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Real Exchange Rate Forecasting and PPP: This Time the Random Walk Loses*

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Abstract

This paper brings four new insights into the Purchasing Power Parity (PPP) debate. First, we show that a half-life PPP (HL) model is able to forecast real exchange rates better than the random walk (RW) model at both short and long-term horizons. Second, we find that this result holds if the speed of adjustment to the sample mean is calibrated at reasonable values rather than estimated. Third, we find that it is preferable to calibrate, rather than to elicit as a prior, the parameter determining the speed of adjustment to PPP. Fourth, for most currencies in our sample, the HL model outperforms the RW also in terms of nominal effective exchange rate forecasting.

JEL codes: C32, F31, F37

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1 Introduction

Exchange rates have long fascinated, challenged and puzzled researchers in international finance. Since the seminal papers by Meese and Rogoff (1983a,b), there has been wide agreement that macroeconomic models are not very helpful for exchange rate forecasting.[1] The exchange rate literature provides, however, at least two reasons for cautious optimism.

First, the dismal forecasting performance of exchange rate models is to some extent explained by estimation and not only misspecification error (Engel et al., 2008). The significant role of estimation error is confirmed, among other things, by the relative good forecasting performance of economic models estimated with a large panels of data (Mark and Sul, 2001; Engel et al., 2008; Ince, 2014) or long time series (Lothian and Taylor, 1996). A number of econometric tests have also been developed to prevent that the random walk (RW) would have an undue advantage relative to estimated forecasting models, which are subject to sampling variance (see Engel, 2013, for a review).

The second reason for being cautiously optimistic about the usefulness of exchange rate models comes from the evidence in favor of the PPP model. According to Taylor and Taylor (2004), the exchange rate literature has turned full circle to the pre-1970s view that PPP holds in the long run. The mean reverting nature of real exchange rates has in particular found support by panel unit root tests, which have higher power than the conventional univariate tests in the case of highly persistent or non-linear processes (Sarno and Taylor, 2002). Only a handful of studies have instead tested whether the mean-reverting properties of the real exchange rate can be exploited in a forecasting setting. In the late 1980s, Meese and Rogoff (1988) extended their classic analysis to reach the conclusion that, like nominal exchange rates, real exchange rates are disconnected from economic fundamentals. Two studies in the mid-1990s argued instead that the RW can be beaten for larger datasets, for example in the case of long-data series (Lothian and Taylor, 1996) or in multivariate frameworks (Jorion and Sweeney, 1996). More recently, Rogoff (2009) suggested that it is worthwhile investigating whether PPP deviations and current account positions may help predict real exchange rate movements.

In this paper we aim to establish if Rogoff's insight on PPP is correct. We reach the

[1] In the mid-1990s Mark (1995) and Chinn and Meese (1995) suggested that the RW model could be beaten at longer horizons. This more optimistic perspective was however short-lived and failed to overturn the previous consensus (Faust et al., 2003, Cheung et al., 2005 or Rogoff, 2009).

conclusion that, to beat the RW, it is crucial to take into account both the role of estimation error and the strong persistence of real exchange rates. The strong persistence in the real exchange rate was first affirmed in a series of studies conducted between the mid-1980s and early 1990s, which employed more than a hundred years of annual data. From an informal meta-analysis of these studies, Rogoff (1996) inferred that it takes between 3 and 5 years to halve real exchange rate deviation from the mean. A number of studies have been more skeptical about what is typically dubbed the "Rogoff consensus". For example, Kilian and Zha (2002) proposed a prior probability distribution based on a survey of professional international economists and derived a posterior probability distribution of the half-life PPP deviation on the basis of a Bayesian autoregressive model. Their results provide very limited support for the view that such statistic ranges between three and five years. In a similar vein Murray and Papell (2002) stressed how univariate methods provide virtually no information on the size of half-lives. Finally, a large cross-country heterogeneity in terms of point estimates and confidence intervals has also been found in the studies by Murray and Papell (2005) and Rossi (2005). There is however another strand of the literature which finds it instead plausible that at the aggregate level half-lives are in the range between 3 and 5 years. It is there argued that aggregation bias both at the time and product dimension helps to reconcile such slow process of adjustment of the real exchange rate with faster convergence at the product and sectoral level (Imbs et al., 2005; Crucini and Shintani, 2008; Mayoral and Dolores Gadea, 2011; Bergin et al., 2013).

In this paper we analyze whether the mean-reversion of real exchange rates can be exploited for forecasting purposes. We test whether a calibrated, half-life PPP (henceforth, HL) model, which sets a gradual linear adjustment of the real exchange rate toward its mean, is able to forecast real exchange rates better than (i) an autoregressive (henceforth, AR) model, where the pace of mean-reversion is estimated and (ii) a RW model. In our baseline the HL model is calibrated so that half of the adjustment of the real exchange rate toward its mean is completed within five years (HL5). We have set this initial value, rather conservatively, i.e. at the top of the range proposed by Rogoff (1996), so that in the short-run the model predictions resemble those of the RW model. As Rogoff's consensus was built on the basis of the data from the pre-1990s and our forecast evaluation sample starts in 1990, what we conduct is a true "out of sample" forecasting exercise. We shall later extend our analysis with a thorough sensitivity analysis to show that, contrary to our initial expectations, all the key results hold true for the whole range of half-lives between 3 and 5 years proposed by Rogoff.

The key findings of our paper are as follows. We show that, exploiting simultaneously the evidence of (i) real exchange rate persistence and (ii) long-term convergence to PPP, leads to a considerable improvement in our ability to forecast real exchange rates even for short samples. To be more specific, we show that the HL model is able to forecast real exchange rates better than the RW for seven out of nine currencies. Particularly persuasive is that this approach beats the RW also at short-horizons.

Another remarkable result of our study is that the forecast accuracy of the estimated HL model is considerably worse than that of the RW. We explain this result both analytically and empirically, emphasizing that this is due to estimation error, even if we have as many as 15 years of monthly data.

Our empirical investigation is then taken a final step forward: we find that the mean reverting nature of the real exchange rate can be exploited to outperform the RW model for forecasting nominal effective exchange rates. This is because, for the majority of the currencies in our sample, the real exchange rate reverts to the mean mainly via the adjustment of the nominal exchange rate (and not via changes in relative prices). This suggests a promising avenue of future research, i.e. to conduct a two-step analysis where the researcher first seeks to forecast real exchange rates and then, as a second step, turns to nominal exchange rate forecasting.

The rest of the paper is structured as follows. Section 2 outlines the alternative models that we shall use in our real exchange rate forecasting competition. In section 3 we report the outcome of our competition. In Section 4 we provide an analytical investigation that sheds some light on our findings. Section 5 shows that the results are robust to several alternative specifications. We also illustrate that our key results are valid for a broad range of half-lives (wider than the range proposed by Rogoff). We conclude by showing that our improved ability to forecast real exchange rates is helpful also in relation to nominal exchange rate forecasting.

2 The models

Let us define the log of the real exchange rate according to the convention that $y_t \equiv s_t + p_t - p_t^*$, where s_t is the log of the nominal exchange rate expressed as the foreign currency price of a unit of domestic currency, and p_t and p_t^* are the logs of home and foreign price levels, respectively.

