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Unifying Macro Elasticities in International Economics*

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Abstract

International trade studies have higher macro (Armington) elasticity measures compared to international finance studies. This observation has evoked not only mixed policy implications regarding tariffs and exchange rates but also mixed welfare gains from trade. Regarding the policy implications, this so-called *international elasticity puzzle* is solved in this paper by distinguishing between elasticities of substitution and price elasticities of demand that are connected to each other through expenditure shares. It is shown theoretically and confirmed empirically that the macro elasticity in international trade is a weighted average of the macro elasticity in international finance and the elasticity of substitution across products of foreign countries. It is implied that one can always find an elasticity of substitution across foreign countries that would be consistent with different macro elasticities in the two literatures; therefore, the puzzle is something artificial due to the way that the foreign products are aggregated at destination countries. Regarding the welfare gains from trade, the two literatures are shown to have the very same implications when international finance studies have a unitary macro elasticity of substitution between home and foreign products or unitary terms of trade. As opposed to the existing literature that has offered many supply-side solutions to the puzzle, the results in this paper are independent of the supply side and thus are consistent with any production structure.

JEL codes: F12, F14, F41

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1 Introduction

International trade studies have higher macro (Armington) elasticity measures compared to international finance studies. Since price changes are converted into quantity changes and thus real effects through these elasticities, this observation has evoked mixed policy implications regarding tariffs and exchange rates in the two literatures (e.g., see Ruhl (2008)). Moreover, since welfare gains from trade are directly connected to these macro elasticity measures (as in Arkolakis, Costinot, and Rodríguez-Clare (2012)), this observation has also evoked mixed welfare gains between the two literatures. Due these mixed implications, this observation has been called the *international elasticity puzzle*.

In order to have a better idea about the magnitude of this puzzle, consider a short summary of studies given in Table 1. Although elasticity measures differ across these studies, international finance studies mostly follow Backus, Kehoe, and Kydland (1994) with a macro elasticity value of about 1.5, while international trade studies mostly follow Anderson and Van Wincoop (2004) or recently Simonovska and Waugh (2014a) and Simonovska and Waugh (2014b) with a macro elasticity value of about 5.[1] It is implied that if we directly employ these numbers in a policy analysis, say, in order to investigate the effects of a foreign price change due to tariffs or exchange rates, international trade studies imply quantity changes that are at least three times the international finance studies. Similarly, if we use the formula for the welfare gains from trade (WGT) as introduced by Arkolakis, Costinot, and Rodríguez-Clare (2012), which is $WGT = (\text{Home Expenditure Share})^{\frac{1}{1-(\text{Macro Elasticity})}}$, international finance studies imply welfare gains (in percentage terms) that are about eight times the international trade studies (for any given home expenditure share).

This paper unifies these macro elasticities that lead into mixed results in the two literatures. Since the upper-tier aggregation is achieved across source countries (including home country) in international trade studies, and it is achieved across home and foreign products in international finance studies, the two literatures are connected to each other by an additional tier of aggregation across different foreign countries in international finance. Although such an additional tier is missing in international finance studies, it is well understood to exist in the background.

Within this framework, we show that there is no connection between the macro elasticities of the two literatures when expenditure shares are negligible in the calculation of

[1] Since the trade elasticity used in new trade models, such as Eaton and Kortum (2002), corresponds to the elasticity of substitution across countries (including home country) minus one in international trade as shown in studies such as by Anderson and Van Wincoop (2004), the commonly used trade elasticity of about 4 suggested by Simonovska and Waugh (2014a) and used by new trade models corresponds to the elasticity of substitution across countries of about 5.

price elasticities. The tables turn when such expenditure shares are taken into account in this paper, which results in having price elasticities of demand connected to elasticities of substitution through such expenditure shares. We show that such a strategy helps us understand several differences across studies in the literature regarding the elasticity measures such as the differences due to simulating versus estimating, differences due to the level of disaggregation (e.g., having different digits of data), and differences between long-run and short-run elasticity measures. More importantly, such a strategy also allows us connect macro elasticities in the two literatures through the elasticity of substitution across different foreign countries in international finance that is newly introduced in this paper. In particular, when expenditure shares are considered, we show that the macro elasticity in international trade is a weighted average of the macro elasticity in international finance and the elasticity of substitution across products of foreign countries. It is implied that one can always find an elasticity of substitution across foreign countries that would be consistent with different macro elasticities in the two literatures; therefore, the puzzle is solved theoretically. Then, how can one make sense of the mixed policy implications and mixed welfare gains from trade implied by the two literatures? We focus on this question next.

From a researcher's or a policy maker's perspective, since policy implications for any individual good coming from an individual foreign country should not depend on how foreign goods are artificially aggregated at the destination country, we equalize the micro price elasticities of demand (depending on expenditure shares) between the two literatures to show theoretically and confirm empirically that the elasticity of substitution across different foreign countries plays an important role in the determination of the puzzle. Therefore, the policy implications at the individual foreign good level are automatically equalized, although the artificially created upper-level elasticities are allowed to change between the two literatures; this inductive approach is different from the deductive approach followed by the existing literature, where the upper-level variables (e.g., utility) are taken as given, while lower-level variables are allowed to change. Since policy implications are equalized between the two literatures for each and every foreign good, the *international elasticity puzzle* disappears at the disaggregated micro level. It is implied that the policy analysis should be first achieved at the disaggregated micro level and then aggregated up to obtain macro implications; such a strategy has also been used by Imbs and Mejean (2015) who have calibrated macro elasticities using a weighted average of sector elasticities.

After solving the puzzle due to its policy implications at the micro level, we continue with focusing on different implications by the two literatures regarding the welfare gains from trade at the macro level (which also correspond to the macro-level effects of a foreign shock in home country). Rather than using the simplified version of the formula given by

Arkolakis, Costinot, and Rodríguez-Clare (2012), we consider the full definition of welfare gains (measured by the costs of autarky) in order to have a comparison between the two literatures by searching for aggregate price indices that would have to adjust to keep the consumer utility the same between the current openness to trade and a hypothetical autarky. We show that the two literatures have the very same welfare gains from trade when there is unitary macro elasticity of substitution between home and foreign products or when there is unitary terms of trade in international finance. Since several international finance studies already employ unitary elasticity of substitution (as in studies such as by Stockman and Tesar (1995) and Corsetti and Pesenti (2001), for which Bergin (2006) has even provided empirical evidence) or unitary terms of trade while defining their steady states (as in studies such as by Devereux and Engel (2002), Corsetti and Dedola (2005) or Gali and Monacelli (2005)), welfare gains are already equalized across the two literatures and the *international elasticity puzzle* is solved at the macro level as well.

The theoretical framework in this paper is closest to the study by Feenstra, Luck, Obstfeld, and Russ (2014) who have three tiers of aggregation in their CES framework; the disaggregation is across goods in the upper-tier, across home and foreign products in the middle-tier, and across foreign sources in the lower tier. They call their middle-tier elasticity (across home and foreign products) as the "macro" elasticity, while they call their lower-tier elasticity (across foreign source countries) as the "micro" elasticity. They consider these "macro" and "micro" elasticities as the elasticities used in international finance and international trade studies, respectively, which they estimate at the good level and show that their "macro" elasticity is higher than their "micro" elasticity only for half of the goods investigated; hence, they have a good-level investigation while comparing their "macro" and "micro" elasticities. However, this aggregation strategy is not consistent with either international trade or international finance studies, where the former aggregates across source countries in the upper-tier (e.g., see Anderson and Van Wincoop (2003), Anderson and Van Wincoop (2004), Head and Ries (2001), Hillberry and Hummels (2013), or Hummels (2001), among many others), and the latter aggregates across home and foreign countries in the upper-tier (e.g., see Backus, Kehoe, and Kydland (1994), Blonigen and Wilson (1999), Corsetti, Dedola, and Leduc (2008), Enders, Müller, and Scholl (2011), or Heathcote and Perri (2002), among many others). Since the international elasticity puzzle is about the comparison of these upper-tier macro elasticities in the two literatures (rather than the middle-tier or the lower-tier), the aggregation strategy used by Feenstra, Luck, Obstfeld, and Russ (2014) is useless to address this puzzle.

