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Exchange Rates Dynamics with Long-Run Risk and Recursive Preferences*

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Abstract

Standard macro models cannot explain why real exchange rates are volatile and disconnected from macro aggregates. Recent research argues that models with persistent growth rate shocks and recursive preferences can solve that puzzle. I show that this result is highly sensitive to the structure of financial markets. When just a bond is traded internationally, then long-run risk generates insufficient exchange rate volatility. A long-run risk model with recursive-preferences can generate realistic exchange rate volatility, if *all* agents efficiently share their consumption risk by trading in complete financial markets; however, this entails massive international wealth transfers, and excessive swings in net foreign asset positions. By contrast, a long-run risk, recursive-preferences model in which only a fraction of households trades in complete markets, while the remaining households lead hand-to-mouth lives, can generate realistic exchange rate and external balance volatility.

JEL codes: F31, F36, F41, F43, F44

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1. Introduction

Real exchange rates among the major currencies are volatile and seem disconnected from macro aggregates. Standard macroeconomics models fail to explain these facts (Obstfeld and Rogoff (2000)). As the exchange rate is one of the key relative prices in an economy, conventional theory predicts that exchange rate movements are closely linked to fluctuations in aggregate demand and supply. For example, standard models predict that households can hedge country-specific output risk, by trading in international financial markets. Under conventional *time-separable* household preferences, the rate of real exchange rate appreciation is, thus, predicted to be perfectly negatively correlated with relative domestic/foreign consumption growth (Kollmann (1991, 1995), Backus and Smith (1993)). Yet, empirically, the real exchange rate is uncorrelated with relative consumption, and also much more volatile than consumption.

Recent research shows that models with recursive (non-separable) preferences of the Epstein and Zin (1989) and Weil (1989, 1990) type can generate realistic exchange rate volatility, *if* agents face 'long-run risk' (persistent growth rate shocks) that is efficiently shared, using complete global financial markets. See, e.g., Kollmann (2009), Colacito and Croce (2011, 2013), Lewis and Liu (2012), Gourio et al. (2013), Caporale et al. (2014) and Sauzet (2014). Recursive preferences allow the coefficient of risk aversion to differ from the inverse of the intertemporal elasticity of substitution, which entails that a household's intertemporal marginal rate of substitution of consumption (IMRS) depends on her expected life-time utility. Efficient risk sharing implies that the ratio of the domestic IMRS to the foreign IMRS is equated to the growth factor of the real exchange rate. With recursive preferences, persistent shocks to output growth generate wide fluctuations in the relative (domestic/foreign) IMRS, and hence in the real exchange rate, when consumption risk is efficiently shared. Importantly, in a world with long-run risk and recursive preferences, exchange rate fluctuations are potentially only weakly correlated with *current* (relative) consumption or output growth.

This paper offers a critical assessment of the role of long-run risk and recursive preferences for exchange rate dynamics. I show that this role is highly sensitive to the structure of international financial markets. I document that when global financial markets are incomplete, in the sense that just an unconditional bond can be traded internationally, as widely assumed in macro theory (e.g., Kollmann (1995, 1996), Baxter and Crucini (1995), Obstfeld and Rogoff (1996), Benigno and Thoenissen (2008)) and in multi-country policy models (e.g., Erceg et al. (2006), in't Veld et al. (2014), Kollmann et al. (2015)), then a model with long-run risk and recursive preferences generates *insufficient* real exchange rate volatility. A model with long-run risk and recursive preferences can generate realistic exchange rate volatility, when *all* agents can share their consumption risk by trading in complete markets. However, I show that this entails that country-specific output shocks trigger huge international wealth transfers, and thus induce vastly excessive swings in countries' net foreign asset positions.

I argue that *within*-country household heterogeneity in access to global financial markets is key for understanding the dynamics of the exchange rate and the external balance. I present a long-run risk, recursive-preferences model, in which only a fraction of households trades in complete markets ('risk-sharers'), while the remaining households lead hand-to-mouth (HTM) lives. The motivation for this structure is that, in reality, there is international trade in a wide array of state-contingent assets (equities, derivatives)—however, only a minority of households holds international assets (Christelis and Georgarakos (2009)). I show that the model with two types of household can generate realistic volatility of the real exchange rate *and* of net foreign assets, if 'risk-sharers' only account for a small share of aggregate output. Redistributive shocks between HTM households and 'risk-sharer' households help to explain why the empirical correlation between relative consumption growth and real exchange rate growth is close to zero.

The paper contributes also to the recent literature on open economy models with recursive preferences by providing (approximate) closed form model solutions and analytical results--the previous literature has relied on numerical simulations.

Section 2 describes the baseline model of a two-country world with recursive preferences. Sect. 3 discusses stylized facts about net foreign asset positions and real exchange rates. Sect. 4 presents simulation results and Sect. 5 concludes.

2. A two-country model with recursive preferences

2.1. Preferences, technologies, risk sharing

To facilitate comparison with the related literature, I consider a baseline structure that closely follows the recent open economy models with recursive preferences.[1] A world with two countries, Home (*H*) and Foreign (*F*) is assumed. The baseline model postulates that each country is inhabited by a representative infinitely-lived household. At date *t*, country $i=H,F$ receives an exogenous endowment of $Y_{i,t}$ units of a perishable tradable output good *i*. The countries have symmetric preferences, and face symmetric endowment processes. The country *i* household combines local and imported output into aggregate consumption, using the technology:

$$C_{i,t} \equiv [\alpha^{1/\phi}(y^i_{i,t})^{(\phi-1)/\phi} + (1-\alpha)^{1/\phi}(y^j_{i,t})^{(\phi-1)/\phi}]^{\phi/(\phi-1)}, \quad j \neq i,$$

where $y^j_{i,t}$ is the amount of input *j* used by country *i*; $\phi>0$ is the substitution elasticity between inputs. There is a local spending bias: $0.5<\alpha<1$. At *t*, country *i*'s consumption price index is:

$$P_{i,t} = [\alpha(p_{i,t})^{1-\phi} + (1-\alpha)(p_{j,t})^{1-\phi}]^{1/(1-\phi)}, \; j \neq i, \tag{1}$$

where $p_{j,t}$ is the price of good *j*. The Home terms of trade and real exchange rate are defined as

$$q_t \equiv p_{H,t}/p_{F,t} \text{ and } RER_t \equiv P_{H,t}/P_{F,t}, \tag{2}$$

respectively, i.e. a rise in *RER* represents an *appreciation* of the Home real exchange rate. Input demands are:

$$y^i_{i,t} = \alpha(p_{i,t}/P_{i,t})^{-\phi} C_{i,t}, \;\; y^j_{i,t} = (1-\alpha)(p_{j,t}/P_{i,t})^{-\phi} C_{i,t} \text{ for } j \neq i. \tag{3}$$

Market clearing requires $y^i_{H,t} + y^i_{F,t} = Y_{i,t}$ for $i=H,F$.

The country *i* household has a recursive intertemporal utility function of the Epstein and Zin (1989) and Weil (1989, 1990) type:

$$U_{i,t} = \{(1-\beta)C^{1-\sigma}_{i,t} + \beta[E_t U^{1-\gamma}_{i,t+1}]^{(1-\sigma)/(1-\gamma)}\}^{1/(1-\sigma)}, \tag{4}$$

[1] The baseline model with efficient risk sharing here is identical to the one used by Kollmann (2009) and Colacito and Croce (2013).

where $U_{i,t}$ is life-time utility at date *t*. $0<\beta<1$ is the subjective discount factor, $1/\sigma$ is the intertemporal elasticity of substitution (IES), while γ is the coefficient of risk aversion. At date *t*, agents know all endogenous and exogenous variables realized at *t* and earlier. E_t is the conditional expectation, given date *t* information.

