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Monetary Rule, Central Bank Loss and Household's Welfare: an Empirical Investigation*

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Abstract

Which monetary policy rule best fits the historical data? Which rule is most effective to reach the central bank's objectives? Is minimizing a central bank loss equivalent to maximizing households' welfare? Are NGDP growth or level targeting good options, and if so, when? Do they perform better than Taylor-type rules? In order to answer these questions, we use Bayesian estimations to evaluate the Smets and Wouters (2007) model under nine monetary policy rules with US data ranging from 1955 to 2017 and over three different sub-periods (among them the zero lower bound period where a shadow rate is introduced). We find that when considering the minimization of the central bank's loss function, the estimates generally indicate the superiority of NGDP level targeting rules. If the behavior of the Fed is expressed in terms of households-welfare, the implications are not necessarily the same.

JEL codes: E52, E58, E32

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1 Introduction

Monetary economists generally contend that central bankers should follow policy rules rather than use their own discretion when devizing monetary policy. Debates held during the 1970s and 1980s suggested nominal income targeting concepts, even if they were not always presented as such.[1] The consensus on Taylor (1993) rules increased during the last two decades.[2] However, criticism of such monetary policy rules also increased,[3] especially during and after the Global Financial Crisis[4] (GFC), arguing that nominal income targeting could be a better way to achieve the central banks' objectives.

An interesting way to compare and evaluate different monetary policy proposals and rules is to introduce them within the framework of a macroeconomic Dynamic Stochastic General Equilibrium (DSGE) model. Because the dynamics are so important and difficult to work through intuitively, these empirical models can provide invaluable clarification of the matter (Taylor, 2013).

Our objective is to use such models in order to evaluate different monetary policy rules and their consequences in terms of central banks and society's objectives. These objectives may differ for various reasons. Hence the need to analyze several hypotheses as to the preferences of the central bank and those of society. Such an approach implies an analysis of the impact of policies on the central bank loss function and on the welfare of the representative household (Taylor and Wieland, 2012; Walsh, 2015).

We compare Taylor-type and nominal income rules through the Smets and Wouters (2007) model, a well-known baseline DSGE model fitted for the US. In this model, both parameters and structural shocks are related to deeper structural parameters describing household preferences and technological and institutional constraints. These micro foundations provide a theoretical framework that could be particularly useful in an econometric analysis of the optimality of various policy strategies.

Our monetary policy rules are of three types: Taylor-type rules; nominal income *growth* targeting rules, and nominal income *level* targeting rules. There are three Taylor-type rules following: (1) a structure *à la* Smets and Wouters (2007), where the nominal interest rate responds to an inflation

[1] See Friedman (1971), Meade (1978), and McCallum (1973, 1987).

[2] See Bernanke and Mishkin (1997), Svensson (1999), and Taylor (1999).

[3] See for instance Hall and Mankiw (1994), Frankel and Chinn (1995), McCallum and Nelson (1999), and Rudebusch (2002a).

[4] Hendrickson (2012), Woodford (2012), Frankel (2014), Sumner (2014, 2015), Belongia and Ireland (2015), and McCallum (2015) for example.

gap, an output gap and output gap growth; (2) a structure *à la* Taylor (1993), where the nominal interest rate responds to an inflation gap and an output gap; and (3) a structure *à la* Galí (2015), where the nominal interest rate responds to an inflation gap, an output gap and a natural interest rate defined as the interest rate in the flexible-price economy. There are also three NGDP growth rules replacing the core functions of the Taylor-type rules with an NGDP growth targeting function. Finally, our last three rules replace the core functions of the Taylor-type rules with an NGDP level targeting rule.

As in An and Schorfheide (2007) and Smets and Wouters (2007), we apply Bayesian techniques to estimate our nine DSGE models (each type is composed of 3 structures) using US data. Note that such approach is in the same vein as Garín et al. (2016). However, the model we use (Smets and Wouters, 2007) is further reaching than theirs and widely accepted by monetary economists and central bankers. As noted below, our research goes further than theirs in four regards. First, we consider a larger set of monetary policy rules. Second, these models are studied over different time periods (in contrast to their estimates that run only from 1984 to 2007, which is just one of our sample periods). Third, we add the analysis of bank losses and welfare compensating variations over these models and periods. Four, we evaluate the rules not only *via* their impact on different central bank loss functions but also *via* their impact on households' welfare. We believe that our analysis and estimates enrich theirs in an informative and innovative way.

Specifically, we estimate all of the parameters over several sample periods: the overall available sample (1955-2017) and three sub-samples, each with different economic environments and monetary policy styles, running from 1955 to 1985, from 1985 to 2007 and from 2007 to 2017.

From December 16, 2008, to December 15, 2015, the effective federal funds rate was in the 0 to 1/4 percent range. In this zero lower bound environment, shadow rate models are being used (Kim and Singleton, 2012; Krippner, 2013).

Monetary policy during the ZLB period can hardly be described by a monetary policy rule in which the monetary policy shock is assumed to be normally distributed. Unconventional policies were implemented (credit easing, quantitative easing, and forward guidance). In these cases, we use the shadow rate[5] developed by Wu and Xia (2016) in order to overcome this sta-

[5] Wu and Xia (2016) devised a shadow Fed funds rate that can be negative, reflecting the Fed's unconventional policies. When quantitative easing or forward guidance is pursued, the current Fed's rate is zero (ZLB) while the shadow rate changes. When rates are above the ZLB, the shadow rate is identical to the Fed fund rate. Once the ZLB is reached, the Wu and Xia (2016) rate uses a Gaussian affine term structure model to generate an effective rate.

tistical problem while taking into consideration most of such unconventional monetary policies. The shadow rate is a version of the federal funds rate that can take negative values ; it is also consistent with a term-structure of interest rates.[6]

From the estimations and simulations of our models, we analyze, among other factors, the following: monetary policy rules' parameters, in-sample fits, central bank's loss functions, households' welfare compensating variations. Estimated parameters, estimated shocks, impulse response functions, and variance decompositions are presented in the Online Appendix.

We find that when considering only the central bank's loss function, the estimates generally indicate the superiority of NGDP level targeting rules, though the Taylor rule leads to nearly similar implications over the Great Moderation period.

A different type of implication can be drawn generally when the Fed seeks to maximize households' welfare compensating variations, even if a Taylor rule performs best during the GM period.

A different central bank function may be more appropriate to achieve the central bank's objectives, for each type of period (stable, crisis, recovery). Policy institutions, that base their forecasts and policy recommendations on such models and rules, should refresh their estimates regularly.

