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Capital Accumulation and Dynamic Gains from Trade*

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Abstract

We compute welfare gains from trade in a dynamic, multi-country Ricardian model where international trade affects capital accumulation. We calibrate the model for 93 countries and examine transition paths between steady-states after a permanent, uniform trade liberalization across countries. Our model allows for both the relative price of investment and the investment rate to depend on the world distribution of trade barriers. Accounting for transitional dynamics, welfare gains are about 60 percent of those measured by comparing only the steady-states, and three times larger than those with no capital accumulation. We extend the model to incorporate adjustment costs to capital accumulation and endogenous trade imbalances. Relative to the model with balanced trade, the gains from trade increase more for small countries because they accumulate capital at faster rates by running trade deficits in the short run.

JEL codes: E22, F11, O11

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1 Introduction

How large are the welfare gains from trade? This is an old and important question. This question has been typically answered in static settings by computing the change in real income from an observed equilibrium to a counterfactual equilibrium. In such computations, the factors of production and technology in each country are held fixed and the change in real income is entirely due to the change in each country's trade share that responds to a change in trade frictions. Recent examples include Arkolakis, Costinot, and Rodríguez-Clare (2012) who compute the welfare cost of autarky and Waugh and Ravikumar (2016) who compute the welfare gains from frictionless trade.

By design, the above computations cannot distinguish between static and dynamic gains from trade. We compute welfare gains from trade in a dynamic multi-country Ricardian model where international trade affects the capital stock in each period. Our environment is a version of Eaton and Kortum (2002) embedded into a two-sector neoclassical growth model. There is a continuum of tradable intermediate goods. The technology for producing the intermediate goods is country-specific and the productivity distribution is Fréchet. Each country is endowed with an initial stock of capital. Investment goods, produced using intermediate goods, augment the stock of capital. Final consumption goods are also produced using intermediate goods. Trade is subject to iceberg costs.

The model features two novel ingredients: (i) endogenous relative price of investment and (ii) endogenous investment rate.

We compute the steady-state of the model for 93 countries and calibrate it to reproduce the observed trade flows across countries, prices, and output per worker in each country in 2011. We use this steady-state as a benchmark and conduct a counterfactual exercise in which trade barriers are reduced simultaneously in every country. We then compute the transition path from the initial steady-state to the new steady-state. With this dynamic path, we compute the welfare gains using a consumption equivalent measure as in Lucas (1987).

We find that (a) comparing only steady states overstates the gains; the gains from trade that include transition are about 60 percent of those measured by only comparing steady-states, (b) static comparisons understate the gains; the dynamic gains with transition are three times larger than than those measured in a static model with capital held fixed, and (c) output increases on impact, but consumption drops since there is a large increase in the marginal product of capital and a fall in the relative price of investment inducing large increases in investment.

We then show the importance of the two main features of our model to analyze dynamic welfare gains from trade. First, the endogenous relative price of investment allows countries to attain permanently higher capital-output ratios, yielding higher output and consumption in the steady-state. Second, the endogenous investment rate yields shorter half lives for capital accumulation induced by temporarily high real rates of return to investment. As a result, the model delivers large dynamic gains from trade.

The model's predictions are consistent with several features of the data. Wacziarg and Welch (2008) identify dates that correspond to a trade liberalization for 118 countries, and show that, after such liberalization, GDP growth increases, the relative price of investment falls fast and real investment rates increase. All these are features of our model. Furthermore, Wacziarg (2001) evaluates empirically several theories of dynamic gains from trade in explaining the effect of trade on economic growth. Consistent with our results, he finds that trade positively affects growth primarily through an increase in investment, and hence capital accumulation.

Our solution method offers an efficient means to compute the transition path. The method generalizes the algorithm of Alvarez and Lucas (2007b) to a dynamic environment by iterating on a small subset of prices using information in excess demand equations. Such an updating rule avoids computing costly gradients and typically converges in a matter of a few hours on a basic laptop computer. This method applies to multi-country models of trade with capital accumulation, CRRA preferences, linear depreciation of capital, and balanced trade.

One limitation of our model, however, is that households cannot borrow against the future, which restricts us to study transition paths in which investment is always positive. To alleviate this limitation, we extend the baseline model by adding adjustment costs to capital accumulation and endogenous trade imbalances. We provide a modified algorithm to compute the transition paths in this economy as well.[1]

Using the extended model with adjustment costs, we consider the same reduction in trade barriers as in the baseline model both when trade is balanced and when there are endogenous trade imbalances. We find that countries that have a higher marginal product of capital in the baseline model, grow faster, and in the extended model experience inflows

[1] Our algorithms for the baseline model and for the extended model rely on gradient-free updating rules. These methods are computationally less demanding than nonlinear solvers used by recent papers that study capital accumulation and endogenous trade imbalances such as Eaton, Kortum, Neiman, and Romalis (2016) and Kehoe, Ruhl, and Steinberg (2016).

of capital and run a trade deficit early on. These countries then converge to a steady-state with a trade surplus. On the contrary, slow-growing countries run a trade surplus early on, but converge to a steady-state with a trade deficit. Welfare is slightly higher in the model with trade imbalances than in the model with balanced trade for all countries. However, relative to a model with balanced trade, countries that run a trade deficit early on exhibit proportionately larger dynamic gains from trade than countries that run a surplus early on.

Our paper relates to several strands of literature. First, it relates to two recent studies that examine dynamic trade models. Eaton, Kortum, Neiman, and Romalis (2016) and Caliendo, Dvorkin, and Parro (2015) compute the transitional dynamics of an international trade model by computing period-over-period change in endogenous variables as a result of changes in parameters (this is the so-called hat-algebra approach). Our approach differs from theirs in several aspects. First, we solve for the transition of our model in levels; we do not use hat algebra. By solving the model in levels, we are able to validate the cross-sectional predictions of our model. In particular, we find that our model is consistent with the cross-sectional distribution of capital and investment rates in the data. Second, computing the initial steady-state in levels allows us to impose discipline on the type of trade liberalization we are interested in, which is not possible without knowing the initial levels. Finally, Eaton, Kortum, Neiman, and Romalis (2016) solve for the planner's problem and assume that the Pareto weights remain constant across counterfactuals, implying that each country's share in world consumption is fixed across counterfactuals. In our computation, however, each country's share in world consumption changes across counterfactuals and along the transition path, a feature that is important when studying welfare.[2]

A second strand of literature has incorporated capital accumulation into trade models to study welfare gains from trade. Alvarez and Lucas (2007a) develop a method that approximates the dynamics by linearizing around the steady-state. Alessandria, Choi, and Ruhl (2015) consider welfare gains from trade in a two-country model with capital accumulation and highlight short run frictions such as fixed and sunk costs to export. Finally, Brooks and Pujolas (2016) compare dynamic welfare gains in a model with endogenous capital accumulation to those in a static model. They do so in the context of a two-country model with balanced trade.

Anderson, Larch, and Yotov (2015) study the transitional dynamics via capital cumula-

[2]Zylkin (2016) uses an approach similar to "hat algebra" to study how China's integration from 1993-2011 has had an effect on investment and capital accumulation in the rest of the world. His "hat algebra" approach differs from other papers in that he computes the change of the variable from its baseline equilibrium value to its counterfactual equilibrium value, rather than computing period-over-period changes.

tion in a multi-country model to measure the welfare gains from trade. Our paper builds on their work by incorporating into the model a relative price of investment and endogenous investment rate that each depend on the world distribution of trade barriers. These features imply that anticipated changes to future trade costs have an impact on current consumption and saving decisions. The endogenous relative price implies that countries can attain higher steady-state capital stocks. The endogenous investment rate implies that countries accumulate capital more quickly in response to a trade liberalization. Both of these features affect the computed gains from trade and are consistent with empirical evidence as shown by Wacziarg (2001) and Wacziarg and Welch (2008).

Our extended model adds to a strand of literature that analyzes dynamics in international trade via endogenous trade imbalances. Reyes-Heroles (2016) studies endogenous trade imbalances in a multi-country model without capital, while Sposi (2012) studies endogenous trade imbalances with an exogenous nominal investment rate for capital. Kehoe, Ruhl, and Steinberg (2016) combine capital accumulation and endogenous trade imbalances into a two-country, general equilibrium model of trade.

Finally, recent studies have used "sufficient statistics" approaches to measure changes in welfare by looking at changes in the home trade share (Arkolakis, Costinot, and Rodríguez-Clare, 2012). In our baseline model the sufficient-statistics formula is only valid across steady-states, but not along the transition path. We show that measuring changes in welfare using changes in consumption along the transition path yields very different implications than one would obtain by using sufficient statistics. Moreover, in our extended model we show that the sufficient-statistics formula breaks down even across steady-states, with systematically larger errors for countries that run steady-state trade deficits compared to countries that run surpluses. That is, when trade imbalances are endogenously determined, changes in the home trade share are not sufficient to characterize the changes in welfare, or in income for that matter, across steady-states.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 describes the quantitative exercise. Section 4 reports the counterfactuals, and section 5 concludes.

2 Model

There are I countries indexed by $i = 1, \dots, I$ and time is discrete, running from $t = 1, \dots, \infty$. There are three sectors: consumption, investment, and intermediates, denoted by c, x, and

m respectively. Neither consumption goods nor investment goods are tradable. There is a continuum of intermediate *varieties* that are tradable. Production of all the goods are carried out by perfectly competitive firms. As in Eaton and Kortum (2002), each country's efficiency in producing each intermediate variety is a realization of a random draw from a country- and time-specific distribution. Trade in intermediate varieties is subject to iceberg costs. Each country purchases each intermediate variety from its lowest-cost supplier and all of the varieties are aggregated into a *composite* intermediate good. The composite intermediate good, which is nontradable, is used as an input along with capital and labor to produce the consumption good, the investment good, and the intermediate varieties.

Each country has a representative household. The representative household owns its country's stock of capital and labor, which it inelastically supplies to domestic firms, and purchases consumption and investment goods from the domestic firms.

2.1 Endowments

In each period, the representative household in country i is endowed with a labor force of size L_i, which is constant over time, and a stock of capital in the initial period, K_{i1}.

2.2 Technology

There is a unit interval of varieties in the intermediates sector. Each variety within the sector is tradable and is indexed by $v \in [0, 1]$.

Composite good Within the intermediates sector, all of the varieties are combined with constant elasticity in order to construct a sectoral composite good according to

$$Q_{it} = \left[\int_0^1 q_{it}(v)^{1-1/\eta} dv \right]^{\eta/(\eta-1)}$$

where η is the elasticity of substitution between any two varieties.[3] The term $q_{it}(v)$ is the quantity of good v used by country i to construct the composite good at time t and Q_{it} is the quantity of the composite good available in country i to be used as an intermediate input.

[3]The value η plays no quantitative role other than satisfying technical conditions which ensure convergence of the integrals.

Varieties Each variety is produced using capital, labor, and the composite intermediate good. The technologies for producing each variety are given by

$$Y_{mit}(v) = z_{mi}(v) \left(K_{mit}(v)^{\alpha} L_{mit}(v)^{1-\alpha}\right)^{\nu_m} M_{mit}(v)^{1-\nu_m}$$

The term $M_{mit}(v)$ denotes the quantity of the composite good used by country i as an input to produce $Y_{mit}(v)$ units of variety v, while $K_{mit}(v)$ and $L_{mit}(v)$ denote the quantities of capital and labor employed.

The parameter $\nu_m \in [0,1]$ denotes the share of value added in total output, while α denotes capital's share in value added. Each of these coefficients is constant both across countries and over time.

The term $z_{mi}(v)$ denotes country i's productivity for producing variety v. Following Eaton and Kortum (2002), the productivity draw comes from independent country-specific Fréchet distributions with shape parameter θ and country-specific scale parameter T_{mi}, for $i = 1, 2, \ldots, I$. The c.d.f. for productivity draws in country i is $F_{mi}(z) = \exp(-T_{mi} z^{-\theta})$.

In country i the expected value of productivity across the continuum is $\gamma^{-1} T_{mi}^{\frac{1}{\theta}}$, where $\gamma = \Gamma(1 + \frac{1}{\theta}(1-\eta))^{\frac{1}{1-\eta}}$ and $\Gamma(\cdot)$ is the gamma function, and $T_{mi}^{\frac{1}{\theta}}$ is the fundamental productivity in country i.[4] If $T_{mi} > T_{mj}$, then on average, country i is more efficient than country j at producing intermediate varieties. The parameter $\theta > 0$ governs the coefficient of variation of the efficiency draws. A larger θ implies more variation in efficiency across countries and, hence, more room for specialization within each sector; i.e., more trade.

Consumption good Each country produces a consumption good using capital, labor, and intermediates according to

$$Y_{cit} = A_{ci} \left(K_{cit}^{\alpha} L_{cit}^{1-\alpha}\right)^{\nu_c} M_{cit}^{1-\nu_c}$$

The terms K_{cit}, L_{cit}, and M_{cit} denote the quantity of capital, labor, and composite intermediate good used by country i to produce Y_{cit} units of consumption at time t. The parameters α and ν_c are constant across countries and over time. The term A_{ci} captures country i's productivity in the consumption goods sector—this term varies across countries.

[4] As discussed in Waugh (2010) and Finicelli, Pagano, and Sbracia (2012), fundamental productivity differs from measured productivity because of selection. In a closed economy, country i produces all varieties in the continuum so its measured productivity is equal to its fundamental productivity. In an open economy, country i produces only the varieties in the continuum for which it has a comparative advantage and imports the rest. So its measured productivity is higher than its fundamental productivity, conditioning on the varieties that it produces in equilibrium.

Investment good Each country produces an investment good using capital, labor, and intermediates according to

$$Y_{xit} = A_{xi}\left(K_{xit}^{\alpha} L_{xit}^{1-\alpha}\right)^{\nu_x} M_{xit}^{1-\nu_x}$$

The terms K_{xit}, L_{xit}, and M_{xit} denote the quantity of capital, labor, and composite intermediate good used by country i to produce Y_{xi} units of investment at time t. The parameters α and ν_x are constant across countries and over time. The term A_{xi} captures country i's productivity in the investment goods sector—this term varies across countries.

2.3 Trade

International trade is subject to barriers that take the iceberg form. Country i must purchase $d_{ij} \geq 1$ units of any intermediate variety from country j in order for one unit to arrive; $d_{ij} - 1$ units *melt* away in transit. As a normalization we assume that $d_{ii} = 1$ for all i.

2.4 Preferences

The representative household values consumption per capita over time, C_{it}/L_i, according to

$$\sum_{t=1}^{\infty} \beta^{t-1} L_i \frac{(C_{it}/L_i)^{1-1/\sigma}}{1-1/\sigma}$$

where $\beta \in (0,1)$ denotes the period discount factor and σ denotes the intertemporal elasticity of substitution. Both parameters are constant across countries and over time.

Capital accumulation Each period the representative household enters with K_{it} units of capital, which depreciates at the rate δ. Investment, X_{it}, adds to future capital.

$$K_{it+1} = (1-\delta)K_{it} + X_{it}$$

Budget constraint The representative household earns income by supplying capital, K_{it}, and labor, L_i, inelastically to domestic firms earning a rental rate r_{it} on each unit of capital and a wage rate w_{it} on each unit of labor. The household purchases consumption at the price P_{cit} per unit and purchases investment at the price P_{xit} per unit. The period budget constraint is given by

$$P_{cit}C_{it} + P_{xit}X_{it} = r_{it}K_{it} + w_{it}L_i$$

2.5 Equilibrium

A competitive equilibrium satisfies the following conditions: i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technology for accumulating capital, ii) taking prices as given, firms maximize profits subject to the available technologies, iii) intermediate varieties are purchased from their lowest-cost provider subject to the trade barriers, and iv) all markets clear. At each point in time, we take world GDP as the numéraire: $\sum_i r_{it}K_{it} + w_{it}L_i = 1$ for all t. We describe each equilibrium condition in more detail in Appendix A.

2.6 Welfare Analysis

We measure welfare using consumption-equivalent units since utility in our model is defined over consumption. This is a departure from much of the literature on static models in which welfare gains are computed as changes in income. In a dynamic model, as income changes along the transition path we need to examine how the income is allocated to consumption and investment.

We follow Lucas (1987) and compute the constant, λ_i^{dyn}:

$$\sum_{t=1}^{\infty} \beta^{t-1} L_i \frac{\left(\left(1 + \frac{\lambda_i^{dyn}}{100}\right) C_i^\star / L_i\right)^{1-1/\sigma}}{1 - 1/\sigma} = \sum_{t=1}^{\infty} \beta^{t-1} L_i \frac{\left(\tilde{C}_{it}/L_i\right)^{1-1/\sigma}}{1-1/\sigma} \tag{1}$$

where $C_i^\star$ is the (constant) consumption in the benchmark steady-state in country i, and $\tilde{C}_{it}$ is the consumption in the counterfactual at time t.[5] We refer to λ_i^{dyn} as "dynamic gains."

In steady-state, this formula can be expressed as

$$\Rightarrow 1 + \frac{\lambda_i^{ss}}{100} = \frac{C_i^{\star\star}}{C_i^\star} \tag{2}$$

where $C_i^{\star\star}$ is the consumption in the the new (counterfactual) steady-state in country i. In

[5] We calculate sums using the counterfactual transition path solved from $t = 1, \ldots, 150$ and then set the counterfactual consumption equal to the new steady-state level of consumption for $t = 151, \ldots, 1000$.

this expression, λ_i^{ss} measures gains from trade across steady-states in country i. In our model consumption is proportional to income across countries in the steady-state.[6]

Dynamic welfare gains require us to compute the entire transition path for consumption, which depends on the transition for capital accumulation. The dynamics of capital are governed by the Euler equation. In particular, combining the Euler equation with the budget constraint and the capital accumulation technology, the equilibrium law of motion for capital must obey the following equation in every country

$$\begin{aligned}
&\left(1+\frac{r_{it+1}}{P_{xit+1}}-\delta\right)\left(\frac{P_{xit+1}}{P_{cit+1}}\right)K_{it+1}+\left(\frac{w_{it+1}}{P_{cit+1}}\right)L_i-\left(\frac{P_{xit+1}}{P_{cit+1}}\right)K_{it+2}\\
&=\beta^{\sigma}\left(1+\frac{r_{it+1}}{P_{xit+1}}-\delta\right)^{\sigma}\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^{\sigma}\\
&\times\left[\left(1+\frac{r_{it}}{P_{xit}}-\delta\right)\left(\frac{P_{xit}}{P_{cit}}\right)K_{it}+\left(\frac{w_{it}}{P_{cit}}\right)L_i-\left(\frac{P_{xit}}{P_{cit}}\right)K_{it+1}\right]
\end{aligned}$$

This is the key equation to analyze dynamics, and it constitutes the main departure from the existing models analyzing welfare gains from trade.[7] Note that the dynamics of capital in country i depend on the capital stocks in all other countries since the prices are determined in the world economy due to trade. Thus, the dynamics are pinned down by the solution to a system of I second-order, nonlinear difference equations. The optimality conditions for the firms combined with the relevant market clearing conditions pin down the prices as a function of the capital stocks across countries.

