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Sustainable International Monetary Policy Cooperation*

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Abstract

We provide new insight on international monetary policy cooperation using a symmetric two-country model based on Benigno and Benigno [2006]. An incentive feasibility problem exists between the policymakers across national borders: Under asymmetric volatilities of shocks among the countries, the home country has an incentive to deviate from an assumed cooperation regime to one with non-cooperation in response to a positive markup shock in the home country. This motivates our study of a constrained cooperation regime which is endogenously sustained by a cross-country, state-contingent contract. We label such a regime sustainable cooperation. Under sustainable cooperation, the responses of inflation and the output gap in both countries are different from those induced by the cooperation and non-cooperation regimes reflecting the endogenous welfare redistribution between countries under the state-contingent contract.

JEL codes: E52, F41, F42

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1 Introduction

The recent global financial crisis has renewed interest in the potential for international policy cooperation. In order for the global economy to recover, the need for international monetary policy cooperation is now a topic of discussion among leaders in major policy institutions. For example, Dr. John Lipsky, Acting Managing Director of the IMF, stated on 21 June 2011:

> "A second success is the remarkable increase in global policy cooperation that has taken place in the wake of the 2008-09 global financial crisis. When the world last faced such grave danger—during the Great Depression—countries acted in their own, perceived self-interest with beggar-thy-neighbor policies that in fact deepened the downturn. This time, countries acted together to tackle the crisis."

The debate on the gains from policy cooperation has been at the heart of international finance and open economy macroeconomics. It goes back as far as Hume [1752] who pointed out the existence of cross-country policy spillover. Cooper [1969] raised the possibility that each country may not be able to maximize its own welfare with increasing interdependence through trade or investment in the Mundell-Fleming model. Hamada [1976] formally analysed the gains from cooperation from a game theoretic perspective. Since then, many have discussed the pros and cons of policy cooperation in theoretical general equilibrium models. Corsetti and Pesenti [2001], Clarida et al. [2002], Benigno and Benigno [2003, 2006], Canzoneri et al. [2005], Corsetti et al. [2010], and Engel [2015] clarified the conditions on when gains from cooperation emerge in the class of New Open Economy Macroeconomics models in the style of Svensson and van Wijnbergen [1989] and Obstfeld and Rogoff [1995].

To date, analyses on international monetary policy cooperation assume that authorities do not have any incentive to deviate from cooperation, or if there is ever any doubt in the modellers' minds, the analyses would proceed to consider the other extreme of (strategic) non-cooperation.[1] In other words, the national central banks are either always equipped with some unspecified commitment technology that ensures international cooperation in all contingencies, or, otherwise under non-cooperation regimes they have no means to improve their individual welfare by undertaking cooperative actions.[2] However, it is unrealistic to assume that each government can

[1] A recent application of such an approach in terms of studying currency unions is Chari et al. [2014].

[2] This point is already acknowledged in previous studies. Benigno and Benigno [2006] wrote "Another important open issue is the enforcement of the proposed targeting rules. We have briefly

fully commit to cooperation under any contingency. Nor is it realistic to assume that non-cooperation fully rationalizes the reality of international monetary policy. A study to inquire into the *endogenous* sustainability of the cooperative regime, and its consequences, is still unexplored. Such a study is of direct relevance to the current state in the global coordination of international monetary policy. In particular, if the needs for policy cooperation become more pressing, as in today's global setting, we must understand under what macroeconomic conditions policy cooperation can be sustained in equilibrium. We also need to understand how such sustainable equilibria may look like, relative to known equilibria under cooperation and non-cooperation regimes.

In this paper, we aim to understand the nature of such endogenously sustainable equilibria. Assuming each monetary policymaker commits to its strategy vis-à-vis its own country, each country may still have the incentive to deviate from a cooperation regime to a non-cooperation regime. In this paper, a possibly distorted version of the cooperation regime is endogenously sustained by a cross-country and state-contingent contract. We name such a regime a *sustainable cooperation regime*. Specifically, we ask two questions: First, when does an assumed cooperation regime fail to be incentive feasible? Second, how does an incentive feasible or sustainable-plans equilibrium look like, or what are its consequences?

To this end, we set up a linear-quadratic (LQ) New Open Economy macroeconomic framework for optimal monetary policy analysis based on Benigno and Benigno [2006, hereinafter BB]. Then we calculate and study the behavior of equilibria, respectively, under each extreme regime of assumed international cooperation and non-cooperation. We use these two versions of well-studied policy regimes to compare with the endogenous sustainable cooperation regime. Under sustainable cooperation, central banks maximize the global social welfare subject to each country's competitive equilibrium restrictions, global resource constraints, and a set of history-contingent *sustainability constraints* (one for each country). These latter constraints encode the following incentive-feasibility requirements: Should a country have the temptation to walk away from international cooperation, the implied contract ensures that it, at best, can only be as well-off as in the equilibrium under a non-cooperation regime. That is, the outside option value to each nation's policymaker is the welfare induced by the non-cooperation regime, which in itself implies a sustainable equilibrium plan. These credible threats, along with other equilibrium

addressed this issue acknowledging that, as in previous contributions in the literature, the cooperation problem is simply shifted at the delegation stage to a supranational authority."

restrictions, induce allocation and pricing processes that are said to be sustainable plans in the language of Chari and Kehoe [1990].

When does an assumed cooperation regime fail to be incentive feasible? Under asymmetric volatilities of markup shocks across the countries, the country with a larger shock volatility has more incentive to stabilize its own output gap at the expense of the other's welfare. We show that in response to a positive markup shock in the home country (under reasonable calibration and setting of parameters), the home country has an incentive to deviate from cooperation to non-cooperation. Since an independent Home policymaker does not internalize its domestic monetary policy effects on its Foreign counterpart's welfare in a non-cooperation setting, the payoff from selfish policymaking can dominate that from remaining in cooperation.

The reason there is a spillover effect of Home's policy outcome onto Foreign's welfare and vice-versa, under non-cooperation, is because of the existence of markets that insure cross-country consumption risk. When one country attempts to stabilize its output gap, this has a direct externality effect on its neighbor's welfare via the equilibrium asset-pricing relation between two countries' output (gap) and the terms of trade. [3] This is conventionally known as the *terms of trade externality*. In our model, this externality is present even in the special case in which the equilibrium policy and private-sector behavior of the economies themselves are "insular".[4]

However, this externality effect can be weakened by additional welfare-relevant feedback effects of the terms of trade movements. These effects—depending on the degree of agents' risk aversion in each country—come from the role of the terms of trade as endogenous markup shock and the resulting retaliation considerations from the other country. Because of this, the Home policymaker will need to consider an outward-looking policy in view of the non-insular equilibrium response from its Foreign counterpart. Therefore, we can show numerically that the incentive to deviate is largest when agents' intertemporal elasticity of substitution becomes closer to unity—viz. when the two countries' equilibrium characterizations become insular.

[3] In this paper, we assume the existence of international markets trading in complete state-contingent consumption claims, as is done in BB. For the purposes of our theoretical analyses this assumption does not matter. More generally, one could instead consider an incomplete markets setting. However, such additional frictions will just cloud the insights on our study of sustainable cooperation plans itself.

[4] This special case arises when the coefficient of relative risk aversion of both countries are unity. By outward-looking or non-insular we mean monetary policies and equilibrium decision rules that are explicit dependent on foreign variables. In the case of competitive equilibrium conditions, non-insularity shows up as explicit terms involving the international terms of trade variable. In terms of the equilibrium monetary policy conditions, non-insularity appears in the form of foreign markup shocks affecting the level of the domestic policymaker's output-inflation trade-off.

This incentive problem is also exacerbated by greater asymmetry in the volatilities of markup shocks.

Given the possible incentive infeasiblity of a given cooperation regime, it is then important to understand the nature of endogenously sustainable monetary policy cooperation and its attendant effects. Under sustainable cooperation, the responses of inflation and output gap in both countries are different from the ones under the cooperation and non-cooperation regimes, reflecting the impact of occassionally binding sustainability constraints. Whenever the sustainability constraint in the home country binds, a history-contingent pseudo-weight on each country's social welfare shifts toward favoring the home country welfare—i.e., the sustainable equilibrium has to redistribute welfare to keep the Home country within the sustainable cooperation regime.

Our main message is as follows: International monetary policy cooperation should not be taken for granted as being incentive feasible. There exists temptation to deviate from cooperation under some macroeconomic conditions. It is important to understand what may make international monetary policy cooperation an untenable proposition, and to what extent are the limits of an assumed cooperation regime. When countries face non-symmetric volatilities in their markup shocks, and when two countries are nearly insular in their structural relationship, the prescriptions of a constrained but endogenously sustainable cooperation regime is of particular importance. In such situations, the best allocation and pricing processes the central banks can achieve—in global welfare terms—are somewhere between those induced under the cooperation and non-cooperation regimes.

The rest of this paper is structured as follows. In Section 2, we describe the model setup (competitive equilibrium) and discuss the policymakers' social welfare measures relevant to the regimes we consider. In Section 3, we consider the two policy regimes standard in the literature, cooperation and non-cooperation, and then we will characterize equilibrium under the regime of sustainable cooperation. We provide explanations for the different equilibrium behaviors under these three regimes. We conclude with Section 4. The optimal trade-offs for the policymakers under each regime, including computational procedure for the equilibrium under sustainable cooperation, are derived in Appendix A. Also, a detailed description of the original model and welfare approximation for the LQ framework is given in Appendix B.

2 LQ Framework

The model is based on BB. For the purposes of our study, we will present the model in terms of its (approximate) LQ characterization.[5] We will first describe the competitive equilibrium of the two country model (for any given feasible monetary policy) in Section 2.1.

The model underlying the competitive equilibrium characterization is as follows: There are two countries—Home and Foreign. In each country, there is a representative household. Each household consumes bundles of differentiated goods produced in Home and Foreign countries. Each household also provides firm-specific labor to firms within the country. Firms in each country produce differentiated goods under monopolistic competition and sticky prices, given the demand function of the households in both countries. There are internationally complete markets for state-contingent consumption claims and the law of one price holds for all goods. As in BB, these two assumptions help to simplify the equilibrium descriptions later: The real exchange rate is unity and consumption is equalized between the two countries. Each country also has a monetary policymaker who maximizes a social welfare function, given the equilibrium conditions of the whole economy. In order to isolate our focus on incentive-feasibility problems in terms of international monetary policy cooperation, we abstract from time-consistency issues within each country. In particular, we assume that each country's policymaker commits to maximizing its own citizen's ex-ante welfare.

To discipline our analyses, we restrict attention to equilibria under the following settings: Consumption is the only component of GDP, the two countries are symmetric in terms of taste, technology and market sizes, and the steady state markup is unity.[6]Also, without loss of generality, we assume that the two countries are also symmetric in terms of their initial levels of assets. We further assume that the elasticity of substitution between domestic and foreign goods is equal to one. Under these assumptions, BB showed that in response to technology shocks, (i) the flexible-price allocation is constrained optimal under cooperation, and (ii) there are no gains by deviating from cooperation to non-cooperation. These results also hold in our model. Given the setting, we can focus on inefficient markup shocks as the only sources of policy incentive to cooperate or not.

[5]This representation provides a connection to the existing literature that also uses the same methodology.

[6]This is achieved by assuming subsidies that eliminate positive rents in the steady state. Thus, a markup shock in this paper can also be interpreted as a structural shock to this subsidy.

As in the case of the closed economy, markup shocks generate a trade-off between inflation and the output gap represented in the NK Phillips curve. In the open economy considered here, the good markets are integrated across countries so that in equilibrium the terms of trade will be a part of the firm's real marginal cost.

In Section 2.2, we will present and discuss the relevant social welfare criteria relevant to the three policy regimes to be considered—cooperation, non-cooperation, and sustainable cooperation. The criterion function will turn out to be the same in the cooperation and sustainable cooperation problems. The social welfare in each setting considered will be representable by a purely quadratic function, which accurately approximates the indirect utility of the representative household in each country up to second order.[7] Here, we will also highlight the terms of trade mechanisms underlying a potential policy externality problem. This externality is what gives rise to a temptation for international monetary policymakers to walk away from a cooperative solution.

2.1 Competitive equilibrium

The equilibrium behavior of households and firms, as far as optimal policy is concerned, is sufficiently summarized by the NK Phillips curve:[8]

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \mu_t + \kappa \left[(\rho + \eta) y_t + (1 - \rho) s_t \right], \tag{1}$$

and

$$\pi_t^* = \beta \mathbb{E}_t \pi_{t+1}^* + \kappa \mu_t^* + \kappa \left[(\rho + \eta) y_t^* + (1 - \rho) s_t^* \right]. \tag{2}$$

All the variables are natural logarithmic transforms of their original levels.[9] The variables with an asterisk ($*$) are of the Foreign country and the ones without asterisk refer to Home country variables. The variable y_t corresponds to output, π_t is net producer price inflation rate, and s_t is the terms of trade. The exogenous variable, μ_t, is the markup shock, which follows a Markov process. The parameters are as follows: $\beta \in (0, 1)$ is the discount factor, $\rho > 0$ is the coefficient of

[7]The competitive equilibrium conditions are approximate linear constraints, representing the optimizing behavior of households and firms (e.g., Benigno and Woodford 2005, 2012), hence the LQ approach. However, when we consider the case of sustainable cooperation, the problem is no longer a standard LQ problem, since the sustainability constraints, albeit involving quadratic forms, will only be ocassionally binding.

[8]Note that consumption Euler equations are redundant, as each country can control the real side of the economy by committing to future policies. Thus we do not present them here.

[9]The steady state equilibrium allocations of output, inflation and terms of trade turn out to be 1. The steady state levels of the markups shocks are also unity.

relative risk aversion, $\eta > 0$ is Frisch elasticity of labor disutility, $\alpha \in (0, 1)$ is the probability of prices being fixed per period [as in Calvo, 1983], and $\sigma > 0$ is the elasticity of substitution among differentiated products. The composite parameter $\kappa = (1 - \alpha)(1 - \alpha\beta) / [\alpha (1 + \sigma\eta)]$ is the slope of the log-linearized NK Phillips curve.[10]

Under internationally complete asset markets and the law of one price, the equilibrium terms of trade is given by

$$s_t = \frac{1}{2}(p_F - p_H) = \frac{1}{2}(y_t - y_t^*), \tag{3}$$

where p_H and p_F are Home and Foreign producer prices. An increase (decrease) in s_t means deterioration (improvements) in the Home terms of trade.