The remainder of the paper is organized as follows. Section 2 describes the theoretical setup. Section 3 describes the empirical methodology. Monetary rule parameters estimates as well as in-sample fit results and analysis are presented in Section 4. Central bank losses and households' welfare measures are presented in Section 5. Our results are interpreted in Section 6. Section 7 draws some policy implications and Section 8 concludes. The Online Appendix presents additional empirical results.

2 The models

The Smets and Wouters (2007) model is the core model used in this paper. Yet, in their article and other working paper versions, those authors do not present the flexible-price economy. We do this work in the detailed description of the log-linearized sticky- and flexible-price economies in our Online Appendix.

This (generic) model, also detailed in the Online Appendix, needs to be completed by adding an ad hoc monetary policy reaction function (Table 1).

[6] The shadow rate allows for meaningful monetary policy analysis and interpretation during low interest rate regimes, without ignoring data from high interest rate periods.

Despite their different formulations, all of these functions include a smoothing process that captures the degree of rule-specific smoothing.

Taylor-type rules

- Model **1** is the original Smets and Wouters (2007) monetary policy rule, which gradually responds to deviations of inflation (π_t) from an inflation objective (normalized to be zero), the output gap, defined as the difference between sticky-price (y_t) and flexible-price (y_t^p) outputs (see the Online Appendix), and deviations of the output gap from the previous period ($\triangle y_t - \triangle y_t^p$).
- Model **2** is the Taylor (1993) monetary policy rule, which gradually responds to deviations of inflation from an inflation objective (normalized to be zero) and of the output gap, as previously defined.[7]
- Model **3** is the Galí (2015) monetary policy rule, which gradually responds to the natural interest rate (r_t^*), as defined in Galí (2015), deviations of inflation from an inflation objective (normalized to be zero) and of the output gap, as previously defined.

Nominal GDP growth rules

- Model **4** is the Adapted NGDP Growth Targeting monetary policy rule, which gradually responds to deviations of nominal output growth ($\pi_t + \triangle y_t$) from an objective, as in McCallum and Nelson (1999), and deviations of the output gap from the previous period (output gap growth, as in model 1).
- Model **5** is the NGDP Growth Targeting monetary policy rule, which gradually responds to deviations of nominal output growth from its flexible-price counterpart.
- Model **6** is the NGDP Growth Targeting monetary policy rule including a natural interest rate (NIR) component, where the policy gradually responds to the NIR, as in Rudebusch (2002a), and deviations of nominal output growth from its flexible-price counterpart.

[7] In the original Taylor rule, the natural interest rate is constant (Taylor, 1993). Log-linearization around the steady-state eliminates this (constant) natural interest rate. Note that rule 1 (Smets and Wouters, 2007) does not either include the natural interest rate.

Nominal GDP level rules

- Model **7** is the Adapted NGDP Level Targeting monetary policy rule, which gradually responds to nominal output level ($p_t + y_t$) deviations from its flexible-price counterpart,[8] as suggested by McCallum (2015), and deviations of the output gap from the previous period (as in model 1).

- Model **8** is the NGDP Level Targeting monetary policy rule, which gradually responds to nominal output level deviations from its flexible-price counterpart.

- Model **9** is the NGDP Level Targeting monetary policy rule including a natural interest rate (NIR) component, where the policy gradually responds to the NIR and to deviations of the nominal output level from its flexible-price counterpart.

As indicated above, there are three categories of rules. The first three (1 to 3) are of the « Taylor-type ». Rules 4 to 6 are nominal GDP rules targeting nominal GDP growth. Rules 7 to 9 target the level of nominal GDP.

Rules 4 and 7 include an output gap growth, as in rule 1 (Smets and Wouters, 2003, 2007). Rules 6 and 9 include the natural interest rate, as in rule 3 (Galí, 2015). Including these variables allows us to compare the various rules with their standard versions as presented by the above-cited authors.

These three categories of rules represent the main policy rules in the contemporary literature.

As these rules are all *ad hoc*, they do not require changes in the specification of the core model. The unique deviating feature of the nine models therefore comes from their respective monetary policy rule. Concerning NGDP Level Targeting rules (models 7 to 9), we add to the core model and the monetary policy rule the definition of prices, derived from (in log form) $\pi_t = p_t - p_{t-1}$, where p_t represents the log-price index at time t.

In addition, we assume that prices do not change over time in the flexible-price economy, that is, (in log form) $p_t^p = p_{t-1}^p$ where p_t^p represents the log-price index in the flexible-price economy at time t. Hence, $\pi_t^p = 0$ and because our core model is computed in deviation from the steady-state, $p_t^p = p_{t-1}^p = 0$. Then, flexible-price nominal income is only defined by $\triangle y_t^p$ (growth) or y_t^p

[8] The level of nominal output is $p_t + y_t$, where prices p_t are deducted from the definition of inflation $\pi_t = p_t - p_{t-1}$.

Models	Sources	Monetary policy rules
1	Smets and Wouters (2007)	$r_t = \rho r_{t-1} + (1-\rho)\left[r_\pi \pi_t + r_y\left(y_t - y_t^p\right)\right] + r_{\triangle y}\left(\triangle y_t - \triangle y_t^p\right) + \varepsilon_t^r$
2	Taylor (1993)	$r_t = \rho r_{t-1} + (1-\rho)\left[r_\pi \pi_t + r_y\left(y_t - y_t^p\right)\right] + \varepsilon_t^r$
3	Galí (2015)	$r_t = \rho r_{t-1} + (1-\rho)\left[r_t^* + r_\pi \pi_t + r_y\left(y_t - y_t^p\right)\right] + \varepsilon_t^r$
4	Adapted NGDP Growth Targeting	$r_t = \rho r_{t-1} + (1-\rho)\left[r_n\left(\pi_t + \triangle y_t - \triangle y_t^p\right)\right] + r_{\triangle y}\left(\triangle y_t - \triangle y_t^p\right) + \varepsilon_t^r$
5	NGDP Growth Targeting	$r_t = \rho r_{t-1} + (1-\rho)\left[r_n\left(\pi_t + \triangle y_t - \triangle y_t^p\right)\right] + \varepsilon_t^r$
6	NGDP Growth Targeting + NIR	$r_t = \rho r_{t-1} + (1-\rho)\left[r_t^* + r_n\left(\pi_t + \triangle y_t - \triangle y_t^p\right)\right] + \varepsilon_t^r$
7	Adapted NGDP Level Targeting	$r_t = \rho r_{t-1} + (1-\rho)\left[r_n\left(p_t + y_t - y_t^p\right)\right] + r_{\triangle y}\left(\triangle y_t - \triangle y_t^p\right) + \varepsilon_t^r$
8	NGDP Level Targeting	$r_t = \rho r_{t-1} + (1-\rho)\left[r_n\left(p_t + y_t - y_t^p\right)\right] + \varepsilon_t^r$
9	NGDP Level Targeting + NIR	$r_t = \rho r_{t-1} + (1-\rho)\left[r_t^* + r_n\left(p_t + y_t - y_t^p\right)\right] + \varepsilon_t^r$

NIR stands for Natural Interest Rate (r_t^*) *à la* Galí (2015).