In contrast, our aggregation strategy successfully employs the upper-tier macro elasticities as they are exactly used in the two literatures.[2]

It is important to emphasize that the results in this paper are independent of the supply side and thus are consistent with any production structure in the literature. In contrast, the existing literature has focused on many solutions to the puzzle based on the supply side. For example, Ruhl (2008) has proposed a solution based on firm-level entry costs and uncertainties on future productivities in a Melitz (2003) framework; Fitzgerald and Haller (2014) have both fixed and sunk costs of export participation, where participation in different export markets are considered as independent decisions after conditioning on a common marginal cost of production; Crucini and Davis (2016) consider the speed of adjustment of capital in the distribution sector; Ramanarayanan (2015) considers intermediate inputs in which heterogeneous producers face a plant-level irreversibility in the structure of inputs used in production; Arkolakis, Eaton, and Kortum (2012) consider the difference between the adjustments in extensive and intensive margins of trade in an Eaton and Kortum (2002) framework. Accordingly, it is implied by the demand-side investigation in this paper that we do not need such supply-side complications in order to understand the puzzling difference between macro elasticities of the two literatures. The puzzle is rather something artificial due to the way that the foreign products are aggregated at destination countries.

The next section introduces the demand-side model where we distinguish between international trade and finance individuals. Section 3 derives price elasticity of demand measures by taking expenditure shares into account, rather than neglecting them as in existing studies. Section 4 connects the two literatures by using the fact that the price elasticities of demand at the micro level should be the same in the two literatures, independent of how foreign products are artificially aggregated at the destination country. Section 5 solves the *international elasticity puzzle* by considering the importance of expenditure shares and searching for conditions under which the two literatures imply the very same welfare gains from trade. Section 6 concludes.

[2] Although it would not be consistent with the literatures considered, even if we would consider the good-level investigation in Feenstra, Luck, Obstfeld, and Russ (2014) as an alternative good-level explanation to the international elasticity puzzle, comparing their "macro" elasticity with their "micro" elastictiy does not correspond to comparing the aggregation across home and foreign countries in international finance with the aggregation across all source countries in international finance. Moreover, since their empirical results highly depend on their production structure, their solution to the puzzle would also depend on their supply side; in constrast, the results in this paper are consistent with any production framework.

2 The Economic Environment

This section introduces a model of international economics consisting of home and foreign countries. It is important to emphasize that the model and its implications are independent of the supply side (i.e., the investigation in this paper is consistent with any supply structure in the literature); therefore, we only focus on the demand side in this paper. In terms of the notation, the superscripts represent the location of consumption, while the subscripts represent the location of production and goods.

2.1 Individuals

At the macro level, there is usually a unique foreign country in international finance studies, while there are multiple foreign countries/regions in trade studies. Accordingly, both of these literatures connect foreign products to home products by using an upper-tier aggregation, although the number of foreign countries are different across the two literatures. Therefore, while products coming from alternative foreign countries are already connected to each other through this upper-tier aggregation in international trade, such alternative foreign countries are not distinguished between each other in international finance. In order to unite these two literatures, we connect the products of each *foreign* country in international trade to the foreign products coming from the rest of the world (ROW) in international finance by using an additional middle tier of aggregation while modeling individual utilities in international finance.

The lower-tier aggregation is achieved across alternative goods coming from a particular country, which is the same between the two literatures, because the products coming from any country does not depend on how they are aggregated at the destination country. In sum, international trade individuals have two tiers of aggregation, representing source countries and goods, while international finance individuals have three tiers of aggregation, representing home versus foreign countries, foreign countries, and goods. The details of each literature are provided in the following subsections, while the definition of variables and parameters are given in Tables 2 and 3.

2.1.1 Individuals in International Trade

International trade studies such as by Anderson and Van Wincoop (2004), Head and Ries (2001), Hillberry and Hummels (2013), or Hummels (2001), among many others, have the following type of CES aggregation, also called the Armington model as in Arkolakis, Costinot,

and Rodríguez-Clare (2012), representing utility C^h in home country h:

$$C^h \equiv \left(\sum_i \left(\beta_i^h\right)^{\frac{1}{\theta}} \left(C_i^h\right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{1}$$

where C_i^h represents products coming from country i (which represents home products when $i = H$), θ is the elasticity of substitution across countries (including both home and foreign countries), and β_i^h is a taste parameter (satisfying $\sum_i \beta_i^h = 1$). C_i^h, which represents consumption of home products when $i = H$, is further given by the following expression:

$$C_i^h \equiv \left(\sum_j \left(\beta_{ij}^h\right)^{\frac{1}{\eta}} \left(C_{ij}^h\right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{2}$$

where C_{ij}^h represents good j coming from country i (which represents good j produced in home country when $i = H$), η is the elasticity of substitution across goods, and β_{ij}^h's represent taste parameters (satisfying $\sum_j \beta_{ij}^h = 1$). The optimal allocation of any given expenditure yields the following demand function:

$$C_{ij}^h = \beta_i^h \beta_{ij}^h \left(\frac{P_{ij}^h}{P_i^h} \right)^{-\eta} \left(\frac{P_i^h}{P^h} \right)^{-\theta} C^h \tag{3}$$

where P_{ij}^h, P_i^h and P^h and are the corresponding prices per units of C_{ij}^h, C_i^h and C^h, respectively, and they are connected to each other through the standard expressions of $P^h \equiv \left(\sum_i \beta_i^h \left(P_i^h\right)^{1-\theta} \right)^{\frac{1}{1-\theta}}$ and $P_i^h \equiv \left(\sum_j \beta_{ij}^h \left(P_{ij}^h\right)^{1-\eta} \right)^{\frac{1}{1-\eta}}$; once again, C_{ij}^h represents good j produced in home country when $i = H$.

2.1.2 Individuals in International Finance

International finance studies such as by Backus, Kehoe, and Kydland (1994), Blonigen and Wilson (1999), Corsetti, Dedola, and Leduc (2008), Enders, Müller, and Scholl (2011), or Heathcote and Perri (2002), among many others, have the following alternative type of CES aggregation representing utility G^h in home country h:

$$G^h \equiv \left(\left(\alpha_H^h\right)^{\frac{1}{\sigma}} \left(C_H^h\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\alpha_H^h\right)^{\frac{1}{\sigma}} \left(G_F^h\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where C_H^h (as in Equation 2 when $i = H$) and G_F^h represent home and foreign/ROW products, respectively, σ is the elasticity of substitution across home and foreign products, and α_H^h is a taste parameter representing the preferences of individuals toward home products (that is

related to home bias). Most international finance studies stop their disaggregation at this level which they use for their investigations. Nevertheless, it is understood that there is an additional tier of aggregation among foreign countries in the background, which we achieve by putting more structure on the index of foreign products G_F^h as follows:

$$G_F^h \equiv \left(\sum_{i \neq h} \left(\alpha_i^h\right)^{\frac{1}{\gamma}} \left(C_i^h\right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{5}$$

where C_i^h represents products imported from foreign country i as in the international trade literature above (as defined in Equation 2), because the products coming from foreign country i do not depend on how they are further aggregated at the destination country; γ is the elasticity of substitution across foreign countries, and α_i^h's (satisfying $\sum_{i \neq h} \alpha_i^h = 1$) represent source-specific taste parameters.

The optimal allocation of any given expenditure yields the following demand function for good j coming from foreign country i:

$$C_{ij}^h = \left(1 - \alpha_H^h\right) \alpha_i^h \beta_{ij}^h \left(\frac{P_{ij}^h}{P_i^h}\right)^{-\eta} \left(\frac{P_i^h}{Q_F^h}\right)^{-\gamma} \left(\frac{Q_F^h}{Q^h}\right)^{-\sigma} G^h \tag{6}$$

where Q_F^h and Q^h represent prices per units of G_F^h and G^h, respectively, and they are connected to each other through the standard expressions of $Q^h \equiv \left(\alpha_H^h \left(P_H^h\right)^{1-\sigma} + \left(1 - \alpha_H^h\right) \left(Q_F^h\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$ and $Q_F^h \equiv \left(\sum_{i \neq h} \alpha_i^h \left(P_i^h\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$; the definitions of β_{ij}^h, P_{ij}^h, P_i^h, P_H^h and η are the same as in the international trade literature. Similarly, the demand for good j produced at home is given by:

$$C_{Hj}^h = \alpha_H^h \beta_{Hj}^h \left(\frac{P_{Hj}^h}{P_H^h}\right)^{-\eta} \left(\frac{P_H^h}{Q^h}\right)^{-\sigma} G^h \tag{7}$$

where the price indices of home products P_H^h's are also the same across the two literatures, since they are independent of how foreign products are aggregated.