Note that under the law of one price, we have $s_t^* = -s_t$. There is a connection between the terms of trade and a notion of *insularity* in terms of each country's competitive equilibrium description—i.e., (1) and (2). How exposed (or insular) a country is to the other's policy and equilibrium outcomes depends crucially on the constant relative risk aversion parameter $\sigma > 0$. When households are more risk averse ($\rho > 1$), an increase in s_t lowers Home marginal costs and hence Home inflation since $(1 - \rho)s_t < 0$. By the same token, it raises Foreign inflation since $(1 - \rho)s_t^* > 0$, and this resembles the effect of a positive markup shock on Foreign inflation. When $\rho < 1$, the opposite is true. When $\rho = 1$, two countries are said to be insular in terms of their competitive equilibrium charaterization.

There are two opposing forces explaining why ρ determines how equilibrium terms of trade feeds back onto Home (and Foreign) inflation. Suppose foreign output falls (rises), holding all else constant. On the one hand, internationally complete asset markets, under the law of one price, imply that in equilibrium the Home terms of trade deteriorates (improves)—see (3). A rise (fall) in s_t means that the purchasing power of Home agent's wages becomes lower (higher) relative to Foreign agents, which tends to raise (lower) firm's real marginal cost as a result of expenditure switching toward Home-produced goods. Therefore Home inflation tends to rise (fall). On the other hand, the rise (fall) in s_t results in a fall (rise) in Home consumption, holding all else constant. (This acts through a market clearing condition and the effect of the terms of trade on Home and Foreign demand for Home goods.[11])

[10] These are the structural parameters related to the households' preference and firms' technology representations in the original model. See Appendix B.1.

[11] A log-linear version of this says $c_t = y_t - s_t$,and for this thought experiment, we have held y_t fixed.

Lower (higher) Home consumption raises (lowers) the shadow value of work and through labor market clearing, that tends to lower (raise) Home firms' real marginal cost, and therefore Home inflation tends to fall (rise).[12] When $\rho > 1$, the second channel dominates the first, hence we see a negative relation between Home terms of trade (s_t) and Home inflation (π_t) in (1). When $\rho < 1$, the first channel dominates the second, so that there is positive relation between Home terms of trade (s_t) and Home inflation (π_t) in (1). The special case is when $\rho = 1$, where the two opposing channels cancel out. The converse logic holds between Foreign terms of trade and inflation in (2).

However, note that the case of $\rho = 1$ does not imply that there is no spillover effects from one country's policy outcomes to another in terms of *welfare*.[13] We will discuss this ever-present welfare relevant *terms of trade externality effect* in the next section.

2.2 Welfare criteria and terms of trade externality

Each country's social welfare function summarizes the households' (competitive equilibrium) value function. These are approximated up to second-order accuracy as:

$$\begin{aligned} V_0 &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u_t, \\ &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} (\eta+\rho)\left(y_t - \frac{1}{\eta+\rho}\mu_t\right)^2 + \frac{\sigma}{\kappa}\pi_t^2 + 2(1-\rho)s_t^2 \\ +(\eta+\rho)\left(y_t^* + \frac{1}{\eta+\rho}\mu_t^*\right)^2 + \frac{\sigma}{\kappa}\left(\pi_t^*\right)^2 \end{array} \right], \end{aligned} \tag{4}$$

for the Home country, and,

$$\begin{aligned} V_0^* &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u_t^*, \\ &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} (\eta+\rho)\left(y_t^* - \frac{1}{\eta+\rho}\mu_t^*\right)^2 + \frac{\sigma}{\kappa}\left(\pi_t^*\right)^2 + 2(1-\rho)\left(s_t^*\right)^2 \\ +(\eta+\rho)\left(y_t + \frac{1}{\eta+\rho}\mu_t\right)^2 + \frac{\sigma}{\kappa}\pi_t^2 \end{array} \right], \end{aligned} \tag{5}$$

[12] In the terminology of Clarida et al. [2002], the former is "the terms of trade effect" while the latter is "the risk sharing effects." For the discussion on this issue in more general setting, see Tille [2001].

[13] Canzoneri et al. [2005] also demonstrate that this arises because countries are interrelated on the consumption side.

for the Foreign country.[14] The social welfare in period 0, V_0, is the expected discounted sum of the fluctuations around the target of these variables. Note that the targets of Home and Foreign output are non-zero and different among countries.

Observe also that each country's welfare depends on the output and markup shock from its foreign counterpart. This arises as there is the terms of trade externality underlying the openness of the economies discussed above. From equations (1) and (2), the output gap is defined as follows:

$$x_t \equiv y_t - \frac{1}{\eta+\rho}\mu_t,$$
$$x_t^* \equiv y_t^* - \frac{1}{\eta+\rho}\mu_t^*.$$

The target outputs are solely functions of markup shocks and setting $x_t = x_t^* = 0$ is consistent with zero inflation $\pi_t = \pi_t^* = 0$ for all $t \geq 0$. On the one hand, the policymakers desire to set output so that the *welfare-relevant* output gaps in equations (4) and (5) are close to zero. On the other, they also want to stabilize domestic and foreign inflation. To fix ideas, suppose that there is a positive Home markup shock. The Home policymaker wishes to set the domestic output $y_t = [1/(\eta+\rho)]\mu_t > 0$ as in Eq. (4) so that x_t is zero. However, doing so harms the Foreign policymaker, as the welfare-relevant gap in the Foreign country becomes $\tilde{x}_t = y_t + [1/(\eta+\rho)]\mu_t = 2[1/(\eta+\rho)]\mu_t$ in equation (5). This illustrates that there is a conflict of interest in the nature of the problem if the central banks behave independently of each other.

One way to expound on this potential externality problem when considering each country's policy problem separately, is as follows: Consider an intermediate step in arriving at the approximating per-period social welfare functions u_t and u_t^*, respectively, found in equations (4) and (5). This intermediate step would produce the following pair of expressions:

$$u_t = s_t - \frac{1-\rho}{2}(y_t - s_t)^2 + \frac{1+\eta}{2}y_t^2 + \frac{\sigma}{\kappa}\pi_t^2,$$
$$u_t^* = -s_t - \frac{1-\rho}{2}(y_t^* + s_t)^2 + \frac{1+\eta}{2}\left(y_t^*\right)^2 + \frac{\sigma}{\kappa}\left(\pi_t^*\right)^2.$$

Of course, one cannot stop at this point in deriving the approximate welfare functions.[15] For our exposition here, this break-down of the steps is nevertheless

[14]See Appendix B.2 for welfare approximation.

[15]If taken at face value, the linear terms in the intermediate welfare approximation step may induce spurious welfare evaluation in the LQ framework [see e.g., Kim and Kim, 2003, 2007].

instructive. Note that a naïve addition of these two terms, $u_t + u_t^*$, yields an expression equivalent to the per-period loss in the global welfare function (6), as the linear terms of s_t are canceled out. That means that under global cooperation of monetary policy, there is no terms of trade externality problem. However, if the social welfare in each country is considered separately under independent national monetary-policy making, s_t is substitutable for a quadratic approximation of the Home and Foreign Phillips curves. This results in the non-zero output targets in equations (4) and (5). In words, and policy-wise, this implies that by manipulating the terms of trade externality on other nations, domestic policymakers can potentially reduce their own losses arising from inflation and output fluctuations, where the latter is aimed at by setting a non-zero output target for their own country. This is because the independent and selfish policymakers know the nexus between their policies and the terms of trade. In turn, they know the net effect of the terms of trade on output gap, all else equal, but they do not care what happens to the other country's welfare.[16]

Consider next the case where Home and Foreign countries face a consolidated or global social welfare function. From equations (4) and (5), the global social welfare

[16]Probing deeper, we can also see that fluctuations in the terms of trade affect welfare via consumption. Denote c_t as log consumption. In a competitive equilibrium consumption fluctuations, respectively in the Home and the Foreign country, are tied to output and the terms of trade movements as:

$$\begin{aligned} c_t &= y_t - s_t, \\ c_t^* &= y_t^* + s_t. \end{aligned}$$

Each country's consumer would prefer to have a smooth consumption outcome across dates and states of the world. However, the terms of trade externality hinders consumption smoothing since selfish policymakers would like to create fluctuations in s_t to minimize their own losses. When $\rho > 1$ ($\rho < 1$), the second order approximation of the utility term involving consumption implies that consumers dislike (prefer) fluctuations in log consumption. Note that it is about consumption in logarithmic terms. Therefore, it is totally consistent with risk averse behavior of the households, or concave utility function, whenever $\rho > 0$.

function is obtained as[17]

$$
\begin{aligned}
V_0^W &= V_0 + V_0^*, \\
&= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u_t + u_t^* \right], \\
&= -2\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(\eta + \rho) y_t^2 + \frac{1-\rho}{2} (y_t - y_t^*)^2 \right. \\
&\qquad \left. + (\eta + \rho) (y_t^*)^2 + \frac{\sigma}{\kappa} \pi_t^2 + \frac{\sigma}{\kappa} (\pi_t^*)^2 \right]. \qquad (6)
\end{aligned}
$$

Here, the targets of Home and Foreign output are zero for both countries, whereas they are non-zero in Eqs. (4) and (5). This echoes our insight earlier that if there is a consolidated global policymaker, or equivalently, the two policymakers somehow cooperative in all contingencies, then the terms of trade externality is not present.

3 Equilibria under Alternative Regimes

We now use the LQ framework to consider three regimes for international monetary policy coordination. In Section 3.1 we first study the familiar cases of an assumed cooperation regime and its polar extreme, the non-cooperation regime. Here we will answer the first question raised in this paper: Under what conditions in our model will an assumed cooperation regime fail to be incentive feasible? Then, we will move on to the next question in Section 3.2: Encoding a threat of reversion of the non-coopeation regime, how would an equilibrium under the sustainable equilibrium regime look like? We will discuss the characterization of such an equilibrium, what it means in terms of global redistribution of welfare, and then study the equilibrium dynamics of sustainable cooperation computationally.

For all of our computational analyses, the model is parametrized by the settings summarized in Table 1. We basically follow BB with some exceptions: The steady state markup is unity, consumption is the only component of GDP, two countries are symmetric and of the same size, and the elasticity of substitution between domestic and foreign goods is equal to one.[18] For the exogenous shock processes, we set $\rho_\mu = \rho_\mu^* = 0.5$, $\sigma_\varepsilon = 1.0$ and $\sigma_\varepsilon^* = 0.2$ —i.e., there is asymmetric volatility between the Home and Foreign countries. Also, where relevant to our questions, we will

[17] As shown in Appendix B.2, naturally, this can be obtained directly by approximating the global welfare.

[18] BB also use different values for Calvo parameters in Home and Foreign countries.

vary parameters such as the coefficient of relative risk aversion ρ to illustrate the intuitions developed earlier. As the equilibrium under sustainable cooperation can be obtained only numerically, we use the same policy function iteration method to solve for the other equlibria under cooperation and non-cooperation and compare the equlibria under different regimes.

Table 1: Parameter values.

	Parameters	Values
β	subjective discount factor	0.99
η	Frisch elasticity	0.47
ρ	coefficient of relative risk aversion	{0.5, 1.0, 1.5}
α	Calvo parameter	0.75
σ	elasticity of substitution among differentiated products	10.0

3.1 Cooperation vs. non-cooperation under markup shocks

Policymakers in cooperation maximize the global social welfare function (6) subject to the Phillips curves (1) and (2). The first-order necessary conditions (FONCs) in the cooperation regime are (1) and (2), appended with the optimal trade-offs for the policymakers:[19]

$$-\sigma\pi_t = y_t - y_{t-1}, \tag{7}$$

$$-\sigma\pi_t^* = y_t^* - y_{t-1}^*. \tag{8}$$

Because of commitment to future policies inherent in both of the cooperation and non-cooperation regimes, lagged output appears in each countries' equation. The policymakers conduct history-dependent policies. Also, as shown by BB, in the cooperation regime, the optimal targeting rules are inward-looking in the sense that the optimal trade-off only involves each policymaker's own-country variables.

A policymaker in the non-cooperation regime maximizes the social welfare function in his country given the other country's outcome. The policymaker in the Home country maximizes (4) subject to the Philips curves (1) and (2), given π_t^*, and the policymaker in the Foreign country maximizes (5) subject to the Philips curves (1) and (2), given π_t.

[19] See Appendix A.1 for the derivation of Eqs. (7)-(10).

The FONCs in the non-cooperation regime are (1) and (2), along with

$$-\sigma\pi_t = y_t - \xi_t - (y_{t-1} - \xi_{t-1}), \tag{9}$$

$$-\sigma\pi_t^* = y_t^* - \xi_t^* - (y_{t-1}^* - \xi_{t-1}^*), \tag{10}$$

where $\xi_t = \frac{(1+\rho+2\eta)\mu_t-(1-\rho)\mu_t^*}{2(1+\eta)(\eta+\rho)}$ and $\xi_t^* = \frac{(1+\rho+2\eta)\mu_t^*-(1-\rho)\mu_t}{2(1+\eta)(\eta+\rho)}$ stem from the Home and Foreign target outputs in equations (4) and (5), and they reflect the terms of trade externality.

Now we look into numerical examples to understand the difference between the equilibria under cooperation and non-cooperation. Figure 1 on page 17 shows the impulse responses of endogenous variables to a one-time positive Home markup shock in period 0. We find: When the countries are insular ($\rho = 1$), Home inflation and output gap responses under cooperation and non-cooperation, respectively, look alike in response to a Home markup shock. The terms of trade s_t responds differently under non-cooperation. When the countries are more risk averse ($\rho > 1$), the terms of trade plays a role like a negative (positive) markup shock to Home (Foreign) inflation. Conditional on shocks, policymakers may have incentive to deviate from cooperation to non-cooperation because under non-cooperation they can manipulate the terms of trade. We further investigate the intuition behind these results as below.

Dynamics and welfare when $\rho = 1$. Consider first the special case of $\rho = 1$. Recall from earlier that when $\rho = 1$, the two countries become insular in the sense that the exogeneous shock in one country does not feedback onto the other. That is, only the Home-country variables respond to the shock. Home inflation and output gap responses in the cooperation and non-cooperation regimes, respectively, are qualitatively similar: The policymaker in either case commits to future deflation and mitigates the trade-off between current inflation and the output gap as in the well-studied closed economy setting. The terms of trade s_t responds differently to a markup shock in Home country under the non-cooperation regime. Under cooperation, the terms of trade responds negatively to markup shocks. Under non-cooperation, the terms of trade responds positively at the impact of the shock, so that the Home output gap ($x_t \equiv y_t - \mu_t/(\eta + \rho)$) response is more attenuated; but this is traded-off with a more aggressive response in Home inflation.