Table 1: Summary of monetary policy rules used in this study

(level). These assumptions are used in rules 4 to 6 (NGDP Growth rules in Table 1) and 7 to 9 (NGDP Level rules in Table 1).

3 Methodology

3.1 Data

The models, with various monetary policy rules, are estimated between 1955 and 2017 and over three different periods within this time interval: from 1955Q1 to 1985Q1, a period when the economy was rather unstable and featured ups and downs and when monetary policy could be characterized as discretionary; from 1985Q1 to 2007Q1, the *Great Moderation era*, when the economy was rather stable and monetary policy more predictable; and from 2007Q1 to 2017Q1, the *GFC/ZLB era*, the crisis and recovery period when monetary policy followed an unusual ZLB track.

During our first sub-sample (1955-1985), monetary policy was rather discretionary and severely criticized in the literature (Friedman, 1982). Since the 1980s, the predictability and stability of monetary policy has improved, with many researchers currently recommending rule-based rather than discretionary monetary policy decisions (Kydland and Prescott, 1977; Taylor, 1986, 1987; Friedman, 1982; Taylor, 1993). Notice that monetary policies occurring during our first sub-sample (1955-1985) were often modeled by a rule in the literature (Smets and Wouters, 2007; Nikolsko-Rzhevskyy and Papell, 2012; Nikolsko-Rzhevskyy et al., 2014).

Our second sub-sample (1985-2007) is inspired by Clarida (2010), describing the period 1985-2007 as the Great Moderation (GM). Although our second sub-sample is in line with the literature (Clarida, 2010; Meltzer, 2012; Taylor, 2012; Nikolsko-Rzhevskyy et al., 2014), we extend it until 2007, to define a sub-sample with a relatively stable economy (despite the Dot-com crisis beginning in the 2000s) that can be compared with the crisis period starting in 2007.

Our third sub-sample (2007-2017) is well documented in the crisis and recovery period literature (Gorton, 2009; Cúrdia and Woodford, 2011; Benchimol and Fourçans, 2017).

The series are quarterly, and data transformations as well as data sources[9]

[9] Data for GDP (Real Gross Domestic Product, GDPC96), inflation (Implicit Price Deflator, GDPDEF), consumption (Personal Consumption Expenditures, PCEC), investment (Fixed Private Investment, FPI), and employment (Civilian Employment, CE16OV) are taken from the Bureau of Economic Analysis (U.S. Department of Commerce) database. Data for population (Civilian Noninstitutional Population, CNP16OV), worked hours (Average Weekly Hours from Nonfarm Business Sector, PRS85006023), and hourly

are exactly the same as in Smets and Wouters (2007).

We estimate our models over the third sub-sample (2007-2017) by using shadow rate data for the US pursuant to Wu and Xia (2016).

3.2 Calibration

To maintain consistency across models for comparison purposes, we calibrate all core model parameters as in Smets and Wouters (2007). A detailed description of this calibration is provided in the Online Appendix.

Except for NGDP targeting rules, monetary policy rule parameters have the same calibration as in Smets and Wouters (2007) in Table 2.

	Law	Mean	Std.
ρ	Beta	0.75	0.10
r_π	Normal	1.50	0.25
r_y	Normal	0.125	0.05
$r_{\Delta y}$	Normal	0.125	0.05
r_n	Normal	$1.5^{(*)}/0.5^{(**)}$	0.25

Table 2: Prior distribution of monetary policy rule parameters. $(*)$ stands for NGDP growth targeting (rules 4, 5 and 6). $(**)$ stands for NGDP level targeting (rules 7, 8 and 9).

Of course, $r_{\triangle y}$ equals zero in models 2, 3, 5, 6, 8, and 9. r_π and r_y are not used in models 4 to 9, and r_n is not used in models 1 to 3.

As explained in Rudebusch (2002a), r_n is higher than one for NGDP growth targeting rules, and positive and smaller than one for NGDP level targeting rules.

3.3 Estimation

As in An and Schorfheide (2007) and Smets and Wouters (2007), we apply Bayesian techniques to estimate our DSGE models with different specifications of monetary policy rules. We estimate all the parameters presented above over the four different periods defined in Section 3.1.

To avoid undue complexity, we do not present all the estimates. We prefer to concentrate on the analysis of the parameters of the different monetary

wages (Compensation Per Hour from Nonfarm Business Sector, COMPNFB) are taken from the Bureau of Labor Statistics (U.S. Department of Labor) database. Data for the nominal interest rate (Effective Federal Funds Rate, FEDFUNDS) are taken from the Board of Governors of the Federal Reserve System database.

rules. All the estimation results are available in the Online Appendix. Other detailed results are available upon request.

To achieve draw acceptance rates between 20% and 40%, we calibrate the tuning parameter on the covariance matrix for each model and each period. Our results, for each model and each period, are based on the standard Monte Carlo Markov Chain (MCMC) algorithm with 6 000 000 draws of 2 parallel chains (where 3 000 000 draws are used for burn-in).

4 Monetary rule parameters and in-sample fit

Parameter estimates are detailed in the Online Appendix with all IRFs and variance decompositions. To draw policy conclusions from our models, we assess monetary policy rule parameters (estimated values) in Section 4.1 and the models' in-sample fit in Section 4.2 (the models' out-sample fit is presented in the Online Appendix).

4.1 Monetary rule parameters

Fig. 1 presents the estimates of the smoothing parameter (ρ), the inflation coefficient (r_π), the output gap coefficient (r_y), the output gap growth coefficient ($r_{\triangle y}$) and the nominal income coefficient (r_n).

As Fig. 1 shows, the smoothing parameter is in line with the literature (Justiniano and Preston, 2010), at approximately 0.8, and rather stable over time, although it appears somewhat smaller for rules 7 and 8, a result in line with Rudebusch (2002a,b).

