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Sudden Stops in Emerging Economies: The Role of World Interest Rates and Foreign Exchange Intervention*

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Abstract

Emerging economies are prone to 'sudden stops', characterized by a collapse in external borrowing and aggregate demand. Sudden stops may be triggered by a spike in world interest rates, which causes rapid private sector deleveraging. In response to a rise in interest rates, deleveraging is individually rational, but in the aggregate, the effect on the real exchange rate may tighten borrowing constraints so much that it precipitates a large crisis. A central bank can intervene by selling foreign reserves when world interest rates are rising, and prevent excess aggregate deleveraging. But the central bank cannot borrow reserves. Then, to intervene during a crisis, the central bank must acquire reserves in advance, which is costly. The optimal reserve management policy trades off the insurance benefits of reserves during a crisis against the welfare costs of accumulating reserves before a crisis.

Keywords: Central Bank; Sudden Stops; Foreign Exchange Intervention

JEL: E50; E30; F40; F30

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1 Introduction

From the Latin American crises of the early 1980's, to the Mexican crisis of 1994, to the Taper Tantrum of 2013, recent history is replete with episodes where increasing interest rates in the U.S. and other developed economies lead to financial crises and sudden stops in emerging markets.[1]

The empirical literature has identified world interest rates as key factor for explaining business cycles in emerging market economies. Neumeyer and Perri (2005) and Uribe and Yue (2006) argue that changes in world interest rates and external borrowing costs are one of the main factors driving emerging market business cycles. Frankel and Rose (1996) show that higher U.S. interest rates increase the probability of emerging market currency crises. Miranda-Agrippino and Rey (2020) argue that U.S. monetary policy is a major driver of the global financial cycle and cycles of capital flow surges and stops in emerging market economies.[2]

We present a model of sudden stops in a small open economy driven by exogenous changes in the world risk-free interest rate. We then show that foreign exchange intervention can be used as a policy instrument to prevent interest rate-driven sudden stops. At first pass it may seem odd that an increase in the world risk-free rate should lead to a borrowing crisis in an emerging market economy, since after all, the increase in the world risk-free rate should reduce foreign borrowing and net external debt in an emerging economy. All else equal this should loosen borrowing constraints. But reduced borrowing also leads to a fall in the exchange rate, which can reduce the value of collateral. If the fall in the value of collateral brought on by this reduction in external debt is greater than the initial reduction in external debt, borrowing constraints can tighten even when external debt is falling.

To capture the idea of a crisis caused by an increase in the world risk-free rate, this paper

[1] See e.g. Diaz-Alejandro (1984), Calvo et al. (1993), Calvo et al. (1996), Dooley et al. (1996), Aizenman et al. (2016), Ahmed et al. (2017)

[2] See e.g. Forbes and Warnock (2012), Fratzscher (2012), Calderon and Kubota (2013), Rey (2015), Ghosh et al. (2014), Bruno and Shin (2015b), Bruno and Shin (2015a), Rey (2016), Eichengreen and Gupta (2016), Fratzscher et al. (2018), Rogers et al. (2018), Jorda et al. (2019).

follows the model of an underborrowing equilibrium from Schmitt-Grohe and Uribe (2020). Schmitt-Grohe and Uribe show how under certain parameterizations, and when external debt passes a high enough threshold, there are multiple equilibria in the external position of a small open economy, driven by self-fulfilling expectations. In one equilibrium, consumption and external debt remains high, sustaining a high value of the real exchange rate, ensuring that the value of collateral is high enough so that the borrowing constraint is slack. But in another equilibrium, a reduction in borrowing leads to a real depreciation, a falling value of collateral, and a binding borrowing constraint.

But aside from the multiple self-fulfilling equilibria there is another feature of the model in Schmitt-Grohe and Uribe (2020). Even if agents beliefs coalesce around the 'best' equilibrium, when external debt reaches a high enough point, the joint effect of pecuniary externalities and non-linearity in the tightness of borrowing constraints can lead to a discontinuous collapse in borrowing capacity, even for very small increases in world interest rates. Our paper highlights this instability, and shows how foreign exchange intervention can be used to avoid such collapses. Our analysis does not rely on the role of self-fulfilling expectations in generating sudden stops. We assume that the best stable equilibrium is always selected. But this equilibrium selection is still associated with large sudden stops, driven by a 'debt-deflation' process as described by Mendoza (2002) and and a pecuniary externality in that agents do not internalize the fact that deleveraging in response to external shocks leads to a fall in the price of collateral and a tightening of borrowing constraints.

This pecuniary externality opens up a role for economic policy. The policy maker internalizes the effect of deleveraging on the value of collateral. At the point where further deleveraging would cause the borrowing constraint to bind, the policy maker will act to prevent further deleveraging and thus keep the value of collateral high. As in Devereux et al. (2019) and others, the policy maker could achieve this through the use of capital controls. To prevent a sudden stop a policy maker could subsidize borrowing (or equivalently tax saving) and thus increase domestic consumption and the price of collateral. However, Eichengreen and Rose (2014) and Fernandez et al. (2015) show that empirically, capital controls are markedly acyclical, and not deployed in response to booms and busts in capital flows. On the other hand, as we describe below, there is ample evidence of countries using foreign exchange intervention as a cyclical policy instrument. Moreover, Davis et al. (2021) show that under certain conditions an optimal tax on capital flows can be exactly replicated by foreign exchange intervention.

We then derive the optimal foreign exchange intervention policy. This requires the central bank to sell foreign bonds in response to an increase in the world interest rate. If the borrowing constraint was slack, and the central bank and private sector had equal access to international capital markets, this intervention policy would have no real effects, since, by Ricardian equivalence, the private sector would fully offset the central bank intervention with a one-for-one purchase of foreign bonds. But a central element in our model is the presence of intermediary frictions in private capital markets, which we adopt from Gabaix and Maggiori (2015). These frictions drive a wedge between domestic and world interest rates, and prevent the private sector from fully offsetting a central bank foreign exchange intervention.

In the presence of these intermediary frictions, a central bank sale of foreign bonds leads to an increase in net external debt, thereby shifting consumption from the future to the present. If foreign exchange intervention were unconstrained, then the central bank would always be able sell enough reserves, ex-post, to prevent a crisis. But there is a natural limit to foreign exchange intervention; central bank foreign exchange reserves cannot fall below zero. If the initial stock of reserves is too low then the central bank may not have sufficient reserves to sustain domestic absorption and the value of collateral. This gives the central bank an incentive to accumulate reserves in advance. By buying reserves in the current period the central bank is shifting consumption from the current period to the future, under the expectation that the extra consumption in the future might be useful for supporting the value of collateral and thus preventing a crisis. The marginal benefit of acquiring an extra unit of reserves and thus shifting a marginal unit of consumption from the present to the future is simply the welfare loss of a sudden stop multiplied by the reduction in the probability of a sudden stop gained by holding that extra unit of reserves.

But the same intermediary friction which gives foreign exchange intervention traction during a crisis makes the accumulation of foreign exchange reserves distortionary before a crisis. Just as the sale of foreign exchange reserves would lead the economy to consume and borrow more, the central bank purchase of foreign exchange reserves would force the economy to consume less and save more. This distortion of agents' optimal consumption plans represents the marginal cost to acquiring reserves. This, together with the marginal benefit of reserves will pinpoint the optimal stock of central bank foreign exchange reserves.[3]

The paper is organized as follows. Section 2 provides a brief review of the empirical and theoretical literature on the role of foreign exchange reserves in preventing currency and financial crises. The model is presented in Section 3. Section 4 discusses the mechanics of a sudden stop following a shock to the world interest rate and the optimal policy response. Numerical results from a global solution of the model and a numerical solution for optimal policy is presented in Section 5. Finally Section 6 concludes.

2 Recent empirical and theoretical literature on foreign exchange intervention

In the empirical literature on foreign exchange intervention, two major themes stand out: one concerns the evidence on the effectiveness of using reserves, either to prevent a crisis or to defend the currency, the second relates to the precautionary accumulation of reserves ex-ante as insurance against a crisis.

[3] Rodrik (2006) uses the spread between domestic interest rates in emerging market economies and the yield on U.S. treasuries to quantify the cost of holding reserves. He concludes that the cost of holding the observed stock of central bank reserves amounted to about 1% of GDP. But he concludes that compared to the cost of a sudden stop, this "insurance premium" is not excessively high.

With respect to the first theme, Fratzscher et al. (2019), use daily data on sterilized foreign exchange intervention. They argue that foreign exchange intervention (FXI) is an effective tool for exchange rate stabilization. Forbes and Klein (2015) conclude that FXI is an effective policy tool to prevent currency depreciation in the face of shocks to the foreign interest rate. Ghosh et al. (2016) estimate a policy reaction function for central bank foreign exchange accumulation and find that emerging market central banks engage in FXI to smooth fluctuations in the real exchange rate. Obstfeld et al. (2009) show that countries with a larger stock of reserves in 2007 had less exchange rate depreciation during the crisis of 2008.

A related literature studies the role of FXI in preventing crises. Frankel and Rose (1996) and Gourinchas and Obstfeld (2012) find that the stock of reserves is negatively associated with increased probability of a crisis. Ahmed et al. (2017) show that emerging market fundamentals, including a higher stock of central bank reserves to GDP and a lower ratio of short-term external debt to reserves, outperformed their emerging market peers on a number of financial indicators during the "taper tantrum" episode of 2013.[4]

Obstfeld et al. (2010) regress reserve stocks on financial openness, and find that countries hold more reserves when they become more financially open and thus more vulnerable to external crises. Aizenman and Hutchison (2012) and Aizenman and Sun (2012) discuss the "fear of losing international reserves" whereby during the crisis of 2008-2009 many emerging market countries chose to allow their exchange rate to depreciate rather than losing reserves.

In Section A of the Appendix, we extend the approach of Frankel and Rose (1996), investigating the influence of world interest rates and foreign exchange reserves on the probability of currency crises for a large group of emerging market countries. We find that increases in the world interest rate significantly increase the probability of currency crises, while lagged reserves to GDP reduce the same probability. More importantly, interacting the two vari-

[4]These findings are related to a number of other papers documenting the early warning signs of an emerging market crisis, including Bussiere and Fratzscher (2006), Rose and Spiegel (2011), Frankel and Saravelos (2012).

ables, we find that the effect of an increase in world interest rates on the probability of a crisis is significantly reduced for countries with a higher stock of foreign exchange reserves.

In the recent theoretical literature on foreign exchange intervention, Jeanne and Ranciere (2011) model reserves as an insurance contract to prevent sudden stops. Durdu et al. (2009) also model reserve accumulation as insurance against a sudden stop resulting from domestic shocks. Chang et al. (2015) and Cavallino (2019) look at optimal foreign exchange intervention in a linear-quadratic New Keynesian model. In these models foreign exchange intervention is an additional tool that helps to stabilize the economy in the presence of portfolio shocks. Fanelli and Straub (2021) model optimal foreign exchange intervention in a setting where the central bank tries to manipulate the price of non-traded goods for distributional considerations. Hur and Kondo (2016) and Bianchi et al. (2018) both consider the use of reserves to mitigate rollover risk in sovereign debt.[5]

In a stylized three-period model, Cespedes et al. (2017) and Bocola and Lorenzoni (2020) develop models with multiple equilibria ex-post. The central bank can eliminate bad equilibria by implementing a lender of last resort policy if it has accumulated a sufficient stock of reserves. In Jeanne and Sandri (2020) both private and central bank foreign assets can serve as insurance against a sudden stop in foreign liabilities, but private agents do not internalize the insurance role of their stock of foreign assets, and thus the central bank, which does internalize the insurance role of their stock of foreign assets, will acquire reserves and as a result the economy will have a higher level of liquid external assets than in the laissez-faire equilibrium. In Cespedes and Chang (2020), the central bank acquires reserves ex-ante to lend to banks following a shock to banks' collateral constraint.

In Arce et al. (2019) the fact that private agents don't internalize the effect of their own external borrowing on the likelihood of a sudden stop crisis represents a pecuniary externality that leads the private sector to borrow more than is efficient. By acquiring foreign exchange

[5]In a different vein, Amador et al. (2020) study foreign exchange intervention as a policy intervention when the nominal interest rate is at the zero lower bound. They use this to explain how central bank foreign exchange intervention at the zero bound was responsible for the deviations from Covered Interest Parity observed during the global financial crisis.

reserves the central bank can force the economy as a whole to save more and thus lead to the efficient level of borrowing. While Arce et al. (2019) model foreign exchange intervention in a different way than in our paper, there are some similarities. We also find that the accumulation of foreign exchange reserves leads to reduced borrowing for the economy as a whole, and we discuss how this is one benefit of the accumulation of reserves. But we focus on the role of reserves as insurance against the risk of a sudden stop caused by a spike in external interest rates, and the way in which the central bank optimally uses foreign exchange intervention to respond to this shock.

3 Model

We construct an infinite horizon model of a small open economy. The economy features a representative household, a financial sector, a central bank. Households derive utility from the consumption of a tradable good y^T and a non-tradable good y^N. Households begin each period with an initial stock of debt. They face a borrowing constraint limiting debt to a fraction of the market value of their endowment in a given period. The only source of exogenous variation in the model is a shock to the country's external borrowing cost.

3.1 Households

Households maximize utility, described as follows:

$$U = E_t \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{1}$$

where $u\left(c\right) = \frac{c^{1-\sigma}}{1-\sigma}$. c_t is defined as

$$c_t = \left[\alpha \left(c_t^T\right)^{\frac{\xi-1}{\xi}} + \left(1-\alpha\right)\left(c_t^N\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \tag{2}$$

where c_t^T (c_t^N) denotes the traded good (non-traded good) consumption. The budget constraint for households is written as follows:

$$c_t^T + p_t c_t^N + B_t = y^T + p_t y^N + R_{t-1} B_{t-1} + T_{t-1} + \Pi_{t-1} \tag{3}$$

where B_t represents the household's holdings of domestic bonds (which are held by households, the financial sector, and the central bank) and R_t is the interest rate on domestic bonds. The central bank earns a net return T_{t-1} on their bond portfolio which is rebated lump-sum to savers (more on this later). The financial sector also earns net interest income on their bond portfolio which is rebated to households in lump sum Π_{t-1}.

