A Theory of the Global Financial Cycle

J. Scott Davis and Eric van Wincoop
A Theory of the Global Financial Cycle*

J. Scott Davis† and Eric van Wincoop‡

August 30, 2021

Abstract

We develop a theory to account for changes in prices of risky and safe assets and gross and net capital flows over the global financial cycle (GFC). The multi-country model features global risk-aversion shocks and heterogeneity of investors both within and across countries. Within-country heterogeneity is needed to account for the drop in gross capital flows during a negative GFC shock (higher global risk-aversion). Cross-country heterogeneity is needed to account for the differential vulnerability of countries to a negative GFC shock. The key vulnerability is associated with leverage. In both the data and the theory, leveraged countries (net borrowers of safe assets) deleverage through negative net outflows of risky assets and positive net outflows of safe assets, experience a rise in the current account and a greater than average drop in risky asset prices. The opposite is the case for non-leveraged countries (net lenders of safe assets).

Keywords: Global Financial Cycle; Capital Flows; Current Account

JEL: F30; F40

*We gratefully acknowledge financial support from the Bankard Fund for Political Economy. This paper represents the views of the authors, which are not necessarily the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.
†J. Scott Davis, Federal Reserve Bank of Dallas, scott.davis@dal.frb.org.
‡Eric van Wincoop, University of Virginia and NBER, ev4n@virginia.edu.
1 Introduction

Rey (2015) has characterized a global financial cycle (GFC) as exhibiting a strong global co-movement in asset prices, gross capital flows, leverage, credit and risk premia. There is also a close relationship between the GFC and measures of risk or risk-aversion. The objective of this paper is to develop a theory for the GFC driven by changes in global risk-aversion. Our focus will be on asset prices (both safe and risky) and capital flows (both gross and net). Other features of the GFC, such as those relating to leverage and risk premia, are natural by-products of our analysis.

Specifically, we aim to understand the following set of facts related to a downturn in the GFC (the opposite applies to an upturn):

1. a fall in risky asset prices across the world and a drop in the real interest rate on safe assets
2. a fall in gross capital inflows and outflows (global retrenchment)
3. countries with a negative net foreign asset position in safe assets experience (i) an increase in net outflows of safe assets, (ii) a decrease in net outflows of risky assets, (iii) a rise in the current account (overall net outflows) and (iv) a larger drop in the price of their risky asset; the opposite is the case for countries with a positive net foreign asset position in safe assets
4. the net foreign asset position in risky assets has no predictive power for the response of net capital flows, or the relative drop in a country’s risky asset price, to the GFC

We will provide evidence for these facts using data for 20 developed countries from 1996 to 2020. Figure 1 provides a graphical illustration by comparing the years 2002, 2007 and 2009. There was an upturn in the GFC from 2002 to 2007 and a downturn from 2007 to 2009. The first two facts are illustrated in the top row of charts: the upturn was associated with a substantial increase in gross capital outflows, stock prices and real interest rate, while the opposite was the case during the downturn. These two facts are well established. Miranda-Agrippino and Rey (2020), from hereon MAR, characterize the GFC as the common component of 858 asset price series around the world. Rey (2015) and MAR show that a drop in the GFC is associated with a drop in the federal funds rate (interest rate on safe asset) and a drop in gross capital inflows and outflows (see further discussion below).

The third fact, as it relates to net capital flows, is illustrated in the remaining charts of Figure 1 for the United States and Japan. The United States is a significant net debtor in safe assets (36 percent of GDP in 2007), while Japan is a large net creditor in safe assets (55 percent of GDP in 2007). The United States saw larger net inflows of safe assets during the
upturn, followed by significantly smaller net inflows during the downturn. The same pattern is visible for total net inflows, while net outflows of risky assets rose during the upturn and fell during the downturn. For Japan all these flows moved in exactly the opposite direction.

The third fact says that a leveraged country (net borrower in safe assets) will deleverage during a downturn in the GFC by selling risky assets and buying safe assets (net inflows of risky assets and net outflows of safe assets). In addition it will experience an increase in the current account (total net outflows) and a larger drop in its risky asset price. While the response to the GFC depends critically on the net foreign asset position in safe assets (fact 3), it does not depend on the net foreign asset position in risky assets (fact 4).

To account for these stylized facts we develop a multi-country model where investors can hold risky assets of all countries as well as a safe asset. Critically, there is heterogeneity both within and across countries. The first stylized fact can be understood without any heterogeneity. A rise in global risk-aversion leads to a portfolio shift from risky to safe assets that lowers the price of risky assets and raises the price of the safe asset (lower interest rate). This also raises the risk premium on risky assets. Within-country heterogeneity is important to understand the second stylized fact. We allow for both risk-aversion and home bias heterogeneity across investors within each country. Cross-country heterogeneity is critical for the last two stylized facts. We allow for both risk-aversion heterogeneity across countries and heterogeneity in expected dividends on the risky assets.

Within-country heterogeneity is such that investors that are more leveraged also tend to be less home biased. A downturn of the GFC lowers risky asset prices, which lowers the relative wealth of the more leveraged investors. These investors will then sell risky assets to non-leveraged investors. However, the portfolio of the more leveraged investors has a larger share of foreign assets. Therefore these leveraged investors will end up selling foreign risky assets to foreigners. This implies a drop in gross capital outflows.

Although many papers have documented a drop in gross capital flows during a decline in the GFC, until now there has been no clear theoretical explanation for this phenomenon. In the absence of within country heterogeneity, asset prices will adjust in such a way that there will be no asset trade and therefore no decline in gross capital flows. Home bias heterogeneity is specifically important as otherwise the leveraged investors simply sell foreign assets to non-leveraged investors in the same country and gross capital flows are unaffected.

The two types of cross-country heterogeneity lead to imbalances in the form of net foreign asset positions in safe and risky assets. We investigate the impact of these imbalances on the vulnerability of a country to the GFC. Empirically, among advanced countries, we show

---

1Facts 3 and 4 are related to findings by Davis et al. (2021b). They show that net capital outflows load positively on the product of the GFC and the net foreign asset position in safe assets, but do not depend on the product of the GFC and the net foreign asset position in risky assets. They only look at total net capital flows, not the breakdown into net capital flows of safe and risky assets.
that the two imbalances have the same sign in some countries and opposite signs in others. The two types of heterogeneity in the model are intended to capture this feature of the data, allowing us to distinguish between these two types of imbalances in terms of a country’s vulnerability to the GFC.

We find, both empirically and theoretically, that what matters in terms of vulnerability of a country to the GFC is the net foreign asset position in safe assets, not the net foreign asset position in risky assets. In other words, what matters is whether a country is leveraged or not. With vulnerability we refer to both the response of net capital flows (safe, risky, total) and the relative drop in a country’s risky asset price. Intuitively, a leveraged country will become more leveraged when risky asset prices fall during a downturn of the GFC. It therefore naturally wants to rebalance, or deleverage, through positive net outflows of safe assets and negative net outflows of risky assets. Since leveraged countries experience a larger drop in wealth than non-leveraged countries, they will also experience a larger than average drop in consumption, and therefore a current account surplus. Finally, as a result of home bias, the larger drop in wealth also leads to a larger than average drop in demand for its domestic risky asset and therefore a larger drop in its risky asset price.

In order to keep the model analytically tractable with both within and cross-country heterogeneity, we simplify in the time dimension. Specifically, we assume that uncertainty about future dividends is resolved at time 2. Therefore risk-aversion only matters at time 1 and there is only a time 1 portfolio allocation problem. With this setup we can consider shocks to global risk-aversion at time 1, but we cannot consider cycles where global risk-aversion goes up and down over time.

The paper touches on various literatures. The global co-movements of asset prices, capital flows, risk premia and leverage have been extensively documented. MAR find that a common component of 858 global asset prices accounts for 20 percent of the variance in the data. Davis et al. (2021b) find that a single global factor accounts for 42 percent of the variance of gross capital flows in advanced countries. This capital flow factor is in turn highly correlated with the MAR asset price factor. Davis et al. (2021b) also find that the same global factor accounts for 21 percent of the variance of net capital flows. There is also plenty of evidence that the global financial cycle accounts for a significant portion of the co-movement of risk

---

2Milesi-Ferretti and Tille (2011) document the sharp fall in global capital flows during the 2008 crisis.

Broner et al. (2013) emphasize the strong co-movement between capital inflows and outflows and show that both decline in the years after a crisis. Forbes and Warnock (2012) find that global factors are significantly associated with extreme capital flow episodes.

3Rey (2015), MAR and Habib and Venditti (2019) all find a close association between the global asset price factor and capital flows.

4Related, Ghosh et al. (2014) show that global factors are key determinants of net capital flow surges to emerging markets. Eichengreen and Gupta (2016) show that sudden stops (drop in net capital inflows) are increasingly driven by global factors. Lane and Milesi-Ferretti (2012) document a current account reversal in many countries as a result of the 2009 global financial crisis.
premums (e.g. Jorda et al. (2019) for equity premiums and Bai et al. (2019), Gilchrist et al. (2021) for sovereign spreads) and is closely tied to a global leverage cycle (e.g. Passari and Rey (2015) and Bruno and Shin (2015b)).

At the more disaggregated level, Forbes and Warnock (2014) find that most episodes of large capital flow changes are dominated by debt flows rather than equity flows. This is in line with the third fact listed above, which says that net capital flows and net flows of safe assets fluctuate together along the GFC cycle, while net flows of risky assets go in the opposite direction. Related also, Shen (2020) provides evidence for 104 countries that net equity flows and net debt flows are negatively correlated. Although we only consider comovements associated with the GFC cycle, this is exactly what we find in both the data and model.

The role of time-varying risk or risk-aversion as a driver of the GFC has been emphasized by many authors. Forbes and Warnock (2012) find that an increase in global risk (proxied by the VIX) predicts a drop in capital outflows (“retrenchment”) and a drop in capital inflows (“sudden stops”). MAR and Habib and Venditti (2019) find that the GFC, as measured by a common component in global asset prices, is closely related to the VIX and the risk premium on corporate bonds. Jorda et al. (2019) find that the increase in stock price synchronization since the early 1990s is associated with a synchronization of risk-appetite across countries. Chari et al. (2020) document the significant impact of changes in global risk-aversion on capital flows to emerging markets and emerging market equity and bond prices. Bai et al. (2019) provide evidence that the GFC is driven by a long-run risk component that can account for the high correlation of sovereign spreads.

At a theoretical level, Maggiori (2017), Mendoza et al. (2009) and Gourinchas et al. (2017) develop models that have several features in common with our model, even though they do not focus on the impact of the GFC. They introduce cross-country heterogeneity that leads the home country (United States) to have a negative net foreign asset position in safe assets and a positive net foreign asset position in risky assets, consistent with the data (e.g. Gourinchas and Rey (2007)). This is exactly what happens in our model in countries that are less risk-averse than average. Gourinchas et al. (2017) also generate this feature through risk-aversion heterogeneity, while Maggiori (2017) and Mendoza et al. (2009) obtain the same outcome by assuming that financial frictions are weaker in the U.S. (financial markets are more developed). One can think of the latter as providing a micro foundation for the assumption of risk-aversion heterogeneity.

Maggiori (2017) and Gourinchas et al. (2017) both emphasize the insurance role that the United States provides to other countries. As a result of its leveraged position, it earns

\footnote{MAR further separate between the price of risk (risk-aversion) and the quantity of risk (stock price volatility) and find the former to be very important.}
higher returns in good times (net wealth transfer from other countries) and experiences worse returns in bad times (net wealth transfer to other countries). In our model this is indeed what happens when comparing an upturn in the GFC (drop in global risk-aversion, leading to high asset prices) to a downturn in the GFC (rise in global risk-aversion, leading to low asset prices). Gourinchas et al. (2017) refer to the higher returns in good times as “exhorbitant privilege” and lower returns in bad times as “exhorbitant duty,” and provide empirical evidence for both.

We should finally emphasize what this paper does not do as the theory is by necessity an abstraction. There is no money in the model. We therefore abstract from the role of monetary policy, exchange rates and the zero lower bound. There are also no frictions or externalities in the model. We therefore do not consider macroprudential policies to mitigate the impacts of the GFC (see Chari et al. (2021) for evidence). The model also does not exhibit liquidity shocks and associated fire sales (see Caballero and Simsek (2020)). We also do not discuss alternative drivers of the GFC beyond changes in global risk-aversion, including monetary policy shocks, changes in global demand for safe assets (see Caballero and Krishnamurthy (2009)) and US business cycle shocks (see Boehm and Kroner (2020)).

The remainder of the paper is organized as follows. Section 2 presents empirical evidence related to the four stylized facts. Section 3 describes the model. Section 4 discusses the impact of a global risk-aversion shock. It starts by considering the simplest case without any within or cross-country heterogeneity, which is sufficient to understand the first stylized fact. It then introduces within-country heterogeneity to shed light on the second stylized fact. Section 5 analyzes the impact of the global risk-aversion shock under cross-country heterogeneity to shed light on the last two stylized facts. Section 6 provides a numerical illustration with both within and cross-country heterogeneity. Section 7 concludes.

2 Empirical Results

We first describe the data used for the analysis. After that we estimate a static factor model of outflows and inflows of safe and risky assets in 20 developed countries over the sample 1996-2020. We show that the first factor is highly correlated with the asset price factor from MAR and refer to it as the GFC factor. We then regress equity prices, the real interest rate, and gross and net capital flows on this GFC factor and the factor interacted with global

---

6Rey (2015) and MAR provide evidence on the role of U.S. monetary policy as a driver of the GFC and argue that it may operate through a global risk-aversion channel. Bekaert et al. (2013) also provide evidence that monetary policy affects both risk and risk-aversion (with the latter being stronger). Bruno and Shin (2015b,2015a) focus on monetary policy transmission through risk-taking, operating through capital flows, leverage and exchange rates. Caballero et al. (2017) discuss policy issues associated with a global shortage of safe assets leading to the zero lower bound.
imbalances. This will tell us to what extent the impact of the GFC on capital flows and risky asset prices varies across countries in a way that depends on the net foreign asset position of safe and risky assets.

### 2.1 Data Description

We use capital flow data from twenty developed countries over the period 1996-2020. Here we present results using annual data, but for robustness we use data at the quarterly frequency in the Online Appendix.

Capital flows are obtained from the IMF Balance of Payments data. For country $i$ in year $t$, outflows and inflows of risky assets, $OF_{i,t}^{risky}$ and $IF_{i,t}^{risky}$, include FDI and portfolio equity flows. Outflows of safe assets, $OF_{i,t}^{safe}$, includes portfolio debt flows, “other” outflows (bank lending and deposits), and central bank foreign exchange reserve accumulation. Inflows of safe assets, $IF_{i,t}^{safe}$, includes portfolio debt flows and “other” inflows. Net flows are denoted $NF_{i,t}^{risky}$ and $NF_{i,t}^{safe}$ and are equal to outflows minus inflows. We use external assets and liabilities from the IMF International Investment Position data to obtain the net foreign asset positions in risky and safe assets, $NFA_{i,t}^{risky}$ and $NFA_{i,t}^{safe}$. We use the same classification of FDI and portfolio equity as risky assets and portfolio debt, “other”, and official reserves as safe assets. We normalize all flows and stock variables in country $i$ and year $t$ by that country’s prior year GDP. Normalized variables are written in lower case.

We believe that this separation between safe and risky assets is reasonable given the available data. Some of the portfolio debt and banking categories include high yield corporate debt, which would be better to add to the “risky” category. We only have a finer level of disaggregation available for inflows, not outflows. For portfolio debt and banking inflows, it is possible to observe the sector that issues the security: government, central bank, deposit taking corporations, and other sectors (other financial corporates, non-financial corporate, households, and non-profits). If we designate the last category (other sectors) as risky, and the others as safe, we show in the Online Appendix that 80 to 90 percent of the variance of portfolio debt and banking inflows involves relatively safe assets. The inability to disaggregate portfolio debt and banking outflows along this line is therefore not likely to be a major concern.

---

7We do not include emerging markets. Davis et al. (2021a) show that central bank foreign exchange reserve flows are one of the largest and most volatile components of the balance of payments in emerging market economies, while being only a minor part of the balance of payments in most advanced countries. Since we do not model foreign exchange reserve flows, it makes sense to focus on advanced countries in the empirical analysis.

8The countries are: United States, Singapore, Australia, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Iceland, Israel, Italy, Japan, Korea, Netherlands, Norway, Portugal and Sweden. When quarterly data are used, Switzerland drops off the list.
Risky asset prices are year-end equity prices from the OECD Main Economic Indicators. The interest rate on the safe asset is the one year U.S. Treasury rate from the Federal Reserve Board, minus the 1 year ahead U.S. inflation expectation from the Survey of Professional Forecasters.

2.2 Factor analysis

Consider the following static factor model with \( k \) factors:

\[
y_{i,t} = \bar{y}_i + F_t \lambda_i + \epsilon_{i,t}\]

(1)

for \( y_{i,t} = o_{i,t}^{\text{risky}}, i_{i,t}^{\text{risky}}, o_{i,t}^{\text{safe}}, i_{i,t}^{\text{safe}} \), where \( \lambda_i \) is a \( k \times 1 \) vector of factor loadings and \( F_t \) is a \( 1 \times k \) vector of global factors, and \( \bar{y}_i \) is simply the average value of \( y_{i,t} \) over the sample period. The factor model includes \( 4n \) series, where \( n \) is the number of countries.

The sample length is \( T \). Define \( y_i \) as a \( T \times 1 \) vector that stacks the country-period scalars \( y_{i,t} - \bar{y}_i \). We can then compactly write the factor model as

\[
y = F \Lambda + \epsilon\]

(2)

where \( y \) is a \( T \times 4n \) matrix that that stacks the \( 4n \) series side by side. \( \Lambda \) is a \( k \times 4n \) matrix that stacks \( k \times 1 \) vectors of factor loadings for each series side-by-side. \( F \) is a \( T \times k \) matrix that contains the factors \( F_t \). The factor analysis gives us a \( k \times 4n \) matrix of loadings \( \Lambda \). We regress the \( 1 \times 4n \) vector of capital inflows and outflows in each period \( t \) on the matrix of loadings \( \Lambda \) to estimate the values of the \( k \) factors in period \( t \).

