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by

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Abstract: This paper focuses on actively managed portfolios of VIX derivatives constructed to reduce portfolio correlation with the equity market. We find that the best results are obtained using Kalman filter-based dynamic CAPM. The portfolio construction method is capable of constructing zero-beta portfolios with positive alpha.

Keywords: Dynamic CAPM, Kalman filter, zero beta, VIX, VIX futures.

Market neutral hedge funds typically try to exploit pricing differences between two or more related securities to generate performance which is uncorrelated with a target benchmark. This may involve creating corresponding long and short positions in the related securities. Often, the benchmark is a broad measure of the equity market. Where that measure is sufficiently broad and the fund's strategy is to construct a portfolio which is uncorrelated with the measure, the fund may be said to pursue a "zero beta" strategy. Such strategies may fail because the related securities used in the strategy do not behave as expected, causing the fund's portfolio to become excessively correlated with the benchmark. We propose an algorithmic remedy to this problem. We apply Kalman filter-based dynamic regression to generate daily updates of portfolio weightings in a zero-beta portfolio. To demonstrate the method, we apply it to VIX futures indexes. These indexes are specified and structured in terms of traded VIX futures contracts. Their specification and structure are public information, which makes them investable. We apply our portfolio construction method to the two most popular indexes, the VIX short-term futures index and the VIX mid-term futures index. Then, we apply the method to combinations of the VIX short-term futures index and other VIX futures indexes to demonstrate the usefulness and generality of the approach.

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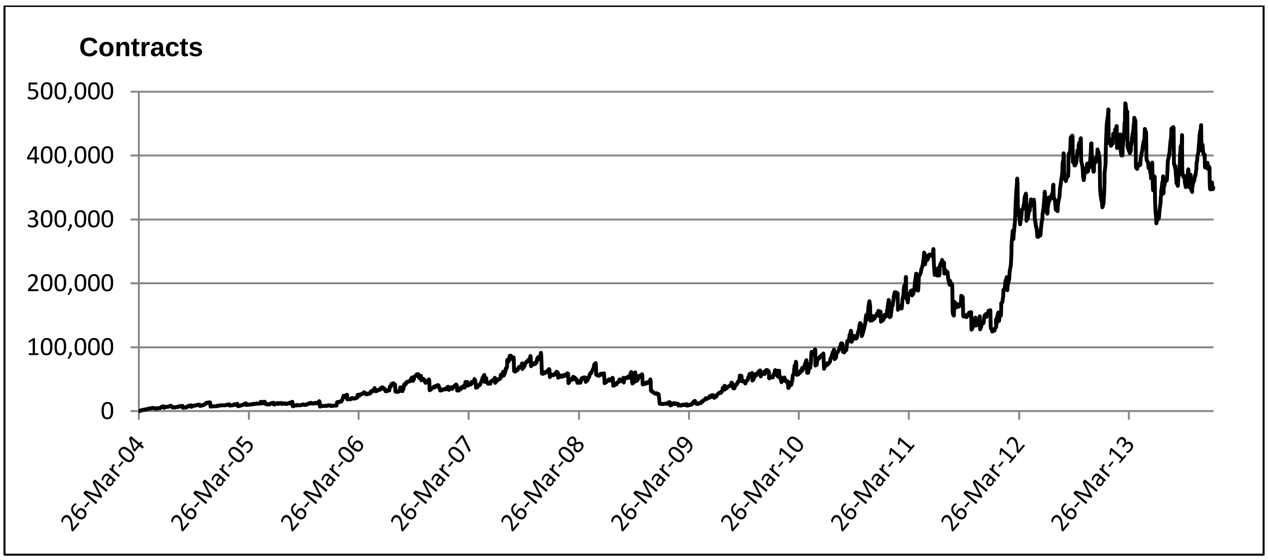
The following sections discuss VIX futures indexes and key statistics, the dynamic CAPM model, an operational zero-beta portfolio construction method, and robustness checks. A final section presents the conclusion.

VIX FUTURES INDEXES AND KEY STATISTICS

The VIX Index was developed by Whaley [1993], and its construction was further developed by the Chicago Board Options Exchange (CBOE) in 2003. It is a weighted blend of prices for a range of options on the Standard & Poor's 500 Index traded on the CBOE. The VIX itself is not tradable, but the CBOE began trading futures contracts on the VIX Index on March 26, 2004. Exhibit 1 shows open interest for VIX futures contracts for all expirations from that date to the end of 2013. In the exhibit, we see a substantial increase in open interest over the period.

Exhibit 1. CBOE VIX Futures Open Interest, March 26, 2004, to December 31, 2013. Source: Chicago Board Options Exchange



Since the introduction of VIX futures, various VIX-related instruments have been introduced to the market (see Alexander and Korovilas [2013]), and researchers and practitioners have become increasingly interested in VIX derivatives in terms of their use in both volatility selling (e.g., see Rhoads [2011]) and portfolio protection strategies (Mencía and Sentana [2013]). VIX futures indexes were developed by Standard & Poor's, which publishes them as daily data beginning December 20, 2005. They are computed as returns of combinations of VIX futures contracts which are rolled daily between contract expiration days. The VIX short-term futures index (Bloomberg: SPVXSP) rolls from the one-month VIX futures contract into the two-month VIX futures contract, maintaining a constant one-month maturity. The VIX two-month futures index (Bloomberg: SPVIX2ME) rolls from the two-month futures into the three-month futures, maintaning a constant two-month maturity, and so on out to the three-month index (Bloomberg: SPVIX3ME) and the four-month index (Bloomberg: SPVIX4ME). The underlying holdings for the VIX mid-term futures index (Bloomberg: SPVXMP) are the four- through seven-month VIX futures. The index continuously rolls out of the four-month VIX futures and into the seven-month futures, maintaining a constant five-month maturity. The VIX six-month futures index (Bloomberg: SPVIX6ME) uses the five- through eight-month VIX futures rolling out of the five-month futures and into the eight-month futures, maintaining a constant six-month maturity.

VIX futures indexes are, of course, highly correlated with the VIX Index. In addition, VIX futures indexes are affected by roll costs or roll gains in rolling from shorter contracts to longer contracts depending on whether the term structure in the VIX futures market is in contango or backwardation, respectively. Exhibit 2 presents summary statistics for the returns of the VIX futures indexes over their history from December 20, 2005, to December 31, 2013.

Exhibit 2. Return Statistics for the S&P 500 Index and VIX Futures Indexes, December 20, 2005, to December 31, 2013.

	SP 500	SPVXSP	SPVIX2ME	SPVIX3ME	SPVIX4ME	SPVXMP	SPVIX6ME
Annual Mean (%)	4.90	-43.78	-28.83	-17.31	-15.70	-12.86	-9.76
Volatility (%)	22.19	62.35	47.33	39.15	34.43	31.53	29.37
Sharpe ratio	0.17	-0.94	-0.74	-0.51	-0.52	-0.47	-0.38
Skewness	-0.31	0.56	0.39	0.44	0.46	0.49	0.52
Excess Kurtosis	9.54	3.16	2.74	2.92	3.18	3.40	3.61
Total Return (%)	46.74	-99.01	-93.45	-78.21	-74.56	-66.83	-56.09
Max Drawdown (%)	56.78	99.54	97.44	92.22	89.97	86.43	81.54

Exhibit 3 presents the correlation matrix for the returns of the S&P 500 Index and the VIX futures indexes from December 20, 2005, to December 31, 2013.

Exhibit 3. Correlation Matrix for the Returns on S&P 500 Index and VIX Futures Indexes, December 20, 2005, to December 31, 2013.

	SP 500	SPVXSP	SPVIX2ME	SPVIX3ME	SPVIX4ME	SPVXMP	SPVIX6ME
SP 500	1.00	-0.76	-0.76	-0.77	-0.77	-0.76	-0.74
SPVXSP	-0.76	1.00	0.96	0.94	0.92	0.90	0.87
SPVIX2ME	-0.76	0.96	1.00	0.97	0.94	0.92	0.90
SPVIX3ME	-0.77	0.94	0.97	1.00	0.98	0.96	0.94
SPVIX4ME	-0.77	0.92	0.94	0.98	1.00	0.99	0.97
SPVXMP	-0.76	0.90	0.92	0.96	0.99	1.00	0.99
SPVIX6ME	-0.74	0.87	0.90	0.94	0.97	0.99	1.00

The six VIX futures indexes are highly correlated with the S&P 500 Index. They are also highly correlated with each other because they are driven by the same underlying VIX fluctuations and similar roll costs/gains from the term structure. In addition, the correlation between pairs of futures indexes with the closest maturity is high because their underlying futures are synchronized in their movements.