3 Price Elasticity of Demand

In any CES aggregation, the elasticity of substitution corresponds to the price elasticity of demand if the expenditure share of components in that aggregation is negligible. However, this may not be the case, especially for the elasticity of substitution across home and foreign

products, where the expenditure share of foreign products may not be negligible in a global world with high levels of trade openness.

Accordingly, while calculating the price elasticity of demand in home country h, on top of the elasticity of substitution, we also consider the effects of disaggregated prices on aggregated prices. In international trade, the price elasticity of demand for good j coming from country i (representing home products when $i = H$) is implied as follows:

$$-\frac{\partial C_{ij}^h}{\partial P_{ij}^h}\frac{P_{ij}^h}{C_{ij}^h} = \varepsilon\left(\eta, \theta, \omega_{ij}^h, \omega_i^h\right) = \eta\left(1-\omega_{ij}^h\right) + \theta\omega_{ij}^h\left(1-\omega_i^h\right) \tag{8}$$

where we have considered the effects of P_{ij}^h on aggregated price indices of P_i^h and P^h as well; $\omega_{ij}^h = \frac{P_{ij}^h C_{ij}^h}{P_i^h C_i^h}$ is the expenditure share of good j imported from country i among all products imported from country i, and $\omega_i^h = \frac{P_i^h C_i^h}{P^h C^h}$ is the expenditure share of country i products in the overall consumption.

Similarly, in international finance, the price elasticity of demand for good j coming from foreign country i is implied as follows:

$$-\frac{\partial C_{ij}^h}{\partial P_{ij}^h}\frac{P_{ij}^h}{C_{ij}^h} = \varepsilon\left(\eta, \gamma, \sigma, \omega_{ij}^h, \chi_i^h, \chi_F^h\right) = \eta\left(1-\omega_{ij}^h\right) + \gamma\omega_{ij}^h\left(1-\chi_i^h\right) + \sigma\chi_i^h\omega_{ij}^h\left(1-\chi_F^h\right) \tag{9}$$

where we have considered the effects of P_{ij}^h on aggregated price indices of P_i^h, Q_F^h and Q^h as well; $\chi_i^h = \frac{P_i^h C_i^h}{Q_F^h G_F^h}$ is the expenditure share of country i products among all foreign products, and $\chi_F^h = \frac{Q_F^h G_F^h}{Q^h G^h} = 1 - \chi_H^h$ is the expenditure share of foreign products (i.e., imports) in the overall consumption, with χ_H^h representing the home expenditure share. Again in international finance, the price elasticity of demand for good j produced at home country is implied as follows:

$$-\frac{\partial C_{Hj}^h}{\partial P_{Hj}^h}\frac{P_{Hj}^h}{C_{Hj}^h} = \varepsilon\left(\eta, \sigma, \omega_{Hj}^h, \chi_H^h\right) = \eta\left(1-\omega_{Hj}^h\right) + \sigma\omega_{Hj}^h\left(1-\chi_H^h\right) \tag{10}$$

where $\omega_{Hj}^h = \frac{P_{Hj}^h C_{Hj}^h}{P_H^h C_H^h}$.

According to these expressions, the price elasticity of demand is connected to the elasticities of substitution through the corresponding expenditure shares, which are different across the two literatures due to the way that foreign products are aggregated. It follows that in a special case in which the corresponding consumption shares all go to zero (i.e., as the expenditure shares become negligible within the aggregation), the price elasticity in both international trade and finance literatures converges to $\varepsilon = \eta$, as is standard in the literature.

It is important to emphasize that according to the model in this paper, the elasticity of substitution measures of η, γ and σ are the ones that are used in simulations in the literature, while the price elasticity measures of $\varepsilon\left(\eta,\theta,\omega_{ij}^{h},\omega_{i}^{h}\right)$ and $\varepsilon\left(\eta,\gamma,\sigma,\omega_{ij}^{h},\chi_{i}^{h},\chi_{F}^{h}\right)$ are the ones that are estimated in empirical international trade and finance studies, respectively. Accordingly, the difference between the elasticity of substitution and the price elasticity of demand can be used to understand several differences across studies in the literature regarding the elasticity measures.

3.1 Implications for Macro-Level Elasticities

A typical macro-level analysis is achieved when the good dimension is ignored in both literatures. Accordingly, in a special case in international trade when $\omega_{ij}^{h}=1$, which suggests that the good dimension is ignored (or there is only one product coming from country i), the price elasticity reduces to $\varepsilon\left(\eta,\theta,\omega_{ij}^{h},\omega_{i}^{h}\right)=\theta\left(1-\omega_{i}^{h}\right)$ that depends on the expenditure share of ω_{i}^{h} (for products coming from i). Similarly, when $\omega_{ij}^{h}=\chi_{i}^{h}=1$ in international finance, which suggests that we only have home and foreign products in the model, the price elasticity reduces to $\varepsilon\left(\eta,\gamma,\sigma,\omega_{ij}^{h},\chi_{i}^{h},\chi_{F}^{h}\right)=\sigma\left(1-\chi_{F}^{h}\right)$ that depends on the foreign expenditure share of χ_{F}^{h}. If we aggregate the expression of price elasticity across source countries in international trade when $\omega_{ij}^{h}=1$, we obtain:

$$\theta=\frac{\sum_{i}\varepsilon\left(\eta,\theta,\omega_{ij}^{h},\omega_{i}^{h}\right)}{N_{i}-1}$$

since $\sum_{i}\omega_{i}^{h}=1$, where N_{i} represents the number of foreign countries. Similarly, if we aggregate the expression of price elasticity across home and foreign countries in international finance when $\omega_{ij}^{h}=\chi_{i}^{h}=1$, we obtain:

$$\sigma=\varepsilon\left(\eta,\gamma,\sigma,\omega_{ij}^{h},\chi_{i}^{h},\chi_{F}^{h}\right)+\varepsilon\left(\eta,\sigma,\omega_{Hj}^{h},\chi_{H}^{h}\right)$$

since $\chi_{H}^{h}+\chi_{F}^{h}=1$. Therefore, when the dimension of goods is ignored in a macro-level investigation (i.e., $\omega_{ij}^{h}=\chi_{i}^{h}=1$), the elasticities of substitution in both literatures are simply the sum of price elasticities (divided by number of foreign countries in the upper-tier minus one). Accordingly, for example, in the case of the elasticity of substitution σ across home and foreign products in international finance, one cannot directly calculate σ by estimating the price elasticity of foreign products $\varepsilon\left(\eta,\gamma,\sigma,\omega_{ij}^{h},\chi_{i}^{h},\chi_{F}^{h}\right)$ but also needs the price elasticity of home products $\varepsilon\left(\eta,\sigma,\omega_{Hj}^{h},\chi_{H}^{h}\right)$. This is consistent with the observations in the literature, where the simulated elasticities of around $\sigma=1.5$ in macro-level studies such as by Backus,

Kehoe, and Kydland (1994) are much higher compared to the estimated price elasticities of foreign products $\varepsilon\left(\eta,\gamma,\sigma,\omega_{ij}^{h},\chi_{i}^{h},\chi_{F}^{h}\right)$ ranging between 0.81 and 0.90 in macro-level studies such as by Blonigen and Wilson (1999), Corsetti, Dedola, and Leduc (2008), or Heathcote and Perri (2002). It is implied that the difference between σ and $\varepsilon\left(\eta,\gamma,\sigma,\omega_{ij}^{h},\chi_{i}^{h},\chi_{F}^{h}\right)$ simply corresponds the price elasticity of home products in this paper $\varepsilon\left(\eta,\sigma,\omega_{Hj}^{h},\chi_{H}^{h}\right)$ that is ignored in the literature due to *negligible* expenditure shares.

3.2 Implications for Micro-Level Elasticities

As discussed in studies such as by Broda and Weinstein (2006), Hummels (2001) and Mc Daniel and Balistreri (2003), among many others, one robust finding in the empirical literature is that more disaggregate analyses correspond to higher elasticities; i.e., as the data used have goods represented at higher-digits, the corresponding elasticity measures increase. Since any disaggregation requires a micro-level investigation, we directly consider our micro price elasticity expressions in Equations 8 and 9 in this subsection.