Figure 2 plots the first factor from the factor model (blue line) together with the Miranda-Agrippino and Rey (2020) asset price factor (red line). The latter is a monthly factor. We normalize their monthly factor to have a standard deviation and mean of respectively 1 and 0, and then annualize by taking the average over a year. Our factor based on capital flow data is clearly closely related to the Miranda-Agrippino and Rey (2020) factor based on asset prices. The correlation between our first factor and the MAR factor is 0.80. Below we refer to the factor based on capital flow data as the GFC factor. While we will use this factor in the analysis, we show in the Online Appendix that very similar results are obtained if we instead use the MAR asset price factor.
2.3 Effect of GFC factor on Asset Prices and Gross Capital Flows

First we consider the effect of the GFC factor on equity prices, the real interest rate and gross capital flows. Consider the following panel regression:

\[ y_{i,t} = c_i + \alpha F_t + \varepsilon_{i,t} \]  

(3)

where \( F_t \) is our estimated GFC factor plotted in Figure 2, and the dependent variable \( y_{i,t} \) is either the log equity price \( q_{i,t} \), capital outflows \( o_{i,t}^{\text{risky}} \), capital inflows \( o_{i,t}^{\text{risky}} \) or their sum \( o_{i,t}^{\text{risky}} + i_{i,t}^{\text{risky}} \). Since equity prices are non-stationary, we remove a least-squares linear trend from the log equity price index before regressing on the GFC factor. Results are very similar if instead we use the change in the log equity price. We also regress the U.S. real interest rate on a constant and \( F_t \). We remove a linear least-squares trend from the real interest rate as it is not stationary in our sample. This is related to the long decline in real interest rates for reasons unrelated to the GFC.

The results from these regressions are presented in Table 1. A one standard deviation decrease in the GFC factor, \( F_t = -1 \), is associated with a 60 basis point fall in the real rate on one-year U.S. treasuries and a 10.4% fall in the average equity price. Gross risky asset flows, \( o_{i,t}^{\text{risky}} + i_{i,t}^{\text{risky}} \), decrease by 4.7% of GDP. Both outflows and inflows of risky assets fall in response to the shock. These results are all statistically significant. We can summarize them in the form of the following stylized facts:

**Stylized Fact 1** A decrease in the GFC factor is associated with a fall in the risk-free interest rate, and for the average country, a fall in risky asset prices.

**Stylized Fact 2** A decrease in the GFC factor is associated with a fall in gross outflows and inflows of risky assets.

2.3.1 Effect of the GFC factor: Role of Global Imbalances

We now ask to what extent global imbalances, in the form of net foreign asset positions in safe and risky assets, affect the impact of the GFC on risky asset prices, net capital flows of safe and risky assets, as well as total net outflows. In all countries, equity prices drop in response to a negative GFC shock. But we will ask whether the extent of the drop is related to the net foreign asset position in safe or risky assets in that country. Net capital flows are zero at a global level, but we can ask whether the increase or decrease in net capital flows in individual countries in response to a negative GFC shock is related to existing global imbalances.

It is useful to first take the look at the size of global imbalances. Figure 3 reports a scatterplot of \( n f \hat{a}_i^{\text{safe}} \) and \( n f \hat{a}_i^{\text{risky}} \) for each of the 20 countries, based on the average net
foreign asset positions in safe and risky assets over the sample. While the data on net foreign asset positions are a panel, most of the variation is in the cross-section. Two points are worth making. First, there is less variation across countries in the net foreign asset position in risky assets than safe assets. Across the 20 countries, the standard deviation of the net foreign asset position in risky assets is 0.27, versus 0.79 for safe assets. Second, there is not a clear relationship between these two imbalances. Regressing $nfa_{i,t}^{\text{risky}}$ on $nfa_{i,t}^{\text{safe}}$ yields a coefficient of -0.05 with a standard error of 0.08. There are multiple countries in all four quadrants.

To determine the role of global imbalances, we expand the panel data regression (3) to include net foreign asset positions, both by themselves and interacted with the GFC factor:

$$y_{i,t} = c_i + \alpha_1 F_t + \alpha_2 \left( nfa_{i,t-1}^{\text{risky}} * F_t \right) + \alpha_3 \left( nfa_{i,t-1}^{\text{safe}} * F_t \right) + \alpha_4 \left( nfa_{i,t-1}^{\text{risky}} \right) + \alpha_5 \left( nfa_{i,t-1}^{\text{safe}} \right) + \varepsilon_{i,t}$$

(4)

where for the dependent variable $y_{i,t}$ we consider $nfa_{i,t}^{\text{safe}}$, $nfa_{i,t}^{\text{risky}}$, $nfa_{i,t}$ and $q_{i,t}$. Here $nfa_{i,t} = nfa_{i,t}^{\text{risky}} + nfa_{i,t}^{\text{safe}}$ is equal to total net outflows. We include one or both of the net foreign asset positions, and their interaction with $F_t$, as regressors.

The results from these regressions are presented in Table 2. The coefficients of the non-interacted $nfa_{i,t-1}^{\text{risky}}$ and $nfa_{i,t-1}^{\text{safe}}$ are not significant and are not included in Table 2. The constant term is omitted as well. The coefficient on $nfa_{i,t-1}^{\text{safe}} * F_t$ is always positive and significant for net outflows of safe assets and total net outflows, while it is negative and significant for net outflows of risky assets. Therefore, in response to a downturn in the GFC factor, a country with a negative foreign asset position in safe assets experiences a rise in net outflows of safe assets, as well as a rise in total net outflows, while it experiences a drop in net outflows of risky assets. In other words, during a downturn in the GFC, countries with a net debt in safe assets pay down their debt both by selling foreign risky assets and by increasing their saving (raising total net outflows).

In all of the net outflow regressions, the coefficient on the interaction term between the GFC factor and the country’s net foreign asset position in risky assets, $nfa_{i,t-1}^{\text{risky}} * F_t$, is insignificant. Moreover, when $nfa_{i,t-1}^{\text{safe}} * F_t$ and $nfa_{i,t-1}^{\text{risky}} * F_t$ are both included in the regression, the goodness of fit of the regression is little changed from the regression when $nfa_{i,t-1}^{\text{safe}} * F_t$ enters alone. We can conclude that the impact of the GFC factor on net capital flows does not depend on the net foreign asset position in risky assets.

The last panel in Table 2 includes the results for the regression of the log equity price. The coefficient on $F_t$ captures the effect of the GFC factor on the average equity price. As we saw in Table 1, a one standard deviation fall in the GFC factor is associated with a drop

---

9This is a somewhat crude way of putting it. Instead of selling foreign risky assets, they may also reduce risky asset liabilities to foreigners. And instead of raising saving, they may also reduce investment.
in the equity price of about 11% in the average country. The coefficient of \( nf_a^{safe}_{t,t-1} \times F_t \) is negative and significant, implying that a country with a negative net foreign asset position in safe assets experiences a greater than average drop in its equity price during a downturn in the global financial cycle.

We can summarize these results in two stylized facts:

**Stylized Fact 3** For a country with a negative net foreign asset position in safe assets, a decrease in the GFC factor is associated with an increase in total net outflows and net outflows of safe assets, a decrease in net outflows of risky assets, and a greater than average fall in risky asset prices. The opposite is the case for a country with a positive net foreign asset position in safe assets. These impacts are bigger the larger the absolute size of the net foreign asset position in safe assets.

**Stylized Fact 4** The impact of a change in the GFC factor on net capital flows of safe and risky assets, and equity prices, is not significantly affected by the net foreign asset position of risky assets of a country.

### 2.4 Robustness Analysis

The empirical results are quite robust to a variety of modifications. Robustness analysis is presented in the Online Appendix. Here we just mention the key results. As already pointed out, the results are robust to using quarterly data instead of annual data. They are also robust to using different measures of the global financial cycle. Specifically, we consider two alternatives. One alternative uses the Miranda-Agrippino and Rey (2020) asset price factor for \( F_t \) in the regressions reported in Tables 1 and 2. The other extends the factor model by adding the equity price and real interest rate to the four capital flow variables (inflows and outflows of safe and risky assets), again using the first factor to measure \( F_t \). Both alternative measures of \( F_t \) yield very similar results.

We also consider alternative measures of the equity price and interest rate. We use the change in the log equity price instead of the detrended log equity price. We consider four measures of the interest rate: the 3-month and one-year U.S. Treasury rate, both nominal and real (subtracting inflation expectations from the Survey of Professional Forecasters). None of this significantly affects the results.

### 3 Model Description

We now turn to a model that can replicate the stylized facts discussed above. There are \( N + 1 \) countries with investors and households, and a single good. Although all agents have
infinite horizons, we effectively collapse the future into a single period by assuming that all uncertainty is resolved in period 2. This simplification allows us to focus on the heterogeneity within and across countries that affects the equilibrium in period 1. In each country there is a continuum of investors on the interval \([0,1]\), who are heterogeneous with respect to risk-aversion and home bias. Investors can hold a safe asset and risky assets from each of the \(N + 1\) countries. Households only hold the safe asset and are all the same. Countries are heterogeneous with respect to risk-aversion and expected dividends of the risky assets. We take period 0 as given. The analysis focuses on the impact of a negative GFC shock in period 1 in the form of a rise in global risk-aversion.

3.1 Assets

The gross interest rate on the safe asset is \(R_t\) from period \(t\) to \(t+1\). Over the same period, the return on the risky asset from country \(n\) is

\[
\frac{Q_{n,t+1} + D_{n,t+1}}{Q_{n,t}}
\]

where \(Q_{n,t}\) is the price of the asset and \(D_{n,t}\) the dividend.

The period 1 dividend on all risky assets is set at 1. There is uncertainty about future dividends, but this uncertainty is resolved at time 2. After that dividends will remain constant: \(D_{n,t} = D_{n,2}\) for \(t \geq 2\). In what follows it is useful to denote

\[D_n = \frac{D_{n,2}}{1 - \beta}\]

where \(\beta\) is the time discount rate. \(D_n\) is the present value of dividends at time 2, which is proportional to \(D_{n,2}\). For simplicity of analysis, we assume that \(D_n\) is uncorrelated across countries.

3.2 Investors

3.2.1 Budget Constraint and Preferences

In period \(t\) investor \(i\) from country \(n\) invests a fraction \(z_{n,m,t}^i\) in the risky asset of country \(m\). A fraction \(1 - \sum_{m=1}^{N+1} z_{n,m,t}^i\) is invested in the safe asset. Wealth of investor \(i\) in country \(n\) evolves according to

\[W_{n,t+1}^i = (W_{n,t}^i - C_{n,t}^i) R_{t+1}^{p,i,n}\]
where $C_{i,n,t}$ is consumption and $R_{t+1}^{p,i,n}$ is the portfolio return from $t$ to $t+1$:

$$R_{t+1}^{p,i,n} = R_t + \sum_{m=1}^{N+1} z_{i,n,m,t} \left( \frac{Q_{m,t+1} + D_{m,t+1}}{Q_{m,t}} - R_t \right)$$  \hspace{1cm} (8)$$

The term in brackets is the excess return of the risky asset from country $m$ over the safe asset.

Investors are assumed to have Rince preferences, which for investor $i$ from country $n$ we can write as

$$\ln(V_{n,t}^i) = \max_{C_{n,t}, z_t} \left\{ (1-\beta) \ln(C_{n,t}^i) + \beta \ln \left( [E_t(V_{n,t+1}^i)]^{1-\gamma_{i,n}} \right) \right\}$$  \hspace{1cm} (9)$$

where $z_t = (z_{i,n,1,t}, \ldots, z_{i,n,1+N,t})$ is the vector of portfolio shares chosen by the investor at time $t$. The investor makes consumption and portfolio decisions. The rate of risk-aversion $\gamma_{i,n}$ will generally vary across investors and countries. Risk-aversion only matters at time 1 as we take period 0 as given and uncertainty is resolved from time 2 onward.

### 3.2.2 Within-Country and Cross-Country Heterogeneity

We introduce two types of within-country heterogeneity, associated with investor home bias and risk-aversion. Their main role in the model is to account for Stylized Fact 2 that gross outflows and inflows fall in response to the rise in global risk-aversion. Home bias heterogeneity is introduced by allowing perceived dividend risk of foreign assets to vary across investors. In period 1 all country $n$ investors perceive the variance of $D_n$ to be $\sigma^2$.

For any foreign asset $m \neq n$, investor $i$ from country $n$ perceives the variance of $D_m$ to be $\sigma^2/\kappa_i$, with $0 < \kappa_i < 1$. When $\kappa_i = 1$, all risky assets are perceived to be equally risky and there will be no home bias. When $\kappa_i < 1$ as a result of information asymmetries, foreign assets are perceived to be riskier, leading to a bias towards the domestic risky asset that varies across investors. The lower $\kappa_i$, the stronger the home bias.

Risk-aversion heterogeneity is introduced as follows. Let

$$\gamma_{i,n} = \frac{1}{\Gamma_i (1 + \epsilon_n^G) G}$$  \hspace{1cm} (10)$$

The higher $\Gamma_i$, $\epsilon_n^G$ or $G$, the less risk-averse the investor is. The term $G$ is common across investors and countries. We will introduce a global risk-aversion shock through a drop in $G$. Variation of $\Gamma_i$ across investors leads to within-country risk-aversion heterogeneity.

We also introduce two types of cross-country heterogeneity: variation across countries in risk-aversion and the expected dividend of risky assets. The parameter $\epsilon_n^G$ in (10) repre-
sents heterogeneity in risk-aversion across countries. For expected dividends, we denote the expectation at time 1 of $D_n$ as $\bar{D}_n$ and assume

$$\bar{D}_n = 1 + a + \frac{\sigma^2_a}{a\bar{\psi}} (1 + \epsilon^D_n)$$  \hspace{1cm} (11)$$

Here $a = \frac{\beta}{1-\beta}$ and $\bar{\psi}$ is the mean across investors of $\psi_i = \Gamma_i (1 + N \kappa_i)$. The parameter $\epsilon^D_n$ represents heterogeneity of expected dividends of risky assets. Both $\epsilon^G_n$ and $\epsilon^D_n$ are assumed to have a zero mean across countries.

These two types of cross-country heterogeneity will be important to account for Stylized Facts 3 and 4. They lead to cross-country variation in the net foreign asset position in safe and risky assets that is critical to these stylized facts. With risk-aversion heterogeneity the net foreign asset positions in safe and risky assets will have opposite signs in each country, while under expected dividend heterogeneity they will have the same sign. By combining them, one can consider the separate roles of both imbalances in the response to the global risk-aversion shock. One can also connect to the data by combining both types of heterogeneity to generate pre-shock net foreign asset positions of safe and risky assets in all countries as in Figure 3.

### 3.2.3 Optimal Consumption and Portfolios

The value function will be proportional to the wealth of the agent: $V^i_{n,1} = \alpha^i_{1,n} W^i_{n,1}$ and $V^i_{n,t} = \alpha^i_{2,n} W^i_{n,t}$ for $t \geq 2$. The coefficients $\alpha^i_{1,n}$ and $\alpha^i_{2,n}$ can be derived from the Bellman equation and depend on structural model parameters (see Appendix A), but are not important to the analysis. Using (7), investors at time $t$ therefore maximize

$$(1 - \beta) \ln(C^i_{n,t}) + \beta ln(W^i_{n,t} - C^i_{n,t}) + \beta \ln \left( \left[ E_t \left( R^{p,i,n}_{t+1} \right) \right]^{1-\gamma_{i,n}} \right)$$  \hspace{1cm} (12)$$

Optimal consumption is then

$$C^i_{n,t} = (1 - \beta) W^i_{n,t}$$  \hspace{1cm} (13)$$

All investors consume a fraction $1 - \beta$ of their wealth during each period. This leaves the investor with financial wealth $\beta W^i_{n,t}$ that is invested in safe and risky assets.

Since uncertainty is resolved at time 2, there is only a portfolio problem at time 1. Therefore the only portfolio return that matters is $R^{p,i,n}_2$, which for simplicity we will denote $R^{p,i,n}$. From (12) optimal portfolio shares are chosen to maximize the certainty equivalent of the portfolio return:

$$[E(R^{p,i,n})^{1-\gamma_{i,n}}]^{\frac{1}{1-\gamma_{i,n}}}$$  \hspace{1cm} (14)$$

Using a second order Taylor expansion of $(R^{p,i,n})^{1-\gamma_{i,n}}$ around the expected portfolio return,
one can approximate this as maximizing

\[ E(R_{p,i,n}) - 0.5\gamma_{i,n}\text{var}(R_{p,i,n}) \]  

(15)

This leads to simple mean-variance portfolios.

We will show below (Theorem 1) that the risky asset prices at time 2 are \( Q_{m,2} = [a/(1 + a)]D_m \), so that period 2 asset payoffs are \( Q_{m,2} + D_{m,2} = D_m \). For ease of notation, from hereon we remove time subscripts from all time 1 variables. The portfolio return then becomes

\[ R_{p,i,n} = R + \sum_{m=1}^{N+1} z_{n,m} \left( \frac{D_m - RQ_m}{Q_m} \right) \]  

(16)

Maximizing (15) leads to the following optimal portfolios

\[ z_{n,n} = Q_n \Gamma_i \left( 1 + \epsilon_n^G \right) G \frac{\bar{D}_n - RQ_n}{\sigma^2} \]  

(17)

\[ z_{n,m} = Q_m \Gamma_i \kappa_i \left( 1 + \epsilon_n^G \right) G \frac{\bar{D}_m - RQ_m}{\sigma^2} \quad m \neq n \]  

(18)

A higher \( \Gamma_i \), \( \epsilon_n^G \) or \( G \) implies lower risk-aversion and therefore larger holdings of risky assets for a given expected excess payoff \( \bar{D}_m - RQ_m \) on risky assets. The portfolio expression for foreign risky assets only differs in that it is scaled down by a factor \( \kappa_i \).