Note that time-separable von Neumann-Morgenstern utility obtains when $\gamma=\sigma$. When $\gamma > \sigma$ holds, then agents have a preference for early resolution of uncertainty over future consumption (Weil (1990)). Country *i*'s intertemporal marginal rate of substitution (IMRS) between aggregate consumption at *t* and t+1 is:

$$\rho_{i,t+1} \equiv \beta \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-\sigma} \left(\frac{U_{i,t+1}}{(E_t U_{i,t+1}^{1-\gamma})^{1/(1-\gamma)}} \right)^{\sigma-\gamma} . \quad (5)$$

The baseline model assumes complete international financial markets, so that consumption risk is efficiently shared between Home and Foreign households. In equilibrium, the ratio of the two household's IMRSs is then equated to the growth factor of the real exchange rate (Kollmann (1991, 1995), Backus and Smith (1993)):

$$RER_{t+1}/RER_t = \rho_{H,t+1}/\rho_{F,t+1} . \quad (6)$$

When $\gamma=\sigma$, then $\rho_{i,t+1}=\beta(C_{i,t+1}/C_{i,t})^{-\sigma}$ holds, i.e. the IMRS depends solely on consumption growth, and risk sharing condition (6) implies: $(C_{H,t})^{-\sigma}=\Lambda\cdot(C_{F,t})^{-\sigma}RER_t$, where Λ is a date- and state invariant quantity that depends on the (relative) wealth of the two countries. Hence, country *H* relative consumption growth is perfectly negatively correlated with the rate of appreciation of the country's real exchange rate, when $\gamma=\sigma$: $\Delta\ln(C_{H,t+1}/C_{F,t+1}) = -(1/\sigma)\Delta\ln(RER_{t+1})$, as was first noted by Kollmann (1991, 1995) and Backus and Smith (1993). These authors document that the correlation between the rate of real exchange rate appreciation and relative consumption growth is close to zero, in data for a range of countries, i.e. the joint hypothesis of time-separable utility and efficient risk sharing is rejected empirically.[2]

[2] See Devereux and Kollmann (2012) and the 'Symposium on international risk sharing' published in 2012 by the Canadian Journal of Economics (Vol. 45, No.2) for detailed references to the risk sharing literature.

When $\gamma \neq \sigma$, then the IMRS also depends on life-time utility (see (5)). This breaks the tight link between relative consumption and the real exchange rate. The finance literature generally assumes $\gamma > \sigma$, as high risk aversion is needed for generating sizable risk premia on risky assets. Good news at date t+1 about future country H output induces an unanticipated rise in the country's life-time utility $U_{H,t+1}$, which lowers H's IMRS $\rho_{H,t+1}$, when $\gamma > \sigma$, and thus the Home real exchange rate depreciates, if markets are complete.[3] As shown below, country H responds to the good news by transferring resources to country F, i.e. country H net exports rise. This triggers a fall in the relative price of the country H output good, which depreciates the country H real exchange rate, as required by the risk sharing condition (6).

Let $NFA_{i,t+1}$ denote country i's net foreign assets at the end of period t. $NFA_{i,t+1}$ equals the present discounted value of i's future net imports. In recursive form: $NFA_{i,t+1} \equiv E_t \rho_{i,t+1}(P_{i,t}/P_{i,t+1})(NFA_{i,t+2} - NX_{i,t+1})$, where $NX_{i,t+1} \equiv p_{i,t+1}Y_{i,t+1} - P_{i,t+1}C_{i,t+1}$ are net exports at t+1. Empirically, $NFA_{i,t+1}$ corresponds to the *market value* of net foreign assets at the end of period t. Below, I report model predictions for country i net foreign assets and net exports, normalized by GDP, $\widetilde{NFA}_{i,t+1} \equiv NFA_{i,t+1}/(p_{i,t}Y_{i,t})$ and $\widetilde{NX}_{i,t} \equiv NX_{i,t}/(p_{i,t}Y_{i,t})$.

2.2. Linearized model

The numerical results presented below are based on a non-linear model solution. However, for building intuition, it is useful to first consider a (log-)linearized model solution--that solution captures the key qualitative features of the non-linear solution. Let $y_t \equiv Y_{H,t}/Y_{F,t}$, $c_t \equiv C_{H,t}/C_{F,t}$ denote date t relative Home output and consumption, respectively. $\hat{x} \equiv \ln(x_t/x)$ denotes the (log) deviation of a variable x_t from its steady state value x. I (log-)linearize the model around a symmetric balanced growth path in which both countries have identical endowments that grow at the constant (log) growth rate μ (i.e. the model is linearized around $RER = q = y = c = 1$). Equations (1),(2) imply:

[3] Country F life-time utility $U_{F,t+1}$ rises too, but less than $U_{H,t+1}$, due to consumption home bias ($\alpha > 0.5$).

$$\widehat{RER}_t = (2\alpha - 1)\widehat{q}_t \,. \tag{7}$$

Thus, a Home terms of trade improvement induces a Home real exchange rate appreciation (as $\alpha > 0.5$). (3) implies that relative world demand for output good *H* (compared to demand for good *F*) is:

$$d_t \equiv \{y^H_{H,t} + y^H_{F,t}\} / \{y^F_{F,t} + y^F_{H,t}\} = q_t^{-\phi} \{\alpha RER_t^{\phi} c_t + 1 - \alpha\} / \{\alpha + (1-\alpha) RER_t^{\phi} c_t\}.$$

Market clearing requires that relative demand equals relative output: $d_t = y_t$. Thus:

$$\widehat{y}_t = -4\alpha(1-\alpha)\phi\widehat{q}_t + (2\alpha - 1)\widehat{c}_t \,. \tag{8}$$

Up to a linear approximation, the Home net exports/GDP ratio obeys $\widetilde{NX}_{H,t} = \eta_q \widehat{q}_t - ((1-\alpha)/(2\alpha - 1))\widehat{y}_t$, with $\eta_q \equiv (1-\alpha)\{1 - \phi 2\alpha/(2\alpha - 1)\}$. Empirical estimate of the price elasticity ϕ of aggregate imports and exports are generally in the range of unity (e.g., Hooper and Marquez (1995), Kollmann (2001)). This implies that $\eta_q < 0$ holds for empirically plausible values of ϕ. Holding constant relative output, a depreciation of the Home real exchange rate is accompanied by a fall in Home relative consumption (see (7),(8)), and by a rise in Home next exports.

Linearizing the risk-sharing condition (6) gives:

$$\Delta\widehat{RER}_{t+1} = -\sigma\Delta\widehat{c}_{t+1} - (\gamma - \sigma)(1 - \widetilde{\beta})(E_{t+1} - E_t)\sum_{s=0}^{\infty} \widetilde{\beta}^s \widehat{c}_{t+1+s}, \tag{9}$$

with $\widetilde{\beta} \equiv \beta \exp(\mu \cdot (1-\sigma))$. (I assume that $\widetilde{\beta} < 1$.) When $\gamma = \sigma$, then this condition gives the standard risk sharing $\Delta\widehat{RER}_{t+1} = -\sigma\Delta\widehat{c}_{t+1}$ (Kollmann (1991, 1996), Backus and Smith (1993)). When $\gamma \neq \sigma$, then $\Delta\widehat{RER}_{t+1} \neq -\sigma\Delta\widehat{c}_{t+1}$, but a conditional version of the standard risk sharing condition holds: $E_t\Delta\widehat{RER}_{t+1} = -\sigma E_t\Delta\widehat{c}_{t+1}$. Thus, under recursive utility, the expected rate of real exchange rate appreciation is perfectly negatively correlated with the expected relative consumption growth rate, up to a first-order approximation. Using (8), it can be shown that this implies:

$$E_t\Delta\widehat{RER}_{t+1} = -H(\sigma)E_t\Delta\widehat{y}_{t+1} \,, \tag{10}$$

where $H(x)\equiv(2\alpha-1)/[(2\alpha-1)^2/x+\phi 4\alpha(1-\alpha)]>0$. Hence, the Home real exchange rate is expected to depreciate between periods *t* and *t*+1, when relative Home GDP is expected to increase, between *t* and *t*+1. (8),(9) and (10) imply:

$$\widehat{RER_{t+1}}-E_t\widehat{RER_{t+1}}=-J(\sigma)\{\widehat{y_{t+1}}-E_t\widehat{y_{t+1}}\}-(H(\gamma)-H(\sigma))(1-\tilde{\beta})(E_{t+1}-E_t)\sum_{s=1}^{\infty}\tilde{\beta}^s\widehat{y_{t+1+s}}, \quad (11)$$

where $J(\sigma)\equiv\tilde{\beta}H(\sigma)+(1-\tilde{\beta})H(\gamma)>0$. (10) and (11) show that the expected rate of real exchange rate appreciation depends on σ (inverse of intertemporal substitution elasticity), but not on the risk aversion coefficient γ; however, γ affects the response of the real exchange rate to output *surprises*.