Under non-cooperation, a positive response of the terms of trade can be deduced from the risk sharing first order condition (3). Note that when $\rho = 1$, the terms of trade s_t has no direct effect on the Phillips curve equilibrium restrictions on policy—

i.e., equations (1) and (2))—nor the policymakers' optimal trade-offs (9) and (10). Given its welfare trade-off with inflation, the output gap will not be completely closed, so then the shock will still imply a negative output gap outcome. Since the Foreign country is insular, y_t^* remains unchanged in Foreign. Thus, the risk sharing condition implies that state-price deflators on complete-market assets must adjust such that the terms of trade rises with the rise in y_t, upon impact of the shock. Since Foreign still behaves in an insular manner and does not react to what Home does, it must absorb the marginal welfare loss involving $\tilde{x}_t \equiv y_t + \mu_t/(\eta + \rho)$. The latter is the terms of trade externality effect discussed earlier. In this example, this is a spillover onto Foreign's welfare that is being exploited by Home.

In contrast, in the cooperation regime, the consolidated policy maximizes the global social welfare function (6). As in the non-cooperation regime, the shock will also yield a negative output gap, but the target output is zero in this case. Thus, the cooperation regime's optimal plan ends up inducing a negative output in response to the Home markup shock. This translates as larger negative Home output gap (which is equal to output itself) and a negative terms of trade response upon impact. In return, Home inflation suffers a smaller fluctuation over time and output gap deviation from zero is shorter lived.

Dynamics and welfare when $\rho \neq 1$. When $\rho \neq 1$, the feedback on or against the plain terms of trade externality effect comes into consideration as well. When the countries are more risk averse ($\rho > 1$), the terms of trade plays a role akin to a negative (positive) markup shock to Home (Foreign) inflation. Since the Foreign country is no longer insular, under non-cooperation Foreign will also react in order to offset Home's desire to manipulate the terms of trade externality. In a Markov perfect equilibrium of the non-cooperation regime, this results in a positive Foreign output gap response which tends to weaken the positive terms of trade response that would have been if $\rho = 1$. Also, from Home's perspective, inducing a rise in the terms of trade acts as "negative markup shock" offsetting the incentive of Home to exploit the terms of trade externality itself. In other words, Home does not need to engineer such a large response in the terms of trade in order to absorb the original positive markup shock at home. Hence, in Figure 1 we can see that the terms of trade response is more attenuated than the reference case of the economy at $\rho = 1$, but the response is still of the same sign. (Note that when the countries are less risk averse $\rho < 1$ but not insular, the terms of trade resembles a positive Home markup shock so that the intuition would be the opposite to the case of $\rho > 1$.)

Incentive (in)feasibility of cooperation. Conditional on shocks, policymakers may have incentive to deviate from cooperation to non-cooperation because under non-cooperation they can manipulate the terms of trade.[20] We now study such possibilities in the model as a function of two things: relative noisiness between the country's markup shocks and risk aversion.

We let $V^c \leq 0$ and $V^n \leq 0$ denote the conditional welfare values to the Home policymaker in the stochastic steady state under the cooperation and non-cooperation regime, respectively.[21] Likewise $V^{*c} \leq 0$ and $V^{*n} \leq 0$ are the corresponding counterparts for Foreign. (Note these values are non-positive since the welfare criteria are negative quadratic.) The statistic, $R_0 \equiv -\max\{V^c/V^n - 1, 0\}$, is less than zero when $V^c < V^n$ holds. If R_0 is negative, then the non-cooperation regime yields a higher welfare to Home than the cooperation regime—i.e. there is an incentive feasibility problem on the part of Home for international cooperation. Otherwise if $R_0 = 0$ then there is no incentive to deviate from cooperation on the part of Home. The interpretation is the same for the Foreign counterpart of this statistic, R_0^*.

Figure 2 on page 18 shows the (relative) conditional social welfare of Home (R_0) and Foreign (R_0^*) under cooperation and non-cooperation. First consider raising the Home's markup shock volatility relative to its Foreign counterpart from $\sigma_\varepsilon/\sigma_\varepsilon^* = 2$ to 5. This exercise shows that policymakers may have incentive to deviate when *asymmetric* markup shock volatility exists across the countries.[22] The higher the ratio of the standard deviation of Home markup shock to its Foreign counterpart $\sigma_\varepsilon/\sigma_\varepsilon^*$, the lower is the Home social welfare under cooperation than under non-cooperation. When one country faces a higher probability of experiencing severe markup shocks, the policymaker has more incentive to deviate from cooperation and behave strategically. That is, only the Home policymaker has more incentive to deviate.[23]

Next consider the statistic, R_0, as a function of risk aversion ρ. The Home policymaker has the largest incentive to deviate at around $\rho = 1$, as the feedback

[20] Coenen et al. [2007] study a similar question in a large-scale model of the European Central Bank (i.e, the New Area Wide Model).

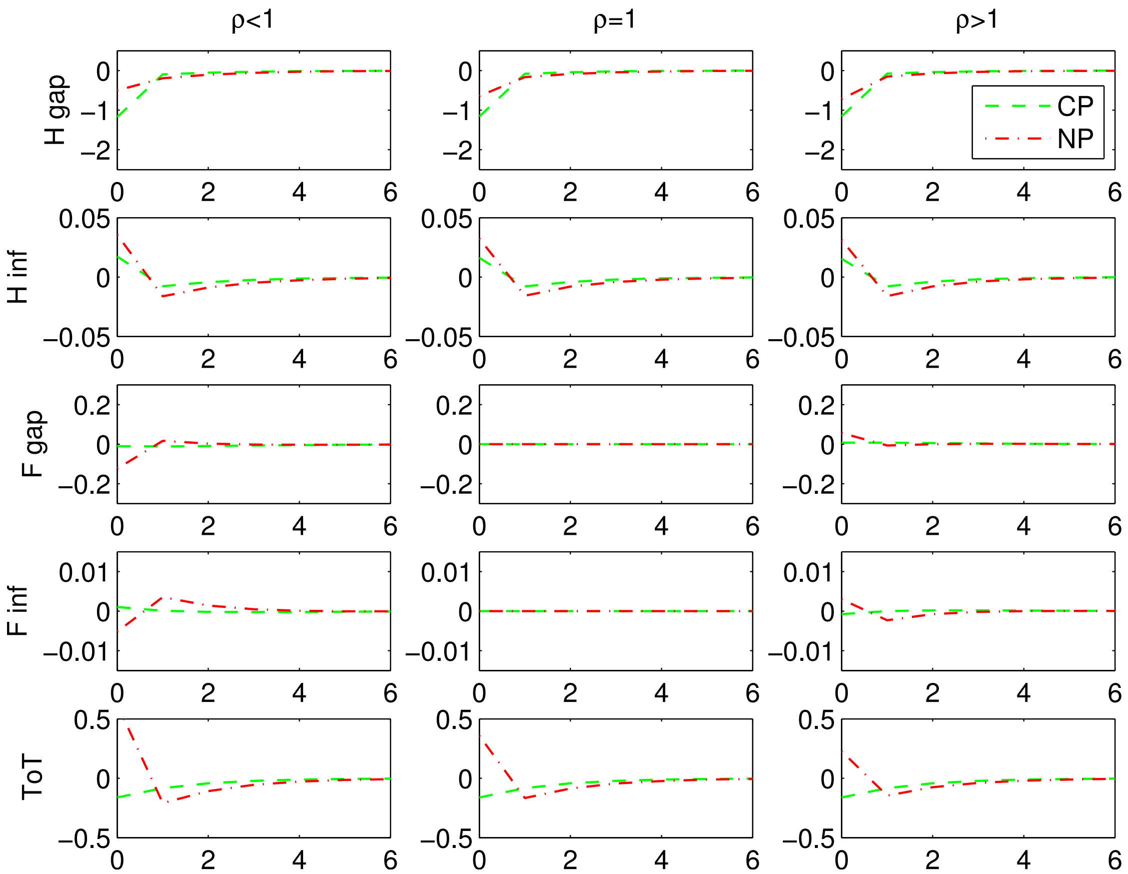
[21] This is called the stochastic or risky steady state [Coeurdacier et al., 2011], as it incorporates the future possibility of shocks hitting the economy.

[22] Corsetti et al. [2010] argue that strategic manipulation of the terms of trade under non-cooperation of a symmetric Nash equilibrium usually is self-defeating. However, we show that the welfare gain from non-cooperation is non-negligible—i.e., the policymaker's has incentive to deviate from the cooperation regime—when there is asymmetric volatility in the markup shock process that each country faces.

[23] Note that, as we adopt the LQ framework, the different value of $\sigma_\varepsilon/\sigma_\varepsilon^*$ only affects the policymakers' incentive to deviate. That is, certainty equivalence holds here.

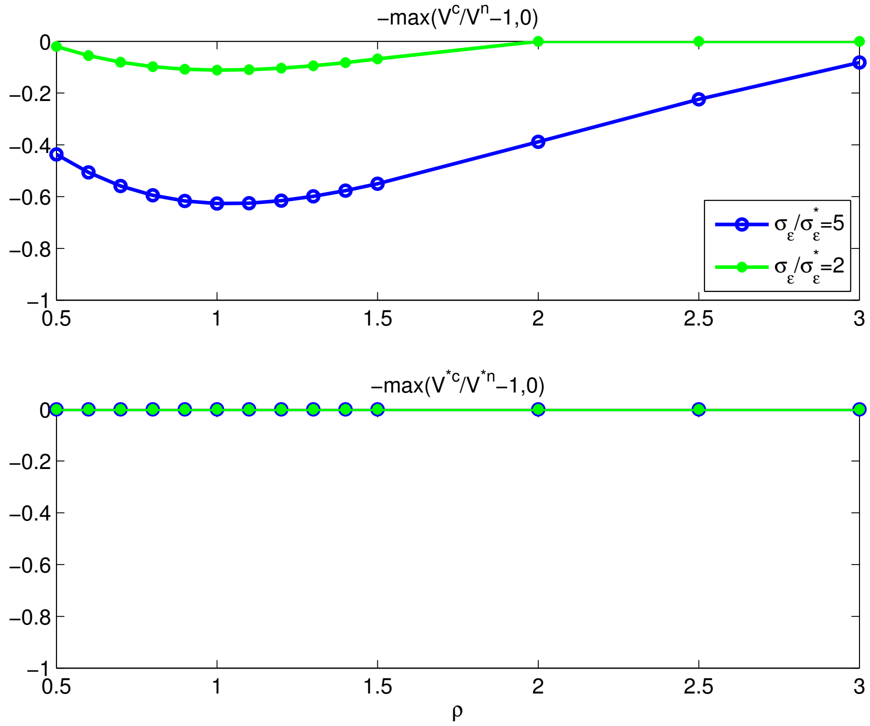
effect from the terms of trade on domestic real marginal cost and from Foreign best responses is non-existent. When $\rho > 1$, the terms of trade fluctuations directly dampens the social welfare. When $\rho < 1$, inflation and output fluctuations increase, as the terms of trade effect in marginal costs aggravates the markup shock in one's own country.

Figure 1: Impulse responses to a positive Home markup shock.



Notes: The left column is for the case of $\rho = 0.5$, the center column is for $\rho = 1.0$, and the right column is for $\rho = 1.5$.

Figure 2: Social welfare under cooperation vs. non-cooperation: with varying ρ



In short, we cannot take the cooperation regime as being always incentive feasible for the independent policymakers. Under arguably realistic settings, e.g., asymmetric volatililities of country specific shocks, the temptation to deviate from cooperation can arise. However, this naturally leads us to ask whether the countries can do better than merely behaving under non-cooperation. We deal with this by considering the endogenous sustainable cooperation regime next.

3.2 Sustainable cooperation

Now we consider the regime of sustainable cooperation. In this setting, a contract between the countries ensures that each policymaker has no incentive to deviate from cooperating as long as the sustainability constraints

$$V_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u_s \geq W(y_{t-1}, y^*_{t-1}, \boldsymbol{\mu}_t), \tag{11}$$

$$V^*_t = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u^*_s \geq W^*(y_{t-1}, y^*_{t-1}, \boldsymbol{\mu}_t), \tag{12}$$

hold. The functions W and W^* are the welfare functions for Home and Foreign, respectively, under the non-cooperation regime. These outside option values depend on past output levels and also $\boldsymbol{\mu}_t = [\mu_t, \mu_{t-1}, \mu_t^*, \mu_{t-1}^*]'$ which is a vector of current and past realizations of exogenous markup shocks. If either country chooses not to cooperate (i.e., either of the sustainability constraints is violated), the non-cooperation equilibrium is triggered.[24] In this regime, policymakers maximize the global welfare function (6) subject to the Phillips curves ((1) and (2)) and sustainability constraints ((11) and (12)) so that neither of the countries has incentive to deviate.

After some algebraic manipulation (see Appendix A.1), we have

$$-\sigma\pi_t = y_t - \zeta_t - z_t(y_{t-1} - \zeta_{t-1}), \tag{13}$$

$$-\sigma\pi_t^* = y_t^* - \zeta_t^* - z_t(y_{t-1}^* - \zeta_{t-1}^*), \tag{14}$$

where $\zeta_t = \frac{(1+\rho+2\eta)\eta_t-(1-\rho)\eta_t^*}{2(1+\eta)(\eta+\rho)}$ and $\zeta_t^* = -\frac{(1+\rho+2\eta)\eta_t^*-(1-\rho)\eta_t}{2(1+\eta)(\eta+\rho)}$. The variable $z_t = \frac{\Psi_{t-1}+\Psi_{t-1}^*}{\Psi_t+\Psi_t^*} \in (0,1]$ is the ratio of the sum of Lagrange multipliers, φ_t and φ_t^*. The summed Lagrange multiplier are cumulative sufficient statistics on past incentive compatibility of the policymakers. In our setting, either of the constraints binds at a time.[25] When either of the sustainability constraints is binding ($\varphi_t > 0$ or $\varphi_t^* > 0$),

$$z_t = \frac{\Psi_{t-1} + \Psi_{t-1}^*}{\Psi_{t-1} + \Psi_{t-1}^* + \varphi_t + \varphi_t^*} < 1$$

holds. Also, η_t and η_t^* are endogenously determined as

$$\begin{aligned}
\eta_t &= (2\nu_t - 1)\mu_t \\
&\quad -(\beta/2)\mathbb{E}_t(z_{t+1}^{-1} - 1)\left[I_{t+1}D_1W(y_t, y_t^*; \boldsymbol{\mu}_{t+1}) + I_{t+1}^*D_1W^*(y_t, y_t^*; \boldsymbol{\mu}_{t+1})\right], \\
\eta_t^* &= (2\nu_t - 1)\mu_t^* \\
&\quad +(\beta/2)\mathbb{E}_t(z_{t+1}^{-1} - 1)\left[I_{t+1}D_2W(y_t, y_t^*; \boldsymbol{\mu}_{t+1}) + I_{t+1}^*D_2W^*(y_t, y_t^*; \boldsymbol{\mu}_{t+1})\right],
\end{aligned}$$

where $\nu_t = \frac{\Psi_t}{\Psi_t+\Psi_t^*} \in (0,1)$ given $\nu_{-1} = 1/2$. The indicator function $I_t = 1$ when the sustainability constraint in Home country is binding in period t; $I_t = 0$ otherwise. The endogenous state variable ν_t is a pseudo-weight, which is related to how countries have temptation to deviate from cooperation—i.e., to behave strategically and

[24]Since the non-cooperation equilibrium studied earlier is Markov perfect, then it can be shown that it is also a sustainable equilibrium.