### 3.3 Households

Households are identical across countries. We therefore omit the country subscript. Households in any one of the countries maximize

\[ \sum_{s=0}^{\infty} \beta^s \frac{(Ch_{t+s})^{1-\frac{1}{\rho}}}{1 - \frac{1}{\rho}} \]  

(19)

They only hold the safe asset, \( B_t^h \) in period \( t \). Households inherit \( B_0^h \) from time 0. They receive an endowment of \( Y \) each period. The budget constraint is

\[ B_t^h = R_{t-1}B_{t-1}^h + Y - C_t^h \]  

(20)

---

\(^{10}\)A second-order Taylor expansion gives \( (R_{p,i,n})^{1-\gamma_{i,n}} = (ER_{p,i,n})^{1-\gamma_{i,n}} + (1 - \gamma_{i,n})(ER_{p,i,n})^{-\gamma_{i,n}}(R_{p,i,n} - ER_{p,i,n}) - 0.5\gamma_{i,n}(1 - \gamma_{i,n})(ER_{p,i,n})^{-\gamma_{i,n}} - 1(R_{p,i,n} - ER_{p,i,n})^2 \). Taking the expectation, we have \( E(R_{p,i,n})^{1-\gamma} = (ER_{p,i,n})^{1-\gamma} - 0.5\gamma_{i,n}(1 - \gamma_{i,n})(ER_{p,i,n})^{-\gamma_{i,n}} - 1\text{var}(R_{p,i,n}) \). Taking this to the power \( 1/(1 - \gamma_{i,n}) \), and linearly expanding around \( ER_{p,i,n} = 1 \) and \( \text{var}(R_{p,i,n}) = 0 \), gives (15).
The first-order condition is

\[ C_t^h = (R_t \beta)^{-\rho} C_{t+1}^h \]  

(21)

Since households are identical across countries, they cannot be a source of equilibrium net capital flows. The importance of introducing a household sector will become clear in the next section.

### 3.4 Asset Market Clearing

The period \( t \) market clearing conditions for risky assets are

\[
\beta \sum_{m=1}^{N+1} \int_0^1 z_{m,n,t}^i \ W_{m,t}^i \, di = Q_{n,t} K_n \quad n = 1, \ldots, N + 1 
\]

(22)

Here \( K_n \) is the supply of risky asset \( n \), which will be discussed further below.

In addition there is a market clearing condition for safe assets. We can also use the aggregate market clearing condition for all assets that equates the demand to the supply of all assets:

\[
(N + 1) B_t^h + \beta \sum_{n=1}^{N+1} \int_0^1 W_{n,t}^i \, di = \sum_n Q_{n,t} K_n 
\]

(23)

Safe assets are in zero net supply. We can also show that (23) corresponds zero world saving. This needs to be the case as there is no investment in the model.

### 3.5 Pre-Shock Equilibrium

We are interested in the impact of a global risk-aversion shock that lowers \( G \). But we first describe the pre-shock equilibrium, for which we assume \( G = 1 \). We make the following assumptions regarding initial conditions at time 0 and risky asset supplies:

**Assumption 1** Assume the following initial conditions for period 0: \( W_{n,0}^i = 1 + a \) for all investors, \( Q_{n,0} = a, R_0 = (1 + a)/a, B_0^h = a \left( \frac{1}{N+1} \sum_{n=1}^{N+1} K_n - 1 \right) \), and

\[
z_{n,n,0}^i = \Gamma_i \frac{1}{\psi} \left( 1 + \epsilon_n^G \right) \left( 1 + \epsilon_n^D \right) 
\]

(24)

\[
z_{m,n,0}^i = \Gamma_i \kappa_i \frac{1}{\psi} \left( 1 + \epsilon_m^G \right) \left( 1 + \epsilon_n^D \right) \quad m \neq n 
\]

(25)

Also assume \( K_n = \left( E(\Gamma) \left( 1 + \epsilon_n^G \right) + E(\Gamma \kappa) \sum_{m \neq n} \left( 1 + \epsilon_m^G \right) \right) \frac{1}{\psi} (1 + \epsilon_n^D) \).

Here the expectation operator refers to the cross-sectional mean, so for example \( E(\Gamma) = \int_0^1 \Gamma_i \, di \). The period 0 assumptions are such that the market clearing conditions (22)-(23)
are satisfied for period 0. Appendix A then proves the following regarding the pre-shock equilibrium:

**Theorem 1** Under Assumption 1 and $G = 1$, there is an equilibrium where in period 1: $Q_n = a$, $W^i_n = 1 + a$, $z^i_{n,n} = z^i_{n,n,0}$ and $z^i_{n,m} = z^i_{n,m,0}$. In all periods $t \geq 1$: $R_t = (1 + a)/a$, $B^h_t = B^h_0$. In all periods $t \geq 2$: $Q_{n,t} = [a/(1 + a)]D_n$, $W^i_{n,t} = W^i_{n,2}$.

Therefore risky asset prices, the interest rate, wealth, portfolio allocation and household bond holding are all the same in period 1 as in period 0. One can think of this as a kind of steady state, which will subsequently be perturbed by a global risk-aversion shock at time 1. Since quantities of asset holdings are also the same in periods 0 and 1, there will be zero gross and net capital flows of safe and risky assets in the pre-shock equilibrium in period 1. Net foreign asset holdings of safe and risky assets will generally be non-zero in the pre-shock equilibrium if there are cross-country asymmetries. Their values will be discussed in Section 5.1.

A couple of comments about the asset supplies $K_n$ and household bond holdings $B^h$ are in order. Without cross-country heterogeneity ($\epsilon^G_n = \epsilon^D_n = 0$ for all $n$), Assumption 1 implies that $K_n = 1$ in all countries. If for example we introduce risk-aversion heterogeneity, with $\epsilon^n > 0$, there will be increased relative demand for country $n$ risky assets due to home bias. We then change asset supplies accordingly to keep risky asset prices identical across countries in period 1 when $G = 1$. In a model with investment, a higher demand for an asset would eventually be accommodated through a higher supply. We think of the pre-shock equilibrium as capturing such an initial state. With regards to household bond holdings $B^h$, it can be checked from Assumption 1 that it is zero at all times, unless both types of cross-country heterogeneity are assumed, which we only do in the numerical application in Section 6.

### 3.6 Period 1 Capital Flows

After the risk-aversion shock, capital flows are generally no longer zero. Time 1 capital outflows $OF^r_n$ are defined as net purchases of foreign risky assets by country $n$ investors, while time 1 inflows $IF^r_n$ are net purchases of country $n$ risky assets by foreign investors.
These are equal to\(^{11}\)

\[
OF_{n}^{\text{risky}} = \frac{a}{1 + a} \int_{0}^{1} \sum_{m \neq n} z_{n,m}^{i} W_{n}^{i} d\bar{t} - \int_{0}^{1} \sum_{m \neq n} Q_{m} z_{n,m,0}^{i} d\bar{t}
\]  

(26)

\[
IF_{n}^{\text{risky}} = \frac{a}{1 + a} \int_{0}^{1} \sum_{m \neq n} z_{n,m,n}^{i} W_{m,n}^{i} d\bar{t} - Q_{n} \int_{0}^{1} \sum_{m \neq n} z_{n,m,0}^{i} d\bar{t}
\]  

(27)

Define the fraction that investor \(i\) from country \(n\) invests in all risky assets as

\[
z_{n}^{i} = \sum_{m=1}^{N+1} z_{n,m}^{i}
\]  

(28)

with its time zero value denoted \(z_{n,0}^{i}\). Period 1 net flows of safe assets are then

\[
NF_{n}^{\text{safe}} = B_{h}^{h} - B_{0}^{h} + \frac{a}{1 + a} \int_{0}^{1} (1 - z_{n}^{i}) W_{n}^{i} d\bar{t} - a \int_{0}^{1} (1 - z_{n,0}^{i}) d\bar{t}
\]  

(29)

Here \(B_{h}^{h} - B_{0}^{h}\) are net flows associated with households, which equals household saving, while the rest captures the change in safe asset holdings from period 0 to 1 by investors. Household saving is equal to \((1/a)B_{0}^{h} + Y - C_{1}^{h}\), where consumption is\(^{12}\)

\[
C_{1}^{h} = \frac{1}{1 + a^{\rho}(1 + a)^{1-\rho} R^{\rho-1}} \left( Y + \frac{1 + a}{R} Y + \frac{1 + a}{a} B_{0}^{h} \right)
\]  

(30)

4 Impact Global Risk-Aversion Shock

We now consider the impact of a negative GFC shock in the form of a rise in global risk-aversion in period 1 (drop in \(G\)). All analytical results consider the derivatives of time 1 endogenous variables with respect to \(G\) at \(G = 1\). In this section we assume that there is no cross-country heterogeneity, so that \(\epsilon_{n}^{G} = \epsilon_{n}^{D} = 0\) for all countries. Then \(K_{n} = 1\), \(B_{0}^{h} = 0\) and \(\bar{D}_{n} = \bar{D} = 1 + a + a^{2} \bar{W}_{n}\) in all countries. All period 1 risky asset prices will be the same and denoted \(Q\). Net capital flows will be zero.

We start with the simplest case where there is also no heterogeneity within countries, so that \(\Gamma_{i} = \Gamma\) and \(\kappa_{i} = \kappa\). This allows us to account for Stylized Fact 1, the drop in risky asset prices and interest rate. After that we introduce within-country heterogeneity, which

---

\(^{11}\)The time 0 portfolio shares correspond to time 0 quantities of assets. By Assumption 1, investors have a financial wealth of \(\beta W_{n,0} = a\) at time 0, so that their nominal investment is \(az_{n,m,0}^{i}\) in the country \(m\) asset. This corresponds to a quantity of \(z_{n,m,0}^{i}\) as \(Q_{m,0} = a\) by Assumption 1.

\(^{12}\)This is derived using \(C_{2}^{h} = Y + (R/(1 + a))B_{1}^{h}\), the first-order condition (21) and the budget constraint (20) for \(t = 1\).
allows us to also account for Stylized Fact 2, a drop in gross capital outflows and inflows of risky assets. In the next section we will consider cross-country heterogeneity, which leads to global imbalances that are needed to account for Stylized Facts 3 and 4.

In both this section and the next, we make the following parameter assumption, which we will argue is quite weak:

**Assumption 2** \( \bar{\psi} (1 + a)^2 > \sigma^2 \)

### 4.1 Risky Asset Prices and Interest Rate

We start with the simplest case, where there is no heterogeneity, either within or across countries. In that case we need to determine the impact of a drop in \( G \) on the risky asset price \( Q \) and the interest rate \( R \). They can be jointly solved from the asset market clearing conditions (22)-(23) in period 1. After substituting optimal portfolio shares (17), investor wealth \( W_n = 1 + Q \), and the household budget constraint (20), these become

\[
(\bar{D} - RQ)(1 + Q) = \frac{1 + a\sigma^2}{\bar{\psi}G} \tag{31}
\]

\[
Y - C^h_1 - \frac{Q-a}{1+a} = 0 \tag{32}
\]

The first equation is the risky asset market equilibrium condition (RAE). The left hand side of (31) shows that demand for risky assets depends on the risky asset price \( Q \) both negatively (first term) and positively (second term). On the one hand, a rise in \( Q \) lowers the expected return on risky assets, lowering its demand. On the other hand, it raises wealth of investors, which raises demand for risky assets. We will assume that the first effect dominates, so that a higher risky asset price reduces demand for risky assets. Taking the derivative of this RAE condition at the pre-shock equilibrium, this is the case when Assumption 2 is satisfied.

Since there is no within-country heterogeneity, Assumption 2 becomes \( \bar{\psi}(1 + a)^2 > \sigma^2 \). This implies that the expected excess return on the risky asset, \( \frac{\bar{D} - R}{Q} \), is less than the square of the gross risk free interest rate. Since the latter is more than 100%, this is clearly a very weak assumption. Assumption 2 then implies a negative relationship between \( Q \) and \( R \) in the risky asset market equilibrium condition. A rise in \( Q \) lowers demand for risky assets, while a drop \( R \) raises it (investors reallocate from safe to risky assets).

The aggregate asset market equilibrium condition (32) is equivalent to a zero saving condition \( (S = 0) \). Since world saving is zero, and there is no cross-country heterogeneity, saving must be zero in each country. The term \( Y - C^h_1 \) is equal to household saving, while the term \( -(Q-a)/(1+a) \) is saving by investors. Household saving depends on the interest
rate, while saving by investors depends on the risky asset price. The higher the risky asset price, the higher the wealth of the investors, which raises consumption and lowers saving. Taking the derivative of (30) at the pre-shock equilibrium, we have

$$\frac{\partial C_h}{\partial R} = -\rho Y \frac{a^2}{(1 + a)^2} \equiv -\lambda$$

(33)

A higher interest rate lowers household consumption, and therefore raises household saving, and more so the higher $\rho$. It follows that (32) implies a positive relationship between $Q$ and $R$.

The equilibrium is illustrated in Figure 4. The zero saving condition $S = 0$ is the upward sloping line and is not affected by a drop in $G$. The risky asset market equilibrium schedule RAE is downward sloping. The pre-shock equilibrium is at point A. From (31) we can see that a rise in global risk-aversion (drop in $G$) shifts the RAE schedule to the left. The new equilibrium is at point C. The increase in global risk-aversion leads to a reallocation from risky to safe assets that lowers both $Q$ and $R$.

How much risky asset prices drop depends on how much the interest rate falls. The more the interest rate falls, the less risky asset prices drop. We can consider two extremes. One is a situation where household saving does not depend on the interest rate, which occurs when the intertemporal elasticity of substitution $\rho$ of households is zero. In that case the $S = 0$ schedule is vertical. The risky asset price $Q$ is then unaffected by the global risk-aversion shock. Although higher risk-aversion lowers demand for risky assets, this is neutralized by a lower interest rate that equally raises demand for risky assets. Since household saving is unaffected by the interest rate, investor saving cannot change either in equilibrium. This can only be the case when the risky asset price does not change.

The other extreme is where household saving is infinitely interest rate elastic, so that $\rho \to \infty$. The $S = 0$ schedule is then horizontal. In this case the interest rate does not change. This leads to an equilibrium at point B, where the drop in $Q$ is largest as there is no reallocation back to risky assets due to a lower interest rate. In general, in order for the risky asset price to drop, household saving must be interest rate elastic. An alternative is to introduce investment that depends negatively on the interest rate, in which case the $S = 0$ schedule becomes the $S = I$ schedule.

Algebraically, the changes in $Q$ and $R$ in response to a change in $G$ are:

$$\frac{dQ}{dG} = \frac{(1 + a) \frac{\sigma^2}{\psi}}{(1 + a)^2 - \frac{\sigma^2}{\psi} + \frac{\sigma^2}{\lambda}}$$

(34)

$$\frac{dR}{dG} = \frac{1}{\lambda(1 + a)} \frac{dQ}{dG}$$

(35)
If Assumption 2 holds, the denominator of (34) is positive. The more sensitive household saving is to the interest rate (larger \( \lambda \)), the larger the drop in the risky asset price and the lower the drop in the interest rate. Also note that both the risky asset price and interest rate drop more when \( \sigma^2/\tilde{\psi} \) is higher, with \( \tilde{\psi} = \Gamma(1 + N\kappa) \). This happens when risk or risk-aversion are higher. A higher \( \sigma^2 \) raises risk of all risky assets, while a lower \( \kappa \) raises the perceived risk of foreign risky assets. The RAE schedule in Figure 4 then shifts further to the left when \( G \) falls.

These findings account for Stylized Fact 1 and are summarized as follows:

**Theorem 2** Assume that there is no heterogeneity of investors within or across countries, and Assumptions 1 and 2 hold. Then a rise in global risk-aversion lowers risky asset prices equally in all countries and also lowers the interest rate on the safe asset.

### 4.2 Gross Capital Flows Risky Assets

We now allow \( \Gamma^i \) and \( \kappa^i \) to vary across investors within countries. Results for this case are derived in Appendix B. We make the following assumption regarding the cross-sectional distribution of \( \Gamma^i \) and \( \kappa^i \):

**Assumption 3** \( \kappa^i - \bar{\kappa} = \epsilon^\kappa_i, \quad \Gamma^i - \bar{\Gamma} = \omega \epsilon^\kappa_i + \epsilon^\Gamma_i, \) where \( \epsilon^\kappa \) and \( \epsilon^\Gamma \) are symmetric, \( E(\epsilon^\kappa) = E(\epsilon^\Gamma) = 0, \) \( \epsilon^\kappa \perp \epsilon^\Gamma, \) \( \omega \geq 0, \Gamma_i > 0, \kappa_i > 0, \) \( \text{var}(\epsilon^\kappa) > 0. \)

Here \( \bar{\Gamma} \) and \( \bar{\kappa} \) are the mean of the cross-sectional distributions of \( \Gamma^i \) and \( \kappa^i \). Appendix B then derives the following result:

**Theorem 3** Assume that there is heterogeneity across investors within countries, but no cross-country heterogeneity, and Assumptions 1, 2, and 3 hold. Then a rise in global risk aversion lowers risky asset prices equally in all countries and lowers the interest rate on the safe asset. Net capital flows of safe and risky assets remain zero, but gross capital outflows and inflows of risky assets fall.

Introducing within-country heterogeneity, we can account not just for Stylized Fact 1, but also for Stylized Fact 2 that gross capital flows drop in response to the global risk-aversion shock. For this to be the case, the conditions in Assumption 3 must hold. Specifically, there must be heterogeneity in the home bias parameter \( \kappa^i \). Heterogeneity in risk-aversion across investors is not critical, but can amplify the drop in gross capital flows when investors with low home bias also tend to have low risk-aversion. While gross capital flows drop, net capital flows remain zero as we have not yet introduced any cross-country heterogeneity.
In order to understand why there is a decline in gross capital flows, it is useful to start with the portfolios of investors prior to the risk-aversion shock. Using (24)-(25), the total share that investor $i$ in any country invests in risky assets is

$$z^i = \frac{\psi^i}{\bar{\psi}}$$

(36)

where $\psi^i = \Gamma_i(1 + N\kappa_i)$. This means that the average portfolio share in risky assets is 1. The share invested in risky assets is higher the lower an investor’s risk-aversion and home bias. When $z^i > 1$, investor $i$ is leveraged and therefore borrows the safe asset. Investors for which $z^i < 1$ are not leveraged. They hold less risky assets, while holding a positive amount of the safe asset.

The period 1 wealth of investor $i$ from any country is

$$W^i = 1 + a + z^i(Q - a)$$

(37)

Clearly, more leveraged investors experience a larger drop in wealth when risky asset prices fall.

Using the equilibrium portfolios (17)-(18) and the risky asset market clearing condition (22), we can show that the quantities of all risky assets that an investor holds go up or down in proportion to their holdings before the shock. The total quantity of risky assets held by investor $i$ is $z^i$ before the shock and

$$\frac{\int_0^1 W^i z^i}{\int_0^1 z^j W^j dj}$$

after the shock. If the wealth of an investor drops relative to the weighted value of the wealth of all investors, it sells risky assets in equilibrium. Those investors whose wealth drops less than the weighted average will buy risky assets.

