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Liquidity Traps in a Monetary Union*

Robert Kollmann†

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Abstract

The closed economy macro literature has shown that a liquidity trap can result from the self-fulfilling expectation that *future* inflation and output will be low (Benhabib et al. (2001)). This paper investigates expectations-driven liquidity traps in a two-country New Keynesian model of a monetary union. In the model here, country-specific productivity shocks induce synchronized responses of domestic and foreign output, while country-specific aggregate demand shocks trigger asymmetric domestic and foreign responses. A rise in government purchases in an individual country *lowers* GDP in the rest of the union. The results here cast doubt on the view that, in the current era of ultra-low interest rates, a rise in fiscal spending by Euro Area (EA) core countries would significantly *boost* GDP in the EA periphery (e.g., Blanchard et al. (2016)).

JEL codes: E3, E4, F2, F3, F4.

Keywords: Zero lower bound, liquidity trap, monetary union, terms of trade, international fiscal spillovers, Euro Area.

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1. Introduction

Since the global financial crisis (2008-9), short-term riskless nominal interest rates in the Euro Area have been close to zero, while Euro Area inflation too has remained low (below the ECB's target). Understanding this 'low rates' environment is one of the key challenges for economic analysis. This paper provides a novel perspective on the effect of a low-rates environment on the transmission of aggregate supply and demand disturbances, in a *monetary union*. The analysis is based on a two-country New Keynesian business cycle model of a monetary union with a zero lower bound (ZLB) constraint for the nominal interest rate.

As is well-known from the *closed economy* macro literature (Benhabib et al. (2001a,b; 2002a,b), the presence of the ZLB can give rise to multiple equilibria that exhibit self-fulfilling fluctuations of inflation and real activity, under the standard assumption that monetary policy follows an 'active' Taylor rule, i.e. a policy rule with a strong interest rate response when the inflation rate deviates from the central bank's inflation target. A liquidity trap, i.e. a situation in which the ZLB binds, can then be due to the self-fulfilling expectation that *future* inflation and output will be low. Mertens and Ravn (2014) and Aruoba et al. (2018) have shown that when a closed economy liquidity trap is caused by pessimistic expectations, then a rise in government purchases has a deflationary effect, which mutes the rise in GDP triggered by the fiscal shock. The mechanism is that a rise in government purchases triggers a rise in the real interest rate. In a pessimism-driven liquidity trap, the rise in the real rate is brought about by a *fall* in the (current and expected future) inflation rate. This deflationary effect dampens the rise in output.

The contribution of the present paper is to study beliefs-driven liquidity traps in a *monetary union*, in order to shed light on key policy issues facing the Euro Area. The model assumes fully integrated goods markets and financial markets. Each country produces a distinct set of goods, while consuming both domestic and imported goods. Due to a spending bias towards locally produced goods, consumer price index (CPI) inflation can differ across countries. The union's central bank follows a Taylor rule that targets *union-wide* inflation.

In this setting, I study beliefs-driven sunspot equilibria that feature an occasionally binding ZLB. The baseline model predicts that, in a liquidity trap, a rise in government purchases in country 'Home' boosts local GDP, while 'Foreign' GDP falls; thus, the cross-country transmission of fiscal spending shocks is *negative*. The domestic government spending multiplier is smaller than unity. The fall in Foreign GDP is smaller (in absolute value) than the rise in

Home GDP. The finding of a weak *domestic* fiscal multiplier resonates with Mertens and Ravn's (2014) prediction of a muted response of output to fiscal spending shocks, in a closed economy liquidity trap. The weak (negative) transmission (of a Home government purchases increase) to Foreign GDP reflects the muted rise in Home GDP, which only generates a weak demand spillover to Foreign output. At the ZLB, a positive shock to Home government purchases triggers a fall in average *union-wide* inflation, which dampens the response of union-wide output. The Home terms of trade improve, which tends to shift private-sector demand towards Foreign goods. However, price rigidity dampens this competitiveness effect, which contributes to the weak (negative) response of Foreign GDP. The model here predicts that country-specific *private* sector demand shocks too induce asymmetric responses of domestic and foreign output. By contrast, a country-specific rise in total factor productivity (TFP) raises both domestic and foreign GDP.

The effect of the expectations-driven liquidity traps considered here differs from that of liquidity traps triggered by large adverse exogenous aggregate demand shocks, such as an autonomous fall in households' subjective discount rates. Liquidity traps of the latter type have been studied by an extensive literature; see, e.g., the closed economy models of Krugman (1998), Eggertsson and Woodford (2003), Christiano et al. (2011), Holden (2016,2019) and Roeger (2015). That literature predicts that fiscal spending multipliers can be much larger at the ZLB than when the ZLB does not bind. A key assumption of that literature is that, once the economy has escaped from a liquidity trap induced by a large adverse demand shock, agents expect that the economy will never again enter a liquidity trap. The large predicted fiscal multipliers during a liquidity trap of this type are due to the fact that, with a forward looking Phillips curve, a small increase in the expected inflation rate, at the date at which the economy permanently emerges from the liquidity trap, has a large positive effect on inflation and output in *earlier* periods. Widely discussed studies by Erceg and Lindé (2010) and Blanchard et al. (2016) have presented models of a monetary union in which a ZLB episode is triggered by a large adverse demand shock. These authors find that, at the ZLB, a country-specific rise in government purchases can have a strong positive effect on domestic and *foreign* output, i.e. the predicted international fiscal spillover is sizable. This theory provides a basis to the view that, in the aftermath of the global financial crisis, fiscal 'austerity' (and weak demand more generally) in Germany contributed to the slump in the rest of the Euro Area (e.g., US Treasury (2013), Krugman (2013), Wolf (2013)).

It also predicts that expansionary fiscal policy in Germany (and other Euro Area core countries) could significantly help the Euro Area 'periphery', in a low interest rate environment. The present paper cautions against the idea of strong cross-border fiscal transmission in a monetary union, during a liquidity trap. If a liquidity trap is caused by self-fulfilling pessimism about future inflation, then cross-country fiscal spending spillovers can be much weaker than in a liquidity trap induced by an adverse aggregate demand shock.

2. Model of a monetary union

I consider a New Keynesian open economy model with a standard structure of goods, labor and financial markets (e.g., Kollmann (2001, 2002, 2004)). There are two countries, referred to as Home (H) and Foreign (F), that belong to a monetary union. A common central bank sets the short-term nominal interest for both countries. In each country there are: (i) a government that makes exogenous purchases which are financed using lump-sum taxes; (ii) a representative infinitely-lived household; (iii) monopolistic firms that produce a continuum of differentiated tradable intermediate goods using domestic labor; (iv) competitive firms that bundle domestic and imported intermediates into composite non-tradable goods that are used for household and government consumption. Intermediate goods prices are sticky; all other prices are flexible. Each country's household owns the domestic firms, and it supplies labor to those firms (labor is immobile internationally). The labor market is competitive. For analytical tractability, the model abstracts from physical capital. The Foreign country is a mirror image of the Home country. The following description focuses on the Home country. Analogous conditions describe the Foreign country.

2.1. Home firms

The Home country's household consumes a composite final consumption good $C_{H,t}$ that is produced using the Cobb-Douglas technology $C_{H,t} \equiv (Y_{H,t}^{H}/\xi)^{\xi}(Y_{H,t}^{F}/(1-\xi))^{1-\xi}$ where $Y_{H,t}^{H}$ and $Y_{H,t}^{F}$ are, respectively, a composite of domestic intermediate goods and a composite of imported intermediates, used by country H. (The superscript on intermediate good quantities denotes the country of origin, while the subscript indicates the destination country.) There is a bias towards using local intermediates, in household consumption: $0.5<\xi<1$. Each country produces a distinct set of intermediates indexed by $s\in[0,1]$. (Intermediate good 's' produced by country H differs from intermediate 's' produced by country F.) The composite intermediate $Y^k_{H,t}$ is given by $Y^k_{H,t}\equiv\{\int_0^1 (y^k_{H,t}(s))^{(\nu-1)/\nu}ds\}^{\nu/(\nu-1)}$ with $\nu>1$, for k=H,F, where $y^k_{H,t}(s)$ is the quantity of the variety s input produced by country k that is sold to country H.

Home government consumption, denoted $G_{H,t}$, too is a composite of intermediary inputs, but government consumption only uses local intermediates (no imports).[1] Specifically, $G_{H,t}\equiv\{\int_0^1 (g^H_{H,t}(s))^{(\nu-1)/\nu}ds\}^{\nu/(\nu-1)}$, where $g^H_{H,t}(s)$ is the quantity of the Home produced variety s input that enters Home government consumption.

Let $p_{k,t}(s)$ be the price of intermediate good s produced by country k. Cost minimization in Home final good production implies: $y^k_{H,t}(s)=(p_{k,t}(s)/P_{H,t})^{-\nu}Y^k_{H,t}$ for k=H,F and $g^H_{H,t}(s)=(p_{H,t}(s)/P_{H,t})^{-\nu}G_{H,t}$, as well as $Y^H_{H,t}=\xi\cdot CPI_{H,t}\cdot C_{H,t}/P_{H,t}$, $Y^F_{H,t}=(1-\xi)\cdot CPI_{H,t}\cdot C_{H,t}/P_{F,t}$ where $P_{k,t}\equiv\{\int_{s=0}^1 p_{k,t}(s)^{1-\nu}ds\}^{1/(1-\nu)}$ and $CPI_{H,t}\equiv(P_{H,t})^{\xi}(P_{F,t})^{1-\xi}$. $P_{k,t}$ is a price index of intermediates produced by country k=H,F. Perfect competition implies that the country H final consumption good price index is $CPI_{H,t}$ (its marginal cost).

The technology of the firm that produces intermediate good s in country H is: $y_{H,t}(s)=\theta_{H,t}L_{H,t}(s)$, where $y_{H,t}(s)$ and $L_{H,t}(s)$ are the firm's output and labor input at date t, while $\theta_{H,t}>0$ is exogenous productivity in country H (all intermediate good producers located in a country have identical productivity). The firm's good is sold domestically and exported: $y_{H,t}(s)=y^H_{H,t}(s)+y^H_{F,t}(s)+g^H_{H,t}(s)$.