3 Quantitative exercise

We describe in Appendix B the details of our algorithm for computing the dynamic equilibrium in the baseline model. Broadly speaking, we first reduce the infinite dimension of the problem down to a finite-time model with $t = 1, \ldots, T$ periods. We make T sufficiently large to ensure convergence to a new steady-state. As such, this requires us to first solve for a terminal steady-state to use as a boundary condition for the path of capital stocks. In addition, we take initial capital stocks as given by the initial steady-state.

In steady-state, all endogenous variables are constant over time. Table B.1 provides the

[6]The formula for to ratio of consumption to income in country i is $\frac{C_i}{y_i L_i} = 1 - \frac{\alpha\delta}{\frac{1}{\beta}-(1-\delta)}$.

[7]Anderson, Larch, and Yotov (2015) use a similar expression to measure dynamic welfare gains from trade, but they impose assumptions on preferences and technologies that yield a fixed investment rate. As a result, their model does not admit an Euler equation.

equilibrium conditions that describe the solution to the steady-state in our model. Our technique for computing the steady-state equilibria are standard, while our method for computing the equilibrium transition path between steady-states is new.[8]

3.1 Calibration

We calibrate the initial parameters of the model to match data in 2011. Our assumption is that the world is in steady-state at this time. Our model covers 93 countries (containing 91 individual countries plus 2 regional country groups). Table F.1 in the Appendix provides a list of the countries along with their 3-digit ISO codes. This set of countries accounts for 90 percent of world GDP as measured by the Penn World Tables, and for 84 percent of world trade in manufactures as measured by the United Nations Comtrade Database. Appendix E provides the details of our data.

Common parameters The values for the common parameters are reported in Table 1. We use recent estimates of the trade elasticity by Simonovska and Waugh (2014) and set $\theta = 4$. The value for η plays no quantitative role in the Eaton-Kortum model of trade other than satisfying the condition that $1 + \frac{1}{\theta}(1 - \eta) > 0$; we set $\eta = 2$.

Table 1: Common parameters

θ	Trade elasticity	4
η	Elasticity of substitution between varieties	2
α	Capital's share in value added	0.33
β	Annual discount factor	0.96
δ	Annual depreciation rate for stock of capital	0.06
σ	Intertemporal elasticity of substitution	0.67
ν_c	Share of value added in final goods output	0.91
ν_x	Share of value added in investment goods output	0.33
ν_m	Share of value added in intermediate goods output	0.28

[8]We solve for the competitive equilibrium of the model. This differs from Eaton, Kortum, Neiman, and Romalis (2016), who solve the planner's problem. In particular, they use the social planner's problem to solve for trade imbalances using fixed weights across counterfactuals. This implies that each countrys share in world consumption expenditures (i.e., the numeraire in their setting) is fixed across counterfactuals. In a decentralized economy, these shares would change, and still be efficient. We see this in our own counterfactuals. The second welfare theorem states that any social planner outcome can be replicated in a decentralized market with the appropriate transfers. In our context, this implies that the social planner weights would need to change in order to generate the same allocation as the decentralized economy without transfers (i.e., in our counterfactuals).

In line with the literature, we set the share of capital in value added to $\alpha = 0.33$ (from Gollin, 2002), the discount factor $\beta = 0.96$, and the intertemporal elasticity of substitution $\sigma = 0.67$.

We compute $\nu_m = 0.28$ by taking the cross-country average of the ratio of value added to gross output of manufactures. We compute $\nu_x = 0.33$ by taking the cross-country average of the ratio of value added to gross output of investment goods. Computing ν_c is slightly more involved since there is not a clear industry classification for consumption goods. Instead, we infer this share by interpreting national accounts data through the lens of our model. We begin by noting that, from combining firm optimization and market clearing conditions for capital and labor we get

$$r_i K_i = \frac{\alpha}{1-\alpha} w_i L_i$$

In steady-state, the Euler equation and the capital accumulation technology imply

$$P_{xi} X_i = \frac{\delta\alpha}{\frac{1}{\beta} - (1-\delta)} \frac{w_i L_i}{1-\alpha} = \phi_x \frac{w_i L_i}{1-\alpha}$$

We compute ϕ_x by taking the cross-country average of the share of gross fixed capital formation in nominal GDP. Given this value, and the relation $\phi_x = \frac{\delta\alpha}{\frac{1}{\beta}-(1-\delta)}$, the depreciation rate for capital is $\delta = 0.06$. The household's budget constraint then implies that

$$P_{ci} C_i = \frac{w_i L_i}{1-\alpha} - P_{xi} X_i = (1-\phi_x) \frac{w_i L_i}{1-\alpha}$$

Consumption in our model corresponds to the sum of private and public consumption, changes in inventories, and net exports. We can use the trade balance condition together with the firm optimality and the market clearing conditions for sectoral output to obtain

$$P_{mi} Q_i = \left[(1-\nu_x)\phi_x + (1-\nu_c)(1-\phi_x)\right] \frac{w_i L_i}{1-\alpha} + (1-\nu_m) P_{mi} Q_i \tag{3}$$

where $P_{mi} Q_i$ is total absorption of manufactures in country i and $\frac{w_i L_i}{1-\alpha}$ is the nominal GDP. We use a standard method of moments estimator to back out ν_c from equation (3).

Country-specific parameters We set the workforce, L_i, equal to the total population. The remaining parameters A_{ci}, T_{mi}, A_{xi} and d_{ij}, for $(i,j) = 1, \ldots, I$, are not directly observable. We parsimoniously back these out by linking structural relationships of the model to observables in the data.

The equilibrium structure relates the unobserved trade barrier for any given country pair directly to the ratio of intermediate-goods prices in the two countries and the trade shares between them as follows

$$\frac{\pi_{ij}}{\pi_{jj}} = \left(\frac{P_{mj}}{P_{mi}}\right)^{-\theta} d_{ij}^{-\theta} \tag{4}$$

Appendix E provides the details for how we construct the empirical counterparts to prices and trade shares. For observations in which $\pi_{ij} = 0$, we set $d_{ij} = 10^8$. We also set $d_{ij} = 1$ if the inferred value of trade cost is less than 1.

Lastly, we derive three structural relationships to pin down the productivity parameters A_{ci}, T_{mi}, and A_{xi}.

$$\frac{P_{ci}/P_{mi}}{P_{cU}/P_{mU}} = \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}/A_{ci}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}/A_{cU}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}}\right)^{\frac{\nu_c-\nu_m}{\nu_m}} \tag{5}$$

$$\frac{P_{xi}/P_{mi}}{P_{xU}/P_{mU}} = \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}/A_{xi}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}/A_{xU}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}}\right)^{\frac{\nu_x-\nu_m}{\nu_m}} \tag{6}$$

$$\frac{y_{mi}}{y_{mU}} = \left(\frac{A_{ci}}{A_{cU}}\right)\left(\frac{A_{xi}}{A_{xU}}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\left(\frac{T_{mU}}{\pi_{UU}}\right)^{\frac{1}{\theta}}}\right)^{\frac{1-\nu_c+\frac{\alpha}{1-\alpha}(1-\nu_x)}{\nu_m}} \tag{7}$$

The three equations relate observables—the price of consumption relative to intermediates, the price of investment relative to intermediates, income per capita, and home trade shares—to the unknown productivity parameters. These derivations are in Appendix D. We set $A_{cU} = T_{mU} = A_{xU} = 1$ as a normalization, where the subscript U denotes the U.S. For each country i, system (5)–(7) yields three nonlinear equations with three unknowns: A_{ci}, T_{mi}, and A_{xi}. Information about constructing the empirical counterparts to $P_{ci}, P_{mi}, P_{xi}, \pi_{ii}$ and y_{mi} is available in Appendix E.

These equations are quite intuitive. The expression for income per capita provides a measure of aggregate productivity across all sectors: higher income per capita is associated with higher productivity levels, on average. The two expressions for relative prices tell us how to allocate the productivity across sectors.

The expressions for relative prices boil down to two components. The first term reflects something akin to the Balassa-Samuelson effect: All else equal, a higher price of capital relative to intermediates suggests a low productivity in capital goods relative to intermediate goods. In our setup, the productivity for the traded intermediate good is endogenous, reflecting the degree of specialization as captured by the home trade share. The second term reflects the extent to which the two goods utilize intermediates with different intensities. If measured productivity is relatively high in intermediates, then the price of intermediate input is relatively low and the sector that uses intermediates more intensively will, all else equal, have a lower relative price.

3.2 Model fit

Our model consists of 8832 country-specific parameters: $I(I-1) = 8556$ bilateral trade barriers, $(I-1) = 92$ consumption-good productivity terms, $(I-1) = 92$ investment-good productivity terms, and $(I-1) = 92$ intermediate-goods productivity terms.

Calibration of the country-specific parameters utilizes 8924 independent data points. The trade barriers use up $I(I-1) = 8556$ data points for bilateral trade shares and $(I-1) = 92$ for ratio of absolute prices of intermediates. The productivity parameters use up $(I-1) = 92$ data points for the price of consumption relative to intermediates, $(I-1) = 92$ for the price of investment relative to intermediates, and $(I-1) = 92$ for income per capita.

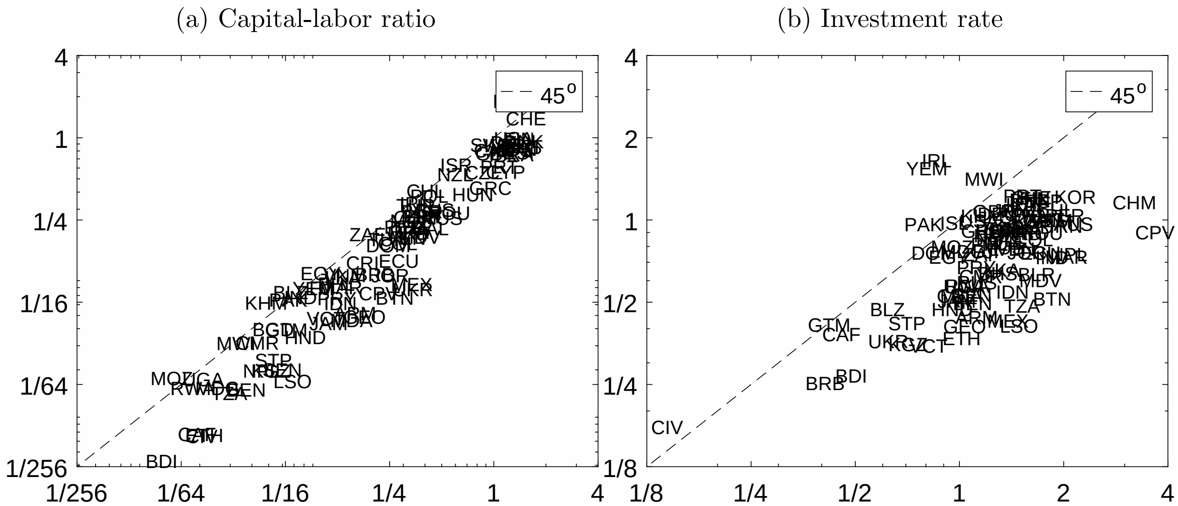
As such, there 92 more data points than parameters so our model does not perfectly replicate the data. Another way to interpret this is that there is one equilibrium condition for each country that we did not impose on our identification:

$$P_{mi} = \gamma \left[\sum_{j=1}^{I} (u_{mj} d_{ij})^{-\theta} T_{mj} \right]^{-\frac{1}{\theta}}$$

The model matches the targeted data well. The correlation between model and data is 0.96 for the bilateral trade shares, 0.97 for the absolute price of intermediates, 1.00 for income per capita, 0.96 for the price of consumption relative to intermediates, and 0.99 for the price of investment relative to intermediates.

Indeed, since we utilized relative prices of consumption and investment, not the absolute prices, matching the absolute prices is a test of the model. The correlation between model and data is 0.93 for the absolute price of consumption, and 0.97 for the absolute price of investment.

Figure 1: Model fit: The vertical axis represents the model and the horizontal axis represents the data



Implication for capital stock In our calibration we targeted income per capita. The burden is on the theory to disentangle what fraction of the cross-country income gap can be attributed to differences in capital and what fraction to differences in TFP.

Figure 1 shows that the model matches the data on capital-labor ratios across countries quite closely: the correlation is 0.93. It also shows that our model captures well the investment rate, $\frac{X_i}{y_i L_i}$, across countries in 2011. Note that we are imposing steady-state in 2011, which implies that the investment rate is proportional to the capital-output ratio. Since our model matches GDP by construction, and also does well explaining capital stocks, our ability to replicate the investment rate is limited to the extent that the steady-state assumption is valid in the data.

4 Counterfactuals

In this section we implement a counterfactual trade liberalization via a one time, permanent reduction in trade barriers. The world begins in the calibrated steady-state. At the beginning of period $t = 1$, trade barriers fall uniformly across all countries such that the ratio of world trade to GDP increases from 50 percent to 100 percent across steady-states. All other parameters are held fixed at their baseline values. This shock is unanticipated prior to time

$t = 1$. This amounts to reducing $d_{ij} - 1$ by 55 percent for each bilateral trade pair.

4.1 Welfare gains from trade

We compute the steady-state gains from trade using equation (2) and the dynamic gains from trade using equation (1). We find that the steady-state gains from trade vary substantially across countries, ranging from 18 percent for the U.S. to 92 percent for Belize. The median gain (Greece) is 53 percent.

Dynamic gains for the median country (Greece) are 32 percent. The differences are large across countries, ranging from 11 percent for the U.S., to 56 percent for Belize.[9]

The distribution of the dynamic gains from trade looks almost identical to the distribution of the steady-state gains. However, the dynamic gains are smaller in each country. The average ratio of dynamic gains to steady-state gains is 60.2 percent across countries, and varies from a minimum of 60.1 percent to a maximum of 60.5 percent.[10]

The proportionality of roughly 60 percent is a result of (i) the initial change in consumption and (ii) the rate at which consumption converges to the new steady-state. If consumption jumped to its new steady-state level on impact then this ratio would be close to 100 percent. If instead consumption declined significantly in the beginning, and then converged to the new steady-state after many years, then the ratio could be closer to 0 percent since there would be consumption losses in earlier periods while higher levels of future consumption would be discounted.

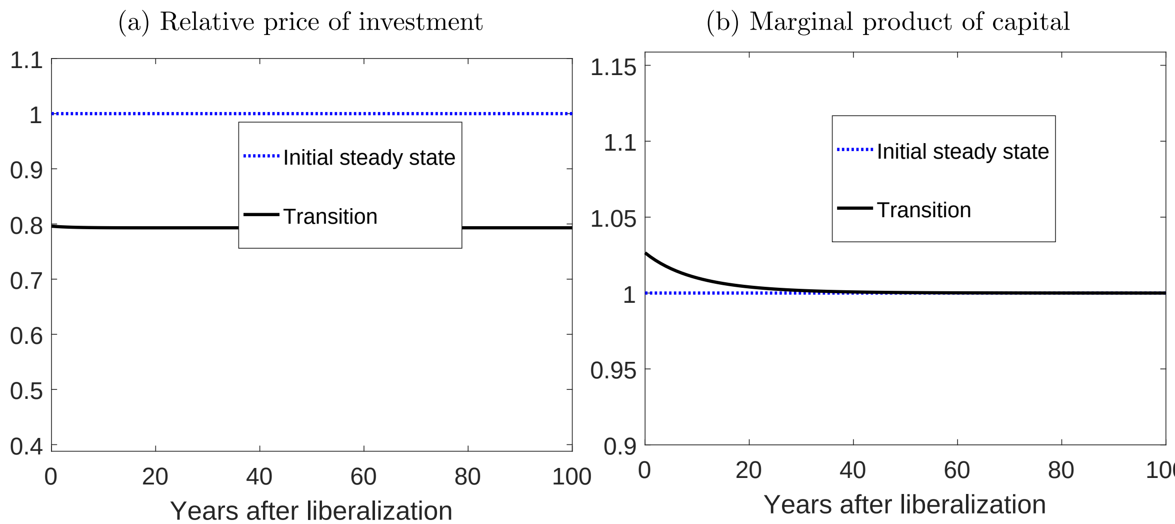
The Euler equation reveals the forces that influence consumption dynamics. Trade liberalization improves each country's terms of trade making more resources available for both consumption and investment. The allocation of output to consumption and investment is determined optimally by the household. The relative price of investment falls, meaning that the household can increase investment by a larger proportion than the increase in output without giving up consumption. In addition, the marginal product of capital (MPK),$\left(1 + \frac{r_{it+1}}{P_{xit+1}} - \delta\right)$, is higher than the steady-state MPK, $\frac{1}{\beta}$. While the MPK is high, households take advantage by investing relatively more. Figure 2 shows the transition paths for the relative price of investment and the MPK in the U.S. Resultantly, consumption falls

[9] The gains from trade are systematically smaller for large countries, rich countries, and countries with smaller average export barriers. All of these findings are consistent with the existing literature (Waugh and Ravikumar, 2016; Waugh, 2010).

[10] Desmet, Nagy, and Rossi-Hansberg (2015) consider, in a model of migration and trade, a counterfactual scenario that increases trade costs by 40% in the first period. They find that welfare decreases by around 34%.

on impact and investment jumps as shown in Figure 3. As capital accumulates the MPK returns to its original steady-state level and investment settles down to its new (higher) steady-state level.[11]

Figure 2: Transition paths for prices in the U.S.



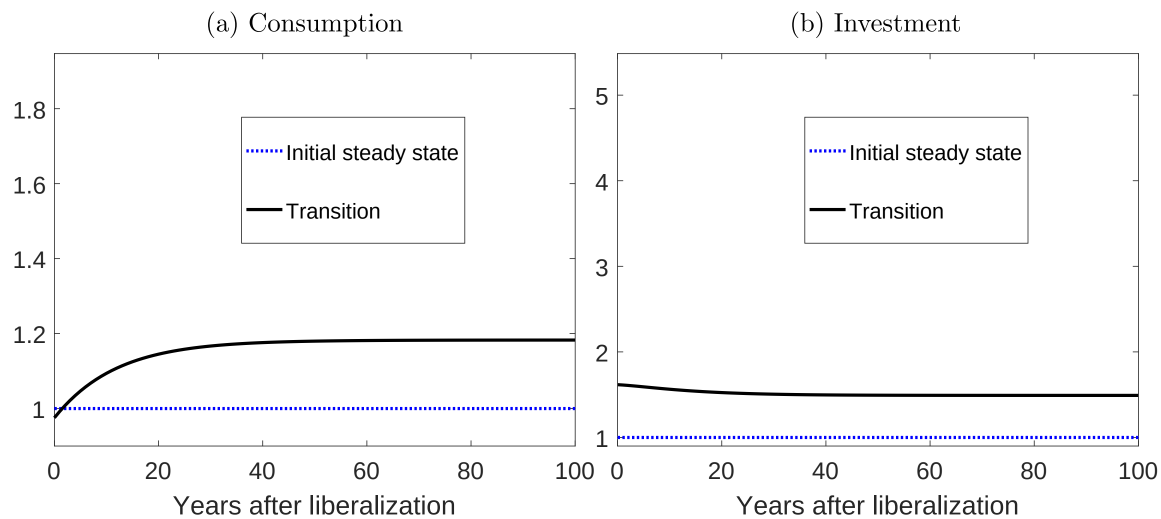
4.2 The mechanism

Some remarks are in order here regarding the importance of two features that distinguish our work from the literature: the endogenous relative price of capital and the endogenous investment rate. In our model, the share of income that the household allocates towards investment expenditures is determined endogenously. The nominal investment rate, $\frac{P_{xit}X_{it}}{L_i P_{cit} y_{it}}$ is not constant along the transition path. Combined with a decline the relative price of investment, the real investment rate, $\frac{X_{it}}{L_i y_{it}}$, increases substantially in response to trade liberalization. Indeed, the real investment rate is permanently higher.