[25]Fuchs and Lippi [2006] studied the situation where more than two constraints simultaneously bind in a multi-country monetary model.

to manipulate the terms of trade as in the non-cooperation regime. Observe that the conditions (13) and (14) are "distorted" versions of the optimal policy trade-offs—their graphs projected onto (y_t, π_t)-space lie in between their corresponding policy trade-off counterparts in the non-cooperation regime (compare the case if $z_t = 1$, $\zeta_t = \xi_t$, $\zeta_t^* = \xi_t^*$ almost everywhere with (9) and (10)) and in the cooperation regime (compare the case if $z_t = 1$, $\zeta_t = 0$, $\zeta_t^* = 0$ almost everywhere with (7) and (8)). We provide more precise conditions for when these observations arise, in the propositions below.

We have the following preliminary property on the behavior of ν_t:

Proposition 1. *$\nu_t = 1 - z_t(1 - \nu_{t-1}) > \nu_{t-1}$ when $\varphi_t > 0$ and $\nu_t = z_t \nu_{t-1} < \nu_{t-1}$ when $\varphi_t^* > 0$, where φ_t and φ_t^* are the Lagrange multipliers on the Home and Foreign sustainability constraints.*

Proof. This can be easily derived from the law of motion of $\Psi_t + \Psi_t^*$. See Appendix A.1. □

This result says that the pseudo-weight is a strictly increasing process whenever Home's sustainability constraint is currently binding. It is strictly decreasing whenever Foreign's incentive constraint is currently binding. Given the dynamics of ν_t, we also have the following limiting cases:

Proposition 2. *(i) When $\nu_t \to 0$, $\zeta_t = -\xi_t$ and $\zeta_t^* = \xi_t^*$ approximately hold;*
(ii) When $\nu_t \to 1$, $\zeta_t = \xi_t$ and $\zeta_t^* = -\xi_t^*$ approximately hold.
(iii) When $\nu_t = 1/2$ for all $t \geq 0$ and all histories, i.e., the sustainability constraint never binds, $\zeta_t = \zeta_t^* = 0$.

Proof. (i) When $\nu_t \to 0$, the sustainability constraint in the Home country ceases to bind; $\eta_t = -\mu_t$, $\eta_t^* = -\mu_t^*$, $\zeta_t = -\xi_t = -\frac{(1+\rho+2\eta)\mu_t - (1-\rho)\mu_t^*}{2(1+\eta)(\eta+\rho)}$ and $\zeta_t^* = \xi_t^* = \frac{(1+\rho+2\eta)\mu_t^* - (1-\rho)\mu_t}{2(1+\eta)(\eta+\rho)}$ approximately hold. (ii) When $\nu_t \to 1$, the sustainability constraint in the Foreign country ceases to bind; $\eta_t = \mu_t$, $\eta_t^* = \mu_t^*$, $\zeta_t = \xi_t$ and $\zeta_t^* = -\xi_t^*$ approximately hold. (iii) is apparent. □

In terms of global welfare, we can deduce that the sustainable cooperation regime is an (endogenously) intermediate case of the two extremes: cooperation and non-cooperation. When the sustainability constraints never bind, $\eta_t = \eta_t^* = 0$ and $\nu_t = 1/2$ hold, and the solution becomes the same as in the cooperation regime. When the Home sustainability constraint binds, for example, the pseudo weight on each country's welfare shifts to keep the Home country within the sustainable cooperation regime. As $\nu_t \to 1$, the home-country dynamics resembles the ones under non-cooperation as $\zeta_t \to \xi_t$ (Eq. (13) becomes Eq. (9)), whereas negative externalities affect the Foreign country as $\zeta_t^* \to -\xi_t^*$. This additional externality may make the situation worse for the Foreign country. When $\rho \neq 1$, there is a spillover effect to the Foreign country more than via the terms of trade in the risk sharing effect, and Home's welfare improves at the sacrifice of Foreign's—i.e., the Foreign policymaker carries no weight strategically as $\nu_t \to 1$.

3.2.1 Computation

The sustainable cooperation problem is nonlinear, despite involving quadratic and linear forms. This is because of the ocassionally binding nature of the sustainability constraints. Thus, the solution for the sustainable cooperation equilibrium can only be obtained numerically.[26] We use a version of the policy function iteration method with occasionally binding constraints [Kehoe and Perri, 2002, Sunakawa, 2015]. Let $s = (y_{-1}, y_{-1}^*, \nu_{-1}, \boldsymbol{\mu}) \in Y \times Y \times N \times \mathcal{M}$ where Y, N and $\mathcal{M}$ are closed sets. A state-space representation is

$$
\begin{aligned}
\pi(s) &= \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y(s) - (1-\rho)\, y^*(s)\right] + \beta \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})\pi(s') + \kappa\mu, \\
\pi^*(s) &= \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y^*(s) - (1-\rho)\, y(s)\right] + \beta \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})\pi^*(s') + \kappa\mu^*, \\
-\sigma\pi(s) &= y(s) - \zeta(s) - z(s)(y_{-1} - \zeta_{-1}), \\
-\sigma\pi^*(s) &= y^*(s) - \zeta^*(s) - z(s)(y_{-1}^* - \zeta_{-1}^*), \\
V(s) &= -u(s) + \beta \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu}) V(s') \geq W(y_{-1}, y_{-1}^*, \boldsymbol{\mu}') \\
V^*(s) &= -u^*(s) + \beta \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu}) V^*(s') \geq W^*(y_{-1}, y_{-1}^*, \boldsymbol{\mu}')
\end{aligned}
$$

where $\zeta(s)$ and $\zeta^*(s)$ depend on $\sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})\Xi(s')$ and $\sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})\Xi^*(s')$ each (see Appendix A.2). The notation $p(\boldsymbol{\mu}'|\boldsymbol{\mu})$ represents the joint transition probability matrix of shocks. This system has a recursive structure with regard to $\pi(s')$, $\pi^*(s')$, $V(s')$, $V(s')$, $\Xi(s')$ and $\Xi^*(s')$. The occasionally binding constraints $V(s) \geq W(y_{-1}, y_{-1}, \boldsymbol{\mu}')$ and $V^*(s) \geq W^*(y_{-1}, y_{-1}^*, \boldsymbol{\mu}')$ must be addressed.[27] $V(s')$, $V(s')$,

[26] Program codes are written in Fortran 90 with OpenMP directives. It takes about 5-10 minutes to converge in a computer with 4 cores of 4.0 Ghz Intel Core-i7 4790K.

[27] In general, these constraints may make the problem non-convex so that the numerical algorithm (based on assuming the existence of a unique functional fixed-point) may end up finding only one of multiple equilibria. We cannot prove the existence nor uniqueness of the equilibrium, but

$\pi(s')$, $\pi^*(s')$, $\Xi(s')$ and $\Xi^*(s')$ need to be approximated by projection onto known families of basis functions, as continuation states $s' = (y(s), y^*(s), \nu(s), \boldsymbol{\mu'})$ may not be on the grid points. Three-dimensional cubic spline bases are used for interpolation. We set $Y = [-5.0, 5.0]$ and $N = (0.0, 1.0)$ and divide them each into five knot points.

Each element in $\boldsymbol{\mu} = (\mu, \mu_{-1}, \mu^*, \mu^*_{-1})$ follows a Markov chain. We set $\mu \in M = \{-\sigma_\varepsilon, 0, \sigma_\varepsilon\}$ and $\mu^* \in M^* = \{-\sigma^*_\varepsilon, 0, \sigma^*_\varepsilon\}$ for each shock, respectively for the Home and Foreign country. Note that $\mathcal{M} = M^2 \times (M^*)^2$. Each Markov chain is independent of the others. The transition matrix is given by $p(\mu'|\mu)_{(1\times3)} = [(1-\rho_\mu)/2, \rho_\mu, (1-\rho_\mu)/2]$ for any μ. Hence, $5^3 \times 3^4 = 10125$ grid points are used to approximate the state space.[28]

3.2.2 Numerical results

Figure 3 on page 24 shows impulse responses to a positive Home markup shock $\mu = \sigma_\varepsilon$ in period 0. By comparing the responses under each regime, it is shown that the responses under sustainable cooperation are intermediate between those under cooperation and non-cooperation. When ρ equals 0.5 or 1.5, i.e., two countries are non-insular, the responses under sustainable cooperation become closer to those under cooperation.[29] Also, when two countries are non-insular, the signs of the Foreign responses under sustainable cooperation are opposite to the ones under cooperation or non-cooperation. Before we interpret these results, let us first discuss the characteristics under sustainable cooperation by looking at the binding pattern of the sustainability constraints.

Figure 4 illustrates how much the sustainability constraint is binding under this shock. The upper row panels in the figure are for the ratio of the sum of Lagrange multipliers, z_t, and lower row panels are for the pseudo weights, ν_t. The Home sustainability constraint binds in period 0, which makes $z_t < 1$ and $\nu_t > \frac{1}{2}$. Again, note that the temptation to deviate from cooperation is the highest when $\rho = 1$. That is, z_t takes the lowest value and ν_t is away from a half and closest to one. Since two countries are insular in this case, Foreign output and inflation rates do

numerically we conduct robustness checks utilizing different initial guesses of the equilibrium policy functions that would have delivered the sustainable cooperation plan. Our numerical results do not seem to suffer from such problems of equilibrium multiplicity.

[28]The number of the grid points for Y and N are increased to check the robustness of our result. As we have seven state variables, this kind of exercise is very time-consuming as it exponentially increases the total number of grid points.

[29]When $\rho = 3$, as in the original calibration in BB, the responses under sustainable cooperation are quite similar to those under cooperation, although the sustainablility contraint is binding.

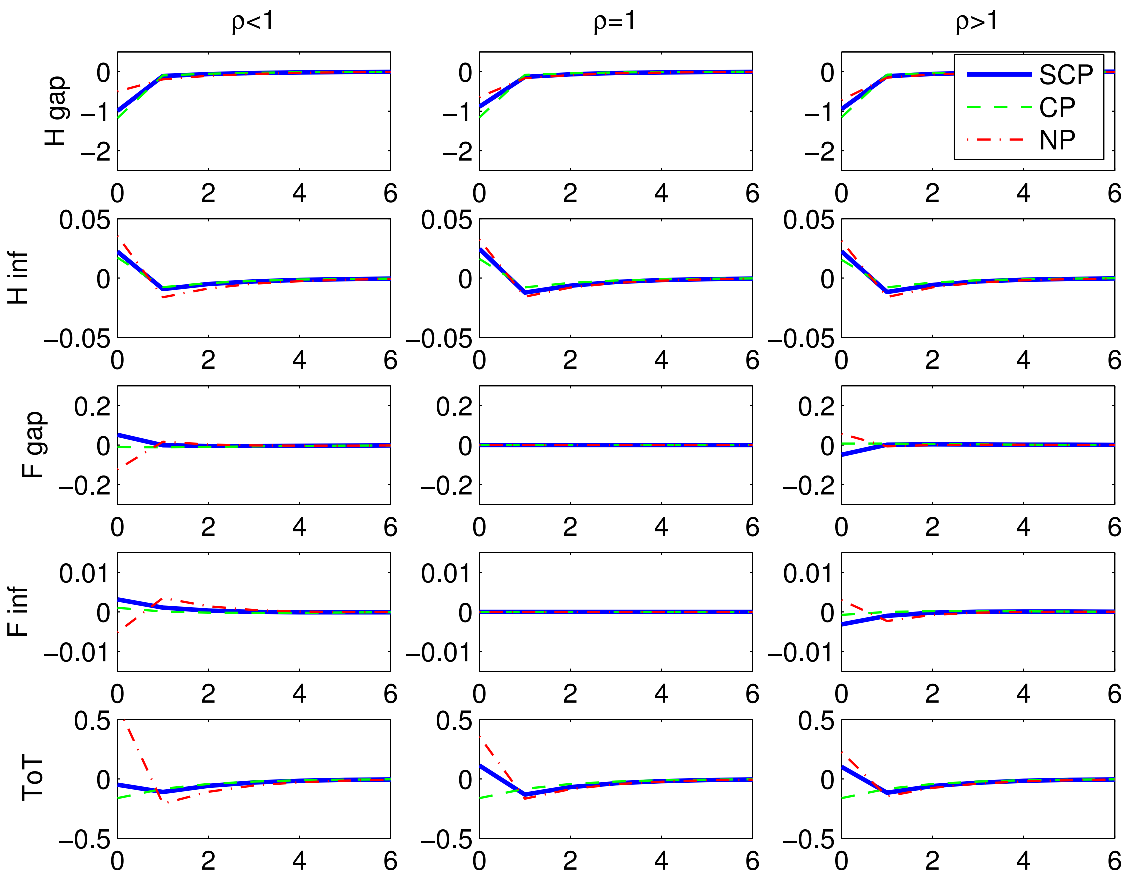
not react to the Home markup shock. The self-centered Home policymaker has every desire to set its own policy to maximize its domestic welfare, ignoring the resulting terms of trade externality on its neighbor. Sustainable cooperation needs to take this temptation for Home policymaker to deviate into account. As a result, the Home sustainability constraint binds and the pseudo weight on each country's social welfare shifts toward the one favoring the Home country—i.e., the sustainable equilibrium has to redistribute welfare from Foreign to Home to keep the Home country within the sustainable cooperation regime.

When ρ is set away from unity, the sustainability constraints bind less aggressively. The pseudo weight goes less away from a half. Less welfare redistribution toward Home is needed to keep the Home country in check under the sustainable cooperation regime.

How binding are the sustainability constraints in relation to risk aversion of agents—i.e., the value of ρ? This can be ascertained from Figure 5. When ρ is around one, the Home policymaker has the biggest temptation to deviate from the cooperation regime. This is consistent with our earlier investigation in Figure 2. When $\rho \neq 1$, as discussed earlier, there is the feedback effect from the terms of trade as markup shock and its accompanying retaliation considerations from Foreign. The Foreign policymaker reacts to offset the terms of trade externality under non-cooperation. However, under sustainable cooperation, the sustainable plan ensures that Home must take into account the sacrifice of Foreign's welfare as well. There is an additional welfare redistribution effect: More welfare will have to be promised to Home to ensure that it stays in cooperation and not exploit the terms of trade externality as much as in the non-cooperation regime. When ρ is around one, the latter effect is largest as there is almost no feedback effect, so the temptation to deviate is largest.