Consider a *transitory* positive innovation to Home relative output at t+1, y_{t+1}, i.e. an innovation that does not change the expected path of output *after* date t+1. It follows from (8) and (11) that this shock triggers a surprise depreciation of the Home real exchange rate at t+1, and a surprise increase in Home relative consumption. Thus, the impact responses of the real exchange rate and of relative consumption to a transitory relative output shock are *negatively* correlated.

New date *t*+1 information about *future* output affects the real exchange rate at *t*+1 when $\gamma\neq\sigma$. A 'pure' news shock that only affects the expected *future* path of relative output, without affecting *current* relative output, triggers *impact* responses of the real exchange rate and of relative consumption that have the same sign (see (8)). (11) suggests that a model with $\gamma\neq\sigma$ has the potential to generate a highly volatile exchange rate, if sufficiently large revisions of expectations about the future output path occur. When $\gamma>\sigma$ is assumed (as in the simulations below), then $H(\gamma)>H(\sigma)$ holds, and an upward revision of the expected path of future relative Home output induces a depreciation of the Home real exchange rate, and a fall in Home relative consumption.

2.3. Calibration

2.3.1. Preference and technology parameters

To facilitate comparison with related studies, I use the same baseline calibration as Colacito and Croce (2013) (that is based on US and UK data). One period represents one calendar year. The subjective discount factor is set at β=0.98. The intertemporal substitution elasticity $(1/\sigma)$ is set at 1.5 consistent with standard estimates of that parameter reported in the macro literature, while the risk aversion coefficient is set $\gamma=8$. A high value of γ (greater than σ) is needed to allow shocks to long-run output growth rates to generate sizable real exchange rate responses. The home bias parameter is set at $\alpha=0.97$, which implies that the steady state trade share (exports/GDP) is 3%.[4] The substitution elasticity between domestic and imported goods is set at $\phi=1$, consistent with the fact that empirical estimate of the price elasticity of aggregate imports and exports are generally in the range of unity (as mentioned above).

2.3.2. Endowment processes

Following Colacito and Croce (2013), the baseline model assumes that log output has a unit root, and is co-integrated across countries:

$$\ln(Y_{i,t})=\mu+\ln(Y_{i,t-1})+z_{i,t-1}-\kappa\cdot[\ln(Y_{i,t-1})-\ln(Y_{j,t-1})]+\varepsilon^{Y}_{i,t},\;\; z_{i,t}=\rho^{z}z_{i,t-1}+\varepsilon^{z}_{i,t} \text{ for } i=H,F \text{ and } j\neq i \quad (12)$$

with $\kappa>0$ and $0<\rho^{z}<1$. $\varepsilon^{Y}_{i,t},\varepsilon^{z}_{i,t}$ are normal white noises. Hence, the growth rate of country *i* output between *t*-1 and *t* is driven by the serially correlated component $z_{i,t-1}$ that is known in period *t*-1, and by the i.i.d. disturbance $\varepsilon^{Y}_{i,t}$. $\mu=0.02$ and $\rho^{z}=0.985$ are assumed, i.e. fluctuations in the (predictable) trend growth rate are highly persistent. A positive date *t* innovation $\varepsilon^{z}_{i,t}$ has no effect on date *t* output, but a permanent positive effect on future output. The error-correction coefficient is set at a very small positive value, $\kappa=0.0005$, which implies that log relative output $\ln(Y_{H,t}/Y_{F,t})$ is stationary, but highly serially correlated. The standard deviations and correlations of the output innovations are set at $Std(\varepsilon^{Y}_{i,t})=1.87\%$ and $Std(\varepsilon^{z}_{i,t})=0.2618\%$ for i=*H*,*F*; $Corr(\varepsilon^{Y}_{H,t},\varepsilon^{Y}_{F,t})=0.05$, $Corr(\varepsilon^{z}_{H,t},\varepsilon^{z}_{F,t})=0.90$ and $Corr(\varepsilon^{Y}_{i,t},\varepsilon^{z}_{j,t})=0$ for *i*,*j*=*H*,*F*. Hence, shocks to the trend growth rate $(\varepsilon^{z}_{i,t})$ are smaller than the transitory growth-rate shocks $(\varepsilon^{Y}_{i,t})$, but markedly more highly correlated across countries. See Colacito and Croce

[4] The total US trade share (0.5*(exports+imports)/GDP) averaged 12% during the period 1990-2013. The key results are robust to setting the steady state trade share at 12% ($\alpha=0.88$).

(2013) for a justification of the output process (12) and its calibration (inspired by the long-run risk literature; e.g. Hoffmann et al. (2013)).

As a sensitivity analysis, I also consider two simpler exogenous processes of the type assumed in the international RBC literature (e.g., Kollmann (1996, 2009)). The first of these processes assumes that log output is first-difference stationary (and cointegrated):

$$\Delta\ln(Y_{i,t})=(1-\rho^{\Delta Y})\mu+\rho^{\Delta Y}\Delta\ln(Y_{i,t-1})-\kappa\cdot[\ln(Y_{i,t})-\ln(Y_{j,t})]+\varepsilon_{i,t}^{\Delta Y},\ 0<\rho^{\Delta Y}<1,\ \text{for } i=H,F \text{ and } j\neq i \quad (13)$$

(again $\mu=0.02, \kappa=0.0005$ is assumed). The empirical autocorrelations of annual US and rest-of-the world (ROW) GDP growth rates 1980-2013 were 0.33 and 0.31, respectively, while the correlation between US and ROW GDP growth rates was 0.39. [5] I set $\rho^{\Delta Y}=0.3$, $Corr(\varepsilon_{1,t}^{\Delta Y},\varepsilon_{2,t}^{\Delta Y})=0.39$ in model versions that assume (13). (Under the baseline process (12), the cross-country correlation of output growth too is 0.39.)

I also consider a trend-stationary output process:

$$\ln(Y_{i,t})=\rho^{Y}\mu+(1-\rho^{Y})\mu\cdot t+\rho^{Y}\ln(Y_{i,t-1})+\varepsilon_{i,t}^{TS},\ 0<\rho^{Y}<1 \quad \text{for } i=H,F. \quad (14)$$

Linearly detrended annual log real GDP in the US and in the ROW during the period 1980-2013 had autocorrelations of 0.89 and 0.87, respectively. In simulations based on (14), I set $\rho^{Y}=0.9$, $Corr(\varepsilon_{1,t}^{TS},\varepsilon_{2,t}^{TS})=0.39$.

For the sake of comparison with the baseline output process (12), I calibrate the standard deviations of the innovations of the alternative processes (13) and (14) so that the implied unconditional standard deviation of the output growth rate equals the standard deviation under (12): 2.41%. Thus, I set $Std(\varepsilon_{i,t}^{\Delta Y})=2.31\%$ and $Std(\varepsilon_{i,t}^{TS})=2.35\%$, respectively.

2.4. Solution method

As trend output growth is positive, I reformulate the model by normalizing each country's date *t* consumption and welfare by its date *t* output (see Appendix I). The

[5] These empirical statistics are based on annual growth rates series from the IMF's World Economic Outlook database. World growth g_t^W is a weighted average of US and ROW growth (g_t^{US}, g_t^{ROW}): $g_t^W=s_t g_t^{US}+(1+s_t)g_t^{ROW}$, where s_t is the share of US GDP in world GDP. I use data on g_t^W, g_t^{US}, s_t provided by the WEO database to construct a time series for g_t^{ROW}.

reformulated model is solved using a third-order approximation around the symmetric deterministic steady state. The Dynare toolbox (version 4.4.3) is used for that purpose (Adjemian et al. (2014)). Simulations are based on the pruned state-space representation of the third-order accurate model solution (Kollmann (2005, 2013), Kim et al. (2008), Andreasen et al. (2013)).

3. Empirical volatility of net foreign assets and real exchange rates

In annual US data for 1980-2013, the standard deviations of $\Delta\widetilde{NFA}$ (first-differenced net foreign assets/GDP ratio) and of $\widetilde{NX}$ (net exports/GDP) were 4.77% and 1.58%, respectively, while the standard deviation of the first-differenced log real effective exchange rate was 5.20%.[6] Note that the empirical standard deviation of US $\Delta\widetilde{NFA}$ is close to the standard deviation of the annual rate of change of the US effective real exchange rate. Kollmann (2006) reports that, across 17 OECD economies during the period 1976-2004, the mean and median standard deviations of $\Delta\widetilde{NFA}$ were 10.97% and 6.94%, respectively, and thus larger than the standard deviation of US $\Delta\widetilde{NFA}$ reported above. Bilateral real exchange rates between the US and individual countries are often more volatile than the effective real exchange rate; for example, Colacito and Croce (2013) report that the 1971-2008 historical standard deviation of bilateral US/UK real annual exchange rate growth rate was 11%.