DYNAMIC CAPM

The portfolio construction method which we present below is rooted in an accurate estimation of the parameters of the capital asset pricing model. CAPM can be expressed as a static OLS regression as follows:

$$r_i - r_f = \alpha_i + \beta_i * (r_M - r_f) + \varepsilon_i \quad (1)$$

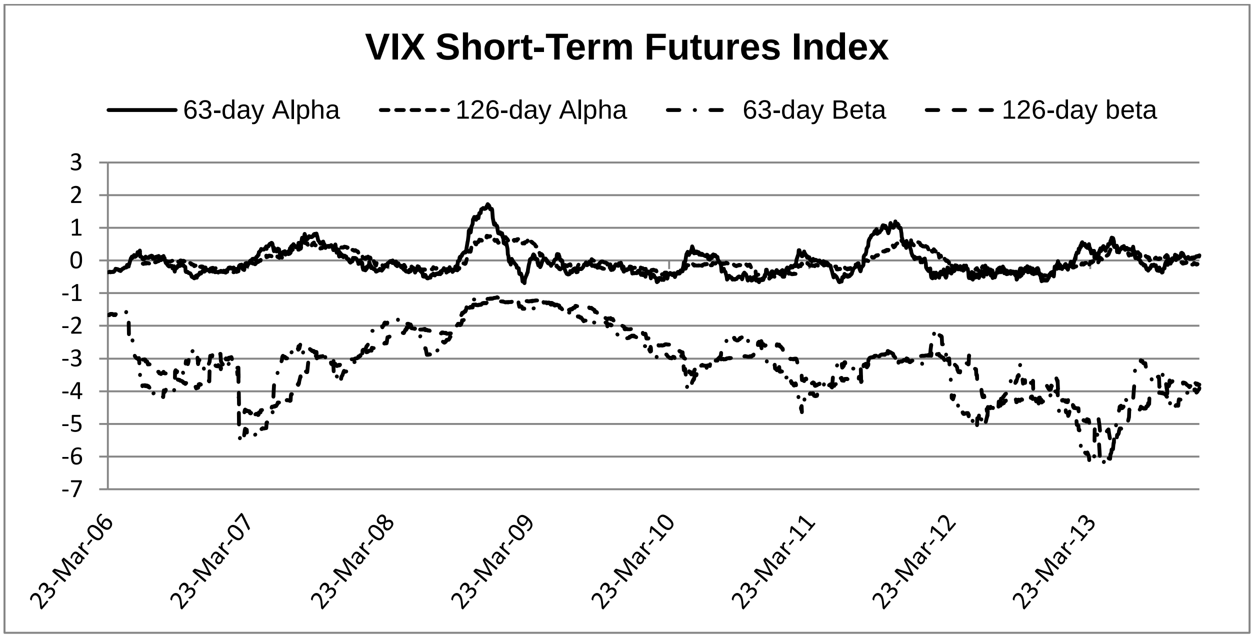
where r_i is the return for security i, r_f is the risk-free rate, and r_M is the market return. Throughout the paper, we use the S&P 500 Index to represent the market portfolio and 90-day Treasury bills to represent the risk-free rate. Exhibit 4 shows the corresponding α and β for each VIX futures index calculated from equation (1) using OLS.

Exhibit 4. Static OLS Results for the VIX Futures Indexes from December 20, 2005, to December 31, 2013.

	SPVXSP	SPVIX2ME	SPVIX3ME	SPVIX4ME	SPVXMP	SPVIX6ME
α (%)	-0.0010	-0.0005	-0.0001	-0.0001	-0.0001	0.0000
P-value for α	0.08	0.22	0.68	0.56	0.69	0.92
β	-2.17	-1.63	-1.37	-1.20	-1.08	-0.98
P-value for β	0.00	0.00	0.00	0.00	0.00	0.00
R^2 (%)	58.14	57.56	59.37	59.38	57.37	54.59

However, α and β are hardly constant for the VIX futures indexes. Rolling regression provides one way to capture the dynamics. Exhibit 5 shows the rolling regression-based α and β for the VIX short-term futures index where the rolling window is 63 market days, which corresponds to about three months, and alternatively 126 days, which corresponds to about six months.

Exhibit 5. 63- and 126-Day Rolling Regression α and β for the VIX Short-Term Futures Indexes, March 23, 2006, to December 31, 2013.



There are two major drawbacks associated with rolling regression in computing α and β. First, as we see in the exhibit, the results are sensitive to the choice of window size, and there is no consensus on choosing the proper window size, a matter which we find below to have a large influence on the performance of zero-beta portfolios. Second, rolling regression is not able to provide instantaneous updates of α and β. The "dynamic" α and β are still averages of α and β over the trailing window.

A dynamic linear model, or Kalman filter, provides an alternative way to estimate dynamic α and β. Here, the state equation is an order-one vector autoregression:

$$\boldsymbol{x}_t = \boldsymbol{\phi}\boldsymbol{x}_{t-1} + \boldsymbol{w}_t. \quad (2)$$

The $p * 1$ state vector $\boldsymbol{x}_t$ is generated from the previous state vector $\boldsymbol{x}_{t-1}$ for $t = 1,2,3,\ldots,n$. The vector $\boldsymbol{w}_t$ is a $p * 1$ independent and identically distributed zero-mean normal vector having a covariance matrix of Q. The starting state vector $\boldsymbol{x}_0$ is assumed to have mean $\boldsymbol{\mu}_0$ and a covariance matrix of Π_0. The state process is a Markov chain, but we do not have direct observation of it. We can only observe a q-dimensional linear transformation of $\boldsymbol{x}_t$ with added noise. Thus, we have the following observation equation:

$$\boldsymbol{y}_t = A_t\boldsymbol{x}_t + \boldsymbol{v}_t, \tag{3}$$

where A_t is a $q * p$ observation matrix, the observed data are in the $q * 1$ vector $\boldsymbol{y_t}$, and $\boldsymbol{v}_t$ is white Gaussian noise with a $q * q$ covariance matrix R. We also assume $\boldsymbol{v}_t$ and $\boldsymbol{w}_t$ are uncorrelated. Our primary interest is to produce the estimator for the underlying unobserved $\boldsymbol{x}_t$ given the data $\boldsymbol{Y}_s = \{\boldsymbol{y}_1, \dots, \boldsymbol{y}_s\}$. In the literature, where $s < t$ this is referred to as forecasting, where $s = t$ this is Kalman filtering, and where $s > t$ this is Kalman smoothing. With the definitions $\boldsymbol{x}_t^s = E(\boldsymbol{x}_t|\boldsymbol{Y}_s)$ and $\boldsymbol{P}_t^s = E[(\boldsymbol{x}_t - \boldsymbol{x}_t^s)(\boldsymbol{x}_t - \boldsymbol{x}_t^s)]$ and the initial conditions $\boldsymbol{x}_0^0 = \boldsymbol{x}_0 = \boldsymbol{\mu}_0$ and $\boldsymbol{P}_0^0 = \Pi_0$, we have the recursive Kalman filter equations as follows:

$$\boldsymbol{x}_t^{t-1} = \boldsymbol{\phi}\boldsymbol{x}_{t-1}^{t-1}, \tag{4}$$

$$\boldsymbol{P}_t^{t-1} = \boldsymbol{\phi}\boldsymbol{P}_{t-1}^{t-1}\boldsymbol{\phi}' + Q, \tag{5}$$

for $t = 1,2,3, \dots, n$ with:

$$\boldsymbol{x}_t^t = \boldsymbol{x}_t^{t-1} + K_t(\boldsymbol{y}_t - A_t\boldsymbol{x}_t^{t-1}), \tag{6}$$