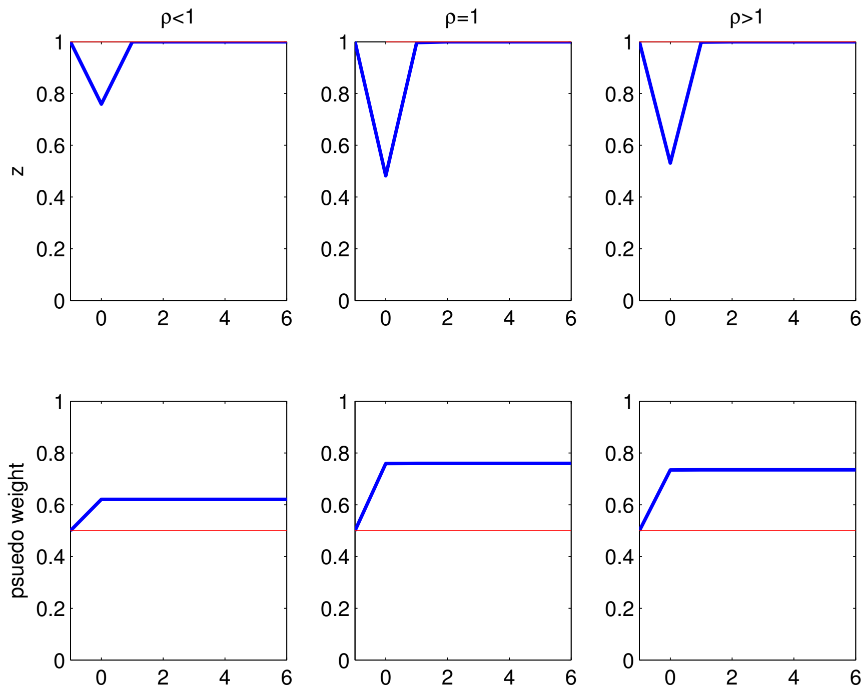
Now, let us return to Figure 3. Since under sustainable cooperation there is a redistribution of welfare from Foreign to Home, we see that the responses of Foreign's variables are different from the ones under cooperation or non-cooperation. In particular, following the explanations above, we can see that when $\rho = 1$ (in the middle column panels of the figure), the response of Home in terms of output gap, inflation and the terms of trade are similar to the non-cooperation case, but they are more attenuated. In fact, they sit in between their corresponding impulse responses under the cooperation and the non-cooperation regimes. Again there is no response of Foreign variables here since it behaves optimally in an insular manner. If $\rho > 1$ (see the right column panels of the figure), the intuition coming from the terms of trade externality and its feedback effects is still present. However, now the responses are much more attenuated than under the non-cooperation regime reflecting the additional welfare redistribution effects. There is a need for shifting welfare weights from Foreign to Home. Also, the signs of the Foreign responses under sustainable cooperation are opposite from the ones under cooperation or non-cooperation. Finally, if $\rho < 1$ (i.e., the left column panels of the figure), our intuition above on the reinforcing threat of welfare redistribution also applies. Interestingly, here the terms of trade response turns out to be negative implying that in this sustainable cooperative regime, Home and Foreign have every incentive to behave very well so that the dynamic outcomes are closer to those under the assumed cooperation case.

Figure 3: Impulse responses to a positive Home markup shock: sustainable cooperation.



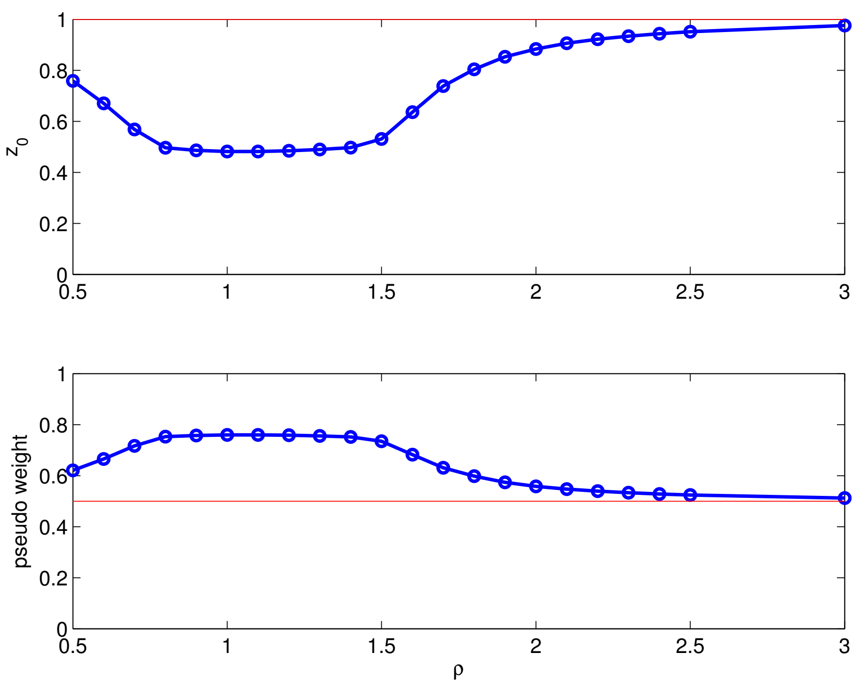
Notes: The left column is for the case of $\rho = 0.5$, the center column is for $\rho = 1.0$, and the right column is for $\rho = 1.5$.

Figure 4: The steadfastness of commitment and pseudo weight.



Notes: The left column is for the case of $\rho = 0.5$, the center column is for $\rho = 1.0$, and the right column is for $\rho = 1.5$.

Figure 5: The steadfastness of commitment and pseudo weight: with varying ρ.



4 Conclusion

We show that the incentive to deviate from a cooperation regime is large when the intertemporal elasticity of substitution is close to unity, and when asymmetric volatility exists among the countries. However, by construction of the long term contract underlying the sustainable cooperation regime, no country has an incentive to deviate from a cooperative outcome. Instead countries in such a program are willing to tolerate welfare redistributions whenever asymmetric shocks hit them, in return for better than no-cooperation equilibrium outcomes. Thus the responses of inflation and output gap in both countries under sustainable cooperation are different from the ones under cooperation and non-cooperation. In particular, for the country faced with the markup shock, inflation, output gap and terms of trade outcomes have dynamics that are intermediate to the cooperation and non-cooperation regimes.

To sum up, monetary cooperation should not be taken for granted to be an always- and everywhere-tenable proposition. When the markup shock is the dominant driver of economic fluctuations, the considerations of sustainable cooperation is particularly important when two countries are insular in structural relationship and face asymmetric volatilities in structural shocks. In such situations, it is important to acknowledge that the best allocations and prices central banks can achieve are somewhere between those under cooperation and non-cooperation.

More can be done following our analyses here. Future extensions include studying conditions with other shocks than the markup shock and a role for fiscal policy under currency unions. Another important issue is to seek for simple operational rules to attain the almost equivalent outcomes to the sustainable cooperation equilibrium. A more challenging extension to the case with policymakers' private information may be interesting as well.[30] These open questions are left for our future research.

[30]For related studies in the closed economy, see for example, Athey et al. [2005], Waki et al. [2015].

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Supplementary (online) Appendix

A LQ Framework: Alternative Policy Equilibria

In this appendix, we will show some analytical results and computation procedure based on the LQ framework presented in Section 2. The model is also explained in detail in Appendix B.

A.1 Analytical results

In this section, we will derive the FONCs in each cooperation, non-cooperation and sustainable cooperation regime.

A.1.1 Cooperation

Lagrangean is

$$\begin{aligned}\mathcal{L}_0^W &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ (\eta+\rho)\, y_t^2 + (\eta+\rho)\,(y_t^*)^2 \\ &\quad + \frac{1-\rho}{2}\,(y_t - y_t^*)^2 + \frac{\sigma}{\kappa}\pi_t^2 + \frac{\sigma}{\kappa}\,(\pi_t^*)^2 \\ &\quad + \phi_t \left(-\pi_t + \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y_t - (1-\rho)\, y_t^*\right] + \kappa\mu_t \right) + \phi_{t-1}\pi_t \\ &\quad + \phi_t^* \left(-\pi_t^* + \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y_t^* - (1-\rho)\, y_t\right] + \kappa\mu_t^* \right) + \phi_{t-1}^*\pi_t^* \Big\}.\end{aligned}$$

Note that $(\rho+\eta)y_t + [(1-\rho)/2]s_t = [(1+\rho+2\eta)/2]y_t - [(1-\rho)/2]y_t^*$ holds. The FONCs are

$$\begin{aligned}\partial y_t &: \quad -2\,(\eta+\rho)\, y_t - (1-\rho)\,(y_t - y_t^*) + \frac{\kappa}{2}(1+\rho+2\eta)\phi_t - \frac{\kappa}{2}(1-\rho)\phi_t^* = 0, \\ \partial y_t^* &: \quad -2\,(\eta+\rho)\, y_t^* + (1-\rho)\,(y_t - y_t^*) - \frac{\kappa}{2}\,(1-\rho)\,\phi_t + \frac{\kappa}{2}(1+\rho+2\eta)\phi_t^* = 0, \\ \partial \pi_t &: \quad -\frac{2\sigma}{\kappa}\pi_t - \phi_t + \phi_{t-1} = 0, \\ \partial \pi_t^* &: \quad -\frac{2\sigma}{\kappa}\pi_t^* - \phi_t^* + \phi_{t-1}^* = 0,\end{aligned}$$

The first two equations are solved for

$$\phi_t = \frac{2}{\kappa} y_t, \qquad \phi_t^* = \frac{2}{\kappa} y_t^*.$$

The equilibrium conditions of cooperation are summarized

$$
\begin{aligned}
-\sigma\pi_t &= y_t - y_{t-1}, \\
-\sigma\pi_t^* &= y_t^* - y_{t-1}^*.
\end{aligned}
$$

A.1.2 Non-cooperation

Lagrangean for the Home country is

$$
\begin{aligned}
\mathcal{L}_0 \;=\; & -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ (\eta+\rho)\left(y_t - \frac{1}{\eta+\rho}\mu_t\right)^2 + (\eta+\rho)\left(y_t^* + \frac{1}{\eta+\rho}\mu_t^*\right)^2 \\
& + \frac{1-\rho}{2}\left(y_t - y_t^*\right)^2 + \frac{\sigma}{\kappa}\pi_t^2 + \frac{\sigma}{\kappa}\left(\pi_t^*\right)^2 \\
& + \varphi_{1,t}\left(-\pi_t + \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y_t - (1-\rho)\, y_t^*\right] + \kappa\mu_t\right) + \varphi_{1,t-1}\pi_t \\
& + \varphi_{2,t}\left(-\pi_t^* + \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y_t^* - (1-\rho)\, y_t\right] + \kappa\mu_t^*\right) + \varphi_{2,t-1}\pi_t^* \Bigg\}.
\end{aligned}
$$

The FONCs are

$$
\begin{aligned}
\partial y_t: &\quad -2(\eta+\rho)\, y_t - (1-\rho)\left(y_t - y_t^*\right) + \frac{\kappa}{2}(1+\rho+2\eta)\,\varphi_{1,t} - \frac{\kappa}{2}(1-\rho)\varphi_{2,t} + 2\mu_t = 0, \\
\partial y_t^*: &\quad -2(\eta+\rho)\, y_t^* + (1-\rho)\left(y_t - y_t^*\right) - \frac{\kappa}{2}(1-\rho)\,\varphi_{1,t} + \frac{\kappa}{2}(1+\rho+2\eta)\,\varphi_{2,t} - 2\mu_t^* = 0, \\
\partial \pi_t: &\quad -\frac{2\sigma}{\kappa}\pi_t - \varphi_{1,t} + \varphi_{1,t-1} = 0.
\end{aligned}
$$

The first two equations are solved for

$$
\varphi_{1,t} = \frac{2}{\kappa}\left(y_t - \frac{(1+\rho+2\eta)\mu_t - (1-\rho)\mu_t^*}{2(1+\eta)(\eta+\rho)}\right), \qquad \varphi_{2,t} = \frac{2}{\kappa}\left(y_t^* + \frac{(1+\rho+2\eta)\mu_t^* - (1-\rho)\mu_t}{2(1+\eta)(\eta+\rho)}\right).
$$

For the Foreign country, the FONCs are

$$
\begin{aligned}
\partial y_t: &\quad -2(\eta+\rho)\left(y_t + \frac{1}{\eta+\rho}\mu_t\right) - (1-\rho)\left(y_t - y_t^*\right) + \frac{\kappa}{2}(1+\rho+2\eta)\,\varphi_{1,t}^* - \frac{\kappa}{2}(1-\rho)\varphi_{2,t}^* = 0, \\
\partial y_t^*: &\quad -2(\eta+\rho)\left(y_t^* - \frac{1}{\eta+\rho}\mu_t^*\right) + (1-\rho)\left(y_t - y_t^*\right) - \frac{\kappa}{2}(1-\rho)\,\varphi_{1,t}^* + \frac{\kappa}{2}(1+\rho+2\eta)\,\varphi_{2,t}^* = 0, \\
\partial \pi_t^*: &\quad -\frac{2\sigma}{\kappa}\pi_t^* - \varphi_{2,t}^* + \varphi_{2,t-1}^* = 0.
\end{aligned}
$$

The first two equations are solved for

$$
\varphi_{1,t}^* = \frac{2}{\kappa}\left(y_t + \frac{(1+\rho+2\eta)\mu_t - (1-\rho)\mu_t^*}{2(1+\eta)(\eta+\rho)}\right), \qquad \varphi_{2,t}^* = \frac{2}{\kappa}\left(y_t^* - \frac{(1+\rho+2\eta)\mu_t^* - (1-\rho)\mu_t}{2(1+\eta)(\eta+\rho)}\right).
$$

The equilibrium conditions of non-cooperation are summarized

$$-\sigma\pi_t = y_t - \xi_t - y_{t-1} + \xi_{t-1},$$
$$-\sigma\pi_t^* = y_t^* - \xi_t^* - y_{t-1}^* + \xi_{t-1}^*,$$

where $\xi_t = \frac{(1+\rho+2\eta)\mu_t-(1-\rho)\mu_t^*}{2(1+\eta)(\eta+\rho)}$ and $\xi_t^* = \frac{(1+\rho+2\eta)\mu_t^*-(1-\rho)\mu_t}{2(1+\eta)(\eta+\rho)}$. Note that the past markup shocks $\{\mu_{t-1}, \mu_{t-1}^*\}$ are included in the state variables. The value functions $V_t = W(y_{t-1}, y_{t-1}^*, \mu_t, \mu_{t-1}, \mu_t^*, \mu_{t-1}^*)$ and $V_t^* = W^*(y_{t-1}, y_{t-1}^*, \mu_t, \mu_{t-1}, \mu_t^*, \mu_{t-1}^*)$ are quadratic function of y_{t-1} and y_{t-1}^*.