Intermediate good producers face quadratic costs to adjusting their prices. The real profit, in units of Home consumption, of the firm that produces Home intermediate good s is:

$$\pi_{H,t}(s)\equiv(p_{H,t}(s)-W_{H,t}/\theta_{H,t})y_{H,t}(s)/CPI_{H,t}-\tfrac{1}{2}\psi\cdot([p_{H,t}(s)-\Pi\cdot p_{H,t-1}(s)]/P_{H,t-1})^2,\quad \psi>0$$

[1] Empirically, the import content of government spending is much lower than that of private consumption (e.g., Bussiere et al. (2013). The main results below do not depend on assuming that the government consumption basket differs from the household consumption basket.

where $W_{H,t}$ is the nominal wage rate in country H. The last term in the profit equation is the real price adjustment cost, where $\Pi > 1$ is the central bank's gross inflation target (see below).[2] The firm sets $p^H_{H,t}(s)$ to maximizes the present value of profits $E_t \sum_{\tau=0}^{\infty} \rho^H_{t,t+\tau} \pi_{H,t+\tau}(s)$, where $\rho^H_{t,t+\tau}$ is the Home household's intertemporal marginal rate of substitution in consumption between periods t and $t+\tau$. All Home intermediate good firms face identical decision problems, and they set identical prices: $p_{H,t}(s)=P_{H,t}$ $\forall$ $s\in[0,1]$. The labor input and output too are equated across all Home intermediate good firms.

The Home terms of trade and the real exchange rate (CPI-based) are $q_t \equiv P_{H,t}/P_{F,t}$ and $RER_t \equiv CPI_{H,t}/CPI_{F,t}$, respectively. Note that $RER_t=(q_t)^{2\xi-1}$. Due to household consumption home bias ($2\xi-1>0$), the real exchange rate is an increasing function of the terms of trade. The real price of the domestic intermediate good, in units of final consumption, too is an increasing function of the terms of trade:

$$P_{H,t}/CPI_{H,t}=(q_t)^{1-\xi}. \tag{1}$$

2.2. Household preferences and labor supply

The intertemporal preferences of the representative Home household are described by $E_0 \sum_{t=0}^{\infty} \beta^t \Psi_{H,t} U(C_{H,t}, L_{H,t})$ where $C_{H,t}$ and $L_{H,t}$ are final consumption and aggregate hours worked, respectively. $0<\beta<1$ is the household's steady state subjective discount factor and $U(C_{H,t},L_{H,t})=\ln(C_{H,t})-\frac{1}{1+1/\eta}(L_{H,t})^{1+1/\eta}$ is the agent's period utility function, where $\eta>0$ is the Frisch labor supply elasticity. $\Psi_{H,t}>0$ is a stationary exogenous preference shock that alters the household's rate of time preference. The household equates the marginal rate of substitution between leisure and consumption to the real wage rate, which implies

$$(1/C_{H,t})(W_{H,t}/CPI_{H,t}) = (L_{H,t})^{1/\eta}. \tag{2}$$

[2] The price adjustment cost is assumed to be a quadratic function of $p^H_{H,t}(s)-\Pi\cdot p^H_{H,t-1}(s)$ (and not of $p^H_{H,t}(s)-p^H_{H,t-1}(s)$, as e.g. in the Rotemberg (1982) cost adjustment original specification). This ensures that the long-run Phillips curve is vertical, if steady state inflation equals the central bank's inflation target.

2.3. Financial markets

The model assumes complete international financial markets, and so consumption risk is efficiently shared across countries. In equilibrium, the ratio of Home to Foreign households' marginal utilities of consumption is, thus, proportional to the Home real exchange rate (Kollmann, 1991, 1995; Backus and Smith, 1993). With log utility, this implies that the relative Home/Foreign marginal utility of consumption is proportional to the Home real exchange rate: $\{\Psi_{H,t}/C_{H,t}\}/\{\Psi_{F,t}/C_{F,t}\}=\Lambda RER_t$, where Λ is a date- and state-invariant term that reflects the (relative) initial wealth of the two countries. I assume that the two countries have the same initial wealth, i.e. $\Lambda=1$. Thus:

$$C_{H,t}/C_{F,t}=(\Psi_{H,t}/\Psi_{F,t})/RER_t. \tag{3}$$

There is also a market for a one-period riskless nominal bond (in zero net supply). The nominal interest rate on that bond is i_{t+1} between periods *t* and *t*+1. The Home household's Euler equation for this bond is:

$$(1+i_{t+1})E_t\beta(\Psi_{H,t+1}/\Psi_{H,t})(C_{H,t}/C_{H,t+1})/\Pi^{CPI}_{H,t+1}=1, \tag{4}$$

where $\Pi^{CPI}_{H,t+1}\equiv CPI_{H,t+1}/CPI_{H,t}$ is the Home gross CPI inflation rate between periods *t* and *t*+1.

2.4. Monetary policy

The monetary union's central bank sets the interest rate i_{t+1} according to a feedback rule that targets the union-wide average CPI inflation rate $\Pi_t\equiv\frac{1}{2}\Pi^{CPI}_{H,t}+\frac{1}{2}\Pi^{CPI}_{F,t}$ subject to the zero lower bound (ZLB) constraint $i_{t+1}\geq 0$. Specifically, the monetary policy rule is

$$1+i_{t+1}=Max\{1,\Pi/\beta+(\gamma_\pi/\beta)\cdot(\Pi_t-\Pi)\},\ \gamma_\pi>1 \tag{5}$$

where $\Pi>1$ is the central bank's gross inflation target. Π/β is the gross nominal interest rate that obtains when the union-wide inflation rate equals the central bank's inflation target. γ_π is a parameter that captures the central bank's policy response to inflation. The 'Taylor principle' $(\gamma_\pi>1)$ is assumed to hold: as long as the ZLB constraint remains slack, a rise in inflation by 1 percentage point (ppt) triggers a rise of the policy rate by more than 1 ppt.

2.5. Market clearing

Market clearing in the country k=H,F labor market requires $L_{k,t}=\int_{s=0}^{1}L_{k,t}(s)\,ds$. Real GDP ($Y_{k,t}$) equals aggregate intermediate good output, $Y_{k,t}=\theta_{k,t}L_{k,t}$. Markets for individual intermediates clear as intermediate good firms meet all demand at posted prices. This implies $Y_{k,t}=Y^{k}_{H,t}+Y^{k}_{F,t}+G_{k,t}$ i.e. aggregate intermediate good output equals the sum of aggregate domestic and foreign intermediate good demand. Using the intermediate good demand functions described above, this condition can be expressed as $Y_{H,t}=\xi CPI_{H,t}C_{H,t}/P_{H,t}+(1-\xi)CPI_{F,t}C_{F,t}/P_{H,t}+G_{H,t}$ and $Y_{F,t}=(1-\xi)CPI_{H,t}C_{H,t}/P_{F,t}+\xi CPI_{F,t}C_{F,t}/P_{F,t}+G_{F,t}$.

2.6. Solving the model

Following much of the previous literature on macro models with a ZLB constraint (see Holden (2016, 2019) for detailed references), I linearize all equations, **with the exception of the interest rate rule (5)**. This allows to capture the macroeconomic effects of the occasionally binding ZLB constraint, while keeping analytical tractability.

I take a linear approximation around a steady state in which (in both countries) the gross inflation rate equals the inflation target Π; the corresponding steady state gross interest rate is $1+i=\Pi/\beta$. Let $\widehat{x_t}\equiv(x_t-x)/x$ denote the relative deviation of a variable x_t from its steady state value $x\neq 0$ (variables without time subscript denote steady state values). To simplify the analytical expressions, I assume that government purchases are zero, in steady state.[3] I define $\widehat{G_{k,t}}\equiv G_{k,t}/Y_k$ as the ratio of government purchases to steady state GDP in country k=H,F.

Linearization of the risk-sharing condition (3) and of the goods market clearing conditions gives:

$$\widehat{C_{H,t}}-\widehat{C_{F,t}}=-(2\xi-1)\widehat{q_t}+\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}}\,, \tag{6}$$

$$\widehat{Y_{H,t}}=\xi\widehat{C_{H,t}}+(1-\xi)\widehat{C_{F,t}}-2\xi(1-\xi)\widehat{q_t}+\widehat{G_{H,t}},\quad \widehat{Y_{F,t}}=(1-\xi)\widehat{C_{H,t}}+\xi\widehat{C_{F,t}}+2\xi(1-\xi)\widehat{q_t}+\widehat{G_{F,t}}\,. \tag{7}$$

[3] None of the substantive results hinge on this assumption. The analysis below will allow for both positive and negative shocks to government purchases. An interpretation of negative government purchases is that government occasionally has an autonomous supply of resources that it distributes to the private sector. Or think of government purchases as an additive technology shock that subtracts (for G>0) or adds (for G<0) resources from/to the private sector technology (Kollmann (1991)).

