Technological Change in Resource Extraction and Endogenous Growth

- Online-Appendix -

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1 The social planner solution

We solve the social planner solution of the model in the following.

Preferences and budget

Household preferences are

$$\int_0^\infty \frac{C(t)^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \ , \tag{1}$$

where ρ is the discount rate and θ is the coefficient of relative risk aversion.

The budget constraint is

$$C + I + M \leq Y = \left[\gamma Z^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) R^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \ , \tag{2}$$

where I is aggregate investment into new machines and M is aggregate R&D expenditure. The R&D expenditure is used for research in the production of Z and R: $M = M_Z + M_R$. Aggregate production uses two inputs, intermediate goods Z and resources R, with elasticity of substitution ε and distribution parameter γ.

The production function of good Z

The production function of Z is

$$Z = \frac{1}{1-\beta} \left(\int_0^{N_Z} x_Z(j)^{1-\beta} dj \right) L^\beta \ , \tag{3}$$

where L is labor. Production inputs are therefore labor and machines $x_Z(j)$ of variety j. The range of machine varieties is denoted N_Z. The social planner chooses the $x_Z(j)$ identical, so that we can write

$$Z = \frac{1}{1-\beta} N_Z x_Z^{1-\beta} L^\beta \ . \tag{4}$$

Intermediates x_Z depreciate fully after use and the marginal cost of production is the same

for all machine varieties and equal to ψ in terms of the final good. Investment in machines is thus given by

$$x_Z \psi = I \ . \tag{5}$$

The range of intermediates expands through investment in R&D by the following production function

$$\dot{N}_L = \eta_Z M_Z \ , \tag{6}$$

where M_Z is spending on R&D and η_Z is a cost parameter. One unit of the final good spent for R&D will generate η_Z new varieties of machines.

Production of the resource

The evolution of the resource stock follows:

$$\dot{S}_t = X_t - R_t \ , \qquad S_t \geq 0, X_t \geq 0, R_t \geq 0 \ . \tag{7}$$

The per unit production cost of the resource is as in equation (42):

$$\frac{1}{\eta_R \delta_1 \delta_2} \ . \tag{8}$$

The cost for R&D in the extractive sector is analogous to R&D in the intermediate goods sector and follows:

$$\dot{N}_R = \eta_R M_R \ . \tag{9}$$

The social planner chooses $X = R$ such that

$$R_t = X_t = \delta_1 \delta_2 \dot{N}_{Rt} = \frac{1}{\eta_R \delta_1 \delta_2} M_R \ . \tag{10}$$

The objective function and first order conditions

The social planner maximizes inter-temporal utility from consumption as defined in equation 1 with respect to the endogenous variables C, N_Z, x_Z, R, and M_Z, and subject to the budget constraint:

$$\left[\gamma Z^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)R^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} - C - I - M_Z - \frac{1}{\eta_R \delta_1 \delta_2} R = 0 \ , \tag{11}$$

where $M = M_Z + \frac{1+\psi\eta_R}{\eta_R\delta_1\delta_2}R$ is the aggregate R&D expenditure and $I = \psi x_Z + \psi x_R$ is the aggregate expenditure on machines.

The Hamiltonian to be maximized by the social planner is therefore

$$\begin{aligned} H \ = \ & \frac{C(t)^{1-\theta} - 1}{1-\theta} \\ & +\lambda\Big(\left[\gamma\left(\frac{1}{1-\beta}N_Z x_Z^{1-\beta}L^{\beta}\right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)R^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} - C - \psi x_Z - \psi x_R - \\ & -M_Z - \frac{1}{\eta_R\delta_1\delta_2}R\Big) \\ & +\mu\eta_Z M_Z \ . \end{aligned}$$