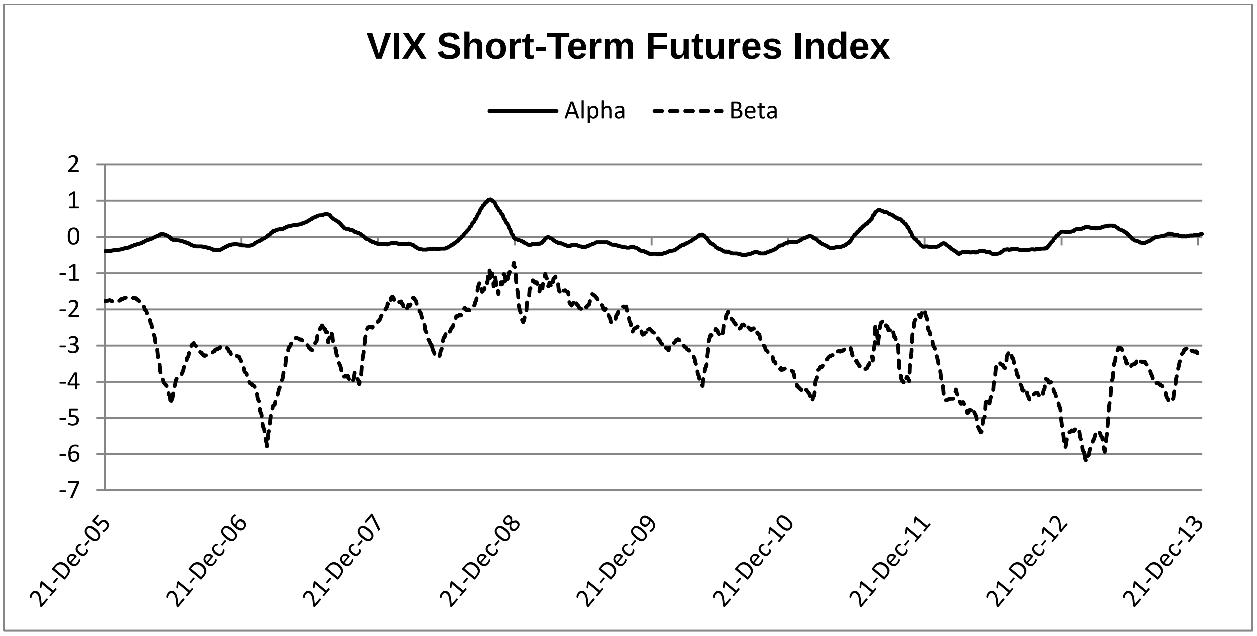
$$\boldsymbol{P}_t^t = [I - K_tA_t]\boldsymbol{P}_t^{t-1}, \tag{7}$$

$$K_t = \boldsymbol{P}_t^{t-1}A_t'[A_t\boldsymbol{P}_t^{t-1}A_t' + \boldsymbol{R}]^{-\mathbf{1}}. \tag{8}$$

Here, $\boldsymbol{y}_t$ is the observed excess returns of a particular VIX futures index at time t, and A_t is the $1 * 2$ row vector representing the intercept and the excess returns of S&P 500 Index. The unobserved state $\boldsymbol{x}_t$ is the vector of time-varying estimates of the parameters of the CAPM equation. As suggested in Lai and Xing [2008], in the case of the dynamic capital asset pricing model, a popular choice is to make $\boldsymbol{\phi}$ the identity matrix and Q diagonal. Then, the Q and R matrices are estimated using maximum likelihood.

Exhibit 6 presents the CAPM results for the VIX short-term futures index using the Kalman smoother.

Exhibit 6. **Kalman Smoother Dynamic α and β for the VIX Short-Term Futures Indexes, December 20, 2005, to December 31, 2013.**



As we see in the exhibit, α and β for the VIX futures index are far from constant during the sample period, which suggests that it would be imprudent to build a portfolio based on static CAPM. In the next section we employ the Kalman filter to build an operational dynamic-CAPM zero-beta portfolio.

AN OPERATIONAL ZERO-BETA PORTFOLIO CONSTRUCTION METHOD

In this section, we create a portfolio combining positions in the VIX short-term futures index and VIX mid-term futures index. These are the most popular VIX futures indexes. Exchange traded products linked to these indexes currently total more than $2 billion, and trading volume is on a commensurate scale. These securities are highly volatile, and thus carry enormous risk. The systematic risk for these securities is high due to the high beta. Meanwhile, the high correlation and similar construction mechanism open ways to hedge the market risks, namely the β. By hedging out the β, it is possible to build a pure α generating portfolio.

We start with the static CAPM model. By assigning proper weights to the securities in the portfolio, the total beta of the portfolio could in principle be reduced to close to zero. And the leftover is the net alpha of the portfolio. Let $r_{1,t}$, $\alpha_{1,t}$, $\beta_{1,t}$, and $w_{1,t}$ denote, respectively, the daily return, alpha, beta, and portfolio weight of the VIX short-term futures index on day t. Let $r_{2,t}$, $\alpha_{2,t}$, $\beta_{2,t}$, and $w_{2,t}$ denote the daily return, alpha, beta, and portfolio weight of the VIX mid-term futures index on day t. Without using leverage, the summation of the absolute values of the portfolio weights equals 1.

$$|w_{1,t}| + |w_{2,t}| = 1 \quad (9)$$

To construct a zero-beta portfolio, we adjust the weight at the end of each day to ensure that:

$$w_{1,t} * \beta_{1,t} + w_{2,t} * \beta_{2,t} = 0. \quad (10)$$

There could be two solutions with opposite signs for the weights, and we use the one with nonnegative portfolio alpha so that:

$$\alpha_{p,t} = w_{1,t} * \alpha_{1,t} + w_{2,t} * \alpha_{2,t}. \quad (11)$$

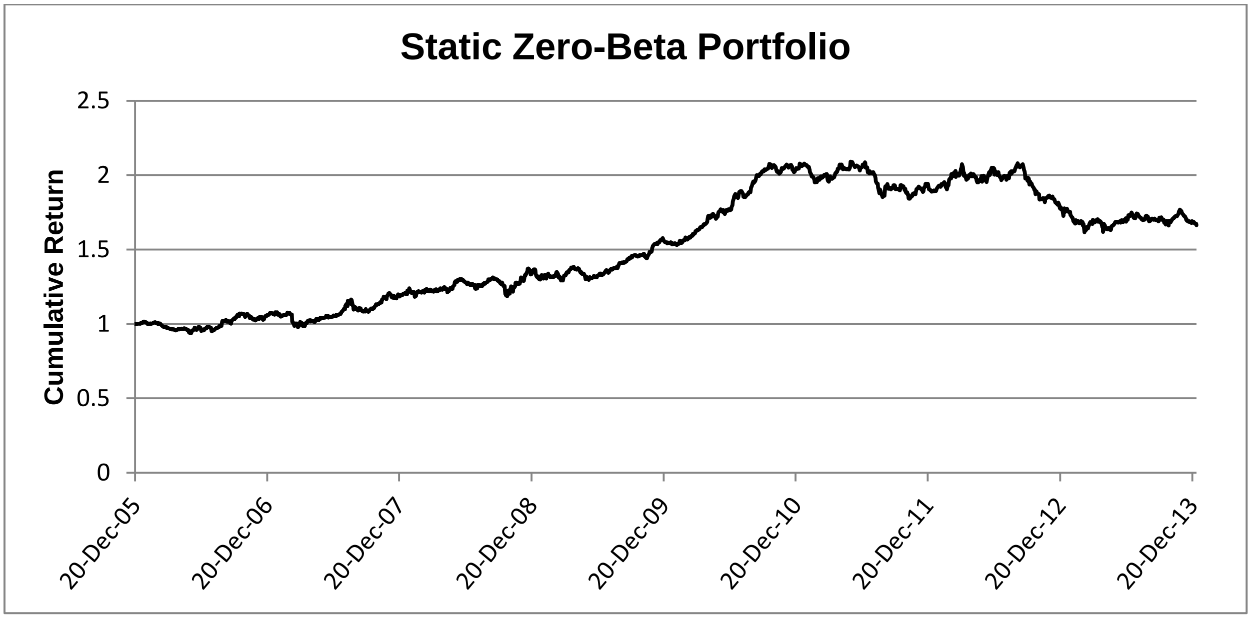
We rebalance the portfolio each day, and the return of the corresponding portfolio on day t+1 thus becomes:

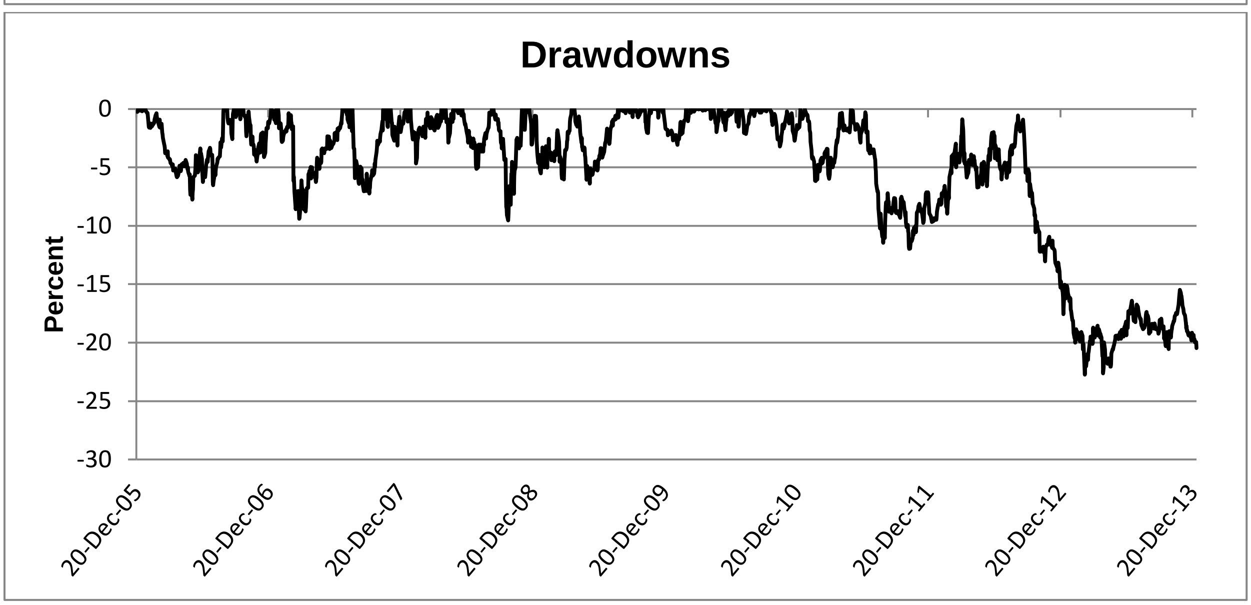
$$r_{p,t+1} = w_{1,t} * r_{1,t+1} + w_{2,t} * r_{2,t+1}. \quad (12)$$

There is an analytical solution to the equations above. So the weights can be easily calculated.

In the static OLS CAPM results, α = -0.00100 and β = -2.16932 for the VIX short-term futures index and α = -0.00018 and β = -1.20313 for VIX mid-term futures index. If we assign a weight of -0.3568 to the VIX short-term futures index and a weight of 0.6432 to VIX mid-term futures index, we have a zero-beta portfolio with a positive alpha of 0.000241024. This ratio is close to the -1:2 ratio employed in UBS XVIX exchange traded product. Alexander and Korovilas [2013] reason that the ratio might come from the fact that the VIX short-term futures index has over time a volatility approximately twice that of the mid-term index, and such a ratio could hedge 95% of the term structure movements. Employing the -1:2 ratio without using leverage, the behavior of the corresponding portfolio, which we call the "static zero-beta portfolio," is shown in Exhibit 7.

Exhibit 7. **Performance of the Static Zero-Beta Portfolio Using the VIX Short-Term Futures Index and VIX Mid-Term Futures Index, December 20, 2005, to December 31, 2013.**

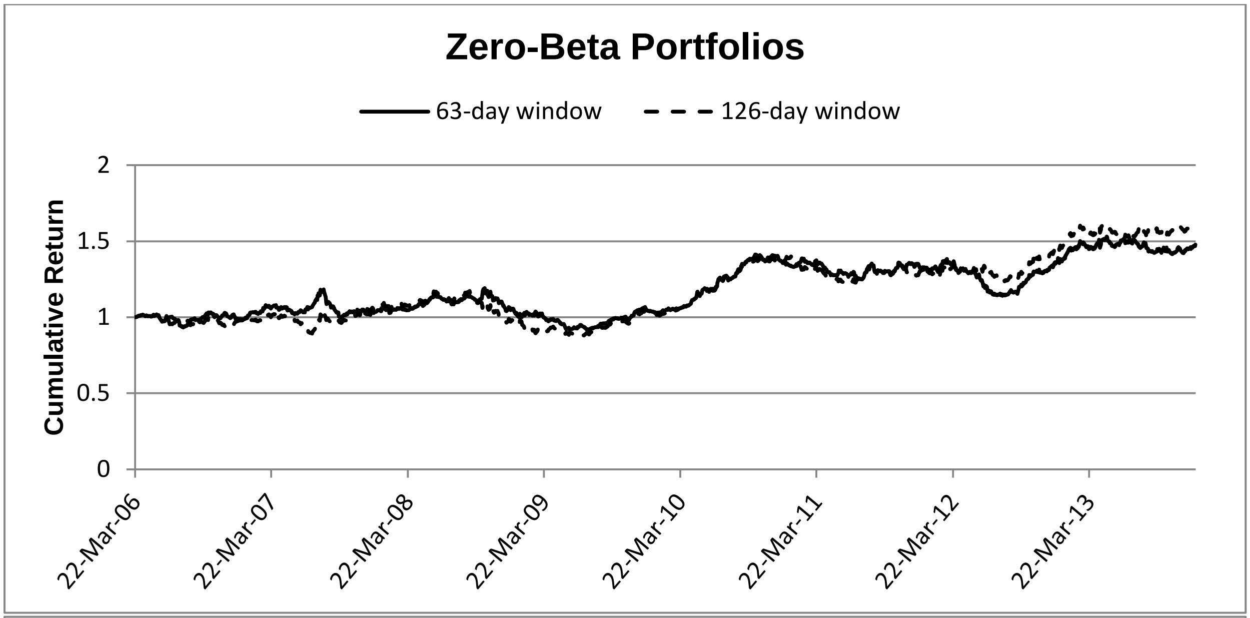


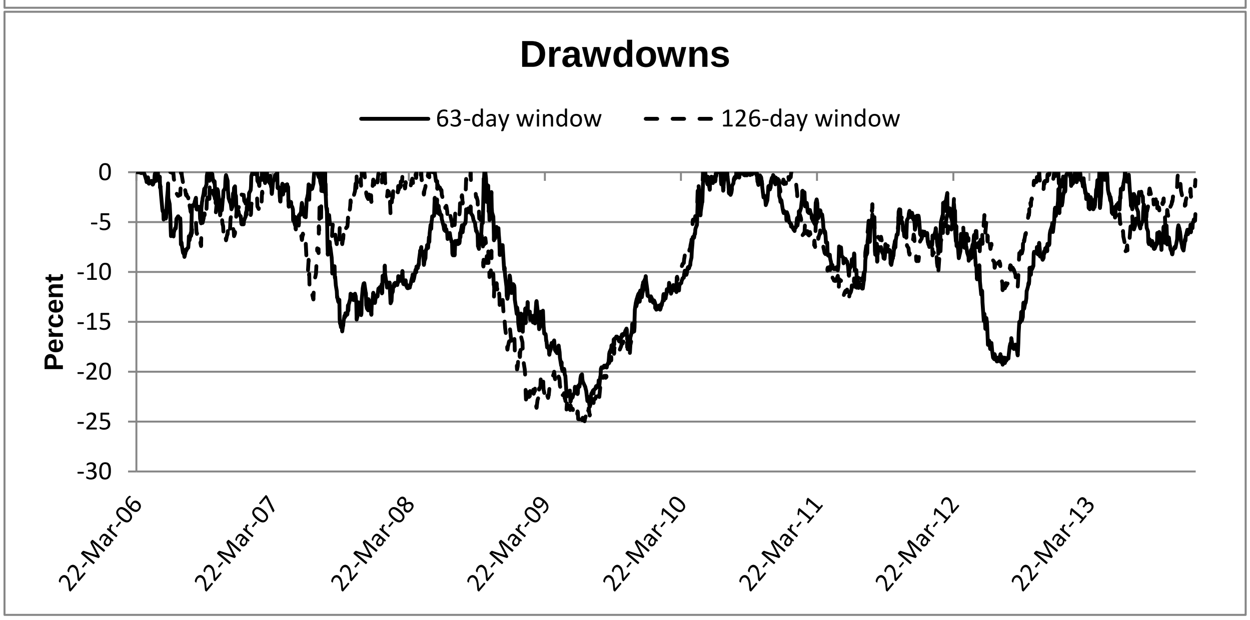


This portfolio had very good performance until the end of 2010. Afterwards, it performed poorly. The problem with this portfolio is the fixed weights. The proposed -1:2 ratio could work well at times, but there is no guarantee that market structure will not change.