The change in the quantity of risky assets held by investor $i$ is then $Z^i dq$, where

$$Z^i = \frac{z^i(z^i - 1 - var(z))}{1 + a}$$

(38)

Here $var(z)$ is the cross-sectional variance of $z^i$. Investor $i$ therefore sells risky assets when $z^i > 1 + var(z)$. The reason that $z^i$ needs to be above $1 + var(z)$ rather than 1 is that what matters is the wealth of investor $i$ relative to a weighted average of the wealth of all investors. Leveraged investors have the largest weight as they hold the most risky assets. Since the cross-sectional mean of $z^i$ is 1, less than half of investors sell risky assets, while more than half buy risky assets.
The impact of the shock on gross capital flows of risky assets depends on the extent of home bias of investors that are buying and selling risky assets. Specifically, let $z_i^F$ be the pre-shock fraction of risky assets allocated to foreign risky assets by investor $i$:

$$z_i^F = \frac{N\kappa_i}{1 + N\kappa_i}$$  \hspace{1cm} (39)

A higher $\kappa_i$ implies less home bias and therefore a larger $z_i^F$. Appendix B shows that gross capital flows drop as a result of the global risk-aversion shock when $E(z_F Z) > 0$. Since $E(Z) = 0$, this happens when $\text{cov}(z_F, Z) > 0$  \hspace{1cm} (40)

Therefore gross capital flows drop in period 1 when investors that sell risky assets ($Z > 0$) tend to invest a relatively large share of risky assets abroad. To see this, we can split investors into two groups: the highly leveraged group that is selling risky assets ($Z^i > 0$) and the other group that is buying risky assets ($Z^i < 0$). Overall the quantity of risky assets sold by the first group is equal to the quantity bought by the second group as $E(Z) = 0$. However, when (40) holds, the first group is less home biased and therefore sells more foreign risky assets than the second group is buying. These foreign risky assets are therefore sold to foreigners, which reduces gross capital flows.

Key to the result is that there is a relationship between leverage and home bias. Investors that are sufficiently leveraged, which will sell risky assets, must also have less home bias. They will then sell more foreign risky assets than the other group of investors is buying. This relationship applies when there is heterogeneity in the home bias parameter $\kappa_i$. A high $\kappa_i$ implies both high leverage and a large fraction invested in foreign assets. The relationship is even stronger when investors with a high $\kappa_i$ also have a low rate of risk-aversion (high $\Gamma_i$). This is the case when $\omega > 0$ and makes the investors that hold a lot of foreign assets even more leveraged.

However, this relationship between leverage and home bias does not apply, and (40) does not hold, when there is only risk-aversion heterogeneity. We know that the overall quantities of risky assets bought and sold by the two groups are equal. When all investors are equally home-biased, this applies to foreign risky assets as well. One group of domestic investors then simply sells foreign risky assets to the other group of domestic investors, leaving gross capital flows unchanged.

Figure 5 provides an illustration. Parameters unrelated to within-country heterogeneity are discussed in Section 6. We assume $\epsilon^\kappa \sim U(-\Delta\kappa, \Delta\kappa)$ and $\epsilon^\Gamma \sim U(-\Delta\Gamma, \Delta\Gamma)$. The left panel considers values of $\Delta\kappa$ from 0 to $\bar{\kappa} = 0.0193$. The blue line shows gross flows of risky assets when $\Delta\Gamma = \omega = 0$. The larger the dispersion of home bias, the larger the drop in gross flows of risky assets. The red line assumes $\omega = 0.25$, so that investors with less home
bias are also less risk-averse. This leads to an even larger drop in gross flows as investors with large holdings of foreign assets are now even more leveraged and therefore sell even more assets. Finally, the green line shows that introducing risk-aversion heterogeneity that is uncorrelated with home bias heterogeneity ($\Delta_\Gamma = \bar{\Gamma} = 0.1$) further increases the drop in gross flows. The largest sale of foreign assets happens by investors that happen to have both low home bias and very low risk-aversion.

The right panel of Figure 5 shows how the increase in global risk-aversion affects outflows of risky assets by individual investors, arranged by their leverage $z^i$. This is done for two cases. The first case (blue line) assumes $\Delta_\kappa = \bar{\kappa} = 0.0193$, $\Delta_\Gamma = 0$ and $\omega = 0.25$. Investors for whom $z^i < 1 + \text{var}(z)$ buy risky assets, while those for whom $z^i > 1 + \text{var}(z)$ sell risky assets. But since the former group does not hold a lot of foreign risky assets, their outflows of risky assets are small. Investors for whom $z^i > 1 + \text{var}(z)$ hold a lot of foreign risky assets, leading their negative outflows of risky assets to be larger, so that overall outflows are negative. The other case (red line) assumes only risk-aversion heterogeneity. Here we see that the negative capital outflows of investors for whom $z^i > 1 + \text{var}(z)$ is equal to the positive capital outflows of investors for whom $z^i < 1 + \text{var}(z)$. This is because there is no home bias heterogeneity. The first group simply sells foreign risky assets to the second group. Overall gross outflows therefore remain unaffected by the risk-aversion shock.

4.3 Discussion

Stepping away from the details of the model for a moment, we can ask more broadly what would be needed to generate a decline in gross capital flows in response to a global risk-aversion shock. Some type of heterogeneity is needed as otherwise we end up with a no trade theorem, where risky asset prices drop after the global risk-aversion shock without any need for trade and therefore no change in gross capital flows. Abstracting from cross-country heterogeneity, as our focus here is on gross and not net flows, it is natural to consider within-country heterogeneity.

Assuming that the risk-aversion shock does not change the supply of assets, asset trade can only happen if some are willing to sell and others willing to buy. This requires that relative demand for risky assets changes across investors. In our model this change in relative demand is generated by a change in relative wealth, which in turn is associated with differences in leverage across agents. One can imagine other ways of generating a change in relative demand. For example, the increase in risk-aversion may vary across investors. Those that experience the largest increase in risk-aversion will have a larger drop in demand for risky assets than others.\(^{13}\)

\(^{13}\)Note that a change in risky asset prices changes all portfolios proportionately. Outside of a change in relative wealth, this therefore does not change relative demand.
But a change in relative demand is not sufficient. If all investors start with the same allocation of their portfolio across domestic and foreign risky assets, investors whose relative demand for risky assets falls the most in equilibrium will simply sell to other domestic investors whose demand falls less. A sufficient drop in the price will make the latter group willing to buy. There would simply be a reshuffling of the holding of risky assets among investors in each country. Gross capital flows are unaffected. This happens in our model when there is only risk-aversion heterogeneity, but no home bias heterogeneity.

It is therefore clear that the risky asset portfolios must vary across investors. Specifically, there must be heterogeneity in home bias and it must be such that investors whose relative demand for risky assets falls also have the lowest home bias. In equilibrium they will then end up selling foreign risky assets to foreign investors, generating the observed decline in gross capital flows. Heterogeneity in home bias is not a big assumption. For example, Hau and Rey (2008) and Coeurdacier and Rey (2013) document significant home bias heterogeneity at the mutual fund level.

In our model the home bias heterogeneity also generates leverage heterogeneity that leads to a change in relative demand for risky assets across investors through a change in relative wealth. It may well be that there are further drivers of such changes in relative demand at the time of the global risk-aversion shock, such as changes in relative risk-aversion across investors or changes in relative informedness of different investors. But it must be the case that those investors whose demand for risky assets falls the most are also the least home biased.

4.4 Leverage, Risk Premia and Credit

So far we have focused on the impact of a global risk-aversion shock on asset prices and gross capital flows. The model also has implications for leverage, risk premia and credit. The implication for risk premia is immediate. The expected excess return on the risky assets is \((\bar{D}/Q) - R\), which rises when \(Q\) and \(R\) drop. Naturally, the premium on risky assets will be higher when global risk-aversion is higher.

The implication for leverage is a bit more subtle. As shown in Appendix C, \(z^i\) drops due to the increase in global risk-aversion as long as \(var(z) < 1/a\), where \(var(z)\) is the cross-sectional variance of \(z^i\) across investors within a country. As long as this cross-sectional variance is not too large, it means that investors that were leveraged before the shock will be less leveraged after a rise in global risk-aversion. This is consistent with for example Rey (2015), who finds that an increase in the VIX leads to reduced leverage.

To understand why this would not be the case with sufficiently large within-country heterogeneity, consider the market clearing condition for risky assets. Demand for risky assets depends on the product of \(z^i\) and \(W^i\), summed across investors. If the heterogeneity
is very large, there is a large drop in wealth $W^i$ exactly for those investors whose portfolio share $z^i$ of risky assets is large. To clear the market, an average increase in $z^i$ is needed, which is accomplished through a larger drop in $Q$ that generates a larger expected excess return on risky assets.

Another important element that affects the response of leverage to a risk-aversion shock is the supply of safe assets. We have assumed that the net supply of safe assets is zero, so that the average portfolio share in risky assets $z^i$ is 1. It is however easy to consider an extension with a positive supply of the safe asset, leading the average portfolio share in risky assets to be less than 1. Average wealth is then less sensitive to the risky asset price. Abstracting for a moment from within-country heterogeneity, the market clearing condition implies that the period 1 portfolio share is $z = Q / [\beta W]$. As $W$ is less sensitive to $Q$, it implies that $z$ drops more when $Q$ falls. Leveraged investors therefore experience a larger drop in leverage in equilibrium with a positive supply of the safe asset.

Finally, bank credit does not explicitly exist in the model as we have not formally modeled banks. But one way to think about this is as the saving of households, intermediated through a banking system. A rise in global risk aversion lowers the interest rate, which lowers household saving and therefore reduces credit under this interpretation of the model. This is again consistent with the findings reported in Rey (2013).

5 Impact Global Risk-Aversion Shock: Role of Global Imbalances

We now analyze what happens in response to the global risk-aversion shock when there is cross-country heterogeneity in either risk-aversion ($\epsilon^G_n$ varies across countries) or the expected dividends of the risky assets ($\epsilon^D_n$ varies across countries). These asymmetries lead to pre-shock global imbalances in the form of non-zero net foreign asset positions of safe and risky assets. We analyze the role that these imbalances play in the impact of the global risk-aversion shock on net capital flows and risky asset prices. To keep the analysis tractable, we only consider one type of heterogeneity at a time and assume that there is no within-country heterogeneity ($\Gamma_i = \bar{\Gamma}$ and $\kappa_i = \bar{\kappa}$). The next section will consider a numerical illustration where all within and cross-country heterogeneities are present simultaneously.
5.1 Net Foreign Asset Positions

The global imbalances are the pre-shock net foreign asset positions of safe and risky assets in period 1, which are equal to those in period 0. For country \( n \) these are

\[
\begin{align*}
NF A_{n}^{safe} & = B^{h} + \left( 1 - \sum_{m=1}^{N+1} z_{n,m} \right) \beta W_{n} \tag{41} \\
NF A_{n}^{risky} & = \sum_{m \neq n} z_{n,m} \beta W_{n} - \sum_{m \neq n} z_{m,n} \beta W_{m} \tag{42}
\end{align*}
\]

Here we have removed the investor superscripts as all investors within a country are identical. The net foreign asset position in safe assets is equal to the safe assets held by households plus investors. The latter is equal to the share \( 1 - \sum_{m=1}^{N+1} z_{n,m} \) invested in safe assets times the financial wealth \( \beta W_{n} \) invested in all assets. The net foreign asset position in risky assets is equal to external assets (the first term) minus liabilities (the second term).

By Theorem 1, before the risk-aversion shock financial wealth is \( \beta W_{n} = a \) in all countries, period 1 portfolio shares correspond to (24)-(25), and \( B^{h} = 0 \). We can substitute these into (41)-(42). First consider the case of risk-aversion heterogeneity across countries. We then have

\[
\begin{align*}
NF A_{n}^{safe} & = -a \epsilon_{n}^{G} \tag{43} \\
NF A_{n}^{risky} & = a \frac{(N + 1) \kappa}{1 + N \kappa} \epsilon_{n}^{G} \tag{44}
\end{align*}
\]

When \( \epsilon_{n}^{G} > 0 \), investors in country \( n \) are less risk averse than average. The country is then leveraged with \( z_{n} = 1 + \epsilon_{n}^{G} > 1 \). It has a positive net foreign asset position in risky assets and a negative net foreign asset position in safe assets. It is also immediate that the overall net foreign asset position is negative.

Next consider cross-country heterogeneity in expected dividends. We then have

\[
\begin{align*}
NF A_{n}^{safe} & = -a \frac{1 - \kappa}{1 + N \kappa} \epsilon_{n}^{D} \tag{45} \\
NF A_{n}^{risky} & = -a \frac{(N + 1) \kappa}{1 + N \kappa} \epsilon_{n}^{D} \tag{46}
\end{align*}
\]

If \( \epsilon_{n}^{D} > 0 \), the risky asset of country \( n \) has a relative high expected dividend. When there is home bias (\( \kappa < 1 \)), this additional appeal of the country \( n \) risky asset will cause it to invest more in risky assets than other countries and therefore be leveraged: \( z_{n} = 1 + [(1 - \kappa)/(1 + N \kappa)] \epsilon_{n}^{D} > 1 \). Leverage is accomplished through a negative net foreign asset position in safe assets. The high expected dividends of the country \( n \) asset will reduce the holding of
foreign risky assets by country $n$ investors and increase the holding of country $n$ risky assets by foreigners. This leads to a negative net foreign asset position in risky assets.

While it is possible to consider other types of cross-country heterogeneity, these two are attractive because of their opposite effects on the sign of net foreign asset positions. In the case of risk-aversion heterogeneity, the net foreign asset positions of safe and risky assets always have opposite signs, while in the case of expected dividend heterogeneity they always have the same sign. Any combination of $NFA^{safe}_n$ and $NFA^{risky}_n$ that we observe in the data can be achieved in the model with a combination of $\epsilon^G_n$ and $\epsilon^D_n$. When presenting the analytical results in this section we focus on only one type of cross-country heterogeneity at a time, but when considering a calibrated numerical model in the next section we set $\epsilon^G_n$ and $\epsilon^D_n$ to match the observed values of $NFA^{safe}_n$ and $NFA^{risky}_n$ in Figure 3.

5.2 Net Capital Flows and Risky Asset Prices

We now consider how cross-country heterogeneity, and the associated global imbalances, leads to heterogeneity across countries in the response of net capital flows and risky asset prices to the global risk-aversion shock.

We assume either risk-aversion or expected dividend heterogeneity. Specifically, assume either $\epsilon^G_n = g_n \epsilon$ and $\epsilon^D_n = 0$ or $\epsilon^D_n = d_n \epsilon$ and $\epsilon^G_n = 0$, where $\sum_{n=1}^{N+1} g_n = \sum_{n=1}^{N+1} d_n = 0$. For example, when $\epsilon > 0$, countries for which $g_n > 0$ are less risk-averse and those for which $g_n < 0$ are more risk-averse.

Now consider a country-specific variable $X_n$, which can be the risky asset price or net capital flows of safe or risky assets, or total net capital flows. Appendix D and E consider the impact of a global risk-aversion shock under respectively risk-aversion heterogeneity and expected dividend heterogeneity. To do so, we compute the second-order derivative

$$\frac{\partial^2 X_n}{\partial G \partial \epsilon} \tag{47}$$

at $\epsilon = 0$ and $G = 1$. We show that it is proportional to $g_n$ (risk-aversion heterogeneity) or $d_n$ (expected dividend heterogeneity), either positively or negatively. This tells us how the response to the global risk-aversion shock will vary across countries.\footnote{For example, when $X_n$ is a net capital flow variable, we have $\partial X_n / \partial G = 0$ at $\epsilon = 0$. When the second-order derivative (F.1) depends positively on $g_n$, it means that countries that are less risk-averse experience a drop in $X_n$ (negative net outflows) in response to a rise in global risk-aversion ($dG < 0$).}

Using the findings from Appendices D and E, Appendix F proves the following Theorem.

Theorem 4 Assume that there is no within-country heterogeneity and there is either cross-country heterogeneity in risk-aversion or expected dividends. Assumptions 1 and 2 hold as well. Then, in response to a rise in global risk-aversion, countries with a negative net
foreign asset position in safe assets experience (1) a larger drop in their risky asset price than average, (2) a positive net outflow in safe assets, (3) a negative net outflow in risky assets, (4) a positive overall net capital outflow. The opposite is the case for countries that have a positive net foreign asset position in safe assets. Moreover, the size of these changes is monotonically related to the size of the net foreign asset position in safe assets.

This is the theoretical counterpart to Stylized Fact 3 documented in the data. We will provide intuition behind Theorem 4 below. But we first discuss the following corollary, which is the theoretical counterpart to Stylized Fact 4.

**Corollary 1** Assume that there is no within-country heterogeneity and there is either cross-country heterogeneity in risk-aversion or expected dividends. Assumptions 1 and 2 hold as well. Then, in response to a rise in global risk-aversion, knowing the sign of the net foreign asset position in risky assets is not informative about whether the country has a relatively large or small drop in its risky asset price or whether it has positive or negative net capital flows of safe or risky assets or total net capital flows.

The reason that the sign of the net foreign asset position in risky assets is not informative is that the outcome will depend on the type of heterogeneity. Countries with a negative net foreign asset position in safe assets are either less risk-averse or have a risky asset whose expected dividend is relatively high. Under expected dividend heterogeneity, the net foreign asset position of risky assets has the same sign as the net foreign asset position of safe assets. Then from Theorem 4, countries with a negative net foreign asset position in risky assets experience a larger drop in their risky asset price, a positive net outflow in safe assets, a negative net outflow in risky assets, and a positive overall net capital outflow. But under risk-aversion heterogeneity, the exact opposite is the case as the sign of the net foreign asset position in risky assets is the opposite of that for safe assets.

We will provide two related types of intuition behind Theorem 4. The first is relatively straightforward, while the second is more complex but perhaps even more intuitive. The simple explanation goes as follows. Consider a country \( n \) that is leveraged and therefore has a negative net foreign asset position in safe assets. We refer to this as the Home country. As a result of the higher leverage, the Home country experiences a larger drop in wealth than the average country when risky asset prices fall. This has three implications. First, the Home country reduces demand for risky assets more than other countries, leading to negative net outflows in risky assets. Second, the relatively large drop in demand for risky assets by Home investors, together with home bias, implies that demand for the Home country risky asset drops more than average. This leads to a larger than average drop in the risky asset price of the Home country. Finally, the larger drop in wealth in the Home country leads to
a larger drop in consumption. This raises saving and leads to a positive overall net capital outflow in the Home country. If the Home country has a negative net outflow in risky assets and a positive total net outflow, it must have a positive net outflow in safe assets.