4. Model predictions

Table 1 reports model-predicted moments of key variables, while Table 2 shows dynamic responses to output innovations. The moments in Table 1 are averages computed across

[6]The empirical measure of US *NFA* used here is the net international investment position reported by the Bureau of Economic Analysis [BEA]. That series is based on market values of gross external assets and liabilities. One can interpret the first difference of *NFA* as the country's 'valuation adjusted' current account. That measure reflects capital gains/losses on external assets and liabilities; thus, it differs from the conventional current account reported in official balance of payments statistics, as the conventional measure equals the net *flow* of assets acquired by a country, and thus does *not* take into account capital gains/losses on external assets/liabilities acquired in the past (e.g., Kollmann (2006) and Coeurdacier et al. (2010)). Annual U.S. GDP data (used for construction of $\widetilde{NFA}$) are also from BEA. The US empirical real effective exchange rate used here is the Federal Reserve Board's 'Price-adjusted Broad Dollar Index', Table H.10 (the published series has a monthly frequency; I construct an annual series by computing the average of the monthly observations in each calendar year).

500 stochastic simulation runs. Each simulation run is initialized at the deterministic steady state and has a length of 100 periods; the reported moments were computed using the last 50 periods only (to reduce dependence on initial conditions).

4.1. Model variants with efficient risk sharing

Panel (a) of Tables 1 shows predicted moments for model variants with efficient risk sharing. Line (a.1) of Table 1 reports moments for the baseline model (i.e. $\gamma > \sigma,$ output process (12) and complete markets are assumed). The predicted standard deviations of the growth rates of the real exchange rate and of relative consumption are 13.31% and 2.10%, respectively, in the baseline model.[7] Thus, that model produces real exchange rate volatility that is broadly in line with the data. However, the predicted standard deviation of the first-differenced net foreign assets/GDP ratio $\Delta\widetilde{NFA}$ (49.10%) is an order of magnitude larger than the corresponding empirical moment for the US (4.77%) reported above.[8]

Also shown in Table 1 are predicted correlations between growth rates of relative consumption and of the real exchange rate $(\Delta\ln(c_{t+1}), \Delta\ln(RER_{t+1})),$ between one-period-ahead expected growth rates of these variables $(E_t\Delta\ln(c_{t+1}),\ E_t\Delta\ln(RER_{t+1})),$ and between Home and Foreign consumption growth (see Columns (5)-(7)). The baseline model

[7] These predicted statistics are close to those reported by Colacito and Croce (2013) (who also compute a third-order model approximation).

[8] Empirically, and in the model, the *level* of the debt/GDP ratio $\widetilde{NFA}$ is highly persistent (Augmented Dickey-Fuller tests fail to reject the hypothesis that historical $\widetilde{NFA}$ has a unit root), which implies that the standard deviation (Std) of $\widetilde{NFA}$ is increasing in the sample length. Thus, I focus on moments of the first-difference $\Delta\widetilde{NFA}.$ Colacito and Croce (2013) [CC], instead, discuss moments of the level $\widetilde{NFA}$ (and not of $\Delta\widetilde{NFA}$). According to CC (Table II), the empirical Std of annual $\widetilde{NFA}$ was 34% in 1971-2008 (16 times the Std of GDP growth rate). CC state that this is the 'simple average of US and UK volatilities' based on the (updated) Lane and Milesi-Ferretti (2007) dataset [LMF]. However, using LMF data, I find that Stds of US and UK $\widetilde{NFA}$ were 10.04% and 12.74%, respectively, 1971-2008 (5.0 and 6.4 times the Std of GDP growth). (Stds of US and UK LMF $\Delta\widetilde{NFA}$: 2.43% and 5.95% in 1971-2013.) In annual BEA data, the Std of US $\widetilde{NFA}$ is 11.79% for 1976-2013. See Appendix II for the $\widetilde{NFA}$ data. CC report that their baseline model predicts that the Std of $\widetilde{NFA}$ is 22 times the Std of GDP growth (i.e. 47%). My baseline model simulations give a 90% Std for $\widetilde{NFA},$ based on runs of 38 periods (the length of the 1971-2008 sample). The model-predicted variability of $\widetilde{NFA}$ is thus *much* greater than the historical variability.

predicts that relative consumption growth is uncorrelated with real exchange rate growth, and that consumption growth is weakly positively correlated across countries. These predictions are in line with the data (e.g., Kollmann (1991, 1995), Backus and Smith (1993)). However, the baseline model also predicts that expected relative consumption growth and expected real exchange rate growth are highly negatively correlated (-0.79). With one exception (a structure with hand-to-mouth households), all other model variants discussed below likewise generate a strong negative correlation between *expected* relative consumption and real exchange rate growth. This is a counterfactual model property: Devereux et al. (2011) document empirically (using surveys of professional forecasts) that predicted relative consumption and real exchange rate growth are essentially uncorrelated.

A model variant with the simple difference-stationary stochastic process for output (13) too generates a highly volatile real exchange rate, and vastly excessive fluctuations of net foreign assets (see Table 1, Line (a.3)). By contrast, a model version with the trend-stationary output process (14) under-predicts the standard deviations of these variables (see Table 1, Line (a.4)).

Line (a.2) of Table 1 considers a model variant that assumes the baseline output process (12), but in which the risk aversion coefficient is set at the inverse of the intertemporal substitution elasticity $(\gamma=\sigma=2/3)$, i.e. that variant assumes standard time-separable utility. In that model variant, the predicted standard deviations of real exchange rate growth (1.68%) and of net exports (0.09%) are much smaller than the corresponding empirical moments (this confirms simulation results reported by Kollmann (2009) and Colacito and Croce (2013) who also consider model variants with long-run risk and $\gamma=\sigma$); the predicted standard deviation of $\Delta\widetilde{NFA}$ (3.73%) too is now smaller than the empirical statistic for the US (1980-2013).

These results confirm the recent literature (see Introduction) that has shown that a model with long-run risk, recursive preferences $(\gamma>\sigma)$ and efficient risk sharing can generate a volatile real exchange rate. However, the simulations here identify a key shortcoming of the proposed mechanism that has not been noted so far, namely that it entails vastly excessive swings in countries' net foreign asset positions, and thus huge cross-country wealth transfers.

The impulse responses reported in Table 2 help to understand these model features. Table 2 reports responses to one-time one-standard deviation positive innovations to Home exogenous forcing processes (assuming that exogenous innovations in all other periods are zero). Responses of the real exchange rate, relative consumption and relative output are expressed as relative deviations from unshocked paths, while the responses of Home net exports and net foreign assets (normalized by GDP) are expressed as differences from the unshocked path.

A one-standard deviation ***transitory*** **Home output growth rate shock** $\varepsilon_{H,t}^{Y}$=1.87% raises Home relative consumption, and depreciates the Home rear exchange rate. This is the case in all model variants. Importantly, in the baseline model (with $\gamma>\sigma$) the impact response of relative consumption (0.78%) is much weaker than the rise in relative output; the responses of net exports (0.25% of GDP), the real exchange rate (-9.12%) and net foreign assets (-8.52% of GDP) are strong and persistent (see Table 2, Panel (a.1)). The transitory growth rate shock $\varepsilon_{H,t}^{Y}$ has a permanent positive effect on Home output, which strongly raises Home welfare. When $\gamma>\sigma$ holds, then efficient risk sharing requires Home to transfer resources to Foreign, which is why Home net exports rise strongly and persistently. This wealth transfer entails the sharp drop in Home net foreign assets.

By contrast, in the model variant with time-separable utility, $\gamma=\sigma$, relative consumption rises roughly by the same amount as relative output, and Home net exports *fall* slightly, which implies a small *increase* in Home net foreign assets, and a modest Home real exchange rate depreciation, -1.22% (see Table 2, Panel (a.2)).