The inflation coefficient (for rules 1 to 3) remains between 1.5 and 2, also in line with the literature (Smets and Wouters, 2007; Adolfson et al., 2011). Note that it is a bit smaller during the GFC and recovery period (GFC/ZLB), suggesting less reaction by the Fed to inflation developments than during more stable periods, notably than during the GM, from 1985 to 2007.

Regarding the coefficient of the output gap, its value varies across the periods. It appears to be higher during the GFC/ZLB period (it remains between 0.15 and 0.20) than between 1955 and 1985 (its value goes from 0.10 to 0.15). This difference is not as significant when we compare the crisis period with the 1985-2007 period (except for rule 3).

These estimates of the Taylor-type rules (rules 1 to 3) imply a Fed that does place greater emphasis (on the margin) on the output gap during the crisis than during the previous, more stable period.

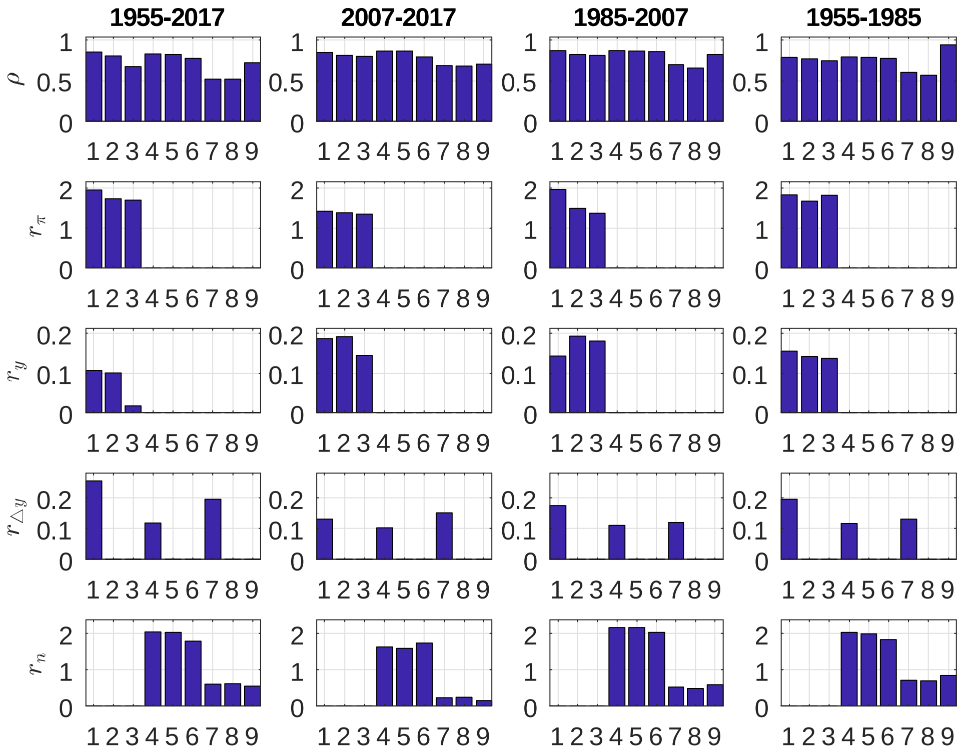

Figure 1: Monetary policy rule parameter values for each model (model 1 to model 9).

Regarding the output gap growth coefficient, it varies somewhat across periods and rules (between 0.10 and 0.23). At least for rule 7, this coefficient appears to be somewhat higher during the GFC/ZLB than during the GM, implying a larger reaction to output growth during the crisis than during the previous, stabler period. For rule 1, this coefficient is highest during the sub-period 1955-1985, yet it remains the smallest, and significantly so, during the crisis sub-period.

The nominal income coefficient associated with the NGDP rules is higher for the growth rules than the level rules, over all periods, a result that is in line with the literature (Rudebusch, 2002a). For the growth and level rules, this coefficient is lower during the GFC/ZLB than otherwise, especially during the GM. The coefficient for the NGDP level rules changes (with time and rule), but is lower during the GFC/ZLB period than during the other periods.

4.2 In-sample fit

Assessing in-sample fit is important to determine whether historical data (sample) are more or less in line with data generated by the estimated model. Table 3 shows the Laplace approximation around the posterior mode (based on a normal distribution), i.e., log marginal densities, for each model and for each sample.

	Rules								
Sample	1	2	3	4	5	6	7	8	9
1955-2017	-1491	-1515	-1512	**-1464**	-1481	-1488	-1563	-1602	-1556
2007-2017	-269	-270	-285	-307	-309	-283	**-258**	-262	-278
1985-2007	**-386**	-428	-408	-406	-404	-396	-393	-395	-405
1955-1985	**-817**	-824	-835	-840	-855	-837	-844	-846	-864

Table 3: Log marginal data densities for each model and each period (Laplace approximation). Best values are in bold.

Table 3 suggests that the first NGDP rule in levels (rule 7) best fits the historical data during the GFC/ZLB period. Rule 1, the Smets and Wouters (2007) rule, performs best during the GM period, while rule 7 is close and rule 2 ranks just after. Furthermore, rule 1 dominates the other rules over the period 1955-1985 whereas rule 4 comes first (rule 5 is almost identical) over the whole sample.

For each period, a different monetary policy rule best fits the historical data, except for rule 1 that comes first twice. Note that standard Taylor-type rules (rules 2 and 3) and NGDP growth targeting rules (rules 4, 5, and 6) are generally inferior to the other rules in explaining historical data, at least over the various sub-periods.

However, note that this result does not imply that models with lower log marginal data densities should be discarded. Whatever the log marginal data density function, it may be argued that each model is designed to capture only certain characteristics of the data. Whether the marginal likelihood is a good measure to evaluate how well the model accounts for particular aspects of the data is an open question (Koop, 2003; Fernández-Villaverde and Rubio-Ramírez, 2004; Del Negro et al., 2007; Benchimol and Fourçans, 2017).

5 Welfare measures

Central banks have objectives that may differ from those of society. The famous conservative banker, for example, may have a higher preference for low and stable inflation than the public at large. Or she may differ on her evaluation of economic outcomes from those of society (Taylor and Wieland, 2012; Walsh, 2015).

Such preferences are generally represented by a loss function that the central bank seeks to minimize. This minimization process is supposed to be the objective of society. Yet in models such as the one we use, the economic outcome can be evaluated in terms of its impact on the welfare of the representative household. The central bank could then use some measures of this welfare as a policy objective.

