the opposite direction of the within-firm effect. In other words, the new effect weakens the classical one. Is it an important result? Based on this result, Luong and Chen (2012) document that skilled labor is responsible for the intensive growth in export, while unskilled labor is responsible for the extensive growth in export. That is because the between firm effect reduces the impact of skilled labor on the extensive margin and enables unskilled labor to influence this margin. By the same token, the between firm effect implies that unskilled labor could reduce the intensive margin while skilled labor now could raise this margin. The between firm effect mitigates the impact of the within firm effect to the extent that skilled labor has the bigger role (than unskilled labor) in explaining export growth along the intensive margin, while unskilled labor influences the extensive margin more. These finding then can be used to explain the importance of the two margins in export, which is crucial to determine the gains from Trade.

Different conclusions about the number of firms and the number of varieties they supply are drawn depending on whether firm heterogeneity is allowed or not. Feenstra and Ma (2008) show that an increase in market size

only leads to more producing firms if they are homogenous, their number stays unchanged when firm heterogeneity is introduced. Firm heterogeneity can also change the conclusion regarding firm scope. Using a nested CES preference utility function Allanson and Montagna (2005) show that in the long run, a firm's scope is independent of the market size. In Mayer, Melitz and Ottaviano (2010), the firms responded to more intense competition by focusing on a few core products and dropping those far from the core. We will show that the degree of firm heterogeneity strengthens the between firm effect, as does the multiple product feature which is the main feature of our paper.

Our paper is related to the New New Trade literature, started from Melitz (2003). We make two new features: the firm is allowed to produce more than one product and they need another type of labor (talent) to extend their scope. A firm introduces a new variety in our model is similar to a firm starting to export to a new destination in Melitz (2003) model. However while in the New New Trade model, lowering costs (in particular the fixed costs) would lower the productivity cut-off as that would make it easier for the unproductive firms to make a profit. We show here that depends on the type of costs: Lowering the fixed costs (or the costs of scope extension) would raise the cut-off instead.

Another interesting result of this paper is the non-uniformity of the impact on the firm's scope: the productive firms extend their scope while the non productive ones shrink to the extent that some of them quit the market. This non-uniform effect is different from that shown by Bernard, Redding and Schott (2010), Mayer, Melitz and Ottaviano (2010). A similar result can be found in Qiu and Zhou (2010) with the alternative linear demand. There is evidence that firms adjust their scope non-uniformly. For instance, some Mexican firms, in particular the more experienced exporters, introduce new varieties while others have to drop the fringe products when foreign markets become more accessible (Iacovone and Javorcik 2008). And in India, the average extensive margin did not seem to change when trade reforms took place in the late 1990s (Goldberg et al. 2010).

When the two countries engage in free trade, if we allow one of the factors (for instance unskilled labor) to be mobile, we can show that the two countries form an integrated economy as if both factors were allowed to move. Different from other trade models, in particular Melitz (2003), trade affects the productivity cut-off not through the market size but rather via

the ratio of the two factors. This is because while more unskilled labor lowers the cut-off, more skilled labor raises it. We also show that trade is welfare enhancing although the gains are not uniformly distributed.

When no factor is mobile, although there is one industry trade pattern is no longer indeterminate. As we discuss above, the industry is decomposed into two seemingly sectors which allow us to open the black box of trade pattern. More skilled labor in one country results in the firms in this country are more productive on average, have bigger scope but smaller scale per variety. Therefore this country will export more of varieties of low prices while import more of varieties of high prices from the other country.

The paper is organized as follows. The next section discusses the case of the country in autarky. The important result in this case is that the market becomes more competitive with more talent. As a result, the price index falls and welfare rises. In section III, the case of open economies is discussed. In particular, we consider two alternatives. The first is when one type of labor, in this case is unskilled labor is mobile across countries. International trade expands the market size, therefore allows the more productive firms in both countries to raise their product portfolio. Therefore, similar to an increase in talent endowment, the toughness in the markets in both countries rises, which is welfare enhancing to the consumers. From the supplier of labor point of view, only talent in the country with a lower talent endowment might lose as their income falls. In the second alternative, neither type of labor is mobile. In this case, the market in the country with more talent is more competitive. This country exports the goods with relatively lower prices and imports the goods with relatively higher prices.

2. The basic framework

Let us assume in the country there are L workers and S managers in an economy. They have different skills that are used in different positions, as will be clarified later. The labor market is assumed to be competitive. All the workers (unskilled) and managers (skilled) are, however, consumers with the same preferences. There are an infinite number of potential firms in a single industry. The firms compete in a monopolistic manner. Each of them, if they decide to produce, can choose how many products and how much of each product they want to supply.

2.1. Consumer's preferences and demand

Our consumers' preferences follow the Dixit-Stiglitz framework. In particular, the consumers can buy products from a certain number M of producing firms, each of them supply n symmetric products². As a result, the utility of a representative consumer is given by:

$$U = \left(\int c(\theta)^\rho Mn(\theta)f(\theta)d\theta \right)^{\frac{1}{\rho}} \quad 0 < \rho < 1.$$

The function $f(\theta)$ is the probability density function of the firms' productivity. Assume that a representative consumer has disposable income E . His consumption equation will be:

$$\max_{\{c(\theta)\}} U \text{ s.t. } \int p(\theta)q(\theta)Mn(\theta)f(\theta)d\theta = E,$$

This yields the demand:

$$q(\theta) = \left(\frac{p(\theta)}{P} \right)^{\frac{-1}{1-\rho}} \frac{E}{P}$$

where P is the price index in the industry:

$$P = \left(\int p(\theta)^{\frac{-\rho}{1-\rho}} Mn(\theta)f(\theta)d\theta \right)^{-\frac{1-\rho}{\rho}}. \quad (1)$$

Since all the consumers have the same preferences, total demand for each variety is given by:

²We can assume that there are heterogeneity within firm across varieties. However, we then need to rank the varieties, either by their costs (as in Arkolakis and Muendler 2010, Mayer Melitz Ottaviano 2009) or by their demand (as in Bernard Redding and Schott 2010). Since the product characteristics are of less importance in this paper, I adopt the industrial organization approach by assuming that all the varieties are symmetric to the consumers.

$$q(\theta) = \left(\frac{p(\theta)}{P} \right)^{\frac{-1}{1-\rho}} \frac{R}{P}, \quad (2)$$

where R is the total revenue (or total spending) in the industry.

