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Abstract

The idea that certain economic variables are roughly constant in the long run is an old one. Kaldor described them as stylized facts, whereas Klein and Kosobud labelled them great ratios. While such ratios are widely adopted in theoretical models in economics as conditions for balanced growth, arbitrage or solvency, the empirical literature has tended to find little evidence for them. We argue that this outcome could be due to episodic failure of cointegration, possible two-way causality between the variables in the ratios, and cross-country error dependence due to latent factors. We propose a new system pooled mean group estimator (SPMG) to deal with these features. Using this new panel estimator and a dataset spanning almost one and a half centuries and seventeen countries, we find support for five out of the seven great ratios that we consider. Extensive Monte Carlo experiments also show that the SPMG estimator with bootstrapped confidence intervals stands out as the only estimator with satisfactory small sample properties.

Keywords: Great ratios; debt, consumption, and investment to GDP ratios; arbitrage conditions; heterogeneous panels; episodic cointegration; two-way long-run causality; error cross-sectional dependence.

JEL Classification: B4, C18, C33, C5.

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1 Introduction

The idea that certain economic variables are roughly constant in the long run is an old one. Kaldor (1957, 1961) described them as *stylized facts* and Klein and Kosobud (1961) labelled them *great ratios*. Kaldor (1957, p591) wrote: "A satisfactory model concerning the nature of the growth process in a capitalist economy must also account for the remarkable historical constancies revealed by recent empirical investigations." He further noted that output per worker and capital per worker have grown at similar rates, so the capital output ratio was roughly constant as was the share of wages in national income and the return on capital. Klein and Kosobud also considered the savings–income ratio, the velocity of money in circulation, capital per worker and the labor force participation rate. Other variables thought to be constant appear in finance including the dividend-price ratio, discussed by Campbell and Shiller (1988).

Although there was some skepticism about these stylized facts,[1] they have been widely adopted in theoretical models in economics and finance, and are often implied by a range of economic theories regarding the conditions required for balanced growth, arbitrage, or debt solvency. Jones and Romer (2010, p225) in a paper called "The New Kaldor facts," take the old ones for granted. They say: "Redoing this exercise nearly 50 years later shows just how much progress we have made. Kaldor's first five facts have moved from research papers to textbooks. There is no longer any interesting debate about the features that a model must contain to explain them."

The great ratios hypothesis has also prompted a large empirical literature, including recent contributions by Müller and Watson (2018), Kapetanios et al. (2020), and Harding (2020), who also provides a survey of the literature. This empirical literature is less supportive of the hypothesis. As Harding notes: "econometric tests reject the great ratios hypothesis but economic growth theorists and quantitative macroeconomic model builders continue to embed that hypothesis in their work." Most, but not all, of these studies focus on individual countries and consider two variables, say y_t and x_t, for instance the logarithms of the numerator and denominator, and investigate if their difference, $z_t = y_t - x_t$, is stationary over a reasonably long period. There are two approaches. Firstly, one might estimate a long run coefficient θ in $y_t = \theta x_t + u_t$ and test whether $\theta = 1$.

[1] For example, Robert Solow in his classic book, Growth Theory, commented that there "is no doubt that they are stylized, though it is possible to question whether they are facts," Solow (1970, p2).

Secondly, one might test whether the difference $y_t - x_t$ is stationary, having neither stochastic nor deterministic trends.[2] Stationarity of $y_t - x_t$ can be tested either using Dickey and Fuller (1979) type tests or the KPSS test proposed by Kwiatkowski et al. (1992). Both of these two testing approaches are known to have important limitations. Testing for unit roots tends to lack power, particularly against highly persistent yet stationary alternatives. The KPSS test involves estimating the long run variance of the partial sum series, $s_t = \sum_{\tau=1}^{t} z_\tau$, which requires quite long time series if the size of the test is to be controlled, particularly when z_t is stationary but highly persistent. A number of other studies treat y_t and x_t as unit root processes, or more generally as first order integrated processes, $I(1)$, and test whether they are cointegrated with a unit long run coefficient, so that z_t becomes stationary, or an $I(0)$ process.[3] The cointegration approach provides a better insight into the reasons behind the failure of the great ratios hypothesis, as it separates the possibility of cointegration between y_t and x_t from the requirement of a unit long run coefficient. For example, it is often found that logs of real income and consumption cointegrate but their long run coefficient is not unity. Cointegration tests are more informative but suffer from similar limitations to unit root testing.

The failure of unit root and cointegration tests in the analysis of long run relations is also examined by Müller and Watson (2018) and Kapetanios et al. (2020). Müller and Watson find that inference on long run covariability of y_t and x_t is complicated and critically depends on the exact form of their long run persistence. Harding (2020) argues that unit root and cointegration tests cannot produce valid inference on the great ratios hypothesis, because of the nature of the stationary distribution of the ratio.

In this paper we accept the critique of applying unit root and cointegration tests to the great ratios hypothesis, but argue that a possible reason for this could be episodic failures of the error correction mechanisms that bring y_t and x_t back to their long run equilibrium relationship. These failures may be associated with major shocks such as wars, depressions, natural disasters, or important policy failures. In consequence, y_t and x_t need not cointegrate over all time periods and across

[2] If there are deterministic trends in x_t and y_t they must be co-trending such that the trend cancels out for the difference not to have a trend.

[3] There can be a long-run relationship whether the variables are both $I(0)$ or both $I(1)$ but it will only be a cointegrating relationship in the $I(1)$ case. When the variables are $I(0)$ they must be co-trending, the trends cancel out, for the great ratios hypothesis to hold.

all countries. Such outcomes, which we call episodic cointegration, make it difficult to estimate the long run coefficients with any precision from the relatively short-span time series that are typically used for a single country. A panel data approach that considers the great ratios hypothesis across many countries seems to be needed. With this in mind we considered most commonly used panel approaches in the literature. We started with the pooled mean group (PMG) estimator of Pesaran, Shin, and Smith (1999) that allows for heterogeneous short run coefficients, whilst restricting the long run coefficient to be the same across countries. This is particularly appropriate for the analysis of the great ratios hypothesis which implies a homogeneous long run coefficient of unity across all countries. The PMG also has the advantage of allowing the country-specific error correction coefficients, that govern adjustments towards the long run equilibrium, to be zero in some countries. The PMG estimators can be applied whether the variables are integrated of order zero, I(0) or I(1). If the variables are I(1) the long run equilibrium corresponds to a cointegrating relationship, which we take to be the case for the great ratios. However, PMG being based on a single equation approach, has the disadvantage that it cannot handle two-way long run causality. Breitung (2005) modified PMG to handle two-way long run causality, but unfortunately his two-step estimator breaks down when there are non-cointegrating episodes in the panel under consideration. Similar limitations also apply to the panel dynamic OLS (PDOLS) estimator proposed by Mark and Sul (2003), and the mean group estimator based on country-specific Müller and Watson (2018) estimates (MGMW estimator, for short) that we shall be considering in this paper.

Using an extensive set of Monte Carlo experiments we show that none of the existing panel estimators are satisfactory for two reasons. First, they all suffer from large size distortions in the case of sample sizes that are typically available, even if their baseline assumptions are met. Second, and more importantly, the assumptions that underlie them are too restrictive for the analysis of great ratios, as they either require cointegration conditions to hold in all countries and/or the direction of long run causality between y_t and x_t to be known. To overcome these drawbacks, we propose a new system PMG estimator, or SPMG for short, that allows for two-way long run causality and does not require the variables to be cointegrated in all countries.[4] Under standard

[4] The SPMG estimator can also be viewed as a panel version of Johansen (1991) maximum likelihood approach where the cointegrating vectors, $\{\boldsymbol{\beta}_i, i = 1, 2, ..., n\}$ are assumed to be the same across all i ($\boldsymbol{\beta}_i = \boldsymbol{\beta}$). In this set up Johansen's reduced rank computational algorithm is no longer applicable and the estimates of the common long run coefficients, $\boldsymbol{\beta}$, and the n unit-specific error correction coefficients must be computed iteratively.

error cross-sectional independence the SPMG estimator continues to be super consistent in T (time period) and converges to its true value at the rate of $T^{-1}n^{-1/2}$, where n is the number of countries. However, cross-sectional independence assumption in unlikely to hold in practice and assuming it could lead to artificially low standard errors. To achieve reliable small sample inference in the presence of cross-sectional dependence of errors, we propose a bootstrap procedure, which performs surprisingly well. Our Monte Carlo experiments show that the SPMG estimator with bootstrapped confidence intervals stands out as the only estimator that meets all three robustness criteria (namely two-way long run causality, episodes of non-cointegration, and error cross-sectional dependence) and at the same time has satisfactory small sample properties.

In the empirical application we use the long-span data covering the period 1870-2016 for a panel of 17 countries that has been made available in the Jordà-Schularick-Taylor (JST) macrohistory database.[5] We focus on the SPMG estimator, but for comparison we also report estimates obtained using PMG, PDOLS, Breitung's estimator, and the MGMW estimator. We present estimates of the mean long run coefficient for seven theoretical relationships. For consumption on GDP, the SPMG estimator of the long run coefficient is 0.907 (0.884-0.930), with the bootstrapped 95% confidence interval in brackets; for investment on GDP the long run coefficient is 1.044 (1.029-1.059); for imports on exports the long run coefficient is 0.967 (0.961-0.973); for government debt on GDP we have 1.051 (0.993-1.108); for short on long interest rates we obtain 1.010 (0.912-1.108); for inflation on long interest rates 0.653 (0.419-0.888); and finally for inflation on money growth 1.227 (1.153-1.300). From a statistical perspective, with the exception of two ratios (debt-GDP and long-short rates), all the remaining long run estimates are significantly different from one at the 5% level. But given that the long run estimates are based on a large number of observations (147 years pooled across 17 countries), perhaps it is not surprising that some of the point estimates that are close to unity are still found to be statistically different from unity. From an economic perspective, all but two (those involving inflation) are quite close to unity, with their point estimates falling in the narrow range of 0.9 to 1.1. Of the seven long run relations considered we can confidently conclude that our empirical results do not support the hypothesis of a unit long run relationship between inflation and the long-term interest rate, and inflation and money supply growth. The

[5] See Jordà, Schularick, and Taylor (2017) and Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019), and the link http://www.macrohistory.net/data.

remaining five long run relations tend to support the great ratio hypothesis, and their inclusion in macroeconomic model seems to us to be justified.

The rest of the paper is set out as follows: Section 2 reviews the theoretical underpinnings of the great ratios hypothesis. Section 3 reviews the econometric issues involved in testing of the great ratios hypothesis, including the issue of episodic cointegration. Section 4 introduces the system pooled mean group estimator and compares it with four other estimators. Section 5 summarizes Monte Carlo evidence on the performance of individual estimators. Section 6 provides empirical evidence for the presence of unit elasticities for seven great ratios using the different estimators, and discusses the findings. Section 7 ends with some concluding remarks. Data Appendix provides additional information on the data.[6]

2 Economics of the great ratios hypothesis

The great ratios hypothesis refers to ratios of non-stationary macro variables and postulates that such ratios are stationary, even though the underlying variables are trended. This could arise when macro variables are driven by common non-stationary factors, which could exhibit deterministic or stochastic trends. The existence of such common drivers is often motivated by resort to long run equilibrium economic theory. Three types of theoretical mechanisms have been suggested in the literature: balanced growth, arbitrage or solvency requirements.

The first mechanism relates to the conditions required for the existence of steady states in growth models. Within the deterministic neoclassical growth models with technical progress assumed to be growing at a constant rate, steady states are defined along balanced growth paths that require output, capital, investment and consumption to grow at the same rate, implying constant consumption-output, capital-output and investment-output ratios. In stochastic settings these ratios vary over time but must be stationary.[7] Attfield and Temple (2010) point out that the equilibrium values of the great ratios depend on the structural parameters of the growth model,

[6] The paper is also accompanied by an online supplement, which provides the details of individual panel estimators, including the proposed bootstrapping procedure and a detailed description of the Monte Carlo experiments with a full set of Monte Carlo results.

[7] Deterministic models of growth were considered by Solow (1956), Swan (1956), and Barro and Sala-i-Martin (1995), among others. Stochastic growth models were developed by Merton (1975), Donaldson and Mehra (1983) and Binder and Pesaran (1999) among others.

which may vary over time, and thus introduce further complications in the empirical analysis.

In the case of the stability of the share of wages discussed by Cette et al. (2019), measurement issues have been of central concern. It is not clear how the earnings of the self employed and top CEOs should be treated, the earnings of the former containing a large wage component the latter a large profit component. The treatment of residential real estate income can also make a difference. When they correct for these biases they do not find a general decline in the labour share in their sample of advanced countries though the behavior of the US labour share after 2000 presents a puzzle. Barro (2021) argues national income accounts double-count investment, which results in GDP overstating sustainable consumption. Barro shows the US labour-income share is reasonably stable when computed using permanent income as opposed to a declining share based on GDP.

A second mechanism is through arbitrage where profitable opportunities due to market misalignments are exploited, thus preventing large price distortions persisting. The arbitrage can operate domestically and across countries through product and asset markets. The central issue is that certain variables, like real wages and labour productivity, that underlie the labour share, or stock prices and dividends, cannot diverge indefinitely. Even small differences in growth rates blow up over the longer term. For example, the well known Gordon (1962) model, derived under the assumption that dividends grow at a constant rate, predicts a constant price-dividend ratio. A stochastic generalization of Gordon model is proposed by Campbell and Shiller (1988).

Another prominent example of arbitrage at work, is the Law of One Price that postulates that prices of similar goods across countries must be the same when denominated in the same currency, although in practice there are price deviations due shipping costs and volatility of exchange rates. Similar arbitrage are expected to take place in the context of international bond markets where in equilibrium interest rate differentials across countries should match expected exchange rate depreciation. However, risks associated with trade and transactions costs may mean that the price divergence has to be above a threshold level before it becomes worthwhile to trade. In addition, the arbitrage conditions would apply to traded goods, while the price indices used include the non-traded services sector leading to the Balassa-Samuelson effect that consumer prices tend to be systematically higher in richer countries because of the larger difference in productivity between the traded and non-traded sectors. Also due to low labour costs, services tend to be much cheaper in poorer countries. In short, whether arbitrage can operate fully and in a timely manner becomes an empirical issue and equilibrium arbitrage conditions need not hold in all countries at all times.

A third mechanism operates through solvency conditions which ensure that variables like balance of payments as a share of income and government debt as a share of output are stationary, otherwise countries could accumulate debt with little likelihood of debt repayments. Solvency conditions impose inter-temporal budget constraints. For governments, the inter-temporal budget constraint requires that the market value of debt is equal to the present discounted value of expected future primary surpluses. However, as argued by Bohn (2007), solvency cannot be inferred from the statistical properties of debt because the inter-temporal budget constraint and transversality condition impose little restriction on the time series properties of the variables. Chudik et al. (2017) show for a panel of countries that, while log public debt and income are I(1), they do not cointegrate for around half of the countries considered. Even for those that cointegrate, there are statistically significant departures from the unit elasticity.[8]

3 Econometrics of the great ratios hypothesis

Kaldor (1957, 1961) did not provide much evidence for stability of the great ratios, beyond a couple of casual references. Klein and Kosobud (1961) were more systematic, fitting linear trends to logarithms of the ratios using U.S. annual data covering the period 1900-1953, and then testing whether the trend coefficient was significant. As it is now well known, such tests are likely to be misleading if the underlying variables are I(1). There is a further problem that there may be long cycles, so that there appears to be a significant upward or downward trends over the estimation sample, but such trends could be reversed if a longer time span is considered. The time period considered by Klein and Kosobud covered two World Wars and the Great Depression and might have biased the results against the great ratios hypothesis.

Pesaran and Smith (1998) note that the evidence for the cointegration of the logarithms of consumption and income or of investment and income in the data from King et al. (1991) is quite weak. Harvey et al. (2003) examine four ratios for the G7 countries. Mills (2009) analyses the

[8]They thus focus on short-run responses and show that the elasticity of debt with respect to output differs depending whether the rise in output is due to a fiscal shock, when it is greater than one, or to a technology shock when it is less than one.

Klein-Kosobud data using modern econometric techniques. Tropimov (2017) considers the stability of the capital output ratio. All these studies question the long run stability of the great ratios.

Müller and Watson (2018) consider the analysis of great ratios from the perspective of long run covariability between y_t and x_t, and argue that the problem arises due to lack of sufficiently long samples and the fact that inference depends on the long run persistence properties of the underlying variables. They argue that since different orders of integration yield statistics with different probability distributions, inference on long run covariability critically depends on the exact form of the long run persistence of the underlying series. But there is limited sample information available to empirically determine the form of persistence. They suggest methods designed to provide reliable inference about long run covariability for a wide range of persistence patterns. Their estimation approach focusses on a relatively small number of low-frequency long averages of the growth rates of the data to measure long run variability and covariability. They apply their methods to various long run relationships. Lunsford and West (2017) and Del Negro et al. (2018) also use Müller and Watson (MW) procedure to examine long run relationships. These studies suggest that in terms of estimation the MW procedure does not add a lot over simpler methods, but it provides methods for inference which are robust to the order of integration. The robustness comes at a price, the confidence intervals attached to their estimates can be quite wide.

There are also well known problems with testing for unit roots, discussed in the introduction. Stock and Watson (2017) following Elliott (1998) emphasize that evidence for cointegration can be very fragile in the case of departures from exact unit roots. Small deviations from a unit root can cause large size distortions in cointegration tests. Harding (2020) also argues that the usual unit root and cointegration tests produce invalid inferences about the great ratios hypothesis because of the nature of the stationary distribution of the ratio. There are further problems with bounded time series, such as ratios that lie between zero and one, Cavaliere and Xu (2014). Testing for deterministic trends can also be problematic as shown recently by Elliott (2020) who proposes a new method for testing the coefficient of the linear time trend in regressions with highly serially correlated errors.

3.1 Episodic cointegration

As noted in the introduction, there may also be episodic failures of the error correction mechanisms that are supposed to bring y_t and x_t back to their long run equilibrium relationship, causing episodic failures of cointegration in particular countries or time periods. This possibility has been discussed by Siklos and Granger (1997) who proposed the concept of regime-sensitive cointegration whereby the variables fall in and out of an equilibrium relationship and the underlying series need not be cointegrated at all times. Cointegration is switched off when a common stochastic trend is added. Psaradakis et al. (2004) consider Markov switching error-correction models where the speed of adjustment to equilibrium could be different in different regimes.

To examine such episodic failure of error correction, consider two variables, x_{it} and y_{it}, which might be the logarithms of the numerator or denominator of one of the great ratios, for country $i = 1, 2, ..., n$ over the period $t = 1, 2, ..., T$. We consider the following general error correction equations for y_{it} and x_{it}:[9]

$$\Delta y_{it} = a_{yi} - \phi_{yit}\xi_{i,t-1} + \sum_{\ell=1}^{p-1} \boldsymbol{\psi}'_{yi\ell}\Delta\mathbf{w}_{i,t-\ell} + u_{yit}, \quad (1)$$

$$\Delta x_{it} = a_{xi} - \phi_{xit}\xi_{i,t-1} + \sum_{\ell=1}^{p-1} \boldsymbol{\psi}'_{xi\ell}\Delta\mathbf{w}_{i,t-\ell} + u_{xit}, \quad (2)$$

where $\Delta y_{it} = y_{it} - y_{i,t-1}$, $\Delta x_{it} = x_{it} - x_{i,t-1}$, $\Delta\mathbf{w}_{it} = (\Delta y_{it}, \Delta x_{it})'$, with the common error correction term defined by

$$\xi_{it}(\theta_i) = y_{it} - \theta_i x_{it} - \mu_{it}. \quad (3)$$

The above system of equations is very general and allows for two-way short run as well as long run feedbacks between y_{it} and x_{it}. Also by allowing the error correction coefficients, ϕ_{yit} and ϕ_{xit}, to vary over time the above specification can also deal with episodic failure of error correction which is the focus of this paper. Specifically, for some country-time episode $\mathcal{T}_i$, error correction fails such that

$$\phi_{sit} = \begin{cases} \phi_{si} \neq 0, & \text{for } t \notin \mathcal{T}_i \\ 0, & \text{for } t \in \mathcal{T}_i \end{cases}, \ s = y, x.$$

[9]Applications with more than two variables are beyond the objective of the current paper, which focusses on the analysis of great ratios.