Consider a simple autoregression (AR) model for the real exchange rate:

$$y_t - \mu = \rho(y_{t-1} - \mu) + \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma^2). \tag{1}$$

For a stationary AR process the parameter ρ measures the speed of reversion to μ, which we interpret as the level of PPP. The half-life of adjustment toward PPP is equal to:

$$hl = \log(0.5)/\log(\rho). \tag{2}$$

For $\rho = 1$ the RER is generated by the RW process.

In the forecasting contest we employ three alternatives of model (1).

1. The first is a RW model, for which we calibrate $\rho = 1$ and $\mu = 0$. The h step ahead forecast is:

$$y^{RW}_{T+h|T} = y_T. \tag{3}$$

2. The second is the HL model for which we assume that the real exchange rate gradually converges to its sample mean ($\bar{\mu}$). The pace of convergence ($\bar{\rho}$) is calibrated with (2) so that the half-life is equal to five years, i.e. at the top of the range proposed by Rogoff[2]. The h step ahead forecast is:

$$y^{HL}_{T+h|T} = \bar{\mu} + \bar{\rho}^h(y_T - \bar{\mu}). \tag{4}$$

3. The third is an autoregressive AR model, whose two parameters are estimated with OLS $\hat{\mu}$ and $\hat{\rho}$, so that the h step ahead forecast is:

$$y^{AR}_{T+h|T} = \hat{\mu} + \hat{\rho}^h(y_T - \hat{\mu}). \tag{5}$$

3 Empirical evidence

To assess the predictability of real exchange rates we gather monthly data for nine major currencies of the following economies: Australia (AUD), Canada (CAD), euro area (EUR), Japan (JPY), Mexico (MXN), New Zealand (NZD), Switzerland (CHF), the United Kingdom (GBP) and the United States (USD) for the period between 1975:1

[2]In the sensitivity analysis we will check the forecasting accuracy of different calibrations for ($\bar{\rho}$).

and 2012:3. For all currencies we take the real effective exchange rates provided by the Bank for International Settlements (Klau and Fung, 2006). The values of the analyzed series are presented in Figure 1.

We assess the out-of-sample forecast performance for horizons ranging from one to sixty months ahead. In our baseline specification the models are estimated using rolling samples of 15 years (R = 180 months). The first set of forecasts is derived with the rolling sample 1975:1-1989:12 for the period 1990:1-1994:12. This procedure is repeated with rolling samples ending in each month from the period 1990:2-2012:2. Since the data available end in 2012:3, the 1-month-ahead forecasts are evaluated on the basis of 267 observations, 2-month-ahead forecasts on the basis of 266 observations, and 60-month-ahead forecasts on the basis of 208 observations.

We measure the forecasting performance of the three competing models with two standard statistics: the mean squared forecast errors (MSFEs) and the correlation coefficient between forecast and realized real exchange rate changes. Table 1 and Figure 2 present the values of MSFEs. For the RW we report the MSFEs values in levels. For the HL and AR models, we report them divided by MSFEs of the RW, so that values below unity indicate that such model outperforms the RW. We also test the null of equal forecast accuracy with the two-sided Diebold and Mariano (1995) test.[3]

In terms of the MSFE criterion the HL model-based forecasts beat the RW for seven out of nine currencies (EUR, MXN, NZD, CHF, GBP, USD, JPY). The MSFEs of the HL model are on average 9% and 23% lower than that of the RW model at the two and five-year horizon, respectively. The HL model-based forecasts are also considerably more precise than those based on the AR model for five currencies (CAD, EUR, JPY, GBP and USD) while are broadly comparable for the other four. At short-horizons the HL model is the best and the AR the worst model. At the one-year horizon the MSFEs of the HL model are on average 3% and 12% lower than those from the RW and AR models, respectively. Further evidence that the HL model beats the other two models can be found using our second criterion, which consists in computing the correlation coefficient between the realized and forecast changes of real exchange rates:

[3]For the rolling scheme the asymptotic test is valid because the estimation error is the same for all forecasting rounds. For the recursive scheme this is not the case as estimation error for the first forecasting round is larger than for the last one. As a result the bootstrapped values would be required only rather for the recursive scheme. As regards the choice of the Diebold-Mariano (DM) vs. Clark-West (CW) tests, our main comparison is between the RW and HL models: in this case the CW cannot be used because these are not-nested models. It could be used to compare instead the RW vs. AR, which is not the main focus of our paper.

$$r_{M,h} = cor(y^M_{T+h|T} - y_T, y_{T+h} - y_T), \tag{6}$$

where M stands for the model name. Note that (3) implies that $r_{RW,h}$ is zero: for that reason in Table 2 we report only the results for HL and the AR models. It shows that the correlation coefficients for the HL model are generally positive for all currencies at all horizons, except for the AUD. The average value of $r_{HL,h}$ also increases with the forecast horizon: from just 0.04 for the one-month ahead forecasts to 0.53 for five-year ahead forecasts. The results do not provide support instead for the AR model: MXN is the only currency with a positive $r_{AR,h}$ throughout the forecast horizon. Moreover, the average value of $r_{AR,h}$ is positive only for horizons above two years. Finally, at all horizons $r_{AR,h}$ is visibly lower than $r_{HL,h}$.

To sum up, the evidence suggests that real exchange rates of major currencies tend to be mean reverting and forecastable, as shown by the good performance of the calibrated HL model. In the next section we provide an analytical explanation of why the estimated AR model performs so poorly.

4 Analytical interpretation of the results

In what follows we show analytically why the finite sample determines a sizable estimation error, which distorts the results in favor of the RW model even when the rolling estimation window covers several years of monthly data. Let us assume that the data generating process (DGP) for y_t is given by (1) so that the unbiased and efficient forecast is:

$$y_{T+h|T} = \mu + \rho^h(y_T - \mu), \tag{7}$$

and the variance of the forecast error:

$$E\{(y_{T+h} - y_{T+h|T})^2\} = \sigma^2 \frac{1-\rho^{2h}}{1-\rho^2}. \tag{8}$$

If the DGP is known, the only source of forecast errors comes from the random term. The variance of the forecast errors generated by our three competing models is however higher than that in (8) because the coefficients μ and ρ are unknown and have to be either estimated or calibrated.

Let us decompose the variance of the forecast error from a generic model $M \in \{RW, HL, AR\}$ into three components:

$$
\begin{aligned}
E\{(y_{T+h} - y^M_{T+h|T})^2\} &= E\{(y_{T+h} - y_{T+h|T})^2\} + \\
&+ 2E\{(y_{T+h} - y_{T+h|T})(y_{T+h|T} - y^M_{T+h|T})\} + \\
&+ E\{(y_{T+h|T} - y^M_{T+h|T})^2\}.
\end{aligned} \tag{9}
$$

The first component, which is given by (8), represents the random error that is common to all models. The second component is equal to zero as future shocks are not forecastable. The third component, which captures the mis-specification and estimation errors, determines the different performance of the three competing models. It is particularly advantageous that the value of this component can be derived analytically for the RW, HL and AR models.