A.1.3 Sustainable cooperation

Lagrangean in Period 0 is

$$\begin{aligned}
\mathcal{L}_0 \;=\; & -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{u_t + u_t^* \\
& +\phi_t \left(-\pi_t + \beta \mathbb{E}_t \pi_{t+1} + \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y_t - (1-\rho)\, y_t^*\right] + \kappa\mu_t\right) \\
& +\phi_t^* \left(-\pi_t^* + \beta \mathbb{E}_t \pi_{t+1}^* + \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y_t^* - (1-\rho)\, y_t\right] + \kappa\mu_t^*\right) \\
& -\varphi_t \left[\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u_s + W(y_{t-1}, y_{t-1}^*, \boldsymbol{\mu}_t)\right] - \varphi_t^* \left[\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u_s^* + W^*(y_{t-1}, y_{t-1}^*, \boldsymbol{\mu}_t)\right]\Bigg\}, \\
=\; & -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{\Psi_t u_t + \Psi_t^* u_t^* \\
& +\phi_t \left(-\pi_t + \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y_t - (1-\rho)\, y_t^*\right] + \kappa\mu_t\right) - \phi_{t-1}\pi_t \\
& +\phi_t^* \left(-\pi_t^* + \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y_t^* - (1-\rho)\, y_t\right] + \kappa\mu_t^*\right) - \phi_{t-1}^*\pi_t^* \\
& -\varphi_t W(y_{t-1}, y_{t-1}^*, \boldsymbol{\mu}_t) - \varphi_t^* W^*(y_{t-1}, y_{t-1}^*, \boldsymbol{\mu}_t)\} ,
\end{aligned}$$

where $\Psi_t = \Psi_{t-1} + \varphi_t$ and $\Psi_t^* = \Psi_{t-1}^* + \varphi_t^*$ given $\Psi_{-1} = \Psi_{-1}^* = 1/2$. The FONCs are

$$\begin{aligned}
\partial y_t: \quad & -2(\eta+\rho)\left[\Psi_t\left(y_t - \frac{1}{\eta+\rho}\mu_t\right) + \Psi_t^*\left(y_t + \frac{1}{\eta+\rho}\mu_t\right)\right] - (1-\rho)(\Psi_t+\Psi_t^*)(y_t - y_t^*) \\
& + \frac{\kappa}{2}(1+\rho+2\eta)\phi_t - \frac{\kappa}{2}(1-\rho)\phi_t^* \\
& -\beta\mathbb{E}_t\left\{\varphi_{t+1}D_1W(y_t, y_t^*; \boldsymbol{\mu}_{t+1}) + \varphi_{t+1}^* D_1W^*(y_t, y_t^*; \boldsymbol{\mu}_{t+1})\right\} = 0, \\
\partial y_t^*: \quad & -2(\eta+\rho)\left[\Psi_t\left(y_t^* + \frac{1}{\eta+\rho}\mu_t^*\right) + \Psi_t^*\left(y_t^* - \frac{1}{\eta+\rho}\mu_t^*\right)\right] + (1-\rho)(\Psi_t+\Psi_t^*)(y_t - y_t^*) \\
& - \frac{\kappa}{2}(1-\rho)\phi_t + \frac{\kappa}{2}(1+\rho+2\eta)\phi_t^* \\
& -\beta\mathbb{E}_t\left\{\varphi_{t+1}D_2W(y_t, y_t^*; \boldsymbol{\mu}_{t+1}) + \varphi_{t+1}^* D_2W^*(y_t, y_t^*; \boldsymbol{\mu}_{t+1})\right\} = 0, \\
\partial \pi_t: \quad & -\frac{2\sigma}{\kappa}(\Psi_t+\Psi_t^*)\pi_t - \phi_t + \phi_{t-1} = 0, \\
\partial \pi_t^*: \quad & -\frac{2\sigma}{\kappa}(\Psi_t+\Psi_t^*)\pi_t^* - \phi_t^* + \phi_{t-1}^* = 0.
\end{aligned}$$

where $\boldsymbol{\mu}_{t+1} = [\mu_{t+1}, \mu_t, \mu_{t+1}^*, \mu_t^*]'$. By normalizing with $\Psi_t + \Psi_t^*$, $((\Psi_t - \Psi_t^*)/(\Psi_t + \Psi_t^*) = 2\Psi_t/(\Psi_t+\Psi_t^*) - 1 = 2\nu_t - 1)$

$$\begin{aligned}
\partial y_t: \quad & -2(\eta+\rho)y_t - (1-\rho)(y_t - y_t^*) + \frac{\kappa}{2}(1+\rho+2\eta)\tilde{\phi}_t - \frac{\kappa}{2}(1-\rho)\tilde{\phi}_t^* \\
& -2(1-2\nu_t)\mu_t - \beta\mathbb{E}_t\Xi_{t+1} = 0, \\
\partial y_t^*: \quad & -2(\eta+\rho)y_t^* + (1-\rho)(y_t - y_t^*) - \frac{\kappa}{2}(1-\rho)\tilde{\phi}_t + \frac{\kappa}{2}(1+\rho+2\eta)\tilde{\phi}_t^* \\
& +2(1-2\nu_t)\mu_t^* - \beta\mathbb{E}_t\Xi_{t+1}^* = 0, \\
\partial \pi_t: \quad & -\frac{2\sigma}{\kappa}\pi_t - (\phi_t - z_t\phi_{t-1}) = 0, \\
\partial \pi_t^*: \quad & -\frac{2\sigma}{\kappa}\pi_t^* - (\phi_t^* - z_t\phi_{t-1}^*) = 0,
\end{aligned}$$

where $\tilde{\phi}_t = \phi_t/(\Psi_t + \Psi_t^*)$, $\tilde{\phi}_t^* = \phi_t^*/(\Psi_t + \Psi_t^*)$, $\nu_t = \Psi_t/(\Psi_t + \Psi_t^*)$, $z_t = (\Psi_{t-1} + \Psi_{t-1}^*)/(\Psi_t + \Psi_t^*)$ and

$$\begin{aligned}
\Xi_{t+1} \quad &\equiv \quad \frac{\varphi_{t+1}}{\Psi_t + \Psi_t^*}D_1W(y_t, y_t^*; \boldsymbol{\mu}_{t+1}) + \frac{\varphi_{t+1}^*}{\Psi_t + \Psi_t^*}D_1W^*(y_t, y_t^*; \boldsymbol{\mu}_{t+1}), \\
\Xi_{t+1}^* \quad &\equiv \quad \frac{\varphi_{t+1}}{\Psi_t + \Psi_t^*}D_2W(y_t, y_t^*; \boldsymbol{\mu}_{t+1}) + \frac{\varphi_{t+1}^*}{\Psi_t + \Psi_t^*}D_2W^*(y_t, y_t^*; \boldsymbol{\mu}_{t+1}).
\end{aligned}$$

We define $\eta_t = -(1-2\nu_t)\mu_t - (\beta/2)\mathbb{E}_t\Xi_{t+1}$ and $\eta^* = -(1-2\nu_t)\mu_t^* + (\beta/2)\mathbb{E}_t\Xi_{t+1}^*$. Then, the first two equations are solved for

$$\tilde{\phi}_t = \frac{2}{\kappa}\left(y_t - \frac{(1+\rho+2\eta)\eta_t - (1-\rho)\eta_t^*}{2(1+\eta)(\eta+\rho)}\right), \qquad \tilde{\phi}_t^* = \frac{2}{\kappa}\left(y_t^* + \frac{(1+\rho+2\eta)\eta_t^* - (1-\rho)\eta_t}{2(1+\eta)(\eta+\rho)}\right).$$

The equilibrium conditions are summarized

$$-\sigma\pi_t = y_t - \zeta_t - z_t(y_{t-1} - \zeta_{t-1}),$$
$$-\sigma\pi_t^* = y_t^* - \zeta_t^* - z_t(y_{t-1}^* - \zeta_{t-1}^*),$$

where $\zeta_t = \frac{(1+\rho+2\eta)\eta_t-(1-\rho)\eta_t^*}{2(1+\eta)(\eta+\rho)}$ and $\zeta_t^* = -\frac{(1+\rho+2\eta)\eta_t^*-(1-\rho)\eta_t}{2(1+\eta)(\eta+\rho)}$.

ν_t is updated as the sustainability constraints are binding. When $I_t = 1$, $z_t < 1$ and

$$\begin{aligned}
\nu_t &= \frac{\Psi_t}{\Psi_t + \Psi_t^*} = 1 - \frac{\Psi_t^*}{\Psi_t + \Psi_t^*} \\
&= 1 - \frac{\Psi_{t-1}^*}{\Psi_{t-1} + \Psi_{t-1}^* + \varphi_t} \\
&= 1 - \frac{\Psi_{t-1} + \Psi_{t-1}^*}{\Psi_{t-1} + \Psi_{t-1}^* + \varphi_t}\frac{\Psi_{t-1}^*}{\Psi_{t-1} + \Psi_{t-1}^*} \\
&= 1 - z_t(1 - \nu_{t-1}) > \nu_{t-1}
\end{aligned}$$

hold. Similarly, when $I_t^* = 1$, $z_t < 1$ and

$$\begin{aligned}
\nu_t &= \frac{\Psi_t}{\Psi_t + \Psi_t^*} \\
&= \frac{\Psi_{t-1}}{\Psi_{t-1} + \Psi_{t-1}^* + \varphi_t^*} \\
&= \frac{\Psi_{t-1} + \Psi_{t-1}^*}{\Psi_{t-1} + \Psi_{t-1}^* + \varphi_t^*}\frac{\Psi_{t-1}}{\Psi_{t-1} + \Psi_{t-1}^*} \\
&= z_t\nu_{t-1} < \nu_{t-1}
\end{aligned}$$

hold. When $I_t = I_t^* = 0$, $z_t = 1$ and $\nu_t = \nu_{t-1}$ hold. Either of the constraints binds at a time.

Ξ_{t+1} and Ξ_{t+1}^* are related to the next period's binding pattern of the sustainability constraints. Note that none or either of the constraints binds. When the sustainability constraint binds in Home country, $\varphi_{t+1} > 0$ and $\varphi_{t+1}^* = 0$ hold. Utilizing $\frac{\varphi_{t+1}}{\Psi_t+\Psi_t^*} = \frac{\Psi_{t+1}+\Psi_{t+1}^*-(\Psi_t+\Psi_t^*)}{\Psi_t+\Psi_t^*} = (z_{t+1}^{-1} - 1)$, we have

$$\Xi_{t+1} = \begin{cases} (z_{t+1}^{-1} - 1)D_1W(y_t, y_t^*; \boldsymbol{\mu}_{t+1}), & \text{when } \varphi_{t+1} > 0, \\ (z_{t+1}^{-1} - 1)D_1W^*(y_t, y_t^*; \boldsymbol{\mu}_{t+1}), & \text{when } \varphi_{t+1}^* > 0, \end{cases}$$

$$\Xi_{t+1}^* = \begin{cases} (z_{t+1}^{-1} - 1)D_2W(y_t, y_t^*; \boldsymbol{\mu}_{t+1}), & \text{when } \varphi_{t+1} > 0, \\ (z_{t+1}^{-1} - 1)D_2W^*(y_t, y_t^*; \boldsymbol{\mu}_{t+1}), & \text{when } \varphi_{t+1}^* > 0. \end{cases}$$

A.2 Computational procedure

A state-space representation is

$$\pi(s) = \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y(s) - (1-\rho)\, y^*(s)\right] + \beta \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})\pi(s') + \kappa\mu,$$
$$\pi^*(s) = \frac{\kappa}{2}\left[(1+\rho+2\eta)\, y^*(s) - (1-\rho)\, y(s)\right] + \beta \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})\pi^*(s') + \kappa\mu^*,$$
$$-\sigma\pi(s) = y(s) - \zeta(s) - z(s)(y_{-1} - \zeta_{-1}(s)),$$
$$-\sigma\pi^*(s) = y^*(s) - \zeta^*(s) - z(s)(y^*_{-1} - \zeta^*_{-1}(s)),$$
$$V(s) = -u(s) + \beta \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})V(s') \geq W(y_{-1}, y_{-1}, \boldsymbol{\mu}'),$$
$$V^*(s) = -u^*(s) + \beta \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})V^*(s') \geq W^*(y_{-1}, y_{-1}, \boldsymbol{\mu}'),$$

where

$$\zeta(s) = \frac{(1+\rho+2\eta)\eta(s) - (1-\rho)\eta^*(s)}{2(1+\eta)(\eta+\rho)}, \quad \zeta^*(s) = -\frac{(1+\rho+2\eta)\eta^*(s) - (1-\rho)\eta(s)}{2(1+\eta)(\eta+\rho)},$$
$$\zeta_{-1}(s) = \frac{(1+\rho+2\eta)\eta_{-1}(s) - (1-\rho)\eta^*_{-1}(s)}{2(1+\eta)(\eta+\rho)}, \quad \zeta^*_{-1}(s) = -\frac{(1+\rho+2\eta)\eta^*_{-1}(s) - (1-\rho)\eta_{-1}(s)}{2(1+\eta)(\eta+\rho)},$$

and

$$\eta(s) = (2\nu(s)-1)\mu - (\beta/2) \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})\Xi(y(s), y^*(s), \nu(s), \boldsymbol{\mu}'),$$
$$\eta^*(s) = (2\nu(s)-1)\mu^* + (\beta/2) \sum_{\boldsymbol{\mu}'} p(\boldsymbol{\mu}'|\boldsymbol{\mu})\Xi(y(s), y^*(s), \nu(s), \boldsymbol{\mu}'),$$
$$\eta_{-1}(s) = (2\nu_{-1}-1)\mu_{-1} - (\beta/2)\Xi(s),$$
$$\eta^*_{-1}(s) = (2\nu_{-1}-1)\mu^*_{-1} + (\beta/2)\Xi(s),$$

and

$$\Xi(s) = (1/z(s)-1)\left[I(s)D_1W(y_{-1}, y^*_{-1}; \boldsymbol{\mu}) + I^*(s)D_1W^*(y_{-1}, y^*_{-1}; \boldsymbol{\mu})\right],$$
$$\Xi^*(s) = (1/z(s)-1)\left[I(s)D_2W(y_{-1}, y^*_{-1}; \boldsymbol{\mu}) + I^*(s)D_2W^*(y_{-1}, y^*_{-1}; \boldsymbol{\mu})\right].$$

B Microfoundations and Welfare Derivations

In this appendix, we will show the model details based on BB. We will also discuss extensively on the Ramsey policies under cooperation and non-cooperation, and their correct LQ approximation.