In the above set up, the great ratios hypothesis applied to $\mathbf{w}_{it} = (y_{it}, x_{it})'$ may fail because (i) the long run coefficient is not unity, $\theta_i \neq 1$; (ii) there are trends or level shifts in the long run relationship, $\mu_{it} \neq 0$; (iii) there is no adjustment, both ϕ_{yit} and ϕ_{xit} are zero ($\phi_{yit} = \phi_{xit} = 0$). For instance, Kapetanios et al. (2020) assume $\theta_i = 1$, and show that for a number of ratios calculated using recent UK data ξ_{it} is $I(1)$ using a constant μ_{it}, but $I(0)$ using a non-parametric estimate of a slowly varying μ_{it}.

4 System pooled mean group estimator

Since the null of the great ratios hypothesis is a homogeneous long run coefficient, namely $\theta_i = \theta = 1$, for all i, we abstract from intercept shifts in the long run relations and set $\mu_{it} = 0$ in (1). But we allow for the short run coefficients to differ across countries. The objective is to estimate θ and test $H_0 : \theta = 1$, allowing for the fact that there may be episodic failure of error correction such that ϕ_{yit} or ϕ_{xit} can be zero in some countries or some time periods. To do this we need an estimator of θ that allows for (i) two-way long run causality between x_{it} and y_{it} (ii) episodic failure of error correction, namely $\phi_{yit} = \phi_{xit} = 0$ for some subperiods, $\mathcal{T}_i$, and (iii) robust inference on testing $\theta = 1$.

Since under the great ratios hypothesis the long run coefficients, θ_i, are the same across countries, then the single-equation pooled mean group (PMG) estimator of Pesaran, Shin, and Smith (1999) is a natural starting choice. However, the PMG estimator applies if direction of long run causality between y_{it} and x_{it} is known, and does not allow for two-way long run causality between y_{it} and x_{it}. The concept of long run causality is discussed in Granger and Lin (1995) and Pesaran, Shin, and Smith (2001), and in the context of (1) and (2) is defined in terms of ϕ_{yit} and ϕ_{xit}. Specifically, x_{it} (resp. y_{it}) is said to long run cause y_{it} (resp. x_{it}) if $\phi_{yit} \neq 0$ and $\phi_{xit} = 0$ (resp. $\phi_{yit} = 0$ and $\phi_{xit} \neq 0$). Two-way long run causality arises when ϕ_{yit} and ϕ_{xit} are both non-zero. It is also worth noting that long run causality does not rule out short-term feedbacks from $\Delta y_{i,t-\ell}$, $\ell = 1, 2, ...p-1$ to Δx_{it} (and *vice versa*). As can be seen from (1) and (2) lagged changes $\Delta x_{i,t-\ell}$, $\ell = 1, 2, ..., p-1$ (resp. $\Delta y_{i,t-\ell}$, $\ell = 1, 2, ..., p-1$) are allowed to influence Δy_{it} (resp. Δx_{it}). In empirical analyses of great ratios it is particularly important that no assumptions are made about the direction of causality, whether short run or long run.

To simplify the analysis we assume the error correction coefficients, ϕ_{yit} and ϕ_{xit}, are time-invariant and denote them by the 2×1 vector $\boldsymbol{\phi}_i = \left(\phi_{yi}, \phi_{xi}\right)'$. We allow for $\boldsymbol{\phi}_i = 0$ for some countries. While our analysis can be extended to scenarios where there are cointegration failures for some sub-periods of the same country, by considering each sub-period as if it relates to a new synthetic country, we decided not to pursue this extension for two reasons. First, identification of such sub-periods is not straightforward, and secondly the time series data available for many countries is rather short and any sample splits can lead to biased estimates due to short sub-samples. We also abstract from structural breaks in growth rates and set $\mu_{it} = 0$. Under these conditions the system of equations (1)-(3) can be written as

$$\Delta \mathbf{w}_{it} = -\boldsymbol{\phi}_i \boldsymbol{\beta}' \mathbf{w}_{i,t-1} + \boldsymbol{\Upsilon}_i \mathbf{q}_{it} + \mathbf{u}_{it}, \tag{4}$$

where $\mathbf{w}_{it} = (y_{it}, x_{it})'$, $\boldsymbol{\beta} = (1, -\theta)'$, $\boldsymbol{\Upsilon}_i = \left(\mathbf{a}_i, \boldsymbol{\Psi}_{i,1}, \boldsymbol{\Psi}_{i,2}, ..., \boldsymbol{\Psi}_{i,p-1}\right)'$, $\mathbf{a}_i = (a_{yi}, a_{xi})'$, $\boldsymbol{\Psi}_{i\ell} = \left(\boldsymbol{\psi}_{yi\ell}, \boldsymbol{\psi}_{xi\ell}\right)'$, for $\ell = 1, 2, ..., p-1$, $\mathbf{q}_{it} = \left(1, \Delta \mathbf{w}'_{i,t-1}, \Delta \mathbf{w}'_{i,t-2}, ..., \Delta \mathbf{w}'_{i,t-p+1}\right)'$, and $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ is a reduced-form error 2×1 vector with $E(\mathbf{u}_{it}) = \mathbf{0}$, and $E(\mathbf{u}'_{it}\mathbf{u}_{it}) = \boldsymbol{\Sigma}_i$, a positive definite covariance matrix.

To deal with two-way long run causality, Breitung (2005) considers the 2×1 vector $\boldsymbol{\gamma}_i = \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\phi}_i$ and assumes that $\boldsymbol{\gamma}'_i \boldsymbol{\phi}_i = \boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\phi}_i \neq 0$, for all i. Then pre-multiplying both sides of (4) by $\boldsymbol{\gamma}'_i$ he obtains[10]

$$\boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \Delta \mathbf{w}_{it} = -\left(\boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\phi}_i\right) \boldsymbol{\beta}' \mathbf{w}_{i,t-1} + \boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\Upsilon}_i \mathbf{q}_{it} + \boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \mathbf{u}_{it},$$

which in turn yields

$$z_{it} = -\boldsymbol{\beta}' \mathbf{w}_{i,t-1} + \boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\Upsilon}_i \mathbf{q}_{it} + \mathbf{v}_{it}, \tag{5}$$

where $z_{it} = \left(\boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\phi}_i\right) \boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \Delta \mathbf{w}_{it}$, and $\mathbf{v}_{it} = \left(\boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\phi}_i\right)^{-1} \boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \mathbf{u}_{it}$. Under the normalization $\boldsymbol{\beta} = (1, -\theta)'$, the above equation can be written equivalently as

$$z^+_{it} = \theta x_{i,t-1} + \boldsymbol{\kappa}'_i \mathbf{q}_{it} + v_{it}, \tag{6}$$

where $z^+_{it} = \left(\boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\phi}_i\right)^{-1} \boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \Delta \mathbf{w}_{it} + y_{i,t-1}$, and $\boldsymbol{\kappa}_i = \boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\Upsilon}_i$.[11] Initial estimates of $\boldsymbol{\phi}_i$ and $\boldsymbol{\Sigma}_i$

[10] Breitung considers a more general set up where there are $k \geq 2$ variables and $0 < r < k$ cointegrating relationships.

[11] Abstracting from higher order lags, and from deterministic terms, equations (5) and (6) correspond to equations

are estimated from the first step regressions (4) for each cross section unit not imposing homogeneity of θ (using Johansen or Engle-Granger approach), while pooled θ is estimated from the second stage (6). The inversion of $\left(\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\phi}}_i\right)$ may however be problematic for some units that do not have a long run relationship, namely when $\hat{\boldsymbol{\phi}}_i$ converges (in probability) to a zero vector. This is why the two-step Breitung's estimator is not robust to absence of cointegration in some cross section units.

To deal with cointegration failure we propose an alternative extension of PMG, that we call system PMG (SPMG), which can handle the two-way long run causality, allows for possible cointegration failure for some countries, as well as being capable of accurate inference even when errors are cross-sectionally dependent and heteroskedastic (across both i and t). Specifically, we allow for two-way causality without needing to use the inverse of $\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\phi}}_i$. The PMG estimator was derived by maximizing the log-likelihood of y_{it} conditional on x_{it}, assuming that $\phi_{ix} = 0$. Here we follow the same likelihood approach but maximize the system log-likelihood function for $\mathbf{w}_{it}$ given by the model (4), allowing for one or both elements of $\boldsymbol{\phi}_i$ to be zero for some i.

Under Gaussian errors, the log-likelihood function for unit i conditional on the initial observations $\mathbf{w}_{i,1}, \mathbf{w}_{i,2}, ..., \mathbf{w}_{i,p}$ is given by

$$\begin{aligned}\mathcal{L}_{i,T}\left(\theta, \boldsymbol{\phi}_i, \boldsymbol{\Sigma}_i\right) &= -\frac{(T-p)}{2}\ln\left(2\pi\right) + (T-p)\ln\left|\boldsymbol{\Sigma}_i^{-1}\right| \\ &\quad -\frac{1}{2}\sum_{t=p+1}^{T}\mathbf{u}_{it}'\boldsymbol{\Sigma}_i^{-1}\mathbf{u}_{it},\end{aligned}$$

where $\mathbf{u}_{it}\left(\theta, \boldsymbol{\phi}_i\right)$ is defined by (4) which we write more compactly as

$$\mathbf{u}_{it} = \Delta\mathbf{w}_{it} + \boldsymbol{\phi}_i\xi_{i,t-1}\left(\theta\right) - \boldsymbol{\Upsilon}_i\mathbf{q}_{it},$$

with the error correction term, $\xi_{it}\left(\theta\right) = y_{it} - \theta x_{it}$. Concentrating out the effects of short-term dynamics (represented by $\mathbf{q}_{it}$) and pooling the individual log-likelihood functions under error cross-

(3) and (6) of Breitung (2005) for $k = 2$, and $r = 1$.

sectional independence, we now obtain the following the concentrated system log-likelihood function

$$\mathcal{L}_{n,T}\left(\theta,\boldsymbol{\phi},\boldsymbol{\Sigma}\right) = -\frac{(T-p)\,n}{2}\ln\left(2\pi\right)+\left(T-p\right)\sum_{i=1}^{n}\ln\left|\boldsymbol{\Sigma}_i^{-1}\right| -\frac{1}{2}\sum_{i=1}^{n}\sum_{t=p+1}^{T}\tilde{\mathbf{u}}_{it}\left(\theta,\boldsymbol{\phi}_i\right)'\boldsymbol{\Sigma}_i^{-1}\tilde{\mathbf{u}}_{it}\left(\theta,\boldsymbol{\phi}_i\right), \tag{7}$$

where $\boldsymbol{\phi}=\left(\boldsymbol{\phi}_1',\boldsymbol{\phi}_2',...,\boldsymbol{\phi}_n'\right)'$, $\boldsymbol{\Upsilon}=\left(\boldsymbol{\Upsilon}_1',\boldsymbol{\Upsilon}_2',...,\boldsymbol{\Upsilon}_n'\right)'$, and $\boldsymbol{\Sigma}=\left(\boldsymbol{\Sigma}_1,\boldsymbol{\Sigma}_2,...,\boldsymbol{\Sigma}_n\right)'$, and $\tilde{\mathbf{u}}_{it}\left(\theta,\boldsymbol{\phi}_i\right)'$are row-vectors of $\tilde{\mathbf{U}}_i=\left[\tilde{\mathbf{u}}_{i,p+1}\left(\theta,\boldsymbol{\phi}_i\right),\tilde{\mathbf{u}}_{i,2}\left(\theta,\boldsymbol{\phi}_i\right),...,\tilde{\mathbf{u}}_{i,T}\left(\theta,\boldsymbol{\phi}_i\right)\right]'$ given by

$$\tilde{\mathbf{U}}_i=\tilde{\mathbf{U}}_i\left(\theta,\boldsymbol{\phi}_i\right)=\mathbf{H}_i\left[\Delta\mathbf{W}_i+\boldsymbol{\xi}_{i,-1}\left(\theta\right)\boldsymbol{\phi}_i'\right], \tag{8}$$

in which $\boldsymbol{\xi}_{i,-1}\left(\theta\right)=\left[\xi_{ip}\left(\theta\right),\xi_{i,p+1}\left(\theta\right),...,\xi_{i,T-1}\left(\theta\right)\right]'$, $\Delta\mathbf{W}_i=\left(\Delta\mathbf{w}_{i,p+1},\Delta\mathbf{w}_{i,p+2},,...,\Delta\mathbf{w}_{iT}\right)'$, $\Delta\mathbf{w}_{it}=\left(\Delta y_{it},\Delta x_{it}\right)'$. $\mathbf{H}_i$ is orthogonal projection matrix given by $\mathbf{H}_i=\mathbf{I}_{T-p}-\mathbf{Q}_i(\mathbf{Q}_i'\mathbf{Q}_i)^{-1}\mathbf{Q}_i'$, where $\mathbf{Q}_i$ is matrix of observations on $\mathbf{q}_{it}$, namely $\mathbf{Q}_i=\left(\mathbf{q}_{i,p+1},\mathbf{q}_{i,p+2},...,\mathbf{q}_{i,T}\right)'$.

Following this approach, the first order conditions for θ, $\boldsymbol{\phi}_i$, and $\boldsymbol{\Sigma}_i$ imply the following implicit solutions for MLE estimators, $\hat{\theta}$, $\hat{\boldsymbol{\phi}}_i$ and $\hat{\boldsymbol{\Sigma}}_i$:

$$\hat{\theta}=-\left[\sum_{i=1}^{n}\left(\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i\right)\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}\right]^{-1}\sum_{i=1}^{n}\mathbf{x}_{i,-1}'\mathbf{H}_i\left(\Delta\mathbf{W}_i+\mathbf{y}_{i,-1}\hat{\boldsymbol{\phi}}_i'\right)\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i, \tag{9}$$

$$\hat{\boldsymbol{\phi}}_i=-\left[\boldsymbol{\xi}_{i,-1}'\left(\theta\right)\mathbf{H}_i\boldsymbol{\xi}_{i,-1}\left(\theta\right)\right]^{-1}\Delta\mathbf{W}_i'\mathbf{H}_i\boldsymbol{\xi}_{i,-1}'\left(\theta\right), \tag{10}$$

and

$$\hat{\boldsymbol{\Sigma}}_i=\left(T-p\right)^{-1}\left[\Delta\mathbf{W}_i+\boldsymbol{\xi}_{i,-1}\left(\theta\right)\hat{\boldsymbol{\phi}}_i'\right]'\mathbf{H}_i\left[\Delta\mathbf{W}_i+\boldsymbol{\xi}_{i,-1}\left(\theta\right)\hat{\boldsymbol{\phi}}_i'\right], \tag{11}$$

where $\mathbf{x}_{i,-1}=\left(x_{ip},x_{i,p+1},...,x_{i,T-1}\right)'$, $\mathbf{y}_{i,-1}=\left(y_{ip},y_{i,p+1},...,y_{i,T-1}\right)'$. Given an initial estimate of θ, say $\hat{\theta}^{(1)}$, initial estimates $\hat{\boldsymbol{\phi}}_i^{(1)}$ and $\hat{\boldsymbol{\Sigma}}_i^{(1)}$ can be computed using (10) and (11). The estimates $\hat{\boldsymbol{\phi}}_i^{(1)}$ and $\hat{\boldsymbol{\Sigma}}_i^{(1)}$, can then be used to updated the estimate of θ, say $\hat{\theta}^{(2)}$, using (9), and so on until convergence.

Note that $\hat{\boldsymbol{\phi}}_i$ (and $\hat{\boldsymbol{\Sigma}}_i^{-1}$) effectively act as weights in (9). Specifically, when $\hat{\boldsymbol{\phi}}_i\to_p\mathbf{0}$ for some i, then the contribution of this unit to the pooled estimate of $\hat{\theta}$ in (9) will be negligible, and tends to zero as $T\to\infty$. This is why SPMG continues to be applicable even if y_{it} and x_{it} do not cointegrate for some (but not all) units. In addition, SPMG is invariant to the ordering of the variables, whilst the two-step version of Breitung (2005) is not. More formally, following the literature, assume that $\mathbf{u}_{it}$ are independently distributed over i and t, with mean $\mathbf{0}$ and finite positive definite covariances, $\boldsymbol{\Sigma}_i$ (specifically $0 < c < \lambda_{min}(\boldsymbol{\Sigma}_i) < \lambda_{max}(\boldsymbol{\Sigma}_i) < C < \infty$), and $\mathbf{w}_{it} \sim I(1)$ for all i, but allow a number of error correction vectors, $\boldsymbol{\phi}_i$, to be zero (non-cointegrating). Specifically, without loss of generality, suppose that

$$\begin{aligned} \boldsymbol{\phi}_i'\boldsymbol{\phi}_i &> 0, \text{ for } i = 1, 2, ..., m, && (12) \\ \boldsymbol{\phi}_i &= \mathbf{0}, \text{ for } i = m+1, m+2, ..., n. && (13) \end{aligned}$$

Using standard results from the literature we have $\hat{\boldsymbol{\phi}}_i = \boldsymbol{\phi}_i + o_p(1)$, and $\hat{\boldsymbol{\Sigma}}_i^{-1} = \boldsymbol{\Sigma}_i^{-1} + o_p(1)$, and[12]

$$\mathbf{H}_i\left(\Delta\mathbf{W}_i + \mathbf{y}_{i,-1}\hat{\boldsymbol{\phi}}_i'\right) = \mathbf{H}_i\left(\mathbf{U}_i - \theta_0\mathbf{H}_i\mathbf{x}_{i,-1}\boldsymbol{\phi}_i'\right) + o_p(1).$$

where θ_0 is the true value of θ. Using these results in (9) it now follows that

$$n^{1/2}T\left(\hat{\theta} - \theta_0\right) = -\ Q_{nT}^{-1}q_{nT} + o_p(1), \quad (14)$$

where

$$Q_{nT} = n^{-1}\sum_{i=1}^{n}\left(\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\right)\left(T^{-2}\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}\right), \text{ and } q_{nT} = n^{-1/2}\sum_{i=1}^{n}\left(T^{-1}\sum_{t=1}^{T}\tilde{x}_{i,t-1}\mathbf{u}_{it}'\right)\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i. \quad (15)$$

Under cross-sectional independence, and certain regularity conditions concerning the moments of $\mathbf{u}_{it}$, it then follows that $n^{1/2}T\left(\hat{\theta} - \theta\right)$ tends to a Gaussian distribution so long as Q_{nT} tends to a non-zero limit as n and $T \to \infty$. Since $\mathbf{w}_{it}$ is a unit root process, then $T^{-2}\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}$ converges to a stochastically bounded and strictly positive random variable, and it is sufficient that $n^{-1}\sum_{i=1}^{n}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i$ also converges to a non-zero limit. Under (12) and (13) we have (note that by

[12] See, for example, Pesaran, Shin, and Smith (1999) and Breitung (2005), and the references cited therein.

assumption $\lambda_{max}(\mathbf{\Sigma}_i) < C < \infty$)

$$n^{-1}\sum_{i=1}^{n}\boldsymbol{\phi}_i'\mathbf{\Sigma}_i^{-1}\boldsymbol{\phi}_i = \left(\frac{m}{n}\right)\left[m^{-1}\sum_{i=1}^{m}\boldsymbol{\phi}_i'\mathbf{\Sigma}_i^{-1}\boldsymbol{\phi}_i\right] > (1/C)\left(\frac{m}{n}\right)\left(m^{-1}\sum_{i=1}^{m}\boldsymbol{\phi}_i'\boldsymbol{\phi}_i\right),$$

and Q_{nT} tends to a non-zero limit as n and $m \to \infty$, if $m/n = 1 - \pi > 0$, where π is the fraction of units that are not cointegrating. Therefore, under regularity conditions typically assumed in the literature, the SPMG estimator of θ can be viewed as a quasi-ML (QML) estimator which is asymptotically normal and converges to its true value at the rate of $T\sqrt{(1-\pi)n}$. The convergence rate of SPMG estimator in terms of n and T is not affected by a non-zero π, so long as π is not too close to unity. However, in practice the effective number of units in the panel is discounted by the proportion of non-cointegrating units.