The first order conditions are

$$\frac{\partial H}{\partial C} = C^{-\theta} - \lambda = 0 \ , \tag{12}$$

$$\frac{\partial H}{\partial N_Z} = \lambda Y^{\frac{1}{\varepsilon}}\gamma Z^{\frac{\varepsilon-1}{\varepsilon}} N_Z^{-1} = \mu\rho - \dot{\mu} \ , \tag{13}$$

$$\frac{\partial H}{\partial x_Z} = \lambda Y^{\frac{1}{\varepsilon}}\gamma Z^{\frac{\varepsilon-1}{\varepsilon}}(1-\beta)x_Z^{-1} - \lambda\psi = 0 \ , \tag{14}$$

$$\frac{\partial H}{\partial R} = \lambda Y^{\frac{1}{\varepsilon}}(1-\gamma)R^{-\frac{1}{\varepsilon}} - \lambda\frac{1}{\eta_R\delta_1\delta_2} = 0 \ , \tag{15}$$

$$\frac{\partial H}{\partial M_Z} = -\lambda + \mu\eta_Z = 0 \ . \tag{16}$$

$$\tag{17}$$

Derivation of the growth rate

The FOC for C in growth rates is

$$g_\lambda = -\theta g_C \ . \tag{18}$$

Substituting the FOC for M_Z into the FOC for N_Z gives

$$g_\mu = \rho - \eta_Z Y^{\frac{1}{\varepsilon}} \gamma Z^{\frac{\varepsilon-1}{\varepsilon}} N_Z^{-1} \tag{19}$$

and

$$g_\lambda = g_\mu \ . \tag{20}$$

From the FOC for x_Z we obtain

$$Y^{\frac{1}{\varepsilon}} \gamma Z^{\frac{\varepsilon-1}{\varepsilon}} = \frac{x_Z \psi}{(1-\beta)} \ . \tag{21}$$

The FOC for R shows that the ratio of resource use and output is constant

$$\frac{R}{Y} = (\eta_R \delta_1 \delta_2 (1-\gamma))^\varepsilon \tag{22}$$

and consequently that they have the same growth rate:

$$g_R = g_Y \ . \tag{23}$$

Substituting (22) into the production function (2) yields

$$\left[1 - (1-\gamma)\left(\eta_R \delta_1 \delta_2 (1-\gamma)\right)^{\varepsilon-1}\right] Y^{\frac{\varepsilon-1}{\varepsilon}} = \gamma Z^{\frac{\varepsilon-1}{\varepsilon}} \tag{24}$$

such that

$$Z = z_1 Y \ , \tag{25}$$

where $z_1 = [\frac{1}{\gamma}(1-(1-\gamma)(\eta_R\delta_1\delta_2(1-\gamma))^{\varepsilon-1})]^{\frac{\varepsilon}{\varepsilon-1}}$ is a constant.

Substituting this into (21) yields

$$x_Z = z_2 Y \ , \tag{26}$$

where $z_2 = \gamma z_1^{\frac{\varepsilon-1}{\varepsilon}}(1-\beta)\frac{1}{\psi}$ is again a constant.

Substituting this in Equation 4 yields

$$N_Z \ = z_1(1-\beta)L^{-\beta}Y^{\beta} \ . \tag{27}$$

Therefore $\frac{x_Z}{N_Z}$ is not constant.

Combining Equations 18, 19, 20, and 21, we obtain the growth rate of the economy

$$g^{opt} = \frac{1}{\theta}\left(\frac{1}{\eta_Z(1-\beta)\psi}\frac{x_Z}{N_Z} - \rho\right) \ . \tag{28}$$

This shows that the market power of technology firms in the intermediate goods sector causes an inefficiency. The growth rate of the economy g^{opt} is not constant, but growing in the optimum as $\frac{x_Z}{N_Z}$ is not constant.

As a result, there is no balanced growth path in the social planner solution. This is in contrast to the decentralized solution, where the growth rate of the economy is constant (see Equation 2). The reason for this difference is that there are efficiency losses in the decentralized solution due to the monopoly power of the technology firms for machines in the intermediate goods sector. In the decentralized solution, the quantity of machines, which is supplied for each variety $x_Z(j)$, is constant as p_Z and $\chi_Z(j)$ are constant (see Equation 48 in the Appendix). In contrast, in the social planner solution x_Z is proportional to aggregate output as $x_Z = z_2 Y$ (see Equation 26). Furthermore, the decentralized solution features constant returns to scale in the production of Z_t (see Equation 31), since firms do not internalize technology in their production technology. For the social planner, however, technology is endogenous so that production has increasing returns in the factors

N_Z and x_Z (see Equation 3).