Alternative models To quantify the importance of the endogenous investment rate and endogenous relative price of investment, we solve versions of the model where we explic-

[11]Two housekeeping remarks are in order here. First, in the figures we index each series to 1 in the initial steady-state. Second, the transition paths for every country exhibit similar characteristics to the U.S., but differ in their magnitudes: Belize is at one extreme and the U.S. is at the other extreme.

Figure 3: Transitions paths for final demand in the U.S.



itly impose that $P_x/P_c = 1$ and/or that the nominal investment rate is exogenous. To do this we change only a couple of equations.

First, to impose an exogenous nominal investment rate, we eliminate the Euler equation from the baseline model and impose $P_{xit}X_{it} = \rho(w_{it}L_{it} + r_{it}K_{it})$, with $\rho = \frac{\alpha\delta}{1/\beta-(1-\delta)} = 0.1948$. That is, the household allocates an exogenous share, ρ, of its income to investment expenditures. The value of ρ corresponds to the nominal investment rate that arises in the fully endogenous model in the steady-state (which is constant across countries and across steady-states).

We implement a similar trade liberalization in which barriers are uniformly reduced by 55 percent in every country. We report the numbers for Greece only, since it is the country that has the median gains from trade. All of the conclusions that we draw from Greece hold in every other country.

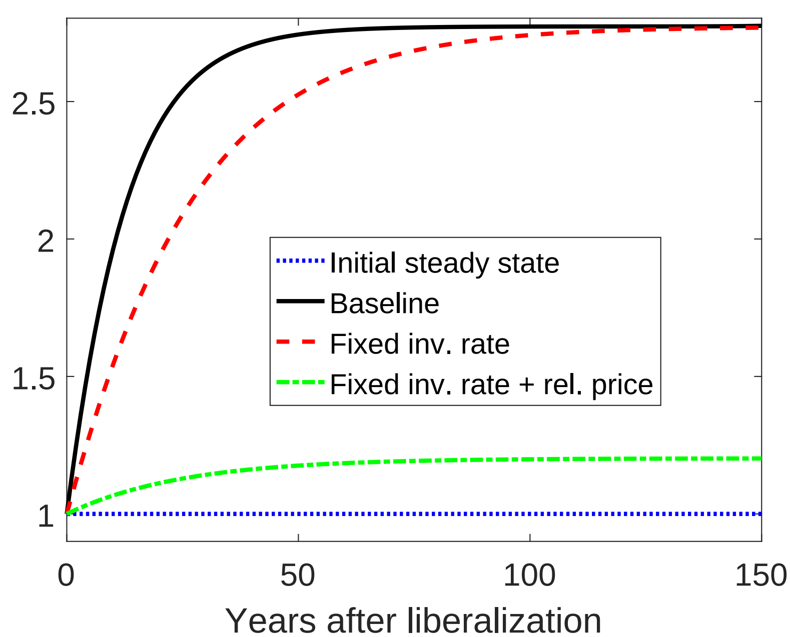
We find that the endogenous investment rate affects the speed of capital accumulation. For instance, Figure 4 shows that capital converges faster to the new steady-state in the model with an endogenous investment rate. Indeed Table 2 reports the half-life for capital accumulation: it is about twice as large in the model with an exogenous investment rate.

Second, in addition to an exogenous nominal investment rate, we fix the relative price of investment to one. To do this we restrict the technologies for consumption and investment goods to be the same. That is, we set $A_{xi} = A_{ci}$ and $\nu_x = \nu_c$. In the calibration we choose

A_x and A_c to match the price of GDP relative to intermediates, and choose $\nu_x = \nu_c = 0.88$ to satisfy the national account equation (3), with all other parameters recalibrated to match the same targets as in the benchmark calibration.

Again, we implement a similar trade liberalization in which barriers are uniformly reduced by 55 percent in every country and report the results for Greece. We find that an endogenous relative price governs the gap in capital between steady-states. For instance, Figure 4 shows that, with a fixed relative price of investment, the capital stock converges to a lower steady-state level. Indeed, having an endogenous relative price allows for higher steady-state capital-output ratio. With the relative price of investment fixed, the real investment rate, $\frac{X_i}{y_i L_i}$, cannot adjust across steady-states since $\rho = \frac{P_{xi} X_i}{P_{ci} y_i L_i} = \frac{\alpha\delta}{1/\beta-(1-\delta)}$ is constant. On the other hand, with an endogenous relative price, the real investment rate converges to a higher steady-state level since the opportunity cost of investing is lower, i.e., the amount of consumption goods the household gives up to acquire additional investment is lower.

Figure 4: Transitional dynamics for capital across alternative models



Notes: Relative price refers to $\frac{P_x}{P_c}$ and nominal investment rate refers to $\frac{P_x X}{P_c y L}$. We consider Greece since it is the country with the median gains from trade.

In sum, an endogenous investment rate allows the economy to transition to the steady-state faster, while an endogenous relative price allows the economy to attain higher steady-state capital stocks. These features have implications for the path of consumption along the transition, and hence, for the ratio of dynamic-to-steady-state gains from trade.

The ratio of dynamic-to-steady-state gains is a function of (i) the initial change in consumption and (ii) the rate of consumption growth, which depends on the half-life for capital. Table 2 shows that the half-life for capital does not depend critically on whether the relative price is fixed or not. For instance, in the model with an exogenous investment rate and fixed relative price the half-life is 18.2 years. In the model with an exogenous investment rate and endogenous relative price the half-life is 19.5 years. However, the initial change in consumption depends on whether the relative price is fixed or not. In particular, when the relative price is fixed, consumption increases by 13.1 percent on impact, whereas it increases by only 9.9 percent when the relative price is not fixed. As a result, the ratio of dynamic-to-steady-state gains is higher in the model with fixed relative price of investment.[12]

Conversely, in the model with endogenous relative price of investment, regardless of whether the investment rate is exogenous of endogenous, the ratio of dynamic-to-steady-state gains are similar. When the nominal investment rate is exogenous, the half-life is twice as large as in the model with endogenous investment rate, but the initial increase in consumption is higher. However, when the nominal investment rate is endogenous, the half-life is shorter, but consumption drops on impact. In other words, with endogenous investment rate, consumption is lower in the beginning of the transition, but converges to the new steady-state faster.

Consequently, the welfare gains from trade that accounts for the whole transition path of the economy in a model where both the investment rate and the relative price of capital are endogenous, are different from models that take one or both of them as exogenous.

Table 2: Outcomes in Greece from global 55% reduction in barriers

	Baseline	Fixed inv. rate	Fixed inv. + rel. price
Half life for capital	9.9 yrs	19.5 yrs	18.2 yrs
Initial change consumption	-5.1 %	9.9 %	13.2 %
Dynamic-to-SS gains	60.4 %	59.6	82.4 %

Notes: We consider Greece since it is the country with the median gains from trade.

[12]Both the dynamic and steady-state gains from trade liberalization are lower in the model with the fixed relative price, but the ratio of the two is higher than in the model with endogenous relative price.

4.3 Putting the dynamic gains in perspective

In this section we perform two exercises. First, we compare dynamic welfare gains from trade in a model with capital accumulation to the static gains that would be obtained in a model with no capital accumulation. Then we relate our methodology to a sufficient-statistics approach in which gains from trade are explained by changes in the home trade share.

Static versus dynamic welfare gains Here we compare our dynamic gains from trade to those that would be obtained in a model with no capital accumulation (i.e., Waugh, 2010). In a static model, the welfare gains from trade are driven entirely by changes in TFP. In Appendix D we show that the steady-state income per capita (which, recall is proportional to consumption per capita) can be expressed as

$$y_i \propto \underbrace{A_{ci}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1-\nu_c}{\theta\nu_m}}}_{\text{TFP contribution}}\underbrace{A_{xi}^{\frac{\alpha}{1-\alpha}}\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{\alpha(1-\nu_x)}{(1-\alpha)\theta\nu_m}}}_{\text{Capital contribution}} \tag{8}$$

Equation (8) allows us to tractably decompose the relative importance of changes in TFP and changes in capital in accounting for the gains. It implies that the log-change in income that corresponds to a log-change in the home trade share is:

$$\frac{\partial\ln(y_i)}{\partial\ln(\pi_{ii})} = -\left(\underbrace{\frac{1-\nu_c}{\theta\nu_m}}_{\text{through TFP}} + \underbrace{\frac{\alpha(1-\nu_x)}{(1-\alpha)\theta\nu_m}}_{\text{through capital}}\right) \tag{9}$$

Based on our calibration, the first term equals 0.08 while the second term equals 0.30. That is, given a change in trade barriers, 79 percent of the resulting change in income per capita across steady-states can be attributed to change in capital, and the remaining 21 percent to change in TFP.[13]

After a trade liberalization, TFP jumps immediately to its new steady-state level. Because the stock of capital does not change on impact, the initial change in TFP is not affected by whether or not there is capital accumulation. Therefore, the initial change in

[13]This number is constant across countries in our model since the elasticities $(\theta, \alpha, \nu_c, \nu_m, \nu_x)$ are all constant across countries. This does not imply that income per capita changes by equal proportions across countries, only that the relative contributions from TFP and capital are the same.

TFP corresponds to the static welfare gains in a model without capital, or one in which capital is exogenous. As a result, the static gains are 21 percent of the steady-state gains. We also know from our counterfactual exercise that dynamic gains are around 60 percent of the steady-state gains in a model with capital accumulation. Therefore, the dynamic gains are almost three times larger than static gains obtained by ignoring changes in capital.

A sufficient-statistics approach We compare, period by period, welfare gains from trade using the same formula as in Arkolakis, Costinot, and Rodríguez-Clare (2012) in our model with capital (augmented ACR), to those resulting from comparing consumption growth period by period. The first measure is a"sufficient statistics" calculation in that it depends only on changes in the home trade share and elasticity parameters (to see why, recall equation (8)).

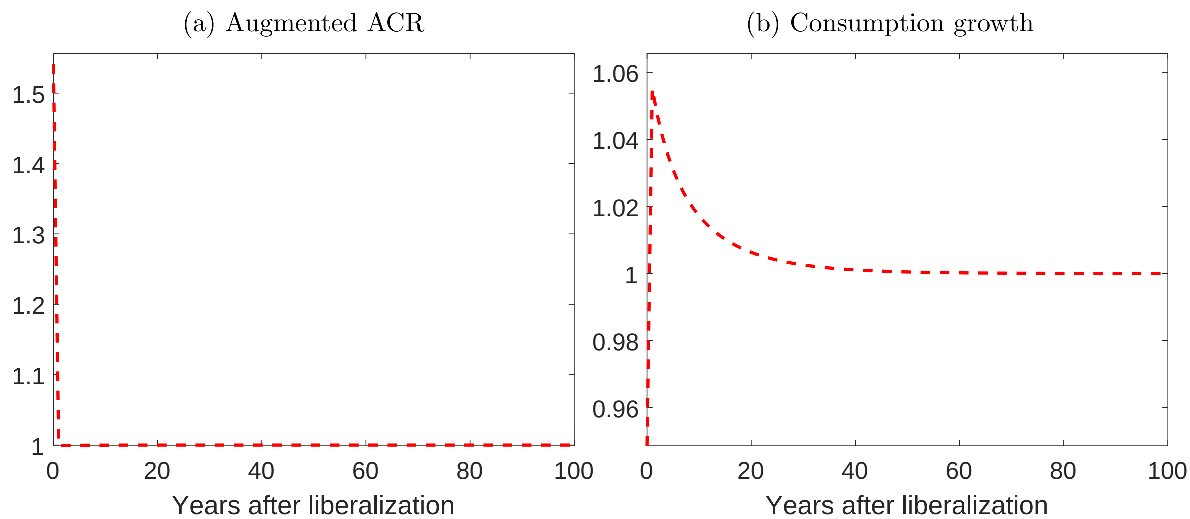
In the steady-state, all the change in income per capita resulting from changes in trade barriers are manifested in the home trade share as in ACR, augmented by the fact that capital is endogenous and it depends on trade barriers as in Anderson, Larch, and Yotov (2015) and Mutreja, Ravikumar, and Sposi (2014). The sufficient-statistics calculation is equivalent to comparing welfare in a series of static exercises.

The second measure captures the effect of capital accumulation on welfare gains from trade, and hence accounts for all of the transitional dynamics following a trade liberalization, which is not reflected in the transition path of the home trade share. Figure 5 plots both measures.

Feeding in the transition path for the home trade share, the augmented ACR formula would imply that all the gains from trade occur in the first period. The reason is that welfare gains occur through a decrease in the home trade share, which jumps upon impact and it reaches its new steady-state immediately. This is consistent with models that measure welfare gains from trade in a static context. If instead we take into account the transitional dynamics and compute consumption growth period by period, we observe that consumption drops upon impact. But, after the initial period, consumption growth is positive, and converges toward zero as the economy reaches the new steady-state.

As a final note, the sufficient-statistics formula is typically applied to assess the welfare costs of moving to autarky, since the home trade share in autarky is 1, and the current home trade share is observed in the data. In moving to free trade, even in a static model, one needs to solve for the home trade shares that arise under free trade. In our model there is no sufficient-statistic to compute the home trade share under free trade, even in steady-state,

Figure 5: Comparison of welfare measures along the transition



since it depends on the world distribution of capital.[14]

This exercise shows that sufficient-statistics approaches can yield a very different picture for the transitional dynamics of the welfare gains from trade, particularly when distinguishing between the short-run gains and the long-run gains.

5 Extended model

Our algorithm for solving the baseline model encompasses multi-country models with CRRA utility, linear depreciation of capital, and balanced trade. The main limitation is that households cannot borrow against the future because physical investment is the only means of intertemporal substitution, and investment must be non-negative. This restricts us to study transition paths in which investment is always positive. To get around the non-negativity constraint we introduce adjustment costs to capital to ensure that the household will optimally choose positive investment every period. To allow the household to borrow against the future we introduce one-period bonds that can be used to finance trade imbalances. The Appendix describes the algorithm to compute the equilibrium transitional dynamics in the extended model.

[14] In a one-sector model with no capital, Waugh and Ravikumar (2016) derive sufficient statistics to measure the gains from moving to free trade. It involves the elasticities, the current home trade share, and current level of output.

5.1 Adjustment costs and endogenous trade imbalances

Adjustment costs are added to the technology for physical capital accumulation as follows

$$K_{it+1} = (1-\delta)K_{it} + \chi X_{it}^{\mu} K_{it}^{1-\mu}$$

where χ denotes an adjustment cost, and μ denotes the rate at which investment goods are converted into future capital stock. We work with the inverse capital accumulation technology for convenience, which is given by

$$X_{it} = \Phi(K_{it+1}, K_{it}) = \left(\frac{1}{\chi}\right)^{\frac{1}{\mu}} \left(K_{it+1} - (1-\delta)K_{it}\right)^{\frac{1}{\mu}} K_{it}^{\frac{\mu-1}{\mu}}$$

The Euler equation for investment in physical capital becomes

$$C_{it+1} = \beta^{\sigma} \left(\frac{\frac{r_{it+1}}{P_{ixt+1}} - \Phi_2(K_{it+2}, K_{it+1})}{\Phi_1(K_{it+1}, K_{it})} \right)^{\sigma} \left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}} \right)^{\sigma} C_{it}$$

where $\Phi_1(\cdot,\cdot)$ and $\Phi_2(\cdot,\cdot)$ denote the first derivatives of the adjustment-cost function with respect to the first and second arguments, respectively:

$$\Phi_1(K_{it+1}, K_{it}) = \left(\frac{1}{\chi}\right)^{\frac{1}{\mu}} \left(\frac{1}{\mu}\right) \left(\frac{K_{it+1}}{K_{it}} - (1-\delta)\right)^{\frac{1-\mu}{\mu}}$$
$$\Phi_2(K_{it+1}, K_{it}) = \left(\frac{1}{\chi}\right)^{\frac{1}{\mu}} \left(\frac{1}{\mu}\right) \left(\frac{K_{it+1}}{K_{it}} - (1-\delta)\right)^{\frac{1-\mu}{\mu}} \left((\mu-1)\frac{K_{it+1}}{K_{it}} - \mu(1-\delta)\right)$$

The second extension we consider is adding endogenous trade imbalances, in addition to adjustment costs to capital accumulation. We allow each country to freely trade one-period bonds that yield a risk-free world interest rate q_t. More specifically, the household's budget constraint becomes

$$P_{cit}C_{it} + P_{xit}X_{it} + B_{it} = w_{it}L_{it} + r_{it}K_{it} + q_t\mathcal{A}_{it}$$

where B_{it} denotes net purchases of one-period bonds (the current account balance) and $\mathcal{A}_{it}$ denotes the net-foreign asset position. Net-foreign assets accumulate according to

$$\mathcal{A}_{it+1} = \mathcal{A}_{it} + B_{it}$$

This adds one more Euler equation to the household's optimization problem, namely

$$\frac{C_{it+1}}{C_{it}} = \beta^{\sigma}\left(\frac{1+q_{t+1}}{P_{cit+1}/P_{cit}}\right)^{\sigma}$$

To close the model, instead of imposing balanced trade period-by-period, we require that the current account equals net exports plus net-foreign income on assets:[15]

$$B_{it} = P_{mit}\left(Y_{it} - M_{it}\right) + q_t A_{it}$$

5.2 Quantitative exercise

We calibrate the adjustment cost parameter, $\chi = \delta^{1-\mu}$ so that there is no cost to mainiatin the level of capital stock in steady-state, that is, $X_i = \delta K_i$. We set the elasticity parameter $\mu = 0.55$ as in Eaton, Kortum, Neiman, and Romalis (2016).

To put the model to work, we assume the world is in steady-state in 2011 with balanced trade and a balanced current account. In addition, we assume that the initial net-foreign asset position is zero in every country: $\mathcal{A}_{i1} = 0$. We use the steady-state level of capital stock that arises from this calibration as the initial level for capital. These assumptions are made so that we can explore the dynamics along the transition path free from any differences in initial conditions and make direct comparisons to the results in a model with balanced trade in every period.