The second intuition is related to a decomposition of net capital flows of safe and risky assets into a portfolio growth, a portfolio rebalancing and a portfolio reallocation component. This decomposition is discussed in the Online Appendix, which derives the effects of the global risk-aversion shock on each of these components. Portfolio growth relates to the fraction invested in a certain class of assets times saving. Portfolio rebalancing relates to asset trade needed to keep portfolio shares the same after changes in relative asset prices. Portfolio reallocation refers to changes in portfolio shares, for example due to changes in expected returns or risk-aversion.

We will see in the next section that portfolio rebalancing is quantitatively by far the dominant component of capital flows in response to a global risk-aversion shock. Moreover, we find in the Online Appendix that the response of net capital flows to the risk-aversion shock in Theorem 4 has the same sign as the response of the portfolio rebalancing component. The other components either have the opposite sign or an ambiguous sign that depends on model parameters and the type of heterogeneity. We will therefore focus on the intuition behind portfolio rebalancing, making only brief remarks on portfolio reallocation.

Again consider a leveraged country, which has a portfolio share in risky assets greater than one and borrows safe assets. When risky asset prices drop as a result of the global risk-aversion shock, leverage automatically increases. The country will then hold an even larger portfolio share in risky assets. In order to rebalance, or deleverage, it needs to sell risky assets and use the proceeds to reduce the debt in safe assets. Net outflows of risky assets is negative and net outflows of safe assets is positive. The opposite is the case for a country that is not leveraged. Rebalancing then happens by buying risky assets and selling safe assets, leading to positive net outflows of risky assets and negative net outflows of safe assets.

In addition there is rebalancing among risky assets as the risky asset price of leveraged countries drops more than average. This causes a leveraged country to rebalance towards its domestic risky asset and away from foreign risky assets, further contributing to negative net outflows of risky assets. The opposite is the case for a non-leveraged country. These are the key mechanisms that give rise to the results in Theorem 4.

One might think that portfolio reallocation is important as well as the rise in global risk-aversion leads to a reallocation from risky to safe assets. But of course equilibrium forces will come into play as not all countries can sell risky assets. The lower price of risky assets raises their expected return, while the lower interest rate on safe assets lowers their return. This leads to a reallocation back from safe to risky assets. There is a non-zero
capital flow component associated with portfolio reallocation, but it is more complex than this and generally ambiguous in sign. Consider for example net capital flows of safe assets. On the one hand, a leveraged country holds a larger portfolio share in risky assets, which causes a larger reallocation towards safe assets when global risk-aversion rises. This implies net outflows of safe assets. On the other hand, the price of its risky asset drops more than average, leading to a higher expected return. This implies a portfolio shift away from safe assets, leading to negative net outflows of safe assets. We leave a full discussion to the Online Appendix.

6 Numerical Illustration

In the analytical results so far we have considered specific cases of within country or cross-country heterogeneity. We now discuss a numerical illustration where we include both types of within-country and cross-country heterogeneity jointly. We first discuss a calibration of the parameters and then consider the impact of the global risk-aversion shock.

6.1 Parameters

The model parameters and their calibrated values are shown in Table 3. The number of countries \( N + 1 \) is set at 20, corresponding to the empirical exercise in Section 2. We set \( a = \frac{\beta}{1-\beta} = 25 \), implying a 4 percent pre-shock interest rate. \( Y \) is set at 2, so that household income (thought of as labor income) is two thirds of GDP. The parameters \( \bar{\Gamma} = 0.1 \) and \( \sigma = 2 \) imply a median risk-aversion of 10 and an average pre-shock premium on risky assets of 4.7 percent.

The home bias parameter \( \bar{\kappa} \) is calibrated such that the average across countries of external assets plus liabilities, as a fraction of GDP, corresponds to the 4.45 average in the data. In the model this includes external assets and liabilities of risky assets plus the absolute value of the net foreign asset position in safe assets. This gives \( \bar{\kappa} = 0.0193 \). On average the countries then invest 73 percent of their risky asset portfolio in domestic risky assets. This is very close to the 74 percent that countries on average invest in domestic equity during the data sample.\(^{15}\)

The household’s intertemporal elasticity of substitution \( \rho \) determines the slope of the \( S = 0 \) schedule in Figure 4, which measures the change in the interest rate on the safe asset relative to the change in the risky-asset price due to the global risk-aversion shock. This is equal to \( \frac{1}{(1+a),\lambda} \), where \( \lambda = \rho Y \frac{a^2}{(1+a)^2} \). The ratio is numerically very similar for large changes

\(^{15}\)For each country and year, this is computed as the ratio of stock market capitalization minus external equity liabilities, divided by stock market capitalization minus external equity liabilities plus external equity assets.
in $G$. Based on the empirical results reported in Table 1, this ratio is 0.60/10.4 in the data. From this we set $\rho = 9$.

From Assumption 3, with regards to within-country heterogeneity we assume that $\kappa_i - \bar{\kappa} = \epsilon_i^\kappa$, $\Gamma_i - \bar{\Gamma} = \omega \epsilon_i^\kappa + \epsilon_i^\Gamma$, where $\epsilon_i^\kappa \sim U(-\Delta_\kappa, \Delta_\kappa)$ and $\epsilon_i^\Gamma \sim U(-\Delta_\Gamma, \Delta_\Gamma)$. We set $\Delta_\kappa = \bar{\kappa}$, $\Delta_\Gamma = \bar{\Gamma}$ and $\omega = 0$. Investor home bias then varies uniformly from holding no foreign assets to holding twice the mean level of foreign assets. Risk-aversion varies from half the mean risk-aversion to infinity.

The two types of cross-country heterogeneity, $\epsilon_G^n$ and $\epsilon_D^n$, jointly determine $NFA_{n}^{\text{risky}}$ and $NFA_{n}^{\text{safe}}$ prior to the shock. We observe these net foreign asset positions of risky and safe assets for all countries in the data, which are reported in Figure 3. In the model we solve for the values of $\epsilon_G^n$ and $\epsilon_D^n$ for each country to match these net foreign asset positions in the data, as a share of GDP.$^{16}$

The size of the global risk-aversion shock is calibrated as follows. In the data, Table 1 shows that the average risky asset price drops by 10.4 percent when the GFC factor goes down by one standard deviation ($\Delta F_t = -1$). We set $G$ to match this in the model. When $G = 1/3$ the average risky asset price drops 10.8 percent in the model as a result of the global risk-aversion shock.

### 6.2 Results

In Section 2, the empirical results were presented in Tables 1 and 2. Table 1 implies that a shock of $\Delta F = -1$ implies a drop in the interest rate of 60 basis points, an average drop in the risky asset price of 10.4% and an average drop of outflows plus inflows of risky assets of 4.7 percent of GDP. We chose $\rho$ and $G$ to match the first two of these in the model. The interest rate drops by 60 basis points, while the average risky asset price drops by 10.8 percent. Outflows plus inflows of risky assets drop by 3.9 percent of GDP. This is close to the 4.7 percent in the data. We could raise it further, matching the data exactly, by assuming a slightly positive value of $\omega$.

Table 2 shows how the response of net capital flows and risky asset prices to the GFC shock depends on the net foreign asset position in safe and risky assets of a country. In the model we cannot estimate a panel regression as we only have one shock. Instead we represent this relationship graphically in Figure 6. Each circle represents an individual country. The horizontal axis has the pre-shock net foreign asset position in safe assets for the 4 charts on the left and the pre-shock net foreign asset position in risky assets for the 4 charts on the right. Going from top to bottom, for each country it shows the response to the global risk-

$^{16}$In the data the average across countries of the net foreign asset positions (as a share of GDP) is not exactly zero. To be consistent with the model, we therefore recenter the net foreign assets positions in the data (as a share of GDP) by subtracting the cross-sectional mean.
aversion shock of net outflows of safe assets, net outflows of risky assets, total net outflows and the risky asset price. As in the data, all net capital flows are as a fraction of GDP.

We see that there is a strong negative relationship between net outflows of safe assets and the net foreign asset position in safe assets. The same is also the case for total net outflows. This is exactly what we documented in the data in Table 2, summarized in Stylized Fact 3. Countries that are leveraged, with a net debt in safe assets, will experience net outflows of safe assets and net outflows overall. This reduces the net debt of safe assets and the net external debt overall. The exact opposite is the case for countries that have a positive net foreign asset position in safe assets. Their net outflows of safe assets, and overall net outflows, turn negative as a result of the shock, reducing their net foreign asset position.

We also see a strong positive relationship between net outflows of risky assets and the net foreign asset position of safe assets. This is again consistent with the data. Leveraged countries, with a negative net foreign asset position in safe assets, experience negative net outflows of risky assets. These countries reduce their net external debt of safe assets both by selling risky assets (negative $NF_{risky}$) and increasing their saving (positive $NF$). The opposite is the case for countries with a positive net foreign asset position in safe assets. For those countries the safe assets are essentially an insurance policy against a bad state in the form of a negative GFC shock. They reduce their positive net external position of safe assets during a bad shock.

Finally, like in the data, the risky asset price drops more for countries with a more negative net foreign asset position in safe assets. It drops less when a country has a positive net foreign asset position in safe assets.

By contrast, the charts on the right show that there is no clear relationship between net capital flows (risky assets, safe assets or total) and the net foreign asset position in risky assets. This is again consistent with the data, summarized in Stylized Fact 4. There does appear to be a slightly positive relationship between the drop in the risky asset price and the net foreign asset position in risky assets. Table 2 documents this in the data as well, though it is less significant than the relationship between the drop in the risky asset price and the net foreign asset position in safe assets.

Figure 7 illustrates that the response of net capital flows of safe and risky assets to the global risk-aversion shock is mostly associated with portfolio rebalancing. The four charts on the left show net outflows of safe assets due to the shock as well as its three components: net outflows due to portfolio growth, portfolio rebalancing and portfolio reallocation. Expressions for these components are derived in the Online Appendix. The four charts on the right show the same for net outflows of risky assets. For both safe and risky assets, it is immediate that almost all the action is from portfolio rebalancing. Countries that are leveraged (negative net foreign asset position of safe assets) become more leveraged when
risky asset prices drop. They deleverage (rebalance) by selling risky assets (negative net outflows of risky assets) and buying safe assets (positive net outflows of safe assets).

We should finally point out that while the model does a good job matching the data qualitatively, it is weaker quantitatively. The drop in the average risky asset price, the interest rate and gross outflows are all quantitatively close to the data. But the slope of the relationship between the net foreign asset position of safe assets and the four variables in Figure 6, while having the correct sign, does not quantitatively match the data. The slope is too large in the model for net outflows of safe and risky assets, while too small for total net outflows and the change in the relative asset price. Expecting these slopes to match exactly is probably too much to ask from such a stylized model with limited dynamics. Enriching the model further could bring us closer to the data. For example, introducing investment that depends on the risky asset price implies that net outflows (saving minus investment) will be more negatively related to the risky asset price. This leads to a more negative relationship between overall net outflows and the net foreign asset position in safe assets.

7 Conclusion

We have developed a theory to shed light on fluctuations in the prices of safe and risky assets and gross and net capital flows over the global financial cycle. In line with extensive evidence of the importance of risk and risk-aversion as drivers of the global financial cycle, we have investigated the impact of a global risk-aversion shock in a multi-country model. We have shown that heterogeneity of investors within countries is required to account for the drop in gross capital flows during a downturn in the GFC cycle. At the same time, cross-country heterogeneity is needed to account for the differential vulnerability of countries to a negative GFC shock. Cross-country heterogeneity leads to imbalances in the net foreign asset position of both safe and risky assets. We have shown that the key vulnerability is associated with the net foreign asset position of safe assets, which determines leverage. In response to a rise in global risk-aversion, leveraged countries (net borrowers of safe assets) deleverage through negative net outflows of risky assets and positive net outflows of safe assets, experience an increase in the current account and a greater than average drop in risky asset prices. The opposite is the case for non-leveraged countries. The same response is documented in the data.

The model can be extended in numerous directions to consider features of the GFC from which we have abstracted here. One direction is to consider the role of monetary policy and associated exchange rate fluctuations. Another is to allow for financial frictions, which would allow us to consider the need for macroprudential policies. Related to that, a third
direction is to more explicitly model financial institutions and the constraints under which they operate. Finally, we have abstracted from the special role that the United States and the dollar play in the international financial system.
Appendix

A Proof of Theorem 1

With period 1 dividends of 1, \( R_0 = (1 + a)/a \) and \( Q_{n,0} = Q_{n,1} = a \), (8) implies that \( R_{t}^{p,i,n} = (1 + a)/a \). We have \( W_{n,0} - C_{n,0} = \beta W_{n,0} = a \), so that from (7) \( W_{n,1} = 1 + a \) for all investors. Substituting \( \bar{D}_n \) from Assumption 1, as well as \( Q_n = a \) and \( R = (1 + a)/a \), into the portfolio expressions (17)-(18) gives time 1 portfolio shares that are the same as the time zero portfolio shares (24)-(25). Substituting these portfolio expressions, as well as \( W_{n,1} = 1 + a \) and \( Q_{n,1} = a \), into the risky asset market clearing conditions (22), the markets clear in period 1 under Assumption 1 about \( K_n \). The aggregate asset market clearing condition (23) also holds in period 1, after substituting \( B_{h}^{0} = B_{h}^{0}, W_{n,1} = 1 + a, Q_{n,1} = a \) and the expression for \( B_{h}^{0} \) in Assumption 1.

Since \( R_t = 1/\beta \) for all \( t \geq 1 \), first-order condition (21) implies that household consumption is constant over time. Since income is constant, this implies \( C_{t}^{h} = Y + B_{h}^{0}/a \). The household budget constraint (20) then implies \( B_{t}^{h} = B_{0}^{h} \) for all \( t \geq 1 \). Since there is no uncertainty starting in period 2, we must have \( R_t = (Q_{n,t+1} + D_{n,2})/Q_{n,t} \) for \( t \geq 2 \). This is satisfied when \( R_t = (1 + a)/a, Q_{n,t} = Q_{n,t+1} = (a/(1 + a))D_n = aD_{n,2} \). Investor wealth remains constant after period 2 since \( W_{n,t+1} = \beta R_t W_{n,t} \) for \( t \geq 2 \) and \( R_t = 1/\beta \).

We finally need to check the aggregate asset market clearing condition (23) for \( t \geq 2 \). Since household safe asset holdings, investor wealth and asset prices remain constant from period 2 onward, we only need to check it for \( t = 2 \). We have

\[
\sum_{n=1}^{N+1} \int_{0}^{1} W_{n,2}^{i} di = \beta (1 + a) \sum_{n=1}^{N+1} \int_{0}^{1} R_{t}^{p,i,n} di = aR(N + 1) + a \sum_{n=1}^{N+1} \sum_{m=1}^{N+1} \int_{0}^{1} z_{n,m} di D_{m} - RQ_{m}/Q_{m}
\]

From (24)-(25), \( \sum_{n=1}^{N+1} \int_{0}^{1} z_{n,m} di = K_m \). Therefore

\[
\sum_{n=1}^{N+1} \int_{0}^{1} W_{n,2}^{i} di = (1 + a)(N + 1) + \sum_{m=1}^{N+1} K_m (D_m - (1 + a))
\]

Using \( B_{2}^{h} = B_{0}^{h} \), the period 2 aggregate asset market equilibrium can then be written as

\[
(N + 1)B_{0}^{h} + a(N + 1) + \frac{a}{1 + a} \sum_{n=1}^{N+1} D_{n}K_n - a \sum_{n=1}^{N+1} K_n = \sum_{n=1}^{N+1} Q_{n,2}K_n
\]

Using \( Q_{n,2} = (a/(1 + a))D_n \) and the expression for \( B_{0}^{h} \) in Assumption 1, it is immediate that
this is satisfied.

We finally point out that the conjectured value functions are correct. We conjectured
\[ V_{i,1}^t = \alpha_1 W_{n,1}^t \text{ and } V_{n,t}^i = \alpha_2 W_{n,t}^i \text{ for } t \geq 2. \]
First substituting the latter into the Bellman equation (9) for \( t \geq 2 \), together with \( C_{i,n}^t = (1 - \beta) W_{n,t}^i \) and \( W_{n,t+1}^i = W_{n,t}^i \), we have \( \alpha_2 = 1 - \beta \). Substituting \( V_{i,1}^t = \alpha_1 W_{n,1}^t \) into the Bellman equation (9) at time 1, together with \( C_{i,n} = (1 - \beta) W_{n,1}^i \) and \( W_{n,2}^i = \beta R_{p,n}^i W_{n,1}^i \), we have
\[
\ln(\alpha_1) = \ln(1 - \beta) + \frac{\beta}{1 - \beta} \ln(\beta) + \frac{1}{1 - \beta} \frac{1}{1 - \gamma_{i,n}} \ln \left( E(R_{p,n}^i)^{1-\gamma_{i,n}} \right)
\]
Substituting the portfolio shares (24)-(25), \( Q_m = a \) and \( R = a/(1 + a) \) into the portfolio return expression (16), \( \alpha_1 \) becomes a function of structural model parameters.

**B Proof of Theorem 3**

The market equilibrium conditions (22)-(23) can be written as
\[
(D - RQ) \left( 1 + a + \frac{E\psi^2}{\tilde{\psi}^2} (Q - a) \right) = \frac{1 + a \sigma^2}{a\tilde{\psi}} \frac{\bar{D}}{G} \tag{B.1}
\]
\[
Y - C_1^h - \frac{Q - a}{1 + a} = 0 \tag{B.2}
\]
These remain very similar to (31)-(32). The only difference is that \( Q - a \) in the risky asset market equilibrium condition is now multiplied by \( E\psi^2/\tilde{\psi}^2 \). Differentiating these equations at the pre-shock levels \( Q = a, R = (1 + a)/a \) and \( G = 1 \), we have
\[
dQ = \frac{(1 + a)(\sigma^2/\tilde{\psi})}{(1 + a)^2 - (\sigma^2 E\psi^2/\tilde{\psi}^3) + \frac{a^2}{\lambda}} dG \tag{B.3}
\]
\[
dR = \frac{(\sigma^2/\tilde{\psi})}{\lambda ((1 + a)^2 - (\sigma^2 E\psi^2/\tilde{\psi}^3)) + \frac{a^2}{\lambda}} dG \tag{B.4}
\]
Assumption 2 implies that the denominator of both expressions is positive, so that a drop in \( G \) (rise in global risk-aversion) leads to a drop in both \( Q \) and \( R \).