In the case of the RW model the forecast error equals:

$$
y_{T+h|T} - y^{RW}_{T+h|T} = (\rho^h - 1)(y_T - \mu) \tag{10}
$$

and thus:

$$
E\{(y_{T+h|T} - y^{RW}_{T+h|T})^2\} = (\rho^h - 1)^2 \times E\{(y_T - \mu)^2\}, \tag{11}
$$

where:

$$
E\{(y_T - \mu)^2\} = \frac{\sigma^2}{1 - \rho^2}.
$$

For the HL model, such error is equal instead to:

$$
y_{T+h|T} - y^{HL}_{T+h|T} = (\rho^h - \bar{\rho}^h)(y_T - \mu) - (1 - \bar{\rho}^h)(\bar{\mu} - \mu). \tag{12}
$$

The first term describes the forecast error caused by the wrong calibration of parameter ρ and the second one is the error related to the estimation of μ. The resulting variance is:

$$
\begin{aligned}
E\{(y_{T+h|T} - y^{HL}_{T+h|T})^2\} = (\rho^h - \bar{\rho}^h)^2 \times E\{(y_T - \mu)^2\} + (1 - \bar{\rho}^h)^2 \times \\
\times E\{(\bar{\mu} - \mu)^2\} - 2(\rho^h - \bar{\rho}^h)(1 - \bar{\rho}^h) \times E\{(y_T - \mu)(\bar{\mu} - \mu)\},
\end{aligned} \tag{13}
$$

where:

$$E\{(\bar{\mu}-\mu)^2\} = \frac{\sigma^2}{1-\rho^2} \times \frac{1}{R^2} \times (R + 2\sum_{j=1}^{R-1}(R-j)\rho^j)$$

$$E\{(y_T-\mu)(\bar{\mu}-\mu)\} = \frac{\sigma^2}{1-\rho^2} \times \frac{1}{R} \times \frac{1-\rho^R}{1-\rho}.$$

Finally, as derived in Fuller and Hasza (1980), for the AR model the second component is approximately equal to:

$$E\{(y_{T+h|T} - y^{AR}_{T+h|T})^2\} \simeq \sigma^2 \times \frac{1}{R} \times \left[h^2\rho^{2(h-1)} + \left(\frac{1-\rho^h}{1-\rho}\right)^2\right] \tag{14}$$

and is entirely caused by estimation error. Given (8)-(14), the assumptions for the DGP coefficients (μ, ρ and σ) and the sample size (R), one can calculate the theoretical value of MSFE for all competing models (RW, HL and AR) at different forecast horizons ($h = 1, 2, \ldots, H$). The theoretical MSFEs of all models do not depend on the value of μ and are proportional to the value of σ^2. The relative MSFEs depend hence only on the convergence coefficient ρ, the sample size R and the forecast horizon h.

Let us now consider values of ρ corresponding to DGPs where the underlying half-life parameter varies from one to ten years. We also postulate the same sample size and forecast horizons as in Section 3. The results are presented in Figure 3, where the theoretical MSFEs of a given model are shown as a ratio of the MSFEs of the RW model.

The analytical results depend on the half-lives of the underlying DGP process. For half-lives above one year, the HL5 model beats the AR model; for values below 10 years it also beats the RW. This means that for a wide range of half-lives, between 1 and 10 years, the calibrated model beats its competitors. For values higher than three years the AR model loses also with the RW model, as the estimation error associated to the autoregressive process is more severe than the model misspecification error of assuming a RW.[4]

The bottom line is that in most univariate applications, unless the sample is very long, the AR model produces likely very imprecise forecasts. It is hence preferable to employ a reasonably calibrated HL model, which assumes a gradual mean reversion to the sample mean.

[4]All the results have also been cross-checked with Monte Carlo simulations.

5 Sensitivity analysis

In this section we show that the HL model remains the best model even when we change the forecast settings in our baseline. We shall then exploit its good performance and extend the analysis to nominal exchange rate forecasting.

Rolling window length

We begin by analyzing whether a change in the length of the rolling window has an impact on our findings. A longer rolling window should, in theory, increase the accuracy of the HL and AR models, as implied by (13) and (14). In the case of the HL model, a longer rolling window helps the modeler to determine with more precision the PPP level. In the case of the AR model, a longer window also helps one to better determine the degree of real exchange rate persistence. A longer rolling window may, however, be counterproductive, if we relax the assumption that the equilibrium value of the real exchange rate is time-invariant (see Rossi, 2006, for a discussion on the importance of parameter instability). As shown by tables 3 and 4, for most currencies in our sample this latter effect seems to play a lesser role, considering that both the HL and AR models tend to become more competitive for longer rolling windows. For a 20 year rolling window as well as in the case of recursive estimation, the HL model outperforms the RW model for 8 out of 9 currencies at almost all horizons.[5] For shorter samples, as in the case of a 10 year rolling window, the HL model continues to generally beat the RW (but this is no longer the case for the US dollar). The AR model instead generates, as expected, inaccurate forecasts, which confirms that the estimation error is the main source of the weak performance of the AR model. To sum up, for the currencies in our sample a rolling window of at least 15-20 years represents a good choice.

Prior on the half-life parameter

A Bayesian autoregressive process may potentially outperform the HL models. To establish this we set the mean-reversion parameter ρ as prior information rather than just impose it as we had done in the calibrated version of the model. To assess the implication of this choice let us consider a Bayesian autoregressive model (BAR), along the line suggested by Kilian and Zha (2002). We use the standard Minnesota setting for

[5]Results for the case of recursive estimation are available upon request and would not change the overall assessment of this paper.

vector autoregressions to elicit our prior on the degree of PPP persistence. In particular, we write down the model (1) in the standard AR form:

$$y_t = \delta + \rho y_{t-1} + \epsilon_t, \tag{15}$$

where $\delta = (1-\rho)\mu$. The prior for $\alpha = [\delta \quad \rho]'$ is assumed to be $\mathcal{N}(\underline{\alpha}, \underline{V})$ with $\underline{\alpha} = [(1-\bar{\rho})\bar{\mu} \quad \bar{\rho}]'$ and $\underline{V} = diag(\lambda\sigma^2, \lambda)$, where σ is the residual standard error from the AR model, $\bar{\rho}$ is the mean-reversion parameter calibrated so that the half-life is five years and λ is the overall tightness hyperparameter. The expected value of the posterior is:

$$\overline{\alpha} = \left(\underline{V}^{-1}\underline{\alpha} + \sigma^{-2}X'X\hat{\alpha}\right),$$

where $\hat{\alpha}$ is the OLS estimate of α, X is the observation matrix and $\overline{V} = (\underline{V}^{-1} + \sigma^{-2}X'X)$. The parameter λ has a very simple intuitive explanation for it allows us to choose an intermediate solution between the calibrated solution ($\lambda = 0$, $\overline{\alpha} = \underline{\alpha}$) and the estimated solution ($\lambda \to \infty$, $\overline{\alpha} = \hat{\alpha}$).

We report in Table 5 the ratios between the MSFEs from the Bayesian autoregressive model (reported as BAR in the table) and the MSFEs from the RW model for λ equal to 0, .1 and ∞. For the intermediate case $\lambda = 0.1$ such ratios are typically higher than those corresponding to the HL model and lower than those corresponding to the AR models. In other words in general the relative MSFEs tend to increase monotonically with the rising of λ. The best solution is therefore to set $\lambda = 0$, i.e. the calibrated solution.