2.2. Production of multiproduct firms

To enter the industry each firm has to pay a fixed entry cost f_e in units of unskilled labor, which is used as the numeraire. Denote M_1 the number of firms that pay the entry cost. Upon the entry cost, they draw their productivity from the usual Pareto distribution function:

$$f(\theta) = k \frac{\theta_0^k}{\theta^{k+1}}.$$

The parameter k indicates the degree of firm heterogeneity. The lower k , the more heterogeneous the firms are.

The firm can draw their products from a continuum of potential varieties, but launching a new variety is costly as documented by Schoar (2002). This assumption is also used in Nocke and Yeaple (2006) where they assume that the marginal cost increases with the number of varieties. Their assumption is however inconsistent with several empirical findings. For example Lee and Tang (2001) report that in Canada firms with more than 500 employees and firms with between 100 and 500 employees are 17 percent and 15 percent respectively more productive than firms with less than 100 employees. The results they found regarding the U.S. were similar. Other evidence can be found in Van Ark and Monnikhof (1996) and Baldwin, Jarmin and Tang (2002). We will assume that the marginal cost is independent of the number varieties the firm supplies but the fixed cost will rise with them, like in Arkolakis and Muendler (2010).

To make this cost explicit, we adopt an approach similar to Grossman-Rossi Hansberg (2008). Production is divided into 2 tasks. There are the tasks fit for low-skilled workers and those fit for high-skilled workers. The tasks that require low-skilled workers are the simple production process: workers with limited qualification can be hired to participate in this process. The unit cost of production, in units of unskilled labor, depends on the productivity θ of the firm:

$$c(\theta) = \frac{1}{\theta}.$$

The tasks that require high-skilled workers are the managerial service: to coordinate the many brands produced by the firm, to ensure that they fit into the strategy of the firm, we need people with sophisticated skills, which we call talent. More specifically, the managerial service has the production function form:

$$n = s^{1/m}, m > 1$$

where s is the talent input. Equivalently the number of managers required to manage n varieties is:

$$F_m(n) = n^m$$

The parameter m represents the managerial "technology" in this country. A low m indicates the firms are more efficient in managerial use. It also indicates the quality of the managers. The more skilled they are, the lower m is³. This parameter controls for the multiple product feature of the model: A very high value of m prevents the firm from increasing the product portfolio, which brings us towards to the case of a single-product firm.

This set-up merits discussions here. First, we assume non-linear management costs, which makes my model different from the ones used in Arkolakis and Muendler (2010), or in Ottaviano and Thisse (2011). In this context when the varieties are symmetric, the linear costs assumption may seem inappropriate. Indeed, with linear costs the firm can supply an infinite number of varieties as long as their profit per variety is higher than the fixed cost. Moreover, the total number of varieties in the economy is entirely determined by the supply of talent. Second, we assume heterogeneity across firms in unskilled labor, but not skilled labor. This is because the firm does

³Another way to justify this management cost is to think the managers as skilled labor in charge of R&D. Assuming no uncertainty, a firm that wants to expand its scope needs to hire a certain number of scientists, depending on the scope and the skills of the scientists.

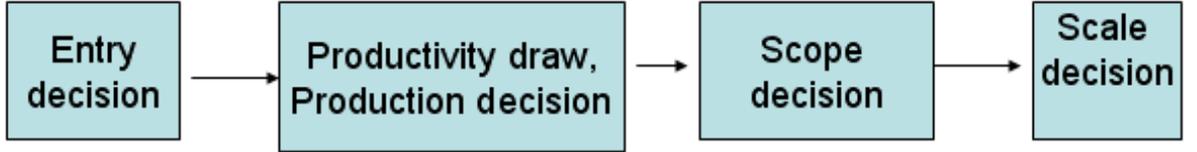


Figure 1: The firm's activities

not own talent, but machines which make the workers (unskilled) more productive. Talent, however, does not belong to the firm (see, for instance, Maria and Verdier 2012). Therefore their productivity is not firm-specific. Since training is absent here, their productivity is the same across firms.

Together with choosing the number of brands to develop, the firm decides how much to produce of each brand. Hence the problem of choosing the quantity to supply for each brand is as follows:

$$\max_p pq - cq.$$

With the demand given from (2), we have the pricing strategy

$$p(\theta) = \frac{c(\theta)}{\rho}, \quad (3)$$

which yields the profit per brand:

$$\pi_1(\theta) = (1 - \rho)P^{\frac{\rho}{1-\rho}}Rc(\theta)^{-\frac{\rho}{1-\rho}}. \quad (4)$$

The sequence of the firm's activities is summarized in Figure 1. It is important at this stage to clarify the notation for the profits of the firms. Upon entry each producing firm needs to pay the variable production costs, the management costs and the fixed cost of production, as illustrated in Figure 2.

We will use the subscripts 1, 2, 3 to denote the profit of the firms after paying each of the different types of cost. The total profit of the industry

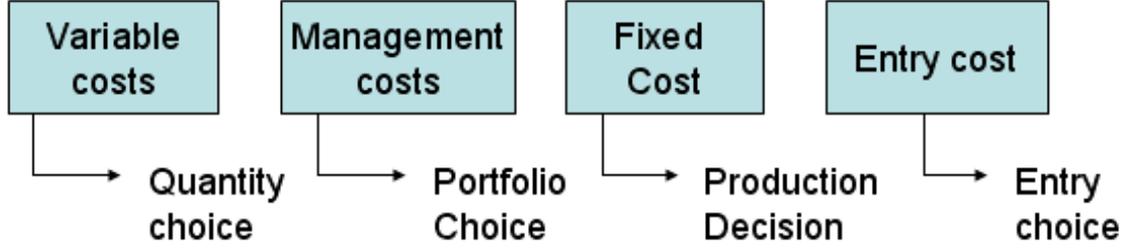


Figure 2: The firm's decisions

after paying the variable cost (payment to workers) is given by:

$$\Pi_1 = \int_{\hat{\theta}}^{\infty} M_1 n(\theta) \pi_1(\theta) k \frac{\theta_0^k}{\theta^{k+1}} d\theta$$

Besides choosing the intensive margin and paying its workers, the firm also has to decide on its optimal scope. If it chooses to supply n brands, it needs to hire n^m managers, each of whom receiving a wage w . Their management cost is then wn^m , which leads to the equation for choosing the optimal scope as:

$$\max_n n \pi_1(\theta) - wn^m$$

This equation yields the following solution:

$$\pi_1(\theta) = wmn^{m-1}(\theta) \quad (5)$$

We see here that the firm's scope increases with its productivity. Unlike in Feenstra and Ma (2009), we do not have an inverted U-shape: this is because the cannibalization effect (within firm) works the same way on every firm, due to the fact that the mark-up is constant which is a feature of the Dixit-Stiglitz framework.