We carry out inference using conventional standart errors (assuming error cross-sectional independence), robust standard errors (allowing for arbitrary error cross-sectional dependence), and bootrstrapped confidence intervals, outlined in Sections S.1 and S.2.3 of the online supplement.

There are also a number of time series estimators of unit-specific cointegrating vectors, such as the original Engle-Granger, fully-modified OLS, dynamic OLS, and ARDL as well as system estimators like Johansen that can be used to investigate the empirical validity of the great ratios hypothesis.[13] These estimators can be averaged across countries to yield corresponding mean group (MG) panel data estimators originally introduced in Pesaran and Smith (1995). This will not only reduces country-specific sampling errors, but also allows the calculation of non-parametric standard errors directly based on the individual estimates, which are robust to serial correlation and heteroskedasticity at the individual country level, avoiding complicated inference problems. Furthermore, as shown in Chudik and Pesaran (2019), the MG group procedure is valid if the error cross-sectional dependence is weak. However, the MG estimator requires that all country-specific estimators are consistent, a condition which will not be met when the cointegration condition does not hold for some of the units under consideration. To ensure that all units being considered are in fact cointegrating involves pre-testing and is subject to further complications. For these reasons we focus on PMG and system PMG estimators, and to deal with the possibility of strong error

[13] See Engle and Granger (1987), Phillips and Hansen (1990), Johansen (1991), Stock and Watson (1993), Phillips (1995), and Pesaran and Shin (1999).

cross-sectional dependence we propose a bootstrap algorithm, with details provided in the online supplement.

5 Monte Carlo evidence

In this section we investigate the finite-sample performance of alternative panel estimators of long run relationships, and accuracy (confidence interval coverage rate) of tests of the common long run coefficient which is of interest in the analysis of great ratios. As noted already, the focus of our Monte Carlo experiments is on (i) robustness of the estimators to possible failure of cointegration in the case of some units, (ii) the ability of the estimator to perform well regardless of the direction of long run causality, and last but not least (iii) the robustness of inference based on the long run estimates to heteroskedasticity and cross-sectional error dependence. This section provides summary of Monte Carlo findings. The full account of these experiments is presented in the online supplement.

5.1 Summary of data generating process

We generate $w_{it} = (y_{it}, x_{it})'$ using a VAR(2) model, which we write in the error-correcting representation as

$$\Delta y_{it} = a_{yi} - \phi_{yit}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{yyi}\Delta y_{i,t-1} + \psi_{yxi}\Delta x_{i,t-1} + u_{yit},$$

$$\Delta x_{it} = a_{xi} - \phi_{xit}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{xyi}\Delta y_{i,t-1} + \psi_{xxi}\Delta x_{i,t-1} + u_{xit}.$$

The coefficient of interest is θ, the long run coefficient, which we set equal to one. When the error-correcting coefficients $\phi_{yit} \neq 0$ and $\phi_{xit} = 0$, there is a long run relationship with long run causality from x to y, which we denote as $x \rightarrow y$. When both $\phi_{yit}, \phi_{xit} \neq 0$ the long run causality runs both ways, which we denote as $x \leftrightarrow y$. Initially, we consider data generating processes (DGP) where the direction of long run causality is from x to y ($x \rightarrow y$) and set $\phi_{xit} = 0$ for all i, t, and generate ϕ_{yit} to be non-zero except for a number of non-cointegrating episodes with durations that vary from a minimum of 10 periods and a maximum of T periods (namely the full sample). This is achieved by setting

$$\phi_{yit} = \begin{cases} 0, & \text{for } t \in \mathcal{T}_i \\ \phi_{yi}, & \text{for } t \notin \mathcal{T}_i \end{cases},$$

where $\phi_{yi} \sim IIDU\left(0.1, 0.25\right)$ and $\mathcal{T}_i$ denotes the set of non-cointegrating episodes for unit i, with each episode having duration T_i. With probability $(1-\pi)$, we set $T_i = 0$ (namely $\mathcal{T}_i = \emptyset$), and with probability π, we draw T_i uniformly from the set of integers $\{10, 11, ..., T\}$. The start of non-cointegrating episodes in $\mathcal{T}_i$ is also generated stochastically. In experiments where $x \leftrightarrow y$, we generate ϕ_{xit} similarly to ϕ_{yit}, namely using the identical index sets $\mathcal{T}_i$, and generate ϕ_{xi} as $\phi_{xi} \sim IIDU\left(-0.15, -0.05\right)$. The parameter π is a key parameter controlling the occurrence of non-cointegrating episodes. We consider $\pi = 0$, the benchmark case without any non-cointegrating episodes, $\pi = 0.05$ when there are non-cointegarting episodes but with low occurrence, and $\pi = 0.2$ where the episodes of non-cointegrating periods occur relatively frequently.

Another key aspect of our design is the strength of error cross-sectional dependence which has importance implications for inference, in particular. We consider three options, the independent case, where $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ is independent of $\mathbf{u}_{jt}$ for all $i \neq j$, and two cross sectionally correlated cases with spatial and latent factor dependence. Specifically, we consider a spatial autoregressive model (SAR) and a mixed spatial factor model. We set the spatial autoregressive parameter to 0.6, and allow the factors to be strong. In all cases, $\mathbf{u}_{it}$ are generated allowing for non-zero contemporaneous covariances, $Cov\left(u_{yit}, u_{xit}\right) \neq 0$, using both Gaussian and non-Gaussian error distributions, to check the robustness of different estimation methods to departures from Gaussianity.[14]

In total we consider 36 different experiments, spanning the choices of (a) π (probability of episodic non-cointegration), (b) the direction of long run causality, (c) error cross section dependence, and (d) error distributions. For each experiment we consider 16 pairs of sample size combinations obtained from $T \in \{50, 100, 150, 200\}$ and $n \in \{30, 50, 100, 200\}$, and use $R_{MC} = 2000$ replications to obtain the results. Among the choices of (n, T), the smaller values are relevant for typical empirical applications in economics, whereas the larger choices of (n, T) are interesting from an econometric perspective as they shed more light on the consistency, validity of asymptotic standard errors, and the relative importance of n and T dimensions. We consider five estimators

[14] Further details are provided in the online supplement, where a full description of DGP for error processes can be found.

outlined below.

5.2 Five estimators

We consider the PMG estimator by Pesaran, Shin, and Smith (1999) (with asymptotic inference or with bootstrapped inference outlined in online supplement), and the system PMG estimator outlined above. The third estimator is mean group estimator based on individual Müller and Watson (2018) estimates. In particular, we split the sample period into q non-overlapping subsamples of (approximately) equal size, where q is treated as fixed as (n, T) changes. We then take simple temporal averages for each of the subsamples, and average individual cross-section specific least squares estimates computed using the sample of q temporally averaged periods. We refer to this estimator as MGMW. Due to temporal averaging, this estimator has the potential to be quite robust. However, the inference based on the conventional heteroskedasticity and serial correlation robust standard errors need not be robust to strong cross-sectional dependence of errors.[15] The fourth estimator included in this study is two-step estimator by Breitung (2005), and the fifth estimator is the off-the-shelf popular panel dynamic OLS (PDOLS) estimator by Mark and Sul (2003). We do not make any modifications to PDOLS estimator and its inclusion in this study is for completeness and for the comparison purposes. PDOLS need not be robust to episodes of non-cointegration and/or cross-sectionally correlated panels, but it is an important benchmark in the literature.[16]

5.3 Summary of Monte Carlo findings

We begin with reporting the results for the baseline case in Tables 1 and 3 where the errors are cross-sectionally independent and Gaussian, the long run causality is known to run from x to y, and there are no episodes of non-cointegration. In this case we expect all five estimators to work reasonably well. Focusing on the sample size combination closest to our empirical applications, namely $T = 100$, and $n = 30$, the best RMSE value of 0.0164 is achieved by PMG estimator, followed closely by SPMG with RMSE of 0.0174, two-step Breitung with RMSE of 0.0190, with

[15] Details of the different estimators and their implementation are provided in the online supplement.

[16] Additional panel estimators in the literature are the panel Fully modified OLS (FMOLS) estimator by Pedroni (2001) and the recently introduced panel Bewley estimator by Chudik, Pesaran, and Smith (2021).

RMSEs of the remaining estimators quite a bit higher, falling in the range 0.0223 to 0.0419. (See Table 1) From an econometric perspective, a difference in RMSE of over 150% between the worst (MGMW) and the best (PMG) estimators is large. From an economic perspective, however, even the worst RMSE is rather small and all five estimators yield reasonably precise estimates of the long run coefficient in the baseline experiments. A similar ordering of the five estimators is obtained when we consider bias, except for the SPMG which now has the smallest bias followed by PMG. The results also show the importance of the T dimension for the performance of all the five estimators.

The 95 per cent coverage rate for the different estimators of the long run coefficient are reported in Tables 1 and Table 3. For ($T = 100$, $n = 30$) sample size the simulated coverage rates vary from 70.9% (for PDOLS, $p = 1$, in Table 1) to 93.8% - 94.3% (bootstrapped confidence intervals of the PMG and the SPMG reported in Table 3). Bootstrapped inference reported in Table 3 appears to be uniformly better than the conventional alternatives reported in Table 1.[17] While for the values of $T > 100$, coverage rates are reasonably good, this is not the case for $T = 50$, where the conventional confidence intervals are in the range of 8.5% to 82.9%.

Consider now the most "demanding" experiment under which we allow for non-Gaussian and cross-sectionally correlated errors with both spatial and factor dependencies (denoted as factor + SAR), two-way long run causality, and $\pi = 0.2$ (a relatively high occurrence of non-cointegrating episodes). The results of this case are summarized in Tables 2 and 3.[18] The SPMG (with bootstrapped confidence intervals in Table 3) emerges as the only reliable estimator and is therefore a clear winner. It is also the only estimator without serious bias. For $T = 100$, and $n = 30$, its bias is only 0.0005 compared with the range of -0.0361 to -0.0087 for the other estimators. Similarly the RMSE value of SPMG estimator at 0.0140 is substantially smaller than the RMSE obtained for the other estimators, which are 0.0172 for PMG, 0.0518 for MGMW, 0.0628 for two-step Breitung estimator, and 0.0537, 0.0551, 0.0617, for PDOLS(1,4,8). The worse performing estimator is

[17] We have also considered a robust alternative to the conventional estimator of SPMG variance, outlined in equation (S.26) in the online supplement, and its thresholding version (S.27). While the robust standard errors work very well for $T = 200$, regardless of cross sectional dependence (and for all our choices of n, see Table S38), they suffer from the same small sample drawbacks as the conventional standard errors. Bootstrap SPMG confidence intervals performed the best in all Monte Carlo experiments.

[18] Summary results for all other experiments (between these two extremes reported in Tables 1-4) are provided in the online supplement. Findings presented in the supplement suggest that non-Gaussianity does not have significant influence on the results. This suggests that the distributional form of the errors is unlikely to be of great important in practice.

the two-step Breitung estimator which is primarily due to the fact that 20% of the cross-section units in these experiments are not cointegrating, and this is not allowed under this estimator. The need to bootstrap for accurate inference is again confirmed by the 92.3% coverage of the SPMG bootstrapped confidence interval compared with the conventional coverage of only 64.3%. Coverage rates of the remaining estimators are poor, and fall in the range 54% to 88%.

While no Monte Carlo exercise, regardless of how extensive or carefully designed, can guarantee the reliability of any particular estimator in real datasets, it can illuminate lack of robustness or other problems in a controlled setting containing features thought to be found in real world data. As is well known, confidence intervals designed for cross sectionally independent errors are invalid when errors are in fact cross sectionally dependent, and the seriousness of this problem is clearly documented by the detailed Monte Carlo results in the online supplement. In addition, while it is not guaranteed that confidence intervals that are robust to cross section dependence will perform well in practice, the bootstrapped confidence intervals adopted in this paper had rather good coverage rates.

Tests indicate that cross section correlation of residuals is clearly present in the data used in this paper. The other two aspects of our design - two way causality and the existence of non-converging episodes - cannot be as easily validated, but are both plausible a priori. Thus it is reassuring to have estimators that are robust to those features.

We also considered the small sample properties of the MG estimators based on Johansen and ARDL individual country estimates, but found that they did not perform well as compared to the pooled approach, particularly when compared to the system PMG estimator. As is well known, MG estimators requires the underlying individual estimates to have finite moments, and this condition does not hold in general. This was found to be the case when we used country-specific Johansen's estimates.

We also considered a number of other approaches to deal with non-cointegrating episodes. Given the long span of our data and the possibility of no cointegration during particular episodes we tried averaging estimates over sub-periods within each cross-section unit. We also tried pre-testing whereby we first tested for cointegration before including the estimate when computing the MG estimator. After considerable investigation, we found out that splitting sample into subperiods, does not seem to be beneficial, because (as the Monte Carlo experiments show), having a large T dimension is crucial for estimation and inference. There was mixed evidence on the value of pre-testing for the existence of level relationships. Pre-testing is not required for PMG and system PMG estimators since the individual estimates are weighted by the magnitude of error-correcting coefficients, which makes them robust to non-cointegrating units.

6 Empirical evidence

In the empirical application, we estimate pooled long run coefficients for seven bivariate relationships using a panel of 17 countries over the years 1870-2016 from the Jordà-Schularick-Taylor macro-history database. The relations are: (1) the logarithms of real consumption per capita and real GDP per capita; (2) the logarithms of investment and GDP; (3) the logarithms of imports and exports; (4) the logarithms of public debt and GDP; (5) short and long interest rates, (6) inflation and long interest rates; and (7) inflation and money growth.[19] In each case we estimate the long run coefficient and its 95% confidence interval. While the dynamics of adjustments might differ across countries, the great ratios hypothesis implies that the long run coefficient will take a common value of unity. Details of the data and variables are given in Data Appendix. Except for the export-import relationship, these pairs overlap with those considered by MW. They use a 68 year post-war US sample and consider data on some other variables including unemployment, total factor productivity, stock returns, dividends and earnings.

We consider the same estimators investigated in the Monte Carlo section. Regarding the PDOLS and its corresponding Monte Carlo evidence on negative consequences of a too short lead/lag order, we only consider the longer lead and lag orders, $p = 4$ and 8. The estimators reported below are (1) PMG, the Pooled Mean Group estimator of Pesaran, Shin, and Smith (1999); (2) SPMG; (3) MGMW, the mean group estimator based on Müller and Watson (2018) country-specific estimates, using temporally aggregated data into $q = 5$ sub-periods; (4) the two-step Breitung (2005) estimator; (5) PDOLS, $p = 4$ is the panel dynamic OLS estimator by Mark and Sul (2003) using 4 leads and lags; and (6) PDOLS, $p = 8$. For the PMG and SPMG two sets of confidence intervals are provided, asymptotic and bootstrap, for the other estimators only

[19] Strictly the last three relationships are not ratios as such, but their long run constancy is often assumed.

asymptotic confidence intervals are given. More details can be found in Sections S.2.1-S.2.4 of the online supplement.

We use the largest available balanced panel for each pair of variables, as described in Data Appendix. The number of countries (n) ranges from 14 (investment) to 17 and the number of time periods (T) from 121 to 143 years.

Other than the SPMG the estimators are not invariant to normalization and like Müller and Watson (2018) we provide estimates assuming under both long run causal ordering, namely $\hat{\theta}_{y.x}$ and $\hat{\theta}_{x.y}$. Only in the case of SPMG one estimator is exactly the reciprocal of the other, namely $\hat{\theta}_{y.x}\hat{\theta}_{x.y} = 1$. This property does not hold for other estimators that depend on whether y_t is regressed on x_t or *vice versa.*

The pooled estimates of the long run coefficients together with their 95% confidence intervals are summarized in Table 4. For each pair of variables (y, x) we report six different estimates of $\hat{\theta}_{y.x}$ and $\hat{\theta}_{x.y}$, namely PMG, SPMG, PMW, Breitung, PDOLS(4), and PDOLS(8). To check for possible error cross-sectional dependence, at the bottom of Table 4 we also report the average pair wise correlation coefficient of residuals from the panel data models and related CD test statistics due to Pesaran (2004, 2015). It is clear that there are significant degrees of error cross-section dependence and for statistical testing it is prudent to focus on bootstrapped confidence intervals reported for PMG and SPMG estimators.

With such a large number of observations and quite small standard errors, in some cases, it is not clear that traditional significance testing is the appropriate criteria for judging closeness to unity. For debt-GDP and long-short interest rates relationships, the estimated long run coefficients are close to unity and not significantly different from it at the 5 per cent level. The SPMG (with bootstrapped 95% confidence interval) gives long run coefficients of $\hat{\theta}_{Debt-GDP} = 1.05$ (0.993-1.108) for debt on GDP and $\hat{\theta}_{Short-Long} = 1.01$ (0.912-1.108) for short on long rates. Similar results are obtained using the other estimators. The bracketed figures refer to 95% confidence intervals.

For investment-GDP and imports-exports, the long run coefficients are also estimated to be close to unity. But due to their high precision the null hypothesis that the long run coefficient is in fact unity gets rejected. The SPMG gives $\hat{\theta}_{INV-GDP} = 1.044$ (1.029-1.059) for investment on GDP and $\hat{\theta}_{IM-EX} = 0.967$ (0.961-0.973) for imports on exports. Again similar results are obtained when other estimators are considered.

The estimates of the long run coefficient for consumption-GDP pair are somewhat away from one. For example, using the SPMG method we obtain $\hat{\theta}_{CON-GDP} = 0.907$ (0.884-0.930), with the other estimates slightly lower ranging from a low of 0.883 when we use PDOLS(4), and 0.900 when we use PMG.

For the remaining two relationships the evidence is more mixed, with different estimators yielding different results. For regressions of inflation on money supply growth the estimates of $\theta_{INF-Money}$ are not significantly different from one in the case of MGMW, Breitung, PDOLS(4), and PDOLS(8) estimators. But an opposite conclusion is reached if the long run coefficient is estimated by running regressions of money supply growth on inflation. This is the case if we consider Breitung, PDOLS(4) and PDOLS(8) estimators. A unit long run relationship between money supply growth and inflation is supported only by MGMW estimator irrespective of which way the regressions are implemented. PMG and SPMG both strongly reject the null of a unit long run relationship between inflation and money supply growth.