In the baseline model ($\gamma>\sigma$), a one-standard deviation **shock to the Home trend output growth rate** $\varepsilon_{H,t}^{z}$=0.26% has an even more powerful effect on the real exchange rate, Home net exports and net foreign assets (than the transitory growth rate shock $\varepsilon_{H,t}^{Y}$). As pointed out above, a positive innovation $\varepsilon_{H,t}^{z}$ only raises Home output with a one-period lag (see (12)). When $\gamma>\sigma$ holds, then efficient risk sharing entails that Home immediately transfers resources to Foreign, in response to news that the future path of Home output will be higher. The trend growth rate innovation $\varepsilon_{H,t}^{z}$=0.26% *lowers* Home relative consumption by 2.99% and raises Home net exports by 0.94% of GDP, on impact, and it depreciates the Home real exchange rate by 22.44%, in the baseline model (Table 2, Panel (a.1)). The strong and persistent rise in Home net exports induced by the shock implies a very sharp and persistent drop in Home net foreign assets: -15.70% of GDP, on impact. By contrast, under time-separable utility $(\gamma=\sigma)$ the trend growth rate shock $\varepsilon^{z}_{H,t}$ has no effect on the real exchange rate, relative consumption and net exports, on impact (Table 2, Panel (a.2)). Responses in subsequent periods are much more muted than in the baseline model variant with $\gamma>\sigma$.

The results discussed so far all pertain to model variants with efficient international risk sharing. I next investigate the role of financial frictions for real exchange rate and external balance dynamics.

4.2. Model variants with financial frictions

4.2.1. Bonds-only economy

Many open economy models assume that global financial markets are incomplete, in the sense that just an unconditional bond can be traded internationally (e.g., Obstfeld and Rogoff (1996), Kollmann (1991, 1996), Baxter and Crucini (1995), Benigno and Thoenissen (2008)). Line (b.1) of Table 1 reports predicted moments generated by a model variant in which the only traded asset is a one-period bond--otherwise this variant is identical to the baseline model with long-run run risk (output process (12)) and $\gamma>\sigma$. To maintain symmetry between the two countries, I assume that the bond is denominated in a basket consisting of half a unit of the Home output good, and half a unit of the Foreign good. Country *i* thus faces the budget constraint:

$$\tfrac{1}{2}(p_{1,t}+p_{2,t})NFA_{i,t+1}+P_{i,t}C_{i,t}=p_{i,t}Y_{i,t}+\tfrac{1}{2}(p_{1,t}+p_{2,t})NFA_{i,t}(1+r_t^{A})\,, \qquad (15)$$

where $NFA_{i,t+1}$ represents bond holdings at the end of period *t*, while r_t^{A} is the bond rate between periods *t*-1 and *t*. Home and Foreign households' optimal intertemporal decisions are governed by these Euler equations:

$$(1+r_{t+1}^{A})E_t(P_{i,t}/P_{i,t+1})((p_{H,t+1}+p_{F,t+1})/(p_{H,t}+p_{F,t}))\rho_{i,t+1}=1 \text{ for } i=H,F. \qquad (16)$$

In the bonds-only set-up, the risk sharing condition (6) fails to hold: the ratio of the Home and Foreign intertemporal marginal rates of substitution is not equated to the growth factor of the real exchange rate on a state-by-state basis. (16) implies merely that, up to a log-linear approximation, the expected ratio of the two countries' intertemporal marginal rates of substitution is equated to expected real exchange rate appreciation: $E_t \Delta \widehat{RER}_{t+1} = E_t(\widehat{\rho_{H,t+1}} - \widehat{\rho_{F,t+1}})$, which implies: $E_t \Delta \widehat{RER}_{t+1} = -\sigma E_t \Delta \widehat{c_{t+1}}$. Thus, *expected* relative consumption and real exchange rate growth rates are perfectly negatively correlated, in the bonds-only world (up to a first-order approximation).

Line (b.1) of Table 1 shows that the bonds-only economy delivers much smaller cross-country wealth transfers than the structure with complete financial markets: the predicted standard deviation of net exports/GDP $\widetilde{NX}$ (0.05%) and of first-differenced net foreign assets/GDP $\Delta\widetilde{NFA}$ (0.07%) are much smaller than in the structure with complete financial markets; real exchange rate growth too is much less volatile (standard deviation: 2.39%). Note that, in the bonds-only structure, these variables are also much less volatile than in the data. Furthermore, the rate of real exchange rate appreciation is now (almost) perfectly negatively correlated with relative consumption growth.[9]

The Impulse responses in Table 2, Panel (b.1) show that, in the bonds-only world, a transitory Home output growth $\varepsilon^{Y}_{H,t} > 0$ shock raises Home (relative) consumption by roughly the same amount as relative output, while net exports and net foreign assets are hardly affected. This reflects the restricted risk sharing in the bonds-only structure. The stronger rise in Home relative consumption implies that the relative price of the Home output good falls much less than under complete markets, and so the Home real exchange rate depreciates much less. A shock to the Home trend growth rate ε^{z}_{H} too triggers only a

[9] Hoffmann et al. (2011, 2013) study the effect of long-run growth shocks in a two-country, bonds-only model with one homogeneous tradable good; these authors show that long-run risk shocks can explain the sizable and persistent US trade balance deficits observed since the 1980s. (See also Equiza (2014) for a related set-up.) That one-good model cannot capture real exchange rate fluctuations. By contrast, the structure here assumes two country-specific output goods. When a high substitution elasticity ϕ between the two goods is assumed in the *bonds-only* model here, then the predicted variability of net exports and of net foreign assets increases, but the predicted variance of the real exchange rate falls, relative to the baseline calibration (where $\phi=1$). E.g., for $\phi=100$, the bonds-only model here (with output process (12) and $\gamma>\sigma$) generates realistic standard deviations of net exports/GDP (2.68%) and first differenced net foreign assets/GDP (3.36%), but the standard deviation of real exchange rate growth drops to 0.12%. Under *complete markets*, a model variant with $\phi=100$ predicts that the standard deviations of net exports, first-differenced net foreign assets and the real exchange rate are 26.13%, 555.36% and 0.39%, respectively.

modest response of the real exchange rate, net exports and net foreign assets, in the bonds-only structure.

4.2.2. Heterogeneous households: risk-sharers and hand-to-mouth agents

The bonds-only structure may seem restrictive as, in reality, there is large-scale international trade in a wide array of state-contingent assets (equities, derivatives)—however, only a minority of households holds international assets (Christelis and Georgarakos (2009)). To simply (and starkly) capture within-country heterogeneity in financial market participation, I now assume that each country is inhabited by two households. These two households have identical recursive preferences $(\gamma > \sigma)$, but differ in their ability to trade in financial markets: one household ('risk-sharer') trades in complete global financial markets, while the other household leads a hand-to-mouth (HTM) life; the HTM household does not participate in asset markets, i.e. each period her consumption spending equals the value of her endowment. The date *t* consumption of the country *i* HTM household is: $C_{i,t}^{HTM} = p_{i,t}\lambda_{i,t}Y_{i,t}/P_{i,t}$ where $\lambda_{i,t}$ is the share of country *i* output received by that household. A risk sharing condition analogous to (6) holds for the Home and Foreign 'risk-sharer' households: $RER_{t+1}/RER_t = \rho_{H,t+1}^{RS}/\rho_{F,t+1}^{RS}$, where $\rho_{i,t+1}^{RS}$ is the intertemporal marginal rate of substitution of the 'risk-sharer' household in country $i=H,F$. Otherwise this model variant is identical to the baseline model (endowment process (12) is assumed). This model version builds on Kollmann (2012) who studied a *static* two-country model in which each country is inhabited by a 'risk-sharer' and by a HTM household.[10]

Table 1, Line (b.2) reports predicted moments generated by a model variant in which HTM households receive a constant 50% share of their country's output in all periods ($\lambda_{i,t} = \bar{\lambda} = 0.5$ for $i=H,F$). Line (b.3) assumes a constant 90% HTM output share. Closed economy models with HTM households typically postulate that those households account for about 50% of aggregate income and consumption, in steady state; e.g., Gali et al. (2007). On the other hand, very few households (directly) trade in *foreign* assets. E.g.,

[10] That static model generates insufficient exchange rate volatility. Also, the static model does not allow to analyze net foreign assets dynamics which is a focus of the paper here.

Christelis and Georgarakos (2004) report that in 2004 only 2.4% of US households held foreign stocks, while merely 0.1% and 2.7% of US households held foreign bonds or foreign liquid accounts, respectively. This is why I also consider the second HTM model variant in which the output share of HTM households is high (90%).