For the one-month horizon we also provide a graphical illustration of what we have just said for values of λ ranging on a continuous scale between zero and ∞ (see Figure 4). On the vertical axis the MSFE is normalized so that it is equal to 100 for $\lambda = 0$. For six currencies (EUR, JPY, NZD, CHF, GBP, USD) the relationship between MSFE and λ is increasing and monotonic, i.e. the more weight one gives to estimation error the worse is the forecasting performance of the Bayesian autoregressive model. For one currency (MXN), the estimated model performs the best. For only two currencies (AUD and CAD) and very specific ranges of λ we find additional gains from using a Bayesian autoregressive model.

Other currencies

As an additional robustness check we evaluate if the results are applicable to other currencies as well. We thus consider the full set of real effective exchange rates indices available in the Bank for International Settlements database. The additional sample consists of eighteen currencies for the following countries: Austria (ATS), Belgium (BEF), Taiwan (TWD), Denmark (DKK), Finland (FIM), France (FRF), Germany (DEM), Greece (GRD), Hong Kong (HKG), Ireland (IEP), Italy (ITL), the South Korea (KRW), the Netherlands (NLG), Norway (NOK), Portugal (PTE), Singapore (SGD), Spain (ESP) and Sweden (SEK). The results are reported in Table 6 and lead to similar conclusions to those reached earlier. The forecasts based on the HL model are better than those based on the RW for 9 of the 18 currencies, comparable for 6 and less accurate for 3. The HL model also delivers more precise forecasts than the AR model for most currencies.

Sensitivity to the HL parameter

We finally evaluate if the performance of the HL model is sensitive to the duration of the adjustment process. Table 7 reports the relative performance of the HL model compared to the RW assuming that half of the adjustment is completed in 1, 3 and 10 years respectively. In the large majority of cases the HL model outperforms the RW regardless of this choice: the HL model beats the RW at the lower bound proposed by Rogoff (HL3) but is also very competitive for half-lives in the broad range of 1 to 10 years. Opting for fast convergence to PPP, such as in the case of the HL1 model, the calibrated half-life model continues to perform satisfactorily for forecast horizons above two years. Opting for a slower pace of convergence, such as in the HL10 model, the HL beats the RW at all horizons. However, at longer horizons the performance of the HL10 model is not as good as the HL3 or HL5 model, suggesting that it is still preferable to select a faster pace of convergence to PPP.

An extension to nominal exchange rate forecasting

The final step in our analysis consists in testing whether the mean reverting nature of the real exchange rate helps us to forecast nominal exchange rates. A simple approach is to assume that the adjustment of the real exchange rate predicted by model M is entirely achieved via changes in nominal exchange rates, while the relative price channel is absent. The predicted change of log nominal exchange rate (s) at horizon h is thus simply equal to the predicted real exchange rate (y) adjustment:

$$s^M_{T+h|T} - s_T = y^M_{T+h|T} - y_T. \tag{16}$$

The results presented in Table 8 are based on the same settings that we had earlier in our baseline for real exchange rate forecasting. The calibrated HL model performs much better than the RW for the same seven currencies as in the case of real exchange rate forecasting. The forecasts generated by the HL model are also generally much more precise than those generated by the AR model. Comparing the numbers in Tables 1 and 8 highlights that our ability to forecast real and nominal exchange rates is similar. For most currencies in our sample, the nominal exchange rate does not follow a RW but contributes to the mean reversion of the real exchange rate.

6 Conclusions

Notwithstanding the recent important progress made in the field of exchange rate economics, we still know very little of what drives currency fluctuations. Numerous studies have shown that exchange rate forecasts tend to be inaccurate both in absolute sense and relative to a naïve RW. Solving the "exchange rate puzzle" has been an endeavor for many economists over the past three decades. The vast exchange rate literature provides, however, at least two reasons for cautious optimism. First, the dismal forecasting performance of exchange rate models is partly due to estimation error, which explains why the RW is less competitive for larger datasets. Second, the literature on PPP has shown that real exchange rates tend to gradually revert to their mean.

In this paper we have illustrated how these two findings can be exploited in relation to real exchange rate forecasting. In particular, we have proposed a simple model that assumes a gradual return of the real exchange rate to its sample mean. From the theoretical perspective this alternative is more appealing than the RW for it takes into account that PPP holds in the long-term horizon. It is also appealing from the empirical perspective as it is consistent with the evidence that real exchange rates are mean reverting but highly persistent.

The key finding of our analysis is that the HL model overwhelmingly beats the RW in terms of real exchange rate forecasting for seven out of nine major world currencies. It is particularly noticeable that it outperforms the RW also at short-horizons, as shown in both the cases of the US dollar and the euro. We believe that our results are intuitive and not trivial: our preferred forecasting model for real exchange rates resembles quite closely the RW in the short-run while it gradually approaches PPP over long term horizons.

A second key finding of our analysis is that if the speed of mean reversion is estimated then the model performs significantly worse than the RW. We explain this result analytically by showing that the estimation forecast error plays an important role even for horizons of 15 to 20 years of monthly data. We have also carried out a comprehensive sensitivity analysis to show the robustness of our results to different rolling windows and the choice of analyzed currencies. The results are also valid for a wide range of half-lives, as long as they are calibrated at reasonable values, rather than estimated, irrespective of whether we use Bayesian techniques. Finally, we have also found that the mean reverting nature of real exchange rates can be exploited to outperform the RW also in terms of nominal exchange rate forecasting. For most currencies in our sample we find that the nominal exchange rate has contributed to the mean reversion process of the real exchange rate rather than just followed a RW.

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Table 1: Mean Squared Forecast Errors (15Y rolling window)

h	RW	HL	AR	RW	HL	AR	RW	HL	AR
		AUD			CAD			EUR	
1	0.05	1.01	1.02*	0.02	1.02	1.03*	0.02	1.00	1.04**
6	0.44	1.03	1.06*	0.23	1.03	1.11**	0.19	0.96	1.12*
12	0.82	1.06	1.09*	0.46	1.06	1.20**	0.43	0.92*	1.14*
24	1.53	1.10*	1.09	0.94	1.10	1.19*	0.89	0.83**	1.18**
36	2.10	1.12	1.06	1.58	1.06	1.20*	1.28	0.77**	1.13*
60	3.00	1.11	1.06	3.02	0.94	1.45**	2.06	0.66**	0.91
		JPY			MXN			NZD	
1	0.06	1.00	1.01	0.12	0.99	0.99	0.03	1.00	1.03*
6	0.59	0.98	1.04	0.83	0.96*	0.92	0.32	0.96	1.06
12	1.00	0.97	1.10*	1.55	0.92*	0.87*	0.72	0.92*	1.03
24	2.34	0.91	1.17**	3.01	0.84**	0.78*	1.59	0.83**	0.89*
36	3.53	0.86	1.19**	3.66	0.78**	0.74**	2.44	0.74**	0.74**
60	3.12	0.89	1.19*	3.56	0.74**	0.72*	3.01	0.64**	0.62**
		CHF			GBP			USD	
1	0.02	1.00	1.06	0.03	1.00	1.02	0.02	1.00	1.03*
6	0.12	0.98	1.22*	0.24	0.97	1.04	0.19	0.96	1.10*
12	0.25	0.97	1.16	0.47	0.95	1.03	0.31	0.93	1.21**
24	0.50	0.88*	0.99	1.06	0.87*	0.98	0.55	0.84*	1.21*
36	0.72	0.80**	0.78**	1.49	0.82**	0.93	0.69	0.72**	1.13
60	0.79	0.72**	0.69**	1.98	0.67**	0.70**	1.41	0.53**	0.91

Notes: For the RW model MSFEs are reported in levels (multiplied by 100), whereas for the remaining methods they appear as the ratios to the corresponding MSFE from the RW model. Asterisks ** and * denote the rejection of the null of the Diebold and Mariano (1995) test, stating that the MSFE from RW are not significantly different from the MSFE of a given model, at 1%, 5% significance level, respectively.