The linearized bond Euler equation (4) of country k=H,F is:

$$\widehat{1+i_{t+1}}=E_t\{\widehat{\Pi^{CPI}_{k,t+1}}+\widehat{C_{k,t+1}}-\widehat{C_{k,t}}+\widehat{\Psi_{kt}}-\widehat{\Psi_{kt+1}}\}\,. \tag{8}$$

Linearizing the first-order condition of the intermediate good firms' decision problem in country k=H,F gives a standard 'forward-looking' Phillips equation:

$$\widehat{\Pi_{k,t}}=\kappa_w\cdot\widehat{mc_{k,t}}+\beta E_t\widehat{\Pi_{k,t+1}}, \tag{9}$$

where $\Pi_{k,t}\equiv P_{k,t}/P_{k,t-1}$ is the gross inflation rate of the *producer price index (PPI)* in country k, while $mc_{k,t}=(W_{H,t}/\theta_{H,t})/P_{H,t}$ is real marginal cost (deflated by the domestic producer price) in the same country's intermediate good sector (e.g., Kollmann (2002)). $\kappa_w>0$ is a coefficient that is a decreasing function of the price adjustment-cost parameter ψ. Using the nominal wage implied by the Home household's labor supply equation (2) (and the analogous Foreign equation) allows to express Home and Foreign real marginal costs as:

$$\widehat{mc_{H,t}}=\widehat{C_{H,t}}+\tfrac{1}{\eta}\widehat{Y_{H,t}}-(1+\tfrac{1}{\eta})\widehat{\theta_{H,t}}-(1-\xi)\widehat{q_t} \text{ and } \widehat{mc_{F,t}}=\widehat{C_{F,t}}+\tfrac{1}{\eta}\widehat{Y_{F,t}}-(1+\tfrac{1}{\eta})\widehat{\theta_{F,t}}+(1-\xi)\widehat{q_t}. \tag{10}$$

Expressing the interest rate rule (5) using 'hatted' variables gives

$$\widehat{(1+i_{t+1})}=Max\{-(\Pi-\beta)/\Pi,\gamma_\pi\cdot\widehat{\Pi_t}\}. \tag{11}$$

Note that the interest rate $\widehat{(1+i_{t+1})}$ is a *non-linear* function of union-wide inflation. The ZLB constraint binds when $\gamma_\pi\widehat{\Pi_t}\le-(\Pi-\beta)/\Pi$.

All exogenous variables follow univariate AR(1) processes. For simplicity, I assume that all exogenous variables have the same autocorrelation ρ, with $0<\rho<1$. Thus, $\widehat{\theta_{k,t+1}}=\rho\widehat{\theta_{k,t}}+\varepsilon^{\theta}_{k,t+1}$, $\widehat{G_{k,t+1}}=\rho\widehat{G_{k,t}}+\varepsilon^{G}_{k,t+1}$ and $\widehat{\Psi_{k,t+1}}=\rho\widehat{\Psi_{k,t}}+\varepsilon^{\Psi}_{k,t+1}$ for k=H,F where $\varepsilon^{\theta}_{k,t+1},\varepsilon^{G}_{k,t+1},\varepsilon^{\Psi}_{k,t+1}$ are exogenous mean-zero innovations.

2.6.1. Union-wide inflation and output

In what follows, variables without country (H or F) subscripts denote union-wide averages. Union-wide consumption, GDP, productivity, government purchases and preference shocks (expressed as deviations from steady state values) are $\widehat{C_t}\equiv\tfrac{1}{2}\widehat{C_{H,t}}+\tfrac{1}{2}\widehat{C_{F,t}}$, $\widehat{Y_t}\equiv\tfrac{1}{2}\widehat{Y_{H,t}}+\tfrac{1}{2}\widehat{Y_{F,t}}$, $\widehat{\theta_t}=\tfrac{1}{2}\widehat{\theta_{H,t}}+\tfrac{1}{2}\widehat{\theta_{F,t}}$, $\widehat{G_t}=\tfrac{1}{2}\widehat{G_{H,t}}+\tfrac{1}{2}\widehat{G_{F,t}}$ and $\widehat{\Psi_t}\equiv\tfrac{1}{2}\widehat{\Psi_{H,t}}+\tfrac{1}{2}\widehat{\Psi_{F,t}}$, respectively. Note that $\widehat{Y_t}=\widehat{C_t}+\widehat{G_t}$

holds (from market clearing condition (7)), i.e. union-wide output equals union-wide consumption plus government purchases. Union-wide CPI inflation equals union-wide PPI inflation: $\widehat{\Pi_t} \equiv \widehat{\Pi_t^{CPI}} = \frac{1}{2}\widehat{\Pi_{H,t}} + \frac{1}{2}\widehat{\Pi_{F,t}}$. Averaging the Home and Foreign Phillips equations (9) (using the expression for real marginal cost (10)) gives a union-wide Phillips curve:

$$\widehat{\Pi}_t = \kappa \cdot (\widehat{C}_t - \widehat{\theta}_t + \tfrac{1}{1+\eta}\widehat{G}_t) + \beta E_t \widehat{\Pi_{t+1}}, \text{ with } \kappa \equiv \kappa_w (1+\eta)/\eta > 0. \tag{12}$$

Averaging the Home and Foreign Euler equations (8) gives:

$$\widehat{1+i_{t+1}} = E_t\{\widehat{\Pi_{t+1}} + \widehat{C_{t+1}} - \widehat{C}_t - (\widehat{\Psi_{t+1}} - \widehat{\Psi}_t)\}. \tag{13}$$

Combining the union-wide Euler equation (13) and the interest rate rule (11), and substituting out union-wide consumption using the union-wide Phillips curve (12) gives the following non-linear expectational difference equation for union-wide inflation:

$$Max\{-(\Pi-\beta)/\Pi, \gamma_\pi \cdot \widehat{\Pi}_t\} = -\tfrac{1}{\kappa}\widehat{\Pi}_t + (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi_{t+1}} - \tfrac{\beta}{\kappa}E_t\widehat{\Pi_{t+2}} + \hat{r}_t, \tag{14}$$

with $\hat{r}_t \equiv E_t\{(\widehat{\theta_{t+1}} - \widehat{\theta}_t) - \frac{1}{1+\eta}(\widehat{G_{t+1}} - \widehat{G}_t) - (\widehat{\Psi_{t+1}} - \widehat{\Psi}_t)\} = (1-\rho)\{-\widehat{\theta}_t + \frac{1}{1+\eta}\widehat{G}_t + \widehat{\Psi}_t\}$.

I will call (14) the union-wide "Euler-Phillips" equation. The term $\hat{r}_t$ reflects expected one-period ahead changes of exogenous variables. If prices were flexible ($\kappa=\infty$), the union-wide expected risk-free gross real interest rate between dates t and t+1 (expressed as a deviation from the steady state rate) would be $\hat{r}_t$. I refer to $\hat{r}_t$ as the (union-wide) natural real interest rate.

Note that the natural real interest rate is a decreasing function of the date t level of (union-wide) productivity and an increasing function of government purchases and of the preference shock. As these forcing variables follow AR(1) processes with autocorrelation ρ, the natural rate too is an AR(1) process with autocorrelation ρ. Because of the assumed mean reversion of productivity, a positive productivity shock reduces the expected future growth rate of productivity; in a flex-prices economy, a positive productivity shock increases consumption on impact; future consumption rises less than current consumption, i.e. the expected growth rate of consumption falls, and hence the real natural interest rate drops. Similar logic explains why positive fiscal spending and preference shocks raise the natural real interest rate.

As first shown by Benhabib et al. (2001a,b; 2002a,b), a non-linear inflation equation of form (14) has multiple bounded solutions (see discussion below). Given a process for union-wide inflation $\{\widehat{\Pi}_t\}$ that solves the Euler-Phillips equation (14), one can determine union-wide consumption, GDP and the nominal interest rate using (12) and (13).

2.6.2. Country-level variables

Country-level inflation, consumption and output depend on the terms of trade. Note that Home and Foreign inflation, consumption and output can be expressed as functions of union-wide variables and of cross-country differences of the corresponding variables:

$$\widehat{\Pi}_{i,t}=\widehat{\Pi}_t+\tfrac{1}{2}(\widehat{\Pi}_{i,t}-\widehat{\Pi}_{j,t}),\ \widehat{C}_{i,t}=\widehat{C}_t+\tfrac{1}{2}(\widehat{C}_{i,t}-\widehat{C}_{j,t}),\ \ \widehat{Y}_{i,t}=\widehat{Y}_t+\tfrac{1}{2}(\widehat{Y}_{i,t}-\widehat{Y}_{j,t})\ \text{ for } i,j=\text{H,F};\ j\neq i. \tag{15}$$

Relative (Home vs. Foreign) gross PPI inflation equals the change of the Home terms of trade: $\widehat{\Pi}_{H,t}-\widehat{\Pi}_{F,t}=\widehat{q}_t-\widehat{q}_{t-1}$. Relative consumption is a function of the terms of trade and of the relative preference shock (see (6)). Relative output can be expressed as a function of the terms of trade, of the relative preference shock and of relative government purchases (from (6) and (7)):

$$\widehat{Y}_{H,t}-\widehat{Y}_{F,t}=-\widehat{q}_t+(2\xi-1)(\widehat{\Psi}_{H,t}-\widehat{\Psi}_{F,t})+(\widehat{G}_{H,t}-\widehat{G}_{F,t})\,. \tag{16}$$

Subtracting the Foreign Phillips curve from the Home Phillips curve (see (9)) gives $\widehat{\Pi}_{H,t}-\widehat{\Pi}_{F,t}=\kappa_w\cdot(\widehat{mc}_{H,t}-\widehat{mc}_{F,t})+\beta E_t\{\widehat{\Pi}_{H,t+1}-\widehat{\Pi}_{F,t+1}\}$. Thus, $\widehat{q}_t-\widehat{q}_{t-1}=\kappa_w\cdot(\widehat{mc}_{H,t}-\widehat{mc}_{F,t})+\beta E_t\{\widehat{q}_{t+1}-\widehat{q}_t\}$. Relative real marginal cost can be written as (using (6),(7),(10)):

$$\widehat{mc}_{H,t}-\widehat{mc}_{F,t}=-\tfrac{1+\eta}{\eta}\widehat{q}_t-\tfrac{1+\eta}{\eta}(\widehat{\theta}_{H,t}-\widehat{\theta}_{F,t})+\tfrac{1}{\eta}(\widehat{G}_{H,t}-\widehat{G}_{F,t})+\tfrac{\eta+2\xi-1}{\eta}(\widehat{\Psi}_{H,t}-\widehat{\Psi}_{F,t})\,.$$