Combining the first order conditions for traded and non-traded goods gives the price of non-traded goods p_t:

$$p_t = \frac{1-\alpha}{\alpha} \left(\frac{c_t^T}{c_t^N} \right)^{\frac{1}{\xi}} \tag{4}$$

Due to limited enforcement of debt contracts, home country borrowers face a borrowing constraint given by:

$$-B_t \leq \kappa \left(y^T + p_t y^N \right) \tag{5}$$

The multiplier on the borrowing constraint is μ_t. The first order condition with respect to B_t is:

$$\lambda_t - \mu_t = E_t \beta \lambda_{t+1} R_t \tag{6}$$

Where λ_t is the marginal utility of traded goods consumption:

$$\lambda_t = c_t^{\frac{1-\sigma\xi}{\xi}} \alpha \left(c_t^T \right)^{\frac{-1}{\xi}} \tag{7}$$

3.2 Financial Intermediaries

A key feature of the model is the presence of frictions in international financial markets. Private households do not directly hold foreign bonds, but must trade with financial intermediaries who can borrow and lend on international financial markets subject to enforcement costs. The financial sector is made up of a continuum of identical atomistic financiers, indexed $i \in [0, 1]$. Financiers issue bonds on the international market, and use the proceeds to buy home bonds from domestic households. Financiers begin each period with zero net worth. They then issue $F_t^{fs}(i)$ international bonds and purchase $B_t^{fs}(i)$ domestic bonds. By intermediating the borrowing from domestic households, financiers thus take a positive position in B_t^{fs} and a negative position in F_t^{fs}, where $B_t^{fs}(i) + F_t^{fs}(i) = 0$. After aggregating across all atomistic financiers the balance sheet for the financial sector is given by:

$$B_t^{fs} + F_t^{fs} = 0 \tag{8}$$

where $F_t^{fs} = \int_0^1 F_t^{fs}(i)\, di$ and $B_t^{fs} = \int_0^1 B_t^{fs}(i)\, di$.

Note that both international bonds and domestic bonds are denominated in units of the traded good. Thus, unanticipated movements in the real exchange rate have no impact on financiers' balance sheets through currency mismatches. But, as we establish below, since financiers act as an intermediary between households and international financial markets, this drives a wedge between the domestic interest rate, R_t, and the exogenous world interest rate R_t^W. Both returns are denominated in traded goods.

The net interest income from the financier's domestic and international bond portfolio is:

$$\Pi_t(i) = R_t^W F_t^{fs}(i) + R_t B_t^{fs}(i) \tag{9}$$

The total net interest income from financiers $\Pi_t = \int_0^1 \Pi_t(i)\, di = R_t^W F_t^{fs} + R_t B_t^{fs}$, is remunerated lump-sum to households.

Each atomistic financier is in operation for a single period, and their objective is to maximize the discounted net interest income from bonds purchased in period t:

$$\beta \Pi_t (i) = \beta \left(R_t^W F_t^{fs} (i) + R_t B_t^{fs} (i) \right) = \beta \left(R_t^W - R_t \right) F_t^{fs} (i) \tag{10}$$

As in Gabaix and Maggiori (2015) we assume that financiers have an incentive to divert the funds they receive from issuing foreign bonds. After taking the position $F_t^{fs} (i) < 0$, the financier can divert a share $\Gamma \left| F_t^{fs} (i) \right|$ of their credit position $\left| F_t^{fs} (i) \right|$, where Γ is a non-negative constant. If the financier diverts the funds their firm is unwound and the proceeds are returned to the creditor. Since creditors correctly anticipate the ability and motivation of the financier to divert funds, financiers are subject to the following incentive compatibility constraint:

$$\beta \Pi_t (i) \geq \Gamma \left| F_t^{fs} (i) \right| \times \left| F_t^{fs} (i) \right| = \Gamma \left(F_t^{fs} (i) \right)^2 \tag{11}$$

The financiers maximization problem is to choose $F_t^{fs} (i)$ to maximize $\Pi_t (i)$ subject to this incentive compatibility constraint. Since the value of the financier's firm, $\Pi_t (i)$, is linear in $F_t^{fs} (i)$ and the right hand side of this constraint is convex in $F_t^{fs} (i)$, the constraint always binds. Thus:

$$R_t^W - R_t = \frac{\Gamma}{\beta} F_t^{fs} (i) \tag{12}$$

If $\Gamma = 0$ then the financial sector is simply a veil and the equilibrium condition for foreign bond holding is $R_t = R_t^W$, exactly as it would be if households could borrow directly from foreigners and faced no frictions. But when $\Gamma > 0$, and $F_t^{fs} < 0$, the domestic interest rate will be higher than the foreign interest rate.[6]

[6] Note that this equilibrium condition in the market for foreign bonds can also be derived in a reduced form by adding a quadratic adjustment cost to holding foreign bonds in the household budget constraint, as in Schmitt-Grohe and Uribe (2003). This reduced form approach was the preferred way of adding intermediary frictions in models with central bank foreign exchange intervention in Chang et al. (2015) and Davis et al. (2021).

3.3 Central Bank and Market Clearing

The central bank also holds a stock of domestic and international bonds. It can vary the composition of that bond portfolio.

$$B_t^{cb} + F_t^{cb} = 0 \tag{13}$$

By the fact that they participate in both the domestic and international bond markets, the central bank is similar to the financial sector. But we assume realistically that the central bank does not face the intermediation friction Γ. However, we impose the constraint that central banks foreign bond holdings cannot be negative, $F_t^{cb} \geq 0$. As witnessed in many episodes of sudden stops in emerging economies, central banks have very limited recourse to international debt markets when reserves are depleted.

The central bank earns a net return T_{t-1} on its portfolio which is rebated lump-sum to savers.

$$T_{t-1} = R_{t-1}^W F_{t-1}^{cb} + R_{t-1} B_{t-1}^{cb} \tag{14}$$

Domestic bonds B are held by households, the financial sector, and the central bank. The domestic bond market clearing condition is given by:

$$B_t + B_t^{fs} + B_t^{cb} = 0 \tag{15}$$

The world interest rate, R_t^W, is taken as given in this small open economy model.

Finally, non-traded goods market clearing implies:

$$c_t^N = y^N \tag{16}$$

3.4 Balance of payments identity

Substituting the financial sector and central bank net interest income in equations (9) and (14), the financial sector and central bank balance sheets in equations (8) and (13), and the domestic bond market clearing and non-traded goods market clearing conditions in equations (15) and (16), into the household's budget constraint in (3), we obtain the economy-wide budget constraint:

$$c_t^T = y^T - F_t^{fs} + R_{t-1}^W F_{t-1}^{fs} - F_t^{cb} + R_{t-1}^W F_{t-1}^{cb} \tag{17}$$

This condition can be rearranged into the familiar balance of payments identity where the current account equals the capital account plus the change in central bank foreign exchange reserves:

$$CA_t = \Delta F_t^{fs} + \Delta F_t^{cb}. \tag{18}$$

The current account, CA_t, equals net exports: $y^T - c_t^T$ plus interest income from international bonds purchased in $t-1$: $F_{t-1}^{cb}\left(R_{t-1}^W - 1\right) + F_{t-1}^{fs}\left(R_{t-1}^W - 1\right)$. The capital and financial account , ΔF_t^{fs}, is equal to net international bond purchases by financiers, $F_t^{fs} - F_{t-1}^{fs}$. The change in reserves, ΔF_t^{cb}, is equal to net international bond purchases by the central bank, $F_t^{cb} - F_{t-1}^{cb}$.

A current account deficit $CA_t < 0$, can be financed either by net private capital inflows, a negative capital and financial account $\Delta F_t^{fs} < 0$, or the sale of central bank foreign bonds $\Delta F_t^{cb} < 0$. The two types of financing, public and private, are not equal, since private financiers face an intermediary friction $\Gamma > 0$. As in Gabaix and Maggiori (2015) this friction allows the central bank to use the purchase or sale of foreign bonds as an instrument to adjust the current account and thus the economy's total external debt. A central bank sale of foreign bonds reduces the current account surplus, while a purchase of foreign bonds has the opposite effect. A formal proof is presented in the appendix, but the intuition is as follows.

Suppose the central bank increases their holdings of foreign bonds, $\Delta F_t^{cb} > 0$. Through the central bank's balance sheet, $\Delta B_t^{cb} < 0$, as the purchase of foreign bonds is financed by issuing domestic bonds. This puts upward pressure on the domestic interest rate and creates an arbitrage opportunity for the financial sector to buy domestic bonds and finance this by selling foreign bonds, $\Delta B_t^{fs} > 0$ and $\Delta F_t^{fs} < 0$.

Absent the intermediary friction, i.e. $\Gamma = 0$ private financiers could fully exploit this arbitrage opportunity and the increase in central bank domestic would be offset one for one by the increase in domestic bond purchases by the financial sector, $\Delta B_t^{cb} = -\Delta B_t^{fs}$. Then the total stock of debt in the economy would be unaffected and the equilibrium condition in the market for foreign bonds would imply that $R_t = R_t^W$. In that case, the financial sector is a veil and central bank foreign exchange intervention has no effect on aggregate macroeconomic variables, as in Obstfeld (1981), Backus and Kehoe (1989), Gabaix and Maggiori (2015), and Davis et al. (2021).

But if the intermediary friction $\Gamma > 0$ then when the central bank purchases foreign bonds, $\Delta F_t^{cb} > 0$, creating an arbitrage opportunity between foreign and domestic bonds, the intermediary friction means that this opportunity is not fully exploited by financiers. As financiers sell foreign bonds to take advantage of the arbitrage opportunity, the intermediary friction tends to push up the domestic interest rate above the world interest rate, reducing the private sector's incentive to sell domestic bonds in the same volume as their purchase of domestic bonds from the central bank.

3.5 Determination of net external assets

Here we examine the determination of the economy's net external debt in the model. Define a country's net external assets as $F_t = F_t^{fs} + F_t^{cb}$, and thus $-F_t$ represents net external debt. For now, abstract from central bank foreign reserves, so we set $F_t^{cb} = 0$. Schmitt-Grohe and Uribe (2020) show how under plausible parameterization of the model, there are multiple equilibria, one steady state equilibrium where the borrowing constraint is not binding, and two equilibria with a lower level of external debt and a binding borrowing constraint.

A steady state is defined by constant values of consumption, domestic and foreign bond holdings, and domestic interest rates. We can describe a steady state assuming a constant value of the world gross interest rate R^W. Here, and for the rest of the paper, we assume that in a steady state $\beta R^W < 1$. So domestic agents are more impatient than the rest of the world.

Using (4) -(7) and (12) a steady state is defined by the conditions:

$$1 = \beta R^W - \Gamma(F) + \frac{\mu}{\lambda} \tag{19}$$

$$-F \leq \kappa \left(y^T + \frac{1-\alpha}{\alpha} \left(y^T + \left(R^W - 1 \right) F \right)^{\frac{1}{\xi}} \right) \tag{20}$$

$$c^T = y^T + (R^W - 1)F \tag{21}$$

where the domestic interest rate $R_t = \frac{\lambda_t - \mu_t}{\beta E_t \lambda_{t+1}}$. For now in this analytical exposition, we make the simplification that the steady state value of the domestic interest rate is constant. [7]

Equations (19)-(21) describe steady state values of μ, F and c^T. The steady state borrowing constraint can be plotted in a chart with total external debt along both the horizontal and vertical axis in Figures 1 and 2. In these figures the right hand side of the inequality in equation (20) is represented by the blue downward sloping line.

Any equilibrium must lie along the 45 degree line. We pick the parameters of the model such that the borrowing constraint is non-binding in the steady state. Specifically, given the subjective discount factor and world interest rate, the financial intermediation parameter Γ determines the steady state level of external debt, $-F = \frac{1}{\Gamma}\left(\beta R^W - 1\right)$, and Γ is set high enough that (20) holds with strict inequality.

[7] Later when presenting the numerical solution to the model, the global solution incorporates the fact that the stochastic steady state value of $E_t\left(\lambda_{t+1}\right)$ is greater than the steady state value of λ_t.

Equation (20) represents the long-run borrowing constraint when $F = F_t = F_{t-1}$. But following Schmitt-Grohe and Uribe (2020) we may also define a short-run borrowing constraint, where unlike the steady-state borrowing constraint in (20), the initial level of external debt F_{t-1} is predetermined:

$$-F_t \leq \kappa \left(y^T + \frac{1-\alpha}{\alpha} \left(y^T + R^W_{t-1} F_{t-1} - F_t \right)^{\frac{1}{\xi}} \right) \tag{22}$$

The right hand side of this short-term borrowing constraint (22) is *increasing* in the choice of debt in period t, $-F_t$. This is due to the fact that given F_{t-1}, increased borrowing in period t raises the price of the non-traded good and thus the value of collateral. The slope of this borrowing constraint with respect to $-F_t$ is $\kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(y^T + R^W_{t-1} F_{t-1} - F_t \right)^{\frac{1}{\xi}-1}$.

This short-term borrowing constraint is given by the red upward sloping convex line in Figures 1 and 2. The difference between the two figures is that the initial debt $-F_{t-1}$ is higher in the second figure. As we can see the short term borrowing constraint in the right hand side of equation (22) is shifted down and to the right as debt carried over from the last period $-F_{t-1}$ increases.

The steady state in each figure corresponds to the level of debt where the short-term and long run borrowing constraints intersect, denoted by point $-F^A$ in both figures. The borrowing constraint is slack, and if initial debt is at $-F^A$, this remains a steady state equilibrium debt level. For a low initial stock of debt the short-term borrowing constraint does not intersect with the 45 degree line. But for a higher level of initial debt the short-term borrowing constraint does intersect the 45 degree line.

Since the short-term borrowing constraint shifts to the right and down when $-F_{t-1}$ increases, for a high enough initial debt level, the constraint intersects the 45 degree line twice and there are three equilibria. These are labeled A, B, and C in Figure 2. The non-binding equilibrium is A, and at B and C, the constraint is binding. Denoting the debt levels corresponding to these equilibria as t, $-F_t = -F^B$ and $-F_t = -F^C$, we see that

$-F^C < -F^B < -F^A$ and:

$$\begin{aligned} -F^B &= \kappa\left(y^T + \frac{1-\alpha}{\alpha}\left(y^T + R^W F^A - F^B\right)^{\frac{1}{\xi}}\right) \\ -F^C &= \kappa\left(y^T + \frac{1-\alpha}{\alpha}\left(y^T + R^W F^A - F^C\right)^{\frac{1}{\xi}}\right) \end{aligned}$$

Schmitt-Grohe and Uribe (2020) present a formal proof to derive under that conditions the short-term borrowing constraint crosses the 45 degree line twice and thus under what conditions will the model have three equilibria.