2.3. Equilibrium

Now after investigating the decisions of the individual firms, we are able to study the aggregate outcome. There are S managers in the country, so the total management cost is wS . The total profit in the industry is then:

$$\Pi_2 = \Pi_1 - wS.$$

Finally, each firm has to pay a fixed cost F in units of labor. From (5) we see that the profit of a firm with productivity θ after paying the management cost is $\frac{m-1}{m}n(\theta)\pi_1(\theta)$.

Since the cost of scope expansion is exponential and the benefit of adding new variety is linear, the marginal firm is the one that breaks even with one variety. Therefore the zero-profit cutoff condition is

$$\frac{m-1}{m}\pi_1(\hat{\theta}) = F. \quad (6)$$

Here $\hat{\theta}$ denotes the cutoff productivity level. The probability that an entrant is active is then $\frac{\theta_0^k}{\hat{\theta}^k}$, which yields the relationship between M_1 and M_2 :

$$M_2 = M_1 \frac{\theta_0^k}{\hat{\theta}^k}. \quad (7)$$

Since only incumbent firms pay the fixed costs, the total net profit in the industry is given by:

$$\Pi_3 = \Pi_2 - M_2F.$$

There are an infinite number of potential entrants who are free to enter the industry if they pay the entry cost, the ex-ante expected profit they would receive upon entry has to be at least equal to the entry cost. The expected profit they receive is $\int_{\hat{\theta}}^{\infty} n(\theta)\pi_3(\theta)k\frac{\theta_0^k}{\theta^{k+1}}d\theta$ where $\pi_3(\theta)$ is the net

profit per brand. We then have the free entry condition:

$$f_e = \int_{\hat{\theta}}^{\infty} n(\theta) \pi_3(\theta) k \frac{\theta_0^k}{\theta^{k+1}} d\theta. \quad (8)$$

In the monopolistic framework, the firms have a constant markup, as shown in the pricing strategy (3). This feature implies that the variable costs (which are the payment to the workers) are proportional to the total revenue in this industry. As a result, the total net profit of the whole industry after paying variable costs is also proportional to total revenue

$$\Pi_1 = (1 - \rho) R. \quad (9)$$

The management cost can be calculated by replacing the number of products per firm by (5) in the talent market clearing condition:

$$wS = \frac{\Pi_1}{m}. \quad (10)$$

From (8) we can show that the total net profit of the active firms is equal to the entry cost that the new entrants have to pay:

$$\Pi_3 = M_1 \int_{\hat{\theta}}^{\infty} n(\theta) \pi_3(\theta) k \frac{\theta_0^k}{\theta^{k+1}} d\theta = M_1 f_e$$

Labor in this economy is used to pay for the entry cost, the variable production cost, and the fixed cost. With the entry cost equal to Π_3 and the management cost proportional to total revenue (see equations 9 and 10), we have:

$$R = \frac{L}{1 - \frac{1-\rho}{m}}. \quad (11)$$

Lemma 1. *Total income R is proportional to the number of unskilled workers and is unchanged when the talent endowment increases.*

This result stems from the CES feature. Indeed, since the mark-up is constant, total revenues is proportional to production costs, which only use unskilled labor.

Then from (10) and (11) we can calculate the salaries of the managers:

$$w = \frac{(1 - \rho) R}{mS} = \frac{(1 - \rho) L}{(m - 1 + \rho) S}. \quad (12)$$

An increase in talent endowment lowers the price of talent. Having more managers also induces more competition in the good market, leading to the following result:

Proposition 2. *More talent relative to unskilled labor makes the competition among firms more intense: the productivity cut-off level is higher.*

Proof. See Appendix A.2. ■

The intuition is as follows. Higher talent endowment reduces talent wages, as shown in (12). As a result, (5) implies that the firms are able to add more varieties. These new varieties result in two effects. On the demand side the revenues of the unproductive firms shrink due to the arrival of the new varieties with lower prices. This effect is similar to a decrease in the market size. However, we have a non-uniform effect here: the least productive firms take the hardest hit because their price is much higher than the others'. This effect is more pronounced when the degree of firm heterogeneity is high (k is low). The second effect is on the supply side. The new varieties added would imply more demand for labor, therefore bids up their wages. The production costs rise as a result. This effect is similar to what is described in Melitz (2003) when the firms are allowed to export: a new foreign market is a new variety added. The extent of this effect depends on how many new varieties are added into the market, which depends on the management technology of the country, as in (5). When the parameter m is low, the firms are efficient in managing their brands, and more varieties will be added. For the unproductive firms, not only they have lower revenues (the demand side effect) but also higher production costs (the supply side effect). It is therefore more difficult to stay in the market: the survival cut-off is higher.

By construction, the use of talent is solely to extend the firm's scope. More talent therefore reduces this fixed cost⁴, which in Melitz's (2003) framework should lower the cut-off. What we see here is the opposite: more talent implies a higher cut-off: in regards of the cut-off, the between firm effect dominates the within firm effect.

The impact on firms' scope has been studied when the market becomes more competitive (Bernard, Redding and Schott 2010, Eckel and Neary 2010, Mayer, Melitz and Ottaviano 2010). In these models, each firm is supposed to be most efficient in their core product and less so in the others. When competition intensifies, the firms drop the marginal products. In other words, the firms react more or less the same way. In our set-up, the effect is non-uniform: the more productive firms add more varieties while the less productive firms drop some varieties. The reason is because all products here are symmetric, the "marginal" products are the ones produced by the least productive firms. As a result, trade liberalization favors the more productive firms. A similar result can be found in Qiu and Zhou (2010) with the alternative linear demand. The hypothesis that the firms respond non uniformly after trade liberalization is supported by what happened in Mexico (Iacovone and Javorcik 2008) and in India (Goldberg et al. 2010).

As the cutoff level is higher and the total income R is a constant, (20) implies that the price index is lower (a formal proof is given in Appendix A.3). This is the result from the usual selection effect: the varieties with high prices are now replaced by these with lower prices. From (7) and (21) we see that after an increase in the number of talent, the industry is more attractive as it is easier to add another variety. As a result, there are more firms who are willing to pay the entry costs. However, the industry is also more competitive. After receiving the productivity draw, only the very productive firms produce. In other words, the survival cut-off is higher. This result does not depend on the assumption that managers are only employed in the management and the fixed cost F are denoted in unskilled labor. What does matter is the fact that a larger talent endowment enables the productive firms to add new varieties into their product portfolio.