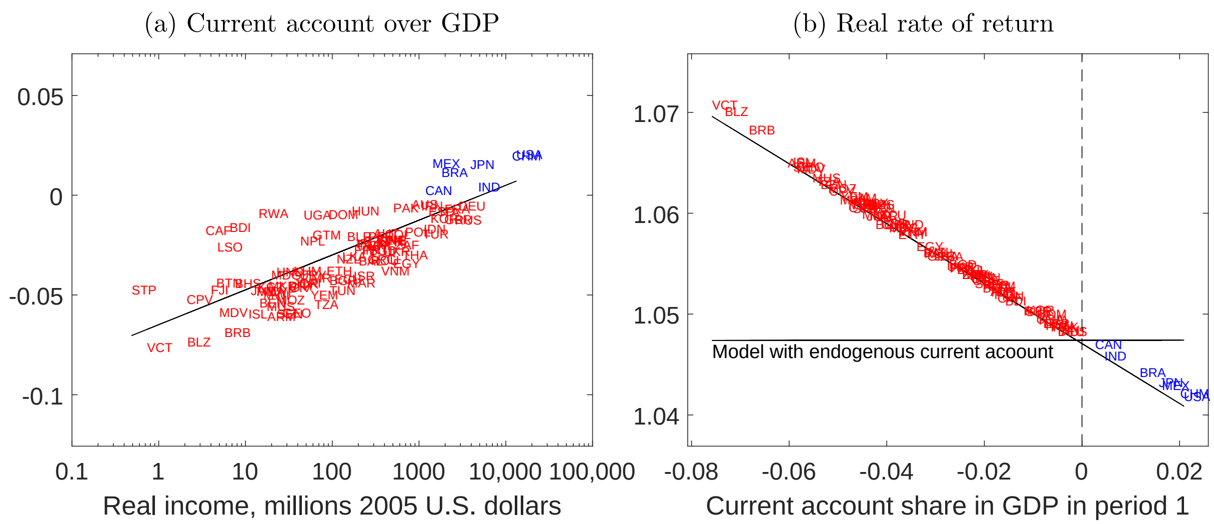
To implement our algorithm, we impose a value for the terminal net-foreign asset positions (NFAPs), $\mathcal{A}_{iT+1}$ for all i; without loss of generality, we set these to zero. We let the model run for 150 periods and discard the last 65. By period 85, the model has converged to a steady-state that is independent from the terminal NFAPs. This is an application of the Turnpike Theorem whereby, regardless of the terminal condition of the $\mathcal{A}_{iT+1}$, if T is large enough, then there is a time $t^{\star}$ at which the model is sufficiently close to the steady-state, i.e., on the Turnpike. This is approach is also used by Kehoe, Ruhl, and Steinberg (2016).[16]

[15] With abuse of notation this implies, equivalently, that $Y = C + I + NX$.

[16] In models with endogenous current accounts, it is known that the transition path and steady-state are determined jointly. The new steady-state is one in which current accounts are balanced in all countries but have potentially permanent trade imbalances. The steady-state depends on the transition path since a country may accumulate bonds early on, financed by a trade surplus, but later on collect income off of the assets and use it to finance a trade deficit. As such, its steady-state trade imbalance depends on what happened along the transition. So net-foreign asset positions need not be zero in steady-state, but they do need to be constant. After 85 periods, the net-foreign asset position begins to regress to its terminal position, i.e., the transition path exits the Turnpike.

Cross-sectional analysis We find that, after a trade liberalization, capital flows from large and more developed countries to small and less-developed countries. The reason is that when there is financial autarky (i.e., trade is balanced), the large countries have a lower real rate of return than the small countries. That is, under balanced trade, there are persistent differences in the real rate of return to capital (RRR) across countries along the transition path, ranging from 4 percent to 7 percent. With trade imbalances, capital flows from countries that have a lower marginal product of capital to countries that have a higher marginal product of capital. This implies that, on impact, small countries run current account deficits while large countries run current account surpluses, see Figure 6a. As a result, RRRs are equalized across countries to $1/\beta = 4.17$ percent (see Figure 6b). Countries that initially run a deficit have a lower RRR relative to the model with balanced trade; the opposite is true for countries in surplus.

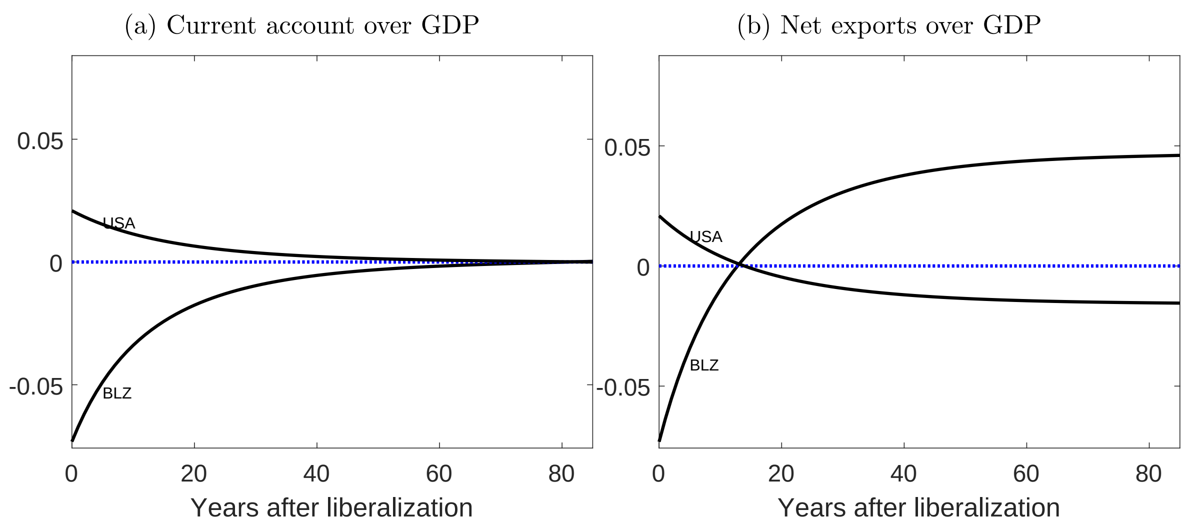
Figure 6: Current account imbalances and real rates of return across countries in period 1



As the world transitions towards the new steady-state, each country's current account converges towards zero. Along the transition, some countries accumulate positive NFAPs by running current account surpluses, while other accumulate negative NFAPs (liabilities) by running current account deficits. To illustrate this point, consider the case of the U.S and Belize. Following the trade liberalization, Figure 7a shows that the U.S. runs a current account surplus. On impact, the current account surplus exactly equals the U.S. trade surplus. In the ensuing periods, the trade surplus shrinks faster than the current account surplus, since the U.S. earns positive net-foreign income off of its existing NFAP. In a matter of 14 periods, the U.S. trade is balanced, meaning that net purchases of bonds (its current account balance) exactly offsets its net-foreign income. After period 14, the U.S. continues to run a current account surplus, although its net exports turn negative. In the new steady-state, its current account is balanced and the U.S. runs a permanent trade deficit that is 1.5 percent of GDP as shown in Figure 7b. The U.S. trade deficit is financed by net-foreign income that accrues off of its permanent and positive NFAP.

The current account dynamics in Belize are the mirror image of those in the U.S. whereas Belize converges to a steady-state with permanent net liabilities offset by a permanent a trade surplus.

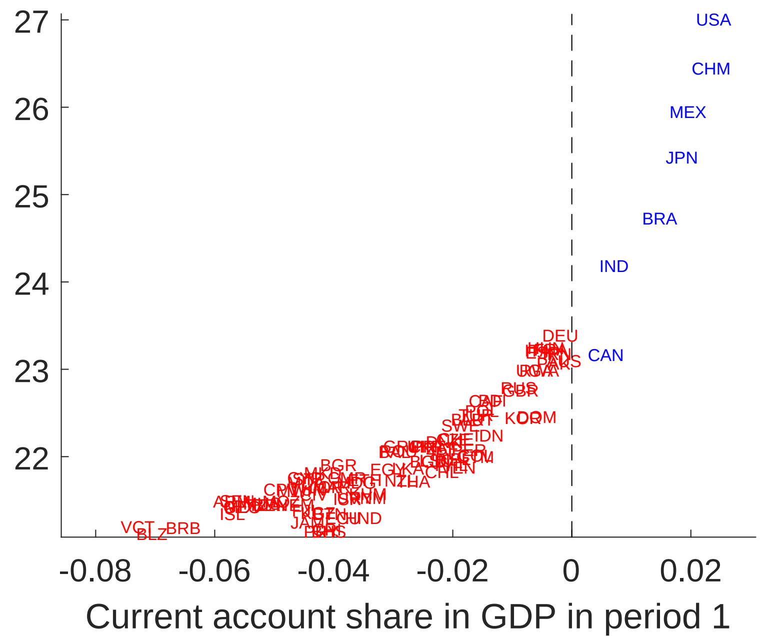
Figure 7: Transition for current account and net exports in the U.S. and in Belize



These results have implications for welfare since they impact the transition path for consumption and capital accumulation. With trade imbalances, countries running a trade deficit consume and invest more than they would if trade were balanced, and the opposite is true for countries running a surplus. As a result, countries like Belize are able to accumulate capital at a faster rate relative to the model with balanced trade, whereas the U.S. accumulates capital at a relatively slower rate. Indeed, Figure 8 shows that the half-life for capital accumulation is shorter for countries that initially run deficits compared to countries that initially run surpluses.

The rate of capital accumulation and the dynamics of trade imbalances govern the tran-

Figure 8: Half-life for capital accumulation

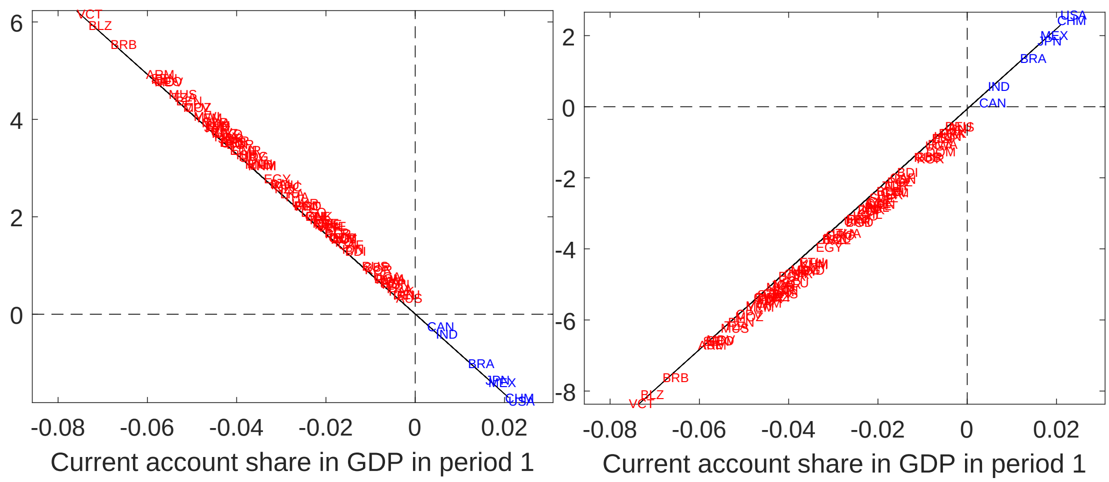


sition for consumption. We find that countries that run trade deficits on impact have higher initial consumption than they would in a model with balanced trade, while countries that run trade surpluses have lower initial consumption, see Figure 9a. The trade-off is that, countries that initially run deficits, like Belize, will eventually run trade surpluses. As a result, the steady-state level of consumption for Belize is lower than in a model with balanced trade. Conversely, countries that initially run surpluses, such as the U.S., will benefit by having higher levels of steady-state consumption than in a model with balanced trade. On net, welfare gains are slightly higher in every country, relative to the gains calculated in the model with balanced trade. However, countries that initially run a deficit gain proportionately more than countries that initially run a surplus.

Implications for sufficient-statistics approach Recall from section 4.3 that the baseline model with balanced trade admits a sufficient-statistics formula to compute the welfare gains from trade across steady-states, but not in the transition. In the model with endogenous trade imbalances, the sufficient-statistics formula no longer applies, even between steady-states. Specifically, equation (9) implies that the elasticity of the change in income with respect to the change in the home trade share is constant across countries in the baseline

Figure 9: Percent change in consumption relative to model with balanced trade

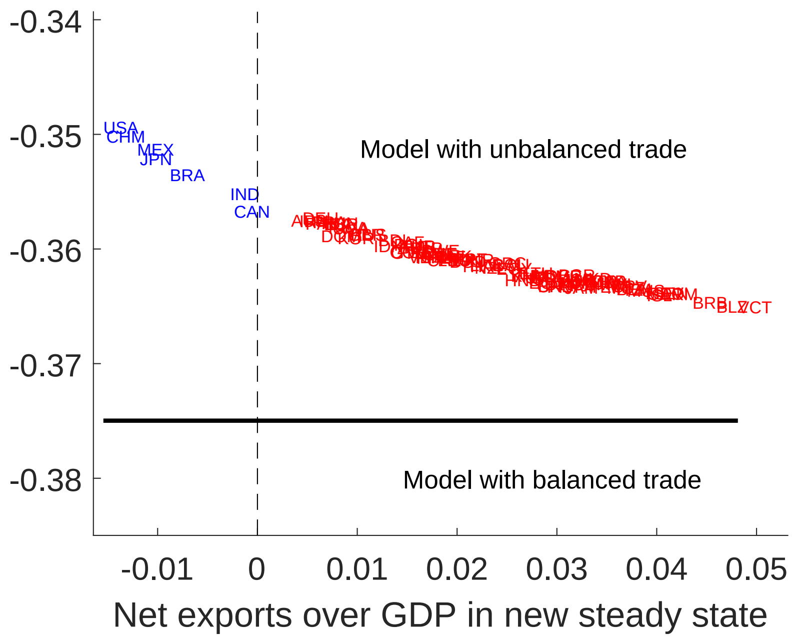
(a) Change in period-1 consumption (b) Change in steady-state consumption



model. Nonetheless, Figure 10 shows that this elasticity systematically varies across countries with the level of the trade imbalance in the extended model.

The real income in a country is affected by its steady-state trade imbalance, a phenomenon that is not fully summarized by the home trade share alone. Countries that run steady-state trade deficits, like the U.S., have lower (in absolute value) elasticities than those that run steady-state trade surpluses, like Belize. This implies that, given the same drop in the home trade share, the steady-state change in income per capita will be higher in Belize than in the U.S. Since the home trade share decreases more in Belize than in the U.S., the steady-state gains are proportionately larger in Belize than in the U.S., relative to a model with trade balance. Since the steady-state outcomes depend on the entire transition in the model with trade imbalances, it is important to take into account to entire transition path when evaluating the gains from trade, even if one is interested in the gains across steady-states only.

Figure 10: Elasticity of change in income with respect to change in home trade share



6 Conclusion

We build a multi-country trade model with capital accumulation to study the welfare gains from trade. The model features endogenous investment rate and endogenous relative price of investment. We then solve for the exact transitional dynamics of a trade liberalization in levels. Our counterfactual suggests that dynamic gains are 60 percent of the gains across steady-states, and three times larger than those implied by a static model with no capital accumulation. Furthermore, endogenous relative price of investment implies higher steady-state capital stocks whereas endogenous investment rate implies faster convergence towards the steady-state.

Our paper adds to the literature on measuring welfare gains from trade, and more generally, to the recent literature addressing dynamics in multi-country models of trade. Many of the static models are based on "sufficient statistics". We find large difference between changes in welfare in a model with endogenous capital accumulation and those measured by "sufficient statistics" along the transition, pointing to the importance on modeling dynamics explicitly when measuring the benefits of openness.

Finally, we extend our model by adding adjustment costs to capital accumulation and endogenous trade imbalances to allow for intertemporal borrowing. This version of the model implies that after a trade liberalization, small countries run trade deficits in the short run and accumulate capital faster than large countries. The small countries, however, run trade surpluses in the long run, but have proportionately larger dynamic welfare gains relative to a model with balanced trade.

Our paper emphasizes the importance of analyzing the entire transitional dynamics of the economies after a trade liberalization to measure welfare gains from trade. These results have implications for the effect of trade policy on economic growth, which we leave for future work.

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A Equilibrium conditions in the baseline model

We describe each equilibrium condition in detail below.

A.0.1 Household optimization

The representative household chooses a path for consumption that satisfies the following Euler equation

$$C_{it+1} = \beta^{\sigma}\left(1 + \frac{r_{it+1}}{P_{xit+1}} - \delta\right)^{\sigma}\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^{\sigma} C_{it} \tag{A.1}$$

where $\Phi_1(\cdot,\cdot)$ and $\Phi_2(\cdot,\cdot)$ denote the first derivatives of the adjustment-cost function with respect to the first and second arguments, respectively.

Combining the representative household's budget constraint together with capital accumulation technology and rearranging, implies the following

$$C_{it} = \left(1 + \frac{r_{it+1}}{P_{xit}+1} - \delta\right)\left(\frac{P_{xit}}{P_{cit}}\right) K_{it} + \left(\frac{w_{it}}{P_{cit}}\right) L_i - \left(\frac{P_{xit}}{P_{cit}}\right) K_{it+1} \tag{A.2}$$

A.1 Firm optimization

Markets are perfectly competitive, so firms set prices equal to marginal costs. Denote the price of variety v, produced in country j and purchased by country i, as $p_{mij}(v)$. Then $p_{mij}(v) = p_{mjj}(v)d_{ij}$, where $p_{mjj}(v)$ is the marginal cost of producing variety v in country

j. Since country i purchases each variety from the country that can deliver it at the lowest price, the price in country i is $p_{mi}(v) = \min_{j=1,\ldots,I}[p_{mjj}(v)d_{mij}]$. The price of the composite intermediate good in country i at time t is then

$$P_{mit} = \gamma \left[\sum_{j=1}^{I} (u_{jt} d_{ij})^{-\theta} T_{mj} \right]^{-\frac{1}{\theta}} \tag{A.3}$$

where $u_{jt} = \left(\frac{r_{jt}}{\alpha\nu_m}\right)^{\alpha\nu_m} \left(\frac{w_{jt}}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m} \left(\frac{P_{jt}}{1-\nu_m}\right)^{1-\nu_m}$ is the unit cost for a bundle of inputs for intermediate-goods producers in country n at time t.

Next we define total factor usage in the intermediates sector by aggregating up across the individual varieties.

$$K_{mit} = \int_0^1 K_{mit}(v)dv, \qquad L_{mit} = \int_0^1 L_{mit}(v)dv,$$
$$M_{mit} = \int_0^1 M_{mit}(v)dv, \qquad Y_{mit} = \int_0^1 Y_{mit}(v)dv$$

The term $L_{mit}(v)$ denotes the quantity of labor employed in the production of variety v at time t. If country i imports variety v at time t, then $L_{mit}(v) = 0$. Hence, L_{mit} is the total quantity of labor employed in sector m in country i at time t. Similarly, K_{mit} is the total quantity of capital used, M_{mit} is the total quantity of intermediates used as an input, and Y_{mit} is the total quantity of output of intermediate goods.

Cost minimization by firms implies that, within each sector $b \in \{c, m, x\}$, factor expenses exhaust the value of output.

$$\begin{aligned} r_{it}K_{bit} &= \alpha\nu_b P_{bit} Y_{bit}, \\ w_{it}L_{bit} &= (1-\alpha)\nu_b P_{bit} Y_{bit}, \\ P_{mit}M_{bit} &= (1-\nu_b) P_{bit} Y_{bit} \end{aligned}$$

That is, the fraction $\alpha\nu_b$ of the value of each sector's production compensates capital services, the fraction $(1-\alpha)\nu_b$ compensates labor services, and the fraction $1-\nu_b$ covers the cost of intermediate inputs; there are zero profits.

A.1.1 Trade flows

The fraction of country i's expenditures allocated to intermediate varieties produced by country j is given by

$$\pi_{ijt} = \frac{(u_{mjt}d_{ijt})^{-\theta}T_{mj}}{\sum_{j=1}^{I}(u_{mjt}d_{ij})^{-\theta}T_{mj}} \tag{A.4}$$

where u_{mjt} is the unit costs of a bundle of factors faced by producers of intermediate varieties in country j.