As we show in the appendix, the key to the existence of multiple equilibria is that the slope of the borrowing constraint is greater than one when the economy is in a non-binding steady state equilibrium with $-F_t = -F^A$. When the slope is greater than one, each additional unit of debt will have a direct effect of tightening the borrowing constraint by one unit but an indirect effect of loosening the borrowing constraint by more than one unit since additional debt leads to higher traded goods consumption and thus a higher relative price of non-traded goods. If in addition, at the intersection point C, the slope of the short run borrowing constraint is positive (as in figure 2), then both B and C represent borrowing-constrained equilibria. A large part of Schmitt-Grohe and Uribe (2020) is devoted to showing under what combinations of parameters this will hold.

In figure 2, each equilibrium can be sustained by self-confirming beliefs. Notice however that equilibrium B is unstable in a traditional sense, since if debt is just below (above) the level indicated by point B, the borrowing constraint would be violated (slack), and debt would be forced to fall (would be increasing).

Which equilibrium will prevail? For this, we need an equilibrium selection rule. In the equilibrium selection rule we use, if the non-binding equilibrium (A) is possible, agents' beliefs will always coalesce around this equilibrium. But if a non-binding equilibrium is not consistent with the borrowing choice of domestic agents given world interest rates (and central bank intervention), then the equilibrium is represented by the highest level of external debt where the borrowing constraint is not violated, which is the stable binding equilibrium, point C.

Note that this selection rule eliminates the possibility of self-fulfilling deleveraging. Movement from a "good" non-binding equilibrium to a "bad" binding equilibrium is driven not by beliefs, but by fundamental shocks, which in this model are represented as shocks to the country's cost of external borrowing, the world interest rate R_t^W.

4 Sudden stops with and without policy intervention

4.1 Competitive equilibrium without intervention

Following on the discussion of the previous section, we can describe how the competitive equilibrium without policy intervention evolves. Beginning from the non-binding steady state with external debt $-F^A$, represented graphically by point A in Figure 2. Following a shock to R_t^W, agents will adjust their desired debt levels according to the first order conditions above. Specifically, for a given sequence of current and expected future world interest rates $\left\{R_t^W\right\}_{t=1}^{\infty}$ agent's will pick a sequence of external borrowing $\{-F_t\}_{t=1}^{\infty}$ to satisfy the first-order condition with respect to external debt in equation (6), subject to the economy wide budget constraint in (17) and the borrowing constraint in (5), taking as given the prices $\{p_t, R_t\}_{t=0}^{\infty}$ that clear the corresponding nontradable goods market and domestic bond market.

Following a positive shock to R_t^W, agents' desired level of external borrowing $-F_t$ will fall. This can be represented as a movement left along the 45-degree line in Figure 2. After the initial shock, if R_t^W is stationary then external borrowing $-F_t$ will gradually converge back to the steady state at point A.

For a small shock, the desired deleveraging leads to a debt level to the right of point B, and so there still exists an equilibrium where the borrowing constraint does not bind. But there is a critical value of R_t^W where the equilibrium level of external debt $-F_t = -F^B$ and the borrowing constraint is just on the margin of binding, so $\mu_t = 0$. This is indicated at point B. Past this, for any further increases in R_t^W, agents will delever to $-\hat{F}$ where $-\hat{F} < -F^B$ and $-\hat{F}$ is in the region where the short-term borrowing constraint lies below the 45 degree line, $-\hat{F} > \kappa\left(y^T + \frac{1-\alpha}{\alpha}\left(y^T + R_{t-1}^W F^A - \hat{F}\right)^{\frac{1}{\xi}}\right)$.

But this point is not an equilibrium, since it violates the borrowing constraint. Agents are forced to delever further to $-\hat{F}' = \kappa\left(y^T + \frac{1-\alpha}{\alpha}\left(y^T + R_{t-1}^W F^A - \hat{F}\right)^{\frac{1}{\xi}}\right)$. But this deleveraging causes further fall in the price of collateral and agents are again forced to delever if the short-term borrowing constraint lies below the 45 degree line at $-\hat{F}'$, where $-\hat{F}' > \kappa\left(y^T + \frac{1-\alpha}{\alpha}\left(y^T + R_{t-1}^W F - \hat{F}'\right)^{\frac{1}{\xi}}\right)$. It is easy to see that this downward spiral will continue until agents delever to the point $-F^C$ where the short run borrowing constraint crosses the 45 degree line, point C in the figure.

In Figure 3 we illustrate this process in a simplified version of the model. In this simplified version of the model there is only a transitory shock to R_t^W in period $t = 1$ and agents know that R_t^W will remain at its steady state level for period $t = 2, \cdots, \infty$. The figure presents the values of time t external debt, time t traded goods consumption, the time t price of non-traded goods, and overall welfare as a function of the size an unexpected transitory shock to R_t^W in period $t = 1$. [8]

The blue solid line presents the case where the central bank holds no foreign exchange reserves, $F_t^{cb} = 0$. The red and green dashed lines consider the cases where the central bank holds foreign exchange reserves before the shock, and will be discussed shortly.

The steady state of the model is where $R_t^W = 1.04$. To the left of this steady state level of R_t^W, when there is a negative shock to the world interest rate, agents respond to the shock by borrowing more, and this leads to an increase in external debt, traded goods consumption and the price of the non-traded good. But the response may be quite asymmetric following a positive shock to R_t^W. For small positive shocks to R_t^W there is gradual deleveraging and a gradual fall in traded goods consumption and the price of the non-traded good. But at a

[8] A more realistic version of the model with persistent shocks where agents know the probability distribution of shocks is solved with a global solution method and presented in the next section. The parameters in this simplified model are the same as those used in the full solution, and are described in the next section.

certain point, which in this model with these parameters occurs when $R_t^W \approx 1.10$, external debt reaches point $-F^B$ in Figure 2 and past this point the non-binding equilibrium no longer exists, and there is a sudden stop. External debt falls to $-F^C$ and traded good consumption falls sharply, along with a large real exchange rate depreciation. The figure also shows that this sudden stop leads to a sharp drop in total welfare. Following the sudden stop, the economy's total external debt will over time gradually return to the original steady state.

4.2 Equilibrium with Policy Intervention

We now allow F_t^{cb} to be the instrument of a benevolent policy maker. The policy maker can either buy or sell reserves, subject to a non-negativity constraint on reserves. Recall from section 3 that, starting from a position where the borrowing constraint is not binding, if the central bank sells foreign exchange reserves, this leads to an increase in the economy's total external debt, while if the central bank were to buy foreign exchange reserves, that will decrease the economy's total external debt. By buying one unit of foreign bonds, the central bank is increasing the economy's total foreign assets by $1 + \frac{\partial F^{fs}}{\partial F^{cb}}$ units, where the intermediary friction $\Gamma > 0$ ensures that $\frac{\partial F^{fs}}{\partial F^{cb}} > -1$. Thus by buying one unit of foreign bonds the central bank reduces period t consumption by $1+\frac{\partial F^{fs}}{\partial F^{cb}}$ but increases next period's consumption by $\left(1 + \frac{\partial F^{fs}}{\partial F^{cb}}\right) R_t^W$. Likewise selling foreign bonds is a way to increase current consumption at the expense of next period's consumption.

Formally, the central bank chooses F_t^{cb} to maximize welfare in equation (1) subject to the economy wide budget constraint in equation (17), financier's incentive compatibility condition in (12), and the borrowing constraint in (5), where $p_t = \frac{1-\alpha}{\alpha}\left(\frac{c_t^T}{y^N}\right)^{\frac{1}{\xi}} = \frac{1-\alpha}{\alpha}\left(\frac{y^T + R_{t-1}^W F_{t-1}^{cb} - F_t^{cb} + R_{t-1}^W F_{t-1}^{fs} - F_t^{fs}}{y^N}\right)^{\frac{1}{\xi}}$. The central bank is subject to the additional constraint that its holdings of foreign bonds can't be negative, $F_t^{cb} \geq 0$. In the appendix we formally present the planner's problem and the first order conditions for optimal policy. Here instead we return to the simplified model and the figures presented in the last subsection.

Recall that the blue solid line in Figure 3 represents the case where prior to the shock the central bank held no foreign bonds, $F^{cb}_{t-1} = 0$. We now plot the responses to the same shock to the world interest rate in a model where prior to the shock the central bank holds a stock of bonds equal to 25% of the traded goods endowment (about 8% of GDP, red dashed line) or 50% of the traded goods endowment (about 16% of GDP, green dashed line).

By acquiring more foreign bonds in period $t-1$ the central bank pushes out the threshold value of R^w_t where the economy falls into a sudden stop. By buying a stock of foreign bonds in period $t-1$, the central bank is shifting consumption from period $t-1$ to period t, and that extra consumption in period t raises the price of the non-traded good t and thus pushes out the point where a sudden stop is triggered.

So acquiring foreign bonds is insurance against a large shock to the world interest rate in period t, but this insurance comes at a cost. When the central bank buys foreign bonds, that forces the economy as a whole to save more and is a distortion to the optimal consumption path of relatively impatient agents. The top half of Figure 4 presents the economy's total external debt and the cutoff value of R^W_t beyond which a sudden stop is triggered as a function of the stock of foreign bonds held by the central bank prior to the shock. The economy's pre-shock external debt is decreasing in the stock of bonds held by the central bank, but the cutoff value of R^W_t is increasing, meaning that a sudden stop is less likely.[9]

Finding the optimal stock of central bank foreign bonds in this model requires imposing a shock process for R^W_t. In this simplified model, we assume that the transitory shock to R^W_t follows a normal distribution, and we calibrate the standard deviation of this normal distribution to ensure that the probability of a sudden stop in the case without foreign exchange intervention is 5%.

Using this shock process for the one-off shock to the world interest rate, the bottom half of Figure 4 plots the probability of a sudden stop and total ex-ante welfare as a function of the

[9]There is a further element to the model that is relevant for a welfare evaluation. Because the central bank can borrow at the world interest rate, and faces no frictions, if the Central Bank were allowed to have negative reserves, it would be optimal for it to fully replace all external debt accrued by the financiers. We rule this possibility out by the assumption that Central Bank cannot have negative reserve holdings.

central bank's stock of foreign bonds. As the stock of bonds increases total ex-ante welfare rises as the diminished probability of a sudden stop outweighs the increasing distortions from forcing relatively impatient agents to save, but at a point the marginal welfare cost of excess saving outweighs the marginal insurance benefit. In this simplified model this point occurs when the central bank's stock of bonds is about 30% of the traded goods endowment, or 9.6% of GDP.

At this point the probability that a shock to the world interest rate would be large enough to trigger a sudden stop is much lower, but it is important to note that as long as the marginal cost to acquiring bonds ex-ante is positive and the probability distribution for R_t^W is continuous, the probability of a crisis at the optimal level of foreign bonds is positive. If the central bank were to acquire so many foreign bonds ex-ante that the probability of a crisis is zero then the marginal benefit of acquiring one extra unit of bonds is zero and yet the marginal cost is positive. Thus as long as there is a marginal cost to acquiring foreign bonds, the optimal stock of central bank foreign bonds will occur where the marginal benefit to acquiring those bonds is positive.

5 Numerical results for the optimal stock of reserves

In this section we move away from the simplified model and present the numerical results from the full stochastic model. We focus on the optimal policy under discretion.

5.1 Parameters and Calibration

The top panel of Table 1 presents the parameter values that we use to calculate the numerical results. Following Schmitt-Grohe and Uribe (2020), $\alpha = 0.31$, $\beta = 0.91$, $R^W = 1.04$, and $\sigma = 2$. One of the key parameters is the elasticity of substitution between traded and non-traded goods. We follow Schmitt-Grohe and Uribe (2020) and set $\xi = 0.5$. Bianchi (2011) uses $\xi = 0.83$, Benigno et al. (2013) follow the empirical estimate from Ostry and

Reinhart (1992) of $\xi = 0.76$. Stockman and Tesar (1995) use $\xi = 0.44$. Akinci (2017) surveys the empirical literature estimating this elasticity and finds that estimates of this elasticity vary between 0.43 and 1.50 depending on the estimation methodology and the countries sampled. Akinci argues that empirical estimates tend to be lower for the emerging markets, and estimates using a few different methodologies put the elasticity around 0.5 in emerging market countries like Argentina and Uruguay.

We make one adjustment to the calibration in Schmitt-Grohe and Uribe (2020) in order to ensure that a sudden stop crisis happens with sufficient frequency. While Schmitt-Grohe and Uribe (2020) focus on endowment shocks, our model is driven by shocks to the world interest rate. With the different shock process the probability of a crisis is different, and in order to ensure that the probability of a crisis is around 5% in the competitive equilibrium without policy intervention, we lower the coefficient in the borrowing constraint from $\kappa = 0.32R^W$ in Schmitt-Grohe and Uribe to $\kappa = 0.27R^W$.

We have no prior for the value of the financial intermediation friction Γ, and thus Γ is calibrated to match a certain value for the steady state level of private external borrowing:

$$\frac{\lambda}{\beta E(\lambda)} = R^W - \frac{\Gamma}{\beta} F^{fs}$$

where $\frac{\lambda}{E(\lambda)} < 1$ depends on the amount of uncertainty in the economy (we will discuss shocks shortly). If the value of Γ is too low then the borrowing constraint binds in the steady state. If it is too high then starting from the steady state equilibrium level of $-F^{fs}$ the short-term borrowing constraint does not cross the 45 degree line, and thus sudden stops are very rare. In order to have a meaningful probability of a sudden stop, we set $\Gamma = 0.05$, which makes the steady state level of external debt high enough to make a sudden stop possible following a sufficiently large interest rate shock, but low enough that the constraint is not binding in the steady state.

The model is driven by exogenous shocks to the world interest rate, R_t^W. These shocks follow an AR(1) process with persistence coefficient 0.572 and standard deviation 0.02. To approximate the equilibrium, we use a time iteration procedure over a discretized state space, and the bottom panel of Table 1 provides information for the discretization of the state space. We discretize the interest rate shock into 11 grid points, the endogenous state $-F_t$ into 300 grid points and central bank reserves F_t^{cb} into 300 grid points. For ease of exposition, we denote the median R^W as the 'steady state R^W'.

There is only one endogenous state variable in the model, the total external debt $-F_{t-1}$. It is important to note that for a given value of the endogenous state in period $t-1$, $-F_{t-1}$, the policy variable F_t^{cb} is a choice variable with the constraint that $F_t^{cb} \geq 0$. The choice of F_t^{cb} will then affect the endogenous state variable in period t, $-F_t$, but in period $t+1$, it is the state variable $-F_t$ that matters.