In particular, for international trade, if we take the summation across i and j in Equation 8, we can obtain an expression for the average price elasticity as follows:

$$\frac{\sum_{i}\sum_{j}\varepsilon\left(\eta,\theta,\omega_{ij}^{h},\omega_{i}^{h}\right)}{\left(N_{i}+1\right)N_{j}}=\frac{\eta\left(N_{j}-1\right)}{N_{j}}+\frac{\theta}{\left(N_{i}+1\right)N_{j}} \tag{11}$$

where N_i represents the number of foreign countries as above (hence, N_i+1 is the total number of source countries including home country), and N_j represents the number of goods. Since having a more disaggregated analysis (i.e., having higher-digits of products) corresponds to having a higher number of goods N_j, we can directly observe how the average price elasticity changes with respect to the number of goods N_j by using the following derivative:

$$\frac{\partial\left(\frac{\sum_{i}\sum_{j}\varepsilon\left(\eta,\theta,\omega_{ij}^{h},\omega_{i}^{h}\right)}{\left(N_{i}+1\right)N_{j}}\right)}{\partial N_{j}}=\frac{\left(N_{i}+1\right)\left(\eta-\theta\right)+\theta}{\left(N_{i}+1\right)\left(N_{j}\right)^{2}} \tag{12}$$

which would take a positive value as long as $\eta \geq \theta$. Therefore, the average price elasticity of demand would increase with the number of goods (representing higher-digits of products) as long as $\eta \geq \theta$. Since the elasticity of substitution η (across goods) measures used in simulations are generally about 10 as in Coibion and Gorodnichenko (2011) or Ferrero (2009) and the elasticity of substitution θ (across source countries) are about 5 as in Anderson and Van Wincoop (2004), Simonovska and Waugh (2014a) or Simonovska and Waugh (2014b), on average across studies, we already have $\eta \geq \theta$ in the literature. Hence, we can safely claim

that the average price elasticity of demand increases in this paper with the number of goods (representing higher-digits of products) due to considering the role of expenditure weights on the price elasticity of demand that is neglected in the literature. A similar investigation can also be achieved by using Equation 9.

3.3 Implications for Time Horizon: Long-run versus Short-run Elasticities

As discussed in studies such as by Drozd and Nosal (2012), Gallaway, McDaniel, and Rivera (2003) and Mc Daniel and Balistreri (2003), another robust finding in the empirical literature is that long-run estimates are higher than short-run estimates. The difference between long-run and short-run is determined by the time horizon that corresponds to using annual, quarterly or monthly data in empirical studies.

It is important to emphasize that the elasticity of substitution measures (η, γ and σ) used in this paper do not depend on the time horizon, however the estimated price elasticity measures of $\varepsilon\left(\eta, \theta, \omega_{ij}^{h}, \omega_{i}^{h}\right)$ and $\varepsilon\left(\eta, \gamma, \sigma, \omega_{ij}^{h}, \chi_{i}^{h}, \chi_{F}^{h}\right)$ may well change due to the time horizon, especially when there are zero-trade observations in high frequency data (such as monthly data). Since the average price elasticity of demand is always given by Equation 11 in international trade, independent of the time horizon, one may expect to have the very same average price elasticity measure independent of the time dimension as well. However, if there are zero-trade observations that are ignored in the calculation of the average price elasticity, certain changes may be observed between long-run and short-run average price elasticity of demand.

In order to see the severity of this problem, consider 3-digit NAICS bilateral CIF imports of the U.S. with its 237 trade partners in 2015 using monthly data.[3] Even at this high level of aggregation, the percentage of monthly zero-trade observations to overall number of monthly observations is about 57%.[4] When monthly data are aggregated across months within each quarter of 2015, the percentage of quarterly zero-trade observations to overall number of quarterly observations is about 49%.[5] When monthly data are aggregated across all months within 2015, the percentage of annual zero-trade observations to overall number of annual observations is about 39%.[6] Therefore, the ratio of zero-trade observations increases with the frequency of the data (or decreases with the time horizon); i.e., there is a higher ratio of zero-trade observations in the short-run.

[3] Such monthly data have been obtained from https://usatrade.census.gov/.

[4] In particular, 54,904 out of 96,696 bilateral trade observations take a value of zero in this monthly sample.

[5] In particular, 15,907 out of 32,232 bilateral trade observations take a value of zero in this annual sample.

[6] In particular, 3,127 out of 8,058 bilateral trade observations take a value of zero in this annual sample.

According to our model, such good-level zero-trade observations correspond to expenditure weights of $\omega_{ij}^h = 0$, according to which the micro price elasticity of demand reduces to η. Therefore, whenever there is a good-level zero-trade observation, the corresponding price elasticity of demand η should enter into the calculation of the average price elasticity expression in Equation 11. However, when zero-trade observations are ignored, such η measures do not enter into this calculation. Since we have $\eta \geq \theta$ on average across studies in the literature (as discussed above), the exclusion of η's (capturing zero-trade observations) would result in a lower average price elasticity in the short-run due to the higher ratio of zero-trade observations in the short-run.

In order to show our claim formally, for international trade, we calculate the micro price elasticity of demand by using Equation 8, where we follow the literature by setting $\eta = 10$ and $\theta = 5$ (as discussed above). We again employ the monthly 3-digit NAICS bilateral CIF imports of the U.S. with its 237 trade partners within 2015 (as introduced above). The empirical results show that when zero-trade observations are taken into account, the average price elasticity is found as 9.85 (for each month, quarter or year), independent of using the monthly, quarterly or annual version of the data.[7] However, when we ignore zero-trade observations, the average price elasticity is calculated as 9.67 (across months), 9.72 (across quarters) and 9.76, for which monthly, quarterly and annual version of the data are used, respectively. Therefore, even at this level of aggregation, the short-run price elasticity measures are in fact lower than long-run price elasticity measures when zero-trade observations are ignored.

When we replicate this analysis by using the 4-digit NAICS version of the very same data set, the average price elasticity is found as 9.95 (for each month, quarter or year) when zero-trade observations are included, independent of using the monthly, quarterly or annual version of the data. Similar as above, when we ignore zero-trade observations, the average price elasticity is calculated as 9.85 (across months), 9.88 (across quarters) and 9.90, for which monthly, quarterly and annual version of the data are used, respectively. This empirical result confirms not only that the short-run price elasticity measures are lower than long-run price elasticity measures when zero-trade observations are ignored at this level of aggregation but also that more disaggregate analyses correspond to higher elasticities (in reference to the previous subsection).

It is implied that the average price elasticity of demand measures taking lower values in the short-run may well be due to ignoring zero-trade observations in the literature, since

[7] Monthly observations of import values are easily converted into quarterly and annual observations by taking the sum across the corresponding months. The good-level average price elasticity is calculated by pooling across foreign countries.

the corresponding studies mentioned above do not even discuss the existence of zero-trade observations. One more time, this paper has achieved this result due to considering the role of expenditure weights on the price elasticity of demand that is neglected in the literature. A similar investigation can also be achieved for international finance by using the implications of Equation 9 for the average price elasticity.

Since the elasticity of substitution measures (η, γ and σ) used in this paper do not depend on the time horizon as we have just shown, for the rest of this paper, we will focus on how international trade and finance literatures are connected to each other regarding these measures by using the static framework that we have introduced so far.[8]

4 Connecting the Two Literatures

Although the two literatures aggregate across foreign products in alternative ways, from the perspective of an empirical researcher (i.e., when models are matched with data), it must be the case that the total expenditure E^h in country h should be the same in each literature, i.e., $E^h = P^h C^h = Q^h G^h$. Similarly, the imported products coming from a certain foreign country should also match up between the two literatures; e.g., the existence of good j coming from country i does not depend on how goods and foreign countries are artificially aggregated in destination country h. Accordingly, for foreign products, the expenditure shares in the two literatures are connected to each other by $\omega_i^h = \chi_i^h \chi_F^h$ and $\chi_F^h = \left(\sum_{k \neq h} \omega_k^h\right)$ for all i, while the expenditure shares for home products in the two literatures are connected to each other by $\omega_H^h = \chi_H^h$.