B.1 Model

B.1.1 Household

There is the representative household in each country. We focus on the household in the Home country. By symmetry, the same results also apply for the household in the Foreign country. The domestic household minimizes the total expenditure

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft},$$

subject to the aggregator $C_t = 2\left(C_{H,t}\right)^{.5}\left(C_{F,t}\right)^{.5}$, where C_t is total consumption, C_H and C_F are bundles of consumption goods produced in Home and Foreign countries. P_t is the consumer price index. P_H and P_F are the price indices for goods produced in Home and Foreign countries. The FONCs are

$$C_{H,t} = \frac{1}{2}\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t,$$
$$C_{F,t} = \frac{1}{2}\left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t.$$

They are the demand function of each bundle of consumption of the domestic household. By substituting them into the aggregator, the price index is given by $P_t = P_{H,t}^{.5} P_{F,t}^{.5}$. The domestic household also minimizes expenditure on bundles of Home and Foreign goods

$$P_{H,t} C_{H,t} = \int_0^{.5} P_{H,t}(h)\, C_{H,t}(h)\, \mathrm{d}h,$$
$$P_{F,t} C_{F,t} = \int_{.5}^{1} P_{F,t}(f)\, C_{F,t}(f)\, \mathrm{d}f,$$

subject to the consumption aggregators $C_{H,t} = \left[2^{\frac{1}{\sigma}} \int_0^{.5} C_{H\,t}(h)^{\frac{\sigma-1}{\sigma}}\, \mathrm{d}h\right]^{\frac{\sigma}{\sigma-1}}$ and $C_{F,t} = \left[2^{\frac{1}{\sigma}} \int_{.5}^{1} C_{F\,t}(f)^{\frac{\sigma-1}{\sigma}}\, \mathrm{d}f\right]^{\frac{\sigma}{\sigma-1}}$, where σ is the elasticity of substitution between differentiated products à la Dixit and Stiglitz [1977]. Each good are produced by firms in both countries. There is an infinite number of firms indexed by $i \in [0,1]$, and index $i = h \in [0, 0.5)$ are for the domestic firms and $i = f \in [0.5, 1.0]$ are for the Foreign firms. The FONCs are

$$
\begin{aligned}
C_{H,t}(h) &= \left[\frac{P_{H,t}(h)}{P_{H,t}}\right]^{-\sigma} C_{H,t}, \\
C_{F,t}(f) &= \left[\frac{P_{F,t}(f)}{P_{F,t}}\right]^{-\sigma} C_{F,t}.
\end{aligned}
$$

By substituting them into the consumption aggregators, the price indices $P_{H,t} = \left\{2\int_0^{.5} [P_{H,t}(h)]^{1-\sigma} \mathrm{d}h\right\}^{\frac{1}{1-\sigma}}$ and $P_{F,t} = \left\{2\int_{.5}^{1} [P_{F,t}(f)]^{1-\sigma} \mathrm{d}f\right\}^{\frac{1}{1-\sigma}}$ are obtained.

Given the above market structure, the domestic household maximizes its life-time utility

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\rho}}{1-\rho} - 2\int_0^{.5} \frac{h_t(h)^{1+\eta}}{1+\eta} \mathrm{d}h\right],
$$

subject to

$$
\mathbb{E}_t [m_{t,t+1} A_{t+1}] + D_t + P_t C_t = A_t + (1 + i_{t-1}) D_{t-1} + \int_0^{.5} W_t(h) h_t(h) \mathrm{d}h + \Pi_t,
$$

where $m_{t,t+1}A_{t+1}$ is the purchase of state-contingent securities by the household, which pays A_{t+1} for each state realized in the next period. D_t is the amount of one-period bond, which pays $(1+i_t)D_t$ for any state in the next period. $W_t(h)$ and $h_t(h)$ are firm-specific nominal wage and hours worked. Π_t is the transfer from firms owned by the household. β is the discount factor, ρ is the coefficient of relative risk aversion, η is Frisch elasticity of labor disutility. The FONCs are given by

$$
\begin{aligned}
h_t(h)^{\eta} &= \frac{W_t(h)}{P_t} C_t^{-\rho}, \\
m_{t,t+1} &= \beta \frac{C_{t+1}^{-\rho} P_t}{C_t^{-\rho} P_{t+1}}, \\
C_t^{-\rho} &= \beta (1 + i_t) \mathbb{E}_t \left\{\frac{P_t}{P_{t+1}} C_{t+1}^{-\rho}\right\}.
\end{aligned}
$$

By symmetry, the same results described above also apply for the household in the Foreign country. We denote the variables in the Foreign country with asterisk $(*)$.

B.1.2 Law of one price, complete risk sharing and the terms of trade

As in BB, we assume that the law of one price $P_{H,t}(i) = E_t P^*_{H,t}(i)$ and $P_{F,t}(i) = E_t P^*_{F,t}(i)$ hold for each good $i \in [0,1]$ produced in both of the Home and Foreign countries, where E_t is the nominal exchange rate. This implies $P_t = E_t P^*_t$, $P_{Ht}/P_t = P^*_{Ht}/P^*_t$ and $P_{Ft}/P_t = P^*_{Ft}/P^*_t$. Also, from the international trade of state-contingent securities,

$$\begin{aligned} m_{t,t+1} &= \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}\frac{P_t}{P_{t+1}} = \frac{\left(C^*_{t+1}\right)^{-\rho}}{\left(C^*_t\right)^{-\rho}}\frac{E_t P^*_t}{E_{t+1}P^*_{t+1}}, \\ \Leftrightarrow \quad \left(\frac{C_t}{C^*_t}\right)^{-\rho}\frac{E_t P^*_t}{P_t} &= \left(\frac{C_{t+1}}{C^*_{t+1}}\right)^{-\rho}\frac{E_{t+1}P^*_{t+1}}{P_{t+1}}. \end{aligned}$$

Without loss of generality, we assume that countries are initially symmetric. This implies that $\left(C_t/C^*_t\right)^{-\rho} E_t P^*_t/P_t = 1$ holds for all states and dates. Combined with the assumption of the law of one price, $C_t = C^*_t$ holds; i.e., complete risk sharing of consumption among countries. Note that in the international economics literature this setting is synonymous with the notion of producer currency pricing (PCP).

Terms of trade for the Home country is defined as $S_t \equiv P_{Ft}/P_t = P_t/P_{Ht}$. The market clearing conditions for both countries imply

$$\begin{aligned} Y_t = C_{Ht} + C^*_{Ht} &= \frac{1}{2}\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t + \frac{1}{2}\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-1} C^*_t = S_t C_t, \\ Y^*_t = C_{Ft} + C^*_{Ft} &= \frac{1}{2}\left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t + \frac{1}{2}\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-1} C^*_t = S_t^{-1} C_t. \end{aligned}$$

Then we have

$$S_t = (Y_t/Y^*_t)^{.5},$$

That is, the terms of trade is determined by the relative output only.

B.1.3 Firms

There is a continuum of firms indexed by $i \in [0,1]$. We focus on the domestic firms $i = h \in [0, 0.5)$. The same results apply for the Foreign firms $i = f \in [0.5, 1.0]$. Each firm has a linear production technology which transfers firm-specific labor into differentiated good, $Y_t(h) = h_t(h)$. The period-by-period profit for firm producing good h is given by

$$\begin{aligned}\Pi_t(h) &= (1-\tau_t) P_t(h) Y_t(h) - W_t(h) Y_t(h), \\ &= [(1-\tau_t) P_t(h) - W_t(h)] Y_t(h),\end{aligned}$$

where τ_t is a subsidy to each firm, which is necessary to eliminate the distortion stemming from monopolistic competition. Note that the market clearing condition for good h implies:

$$\begin{aligned}Y_t(h) &= C_{H,t}(h) + C^*_{H,t}(h), \\ &= \left[\frac{P_{H\,t}(h)}{P_{H,t}}\right]^{-\sigma} C_{H,t} + \left[\frac{P^*_{H\,t}(h)}{P^*_{H,t}}\right]^{-\sigma} C^*_{H,t}, \\ &= \frac{1}{2}\left[\frac{P_{H\,t}(h)}{P_{H,t}}\right]^{-\sigma}\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t + \frac{1}{2}\left[\frac{P^*_{H\,t}(h)}{P^*_{H,t}}\right]^{-\sigma}\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-1} C^*_t, \\ &= \left[\frac{P_{H\,t}(h)}{P_{H,t}}\right]^{-\sigma}\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t.\end{aligned}$$

Given the demand function, the firm h chooses $\bar{P}_{H,t} = P_{H,t+i}(h)$ for $i > 0$ so as to maximize

$$\mathbb{E}_t \sum_{i=0}^{\infty} \alpha^i m_{t,t+i}\left[(1-\tau_{t+i})\bar{P}_{H,t} - W_{t+i}(h)\right]\left[\frac{\bar{P}_{H,t}}{P_{H,t+i}}\right]^{-\sigma}\left(\frac{P_{H,t+i}}{P_{t+i}}\right)^{-1} C_{t+i},$$

where α is the probability of fixing prices a la Calvo [1983]. The random variable $m_{t,t+i} = \beta^i C^{-\rho}_{t+i} P_t / (C^{-\rho}_t P_{t+i})$ is the stochastic discount factor. Note that $W_{t+i}(h)$ is given for the firm. The optimality condition is

$$\mathbb{E}_t \sum_{i=0}^{\infty} \alpha^i m_{t,t+i}\left(\frac{P_{H,t+i}}{P_{t+i}}\right)^{-1} C_{t+i} P^{\sigma}_{H,t+i}\left[(1-\tau_{t+i})\bar{P}_{H,t} - \frac{\sigma}{\sigma-1} W_{t+i}(h)\right] = 0.$$

This can be further transformed in a recursive fashion

$$\begin{aligned}&\left(\frac{\bar{P}_{H,t}}{P_{H,t}}\right)^{1+\eta\sigma} F_t = K_t, \\ &F_t = M_t^{-1} C_t^{1-\rho} + \alpha\beta\mathbb{E}_t \Pi^{\sigma-1}_{H,t+1} F_{t+1}, \\ &K_t = Y_t^{1+\eta} + \alpha\beta\mathbb{E}_t \Pi^{\sigma(1+\eta)}_{H,t+1} K_{t+1},\end{aligned}$$

where $M_t = (1-\tau_t)\,\sigma/(\sigma-1)$ is a markup shock as a function of exogenous variations τ in the subsidy to firms; and $\Pi_{H,t} = P_{H,t}/P_{H,t-1}$ is the gross domestic inflation rate.

Furthermore, the Home price index $P_{H,t} = \left\{2\int_0^{.5}\left[P_{H,t}\left(h\right)\right]^{1-\sigma}\mathrm{d}h\right\}^{\frac{1}{1-\sigma}}$ can be written as

$$P_{H,t}^{1-\sigma} = \alpha P_{H\,t-1}^{1-\sigma} + (1-\alpha)\bar{P}_{H,t}^{1-\sigma}.$$

That is, only the $1-\alpha$ fraction of the domestic firms can set the new price $\bar{P}_{H,t}$. It can be further arranged as

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \left[\frac{1-\alpha\left(\frac{P_{H\,t-1}}{P_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{1}{1-\sigma}}.$$

Using the demand function of good h, $Y_t(h) = \left[\frac{P_{H\,t}(h)}{P_{H,t}}\right]^{-\sigma}\left(\frac{P_{H,t}}{P_t}\right)^{-1}C_t = \left[\frac{P_{H\,t}(h)}{P_{H,t}}\right]^{-\sigma}Y_t$ and the linear production technology of firm h, $Y_t(h) = h_t(h)$, the disutility from working is

$$\begin{aligned} & 2\int_0^{.5}\frac{h_t\left(h\right)^{1+\eta}}{1+\eta}\mathrm{d}h, \\ = & \frac{Y_t^{1+\eta}}{1+\eta}2\int_0^{.5}\left[\frac{P_{H,t}\left(h\right)}{P_{H,t}}\right]^{-\sigma(1+\eta)}\mathrm{d}h, \\ = & \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t. \end{aligned}$$

Also, the Home price dispersion $\Delta_t \equiv 2\int_0^{.5}\left[\frac{P_{H,t}(h)}{P_{H,t}}\right]^{-\sigma(1+\eta)}\mathrm{d}h$ can be further transformed into

$$\begin{aligned} \Delta_t &= \alpha 2\int_0^{.5}\left[\frac{P_{H,t-1}\left(h\right)}{P_{H,t}}\right]^{-\sigma(1+\eta)}\mathrm{d}h + (1-\alpha)\left(\frac{\bar{P}_{H,t}}{P_{H,t}}\right)^{-\sigma(1+\eta)} \\ &= \alpha\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\sigma(1+\eta)}\Delta_{t-1} + (1-\alpha)\left[\frac{1-\alpha\left(\frac{P_{H\,t-1}}{P_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{\sigma(1+\eta)}{\sigma-1}}. \end{aligned}$$

B.1.4 Equilibrium conditions

The equilibrium conditions in the model described above are

$$Y_t = S_t C_t, \tag{15}$$

$$\frac{Y_t}{Y_t^*} = S_t^2, \tag{16}$$

$$\left[\frac{1-\alpha\left(\frac{1}{\Pi_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{1+\eta\sigma}{1-\sigma}} F_t = K_t, \tag{17}$$

$$F_t = M_t^{-1} C_t^{1-\rho} + \alpha\beta \mathbb{E}_t \Pi_{H,t+1}^{\sigma-1} F_{t+1}, \tag{18}$$

$$K_t = Y_t^{1+\eta} + \alpha\beta \mathbb{E}_t \pi \Pi_{H,t+1}^{\sigma(1+\eta)} K_{t+1}, \tag{19}$$

$$\Delta_t = \alpha\left(\frac{1}{\Pi_{H,t}}\right)^{-\sigma(1+\eta)} \Delta_{t-1} + (1-\alpha)\left[\frac{1-\alpha\left(\frac{1}{\Pi_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{\sigma(1+\eta)}{\sigma-1}}, \tag{20}$$

$$\left[\frac{1-\alpha\left(\frac{1}{\Pi^*_{F,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{1+\eta\sigma}{1-\sigma}} F^*_t = K^*_t, \tag{21}$$

$$F^*_t = M_t^{*-1} C_t^{1-\rho} + \alpha\beta \mathbb{E}_t \left(\Pi^*_{F,t+1}\right)^{\sigma-1} F^*_{t+1}, \tag{22}$$

$$K^*_t = (Y^*_t)^{1+\eta} + \alpha\beta \mathbb{E}_t \left(\Pi^*_{F,t+1}\right)^{\sigma(1+\eta)} K^*_{t+1}, \tag{23}$$

$$\Delta^*_t = \alpha\left(\frac{1}{\Pi^*_{F,t}}\right)^{-\sigma(1+\eta)} \Delta^*_{t-1} + (1-\alpha)\left[\frac{1-\alpha\left(\frac{1}{\Pi^*_{F,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{\sigma(1+\eta)}{\sigma-1}}. \tag{24}$$

We have 12 endogenous variables $\{F_t, K_t, \Delta_t, F^*_t, K^*_t, \Delta^*_t, C_t, Y_t, Y^*_t, \Pi_{H,t}, \Pi^*_{F,t}, S_t\}$ and 10 Eqs. (15)-(24). Note that consumption Euler equation is redundant as it determines the nominal interest rate. Note that the equilibrium is indeterminate without any policies, due to the lack of 2 ($= 12 - 10$) equilibrium conditions. Monetary policy must be defined to pin down the equilibrium.