The comparison between the decentralized and the social planner solution illustrates the difference between our model based on Acemoglu (2002) and the Schumpeterian model with a non-renewable resource presented by Aghion and Howitt (1998). Aghion and Howitt (1998) make the assumption that "succeeding vintages of goods are increasingly capital intensive" (p. 153) in order to explain an exponent smaller than one on technology. This leads to constant returns to scale in their social planner solution. The idea of the model by Acemoglu (2002) is that there are increasing returns to scale, but these are not exploited due to the inefficiency in the decentralized solution.

The extractive sector does not make a difference to the two solutions. Technology firms are not able to obtain a monopoly price for machines, because machines are linked to the extraction of one specific occurrence, while the produced resource is a homogeneous good. There is therefore no efficiency loss in the extractive sector of the decentralized model. The resource production in the decentralized and in the social planner solution functions in the same efficient way. Comparing the respective first order conditions, the first order condition of the decentralized solution is given by the demand of the final good producer for the resource (see Equation 41). When substituting the price from Equation 44 to Equation 41, it becomes identical to the respective first order condition in the social planner solution in Equation 15.

There is no straightforward way to correct for the inefficiency in the decentralized model. Technology firms obtain patents for machines. The property right of the patent ensures that only the respective firm is able to produce the machine. However, the patent also entails market power in the intermediate goods sector, such that the provided quantity of machines is below the social optimum. There is demand for each variety and each variety is supplied by a single firm. A subsidy on the sale of machines in the intermediate goods sector affects the supply of machines, but does not have an impact on the growth rate of machine supply. To do so, the government needs to apply policy instruments like a subsidy on sales that increases with time, or modify the market structure by disconnecting R&D

investment from the market power created by patents. The latter solution would require government compensation to inventors or some other incentive device. Finally, we have to keep in mind that this inefficiency has also been introduced by Acemoglu (2002) to obtain a balanced growth path in the decentralized solution.

2 The growth model with machine varieties in the extraction sector

In this part of the online appendix we provide an alternative model to the one Section 4 in the paper. In contrast to the model in the main part of the paper, this version explicitly models "machine varieties" in the extractive sector. This is in analogy to the "machine varieties" used in the intermediate good sector. This version of the growth model emphasizes making the formal approach between the two sectors more analogues. It is technically and notationally more intensive and does not provide a better insight in the mechanism that we would like to highlight. We therefore decided to include the more concise version in the main body of the paper and to present this version in the online appendix.

2.1 The setup

We consider an economy with a representative consumer that has constant relative risk aversion preferences:

$$\int_0^\infty \frac{C_t^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \ . \tag{29}$$

The variable C_t denotes the consumption of aggregate output at time t, ρ is the discount rate, and θ is the coefficient of relative risk aversion. The budget constraint of the consumer is

$$C_t + I_t + M_t \leq Y_t \equiv \left[\gamma Z_t^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) R_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \ , \tag{30}$$

where I_t is aggregate investment in machines by the two sectors, and M_t denotes aggregate R&D investment in developing new varieties of machines. The usual no-Ponzi game condition applies. According to the right hand side of Equation 30, aggregate output production uses two inputs, intermediate goods Z_t and the non-renewable resource R_t. There are two sectors in the economy that produce the inputs to aggregate output production:

the intermediate goods sector and the extractive sector. The distribution parameter γ indicates the respective importance in producing aggregate output Y_t. The R&D expenditure is the sum of R&D expenditure in the intermediate sector and in the extractive sector: $M_t = M_{Zt} + M_{Rt}$.

The elasticity of substitution is $\varepsilon > 0$. Inputs Z_t and R_t are substitutes for $\varepsilon > 1$. In this case, the resource is not essential for aggregate production (see Dasgupta and Heal, 1980). The Cobb-Douglas case is $\varepsilon = 1$. For $0 < \varepsilon < 1$ the two inputs are complements.