The model variant with the 50% HTM output share generates roughly the same exchange rate standard deviation (12.08%) as the baseline model (without HTM households); the predicted standard deviation of first-differenced net foreign assets/GDP (31.35%) is smaller than in the baseline model, but it remains excessively high, when compared to the data. The model variant with a constant 90% HTM output share generates a 6.65% real exchange rate standard deviation; importantly, that variant produces a standard deviation of first-differenced net foreign assets (6.87%) that is much closer to the US empirical moment. (Table 2, Panel (b.3) shows that this model variant (constant 90% HTM output share) generates much weaker responses of net exports to output shocks than the baseline model with full risk sharing.) Hence, a model in which only a small fraction of households participates in complete markets, while the remaining household lead hand-to-mouth lines, is much better suited for generating realistic volatility of the real exchange rate *and* of the external balance.

However, the model variant with the constant 90% HTM output share predicts that the correlations between growth rates of relative consumption and of the real exchange rate (-0.67) and between expected growth rates of these variables (-0.94) are strongly negative, while empirical correlations are in the range of zero. This limitation can be addressed by assuming shocks to the HTM output shares, as those shocks trigger positively correlated responses of relative consumption and of the real exchange rate. E.g., a rise in the Home HTM output share increases relative Home consumption, which *appreciates* the Home real exchange rate.[11] Line (b.4) of Table 1 shows illustrative simulations of a model variant with shocks to HTM output shares that are independent of output and independent across countries: $\lambda_{i,t}-\overline{\lambda}=\rho^{\lambda}(\lambda_{i,t-1}-\overline{\lambda})+\varepsilon_{i,t}^{\lambda}$ with mean HTM output

[11]The shock raises the consumption of the Home HTM agent; the endowment of the Home 'risk-sharer' falls, but this is partly off-set by a transfer from the Foreign risk-sharer, so relative Home consumption rises.

share $\overline{\lambda}$=0.90, ρ^{λ}=0.95 and $Std(\varepsilon_{i,t}^{\lambda})$=0.59% for i=H,F.[12] Naturally, that variant generates greater standard deviations of real exchange rate growth (9.26%) and of first-differenced net foreign assets $\Delta\widetilde{NFA}$ (11.15%) than the model with a constant 90% HTM output share. However, $\Delta\widetilde{NFA}$ volatility remains markedly smaller than in the baseline model with full risk sharing. The HTM model with redistributive shocks predicts that correlations between growth rates of relative consumption and of the real exchange rate, and between expected growth rates of these variables are close to zero.

5. Conclusion

Recent research has argued that models with 'long-run risk' (persistent growth rate shocks) and recursive preferences can generate realistic exchange rate volatility, and solve other international finance puzzles. I have shown that this result hinges on the assumption that long-run consumption risk is efficiently shared among all (domestic and foreign) households. When financial markets are incomplete, in the sense that only an unconditional bond can be traded internationally, then long-run risk generates insufficient exchange rate volatility. I also document that a recursive preferences model, in which *all* households have access to complete global financial markets, entails implausibly large international wealth transfers in response to country-specific output growth rate shocks. By contrast, a long-run risk, recursive-preferences model in which only a small fraction of households trades in complete markets, while the remaining households lead hand-to-mouth lines, can generate realistic volatility of the real exchange rate *and* of net foreign assets.

[12]Empirically, participation in financial markets is highly positively correlated with household wealth; households whose main source of income is labor income are much less likely to hold international assets (Christelis and Georgarakos (2009)). Kollmann (2012) argues that, thus, *fluctuations* in the labor share may be taken as a proxy for *movements* in the fraction of GDP received by HTM households. I regressed the US labor share (compensation of employees/GDP) on a constant and the lagged share, using annual BEA data for 1980-2013 (NIPA Table 1.10). The coefficient of the lagged share is 0.95, the Std of the regression residual is 0.59%.

Appendix I: The reformulated model

The numerical solution uses a reformulated model in which consumption and welfare are scaled by domestic output. Let $\widetilde{C_{i,t}} \equiv C_{i,t}/Y_{i,t}$ and $\widetilde{U_{i,t}} \equiv U_{i,t}/Y_{i,t}$ be scaled consumption and welfare in country *i*, and let $G^Y_{i,t} \equiv (Y_{i,t}/Y_{i,t-1})/\exp(\mu)$ be the growth factor of country *i* output between periods *t*-1 and *t*, divided by the steady state growth factor. (4) implies

$$\widetilde{U_{i,t}} = \{(1-\beta)(\widetilde{C_{i,t}})^{1-\sigma} + \widetilde{\beta}[E_t(\widetilde{U_{i,t+1}}G^Y_{i,t+1})^{1-\gamma}]^{(1-\sigma)/(1-\gamma)}\}^{1/(1-\sigma)}, \quad \text{(A.1)}$$

with $\widetilde{\beta} \equiv \beta\exp(\mu\cdot(1-\sigma))$. Country *i*'s intertemporal marginal rate of substitution (IMRS) between periods *t* and *t*+1 (see (5)) can be written as:

$$\rho_{i,t+1} \equiv \beta\exp(-\mu\sigma)(G^Y_{i,t+1})^{-\gamma}\left(\frac{\widetilde{C_{i,t+1}}}{\widetilde{C_{i,t}}}\right)^{-\sigma}\left(\frac{\widetilde{U_{i,t+1}}}{\{E_t(\widetilde{U_{i,t+1}}G^Y_{i,t+1})^{1-\gamma}\}^{1/(1-\gamma)}}\right)^{\sigma-\gamma}. \quad \text{(A.2)}$$

Country *i's* demand functions for the two output goods are

$$\widetilde{y^i_{i,t}} = \alpha(p_{i,t}/P_{i,t})^{-\phi}\widetilde{C_{i,t}}, \quad \widetilde{y^j_{i,t}} = (1-\alpha)(p_{j,t}/P_{i,t})^{-\phi}\widetilde{C_{i,t}} \text{ for } j \neq i, \quad \text{(A.3)}$$

where $\widetilde{y^j_{i,t}} \equiv y^j_{i,t}/Y_{i,t}$ is country *i*'s demand for good *j*, normalized by *i*'s output.

The market clearing conditions for goods *H* and *F* can be expressed as

$$1 = \widetilde{y^H_{H,t}} + \widetilde{y^H_{F,t}}/y_t \quad \text{and} \quad 1 = \widetilde{y^F_{H,t}}\cdot y_t + \widetilde{y^F_{F,t}}, \quad \text{(A.4)}$$

where $y_t \equiv Y_{H,t}/Y_{F,t}$ is relative country *H* output. Home net exports/GDP are given by:

$$\widetilde{NX_{H,t}} = 1-(P_{H,t}/p_{h,t})\widetilde{C_{H,t}}. \quad \text{(A.5)}$$

Without loss of generality, I set

$$\tfrac{1}{2}(p_{H,t}+p_{F,t}) = 1, \quad \text{(A.6)}$$

i.e. a basket consisting of half a unit of good *H* and of good *F* is used as numéraire.