Combining these expressions, we see that the terms of trade are governed by the difference equation

$$E_t\widehat{q}_{t+1}=\tfrac{1+\kappa+\beta}{\beta}\widehat{q}_t-\tfrac{1}{\beta}\widehat{q}_{t-1}+\tfrac{\kappa}{\beta}(\widehat{\theta}_{H,t}-\widehat{\theta}_{F,t})-\tfrac{1}{1+\eta}\tfrac{\kappa}{\beta}(\widehat{G}_{H,t}-\widehat{G}_{F,t})-\tfrac{\kappa}{\beta}\tfrac{\eta+2\xi-1}{1+\eta}(\widehat{\Psi}_{H,t}-\widehat{\Psi}_{F,t})\,. \tag{17}$$

$0<\beta<1$ and $\kappa>0$ ensure that equation (17) has a unique non-explosive solution

$$\widehat{q}_t=\Xi\cdot\widehat{q}_{t-1}-a_\theta(\widehat{\theta}_{H,t}-\widehat{\theta}_{F,t})+a_G(\widehat{G}_{H,t}-\widehat{G}_{F,t})+a_\Psi(\widehat{\Psi}_{H,t}-\widehat{\Psi}_{F,t})\,, \tag{18}$$

with $0<\Xi<1$ and $a_\theta,a_G,a_\Psi>0$. Ξ is a root of the polynomial

$$\Gamma(z)\equiv-z^2+\tfrac{1+\kappa+\beta}{\beta}z-\tfrac{1}{\beta}=0.\ ^{4} \tag{19}$$

[4] $\Xi=a/2-((a/2)^2-1/\beta)^{0.5}$ with $a\equiv(1+\kappa+\beta)/\beta$; $a_\theta\equiv-(\kappa/\beta)/\zeta$, $a_G\equiv-(1/(1+\eta))(\kappa/\beta)/\zeta$, $a_\Psi\equiv-((\eta+2\xi-1)/(1+\eta))(\kappa/\beta)/\zeta$, where $\zeta\equiv\Xi+\rho-a<0$.

(18) shows that the terms of trade are a function of current and lagged relative (Home/Foreign) productivity, government purchases and preference shocks. A rise in relative Home productivity worsens the Home terms of trade, while a rise in Home government purchases and in the Home preference parameter Ψ_H improves the Home terms of trade. Price stickiness affects the dynamics of the terms of trade: when prices are more sticky (lower Phillips-curve slope κ), the terms of trade respond more sluggishly to exogenous shocks.

Given the terms of trade process (18), relative (Home/Foreign) output and consumption are uniquely pinned down by (6) and (16). Given a process for union-wide inflation and output, the country-level variables can then be determined using (15). Importantly, monetary policy affects union-wide output and inflation, but it does *not* affect the terms of trade and relative (Home vs. Foreign) variables. While union-wide inflation and output are indeterminate, relative variables are *determinate*.

2.7. Flex-prices model

A flex-prices (Real Business Cycle, RBC) model provides a useful benchmark for understanding the dynamics of real variables in the sticky-prices economy. (The flex-prices model is a special case of the model outlined above in which price adjustment costs are zero, ψ=0, and the slope of the Phillips curve is infinite, κ=∞.) The flex-prices equilibrium allocation is described by:

$$\widehat{Y_{i,t}}=\widehat{\theta_{i,t}}+\tfrac{\eta}{1+\eta}\widehat{G_{i,t}}-\tfrac{(1-\xi)\eta}{1+\eta}\cdot(\widehat{\Psi_{i,t}}-\widehat{\Psi_{j,t}}), \quad \text{for } i\in\{\mathrm{H,F}\};$$

$$\widehat{C_{i,t}}=\xi\widehat{\theta_{i,t}}+(1-\xi)\widehat{\theta_{j,t}}-\tfrac{\xi}{1+\eta}\widehat{G_{i,t}}-\tfrac{1-\xi}{1+\eta}\widehat{G_{j,t}}+\tfrac{(1-\xi)(2\xi+\eta)}{1+\eta}\cdot(\widehat{\Psi_{i,t}}-\widehat{\Psi_{j,t}}) \quad \text{for } i,j\in\{\mathrm{H,F}\},\ i\neq j;$$

$$\widehat{q_t}=\tfrac{2\xi-1+\eta}{1+\eta}\cdot(\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}})-(\widehat{\theta_{H,t}}-\widehat{\theta_{F,t}})+\tfrac{1}{1+\eta}\cdot(\widehat{G_{H,t}}-\widehat{G_{F,t}}).$$

Given log-utility preferences and a unit substitution elasticity between domestic and imported inputs in private consumption, output in country *i* only depends (positively) on domestic productivity and on domestic government purchases, in a flex-prices world; by contrast, consumption in a given country depends positively on both domestic and foreign productivity, and negatively on domestic and foreign government purchases. The Home terms of trade are a decreasing function of relative (Home vs. Foreign) productivity and an increasing function of relative government purchases, under flexible prices. A positive shock to country *i*'s relative preference shock raises *i*'s consumption, and it lowers *i*'s output (as the rise in consumption triggers a fall in labor supply). The terms of trade too are an increasing function of a country's relative preference shock, under flexible prices.

3. Expectations-driven liquidity traps

3.1. Constant productivity, government purchases and preferences

As pointed out above, the model has multiple bounded solutions. In order to see this in the simplest possible way, consider first an economy in which productivity, government purchases and the preference parameter Ψ are constant, so that the natural real interest rate is likewise constant: $\widehat{r_{t+1}}=0\ \forall t$. Given our assumption that the Taylor principle holds ($\gamma_\pi>1$), the union-wide Euler-Phillips equation (14) is then solved by two steady state (constant) inflation rates: $\widehat{\Pi}=0$ and $\widehat{\Pi}=-(\Pi-\beta)/\Pi$. The ZLB binds in the latter steady state. This finding is in line with Benhabib et al. (2001a,b) who shows (in a simpler model) that the combination of the ZLB and the Taylor rule produces two steady states, and that the ZLB binds in one of these steady states.

There also exist stochastic equilibria with an *occasionally* binding ZLB. Those equilibria seem especially relevant empirically. Arifovic et al. (2018) and Aruoba et al. (2018) shows that there exist equilibria in which an exogenous random sunspot variable determines whether the ZLB constraint is slack or whether it binds. Assume that union-wide inflation follows a Markov chain with two possible values: $\widehat{\Pi_t}\in\{\widehat{\Pi^S},\widehat{\Pi^B}\}$ such that $\widehat{\Pi^B}<\widehat{\Pi^S}$ and

$$\gamma_\pi\widehat{\Pi^B}\leq-(\Pi-\beta)/\Pi<\gamma_\pi\widehat{\Pi^S}. \tag{20}$$

Thus, the ZLB constraint binds when low inflation $\widehat{\Pi^B}$ is realized; the ZLB constraint is slack when high inflation $\widehat{\Pi^S}$ obtains. Denote the transition probabilities between union-wide inflation at dates t and at t+1 by $p_{ij}\equiv\text{Prob}(\widehat{\Pi_{t+1}}=\widehat{\Pi^j}|\widehat{\Pi_t}=\widehat{\Pi^i})$ for $i,j\in\{\text{S;B}\}$, with $0\leq p_{ij}\leq 1$ and $p_{iS}+p_{iB}=1$, and let $\Phi\equiv\begin{bmatrix}p_{SS} & p_{SB}\\ p_{BS} & p_{BB}\end{bmatrix}$ be the matrix of transition probabilities. Let $\vec{\widehat{\Pi}}\equiv[\widehat{\Pi^S};\widehat{\Pi^B}]$ (column vector). Expected future inflation is then $E(\widehat{\Pi_{t+1}}|\widehat{\Pi_t}=\widehat{\Pi^S})=\Phi(1,:)\vec{\widehat{\Pi}}$, $E(\widehat{\Pi_{t+1}}|\widehat{\Pi_t}=\widehat{\Pi^B})=\Phi(2,:)\vec{\widehat{\Pi}}$, $E(\widehat{\Pi_{t+2}}|\widehat{\Pi_t}=\widehat{\Pi^S})=\widetilde{\Phi}(1,:)\vec{\widehat{\Pi}}$, $E(\widehat{\Pi_{t+2}}|\widehat{\Pi_t}=\widehat{\Pi^B})=\widetilde{\Phi}(2,:)\vec{\widehat{\Pi}}$, where $\widetilde{\Phi}\equiv\Phi\times\Phi$.

An equilibrium with an occasionally binding ZLB is defined by inflation rates $\widehat{\Pi}^S, \widehat{\Pi}^B$ and transition probabilities p_{SS}, p_{BB} such that the inequalities in (20) and the Euler-Phillips equation (14) hold:

$$\gamma_\pi \widehat{\Pi}^S = -\tfrac{1}{\kappa}\widehat{\Pi}^S + (1+\tfrac{1+\beta}{\kappa})\Phi(1,:)\vec{\widehat{\Pi}} - \tfrac{\beta}{\kappa}\widetilde{\Phi}(1,:)\vec{\widehat{\Pi}}\,, \tag{21}$$

$$-(\Pi-\beta)/\Pi = -\tfrac{1}{\kappa}\widehat{\Pi}^B + (1+\tfrac{1+\beta}{\kappa})\Phi(2,:)\vec{\widehat{\Pi}} - \tfrac{\beta}{\kappa}\widetilde{\Phi}(2,:)\vec{\widehat{\Pi}}\,. \tag{22}$$

Equations (21) and (22) are, respectively, the Euler-Phillips equation if the ZLB constraint is slack and if it binds.