As discussed in section 4 above, while the model admits multiple expectational equilibrium, we maintain a particular equilibrium selection criterion, and focus on the role of shocks to fundamentals to generate sudden stops. Following a shock to R_t^W, if agents' first-order conditions and the other equilibrium conditions in the model are satisfied at a level of external debt $-\hat{F}$ where the borrowing constraint is not binding (to the right of point B in Figure 2), we pick this as the equilibrium. If on the other hand agents' first-order conditions and the other equilibrium conditions in the model are satisfied at a level of external debt $-\hat{F}$ where the borrowing constraint is binding (to the left of point B), the equilibrium is the lower level of external debt (point C).

5.2 Numerical results

5.2.1 Policy function

The policy function for the optimal choice of F_t^{cb} as a function of the endogenous state, $-F_{t-1}$, and the exogenous state, R_t^W, is presented in Figure 5. The blue solid line shows the central bank's optimal choice of F_t^{cb}, and the red dashed line plots the multiplier on the borrowing constraint, which changes from 0 to a positive number when the sudden stop occurs. All quantity variables such as total external debt and the stock of central bank reserves are presented as a percent of GDP.

Begin with the middle panel in the figure, this panel plots the optimal choice of F_t^{cb} as a function of $-F_{t-1}$ when R_t^W is equal to its steady state value. The figure shows that as long as the initial stock of external debt is less than 28.5 percent of GDP, a crisis does not occur in period t. Intuitively, this implies that the equilibrium $-F_t$ along the 45 degree line in Figure 2 remains to the right of point B when the initial stock of external debt is less than 28.5% of GDP. Then the policy function for F_t^{cb} for a low stock of initial external debt is zero. When external debt is this low, the probability of a sudden stop in period $t+1$ is zero and thus the central bank sees no need to distort the economy today by buying reserves as insurance against a possible crisis tomorrow. But the figure shows that for an external debt greater than 28% but less than 28.5% the central bank will start acquiring reserves F_t^{cb} as precaution against a large positive shock to the world interest rate in period $t+1$. As the initial stock of external debt increases the probability of a sudden stop increases, and thus the marginal benefit of F_t^{cb} increases. The central bank's optimal F_t^{cb} increases right up to the point where the debt is high enough that a sudden stop crisis would have happened in period t. At the highest point, central bank reserves are about 10% of GDP.

The top panel of the figure shows the same policy function when R_t^W is below its steady state value, and the bottom panel shows the same policy functions when R_t^W is above its steady state value. The policy function for F_t^{cb} is much higher when the current shock is low than when it is high. Recall that the exogenous world interest rate R_t^W follows an AR(1) process. So a low value of R_t^W today implies that is is likely to be higher in the next period, and a high value of R_t^W implies that it is likely to be lower in the next period. When the shock is currently low an initial level of external debt in excess of 29% may not lead to a crisis in period t, but it may in the future when the interest rate mean reverts. Thus the central bank will seek to reduce the probability of a crisis next period by acquiring reserves, and at its height, when R_t^W is low, the central bank will buy reserves F_t^{cb} up to 13% of GDP. Likewise when the current world interest rate is high, if the initial level of external debt is low enough that a crisis was not triggered in period t even when the interest rate was high, the probability of a crisis next period as the interest rate mean reverts is low, and as a consequence the policy function for F_t^{cb} remains close to zero.

5.2.2 Density of External Debt

Figure 6 plots the density of external debt in simulations of the model. In this figure we simulate the model over $T = 10^6$ periods and plot the density of the distribution of total external debt, $-F_t$, over these simulations. The blue solid line plots the density when the central bank does not engage in foreign exchange intervention and $F^{cb} = 0$, the red dashed line plots the density when the central bank engages in optimal foreign exchange intervention, described by the policy functions in Figure 5.

When the central bank does not engage in foreign exchange intervention the density of $-F_t$ has a large mass around 28.5 percent of GDP and a long left tail. With no policy intervention the economy, after a long string of shock realizations of zero, the economy would settle to a 'steady state' level of external debt a little less than 29%.[10] Negative shocks to R_t^W would lead agents to hold more debt and positive shocks to R_t^W would lead agents to hold a little less debt. But as the density figure shows, at a point around an external debt level of 28% the density drops. This is where the sudden stop occurs at point B in Figure 2. If the shock is large enough to trigger a sudden stop then the economy's total external debt falls to less than 18 percent of GDP and then begins a slow process of releveraging as the economy returns to the steady state.

The density of external debt under optimal foreign exchange intervention shows that optimal policy nearly eliminates the probability of a sudden stop. While it is difficult to see the scale on this graph, but there is a small weight in the left tail in the optimal FXI density,

[10] The dynamic models have a stationary distribution of external debts. To facilitate comparison with the deterministic steady state without shocks, we still use the term 'steady state' to describe the situation in which the economy stays when world interest rate is at its middle level for a long time given policy functions obtained in a dynamic model. This situation is sometimes called 'risky steady state' (Coeurdacier et al. (2011)).

but the weight is very close to zero. As discussed earlier, this probability is never zero, but in these simulations the probability is small.

But the density plots in Figure 6 show that the density in the non-binding region, where the economy is not in a sudden stop, is shifted to the left under optimal FXI. As discussed earlier, optimal foreign exchange accumulation is insurance against a crisis, but the cost is that it forces the economy to save more than it otherwise would.

5.2.3 Event Analysis

We now turn to event analysis to examine the dynamics of a typical financial crisis in the model to see how foreign exchange intervention reduces the probability of a sudden stop.

We construct an event analysis as follows. In a simulation path with length $T = 10^6$, a crisis is defined as a binding credit constraint $\mu_t > 0$ in period $t = 0$ in the competitive equilibrium with no foreign exchange intervention, and an event is a window of 11 periods from $t = -5$ to $t = 5$. We average all such events along the simulated path above. Figure 7 presents the path of the world interest rate, total external debt, the central bank's stock of foreign exchange reserves, tradable consumption, the price of non-tradables, the current account, the domestic interest rate, the multiplier on the borrowing constraint, and the spread between the domestic interest rate and the world interest rate during the average of these events. The solid blue line in the figure plots the event in economy with no foreign exchange intervention, and the dashed red line presents the responses of the same variables to the same path of the exogenous world interest rate and conditional on identical external debt in the first period of the event in the economy with optimal foreign exchange intervention.

The exogenous shock that triggers a crisis event is an increase in the world interest rate. The figure shows that the average crisis is triggered by an increase in the world interest rate from 3% to 7%. It is interesting to note that in the period before the crisis the world interest rate was slightly less than the steady state world interest rate of 4%. This indicates that at least some of the crisis episodes occur following a period of low interest rates which lead to a build-up external debt. This shock to the world interest rate is enough to cause agents to delever to a point to the left of point B in Figure 2, triggering a sudden stop and leading to a rapid deleveraging to a much lower level of external debt. This deleveraging implies a sharp fall in traded goods consumption a large depreciation of the real exchange rate, and a sharp increase in the current account.

The red dashed line plots the same variables in the same events (the window of ± 5 periods around a crisis in the economy without foreign exchange intervention), but now allowing the central bank to implement the optimal foreign exchange intervention, as show in the policy functions in Figure 5. Under optimal foreign exchange intervention, external debt is lower in the periods leading up to the crisis. The same size foreign interest rate shock now causes some deleveraging, but not enough to trigger a crisis. Thus there is some fall in traded goods consumption and real exchange rate depreciation, but substantially less depreciation relative to the outcome without policy intervention.

The response of the domestic interest rate and the spread over the world interest rate provides an interesting insight into the optimal FXI response. Prior to the crisis there is a spread between the domestic interest rate and the world interest rate of about 4.5% in the economy without FXI and 6% in the economy with FXI. By holding foreign bonds and forcing relatively impatient households to save, the central bank is keeping the domestic interest rate higher.

Finally, the graph of the central bank's optimal stock of bonds in the bottom right hand corner shows that in the periods leading up to the crisis the central bank adds to the stock of bonds as a precautionary measure in the face of low world interest rates. Following the interest rate shock the central bank does not roll over these bonds in order to maximize consumption in the period of the shock.

6 Summary and Conclusion

This paper presents a simple model where a sudden stop can arise in a small open economy. There is a pecuniary externality in that agents do not internalize the effect of their deleveraging on the price of collateral, and because of this, for even a relatively small increase in the world risk-free rate, the economy can fall into a sudden stop equilibrium. A policy maker that does internalize the effect of deleveraging on the price of collateral will set policy to keep the economy out of this sudden stop equilibrium.

When the tools available to the policy maker are the central bank purchases and sales of foreign bonds some interesting questions arise. The central bank's holdings of foreign bonds can't be negative, so in order to reduce the stock of bonds purchased following the shock the central bank must buy a positive stock of bonds before the shock. This central bank purchase of foreign bonds is distortionary since it forces economy wide savings to be higher than it otherwise would be. This paper attempts to provide a framework for assessing the marginal costs and marginal benefits of holding a stock of reserves in order to pinpoint the optimal stock of foreign bonds held by the central bank to safeguard against a sudden stop driven by an increase in the world risk-free interest rate.

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Figure 1: Single Non-binding equilibrium for a low level of debt.

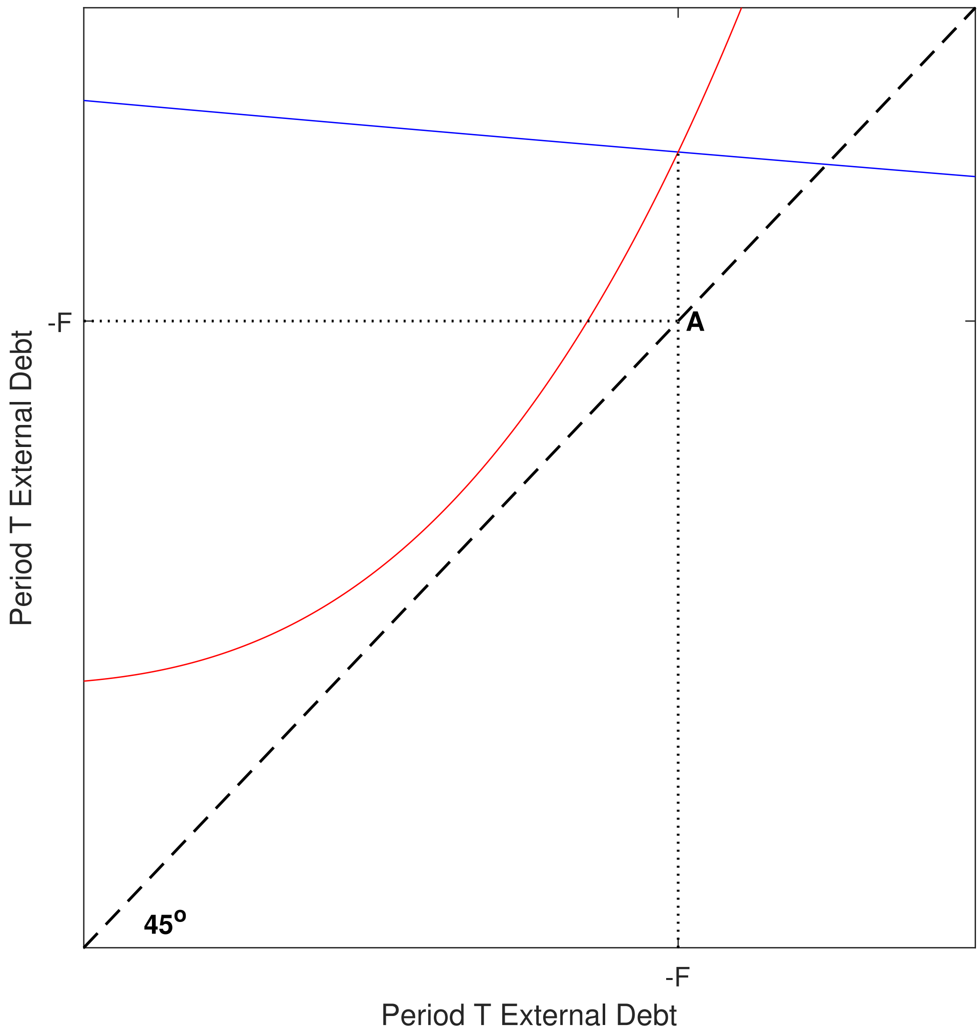


Figure 2: Multiple equilibria for a higher level of debt.