With the wage taken as the numeraire, the workers benefits from an

⁴In fact, it is not important whether this is a fixed cost or a variable cost. In Melitz (2003) framework, lower costs allow higher profit for every firm, allowing the unproductive firms in particular to stay.

increase in talent endowment since their purchasing power is higher. The managers, however, may be worse off because their wage is lower than before. Indeed, their welfare depends on how many of them in the market: if there are not many of them then they are better off when there is additional talent; but when there are already many managers in the market, a few more of them means a reduction in their welfare (the proof is given in Appendix A.3). This result is intuitive: if there are many managers in the market, it becomes more difficult for the firms to recruit the new managers as they bring little profit to the firms. This lowers the compensation of the current managers, making them worse off. We summarize those results by the following proposition:

Proposition 3. *A relative increase in the supply of talent lowers the price index, which provides welfare gains for workers. Managers are only better off when the skilled-unskilled ratio is small and are worse off otherwise.*

Proof. See Appendix A.3. ■

As discussed above, the extent to which talent endowment influences economic variables such as the price index, the firm's scope, the consumers' welfare, etc. depends on the parameter m . In particular, when this parameter takes a high value (we converge to the case of a single-product firm), the impact of talent endowment falls: In other words, the multiple-product feature of the model is the framework based on which we can study how factor endowment influence the competitiveness of the market.

3. Economic integration

In this section I consider two countries, Home and Foreign (denoted by an asterisk) who integrate into a common market: consumers will have access to the same pools of goods supplied by all the firms in both markets. The two countries differ only in their talent endowment: the Home country is assumed to have more talent than the Foreign country. A firm's nationality is defined as the location of its headquarters. This model also assumes that managers cannot move across borders; labor, however, may or may not move across the border ⁵. The two possibilities will be discussed below.

⁵We will see later that the immobility example leads to factor price equalization, therefore the assumption of labor mobility can be made without loss of generality.

3.1. Labor mobility

In this case, unskilled labor can move freely across borders. An example of this case is the European Union, in which people are free to work in the country of their choice ⁶. With a competitive labor market, the wages for workers have to be the same across countries. From now on we will take this wage level as the numeraire.

We will use the subscripts H and F to distinguish the variables in the Home and in the Foreign country respectively. However, the subscripts will be dropped if there is no confusion. Due to the trade balance condition, total revenues of the firms and total incomes of the workers and managers in each country have to be equal:

$$R = L + wS$$

Again due to constant markup total income of each country is proportional to the number of workers in that country:

$$R = \frac{L}{1 - \frac{1-\rho}{m}},$$

As a result, the wages of managers in each country are given by:

$$w = \frac{(1 - \rho) L}{(m - 1 + \rho) S} \quad (13)$$

$$w^* = \frac{(1 - \rho) L^*}{(m - 1 + \rho) S^*}. \quad (14)$$

Because of labor mobility, the wages of workers are equal across countries. Without loss of generality, assume that the salaries of managers in

⁶One can think of some moving costs that restrict people from moving. Low skill workers, in particular from Eastern Europe, might have low moving costs and are willing to move to where they are offered better wages. Skilled labor in this model are the managers who have such a high moving costs that they do not want to reallocate their family.

the Home country are lower than in the Foreign country. Since the wages of workers are equal, the marginal costs of two firms with equal productivity⁷, one in each country, are also equal. Consequently, they have the same sales, as well as the same profit per product. As the salaries of managers in the Home country are lower, the Home firm develops more products than the Foreign firm.

This is, however, not a stable equilibrium. In fact, for any firm in the Foreign country, there is at least one firm in the Home country that can use the one additional unit of labor more efficiently. Indeed, with one additional unit of labor, the Home firm can produce θ more products, or an increase of $\frac{\theta}{n_H(\theta)}$ per variety. The Foreign firm with the same productivity level would produce $\frac{\theta}{n_F(\theta)}$ more products per variety. Since $n(\theta) > n^*(\theta)$, the supply effect on the Home firm is less severe: its price drops less than that of the Foreign firm. As a result, its total sales are higher than the Foreign firm's. Consequently, the Home firm can pay more to one additional unit of labor than the Foreign firm can, which results in a movement of labor from the Foreign country to the Home country. This labor movement stops when we have factor price equalization.

The consumers in both countries can buy from the same basket of goods, therefore there will be the common price index:

$$P = \left[\int_{\hat{\theta}} M_1 p^{-\frac{\rho}{1-\rho}}(\theta) n(\theta) k \frac{\theta_0^k}{\theta^{k+1}} d\theta + \int_{\hat{\theta}^*} M_1^* p^{-\frac{\rho}{1-\rho}}(\theta) n^*(\theta) k \frac{\theta_0^k}{\theta^{k+1}} d\theta \right]^{-\frac{\rho}{1-\rho}},$$

where $\hat{\theta}_H$ and $\hat{\theta}_F$ denote the Home and Foreign cutoffs. With this same price index and the same pricing strategy due to the monopolistic competition, two firms with the same productivity level in the Home and Foreign countries will have the same gross profit (after paying the labor production cost) per variety:

$$\pi_1(\theta) = \pi_1^*(\theta).$$

⁷From now on, I will compare the firms in the Home country with the firms in the Foreign country with the same productivity level. They will be called comparable firms.

Formula (6) that determines the cut-off level then implies that the cut-offs in both countries are the same. As workers are paid equally, if the firms have the same productivity level in both countries, they will have the same marginal cost, or the same unit price as in (3). From the demand function (2), for each brand they develop they have the same sales, the same revenues and the same profit π_1 . Since management costs are equal due to factor price equalization, from the optimal scope condition (5) the firms in the Home country have the same number of brands as the comparable ones in the Foreign country. As a result, we have an integrated economy in which national welfare is improved.

Proposition 4. *With one factor mobile, the two countries form an integrated economy: the two countries have the same survival cut-off with factor price equalization.*

With this integrated economy, we can apply all the results in the previous section. In particular, there are three important results in this case. The first one is that, compared to autarky the survival cut-off is higher in the Foreign country while that in the Home country is lower. Economic integration only changes the cut-off through the ratio of skilled to unskilled labor, not the market size. In particular if two identical countries integrate, the cut-off will be the same. This is because while more skilled labor raises the cut-off, more unskilled labor lowers it. Economic integration, however, has a clear impact on welfare.