5.1 Central bank losses

In this section, we present loss measures based on the variance of the variables of interest from the central bank's perspective. These variances are estimated for each model and for each period.

Many ad hoc central bank loss functions appear in the literature (Svensson and Williams, 2009; Taylor and Wieland, 2012; Adolfson et al., 2014). Our methodology intends to summarize all standard possibilities. For various sets of weights defining these functions, we compute the ex post optimal rule, consistent with the estimated DSGE model. This approach is used extensively in the literature to investigate monetary policy rules (Taylor, 1979; Fair and Howrey, 1996; Taylor, 1999).

Non-separability between consumption and labor (worked hours) in Smets and Wouters (2007) household's utility function (see Section 5.2) introduces labor-related variables into the inflation and output equations. By minimizing its loss function with respect to these two equations, the central bank must also consider labor-related variables, such as wages (price of worked hours).

Our general central bank loss function, L_t, is defined in a traditional way, as[10]

$$L_t = var\left(\pi_t\right) + \lambda_y var\left(y_t - y_t^p\right) + \lambda_{\Delta r} var\left(\Delta r_t\right) + \lambda_w var\left(w_t\right) \tag{2}$$

where λ_y is the weight on output gap variances, $\lambda_{\Delta r}$ the weight on nominal interest rate differential variance, and λ_w the weight on wage inflation variance. The weight on price inflation variance is normalized to unity, and $var\left(.\right)$ is the variance operator. π_t is price inflation, $y_t - y_t^p$ the output gap, Δr_t nominal interest rate differential, and w_t wage inflation.[11]

First, in Fig. 2, we present the estimated variances of each variable (inflation, output gap, nominal interest rate differential, and wage inflation) entering the central bank loss functions.

The variances of all variables under consideration are significantly higher before 1985 and over the full sample. Even during the 2007-2017 period, these variances were lower than before 1985 and little different than during the GM period. The fact that estimated variances over the GFC/ZLB period are comparable across the models with those of the GM period does not mean that variances of historical data during the GFC/ZLB and GM are comparable. Indeed, the variances presented in Fig. 2 are estimated from the models while assuming that the Fed followed various rules and the US economy behaved as in the Smets and Wouters (2007) model. The high inflation period *cum* various significant ups and downs in economic activity and interest rates explain the high values observed between 1955 and 1985.

However, changes in the Fed's monetary policy and the stabilization period that occurred during the 1990s explain the low variance of the GM period relative to the 1955-1985 period. Output variances are a bit higher during the GFC/ZLB period than during the GM period, while those of the inflation rate are close. The low interest rates of the GFC/ZLB period lead to lower variances of the shadow interest rate differentials during the GFC/ZLB than during the GM period, even though the difference is not high. The variances of wages were also smaller during the GFC/ZLB period than during the GM period.

Second, we compute *ad hoc* loss functions based on Eq. 2. Fig. 3 to Fig. 6 present central bank losses when a moderate weight is applied on

[10] See Galí (2015) for more details. Another loss measure based on the squared distance of variables generated by the models can be defined:

$$L_t = \pi_t^2 + \lambda_y y_t^2 + \lambda_r \left(\Delta r_t\right)^2 \tag{1}$$

Empirically, this type of formulation leads to similar results to those given by Eq. 2. See the Online Appendix for loss functions using Eq. 1.

[11] See the Online Appendix for more details about the variables of the models.

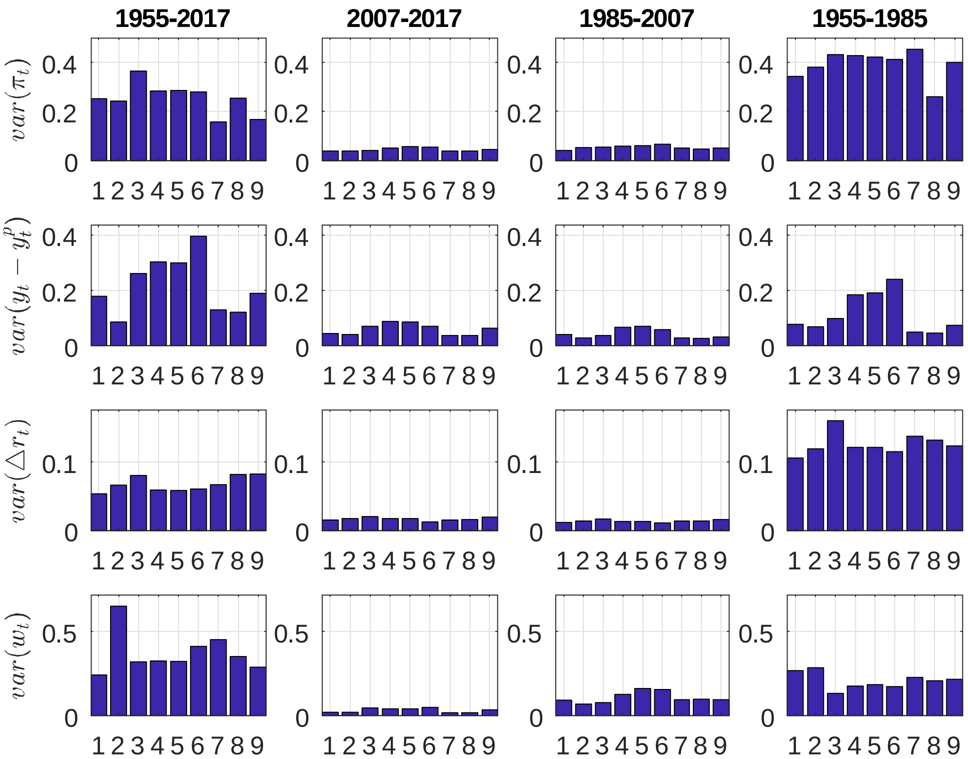

Figure 2: Estimated variances of central bank loss function variables, for each period and each rule.

wage variance.[12] Rules 7 and 8 lead to the lowest losses over the GFC/ZLB and GM periods. As before, the results vary somewhat when considering the 1955-1985 period where rule 8 dominates.

Rules 7 and 8 lead to the lowest losses over the GFC/ZLB and GM periods. Again, the results vary somewhat when considering the 1955-1985 period where rule 8 dominates.

As observed in the Online Appendix, these results do not modify in any significant way the results obtained without including the variance of wage ($\lambda_w = 0$), as well as with high weight on wage variance ($\lambda_w = 1$) in the loss functions.