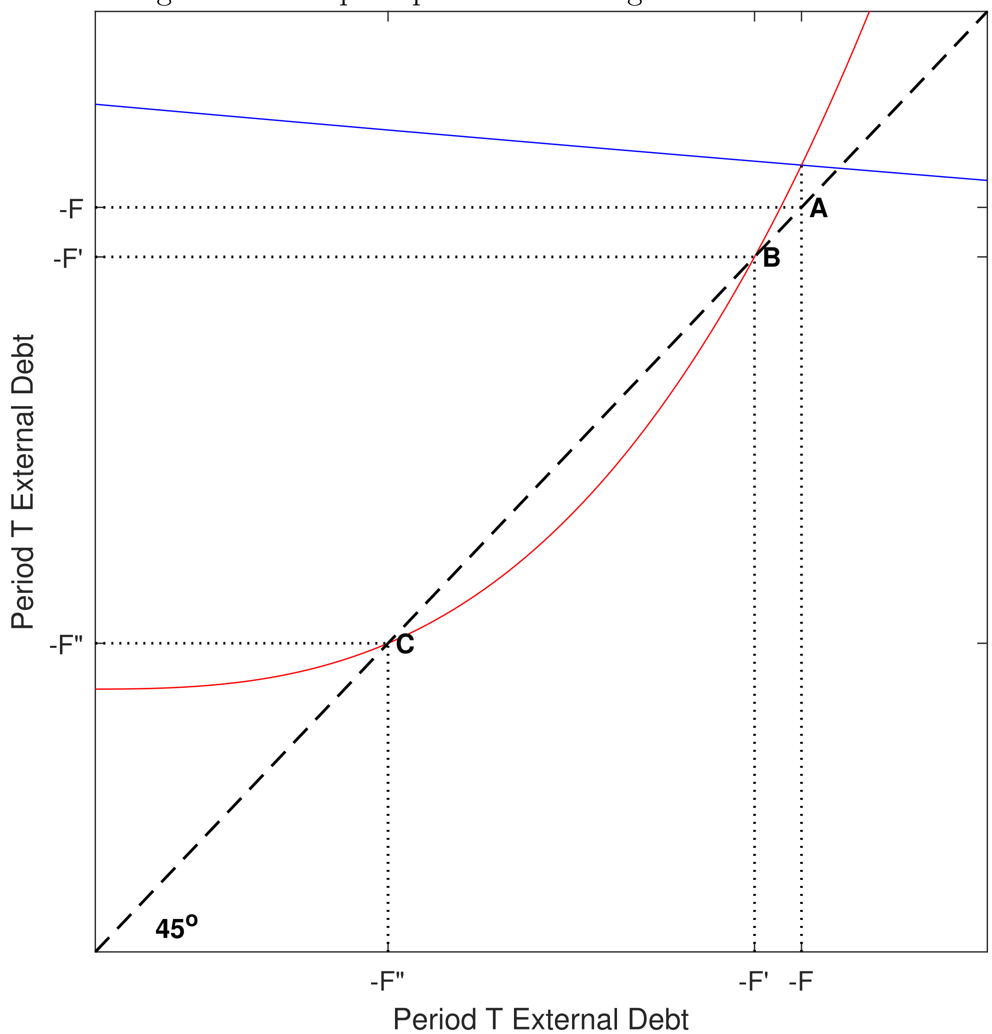
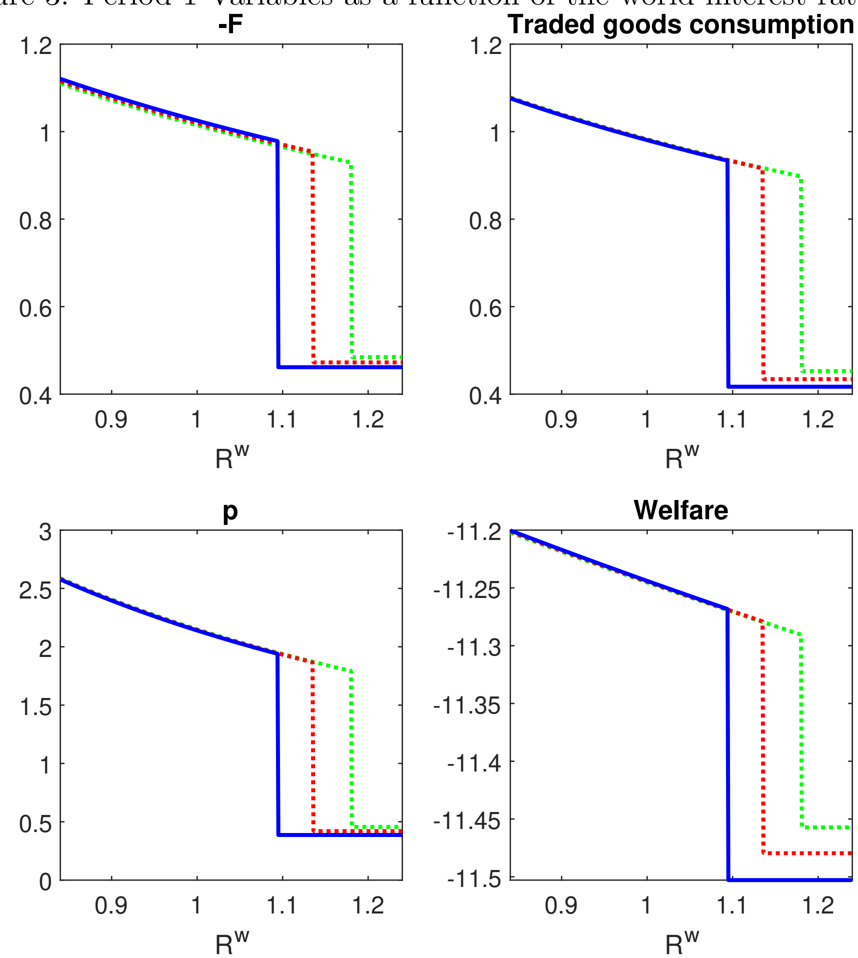
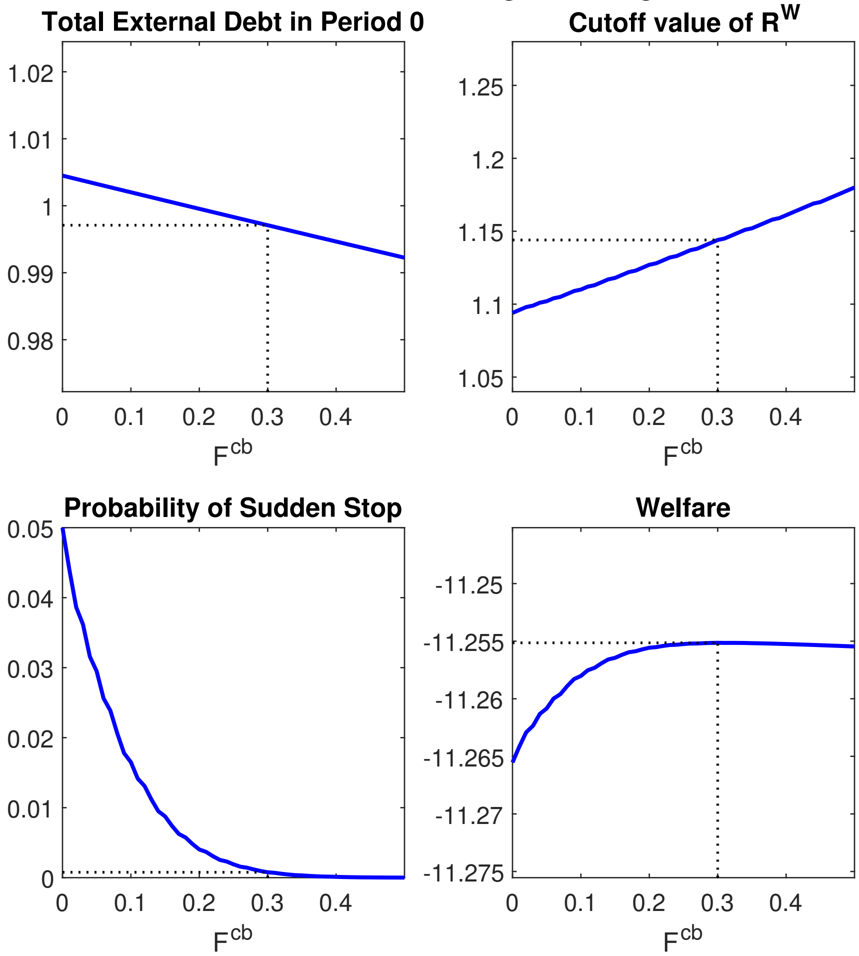


Figure 3: Period 1 Variables as a function of the world interest rate Rw



Notes: The blue line is the equilibrium when the central bank acquires no foreign bonds in period 0 and thus there is no foreign exchange intervention in period 1. The red line is when the central bank acquires foreign exchange reserves of 25% of the traded goods endowment. The green line is where the central bank acquires foreign exchange reserves of 50% of the traded good endowment.

Figure 4: Period 0 external debt, the cutoff value of Rw, the probability of a sudden stop, and total welfare as a function of the stock of foreign exchange reserves



Notes: Foreign exchange reserves F^{cb} expressed as a share of the traded goods endowment.

Figure 5: The policy function for reserve accumulation (blue solid line) and the multiplier on the borrowing constraint (red dashed line) as a function of external borrowing and the exogenous state. Low, Mid and High R_t^W denote the lowest level, middle level and highest level of world interest rate, respectively.

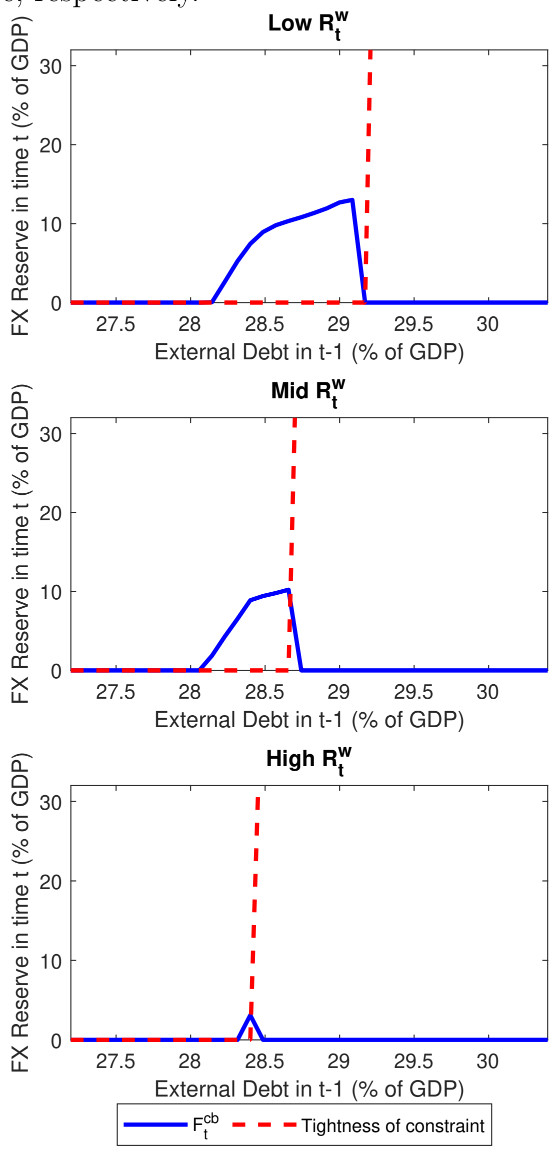


Figure 6: The density of external debt in period t in the model without foreign exchange intervention and in the model with optimal foreign exchange intervention. The smooth parameter is 20 (average of between period $t-19$ and t).

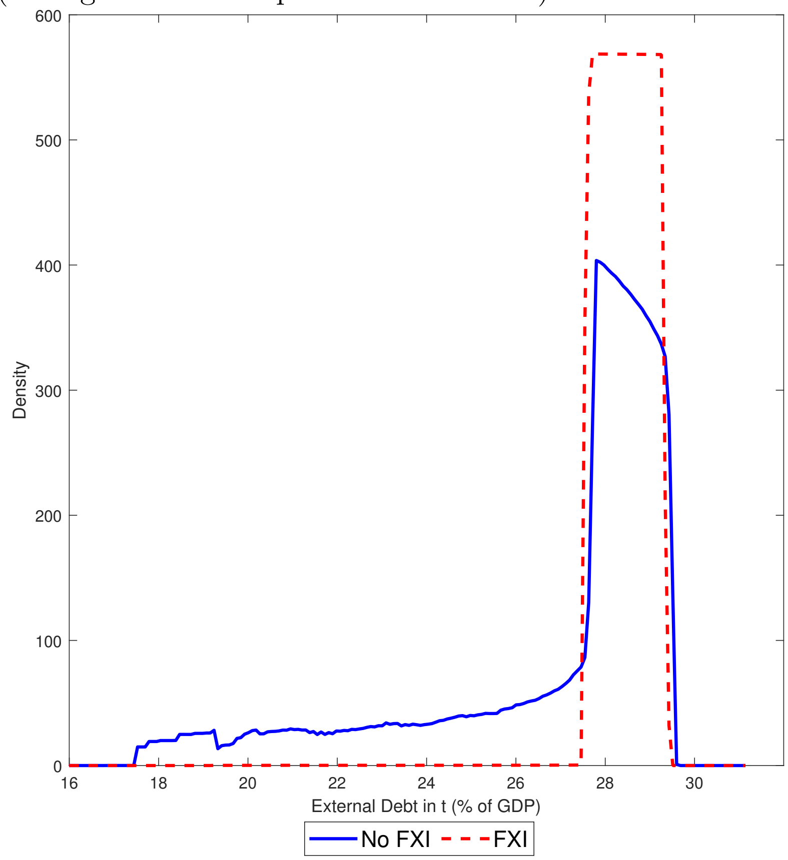
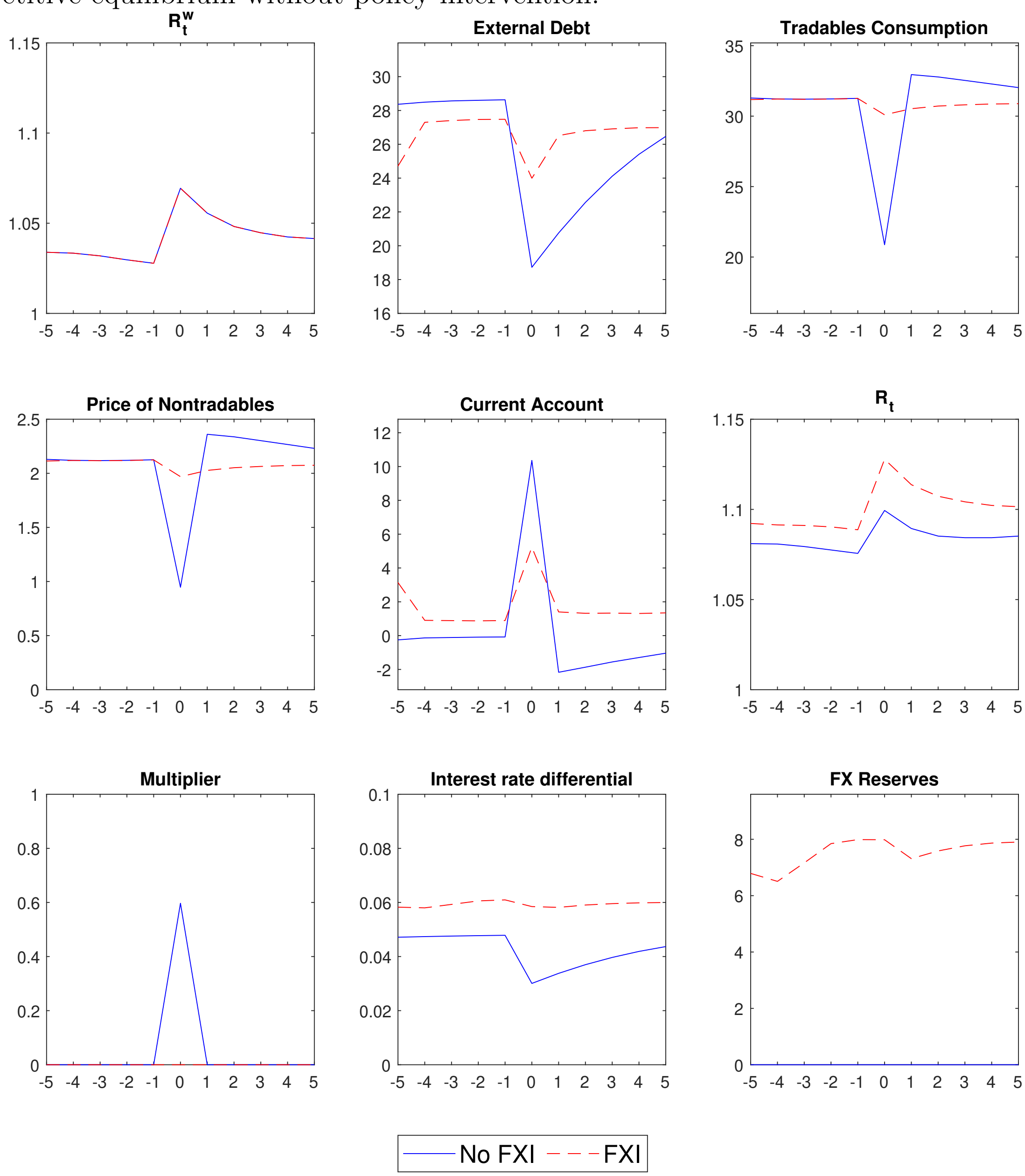


Figure 7: An event in a typical financial crisis when there is no foreign exchange intervention (blue solid lines) and the corresponding event when there's foreign exchange intervention (red dashed lines) along a simulated period of $T = 10^6$. A crisis occurs at period $t = 0$ in the competitive equilibrium without policy intervention.