Model calibration

One period in the model represents one quarter in calendar time. I set β=0.9975, which implies a 1% per annum steady state riskless real interest rate. The Frisch labor supply elasticity is set at unity, η=1, a conventional value in macro models. The local content of private consumption spending is set at ξ=0.8. [5] The Central Bank's quarterly gross inflation target is set at Π=1.005, in line with the (roughly) 2% annual inflation target of the ECB. The inflation coefficient of the interest rate rule (5) is set at the conventional value γ_π=1.5. The slope coefficient κ_w of the Phillips curve (9) is set at a value such that the observationally equivalent Phillips curve implied by Calvo (1983) staggered price setting entails an average duration between price changes of 4 quarters. This mean duration is consistent with empirical evidence on price setting in the Euro Area (see Kollmann (2001), Alvarez et al. (2006), Giovannini et al. (2019)).[6] *The preceding parameters are used in all simulations below.*

The existence of an equilibrium with an occasionally binding ZLB constraint requires persistent inflation regimes, i.e. the probabilities p_{SS} and p_{BB} have to be close to unity. Assume, for example, $p_{SS}=p_{BB}$=0.95; thus, if the economy is in a liquidity trap at date *t*, then there is a 95% probability that the liquidity trap will still be in effect at *t*+1; the average duration of a liquidity trap is 20 quarters. Then (20),(21) and (22) are solved by $\widehat{\Pi}_B=-0.0080$ and

[5] This value of ξ matches the fact that, empirically, the ratio of within-Euro Area trade to EA GDP is about 20%.

[6] Under Calvo price setting, the slope of the Phillips curve (9) is $\kappa_w=(1-D)(1-\beta D)/D$, where 1-*D* is the probability that an individual firm can change its price in a given period, so that the average duration between price changes is 1/(1-*D*). I set *D*=0.75 (average stickiness of 4 periods), which implies κ_w=0.08395.

$\widehat{\Pi_S}$= -0.0011. This corresponds to annualized union-wide inflation rates in the states with binding and slack ZLB constraints of -1.22% and 1.55%, respectively. Union-wide GDP $\hat{Y}$ in these two states is -0.22% and 0.20%, respectively. Unconditional mean output (-0.01%) is thus slightly below steady state, in the model with an occasionally binding ZLB. The nominal interest rate is 2.33% per annum in the non-ZLB state.[7]

3.2. Time-varying productivity, government purchases and preference shocks

I next consider an economy with time-varying productivity, government purchases and preference shocks. Interest centers on how the transmission of exogenous shocks depends on whether the ZLB constraint is slack or binding. I continue to assume that a Markov chain with transition probability matrix Φ determines in which periods the ZLB binds. That Markov chain is assumed independent of shocks to productivity, government purchases and preferences.

I construct equilibria in which union-wide inflation follows state-dependent linear policy functions of the (union-wide) natural real interest rate, $\hat{r}_t$:

$$\widehat{\Pi_t^B} = \mu^B + \lambda^B \hat{r}_t \quad \text{if the ZLB constraint binds at } t; \tag{23}$$

$$\widehat{\Pi_t^S} = \mu^S + \lambda^S \hat{r}_t \quad \text{if the ZLB constraint is slack at } t, \tag{24}$$

$$\text{with} \quad \gamma_\pi \widehat{\Pi_t^B} \leq -(\Pi - \beta)/\Pi < \gamma_\pi \widehat{\Pi_t^S}. \tag{25}$$

The coefficients $\mu^B, \lambda^B, \mu^S, \lambda^S$ can be solved for using the method of undetermined coefficients (after substituting (23)-(24) into the Euler-Phillips equation (14)).

3.2.1. Permanent liquidity trap

To build intuition for equilibria with time-varying forcing variables and an occasionally binding ZLB constraint (discussed below), I first consider an economy that is in a permanent liquidity trap (so that the liquidity trap is an absorbing state, p_{BB}=1). For that case, a closed-form model solution with time-varying forcing variables can readily be derived.

[7]With constant productivity and preference parameters, the terms of trade in the initial period, say t=0, q_0 pins down the path of the terms of trade in all subsequent periods. In the long-run, the terms of trade asymptotes to the steady state terms of trade. Assume that, by period t, the terms of trade in have converged (sufficiently close) to steady state. Then, output and prices are equated across countries, and equal to union-wide averages of those variables.

In a permanent liquidity trap, we have $\widehat{\Pi}_t = \widehat{\Pi}_t^B \leq -\{(\Pi-\beta)/\Pi\}/\gamma_\pi \quad \forall t$ (see (25)), and the Euler-Phillips equation (14) becomes:

$$-(\Pi-\beta)/\Pi = -\tfrac{1}{\kappa}\widehat{\Pi}_t^B + (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi}_{t+1}^B - \tfrac{\beta}{\kappa}E_t\widehat{\Pi}_{t+2}^B + \hat{r}_t. \tag{26}$$

Substitution of the policy function (23) into (26) gives:

$$-(\Pi-\beta)/\Pi = -\tfrac{1}{\kappa}\{\mu^B+\lambda^B\hat{r}_t\} + (1+\tfrac{1+\beta}{\kappa})\{\mu^B+\lambda^B\rho\hat{r}_t\} - \tfrac{\beta}{\kappa}\{\mu^B+\lambda^B\rho^2\hat{r}_t\} + \hat{r}_t, \tag{27}$$

where I use the fact that $\mathrm{E}_t\widehat{\Pi}_{t+s}^B = \mu^B + \lambda^B\rho^s\hat{r}_t$ for $s\geq 0$.

(27) holds for arbitrary values of $\hat{r}_t$ iff $\mu^B = -(\Pi-\beta)/\Pi$ and $\{-\tfrac{1}{\kappa} + (1+\tfrac{1+\beta}{\kappa})\rho - \tfrac{\beta}{\kappa}\rho^2\}\lambda^B + 1 = 0$. Thus, the slope of the policy function in a permanent liquidity trap is: $\lambda^B = -1/\{-\tfrac{1}{\kappa} + (1+\tfrac{1+\beta}{\kappa})\rho - \tfrac{\beta}{\kappa}\rho^2\}$. This can be written as $\lambda^B = -(\kappa/\beta)/\Gamma(\rho)$, where $\Gamma(\rho)$ is the polynomial defined in (19). Note that $\Gamma(0) = -\tfrac{1}{\beta} < 0$ and $\Gamma(1) = \tfrac{\kappa}{\beta} > 0$; furthermore $\Gamma'(\rho) > 0$ for $0 \leq \rho \leq 1$. Therefore, $\Gamma(\rho) > 0$ holds for $0 < \Xi < \rho \leq 1$ where Ξ is the root of the polynomial $\Gamma(\Xi) = 0$ that corresponds to the autoregressive coefficient of the law of motion of the terms of trade (18). For the values of β, κ assumed in the model calibration (see above), we have $\Xi = 0.67$. Empirical estimates of the quarterly autocorrelation of productivity, government purchases (and other macroeconomic shocks) are typically in the range of 0.95. This implies that $\Gamma(\rho) > 0$ holds for a plausible autocorrelation ρ. Throughout the subsequent discussion, it will be assumed that this condition is met. (The simulations below set $\rho = 0.95$.) Thus, for plausible persistence of the forcing variables, $\lambda^B < 0$ holds, i.e. a rise in the natural interest rate lowers the union-wide inflation rate, in a permanent liquidity trap. As the natural real interest rate is a decreasing function of productivity, and an increasing function of government purchases and of the household demand (preference) shifter Ψ, we find that inflation in a permanent liquidity trap is *increasing* in the aggregate supply (productivity) shock and decreasing in the aggregate demand (government purchases and preference shift) shocks.

A persistent rise in the union-wide natural real interest rate triggers a rise in the expected real interest rate. In a permanent liquidity trap, the nominal interest rate is stuck at zero, and the rise in the real interest rate is brought about by a fall in the union-wide inflation rate. This can be seen most easily when ρ is very close to (but below) unity. Then $\widehat{\Pi}_t^B \approx E_t\widehat{\Pi}_{t+1}^B \approx E_t\widehat{\Pi}_{t+2}^B$, and (26) gives $\widehat{\Pi_t^B} \approx -(\Pi-\beta)/\Pi - \hat{r}_t$, so that a positive shock to the natural real rate triggers (approximately) a one-to-one negative response of the current and expected future union-wide inflation rate.

I assume henceforth that ρ=0.95. Then the policy function for union-wide inflation in a permanent liquidity trap is

$$\widehat{\Pi_t^B} = -0.0074 - 1.070\,\hat{r}_t = -0.0074 + 0.05\,\widehat{\theta}_t - 0.03\widehat{G}_t - 0.05\,\widehat{\Psi}_t\,. \tag{28}$$

Thus, a 1% percent increase in union-wide productivity raises union-wide (gross) inflation by 0.05% (this corresponds to a rise of the annualized inflation rate by 0.2 percentage points); while a 1% increase in union-wide government purchases lowers gross inflation by 0.03%. [8]

In a permanent liquidity trap, the policy rule for union-wide output is

$$\widehat{Y_t^B} = -0.0001 + 1.02\,\widehat{\theta}_t + 0.49\,\widehat{G}_t - 0.02\,\widehat{\Psi}_t\,, \tag{29}$$

i.e. the union-wide government purchases multiplier is 0.49. Although the rise in government purchases lowers union-wide inflation (see above), the government purchases multiplier is positive, because a rise in government purchases lowers union-wide consumption, which raises union-wide labor supply.

3.2.2. Permanently slack ZLB constraint

I next consider an equilibrium in which the economy stays forever away from the ZLB. Then $\widehat{\Pi}_t = \widehat{\Pi_t^S} > -\{(\Pi-\beta)/\Pi\}/\gamma_\pi \quad \forall t$ (see (25)), and the union-wide inflation rate is governed by the following Euler-Phillips equation (from (14)):

$$\gamma_\pi \widehat{\Pi_t^S} = -\tfrac{1}{\kappa}\widehat{\Pi_t^S} + (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi_{t+1}^S} - \tfrac{\beta}{\kappa}E_t\widehat{\Pi_{t+2}^S} + \hat{r}_t. \tag{30}$$

Substitution of policy rule (24) into (30) shows that the coefficients of the policy rule are

$$\mu^S = 0 \text{ and } \lambda^S = -(\kappa/\beta)/\{\Gamma(\rho) - \gamma_\pi \cdot (\kappa/\beta)\}. \tag{31}$$

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[8] The existence of an equilibrium with a permanent liquidity trap requires that the forcing variables are bounded, to ensure that $\gamma_\pi \widehat{\Pi_t^B} \leq -(\Pi-\beta)/\Pi$ holds $\forall t$. For example, if productivity and the preference shifter take steady state values, then $\hat{G}_t \geq -9\%$ has to hold: when government purchases fall below this lower bound, then the inflation rate defined by (28) rises to a level which is such that the Taylor rule prescribes a strictly positive nominal interest rate, which violates the first inequality shown in (25). The subsequent analysis assumes that all exogenous forcing variables remain sufficiently close to steady state values, so that the inequalities in (25) are satisfied.