Table 2: Correlation of forecast and realized changes of real exchange rates

h	AUD	CAD	EUR	JPY	MXN	NZD	CHF	GBP	USD	mean
	HL model									
1	-0.04	-0.03	0.06	0.04	0.11	0.06	0.08	0.06	0.07	0.04
6	-0.01	0.02	0.21	0.15	0.30	0.23	0.21	0.19	0.22	0.17
12	-0.05	0.01	0.32	0.18	0.42	0.36	0.25	0.26	0.29	0.23
24	-0.10	0.02	0.46	0.31	0.60	0.56	0.50	0.43	0.43	0.36
36	-0.12	0.11	0.55	0.38	0.66	0.73	0.66	0.53	0.56	0.45
60	-0.20	0.26	0.71	0.31	0.61	0.81	0.74	0.71	0.78	0.53
	AR model									
1	-0.04	-0.09	-0.19	-0.04	0.13	-0.07	-0.13	-0.03	-0.09	-0.06
6	-0.07	-0.22	-0.29	-0.10	0.33	-0.05	-0.21	0.02	-0.17	-0.08
12	-0.06	-0.31	-0.24	-0.22	0.44	0.05	-0.04	0.08	-0.24	-0.06
24	0.00	-0.14	-0.22	-0.33	0.60	0.35	0.22	0.19	-0.13	0.06
36	0.07	-0.09	-0.06	-0.32	0.64	0.61	0.52	0.26	0.05	0.19
60	0.10	-0.43	0.32	0.04	0.63	0.78	0.64	0.58	0.34	0.33

Table 3: Mean Squared Forecast Errors (10Y rolling window)

h	RW	HL	AR	RW	HL	AR	RW	HL	AR
	AUD			CAD			EUR		
1	0.05	1.00	1.05	0.02	1.03	1.02	0.02	1.01	1.04*
6	0.44	0.99	1.15	0.23	1.07	1.10*	0.19	0.99	1.14**
12	0.82	0.98	1.10	0.46	1.14*	1.21	0.43	0.96	1.24**
24	1.53	0.96*	1.09	0.94	1.23**	1.24**	0.89	0.92	1.55**
36	2.10	0.95	1.20	1.58	1.21**	1.32*	1.28	0.88	1.96**
60	3.00	1.02	1.63	3.02	1.09	1.26*	2.06	0.77*	2.34*
	JPY			MXN			NZD		
1	0.06	1.00	1.02	0.12	0.99	1.02	0.03	1.00	1.00
6	0.59	0.98	1.03	0.83	0.95	1.06	0.32	0.96	0.99
12	1.00	0.99	1.08*	1.55	0.90**	1.15	0.72	0.92**	0.97
24	2.34	0.92	1.10	3.01	0.81**	1.26*	1.59	0.84**	0.92
36	3.53	0.87	1.08	3.66	0.73**	1.10	2.44	0.75**	0.96
60	3.12	0.95	1.14	3.56	0.68**	0.89	3.01	0.64**	1.26
	CHF			GBP			USD		
1	0.02	1.00	1.05	0.03	1.00	1.05	0.02	1.01	1.04**
6	0.12	0.99	1.24	0.24	0.98	1.25	0.19	1.01	1.17**
12	0.25	0.99	1.09	0.47	0.98	1.52	0.31	1.04	1.38**
24	0.50	0.92	1.02	1.06	0.93	3.35	0.55	1.05	1.55**
36	0.72	0.84**	0.88**	1.49	0.91	13.45	0.69	1.03	1.78**
60	0.79	0.76**	0.86	1.98	0.78**	0.82*	1.41	0.75*	1.87**

Notes: As in Table 1.

Table 4: Mean Squared Forecast Errors (20Y rolling window)

h	RW	HL	AR	RW	HL	AR	RW	HL	AR
		AUD			CAD			EUR	
1	0.05	1.00	1.05	0.02	1.02	1.01	0.02	1.00	1.00
6	0.44	0.99	1.15	0.23	1.05	1.05*	0.19	0.95	0.98
12	0.82	0.98*	1.10*	0.46	1.09	1.11**	0.43	0.90*	0.94
24	1.53	0.96*	1.09**	0.94	1.15*	1.17**	0.89	0.82**	0.90*
36	2.10	0.95*	1.20*	1.58	1.11	1.18**	1.28	0.77**	0.86**
60	3.00	1.02	1.63	3.02	0.98	1.35**	2.06	0.67**	0.75**
		JPY			MXN			NZD	
1	0.06	1.00	1.01	0.12	0.99	1.00	0.03	1.00	1.01
6	0.59	0.99	1.05*	0.83	0.94 *	0.94	0.32	0.97	0.99
12	1.00	1.00	1.12**	1.55	0.89**	0.87	0.72	0.92*	0.94
24	2.34	0.94	1.17**	3.01	0.79**	0.72**	1.59	0.82**	0.82**
36	3.53	0.89	1.16**	3.66	0.72**	0.67**	2.44	0.72**	0.69**
60	3.12	0.90	1.13	3.56	0.68 **	0.69*	3.01	0.59**	0.55**
		CHF			GBP			USD	
1	0.02	1.00	1.02	0.03	0.99	1.00	0.02	1.00	0.99
6	0.12	0.98	1.06	0.24	0.96	0.96	0.19	0.96	0.96*
12	0.25	0.97	1.02	0.47	0.93	0.91	0.31	0.94	0.92**
24	0.50	0.89*	0.87	1.06	0.84**	0.85**	0.55	0.86*	0.87**
36	0.72	0.82**	0.74**	1.49	0.79**	0.77**	0.69	0.78*	0.86**
60	0.79	0.78*	0.66**	1.98	0.65**	0.62**	1.41	0.59**	0.82**

Notes: As in Table 1.