A.1.2 Market clearing conditions

We begin by describing the domestic factor market clearing conditions.

$$\sum_{b\in\{c,m,x\}} K_{bit} = K_{it}, \qquad \sum_{b\in\{c,m,x\}} L_{bit} = L_i, \qquad \sum_{b\in\{c,m,x\}} M_{bit} = Q_{it}$$

The first two conditions impose that the capital and labor market clear in country i at each time t. The third condition requires that the use of composite intermediate good equal its supply. It's use consists of intermediate demand by firms in each sector. Its supply is the quantity of the composite good which consists of both domestically- and foreign-produced varieties.

The next conditions require that goods markets clear.

$$C_{it} = Y_{cit}, \qquad X_{it} = Y_{xit}, \qquad \sum_{j=1}^{I} P_{mjt}\left(M_{cjt} + M_{mjt} + M_{xjt}\right)\pi_{jit} = P_{mit}Y_{mit}$$

The first condition states that the quantity of consumption demanded by the representative household in country i must equal the quantity produced by country i. The second condition says the same for the investment good. The third condition imposes that the value of intermediates produced by country i has to be absorbed globally. Recall that $P_{mjt}M_{bjt}$ is the value of intermediate inputs that country i uses in production in sector b. The term π_{jit} is the fraction of country j intermediate-good expenditures sourced from country i. Therefore, $P_{mjt}M_{bjt}\pi_{jit}$ denotes the total value of trade flows from country i to country j.

Finally, we impose an aggregate resource constraint in each country: net exports equal zero. Equivalently, gross output equals gross absorption.

$$P_{mit}Y_{mit} = P_{mit}Q_{it}$$

The left-hand side denotes the gross output of intermediates in country i and the right-hand side denotes total expenditures on intermediates.

B Solution algorithm for the baseline model

In this section of the Appendix we describe the algorithm for computing 1) the steady-state and 2) the transition path. Before going further into the algorithms, we introduce some notation. We denote the steady-state objects using the $\star$ as a superscript, i.e., $K_i^\star$ is the steady-state stock of capital in country i. We denote the cross-country vector of capital at a point in time using vector notation; $\vec{K}_t = \{K_{it}\}_{i=1}^I$ is the vector of capital stocks across countries at time t.

B.1 Computing the steady-state equilibrium in the baseline model

The steady-state equilibrium consists of 23 objects: $\vec{w}^\star$, $\vec{r}^\star$, $\vec{P}_c^\star$, $\vec{P}_m^\star$, $\vec{P}_x^\star$, $\vec{C}^\star$, $\vec{X}^\star$, $\vec{K}^\star$, $\vec{Q}^\star$, $\vec{Y}_c^\star$, $\vec{Y}_m^\star$, $\vec{Y}_x^\star$, $\vec{K}_c^\star$, $\vec{K}_m^\star$, $\vec{K}_x^\star$, $\vec{L}_c^\star$, $\vec{L}_m^\star$, $\vec{L}_x^\star$, $\vec{M}_c^\star$, $\vec{M}_m^\star$, $\vec{M}_x^\star$, $\vec{\vec{\pi}}^\star$. Table B.1 provides a list of equilibrium conditions that these objects must satisfy.

We use the technique from Mutreja, Ravikumar, and Sposi (2014), which builds on Alvarez and Lucas (2007b), to solve for the steady-state. The idea is to guess at a vector of wages, then recover all remaining prices and quantities using optimality conditions and market clearing conditions, excluding the trade balance condition. We then use departures from the the trade balance condition in each country to update our wage vector and iterate until we find a wage vector that satisfies the trade balance condition. The following steps outline our procedure in more detail.

1. We guess a vector of wages $\vec{w} \in \Delta = \{w \in \mathbb{R}_+^I : \sum_{i=1}^I \frac{w_i L_i}{1-\alpha} = 1\}$; that is, with world GDP as the numéraire.

2. We compute prices $\vec{P}_c, \vec{P}_x, \vec{P}_m$, and $\vec{r}$ simultaneously using conditions 16, 17, 18, and 23 in Table B.1. To complete this step, we compute the bilateral trade shares $\vec{\vec{\pi}}$ using condition 19.

3. We compute the aggregate capital stock as $K_i = \frac{\alpha}{1-\alpha}\frac{w_i L_i}{r_i}$, for all i, which derives easily from optimality conditions 1 & 4, 2 & 5, and 3 & 6, coupled with market clearing conditions for capital and labor 10 &11 in Table B.1.

Table B.1: Equilibrium conditions in steady-state

1	$r_i^\star K_{ci}^\star = \alpha\nu_c P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
2	$r_i^\star K_{mi}^\star = \alpha\nu_m P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
3	$r_i^\star K_{xi}^\star = \alpha\nu_x P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
4	$w_i^\star L_{ci}^\star = (1-\alpha)\nu_c P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
5	$w_i^\star L_{mi}^\star = (1-\alpha)\nu_m P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
6	$w_i^\star L_{xi}^\star = (1-\alpha)\nu_x P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
7	$P_{mi}^\star M_{ci}^\star = (1-\nu_c) P_{ci}^\star Y_{ci}^\star$	$\forall(i)$
8	$P_{mi}^\star M_{mi}^\star = (1-\nu_m) P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
9	$P_{mi}^\star M_{xi}^\star = (1-\nu_x) P_{xi}^\star Y_{xi}^\star$	$\forall(i)$
10	$K_{ci}^\star + K_{mi}^\star + K_{xi}^\star = K_i^\star$	$\forall(i)$
11	$L_{ci}^\star + L_{mi}^\star + L_{xi}^\star = L_i$	$\forall(i)$
12	$M_{ci}^\star + M_{mi}^\star + M_{xi}^\star = Q_i^\star$	$\forall(i)$
13	$C_i^\star = Y_{ci}^\star$	$\forall(i)$
14	$\sum_{j=1}^{I} P_{mj}^\star \left(M_{cj}^\star + M_{mj}^\star + M_{xj}^\star\right) \pi_{ji} = P_{mi}^\star Y_{mi}^\star$	$\forall(i)$
15	$X_i^\star = Y_{xi}^\star$	$\forall(i)$
16	$P_{ci}^\star = \left(\frac{1}{A_{ci}}\right)\left(\frac{r_i^\star}{\alpha\nu_c}\right)^{\alpha\nu_c}\left(\frac{w_i^\star}{(1-\alpha)\nu_c}\right)^{(1-\alpha)\nu_c}\left(\frac{P_{mi}^\star}{1-\nu_c}\right)^{1-\nu_c}$	$\forall(i)$
17	$P_{mi}^\star = \gamma\left[\sum_{j=1}^{I}(u_{mj}^\star d_{ij})^{-\theta} T_{mj}\right]^{-\frac{1}{\theta}}$	$\forall(i)$
18	$P_{xi}^\star = \left(\frac{1}{A_{xi}}\right)\left(\frac{r_i^\star}{\alpha\nu_x}\right)^{\alpha\nu_x}\left(\frac{w_i^\star}{(1-\alpha)\nu_x}\right)^{(1-\alpha)\nu_x}\left(\frac{P_{mi}^\star}{1-\nu_x}\right)^{1-\nu_x}$	$\forall(i)$
19	$\pi_{ij}^\star = \frac{(u_{mj}^\star d_{ij})^{-\theta} T_{mj}}{\sum_{j=1}^{I}(u_{mj}^\star d_{ij})^{-\theta} T_{mj}}$	$\forall(i,j)$
20	$P_{mi}^\star Y_{mi}^\star = P_{mi}^\star Q_i^\star$	$\forall(i)$
21	$P_{ci}^\star C_i^\star + P_{xi}^\star X_i^\star = r_i^\star K_i^\star + w_i^\star L_i^\star$	$\forall(i)$
22	$X_i^\star = \delta K_i^\star$	$\forall(i)$
23	$r_i^\star = \left(\frac{1}{\beta} - (1-\delta)\right) P_{xi}^\star$	$\forall(i)$

Note: $u_{mj}^\star = \left(\frac{r_j^\star}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_j^\star}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{mj}^\star}{1-\nu_m}\right)^{1-\nu_m}$.

4. We use condition 22 to solve for steady-state investment $\vec{X}$. Then we use condition 21 to solve for steady-state consumption $\vec{C}$.

5. We combine conditions 4 & 13 to solve for $\vec{L}_c$, combine conditions 5 & 14 to solve for $\vec{L}_x$, and use condition 11 to solve for $\vec{L}_m$. Next we combine conditions 1 & 4 to solve for $\vec{K}_c$, combine conditions 2 & 5 to solve for $\vec{K}_M$, and combine conditions 3 & 6 to solve for $\vec{K}_x$. Similarly, we combine conditions 4 & 7 to solve for $\vec{M}_c$, combine conditions 5 & 8 to solve for $\vec{M}_m$, and combine conditions 6 & 9 to solve for $\vec{M}_x$.

6. We compute $\vec{Y}_c$ using condition 13, compute $\vec{Y}_m$ using condition 14, and compute $\vec{Y}_x$ using condition 15.

7. We compute an excess demand equation as in Alvarez and Lucas (2007b) defined as

$$Z_i(\vec{w}) = \frac{P_{mi}Y_{mi} - P_{mi}Q_i}{w_i}$$

(the trade deficit relative to the wage). Condition 20 requires that $Z_i(\vec{w}) = 0$ for all i. If the excess demand is sufficiently close to zero then we have an equilibrium. If not, we update our guess at the equilibrium wage vector using the information in the excess demand as follows.

$$\Lambda_i(\vec{w}) = w_i \left(1 + \psi \frac{Z_i(\vec{w})}{L_i}\right)$$

is be the updated guess to the wage vector, where ψ is chosen to be sufficiently small so that $\Lambda > 0$. Note that $\sum_{i=1}^{I} \frac{\Lambda_i(\vec{w})L_i}{1-\alpha} = \sum_{i=1}^{I} \frac{w_i L_i}{1-\alpha} + \psi \sum_{i=1}^{I} w_i Z_i(\vec{w})$. As in Alvarez and Lucas (2007b), it is easy to show that $\sum_{i=1}^{I} w_i Z_i(\vec{w}) = 0$ which implies that $\sum_{i=1}^{I} \frac{\Lambda_i(\vec{w})L_i}{1-\alpha} = 1$, and hence, $\Lambda : \Delta \to \Delta$. We return to step 2 with our updated wage vector and repeat the steps. We iterate through this procedure until the excess demand is sufficiently close to zero. In our computations we find that our preferred convergence metric:

$$\max_{i=1}^{I} \{|Z_i(\vec{w})|\}$$

converges roughly monotonically towards zero.

Table B.2: Equilibrium conditions along the transition

1	$r_{it}K_{cit} = \alpha\nu_c P_{cit}Y_{cit}$	$\forall(i,t)$
2	$r_{it}K_{mit} = \alpha\nu_m P_{mit}Y_{mit}$	$\forall(i,t)$
3	$r_{it}K_{xit} = \alpha\nu_x P_{xit}Y_{xit}$	$\forall(i,t)$
4	$w_{it}L_{cit} = (1-\alpha)\nu_c P_{cit}Y_{cit}$	$\forall(i,t)$
5	$w_{it}L_{mit} = (1-\alpha)\nu_m P_{mit}Y_{mit}$	$\forall(i,t)$
6	$w_{it}L_{xit} = (1-\alpha)\nu_x P_{xit}Y_{xit}$	$\forall(i,t)$
7	$P_{mit}M_{cit} = (1-\nu_c)P_{cit}Y_{cit}$	$\forall(i,t)$
8	$P_{mit}M_{mit} = (1-\nu_m)P_{mit}Y_{mit}$	$\forall(i,t)$
9	$P_{mit}M_{xit} = (1-\nu_x)P_{xit}Y_{xit}$	$\forall(i,t)$
10	$K_{cit} + K_{mit} + K_{xit} = K_{it}$	$\forall(i,t)$
11	$L_{cit} + L_{mit} + L_{xit} = L_i$	$\forall(i,t)$
12	$M_{cit} + M_{mit} + M_{xit} = Q_{it}$	$\forall(i,t)$
13	$C_{it} = Y_{cit}$	$\forall(i,t)$
14	$\sum_{j=1}^{I} P_{mjt}\left(M_{cjt} + M_{mjt} + M_{xjt}\right)\pi_{jit} = P_{mit}Y_{mit}$	$\forall(i,t)$
15	$X_{it} = Y_{xit}$	$\forall(i,t)$
16	$P_{cit} = \left(\frac{1}{A_{ci}}\right)\left(\frac{r_{it}}{\alpha\nu_c}\right)^{\alpha\nu_c}\left(\frac{w_{it}}{(1-\alpha)\nu_c}\right)^{(1-\alpha)\nu_c}\left(\frac{P_{mit}}{1-\nu_c}\right)^{1-\nu_c}$	$\forall(i,t)$
17	$P_{mit} = \gamma\left[\sum_{j=1}^{I}(u_{mjt}d_{ij})^{-\theta}T_{mj}\right]^{-\frac{1}{\theta}}$	$\forall(i,t)$
18	$P_{xit} = \left(\frac{1}{A_{xi}}\right)\left(\frac{r_{it}}{\alpha\nu_x}\right)^{\alpha\nu_x}\left(\frac{w_{it}}{(1-\alpha)\nu_x}\right)^{(1-\alpha)\nu_x}\left(\frac{P_{mit}}{1-\nu_x}\right)^{1-\nu_x}$	$\forall(i,t)$
19	$\pi_{ijt} = \frac{(u_{mjt}d_{ij})^{-\theta}T_{mj}}{\sum_{j=1}^{I}(u_{mjt}d_{ij})^{-\theta}T_{mj}}$	$\forall(i,j,t)$
20	$P_{mit}Y_{mit} = P_{mit}Q_{it}$	$\forall(i,t)$
21	$P_{cit}C_{it} + P_{xit}X_{it} = r_{it}K_{it} + w_{it}L_i$	$\forall(i,t)$
22	$K_{it+1} = (1-\delta)K_{it} + X_{it}$	$\forall(i,t)$
23	$\left(\frac{C_{it+1}}{C_{it}}\right) = \beta^{\sigma}\left(1 + \frac{r_{it+1}}{P_{xit+1}} - \delta\right)^{\sigma}\left(\frac{P_{xt+1}/P_{ct+1}}{P_{xt}/P_{ct}}\right)^{\sigma}$	$\forall(i,t)$

Note: $u_{mjt} = \left(\frac{r_{jt}}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_{jt}}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{mjt}}{1-\nu_m}\right)^{1-\nu_m}$.

B.2 Computing the equilibrium transition path in the baseline model

The equilibrium transition path consists of 23 objects: $\{\vec{w}_t\}_{t=1}^{\infty}$, $\{\vec{r}_t\}_{t=1}^{\infty}$, $\{\vec{P}_{ct}\}_{t=1}^{\infty}$, $\{\vec{P}_{mt}\}_{t=1}^{\infty}$, $\{\vec{P}_{xt}\}_{t=1}^{\infty}$, $\{\vec{C}_t\}_{t=1}^{\infty}$, $\{\vec{X}_t\}_{t=1}^{\infty}$, $\{\vec{K}_t\}_{t=1}^{\infty}$, $\{\vec{Q}_t\}_{t=1}^{\infty}$, $\{\vec{Y}_{ct}\}_{t=1}^{\infty}$, $\{\vec{Y}_{mt}\}_{t=1}^{\infty}$, $\{\vec{Y}_{xt}\}_{t=1}^{\infty}$, $\{\vec{K}_{ct}\}_{t=1}^{\infty}$, $\{\vec{K}_{mt}\}_{t=1}^{\infty}$, $\{\vec{K}_{xt}\}_{t=1}^{\infty}$, $\{\vec{L}_{ct}\}_{t=1}^{\infty}$, $\{\vec{L}_{mt}\}_{t=1}^{\infty}$, $\{\vec{L}_{xt}\}_{t=1}^{\infty}$, $\{\vec{M}_{ct}\}_{t=1}^{\infty}$, $\{\vec{M}_{mt}\}_{t=1}^{\infty}$, $\{\vec{M}_{xt}\}_{t=1}^{\infty}$, $\{\vec{\vec{\pi}}_t\}_{t=1}^{\infty}$ (we use the double-arrow notation on $\vec{\vec{\pi}}_t$ to indicate that this is an $I \times I$ matrix in each period t). Table B.2 provides a list of equilibrium conditions that these objects must satisfy.

We reduce the infinite-dimensionality down to a finite-time problem from $t = 1, \ldots, T$, with T sufficiently large to ensure that the endogenous variables settle down to a steady-state by T. As such, solving the transition first requires solving the terminal steady-state. Also, it requires taking an initial stock of capital as given (either by computing an initial steady-state or just taking it from data, for instance).

Our solution procedure mimics the idea of that for the steady-state, but slightly modified to take into account the dynamic aspect as in Sposi (2012). Basically, we start with an initial guess for the entire sequence of wage vectors and rental rates (across countries and over time). Form these two objects we can recover all prices and quantities, across countries and throughout time, using optimality conditions and market clearing conditions, excluding the trade balance condition and the market clearing condition for the stock of capital. We then use departures from the the trade balance condition and the market clearing condition for the stock of capital at each point in time and in each country to update our wages and rental rates. Then we iterate until we find wages and rental rates that satisfy the trade balance condition and the market clearing condition for the stock of capital. We describe the steps to our procedure in more detail below.

1. We guess the entire path for wages $\{\vec{w}_t\}_{t=1}^{T}$ and rental rates $\{\vec{r}_t\}_{t=2}^{T}$ across countries, such that $\sum_i \frac{w_{it}L_i}{1-\alpha} = 1\,(\forall t)$. In period 1 set $\vec{r}_1 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\vec{w}_1\vec{L}}{\vec{K}_1}\right)$ since the initial stock of capital is predetermined.

2. We compute prices $\{\vec{P}_{ct}\}_{t=1}^{T}, \{\vec{P}_{xt}\}_{t=1}^{T}$, and $\{\vec{P}_{mt}\}_{t=1}^{T}$ simultaneously using conditions 16, 17, and 18, in Table B.2. To complete this step, we compute the bilateral trade shares $\{\vec{\vec{\pi}}_t\}_{t=1}^{T}$ using condition 19.

3. This step is slightly more involved. We show how we compute the path for consumption and investment by solving the intertemporal problem of the household. We do this in three parts. First we derive the lifetime budget constraint, second we derive the fraction of lifetime wealth allocated to consumption at each period t, and third we recover the sequence for investment and the stock of capital.