Next, we apply rolling regression to calculate the α and β, then build a zero-beta portfolio. On day t we estimate the CAPM using the past n days of data. Again, we examine two cases, n = 63 days and n = 126 days. The corresponding weights to generate a zero-beta portfolio are calculated. The historical performance of the zero-beta portfolios computed with the 63- and 126-day windows is presented in Exhibit 8.

Exhibit 8. 63- and 126-Day Rolling Window Zero-Beta Portfolios Using the VIX Short-Term Futures Index and VIX Mid-Term Futures Index, March 22, 2006, to December 31, 2013.

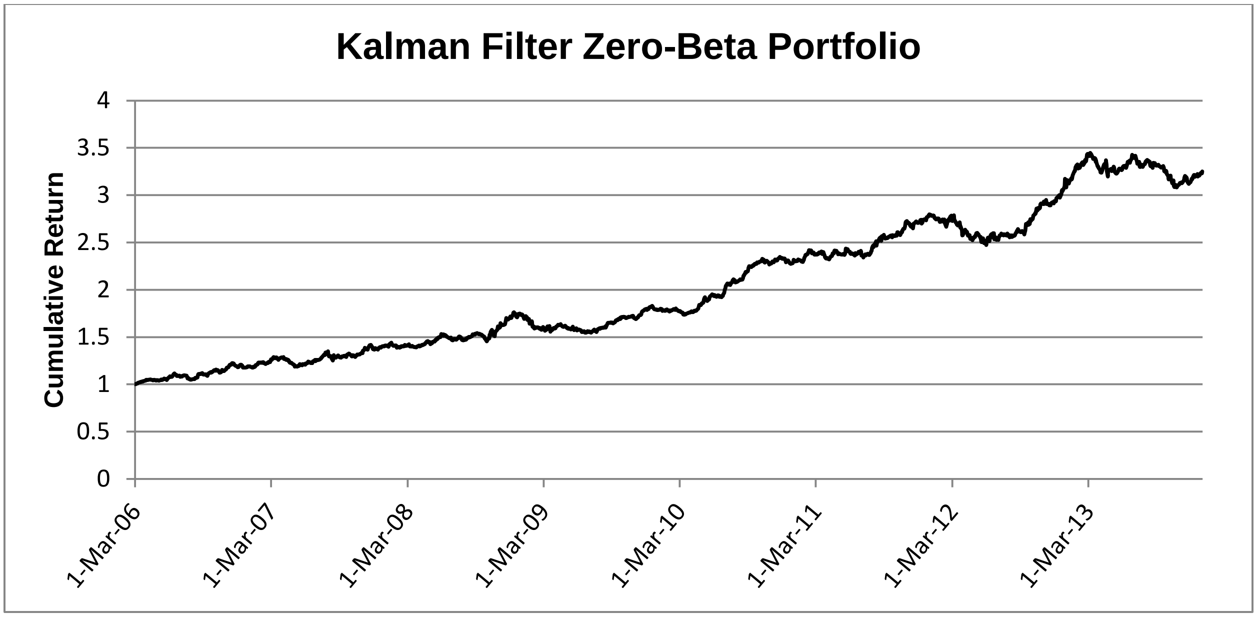


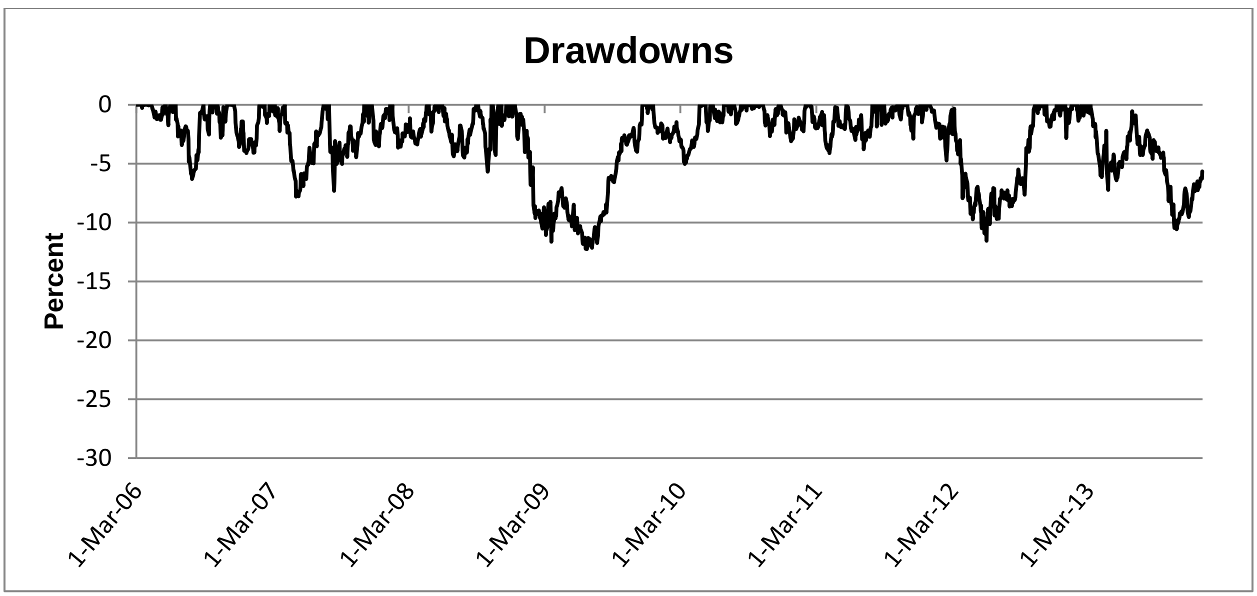


The two choices of window size yield different results, especially in 2012. The performance of the portfolio computed from the 126-day window is better in the second half of 2012. Such differences shed further light on the problem of choice of window size using rolling regression. The rolling window CAPM provides some self-adjusting capability to the portfolio. This is evident in the fact that performance did not eventually collapse as it did using static regression. We suspect the major drawdowns are caused by changes in market structure. The rolling approach could adjust itself to suit the changing environment and generate positive returns again. But the problem lies in part in the choice of window size. If the window size is too short, the estimation is susceptible to the effects of noise, and the uncertainty in the estimated parameters could greatly degrade portfolio performance. On the other hand, if the window size is too large, it will smooth over key changes in market structure, subjecting dynamic hedging to the effects of substantial lags which degrade performance. Complicating matters, market structure may be changing at varying frequencies. These considerations argue for a more dynamic approach.

Finally, we resort to dynamic CAPM using the Kalman filter. Cooper [2013] employed the technique using the non-investable VIX Index as the independent variable in the regression. We use S&P 500 index excess returns as the independent variable. VIX is mean reverting and, even during a sustained bull market, it could have many ups and downs which could complicate our view of market direction. At each day, we use the information from day 0 to day t to estimate the dynamic α and β. The corresponding weights are applied as in the rolling regression case. The performance of the resulting Kalman filter zero-beta portfolio is presented in Exhibit 9. In the exhibit, the data from December 20, 2005, to February 28, 2006, are used to stabilize the Kalman filter estimation.

Exhibit 9. **Kalman Filter Zero-Beta Portfolio Using the VIX Short-Term Index and VIX Mid-Term Index, March 1, 2006, to December 31, 2013.**





Overall, the log-cumulative return has a linear trend which is a good indication of market neutrality of the portfolio. This is because the Kalman filter zero-beta portfolio is based on more timely estimation of the dynamic α and β. It is designed to update and adjust the portfolio weight dynamically and rapidly. The Kalman filter estimation is not rooted in any analysis of contango/backwardation in the term structure of VIX futures or considerations of changes in the volatility of VIX. It only assesses how such changes are influencing α and β. With its random walk assumptions, it has some persistence in its estimations. Still, it is able to tune itself to changes in the market environment relatively fast.