Almost all estimates of the long run coefficient of inflation on long term interest rate are significantly below unity - the exception being when the long run coefficient is estimated by PDOLS(8) using the regression of inflation on the long term interest rate. Even in this case the long run coefficient is poorly estimated and an opposite conclusion is reached if the long run estimate is computed from the reverse regression of the long term rate on inflation.

Overall, it is quite encouraging that five out of the seven long run coefficients are quite close to unity, with substantial empirical evidence in support of Debt to GDP and Imports to Exports as being great ratios, and the difference between long and short interest rates being stationary. The evidence on consumption-GDP and investment-GDP as great ratios is less overwhelming. This is particularly problematic for consumption-GDP ratio where the largest estimate obtained for the long run elasticity of consumption to GDP is 0.907 (using SPMG) which is difficult to rationalize. At the level of the cross section of individual households this could correspond to the well established pattern that savings as a proportion of income increase with income, the rich save more. At the level of the time series for a country, this could correspond to the fact that the measured private consumption is only a part of total consumption and with increasing income, government consumption has accounted for a growing part.

7 Concluding comments

By using long span panel data and a robust estimator we provide more evidence for close to unit elasticities for two balanced growth conditions, two solvency conditions and a stable term structure, but found evidence against a unit long run coefficient in the case of the Fisher relationship and the inflation money growth relationship.

We relied on long-span panel data to overcome the drawbacks of the single-country regressions. However, panel analysis of great ratios presents its own challenges - namely a possibility that some periods in some countries are not cointegrating, the unknown direction of long run causality, and cross-sectionally correlated observations. To overcome these challenges, we have proposed a new system pooled mean group estimator which is shown to perform well in small samples even in the presence of two-way long run causality, episodes of non-cointegration, and error cross-sectional dependence.

Table 1: MC results for the estimation of long run coefficient $\theta_0 = 1$ in the baseline experiments

	Bias (×100)				RMSE (×100)				Coverage rate (x100)			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator											
30	-0.86	-0.19	-0.09	-0.05	4.22	1.64	1.03	0.74	67.7	85.1	88.2	90.3
50	-1.03	-0.26	-0.10	-0.05	3.21	1.32	0.79	0.57	69.9	83.6	88.8	89.5
100	-0.91	-0.22	-0.08	-0.05	2.26	0.89	0.55	0.39	68.2	85.0	88.8	91.5
200	-0.98	-0.21	-0.09	-0.05	1.76	0.64	0.38	0.28	62.0	84.0	89.4	91.2
	SPMG estimator											
30	0.24	0.07	0.02	0.01	4.95	1.74	1.06	0.76	59.1	80.5	86.8	88.7
50	-0.01	-0.04	-0.01	0.00	3.59	1.38	0.80	0.59	59.9	79.6	87.0	87.8
100	0.10	0.01	0.01	0.00	2.46	0.93	0.57	0.40	61.2	81.0	86.0	89.6
200	0.00	0.01	0.01	0.00	1.71	0.65	0.39	0.28	61.3	81.4	87.2	88.6
	MGMW estimator, $q = 5$											
30	-5.64	-1.84	-0.87	-0.49	8.92	4.19	2.80	2.04	82.2	90.7	92.5	92.3
50	-5.64	-2.09	-0.99	-0.55	7.63	3.52	2.18	1.56	76.8	87.2	90.3	92.3
100	-5.59	-1.91	-0.96	-0.55	6.76	2.77	1.66	1.15	63.4	82.4	89.0	91.8
200	-5.55	-1.92	-0.93	-0.54	6.14	2.38	1.36	0.90	40.7	71.4	82.2	88.2
	two-step Breitung's estimator											
30	-2.25	-0.57	-0.26	-0.13	4.46	1.90	1.14	0.85	74.6	85.7	90.5	91.1
50	-2.34	-0.65	-0.28	-0.15	3.71	1.54	0.91	0.66	68.9	84.5	89.7	91.8
100	-2.30	-0.62	-0.28	-0.15	3.08	1.14	0.66	0.47	57.2	81.8	88.9	90.7
200	-2.31	-0.62	-0.27	-0.15	2.72	0.92	0.51	0.35	39.2	75.3	85.2	87.9
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.65	-2.30	-1.52	-1.11	5.94	2.93	1.91	1.42	69.2	70.9	71.8	73.1
50	-4.73	-2.38	-1.54	-1.12	5.50	2.76	1.79	1.31	56.0	54.1	55.9	59.4
100	-4.65	-2.31	-1.52	-1.11	5.07	2.51	1.65	1.21	31.7	30.4	31.4	32.8
200	-4.63	-2.30	-1.50	-1.10	4.83	2.40	1.57	1.15	8.5	7.1	7.6	7.8
	PDOLS estimator, leads and lags order $p = 4$											
30	-2.47	-1.09	-0.71	-0.51	5.23	2.23	1.36	1.01	82.9	87.0	89.3	88.8
50	-2.49	-1.16	-0.74	-0.52	4.18	1.88	1.17	0.84	80.8	83.9	84.6	85.8
100	-2.50	-1.14	-0.73	-0.52	3.48	1.55	0.97	0.70	72.3	75.0	77.4	77.2
200	-2.48	-1.14	-0.72	-0.52	3.01	1.35	0.85	0.62	59.7	61.0	61.2	60.8
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.48	-0.46	-0.30	-0.20	8.14	2.41	1.32	0.95	71.1	89.6	92.1	92.2
50	-1.42	-0.52	-0.31	-0.21	5.95	1.83	1.05	0.73	75.2	89.5	90.8	92.4
100	-1.33	-0.49	-0.31	-0.21	4.35	1.34	0.77	0.54	75.1	88.6	90.0	90.9
200	-1.30	-0.52	-0.31	-0.22	3.24	1.02	0.59	0.42	76.1	85.6	87.8	88.2

Notes: Coverage rate is 95% confidence interval coverage rate. This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0$, and no cross section dependence of errors. See Section S.3.1 of the online supplement for full details of the data generating process. Description of the PMG, SPMG, MGMW, and 2-step Breitung estimators, and the description of bootstrapping procedures are provided in Sections S.2.1-S.2.3 of the online Supplement. PDOLS is the panel dynamic OLS estimator by Mark and Sul (2003). The number of Monte Carlo replications is $R_{MC} = 2000$.

Table 2: MC results for the estimation of long run coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, $x \leftrightarrow y$, $\pi = 0.2$ and CS dependence of errors.

	Bias (×100)				RMSE (×100)				Coverage rate (x100)			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator											
30	-1.74	-0.87	-0.62	-0.45	3.40	1.72	1.12	0.83	50.4	57.8	58.0	59.7
50	-1.77	-0.85	-0.58	-0.42	3.01	1.43	0.93	0.70	45.2	50.7	52.9	55.0
100	-1.62	-0.85	-0.56	-0.41	2.44	1.23	0.80	0.60	35.9	38.7	39.7	43.4
200	-1.59	-0.82	-0.55	-0.41	2.20	1.11	0.73	0.54	26.9	27.6	27.5	28.8
	SPMG estimator											
30	0.12	0.05	-0.02	0.00	3.23	1.40	0.88	0.64	53.0	64.3	67.5	69.8
50	0.00	0.05	0.03	0.02	2.43	1.05	0.66	0.48	53.0	64.0	68.6	71.4
100	0.10	-0.01	0.00	0.01	1.85	0.74	0.46	0.34	51.2	63.2	69.9	71.4
200	0.05	0.01	0.01	0.01	1.24	0.53	0.33	0.24	50.1	63.0	67.5	70.3
	MGMW estimator, $q = 5$											
30	-2.04	-1.62	-1.23	-1.11	7.06	5.18	4.74	4.40	83.5	87.6	89.3	90.1
50	-2.41	-1.48	-1.21	-0.95	6.02	4.08	3.66	3.36	81.3	84.1	87.6	88.5
100	-1.95	-1.40	-1.21	-0.95	4.59	3.22	2.87	2.55	78.5	82.7	83.8	85.9
200	-2.04	-1.32	-1.09	-1.01	4.08	2.67	2.30	2.17	72.3	78.1	79.6	80.3
	two-step Breitung's estimator											
30	-4.04	-2.69	-2.21	-1.94	8.42	6.28	5.31	4.90	49.0	54.2	58.8	60.9
50	-4.19	-2.53	-2.18	-1.83	7.14	5.13	4.76	4.28	41.0	52.6	55.8	59.5
100	-4.04	-2.47	-2.16	-1.99	6.13	4.56	3.78	4.20	33.2	45.5	48.1	49.9
200	-4.04	-2.51	-2.11	-1.85	5.28	3.69	3.10	2.82	22.9	35.6	38.0	41.6
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.97	-3.61	-2.95	-2.59	7.31	5.37	4.64	4.15	74.5	79.3	81.2	84.6
50	-5.14	-3.39	-2.88	-2.51	6.81	4.64	3.98	3.62	65.2	75.0	74.2	78.7
100	-4.97	-3.33	-2.90	-2.54	6.03	4.10	3.62	3.24	56.4	62.3	62.7	64.4
200	-4.89	-3.35	-2.82	-2.51	5.65	3.85	3.29	2.95	44.8	44.2	45.0	50.0
	PDOLS estimator, leads and lags order $p = 4$											
30	-4.27	-3.27	-2.69	-2.37	7.91	5.51	4.68	4.16	73.3	83.0	84.8	88.2
50	-4.52	-3.02	-2.60	-2.30	7.04	4.59	3.93	3.57	70.9	81.5	82.0	84.5
100	-4.38	-2.96	-2.65	-2.33	5.99	3.94	3.50	3.14	63.7	74.5	75.1	74.1
200	-4.35	-3.01	-2.57	-2.30	5.48	3.64	3.11	2.79	56.9	62.1	60.7	63.9
	PDOLS estimator, leads and lags order $p = 8$											
30	-3.62	-3.23	-2.65	-2.32	10.63	6.17	5.00	4.38	56.5	81.0	84.2	88.0
50	-3.93	-2.92	-2.54	-2.23	8.74	4.95	4.15	3.69	57.5	80.7	82.7	86.1
100	-3.88	-2.84	-2.62	-2.29	7.12	4.14	3.64	3.22	52.9	75.5	77.2	77.4
200	-4.04	-2.91	-2.53	-2.27	6.24	3.77	3.18	2.83	48.2	66.7	67.1	69.0

Notes: Coverage rate is 95% confidence interval coverage rate. This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table 1.

Table 3: **Robust bootstrapped 95% confidence interval coverage rates (x100) for PMG and SPMG estimators in MC experiments.**

$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	Baseline experiments							
	PMG estimator				SPMG estimator			
30	89.0	94.3	94.0	94.9	86.1	93.8	93.7	94.6
50	89.0	92.7	94.7	94.2	86.2	92.1	94.2	94.2
100	89.9	93.5	94.5	94.8	86.0	93.5	93.9	93.9
200	85.8	93.3	94.9	95.0	87.5	93.1	94.7	95.0
	Experiments with non-Gaussian errors, $x \leftrightarrow y$, $\pi = 0.2$ and CS dependence of errors							
	PMG estimator				SPMG estimator			
30	87.6	87.9	88.2	88.1	89.4	92.3	93.6	93.2
50	83.9	86.1	86.4	87.3	89.5	92.4	93.5	94.3
100	78.3	78.8	79.7	81.2	87.5	92.4	94.0	94.1
200	71.8	71.5	71.8	71.9	88.7	92.4	93.6	94.7

Notes: Bootstrapped critical values are computed in each of the Monte Carlo replication as described in Sections S.2.1-S.2.3 of the online supplement, based on $R_b = 2000$ bootstrap replications. See notes to Table 1 and 2.

Table 4: Direct and reverse estimates of long run coefficient θ and 95% confidence intervals (CI)

y_{it} :	Real consumption per capita		Investment		Imports		Debt	
x_{it} :	Real GDP per capita		GDP		Exports		GDP	
	y on x	x on y	y on x	x on y	y on x	x on y	y on x	x on y
	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$
PMG	**0.900**	**1.109**	**1.043**	**0.965**	**0.963**	**1.033**	**1.054**	**1.002**
asymptotic CI	(0.884-0.915)	(1.083-1.134)	(1.034-1.052)	(0.955-0.975)	(0.957-0.969)	(1.028-1.039)	(1.026-1.082)	(0.982-1.022)
bootstrap CI	(0.876-0.923)	(1.063-1.154)	(1.027-1.058)	(0.950-0.980)	(0.954-0.972)	(1.026-1.041)	(0.986-1.122)	(0.969-1.035)
SPMG	**0.907**	**1.103**	**1.044**	**0.958**	**0.967**	**1.035**	**1.051**	**0.952**
asymptotic CI	(0.893-0.921)	(1.086-1.120)	(1.035-1.052)	(0.950-0.966)	(0.962-0.971)	(1.029-1.040)	(1.026-1.075)	(0.929-0.974)
bootstrap CI	(0.884-0.930)	(1.074-1.132)	(1.029-1.059)	(0.945-0.971)	(0.961-0.973)	(1.028-1.041)	(0.993-1.108)	(0.905-0.998)
MGMW	**0.885**	**1.128**	**1.070**	**0.934**	**0.977**	**1.025**	**1.046**	**0.956**
asymptotic CI	(0.844-0.926)	(1.080-1.176)	(1.050-1.090)	(0.917-0.952)	(0.956-0.997)	(1.004-1.046)	(0.975-1.117)	(0.893-1.018)
Breitung	**0.898**	**1.107**	**1.035**	**0.966**	**0.983**	**1.017**	**1.012**	**0.980**
asymptotic CI	(0.872-0.923)	(1.076-1.138)	(1.027-1.042)	(0.959-0.973)	(0.979-0.986)	(1.013-1.021)	(0.987-1.037)	(0.956-1.004)
PDOLS, p=4	**0.883**	**1.109**	**1.036**	**0.962**	**0.983**	**1.017**	**1.002**	**0.984**
asymptotic CI	(0.846-0.919)	(1.069-1.150)	(1.021-1.052)	(0.945-0.978)	(0.975-0.991)	(1.008-1.025)	(0.969-1.035)	(0.948-1.020)
PDOLS, p=8	**0.887**	**1.110**	**1.035**	**0.963**	**0.982**	**1.019**	**0.996**	**0.995**
asymptotic CI	(0.848-0.925)	(1.070-1.150)	(1.023-1.048)	(0.950-0.977)	(0.975-0.989)	(1.008-1.029)	(0.966-1.026)	(0.963-1.026)
n	15	15	14	14	17	17	16	16
T	143	143	131	131	131	131	121	121
Cross section dependence of residuals								
	u_{yt}	u_{xt}	u_{yt}	u_{xt}	u_{yt}	u_{xt}	u_{yt}	u_{xt}
CD test	22.16	21.32	16.21	14.98	37.77	33.39	3.72	16.22
Ave. pair-wise corr.	0.182	0.175	0.150	0.138	0.285	0.252	0.031	0.136

Notes: Description of the PMG, SPMG, MGMW and Breitung estimators is provided in Sections S.2.1-S.2.3 of the online supplement. PDOLS is the panel dynamic OLS estimator by Mark and Sul (2003). Bootstrapped confidence intervals are based on $R_b = 2000$ bootstrap replications. The source for all variables is the Jordà-Schularick-Taylor (JST) macro-history database available at http://www.macrohistory.net/data/, see, Jordà, Schularick, and Taylor (2017) and Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019). See Table A1 in Data Appendix for variable description. Estimations are conducted using the largest balanced panel as described in Data Appendix.

Table 4(Continued): Direct and reverse estimates of long run coefficient θ and 95% confidence intervals (CI)

y_{it} :	Short IR		Inflation		Inflation	
x_{it} :	Long IR		Long IR		Money	
	y on x	x on y	y on x	x on y	y on x	x on y
	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$
PMG	**0.971**	**0.865**	**0.521**	**0.567**	**0.677**	**0.650**
asymptotic CI	(0.920-1.022)	(0.815-0.915)	(0.356-0.685)	(0.455-0.679)	(0.619-0.736)	(0.606-0.695)
bootstrap CI	(0.867-1.075)	(0.755-0.975)	(0.303-0.739)	(0.380-0.754)	(0.601-0.753)	(0.595-0.706)
SPMG	**1.010**	**0.990**	**0.653**	**1.530**	**1.227**	**0.815**
asymptotic CI	(0.963-1.057)	(0.947-1.032)	(0.498-0.809)	(1.418-1.641)	(1.169-1.284)	(0.775-0.854)
bootstrap CI	(0.912-1.108)	(0.899-1.081)	(0.419-0.888)	(1.256-1.803)	(1.153-1.300)	(0.763-0.866)
MGMW	**1.020**	**0.857**	**0.878**	**0.429**	**0.943**	**0.952**
asymptotic CI	(0.929-1.110)	(0.774-0.939)	(0.758-0.999)	(0.273-0.585)	(0.805-1.082)	(0.829-1.074)
Breitung	**1.001**	**0.945**	**0.665**	**0.239**	**1.041**	**0.696**
asymptotic CI	(0.932-1.069)	(0.884-1.005)	(0.380-0.949)	(0.179-0.299)	(0.964-1.118)	(0.649-0.743)
PDOLS, p=4	**0.986**	**0.868**	**0.630**	**0.127**	**1.051**	**0.756**
asymptotic CI	(0.892-1.079)	(0.779-0.956)	(0.309-0.950)	(0.029-0.226)	(0.933-1.169)	(0.675-0.837)
PDOLS, p=8	**0.960**	**0.901**	**0.747**	**0.194**	**1.025**	**0.795**
asymptotic CI	(0.862-1.058)	(0.798-1.004)	(0.369-1.126)	(0.062-0.326)	(0.884-1.166)	(0.685-0.906)
n	16	16	17	17	16	16
T	120	120	137	137	133	133
Cross section dependence of residuals						
	u_{yt}	u_{xt}	u_{yt}	u_{xt}	u_{yt}	u_{xt}
CD test	33.15	40.34	31.58	50.96	19.69	13.47
Ave. pair-wise corr.	0.279	0.339	0.233	0.376	0.157	0.107

See notes on the previous page.

A Data Appendix

The data are taken from the Jordà-Schularick-Taylor (JST) Macrohistory Database (available at http://www.macrohistory.net/data/),[20] see Jordà, Schularick, and Taylor (2017). Jordà et al. (2019) provides further discussion of the rate of return data. JST provide data for 17 countries over the period 1870-2016. There are clearly issues with the measurement of economic variables over such a long span. However, JST is a carefully compiled database which has been widely used. The data are assembled from a wide variety of sources with different definitions of variables and countries vary because of boundary changes.[21] For instance, the long interest rate is on government bonds, with a maturity typically around 10 years, but sometimes longer like the British Consols which were perpetuals. From about 1950 the maturity is fairly accurately defined at about 10 years. While the series may be noisy, there is a lot of variation, so the signal-noise ratio may be high.

Table A1 lists the series we use together with their availability. We use the largest balanced panel for estimations. For each country i, we omit gap years (if any) and compute the number of available time periods for each country, denoted as T_i. Then we re-order countries so that $T_1 \geq T_2 \geq ... \geq T_{n_{\max}}$. Note that T_1 is the largest time dimension (and the largest number of observations) if only one country was to be used for estimation, $2T_2$ is the largest number of observations for a balanced panel if two countries were chosen for estimation, and so on. We find $n^* = \max_{1 \leq n \leq n_{\max}} \{nT_n\}$, and the largest balanced panel features n^* countries and T_{n^*} periods.

[20] We downloaded version JSTdatasetR4 (Release 4, May 2019), in particular we have downloaded datafile http://www.macrohistory.net/JST/JSTdatasetR4.xlsx.

[21] There is detailed documentation of the sources at http://www.macrohistory.net/data/ .