   Deriving the lifetime budget constraint To begin, we compute the lifetime budget constraint for the representative household in country i. Begin with the period budget constraint from condition 21 and combine it with the capital accumulation technology in condition 22 to get

$$K_{it+1} = \left(\frac{w_{it}}{P_{xit}}\right) L_i + \left(1 + \frac{r_{it}}{P_{xit}} - \delta\right) K_{it} - \left(\frac{P_{cit}}{P_{xit}}\right) C_{it}.$$

We will iterate the period budget constraint forward through time and derive a lifetime budget constraint. At time $t = 1$ the stock of capital, $K_{i1} > 0$, is given. Next, compute the stock of capital at time $t = 2$.

$$K_{i2} = \left(\frac{w_{i1}}{P_{xi1}}\right) L_i + \left(1 + \frac{r_{i1}}{P_{xi1}} - \delta\right) K_{i1} - \left(\frac{P_{ci1}}{P_{xi1}}\right) C_{i1}$$

Similarly, compute the stock of capital at time $t = 3$, but do it so that it is in terms the initial stock of capital.

$$\begin{aligned}
K_{i3} &= \left(\frac{w_{i2}}{P_{xi2}}\right) L_i + \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) K_{i2} - \left(\frac{P_{ci2}}{P_{xi2}}\right) C_{i2} \\
\Rightarrow K_{i3} &= \left(\frac{w_{i2}}{P_{xi2}}\right) L_{i2} + \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(\frac{w_{i1}}{P_{xi1}}\right) L_i \\
&+ \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(1 + \frac{r_{i1}}{P_{xi1}} - \delta\right) K_{i1} \\
&- \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(\frac{P_{ci1}}{P_{xi1}}\right) C_{i1} - \left(\frac{P_{ci2}}{P_{xi2}}\right) C_{i2}
\end{aligned}$$

Continue to period 4 in a similar way

$$\begin{aligned}
K_{i4} &= \left(\frac{w_{i3}}{P_{xi3}}\right) L_i + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) K_{i3} - \left(\frac{P_{ci3}}{P_{xi3}}\right) C_{i3} \\
\Rightarrow K_{i4} &= \left(\frac{w_{i3}}{P_{xi3}}\right) L_i + \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(\frac{w_{i2}}{P_{xi2}}\right) L_i \\
&+ \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(\frac{w_{i1}}{P_{xi1}}\right) L_i \\
&+ \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(1 + \frac{r_{i1}}{P_{xi1}} - \delta\right) K_{i1} \\
&- \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(1 + \frac{r_{i2}}{P_{xi2}} - \delta\right) \left(\frac{P_{ci1}}{P_{xi1}}\right) C_{i1} \\
&- \left(1 + \frac{r_{i3}}{P_{xi3}} - \delta\right) \left(\frac{P_{ci2}}{P_{xi2}}\right) C_{i2} - \left(\frac{P_{ci3}}{P_{xi3}}\right) C_{i3}
\end{aligned}$$

Before we continue, it will be useful to define $(1+R_{it}) = \prod_{n=1}^{t}\left(1+\frac{r_{in}}{P_{xin}}-\delta\right)$.

$$\begin{aligned}
\Rightarrow K_{i4} &= \frac{(1+R_{i3})\left(\frac{w_{i3}}{P_{xi3}}\right)L_i}{(1+R_{i3})} + \frac{(1+R_{i3})\left(\frac{w_{i2}}{P_{xi2}}\right)L_{i2}}{(1+R_{i2})} + \frac{(1+R_{i3})\left(\frac{w_{i1}}{P_{xi1}}\right)L_i}{(1+R_{i1})} \\
&\quad + (1+R_{i3})K_{i1} \\
&\quad - \frac{(1+R_{i3})\left(\frac{P_{ci3}}{P_{xi3}}\right)C_{i3}}{(1+R_{i3})} - \frac{(1+R_{i3})\left(\frac{P_{ci2}}{P_{xi2}}\right)C_{i2}}{(1+R_{i2})} - \frac{(1+R_{i3})\left(\frac{P_{ci1}}{P_{xi1}}\right)C_{i1}}{(1+R_{i1})} \\
\Rightarrow K_{i4} &= \sum_{n=1}^{3}\frac{(1+R_{i3})\left(\frac{w_{in}}{P_{xin}}\right)L_{in}}{(1+R_{in})} - \sum_{n=1}^{3}\frac{(1+R_{i3})\left(\frac{P_{cin}}{P_{xin}}\right)C_{in}}{(1+R_{in})} + (1+R_{i3})K_{i1}
\end{aligned}$$

By induction, for any time t,

$$\begin{aligned}
K_{it+1} &= \sum_{n=1}^{t}\frac{(1+R_{it})\left(\frac{w_{in}}{P_{xin}}\right)L_i}{(1+R_{in})} - \sum_{n=1}^{t}\frac{(1+R_{it})\left(\frac{P_{cin}}{P_{xin}}\right)C_{in}}{(1+R_{in})} + (1+R_{it})K_{i1} \\
\Rightarrow K_{it+1} &= (1+R_{it})\left(\sum_{n=1}^{t}\frac{\left(\frac{w_{in}}{P_{xin}}\right)L_i}{(1+R_{in})} - \sum_{n=1}^{t}\frac{\left(\frac{P_{cin}}{P_{xin}}\right)C_{in}}{(1+R_{in})} + K_{i1}\right)
\end{aligned}$$

Finally, observe the previous expression as of $t = T$ and rearrange terms to derive the lifetime budget constraint.

$$\sum_{n=1}^{T}\frac{P_{cin}C_{in}}{P_{xin}(1+R_{in})} = \underbrace{\sum_{n=1}^{T}\frac{w_{in}L_i}{P_{xin}(1+R_{in})} + K_{i1} - \frac{K_{iT+1}}{(1+R_{iT})}}_{W_i} \tag{B.1}$$

In the lifetime budget constraint (C.1), we use W_i to denote the net present value of lifetime wealth in country i, and we take the capital stock at the end of time, K_{iT+1}, as given; in our case it will be the capital stock in the new steady-state with T sufficiently large. Note that by imposing the terminal condition that $K_{iT+1} = K_i^\star$, the transversality condition is automatically satisfied since $\lim_{T\to\infty}(1+R_{iT}) = \infty$ and $\lim_{T\to\infty} K_{iT+1} = K_i^\star$.

Solving for the path of consumption Next we compute how the lifetime consumption expenditures will be allocated throughout time. The Euler equation (condition 23) implies the following relationship between consumption in any two periods t and

n:

$$C_{in} = \beta^{\sigma(n-t)} \left(\frac{(1+R_{in})}{(1+R_{it})} \right)^{\sigma} \left(\frac{P_{xin}}{P_{xit}} \right)^{\sigma} \left(\frac{P_{cit}}{P_{cin}} \right)^{\sigma} C_{it}$$

$$\Rightarrow \frac{P_{cin}C_{in}}{P_{xin}(1+R_{in})} = \beta^{\sigma(n-t)} \left(\frac{P_{xin}(1+R_{in})}{P_{xit}(1+R_{it})} \right)^{\sigma-1} \left(\frac{P_{cin}}{P_{cit}} \right)^{1-\sigma} \left(\frac{P_{cit}C_{it}}{P_{xit}(1+R_{it})} \right)$$

Since equation (C.1) implies that $\sum_{n=1}^{T} \frac{P_{cin}C_{in}}{P_{xin}(1+R_{in})} = W_i$, then we can rearrange the previous expression to obtain

$$\frac{P_{cit}C_{it}}{P_{xit}(1+R_{it})} = \underbrace{\left(\frac{\beta^{\sigma t} P_{xit}^{\sigma-1}(1+R_{it})^{\sigma-1} P_{cit}^{1-\sigma}}{\sum_{n=1}^{T} \beta^{\sigma n} P_{xin}^{\sigma-1}(1+R_{in})^{\sigma-1} P_{cin}^{1-\sigma}} \right)}_{\xi_{it}} W_i \tag{B.2}$$

That is, each period the household spends a share ξ_{it} of lifetime wealth on consumption, with $\sum_{t=1}^{T} \xi_{it} = 1$ for all i. Note that ξ_{it} depends only on prices.

Computing investment and the sequence of capital stocks Given paths of consumption, solve for investment $\{\vec{X}_t\}_{t=1}^{T}$ using the period budget constraint in condition 21. The catch here is that there is no restriction that household investment be non-negative up to this point. Looking ahead, there is no way that negative investment can satisfy market clearing conditions together with firm optimality conditions. As such, we restrict our attention to transition paths for which investment is always positive, which we find is the case for the equilibrium outcomes in our paper. However, off the equilibrium path, if during the course of the iterations any given value of X_{it} is negative, then set we it equal to a small positive number.

The last part of this step is to use condition 22 to compute the path for the stock of capital. $\{\vec{K}_t\}_{t=2}^{T+1}$. Note that $\vec{K}_1$ is taken as given and that $\vec{K}_{T+1}$ is by construction equal to the terminal steady-state value.

4. We combine conditions 4 & 13 to solve for $\{\vec{L}_{ct}\}_{t=1}^{T}$, combine conditions 5 & 14 to solve for $\{\vec{L}_{xt}\}_{t=1}^{T}$, and use condition 11 to solve for $\{\vec{L}_{mt}\}_{t=1}^{T}$. Next we combine conditions 1 & 4 to solve for $\{\vec{K}_{ct}\}_{t=1}^{T}$, combine conditions 2 & 5 to solve for $\{\vec{K}_{mt}\}_{t=1}^{T}$, and combine conditions 3 & 6 to solve for $\{\vec{K}_{xt}\}_{t=1}^{T}$. Similarly, we combine conditions 4 & 7 to solve for $\{\vec{M}_{ct}\}_{t=1}^{T}$, combine conditions 5 & 8 to solve for $\{\vec{M}_{mt}\}_{t=1}^{T}$, and combine conditions 6 & 9 to solve for $\{\vec{M}_{xt}\}_{t=1}^{T}$.

5. We compute $\{\vec{Y}_{ct}\}_{t=1}^{T}$ using condition 13, compute $\{\vec{Y}_{mt}\}_{t=1}^{T}$ using condition 14, and compute $\{\vec{Y}_{xt}\}_{t=1}^{T}$ using condition 15.

6. Until now we have imposed all equilibrium conditions except for two: The first being the trade balance condition 20, and the second being the capital market clearing condition 10.

Trade balance condition We compute an excess demand equation as in Alvarez and Lucas (2007b) defined as

$$Z_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = \frac{P_{mit}Y_{mit} - P_{mit}Q_{it}}{w_{it}}$$

(the trade deficit relative to the wage). Condition 20 requires that $Z_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = 0$ for all i. If this is different from zero in at least some country at some point in time we update our guess at the wages as follows.

$$\Lambda_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = w_{it}\left(1 + \psi \frac{Z_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right)}{L_i}\right)$$

is the updated guess to the wages, where ψ is chosen to be sufficiently small so that $\Lambda^{w} > 0$.

Market clearing condition for the stock of capital We compute an excess demand equation defined as

$$Z_{it}^{r}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = \frac{w_{it}L_i}{1-\alpha} - \frac{r_{it}K_{it}}{\alpha}$$

We have imposed, using conditions 1-6, that within each sector $\frac{r_{it}K_{bit}}{\alpha} = \frac{w_{it}L_{bit}}{1-\alpha}$. We have also imposed condition 11 that the labor market clear. Hence, the market for capital is in excess demand (i.e., $K_{cit} + K_{mit} + K_{xit} > K_{it}$) in country i at time t if and only if $\left(\frac{w_{it}L_i}{1-\alpha}\right) > \left(\frac{r_{it}K_{it}}{\alpha}\right)$ (it is in excess supply if and only if the inequality is $<$). If this condition does not hold with equality in some country at some point in time then we update our guess for rental rates as follows. Let

$$\Lambda_{it}^{r}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{L_i}{K_{it}}\right)\Lambda_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right)$$

be the updated guess to the rental rates (taking into account the updated guess for wages).

We return to step 2 with our updated wages and rental rates and repeat the steps. We iterate through this procedure until the excess demand is sufficiently close to zero. In our computations we find that our preferred convergence metric:

$$\max_{t=1}^{T}\left\{\max_{i=1}^{I}\left\{\left|Z_{it}^{w}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)\right|+\left|Z_{it}^{r}\left(\{\vec{w}_t,\vec{r}_t\}_{t=1}^{T}\right)\right|\right\}\right\}$$

converges roughly monotonically towards zero.

Along the equilibrium transition, $\sum_i w_{it}L_i + r_{it}K_{it} = 1$ $(\forall t)$; that is, we have chosen world GDP as the numéraire at each point in time.

The fact that $\vec{K}_{T+1} = \vec{K}^{\star}$ at each iteration is a huge benefit of our algorithm compared to algorithms that rely on shooting procedures or those that rely on using the Euler equation for updating. Such algorithms inherit the instability (saddle-path) properties of the Euler equation and generate highly volatile terminal stocks of capital with respect to the initial guess. Instead, we impose the Euler equation and the terminal condition for $\vec{K}_{T+1} = \vec{K}^{\star}$ at each iteration and use excess demand equations for our updating rules, just as in the computation of static models such as Alvarez and Lucas (2007b). Another main advantage of using excess-demand iteration is that we do not need to compute gradients to choose step directions or step size, as is the case of most nonlinear solvers such as the ones used by Eaton, Kortum, Neiman, and Romalis (2016) and Kehoe, Ruhl, and Steinberg (2016). This saves a tremendous amount of computational time, particularly as the number of countries or the number of time periods is increased.

C Solution algorithm for the extended model with endogenous trade imbalances and adjustment costs to capital

In this section of the Appendix we describe the algorithm for computing the equilibrium transition path in the model with adjustment costs to capital and endogenous trade imbalances. We first take care a some housekeeping remarks regarding the capital accumulation technology.

We will work with in inverse capital accumulation technology for convenience, which is given by

$$X_{it} = \Phi(K_{it+1}, K_{it}) = \left(\frac{1}{\chi}\right)^{\frac{1}{\mu}} (K_{it+1} - (1-\delta)K_{it})^{\frac{1}{\mu}} K_{it}^{\frac{\mu-1}{\mu}}$$

We will make use of the derivative of the investment function, with respect to future and current capital, as given by

$$\Phi_1(K_{it+1}, K_{it}) = \left(\frac{1}{\chi}\right)^{\frac{1}{\mu}} \left(\frac{1}{\mu}\right) \left(\frac{K_{it+1}}{K_{it}} - (1-\delta)\right)^{\frac{1-\mu}{\mu}}$$

$$\Phi_2(K_{it+1}, K_{it}) = \left(\frac{1}{\chi}\right)^{\frac{1}{\mu}} \left(\frac{1}{\mu}\right) \left(\frac{K_{it+1}}{K_{it}} - (1-\delta)\right)^{\frac{1-\mu}{\mu}} \left((\mu-1)\frac{K_{it+1}}{K_{it}} - \mu(1-\delta)\right)$$

C.1 Computing the steady-state equilibrium in the extended model

The solution to the steady-state equilibrium is identical to the baseline model, with a modification to condition 23 in table B.1. In particular, the real rate of return to capital has to account for the cost of adjusting the capital stock so that it becomes

$$r_i = \left(\frac{\Phi_{1i}}{\beta} + \Phi_{2i}\right) P_{xi}$$

Note that in steady-state, when we set $\chi = \delta^{1-\mu}$, so that there are no adjustment costs in steady-state (i.e., $X_i = \delta K_i$), then $\Phi_1 = \frac{1}{\mu}$ and $\Phi_2 = \delta - \frac{1}{\mu}$.

In general, in models with trade imbalances, the steady-state is not independent from the transition path that leads up to that steady-state. In our model, we will treat the initial steady-state as independent from the prior transition, and compute the transition from that steady-state. As a result, the new steady-state will be determined jointly with the equilibrium transition path.

C.2 Computing the equilibrium transition path in the extended model

The equilibrium transition path consists of the following objects: $\{\vec{w}_t\}_{t=1}^T$, $\{\vec{r}_t\}_{t=1}^T$, $\{q_t\}_{t=1}^T$, $\{\vec{P}_{ct}\}_{t=1}^T$, $\{\vec{P}_{mt}\}_{t=1}^T$, $\{\vec{P}_{xt}\}_{t=1}^T$, $\{\vec{C}_t\}_{t=1}^T$, $\{\vec{X}_t\}_{t=1}^T$, $\{\vec{K}_t\}_{t=1}^{T+1}$, $\{\vec{B}_t\}_{t=1}^T$,$\{\vec{\mathcal{A}}_t\}_{t=1}^{T+1}$, $\{\vec{Y}_{ct}\}_{t=1}^T$, $\{\vec{Y}_{mt}\}_{t=1}^T$, $\{\vec{Y}_{xt}\}_{t=1}^T$, $\{\vec{K}_{ct}\}_{t=1}^T$, $\{\vec{K}_{mt}\}_{t=1}^T$, $\{\vec{K}_{xt}\}_{t=1}^T$, $\{\vec{L}_{ct}\}_{t=1}^T$, $\{\vec{L}_{mt}\}_{t=1}^T$, $\{\vec{L}_{xt}\}_{t=1}^T$, $\{\vec{M}_{ct}\}_{t=1}^T$, $\{\vec{M}_{mt}\}_{t=1}^T$, $\{\vec{M}_{xt}\}_{t=1}^T$, $\{\vec{\vec{\pi}}_t\}_{t=1}^T$ (I use the double-arrow notation on $\vec{\vec{\pi}}_t$ to indicate that this is an $I \times I$ matrix in each period t). Table C.1 provides a list of equilibrium conditions that these objects must satisfy.

In this environment, the world interest rate is strictly nominal. That is, in the model, the prices map into current units, as opposed to constant units. In other words, the model can be rewritten so that all prices are quoted in time-1 units (like an Arrow-Debreu world) with the world interest rate of zero and the equilibrium would yield identical quantities. Since our choice of numéraire is world GDP in each period, the world interest rate reflects the relative valuation of world GDP at two points in time. This interpretation is useful in guiding the solution procedure.

The solution procedure boils down to two iterations. First, we guess a set of nominal investment rates at each point in time for every country. Given these investment rates, we adapt the algorithm of Sposi (2012) and iterate on the wages and the world interest rate to pin down the solution to the endogenous trade imbalances. Then we go back and update the nominal investment rates that satisfy the Euler equation for the optimal rate of capital accumulation.

To begin, we take the initial capital stock, K_{i1} as given in each country.

1. Guess a path for nominal investment rates $\{\vec{\rho}_t\}_{t=1}^{T}$.

2. Guess the entire path for wages $\{\vec{w}_t\}_{t=1}^{T}$ across countries and the world interest rate $\{q_t\}_{t=2}^{T}$, such that $\sum_i \frac{w_{it}L_{it}}{1-\alpha} = 1\,(\forall t)$.

3. In period 1 set $\vec{r}_1 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\vec{w}_1\vec{L}}{\vec{K}_1}\right)$ since the initial stock of capital is predetermined. Compute prices P_{c1}, P_{x1}, and P_{m1} simultaneously using conditions 16, 17, and 18, in Table C.1. Solve for physical investment, X_1, using

$$X_{it} = \rho_{it}\frac{w_{it}L_{it} + r_{it}K_{it}}{P_{xit}}$$

and then solve for the next-period capital stock, K_2, using condition 22. Repeat this set of calculations for period 2, then for period 3, and continue all the way through period T. To complete this step, compute the bilateral trade shares $\{\vec{\vec{\pi}}_t\}_{t=1}^{T}$ using condition 19.