Table 5: Mean Squared Forecast Errors (MSFEs) – BAR model

h	HL	BAR	AR	HL	BAR	AR	HL	BAR	AR
λ	0	.1	∞	0	.1	∞	0	.1	∞
	AUD			CAD			EUR		
1	1.01	1.02*	1.02*	1.02	1.03*	1.03*	1.00	1.04**	1.04**
6	1.03	1.05*	1.06*	1.03	1.11**	1.11**	0.96**	1.12*	1.12*
12	1.06	1.09*	1.09*	1.06	1.19**	1.20**	0.92*	1.14*	1.14*
24	1.10*	1.09	1.09	1.10	1.19*	1.19*	0.83**	1.17*	1.18*
36	1.12	1.06	1.06	1.06	1.20*	1.20*	0.77**	1.12*	1.13*
60	1.11	1.06	1.06	0.94	1.44**	1.45**	0.66**	0.91	0.91
	JPY			MXN			NZD		
1	1.00	1.01	1.01	0.99	0.99	0.99	1.00	1.03*	1.03*
6	0.98	1.04	1.04	0.96*	0.92	0.92	0.96	1.05	1.06
12	0.97	1.10*	1.10*	0.92*	0.87*	0.87*	0.92*	1.03	1.03
24	0.91	1.16**	1.17**	0.84**	0.78*	0.78*	0.83**	0.89**	0.89**
36	0.86	1.19**	1.19**	0.78**	0.74**	0.74**	0.74**	0.75**	0.74**
60	0.89	1.19*	1.19*	0.74**	0.72*	0.72*	0.64**	0.62**	0.62**
	CHF			GBP			USD		
1	1.00	1.05	1.06	1.00	1.02	1.02	1.00	1.03*	1.03*
6	0.98	1.21*	1.22*	0.97	1.04	1.04	0.96	1.10*	1.10*
12	0.97	1.15	1.16	0.95	1.03	1.03	0.93	1.21*	1.21*
24	0.88*	0.99	0.99	0.87*	0.98	0.98	0.84*	1.21*	1.21*
36	0.80**	0.79**	0.78**	0.82**	0.93	0.93	0.72**	1.12	1.13
60	0.72**	0.70**	0.69**	0.67**	0.70**	0.70**	0.53**	0.91	0.91

Notes: For all models MSFEs are reported in the ratios to the RW model. Asterisks ** and * denote the rejection of the null of the Diebold and Mariano (1995) test, stating that the MSFE from RW are not significantly different from the MSFE of a given model, at 1%. 5% significance level, respectively.

Table 6: Mean Squared Forecast Errors for other currencies

h	RW	HL	AR	RW	HL	AR	RW	HL	AR
		ATS			BEF			TWD	
1	0.05	1.00	1.05	0.00	1.00	1.04	0.02	1.01	1.01
6	0.44	0.99	1.15	0.03	0.96	1.09	0.13	1.07	1.06
12	0.82	0.98	1.10	0.07	0.92	1.07	0.25	1.15*	1.13
24	1.53	0.96	1.09	0.14	0.83*	1.00	0.43	1.39**	1.34**
36	2.10	0.95	1.20	0.21	0.74**	0.94	0.56	1.69**	1.49**
60	3.00	1.02*	1.63	0.38	0.60**	0.64**	1.05	1.83**	1.57**
		DKK			FIM			FRF	
1	0.00	1.00	1.02	0.01	1.01	1.01	0.00	1.00	1.02
6	0.04	0.97	1.07	0.15	0.99	1.03	0.03	0.97	1.06
12	0.08	0.94	1.10	0.40	0.94	1.06**	0.08	0.92	1.05
24	0.12	0.94	1.19*	1.07	0.85*	1.12**	0.15	0.83**	1.10
36	0.14	1.06	1.26*	1.34	0.83	1.19**	0.21	0.76**	1.11
60	0.18	1.23*	1.43**	0.89	1.02	1.60**	0.33	0.67**	0.89
		DEM			GRD			HKD	
1	0.01	0.99	1.03	0.02	1.01	1.03*	0.02	1.04	1.05*
6	0.06	0.95	1.12	0.05	1.17**	1.35**	0.23	1.11	1.23**
12	0.13	0.89*	1.11	0.08	1.41**	1.76**	0.48	1.20	1.50**
24	0.28	0.76**	1.03	0.23	1.52**	1.80**	1.23	1.18	1.84**
36	0.41	0.65**	0.91	0.41	1.55**	1.66**	2.06	1.14	2.13**
60	0.64	0.47**	0.75**	0.77	1.65**	1.60**	4.56	0.92	2.94**
		IEP			ITL			KRW	
1	0.01	1.01	1.03	0.02	1.00	1.04**	0.10	0.99	1.01
6	0.12	1.00	1.06	0.13	0.95	1.15**	0.82	0.94*	1.02
12	0.26	0.99	1.13	0.30	0.88	1.26**	1.48	0.89**	0.99
24	0.56	0.95	1.33**	0.63	0.74**	1.35**	2.65	0.81**	0.99
36	0.92	0.90	1.32	1.08	0.61**	1.38**	3.00	0.77**	1.17
60	1.33	0.91	0.94	1.09	0.46**	1.65**	3.11	0.83	3.76
		NLG			NOK			PTE	
1	0.01	1.00	1.02	0.02	0.99	1.01	0.01	1.08*	1.20**
6	0.05	0.97	1.05	0.12	0.96	1.00	0.05	1.29**	1.75**
12	0.10	0.93	0.98	0.22	0.92*	0.97	0.12	1.39**	1.94**
24	0.20	0.83**	0.86	0.25	0.93	1.07	0.23	1.72**	1.94**
36	0.26	0.79*	0.83	0.29	0.97	1.12	0.23	2.44**	2.14**
60	0.40	0.74**	0.72*	0.28	1.08	1.28*	0.38	3.17**	2.19**
		SGD			ESP			SEK	
1	0.01	1.00	1.02*	0.01	1.01	1.04**	0.02	1.00	1.01
6	0.08	1.00	1.08*	0.06	0.99	1.17**	0.24	0.99	0.99
12	0.19	0.96	1.09	0.16	0.91	1.20**	0.52	0.97	0.96
24	0.52	0.84**	1.04	0.37	0.76*	1.18*	0.95	0.97	0.96
36	0.85	0.73**	0.97	0.55	0.61**	1.03	1.09	1.10	0.96
60	1.53	0.57**	0.83**	0.84	0.52**	0.82*	1.18	1.48**	1.03

Notes: As in Table 1.