The production function of the intermediate goods sector

The intermediate goods sector follows the basic setup of Acemoglu (2002). It produces intermediate goods Z_t according to the following production function[1]:

$$Z_t = \frac{1}{1-\beta}\left(\int_0^{N_{zt}} x_{zt}(j)^{1-\beta} dj\right) L^{\beta} \ , \tag{31}$$

where $\beta \in (0,1)$. The intermediate goods sector uses labor L_t, which has a fixed supply, and machines as inputs to production. $x_{Zt}(j)$ refers to the number of machines that are used for each machine variety j at time t. Machines depreciate fully after use within one period. We denote the number of varieties of machines as N_{zt}. Profits for the firm producing good Z are simply the difference between revenues and the expenses for labor, as well as for the intermediates $x_Z(j)$,

$$\pi_Z = p_Z Z - w_Z L - \int_0^{N_Z} \chi_Z(j) x_Z(j) dj \ . \tag{32}$$

Sector-specific technology firms invent new technologies for which they hold a fully enforceable patent. They exploit the patent by producing a machine type which corresponds uniquely to their technology. The uniqueness gives them market power which they can use to set a price $\chi_{Z_t}(j)$ above marginal cost. The marginal cost of production in terms of the final good is the same for all machines. Machines depreciate fully after each period,

[1]Like Acemoglu (2002) we assume that the firm level production functions of the two sectors exhibit constant returns to scale, so there is no loss of generality in focusing on the aggregate production functions.

so that the technology owner has to produce the corresponding machines each period.

The range of machines expands through R&D expenditure by

$$\dot{N}_{Zt} = \eta_Z M_{Zt} \ , \tag{33}$$

where M_{Zt} is R&D investment by the technology firms for machines in the intermediate goods sector in terms of the final product, and η_Z is a cost parameter. One unit of final good spent for R&D will generate η_Z new varieties of machines. A technology firm that discovers a new machine receives a patent and becomes its sole supplier.

The production function of the extractive sector

The extractive sector differs from the intermediate goods sector in the production function and in the way technological change takes place. Whereas the intermediate goods sector is labor intensive, the extractive sector is resource intensive, so that we do not model labor input explicitly.

The extractive sector faces stock constraints which the intermediate goods sector does not. The stock of the non-renewable resource at time t is noted $S_t \geq 0$. R_t notifies the quantity of the non-renewable resource that is sold for aggregate output production. Investing in new machines makes occurrences of lower grades extractable and expands the resource stock by X_t. The evolution of the stock follows:

$$\dot{S}_t = X_t - R_t \ , \qquad S_t \geq 0, X_t \geq 0, R_t \geq 0 \ , \tag{34}$$

where $\dot{S}_t$ is the change in the stock in period t, X_t is the inflow through investment of new machines, and R_t is the outflow by extracting and selling the resource. Note that for $X_t = 0$, this formulation is the standard Hotelling (1931) setup.

Extractive firms increase the resource stock by

$$X_t = \delta_1 \delta_2 \dot{N}_{Rt} x_R(j) \ , \tag{35}$$

which is equal to Equation (10) in Proposition 1, with the number of machines $x_R(j)$ that extractive firms purchase from sector specific technology firms added. Each machine $x_R(j)$ makes a specific additional mineral occurrence with a lower grade extractable. In contrast to the intermediate goods sector, the use of the machine variety j is bound to a specific deposit. Each mineral occurrence has the same quantity of the resource, but each at a different grade. Once a firm has extracted the resource from a specific mineral occurrence by use of machine variety j, the next deposit - with a lower grade - is not extractable any more by machine variety j. A new machine of a new variety needs to be bought from the sector specific technology firms. As a result, each variety of machines in the extractive sector can only be used once, whereas in the intermediate goods sector, each machine variety is used infinitely often. We normalize the size of R&D investment to one, $xR(j) = 1$. This is mathematically not exactly the same as in the intermediate goods sector, but it provides a comparable micro-foundation by subdividing the technology growth into units. In the intermediate goods sector, a machine is an infinitesimally small variety, whereas in the extractive sector it is a normalized fraction of R&D investment.

The term $\dot{N}_{Rt}$ denotes the range of the new machine varieties invented by the sector specific technology firms. The extractive sector is consistently under pressure to buy newly developed machines as once developed machines are not able to extract the resource from declining grades. This is in contrast to the intermediate goods sector (see Equation 31), which produces from all machine varieties that have been developed.

The sector specific technology firms develop $\dot{N}_{Rt}$ new patents for machines of the extractive sector analogously to the intermediate goods sector according to:

$$\dot{N}_{Rt} = \eta_R M_{Rt} \ , \tag{36}$$

where M_{Rt} is spending on R&D in the extractive sector in terms of the final product, and η_R is a cost parameter.