4. This step is slightly more involved. We show how to compute the path for consumption and bond purchases by solving the intertemporal problem of the household. This is done in three parts. First we derive the lifetime budget constraint, second we derive

Table C.1: Equilibrium conditions along the transition

1	$r_{it}K_{cit} = \alpha\nu_c P_{cit}Y_{cit}$	$\forall(i,t)$
2	$r_{it}K_{mit} = \alpha\nu_m P_{mit}Y_{mit}$	$\forall(i,t)$
3	$r_{it}K_{xit} = \alpha\nu_x P_{xit}Y_{xit}$	$\forall(i,t)$
4	$w_{it}L_{cit} = (1-\alpha)\nu_c P_{cit}Y_{cit}$	$\forall(i,t)$
5	$w_{it}L_{mit} = (1-\alpha)\nu_m P_{mit}Y_{mit}$	$\forall(i,t)$
6	$w_{it}L_{xit} = (1-\alpha)\nu_x P_{xit}Y_{xit}$	$\forall(i,t)$
7	$P_{mit}M_{cit} = (1-\nu_c)P_{cit}Y_{cit}$	$\forall(i,t)$
8	$P_{mit}M_{mit} = (1-\nu_m)P_{mit}Y_{mit}$	$\forall(i,t)$
9	$P_{mit}M_{xit} = (1-\nu_x)P_{xit}Y_{xit}$	$\forall(i,t)$
10	$K_{cit} + K_{mit} + K_{xit} = K_{it}$	$\forall(i,t)$
11	$L_{cit} + L_{mit} + L_{xit} = L_{it}$	$\forall(i,t)$
12	$M_{cit} + M_{mit} + M_{xit} = M_{it}$	$\forall(i,t)$
13	$C_{it} = Y_{cit}$	$\forall(i,t)$
14	$\sum_{j=1}^{I} P_{mjt}M_{jt}\pi_{jit} = P_{mit}Y_{mit}$	$\forall(i,t)$
15	$X_{it} = Y_{xit}$	$\forall(i,t)$
16	$P_{cit} = \left(\frac{1}{Z_{ci}}\right)\left(\frac{r_{it}}{\alpha\nu_c}\right)^{\alpha\nu_c}\left(\frac{w_{it}}{(1-\alpha)\nu_c}\right)^{(1-\alpha)\nu_c}\left(\frac{P_{mit}}{1-\nu_c}\right)^{1-\nu_c}$	$\forall(i,t)$
17	$P_{mit} = \gamma\left[\sum_{j=1}^{I}(u_{mjt}d_{ijt})^{-\theta}T_{mjt}\right]^{-\frac{1}{\theta}}$	$\forall(i,t)$
18	$P_{xit} = \left(\frac{1}{Z_{xi}}\right)\left(\frac{r_{it}}{\alpha\nu_x}\right)^{\alpha\nu_x}\left(\frac{w_{it}}{(1-\alpha)\nu_x}\right)^{(1-\alpha)\nu_x}\left(\frac{P_{mit}}{1-\nu_x}\right)^{1-\nu_x}$	$\forall(i,t)$
19	$\pi_{ijt} = \frac{(u_{mjt}d_{ijt})^{-\theta}T_{mjt}}{\sum_{j=1}^{I}(u_{mjt}d_{ijt})^{-\theta}T_{mjt}}$	$\forall(i,j,t)$
20	$P_{cit}C_{it} + P_{xit}X_{it} + B_{it} = r_{it}K_{it} + w_{it}L_{it} + q_t\mathcal{A}_{it}$	$\forall(i,t)$
21	$\mathcal{A}_{it+1} = \mathcal{A}_{it} + B_{it}$	$\forall(i,t)$
22	$K_{it+1} = (1-\delta)K_{it} + \chi X_{it}^{\mu}K_{it}^{1-\mu}$	$\forall(i,t)$
23	$\frac{C_{it+1}}{C_{it}} = \beta^{\sigma}\left(\frac{\frac{r_{it+1}}{P_{xit+1}} - \Phi_2(K_{it+1},K_{it})}{\Phi_1(K_{it+1},K_{it})}\right)^{\sigma}\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^{\sigma}$	$\forall(i,t)$
24	$\frac{C_{it+1}}{C_{it}} = \beta^{\sigma}\left(\frac{1+q_{t+1}}{P_{cit+1}/P_{cit}}\right)^{\sigma}$	$\forall(i,t)$
25	$B_{it} = P_{mit}Y_{mit} - P_{mit}M_{it} + q_t\mathcal{A}_{it}$	$\forall(i,t)$

Note: The term $u_{mjt} = \left(\frac{r_{jt}}{\alpha\nu_m}\right)^{\alpha\nu_m}\left(\frac{w_{jt}}{(1-\alpha)\nu_m}\right)^{(1-\alpha)\nu_m}\left(\frac{P_{mjt}}{1-\nu_m}\right)^{1-\nu_m}$. The function $\Phi(K_{it+1}, K_{it}) = \left(\frac{1}{\chi}\right)^{\mu}(K_{it+1} - (1-\delta)K_{it})^{\frac{1}{\mu}} K_{it}^{\frac{\mu-1}{\mu}}$ represents the inverse of the capital accumulation technology, and hence the level of investment. Also, $\Phi_1(\cdot,\cdot)$ and $\Phi_2(\cdot,\cdot)$ denote the derivatives w.r.t. the first and second arguments, respectively.

the fraction of lifetime wealth allocated to consumption at each period t, and third we recover the sequences for bond purchases and the stock of net-foreign assets.

Deriving the lifetime budget constraint To begin, compute the lifetime budget constraint for the representative household (omitting country subscripts for now). Begin with the period budget constraint from condition 20 and combine it with the net-foreign asset accumulation technology in condition 21 to get

$$\mathcal{A}_{t+1} = r_t K_t + w_t L_t - P_{ct} C_t - P_{xt} X_t + (1 + q_t)\mathcal{A}_t$$

Iterate the period budget constraint forward through time and derive a lifetime budget constraint. At time $t = 1$ the net-foreign asset position (NFAP), $A_{i1} > 0$, is given. Next, compute the NFAP at time $t = 2$.

$$\mathcal{A}_2 = r_1 K_1 + w_1 L_1 - P_{c1} C_1 - P_{x1} X_1 + (1 + q_1)\mathcal{A}_1$$

Similarly, compute the NFAP at time $t = 3$, but do it so that it is in terms the initial NFAP.

$$\begin{aligned}
\mathcal{A}_3 &= r_2 K_2 + w_2 L_2 - P_{c2} C_2 - P_{x2} X_2 + (1 + q_2)\mathcal{A}_2 \\
\Rightarrow \mathcal{A}_3 &= r_2 K_2 + w_2 L_2 - P_{x2} X_2 + (1 + q_2)(r_1 K_1 + w_1 L_1 - P_{x1} X_1) \\
&\quad - P_{c2} C_2 - (1 + q_2) P_{c1} C_1 + (1 + q_2)(1 + q_1)\mathcal{A}_{i1}
\end{aligned}$$

Continue to period 4 in a similar way

$$\begin{aligned}
\mathcal{A}_4 &= r_3 K_3 + w_3 L_3 - P_{c3} C_3 - P_{x3} X_3 + (1 + q_3)\mathcal{A}_3 \\
\Rightarrow \mathcal{A}_4 &= r_3 K_3 + w_3 L_3 - P_{x3} X_3 \\
&\quad + (1 + q_3)(r_2 K_2 + w_2 L_2 - P_{x2} X_2) \\
&\quad + (1 + q_3)(1 + q_2)(r_1 K_1 + w_1 L_1 - P_{x1} X_1) \\
&\quad - P_{c3} C_3 - (1 + q_3) P_{c2} C_2 - (1 + q_3)(1 + q_2) P_{c1} C_1 + (1 + q_3)(1 + q_2)(1 + q_1)\mathcal{A}_1
\end{aligned}$$

Before proceeding, it will be useful to define $(1+\mathcal{Q}_t) = \prod_{n=1}^{t}(1+q_n)$.

$$\begin{aligned}
\Rightarrow \mathcal{A}_4 = &\frac{(1+\mathcal{Q}_3)(r_3K_3 + w_3L_3 - P_{x3}X_3)}{(1+\mathcal{Q}_3)} \\
&+ \frac{(1+\mathcal{Q}_3)(r_2K_2 + w_2L_2 - P_{x2}X_2)}{(1+\mathcal{Q}_2)} \\
&+ \frac{(1+\mathcal{Q}_3)(r_1K_1 + w_1L_1 - P_{x1}X_1)}{(1+\mathcal{Q}_1)} \\
&- \frac{(1+\mathcal{Q}_3)P_{c3}C_3}{(1+\mathcal{Q}_3)} \\
&- \frac{(1+\mathcal{Q}_3)P_{c2}C_2}{(1+\mathcal{Q}_2)} \\
&- \frac{(1+\mathcal{Q}_3)P_{c1}C_1}{(1+\mathcal{Q}_1)} \\
&+ (1+\mathcal{Q}_3)\mathcal{A}_1
\end{aligned}$$

By induction, for any time t,

$$\begin{aligned}
\mathcal{A}_{t+1} &= \sum_{n=1}^{t} \frac{(1+\mathcal{Q}_t)(r_nK_n + w_nL_n - P_{xn}X_n)}{(1+\mathcal{Q}_n)} - \sum_{n=1}^{t} \frac{(1+\mathcal{Q}_t)P_{cn}C_n}{(1+\mathcal{Q}_n)} + (1+\mathcal{Q}_t)\mathcal{A}_1 \\
\Rightarrow \mathcal{A}_{t+1} &= (1+\mathcal{Q}_t)\left(\sum_{n=1}^{t} \frac{r_nK_n + w_nL_n - P_{xn}X_n}{(1+\mathcal{Q}_n)} - \sum_{n=1}^{t} \frac{P_{cn}C_n}{(1+\mathcal{Q}_n)} + \mathcal{A}_1\right)
\end{aligned}$$

Finally, observe the previous expression as of $t = T$ and rearrange terms to derive the lifetime budget constraint.

$$\sum_{n=1}^{T} \frac{P_{cn}C_n}{(1+\mathcal{Q}_n)} = \underbrace{\sum_{n=1}^{T} \frac{r_nK_n + w_nL_n - P_{xn}X_n}{(1+\mathcal{Q}_n)} + \mathcal{A}_1 - \frac{\mathcal{A}_{T+1}}{(1+\mathcal{Q}_T)}}_{W} \tag{C.1}$$

In the lifetime budget constraint (C.1), W denotes the net present value of lifetime wealth, taking both the initial and terminal NFAPs as given.

Solving for the path of consumption Next, compute how the net-present value of lifetime wealth is optimally allocated throughout time. The Euler equation (condition 24) implies the following relationship between consumption in any two periods t and

n:

$$C_n = \left(\frac{L_n}{L_t}\right) \beta^{\sigma(n-t)} \left(\frac{\psi_n}{\psi_t}\right)^{\sigma} \left(\frac{1+\mathcal{Q}_n}{1+\mathcal{Q}_t}\right)^{\sigma} \left(\frac{P_{ct}}{P_{cn}}\right)^{\sigma} C_t$$
$$\Rightarrow \frac{P_{cn}C_n}{1+\mathcal{Q}_n} = \left(\frac{L_n}{L_t}\right) \beta^{\sigma(n-t)} \left(\frac{\psi_n}{\psi_t}\right)^{\sigma} \left(\frac{1+\mathcal{Q}_n}{1+\mathcal{Q}_t}\right)^{\sigma-1} \left(\frac{P_{ct}}{P_{cn}}\right)^{\sigma-1} \frac{P_t C_t}{1+\mathcal{Q}_t}$$

Since equation (C.1) implies that $\sum_{n=1}^{T} \frac{P_{cin}C_{in}}{1+\mathcal{Q}_n} = W$, rearrange the previous expression (putting country subscripts back in) to obtain

$$\frac{P_{cit}C_{it}}{1+\mathcal{Q}_{it}} = \underbrace{\left(\frac{L_{it}\beta^{\sigma t}\psi_{it}^{\sigma}(1+\mathcal{Q}_{it})^{\sigma-1}P_{cit}^{1-\sigma}}{\sum_{n=1}^{T} L_{in}\beta^{\sigma n}\psi_{in}^{\sigma}(1+\mathcal{Q}_{in})^{\sigma-1}P_{cin}^{1-\sigma}}\right)}_{\xi_{it}} W_i \tag{C.2}$$

That is, each period the household spends a share ξ_{it} of lifetime wealth on consumption, with $\sum_{t=1}^{T} \xi_{it} = 1$ for all i. Note that ξ_{it} depends only on prices.

Computing bond purchases and the net-foreign asset positions In period 1 take as given consumption spending, investment spending, capital income, labor income, and net income from the initial NFAP, to solve for net bond purchases $\{\vec{B}_t\}_{t=1}^{T}$ using the period budget constraint in condition 20. Solve for the NFAP in period 2 using condition 21. Then given income and spending in period two, recover the net bond purchases in period two, and compute the NFAP for period three. Continue doing this through all points in time.

Trade balance condition We compute an excess demand equation as in Alvarez and Lucas (2007b), but instead of imposing that net exports equal zero in each country, we impose that net exports equal the current account less net-foreign income from asset holding. That is,

$$Z_{it}^{w}\left(\{\vec{w}_t, q_t\}_{t=1}^{T}\right) = \frac{P_{mit}Y_{mit} - P_{mit}M_{it} - B_{it} + q_t\mathcal{A}_{it}}{w_{it}}$$

Condition 25 requires that $Z_{it}^{w}\left(\{\vec{w}_t, \vec{r}_t\}_{t=1}^{T}\right) = 0$ for all (i,t) in equilibrium. If this is different from zero in at least some country at some point in time update the wages as follows.

$$\Lambda^w_{it}\left(\{\vec{w}_t, q_t\}_{t=1}^T\right) = w_{it}\left(1+\psi\frac{Z^w_{it}\left(\{\vec{w}_t, q_t\}_{t=1}^T\right)}{L_{it}}\right)$$

is the updated guess to the wages, where ψ is chosen to be sufficiently small so that $\Lambda^w > 0$.

Normalizing model units The next part of this step is updating the equilibrium world interest rate. Recall that the numéraire is defined to be world GDP at each point in time: $\sum_{i=1}^I(r_{it}K_{it} + w_{it}L_{it}) = 1$ $(\forall t)$. For an arbitrary sequence of $\{q_{t+1}\}_{t=1}^T$, this condition need not hold. As such, update the the world interest rate as

$$1 + q_t = \frac{\sum_{i=1}^I(r_{it-1}K_{it-1} + \Lambda^w_{it-1}L_{it-1})}{\sum_{i=1}^I(r_{it}K_{it} + \Lambda^w_{it}L_{it})} \text{ for } t = 2, \ldots, T \tag{C.3}$$

The inputs for capital and the rental rate of capital are computed in step 2, while the input for wages is the updated values Λ^w above. The world interest rate in the initial period, q_1 has no influence on the model other than scaling the initial NFAP q_1A_{i1}, i.e., it is purely nominal. As such, we set $q_1 = \frac{1-\beta}{\beta}$ (the interest rate that prevails in a steady-state) and chose A_{i1} so that q_1A_{i1} matches the desired initial NFAP in current prices.

Having updated the wages and the world interest rate, return to step 2 and perform each step again. Iterate through this procedure until the excess demand is sufficiently close to zero. In the computations we find that our preferred convergence metric:

$$\max_{t=1}^T\left\{\max_{i=1}^I\left\{\left|Z^w_{it}\left(\{\vec{w}_t, q_t\}_{t=1}^T\right)\right|\right\}\right\}$$

converges roughly monotonically towards zero. This provides a the solution to a "sub equilibrium" for an exogenously specified nominal investment rate.

5. The last step of the algorithm is to update the nominal investment rate. Until now, the Euler equation for investment in physical capital, condition 23, has not been used. As such, we compute an "Euler-equation residual" as

$$Z^r_{it}\left(\{\vec{\rho}_t\}_{t=1}^T\right) = \beta^\sigma\left(\frac{\frac{r_{it+1}}{P_{xit+1}} - \Phi_2(K_{it+2}, K_{it+1})}{\Phi_1(K_{it+1}, K_{it})}\right)^\sigma\left(\frac{P_{xit+1}/P_{cit+1}}{P_{xit}/P_{cit}}\right)^\sigma - \left(\frac{C_{it+1}}{C_{it}}\right) \tag{C.4}$$

Condition 23 requires that $Z_{it}^{r}\left(\{\vec{\rho}_t\}_{t=1}^{T}\right)=0$ for all (i,t) in equilibrium. We update the nominal investment rates as

$$\Lambda_{it}^{r}\left(\{\vec{\rho}_t\}_{t=1}^{T}\right)=\rho_{it}\left(1+\psi Z_{it}^{r}\left(\{\vec{\rho}_t\}_{t=1}^{T}\right)\right) \tag{C.5}$$

In order to update ρ_{iT}, we need to define $\Phi_2(K_{iT+2},K_{iT+1})$, which is simply its steady-state value, $\Phi_2^{\star}=\delta-\frac{1}{\mu}$, which serves as a boundary condition for the transition path of capital stocks.

Given the updated sequence of nominal investment rates, return to step 1 an repeat. Continue the iterations until $\max_{t=1}^{T}\left\{\max_{i=1}^{I}\left\{|Z_{it}^{r}\left(\{\vec{\rho}_t\}_{t=1}^{T}\right)|\right\}\right\}$ is sufficiently close to zero.

D Derivations

This section of the Appendix shows the derivations of key structural relationships. We refer to Table B.2 for the basis of the derivations and omit time subscripts to ease notation. We begin by deriving an expression for $\frac{w_i}{P_{mi}}$ that will be used repeatedly.

Combining conditions 17 and 19 we obtain

$$\pi_{ii}=\gamma^{-\theta}\left(\frac{u_{mi}^{-\theta}T_{mi}}{P_{mi}^{-\theta}}\right)$$

Use the fact that $u_{mi}=B_m r_i^{\alpha\nu_m} w_i^{(1-\alpha)\nu_m} P_{mi}^{1-\nu_m}$, where B_m is a collection of constants, then rearrange to obtain

$$P_{mi}=\left(\frac{T_{mi}}{\pi_{ii}}\right)^{-\frac{1}{\theta}}\left(\frac{r_i}{w_i}\right)^{\alpha\nu_m}\left(\frac{w_i}{P_{mi}}\right)^{\nu_m}P_{mi}$$

$$\Rightarrow \frac{w_i}{P_{mi}}=\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1}{\nu_m}}\left(\frac{w_i}{r_i}\right)^{\alpha} \tag{D.1}$$

Note that this relationship holds in both the steady-state and along the transition.

Relative prices We show how to derive the price of consumption relative to intermediates; the relative price of investment is analogous. Begin with condition 16

$$P_{ci} = \left(\frac{B_c}{A_{ci}}\right)\left(\frac{r_i}{w_i}\right)^{\alpha\nu_c}\left(\frac{w_i}{P_{mi}}\right)^{\nu_c} P_{mi}$$

where B_c is a collection of constants. Substitute equation (D.1) into the previous expression and rearrange to obtain

$$\frac{P_{ci}}{P_{mi}} = \left(\frac{B_c}{A_{ci}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{\nu_c}{\nu_m}} \tag{D.2}$$

Analogously,

$$\frac{P_{xi}}{P_{mi}} = \left(\frac{B_x}{A_{xi}}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{\nu_x}{\nu_m}} \tag{D.3}$$

Note that these relationships hold in both the steady-state and along the transition.