Table A1: Variable description and data availability*

Name	Gap years**	Description	Variable construction***
Real consumption per capita	88 (3.5%)	log of real consumption per capita index	log(rconpc)
Real GDP per capita	0 (0%)	log of real GDP per capita index	log(rgdppc)
GDP	25 (1%)	log of nominal GDP (local ccy)	log(gdp)
Investment	220 (8.8%)	log of nominal investment (local ccy)	log(iy*gdp)
Imports	41 (1.6%)	log of nominal imports (local ccy)	log(imports)
Exports	41 (1.6%)	log of nominal exports (local ccy)	log(exports)
Public Debt	184 (7.4%)	log of public debt (local ccy)	log(debtgdp*gdp)
Short IR	148 (5.9%)	short nominal interest rate, log(1+r/100), r is in percent per year	log(1+stir/100)
Long IR	35 (1.4%)	long nominal interest rate, log(1+r/100), r is in percent per year	log(1+ltrate/100)
Inflation	17 (0.7%)	Annual Consumer Price Inflation	log(cpi/cpi(-1))
Money	172 (6.9%)	Annual nominal broad money growth	log(money/money(-1))

Notes: (*) The source for all variables is the Jordà-Schularick-Taylor (JST) macrohistory database available at http://www.macrohistory.net/data/, see, Jordà, Schularick, and Taylor (2017) and Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019). We have downloaded the latest version available at the beginning of September 2020, which is "JSTdatasetR4" (Release 4, May 2019).

(**) The full sample covers $n_{\max} = 17$ countries and $T = 147$ years, together $n_{\max} T_{\max} = 2499$ country-year datapoints. Countries are: Australia, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, UK, Italy, Japan, Netherlands, Norway, Portugal, Sweden, and USA. Time is 1870-2016. The column 'Gap years' reports the number of country-year data points with missing data. The shares of the gap years in the overall sample are reported in the parentheses.

(***) The column 'Variable construction' shows variable transformations referencing the underlying variable codes in the JST database.

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Online Supplement to
"Revisiting the Great Ratios Hypothesis"

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This online supplement consists of three sections. Section S.1 provides likelihood function, first order conditions, and asymptotic variance of SPMG estimator. Section S.2 describes implementation of PMG, SPMG, 2-step Breitung, and MGMW estimators, including bootstrapping procedures. Section S.3 provides full account of Monte Carlo (MC) experiments, including a detailed description of MC design, and the full set of MC findings.

S.1 Likelihood function, first order conditions, and asymptotic variance of SPMG estimator

We assume observations on $\mathbf{w}_{it} = (y_{it}, x_{it})'$ are available for $i = 1, 2, ..., n$ and $t = 1, 2, ..., T$. Our model is given by (1)-(3), which can be conveniently written as (4), namely:

$$\Delta \mathbf{w}_{it} = -\boldsymbol{\phi}_i \boldsymbol{\beta}' \mathbf{w}_{i,t-1} + \boldsymbol{\Upsilon}_i \mathbf{q}_{it} + \mathbf{u}_{it}, \tag{S.1}$$

for $i = 1, 2, ..., n$ and $t = p + 1, p + 2, ..., T$, where $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ is a reduced-form error vector with $E(\mathbf{u}_{it}) = \mathbf{0}$, and $E(\mathbf{u}_{it}' \mathbf{u}_{it}) = \boldsymbol{\Sigma}_i$, a positive definite covariance matrix. Also, $\boldsymbol{\beta} = (1, -\theta)'$, $\mathbf{q}_{it} = \left(1, \Delta \mathbf{w}_{i,t-1}', \Delta \mathbf{w}_{i,t-2}', ..., \Delta \mathbf{w}_{i,t-p+1}'\right)'$, and $\boldsymbol{\Upsilon} = (\mathbf{a}_i, \boldsymbol{\Psi}_{i,1}, \boldsymbol{\Psi}_{i,2}, ..., \boldsymbol{\Psi}_{i,p-1})'$, with $\mathbf{a}_i = (a_{yi}, a_{xi})'$ and $\boldsymbol{\Psi}_{i\ell} = \left(\boldsymbol{\psi}_{yi\ell}, \boldsymbol{\psi}_{xi\ell}\right)'$ for $\ell = 1, 2, ..., p-1$.

For notational convenience, we also define $\boldsymbol{\phi} = \left(\boldsymbol{\phi}_1', \boldsymbol{\phi}_2', ..., \boldsymbol{\phi}_n'\right)'$, $\boldsymbol{\Upsilon} = \left(\boldsymbol{\Upsilon}_1', \boldsymbol{\Upsilon}_2', ..., \boldsymbol{\Upsilon}_n'\right)'$, and $\boldsymbol{\Sigma} = \left(\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2, ..., \boldsymbol{\Sigma}_n\right)'$. In addition, let

$$\boldsymbol{\beta}'\mathbf{w}_{i,t-1} = y_{it} - \theta x_{it} = \xi_{i,t-1}(\theta).$$

Assuming $\mathbf{u}_{it} \sim IIDN(\mathbf{0}, \boldsymbol{\Sigma}_i)$, independently distributed over i, the log-likelihood function conditional on the initial observations $\mathbf{w}_{i,1}, \mathbf{w}_{i,2}, ..., \mathbf{w}_{i,p}$, is given by:

$$\begin{aligned}\mathcal{L}_{n,T}(\theta, \boldsymbol{\phi}, \boldsymbol{\Upsilon}, \boldsymbol{\Sigma}) &= -\frac{(T-p)n}{2}\ln(2\pi) + \frac{(T-p)}{2}\sum_{i=1}^{n}\ln\left|\boldsymbol{\Sigma}_i^{-1}\right| \\ &\quad -\frac{1}{2}\sum_{i=1}^{n}\sum_{t=p+1}^{T}\left(\Delta\mathbf{w}_{it} + \boldsymbol{\phi}_i\xi_{i,t-1}(\theta) - \boldsymbol{\Upsilon}_i\mathbf{q}_{it}\right)'\boldsymbol{\Sigma}_i^{-1}\left(\Delta\mathbf{w}_{it} + \boldsymbol{\phi}_i\xi_{i,t-1}(\theta) - \boldsymbol{\Upsilon}_i\mathbf{q}_{it}\right).\end{aligned}$$

Consider the projection matrix $\mathbf{H}_i = \mathbf{I}_{T-p} - \mathbf{Q}_i(\mathbf{Q}_i'\mathbf{Q}_i)^{-1}\mathbf{Q}_i'$, where $\mathbf{Q}_i$ is a matrix of observations on $\mathbf{q}_{it} = \left(1, \Delta\mathbf{w}_{i,t-1}', \Delta\mathbf{w}_{i,t-2}', ..., \Delta\mathbf{w}_{i,t-p+1}'\right)'$, namely $\mathbf{Q}_i = \left(\mathbf{q}_{i,p+1}, \mathbf{q}_{i,p+2}, ..., \mathbf{q}_{i,T}\right)'$. In addition, let $\mathbf{x}_{i,-1} = \left(x_{i,p}, x_{i,p+1}, ..., x_{i,T-1}\right)'$, $\mathbf{y}_{i,-1} = \left(y_{i,p}, y_{i,p+1}, ..., y_{i,T-1}\right)'$, $\boldsymbol{\xi}_{i,-1} = \left(\xi_{ip}, \xi_{i,p+1}, ..., \xi_{i,T-1}\right)'$, $\Delta\mathbf{W}_i = \left(\Delta\mathbf{w}_{i,p+1}, \Delta\mathbf{w}_{i,p+2,}, ..., \Delta\mathbf{w}_{iT}\right)'$, and define

$$\Delta\tilde{\mathbf{W}}_i = \mathbf{H}_i\Delta\mathbf{W}_i,\ \tilde{\boldsymbol{\xi}}_{i,-1}(\theta) = \mathbf{H}_i\boldsymbol{\xi}_{i,-1}(\theta),$$

where row vectors of $\Delta\tilde{\mathbf{W}}_i$ and elements of $\tilde{\boldsymbol{\xi}}_{i,-1}(\theta)$ are denoted as $\Delta\tilde{\mathbf{W}}_i = \left(\Delta\tilde{\mathbf{w}}_{i,p+1}, \Delta\tilde{\mathbf{w}}_{i,p+2,}, ..., \Delta\tilde{\mathbf{w}}_{iT}\right)'$ and $\tilde{\boldsymbol{\xi}}_{i,-1}(\theta) = \left[\tilde{\xi}_{ip}(\theta), \tilde{\xi}_{i,p+1}(\theta), ..., \tilde{\xi}_{i,T-1}(\theta)\right]'$.

Concentrating $\boldsymbol{\Upsilon}$ out, the concentrated log-likelihood function is given by

$$\begin{aligned}\mathcal{L}_{n,T}(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma}) &= -\frac{(T-p)n}{2}\ln(2\pi) + (T-p)\sum_{i=1}^{n}\ln\left|\boldsymbol{\Sigma}_i^{-1}\right| \\ &\quad -\frac{1}{2}\sum_{i=1}^{n}\sum_{t=p+1}^{T}\left[\Delta\tilde{\mathbf{w}}_{it} + \boldsymbol{\phi}_i\tilde{\xi}_{i,t-1}(\theta)\right]'\boldsymbol{\Sigma}_i^{-1}\left[\Delta\tilde{\mathbf{w}}_{it} + \boldsymbol{\phi}_i\tilde{\xi}_{i,t-1}(\theta)\right],\end{aligned}$$

which corresponds to equation (7) in the paper.

Partial derivatives of $\mathcal{L}_{n,T}(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma})$ with respect to θ and $\boldsymbol{\phi}$ are

$$\begin{aligned}\frac{\partial\mathcal{L}_{n,T}(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma})}{\partial\theta} &= \sum_{i=1}^{n}\sum_{t=p+1}^{T}\left(\tilde{x}_{i,t-1}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\Delta\tilde{\mathbf{w}}_{it} + \boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\tilde{x}_{i,t-1}\tilde{y}_{i,t-1}\right) \\ &\quad -\sum_{i=1}^{n}\sum_{t=p+1}^{T}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\tilde{x}_{i,t-1}^2, \qquad \text{(S.2)}\end{aligned}$$

and

$$\frac{\partial \mathcal{L}_{n,T}(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma})}{\partial \boldsymbol{\phi}_i} = \sum_{t=p+1}^{T} \left[\tilde{\xi}_{i,t-1}(\theta) \boldsymbol{\Sigma}_i^{-1} \Delta \tilde{\mathbf{w}}_{it} + \tilde{\xi}_{i,t-1}^2(\theta) \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\phi}_i \right], \tag{S.3}$$

respectively. Setting partial derivatives (S.2)-(S.3) equal to zero gives implicit solutions (9)-(10) in the paper. Similarly, setting partial derivatives of $\mathcal{L}_{n,T}(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma})$ with respect to elements of $\boldsymbol{\Sigma}_i$ equal to zero yields implicit solution (11) for $\hat{\boldsymbol{\Sigma}}_i$ in the paper.

Using (14)-(15), asymptotic variance of the SPMG estimator is given by $p\lim_{n,T\to\infty} Q_{nT}^{-1} V_{qnT} Q_{nT}^{-1}$, where $V_{qnT} = E(q_{nT} q_{nT}') = E(q_{nT}^2)$. Under error cross-sectional independence, we have $V_{qnT} = Q_{nT}$, in which case the asymptotic variance reduces to $p\lim_{n,T\to\infty} Q_{nT}^{-1}$, and it can be consistently estimated by $\hat{Q}_{nT}^{-1}$, where

$$\hat{Q}_{nT} = n^{-1} \sum_{i=1}^{n} \left(\hat{\boldsymbol{\phi}}_i' \hat{\boldsymbol{\Sigma}}_i^{-1} \hat{\boldsymbol{\phi}}_i \right) \left(T^{-2} \mathbf{x}_{i,-1}' \mathbf{H}_i \mathbf{x}_{i,-1} \right), \tag{S.4}$$

with $\hat{\boldsymbol{\phi}}_i$ and $\hat{\boldsymbol{\Sigma}}_i$ denoting the SPMG estimates of $\boldsymbol{\phi}_i$ and $\boldsymbol{\Sigma}_i$, respectively, as given by (10) and (11). Under weak error cross-sectional dependence, it is no longer the case that $V_{qnT} = Q_{nT}$, but it is possible to consistently estimate V_{qnT} as

$$\hat{V}_{qnT} = \frac{1}{nT^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=2}^{T} \hat{\zeta}_{it} \hat{\zeta}_{jt}, \tag{S.5}$$

where $\hat{\zeta}_{it} = \tilde{x}_{i,t-1} \hat{\tilde{\mathbf{u}}}_{it}' \hat{\boldsymbol{\Sigma}}_i^{-1} \hat{\boldsymbol{\phi}}_i$, $\hat{\tilde{\mathbf{u}}}_{it}'$, for $t = p+1, p+2, ..., T$, are the rows of $\hat{\tilde{\mathbf{U}}}_i \left(\hat{\theta}, \hat{\boldsymbol{\phi}}_i \right) = \mathbf{H}_i \left[\Delta \mathbf{W}_i + \boldsymbol{\xi}_{i,-1} \left(\hat{\theta} \right) \hat{\boldsymbol{\phi}}_i' \right]$, see (8). A consistent estimator of the asymptotic variance of SPMG estimator $\hat{\theta}$, regardless of error cross-sectional dependence, is given by $\hat{Q}_{nT}^{-1} \hat{V}_{qnT} \hat{Q}_{nT}^{-1}$.

S.2 Implementation of individual estimators and bootstrapping procedures

This section of the online supplement provide detailed description of the implementation of PMG, 2-step Breitung, SPMG, and MGMW estimators. Computation of bootstrapped confidence intervals for PMG and SPMG estimators is outlined as well.

S.2.1 PMG estimator

PMG estimator of the level coefficient θ is based on the (conditional) ARDL specification

$$y_{it} = c_i + \sum_{\ell=1}^{p} \alpha_{i,\ell} y_{i,t-1} + \sum_{\ell=0}^{q} \beta_{i,\ell} x_{i,t-\ell} + \varepsilon_{it}, \tag{S.6}$$

where

$$\theta = \frac{\sum_{\ell=0}^{q} \beta_{i,\ell}}{1 - \sum_{\ell=1}^{p} \alpha_{i,\ell}},$$

is homogenous across i. Conditional model (S.6) can be obtained from (1)-(3), assuming that $E\left(u_{xit}u_{yit}\right)/E\left(u_{xit}^2\right)$ is time invariant (see, for instance, Section 5 of Chudik and Pesaran, 2021).

For expositional convenience, we set $p = q = 2$ below. The same choices of lag orders are used in the Monte Carlo section, and in the empirical section of the main paper. For $p = q = 2$, (S.6) can be written as

$$\Delta y_{it} = c_i - \phi_i \xi_{i,t-1}\left(\theta\right) + \omega_{yi}\Delta y_{i,t-1} + \omega_{xi,0}\Delta x_{it} + \omega_{xi,1}\Delta x_{i,t-1} + \varepsilon_{it}, \tag{S.7}$$

where

$$\xi_{i,t-1}\left(\theta\right) = y_{i,t-1} - \theta x_{i,t-1}. \tag{S.8}$$

We continue to assume observations on (x_{it}, y_{it}) for $t = 1, 2..., T$ time periods and $i = 1, 2, ..., n$ cross-section units are available for estimation. The PMG estimator is computed by a back-substitution algorithm. Assuming $\varepsilon_{it} \sim IIDN\left(0, \sigma_{\varepsilon i}^2\right)$, and setting the first derivatives of the concentrated log-likelihood function

$$\ell_{nT}(\boldsymbol{\varphi}) = -\frac{T-2}{2}\sum_{i=1}^{n} \ln 2\pi\sigma_i^2 - \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\sigma_{\varepsilon i}^2}\left[(\Delta \mathbf{y}_i + \phi_i \boldsymbol{\xi}_{i,-1}(\theta)\right]' \mathbf{H}_i^{\Diamond}\left[\Delta \mathbf{y}_i + \phi_i \boldsymbol{\xi}_{i,-1}(\theta)\right], \tag{S.9}$$

with respect to $\boldsymbol{\varphi} = \left(\theta, \phi_1, \phi_2, ..., \phi_n, \sigma_{\varepsilon i}^2, \sigma_{\varepsilon 2}^2, ..., \sigma_{\varepsilon n}^2\right)'$ to $\mathbf{0}$ yields the following implicit expressions for $\hat{\theta}^{PMG}$, $\hat{\phi}_i^{PMG}$, and $\hat{\sigma}_{\varepsilon i}^2$ which we solve iteratively:

$$\hat{\theta}^{PMG} = -\left\{\sum_{i=1}^{n}\frac{\left(\hat{\phi}_i^{PMG}\right)^2}{\hat{\sigma}_{\varepsilon i}^2}\mathbf{x}_{i,-1}'\mathbf{H}_i^{\Diamond}\mathbf{x}_{i,-1}\right\}^{-1}\left\{\sum_{i=1}^{n}\frac{\hat{\phi}_i^{PMG}}{\hat{\sigma}_{\varepsilon i}^2}\mathbf{x}_{i,-1}'\mathbf{H}_i^{\Diamond}\left(\Delta \mathbf{y}_i + \hat{\phi}_i^{PMG}\mathbf{y}_{i,-1}\right)\right\}, \tag{S.10}$$

$$\hat{\phi}_i^{PMG} = -\left[\boldsymbol{\xi}_{i,-1}'\left(\hat{\theta}^{PMG}\right)\mathbf{H}_i^{\Diamond}\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{PMG}\right)\right]^{-1}\boldsymbol{\xi}_{i,-1}'\left(\hat{\theta}^{PMG}\right)\mathbf{H}_i^{\Diamond}\Delta\mathbf{y}_i,\ i = 1, 2, ..., n, \tag{S.11}$$

$$\hat{\sigma}_{\varepsilon i}^2 = (T-2)^{-1}\left[\Delta\mathbf{y}_i + \hat{\phi}_i^{PMG}\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{PMG}\right)\right]'\mathbf{H}_i^{\Diamond}\left[\Delta\mathbf{y}_i + \hat{\phi}_i^{PMG}\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{PMG}\right)\right],\ i = 1, 2, ..., n, \tag{S.12}$$

where $\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{PMG}\right) = \mathbf{y}_{i,-1} - \hat{\theta}^{PMG}\mathbf{x}_{i,-1}$, and $\mathbf{H}_i^{\Diamond} = \mathbf{I}_{T-2} - \mathbf{Q}_i^{\Diamond}(\mathbf{Q}_i^{\Diamond\prime}\mathbf{Q}_i^{\Diamond})^{-1}\mathbf{Q}_i^{\Diamond\prime}$, $\mathbf{Q}_i^{\Diamond} = (\Delta\mathbf{y}_{i,-1}, \Delta\mathbf{x}_i, \Delta\mathbf{x}_{i,-1}, \boldsymbol{\tau}_{T-2})$, $\Delta\mathbf{y}_{i,-1} = (\Delta y_{i,2}, \Delta y_{i,3}, ..., \Delta y_{i,T-1})'$, $\mathbf{y}_{i,-1} = (y_{i,2}, y_{i,3}, ..., y_{i,T-1})'$, $\mathbf{x}_{i,-1} = (x_{i,2}, x_{i,3}, ..., x_{i,T-1})'$, $\Delta\mathbf{x}_i = (\Delta x_{i,3}, \Delta x_{i,4}, ..., \Delta x_{i,T})'$, $\Delta\mathbf{x}_{i,-1} = (\Delta x_{i,2}, \Delta x_{i,3}, ..., \Delta x_{i,T-1})'$, and $\boldsymbol{\tau}_{T-2}$ is $T-2$ dimensional column vector of ones. Higher order lags of Δy_{it} and Δx_{it} in (S.7) could be accommodated by augmenting $\mathbf{Q}_i^{\Diamond}$ by these terms.