Once the patent has been developed, the technology firms produce the new machine variety j at a unit cost of Ψ in terms of the final good. Technology firms can only

produce one machine for each patent. They sell machines to the extractive sector in perfect competition, because the machines are perfect substitutes for producing the resource. This implies that the firm that buys the machines from the technology firms is entirely indifferent between the machines. Since sector specific technology firms have no market power, they obtain a price of the machine equal to marginal cost.

As the extractive firms can only use each machine variety once, the price of each machine is

$$\chi_R(j) = \frac{1}{\eta_R} + \Psi \ . \tag{37}$$

The first term on the right hand side, $\frac{1}{\eta_R}$, is the marginal R&D expenditure for developing one patent. This results from the equation $\eta_R M_R = \dot{N}_R$. Setting $\dot{N}_R = 1$ and solving for M_R, yields $M_R = \frac{1}{\eta_R}$. The second term, Ψ, notifies the cost of producing the machine in terms of aggregate output.

Profits for resource firms are thus given by revenues from selling the resources less the amount of $M_R = \frac{1}{\eta_R}\dot{N}_R = \frac{1}{\delta_1\delta_2}X_t$ at the price from equation (37):

$$\pi_{Rt} = p_R R_t - \frac{1 + \psi\eta_R}{\eta_R\delta_1\delta_2}X_t \ . \tag{38}$$

The production function of the extractive sector is equal to the outflows from the resource stock R_t:

$$R_t = \delta_1\delta_2\dot{N}_{Rt}x_R(j) - \dot{S}_t \ . \tag{39}$$

It illustrates the fundamental difference between the intermediate goods sector and the extractive sector in the relationship between technological change and the respective production. If technology firms stop investing in R&D in the intermediate goods sector, the intermediate goods sector will still be able to produce the good Z_t by buying machines based on the existing patents. However, if investment in R&D of the extraction technology stops at time T, the quantity of the resource that will still be extractable with the machines from the existing technology is limited to the existing stock.

2.2 Results

We begin the formal analysis with the optimization of the extractive firms. Firms have full control over inflows and outflows from their resource stock. Inflows X_t depend on R&D investment in the extractive sector, and outflows R_t are the sales of the resource to the final good producer. Since the marginal cost for R&D is constant, we obtain the typical result of stock management: inflows and outflows have to balance over time.

Proposition 1 *The quantity of the resource used in aggregate production equals the quantity of newly acquired resources through R&D:* $R_t = X_t$.

Proof of proposition 1

The final good producer demands the resource for aggregate production. The price of the final good is the numeraire. The first order condition with respect to the resource from production (see Equation 30) is

$$Y^{\frac{1}{\varepsilon}}(1-\gamma)R^{-\frac{1}{\varepsilon}} - p_R = 0 \ , \tag{40}$$

so that the demand for the resource is

$$R = \frac{Y(1-\gamma)^{\varepsilon}}{p_R^{\varepsilon}} \ . \tag{41}$$

Assume that initially, the resource stock available to the extractive firms is zero, $S_t = 0$. Revenues are given by $p_R R$ and expenses are given by $M_R = \frac{1}{\eta_R}\dot{N}_R$ in terms of the final good. Given the machine price from Equation 37, the per-unit production cost of the resource is

$$\left(\frac{1}{\eta_R} + \psi\right)\frac{1}{\delta_1\delta_2} = \frac{1+\psi\eta_R}{\eta_R\delta_1\delta_2} \ . \tag{42}$$

The extractive firms make profits

$$\pi_{Rt} = p_R R_t - \frac{1+\psi\eta_R}{\eta_R\delta_1\delta_2}X_t \ . \tag{43}$$

Since the stock of the resource S cannot be negative, newly acquired resources cannot be less than the resources sold to the final good producer: $X_t \geq R_t$. Newly acquired resources in excess of those sold could be stored. In a world without uncertainty, however, this would not be profitable. The price therefore must be equal to marginal cost:

$$p_R = \frac{1 + \psi \eta_R}{\eta_R \delta_1 \delta_2} \ . \tag{44}$$

It remains to consider the case of a positive initial stock of the resource, $S_t > 0$. Under perfect competition, this stock is immediately sold off to the final good producer such that the case of $S_t = 0$ returns. □