Proposition 5. *Economic integration with labor mobility is welfare enhancing for both countries. However the gains from trade are not uniform across people. Almost all consumers are better off, except for the managers in the Foreign country who could be worse off when talent endowments are greatly different and the degree of firm heterogeneity is small.*

Proof. See Appendix A.4 ■

This lower price index implies that both countries gain from trade (note that the total revenues are proportional to the number of workers). However, the gains are not distributed equally among consumers. The workers in both countries are the clear winners because the price index, in terms

of their wages, decreases. The welfare effect on managers is less clear-cut. Talent in the Home country is the winner with the largest gains because not only is the price index lower, their salaries also increase since the relative factor endowment favors them in free trade ($\frac{L}{S} < \frac{2L}{S+S^*}$) according to equations (12) and (13). Talent in the Foreign country, however, can find themselves disadvantaged compared to under autarky. Indeed, their salaries are lower since the relative factor endowment does not favor them in free trade ($\frac{L}{S^*} > \frac{2L}{S+S^*}$). Therefore, if the decrease in their salaries is more than that of the price index, they are worse-off. This happens when the talent endowments are sufficiently different as this reduces significantly the salaries of Foreign managers; and when the firms are less heterogeneous (note that the selection effect here, which results in the price index drop, depends on the degree of firm heterogeneity). This result is in line with Krugman (1994) when he shows that the scarce factor (which is talent in our case) loses when the two countries are dissimilar in their factor endowment and when the products are less differentiated.

3.2. Labor immobility

We have seen in the previous section that when labor is free to move across borders the two countries form an integrated economy where the wages are the same. As a result, the firms in both countries behave the same way: with the same productivity level, they develop the same number of products and sell the same amount of units per product. In this section, labor is immobile. We will see that the production costs, in particular the salaries of workers and managers are different, which implies that the comparable firms will not produce the same number of goods. First, we will prove the following result:

Lemma 6. *If the firms in one country produce more products than the comparable firms in the other country, the former will produce less units per product than the latter. In other words, if $n(\theta) > n^*(\theta)$ then we must have $q(\theta) < q^*(\theta)$ and vice versa.*

Proof. Without loss of generality, let us assume that the Home firm both supply more varieties and produce more per variety than the comparable firm in the Foreign country. Since the scale of the Home firm is higher, its

marginal cost has to be lower than that of a foreign firm with the same productivity level, which implies that the wage of workers in the Home country is lower. These results imply that the Home firm profit is higher:

$$\begin{aligned}
n(\theta)\pi_3(\theta) &= n(\theta)\pi_2(\theta) - F \\
&= \frac{m-1}{m}n(\theta)\pi_1(\theta) - F \\
&> \frac{m-1}{m}n^*(\theta)\pi_1^*(\theta) - Fw_o \\
&= n^*(\theta)\pi_3^*(\theta).
\end{aligned}$$

We can also show that $\pi_1(\hat{\theta}) < \pi_1^*(\hat{\theta}^*)$ because $\frac{m-1}{m}\pi_1(\hat{\theta}) = F = \frac{m-1}{m}\frac{\pi_1^*(\hat{\theta}^*)}{w_o}$ and $w_o > 1$. Hence $\pi_1^*(\hat{\theta}) < \pi_1(\hat{\theta}) < \pi_1^*(\hat{\theta}^*)$, which implies $\hat{\theta} < \hat{\theta}^*$. Because the cutoff is lower, and $n(\theta)\pi_3(\theta) > n^*(\theta)\pi_3^*(\theta)$, the average profit in the Home country is higher:

$$\int_{\hat{\theta}}^{\infty} n(\theta)\pi_3(\theta)k\frac{\theta_0^k}{\theta^{k+1}}d\theta > \int_{\hat{\theta}^*}^{\infty} n^*(\theta)\pi_3^*(\theta)k\frac{\theta_0^k}{\theta^{k+1}}d\theta.$$

This result leads us to a contradiction because the free entry condition is violated: the entry cost in the Home country is lower but expected profit is higher than in the Foreign country. ■

Since managers are more numerous in the Home country, it can be shown that the salaries of the Home managers, relative to the wages of the workers, are lower than in the Foreign country. Indeed, with the same argument as in the case of labor mobility (the proof can be found in Appendix A5), the relative salaries for managers in the Home and Foreign countries are given by:

$$w = \frac{1-\rho}{m} \frac{L/S}{\frac{m-1}{m}(1-\rho) + \rho}. \quad (15)$$

$$\frac{w^*}{w_o} = \frac{1-\rho}{m} \frac{L/S^*}{\frac{m-1}{m}(1-\rho) + \rho}. \quad (16)$$

These two formulas yield:

$$w < \frac{w^*}{w_o}.$$

The lemma below shows that the wages of workers in the Foreign country have to be lower than in the Home country.

Lemma 7. *In the country with more talent workers have higher wages.*

Proof. By contradiction, let me assume the inverse. This implies that the marginal cost is higher in the Foreign country, and hence the unit price is higher:

$$p^*(\theta) = \frac{w_o}{\theta\rho} > \frac{1}{\theta\rho} = p(\theta).$$

The monopolistic demand (2) implies that the sale per product is therefore higher in the Home country. As a result, Home country firms have more profit per brand than those in the Foreign country:

$$\pi_1(\theta) > \pi_1^*(\theta).$$

The number of brands per firm in the Home country is given by

$$n(\theta) = \left(\frac{1}{w} \frac{\pi_1(\theta)}{m} \right)^{\frac{1}{m-1}}. \quad (17)$$

Similarly, the number of brands per firm in the Foreign country is

$$n^*(\theta) = \left(\frac{w_o}{w^*} \frac{\pi_1^*(\theta)}{m} \right)^{\frac{1}{m-1}}. \quad (18)$$

Since $\pi_1(\theta) > \pi_1^*(\theta)$ and $\frac{1}{w} > \frac{w_o}{w^*}$ then we have $n(\theta) > n^*(\theta)$. This is a contradiction, according to Lemma 6, because the Home firms can not

develop more products and sell more units per product than the comparable firms in the Foreign country. ■

The two lemmas above lead us to the following result:

Proposition 8. *The firms in the country with more talent develop more products but sell less units per product.*

Proof. From Lemma 7, the pricing rule (3) and the demand function (2), it is clear that the Home firm produce less per product than the comparable firm in the Foreign country. Lemma 6 then implies that the Home firm supplies more varieties. ■

Having access to a larger talent endowment allows the Home firms to focus on developing more products and also creates a more competitive environment where only the productive firms can survive:

Proposition 9. *In the country with a larger talent endowment, the competition is fiercer, represented by a higher productivity cut-off.*

Proof. Since the wages of workers in the Home country are higher, it is clear that $\pi_1^*(\theta) > \pi_1(\theta)$. From (6) we have

$$\frac{m-1}{m}\pi_1^*(\widehat{\theta}^*) = w_o F < F = \frac{m-1}{m}\pi_1(\widehat{\theta}).$$

We can rewrite this as

$$\pi_1(\widehat{\theta}^*) < \pi_1^*(\widehat{\theta}_F) < \pi_1(\widehat{\theta}).$$

The formula above implies that the cutoff in the Home country is higher than that in the Foreign country. ■

This result is different from what was shown in the previous section where labor is mobile: the cutoffs in both countries are no longer the same.