The performance statistics for the four zero-beta portfolios are summarized in Exhibit 10.

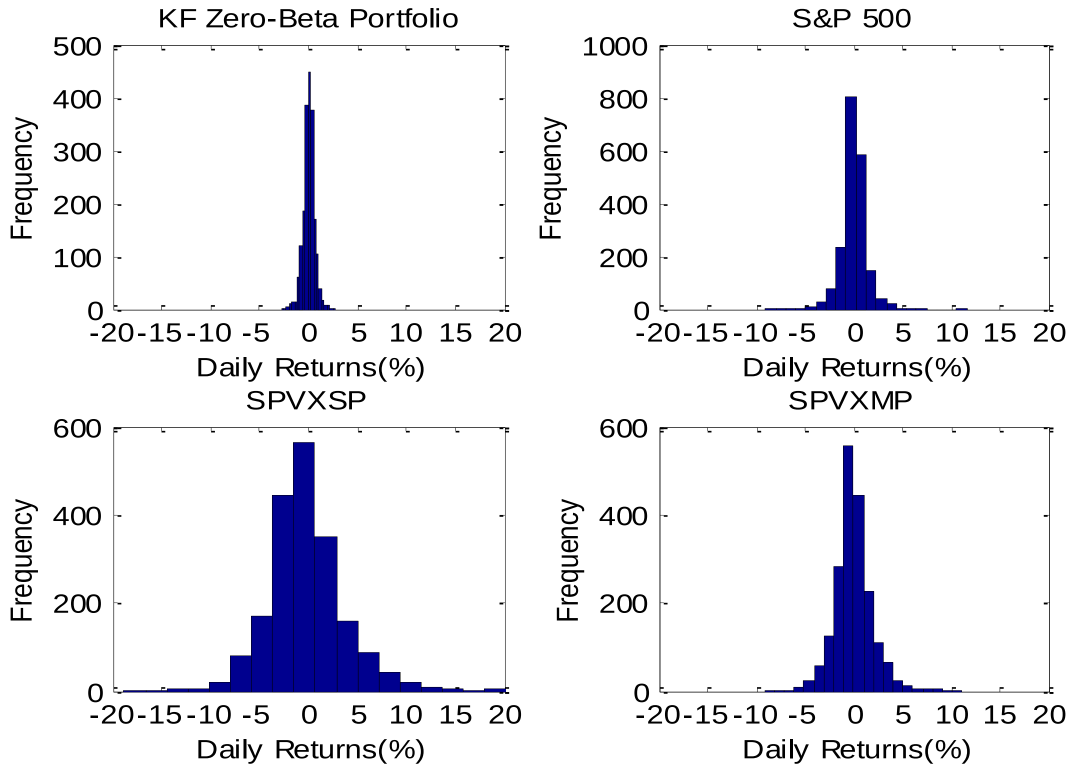
Exhibit 10. Performance Summary Statistics of the Alternative Approaches for Constructing Zero-Beta Portfolios, June 21, 2006, to December 31, 2013.

	Static	OLS-63	OLS-126	Kalman Filter
Annual Mean (%)	8.09	6.47	7.01	16.19
Volatility (%)	9.66	9.43	9.54	9.41
Sharpe ratio	0.68	0.54	0.59	1.47
Skewness	-0.25	-0.04	0.22	-0.06
Excess Kurtosis	4.70	2.22	4.06	1.95
Total Return (%)	79.51	60.20	66.36	208.81
Max Drawdown (%)	22.75	25.24	23.53	12.27
Correlation with SP 500	0.00	-0.01	0.01	-0.06
Correlation with SPVXSP	-0.22	0.03	0.00	0.06
Correlation with SPVXMP	0.23	0.08	0.08	0.11
α (%)	0.03	0.02	0.02	0.06
P-value for α	0.04	0.11	0.08	0.00
β	0.00	0.00	0.01	-0.02
P-value for β	0.89	0.66	0.55	0.02
R^2 (%)	0.00	0.01	0.02	0.30

All four zero-beta portfolios have low correlations with the S&P 500 Index, and their betas in the traditional CAPM mode are also very low. We note that the p-value for the Kalman filter portfolio indicates that its beta of -0.02 is statistically different from zero, but that value of -0.02 means that financially the Kalman filter produces effectively a zero-beta portfolio. In addition, the Kalman filter portfolio has superior performance. It has the best total return, the best Sharpe ratio and the smallest drawdown, and its correlation with the S&P 500 Index is very low. We suspect that this superior performance comes from the Kalman filter's capability to identify the current market structure.

Besides the return statistics, it would be beneficial to look at the distribution of the Kalman filter zero-beta portfolio daily returns. This is presented in the form of histograms in Exhibit 11.

Exhibit 11. Histograms of Daily Returns for the Kalman Filter Zero-Beta Portfolio, S&P 500 Index, VIX Short-Term Index, and VIX Mid-Term Index from March 1, 2006, to December 31, 2013.



In spite of the financial crisis and the subsequent bull market, the Kalman filter zero-beta portfolio has a very narrow distribution. Due to its portfolio construction, maximum daily swings are less than 3%. Compared with the S&P 500 Index, VIX short-term futures index, and VIX mid-term futures index, the Kalman filter zero-beta portfolio has negligible skewness.

Overall, the Kalman filter zero-beta portfolio offers an investment enjoying low correlation with the broad market, low volatility, and excellent total return during both the financial crisis and the subsequent bull market. The resulting portfolio is not only mean neutral, which implies zero beta exposure to the market index, but also variance neutral, which implies that the returns are not correlated with market risk movement including the 2008 market crisis.

ROBUSTNESS CHECKS

Robustness and Randomization of Parameters

The Kalman filter parameters are estimated by MLE using historical data. The estimations could vary using different optimization techniques, which raises the question whether the methodology yields the "best" parameters for purposes of portfolio construction. To examine this issue, we introduce randomness in the Kalman filter parameters. We allow all six estimated parameters in the Q and R matrices in the Kalman filter equations to vary independently and randomly within two standard deviations of the MLE estimated values. Then, we recompute the zero-beta portfolio with the new sets of "noisy" parameters. This exercise is repeated 100 times. The results are shown in Exhibit 12. Here, we can see strong similarities in portfolio performance despite the randomness in the parameters. Each run shows a steady increase in portfolio value, and in each run we see that the portfolios adjust themselves to changing market conditions.

Exhibit 12. Performance for 100 Portfolios Using Randomized Kalman Filter Parameters, March 1, 2006, to December 31, 2013.

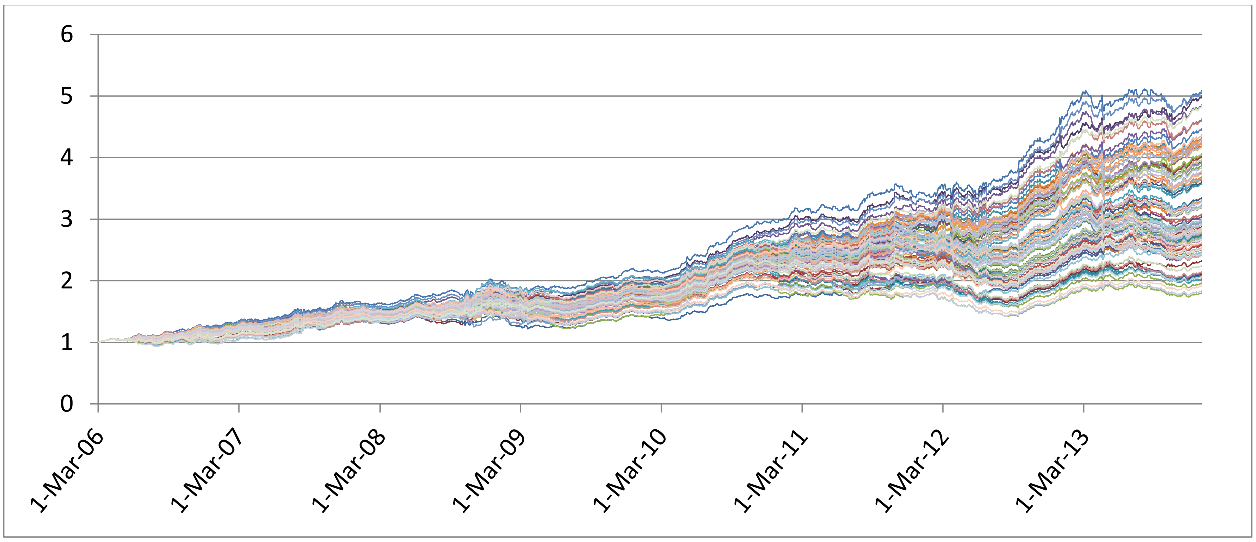
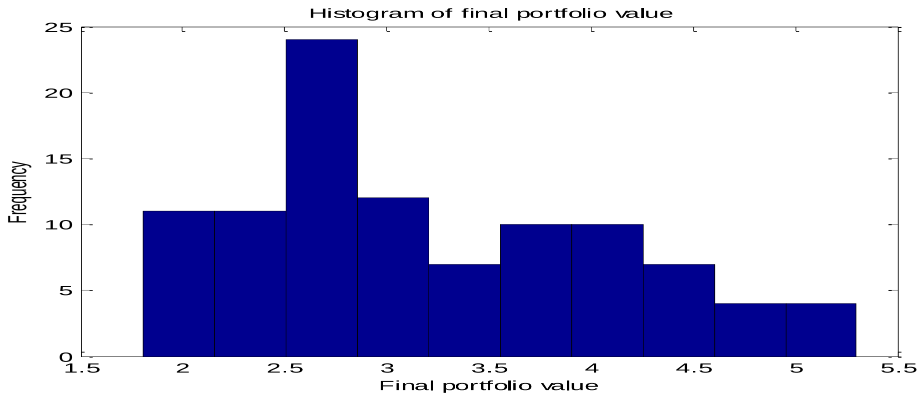