$\gamma_\pi > 1$ (Taylor principle) implies that $\Gamma(\rho) - \gamma_\pi \cdot (\kappa/\beta) < 0 \;\; \forall \;\; 0 \le \rho \le 1$, and so $\lambda^S > 0$: when the ZLB is always slack, then a rise in the natural real interest rate triggers a rise in the inflation rate, and thus an increase in the nominal interest rate.

When the ZLB never binds, then a rise in productivity (which reduces the natural interest rate) lowers thus union-wide inflation, while positive aggregate demand shocks raise inflation. For ρ=0.95, the policy rule for union-wide inflation, under a permanently slack ZLB constraint is

$$\widehat{\Pi_t^S} = 1.77\,\hat{r}_t = -0.09\,\widehat{\theta_t} + 0.04\,\widehat{G_t} + 0.09\,\widehat{\Psi_t}, \tag{32}$$

while the policy rule for union-wide output is:

$$\widehat{Y_t^S} = 0.97\,\widehat{\theta_t} + 0.51\,\widehat{G_t} + 0.03\,\widehat{\Psi_t}. \tag{33}$$

Quantitatively, the union-wide output responses to shocks are similar across the regimes with permanently binding/slack ZLB constraints. With a permanently slack ZLB constraint, union-wide output is slightly less responsive to productivity shocks, but slightly more responsive to government purchase shocks than in the permanent liquidity trap (see (29),(33)). The shocks considered here are persistent. Thus, these shocks have a muted effect on the natural real interest rate, and hence the response of the inflation rate to these shocks is likewise muted. This helps to understand why, although the response of union-wide inflation to shocks differs *qualitatively* across the two regimes, the response of output is so similar across regimes.

3.2.3. Occasionally binding ZLB constraint

I next consider an equilibrium with random switches in/out of the liquidity trap. Assume as in Sect. 3.1. that the persistence of each regime is $p_{SS} = p_{BB} = 0.95$. In this case, the policy rules for union-wide inflation and output in the regime with a binding ZLB constraint ('B') are

$$\widehat{\Pi_t^B} = -0.0080 - 1.36\,\hat{r}_t = -0.0080 + 0.07\,\widehat{\theta_t} - 0.03\widehat{G_t} - 0.07\,\widehat{\Psi_t}. \tag{34}$$

$$\widehat{Y_t^B} = -0.0022 + 1.06\,\widehat{\theta_t} + 0.47\,\widehat{G_t} - 0.06\,\widehat{\Psi_t}. \tag{35}$$

The corresponding policy rules in the regime with a slack ZLB ('S') are

$$\widehat{\Pi_t^S} = -0.0011 + 1.28\,\hat{r}_t = -0.0011 - 0.06\,\widehat{\theta_t} + 0.03\,\widehat{G_t} + 0.06\,\widehat{\Psi_t}, \tag{36}$$

$$\widehat{Y_t^S} = 0.0020 + 0.94\,\widehat{\theta_t} + 0.53\,\widehat{G_t} + 0.06\,\widehat{\Psi_t}. \tag{37}$$

Given that the two regimes are persistent, it is not surprising that the policy functions (34)-(37) are similar to the ones that obtain when each regime is permanent ((28),(29),(32),(33)). In particular, it remains true that, in a liquidity trap, a positive aggregate supply shock raises union-wide inflation, while a positive aggregate demand shock lowers union-wide inflation. It is again found that the responses of union-wide output to productivity and government purchases shocks are similar across the two regimes. The government purchases multiplier is close to 0.5 in both regimes.

The effect of a regime shift on union-wide inflation and output depends on the level of the forcing variables. Note that $\widehat{Y_t^B} - \widehat{Y_t^S} = -0.0042 + 0.12\,\widehat{\theta}_t - 0.06\,\widehat{G}_t - 0.12\,\widehat{\Psi}_t$. Thus, entry into a liquidity trap has a slightly more detrimental effect on union-wide output when productivity is low and government purchases are high.

3.2.4. Country-level variables

As discussed above, country-level inflation, consumption and output can be computed by adding (or subtracting) relative (Home vs. Foreign) variables from the union-wide aggregates (see (15)). Relative quantities depend on (current and lagged) relative forcing variables; importantly, relative quantities do not depend on monetary policy or on whether the economy is in a liquidity trap.

3.3. Dynamic responses

Table 1 reports dynamic shock responses for the baseline New Keynesian model with an occasionally binding ZLB constraint, and regime persistence $p_{SS}=p_{BB}$=0.95. 1% shocks to Home productivity, Home government purchases and to the Home preference shifter are considered. Responses after 0, 5 and 10 periods are reported; see Column labelled 'Horizon'. Panel (a) of Table 1 shows responses that obtain when the ZLB constraint binds, while Panel (b) shows responses under a slack ZLB constraint. For comparison purposes, Table 2 shows dynamic responses for a New Keynesian model in which the ZLB constraint is always slack (p_{SS}=1). Table 3 shows dynamic response in an RBC (flex-prices) world.

In the **RBC model**, a positive shock to Home productivity raises Home output while Foreign output is unaffected. Home and Foreign consumption both rise. The rise in Home consumption exceeds the increase in Foreign consumption. The Home terms of trade deteriorate. In the RBC model, a positive shock to Home government purchases raises union-wide output, but it lowers union-wide consumption. Home output rises, while Foreign output is unaffected. Private consumption falls in both countries—the reduction in consumption is stronger in country 'Home'; the Home terms of trade improve. Finally, in the RBC model a positive Home preference shock has zero effect on union-wide output and consumption; however, the shock raises Home consumption, as the shock amounts to a fall in the Home subjective discount factor; as union-wide output does not change, Foreign consumption falls; the rise in Home consumption lowers Home labor supply, and thus Home output falls, while Foreign output rises; this is accompanied by an improvement of the Home terms of trade.

Note that the dynamic equations that govern union-wide variables are isomorphic to a closed economy model. Tables 1 and 2 show that, in the sticky-prices economies, the responses of union-wide output and consumption are close to the responses in the RBC economy. This is consistent with the well-known fact that, in a *closed* economy with sticky prices, a monetary policy rule that stabilizes the inflation rate can imply aggregate dynamics that mimic a flex-prices economy, and that in the face of a both aggregate demand and aggregate supply shocks (e.g., Kollmann (2008)). For example, a rise in Home government purchases worth 1% of steady state Home output raises union-wide output by 0.25%, on impact, in the RBC model, compared to a rise of 0.26% in the sticky-prices model without ZLB constraint (Table 2); in the sticky-prices model with an occasionally binding ZLB constraint, the corresponding increases of union-wide output are 0.24% when the ZLB constraint binds, and 0.26% when the ZLB constraint is slack (see Panels (a.2) and (b.2) of Table 1).

However, the adjustment of relative (Home vs. Foreign) variables to country-specific shocks is distorted significantly, in the short term, by price stickiness, and thus it differs from the response of in a RBC world. This is because price stickiness slows the adjustment of the terms of trade to country-specific shocks. Monetary policy in a currency union cannot undo this distortion. However, importantly, the adjustment of country-level does not depend much on whether the ZLB binds or not.

Consider the effects of a 1% Home productivity increase. In the sticky-prices model, the Home terms of trade deteriorate much less (-0.30%), on impact, than in the RBC world (-1%). This means that, under sticky prices, *relative* demand for Home output rises less (see (16)), while the risk sharing condition (6) implies a more muted rise in Home relative consumption. This explains why, under sticky prices, Home output and consumption rise less, while Foreign output and consumption rise more than in the flex-prices world. (In the flex-prices world, the 1% Home productivity increase raises Home output by 1%, on impact, while Foreign output is unchanged; in the sticky-prices model, Home and Foreign output rise by 0.68% and 0.38% on impact, respectively, when the ZLB binds; Home and Foreign output rise by 0.62% and 0.32%, respectively when the ZLB is slack.)

Price rigidity weakens the appreciation of the Home terms of trade triggered by a positive shock to Home government purchases. This implies that, under nominal rigidities, the shock triggers a stronger rise in Home output (than in the RBC world), and a *fall* in Foreign output. The international transmission of country-specific fiscal spending shocks is, thus, negative in the sticky-prices model. This is the case both at the ZLB and away from the ZLB (see Table 1, Panels (a.2) and (b.2)). Under a binding ZLB, a rise in Home government purchases worth 1% of steady state Home output raises Home GDP by 0.66 on impact, while Foreign GDP drops by 0.19%. When the ZLB does not bind, then Home and Foreign output respond by 0.69% and -0.16%, respectively, on impact. Because the union-wide fiscal spending multiplier is slightly smaller at the ZLB than away from the ZLB (due to the fact that the fiscal shock triggers a fall in union-wide inflation at the ZLB, but a rise in union-wide inflation away from the ZLB), the fall in Foreign output is slightly greater at the ZLB than away from the ZLB.

A similar mechanism also operates in response to a Home household demand (preference) shock. Price stickiness dampens the appreciation of the Home terms of trade, after a positive Home preference shock. This helps to understand why, under nominal rigidities, a positive Home preference shock *raises* Home output and lowers Foreign output, on impact. This is the case both when the ZLB constraint binds and when it is slack. (By contrast, in the RBC world, Home output falls, while Foreign output rises; see above.)