Notes: Interest rates, Lagrange multiplier and nontraded good price are expressed in level and other variables are expressed as percentage point of GDP.

Table 1: Parameter values

Parameter	Value	Description
		Structural Parameters
R^W	1.04	Annual gross world interest rate
σ	2	Inverse of intertemporal elasticity of consumption
κ	$0.27R^w$	Parameter in borrowing constraint
β	0.91	Subjective discount factor
x	1	endowment of traded goods
y	1	endowment of non-traded goods
α	0.31	Weight on traded goods in CES aggregator
ξ	0.5	Elasticity of substitution between traded/non-traded goods
Γ	0.05	Financial intermediation friction
		Discretization of State Space
$\left[\ln \frac{R^W_{\min}}{R^W} \;\; \ln \frac{R^W_{\max}}{R^W}\right]$	$[-0.071, 0.071]$	Range for the world interest rate
$[-F_{\min} \;\; -F_{\max}]$	$[0.2, 1.0]$	Range for total external debt
$\left[F^{cb}_{\min} \;\; F^{cb}_{\max}\right]$	$[0, 0.5]$	Range for foreign exchange reserves
n_{Rw}	11	number of grid points for $\ln \frac{R^W_t}{R^W}$, equally spaced
n_F	300	number of grid points for $-F_t$, equally spaced
n_{Fcb}	300	number of grid points for F^{cb}_t, equally spaced

Not For Publication Technical Appendix

A Empirical motivation

Our paper focuses on how an increase in the world interest rate can trigger a sudden stop and consequently lead to a financial crisis. Central banks in small economies may make use of foreign exchange reserves to prevent future financial crisis. Here we provide a supportive evidence to our theoretical model specification and the use of foreign exchange reserves by central banks. We specify an empirical analysis as follows,

$$\text{Prob.}(crisis = 1)_{it} = \beta_1 \Delta R_t^W + \beta_2 \frac{\text{FX reserves}_{i,t-1}}{\text{GDP}_{i,t-1}} + \beta_3 \Delta R_t^W \times \frac{\text{FX reserves}_{i,t-1}}{\text{GDP}_{i,t-1}} + \beta_4' X_{it-1} + \beta_5' Z_t + \epsilon_{it} \tag{A.1}$$

The dependent variable here is an indicator of a financial crisis in country i period t, which is taken from Laeven and Valencia (2020). Independent variables include the change in world interest rate ΔR_t^W, foreign exchange reserves over GDP, $\frac{\text{FX reserves}_{i,t-1}}{\text{GDP}_{i,t-1}}$ in country i period $t-1$, other country specific control variables are stacked in vector $X_{i,t-1}$ and other global control variables are stacked in vector Z_t. The last term ϵ_{it} is an error term. The world interest rate is the US 1-Year real treasury constant maturity rate. Definitions of variables and data sources are presented in table 1.

Table 2 reports regression results for our empirical specification (A.1) by using a linear probability model and a Probit model. Results show that a rise in US interest rate leads to a significant increase in currency crisis incidence $\beta_1 > 0$, particularly for emerging and developing economies, after controlling for country fixed effects and other domestic and global factors. Movement in the world interest rate is one of key driving forces of a financial crisis. At the same time, a higher stock of foreign exchange reserves before the onset of a financial crisis reduces the incidence of a crisis both for emerging and advanced economies $\beta_2 < 0$. More importantly, the coefficient for the interaction term between the change of world interest rate and the lagged reserves-GDP ratio β_3 is significantly negative for emerging and developing economies.

Figure 1 displays how the lagged reserves-GDP ratios affect the average marginal effects of the change of world interest rate on crisis incidence $\frac{\partial \text{Prob.}(crisis=1)_{it}}{\partial \Delta R_t^W}$. The figures show that higher reserve-GDP ratios before a crisis leads to a lower crisis incidence. When the lagged reserves-GDP ratio is at its medium level (reserves/GDP=0.08), the average marginal effect is 0.006. Note that the standard deviation of percentage change of R_t^W is 2.3, and therefore the currency crisis will increase by $0.006 \times 2.3 = 0.014$ when the world interest rate increases by a standard deviation 2.3. This magnitude is economically significant compared to the unconditional currency crisis incidence 0.04 in the data sample. When reserves-GDP ratios move from 50 percentile (reserves/GDP=0.08) to bottom 5 percentile (reserves/GDP=0.01), currency crisis probability increases by $(0.008 - 0.006) \times 2.3 = 0.0046$. When foreign reserves increases from the bottom 5 percentile (reserves/GDP $= 0.01$) to the upper 95 percentile (reserves/GDP=0.37), currency crisis incidence declines by $(0.008 - 0.0005) \times 2.3 = 0.017$ when responding to a standard deviation rise in the world interest rate. This reduction in crisis due to reserve accumulation could account for almost 40% decline in the unconditional probability of currency crisis.

Table 1: Variable definitions and data sources

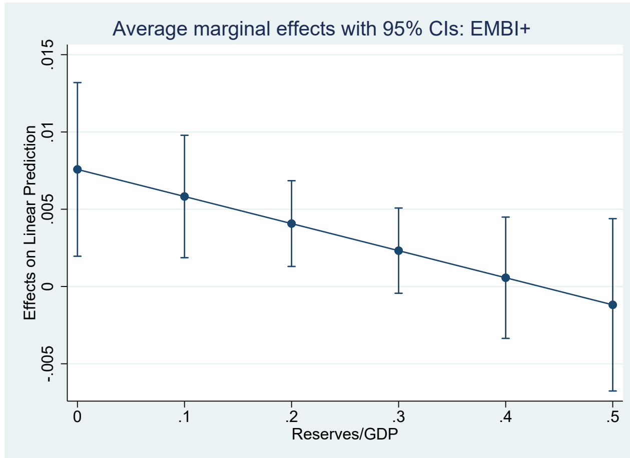
Variables	Description and data source
Data coverage	1970 – 2015 unbalanced panel data
Currency crisis	from Laeven and Valencia (2020)
R_t^W	US 1-Year Treasury Constant Maturity Rate (Percent) mimus CPI inflation expeaction. Interest rate is taken from Federal Reserve Economic Data and inflation expectation is from the Livingston Survey by Federal Reserve Bank of Philadelphia.
FX reserves/GDP	FX Reserves minus gold/GDP, from Lane and Milesi-Ferretti (2007, 2018)
FI/GDP	Financial integration/GDP. Financial integration is the sum of external assets and liabilities for a given country. External assets and liabilities are taken from Lane and Milesi-Ferretti (2007, 2018)
CA/GDP	Current account/GDP, from Lane and Milesi-Ferretti (2007, 2018)
Real GDP per capita	from IMF International Financial Statistics
Energy	Energy price index, from World Bank Commodity Price Data (The Pink Sheet)
Nonenergy	Non-energy price index, from World Bank Commodity Price Data (The Pink Sheet)
Economies covered by EMBI+	Algeria, Argentina, Azerbaijan, Belarus, Belize, Brazil, Bulgaria, Chile, China, Colombia, Cote d'Ivoire, Croatia, Dominican Republic, Ecuador, Egypt Arab Rep., El Salvador, Gabon, Georgia, Ghana, Greece, Guatemala, Hungary, India, Indonesia, Iraq, Jamaica, Jordan, Kazakhstan, Korea Rep., Lebanon, Lithuania, Malaysia, Mexico, Mongolia, Morocco, Namibia, Nigeria, Pakistan, Panama, Peru, Philippines, Poland, Russian Federation, Senegal, South Africa, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, Venezuela RB, Vietnam, Zambia
Advanced economies	Austria, Australia, Canada, Czech Republic, Denmark, Estonia, Finland, France, Germany, Iceland, Ireland, Israel, Italy, Japan, Latvia, the Netherlands, New Zealand, Norway, Portugal, Slovak Republic, Slovenia, Spain, Sweden

Table 2: US interest rate, foreign exchange reserves and currency crisis incidence

	(1)	(2)	(3)	(4)
	OLS All	OLS EMBI+	Probit All	Probit EMBI+
ΔR_t^W	0.005**	0.008**	0.045*	0.063**
	(2.017)	(2.643)	(1.815)	(2.198)
Lagged FX reserves/GDP	-0.180***	-0.228***	-3.487***	-3.949***
	(-3.316)	(-2.932)	(-2.892)	(-2.962)
$\Delta R_t^W \times$ Lagged FX reserves/GDP	-0.013	-0.018*	-0.143	-0.031
	(-1.552)	(-1.729)	(-0.555)	(-0.162)
Lagged FI/GDP	-0.004	0.008	-0.046	0.082
	(-0.466)	(0.494)	(-0.496)	(0.686)
Lagged CA/GDP	-0.106	-0.073	-1.508**	-1.071
	(-1.522)	(-1.114)	(-2.074)	(-1.423)
Lagged Real GDP per capita	0.001	0.009**	-0.012*	0.040*
	(1.587)	(2.620)	(-1.789)	(1.823)
Energy price	-0.001	-0.001**	-0.008*	-0.011**
	(-1.628)	(-2.408)	(-1.874)	(-2.408)
Nonenergy price	0.001	0.001	0.011	0.012
	(0.960)	(1.296)	(1.500)	(1.531)
Observations	2,729	1,917	2,729	1,917
Number of countries	78	55	78	55
Adjusted R-squared	0.014	0.023		

Notes: Currency crisis is a dummy variable, equals to one when there is a currency crisis in year t and zero otherwise. FX reserves are foreign exchange reserves excluding gold. FI denotes financial integration, which is the sum of external assets and liabilities. CA stands for current account. All is for the whole data sample, $EMBI+$ for economies covered by J.P. Morgan Emerging Markets Bond Indix Global, and Advanced for other higher income advanced economies. Significance is denoted as $***\ p < 0.01$, $**\ p < 0.05$, $*\ p < 0.1$. Standard errors are clustered at country code and robust t-statistics is reported in parentheses. OLS denotes a linear probability model using a panel regression with country fixed effects. Probit denotes a panel Probit model. The data sample covers 78 emerging, developing and advanced economies from $1970 - 2015$. We drop two years immediately after a currency crisis and treat non-crisis years as normal times.

Figure 1: This figure shows the average marginal effects of the change of world interest rate, ΔR_t^W, on currency crisis for countries covered by EMBI+, conditional on different reserves/GDP ratio in the linear probability models.



B Variables and Equations

Variables in the model:
c_t^T, c_t^N, R_t, B_t^{cb}, B_t, B_t^{fs}, F_t^{fs}, λ_t, μ_t, p_t, T_t, Π_t, F_t^{cb}, τ_t taking B_0, F_0^{fs}, F_0^{cb} and R_t^W as given
Equations:
c_t^T:

$$\left(\left[\alpha\left(c_t^T\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_t^N\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}\right)^{-\sigma}\times$$

$$\left[\alpha\left(c_t^T\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_t^N\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}-1}\alpha\left(c_t^T\right)^{\frac{-1}{\xi}}=\lambda_t$$

c_t^N:

$$\left(\left[\alpha\left(c_t^T\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_t^N\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}\right)^{-\sigma}\times$$

$$\left[\alpha\left(c_t^T\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_t^N\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}-1}(1-\alpha)\left(c_t^N\right)^{\frac{-1}{\xi}}=p_t\lambda_t$$

$R_t : B_t + B_t^{cb} + B_t^{fs} = 0$
$B_t^{cb} : B_t^{cb} + F_t^{cb} = 0$
$B_t : \frac{\lambda_t - \mu_t}{R_t} = \beta\lambda_{t+1}$
$B_t^{fs} : B_t^{fs} + F_t^{fs} = 0$
$F_t^{fs} : R_t = R_t^W - \frac{\Gamma}{\beta}F_t^{fs}$

$\lambda_t : c_t^T + p_t c_t^N + B_t = y^T + p_t y^N + R_{t-1} B_{t-1} + T_t + \Pi_t$
$\mu_t : \mu_t = 0$ or $-B_t = \kappa \left(y^T + p_t y^N\right)$
$p_t : c_t^N = y^N$
$T_t : T_t = R_{t-1}^W F_{t-1}^{cb} + R_{t-1} B_{t-1}^{cb} - F_t^{cb} - B_t^{cb}$
$\Pi_t : \Pi_t = R_{t-1}^W F_{t-1}^{fs} + R_{t-1} B_{t-1}^{fs} - F_t^{fs} - B_t^{fs}$
$F_t^{cb} : policy$

C Proof of effectiveness of foreign exchange intervention when $\Gamma > 0$

To see how central bank foreign exchange intervention can have a macroeconomic effect when the economy is in a non-binding equilibrium and $\Gamma > 0$, start from the non-binding equilibrium where private sector demand for international bonds is given by F_t^{fs}, the central bank does not change their stock of international bonds $F_t^{cb} = F_{t-1}^{cb}$, and thus traded goods consumption in periods t and $t+1$ is given by:

$$\begin{aligned} c_t^T &= y^T - F_t^{fs} + R_{t-1}^W F_{t-1}^{fs} - F_t^{cb} + R_{t-1}^W F_{t-1}^{cb} \\ c_{t+1}^T &= y^T - F_{t+1}^{fs} + R_t^W F_t^{fs} - F_{t+1}^{cb} + R_t^W F_t^{cb} \end{aligned}$$

Now suppose instead the central bank engages in a sale of some of their stock of international bonds, $F_t^{cb\prime} < F_{t-1}^{cb}$. Following this central bank intervention the private sector changes their demand for international bonds from F_t^{fs} to $F_t^{fs\prime}$. The new consumption level following this central bank intervention and private sector response is:

$$\begin{aligned} c_t^{T\prime} &= y^T - F_t^{fs\prime} + R_{t-1}^W F_{t-1}^{fs} - F_t^{cb\prime} + R_{t-1}^W F_{t-1}^{cb} \\ c_{t+1}^{T\prime} &= y^T - F_{t+1}^{fs} + R_t^W F_t^{fs\prime} - F_{t+1}^{cb} + R_t^W F_t^{cb\prime} \end{aligned}$$

We want to prove that when $\Gamma > 0$, the central bank sale of international bonds led to higher external debt, $-F_t^{fs} - F_t^{cb} < -F_t^{fs\prime} - F_t^{cb\prime}$ and thus by extension, higher traded goods consumption in period t.