When the resource stock is zero, $S_t = 0$, it is not possible to extract the non-renewable resource without additional R&D in the extractive sector. An extractive firm needs to buy a new machine and hence, trigger investment in R&D by the technology firms. The resulting resource stock can then be extracted and sold to the final goods producer. However, another extractive firm may also invest in R&D, and also extract and sell the resulting resource. This situation of perfect competition means that resource prices are equal to marginal costs, which is the cost of extraction. This also highlights why the case $S_t > 0$ never occurs under the assumption of no uncertainty: An extractive firm investing in R&D will always extract and sell the newly available resource stock, because the selling price will remain constant.

The result is of course affected by the assumption of no uncertainty. Following the standard in growth models, we have assumed in equation 36 that patents for new machines result in a deterministic way from the respective R&D investments. This reflects a long-term perspective. The model could be made more sophisticated by assuming that R&D is stochastic. Extractive firms would then keep a positive stock of the resource S_t to be on the safe side in the case of a series of bad draws in R&D. This stock would grow over time as the economy grows. But in essence, the result above would remain the same: In the long term, resources used in aggregate production equal those added to the resource

stock through R&D.

We turn to the solution of the model:

Proposition 2 *The growth rate in the balanced growth path of the economy is constant and given by*

$$g = \theta^{-1}\left(\beta\eta_Z L\left[1-\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{1-\varepsilon}}\frac{1+\psi\eta_R}{\eta_R\delta_1\delta_2}\right]^{\frac{1}{\beta}}-\rho\right) .$$

Proof of Proposition 2

The first order conditions (FOC) of the final good producer for the optimal input of Z and R are $Y^{\frac{1}{\varepsilon}}\gamma Z^{-\frac{1}{\varepsilon}} - p_Z = 0$ and $Y^{\frac{1}{\varepsilon}}(1-\gamma)R^{-\frac{1}{\varepsilon}} - p_R = 0$, where the final good is the numeraire. From this the relative price is

$$p = \frac{p_R}{p_Z} = \frac{1-\gamma}{\gamma}\left(\frac{R}{Z}\right)^{-\frac{1}{\varepsilon}} . \tag{45}$$

Setting the price of the final good as the numeraire gives (for the derivation of the price index see the derivation of Equation (12.11) in Acemoglu (2009)):

$$\left[\gamma^{\varepsilon}p_Z^{1-\varepsilon} + (1-\gamma)^{\varepsilon}p_R^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = P = 1 . \tag{46}$$

The intermediate goods sector

As in Acemoglu (2009), the maximization problem in the intermediate goods sector is:

$$\max_{L,\{x_Z(j)\}} p_Z Z - w_Z L - \int_0^{N_Z} \chi_Z(j)x_Z(j)dj . \tag{47}$$

The FOC with respect to $x_Z(j)$ is $p_Z x_Z(j)^{-\beta}L^{\beta} - \chi_Z(j) = 0$ so that

$$x_Z(j) = \left(\frac{p_Z}{\chi_Z(j)}\right)^{\frac{1}{\beta}} L . \tag{48}$$

From the FOC with respect to L we obtain the wage rate

$$w_Z = \frac{\beta}{1-\beta} p_Z \left(\int_0^{N_Z} x_Z(j)^{-\beta} dj \right) L^{\beta-1} \ . \tag{49}$$

The profits of the technology firms are:

$$\pi_Z(j) = (\chi_Z(j) - \psi) x_Z(j) \ . \tag{50}$$

Substituting Equation 48 into Equation 50 we calculate the FOC with respect to the price of a machine $\chi_Z(j)$: $\left(\frac{p_Z}{\chi_Z(j)}\right)^{\frac{1}{\beta}} L - (\chi_Z(j) - \psi) p_Z^{\frac{1}{\beta}} \frac{1}{\beta} \chi_Z(j)^{\frac{1}{\beta}-1} L = 0$. Solving this for $\chi_Z(j)$ yields $\chi_Z(j) = \frac{\psi}{1-\beta}$. Following Acemoglu (2002) we normalize $\psi = 1-\beta$ so that $\chi_Z(j) = 1$. Combining this result with Equations 48 and 50 we write profits as