4. Fundamental liquidity traps driven by a large negative demand shock

This paper focuses on beliefs-driven, self-fulfilling liquidity traps. As discussed in the Introduction, an extensive literature assumes that liquidity traps are triggered by large adverse demand shocks, such as an autonomous fall in households' subjective discount rates. Mertens and Ravn (2014) refer to liquidity traps of this type as "fundamental" liquidity traps. A key assumption in fundamental liquidity trap models is that the liquidity trap ends when the adverse aggregate demand shock subsides. Once the economy has escaped from a liquidity trap, agents expect that the economy will never again enter a liquidity trap. See Holden (2016, 2019) for a comprehensive analysis of deterministic equilibria that feature a permanent exit from a liquidity trap. The literature shows that, in a fundamental liquidity trap, fiscal spending multipliers can be markedly higher than when the ZLB does not bind; also, a positive technology shock can trigger an output *contraction* (e.g., Eggertsson and Woodford (2003)).

Widely discussed studies by Erceg and Lindé (2010) and Blanchard et al. (2016) have presented quantitative models of a two-country monetary union that experiences a fundamental liquidity trap driven by a large adverse household demand (preference) shock; these authors show that, in such a liquidity trap, a country-specific rise in government purchases can have a strong positive effect on domestic and *foreign* output, i.e. the predicted international fiscal spillover is sizable. [9]

For comparison purposes with the expectations-driven liquidity traps analyzed in this paper, I now discuss a fundamental liquidity trap, in the stylized two-country model of a monetary union used above. Following Blanchard et al. (2016), I consider a scenario in which a liquidity trap is brought about by an unanticipated one-time shock at some initial date t=0 that depresses the union-wide natural real interest rate below its steady level, so that $\hat{r}_0<0$. Except for shocks at date t=0, there are no random disturbances. Thus the economy evolves deterministically (perfect foresight), after t=0. There exists a unique deterministic equilibrium in which the economy permanently leaves the liquidity trap after a finite time interval.[10]

As that there are no exogenous innovations after date t=0, the natural real interest rate at $t\geq 0$ is: $\hat{r}_t=\rho^t \cdot \hat{r}_0 < 0,$ where $0<\rho<1$ is the autocorrelation of the exogenous forcing processes and of the natural rate (see (14)).

In a deterministic equilibrium *without* ZLB constraint, the union-wide inflation rate and the nominal interest rate at dates $t\geq 0$ would be

[9] Open economy models of liquidity traps induced by large adverse demand shock are also studied by, e.g., Jeanne (2009), Cook and Devereux (2013), Fujiwara and Ueda (2013), Farhi and Werning (2016), Acharya and Bengui (2018) and Fornaro and Romei (2019).

[10] In the model here, the Phillips-Euler equation for union-wide inflation (see (14)) does not include lagged endogenous state variables. As shown by Holden (2016, 2019), this ensures that an equilibrium featuring eventual exit from the liquidity trap is unique; models with endogenous state variables may have multiple deterministic equilibria that eventually escape from the liquidity trap.

$$\widehat{\Pi_t^*}=\lambda^S \rho^t \hat{r}_0 \text{ and } \widehat{i_{t+1}^*} = \gamma_\pi \lambda^S \rho^t \hat{r}_0, \tag{38}$$

respectively, where $\lambda^S>0$ is the policy rule coefficient for a regime with a permanently slack ZLB constraint defined in (31). A fundamental liquidity trap occurs when the unconstrained nominal interest rate is negative at date t=0, i.e. when (expressing the interest rate in deviation from steady state):

$$\hat{i}_1^* < -(\Pi-\beta)/\Pi. \tag{39}$$

This inequality holds when the real natural rate at date t=0 is sufficiently low. Assume that (39) applies, and let T^* be the smallest value of t≥0 for which $\widehat{i_{t+1}^*} \geq -(\Pi-\beta)/\Pi.$ Thus,

$$\widehat{i_{T^*}^*} < -(\Pi-\beta)/\Pi \text{ and } \widehat{i_{T^*+1}^*} \geq -(\Pi-\beta)/\Pi. \tag{40}$$

The fundamental liquidity trap equilibrium studied by Blanchard et al. (2016) has the property that the ZLB constraint binds until period T^*-1, and that the ZLB does not bind in $t \geq T^*$. Thus, $\widehat{\Pi_t}=\widehat{\Pi_t^*}$ and $\widehat{i_{t+1}}=\widehat{i_{t+1}^*}$ hold for $t \geq T^*$ (where $\widehat{\Pi_t^*}$ and $\widehat{i_{t+1}^*}$ are defined in (38)). In periods $t < T^*$, the nominal interest rate is zero, i.e. $\widehat{i_{t+1}}= -(\Pi-\beta)/\Pi.$ From the Euler-Phillips equation (14), we see that union-wide inflation for $0 \leq t<T^*$ has to obey the condition $-(\Pi-\beta)/\Pi = -\frac{1}{\kappa}\widehat{\Pi}_t + (1+\frac{1+\beta}{\kappa})\widehat{\Pi_{t+1}} - \frac{\beta}{\kappa}\widehat{\Pi_{t+2}} + \hat{r}_t.$ Thus,

$$\widehat{\Pi}_t = \kappa(\Pi-\beta)/\Pi + (1+\beta+\kappa)\widehat{\Pi_{t+1}} - \beta\widehat{\Pi_{t+2}} + \kappa\hat{r}_t \quad \text{for } 0 \leq t < T^*. \tag{41}$$

Iterating backward in time using (41) allows to compute union-wide inflation in periods $0 \leq t < T^*$.[11] Importantly, the largest root of the "backward" inflation equation (41) exceeds unity. Thus, the backward inflation loop is explosive. This implies that the impact inflation response (at t=0) of the shock that triggers a fundamental liquidity trap can be large, as confirmed by the simulations below. Also, shocks to the natural real interest rate that induce small changes in the inflation rate in periods T^* and T^*+1, i.e. when the economy emerges from the liquidity trap, may have a big effect on inflation, and thus on output, at the start of the liquidity trap. This explains why fiscal multipliers in a fundamental liquidity trap can be (very) large.

[11] Inflation in T^*-1 (last period of the liquidity trap) is $\widehat{\Pi_{T^*-1}}=\kappa(\Pi-\beta)/\Pi+(1+\beta+\kappa)\widehat{\Pi_{T^*}^*}-\beta\widehat{\Pi_{T^*+1}^*}+\kappa\widehat{r_{T^*-1}}$ etc.

Table 4 reports dynamic responses to shocks, in a fundamental liquidity trap. All preference, technology and price stickiness parameters are set at the values used in previous Sections. The exogenous forcing processes again have autocorrelation ρ=0.95.

Following Blanchard et al. (2016), I consider a baseline fundamental liquidity trap scenario that starts in period t=0, and that that lasts 12 quarters. In the model here, that baseline scenario is brought about by an unanticipated one-time -9.90% innovation to Home and Foreign preference parameters (Ψ) at t=0 that depresses the union-wide natural real interest rate by 49.50 basis points, on impact.

Panel (a) of Table 4 reports the adjustment process, under the baseline scenario. Panels (b.1)-(b.3) report dynamics under alternative scenarios that obtain when positive 1% date t=0 innovations to Home productivity, Home government purchases and the Home preference parameter Ψ_H are *added* to the baseline scenario. Effects of those Home shocks are shown as deviations from the baseline scenario. [12]

The baseline liquidity trap scenario (Panel (a)) exhibits a sharp, but short-lived, contraction in union-wide inflation and output. Union-wide inflation drops by 28.56 ppt, 19.42 ppt, 13.36 ppt and 6.71 ppt (per annum) below steady state, in quarters 0 to 3 after the baseline shock, while union-wide output is 13.68%, 9.07%, 6.00% and 2.61% below steady state in the same periods.

During the liquidity trap, exogenous Home productivity, government purchases and preference shocks have a strong effect on output and inflation, in both countries, as can be seen in Panel (b) of Table 4. A 1% positive innovation (at t=0) to Home productivity lowers union-wide inflation and output by 13.14 ppt and 6.10%, respectively, on impact. Given the muted short-run response of the terms of trade, due to price stickiness, Home and Foreign output both drop sharply on impact, by 5.95% and 6.26%, respectively (see Panel (b.1)). An innovation to Home government purchases worth 1% of Home steady state output raises union-wide inflation and output by 4.64 ppt and 2.59%, on impact (Panel (b.2)). This very strong rise in union-wide output and the muted terms of trade response imply that Home *and* Foreign output both rise strongly, in the short run. On impact, Home and Foreign output go up by 3.01% and 2.16%, respectively. Thus, the cross-country spillover of government purchases is positive and sizable.

[12] Shock responses, *relative* to a given baseline scenario, do not depend on the combination of changes in $\Psi_{H,t}$, $\Psi_{F,t}$ that generates that baseline scenario.

4. Conclusion

The paper investigates expectations-driven liquidity traps in a two-country New Keynesian model of a monetary union. In the model here, country-specific productivity shocks induce synchronized responses of domestic and foreign output, while country-specific public and private demand shocks trigger asymmetric domestic and foreign responses. In particular, a rise in government purchases in an individual country *lowers* GDP in the rest of the union. The present paper cautions against the idea of strong cross-border fiscal transmission in a monetary union, during a liquidity trap.