In order to have the very same micro implications, we bridge the gap between the two literatures by considering the equality of the price elasticity of demand for individual goods coming from individual foreign countries. In particular, since the left hand sides of Equations 8 and 9 are the same due to their definitions, it is implied that:

$$\theta = \gamma \phi_i^h + \sigma \left(1 - \phi_i^h\right) \tag{13}$$

[8] Independent of the implications of our model regarding the elasticity of substitution not depending on the time horizon, both literatures have considered alternative time horizons anyways. For instance, while some international finance studies such as by Backus, Kehoe, and Kydland (1994), Bergin (2006), or Blonigen and Wilson (1999) employ quarterly data, other international finance studies such as by Corsetti, Dedola, and Leduc (2008) or Crucini and Davis (2016) employ annual data. Similarly, while some international trade studies such as by Anderson and Van Wincoop (2003), Eaton and Kortum (2002), Head and Ries (2001), Simonovska and Waugh (2014a) or Simonovska and Waugh (2014b) consider annual data, other international trade studies such as by Reinert and Roland-Holst (1992) or Shiells and Reinert (1993) consider quartlerly data.

where $\phi_i^h = \frac{1-\chi_i^h}{1-\chi_i^h\chi_F^h}$. It is important to emphasize that the elasticity of substitution across goods η has been effectively eliminated; therefore, the rest of our investigation does not depend on the value or the determination of η. It is implied that no matter what type of a production structure we have on the supply side, the results in the rest of this paper are not affected.

In Equation 13, if we take the summation across i on both sides, we obtain:

$$\theta = \gamma\Omega + \sigma\left(1-\Omega\right) \tag{14}$$

where $\Omega = \frac{N_i-1}{N_i-1+\chi_H^h}$ with N_i representing the number of foreign countries (as introduced, above). Therefore, the elasticity of substitution across countries in international trade θ is a weighted average of the elasticity of substitution across foreign countries in international finance γ and the elasticity of substitution across home and foreign countries in international finance σ, where weights are determined by the number of foreign countries N_i and the home expenditure share of χ_H^h.

It is easy to show that, when the number of foreign countries is one in the upper-tier, $N_i = 1$, as in the case of international finance literature, θ reduces to $\theta = \sigma$, independent of χ_H^h. On the other hand, as the number of foreign countries increases, as in the case of international trade where several bilateral-trade relationships are considered at the same time, θ approaches $\theta = \gamma$, again independent of χ_H^h. In another special case in which the home expenditure share goes to zero, $\chi_H^h = 0$, as is standard in the existing literature that neglects expenditure shares, θ approaches $\theta = \gamma$. The latter special case is essential to understand in order to see the contribution of this paper, because it directly suggests that the trade elasticity of substitution θ and the finance elasticity of substitution σ are independent of each other when expenditure shares are neglected; nevertheless, they are connected to each other through χ_H^h, γ and N_i when expenditure shares are taken into account as in this paper. It is implied that if we are not in one of the special cases that we just covered, as long as the elasticity of substitution across foreign countries in international finance γ is a number bigger than σ, we have $\theta > \sigma$, which would solve the international elasticity puzzle that we focus on next.

5 Solving International Elasticity Puzzle

In order to have a better idea about the severity of the international elasticity puzzle, we consider an alternative version of Equation 14 that is obtained after dividing both sides by

σ as follows:

$$\frac{\theta}{\sigma} = \frac{\gamma}{\sigma}\left(\frac{N_i - 1}{N_i - 1 + \chi_H^h}\right) + \left(\frac{\chi_H^h}{N_i - 1 + \chi_H^h}\right)$$

where the magnitude of the puzzle is measured by the left hand side ratio of θ/σ. As is evident, the magnitude of the puzzle depends on not only the value of γ, but also the values of N_i and χ_H^h. Accordingly, for given values of N_i and χ_H^h, the effects of γ on the puzzle can be investigated by considering the following derivative:

$$\frac{\partial\left(\theta/\sigma\right)}{\partial\gamma} = \frac{1}{\sigma}\left(\frac{N_i - 1}{N_i - 1 + \chi_H^h}\right)$$

which takes a value of zero when $N_i = 1$ (as in the case of upper-tier in international finance literature) and has a positive value when $N_i > 1$ (as in the case of upper-tier in international trade literature). Hence, the severity of the puzzle increases with the value of γ. Similarly, for given values of γ and N_i, the effects of home expenditure share χ_H^h is given by:

$$\frac{\partial\left(\theta/\sigma\right)}{\partial\chi_H^h} = -\left(\frac{\gamma}{\sigma} + 1\right)\left(\frac{N_i - 1}{\left(N_i - 1 + \chi_H^h\right)^2}\right)$$

which implies that the severity of the puzzle decreases with trade openness. Finally, for given values of γ and χ_H^h, the effects of the number of foreign countries N_i is given by:

$$\frac{\partial\left(\theta/\sigma\right)}{\partial N_i} = \left(\frac{\gamma}{\sigma} - 1\right)\left(\frac{\chi_H^h}{\left(N_i - 1 + \chi_H^h\right)^2}\right)$$

which takes a positive value when $\gamma > \sigma$ and a negative value when $\gamma < \sigma$. Accordingly, N_i is ineffective when $\gamma = \sigma$, but it increases (decreases) the severity of the puzzle when $\gamma > \sigma$ ($\gamma < \sigma$).

We now continue by focusing on the two dimensions of the puzzle.

5.1 Solution for Policy Implications

We have just shown that the international elasticity puzzle highly depends on the relationship between γ and σ. In order to determine the magnitude of this relationship, we consider the implications of Equation 13 in our theoretical and empirical investigations, which can be rewritten as follows:

$$\frac{\gamma - \theta}{\theta - \sigma} = \left(\frac{\chi_i^h}{1 - \chi_i^h}\right)\chi_H^h \tag{15}$$

where the knowledge of expenditure shares, χ_i^h and χ_H^h, would be enough to calculate the ratio of the left hand side. In particular, if $\theta > \sigma$, as suggested by the international elasticity puzzle, having $\gamma > \sigma$ would be enough to solve the puzzle. According to this expression, since $\chi_i^h > 0$ and $\chi_H^h > 0$ for any open economy, we can safely claim that the right hand side is a positive number, which implies that $\gamma > \theta$. Since $\theta > \sigma$ according to the international elasticity puzzle, it is further implied that $\gamma > \sigma$ and thus the puzzle is solved.

Although the puzzle is solved theoretically, we can still investigate the implications of our model for the value of γ for given values of θ and σ. In particular, Equation 15 can be used to find a value for γ when θ, σ, χ_i^h and χ_H^h is known. Accordingly, we will borrow the value of $\theta = 5$ from the international trade literature and the value of $\sigma = 1.5$ from the international finance literature in order to combine them with the corresponding trade data for $\chi_i^h \left(= \left(P_i^h C_i^h\right) / \left(Q_F^h G_F^h\right)\right)$ and $\chi_H^h \left(= 1 - \sum_{i \neq h} \left(P_i^h C_i^h\right) / \left(P^h C^h\right)\right)$.

We borrow the bilateral CIF imports data from Glick and Rose (2016) for the years between 1948-2013, which we use to obtain the measures of $P_i^h C_i^h$. For the overall home expenditure of $E^h = P^h C^h$, we use Gross National Expenditure (GNE) data obtained from World Development Indicators for the years between 1960-2013. The intersection of the two data sets corresponds to the annual period of 1960-2013 for 194 countries. We pool the data across all countries and all years in the calculation, subject to some missing observations due to the lack of GNE data in certain years for certain countries. The final data used in the calculation has 702,406 bilateral observations (representing i, h pairs in Equation 15) as the sample size. It is important to emphasize that zero-trade observations are also included in the investigation without any problems, since χ_i^h can easily take a value of zero in the calculation.

Since we have multiple countries and multiple years in the data set, after representing the time dimension by t, we can take the summation across t, h, i to have the following version of Equation 15 in our calculations:

$$\frac{\gamma - \theta}{\theta - \sigma} = \frac{\sum_t \sum_h \sum_i \left(\frac{\chi_i^h(t)}{1 - \chi_i^h(t)}\right) \chi_H^h(t)}{\text{Sample Size}} = 0.01 \tag{16}$$

which results in a value of 0.01. For robustness, we also consider a formal estimation strategy for the determination of $(\gamma - \theta) / (\theta - \sigma)$ of which details are given in the Appendix; the estimation results in the very same value of $(\gamma - \theta) / (\theta - \sigma) = 0.01$ which is significant at the 0.1% level.