Exhibit 13 shows the results in the form of a histogram of the final portfolio values. The final portfolios each show positive returns, which further corroborates the robustness of the algorithm. The robustness stems from the way the Kalman filter controls the drift of the underlying random walk. Even with randomized changes of the presumed drift rate, the computation of α and β is maintained with sufficient precision to achieve good zero-beta portfolio performance.

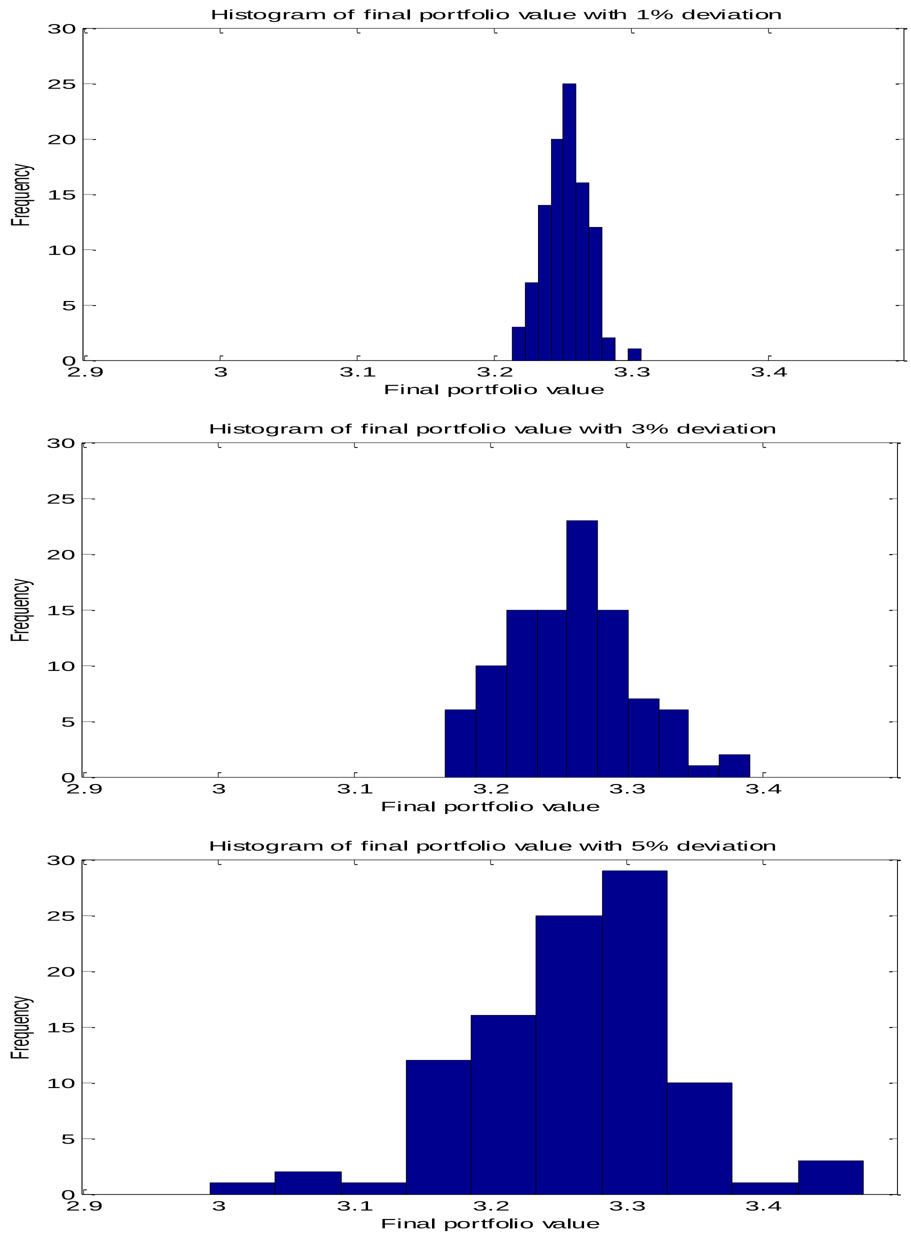
Exhibit 13. Histogram of Final Portfolio Values Using Randomized Parameters.



Robustness and the Effects of Intraday Price Volatility

As another robustness check, we examine whether the portfolio construction method is sensitive to the effects on trade execution of intraday volatility in the prices of VIX futures contracts. That is, the portfolio construction method is based on the assumption that trades take place at the close of the market. However, it may not be possible to transact at the closing price. The difference between the transaction price and closing price would be expressed as deviations in the portfolio weights $w_{1,t}$ and $w_{2,t}$ from their targeted values. So, to capture the effect of these price deviations, we introduce randomness into the daily portfolio weights $w_{1,t}$ and $w_{2,t}$. We allow the weights to vary independently and randomly each day within y percent plus/minus of the ideal values where y is set alternately at 1, 3, and 5 percent. For each y, we recompute the zero-beta portfolios 100 times with the sets of "noisy" weights. Exhibit 14 shows the performance outcomes in the form of histograms. Here, we see that the dispersions of the histograms increase as y increases. However, for each y, performance is always positive, and the means of the histograms are approximately the same.

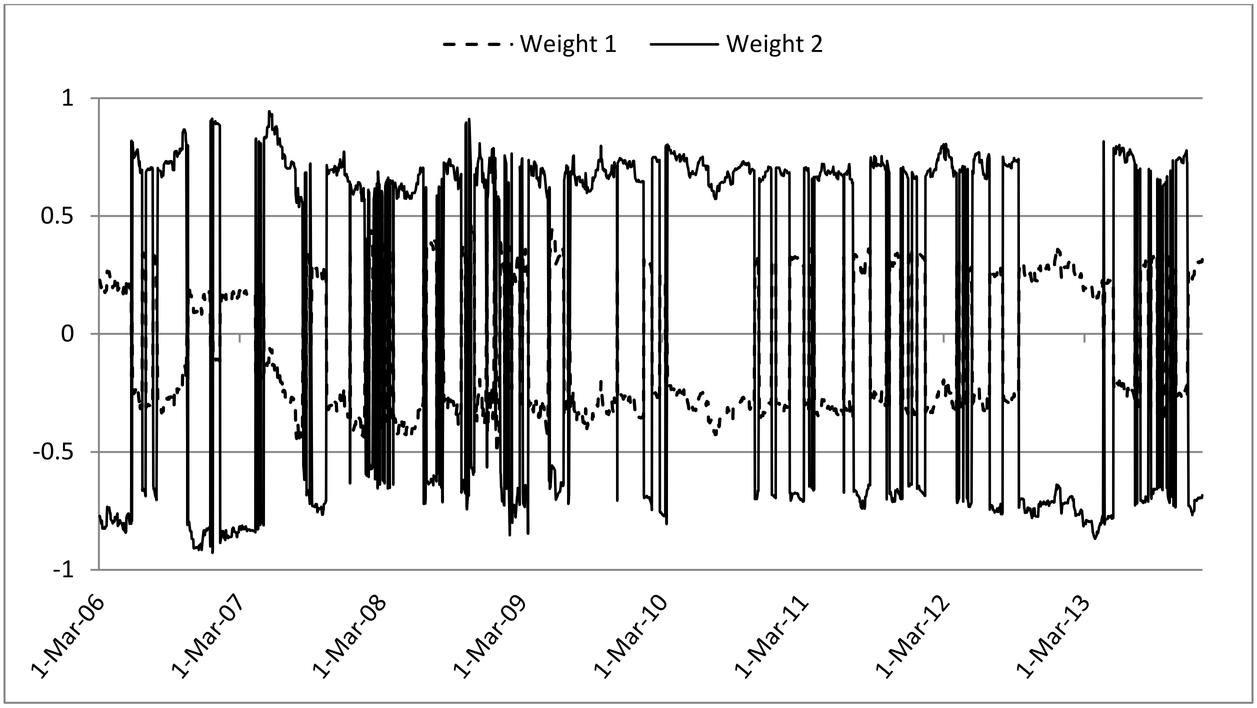
Exhibit 14. **Histograms of Final Portfolio Values Using Randomized Weights.**