B.1.5 Cooperation and non-cooperation policies

The policymakers under cooperation jointly maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ .5\left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t\right] + .5\left[\frac{C_t^{*1-\rho}}{1-\rho} - \frac{Y_t^{*1+\eta}}{1+\eta}\Delta^*_t\right]\right\}$$

where $2\int_0^{.5} \frac{h_t(h)^{1+\eta}}{1+\eta} dh = \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t$ and $2\int_{.5}^{1} \frac{h_t(f)^{1+\eta}}{1+\eta} df = \frac{Y_t^{*1+\eta}}{1+\eta}\Delta^*_t$, subject to the equilibrium conditions above. Under non-cooperation, the domestic policymaker maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t\right],$$

subject to the model above, given $\Pi_{F,t}^{*}$; On the other hand, the Foreign policymaker maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{*1-\rho}}{1-\rho} - \frac{Y_t^{*1+\eta}}{1+\eta} \Delta_t^{*} \right],$$

subject to the model above, given $\Pi_{H,t}$.

We need to compute the steady state under cooperation and non-cooperation. Yet, we know that $\Pi_H = \Pi_F^* = 1$ in the steady state. We also assume that $M = M^* = 1$. Therefore, the model must be approximated around the steady state: $K = F = K^* = F^* = (1-\alpha\beta)^{-1}$ and $\Delta = \Delta^* = C = Y = Y^* = S = 1$.

B.1.6 Log-linearization

The log-linearized equilibrium conditions around the steady state are

$$y_t - s_t = c_t,$$

$$s_t = \frac{1}{2}\left(y_t - y_t^*\right),$$

$$\frac{\alpha\left(1+\eta\sigma\right)}{1-\alpha}\pi_{H,t} + f_t = k_t,$$

$$f_t = \left(1-\alpha\beta\right)\left(1-\rho\right)c_t - \left(1-\alpha\beta\right)\mu_t + \alpha\beta\left(\sigma-1\right)\mathbb{E}_t\pi_{H,t+1} + \alpha\beta\mathbb{E}_t f_{t+1},$$

$$k_t = \left(1-\alpha\beta\right)\left(1+\eta\right)y_t + \alpha\beta\sigma\left(1+\eta\right)\mathbb{E}_t\pi_{H,t+1} + \alpha\beta\mathbb{E}_t k_{t+1},$$

$$\frac{\alpha\left(1+\eta\sigma\right)}{1-\alpha}\pi_{F,t}^* + f_t^* = k_t^*,$$

$$f_t^* = \left(1-\alpha\beta\right)\left(1-\rho\right)c_t^* - \left(1-\alpha\beta\right)\mu_t^* + \alpha\beta\left(\sigma-1\right)\mathbb{E}_t\pi_{F,t+1}^* + \alpha\beta\mathbb{E}_t f_{t+1}^*,$$

$$k_t^* = \left(1-\alpha\beta\right)\left(1+\eta\right)y_t^* + \alpha\beta\sigma\left(1+\eta\right)\mathbb{E}_t\pi_{F,t+1}^* + \alpha\beta\mathbb{E}_t k_{t+1}^*,$$

Note that the log deviation of a variable X_t from the steady state X is defined in lowercase as $x_t \equiv \log(X_t/X)$ and the Taylor approximation of X_t up to the first order is $X_t \approx X(1+x_t)$.[31] Given $\Pi_H = \Pi_F = 1$ in the steady state, $\pi_t \equiv \pi_{H,t} = \log(\Pi_{H,t}) \approx \Pi_{H,t} - 1$ and $\pi_{F,t} \equiv \pi_{F,t} = \log(\Pi_{F,t}) \approx \Pi_{F,t} - 1$ are the net domestic and

[31] We will also use the same lowercase Greek convention for Greek-lettered variables—e.g., x is to X , as δ is to Δ , or, as μ is to M.

Foreign inflation rates. Note that $\delta_t = \delta_t^* = 0$, i.e., the price dispersion terms have no effect at the first order. These equations are summarized as follows:

$$\pi_{H,t} = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\eta\sigma)}\left[\mu_t + (\rho+\eta)\,y_t + (1-\rho)\,s_t\right] + \beta\mathbb{E}_t\pi_{H,t+1}, \tag{25}$$

$$\pi_{F,t}^* = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\eta\sigma)}\left[\mu_t + (\rho+\eta)\,y_t^* - (1-\rho)\,s_t\right] + \beta\mathbb{E}_t\pi_{F,t+1}^*, \tag{26}$$

and

$$s_t = \frac{1}{2}\left(y_t - y_t^*\right), \tag{27}$$

which corresponds to Eqs. (1)-(2).

B.2 Welfare approximation for LQ framework

Instead of the nonlinear cooperation and non-cooperation policies explained above, we will consider the LQ framework by following to BB. For that purpose, the objective functions must be correctly approximated so that the Ramsey policy in the LQ framework yields exactly the same results, i.e., impulse responses to markup shocks, as in the nonlinear Ramsey policy does up to the first order [Benigno and Woodford, 2012, Debortoli and Nunes, 2006, Levine et al., 2008, Bodenstein et al., 2014].

Let x_t denote the percentage deviation of the level of a variable X_t from its deterministic steady-state point X. Note that for a variable X_t, the Taylor approximation up to the second order is $X_t - X \approx X\left(x_t + \frac{1}{2}x_t^2\right)$. Thus,

$$\begin{aligned}
\frac{C_t^{1-\rho}}{1-\rho} &\approx \frac{C^{1-\rho}}{1-\rho} + C^{-\rho}\left(C_t - C\right) - \frac{\rho C^{-\rho-1}}{2}\left(C_t - C\right)^2, \\
&\approx C^{1-\rho}\left(\frac{C_t - C}{C}\right) - \frac{\rho C_t^{1-\rho}}{2}\left(\frac{C_t - C}{C}\right)^2 + \text{t.i.p.}, \\
&= c_t + \frac{1-\rho}{2}c_t^2 + \text{t.i.p.}
\end{aligned}$$

where t.i.p. stands for terms independent of policy. Similarly,

$$\frac{Y_t^{1+\eta}}{1+\eta} \approx y_t + \frac{1+\eta}{2}y_t^2 + \text{t.i.p.}$$

Note that $C^{1-\rho} = Y^{1+\eta} = 1$.

B.2.1 Cooperation

$$
\begin{aligned}
& \frac{C_t^{1-\rho}}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t - \frac{(Y_t^*)^{1+\eta}}{1+\eta}(\Delta_t^*), \\
\approx\ & c_t + \frac{1-\rho}{2}c_t^2 + c_t^* + \frac{1-\rho}{2}(c_t^*)^2 \\
& -y_t - \frac{1+\eta}{2}y_t^2 - y_t^* - \frac{1+\eta}{2}(y_t^*)^2 - \delta_t - \delta_t^* + \text{t.i.p.}, \\
=\ & \frac{1-\rho}{2}(y_t - s_t)^2 + \frac{1-\rho}{2}(y_t^* + s_t)^2 - \frac{1+\eta}{2}y_t^2 - \frac{1+\eta}{2}(y_t^*)^2 \\
& -\Delta_t - \Delta_t^* + \text{t.i.p.}, \\
=\ & -\frac{\rho+\eta}{2}y_t^2 - \frac{\rho+\eta}{2}(y_t^*)^2 - (1-\rho)s_t^2 - \delta_t - \delta_t^* + \text{t.i.p.},
\end{aligned}
$$

where we use $s_t = \frac{1}{2}(y_t - y_t^*)$, and $y_t - s_t = c_t$, $y_t^* + s_t^* = c_t^*$, noting that $s = -s^*$. Note that the second order approximation of

$$
\Delta_t = \alpha\left(\frac{1}{\Pi_{H,t}}\right)^{-\sigma(1+\sigma\eta)}\Delta_{t-1} + (1-\alpha)\left[\frac{1-\alpha\left(\frac{1}{\Pi_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{\sigma(1+\eta\sigma)}{\sigma-1}}
$$

leads to

$$
\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Delta_t \approx \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{\alpha\sigma(1+\eta\sigma)}{2(1-\alpha)(1-\alpha\beta)}\pi_{H,t}^2.
$$

Therefore, the terms of price dispersion can be considered as the second order ones. Thus, the welfare under cooperation is now approximated up to the second order as

$$
\begin{aligned}
& -\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\delta_t + \delta_t^* + \frac{\rho+\eta}{2}y_t^2 + \frac{\rho+\eta}{2}(y_t^*)^2 + (1-\rho)s_t^2\right], \\
=\ & -.5\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c} \frac{\alpha\sigma(1+\eta\sigma)}{(1-\alpha)(1-\beta\alpha)}\pi_{H,t}^2 + \frac{\alpha\sigma(1+\eta\sigma)}{(1-\alpha)(1-\beta\alpha)}\left(\pi_{F,t}^*\right)^2 \\ +(\rho+\eta)y_t^2 + (\rho+\eta)(y_t^*)^2 \\ +\frac{1-\rho}{2}(y_t - y_t^*)^2 \end{array}\right].
\end{aligned}
$$

This corresponds to (6) in Section 2.

B.2.2 Non-cooperation

Domestic instantaneous welfare is

$$
\begin{aligned}
& \frac{C_t^{1-\rho}}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta}\Delta_t, \\
\approx \;& c_t + \frac{1-\rho}{2}c_t^2 - y_t - \frac{1+\eta}{2}y_t^2 - \delta_t + \text{t.i.p.}, \\
= \;& -s_t + \frac{1-\rho}{2}c_t^2 - \frac{1+\eta}{2}\left(c_t + s_t\right)^2 - \delta_t + \text{t.i.p.},
\end{aligned} \tag{28}
$$

where δ is the percentage deviation of Δ from its steady state. Similarly, for the Foreign country we have

$$
\begin{aligned}
& \frac{\left(C_t^*\right)^{1-\rho}}{1-\rho} - \frac{\left(Y_t^*\right)^{1+\eta}}{1+\eta}\Delta_t^*, \\
\approx \;& c_t^* + \frac{1-\rho}{2}\left(c_t^*\right)^2 - y_t^* - \frac{1+\eta}{2}\left(y_t^*\right)^2 - \delta_t^* + \text{t.i.p.}, \\
= \;& s_t + \frac{1-\rho}{2}c_t^2 - \frac{1+\eta}{2}\left(c_t - s_t\right)^2 - \delta_t^* + \text{t.i.p.},
\end{aligned} \tag{29}
$$

Note that each approximation includes the log-linear term of s_t. The linear terms in the approximated welfare induce spurious welfare evaluation in the LQ framework. The correct LQ approximation must be derived with a purely quadratic welfare function [Kim and Kim, 2003, 2007, Benigno and Woodford, 2005, 2012, Benigno and Benigno, 2006, Fujiwara and Teranishi, 2013]. We need to substitute out the linear terms of s_t. For this purpose, we approximate the NK Phillips curve up to the second order.

Second order approximation of the NK Phillips Curve By following Benigno and Benigno [2006], the second order approximation of the NK Phillips curve leads to

$$
\begin{aligned}
& \kappa \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\eta y_t + \rho c_t - p_{H,t} + \mu_t\right) \\
\approx \;& K_0 - \frac{\kappa}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\eta y_t + \rho c_t - p_{H,t} + \mu_t\right)\left(2c_t - \rho c_t + \eta y_t - p_{H,t} + \mu_t\right) \\
& - \frac{\sigma\left(1+\eta\right)}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2.
\end{aligned}
$$

where, $\hat{\mu}$ is the percentage deviation of μ from steady state, $\kappa = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\eta\sigma)}$ and K_0 is given and without loss, assumed to be zero. Therefore, we can have the approximated condition:

$$\begin{aligned} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(\eta+\rho) c_t + (1+\eta) s_t \right] \\ \approx \quad & -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left[(\eta+\rho) c_t + (1+\eta) s_t + \mu_t \right] \\ \times \left[(2-\rho+\eta) c_t + (1+\eta) s_t + \mu_t \right] \\ -\frac{\sigma(1+\eta)}{2\kappa} \pi_{H,t}^2 \end{array} \right\}, \end{aligned}$$

Similarly for the Foreign Phillips curve, we have

$$\begin{aligned} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(\eta+\rho) c_t - (1+\eta) s_t \right] \\ \approx \quad & -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left[(\eta+\rho) c_t - (1+\eta) s_t + \mu_t^* \right] \\ \times \left[(2-\rho+\eta) c_t - (1+\eta) s_t + \mu_t^* \right] \\ -\frac{\sigma(1+\eta)}{2\kappa} \left(\pi_{F,t}^*\right)^2 \end{array} \right\}. \end{aligned}$$

From these approximations, we have the linear terms replaced by quadratic terms:

$$\begin{aligned} 2\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t s_t \quad \approx \quad & -\frac{1}{4(1+\eta)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(\eta+\rho) c_t + (1+\eta) s_t + \mu_t \right] \times \\ & \left[(2-\rho+\eta) c_t + (1+\eta) s_t + \mu_t \right] - \frac{\sigma}{4\kappa} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2 \\ & + \frac{1}{4(1+\eta)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\eta+\rho) c_t - (1+\eta) s_t + \mu_t^* \right) \times \\ & \left[(2-\rho+\eta) c_t - (1+\eta) s_t + \mu_t^* \right] + \frac{\sigma}{4\kappa} \sum_{t=0}^{\infty} \beta^t \left(\pi_{F,t}^*\right)^2 . \end{aligned} \tag{30}$$

By substituting (30) into (28), we have

$$\begin{aligned} L_t \quad = \quad & -(\rho+\eta) \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(y_t - \frac{1}{\rho+\eta} \mu_t \right)^2 - (\rho+\eta) \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(y_t^* + \frac{1}{\rho+\eta} \mu_t^* \right)^2 \\ & -\frac{1-\rho}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(y_t - y_t^* \right) - \frac{\sigma}{\kappa} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_{H,t}^2 - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\sigma}{\kappa} \left(\pi_{F,t}^*\right). \end{aligned}$$

This corresponds to (4) in Section 2. Similarly we can derive (5) as well.