The dynamics of output growth and of relative output depend on the assumed exogenous output process. Under the baseline output process (12) we have

$$\ln(G^Y_{H,t}) = z_{H,t-1} - \kappa\cdot\ln(y_{t-1}) + \varepsilon^Y_{H,t}, \quad \ln(G^Y_{F,t}) = z_{F,t-1} + \kappa\cdot\ln(y_{t-1}) + \varepsilon^Y_{F,t},$$

where $z_{i,t} = \rho^z z_{i,t-1} + \varepsilon^z_{i,t}$ for $i=H,F$ and $\ln(y_t) = (1-2\kappa)\ln(y_{t-1}) + z_{H,t-1} - z_{F,t-1} + \varepsilon^Y_{H,t} - \varepsilon^Y_{F,t}$. (A.7)

When the first-difference stationary output process (13) is assumed, then

$$\ln(G^Y_{H,t}) = \rho^{\Delta Y}\ln(G^Y_{H,t-1}) - \kappa\cdot\ln(y_t) + \varepsilon^{\Delta Y}_{H,t}, \quad \ln(G^Y_{F,t}) = \rho^{\Delta Y}\ln(G^Y_{F,t-1}) + \kappa\cdot\ln(y_t) + \varepsilon^{\Delta Y}_{F,t},$$

$$\Delta\ln(y_t) = \rho^{\Delta y}\Delta\ln(y_{t-1}) - 2\kappa\ln(y_{t-1}) + \varepsilon^{\Delta y}_{H,t} - \varepsilon^{\Delta y}_{F,t}. \quad \text{(A.8)}$$

Finally, under the trend-stationary endowment process (14) we have

$\ln(G^Y_{i,t}) = \xi_{i,t} - \xi_{i,t-1}$ and $\ln(y_t) = \xi_{H,t} - \xi_{F,t}$, where $\xi_{i,t} \equiv \ln(Y_{i,t}) - \mu\cdot t$ obeys $\xi_{i,t} = \rho\xi_{i,t-1} + \varepsilon^{TS}_{i,t}$. (A.9)

● In <u>model variants with efficient risk sharing</u>, the net foreign assets/GDP ratio obeys

$$\widetilde{NFA_{H,t+1}} = E_t\rho_{H,t+1}\{\widetilde{NFA_{H,t+2}} - 1 + (P_{H,t+1}/p_{H,t+1})\widetilde{C_{H,t+1}}\}(P_t/P_{t+1})(p_{H,t+1}/p_{H,t})G^Y_{H,t+1}\exp(\mu). \quad \text{(A.10)}$$

Equations (1),(2),(6), (A.1)-(A.6) and the exogenous output process ((A.7), (A.8) or (A.9)) determine

$$\{\widetilde{U_{H,t}},\widetilde{U_{F,t}},\widetilde{C_{H,t}},\widetilde{C_{F,t}},\widetilde{y^H_{H,t}},\widetilde{y^H_{F,t}},\widetilde{y^F_{H,t}},\widetilde{y^F_{F,t}},G^Y_{H,t},G^Y_{F,t},y_t,p_{H,t},p_{F,t},P_{H,t},P_{F,t},q_t,RER_t,\widetilde{NX_{H,t}},\widetilde{NFA_{H,t+1}}\},$$

in the model variants with efficient risk sharing. Once these variables have been solved for, it is easy to determine other variables of interest. E.g., the growth rate of consumption is $\Delta\ln C_{i,t+1}=\Delta\ln\widetilde{C_{i,t+1}}+\Delta\ln Y_{i,t+1}$ etc.

● In the bonds-only model variant, the country H budget constraint can be written as

$$\widetilde{NFA_{H,t+1}}+(P_{H,t}/p_{H,t})\widetilde{C_{i,t}}=1+\widetilde{NFA_{H,t}}(1+r_t^A)/\{(p_{H,t}/p_{H,t-1})G_{H,t}^Y\exp(\mu)\}, \qquad \text{(A.11)}$$

given the choice of numéraire (A.6). The Euler equation (15) then gives

$$(1+r_{t+1}^A)E_t(P_{i,t}/P_{i,t+1})\rho_{i,t+1}=1. \qquad \text{(A.12)}$$

Equations (1),(2), (A.1)-(A.6), (A.11),(A.12) and the law of motion of output determine $\{\widetilde{U_{H,t}},\widetilde{U_{F,t}},\widetilde{C_{H,t}},\widetilde{C_{F,t}},\widetilde{y_{H,t}^H},\widetilde{y_{F,t}^H},\widetilde{y_{H,t}^F},\widetilde{y_{F,t}^F},G_{H,t}^Y,G_{F,t}^Y,y_t,p_{H,t},p_{F,t},P_{H,t},P_{F,t},q_t,RER_t,\widetilde{NX_{H,t}},\widetilde{NFA_{H,t+1}},r_{t+1}^A\}$, in the bonds-only economy.

● In the model variants with hand-to-mouth (HTM) households, the scaled consumption of the country i HTM household is given by

$$\widetilde{C_{i,t}^{HTM}}\equiv C_{i,t}^{HTM}/Y_{i,t}=\lambda_{i,t}p_{i,t}/P_{i,t}. \qquad \text{(A.13)}$$

Note that

$$\widetilde{C_{i,t}}=\widetilde{C_{i,t}^{HTM}}+\widetilde{C_{i,t}^{RS}}, \text{ where } \widetilde{C_{i,t}^{RS}}\equiv C_{i,t}^{RS}/Y_{i,t} \qquad \text{(A.14)}$$

is the scaled consumption of the country's 'risk-sharer' household. The scaled welfare of the 'risk-sharer' household $\widetilde{U_{i,t}^{RS}}\equiv U_{i,t}^{RS}/Y_{i,t}$ obeys

$$\widetilde{U_{i,t}^{RS}}=\{(1-\beta)(\widetilde{C_{i,t}^{RS}})^{1-\sigma}+\widetilde{\beta}[E_t(\widetilde{U_{i,t+1}^{RS}}G_{i,t+1}^Y)^{1-\gamma}]^{(1-\sigma)/(1-\gamma)}\}^{1/(1-\sigma)}, \qquad \text{(A.15)}$$

and her IMRS is:

$$\rho_{i,t+1}^{RS}\equiv\beta\exp(-\mu\sigma)(G_{i,t+1}^Y)^{-\gamma}\left(\frac{\widetilde{C_{i,t+1}^{RS}}}{\widetilde{C_{i,t}^{RS}}}\right)^{-\sigma}\left(\frac{\widetilde{U_{i,t+1}^{RS}}}{\{E_t(\widetilde{U_{i,t+1}^{RS}}G_{i,t+1}^Y)^{1-\gamma}\}^{1/(1-\gamma)}}\right)^{\sigma-\gamma}. \qquad \text{(A.16)}$$

Efficient risk sharing among the Home and Foreign 'RS' households implies:

$$RER_{t+1}/RER_t=\rho_{H,t+1}^{RS}/\rho_{F,t+1}^{RS}, \qquad \text{(A.17)}$$

Equations (1),(2), (A.3)-(A.6), (A.13)-(A.17), and the law of motion of output determine $\{\widetilde{U_{H,t}^{RS}},\widetilde{U_{F,t}^{RS}},\widetilde{C_{H,t}^{RS}},\widetilde{C_{F,t}^{RS}},\widetilde{C_{H,t}^{HTM}},\widetilde{C_{F,t}^{HTM}},\widetilde{C_{H,t}},\widetilde{C_{F,t}},\widetilde{y_{H,t}^H},\widetilde{y_{F,t}^H},\widetilde{y_{H,t}^F},\widetilde{y_{F,t}^F},G_{H,t}^Y,G_{F,t}^Y,y_t,p_{H,t},p_{F,t},P_{H,t},P_{F,t},q_t,RER_t,$
$\widetilde{NX_{H,t}},\widetilde{NFA_{H,t+1}}\}$, in the HTM model variants.

Appendix II: Net foreign assets/GDP ratio of US and UK

The Table below provides annual data on the net foreign assets/GDP ratio for the US and the UK (NFA measured at end of year).

Col. 1: year; Cols. 2 and 3: US and UK NFA/GDP ratios as reported in the updated and extend version of the Lane and Milesi-Ferretti (2007) dataset [LMF] (http://www.philiplane.org/EWN.html).

Col. 4: US NFA/GDP series computed by dividing the Bureau of Economic Analysis [BEA] series 'U.S. net international investment position' (IIP Table 1.1, in current dollars) by the BEA GDP series (current dollars, NIPA Table 1.1.5).