For a given weight on the variance of the interest rate differential ($\lambda_{\Delta r}$), the loss diminishes for all rules and for all periods when the weight on the variance of the output gap diminishes (vertical observation). For a given weight on the variance of the output gap (λ_y), the loss diminishes, albeit to a limited extent, for all rules and for all periods when the weight on the

[12] In this section, we only present central bank losses with $\lambda_w = 0.5$. The Online Appendix presents central bank losses with $\lambda_w = 0$ and $\lambda_w = 1$.

variance of the interest rate differential diminishes (horizontal observation). These results are directly related to the simple (linear) functional form of the central bank loss function.

Interestingly, the change in the loss is very minor for a given λ_y (horizontal observation) compared to the change in the loss for a given $\lambda_{\Delta r}$ (vertical observation). This result would imply that a central bank gains almost nothing by including the interest rate differential in its loss function.

One can interpret this result in light of the interest rate smoothing assumption. Most of the monetary policy rules used in the literature assume interest rate smoothing, as we do. This smoothing implies that the central bank already minimizes the variances in the interest rate differential over time, hence the small gain generated by changing the interest rate differential coefficient in the central bank loss function for a given λ_y (horizontal observation).

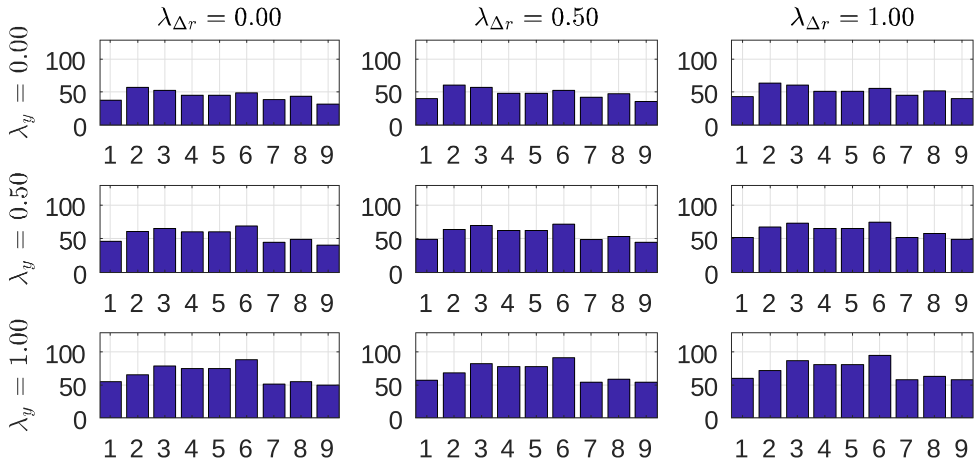

Figure 3: Central bank losses, for each rule, between 1955 and 2017 ($\lambda_w = 0.5$).

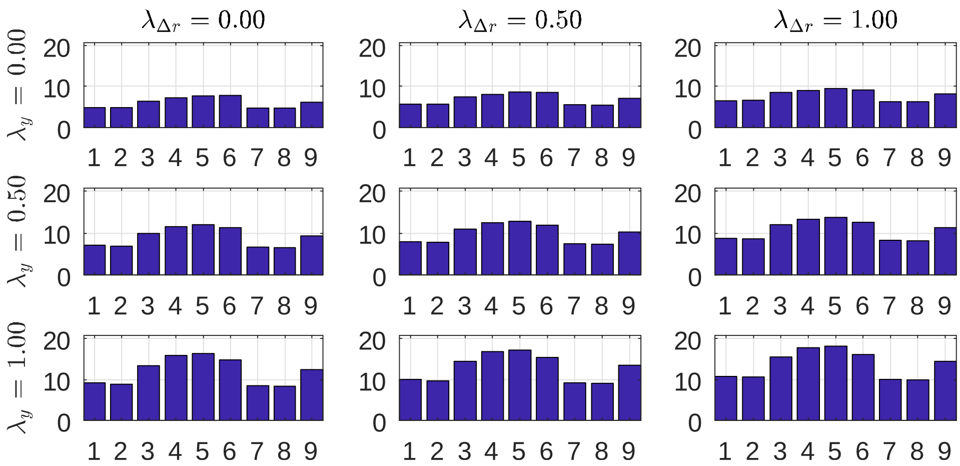

Figure 4: Central bank losses, for each rule, between 2007 and 2017 ($\lambda_w = 0.5$).

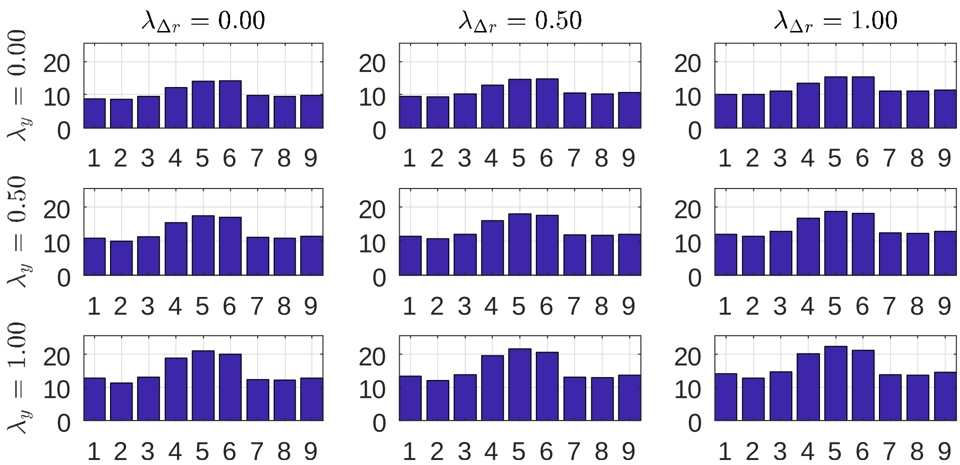

Figure 5: Central bank losses, for each rule, between 1985 and 2007 ($\lambda_w = 0.5$).