It is implied that if $\theta > \sigma$ as suggested by the international elasticity puzzle, $\gamma > \theta$ and thus $\gamma > \sigma$ which means that the puzzle is solved. Therefore, the elasticity of substitution

γ across foreign countries is higher than the elasticity of substitution σ between home and foreign countries or the elasticity of substitution θ between all countries (including home country). Regarding the magnitude of γ, by using $\theta = 5$ to be consistent with the international trade literature and $\sigma = 1.5$ to be consistent with the the international finance literature, it is implied that $\gamma = 5.03$. Similar γ values can be found for alternative values of θ and σ according to the following implication:

$$\gamma = \theta + 0.01 \times (\theta - \sigma)$$

where, due to the low value of 0.01, γ is much closer to the trade elasticity of substitution θ. It is important to emphasize that this expression would reduce to $\gamma = \theta$ if the expenditure shares were neglected, as in the existing literature, and there would be no connection between γ, θ and σ in such a case. Nevertheless, by considering the corresponding expenditure shares, we are able to show such connections in this paper, both theoretically or empirically, independent of the supply-side.

Overall, according to Equations 13 and 14, the policy implications at the individual foreign good level are automatically equalized between the two literatures, although the artificially created upper-level elasticities θ, σ and γ are allowed to change between the two literatures. Since policy implications are equalized between the two literatures for each and every foreign good, the *international elasticity puzzle* disappears at the micro level.

It is implied that any policy analysis should be achieved at the disaggregated micro level and then aggregated up to obtain macro implications. Such an inductive approach has also been used by Imbs and Mejean (2015) who have calibrated macro elasticities using a weighted average of sector elasticities; however, they do not have any formal investigation to connect international trade to international finance at the macro level in order to have the very same effects on micro-level variables. Nevertheless, in this paper, we contribute on top of Imbs and Mejean (2015) by showing how macro elasticities are connected to each other between the two literatures in order to have the very same micro implications, especially through Equation 14, above, where we have formally shown that the macro elasticity of trade is a weighted average of the macro elasticity of finance and the elasticity of substitution across foreign countries. It is directly implied that as long as Equation 14 holds, the two literatures have the very same micro implications; however, this result does not say anything at the macro level. In particular, how do the two literatures behave after a macro-level foreign shock? In order to answer this question, we investigate the conditions under which the two literatures imply the very same macro implications after a foreign shock in home country, which we achieve next.

5.2 Solution for Welfare Gains from Trade

After solving the international elasticity puzzle from the perspective of policy implications at the micro level, in this section, we would like to focus on the second part of the puzzle by investigating the conditions under which the two literatures imply the very same welfare gains from trade at the macro level (in terms of welfare costs of autarky) as described in Arkolakis, Costinot, and Rodríguez-Clare (2012). Therefore, in both literatures, we search for the aggregate price indices P^h and Q^h that would have to adjust to keep the consumer utility the same between the current openness to trade and a hypothetical autarky. Accordingly, in international trade, welfare gains from trade $WGT^h(T)$ are given as follows:

$$WGT^h(T) = \frac{C^h}{C^{h,A}} = \frac{E^h}{E^{h,A}} \frac{P^{h,A}}{P^h} \tag{17}$$

while, in international finance, welfare gains from trade $WGT^h(F)$ are given as follows:

$$WGT^h(F) = \frac{G^h}{G^{h,A}} = \frac{E^h}{E^{h,A}} \frac{Q^{h,A}}{Q^h} \tag{18}$$

where $E^h \left(= P^h C^h = Q^h G^h\right)$ reprsents total expenditure in both literatures (as introduced above), and superscript A stands for autarky.

We are interested in how the welfare gains of $WGT^h(T)$ and $WGT^h(F)$ are connected to each other. Accordingly, we start with investigating the autarky prices of $P^{h,A}$ and $Q^{h,A}$, which are connected to expenditure share of home goods according to the following expression in international trade:

$$\omega_H^{h,A} = \beta_H^{h,A} \left(\frac{P_H^{h,A}}{P^{h,A}}\right)^{1-\theta}$$

and the following expression in international finance:

$$\chi_H^{h,A} = \alpha_H^{h,A} \left(\frac{P_H^{h,A}}{Q^{h,A}}\right)^{1-\sigma}$$

which are typical expressions in autarky obtained from the optimization of Equations 1 and 4, respectively. Autarky is defined as the case in which home expenditure share is equal to one; i.e., $\omega_H^{h,A} = \chi_H^{h,A} = 1$. According to the last two expressions, the autarky aggregate prices are implied as $P^{h,A} = P_H^{h,A} \left(\beta_H^{h,A}\right)^{\frac{1}{1-\theta}}$ in international trade and $Q^{h,A} = P_H^{h,A} \left(\alpha_H^{h,A}\right)^{\frac{1}{1-\sigma}}$ in international finance. According to Equations 17 and 18, the ratio of welfare gains in the

two literatures are further implied as follows:

$$\frac{WGT^h(F)}{WGT^h(T)} = \frac{P^h}{Q^h}\frac{Q^{h,A}}{P^{h,A}} = \frac{\left(\alpha_H^{h,A}\right)^{\frac{1}{1-\sigma}}}{\left(\beta_H^{h,A}\right)^{\frac{1}{1-\theta}}}\frac{P^h}{Q^h}$$

where E^h, $E^{h,A}$, and $P_H^{h,A}$ have been effectively eliminated. Since total expenditure is the same between the two literatures, $P^hC^h = Q^hG^h = E^h$, we can rewrite this expression by using Equations 3 and 6 as follows:

$$\frac{WGT^h(F)}{WGT^h(T)} = \frac{\left(\alpha_H^{h,A}\right)^{\frac{1}{1-\sigma}}\beta_i^h\beta_{ij}^h\left(\frac{P_{ij}^h}{P_i^h}\right)^{-\eta}\left(\frac{P_i^h}{P^h}\right)^{-\theta}}{\left(\beta_H^{h,A}\right)^{\frac{1}{1-\theta}}\left(1-\alpha_H^h\right)\alpha_i^h\beta_{ij}^h\left(\frac{P_{ij}^h}{P_i^h}\right)^{-\eta}\left(\frac{P_i^h}{Q_F^h}\right)^{-\gamma}\left(\frac{Q_F^h}{Q^h}\right)^{-\sigma}}$$

We are interested in the conditions under which the two literatures imply the very same welfare gains from trade; therefore, we directly consider the special case of $WGT^h(F) = WGT^h(T)$ which implies (after simple manipulations) that:

$$\chi_i^h = \alpha_i^h\left(\frac{\left(\alpha_H^{h,A}\right)^{\frac{1-\theta}{1-\sigma}}}{\beta_H^{h,A}}\right)^{\frac{\gamma}{\gamma-\theta}}\left(\frac{1-\chi_H^h}{1-\alpha_H^h}\right)^{\frac{\gamma(\theta-\sigma)}{\sigma(\gamma-\theta)}}$$

where we have used the definition of expenditure shares. When we take the sum of both sides across foreign countries, we obtain:

$$\left(\beta_H^{h,A}\right)^{1-\sigma} = \left(\alpha_H^{h,A}\right)^{1-\theta}\left(\frac{1-\chi_H^h}{1-\alpha_H^h}\right)^{\frac{(\theta-\sigma)(1-\sigma)}{\sigma}} \tag{19}$$

where we used $\sum_{i\neq h}\chi_i^h = \sum_{i\neq h}\alpha_i^h = 1$ according to their definitions. Therefore, when this expression holds, the two literatures imply the very same welfare gains from trade; we investigate the details of this expression next.