$$\pi_Z(j) = \beta p_Z^{\frac{1}{\beta}} L \ . \tag{51}$$

The present discounted value is:

$$r V_Z - \dot{V}_Z = \pi_Z \ . \tag{52}$$

The steady state ($\dot{V} = 0$) is:

$$V_Z = \frac{\beta p_Z^{\frac{1}{\beta}} L}{r} \ . \tag{53}$$

Substituting Equation 48 into Equation 31 yields

$$Z = \frac{1}{1-\beta} p_Z^{\frac{1-\beta}{\beta}} N_Z L \ . \tag{54}$$

Solving for the variables of the intermediate goods sector

Solving Equation 46 for p_Z yields

$$p_Z = \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma} \right)^{\varepsilon} p_R \right)^{\frac{1}{1-\varepsilon}} \ . \tag{55}$$

This can be used, together with the expression for R from Equation 41 and the expression for p_R from Equation 44 to determine Z as a function of Y from Equation 45. We obtain the range of machines N_Z as a function of Y from Equation 54.

The growth rate of consumption

The consumer earns wages from working in the sector which produces good Z and earns interest on investing in the technology N_Z. The budget constraint thus is $C = w_Z L + rM$. Maximizing utility in Equation 29 with respect to consumption and investments yields the first order conditions $C^{-\theta} e^{-\rho t} = \lambda$ and $\dot{\lambda} = -r\lambda$ so that the growth rate of consumption is

$$g_c = \theta^{-1}(r - \rho) \ . \tag{56}$$

This will be equal to output growth on the balanced growth path. We can thus solve for the interest rate and obtain $r = \theta g + \rho$. The free entry condition for the technology firms imposes that profits from investing in patents must be zero. Revenue per unit of R&D investment is given by V_Z, cost is equal to $\frac{1}{\eta_Z}$. Consequently, we have $\eta_Z V_Z = 1$. Substituting Equation 53 into it we obtain $\frac{\eta_Z \beta p_Z^{\frac{1}{\beta}} L}{r} = 1$. Solving this for r and substituting into Equation 56 we obtain

$$g = \theta^{-1}(\beta \eta_Z L p_Z^{\frac{1}{\beta}} - \rho) \ . \tag{57}$$

Together with Equations 44 and 55 this yields the growth rate. □

A higher rate of return to R&D investment in new machines of the labor sector, η_Z, increases the growth rate of the economy. We discuss the effects of parameters η_R, δ_1, and δ_2 on the growth rate in Proposition 4.

Since the growth model employed in this paper is a modification of Acemoglu (2002), we can point out the difference to the growth rates in that model. The Euler equation, $g = \theta^{-1}(r - \rho)$, is clearly visible. The interest rate, however, differs. In Acemoglu (2002) there are exogeneously given resources in both sectors. In our model there is only one exogenous

resource, L, and the other sector generates the resource R endogenously. Therefore, instead of the two exogeneously given resources, our growth rates features the one exogenous resource and the price for obtaining the endogenous resource R, $p_R = \frac{1+\psi\eta_R}{\eta_R\delta_1\delta_2}$.

In order to understand the role of the non-renewable resource in the economy, we determine its relative importance:

Proposition 3 *The resource intensity of the economy is given by*

$$\frac{R}{Y} = \left[(1-\gamma)\frac{\eta_R\delta_1\delta_2}{1+\psi\eta_R}\right]^{\varepsilon} . \tag{58}$$

It depends positively on the distribution parameter for the resource γ.

Proof of Proposition 3

Substitute Equation 44 into Equation 41. □

The distribution parameter γ indicates the importance of the resource for the economy, as shown in the production function in Equation 30.

Extractive firms face constant marginal costs of extracting the non-renewable resource, since the resource stock can be expanded due to R&D in extraction technology. The price thus remains constant over time as well:

Proposition 4 *The resource price is*

$$p_{Rt} = \frac{1+\psi\eta_R}{\eta_r\delta_1\delta_2} .$$

A higher resource price has the following effects: (i) it decreases the resource intensity, and (ii) it decreases the growth rate of the economy.

Proof of Proposition 4

The total cost of extracting resources can be split into the price of the new machine and the extraction cost. The technology costs have been derived in Proposition 1 as

proportional to R&D in extraction technology. The extraction cost is given by the constant E. Since the extraction cost is constant and this model focusses on the innovation side, we make the simplifying assumption of zero extraction cost, $E = 0$. Therefore the total cost is given by the cost for the new machine.