Year	US (LMF)	UK (LMF)	US (BEA)
1970	0.085	0.048	
1971	0.071	0.072	
1972	0.067	0.073	
1973	0.075	0.047	
1974	0.076	0.005	
1975	0.077	0.002	
1976	0.069	0.014	0.042895
1977	0.061	0.024	0.04726
1978	0.062	0.049	0.054431
1979	0.07	0.023	0.088238
1980	0.078	0.059	0.103707
1981	0.08	0.105	0.070692
1982	0.065	0.125	0.07126
1983	0.057	0.157	0.071877
1984	0.022	0.192	0.034682
1985	-0.006	0.205	0.023991
1986	-0.029	0.245	0.023797
1987	-0.042	0.121	0.012241
1988	-0.054	0.095	0.004089
1989	-0.064	0.088	-0.00596
1990	-0.057	-0.031	-0.02501
1991	-0.064	-0.011	-0.03941
1992	-0.079	0.012	-0.06608
1993	-0.058	0.039	-0.0177
1994	-0.056	0.029	-0.01509
1995	-0.072	-0.023	-0.03622
1996	-0.071	-0.082	-0.04053
1997	-0.103	-0.07	-0.09156
1998	-0.106	-0.137	-0.11374
1999	-0.086	-0.204	-0.10375
2000	-0.142	-0.1	-0.14943
2001	-0.189	-0.137	-0.21607
2002	-0.201	-0.123	-0.21963
2003	-0.198	-0.116	-0.19921
2004	-0.2	-0.196	-0.19254
2005	-0.164	-0.191	-0.14189
2006	-0.176	-0.31	-0.13052
2007	-0.144	-0.231	-0.08838
2008	-0.244	-0.059	-0.27145
2009	-0.183	-0.229	-0.18224
2010	-0.181	-0.252	-0.16785
2011	-0.274	-0.174	-0.28709
2012			-0.28325
2013			-0.29788

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Table 1. Predicted Moments

Standard deviations (%)				Consumption correlations		
$\Delta\ln RER$	$\Delta\ln c$	$\widetilde{NX}_H$	$\Delta\widetilde{NFA}_H$	$\rho(\Delta\ln c_{t+1}, \Delta\ln RER_{t+1})$	$\rho(E_t\Delta\ln c_{t+1}, E_t\Delta\ln RER_{t+1})$	$\rho(\Delta\ln C_H, \Delta\ln C_F)$
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
(a) Model variants with efficient risk sharing						
(a.1) Baseline output process						
13.31	2.10	2.30	49.10	0.03	-0.79	0.42
(a.2) Baseline output process, time-separable utility $(\gamma=\sigma)$						
1.68	2.53	0.09	3.73	-1.00	-1.00	0.16
(a.3) AR(1) difference-stationary output						
14.00	1.02	1.31	35.78	-0.68	-0.97	0.91
(a.4) Trend-stationary output						
2.79	2.45	0.07	2.49	-0.99	-1.00	0.46
(b) Model variants with financial frictions						
(b.1) Bonds-only world						
2.39	2.45	0.05	0.07	-0.97	-0.98	0.20
(b.2) 50% of endowment received by hand-to-mouth households						
12.08	1.85	1.55	31.35	-0.16	-0.59	0.48
(b.3) 90% of endowment received by hand-to-mouth households						
6.65	2.05	0.47	6.87	-0.67	-0.94	0.36
(b.4) Economy with hand-to-mouth households and redistributive shocks						
9.26	2.25	0.73	11.15	-0.16	-0.16	0.28

Note: The Table reports predicted moments generated by different model variants (see main text). A third-order approximation is used to solve the model. Moments are averages computed across 500 stochastic simulation runs (each simulation run is initialized at the deterministic steady state and has a length of 100 periods; moments are computed using the last 50 periods only). Columns *(1)*-*(2)* report % standard deviations of the log growth rates of the Home real exchange rate (*RER*) and of relative Home/Foreign consumption $(c \equiv C_H/C_F)$, respectively. Cols. (3) and (4) show % standard deviations of the Home net exports/GDP ratio $(\widetilde{NX}_H)$ and of the first-differenced Home net foreign assets/GDP ratio $(\Delta\widetilde{NFA}_H)$. Col. (5): correlation between log growth rates of relative consumption and of the real exchange rate; Col. (6): correlation between one-period-ahead expectations of log growth rates of relative consumption and of the real exchange rate; Col. (7): correlation between log growth rates of Home and Foreign consumption.

Table 2. Dynamic responses to Home country innovations (1 standard deviation)

Horizon	*RER*	*c*	*y*	$\widetilde{NX}_H$	$\widetilde{NFA}_H$
	(1)	*(2)*	*(3)*	*(4)*	*(5)*
(a) Model variants with efficient risk sharing					
(a.1) Baseline output process					
Transitory shock to Home growth rate ε_H^Y (1.87%)					
0	-9.12	0.78	1.87	0.25	-8.52
4	-9.03	0.79	1.87	0.25	-8.32
20	-8.68	0.80	1.85	0.24	-7.49
Shock to Home trend growth rate ε_H^z (0.26%)					
0	-22.44	-2.99	0.00	0.94	-15.70
4	-23.43	-2.03	1.02	0.94	-15.93
20	-26.55	1.26	4.61	0.92	-16.12
(a.2) Baseline output process, time-separable utility, $\gamma = \sigma$					
Transitory shock to Home growth rate ε_H^Y (1.87%)					
0	-1.22	1.83	1.87	-0.02	1.29
4	-1.21	1.82	1.87	-0.02	1.29
20	-1.19	1.79	1.85	-0.02	1.27
Shock to Home trend growth rate ε_H^z (0.26%)					
0	0.00	0.00	0.00	0.00	6.76
4	-0.66	1.00	1.02	-0.01	7.07
20	-2.94	4.41	4.61	-0.04	8.08
(a.2) AR(1) difference-stationary output					
Shock to Home growth rate $\varepsilon_H^{\Delta Y}$ (1.87%)					
0	-13.08	0.73	2.31	0.39	-25.94
4	-13.65	1.68	3.33	0.37	-25.72
20	-13.36	1.65	3.26	0.37	-25.16
(a.3) Trend-stationary output					
Shock to Home output ε_H^{TS} (2.38%)					
0	-2.52	2.17	2.35	0.01	-2.27
4	-1.99	1.38	1.55	0.02	-2.34
20	-1.17	0.15	0.29	0.03	-2.44

Table 2.—ctd.

Dynamic responses to Home country innovations (1 standard deviation)

Horizon	*RER*	*c*	*y*	$\widetilde{NX}_H$	$\widetilde{NFA}_H$
	(1)	*(2)*	*(3)*	*(4)*	*(5)*
(b) Model variants with financial frictions					
(b.1) Bonds-only world					
Transitory shock to Home growth rate ε_H^Y (1.87%)					
0	-1.70	1.76	1.87	-0.00	0.00
4	-1.70	1.76	1.87	-0.00	-0.01
20	-1.73	1.72	1.85	0.00	-0.04
Shock to Home trend growth rate ε_H^z (0.26%)					
0	-1.10	-0.14	0.00	0.04	0.04
4	-1.75	0.85	1.02	0.03	0.17
20	-4.07	4.25	4.61	-0.00	0.57
(b.2) 50% of endowment received by hand-to-mouth households					
Transitory shock to Home growth rate ε_H^Y (1.87%)					
0	-8.15	0.91	1.87	0.22	-7.91
4	-8.05	0.92	1.87	0.22	-7.67
20	-7.64	0.94	1.85	0.22	-6.71
Shock to Home trend growth rate ε_H^z (0.26%)					
0	-20.57	-2.73	0.00	0.81	-16.06
4	-21.36	-1.74	1.02	0.80	-15.85
20	-23.79	1.64	4.61	0.76	-14.48
(b.3) 90% of endowment received by hand-to-mouth households					
Transitory shock to Home growth rate ε_H^Y (1.87%)					
0	-4.35	1.41	1.87	0.09	-3.21
4	-4.29	1.41	1.87	0.09	-3.09
20	-4.08	1.41	1.85	0.08	-2.63
Shock to Home trend growth rate ε_H^z (0.26%)					
0	-8.96	-1.18	0.00	0.32	-5.45
4	-9.73	-0.19	1.02	0.31	-5.18
20	-12.19	3.18	4.61	0.28	-4.04
(b.4) Economy with hand-to-mouth households and redistributive shocks					
Shock to HTM output share ε_H^λ (1.00%)					
0	4.71	0.62	0.00	-0.16	4.85
4	4.13	0.54	0.00	-0.14	4.41
20	2.69	0.35	0.00	-0.09	3.23

Note: The Table reports dynamic effects of one-standard deviation innovations to Home country exogenous variables after 0, 4 and 20 years (see left-most Column labeled 'Horizon'). In each case, a one-time innovation is considered, assuming that all other exogenous innovations (in all periods) are zero. Predetermined state variables in the period of the shock are assumed to equal steady state values; the responses of the Home real exchange rate (*RER*), relative Home consumption ($c \equiv C_H/C_F$) and relative Home output ($y \equiv Y_H/Y_F$) are expressed as relative deviations from unshocked paths, while the responses of Home net exports/GDP ($\widetilde{NX}_H$) and of end-of-period Home net foreign assets/GDP ($\widetilde{NFA}_H$) are expressed as differences from the unshocked path.