As discussed above, the bigger common market allows the more productive firms in both countries to expand their production, both the scale (the market size effect, see equation 4) and the scope (more profit per brand enables the firms to pay for the managers to develop more brands, as in equation 5). Scale expansion results in more demand for labor, whereas scope expansion leads to higher demand for talent. As a result the costs are higher and it becomes more difficult to break it even in the markets. The difference between the case of labor mobility and this case here is that in the former case, the unproductive firms in the Home country compete with all the other firms, in particular the unproductive firms in the Foreign country, for the same pool of labor. In the latter case, however, their competitors are the more productive firms in the Home country. Compared to the Foreign country, high talent endowment enables the more productive firms in the Home country to produce more varieties, and therefore, acquire more labor than the firms with the same productivity level in the Foreign country. Consequently, the unproductive firms in the Home country face more competition than their comparable firms in the Foreign country, which explains why the cut-off in the Home country is higher.

By construction the industry is decomposed into 2 "seemingly" sectors: the "extensive" and the "intensive" sectors. As a result, unlike in the New Trade model trade pattern is no longer indeterminate. Indeed, with two factors and two "seemingly" sectors, it brings the Heckscher-Ohlin to one's mind: the country that is abundant in talent (Home) should export the "extensive" sector and the other country exports the "intensive sector". We need to be careful here: it is not always the case that the former country exports more varieties because there is the between firm effect that plays a role here. Propositions 8 and 9 yield an interpretation for this type of pattern of trade. Since the cut-off level in the Foreign country is lower, this country exports the varieties supplied by the unproductive firms. These varieties have high prices. With more talent in the Home country, the productive firms in this country develop more varieties than the comparable firms in the Foreign country. As a result, the Home country exports the varieties with low prices. It should be noted that the Foreign country also has the productive firms produce the varieties with low prices and export to the Home country. However, these firms produce less varieties than their counterparts in the Home country. This pattern of trade is in line with the evidence that high income countries, which presumably have relatively more talents, export high quality products (Hummels and Klenow 2002, Hallak

and Schott 2011).

It can be shown (see Appendix A.5) that the wages of the managers, relative to the wages of the workers, do not change from autarky to free trade in both countries. Therefore it is possible that free trade worsen welfare for one of the countries. Indeed, while free trade enhances welfare for the Home country, the Foreign country can be worse off. This is because with less talent, competition in the Foreign country is less fierce than in the Home country (the cut-off is lower). As a result, there are more unproductive firms in the Foreign country. When the Foreign country opens up to trade with the Home country, the Foreign firms have to lower their prices in order to compete with the more productive firms in the Home country. The result is that wages in the Foreign country are lower than in autarky.

To show that such a scenario is possible, we just need to find a numerical example. For the national factor endowments, we choose $L = 1$, talent supply in the Home country to be 10 percent of that of unskilled labor ($S = 0.1$) while it is slightly lower in the Foreign country ($S^* = 0.09$). The costs parameters are chosen as $f_e = 10$, $F = 1$. The elasticity of substitution among varieties is chosen as $\rho = 0.75$ to be consistent with other studies, in particular Broda and Weinstein 2006). More importantly are the two new parameters of interest: the efficiency of the managerial service m and the degree of firm heterogeneity γ . we choose the intermediate values $m = 5, k = 5$. In this case, the price index in the Home country in autarky is 3.34 and the wage of the managers is 0.53. In the Foreign country, the corresponding values are 3.37 and 0.59. When the two countries engage in free trade, the price index is 2.65. Since the wages of workers and salaries of managers in the Home country do not change, they are all better off. However, the wages of workers and managers in the Foreign country are now 0.30 and 0.17 respectively. They are all worse off as their wages fall more than the change in the price index.

Unlike the previous case, the workers in the Foreign country can not move to the Home country where labor is more needed. In this case, compared to the case with factor mobility, the Home managers and the Foreign workers lose: while the Home managers are no longer the winners with the largest gains in free trade, the Foreign workers now may become the losers.

4. Conclusion

The emergence of multi-product firms require thorough investigation. Combining the multiple product feature with the main ingredients of the international trade workhorse models enables us to generate some new and interesting results. Indeed, in our set-up there are two factors, unskilled and skilled labor, two "seemingly" sectors (the main features of the Heckscher-Ohlin model); monopolistic competition (the New Trade model); firm heterogeneity (the New New Trade model). In the Heckscher-Ohlin model, factor endowment implies the within-firm effect: more skilled labor increases the relative supply of the "extensive" sector, which uses intensively (or uniquely) this factor. We show here that this within-firm effect creates two implications: on the demand side, new varieties cannibalize the existing varieties; on the supply side new varieties require more (unskilled) labor, bid up their wages and therefore raise the production costs. In fact, we can think of one new variety introduced as similar as a firm starts to export in Melitz (2003) model. Consequently, the unproductive firms have less profit: some of them no longer have enough profit to cover the fixed cost of production. This between firm and within firm effects result in the non-uniform impact on the firms: some of them (due to the productive ones) expand their scope (the within firm effect) while some others (the unproductive ones) reduce their scope (due to the between firm effect). At the end, the productivity cut-off is higher than before.

When engaging in free trade, two identical countries, differing only in their talent endowment can replicate an integrated economy if labor is mobile. This economic integration influences the productivity cut-off only through the ratio of the two factors, not the market size. In particular, one country will see their cut-off rise, while the other sees their cut-off drop. In general, national welfare in both countries improve, although certain people might lose. When labor mobility is now allowed, trade pattern is no longer indeterminate as in the New Trade model. However, due to the between firm effect above, this trade pattern is not exactly what the Heckscher-Ohlin would predict: it is not always the case that the country abundant in skilled labor exports more varieties. In fact, this country exports more varieties of low prices while the other country exports the varieties of high prices.