If $F_t^{fs\prime} < F_t^{fs}$, the proof is trivial (this would be where the central bank sold foreign bonds, and as a result the private sector also sold foreign bonds). Turning to the case where $F_t^{fs\prime} > F_t^{fs}$: combine the household's first order condition with respect the F in equation (6) with the equilibrium condition for foreign borrowing in equation (12).

$$\frac{\lambda_t - \mu_t}{\beta \lambda_{t+1}} = R_t^W - \frac{\Gamma}{\beta} \left(F_t^{fs}\right) \tag{C.1}$$

Thus in a non-binding equilibrium:

$$\frac{\lambda_t}{\beta \lambda_{t+1}} + \frac{\Gamma}{\beta} \left(F_t^{fs}\right) = \frac{\lambda_t'}{\beta \lambda_{t+1}'} + \frac{\Gamma}{\beta} \left(F_t^{fs\prime}\right) \tag{C.2}$$

where λ_t and λ_{t+1} are the marginal utilities of consumption when traded goods consumption is c_t^T and c_{t+1}^T, and λ_t' and λ_{t+1}' are the marginal utilities of consumption when traded goods consumption is $c_t^{T\prime}$ and $c_{t+1}^{T\prime}$. If $F_t^{fs\prime} > F_t^{fs}$ and $\Gamma > 0$ then the above equality becomes:

$$\frac{\lambda_t}{\beta\lambda_{t+1}} > \frac{\lambda_t'}{\beta\lambda_{t+1}'} \tag{C.3}$$

which implies that the central bank intervention by selling international bonds lowers the domestic interest rate. This implies:

$$c_t^T - c_{t+1}^T < c_t^{T\prime} - c_{t+1}^{T\prime}$$

which implies that $-F_t^{fs} - F_t^{cb} < -F_t^{fs\prime} - F_t^{cb\prime}$. Note that the switch from the equality in equation (C.2) to the inequality in equation (C.3) relied on the fact that $\Gamma > 0$. If there were no intermediary frictions $\Gamma = 0$, then the inequality in equation (C.3) would be an equality and $-F_t^{fs} - F_t^{cb} = -F_t^{fs\prime} - F_t^{cb\prime}$. Any central bank sale of international bonds would be exactly offset by a private sector purchase of international bonds, leaving total external debt unchanged.

D Proof of multiple equilibria

The initial level of debt $-\tilde{F}$, which separates the low and high levels of initial debt is represented by the point where the short-term borrowing constraint is tangent to the 45 degree line. At this point, the slope of the short-term borrowing constraint is unity, and the short-term borrowing constraint is binding, so that a) $\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(y^T + R^W\tilde{F} - F_t\right)^{\frac{1-\xi}{\xi}} = 1$ and b) $-F_t = \kappa\left(y^T + \frac{1-\alpha}{\alpha}\left(y^T + R^W\tilde{F} - F_t\right)^{\frac{1}{\xi}}\right)$.

A little algebra shows that these two conditions are satisfied when:

$$-\tilde{F} = \frac{1}{R^W}\left(y^T(1+\kappa) + (\frac{\kappa(1-\alpha)}{\alpha\xi})^{\frac{\xi}{\xi-1}}(\xi - 1)\right)$$

We want to prove that for an initial level of external debt of $-F^A$, where $-\tilde{F} < -F^A < -\bar{F}$. The short term borrowing constraint given by:

$$-F_t \le \kappa\left(y^T + \frac{1-\alpha}{\alpha}\left(y^T + R_{t-1}^W F^A - F_t\right)^{\frac{1}{\xi}}\right)$$

will have two points $-F^B$ and $-F^C$ where:

$$\begin{aligned} -F^B &= \kappa\left(y^T + \frac{1-\alpha}{\alpha}\left(y^T + R^W F^A - F^B\right)^{\frac{1}{\xi}}\right) \\ -F^C &= \kappa\left(y^T + \frac{1-\alpha}{\alpha}\left(y^T + R^W F^A - F^C\right)^{\frac{1}{\xi}}\right) \end{aligned}$$

We know that when $-F^A < -\bar{F}$, the borrowing constraint is not binding when $-F_t = -F^A$

$$-F^A < \kappa \left(y^T + \frac{1-\alpha}{\alpha} \left(y^T + R^W F^A - F^A \right)^{\frac{1}{\xi}} \right)$$

We know that if $-\tilde{F} < -F^A$, the point $-\hat{F}$ where the slope of the borrowing constraint is equal to one, $\kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(y^T + R^W_{t-1} F^A - \hat{F} \right)^{\frac{1}{\xi}-1} = 1$ must violate the borrowing constraint:

$$-\hat{F} > \kappa \left(y^T + \frac{1-\alpha}{\alpha} \left(y^T + R^W F^A - \hat{F} \right)^{\frac{1}{\xi}} \right)$$

Furthermore we know that when $\xi < 1$, the borrowing constraint, $\kappa \left(y^T + \frac{1-\alpha}{\alpha} \left(y^T + R^W_{t-1} F^A - F_t \right)^{\frac{1}{\xi}} \right)$ it convex. If at the point $-F_t = -F^A$, the slope of the borrowing constraint is greater than one, $\kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(y^T + R^W_{t-1} F^A - F^A \right)^{\frac{1}{\xi}-1} > 1$, and the slope of the same borrowing constrain is equal to one when $-F_t = -\hat{F}$, then it must be that $-\hat{F} < -F^A$. Thus is the borrowing constraint is non-binding when $-F_t = -F^A$ and yet it is violated when $-F_t = -\hat{F}$, then by the intermediate value theorem there must be a point $-F_t = -F^B$ where $-\hat{F} < -F^B < -F^A$ where the borrowing constraint holds with equality.

At the other end, at the maximum amount of deleveraging (i.e. the minimum $-F_t$), traded goods consumption in period t is zero, $y^T + R^W F^A - F_t = 0$. Thus at the point where $-F_t = -y^T - R^W F^A$, the short term collateral constraint is equal to κy^T, under what parameterization is the constraint not binding at the minimum amount of traded goods consumption, $\kappa y^T > -y^T - R^W F^A$. Thus we ask, what is the minimum set of parameter values where $-R^W F^A < (1+\kappa) y^T$ holds?

We know that $-F^A < -\bar{F}$ where:

$$-\bar{F} = \kappa \left(y^T + \frac{1-\alpha}{\alpha} \left(y^T + R^W_{t-1} \bar{F} - \bar{F} \right)^{\frac{1}{\xi}} \right)$$

Define $a = \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(y^T + R^W_{t-1} \bar{F} - \bar{F} \right)^{\frac{1}{\xi}-1}$, then we can rearrange the above equality $-\bar{F} = \kappa y^T + a\xi \left(y^T + R^W_{t-1} \bar{F} - \bar{F} \right)$, or $-\bar{F} = \frac{(a\xi+\kappa) y^T}{\left(a\xi R^W_{t-1} + (1-a\xi) \right)}$. Since $-F^A < -\bar{F}$ then $-F^A < \frac{(a\xi+\kappa) y^T}{\left(a\xi R^W_{t-1} + (1-a\xi) \right)}$. From this it should be obvious that $-R^W F^A < (1+\kappa) y^T$ will also hold as long as $a\xi < 1$, where $a\xi = \kappa \frac{1-\alpha}{\alpha} \left(y^T + R^W_{t-1} F^A - F^A \right)^{\frac{1}{\xi}-1}$. We know that $y^T + R^W_{t-1} F^A - F^A < 1$, so as long as $\kappa < \frac{\alpha}{1-\alpha} \left(y^T + R^W_{t-1} F^A - F^A \right)^{1-\frac{1}{\xi}} < \frac{\alpha}{1-\alpha}$ in our benchmark parameterization $\alpha = 0.31$, so as long as $\kappa < 0.44$ the constraint is non-binding at point where traded goods consumption is zero. (we use $\kappa = 0.27 R^W$ where $R^W = 1.04$)

Furthermore we know that at this point since traded goods consumption is zero, $y^T + R^W F^A - F_t = 0$, the slope of the borrowing constraint $\kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(y^T + R^W_{t-1} F^A - F_t \right)^{\frac{1}{\xi}-1} = 0$. Since the slope of the borrowing constraint is one when $-F_t = -\hat{F}$, then $-y^T - R^W F^A < -\hat{F}$. By the intermediate value theorem there must be a point $-y^T - R^W F^A < -F^C < -\hat{F}$ where the borrowing constraint holds with equality.

E Social planner's problem

The central bank's problem can be written as

$$\begin{aligned}\mathcal{L}_{\{c_t^T, F_t, F_t^{cb}, \theta_t^p\}} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \{ & U(c_t^T) + \eta_t \left(y^T + R_{t-1}^W F_{t-1} - F_t - c_t^T\right) \\ & + \gamma_t \left[\lambda_t - \beta \lambda_{t+1} \left(R_t^W - \frac{\Gamma}{\beta}(F_t - F_t^{cb})\right) - \mu_t^p\right] \\ & + \varphi_t \left[\kappa \left[y^T + y^N \frac{1-\alpha}{\alpha} \left(\frac{c_t^T}{y}\right)^{\frac{1}{\xi}}\right] + F_t - m_t^p\right] \\ & + \mu_t^s \left[\kappa \left[y^T + y^N \frac{1-\alpha}{\alpha} \left(\frac{c_t^T}{y}\right)^{\frac{1}{\xi}}\right] + F_t\right] \\ & + \phi_t \left(F_t^{cb}\right)\end{aligned}$$

where $\mu_t^p \equiv (\max(0, \theta_t^p))^L$ standards for the Lagrange multiplier in the competitive equilibrium and m_t^p is the excess borrowing capacity $m_t^p \equiv (\max(0, -\theta_t^p))^L = \kappa \left[y^T + y^N \frac{1-\alpha}{\alpha} \left(\frac{c_t^T}{y}\right)^{\frac{1}{\xi}}\right] + F_t$, with $\theta_t^p \in \mathbb{R}$ and L is a positive integer larger than 2. Given the initial external asset position F_{-1} and shocks $\{R_t^W\}_0^\infty$, and Lagrange multipliers $\eta_t \geq 0$, $\gamma_t \geq 0$, $\mu_t^s \geq 0$ and $\phi_t \geq 0$, the central bank chooses optimal paths for $\{c_t^T, F_t, F_t^{cb}, \theta_t^p\}$.

Note that policy functions in the competitive equilibrium for a given central bank policy path under the parameterization of Schmitt-Grohe and Uribe (2020) are discontinuous. Therefore the standard marginal analysis for an optimal policy based on the first-order optimality conditions doesn't apply in this situation. The main text instead uses a global solution method to find optimal policy functions. Nevertheless, when taking use of the parameterization of Bianchi (2011), policy functions in the competitive equilibrium are continuous. Therefore, in order to compare with the literature and develop the key logic of optimal foreign exchange intervention, the following analysis assumes that policy functions are continuous given any path of foreign exchange intervention.

The optimality conditions can be written as follows. The optimal reserve F_t^{cb} satisfies:

$$\phi_t = \Gamma \gamma_t \mathbb{E}_t \lambda_{t+1} \tag{E.1}$$

For the optimal level of θ_t^p:

$$-\gamma_t \frac{\partial \mu_t^p}{\partial \theta_t^p} - \varphi_t \frac{\partial m_t^p}{\partial \theta_t^p} = 0$$

When $\theta_t^p > 0$, the credit constraint binds, $\mu_t^p > 0$, $m_t^p = 0$, $\mu_t^s > 0$, then $\gamma_t = 0$ and $\phi_t = 0$. When the credit constraint doesn't bind, $\theta_t^p < 0$, $m_t^p > 0$, $\mu_t^p = \mu_t^s = 0$, then $\varphi_t = 0$.

Combining both cases, we always have $\gamma_t \mu_t^p + \varphi_t m_t^p = 0$. Let $\mu_t \equiv \varphi_t + \mu_t^s$. The following analysis will use this simplified notation.

For traded consumption good c_t^T:

$$\eta_t = U_T(c_t^T) + \mu_t \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(\frac{c_t^T}{y} \right)^{\frac{1}{\xi}-1} + \gamma_t \frac{\partial \lambda_t}{\partial c_t^T} - \gamma_{t-1} R_{t-1} \frac{\partial \lambda_t}{\partial c_t^T} \tag{E.2}$$

where $U_T(c_t^T)$ denotes marginal utility of traded consumption.

For net external asset position F_t:

$$\eta_t = \mathbb{E}_t \beta R_t^W \eta_{t+1} + \mu_t + \Gamma \gamma_t \mathbb{E}_t \lambda_{t+1} \tag{E.3}$$

And complementarity conditions:

$$\eta_t \left(y^T + R_{t-1}^W F_{t-1} - F_t - c_t^T \right) = 0, \eta_t \geq 0 \tag{E.4}$$

$$\gamma_t \left[\lambda_t - \beta \mathbb{E}_t \lambda_{t+1} \left(R_t^W - \frac{\Gamma}{\beta}(F_t - F_t^{cb}) \right) - \mu_t^p \right] = 0, \gamma_t \geq 0 \tag{E.5}$$

$$\mu_t \left[\kappa \left[y^T + y^N \frac{1-\alpha}{\alpha} \left(\frac{c_t^T}{y} \right)^{\frac{1}{\xi}} \right] + F_t \right] = 0, \mu_t \geq 0 \tag{E.6}$$

$$\phi_t \left(F_t^{cb} \right) = 0, \phi_t \geq 0 \tag{E.7}$$

Consider the central bank's policy under discretion, therefore $\gamma_{t-1} = 0$. By definition, $\lambda_t = U_T(c_t^T)$.