Next consider capital flows. (26)-(27) can be written as
\[
OF_{n}^{\text{risky}} = IF_{n}^{\text{risky}} = \frac{a}{1 + a} Q \bar{\psi} G \frac{D - RQ}{\sigma^2} \int_0^1 z_i^i (1 + a + z_i^i (Q - a)) di - QE(z_F z) \tag{B.5}
\]
with \( z \) and \( z_F \) defined in (36) and (39). First note that net flows of risky assets clearly remain zero. Since saving is also zero (see (B.2)), total net capital flows are zero and therefore net
flows in safe assets are zero as well. Substituting (B.1), we have
\[ OF_{n}^{\text{risky}} = IF_{n}^{\text{risky}} = Q \frac{(1 + a)E(z_{F}z) + (Q - a)E(z_{F}z^{2})}{1 + a + (1 + \text{var}(z))(Q - a)} - QE(z_{F}) \] (B.6)

This uses that \( 1 + \text{var}(z) = Ez^{2} = E(\psi/\bar{\psi})^{2} = (E\psi^{2})/\bar{\psi}^{2} \). Differentiating with respect to \( Q \) at \( Q = a \) gives
\[ dOF_{n}^{\text{risky}} = dIF_{n}^{\text{risky}} = \frac{a}{1 + a} (Ez_{F}z(z - 1 - \text{var}(z))) dQ \] (B.7)

It follows that both outflows and inflows of risky assets go down equally in response to an increase in global risk-aversion as long as \( \text{cov}(z_{F}z, z) - (Ez_{F}z)\text{var}(z) > 0 \). Substituting \( z = \psi/E(\psi) \) and \( \eta = \bar{\psi}/E(\bar{\psi}) \), where \( \eta = N\bar{\psi} \), we need to show that
\[ \text{cov}(\eta, \psi)E(\psi) - \text{var}(\psi)E\eta > 0 \] (B.8)

Using that \( \kappa = \bar{\kappa} + \epsilon^\kappa \) and \( \Gamma = \bar{\Gamma} + \omega\epsilon^\kappa + \epsilon^\Gamma \), we have
\[ \psi = \bar{\Gamma}(1 + N\bar{\kappa}) + (\omega + N\omega\bar{\kappa} + N\bar{\Gamma})\epsilon^\kappa + (1 + N\bar{\kappa})\epsilon^\Gamma + N\epsilon^\kappa\epsilon^\Gamma + \omega N (\epsilon^\kappa)^2 \] (B.9)
\[ \eta = N\Gamma\bar{\kappa} + N(\omega\bar{\kappa} + \bar{\Gamma})\epsilon^\kappa + N\kappa\epsilon^\Gamma + N\epsilon^\kappa\epsilon^\Gamma + \omega N (\epsilon^\kappa)^2 \] (B.10)

Using the assumed properties of \( \epsilon^\kappa \) and \( \epsilon^\Gamma \) (independent, symmetric distributions), it follows that
\[ \text{var}(\psi) = \omega + N\omega\bar{\kappa} + N\bar{\Gamma})^2\text{var}(\epsilon^\kappa) + (1 + N\bar{\kappa})^2\text{var}(\epsilon^\Gamma) \]
\[ + N^2\text{var}(\epsilon^\Gamma)\text{var}(\epsilon^\kappa) + \omega^2 N^2 E(\epsilon^\kappa)^4 - \omega^2 N^2 [\text{var}(\epsilon^\kappa)]^2 \]
\[ \text{cov}(\eta, \psi) = (\omega + N\omega\bar{\kappa} + N\bar{\Gamma})N(\omega\bar{\kappa} + \bar{\Gamma})\text{var}(\epsilon^\kappa) + (1 + N\bar{\kappa})N\bar{\kappa}\text{var}(\epsilon^\Gamma) \]
\[ + N^2\text{var}(\epsilon^\Gamma)\text{var}(\epsilon^\kappa) + \omega^2 N^2 E(\epsilon^\kappa)^4 - \omega^2 N^2 [\text{var}(\epsilon^\kappa)]^2 \]
\[ E(\psi) = \bar{\Gamma}(1 + N\bar{\kappa}) + \omega N \text{var}(\epsilon^\kappa) \]
\[ E(\eta) = N\Gamma\bar{\kappa} + \omega N \text{var}(\epsilon^\kappa) \]

Then collecting terms, we have
\[ \text{cov}(\eta, \psi)E(\psi) - \text{var}(\psi)E\eta = (\omega + N\omega\bar{\kappa} + N\bar{\Gamma})N\Gamma^2\text{var}(\epsilon^\kappa) + \bar{\Gamma} N^2\text{var}(\epsilon^\Gamma)\text{var}(\epsilon^\kappa) + \bar{\Gamma} \omega^2 N^2 E(\epsilon^\kappa)^4 \]
\[ - \omega^2 N(\omega + N\omega\bar{\kappa} + 2N\bar{\Gamma}) [\text{var}(\epsilon^\kappa)]^2 - \omega N(1 + N\bar{\kappa})\text{var}(\epsilon^\Gamma)\text{var}(\epsilon^\kappa) \]
Rewrite this as

$$\text{cov}(\eta, \psi)E(\psi) - \text{var}(\psi)E\eta = \bar{\Gamma}N^2\text{var}(\epsilon^\Gamma)\text{var}(\epsilon^\kappa) + (\bar{\Gamma}^2 - \text{var}(\epsilon^\Gamma) - \omega^2\text{var}(\epsilon^\kappa)) (1 + N\bar{\kappa})N\omega\text{var}(\epsilon^\kappa)$$

$$N^2\text{var}(\epsilon^\kappa) (\omega^2E(\epsilon^\kappa)^4 - 2\omega^2[\text{var}(\epsilon^\kappa)]^2 + \bar{\Gamma}^2\text{var}(\epsilon^\kappa)) N^2\bar{\Gamma}$$

(B.11)

Assumption 3 implies that $\text{var}(\epsilon^\kappa) > 0$, $\text{var}(\epsilon^\Gamma) \geq 0$ and $\omega \geq 0$. This means that the first term of (B.11) is greater than or equal to zero. Next consider the second term in the first line of (B.11). $\Gamma > 0$ in Assumption 3, which assures that risk-aversion (which depends inversely on $\Gamma$) is well defined. Therefore $\omega\epsilon^\kappa + \epsilon^\Gamma > -\bar{\Gamma}$. Since Assumption 3 states that $\epsilon^\kappa$ and $\epsilon^\Gamma$ are symmetrically distributed, it follows that $(\omega\epsilon^\kappa + \epsilon^\Gamma)^2 < \bar{\Gamma}^2$ and therefore

$$\omega^2\text{var}(\epsilon^\kappa) + \text{var}(\epsilon^\Gamma) < \bar{\Gamma}^2$$

(B.12)

This implies that the second term of the first line of (B.11) is positive. Finally consider the last term of (B.11). We have

$$E(\epsilon^\kappa)^4 = \text{var}((\epsilon^\kappa)^2) + [\text{var}(\epsilon^\kappa)]^2 \geq [\text{var}(\epsilon^\kappa)]^2$$

Therefore the term in brackets in the last term of (B.11) is

$$\omega^2E(\epsilon^\kappa)^4 - 2\omega^2[\text{var}(\epsilon^\kappa)]^2 + \bar{\Gamma}^2\text{var}(\epsilon^\kappa) \geq (\bar{\Gamma}^2 - \omega^2\text{var}(\epsilon^\kappa)) \text{var}(\epsilon^\kappa)$$

From (B.12) this is positive, which completes the proof that $\text{cov}(\eta, \psi)E(\psi) - \text{var}(\psi)E\eta > 0$. Therefore gross capital flows must fall in response to the increase in global risk aversion.

C Leverage

First consider the model with only within country heterogeneity. From (17)-(18) we have

$$z^i = z^i_{n,n} + \sum_{m \neq n} z^i_{n,m} = \Gamma_i(1 + N\kappa_i)\frac{1}{\sigma^2}GQ(\bar{D} - RQ)$$

(C.1)

Clearly, $z^i$ changes proportionately for all investors, whether leveraged or not. Define $H = GQ(\bar{D} - RQ)$. To determine the direction of the change in the portfolio shares, we need to take the derivative of $H$ with respect to $G$. This is

$$\frac{\partial H}{\partial G} = (\bar{D} - (1 + a)) \left( a + \frac{\partial Q}{\partial G} \right) - a^2 \frac{\partial R}{\partial G} - (1 + a) \frac{\partial Q}{\partial G}$$

(C.2)
Now substitute $\bar{D} = 1 + a + \frac{a^2}{a\bar{\psi}}$, the derivatives (B.3)-(B.4), and multiply by the denominator of the expression for $\partial Q/\partial G$ times $\bar{\psi}/\sigma^2$. This gives

$$-\frac{\sigma^2 E\psi^2}{\bar{\psi}^3} + \frac{1 + a}{a}(\sigma^2/\bar{\psi})$$  \hspace{1cm} (C.3)

Using that $z^i = \psi_i/\bar{\psi}$, this is positive when

$$\text{var}(z) < \frac{1}{a}$$ \hspace{1cm} (C.4)

In that case leverage falls when $G$ falls (global risk-aversion rises).

Next consider the model with only cross-country heterogeneity. (17)-(18) then give

$$z_n = z_{n,n} + \sum_{m\neq n} z_{n,m} =$$

$$\Gamma G \frac{1 + \epsilon^G_n}{\sigma^2} \left[ Q_n \left( 1 + a + \frac{\sigma^2}{a\bar{\psi}}(1 + \epsilon^D_n) - RQ_n \right) + \kappa \sum_{m\neq n} Q_m \left( 1 + a + \frac{\sigma^2}{a\bar{\psi}}(1 + \epsilon^D_m) - RQ_m \right) \right]$$

Before the shock leverage is

$$z_n = (1 + \epsilon^G_n) \left( 1 + \frac{1 - \kappa}{1 + N\kappa} \epsilon^D_n \right)$$  \hspace{1cm} (C.6)

Clearly, a country is leveraged if it has lower than average risk-aversion ($\epsilon^G_n > 0$) or higher than average expected dividends ($\epsilon^D_n > 0$). Evaluated at $G = 1$ and $\epsilon^G_n = \epsilon^D_n = 0$, we have

$$\frac{\partial z_n}{\partial G} > 0$$ \hspace{1cm} (C.7)

This follows from the results above with $\text{var}(\psi) = 0$. Therefore raising global risk-aversion lowers $z_n$ for all countries. Countries that are leveraged then become less leveraged.

### D Cross Country Heterogeneity in Risk Aversion

Appendix F will proof Theorem 4. To do so, we first need to to derive the second order derivatives of risky asset prices and net capital flows (safe, risky, total) with respect to $G$ and $\epsilon$. In this section we do so for cross-country risk aversion heterogeneity. In Section E we do so for heterogeneity in expected dividends. We start by describing the market clearing conditions. After that we derive the second-order derivatives for risky asset prices, total net capital outflows and net outflows of risky assets as linear functions of $g_n$. The last two also give us the second-order derivative for net flows of safe assets. We remove superscripts for
individual agents as there is no within-country heterogeneity.

D.1 Market Clearing Conditions

The market clearing conditions are

\[
\frac{a}{1+a} \sum_{m=1}^{N+1} z_{m,n} W_m = Q_n^h K_n \quad n = 1, \ldots, N + 1 \tag{D.1}
\]

\[
(N + 1) B_1^h + \frac{a}{1+a} \sum_{n=1}^{N+1} W_n = \sum_n Q_n^h K_n \tag{D.2}
\]

First consider wealth. Using the expressions for portfolio shares (24)-(25) in the pre-shock equilibrium, we have

\[
W_n = 1 + a + (1 + \epsilon_n^G) \frac{1 - \kappa}{1 + N\kappa} (Q_n - a) + (1 + \epsilon_n^G) \frac{\kappa}{1 + N\kappa} \sum_m (Q_m - a) \tag{D.3}
\]

From Assumption 1 we have

\[
K_n = \frac{1 - \kappa}{1 + N\kappa} (1 + \epsilon_n^G) + \frac{(N + 1) \kappa}{1 + N\kappa} \tag{D.4}
\]

Since \(\sum_n (1 + \epsilon_n^G) = N + 1\), it follows that \(\sum_n K_n = N + 1\), so that from Assumption 1 \(B_0^h = 0\). Therefore \(B^h = Y - C_1^h\). Together with the expressions for \(K_n^h\) and \(W_n\), we can then write the aggregate asset market clearing condition (D.2) as

\[
\frac{1 - \kappa}{1 + N\kappa} \sum_{m=1}^{N+1} (Q_m - a) \epsilon_m^G + \sum_{m=1}^{N+1} (Q_m - a) = (N + 1)(1 + a)(Y - C_1^h) \tag{D.5}
\]

Taking the derivative with respect to \(G\), this implies:

\[
\frac{\partial R}{\partial G} = \frac{1}{(N + 1)(1 + a)} \frac{1 - \kappa}{1 + N\kappa} \sum_{m=1}^{N+1} \frac{\partial Q_m}{\partial G} \epsilon_m^G + \frac{1}{(N + 1)(1 + a)\lambda} \sum_{m=1}^{N+1} \frac{\partial Q_m}{\partial G} \tag{D.6}
\]

Next consider the market clearing conditions for risky assets (D.1). Substituting the portfolio shares (17)-(18) and wealth expressions (D.3) into (D.1), the market clearing conditions
for risky assets are

\[
(1 + a) \left( 1 + \epsilon_n^G \right) (1 - \kappa) + \kappa (N + 1)(1 + a) + \frac{(1 - \kappa)^2}{1 + N\kappa} (1 + \epsilon_n^G)^2 (Q_n - a) \\
+ \frac{(1 - \kappa)\kappa}{1 + N\kappa} (1 + \epsilon_n^G)^2 \sum_m (Q_m - a) + \frac{(1 - \kappa)\kappa}{1 + N\kappa} \sum_m (1 + \epsilon_m^G)^2 (Q_m - a) \\
+ \frac{\kappa^2}{1 + N\kappa} \left( \sum_m (1 + \epsilon_m^G)^2 \right) \left( \sum_m (Q_m - a) \right) = 1 + \frac{a}{\kappa} K_n \frac{1}{\Gamma G \bar{D} - RQ_n} \tag{D.7}
\]

Differentiating (D.7) and substituting (D.6) gives

\[
\frac{(1 - \kappa)^2}{1 + N\kappa} (1 + \epsilon_n^G)^2 \frac{\partial Q_n}{\partial G} + \frac{(1 - \kappa)\kappa}{1 + N\kappa} (1 + \epsilon_n^G)^2 \sum_m \frac{\partial Q_m}{\partial G} + \frac{(1 - \kappa)\kappa}{1 + N\kappa} \sum_m (1 + \epsilon_m^G)^2 \frac{\partial Q_m}{\partial G} \\
+ \frac{\kappa^2}{1 + N\kappa} \left( \sum_m (1 + \epsilon_m^G)^2 \right) \left( \sum_m \frac{\partial Q_m}{\partial G} \right) = -\frac{1 + a}{\kappa} K_n \frac{1}{\Gamma G^2 \bar{D} - RQ_n} \sigma^2 \\
+ \frac{1 + a}{\kappa} RK_n \frac{1}{\Gamma G (\bar{D} - RQ_n)^2} \frac{\partial Q_n}{\partial G} + Q_n K_n \frac{1}{\Gamma G (\bar{D} - RQ_n)^2} \frac{1}{(N + 1)\lambda} \sum_{m=1}^{N+1} \frac{\partial Q_m}{\partial G} \\
+ Q_n K_n \frac{1}{\Gamma G (\bar{D} - RQ_n)^2} \frac{1}{(N + 1)\lambda} (1 - \kappa) \sum_{m=1}^{N+1} \frac{\partial Q_m}{\partial G} \epsilon_m \tag{D.8}
\]

### D.2 Impact on Relative Prices Risky Assets

We first consider the impact of the global risk-aversion shock on relative prices of risky assets. We set \( \epsilon_n^G = g_n \epsilon \), with \( \sum_n g_n = 0 \). To show that the risky asset price \( Q_n \) drops more the lower risk-aversion in country \( n \), and therefore the higher \( g_n \) when \( \epsilon > 0 \), we need to show that

\[
\frac{\partial^2 Q_n}{\partial G \partial \epsilon} \tag{D.9}
\]

depends positively on \( g_n \).

To this end we need to differentiate (D.8) with respect to \( \epsilon \) and evaluate at \( \epsilon = 0 \) and \( G = 1 \). At that point \( \epsilon_n^G = 0, Q_m = a, R = (1 + a)/a, \bar{D} - RQ = \sigma^2/(a\Gamma(1 + N\kappa)) \) and \( K_n = 1 \). We also have from (D.4) that \( \partial K_n/\partial \epsilon = (1 - \kappa)g_n/(1 + N\kappa) \). We use that the pre-shock equilibrium for risky asset prices and the interest rate do not depend on \( \epsilon \), so that \( \partial Q_m/\partial \epsilon = \partial R/\partial \epsilon = 0 \). Since all first order derivatives of risky asset prices with respect to
$G$ will be the same, we simply denote them $\partial Q/\partial G$ (see (34)). This gives

$$
(1 - \kappa)^2 \frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \frac{2(1 - \kappa + N \kappa)\kappa}{1 + N \kappa} \sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon} + 2(1 - \kappa)g_n \frac{\partial Q}{\partial G} = -(1 + a)(1 - \kappa)g_n
$$

$$
+(1 + a)^2 g_n (1 + N \kappa) \Gamma (1 - \kappa) \frac{1}{\sigma^2} \frac{\partial Q}{\partial G} + (1 + a)^2 (1 + N \kappa)^2 \Gamma \frac{1}{\sigma^2} \frac{\partial^2 Q_n}{\partial G \partial \epsilon}
$$

$$
+ a^2 (1 - \kappa) \Gamma (1 + N \kappa) g_n \frac{\partial Q}{\partial G} + a^2 \Gamma (1 + N \kappa)^2 \sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon} = (D.10)
$$

Taking the sum over all $n$, using that $\sum_n g_n = 0$, it follows that $\sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon} = 0$. Therefore

$$
\frac{\partial^2 Q}{\partial G \partial \epsilon} = \frac{(1 - \kappa) g_n}{1 + N \kappa} \frac{(1 + a) + (2 - (1 + a)^2 \tilde{\psi} \frac{1}{\sigma^2} - a^2 \tilde{\psi} \frac{1}{\sigma^2})}{(1 + a)^2 \tilde{\psi} \frac{1}{\sigma^2} - (1 - \kappa)^2 \frac{1}{(1 + N \kappa)^2}}
$$

(D.11)

Assumption 2 says $\tilde{\psi}(1 + a)^2 > \sigma^2$. It is immediate from this condition that the denominator of (D.11) is positive. To see that the numerator is positive, we can substitute the solution for $\partial Q/\partial G$ from (34). Multiplying through by the denominator of (34), which is positive, the numerator of the large ratio in (D.11) becomes

$$
(1 + a)^3 - (1 + a)(\sigma^2/\tilde{\psi}) + \frac{a^2(1 + a)}{\lambda} + 2(1 + a)(\sigma^2/\tilde{\psi}) - (1 + a)^3 - (1 + a)a^2 \frac{1}{\lambda}
$$

This is equal to $(1 + a)\sigma^2/\tilde{\psi}$, which is positive. It follows that (D.9) is a positive linear function of $g_n$, which implies that the risky asset price drops more in countries with lower risk-aversion, which are more leveraged.