References

- Allanson, P., Montagna, C., 2005. Multiproduct firms and market structure: An explorative application to the product life cycle. *International Journal of Industrial Organization* 23, 587–97.
- Arkolakis, C., Muendler, M.A., 2010. The extensive margin of exporting goods: A firm-level analysis. Working paper.
- Baldwin, J., Gu, W., 2009. The impact of trade on plant scale, production-run length and diversification, in: T. Dunne, J.B.J., Roberts, M.J. (Eds.), *Producers Dynamics: New Evidence from Micro Data*. University of Chicago Press.
- Baldwin, J.R., Jarmin, R., Tang, J., 2002. The trend to smaller producers in manufacturing: A Canada/u.s. comparison. *Economic Analysis Research Paper Series* 3.
- Bernard, A.B., Redding, S.J., Schott, P.K., 2009. Products and productivity. Working paper.
- Bernard, A.B., Redding, S.J., Schott, P.K., 2010. Multiproduct firms and product switching. *American Economic Review* 100, 70–97.
- Bernard, A.B., Redding, S.J., Schott, P.K., 2011. Multi-product firms and trade liberalization. *Quarterly Journal of Economics* 126, 1271–1318.
- Broda, C., Weinstein, D.E., 2006. Globalization and the gains from variety. *Quarterly Journal of Economics* 121, 541–85.
- De Loecker, J., 2010. Product differentiation, multi-product firms and estimating the impact of trade liberalization on productivity. Preprint, available at <http://www.nber.org/papers/w13155>.
- Dhingra, S., 2010. Trading away wide brands for cheap brands. Preprint, available at www.sdhingra.com/innovation.pdf.
- Eckel, C., Neary, J.P., 2010. Multi-product firms and flexible manufacturing in the global economy. *Review of Economic Studies* 77, 188–217.
- Feenstra, R.C., Ma, H., 2008. Optimal choice of product scope for multiproduct firms under monopolistic competition, in: Helpman, Elhanan, D.M., Verdier, T. (Eds.), *The Organization of Firms in a Global Economy*. Harvard University Press.
- Goldberg, P.K., Khandelwal, A., Pavcnik, N., Topalova, P., 2010. Multi-product firms and product turnover in the developing world: Evidence from India. *Review of Economics and Statistics* 92, 1042–49.
- Hallak, J.C., Schott, P.K., 2011. Estimating cross-country differences in product quality. *Quarterly Journal of Economics* 126, 417–74.
- Hummels, D., Klenow, P.J., 2005. The variety and quality of a nation’s exports. *American Economic Review* 95, 704–23.
- Iacovone, L., Javorcik, B., 2008. Multiproduct exporters: Diversification and micro-level dynamics. Working paper.
- Krugman, P., 1994. *Rethinking International Trade*. MIT press.
- Lee, F., Tang, J., 2001. Multifactor productivity disparity between Canadian and U.S. manufacturing firms. *Journal of Productivity Analysis* 15, 115–28.
- Luong, T.A., Chen, W.C., 2012. Trade margins and labor heterogeneity. Working paper.
- Marin, D., Verdier, T., 2012. Globalization and the empowerment of talent. *Journal of International Economics* 86, 209–23.
- Mayer, T., Melitz, M.J., Ottaviano, G.I.P., 2010. Market size, competition and the product mix of exporters. Working paper.

- Melitz, M., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* 71, 1695–725.
- Nocke, V., Yeaple, S., 2006. Globalization and endogenous firm scope. Preprint, available at <http://www.nber.org/papers/w12322>.
- Ottaviano, G., Thisse, J.F., 2011. Monopolistic competition, multiproduct firms and product diversity. *The Manchester School* 79, 938–51.
- Qiu, L.D., Zhou, W., 2010. Trade, capital redistribution and firm structure. Working paper.
- Schoar, A., 2002. The effect of corporate diversification on firm productivity. *The Journal of Finance* 62, 2379–403.
- Van, A.B., Monnikhof, E., 1996. Size distribution of output and employment: a data set for manufacturing industries in five oecd countries, 1960s-1990. Preprint, available at <http://www.oecd.org/dataoecd/33/37/1863515.pdf>.

A. Appendix

A.1. *What is a product?*

A product, by definition, is to be offered in the market to satisfy certain need or want. A brand, which is a special form of product: the producers add some attributes or characteristics to differentiate your products from others. In this paper, there is not necessarily any difference between a product or a brand. we use the terms brands, products and varieties interchangeably.

A.2. *Proof of proposition 2*

From (1) we have:

$$\begin{aligned}
P^{-\frac{\rho}{1-\rho}} &= \int_{\hat{\theta}} M_1 p^{-\frac{\rho}{1-\rho}}(\theta) n(\theta) df(\theta) \\
&= \int_{\hat{\theta}} M_1 p^{-\frac{\rho}{1-\rho}}(\theta) \left(\frac{\pi_1(\theta)}{wm} \right)^{\frac{1}{m-1}} k \frac{\theta_0^k}{\theta^{k+1}} d\theta \\
&= \rho^{\frac{\rho m}{(1-\rho)(m-1)}} (1-\rho)^{\frac{1}{m-1}} R^{\frac{1}{m-1}} (mw)^{-\frac{1}{m-1}} P^{\frac{\rho}{(1-\rho)(m-1)}} M_1 \int_{\hat{\theta}} k \theta_0^k \theta^{\left(\frac{\rho m}{(1-\rho)(m-1)} - k - 1\right)} d\theta \\
&= \frac{\rho^{\frac{\rho m}{(1-\rho)(m-1)}} (1-\rho)^{\frac{1}{m-1}} R^{\frac{1}{m-1}} P^{\frac{\rho}{(1-\rho)(m-1)}} M_1 \theta_0^k \hat{\theta}^{\left(\frac{\rho m}{(1-\rho)(m-1)} - k\right)}}{(mw)^{\frac{1}{m-1}} \left(1 - \frac{\rho m}{k(1-\rho)(m-1)}\right)},
\end{aligned}$$

or

$$P = \frac{(mw)^{\frac{1-\rho}{\rho m}} \left(1 - \frac{\rho m}{k(1-\rho)(m-1)}\right)^{\frac{(1-\rho)(m-1)}{\rho m}}}{\rho (1-\rho)^{\frac{1-\rho}{\rho m}} R^{\frac{1-\rho}{\rho m}} M_1^{\frac{(1-\rho)(m-1)}{\rho m}} \theta_0^k \hat{\theta}^{\frac{k(1-\rho)(m-1)}{\rho m} - 1}}. \quad (19)$$