From equation (E.2), the social marginal utility of traded consumption consists of three parts: marginal utility of traded goods consumption λ_t, the benefit of relaxing credit constraint when credit constraint is binding $\mu_t > 0$, and the intertemporal reallocation of consumption $\gamma_t \frac{\partial \lambda_t}{\partial c_t^T} \leq 0$.

Equation (E.3) shows the social optimal Euler equation for external saving/borrowing. Social marginal cost of saving is the foregone consumption η_t, while the social marginal benefit of saving consists of relaxing current credit constraint, increasing future consumption via saving in abroad and the intertemporal shifting of consumption, captured by γ_t.

Changes in reserves F_t^{cb} will change domestic interest rate, $\partial R_t / \partial F_t^{cb} = \Gamma / \beta$. Equation (E.1) shows that when the reserve hits its lower bound below and reserves are decreased by one unit, the marginal cost is captured by the shadow price ϕ_t (public borrowing when negative), while the marginal benefit is the product of the change of domestic interest rate (Γ), private marginal utility of consumption λ_{t+1}, and the shadow price of shifting consumption intertemporally (λ_t).

Substituting out $\gamma_t = \frac{\phi_t}{\Gamma \mathbb{E}_t \lambda_{t+1}}$ from the FOCs, and the social Euler equation can be written as

$$\begin{aligned}
&\lambda_t + \mu_t \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(\frac{c_t^T}{y} \right)^{\frac{1}{\xi}-1} + \frac{\phi_t}{\Gamma \mathbb{E}_t \lambda_{t+1}} \frac{\partial \lambda_t}{\partial c_t^T} \\
= \;& \mathbb{E}_t \beta R_t^W \left\{ \lambda_{t+1} + \mu_{t+1} \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(\frac{c_{t+1}^T}{y} \right)^{\frac{1}{\xi}-1} + \frac{\phi_{t+1}}{\Gamma \mathbb{E}_{t+1} \lambda_{t+2}} \frac{\partial \lambda_{t+1}}{\partial c_{t+1}^T} \right\} + \mu_t + \phi_t \quad \text{(E.8)}
\end{aligned}$$

The optimality conditions (E.1), (E.4)-(E.8) determine the optimal path of reserve accumulation F_t^{cb}, consumption c_t^T and total external borrowing F_t. Given the optimal allocation, the corresponding Euler equation for households' borrowing in the competitive equilibrium reads

$$\lambda_t = \beta \mathbb{E}_t \lambda_{t+1} R_t + \mu_t^p \quad \text{(E.9)}$$

Note that μ_t and μ_t^p measure the shadow prices of borrowing faced by the central bank and households respectively, and the only difference is that the social planner will internalize households' borrowing decision on non-traded good price and consequently borrowing capacity. Therefore, $\mu_t \geq \mu_t^p \geq 0$, and the Lagrange multipliers are either positive, $\mu_t^p > 0$, $\mu_t > 0$ or zero $\mu_t^p = \mu_t = 0$.

Subtracting households' Euler equation (E.9) from the central bank's Euler equation (E.8), yields

$$\begin{aligned}
\mathbb{E}_t \beta \left\{ (R_t - R_t^W) \lambda_{t+1} \right\} &= -\mu_t \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(\frac{c_t^T}{y} \right)^{\frac{1}{\xi}-1} + \phi_t \left(1 - \frac{1}{\Gamma \mathbb{E}_t \lambda_{t+1}} \frac{\partial \lambda_t}{\partial c_t^T} \right) \\
&+ \mathbb{E}_t \beta R_t^W \left\{ \mu_{t+1} \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(\frac{c_{t+1}^T}{y} \right)^{\frac{1}{\xi}-1} + \frac{\phi_{t+1}}{\Gamma \mathbb{E}_{t+1} \lambda_{t+2}} \frac{\partial \lambda_{t+1}}{\partial c_{t+1}^T} \right\} + \mu_t - \mu_t^p \quad \text{(E.10)}
\end{aligned}$$

First of all, consider a special case in which F_t^{cb} is unbounded; that is, the central bank could directly borrow from international capital markets but doesn't suffer any intermediation frictions. A specific example of this scenario would be that the central bank engages in a currency swap agreement with international liquidity providers. Suppose that currency swap agreements don't incur any cost and are unlimited to the central bank, then $\phi_t = \gamma_t = 0$ in all periods. The central bank's problem above is degenerated to the constrained efficient allocation as in Bianchi (2011) but uses foreign reserves (currency swaps when $F_t^{cb} < 0$) as the policy instrument to implement the constrained efficient allocation. Due to financial intermediation frictions, the central bank is able to move households' borrowing cost R_t by intervening the foreign exchange market.

When home economy has a very low leverage and stays far away from the crisis region, $\mu_t = 0$, $E_t \mu_{t+1} = 0$, the central bank's Euler equation implies that $R_t = R_t^W$, that is, households directly borrow from the central bank, which in turn borrows from broad and financiers don't channel any fund across borders. Since households are less patient $\beta R_t^W < 1$ and would front-load consumption by borrowing, households' debt would rise over time. When current debt level becomes high enough, a further increase in borrowing and future adverse shocks may trigger a deleveraging crisis in the near future. Consider a case when current borrowing constraint doesn't bind but it may bind in the near future, $\mu_t = 0$, $E_t\mu_{t+1} > 0$, the central bank's Euler equation can be simplified as

$$\mathbb{E}_t\beta\left\{(R_t - R_t^W)\lambda_{t+1}\right\} = \mathbb{E}_t\beta R_t^W\left\{\mu_{t+1}\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t+1}^T}{y}\right)^{\frac{1}{\xi}-1}\right\}$$

The left hand side of expression, $\mathbb{E}_t\beta\left\{(R_t - R_t^W)\lambda_{t+1}\right\}$, represents the marginal cost of acquiring one extra unit of reserves. When the economy has a positive external debt and international financiers hold a negative position in foreign bonds F_t^{fs}, R_t will exceed R_t^W. For the central bank, buying foreign bonds involves the central bank borrowing at rate R_t and lending a lower rate. Equivalently, buying reserves distorts agents' optimal consumption paths. Buying reserves in period t drives up the interest rate spread $R_t - R_t^W$ and reduces the economy's total external debt, thus forcing agents to save more than they otherwise would have. The right hand side of the expression above represents a social benefit of a marginal increase in next period's consumption when future borrowing constraint might bind. The responses of reserves F_t^{cb} and interest rate R_t depend on the right hand side of the expression above $\mu_{t+1}\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t+1}^T}{y}\right)^{\frac{1}{\xi}-1}$, which captures the impact on expected household welfare of the central bank buying reserves at time t, through its effect on the price of future non-traded goods, when future collateral constraint will be binding. While private agents take the price of non-traded goods as given when making their borrowing decisions, the central bank internalizes the effect of its intervention on the non-traded goods price and its effect on the borrowing constraint.

When the economy enters a crisis, $\mu_t > 0$, and it's possible that $E_t\mu_{t+1} = 0$. Then the central bank's Euler equation can be written as

$$\mathbb{E}_t\beta\left\{(R_t - R_t^W)\lambda_{t+1}\right\} = -\mu_t\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_t^T}{y}\right)^{\frac{1}{\xi}-1} + \mu_t - \mu_t^p$$

When current borrowing constraint binds, the central bank will sell reserves to lower interest rate, and therefore shift consumption from period $t+1$ to period t. As analyzed above, the pecuniary externality is captured by $\mu_t\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_t^T}{y}\right)^{\frac{1}{\xi}-1}$. The central bank would internalize such an externality by decumulating F_t^{cb} to lower domestic interest rate R_t in a crisis. We summarize the results below

Result 1. *When reserves are unbounded below, the central bank can use foreign exchange intervention to implement the constrained efficient allocation. In addition, the central bank accumulates reserves to reduce households' borrowing in normal times when collateral constraint might bind in the future, and decumulate reserves in crisis.*

Nevertheless, currency swap agreements between emerging economies and advanced economies are quite limited. In the main text, we focus on the case of $F_t^{cb} \geq 0$. In the competitive equilibrium without policy intervention, all households' borrowing is channelled through international financiers, implying that $R_t > R_t^W$. When they make borrowing decisions, atomistic private agents take non-traded good price and domestic interest rate as given. But the central bank will internalize borrowing and saving decisions on nontraded good prices. Consider first the case in which collateral constraint doesn't bind in period t but may bind in next period $t+1$, the Euler equation under optimal foreign exchange intervention reads,

$$\mathbb{E}_t\beta\left\{(R_t - R_t^W)\lambda_{t+1}\right\} = \phi_t\left(1 - \frac{1}{\Gamma\mathbb{E}_t\lambda_{t+1}}\frac{\partial\lambda_t}{\partial c_t^T}\right)$$

$$+\mathbb{E}_t\beta R_t^W\left\{\mu_{t+1}\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t+1}^T}{y}\right)^{\frac{1}{\xi}-1} + \frac{\phi_{t+1}}{\Gamma\mathbb{E}_{t+1}\lambda_{t+2}}\frac{\partial\lambda_{t+1}}{\partial c_{t+1}^T}\right\}$$

Similar to the unbounded situation, the central bank will accumulate reserves (therefore $\phi_t = 0$ when $F_t^{cb} > 0$) when future borrowing constraint may bind. But the central bank also has to take into account the effect of future reserves being bounded below, captured by $\frac{\phi_{t+1}}{\Gamma\mathbb{E}_{t+1}\lambda_{t+2}}\frac{\partial\lambda_{t+1}}{\partial c_{t+1}^T} \leq 0$. This non-positive sign reflects a fact that if the future crisis is severe and the central bank might use up all reserves $\phi_{t+1} > 0$, the central bank might not aggressively accumulate reserves in the current period since the cost of accumulating reserves outweighs the benefit of relaxing future collateral constraint.

Now consider the case in which current collateral constrain is binding but future constraint doesn't bind and also central bank has accumulated a pile of positive reserves. The central bank's Euler equation can be written as,

$$\mathbb{E}_t\beta\left\{(R_t - R_t^W)\lambda_{t+1}\right\} = -\mu_t\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_t^T}{y}\right)^{\frac{1}{\xi}-1} + \phi_t\left(1 - \frac{1}{\Gamma\mathbb{E}_t\lambda_{t+1}}\frac{\partial\lambda_t}{\partial c_t^T}\right) + \mu_t - \mu_t^p$$

Following a similar logic as in the unbounded scenario, the central bank here would decumulate reserves to lower domestic interest rate R_t. ϕ_t captures the shadow value of one more unit of reserves deployed by the central bank when the level of reserves is at its lower bound. Whether the central bank sells off all its reserves to purchase domestic bond may depend on the severity of a crisis. For a moderate crisis, the central bank may reduce a little bit of reserves, but for a severe crisis, the central bank may deploy all reserves to push down domestic interest rate. When the world interest rate spikes dramatically, the economy may be pushed into a severe crisis, and the Lagrange multiplier for the collateral constraint μ_t would rise substantially. But reserves are bounded below, and the central bank has no other ways to further reduce domestic interest rate. Therefore, the decrease of domestic interest rate in this case would be lower than that when reserves are unbounded, and the difference is captured by $\phi_t\left(1 - \frac{1}{\Gamma\mathbb{E}_t\lambda_{t+1}}\frac{\partial\lambda_t}{\partial c_t^T}\right) > 0$ (note that $\frac{\partial\lambda_t}{\partial c_t^T} < 0$). Similarly, we summarize the result as follows.

Result 2. *When reserves are bounded below (i.e., reserves are nonnegative), the central bank will accumulate reserves in normal times when future collateral constraints might bind and decumulate reserves in crisis. The optimal foreign exchange intervention depends on the trade-off between the cost of holding reserves and the benefit of reducing the incidence of crisis.*

F Optimal FXI in an infinite horizon model

This section presents the policy maker's problem under discretion in an infinite horizon model. As usual, the policy maker's problem is defined as follows: a path of reserve holding $\{F_t^{cb}\}_{t=0}^{\infty}$ that maximizes the representative domestic household's objective function, subject to the constraints in the competitive equilibrium defined in the main text. In order to make the expressions of constraints more concisely, we make the following equivalent changes of variables. Let the exogenous variable be $s_t \equiv R_t^w$, total external borrowing at the beginning of period t, $f_{t-1} \equiv -R_{t-1}^W F_{t-1}$ and reserve holding at the beginning of period t, $f_{t-1}^{cb} \equiv F_{t-1}^{cb} R_{t-1}^W$. Therefore the end-of-period portfolio can be written as $F_t = -f_t / R_t^W$ and $F_t^{cb} = f_t^{cb} / R_t^W$. The Bellman equation for the policy maker's problem can be written as,

$$\mathbf{P_1} : v(f_{t-1}, s_t) = \max_{\{c_t^T, f_t, f_t^{cb}\}} \{u(c_t) + \beta \mathbb{E}_t [v(f_t, s_{t+1})]\} \tag{F.1}$$

subject to

$$c_t^T = y^T - f_{t-1} + \frac{f_t}{R_t^w} \tag{F.2}$$

$$u_{c^T}(c_t)(1 - \mu_t) = \beta R_t \mathbb{E}_t u_{c^T}(c_{t+1}) \tag{F.3}$$

$$\frac{f_t}{R_t^w} \leq \kappa \left(y^T + p_t y^N\right) \tag{F.4}$$

$$\mu_t \left[\kappa \left(y^T + p_t y^N\right) - \frac{f_t}{R_t^w}\right] = 0, 0 \leq \mu_t < 1$$

$$p_t = \frac{1-\alpha}{\alpha} \left(\frac{c_t^T}{y^N}\right)^{\frac{1}{\xi}} \tag{F.5}$$

$$R_t = R_t^w + \frac{\Gamma}{\beta} \frac{f_t + f_t^{cb}}{R_t^w} \tag{F.6}$$

where aggregate consumption and marginal utility can be written as

$$c_t = \left[\alpha (c_t^T)^{\frac{\xi-1}{\xi}} + (1-\alpha)(y^N)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

$$u_{c^T}(c_t) = \alpha c_t^{-\sigma} \left(c_t / c_t^T\right)^{\frac{1}{\xi}}$$

Note that central bank policy f_t^{cb} only appears in equation (F.6) as a control variable and could change domestic bond interest rate R_t, which can facilitate computing an optimal allocation substantially.