Table 7: Mean Squared Forecast Errors for other HL duration

h	HL1	HL3	HL10	HL1	HL3	HL10	HL1	HL3	HL10
	AUD			CAD			EUR		
1	1.18**	1.03	1.00	1.30**	1.04	1.01	1.09	1.00	1.00
6	1.47**	1.07	1.01	1.74**	1.10	1.01	1.17	0.96	0.97
12	1.71**	1.13	1.02	2.02**	1.17	1.01	1.11	0.92*	0.95*
24	1.79**	1.22**	1.04	2.05**	1.24*	1.02	0.92	0.83**	0.89**
36	1.72**	1.25*	1.04	1.68**	1.18	1.00	0.76	0.77**	0.85**
60	1.46**	1.22	1.03	1.13	0.99	0.93	0.52**	0.66**	0.78**
	JPY			MXN			NZD		
1	1.10*	1.01	1.00	1.01	0.99	1.00	1.07	1.00	1.00
6	1.22	0.99	0.98	0.97	0.94	0.98*	1.06	0.96	0.98
12	1.41*	1.00	0.97	0.92	0.89*	0.95*	0.96	0.90*	0.95*
24	1.19	0.92	0.93	0.76	0.79**	0.90**	0.72*	0.77**	0.90**
36	1.04	0.86	0.90*	0.71*	0.72**	0.86**	0.54**	0.65**	0.84**
60	1.20	0.97	0.88*	0.79	0.73**	0.81**	0.45**	0.55**	0.77**
	CHF			GBP			USD		
1	1.04	1.00	1.00	1.08*	1.00	1.00	1.10*	1.00	1.00
6	1.10	0.98	0.98	1.16	0.97	0.98	1.19	0.96	0.98
12	1.09	0.97	0.98	1.19	0.95	0.97*	1.33	0.94	0.95*
24	0.84	0.84*	0.93**	0.94	0.85*	0.92**	1.18	0.84	0.89**
36	0.66**	0.73**	0.88**	0.78	0.77**	0.88**	0.97	0.71*	0.81**
60	0.61**	0.65**	0.81**	0.53**	0.59**	0.78**	0.42**	0.45**	0.68**

Notes: As in Table 1.

Table 8: Mean Squared Forecast Errors for NEERs

h	RW	HL	AR	RW	HL	AR	RW	HL	AR
		AUD			CAD			EUR	
1	0.05	1.01	1.02*	0.02	1.02	1.03*	0.02	1.00	1.04*
6	0.44	1.02	1.06*	0.24	1.03	1.11**	0.19	0.97	1.12*
12	0.78	1.06	1.11**	0.47	1.06	1.22**	0.40	0.93	1.15*
24	1.30	1.12 *	1.13*	0.86	1.10	1.26**	0.81	0.85*	1.20**
36	1.70	1.16 *	1.10	1.42	1.05	1.29**	1.13	0.78**	1.16**
60	2.23	1.21 *	1.09	2.72	0.91	1.56**	1.78	0.66**	0.93
		JPY			MXN			NZD	
1	0.06	1.00	1.01	0.12	1.00	1.00	0.03	1.00	1.03*
6	0.57	0.99	1.03	1.23	0.98	0.95	0.32	0.96	1.06
12	1.03	0.99	1.08*	2.92	0.95*	0.91	0.69	0.91*	1.03
24	2.55	0.93	1.11*	7.19	0.90**	0.84*	1.41	0.81**	0.89*
36	3.98	0.88	1.11*	11.86	0.87**	0.81**	2.17	0.70**	0.74**
60	3.51	0.99	0.95	25.53	0.88**	0.82**	2.69	0.60**	0.59**
		CHF			GBP			USD	
1	0.02	1.00	1.06 *	0.03	1.00	1.02*	0.03	1.00	1.03*
6	0.13	1.00	1.22 *	0.23	0.99	1.06	0.25	0.97	1.09 *
12	0.29	0.99	1.19	0.49	0.98	1.06	0.45	0.95	1.17**
24	0.62	0.93*	1.01	1.12	0.92	1.02	0.90	0.88*	1.15**
36	0.90	0.87**	0.85*	1.69	0.88*	0.98	1.26	0.81*	1.05
60	1.12	0.87**	0.83*	2.42	0.74**	0.79**	2.52	0.67*	0.87**

Notes: As in Table 1.

Figure 1: Real exchange rates (2010 = 100)