### D.3 Impact on Total Net Flows

We now consider the impact of the shock on total net capital flows (risky plus safe assets), which is equal to the current account, which is equal to saving. Therefore net flows of country $n$ are

$$
NF_n = Y - C_1^n + 1 - \frac{1}{1 + a} W_n
$$

(D.12)

Here $Y - C_1^n$ is saving by households and $1 - W_n/(1 + a)$ is saving by investors. They earn dividend and interest income equal to 1 and consume $W_n/(1 + a)$. Using (D.3), $\sum_m CA_m = 0$, and that household consumption is the same in all countries, we can write

$$
CA_n = \frac{1}{1 + N} \sum_m (CA_n - CA_m) = -\frac{1}{1 + N} \frac{1}{1 + a} \sum_m (W_n - W_m) =
$$

$$
-\frac{1}{1 + a} \frac{1}{1 + N} \frac{1 - \kappa}{1 + N\kappa} \sum_m ((1 + \epsilon_n^n)(Q_n - a) - (1 + \epsilon_m^n)(Q_m - a)) - \frac{1}{1 + a} \frac{\kappa}{1 + N\kappa} \epsilon_n^n \sum_m (Q_m - a)
$$

(D.13)
The effect of a risk aversion shock is

\[
\frac{\partial CA_n}{\partial G} = -\frac{1}{1+a} \frac{1}{1+N} \frac{1-\kappa}{1+N\kappa} \sum_m \left( (1+\epsilon_n^G) \frac{\partial Q_n}{\partial G} - (1+\epsilon_m^G) \frac{\partial Q_m}{\partial G} \right) - \frac{1}{1+a} \frac{\kappa}{1+N\kappa} \epsilon_n^G \sum_m \frac{\partial Q_m}{\partial G}
\] (D.14)

Next take the derivative with respect to \( \epsilon \) and evaluate at \( \epsilon = 0 \) and \( G = 1 \). Using that \( \sum_{m=1}^{N+1} \frac{\partial^2 Q_m}{[\partial G\partial \epsilon]} = 0 \), we have

\[
\frac{\partial^2 CA_n}{\partial G \partial \epsilon} = -\frac{1}{1+a} \frac{1-\kappa}{1+N\kappa} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} - \frac{1}{1+a} g_n \frac{\partial Q}{\partial G}
\] (D.15)

Since \( \partial Q/\partial G > 0 \), the last term is a negative linear function of \( g_n \). The same is the case for the first term as we have already established that \( \partial^2 Q_n/\partial G \partial \epsilon \) is a positive linear function of \( g_n \). It therefore follows that lower risk-aversion (higher \( g_n \) with positive \( \epsilon \)) implies a higher current account when \( G \) falls. Net capital outflows will therefore be higher in response to a global risk-aversion shock in countries that are less risk-averse.

### D.4 Net Outflows Risky Assets

From (26) and (27), outflows and inflows of risky assets are

\[
OF_{n}^{\text{risky}} = \frac{a}{1+a} \sum_{m \neq n} z_{n,m} W_n - (1+\epsilon_n^G) \frac{\kappa}{1+N\kappa} \sum_{m \neq n} Q_m
\] (D.16)

\[
IF_{n}^{\text{risky}} = \frac{a}{1+a} \sum_{m \neq n} z_{m,n} W_m - Q_n \frac{\kappa}{1+N\kappa} \sum_{m \neq n} (1+\epsilon_m^G)
\] (D.17)

This uses (25) for \( z_{n,m,0} \) and \( z_{m,n,0} \). Substituting the time 1 portfolio shares in (18), net outflows of risky assets are

\[
NF_{n}^{\text{risky}} = \frac{a}{1+a} \left( 1+\epsilon_n^G \right) W_n G T \kappa \sum_m Q_m \frac{\bar{D} - RQ_m}{\sigma^2}
\] (D.18)

\[
-\frac{a}{1+a} G T \kappa Q_n \frac{\bar{D} - RQ_n}{\sigma^2} \sum_m \left( 1+\epsilon_m^G \right) W_m + (N+1)Q_n \frac{\kappa}{1+N\kappa} - (1+\epsilon_n^G) \frac{\kappa}{1+N\kappa} \sum_m Q_m
\]
Taking the derivative with respect to \( G \), we have

\[
\frac{\partial N F_{\text{risky}}^n}{\partial G} = \frac{a}{1 + a} \left( 1 + \epsilon_n^G \right) \Gamma \kappa \left( \sum_m Q_m \frac{D - RQ_m}{\sigma^2} \right) \left( G \frac{\partial W_n}{\partial G} + W_n \right) + \frac{a}{1 + a} \left( 1 + \epsilon_n^G \right) W_n G \Gamma \kappa \sum_m \left( (D - 2RQ_m) \frac{\partial Q_m}{\partial G} - Q_m^2 \frac{\partial R}{\partial G} \right)
\]

\[
- \frac{a}{1 + a} \Gamma \kappa \sum_m \left( (1 + \epsilon_m^G) W_m + G \sum_m \left( 1 + \epsilon_m^G \right) \frac{\partial W_m}{\partial G} \right) - \frac{a}{1 + a} \Gamma \kappa \sum_m \left( (\bar{D} - 2RQ_m) \frac{\partial Q_m}{\partial G} - Q_m^2 \frac{\partial R}{\partial G} \right)
\]

\[
+ \frac{\kappa(N + 1)}{1 + N \kappa} \frac{\partial Q_n}{\partial G} - (1 + \epsilon_n^G) \frac{\kappa}{1 + N \kappa} \sum_m \frac{\partial Q_m}{\partial G}
\]

(D.19)

Next we take the derivative with respect to \( \epsilon \) at the starting point where \( \epsilon = 0 \) and \( G = 1 \). It is useful to first compute the derivatives involving wealth, using (D.3). Since the first-order derivatives of risky asset prices with respect to \( \epsilon \) are zero, so is the first-order derivative of \( W_n \) with respect to \( \epsilon \). It is also useful to derive an expression for \( \frac{\partial^2 W_n}{\partial G \partial \epsilon} \).

We have

\[
\frac{\partial W_n}{\partial G} = (1 + \epsilon_n^G) \frac{1 - \kappa}{1 + N \kappa} \frac{\partial Q_n}{\partial G} + (1 + \epsilon_n^G) \frac{\kappa}{1 + N \kappa} \sum_m \frac{\partial Q_m}{\partial G}
\]

(D.20)

Evaluated at the initial point, this is equal to \( \frac{\partial Q}{\partial G} \). The second order derivative is

\[
\frac{\partial^2 W_n}{\partial G \partial \epsilon} = g_n \frac{\partial Q}{\partial G} + \frac{1 - \kappa}{1 + N \kappa} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \frac{\kappa}{1 + N \kappa} \sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon}
\]

(D.21)

We also use that \( \frac{\partial R}{\partial G} = \frac{(\partial Q/\partial G)}{(1 + (1 + a) \lambda)} \) from (D.6).

Using this, taking the derivative of (D.19) with respect to \( \epsilon \), and subtracting the same expression for country \( k \), gives

\[
\frac{\partial^2 \left( N F_{\text{risky}}^n - N F_{\text{risky}}^k \right)}{\partial G \partial \epsilon} = 2 \frac{a}{1 + a} \frac{\kappa(N + 1)}{1 + N \kappa} \left( \bar{D} - 2RQ_n - \bar{D} - 2RQ_k \right) \frac{\partial Q}{\partial G} + a \left( g_n - g_k \right) \frac{\kappa(N + 1)}{1 + N \kappa} \left( 1 - \frac{a(1 + a)(1 + N \kappa) \Gamma}{\sigma^2} \right) \frac{\partial Q}{\partial G}
\]

\[
+ \frac{a}{1 + a} \frac{\kappa(1 - \kappa)(N + 1)}{(1 + N \kappa)^2} \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon} + \left( g_n - g_k \right) \frac{\kappa(N + 1)}{1 + N \kappa} \left( 1 - \frac{a(1 + a)(1 + N \kappa) \Gamma}{\sigma^2} \right) \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon}
\]

\[
- \frac{a^3(N + 1) \Gamma \kappa}{(1 + a) \lambda \sigma^2} \left( g_n - g_k \right) \frac{\partial Q}{\partial G} - \frac{\kappa(N + 1)}{1 + N \kappa} \left( g_n - g_k \right) \frac{\partial Q}{\partial G}
\]
Aggregating across $k$ and using that $\sum_k N F_{risky}^k = 0$, we have

$$\frac{\partial^2 N F_{risky}^n}{\partial G \partial \epsilon} = 2 \frac{\kappa(N + 1)}{1 + a} \frac{\partial Q}{\partial G} + a g_n \frac{\kappa(N + 1)}{1 + N \kappa}$$

$$+ \frac{a}{1 + a(1 + N \kappa)^2} \sum_k \frac{\partial^2(Q_n - Q_k)}{\partial G \partial \epsilon} + g_n \frac{\kappa(N + 1)}{1 + N \kappa} \left( 1 - \frac{a(1 + a)(1 + N \kappa)}{\sigma^2} \right) \frac{\partial Q}{\partial G}$$

$$- \frac{a^3(N + 1) \Gamma \kappa}{(1 + a) \lambda \sigma^2} g_n \frac{\partial Q}{\partial G} \left[ \frac{\kappa(N + 1)}{1 + N \kappa} \frac{\partial Q}{\partial G} \right]$$

$$+ \frac{\kappa}{1 + N \kappa} \sum_k \frac{\partial^2(Q_n - Q_k)}{\partial G \partial \epsilon} - \frac{\kappa(N + 1)}{1 + N \kappa} g_n \frac{\partial Q}{\partial G} \left( \frac{\partial^2 Q_n}{\partial G \partial \epsilon} \right) \left( \frac{\partial^2 Q_n}{\partial G \partial \epsilon} \right) \left( \frac{\partial^2 Q_n}{\partial G \partial \epsilon} \right)$$

We need to show that (D.22) is a positive linear function of $g_n$. If so, it follows that countries with lower risk-aversion (higher $g_n$ when $\epsilon > 0$) have lower net outflows of risky assets when global risk aversion rises ($G$ falls). Using that $\sum_{k=1}^{N+1} \frac{\partial^2 Q_k}{\partial G \partial \epsilon} = 0$, collecting terms gives

$$\frac{\partial^2 N F_{risky}^n}{\partial G \partial \epsilon} = \frac{\kappa(N + 1)}{1 + N \kappa} \left( \frac{a}{1 + a} \frac{1 - \kappa}{1 + a + N \kappa} + a(1 + a)(1 + N \kappa) \frac{\Gamma}{\sigma^2} \right) \frac{\partial^2 Q_n}{\partial G \partial \epsilon}$$

$$+ \frac{\kappa(N + 1)}{1 + N \kappa} g_n \left[ a + \left( \frac{2}{1 + a} - a(1 + a)(1 + N \kappa) \frac{\Gamma}{\sigma^2} \right) - \frac{a^3 \Gamma(1 + N \kappa)}{(1 + a) \lambda \sigma^2} \right] \frac{\partial Q}{\partial G}$$

The first line is clearly a positive linear function of $g_n$ as we have already shown that $\frac{\partial^2 Q_n}{\partial G \partial \epsilon}$ is a positive linear function of $g_n$. Substituting the expression for $\frac{\partial Q}{\partial G}$ in (34), the second line becomes

$$\frac{\kappa(N + 1)}{1 + N \kappa} g_n \frac{a \sigma^2}{(1 + a)^2 \Gamma(1 + N \kappa) - \sigma^2 + \frac{a^2 \Gamma(1 + N \kappa)}{\lambda}}$$

This is also a positive linear function of $g_n$. The denominator is positive by Assumption 2 that $\sigma^2 < \Gamma(1 + N \kappa)(1 + a)^2$.

Since the second-order derivative of total net outflows is a negative function of $g_n$, and the second-order derivative of net outflows of risky assets is a positive function of $g_n$, it follows that the second-order derivative of net outflows of safe assets is a negative function of $g_n$. Therefore a country with lower than average risk-aversion will have negative net outflows of risky assets due to the global risk-aversion shock, and positive total net outflows and net outflows of safe assets.
E  Cross Country Heterogeneity in Expected Dividends

Following the same steps as in Appendix D, we now consider the impact of heterogeneity across countries in expected dividends.

E.1  Market Clearing Conditions

The market clearing conditions remain the same as (22)-(23). Using the period 0 portfolio shares, which correspond to (24)-(25), wealth is

\[ W_n = 1 + a + \frac{1 - \kappa}{1 + N\kappa}(1 + d_n\epsilon)(Q_n - a) + \frac{\kappa}{1 + N\kappa} \sum_m (1 + d_m\epsilon)(Q_m - a) \]  

(E.1)

From Assumption 1 we have \( K_n = 1 + d_n\epsilon \). Since \( \sum_n d_n = 0 \), it follows that \( \sum_n K_n = N + 1 \), so that from Assumption 1 \( B_0^h = 0 \). Therefore \( B_1^h = Y - C_1^h \). Together with the expressions for \( K_n \) and \( W_n \), we can then write the aggregate asset market clearing condition (23) as

\[ \sum_{m=1}^{N+1} (1 + d_m\epsilon)(Q_m - a) = (N + 1)(1 + a)(Y - C_1^h) \]  

(E.2)

Taking the derivative with respect to \( G \), we have

\[ \frac{\partial R}{\partial G} = \frac{1}{(N + 1)(1 + a)\lambda} \sum_{m=1}^{N+1} (1 + d_m\epsilon) \frac{\partial Q_m}{\partial G} \]  

(E.3)

Next consider the market clearing conditions for risky assets (22). Substituting the portfolio shares (17)-(18) and wealth expressions (E.1) into (22), the market clearing conditions for risky assets are

\[ (1 + a)(1 + N\kappa) + \frac{(1 - \kappa)^2}{1 + N\kappa}(1 + d_n\epsilon)(Q_n - a) + \kappa \left( 1 + \frac{1 - \kappa}{1 + N\kappa} \right) \sum_m (1 + d_m\epsilon)(Q_m - a) \]

\[ = \frac{1 + a}{a} (1 + d_n\epsilon) \frac{1}{FG} \frac{\sigma^2}{D_n - RQ_n} \]  

(E.4)
Differentiating with respect to $G$ and substituting (E.3), we have

$$
\frac{(1 - \kappa)^2}{1 + N \kappa} (1 + d_n \epsilon) \frac{\partial Q_n}{\partial G} + \kappa \left(1 + \frac{1 - \kappa}{1 + N \kappa}\right) \sum_m (1 + d_m \epsilon) \frac{\partial Q_m}{\partial G} \\
= -\frac{1 + a}{a}(1 + d_n \epsilon) \frac{1}{\Gamma G^2} \frac{\sigma^2}{D_n - RQ_n} + \frac{1 + a}{a}(1 + d_n \epsilon) \frac{1}{\Gamma G} \frac{\sigma^2}{(D_n - RQ_n)^2 R} \frac{\partial Q_n}{\partial G} \\
+ \frac{1}{(N + 1) a \lambda} (1 + d_n \epsilon) \frac{1}{\Gamma G} \frac{\sigma^2}{(D_n - RQ_n)^2 Q_n} \sum_{m=1}^{N+1} (1 + d_m \epsilon) \frac{\partial Q_m}{\partial G}
$$

(E.5)

### E.2 Impact on Relative Prices Risky Assets

We first consider the impact of the global risk-aversion shock on relative prices of risky assets. To show that the risky asset price $Q_n$ drops more in countries with a higher expected dividend, and therefore a higher $d_n$ when $\epsilon > 0$, we need to show that $\partial^2 Q_n / [\partial G \partial \epsilon]$ depends positively on $d_n$.