The extractive firms sell the resource R, to the final good producer at price p_R. Its total revenues are thus Rp_R. The expenses are given by the price of a machine, $\chi_R(j)$ times the number of machines bought, $\dot{N}_{Rt}x_R(j)$ with $x_R(j) = 1$. Total expenses are thus $\frac{1}{\chi_R(j)}\dot{N}_{Rt}$. The extraction firms are in perfect competition, just like firms in the intermediate goods sector. Therefore profits are zero, its revenues must equal expenses: $Rp_R = \chi_R(j)\dot{N}_{Rt}$. Inserting Equation 35 we obtain $\delta_1\delta_2\dot{N}_{Rt}x_R(j)p_R = \chi_R(j)\dot{N}_{Rt}$, so that $p_R = \frac{1+\psi\eta_R}{\eta_R\delta_1\delta_2}$. □

This proposition shows that the resource price plays a central role in the model. To understand it, we first consider its determinants and then focus on its effects.

The determinants of the price are given by the parameters η_R, δ_1, and δ_2. The productivity of R&D in the extractive sector, defined in Equation 36, and given by η_R, determines the number of new machine varieties that are developed by the sector specific technology firms per unit of aggregate output. The higher this parameter, the higher the resource use in the economy. δ_1, defined in Equation (6), is a productivity parameter for the marginal effect of R&D investment on the extractability of occurrences of lower grades. δ_2, defined in Equation (8), determines the steepness of the distribution of elements over mineral occurrences of various grades in the earth's crust. If the quantity of the extractable resource strongly increases as the grade of occurrences decreases, the return on investments in R&D for the extraction technology increases, and the economy uses a larger quantity of the resource in proportion to aggregate output.

The resource price is constant, but Proposition 4 shows that the resource price is high when the productivity parameters are low and vice versa. It states quite intuitively that the selling price of the resource is low, if the productivity parameters are high.

Moreover, Proposition 4 in combination with Propositions 2 and 3 shows the effect of a lower resource price on the growth rate and the resource intensity of the economy. Both depend negatively on the resource price. When the price is low, the non-renewable resource is used intensively and the resource constraint on growth is weak. When the price is high, the economy uses substitutes, but this reduces growth.

We compare the growth rates of technology in the two sectors.

Proposition 5 *The level of technology in the intermediate goods sector is*

$$N_Z = \left(\frac{1-\gamma}{\gamma}\right)^{-\varepsilon} \left(\frac{\eta_R \delta_1 \delta_2}{1+\psi\eta_R}\right)^{\varepsilon} \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^{\varepsilon} \frac{\eta_R \delta_1 \delta_2}{1+\psi\eta_R}\right)^{\left(\frac{1}{1-\varepsilon}\right)\left(-\varepsilon+\frac{1-\beta}{\beta}\right)} (1-\gamma)^{\varepsilon} L^{-1} Y \ .$$

The growth rate of technology in the extractive sector is

$$\dot{N}_R = (1-\gamma)^{\varepsilon} \frac{\eta_R}{1+\psi\eta_R} Y \ .$$

Proof of Proposition 5

We use Equation 55, together with the expression for R from Equation 41 and the expression for p_R from Equation 44 to determine Z as a function of Y from Equation 45. This can then be used to obtain the range of machines N_Z as a function of Y from Equation 54.

The expression for $\dot{N}_R$ follows from equation (10), Proposition 1 as well as equation 41. □

There is thus a qualitative difference in the growth rate of the two sectors. While the *level* of technology in the intermediate goods sector is proportional to output, the *growth rate* of technology in the extractive sector is proportional to output. N_Z therefore has the *constant* growth rate g, as given in Proposition 2. N_R has an *increasing* growth rate. It is the second derivative $\frac{\partial^2 N_R}{\partial^2 t}$ which is equal to g.

Proposition 5 also shows how investments into technology M_{Rt} depend on the elasticity of substitution ϵ, since $\dot{N}_R$ and M_{Rt} are closely linked through equation 36. Since $1-\gamma < 1$, the elasticity of substitution and investments into technology M_{Rt} are negatively linked.

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