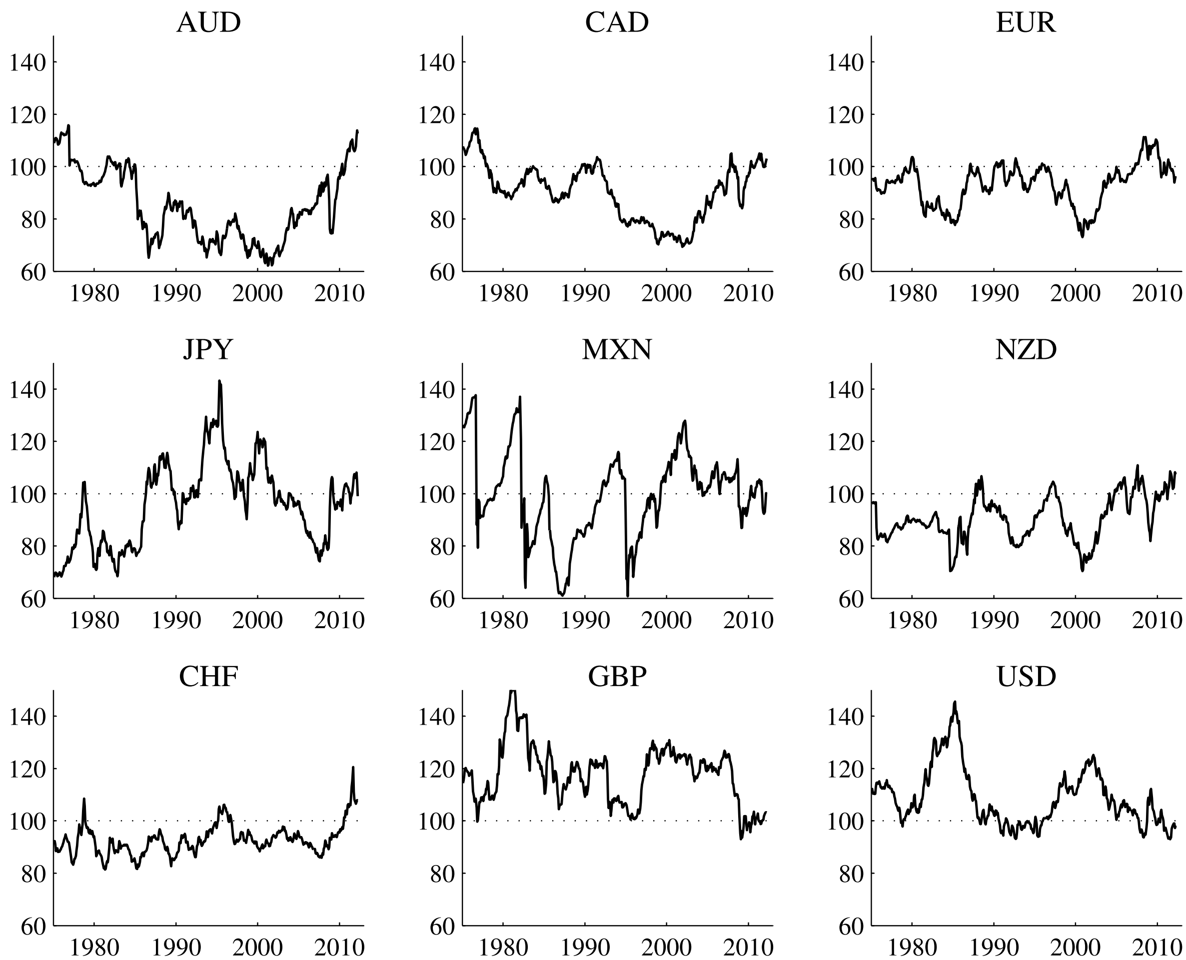
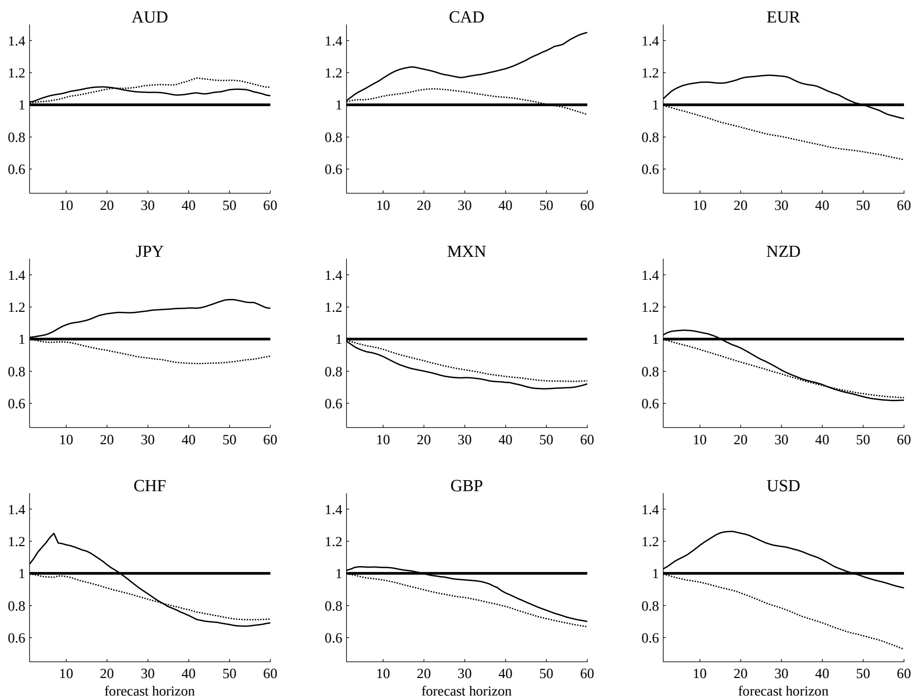
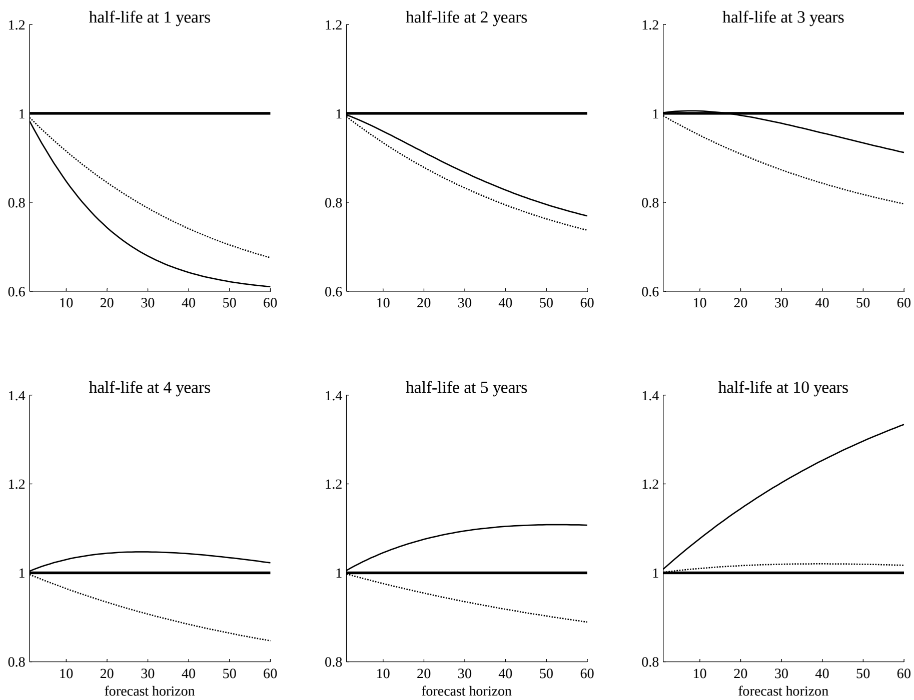


Figure 2: Mean Squared Forecast Errors



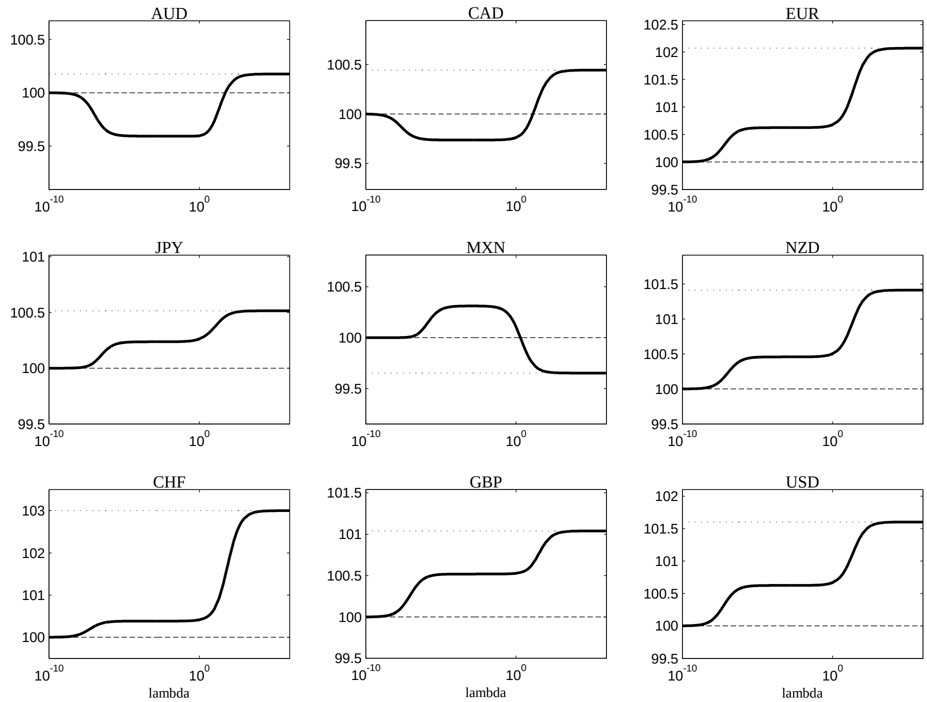
Notes: Each line represents the ratio of MSFE from a given method to MSFE from the random walk, where values below unity indicate better accuracy of point forecasts. The straight and dotted lines stand for AR and HL5, respectively. The forecast horizon is expressed in months.

Figure 3: Theoretical Mean Squared Forecast Errors



Notes: Each line represents the ratio of MSFE from a given method to MSFE from the random walk, where values below unity indicate better accuracy of point forecasts. The straight and dotted lines stand for AR and HL, respectively. The forecast horizon is expressed in months.

Figure 4: Sensitivity analysis of MSFE on the λ (forecast horizon: 1 month)



Notes: Each line represents the ratio of MSFE from a given method to MSFE from the HL5 multiplied by 100, where values 100 unity indicate better accuracy of point forecasts. The straight, dashed and dotted lines stand for BAR, HL and AR, respectively. The value of λ parameter is expressed using the logarithmic scale