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Table 1. New Keynesian model with occasionally binding ZLB constraint: dynamic responses to exogenous shocks

Horizon	*Y*	*C*	*Π*	*i*	Y_H	Y_F	C_H	C_F	$Π_H$	$Π_F$	*q*	*Exogenous variables*
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*	*(12)*
(a) Binding ZLB constraint												
(a.1) Home productivity increase												θ_H
0	0.53	0.53	0.14	0.00	0.68	0.38	0.62	0.44	-0.47	0.75	-0.30	1.00
5	0.43	0.43	0.11	0.00	0.78	0.09	0.64	0.23	0.06	0.16	-0.69	0.81
10	0.32	0.32	0.08	0.00	0.62	0.02	0.50	0.14	0.13	0.03	-0.60	0.60
(a.2) Home government purchases increase												G_H
0	0.24	-0.27	-0.07	0.00	0.66	-0.19	-0.31	-0.22	0.24	-0.37	0.15	1.00
5	0.19	-0.22	-0.06	0.00	0.43	-0.04	-0.32	-0.11	-0.03	-0.08	0.34	0.81
10	0.14	-0.16	-0.04	0.00	0.29	-0.01	-0.25	-0.07	-0.07	-0.02	0.30	0.60
(a.3) Home private demand increase (preference shock)												Ψ_H
0	-0.03	-0.03	-0.14	0.00	0.15	-0.21	0.40	-0.46	0.35	-0.62	0.24	1.00
5	-0.02	-0.02	-0.11	0.00	-0.06	0.01	0.22	-0.27	-0.07	-0.16	0.55	0.81
10	-0.02	-0.02	-0.08	0.00	-0.08	0.04	0.14	-0.17	-0.12	-0.04	0.48	0.60
(b) Slack ZLB constraint												
(b.1) Home productivity increase												θ_H
0	0.47	0.47	-0.13	-0.19	0.62	0.32	0.56	0.38	-0.74	0.48	-0.30	1.00
5	0.38	0.38	-0.11	-0.16	0.73	0.04	0.59	0.18	-0.16	-0.05	-0.69	0.81
10	0.28	0.28	-0.08	-0.12	0.58	-0.02	0.46	0.10	-0.03	-0.13	-0.60	0.60
(b.2) Home government purchases increase												G_H
0	0.26	-0.24	0.06	0.10	0.69	-0.16	-0.28	-0.19	0.37	-0.24	0.15	1.00
5	0.22	-0.19	0.05	0.08	0.45	-0.02	-0.30	-0.09	0.08	0.03	0.34	0.81
10	0.16	-0.14	0.04	0.06	0.31	0.01	-0.23	-0.05	0.01	0.07	0.30	0.60
(b.3) Home private demand increase (preference shock)												Ψ_H
0	0.03	0.03	0.13	0.19	0.21	-0.15	0.46	-0.40	0.62	-0.36	0.24	1.00
5	0.02	0.02	0.11	0.16	-0.01	0.05	0.27	-0.22	0.15	0.06	0.55	0.81
10	0.02	0.02	0.08	0.12	-0.04	0.08	0.17	-0.14	0.04	0.12	0.48	0.60

Notes: Dynamic responses to 1% innovations to exogenous variables are reported. Responses 0, 5 and 10 periods after the shock are shown (see Column labelled 'Horizon'). Panel (a) shows responses that obtain when the ZLB constraint binds (liquidity trap); Panel (b) assumes that the ZLB constraint is slack. Variables: union-wide output (*Y*), union-wide consumption (*C*), union-wide inflation (*Π*), nominal interest rate (*i*); Home (H) and Foreign (F) output (Y_H,Y_F), consumption (C_H,C_F), inflation ($Π_H$, $Π_F$) and Home terms of trade (*q*). Shocks to Home productivity (θ_H), Home government purchases (G_H) and to the Home intertemporal preference shifter (Ψ_H) are considered. Output, consumption and the terms of trade are reported as % deviations from steady state; the interest rate and inflation are reported as percentage point (ppt) per annum differences from steady state.

Table 2. New Keynesian model in which the ZLB never binds: dynamic responses to exogenous shocks

Horizon	Y	C	Π	i	Y_H	Y_F	C_H	C_F	Π_H	Π_F	q
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
(1) Home productivity increase											
0	0.49	0.49	-0.18	-0.27	0.64	0.33	0.58	0.40	-0.79	0.43	-0.30
5	0.40	0.40	-0.15	-0.22	0.74	0.05	0.60	0.19	-0.20	-0.09	-0.69
10	0.29	0.29	-0.11	-0.16	0.59	-0.01	0.47	0.11	-0.05	-0.16	-0.60
(2) Home government purchases increase											
0	0.26	-0.24	0.09	0.13	0.68	-0.17	-0.29	-0.20	0.39	-0.22	0.15
5	0.21	-0.20	0.07	0.11	0.45	-0.03	-0.30	-0.10	0.09	0.04	0.34
10	0.15	-0.15	0.05	0.08	0.30	0.00	-0.24	-0.06	0.03	0.08	0.30
(3) Home private demand increase (preference shock)											
0	0.01	0.01	0.18	0.27	0.19	-0.17	0.44	-0.41	0.67	-0.31	0.24
5	0.01	0.01	0.15	0.22	-0.02	0.04	0.25	-0.23	0.19	0.10	0.55
10	0.01	0.01	0.11	0.16	-0.05	0.07	0.16	-0.15	0.07	0.15	0.48

Notes: Dynamic responses to 1% innovations to exogenous variables are reported. See Table 1 for definition of variables and other information.

Table 3. RBC model: dynamic responses to exogenous shocks

Horizon	Y	C	Π	i	Y_H	Y_F	C_H	C_F	Π_H	Π_F	q
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
(1) Home productivity increase											
0	0.50	0.50	---	---	1.00	0.00	0.80	0.20	---	---	-1.00
5	0.41	0.41	--	---	0.81	0.00	0.65	0.16	---	---	-0.81
10	0.30	0.30	---	---	0.60	0.00	0.48	0.12	---	---	-0.60
(2) Home government purchases increase											
0	0.25	-0.25	---	---	0.50	0.00	-0.40	-0.10	---	---	0.50
5	0.20	-0.20	---	---	0.41	0.00	-0.33	-0.08	---	---	0.41
10	0.15	-0.15	---	---	0.30	0.00	-0.24	-0.06	---	---	0.30
(3) Home private demand increase (preference shock)											
0	0.00	0.00	---	---	-0.10	0.10	0.26	-0.26	---	---	0.80
5	0.00	0.00	---	---	-0.08	0.08	0.21	-0.21	---	---	0.65
10	0.00	0.00	---	---	-0.06	0.06	0.16	-0.16	---	---	0.48

Notes: Dynamic responses to 1% innovations to exogenous variables are reported. See Table 1 for definition of variables and other information.

Table 4. New Keynesian model with a fundamental liquidity trap (12 periods): baseline liquidity trap scenario and dynamic responses to exogenous shocks

Horizon	*Y*	*C*	*Π*	*i*	Y_H	Y_F	C_H	C_F	$Π_H$	$Π_F$	*q*	*Exogenous variables*
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*	*(12)*
(a) Baseline scenario (triggered by adverse shocks to Home and Foreign private demand)												Ψ_H & Ψ_F
0	-13.68	-13.68	-28.56	0.00	-13.68	-13.68	-13.68	-13.68	-28.56	-28.56	0.00	-9.90
5	-2.61	-2.61	-6.71	0.00	-2.61	-2.61	-2.61	-2.61	-6.71	-6.71	0.00	-7.66
10	-0.20	-0.20	-2.12	0.00	-0.20	-0.20	-0.20	-0.20	-2.12	-2.12	0.00	-5.93
11	-0.16	-0.16	-1.99	0.00	-0.16	-0.16	-0.16	-0.16	-1.99	-1.99	0.00	-5.63
12	-0.15	-0.15	-1.89	0.15	-0.15	-0.15	-0.15	-0.15	-1.89	-1.89	0.00	-5.35
(b) Dynamic responses to shocks (shown as difference relative to baseline scenario)												
(b.1) Home productivity increase												θ_H
0	-6.10	-6.10	-13.14	0.00	-5.95	-6.26	-6.01	-6.19	-13.74	-12.53	-0.30	1.00
5	-0.88	-0.88	-2.53	0.00	-0.54	-1.23	-0.68	-1.09	-2.58	-2.47	-0.69	0.81
10	0.21	0.21	-0.19	0.00	0.51	-0.09	0.39	0.03	-0.14	-0.24	-0.60	0.60
11	0.24	0.24	-0.13	0.00	0.52	-0.05	0.41	0.07	-0.07	-0.18	-0.57	0.57
12	0.26	0.26	-0.10	-0.14	0.54	-0.01	0.43	0.10	-0.04	-0.15	-0.54	0.54
(b.2) Home government purchases increase												G_H
0	2.59	2.09	4.64	0.00	3.01	2.16	2.04	2.13	4.94	4.33	0.15	1.00
5	0.66	0.25	0.88	0.00	0.90	0.43	0.15	0.36	0.91	0.86	0.34	0.81
10	0.17	-0.13	0.07	0.00	0.32	0.02	-0.21	-0.04	0.04	0.09	0.30	0.60
11	0.15	-0.14	0.05	0.07	0.29	0.01	-0.22	-0.05	0.02	0.08	0.29	0.57
12	0.14	-0.13	0.05	0.07	0.27	0.01	-0.21	-0.05	0.02	0.07	0.27	0.54
(b.3) Home private demand increase (preference shock)												Ψ_H
0	4.65	4.65	9.23	0.00	4.83	4.47	5.08	4.23	9.71	8.74	0.24	1.00
5	0.91	0.91	1.76	0.00	0.88	0.94	1.15	0.67	1.80	1.71	0.55	0.81
10	0.05	0.05	0.13	0.00	-0.01	0.11	0.21	-0.11	0.09	0.18	0.48	0.60
11	0.01	0.01	0.10	0.15	-0.05	0.07	0.16	-0.14	0.06	0.14	0.46	0.57
12	0.01	0.01	0.10	0.14	-0.05	0.06	0.15	-0.13	0.05	0.14	0.44	0.54

Notes: Panel (a) reports the baseline scenario of a "fundamental" liquidity trap equilibrium lasting 12 quarters. Panel (b) reports dynamic responses of 1% innovations to exogenous variables (the responses are shown as differences compared to the baseline scenario). Output, consumption and the terms of trade are reported as % deviations from steady state; the interest rate and inflation are reported as percentage point (ppt) per annum differences from steady state. See Table 1 for definition of variables.