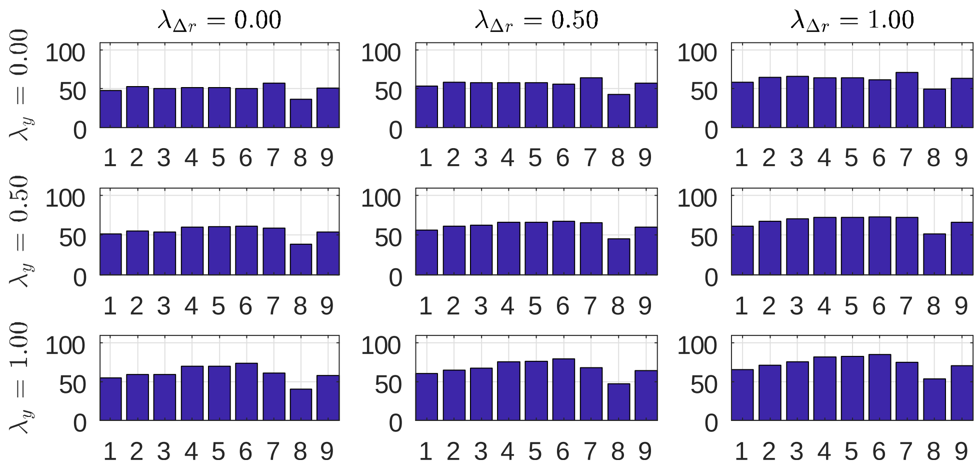

Figure 6: Central bank losses, for each rule, between 1955 and 1985 ($\lambda_w = 0.5$).

Note also that whatever the values of λ_y and $\lambda_{\Delta r}$ during the GFC/ZLB, rules 7 and 8 dominate the others, but rules 1 and 2 lead to nearly similar values.

For the GM period (1985-2007), the NGDP rules in levels, rules 7 and 8, as well as rule 2, dominate the other policy reaction functions.

During the 1955-1985 period, NGDP level targeting rule 8 leads to the lowest loss. From the full sample estimates, it appears that NGDP level targeting rule 7 is the best to minimize losses.

From these observations, we infer that during the exceptional GFC/ZLB period, the Fed would have minimized its loss by following an NGDP rule in levels, especially rules 7 and 8. However, had it employed Taylor-type rules 1 and 2, the difference in terms of loss would have been minor.

5.2 Households' welfare

As the model we use is micro-founded, maximizing a measure of welfare of the representative household may be an appropriate instrumental objective for the central bank. Two main welfare measures can be used. The *welfare levels*, presented in Section 5.2.1, and the *welfare compensating variations* (CV) that result from the difference in welfare levels between sticky and flexible-price and wage economies, presented in Section 5.2.2..

5.2.1 Welfare level

This section presents the estimated sticky-price (s) and the flexible-price (f) welfare levels. The Smets and Wouters (2007) model has both price and wage rigidities that emerge from imperfect substitution between goods and between labor. When all households receive identical wages, aggregate labor supply (the sum of differentiated labor supply across households) equals aggregate labor supplied by the labor aggregating firm. However, when households receive different wages, aggregate labor supply and labor used in production diverge due to wage dispersion.[13]

We rely on a second order approximation of the equilibrium conditions around the non-stochastic steady-state in order to obtain reliable welfare level measures.[14]

Households' welfare in the economy $k \in \{f, s\}$ is measured, to a second-order approximation, as the discounted expected sum of each quarter's utility value over each sample-period and for each model $m \in [1, ..., 9]$, such as

$$W_{t,k,m} = U_{t,k,m} + \beta_m E_t \left[W_{t+1,k,m} \right] \tag{3}$$

where the utility function, $U_{t,k,m}$, has the same functional form as in Smets and Wouters (2007) but differs with the economy (flexible-price, f, or sticky-price, s) and the model (β_m is the model-specific discount factor).

Aggregate households' welfare in economy k, $W_{k,m}$, is estimated at a second-order approximation for each model.[15]

5.2.2 Welfare compensating variations

Welfare level measures lead to quantitative statements regarding the relative losses of pursuing a given policy in an economy with sticky-prices and wages, or in an economy with flexible-prices and wages. Assessing policy rules necessitates a comparison of welfare between these two types of economies. To be able to implement this comparison, we compute the consumption level that would make the household indifferent between a world with flexible-prices and wages and a world with price and wage rigidities.

This can be addressed by calculating *welfare compensating variations*, as in Garín et al. (2016), i.e., by computing the deviation between aggregate welfare level in flexible ($W_{f,m}$) and sticky ($W_{s,m}$) prices and wages economies for each model m.

[13] This situation exists at higher order approximations of the model, with no trend inflation and no trend growth.

[14] See Woodford (2003) for a discussion about a first-order and higher order approximations for welfare analysis. Kim and Kim (2003) show that welfare evaluations based on a first order approximation of the equilibrium policy functions could be erroneous and the appropriate second-order approximations provide correct welfare rankings.

[15] These estimates under flexible and sticky-prices and wages for each monetary policy regime (m) are available upon request.

To analyze relative merits of a particular rule, we calculate these compensating variations.[16] We define ϕ_m as the model-specific percent of household's consumption that would equate the welfare under flexible-price, $W_{f,m}$, to the level of welfare under the rule being scrutinized, $W_{s,m}$.

ϕ_m is the model-specific percentage of consumption that would make the household indifferent between a world with flexible-prices and a world with price and wage rigidities.

Taking the same definition of the utility function as in Smets and Wouters (2007) and solving for ϕ_m, the welfare compensating variation, we obtain[17]

$$\phi_m = W_m^{\frac{1}{1-\sigma_{c,m}}} \tag{4}$$

where $W_m = W_{f,m}/W_{s,m}$ is the model-specific simple welfare loss, that is, the welfare loss associated with nominal rigidities under a particular rule m, and $\sigma_{c,m}$ the model-specific households' relative risk aversion.

More desirable monetary policies are associated with lower levels of compensating variations.

Table 4 presents welfare compensating variations (CV) estimates for each rule and each period.

	Rules								
Sample	1	2	3	4	5	6	7	8	9
1955-2017	**.1232**	.1341	.1468	.1486	.1545	.1509	.5521	.1389	.1433
2007-2017	.0102	.0266	.1011	.0123	.0527	**.0002**	.0005	.0042	.0969
1985-2007	.2087	**.1513**	.1807	.5446	.3943	.2858	.1726	.1655	.2231
1955-1985	.1114	.1149	**.0664**	.1453	.1355	.1265	.1248	.1287	.1278

Table 4: Estimated households' welfare compensating variations (ϕ_m). Best values are in bold.