To this end we need to differentiate (E.5) with respect to $\epsilon$ and evaluate at $\epsilon = 0$ and $G = 1$. At that point $Q_m = a$, $R = (1 + a)/a$ and $D_n - RQ = \sigma^2/(a \Gamma(1 + N \kappa))$. From the expression for $D_n$ in Assumption 1 we have that $\partial D_n / \partial \epsilon = d_n \sigma^2 / [a \Gamma(1 + N \kappa)]$. We use that the pre-shock equilibrium for risky asset prices and the interest rate does not depend on $\epsilon$, so that $\partial Q / \partial \epsilon = \partial R / \partial \epsilon = 0$. Since all first order derivatives of risky asset prices with respect to $G$ will be the same, we simply denote them $\partial Q / \partial G$ (see (34)). This gives

$$
\frac{(1 - \kappa)^2}{1 + N \kappa} d_n \frac{\partial Q}{\partial G} + \frac{(1 - \kappa)^2}{1 + N \kappa} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \kappa \left(1 + \frac{1 - \kappa}{1 + N \kappa}\right) \sum_m \frac{\partial^2 Q_m}{\partial G \partial \epsilon} = \\
+(1 + a)^2 \frac{\Gamma(1 + N \kappa)^2}{\sigma^2} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} - \frac{\Gamma(1 + N \kappa)^2}{\sigma^2} d_n \left((1 + a)^2 + \frac{a^2}{\lambda}\right) \frac{\partial Q}{\partial G} + \frac{\Gamma(1 + N \kappa)^2 a^2}{(N + 1) \lambda \sigma^2} \sum_{m=1}^{N+1} \frac{\partial^2 Q_m}{\partial G \partial \epsilon}
$$

(E.6)

Taking the sum across $n$, using that $\sum_{n=1}^{N+1} d_n = 0$, gives $\sum_{n=1}^{N+1} \partial^2 Q_n / [\partial G \partial \epsilon] = 0$. We then have

$$
\frac{\partial^2 Q_n}{\partial G \partial \epsilon} = d_n \frac{(1 - \kappa)^2}{1 + N \kappa} + \frac{1}{\sigma^2} \frac{\Gamma(1 + N \kappa)^2}{\Gamma(1 + N \kappa)^2} \left((1 + a)^2 + \frac{a^2}{\lambda}\right) \frac{\partial Q}{\partial G}
$$

(E.7)

The numerator of this ratio is positive. The denominator is positive as well since from Assumption 2 we have $\sigma^2 < (1 + a)^2 \Gamma(1 + N \kappa)$. Since $\partial Q / \partial G$ is positive, it follows that (E.7) is a positive linear function of $d_n$, which implies that the risky asset price drops more in countries with a higher expected dividend.
E.3 Impact on Total Net Flows

We now consider the impact of the shock on total net capital flows, which is equal to the current account as in (D.13). As in (D.13), we have

\[ CA_n = -\sum_m (W_n - W_m)/(1+a)(1+N) \]

Using the wealth expression (E.1), we have

\[ CA_n = \frac{1}{1+a} \left(1 - \frac{1}{N\kappa}\right) \left(-1 + d_n \epsilon (Q_n - a)\right) \]

The effect of a risk aversion shock is

\[ \frac{\partial CA_n}{\partial G} = \frac{1}{1+a} \left(1 - \frac{1}{N\kappa}\right) \left(-1 + d_n \epsilon \frac{\partial Q_n}{\partial G}\right) \]

Next take the derivative with respect to \(\epsilon\) and evaluate at \(\epsilon = 0\) and \(G = 1\). This gives

\[ \frac{\partial^2 CA_n}{\partial G \partial \epsilon} = -\frac{1}{1+a} \left(1 - \frac{1}{N\kappa}\right) \left(d_n \frac{\partial Q}{\partial G} + \frac{\partial^2 Q_n}{\partial G^2}\right) \] (E.8)

Since \(\partial Q/\partial G > 0\) and we have already shown that \(\partial^2 Q_n/\partial G \partial \epsilon\) is a positive linear function of \(d_n\), it follows that this second derivative is a negative linear function of \(d_n\). Therefore countries with higher expected dividends (higher \(d_n\) when \(\epsilon > 0\)) have larger net capital outflows when \(G\) falls.

E.4 Net Outflows Risky Assets

From (26) and (27), substituting the portfolio expressions (18) and (25), net outflows of risky assets are

\[ N F_{n}^{\text{risky}} = \frac{a}{1+a} W_n G \Gamma \kappa \sum_m \bar{D}_m - R Q_m \sigma^2 - \frac{a}{1+a} G \Gamma \kappa Q_n \bar{D}_n - R Q_n \sigma^2 \sum_m W_m \]

\[ + \frac{\kappa (N+1)}{1+N\kappa} (1+d_n \epsilon) Q_n - \frac{\kappa}{1+N\kappa} \sum_m (1+d_m \epsilon) Q_m \] (E.9)

Subtracting the same equation for country \(k\), we have

\[ N F_{n}^{\text{risky}} - N F_{k}^{\text{risky}} = \frac{a}{1+a} (W_n - W_k) G \Gamma \kappa \sum_m \bar{D}_m Q_m - R Q_m^2 \sigma^2 \]

\[ - \frac{a}{1+a} G \Gamma \kappa \left(\bar{D}_n Q_n - R Q_n^2\right) - \frac{a}{1+a} \sum_m W_m + \frac{\kappa (N+1)}{1+N\kappa} ((1+d_n \epsilon) Q_n - (1+d_k \epsilon) Q_k) \] (E.10)
Taking the derivative with respect to $G$, we have

$$\frac{1 + a}{a} \frac{\partial \left( N F_{risk} - N F_{risk}^k \right)}{\partial G} = (W_n - W_k) \frac{\Gamma_k}{\sigma^2} \sum_m (D_m Q_m - R Q_m^2)$$  \hspace{1cm} (E.11)

$$+ \frac{\partial (W_n - W_k)}{\partial G} \frac{G \Gamma_k}{\sigma^2} \sum_m (D_m Q_m - R Q_m^2) + (W_n - W_k) \frac{G \Gamma_k}{\sigma^2} \sum_m \left[ (D_m - 2R Q_m) \frac{\partial Q_m}{\partial G} - Q_m^2 \frac{\partial R}{\partial G} \right]$$

$$- \frac{\Gamma_k}{\sigma^2} \left[ (\bar{D}_n Q_n - R Q_n^2) - (\bar{D}_k Q_k - R Q_k^2) \right] \sum W_m$$

$$- \frac{G \Gamma_k}{\sigma^2} \left[ (D_n - 2R Q_n) \frac{\partial Q_n}{\partial G} - (D_k - 2R Q_k) \frac{\partial Q_k}{\partial G} \right] \sum \frac{\partial W_m}{\partial G}$$

$$+ \frac{G \Gamma_k}{\sigma^2} (Q_n^2 - Q_k^2) \frac{\partial R}{\partial G} + \frac{1 + a \kappa (N + 1)}{a} \frac{\partial Q_n}{\partial G} (1 + d_n \epsilon) - \frac{\partial Q_k}{\partial G} (1 + d_k \epsilon)$$

We will evaluate this derivative with respect to $\epsilon$ at the starting point where $\epsilon = 0$ and $G = 1$. Use that the derivative of any asset price $Q_m$ and wealth $W_m$ with respect to $\epsilon$ is zero at this point. It is also useful to derive an expression for $\frac{\partial^2 W_n}{\partial G \partial \epsilon}$. We have

$$\frac{\partial W_n}{\partial G} = \frac{1 - \kappa}{1 + N \kappa} (1 + d_n \epsilon) \frac{\partial Q_n}{\partial G} + \frac{\kappa}{1 + N \kappa} \sum m (1 + d_m \epsilon) \frac{\partial Q_m}{\partial G}$$  \hspace{1cm} (E.12)

Evaluated at the initial point, this is equal to $\frac{\partial W_n}{\partial G} = \frac{\partial Q}{\partial G}$. The second order derivative is

$$\frac{\partial^2 W_n}{\partial G \partial \epsilon} = \frac{1 - \kappa}{1 + N \kappa} d_n \frac{\partial Q}{\partial G} + \frac{1 - \kappa}{1 + N \kappa} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \frac{\kappa}{1 + N \kappa} \sum \frac{\partial^2 Q_m}{\partial G \partial \epsilon}$$  \hspace{1cm} (E.13)

Then

$$\frac{\partial^2 (W_n - W_k)}{\partial G \partial \epsilon} = (d_n - d_k) \frac{1 - \kappa}{1 + N \kappa} \frac{\partial Q}{\partial G} + \frac{1 - \kappa}{1 + N \kappa} \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon}$$  \hspace{1cm} (E.14)

Taking the derivative of (E.11) with respect to $\epsilon$ then gives

$$\frac{\partial^2 \left( N F_{risk} - N F_{risk}^k \right)}{\partial G \partial \epsilon} = \frac{a \kappa (1 - \kappa)(N + 1)}{1 + a} \frac{\partial Q}{\partial G} + \frac{a \kappa (1 - \kappa)(N + 1)}{1 + a} \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon}$$

$$- \frac{\kappa (N + 1)}{1 + N \kappa} (d_n - d_k) \frac{\partial Q}{\partial G} + \frac{\kappa (N + 1)}{1 + N \kappa} \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon}$$

$$- \frac{a \kappa (N + 1)}{1 + a} \frac{\partial Q}{\partial G} + \frac{\kappa (N + 1)}{1 + N \kappa} \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon}$$

$$- \frac{\kappa (N + 1)}{1 + N \kappa} (d_n - d_k) \frac{\partial Q}{\partial G} + \frac{\kappa (N + 1)}{1 + N \kappa} \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon}$$

49
Collecting terms and summing over \( k \), we have

\[
\sum_k \frac{\partial^2 \left( N F_n^{\text{risky}} - N F_k^{\text{risky}} \right)}{\partial G \partial \epsilon} = -a \kappa (N + 1)^2 d_n - \frac{a}{1 + a (1 + N \kappa)^2} \frac{\partial Q}{\partial G} \left( 1 + N \kappa \right) \sum_k \frac{\partial^2 (Q_n - Q_k)}{\partial G \partial \epsilon}
\]

Using that \( \sum_k N F_k^{\text{risky}} = 0 \), \( \sum_{k=1}^{N+1} \partial^2 Q_k / [\partial G \partial \epsilon] = 0 \), substituting (34) and (E.7) and collecting terms, we have

\[
\frac{\partial^2 N F_n^{\text{risky}}}{\partial G \partial \epsilon} = \frac{a \kappa (N + 1)}{(1 + N \kappa)} \left( 2 (N - 1) \kappa (1 - \kappa) \Gamma \left( 2 (1 + a)^2 + \frac{a^2}{\kappa} \right) d_n \right)
\]

This is clearly a positive linear function of \( d_n \). The terms in the denominator are positive since \( \sigma^2 < (1 + a)^2 \Gamma(1 + N \kappa) \).

Since the second-order derivative of total net outflows is a negative function of \( d_n \), and the second-order derivative of net outflows of risky assets is a positive function of \( d_n \), it follows that the second-order derivative of net outflows of safe assets is a negative function of \( d_n \). Therefore a country with a higher than average expected dividends will have negative net outflows of risky assets due to the global risk-aversion shock, and positive total net outflows and net outflows of safe assets.

\section*{F Proof of Theorem 4}

Given the results in Appendix D and E, Theorem 4 is now easy to proof. Let \( X_n \) be either \( Q_n \), \( N F_n \), \( N F_n^{\text{risky}} \) or \( N F_n^{\text{safe}} \). We have seen that for risk-aversion heterogeneity

\[
\frac{\partial^2 X_n}{\partial G \partial \epsilon}
\]

is a positive linear function of \( g_n \) when \( X_n \) is \( Q_n \) or \( N F_n^{\text{risky}} \), while it is a negative linear function of \( g_n \) when \( X_n \) is \( N F_n \) or \( N F_n^{\text{safe}} \). Similarly, under expected dividend heterogeneity (F.1) is a positive linear function of \( d_n \) when \( X_n \) is \( Q_n \) or \( N F_n^{\text{risky}} \), while it is a negative linear function of \( d_n \) when \( X_n \) is \( N F_n \) or \( N F_n^{\text{safe}} \).

Assume without loss of generality that \( \epsilon > 0 \). First assume that there is risk-aversion heterogeneity. From Section 5.1, a country for which \( g_n > 0 \) is then leveraged (has a negative
net foreign asset position in safe assets). The results then imply that as a result of a rise in global risk-aversion $Q_n$ and $NF_n^{risky}$ are lower than in the average country, while $NF_n$ or $NF_n^{safe}$ are higher than in the average country. This means that a leveraged country has a larger than average drop in the risky asset price, negative net outflows of risky assets and positive total net outflows and net outflows of safe assets. This uses that the first-order derivatives of all net outflow variables with respect to $G$ are zero. Since (F.1) is linear in $g_n$, the opposite will be the case for countries with a positive net foreign asset position in safe assets. It also follows that the size of these changes (in the relative risky asset price and net capital flow variables) is larger the larger the absolute size of the net foreign asset position in safe assets. The exact same results apply under expected dividend heterogeneity, using from Section 5.1 that a country for which $d_n > 0$ is leveraged.
References


https://doi.org/10.1016/j.jmoneco.2013.06.003


https://doi.org/10.1016/j.jmoneco.2012.12.004

https://doi.org/10.1016/j.jmoneco.2014.11.011

https://doi.org/10.1093/restud/rdu042

https://doi.org/10.1257/jep.31.3.29

https://doi.org/10.1257/aer.99.2.584

https://doi.org/10.1086/705719


https://doi.org/10.3386/w27927

https://doi.org/10.1257/jel.51.1.63

https://doi.org/10.1016/j.jmoneco.2020.02.006
https://doi.org/10.1016/j.jinteco.2020.103397

https://doi.org/10.1596/1813-9450-7639

https://doi.org/10.1016/j.jinteco.2012.03.006


https://doi.org/10.1016/j.jinteco.2013.12.007


https://doi.org/10.1257/aer.98.2.333

https://doi.org/10.1057/s41308-019-00077-1


https://doi.org/10.1257/aer.20130479

https://doi.org/10.1086/599706


Table 1: Regression Real Interest Rate, Stock Prices, Gross Risky Asset Flows on First Factor

<table>
<thead>
<tr>
<th></th>
<th>$r_t$</th>
<th>$q_{i,t}$</th>
<th>$of_{i,t}^{\text{risky}}$</th>
<th>$if_{i,t}^{\text{risky}}$</th>
<th>$of_{i,t}^{\text{risky}} + if_{i,t}^{\text{risky}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_t$</td>
<td>0.603***</td>
<td>10.431***</td>
<td>2.801**</td>
<td>1.911***</td>
<td>4.712***</td>
</tr>
<tr>
<td></td>
<td>(0.231)</td>
<td>(0.789)</td>
<td>(1.148)</td>
<td>(0.751)</td>
<td>(1.841)</td>
</tr>
</tbody>
</table>

$R^2$ 0.237 0.192 0.362 0.360 0.375

Notes: A linear trend is removed from the real interest rate series and each stock price series.

Table 2: Panel regression Net Capital Flows and Stock Prices on First Factor

<table>
<thead>
<tr>
<th>Dep. Var:</th>
<th>$n_{\text{f, safe}}^t$</th>
<th>$n_{\text{f, risky}}^t$</th>
<th>$n_{\text{f, safe}}^t$</th>
<th>$n_{\text{f, risky}}^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_t$</td>
<td>-0.786</td>
<td>-0.623</td>
<td>-0.547</td>
<td>0.649*</td>
</tr>
<tr>
<td></td>
<td>(0.590)</td>
<td>(0.653)</td>
<td>(0.388)</td>
<td>(0.389)</td>
</tr>
</tbody>
</table>

$nf_{a, i, t-1}^{\text{safe}} * F_t$ 0.031*** 0.028*** -0.012*** -0.009***

|           | (0.010)                | (0.005)                  | (0.003)                | (0.002)                |

$nf_{a, i, t-1}^{\text{risky}} * F_t$ -0.056 -0.026 0.030 0.019

|           | (0.045)                | (0.024)                  | (0.026)                | (0.021)                |

$R^2$ 0.519 0.413 0.537 0.266 0.240 0.297

<table>
<thead>
<tr>
<th>Dep. Var:</th>
<th>$n_{\text{f}}^t$</th>
<th>$n_{\text{t}}^t$</th>
<th>$q_{\text{t}}$</th>
<th>$q_{\text{t}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_t$</td>
<td>-0.137</td>
<td>-0.067</td>
<td>-0.062</td>
<td>10.440***</td>
</tr>
<tr>
<td></td>
<td>(0.271)</td>
<td>(0.407)</td>
<td>(0.235)</td>
<td>(0.673)</td>
</tr>
</tbody>
</table>

$nf_{a, i, t-1}^{\text{safe}} * F_t$ 0.020*** 0.019*** -0.020*** -0.017***

|           | (0.007)        | (0.006)        | (0.005)      | (0.005)      |

$nf_{a, i, t-1}^{\text{risky}} * F_t$ -0.026 -0.006 -0.057* -0.066**

|           | (0.020)        | (0.006)        | (0.030)      | (0.027)      |

$R^2$ 0.594 0.536 0.595 0.197 0.212 0.215

Notes: A linear trend is removed from each stock price series. $nf_{a, \text{safe}}$ and $nf_{a, \text{risky}}$ are a country’s net foreign asset positions in safe and risky assets.
Table 3: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>19</td>
<td>Number of countries in empirical section</td>
</tr>
<tr>
<td>$a$</td>
<td>25</td>
<td>4 percent annual interest rate</td>
</tr>
<tr>
<td>$Y$</td>
<td>2</td>
<td>Household income two-thirds of GDP</td>
</tr>
<tr>
<td>$\bar{\Gamma}$</td>
<td>10</td>
<td>Median risk aversion of 10</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Equity risk premium 4.6 percent</td>
</tr>
<tr>
<td>$\bar{k}$</td>
<td>0.0229</td>
<td>Cross-country average of $GFA^{risky}_i$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>9</td>
<td>Observed relative variation in risky and safe asset prices</td>
</tr>
<tr>
<td>$\Delta \Gamma$</td>
<td>$\bar{\Gamma}$</td>
<td>Investor leverage varies from 0% to 200%</td>
</tr>
<tr>
<td>$\Delta \kappa$</td>
<td>$\bar{k}$</td>
<td>Investor home bias varies from 0 to twice the mean</td>
</tr>
<tr>
<td>$\omega$</td>
<td></td>
<td>Varies in text</td>
</tr>
</tbody>
</table>
Notes: Gross risky asset outflows measures risky asset capital outflows across a sample of 20 advanced economies normalized by GDP. The stock index is the GDP weighted average of the stock index of the same 20 countries, the real 1yr T-bill rate is the interest rate on the 1yr US treasury minus the 1yr-ahead inflation expectation from the Survey of Professional Forecasters. Net flows in safe assets measure capital outflows minus capital inflows in portfolio debt and other debt flows. Net flows in risky assets measure capital outflows minus capital inflows in portfolio equity and direct investment. Total net flows are the sum of net flows in safe assets and net flows in risky assets.
Figure 2: First Factor from Capital Flow Factor Model and MAR factor
Figure 3: NFA Debt and NFA Equity all 20 countries (percent of GDP, average 1996-2015)
Figure 4: Asset Market Equilibrium following Global Risk Aversion Shock
Figure 5: Response Gross Flows to Risk Aversion Shock: Role of Within-Country Heterogeneity
Figure 6: Net Capital Flows and Risky Asset Prices following Risk-Aversion Shock
Figure 7: Net Capital Flows and their Decomposition following Risk-Aversion Shock