Starting with a consistent initial estimate of $\boldsymbol{\theta}$, say $\hat{\boldsymbol{\theta}}_{(0)}^{PMG}$, estimates of ϕ_i and σ_i^2 are computed using

(S.11) and (S.12), which can then be substituted in (S.10) to obtain a new estimate of $\boldsymbol{\theta}$, say $\hat{\boldsymbol{\theta}}_{(1)}^{PMG}$, and so on until convergence is achieved. Our criterion for convergence is $\left\|\hat{\boldsymbol{\theta}}_{(j)}^{PMG} - \hat{\boldsymbol{\theta}}_{(j-1)}^{PMG}\right\| < 10^{-4}$. Convergence is usually achieved very fast with average number of iterations generally quite small (<10 in most cases).

The initial estimate $\hat{\boldsymbol{\theta}}_{(0)}^{PMG}$ is taken to be the FE estimator of the coefficient $\boldsymbol{\theta}$ in the following panel Engle-Granger regression,

$$y_{it} = \mu_i + \boldsymbol{\theta} x_{it} + e_{it}, \tag{S.13}$$

for $t = 1, 2, ..., T$, $i = 1, 2, ..., n$.

Inference for the PMG estimator $\hat{\boldsymbol{\theta}}^{PMG}$ is conducted using

$$\widehat{Var}\left(\hat{\boldsymbol{\theta}}^{PMG}\right) = \hat{\Omega}_{PMG} = \left(\sum_{i=1}^{n} \frac{\left(\hat{\phi}_i^{PMG}\right)^2}{\hat{\sigma}_{\varepsilon i}^2} \mathbf{x}_{i,-1}' \mathbf{H}_i^{\Diamond} \mathbf{x}_{i,-1}\right)^{-1}. \tag{S.14}$$

S.2.1.1 Bootstrapping critical values

In addition to asymptotic critical values, based on (S.14) we also consider bootstrap critical values of the test statistics

$$t_{\hat{\theta}}\left(\theta_0\right) = \frac{\hat{\theta}^{PMG} - \theta_0}{\widehat{s.e.}\left(\hat{\theta}^{PMG}\right)}, \tag{S.15}$$

where

$$\widehat{s.e.}\left(\hat{\theta}^{PMG}\right) = \hat{\Omega}_{PMG}^{1/2}, \tag{S.16}$$

and $\hat{\Omega}_{PMG}$ is given by (S.14). We consider two bootstrapping procedures: (i) conditional on x_{it} and (ii) unconditional, where model for x_{it} is also utilized. Conditional procedure in Subsection S.2.1.1 is valid in the case of strict exogeneity of x_{it}, whereas the unconditional procedure outlined in Subsection S.2.1.1 is valid also in the case of short run feedbacks from y_{it} to x_{it}. However, in all of our MC experiments, these two procedures performed very similarly, regardless of the presence of the short run feedbacks from y_{it} to x_{it}. In the empirical section, we reported confidence intervals based on the unconditional procedure. Both types of confidence intervals are available in our codes package.

Bootstrapping critical values (conditional on x_{it}) The following procedure is adopted.

1. Using data $y_{i1}, y_{i2}, ..., y_{iT}$ and $x_{i1}, x_{i2}, ..., x_{iT}$ $(i = 1, 2, ..., n)$ estimate all parameters of

$$\Delta y_{it} = c_i - \phi_i (y_{i,t-1} - \theta x_{i,t-1}) + \omega_{yi} \Delta y_{i,t-1} + \omega_{xi,0} \Delta x_{it} + \omega_{xi,1} \Delta x_{i,t-1} + \varepsilon_{it}. \tag{S.17}$$

Denote the corresponding PMG estimates of the unknown coefficients $\hat{\theta}$, $\left\{\hat{c}_i\right\}_{i=1}^n$, $\left\{\hat{\phi}_i\right\}_{i=1}^n$, $\left\{\hat{\omega}_{yi}\right\}_{i=1}^n$,

$\{\hat{\omega}_{xi,0}\}_{i=1}^{n}$, $\{\hat{\omega}_{xi,1}\}_{i=1}^{n}$. We omit superscript "PMG" in this procedure to simplify notations.

2. Obtain residuals, denoted as $\hat{\varepsilon}_{it}$.

3. Repeat for $b = 1, 2, ..., B$:

   (a) Generate $\varepsilon_{it}^{(b)} = \varkappa_t^{(b)} \hat{\varepsilon}_{y,it}$, where $\varkappa_t$ is randomly drawn from Rademacher distribution (Davidson and Flachaire, 2008)
$$\varkappa_t^{(b)} = \begin{cases} -1, & \text{with probability } 1/2 \\ 1, & \text{with probability } 1/2 \end{cases}.$$

   (b) Generate
$$y_{it}^{(b)} = y_{i,t-1}^{(b)} + \hat{c}_i - \hat{\phi}_i \left(y_{i,t-1}^{(b)} - \hat{\theta} x_{i,t-1} \right) + \hat{\omega}_{y,i} \Delta y_{i,t-1}^{(b)} + \hat{\omega}_{xi,0} \Delta x_{it} + \hat{\omega}_{xi,1} \Delta x_{i,t-1} + \varepsilon_{it}^{(b)}, \quad \text{(S.18)}$$
   for $t = 3, 4, ..., T$, with initial values $y_{i1}^{(b)} \equiv y_{i1}$ and $y_{i2}^{(b)} \equiv y_{i2}$.

   (c) Using the bootstrap sample $\mathbf{y}_i^{(b)} = \left(y_{i1}, y_{i2}, y_{i3}^{(b)}, y_{i4}^{(b)}, ..., y_{iT}^{(b)} \right)'$ and $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{iT})'$ $(i = 1, 2, ..., n)$, compute PMG estimate $\hat{\theta}^{(b)}$, and the associated test statistics
$$t_{\hat{\theta}}^{(b)} = \frac{\hat{\theta}^{(b)} - \hat{\theta}}{\widehat{s.e.}\left(\hat{\theta}^{(b)}\right)}, \quad \text{(S.19)}$$
   where $\widehat{s.e.}\left(\hat{\theta}^{(b)}\right) = \left(\hat{\Omega}_{PMG}^{(b)}\right)^{1/2}$.

4. Inference (at 5% nominal level) is conducted using the 95 percent quantile of $\left\{ \left| t_{\hat{\theta}}^{(b)} \right|, b = 1, 2, ..., B \right\}$ as critical value.

Bootstrapping critical values (unconditional) This procedure is the same as above, but we allow for feedback from y_{it} to x_{it}. Specifically, we estimate the conditional model
$$\Delta y_{it} = c_i - \phi_i (y_{i,t-1} - \theta x_{i,t-1}) + \omega_{yi} \Delta y_{i,t-1} + \omega_{xi,0} \Delta x_{it} + \omega_{xi,1} \Delta x_{i,t-1} + \varepsilon_{it}.$$
and the marginal model
$$\Delta x_{it} = d_i + \delta_{xi} \Delta x_{i,t-1} + \delta_{yi} \Delta y_{i,t-1} + v_{it}. \quad \text{(S.20)}$$
Bootstrap samples are consequently generated as
$$x_{it}^{(b)} = x_{i,t-1}^{(b)} + \hat{d}_i + \hat{\delta}_{xi} \Delta x_{i,t-1}^{(b)} + \hat{\delta}_{yi} \Delta y_{i,t-1}^{(b)} + v_{it}^{(b)},$$

and

$$y_{it}^{(b)} = y_{i,t-1}^{(b)} + \hat{c}_i - \hat{\phi}_i \left(y_{i,t-1}^{(b)} - \hat{\theta} x_{i,t-1}^{(b)} \right) + \hat{\omega}_{y,i} \Delta y_{i,t-1}^{(b)} + \hat{\omega}_{xi,0} \Delta x_{it}^{(b)} + \hat{\omega}_{xi,1} \Delta x_{i,t-1}^{(b)} + \varepsilon_{it}^{(b)},$$

for $t = 3, 4, ..., T$ using the initial values $y_{it}^{(b)} = y_{it}$, for $t = 1, 2$ and $x_{it}^{(b)} = x_{it}$, for $t = 1, 2$, in which $v_{it}^{(b)}$ is generated similarly to $\varepsilon_{it}^{(b)}$.

S.2.2 2-step Breitung's (2005) panel data estimator

Breitung's 2-step panel data estimator requires an initial consistent estimator. We follow the same choices for the implementation of Breitung's estimator as in the original paper Breitung (2005), and in the accompanying Gauss codes we received from Joerg Breitung, which we gratefully acknowledge. For each unit $i = 1, 2, ..., n$, we compute

$$\hat{\theta}_i = \left(\tilde{\mathbf{x}}_i' \tilde{\mathbf{x}}_i \right)^{-1} \tilde{\mathbf{x}}_i' \tilde{\mathbf{y}}_i, \tag{S.21}$$

where $\tilde{\mathbf{x}}_i = \mathbf{M}_\tau \mathbf{x}_i$, $\tilde{\mathbf{y}}_i = \mathbf{M}_\tau \mathbf{y}_i$, $\mathbf{x}_i = (x_{i1}, x_{i1}, ..., x_{iT})'$, $\mathbf{y}_i = (y_{i1}, y_{i1}, ..., y_{iT})'$, $\mathbf{M}_\tau = \mathbf{I}_T - \boldsymbol{\tau}\boldsymbol{\tau}'/T$, $\mathbf{I}_T$ is $T \times T$ identity matrix and $\boldsymbol{\tau}$ is $T \times 1$ vector of ones. Next, we compute

$$\xi_{it} \left(\hat{\theta}_i \right) = y_{it} - \hat{\theta}_i x_{it}.$$

We continue to assume $p = q = 2$. Define the following data vectors

$$\begin{aligned}
\Delta \mathbf{x}_i &= \left(\Delta x_{i,3}, \Delta x_{i,4}, ..., \Delta x_{i,T} \right)' \\
\Delta \mathbf{y}_i &= \left(\Delta y_{i,3}, \Delta y_{i,4}, ..., \Delta y_{i,T} \right)', \\
\mathbf{x}_{i-1} &= \left(x_{i,2}, x_{i,3}, ..., x_{i,T-1} \right)', \\
\mathbf{y}_{i-1} &= \left(y_{i,2}, y_{i,3}, ..., y_{i,T-1} \right)', \\
\boldsymbol{\xi}_{i,-1} \left(\hat{\theta}_i \right) &= \left[\xi_{i,2} \left(\hat{\theta}_i \right), \xi_{i,3} \left(\hat{\theta}_i \right), ..., \xi_{i,T-1} \left(\hat{\theta}_i \right) \right]' \\
\mathbf{q}_{i,t-1} &= \left(1, \Delta x_{i,t-1}, \Delta y_{i,t-1} \right)',
\end{aligned}$$

and the following data matrices

$$\Delta \mathbf{W}_i = \left(\Delta \mathbf{y}_i, \Delta \mathbf{x}_i \right), \ \mathbf{W}_{i,-1} = \left(\mathbf{y}_{i,-1}, \mathbf{x}_{i,-1} \right),$$

$$\mathbf{Q}_i = \left(\mathbf{q}_{i,2}, \mathbf{q}_{i,3}, ..., \mathbf{q}_{i,T-1} \right)'.$$

Let $\mathbf{H}_i = \mathbf{I}_{T-2} - \mathbf{Q}_i\left(\mathbf{Q}_i'\mathbf{Q}_i\right)^{-1}\mathbf{Q}_i$, and

$$\Delta\tilde{\mathbf{W}}_i = \mathbf{H}_i\Delta\mathbf{W}_i,\ \tilde{\mathbf{W}}_{i,-1} = \mathbf{H}_i\mathbf{W}_{i,-1},\ \tilde{\boldsymbol{\xi}}_{i,-1}\left(\hat{\theta}_i\right) = \mathbf{H}_i\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}_i\right).$$

In addition, let

$$\begin{aligned}
\hat{\boldsymbol{\phi}}_i &= \left[\tilde{\boldsymbol{\xi}}_{i,-1}'\left(\hat{\theta}_i\right)\tilde{\boldsymbol{\xi}}_{i,-1}\left(\hat{\theta}_i\right)\right]^{-1}\tilde{\boldsymbol{\xi}}_{i,-1}'\left(\hat{\theta}_i\right)\Delta\tilde{\mathbf{W}}_i, \\
\hat{\mathbf{U}}_i &= \Delta\tilde{\mathbf{W}}_i + \tilde{\boldsymbol{\xi}}_{i,-1}\left(\hat{\theta}_i\right)\hat{\boldsymbol{\phi}}_i', \\
\hat{\boldsymbol{\Sigma}}_i &= \hat{\mathbf{U}}_i'\hat{\mathbf{U}}_i/\left(T-3\right), \\
\tilde{\mathbf{z}}_i^{+} &= \tilde{\mathbf{W}}_{i,-1}\mathbf{s}_1 + \left(\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\phi}}_i\right)^{-1}\Delta\tilde{\mathbf{W}}_i\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i
\end{aligned}$$

$\mathbf{s}_1 = (1,0)'$, and $\mathbf{s}_2 = (0,1)'$. Then 2-step Breitung (2005) estimator of θ is computed as

$$\hat{\theta}^{Br} = \left(\sum_{i=1}^{n}\mathbf{s}_2'\tilde{\mathbf{W}}_{i,-1}'\tilde{\mathbf{W}}_{i,-1}\mathbf{s}_2\right)^{-1}\sum_{i=1}^{n}\mathbf{s}_2'\tilde{\mathbf{W}}_{i,-1}'\Delta\tilde{\mathbf{z}}_i^{+}.$$

Inference is conducted using the "2S-OLS" and "2S-robust" standard errors as described in Breitung (2005).

S.2.3 SPMG estimator

SPMG estimator avoids inversion of $\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\phi}}_i$, since it can be the case that $\hat{\boldsymbol{\phi}}_i = o_p(1)$ for some units that do not have long run relationship (or $\hat{\boldsymbol{\phi}}_i$ can converge to a rank-deficient matrix in probability in a more general case). SPMG estimator of θ is solved iteratively using

$$\hat{\theta}^{\text{SPMG}} = -\left(\sum_{i=1}^{n}\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}\right)^{-1}\sum_{i=1}^{n}\mathbf{x}_{i,-1}'\mathbf{H}_i\left(\Delta\mathbf{W}_i + \mathbf{y}_{i,-1}\hat{\boldsymbol{\phi}}_i'\right)\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i, \tag{S.22}$$

$$\hat{\boldsymbol{\phi}}_i^{\text{SPMG}} = -\left[\boldsymbol{\xi}_{i,-1}'\left(\hat{\theta}^{\text{SPMG}}\right)\mathbf{H}_i\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{\text{SPMG}}\right)\right]^{-1}\boldsymbol{\xi}_{i,-1}'\left(\hat{\theta}^{\text{SPMG}}\right)\mathbf{H}_i\Delta\mathbf{W}_i, \tag{S.23}$$

$$\hat{\boldsymbol{\Sigma}}_i^{\text{SPMG}} = (T-2)^{-1}\left[\Delta\mathbf{W}_i + \boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{\text{SPMG}}\right)\hat{\boldsymbol{\phi}}_i^{\text{SPMG}\prime}\right]'\mathbf{H}_i\left[\Delta\mathbf{W}_i + \boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{\text{SPMG}}\right)\hat{\boldsymbol{\phi}}_i^{\text{SPMG}\prime}\right], \tag{S.24}$$

in which $\mathbf{x}_{i,-1} = (x_{i,1}, x_{i,2}, ..., x_{i,T-1})'$, $\mathbf{y}_{i,-1} = (y_{i,1}, y_{i,2}, ..., y_{i,T-1})'$, $\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{\text{SPMG}}\right) = \left[\xi_{i1}\left(\hat{\theta}^{\text{SPMG}}\right), \xi_{i2}\left(\hat{\theta}^{\text{SPMG}}\right), ..., \xi_{iT-1}\left(\hat{\theta}^{\text{SPMG}}\right)\right]$, $\boldsymbol{\xi}_{it}\left(\hat{\theta}^{\text{SPMG}}\right) = y_{it} - \hat{\theta}^{\text{SPMG}}x_{it}$, $\Delta\mathbf{W}_i = (\Delta\mathbf{w}_{i2,}\Delta\mathbf{w}_{i3,}, ..., \Delta\mathbf{w}_{iT})'$,and $\Delta\mathbf{w}_{it} = (\Delta y_{it}, \Delta x_{it})'$. $\mathbf{H}_i = \mathbf{I}_{T-2} - \mathbf{Q}_i\left(\mathbf{Q}_i'\mathbf{Q}_i\right)^{-1}\mathbf{Q}_i$, $\mathbf{Q}_i = (\mathbf{q}_{i,2}, \mathbf{q}_{i,3}, ..., \mathbf{q}_{i,T-1})'$ and $\mathbf{q}_{i,t-1} = (1, \Delta x_{i,t-1}, \Delta y_{i,t-1})'$.

The conventional estimator of the variance of $\hat{\theta}^{\text{SPMG}}$ is computed, using (S.4), as

$$\widehat{Var}\left(\hat{\theta}^{\text{SPMG}}\right) = \hat{\Omega}_{\text{SPMG}} = n^{-1}T^{-2}\hat{Q}_{nT} = \left[\sum_{i=1}^{n} \hat{\boldsymbol{\phi}}_i^{\text{SPMG}\prime} \left(\hat{\boldsymbol{\Sigma}}_i^{\text{SPMG}}\right)^{-1} \hat{\boldsymbol{\phi}}_i^{\text{SPMG}} \mathbf{x}_{i,-1}' \mathbf{H}_i \mathbf{x}_{i,-1}\right]^{-1}. \tag{S.25}$$

Estimators of $Var\left(\hat{\theta}^{\text{SPMG}}\right)$ that are robust to error cross-sectional dependence are computed, using (S.4)-(S.5), as

$$\widetilde{Var}\left(\hat{\theta}^{\text{SPMG}}\right) = n^{-1}T^{-2}\hat{Q}_{nT}^{-1}\hat{V}_{qnT}\hat{Q}_{nT}^{-1} = \hat{\Omega}_{\text{SPMG}}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=2}^{T}\hat{\zeta}_{it}\hat{\zeta}_{jt}\right)\hat{\Omega}_{\text{SPMG}}, \tag{S.26}$$

where

$$\hat{\zeta}_{it} = \tilde{x}_{i,t-1}\widehat{\tilde{\mathbf{u}}}_{it}' \left(\hat{\boldsymbol{\Sigma}}_i^{\text{SPMG}}\right)^{-1} \hat{\boldsymbol{\phi}}_i^{\text{SPMG}},$$

and $\widehat{\tilde{\mathbf{u}}}_{it}'$ for $t = 2, 3, ..., T$, are the rows of $\mathbf{H}_i\hat{\mathbf{U}}_i$, in which $\hat{\mathbf{U}}_i = (\hat{\mathbf{u}}_{i,2}, \hat{\mathbf{u}}_{i,3}, ..., \hat{\mathbf{u}}_{i,T})'$ is the matrix of SPMG residuals. We consider inference based on (S.25) using asymptotic critical values, inference based on (S.26) with asymptotic critical values (reported in Table S38), and inference based on (S.25) using bootstrapped critical values outlined below. Our bootstrapping procedure is robust to error cross sectional dependence, and it outperforms the other alternatives in our Monte Carlo experiments in terms of accuracy of the confidence intervals coverage rates. In situations where n is large relative to T, one could also consider a threshold version of (S.26), given by:

$$\widetilde{Var}\left(\hat{\theta}^{\text{SPMG}}\right) = \hat{\Omega}_{\text{SPMG}}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=2}^{T}\hat{\zeta}_{it}\hat{\zeta}_{jt} I\left(\left|\hat{\rho}_{\zeta ij}\right| > \sqrt{T}c_p(n)\right)\right]\hat{\Omega}_{\text{SPMG}}, \tag{S.27}$$

where $\hat{\rho}_{\zeta ij}$ is the sample correlation of $\hat{\zeta}_{it}$ and $\hat{\zeta}_{jt}$, and $c_p(n)$ is a suitably chosen thresholding critical value, as considered by Bailey et al. (2019), $c_p(n) = \Phi^{-1}\left(1 - pn^{-\delta}/2\right)$, with $p = 0.05$ and $\delta = 2$. Our Monte Carlo findings suggest thresholding is not required for the sample sizes we consider.