After assuming that $\theta \neq \sigma$ (which is the definition of international elasticity puzzle), in order to go one step further, we have to connect the autarky preferences of $\beta_H^{h,A}$ and $\alpha_H^{h,A}$ to the corresponding preferences under trade openness, β_H^h and α_H^h. For robustness, we follow two different approaches. The first approach, as we call it the *liberal* approach, assumes that the taste parameters assigned for home goods in autarky are given by $\beta_H^{h,A} = \alpha_H^{h,A} = 1$, since we have only home products in autarky; accordingly, $\beta_i^{h,A}$ for all $i \neq H$ and $1-\alpha_H^{h,A}$ are set equal to zero in autarky. Although this liberal approach is consistent with the definition

of autarky, where there are no foreign products and expenditure shares add up to one, it also corresponds to a change in taste parameters of $\beta_H^{h,A}$ and $\alpha_H^{h,A}$ between trade openness and autarky; however, one may also want to have a more conservative approach by keeping the taste parameters the same between trade openness and autarky. Accordingly, the second approach, as we call it the *conservative* approach, assumes that the preferences remain the same between autarky and trade openness, $\beta_H^{h,A} = \beta_H^h$ and $\alpha_H^{h,A} = \alpha_H^h$.

5.2.1 Implications of the Liberal Approach

According to Equation 19, the liberal approach ($\beta_H^{h,A} = \alpha_H^{h,A} = 1$) implies the following expression:

$$\left(\frac{1-\chi_H^h}{1-\alpha_H^h}\right)^{\frac{(\theta-\sigma)(1-\sigma)}{\sigma}} = 1 \tag{20}$$

which can be achieved either when $\alpha_H^h = \chi_H^h$ and/or when $\sigma = 1$ (still assuming that $\theta \neq \sigma$). Since the home expenditure share in international finance is given by $\chi_H^h = \alpha_H^h \left(\frac{P_H^h}{Q^h}\right)^{1-\sigma}$ due to the optimization of Equation 4 and the corresponding expenditure share definitions, in order to have $\chi_H^h = \alpha_H^h$, one needs to have either $\sigma = 1$ or $P_H^h = Q_F^h = Q^h$. While the former implication of $\sigma = 1$ corresponds *unitary macro elasticity* between home and foreign products in international finance, the latter implication of $P_H^h = Q_F^h = Q^h$ corresponds to *unitary terms of trade* in international finance.

Since unitary macro elasticity implies Cobb-Douglas aggregation between home and foreign products in international finance, we have to make sure that the corresponding aggregation/utility is consistent with our investigation. In particular, since $\beta_H^{h,A} = \alpha_H^{h,A} = 1$, and $\beta_i^{h,A}$ for all $i \neq H$ and $1 - \alpha_H^{h,A}$ are set equal to zero according to the liberal approach, the multiplicative Cobb-Douglas aggregation in autarky is identified (i.e., it takes a positive value), since the foreign products taking values of zero are raised to a zero power (which results in $0^0 = 1$). Therefore, according to the liberal approach, international trade and finance studies may have alternative macro elasticities of θ and σ, but as long as $\sigma = 1$, they imply the very same welfare gains from trade. It is important to emphasize that the case of $\sigma = 1$ is very close to the elasticity measures used in international finance studies covered in Table 1. Moreover, the case of $\sigma = 1$ is the exact elasticity measure used by Stockman and Tesar (1995) and Corsetti and Pesenti (2001), for which Bergin (2006) has even provided empirical evidence. Hence, according to the liberal approach, there is no *international elasticity puzzle* from the perspective of welfare gains from trade as long as there is *unitary macro elasticity* in international finance and as long as we do not make any simplifying assumptions regard-

ing the implications of our models for welfare gains from trade; one should rather derive model-specific formulas.

The other implication of $P_H^h = Q_F^h = Q^h$ (to have $\chi_H^h = \alpha_H^h$ in Equation 20), which corresponds to *unitary terms of trade,* is also consistent with our utility functions. Therefore, as long as international finance studies have *unitary terms of trade,* international trade and finance studies again imply the very same welfare gains from trade. Regarding the literature, the unitary terms of trade (that also corresponds to the purchasing power parity, PPP, condition) is typically used in many international finance studies such as by Devereux and Engel (2002), Corsetti and Dedola (2005) or Gali and Monacelli (2005) as a part of the symmetric steady-state definition (that is comparable with the static nature of the model in this paper).

In sum, according to the liberal approach, international trade and finance studies imply the very same welfare gains from trade as long as there is unitary macro elasticity of substitution between home and foreign products (represented by a Cobb-Douglas aggregation) or there is unitary terms of trade (implying PPP condition) in international finance. Since many international finance studies (as covered above) already have such ingredients, it is implied that the two literatures already have the same welfare gains from trade and thus *international elasticity puzzle* is solved.

5.2.2 Implications of the Conservative Approach

According to Equation 19, the conservative approach ($\beta_H^{h,A} = \beta_H^h$ and $\alpha_H^{h,A} = \alpha_H^h$) implies the following expression:

$$\left(\beta_H^h\right)^{1-\sigma} = \left(\alpha_H^h\right)^{1-\theta} \left(\frac{1-\chi_H^h}{1-\alpha_H^h}\right)^{\frac{(\theta-\sigma)(1-\sigma)}{\sigma}} \tag{21}$$

Since this expression cannot be simplified any further without making additional assumptions, we focus on a special case in which $\left(\beta_H^h\right)^{1-\sigma} = \left(\alpha_H^h\right)^{1-\theta}$ that is consistent with the corresponding definitions that we have in this paper (implying the same autarky aggregate prices between the two literatures, $Q^{h,A} = P^{h,A}$). Such a special case reduces Equation 21 to Equation 20, and hence we again have the implications of $\sigma = 1$ or $P_H^h = Q_F^h = Q^h$, although we still have to make sure that the corresponding aggregation/utility is consistent with these implications.

We start with investigating the implication of $\sigma = 1$, which corresponds to Cobb-Douglas aggregation between home and foreign products in international finance. Since $\alpha_H^{h,A} = \alpha_H^h$, which are both supposedly positive numbers, a value of $\sigma = 1$ in Equation 21 would imply $\alpha_H^h = 1$, which is not consistent with the concept of trade openness. In terms of technical

intuition, the Cobb-Douglas aggregation is not defined in autarky (i.e., results in zero utility) due to foreign products taking values of zero. Hence, the implication of $\sigma = 1$ is not consistent with the conservative approach.

When we continue with the other implication of $P_H^h = Q_F^h = Q^h$, we realize that it does not contradict with either Equation 21 or any other implications of our model; therefore, according to both the liberal approach and a special case of the conservative approach, as long as international finance studies have *unitary terms of trade,* international trade and finance studies imply the very same welfare gains from trade and thus *international elasticity puzzle* is solved.

6 Conclusion

International trade studies have higher macro elasticity measures compared to international finance studies. This observation has been puzzling for many researchers mostly due to two reasons. The first reason is that price changes are transferred into quantity changes through elasticities, and having alternative elasticity measures correspond to alternative policy implications; therefore, it a matter of scale (as in Ruhl (2008)). The second reason is that since welfare gains from trade are approximated by home expenditure shares and the upper-tier elasticities of substitution in individual utilities (as introduced by Arkolakis, Costinot, and Rodríguez-Clare (2012)), having alternative elasticity measures correspond to alternative welfare gains from trade.

By connecting the two literatures through an additional tier of aggregation across foreign countries, which is missing in international finance studies, this paper first solves the puzzle due to scale effects. In particular, when the elasticities of substitution and price elasticities of demand are distinguished by considering expenditure shares, which are assumed to be negligible in existing studies, it is shown theoretically and confirmed empirically that one can always find an elasticity of substitution across foreign countries that would be consistent with the macro elasticities in the two literatures. Since micro-level price elasticities of demand are equalized independent of how they are artificially aggregated at the destination country, the *international elasticity puzzle* disappears when policy implications are first calculated at the disaggregated micro level and then aggregated to obtain macro implications. Since the investigation is conducted by only using expenditure shares, the results are also robust to the consideration of zero-trade observations.

Considering the role of expenditure shares on the price elasticity of demand also helps us understand several differences across studies in the literature regarding the elasticity measures; these include the differences due to simulating versus estimating, differences due to

using alternative digits of data in estimations (i.e., the level of disaggregation), and differences between long-run and short-run elasticity measures.