During the GFC/ZLB, rule 6 (targeting the growth of NGDP with the natural rate in the function) leads to the lowest welfare compensating varia-

[16] Garín et al. (2016) evaluate different policy rules by computing the unconditional mean of welfare for a particular policy rule and comparing that to the unconditional mean of welfare in a hypothetical economy where prices and wages are both flexible. They compute these unconditional means by solving the model using a second order approximation of the equilibrium conditions about the non-stochastic steady-state. Then, they calculate a compensating variation, computing for eacch period the percent of consumption which would make a household indifferent between the flexible-price and wage economy and the sticky-price and wage economy. This compensating variation is interpreted as a welfare loss from price and wage rigidity. More desirable monetary policy regimes coincide with lower values of the compensating variation.

[17] For further details, see the Online Appendix.

tion. It is rule 2 that dominates during the GM period. From 1955 to 1985, rule 3 leads to the lower welfare compensating variations.

One can tentatively infer from these observations that except during the GFC/ZLB period, Taylor-type rules were best in terms of welfare compensating variations.

6 Interpretation

Table 5 summarizes our results to capture the essential facts of our exercise.

	1955-2017	2007-2017	1985-2007	1955-1985
Fitting				
Marginal density	4	7	1	1
Central bank losses				
Loss function[1]	7,9	7,8	2,7,8	8
Households				
Welfare (CV)	1	6	2	3

[1]Loss function presented in Eq. 2

Table 5: Summary of the best rule(s) for each criterion

In terms of fitting the data, the marginal density values show that rule 1 performs better than all others during the GM and the 1955-1985 periods. Rule 7 is best during the GFC/ZLB.

Yet, for reasons explained in Section 4.2, the values of the marginal densities are not a definitive proof that we have the correct ranking of rules. These values constitute an indication as to which rules were more or less followed during the various periods, assuming that the Fed followed a policy rule and that the economy behaved as in the Smets and Wouters (2007) model.

It is worth noting that during the GFC/ZLB, the NGDP level targeting rule best fits the data while during the other sub-periods the Smets and Wouters (2007) monetary policy rule performs best.

An analysis of the losses of the central bank leads to the general superiority of NGDP level targeting rules for all periods (with rule 2 leading to very similar results during the GM).

Welfare compensating variations are in favor of Taylor type rules other than during the GFC/ZLB period where a NGDP growth rule performs better.

From Table 5, it can be inferred that during the GFC/ZLB, in-sample fitting, central bank loss functions and welfare analysis lead to the best performance using some NGDP rules. This is not the case during the other periods.

These results are not intended to prove that the Fed followed any given type of rule depending on each period. An explicit rule is only a model that attempts to capture some monetary policy parameters explaining the methodology whereby the central bank determines its interest rate.

The estimates show that an NGDP level targeting rule would be best to minimize the loss function of the central bank over the various periods (with the Taylor rule leading to very similar results during the GM period).

It is worth noting that the generally assumed direct theoretical link between central bank loss functions and household's welfare (Woodford, 2003; Walsh, 2010; Galí, 2015) is not clear in practice. Table 5 shows that during almost all sub-periods, minimizing the central bank loss function does not necessarily lead to a minimization of household's welfare compensating variations, even if we do not take into consideration wage inflation and interest rate smoothing variances in central bank losses.

7 Policy implications

Irrespective of the period in question, central bank's objectives are not achieved by one single rule. For each period, there is a preferred monetary policy reaction function. In other words, for each type of period (more or less stable, crisis, recovery), a given central bank reaction function performs better than others. Yet, in general, if we only consider the loss function of the central bank, the results indicate the superiority of NGDP rules in level (even if the Taylor rule leads to nearly similar conclusions over the GM period). If the central bank wants to minimize household's welfare compensating variations, the implications are different.

Parameter estimates change with respect to the period considered, for any given monetary policy rule. Policy institutions, which base their forecasts and policy recommendations on such models and rules, should refresh their estimates regularly to avoid inaccurate policy conclusions. This policy recommendation is also reinforced by Kolasa (2015).

Our paper also demonstrates that policy implications depend on the chosen monetary policy rule. Central banks should use several monetary policy rules in order to base their policy on a broader scope of results than those obtained by only one model or monetary policy rule. Our monetary policy rule analyzes produce insights such as the use of Taylor or NGDP rules comparison, an objective that is in line with Wieland et al. (2012).

It is standard practice to assume that a central bank seeks to minimize a loss function that includes, at least, inflation and output variances. Would this minimization process necessarily leads to the best choice in terms of households' welfare compensating variations ? Our results show that this is not necessarily the case.

Both of these strategies may still sometimes be (or close to be) compatible as is the case during the GFC/ZLB period.

8 Conclusion

The purpose of this paper is to shed light on the effects of different monetary policy rules on the macroeconomic equilibrium. Specifically, we seek to determine, first, which of the various monetary policy rules is most in line with the historical data for the US economy and, second, what policy rule would work best to assist the central bank reach its objectives via a loss function or via some measure of households' welfare. The first consideration is positive, the second is normative.

To conduct this type of analysis, we compare Taylor-type and nominal income rules through the well-known Smets and Wouters (2007) DSGE model.

We consider nine monetary policy rules. Three are of the "Taylor-type" and six are of the nominal income targeting type (NGDP), either in growth or levels. We test the model with these various rules through Bayesian estimations from 1955 to 2017, over three different periods: 1955-1985, 1985-2007, and 2007-2017. These sub-periods are selected to capture the impact of policy rules given different economic environments (more or less stable periods, crisis and recovery).

In terms of fit with historical data, the marginal density values suggest that one NGDP level targeting rule exhibits the best fit during the GFC/ZLB – and a NGDP growth targeting rule over the whole sample. A Taylor type rule is best during the GM period as well as over the 1955-1985 period.

The results regarding the losses of the central bank suggest the superiority of NGDP level targeting rules, whatever the sub-period (a Taylor type rule leading to similar results over the GM period).

When households' welfare compensating variations are taken into consideration, the results are in favor of some Taylor type rule, except during the GFC/ZLB period.

Several policy implications can be drawn.

First, central bank's objectives cannot be achieved by one single rule over all time frames. For each type of period (more or less stable, crisis, recovery), a specific central bank reaction function performs better than others.

Second, central banks, which base their forecasts and policy recommendations on such models and rules, should refresh their estimates regularly to avoid inaccurate policy decisions.

Third, policy makers should estimate central bank losses and welfare compensating variations through several monetary policy rules and models in order to better assess their interest rate decisions.

Fourth, central bankers must be aware that minimizing their loss function does not necessarily lead to the minimization of households' welfare compensating variations. They may have to choose between the two strategies.

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