Substitute the price index by (19) in (6):

$$\begin{aligned}
F &= \frac{m-1}{m} \pi_1(\hat{\theta}) = \frac{m-1}{m} (1-\rho) P^{\frac{\rho}{1-\rho}} R c(\hat{\theta})^{-\frac{\rho}{1-\rho}} \\
&= \frac{m-1}{m} \frac{(1-\rho) (mw)^{\frac{1}{m}} \left(1 - \frac{\rho m}{k(1-\rho)(m-1)}\right)^{\frac{m-1}{m}} R^{\frac{(m-1)k}{m}} \hat{\theta}^{\frac{k(m-1)}{m}}}{\rho^{\frac{\rho}{1-\rho}} (1-\rho)^{\frac{1}{m}} R^{\frac{1}{m}} M_1^{\frac{m-1}{m}}}.
\end{aligned} \quad (20)$$

Finally from the free-entry condition (8) we have:

$$f_e = \frac{\Pi_3}{M_1} = \frac{\Pi_1 - wS - M_2 F}{M_1} = \frac{(1-\rho) \left(1 - \frac{1}{m}\right) R - \frac{\theta_0^k}{\hat{\theta}^k} M_1 F}{M_1},$$

or

$$M_1 = \frac{(1 - \rho) \left(1 - \frac{1}{m}\right) R}{f_e + F \frac{\theta_0^k}{\widehat{\theta}^k}}. \quad (21)$$

Substituting M_1 by (21) into (20) we have:

$$F = \frac{(m-1)^{\frac{1}{m}}}{\rho^{\frac{\rho}{1-\rho}}} \left(1 - \frac{\rho m}{k(1-\rho)(m-1)}\right)^{\frac{m-1}{m}} w^{\frac{1}{m}} \left(f_e \widehat{\theta}^k + F \theta_0^k\right)^{\frac{m-1}{m}} \theta_0^{\frac{k(m-1)}{m}} \quad (22)$$

The formula above shows that when the wage of managers w is lower (due to an increase in the talent endowment), the cut-off $\widehat{\theta}$ is higher.

A.3. Proof of proposition 3

From (4) and (6):

$$P = \left[\frac{mF}{(m-1)(1-\rho)R} \right]^{\frac{1-\rho}{\rho}} \frac{1}{\widehat{\theta}}$$

Since total revenue R is proportional to L (see equation 11), it is clear that more skilled labor raises the cut-off $\widehat{\theta}$ which in turn reduces the price index P . From the wage formula (12) and (22) we then have the real salaries of the managers are proportional to $L^{\frac{1-\rho}{\rho}} \left[\left(\frac{L}{S}\right)^{k-\frac{1}{m-1}} - B \left(\frac{L}{S}\right)^k \right]^{\frac{1}{k}}$. Since $k - \frac{1}{m-1} < k$, when $\frac{L}{S}$ is higher than a certain threshold, the manager real income will be proportional to $L^{\frac{1-\rho}{\rho}} \frac{S}{L}$ in which case it decreases with $\frac{L}{S}$. In other words, the managers can lose (i.e. their real income falls) if the relative number of skilled labor is high enough.

A.4. Proof of proposition 5

From (6) we have:

$$F = (1 - \rho) P^{\frac{\rho}{1-\rho}} R c (\hat{\theta})^{-\frac{\rho}{1-\rho}},$$

or

$$\begin{aligned} \left(\frac{F}{1 - \rho} \right)^{\frac{1-\rho}{\rho}} &= P R^{\frac{1-\rho}{\rho}} e^{\alpha \hat{\theta}} \\ &= P \left(\frac{2L}{1 - \frac{1-\rho}{m}} \right)^{\frac{1-\rho}{\rho}} \left(B w^{-\frac{1}{m-1}} - F \right)^{\frac{1}{\gamma}} \\ &\propto P (2L)^{\frac{1-\rho}{\rho}} \left(\left(\frac{S_H + S_F}{2L} \right)^{\frac{1}{m-1}} - C \right)^{\frac{1}{\gamma}} \\ &\propto P L^{\frac{1-\rho}{\rho}} \left(\left(\frac{S_H + S_F}{L} 2^{\frac{(1-\rho)(m-1)\gamma}{\rho} - 1} \right)^{\frac{1}{m-1}} - C \right)^{\frac{1}{\gamma}}. \end{aligned}$$

C here is a constant. The price index in the integrated economy is lower than that of the Foreign country in autarky because $\frac{S_H + S_F}{2L} > \frac{S_F}{L}$. It is also lower than that of the Home country because $\frac{S_H + S_F}{L} > \frac{S_H}{L}$ and $2^{\frac{(1-\rho)(m-1)\gamma}{\rho} - 1} > 1$. Therefore economic integration in this case (labor mobility) is welfare enhancing.

A.5. *The relative wages of managers when labor is immobile*

We can prove that the relative wages of the managers are proportional to the ratio of unskilled labor to skilled labor by using the labor market clearing conditions. Indeed, in the unskilled labor market (we drop the country notations because the argument below applies to both countries):

$$\begin{aligned}
L_H^D &= M_1 f_e + M_2 F + \rho \frac{\Pi_1 w}{1 - \rho} \\
&= \Pi_3 + M_2 F + \rho \frac{\Pi_1 w}{1 - \rho} \\
&= \Pi_1 - w_H S_H + \rho \frac{\Pi_1 w}{1 - \rho} \\
&= \left[\frac{m-1}{m} + \frac{\rho}{1-\rho} \right] \Pi_1 w
\end{aligned} \tag{23}$$

and in the skilled labor market:

$$S = \frac{\Pi_1}{m w_S} \tag{24}$$

therefore:

$$\frac{w_S}{w} = \frac{1}{\left[\frac{m-1}{m} + \frac{\rho}{1-\rho} \right] m} \frac{L}{S}$$

A.6. Micro-foundation for the consumer preferences

The consumer preferences could be found when all the varieties within a firm are symmetric. When the varieties are heterogenous, if the elasticity of substitution within firm is equal to the elasticity of substitution across firm $\sigma = \rho$ (similar to Arkolakis and Muendler 2009) then we have:

$$\begin{aligned}
U &= \left[\int M \left(\int q(i, j)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma-1}{\sigma-1} \frac{\rho-1}{\rho}} f(i) di \right]^{\frac{\rho}{\rho-1}} \\
&= \left[\int M \int q(i, j)^{\frac{\sigma-1}{\sigma}} dj f(i) di \right]^{\frac{\sigma}{\sigma-1}} \\
&= \left[\int MN(i) q(i, j)^{\frac{\sigma-1}{\sigma}} f(i) di \right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

because in equilibrium, the varieties are symmetric.