When the puzzle is investigated from the perspective of welfare gains from trade, which corresponds to the macro-level effects of a foreign shock in home country, independent of the difference between elasticities of substitution and price elasticities of demand, it is shown that the two literatures imply the very same welfare gains from trade when there is unitary elasticity of substitution between home and foreign products or when there is unitary terms of trade. Since many international finance studies already employ unitary elasticity of substitution (as in studies such as by Stockman and Tesar (1995) and Corsetti and Pesenti (2001), for which Bergin (2006) has even provided empirical evidence) or unitary terms of trade while defining their steady states (as in studies such as by Devereux and Engel (2002), Corsetti and Dedola (2005) or Gali and Monacelli (2005)), welfare gains are already equalized across the two literatures and the *international elasticity puzzle* is solved at the macro level as well.

Overall, the literature does not need any complicated solutions to the international elasticity puzzle as opposed to studies such as by Ruhl (2008), Fitzgerald and Haller (2014), Crucini and Davis (2016), Ramanarayanan (2015) or Arkolakis, Eaton, and Kortum (2012). The puzzle is rather something artificial due to the way that the foreign products are aggregated at destination countries. It is important to emphasize that this result does not depend on the production/supply side of the model that has been extensively used to explain the puzzle in the literature; therefore, the results in this paper are consistent with any supply structure in the literature, including any type of firm-level investigation that would become ineffective when the implications for the two literatures are compared, where micro-level variables would effectively disappear in the comparison as we have shown in this paper.

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7 Appendix: Alternative Empirical Investigation

In order to determine the relationship between γ and σ, we can also consider an alternative implication of Equation 13 in our empirical investigation as follows:

$$\underbrace{\omega_i^h}_{\substack{\text{Expenditure Share}\\ \text{of Country } i\\ \text{among Home and}\\ \text{Foreign Products}}} = \rho_0 + \rho_1 \underbrace{\chi_i^h}_{\substack{\text{Expenditure Share}\\ \text{of Country } i\\ \text{among Foreign}\\ \text{Products}}} \tag{22}$$

where the coefficients of $\rho_0 = (\theta - \gamma) / (\theta - \sigma)$ and $\rho_1 = (\gamma - \sigma) / (\theta - \sigma)$ can be estimated by using data on expenditure shares of $\omega_i^h \left(= P_i^h C_i^h / P^h C^h\right)$ and $\chi_i^h \left(= P_i^h C_i^h / Q_F^h G_F^h\right)$.

In particular, if $\theta > \sigma$, as suggested by the international elasticity puzzle, we expect to find $\gamma > \sigma$ in order to solve the puzzle; accordingly, we expect to have a positive ρ_1. Before moving to the formal estimation, we can also put more structure on ρ_1 by using Equation 14 as follows:

$$\rho_1 = \frac{1}{\Omega} = 1 + \frac{\chi_H^h}{N_i - 1}$$

Since $\rho_0 + \rho_1 = 1$ (due to their definitions), it is implied that:

$$\rho_0 = -\frac{\chi_H^h}{N_i - 1}$$

Therefore, the knowledge of χ_H^h and N_i are enough to estimate ρ_0 and ρ_1.

Using the very same data as in the main text, we start with calculating the average values of ρ_0 and ρ_1 according to their theoretical definitions as follows:

$$\rho_0 = -\frac{\sum_t \sum_h \sum_i \left(\frac{\chi_H^h(t)}{N_i(t)-1}\right)}{\text{Sample Size}} = -0.01$$

and

$$\rho_1 = 1 + \frac{\sum_t \sum_h \sum_i \left(\frac{\chi_H^h(t)}{N_i(t)-1}\right)}{\text{Sample Size}} = 1.01$$

where t represents the time dimension, and we pool across countries and years. The results imply that if $\theta > \sigma$, as suggested by the international elasticity puzzle, $\gamma > \theta$ (since $\rho_0 < 1$) and $\gamma > \sigma$ (since $\rho_1 > 1$) which means that the puzzle is solved. Therefore, the elasticity of substitution γ across foreign countries is higher than the elasticity of substitution σ between

home and foreign countries or the elasticity of substitution θ between all countries (including home country).

When we move to the formal estimation, we use restricted least squares since $\rho_0 + \rho_1 = 1$ (due to their definitions). The results are given as follows:

$$\begin{array}{rccc} \omega_i^h = & \underbrace{\rho_0} & + & \underbrace{\rho_1} \ \chi_i^h \\ & -0.01 & & 1.01 \\ & (0.00) & & (0.00) \\ & [0.00] & & [0.00] \end{array} \tag{23}$$

where the standard errors are given in parenthesis, while the corresponding p-values are given in brackets. As is evident, the estimation results are the same with the average calculations based on theoretical definitions. Both coefficients of ρ_0 and ρ_1 are highly significant according to these estimation results, which support our overall investigation through which we have solved the international elasticity puzzle.

Table 1 - Elasticities in Selected Studies

Paper	Trade Between	Elasticity
International Finance		
Backus, Kehoe, and Kydland (1994)	Home versus Foreign	1.5
Bergin (2006)	Home versus Foreign	1
Blonigen and Wilson (1999)	Home versus Foreign	0.81
Corsetti and Pesenti (2001)	Home versus Foreign	1
Corsetti, Dedola, and Leduc (2008)	Home versus Foreign	0.85
Heathcote and Perri (2002)	Home versus Foreign	0.9
Stockman and Tesar (1995)	Home versus Foreign	1
Average in International Finance	Home versus Foreign	1.01
International Trade		
Anderson and Van Wincoop (2003)	Source Countries	$[5, 10]$
Anderson and Van Wincoop (2004)	Source Countries	$[5, 10]$
Clausing (2001)	Source Countries	$[9.86, 12.07]$
Eaton and Kortum (2002)	Source Countries	9.28
Head and Ries (2001)	Source Countries	$[7.9, 11.4]$
Hummels (2001)	Source Countries	$[2.00, 5.26]$
Simonovska and Waugh (2014a)	Source Countries	$[3.79, 5.46]$
Simonovska and Waugh (2014b)	Source Countries	5.63
Average in International Trade	Source Countries	7.35

Notes: This is a very brief summary studies selected among many others. For international trade studies, we have considered the elasticity measures at the macro (rather than the micro) level; see Simonovska and Waugh (2014b) for a nice discussion based on the difference between micro and macro elasticities in new international trade models.

Table 2 - Definition of Variables and Parameters in International Trade

Variables	Definition
C^h	Aggregate consumption in home country
C_i^h	Aggregate consumption of products from country i
C_{ij}^h	Consumption of good j imported from country i
P^h, P_i^h, P_{ij}^h	Prices per unit of C^h, C_i^h, C_{ij}^h, respectively
β_i^h, β_{ij}^h	Demand shifters of C_i^h, C_{ij}^h, respectively
E^h	Total expenditure in home country
Parameters	
θ	Elasticity of substitution across all countries
η	Elasticity of substitution across goods
$\varepsilon\left(\eta, \theta, \omega_{ij}^h, \omega_i^h\right)$	Price elasticity of demand for good j from country i
ω_i^h	Expenditure share of products from country i
ω_{ij}^h	Expenditure share of good j within country i products

Table 3 - Definition of Variables and Parameters in International Finance

Variables	Definition
G^h	Aggregate consumption in home country
C_H^h, G_F^h	Aggregate consumption of home and foreign products, respectively
C_i^h	Aggregate consumption of products from foreign country i
C_{ij}^h	Consumption of good j imported from foreign country i
$P_{ij}^h, P_i^h, P_H^h, Q_F^h, Q^h$	Prices per unit of $C_{ij}^h, C_i^h, C_H^h, G_F^h, G^h$, respectively
$1-\alpha_H^h, \alpha_H^h, \alpha_i^h, \beta_{ij}^h$	Demand shifters of $G_F^h, C_H^h, C_i^h, C_{ij}^h$, respectively
E^h	Total expenditure in home country
Parameters	
σ	Elasticity of substitution across home and foreign countries
γ	Elasticity of substitution across foreign countries
η	Elasticity of substitution across goods
$\varepsilon\left(\eta, \gamma, \sigma, \omega_{ij}^h, \chi_i^h, \chi_F^h\right)$	Price elasticity of demand for good j from foreign country i
$\varepsilon\left(\eta, \sigma, \omega_{Hj}^h, \chi_H^h\right)$	Price elasticity of demand for good j produced at home country
χ_H^h	Expenditure share of home products
χ_F^h	Expenditure share of foreign products
χ_i^h	Expenditure share of products from foreign country i
χ_{ij}^h	Expenditure share of good j within foreign country i products