S.2.3.1 Bootstrapping critical values

We consider bootstrap critical values of the test statistics

$$t_{\hat{\theta}^{\text{SPMG}}}(\theta_0) = \frac{\hat{\theta}^{\text{SPMG}} - \theta_0}{\widehat{s.e.}\left(\hat{\theta}^{\text{SPMG}}\right)}, \tag{S.28}$$

where

$$\widehat{s.e.}\left(\hat{\theta}^{\text{SPMG}}\right) = \hat{\Omega}_{\text{SPMG}}^{1/2}, \tag{S.29}$$

and $\hat{\Omega}_{\text{SPMG}}$ is given by (S.25).

The following procedure is adopted.

1. Using data $y_{i1}, y_{i2}, ..., y_{iT}$ and $x_{i1}, x_{i2}, ..., x_{iT}$ $(i = 1, 2, ..., n)$ estimate all parameters of

$$\Delta y_{it} = a_{yi} - \phi_{yi}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{yyi}\Delta y_{i,t-1} + \psi_{yxi}\Delta x_{i,t-1} + u_{yit}. \tag{S.30}$$

$$\Delta x_{it} = a_{xi} - \phi_{xi}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{xxi}\Delta x_{i,t-1} + \psi_{xyi}\Delta y_{i,t-1} + u_{xit}. \tag{S.31}$$

   Denote the corresponding SPMG estimates of the unknown coefficients as $\hat{\theta}$, $\{\hat{a}_{yi}\}_{i=1}^{n}$, $\{\hat{a}_{xi}\}_{i=1}^{n}$, $\left\{\hat{\phi}_{yi}\right\}_{i=1}^{n}$, $\left\{\hat{\phi}_{xi}\right\}_{i=1}^{n}$, $\{\psi_{yyi}\}_{i=1}^{n}$, $\{\psi_{yxi}\}_{i=1}^{n}$, $\{\psi_{xyi}\}_{i=1}^{n}$, and $\{\psi_{xxi}\}_{i=1}^{n}$. We omit superscript "SPMG" in this procedure to simplify notations.

2. Obtain residuals, denoted as $\hat{\mathbf{u}}_{it} = (\hat{u}_{yit}, \hat{u}_{xit})$.

3. Repeat for $b = 1, 2, ..., B$:

   (a) Generate $\mathbf{u}_{it}^{(b)} = \left(u_{yit}^{(b)}, u_{xit}^{(b)}\right)' = \varkappa_t^{(b)}\hat{\mathbf{u}}_{it}$, where $\varkappa_t$ is randomly drawn from Rademacher distribution (Davidson and Flachaire, 2008)

$$\varkappa_t^{(b)} = \begin{cases} -1, & \text{with probability } 1/2 \\ 1, & \text{with probability } 1/2 \end{cases}.$$

   (b) Generate

$$y_{it}^{(b)} = y_{i,t-1}^{(b)} + \hat{a}_{yi} - \hat{\phi}_{yi}\left(y_{i,t-1}^{(b)} - \hat{\theta}x_{i,t-1}^{(b)}\right) + \hat{\psi}_{yyi}\Delta y_{i,t-1}^{(b)} + \hat{\psi}_{yxi}\Delta x_{i,t-1}^{(b)} + u_{yit}^{(b)}, \tag{S.32}$$

$$x_{it}^{(b)} = x_{i,t-1}^{(b)} + \hat{a}_{xi} - \hat{\phi}_{xi}(y_{i,t-1}^{(b)} - \hat{\theta}x_{i,t-1}^{(b)}) + \hat{\psi}_{xxi}\Delta x_{i,t-1}^{(b)} + \hat{\psi}_{xyi}\Delta y_{i,t-1}^{(b)} + u_{xit}^{(b)}, \tag{S.33}$$

   for $t = 3, 4, ..., T$, with initial values $y_{i1}^{(b)} \equiv y_{i1}$, $y_{i2}^{(b)} \equiv y_{i2}$, $x_{i1}^{(b)} \equiv x_{i1}$, and $x_{i2}^{(b)} \equiv x_{i2}$.

   (c) Using the bootstrap sample $\mathbf{y}_i^{(b)} = \left(y_{i1}, y_{i2}, y_{i3}^{(b)}, y_{i4}^{(b)}, ..., y_{iT}^{(b)}\right)'$ and $\mathbf{x}_i^{(b)} = \left(x_{i1}, x_{i2}, x_{i3}^{(b)}, x_{i4}^{(b)}, ..., x_{iT}^{(b)}\right)'$ $(i = 1, 2, ..., n)$, compute SPMG estimate $\hat{\theta}^{(b)}$, $\hat{\Omega}_{\text{SPMG}}^{(b)}$ (based on (S.25)), and the associated test statistics

$$t_{\hat{\theta}}^{(b)} = \frac{\hat{\theta}^{(b)} - \hat{\theta}}{\widehat{s.e.}\left(\hat{\theta}^{(b)}\right)}, \tag{S.34}$$

   where $\widehat{s.e.}\left(\hat{\theta}^{(b)}\right) = \sqrt{\hat{\Omega}_{\text{SPMG}}^{(b)}}$.

4. Inference (at 5% nominal level) is conducted using the 95 percent quantile of $\left\{\left|t_{\hat{\theta}}^{(b)}\right|, b = 1, 2, ..., B\right\}$ as critical value.

S.2.4 Mean Group MW estimator

Our "MGMW" estimator is a mean group estimator that utilizes cross-section specific estimates by Müller and Watson (2018). We split the T time period into q subsamples of (approximately) equal size. In the paper we set $q = 5$ both in the Monte Carlo and empirical sections. Specifically, let m be the integer part of T/q, denoted as $m = floor\,(T/q)$, and let $r = (T/q - m)\,q$ be the remainder of the division of T by q. Let $\mathcal{H}_s$ be the index set of time periods belonging to the sub-period s, for $s = 1, 2, ..., q$, defined as

$$\mathcal{H}_s = \{H_{s-1} + 1, H_{s-1} + 2, ..., H_s\},$$

where H_s is the last period of the subsample s, and it is defined recursively as

$$\begin{aligned} H_0 &= 0, \\ H_s &= H_{s-1} + T_s, \text{ for } s = 1, 2, ..., q, \end{aligned}$$

in which T_s is the number of time periods in the subsample s, defined as

$$T_s = \begin{cases} m+1 & \text{for } s \leq r \\ m & \text{for } s > r \end{cases}, \; s = 1, 2, ..., q.$$

Note that $H_q = T$. Define the temporal aggregates:

$$\bar{y}_{i,s} = \frac{1}{T_s} \sum_{s \in \mathcal{H}_s} y_{it}, \text{ and similarly } \bar{x}_{i,s} = \frac{1}{T_s} \sum_{s \in \mathcal{H}_s} x_{it},$$

for $i = 1, 2, ..., n$, and $s = 1, 2, ..., q$. MGMW estimator is the average of cross-section specific Least Squares estimates of θ_i in the regression

$$\bar{y}_{is} = \mu_i + \theta_i \bar{x}_{is} + \epsilon_{is}, \tag{S.35}$$

for $i = 1, 2, ..., n$, and $s = 1, 2, ..., q$. Inference is conducted using the conventional nonparametric standard errors, as outlined in Chapter 28.5 of Pesaran (2015).

S.3 Monte Carlo evidence

S.3.1 Design

We generate $\Delta \mathbf{w}_{it} = (\Delta y_{it}, \Delta x_{it})'$ based on the following panel vector error correction model, for $i = 1, 2, ..., n$ and $t = 1, 2, ..., T$,

$$\Delta y_{it} = a_{yi} - \phi_{yit}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{yyi}\Delta y_{i,t-1} + \psi_{yxi}\Delta x_{i,t-1} + u_{yit}, \tag{S.36}$$

and

$$\Delta x_{it} = a_{xi} - \phi_{xit}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{xxi}\Delta x_{i,t-1} + \psi_{xyi}\Delta y_{i,t-1} + u_{xit}, \tag{S.37}$$

which can be equivalently written using a vector notation as

$$\Delta \mathbf{w}_{it} = \mathbf{a}_i - \boldsymbol{\phi}_{it}\boldsymbol{\beta}'\mathbf{w}_{i,t-1} + \boldsymbol{\Psi}_i \Delta \mathbf{w}_{i,t-1} + \mathbf{u}_{it}, \tag{S.38}$$

for $i = 1, 2, ..., n$ and $t = 1, 2, ..., T$ with initial values $\boldsymbol{\beta}'\mathbf{w}_{i0}$ and $\Delta \mathbf{w}_{i0}$, where $\mathbf{u}_{it} = (u_{yit}, u_{xit})' \sim (\mathbf{0}, \boldsymbol{\Sigma}_i)$, $\mathbf{a}_i = (a_{yi}, a_{xi})'$, $\boldsymbol{\phi}_{it} = \left(\phi_{yit}, \phi_{xit}\right)'$, $\boldsymbol{\beta} = (1, -\theta)'$, we set the long-run coefficient $\theta = 1$, and

$$\boldsymbol{\Psi}_i = \begin{pmatrix} \psi_{yyi} & \psi_{yxi} \\ \psi_{xyi} & \psi_{xxi} \end{pmatrix}.$$

We generate $\psi_{yyi}, \psi_{xxi} \sim IIDU\,(0, 0.4)$, and $\psi_{yxi}, \psi_{xyi} \sim IIDU\,(-0.1, 0.2)$, for $i = 1, 2, ..., n$.

Initial values: To generate the initial values $\boldsymbol{\beta}'\mathbf{w}_{i0}$ and $\Delta \mathbf{w}_{i0}$ we assume that initially up to date $t = 1$, $\boldsymbol{\phi}_{it} = \boldsymbol{\phi}_i$ for all i and all $t < 1$. In such a case $\boldsymbol{\beta}'\mathbf{w}_{it}$ and $\Delta \mathbf{w}_{it}$ are covariance stationary for $t < 1$, and we can use the the Granger representation theorem (Engle and Granger (1987), Johansen (1991), Hansen (2005)), to obtain a moving average representation for $\boldsymbol{\beta}'\mathbf{w}_{it}$ and $\Delta \mathbf{w}_{it}$. By Corollary 1 of Hansen (2005), we have the following representations

$$\boldsymbol{\beta}'\mathbf{w}_{it} = \boldsymbol{\beta}'\mathbf{C}_i^*\left(L\right)\left(\mathbf{u}_{it} + \mathbf{a}_i\right) = d_i + \boldsymbol{\beta}'\mathbf{C}_i^*\left(L\right)\mathbf{u}_{it},$$

and

$$\Delta \mathbf{w}_{it} = \mathbf{C}_i\left(L\right)\left(\mathbf{u}_{it} + \mathbf{a}_i\right) = \boldsymbol{\tau}\gamma_i + \mathbf{C}_i\left(L\right)\mathbf{u}_{it},$$

for $i = 1, 2, ..., n$ and $t < 1$, where $\boldsymbol{\tau} = (1, 1)'$, $\mathbf{C}_i\left(L\right) = \sum_{\ell=0}^{\infty}\mathbf{C}_{i\ell}$, $\mathbf{C}_i(L)$ can be partitioned as $\mathbf{C}_i(L) = \mathbf{C}_i\left(1\right) + (1-L)\mathbf{C}_i^*(L)$, $\mathbf{C}_i^*\left(L\right) = \sum_{\ell=0}^{\infty}\mathbf{C}_{i\ell}^* L^\ell$, matrices $\mathbf{C}_{i\ell}$ are defined recursively by (S.44)-(S.46), matrices

$\mathbf{C}_{i\ell}^{*}$ are defined by (S.47)-(S.48), $d_i = \boldsymbol{\beta}'\mathbf{C}_i^{*}(1)\,\mathbf{a}_i$, and $\mathbf{C}_i(1)\,\mathbf{a}_i = \boldsymbol{\tau}\gamma_i$.[1] We generate the initial values $\boldsymbol{\beta}'\mathbf{w}_{i0}$ and $\Delta\mathbf{w}_{i0}$ as

$$\boldsymbol{\beta}'\mathbf{w}_{i0} = d_i + \sum_{\ell=0}^{M} \boldsymbol{\beta}'\mathbf{C}_{i,-\ell}^{*}\mathbf{u}_{i,-\ell},$$

and

$$\Delta\mathbf{w}_{i0} = \tau\gamma_i + \sum_{\ell=0}^{M} \mathbf{C}_{i,-\ell}\mathbf{u}_{i,-\ell},$$

where we set $M = 50$. The slopes of the linear trends in levels, γ_i, and the means $d_i = E\left(\boldsymbol{\beta}'\mathbf{w}_{i0}\right)$ are generated as $\gamma_i, d_i \sim IIDN\left(0.02, 0.01^2\right)$. Given (d_i, γ_i) we recover the corresponding values for $\mathbf{a}_i$ in (S.38).[2] Using generated initial values, $\boldsymbol{\beta}'\mathbf{w}_{i0}$ and $\Delta\mathbf{w}_{i0}$, we then generate $\Delta\mathbf{w}_{it}$ for $t = 1, 2, ..., T$ and $i = 1, 2, ..., n$, using (S.38).

Three options are considered for the innovations $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ regarding the cross sectional dependence:

1. Cross sectionally independent errors. $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ is generated as

$$u_{yit} = \kappa u_{xit} + \sigma_{yi}\varepsilon_{yit},\ u_{xit} = \sigma_{xi}\varepsilon_{xit}, \tag{S.39}$$

where $\kappa = 0.5$, σ_{yi}^2,$\sigma_{xi}^2 \sim 0.1 + 0.1 \cdot IID\chi^2(2)$. We consider Gaussian and non-Gaussian cases to generate ε_{yit} and ε_{xit} (to illustrate robustness from departures from Gaussianity). In the Gaussian case, we generate $\varepsilon_{yit}, \varepsilon_{xit} \sim IIDN(0,1)$. In the non-Gaussian case, we generate

$$\varepsilon_{yit}, \varepsilon_{xit} \sim \frac{IIDN\chi^2(1) - 1}{\sqrt{2}}. \tag{S.40}$$

2. "SAR" errors. We generate $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ based on (S.39) with (as before) $\kappa = 0.5$, and σ_{yi}^2,$\sigma_{xi}^2 \sim 0.1 + 0.1 \cdot IID\chi^2(2)$, but we generate ε_{xit} and ε_{yit} based on the following spatial autoregressive model,

$$\varepsilon_{hit} = \delta \sum_{j=1}^{n} d_{ij}\varepsilon_{hjt} + v_{hit},\ \text{for } h = x, y, \tag{S.41}$$

where $\delta = 0.6$. Similarly to the cross sectionally independent case, we consider Gaussian and non-Gaussian cases to generate v_{yit} and v_{xit}. In the Gaussian case, $v_{yit}, v_{xit} \sim IIDN(0,1)$. In the non-Gaussian case, $v_{yit}, v_{xit} \sim 2^{-1/2}\left[IIDN\chi^2(1) - 1\right]$. d_{ij} are the elements of the $n \times n$ spatial weights matrix $\mathbf{D} = (d_{ij})$. We follow Kelejian and Prucha (2007) and assume units are set out on a rectangular grid at locations (s, r), for

[1] γ_i is the common slope of the linear trend of x_{it} and y_{it}. Model (S.38) features unrestricted intercepts $\mathbf{a}_i$, and therefore the level variables x_{it} and y_{it} can feature linear trends. The slope of the linear trend of x_{it} and y_{it} is common in this design because $\boldsymbol{\beta} = (1, -1)'$ and linear trends are not allowed in (S.38), which also implies the error-correcting term $\boldsymbol{\beta}'\mathbf{w}_{it} = y_{it} - x_{it}$ does not feature a linear trend.

[2] Specifically, $\mathbf{a}_i = \boldsymbol{\Upsilon}_i^{-1}(d_i, \gamma_i)'$, where $\boldsymbol{\Upsilon}_i = [\boldsymbol{\phi}_i, (\mathbf{I} - \boldsymbol{\Psi}_i)\,\boldsymbol{\tau}]$.

$r = 1, 2, ..., m_1$ and $s = 1, 2, ..., m_2$ such that $n = m_1 m_2$.[3] $\mathbf{W}$ is a rook type matrix, where two units are neighbors if their Euclidean distance is less than or equal to one. The weights matrix is normalized such that rows sum to one.

3. "Factor + SAR" errors. Errors are generated as

$$u_{yit} = \gamma_{yi} f_t + u^*_{xit}, \text{ and } u_{xit} = \gamma_{xi} f_t + u^*_{xit},$$

where $\gamma_{yi}, \gamma_{xi} \sim IIDN\left(1, 0.25^2\right)$, $f_t \sim IIDN\left(0, 1\right)$, and u^*_{yit} and u^*_{xit} are generated in the same way as the SAR errors outlined above.

We initially consider the direction of long-run causality from x to y ($x \to y$) by setting $\phi_{xit} = 0$ in (S.36)-(S.37). We generate ϕ_{yit} based on the two episodes,

$$\phi_{yit} = \begin{cases} \phi_{yi}, & \text{for } t \notin \mathcal{T}_i \\ 0, & \text{for } t \in \mathcal{T}_i \end{cases}, \tag{S.42}$$

where $\phi_{yi} \sim IIDU\left(0.1, 0.25\right)$, and $\mathcal{T}_i$ defines the sample index set for the episode with no convergence towards the long-run.

We also consider experiments with the two-way long-run causality, where ϕ_{yit} is generated according to (S.42), and ϕ_{xit} is generated similarly as

$$\phi_{xit} = \begin{cases} \phi_{xi}, & \text{for } t \notin \mathcal{T}_i \\ 0, & \text{for } t \in \mathcal{T}_i \end{cases}, \tag{S.43}$$

where $\phi_{xi} \sim IIDU\left(-0.15, -0.05\right)$.

The episode sample index $\mathcal{T}_i$ defines the non-equilibrating episodes. It is stochastically generated as follows. For each $i = 1, 2, ..., n$, we draw indicator s_i from the Bernoulli distribution with probability parameter π, hence $s_i = 1$ with probability π and $s_i = 0$ with probability $1 - \pi$. We consider three options for π:

1. $\pi = 0$ no non-equilibrating episodes (benchmark case),
2. $\pi = 0.05$ (moderate occurrence of non-equilibrating episodes), and
3. $\pi = 0.2$ (high occurrence of non-equilibrating episodes).

[3] We consider $(m_1, m_2) \in \{(6, 5), (10, 5), (10, 10), (20, 10), (20, 25)\}$, for $n = 30$, 50, 100, 200, and 300, respectively.