Robustness and Trade Avoidance

As another robustness check, we note that hedge funds may seek to avoid trades where day-to-day changes in computed weights are small. The portfolio construction method can produce rapid changes in the weights $w_{1,t}$ and $w_{2,t}$ as seen in Exhibit 15. Still, the weights tend to be little changed for days at a time, which opens the possibility that small changes in the weights in such periods could be ignored without appreciably sacrificing performance.

Exhibit 15. **Daily Portfolio Weights, March 1, 2006, to December 31, 2013.**



To examine this issue, we note that, since the absolute values of the weights sum to 1, where one weight deviates in fixed absolute amount D, so does the other weight, and we compute performance where the weights are not adjusted until each differs absolutely by more than D from the last established weight. Exhibit 16 shows the relationship between values of D and final portfolio value. Here, we see that the imposition of the trading threshold does not generally impair performance. As D increases, more and more trades, those involving relatively small changes in weights, are eliminated, but the trades involving relatively large changes in weights are not eliminated, and these trades drive performance.

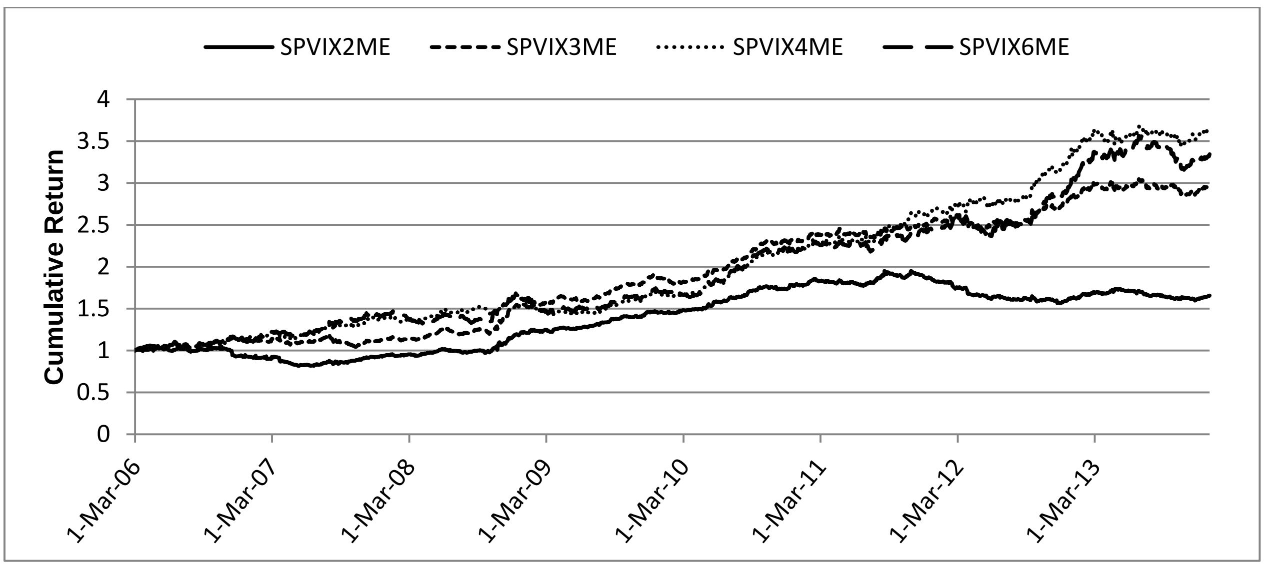
Exhibit 16. **Final Portfolio Value Where Trades Are Subject to Threshold D.**

D	Trades	Final Portfolio Value	Maximum Drawdown
0.00	1973	3.37	12.27
0.05	293	3.42	10.20
0.10	190	3.26	14.23
0.15	167	3.13	13.15
0.20	160	3.21	15.00
0.25	156	4.02	13.23
0.30	153	3.46	13.23

Robustness Using Alternative VIX Indexes

The Kalman filter zero-beta portfolio using the combination of the VIX short-term futures index and VIX mid-term futures index looks very attractive, but the question arises whether this performance is simply fortuitous. That is, are there deep connections among all the VIX futures indexes? Using the Kalman filter algorithms, we construct four additional zero-beta portfolios which combine the VIX short-term futures index and, one by one, the four other VIX futures indexes. The results are presented in Exhibit 17.

Exhibit 17. Performance Results for Kalman Filter Zero-Beta Portfolios Using the VIX Short-Term Futures Index Combined with Alternative VIX Futures Indexes, March 1, 2006, to December 31, 2013.



Performance results are summarized in Exhibit 18 for the original portfolio and the four additional portfolios. All five portfolios have suppressed volatility and betas of almost zero. With the exception of the portfolio containing SPVIX2ME, all portfolios have good Sharpe ratios and superior return characteristics. The results in the table seem to indicate that combining the VIX short-term futures index with the relatively longer maturities may provide better hedging capability in terms of portfolio skewness and kurtosis.

Exhibit 18. **Summary of Performance Results with Alternative VIX Futures Indexes.**

	SPVIX2ME	SPVIX3ME	SPVIX4ME	SPVXMP	SPVIX6ME
Annual Mean (%)	6.99	15.42	18.52	16.77	17.26
Volatility (%)	7.99	8.35	8.99	9.36	10.08
Sharpe ratio	0.68	1.56	1.75	1.52	1.45
Skewness	-0.42	0.13	0.01	-0.06	0.04
Excess Kurtosis	11.30	4.35	2.05	1.99	2.32
Total Return (%)	69.75	207.24	278.18	236.74	247.85
Max Drawdown (%)	22.27	12.87	14.51	12.27	15.34
Correlation with SP 500	-0.06	-0.02	-0.04	-0.06	-0.11
α (%)	0.02	0.05	0.06	0.06	0.06
P-value for α	0.04	0.00	0.00	0.00	0.00
β	-0.02	-0.01	-0.01	-0.02	-0.05
P-value for β	0.01	0.30	0.10	0.01	0.00
R^2 (%)	0.30	0.05	0.13	0.33	1.15

CONCLUSION

VIX instruments have been gaining in popularity among market participants, but their large swings pose risks. In this paper, we propose using the Kalman filter to hedge risks dynamically. Unlike conventional approaches, such as static OLS or rolling regression, the Kalman filter responds more promptly to changes in market conditions, allowing the user to adjust hedging ratios more rapidly, thus reducing portfolio volatility. While maintaining a portfolio with zero beta, our approach generates positive alpha, which in essence means that the dynamic system is long the winner and short the loser.

Traditional strategies are based on the speculation that a pricing relationship that appears to be out of line with theory or past behavior will revert to the perceived normal relationship. In contrast, by not betting on such convergence, which could widen dramatically during periods of market stress, our approach may avoid the substantial losses which occur often in traditional strategies. It may be possible to extend the zero-beta portfolio concept to multi-asset portfolios and to other highly correlated and volatile markets.

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