Capital-labor ratio We derive a structural relationship for the capital-labor ratio in the steady-state only and make reference to conditions in Table B.1. Conditions 1-6 together with conditions 10 and 11 imply that

$$\frac{K_i}{L_i} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_i}{r_i}\right)$$

Using condition 23 we know that

$$r_i = \left(\frac{1}{\beta} - (1-\delta)\right) P_{xi}$$

which, by substituting into the prior expression implies that

$$\frac{K_i}{L_i} = \left(\frac{\alpha}{(1-\alpha)\left(\frac{1}{\beta} - (1-\delta)\right)}\right)\left(\frac{w_i}{P_{xi}}\right)$$

which leaves the problem of solving for $\frac{w_i}{P_{xi}}$. Equations (D.1) and (D.3) imply

$$\begin{aligned}\frac{w_i}{P_{xi}} &= \left(\frac{w_i}{P_{mi}}\right)\left(\frac{P_{mi}}{P_{xi}}\right)\\ &= \left(\frac{A_{xi}}{B_x}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_x}{\nu_m}}\left(\frac{w_i}{r_i}\right)^{\alpha}\end{aligned}$$

Substituting in once more for $\frac{w_i}{r_i}$ in the previous expression yields

$$\left(\frac{w_i}{P_{xi}}\right)^{1-\alpha} = \left(\frac{1}{\beta} - (1-\delta)\right)^{-\alpha}\left(\frac{A_{xi}}{B_x}\right)\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_x}{\nu_m}}$$

Solve out for the aggregate capital-labor ratio

$$\frac{K_i}{L_i} = \left(\frac{\frac{\alpha}{1-\alpha}}{\left(\frac{1}{\beta} - (1-\delta)\right)^{-\frac{1}{1-\alpha}}}\right)\left(\frac{A_{xi}}{B_x}\right)^{\frac{1}{1-\alpha}}\left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_x}{(1-\alpha)\nu_m}} \tag{D.4}$$

Note that we invoked steady-state conditions so this expression does not necessarily hold along the transition path.

Income per capita We define (real) income per capita in our model as

$$y_i = \frac{r_i K_i + w_i L_i}{L_i P_{ci}}$$

We invoke conditions from Table B.2 for the remainder of this derivation. Conditions 1-6, 10, and 11 imply that

$$\begin{aligned} r_i K_i + w_i L_i &= \frac{w_i L_i}{1-\alpha}\\ \Rightarrow y_i &= \left(\frac{1}{1-\alpha}\right)\left(\frac{w_i}{P_{ci}}\right)\end{aligned}$$

To solve for $\frac{w_i}{P_{ci}}$ we use condition 16

$$\begin{aligned} P_{ci} &= \frac{B_c}{A_{ci}} \left(\frac{r_i}{w_i}\right)^{\alpha\nu_c} \left(\frac{w_i}{P_{mi}}\right)^{\nu_c} P_{mi} \\ \Rightarrow \frac{P_{ci}}{w_i} &= \frac{B_c}{A_{ci}} \left(\frac{r_i}{w_i}\right)^{\alpha\nu_c} \left(\frac{w_i}{P_{mi}}\right)^{\nu_c - 1} \end{aligned}$$

Substituting equation (D.1) into the previous expression, and exploiting the fact that $\frac{w_i}{r_i} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{K_i}{L_i}\right)$ yields

$$\begin{aligned} y_i &= \left(\frac{1}{1-\alpha}\right)\left(\frac{w_i}{P_{ci}}\right) \\ &= \alpha^{-\alpha} (1-\alpha)^{\alpha-1} \left(\frac{A_{ci}}{B_c}\right) \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_c}{\theta\nu_m}} \left(\frac{K_i}{L_i}\right)^{\alpha} \end{aligned} \tag{D.5}$$

Note that this expression holds both in the steady-state and along the transition path.

The steady-state income per capita can be expressed more fundamentally by invoking equation (D.4) as

$$y_i = \left(\frac{\left(\frac{1}{\beta} - (1-\delta)\right)^{-\frac{\alpha}{1-\alpha}}}{1-\alpha}\right) \left(\frac{A_{ci}}{B_c}\right) \left(\frac{A_{xi}}{B_x}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\left(\frac{T_{mi}}{\pi_{ii}}\right)^{\frac{1}{\theta}}}{\gamma B_m}\right)^{\frac{1-\nu_c+\frac{\alpha}{1-\alpha}(1-\nu_x)}{\nu_m}} \tag{D.6}$$

E Data

This section of the Appendix describes the sources of data as well as any adjustments we make to the data to map it to the model.

E.1 Production and trade data

Mapping the trade dimension of our model to the data requires data on both production and international trade flows. Our focus is on manufactured intermediate goods. We interpret manufacturing broadly as defined by the International Standard Industrial Classification (ISIC).

We obtain production data from multiple sources. First, we utilize value added and gross output data from the (INDSTAT) which is reported at the two-digit Ievel using ISIC. This data countries extends no further than 2010, and even less for many countries. We turn to data on value added output in (UNIDO MEI) which reports value added output for 2011. For countries that report both value added and gross output in INDSTAT, we use the ratio from the year that is closet to 2011, and apply that ratio to the value added from UNIDO to recover gross output. For countries that are have no data on gross output in INDSTAT for any years, we apply the average ratio of value-added-to-gross output across all countries, and apply that ratio to the value added figure in UNIDO for 2011. In our data set, the ratio of value-added-to-gross output does not vary significantly over time, and is also not correlated with level of development or country size.

Our source of trade data is the UN Comtrade Database http://comtrade.un.org. Trade is reported for goods using revision 2 Standard International Trade Classification (SITC2) at the four-digit level. We make use of the correspondence tables created by Affendy, Sim Yee, and Satoru (2010) to map SITC2 to ISIC. We also omit any petroleum-related products from the trade data.

Using the trade and production data, we construct bilateral trade shares for each country pair by following Bernard, Eaton, Jensen, and Kortum (2003) as follows:

$$\pi_{ij} = \frac{X_{ij}}{ABS_{bi}},$$

where i denotes the importer and j denotes the exporter. X_{ij} denotes manufacturing trade flows from j to i, and ABS_i is country i's absorption defined as gross output less net exports of manufactures.

E.2 National accounts and price data

PPP GDP and population For our baseline calibration, we collect data on output-side real GDP at current PPPs (2005 U.S. dollars) from version 8.1 of the Penn World Tables (see Feenstra, Inklaar, and Timmer, 2015, (PWT from now on)) using the variable `cgdpo`.

We use the variable `pop` from PWT to measure the population in each country. The ratio $\frac{\texttt{cgdpo}}{\texttt{pop}}$ corresponds to GDP per capita, y, in our model.

In our counterfactuals, we compare changes over time to past trade liberalization episodes using national accounts data from the PWT: *rgdpna*, *rkna*, and *rtfpna*.

We take the price level of household consumption and the price level of capital formation (both relative to the price of output-side GDP in the U.S. in constant prices) from PWT using variables `pl_c` and `pl_i` respectively. These correspond to P_c and P_x in our model.

We construct the price of intermediate goods (manufactures) by combining disaggregate price data from the World Bank's 2011 International Comparison Program (ICP): http://siteresources.worldbank.org/ICPEXT/Resources/ICP_2011.html. The data has several categories that fall under what we classify as manufactures: "Food and nonalcoholic beverages", "Alcoholic beverages, tobacco, and narcotics", "Clothing and foot wear", and "Machinery and equipment". The ICP reports expenditure data for these categories in both nominal U.S. dollars and real U.S. dollars. The conversion from nominal to real uses the PPP price, that is: the PPP price equals the ratio of nominal expenditures to real expenditures. As such, we compute the PPP for manufactures as a whole of manufactures for each country as the sum of nominal expenditures across categories divided by the sum of real expenditures across categories. For the RoW aggregate, we simply sum the expenditure across all of the countries that are not part of the 40 individual countries.

There is one more step before we take these prices to the model. The data correspond to expenditures, thus include additional margins such as distribution. In order to adjust for this this, we first construct a price for distribution services. We assume that the price of distribution services is proportional to the overall price of services in each country and use the same method as above to compute the price across the following categories: "Housing, water, electricity, gas, and other fuels", "Health", "Transport", "Communication", "Recreation and culture", "Education", "Restaurants and hotels", and "Construction".

Now that we have the price of services in hand, we strip it away from the price of goods computed above to arrive at a measure of the price of manufactures that better corresponds to our model. In particular, let P_d denote the price of distribution services and let P_g denote the price of goods that includes the distribution margin. We assume that $P_g = P_d^{\psi} P_m^{1-\psi}$, where P_m is the price of manufactures. We set $\psi = 0.45$ which is a value commonly used in the literature.

F Additional figures and tables

Table F.1: Gains from trade (%) following uniform reduction in barriers by 55%

		Model 1		Model 2		Model 3	Model 4	Model 5	
Country	**ISO**	**Dyn**	**SS**	**Dyn**	**SS**	**Dyn**	**Dyn**	**Dyn**	**SS**
Armenia	ARM	24.2	29.3	46.9	77.8	47.2	35.8	36.0	61.7
Australia	AUS	10.9	13.2	20.5	34.5	20.8	16.2	16.2	31.6
Austria	AUT	14.3	17.3	26.5	44.6	26.9	20.8	20.8	38.7

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Table F.1 – Continued

Country	ISO	Model 1 Dyn	Model 1 SS	Model 2 Dyn	Model 2 SS	Model 3 Dyn	Model 4 Dyn	Model 5 Dyn	Model 5 SS
Bahamas	BHS	21.7	26.2	40.9	67.2	40.8	31.3	31.4	54.9
Bangladesh	BGD	16.7	20.2	30.8	51.5	31.1	24.0	24.1	43.6
Barbados	BRB	26.6	32.3	52.6	86.6	52.7	39.8	40.0	67.9
Belarus	BLR	14.2	17.2	26.9	45.3	27.3	21.0	21.1	39.1
Belize	BLZ	28.6	34.7	55.8	91.6	55.7	42.0	42.3	71.4
Benin	BEN	22.3	27.1	43.4	72.1	43.7	33.2	33.4	57.7
Bhutan	BTN	21.4	26.0	40.4	66.5	40.4	31.0	31.0	54.4
Brazil	BRA	7.6	9.2	14.3	24.3	14.6	11.4	11.4	24.4
Bulgaria	BGR	19.6	23.8	36.8	61.6	37.3	28.4	28.6	50.5
Burundi	BDI	13.1	15.9	24.8	42.0	25.3	19.5	19.6	36.8
Cabo Verde	CPV	21.8	26.5	42.2	70.4	42.7	32.4	32.6	56.6
Cambodia	KHM	19.7	23.8	36.3	60.1	36.4	28.0	28.1	49.8
Cameroon	CMR	19.3	23.4	37.1	61.8	37.4	28.6	28.7	50.7
Canada	CAN	10.3	12.4	18.8	31.5	19.0	14.8	14.8	29.7
Central African Rep.	CAF	13.3	16.2	25.4	42.8	25.8	19.9	20.0	37.3
Chile	CHL	16.1	19.5	30.0	49.9	30.2	23.3	23.4	42.5
China, Hong Kong, Macao	CHM	6.5	7.9	11.1	19.1	11.5	8.9	8.9	20.7
Colombia	COL	14.6	17.7	27.6	45.9	27.8	21.5	21.5	39.8
Costa Rica	CRI	21.2	25.7	40.8	67.0	40.7	31.2	31.3	54.8
Cyprus	CYP	20.3	24.6	39.5	66.0	40.0	30.4	30.6	53.4
Czech Rep.	CZE	15.4	18.6	28.5	47.8	28.9	22.3	22.4	41.0
Cte d'Ivoire	CIV	21.0	25.4	40.0	66.3	40.2	30.7	30.8	53.9
Denmark	DNK	15.7	19.0	29.2	48.9	29.6	22.8	22.8	41.7
Dominican Rep.	DOM	13.2	16.0	23.5	39.2	23.7	18.4	18.4	35.1
Ecuador	ECU	20.6	25.0	39.0	64.4	39.0	30.0	30.0	52.8
Egypt	EGY	17.9	21.7	33.6	56.1	33.9	26.0	26.1	46.7
Ethiopia	ETH	18.1	21.9	35.4	59.1	35.7	27.3	27.5	48.8
Fiji	FJI	21.3	25.8	40.9	67.7	41.0	31.4	31.5	54.9
Finland	FIN	16.3	19.7	30.2	50.6	30.6	23.5	23.6	42.9
France	FRA	11.3	13.7	21.0	35.6	21.4	16.6	16.6	32.3
Georgia	GEO	24.0	29.2	46.4	76.8	46.6	35.4	35.6	61.1
Germany	DEU	11.1	13.4	20.3	34.4	20.7	16.0	16.1	31.5
Greece	GRC	16.7	20.2	31.8	53.5	32.3	24.7	24.9	44.8
Guatemala	GTM	15.0	18.2	27.6	46.0	27.8	21.5	21.6	39.8
Honduras	HND	20.0	24.2	37.8	62.1	37.7	29.0	29.0	51.3
Hungary	HUN	11.5	13.9	21.2	35.9	21.6	16.7	16.8	32.5
Iceland	ISL	24.5	29.8	47.2	78.1	47.5	36.0	36.2	62.1
India	IND	9.1	11.0	16.6	28.3	17.0	13.2	13.3	27.2
Indonesia	IDN	13.6	16.5	25.9	43.4	26.2	20.3	20.3	37.9
Iran	IRN	11.9	14.4	20.7	34.9	21.0	16.3	16.4	31.9
Ireland	IRL	16.2	19.6	29.4	49.0	29.6	22.9	22.9	41.9
Israel	ISR	19.9	24.0	37.6	62.2	37.7	28.9	29.0	51.2
Italy	ITA	11.6	14.1	21.4	36.4	21.9	16.9	17.0	32.9
Jamaica	JAM	21.6	26.1	41.8	68.9	41.8	32.0	32.1	55.9
Japan	JPN	7.1	8.6	12.8	21.8	13.1	10.2	10.3	22.6
Jordan	JOR	20.1	24.4	38.5	64.0	38.8	29.6	29.8	52.3
Kyrgyzstan	KGZ	21.1	25.6	40.2	66.7	40.4	30.8	31.0	54.1
Lesotho	LSO	16.0	19.3	30.5	50.6	30.6	23.6	23.7	43.1
Madagascar	MDG	19.1	23.2	36.5	60.9	36.9	28.2	28.3	50.1
Malawi	MWI	22.1	26.9	40.8	68.0	41.2	31.3	31.5	54.8
Maldives	MDV	25.4	30.9	45.8	76.1	46.2	35.0	35.2	60.5
Mauritius	MUS	23.5	28.5	44.4	73.8	44.8	34.0	34.2	59.0
Mexico	MEX	6.5	7.9	12.0	20.8	12.5	9.7	9.7	21.8
Morocco	MAR	20.0	24.3	38.4	63.9	38.7	29.6	29.7	52.2
Mozambique	MOZ	22.1	26.8	42.7	71.0	43.1	32.7	32.9	57.1
Nepal	NPL	15.8	19.2	29.3	48.8	29.5	22.8	22.8	41.8
New Zealand	NZL	17.4	21.1	32.9	54.7	33.1	25.5	25.6	45.8
Pakistan	PAK	12.0	14.5	21.1	35.6	21.5	16.6	16.7	32.4

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Table F.1 – Continued

Country	ISO	Model 1		Model 2		Model 3	Model 4	Model 5	
		Dyn	**SS**	**Dyn**	**SS**	**Dyn**	**Dyn**	**Dyn**	**SS**
Paraguay	PRY	21.5	26.0	40.8	67.4	40.9	31.3	31.4	54.8
Peru	PER	14.9	18.0	27.6	46.2	27.9	21.5	21.6	39.8
Philippines	PHL	16.0	19.3	29.3	48.8	29.5	22.8	22.9	41.8
Poland	POL	14.0	17.0	26.0	43.8	26.4	20.4	20.4	38.1
Portugal	PRT	16.3	19.7	30.3	50.7	30.7	23.6	23.7	43.0
Rep. of Korea	KOR	13.3	16.1	23.9	40.0	24.2	18.8	18.8	35.6
Rep. of Moldova	MDA	20.9	25.3	40.0	66.8	40.5	30.8	31.0	54.0
Romania	ROU	17.4	21.1	32.4	54.2	32.8	25.1	25.3	45.4
Russian Federation	RUS	12.5	15.2	23.4	39.6	23.9	18.4	18.5	35.1
Rwanda	RWA	11.8	14.3	22.0	37.3	22.5	17.4	17.4	33.5
Saint Vincent and the Grenadines	VCT	28.1	34.2	56.0	92.7	56.4	42.3	42.7	71.9
Sao Tome and Principe	STP	20.8	25.3	39.4	65.8	39.9	30.3	30.5	53.4
Senegal	SEN	23.5	28.5	46.2	76.8	46.6	35.3	35.5	61.0
South Africa	ZAF	15.3	18.6	29.3	49.0	29.6	22.8	22.9	41.8
Southeast Europe	SEE	17.4	21.1	32.4	54.4	32.9	25.2	25.3	45.5
Spain	ESP	11.7	14.2	21.4	36.3	21.9	16.9	17.0	32.8
Sri Lanka	LKA	17.4	21.1	32.1	53.6	32.4	24.9	25.0	45.1
Sweden	SWE	14.8	17.9	27.7	46.5	28.1	21.6	21.7	40.0
Switzerland	CHE	15.4	18.7	28.4	47.5	28.7	22.1	22.2	40.8
TFYR of Macedonia	MKD	20.3	24.6	38.4	64.2	38.9	29.6	29.8	52.3
Thailand	THA	17.4	21.0	32.2	53.6	32.4	25.0	25.0	45.1
Tunisia	TUN	21.2	25.7	40.3	67.1	40.7	31.0	31.1	54.4
Turkey	TUR	14.4	17.5	26.3	44.4	26.8	20.6	20.7	38.5
USA	USA	5.7	6.9	10.6	18.3	11.0	8.5	8.5	20.0
Uganda	UGA	11.8	14.3	22.0	37.4	22.5	17.4	17.5	33.5
Ukraine	UKR	15.9	19.3	30.3	50.9	30.7	23.6	23.7	43.0
United Kingdom	GBR	12.7	15.4	23.4	39.5	23.8	18.4	18.5	35.1
United Rep. of Tanzania	TZA	22.5	27.3	43.7	72.7	44.1	33.5	33.7	58.1
Uruguay	URY	19.0	23.0	36.6	60.9	36.9	28.3	28.4	50.1
Venezuela	VEN	15.5	18.7	29.1	48.4	29.2	22.6	22.7	41.5
Viet Nam	VNM	19.7	23.9	36.3	60.2	36.5	28.0	28.1	49.7
Yemen	YEM	22.1	26.9	41.9	69.3	42.0	32.1	32.2	56.0

Note: "Dyn" refers to dynamic gains and "SS" refers to steady-state gains. Model 1 is the model with exogenous nominal investment rate, fixed relative price of investment. Model 2 adds the endogenous relative price of investment to model 1. Model 3 (baseline) adds the endogenous nominal investment rate to Model 2. Model 4 adds adjustment costs to capital accumulation to Model 3. Model 5 adds the endogenous trade imbalances to Model 4. Steady-state gains are identical in Models 2,3, and 4. The group "Southeast Europe" is an aggregate of Albania, Bosnia and Herzegovina, Croatia, Montenegro, and Serbia.