If $s_i = 0$ then we set $\mathcal{T}_i = \emptyset$. If $s_i = 1$, we generate the starting point and the duration of non-equilibrating episode randomly. We first draw the duration d_i from uniform distribution from $\{T_{\min}, T_{\min}+1, ..., T\}$, with $T_{\min} = 10$, ensuring non-equilibrating episode is at least 10 periods long (the average duration is $T/2 - 5$ periods, and the max duration is $T - 9$). Then we draw a starting period t_i^s from uniform distribution on $\{1, 2, ..., T - d_i\}$. We set $\mathcal{T}_i = \{t_i^s, t_i^s + 1, ..., t_i^s + d_i - 1\}$.

To obtain variable in levels ($\mathbf{w}_{it}$), we generate initial level values $x_{i,0} \sim N(1, 1)$ for $i = 1, 2, ..., n$. We then compute $y_{i,0}$ using $x_{i,0}$ and the initial value $\boldsymbol{\beta}'\mathbf{w}_{i0}$.[4] Given $\mathbf{w}_{i0}$, and $\Delta\mathbf{w}_{it}$ for $t = 1, 2, ..., T$, we compute $\mathbf{w}_{it} = \mathbf{w}_{i0} + \sum_{\ell=1}^{t} \Delta\mathbf{w}_{i\ell}$ for $t = 1, 2, ..., T$. Sample of n cross section units and T time periods, $\{\mathbf{w}_{it}, \, i = 1, 2, ..., n, \, t = 1, 2, ..., T\}$ is used for estimation of $\theta = 1$ and the corresponding inference. The following choices for sample size are considered: $T \in \{50, 100, 150, 200\}$, and $n \in \{30, 50, 100, 200\}$. $R_{MC} =$ 2 000 replications are conducted for each experiment.

S.3.1.1 Definition of $\mathbf{C}_{i\ell}$ and $\mathbf{C}_{i\ell}^*$

The coefficient matrices $\mathbf{C}_{i\ell}$ and $\mathbf{C}_{i\ell}^*$ are given recursively by

$$\mathbf{C}_{i0} = \mathbf{I}_2 \quad \text{(S.44)}$$

$$\mathbf{C}_{i1} = \mathbf{C}_{i0}\boldsymbol{\Phi}_{i1} - \mathbf{I}_2 \quad \text{(S.45)}$$

$$\mathbf{C}_{i\ell} = \mathbf{C}_{i,\ell-1}\boldsymbol{\Phi}_{i1} + \mathbf{C}_{i,\ell-2}\boldsymbol{\Phi}_{i2}, \, \ell = 2, 3, ... \quad \text{(S.46)}$$

and

$$\mathbf{C}_{i0}^* = \mathbf{I}_2 - \mathbf{C}_i(1) \quad \text{(S.47)}$$

$$\mathbf{C}_{ij}^* = \mathbf{C}_{i,j-1}^* + \mathbf{C}_{ij}, \text{ for } j = 1, 2, ..., \quad \text{(S.48)}$$

where $\mathbf{C}_i(1) = \sum_{\ell=0}^{\infty} \mathbf{C}_{i\ell}$,

$$\boldsymbol{\Phi}_{i1} = \mathbf{I}_2 - \boldsymbol{\phi}_i\boldsymbol{\beta}' + \boldsymbol{\Psi}_i, \quad \text{(S.49)}$$

$$\boldsymbol{\Phi}_{i2} = -\boldsymbol{\Psi}_i, \quad \text{(S.50)}$$

and $\boldsymbol{\phi}_i = \left(\phi_{yi}, \phi_{xi}\right)'$.

[4] Noting that $\boldsymbol{\beta}'\mathbf{w}_{i0} = y_{i,0} - x_{i0}$, it follows $y_{i,0} = \boldsymbol{\beta}'\mathbf{w}_{i0} + x_{i,0}$.

S.3.2 Summary of Monte Carlo experiments

We consider 36 experiments in total, based on the individual choices for:

1. distribution of errors (Gaussian and non-Gaussian),
2. direction of long run causality ($x \rightarrow y$, or $x \leftrightarrow y$),
3. the choice of the episodic probability parameter π ($0, 0.05$ or 0.2), and
4. the choice of cross-sectional dependence of errors (none, SAR, or factor+SAR).

The following summary table provides summary of all experiments:

Table S1: Summary of Monte Carlo experiments

Experiment No.	Results reported in	Error distribution	LR causality	π	CS dependence of errors
1	Table S2	Gaussian	$x \rightarrow y$	0	none
2	Table S3	Gaussian	$x \rightarrow y$	0	SAR
3	Table S4	Gaussian	$x \rightarrow y$	0	Factor + SAR
4	Table S5	Gaussian	$x \rightarrow y$	0.05	none
5	Table S6	Gaussian	$x \rightarrow y$	0.05	SAR
6	Table S7	Gaussian	$x \rightarrow y$	0.05	Factor + SAR
7	Table S8	Gaussian	$x \rightarrow y$	0.2	none
8	Table S9	Gaussian	$x \rightarrow y$	0.2	SAR
9	Table S10	Gaussian	$x \rightarrow y$	0.2	Factor + SAR
10	Table S11	Gaussian	$x \leftrightarrow y$	0	none
11	Table S12	Gaussian	$x \leftrightarrow y$	0	SAR
12	Table S13	Gaussian	$x \leftrightarrow y$	0	Factor + SAR
13	Table S14	Gaussian	$x \leftrightarrow y$	0.05	none
14	Table S15	Gaussian	$x \leftrightarrow y$	0.05	SAR
15	Table S16	Gaussian	$x \leftrightarrow y$	0.05	Factor + SAR
16	Table S17	Gaussian	$x \leftrightarrow y$	0.2	none
17	Table S18	Gaussian	$x \leftrightarrow y$	0.2	SAR
18	Table S19	Gaussian	$x \leftrightarrow y$	0.2	Factor + SAR
19	Table S20	Non-Gaussian	$x \rightarrow y$	0	none
20	Table S21	Non-Gaussian	$x \rightarrow y$	0	SAR
21	Table S22	Non-Gaussian	$x \rightarrow y$	0	Factor + SAR
22	Table S23	Non-Gaussian	$x \rightarrow y$	0.05	none
23	Table S24	Non-Gaussian	$x \rightarrow y$	0.05	SAR
24	Table S25	Non-Gaussian	$x \rightarrow y$	0.05	Factor + SAR
25	Table S26	Non-Gaussian	$x \rightarrow y$	0.2	none
26	Table S27	Non-Gaussian	$x \rightarrow y$	0.2	SAR
27	Table S28	Non-Gaussian	$x \rightarrow y$	0.2	Factor + SAR
28	Table S29	Non-Gaussian	$x \leftrightarrow y$	0	none
29	Table S30	Non-Gaussian	$x \leftrightarrow y$	0	SAR
30	Table S31	Non-Gaussian	$x \leftrightarrow y$	0	Factor + SAR
31	Table S32	Non-Gaussian	$x \leftrightarrow y$	0.05	none
32	Table S33	Non-Gaussian	$x \leftrightarrow y$	0.05	SAR
33	Table S34	Non-Gaussian	$x \leftrightarrow y$	0.05	Factor + SAR
34	Table S35	Non-Gaussian	$x \leftrightarrow y$	0.2	none
35	Table S36	Non-Gaussian	$x \leftrightarrow y$	0.2	SAR
36	Table S37	Non-Gaussian	$x \leftrightarrow y$	0.2	Factor + SAR

Notes: See Section S.3.1 for details of the Monte Carlo design.

S.3.3 Monte Carlo results

This section presents Monte Carlo results, as outlined in Section S.3.2.

Table S2: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.86	-0.19	-0.09	-0.05	4.22	1.64	1.03	0.74	67.7	85.1	88.2	90.3	89.0	94.3	94.0	94.9
50	-1.03	-0.26	-0.10	-0.05	3.21	1.32	0.79	0.57	69.9	83.6	88.8	89.5	89.0	92.7	94.7	94.2
100	-0.91	-0.22	-0.08	-0.05	2.26	0.89	0.55	0.39	68.2	85.0	88.8	91.5	89.9	93.5	94.5	94.8
200	-0.98	-0.21	-0.09	-0.05	1.76	0.64	0.38	0.28	62.0	84.0	89.4	91.2	85.8	93.3	94.9	95.0
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-2.25	-0.57	-0.26	-0.13	4.46	1.90	1.14	0.85	74.6	85.7	90.5	91.1	72.2	84.4	89.5	91.1
50	-2.34	-0.65	-0.28	-0.15	3.71	1.54	0.91	0.66	68.9	84.5	89.7	91.8	66.5	83.4	89.0	91.5
100	-2.30	-0.62	-0.28	-0.15	3.08	1.14	0.66	0.47	57.2	81.8	88.9	90.7	56.0	80.5	88.8	90.3
200	-2.31	-0.62	-0.27	-0.15	2.72	0.92	0.51	0.35	39.2	75.3	85.2	87.9	38.0	74.5	84.4	86.9
	SPMG estimator								Standard				Robust bootstrapped			
30	0.24	0.07	0.02	0.01	4.95	1.74	1.06	0.76	59.1	80.5	86.8	88.7	86.1	93.8	93.7	94.6
50	-0.01	-0.04	-0.01	0.00	3.59	1.38	0.80	0.59	59.9	79.6	87.0	87.8	86.2	92.1	94.2	94.2
100	0.10	0.01	0.01	0.00	2.46	0.93	0.57	0.40	61.2	81.0	86.0	89.6	86.0	93.5	93.9	93.9
200	0.00	0.01	0.01	0.00	1.71	0.65	0.39	0.28	61.3	81.4	87.2	88.6	87.5	93.1	94.7	95.0

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.65	-2.30	-1.52	-1.11	5.94	2.93	1.91	1.42	69.2	70.9	71.8	73.1
50	-4.73	-2.38	-1.54	-1.12	5.50	2.76	1.79	1.31	56.0	54.1	55.9	59.4
100	-4.65	-2.31	-1.52	-1.11	5.07	2.51	1.65	1.21	31.7	30.4	31.4	32.8
200	-4.63	-2.30	-1.50	-1.10	4.83	2.40	1.57	1.15	8.5	7.1	7.6	7.8
	PDOLS estimator, leads and lags order $p = 4$											
30	-2.47	-1.09	-0.71	-0.51	5.23	2.23	1.36	1.01	82.9	87.0	89.3	88.8
50	-2.49	-1.16	-0.74	-0.52	4.18	1.88	1.17	0.84	80.8	83.9	84.6	85.8
100	-2.50	-1.14	-0.73	-0.52	3.48	1.55	0.97	0.70	72.3	75.0	77.4	77.2
200	-2.48	-1.14	-0.72	-0.52	3.01	1.35	0.85	0.62	59.7	61.0	61.2	60.8
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.48	-0.46	-0.30	-0.20	8.14	2.41	1.32	0.95	71.1	89.6	92.1	92.2
50	-1.42	-0.52	-0.31	-0.21	5.95	1.83	1.05	0.73	75.2	89.5	90.8	92.4
100	-1.33	-0.49	-0.31	-0.21	4.35	1.34	0.77	0.54	75.1	88.6	90.0	90.9
200	-1.30	-0.52	-0.31	-0.22	3.24	1.02	0.59	0.42	76.1	85.6	87.8	88.2
	MGMW estimator, $q = 5$											
30	-5.64	-1.84	-0.87	-0.49	8.92	4.19	2.80	2.04	82.2	90.7	92.5	92.3
50	-5.64	-2.09	-0.99	-0.55	7.63	3.52	2.18	1.56	76.8	87.2	90.3	92.3
100	-5.59	-1.91	-0.96	-0.55	6.76	2.77	1.66	1.15	63.4	82.4	89.0	91.8
200	-5.55	-1.92	-0.93	-0.54	6.14	2.38	1.36	0.90	40.7	71.4	82.2	88.2

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0$, and no cross section dependence of errors. See Section S.3.1 for details of the data generating process. Description of the PMG, 2-step Breitung, SPMG, and MGMW estimators, and the description of bootstrapping procedures are provided in Sections S.2.1-S.2.3 of the online supplement. PDOLS is the panel dynamic OLS estimator by Mark and Sul (2003). The number of Monte Carlo replications is $R_{MC} = 2000$. Bootstrapped critical values are computed in each of the Monte Carlo replication as described in Sections S.2.1-S.2.3 of the online supplement, based on $R_b = 2000$ bootstrap replications.

Table S3: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.95	-0.23	-0.10	-0.07	4.90	2.12	1.34	0.98	62.7	75.7	79.4	80.1	89.2	93.8	94.4	94.2
50	-1.07	-0.24	-0.09	-0.06	3.77	1.62	1.00	0.77	61.8	74.4	79.5	79.1	88.5	92.3	94.4	93.5
100	-0.97	-0.21	-0.09	-0.06	2.71	1.10	0.69	0.50	60.1	75.7	79.9	82.3	87.2	93.1	94.8	95.7
200	-0.98	-0.20	-0.08	-0.05	2.01	0.80	0.50	0.36	56.0	73.6	79.3	81.7	86.8	93.0	94.1	94.4
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-2.36	-0.63	-0.29	-0.16	5.33	2.48	1.55	1.15	66.6	76.6	79.7	81.3	73.2	85.7	88.9	89.9
50	-2.50	-0.65	-0.28	-0.16	4.43	1.90	1.19	0.90	60.2	75.8	79.7	79.4	67.2	83.9	88.4	89.9
100	-2.39	-0.64	-0.30	-0.17	3.47	1.39	0.84	0.60	52.8	72.6	80.0	82.9	60.7	82.2	88.0	91.6
200	-2.32	-0.61	-0.28	-0.15	2.94	1.07	0.63	0.45	40.4	67.4	77.8	78.9	47.7	77.9	86.7	88.9
	SPMG estimator								Standard				Robust bootstrapped			
30	0.15	0.05	0.02	0.00	5.68	2.26	1.39	1.01	53.9	70.3	76.5	78.9	86.3	92.6	93.7	93.8
50	-0.03	0.00	0.00	-0.01	4.36	1.69	1.03	0.79	52.5	70.4	75.5	76.7	84.4	91.7	94.3	93.2
100	0.04	0.02	0.00	-0.01	2.97	1.14	0.70	0.51	53.0	71.9	77.9	80.3	85.8	93.0	94.2	95.2
200	0.05	0.04	0.02	0.01	2.03	0.81	0.51	0.37	54.0	72.3	76.8	79.1	86.5	92.2	93.7	93.6

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.73	-2.40	-1.62	-1.20	6.58	3.40	2.25	1.69	64.5	65.1	65.0	66.0
50	-4.92	-2.43	-1.59	-1.19	6.03	3.01	1.99	1.50	52.1	52.4	54.1	54.8
100	-4.76	-2.36	-1.58	-1.17	5.35	2.68	1.78	1.32	33.0	33.4	33.0	34.2
200	-4.67	-2.34	-1.54	-1.14	4.98	2.50	1.65	1.22	14.5	12.8	12.3	12.6
	PDOLS estimator, leads and lags order $p = 4$											
30	-2.46	-1.15	-0.76	-0.56	6.21	2.82	1.76	1.30	74.3	79.0	78.1	79.6
50	-2.68	-1.18	-0.76	-0.56	4.95	2.20	1.42	1.05	73.4	76.8	76.2	75.7
100	-2.55	-1.16	-0.76	-0.56	3.92	1.77	1.12	0.82	67.6	69.3	69.9	71.8
200	-2.47	-1.15	-0.74	-0.54	3.28	1.48	0.94	0.69	56.8	57.4	58.1	57.2
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.35	-0.49	-0.32	-0.24	9.37	3.10	1.77	1.27	65.6	81.3	82.5	83.3
50	-1.66	-0.49	-0.30	-0.22	7.05	2.27	1.35	0.98	68.8	81.8	82.0	81.2
100	-1.28	-0.48	-0.32	-0.23	5.11	1.64	0.96	0.67	66.8	80.7	81.8	83.7
200	-1.29	-0.53	-0.31	-0.22	3.70	1.23	0.72	0.51	68.4	78.0	79.6	81.4
	MGMW estimator, $q = 5$											
30	-5.76	-1.94	-0.96	-0.57	9.58	4.82	3.21	2.35	79.1	84.3	85.1	85.9
50	-5.78	-2.10	-1.00	-0.56	8.27	3.80	2.42	1.80	72.7	83.4	86.6	87.1
100	-5.73	-1.92	-0.98	-0.61	7.11	2.97	1.86	1.33	59.9	78.6	84.3	87.0
200	-5.58	-1.94	-0.97	-0.55	6.37	2.57	1.50	1.00	42.5	67.0	78.9	83.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0$, and SAR cross section dependence of errors. See notes to Table S1.

Table S4: MC findings for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias ($\times$100)				**RMSE ($\times$100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.32	-0.08	0.00	-0.04	3.69	1.69	1.05	0.79	53.8	63.8	68.7	67.8	89.8	92.6	94.5	93.9
50	-0.36	-0.07	-0.03	0.02	2.97	1.29	0.83	0.62	50.9	62.9	64.9	67.4	89.8	94.3	94.1	94.2
100	-0.34	-0.02	-0.01	-0.01	2.14	0.99	0.61	0.44	48.3	58.6	63.9	65.2	88.8	93.7	95.0	94.7
200	-0.22	-0.04	-0.03	-0.01	1.59	0.74	0.45	0.34	47.5	54.6	60.0	60.5	90.4	94.4	95.6	95.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-1.17	-0.32	-0.10	-0.09	4.16	1.95	1.22	0.92	58.5	66.2	69.5	69.3	79.0	87.5	91.6	92.1
50	-1.16	-0.33	-0.16	-0.05	3.24	1.52	1.00	0.72	55.6	64.5	64.9	67.8	79.4	90.3	91.8	93.6
100	-1.21	-0.26	-0.13	-0.08	2.60	1.14	0.74	0.53	49.0	60.0	62.4	65.7	79.0	91.3	94.2	94.6
200	-1.04	-0.28	-0.14	-0.07	2.06	0.92	0.56	0.41	45.0	54.3	60.4	60.5	84.1	93.6	95.9	96.0
	SPMG estimator								Standard				Robust bootstrapped			
30	0.05	0.02	0.06	0.00	5.17	1.84	1.10	0.81	44.4	58.8	64.0	65.2	83.9	90.8	93.4	93.7
50	-0.07	0.02	0.01	0.04	4.30	1.38	0.88	0.65	43.8	58.8	61.5	64.6	84.4	92.6	92.8	93.4
100	-0.11	0.08	0.03	0.01	4.42	1.08	0.63	0.46	40.9	53.2	60.0	62.8	83.3	91.3	94.1	94.1
200	0.16	0.05	0.01	0.01	2.34	0.84	0.48	0.34	40.6	51.1	58.6	60.4	83.8	91.6	94.3	94.6

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias ($\times$100)				**RMSE ($\times$100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-2.53	-1.41	-0.96	-0.73	4.55	2.43	1.63	1.22	65.2	64.3	64.4	64.5
50	-2.70	-1.43	-1.00	-0.73	3.94	2.10	1.47	1.09	56.7	56.1	55.8	56.3
100	-2.67	-1.38	-0.99	-0.74	3.45	1.80	1.28	0.96	44.3	45.1	42.0	40.5
200	-2.51	-1.40	-0.94	-0.71	3.04	1.69	1.12	0.86	29.4	25.3	23.9	25.0
	PDOLS estimator, leads and lags order $p = 4$											
30	-1.74	-0.88	-0.55	-0.42	4.98	2.29	1.38	1.03	66.9	68.4	69.8	69.6
50	-1.85	-0.86	-0.59	-0.41	4.11	1.79	1.18	0.85	63.2	66.1	64.8	65.5
100	-1.79	-0.81	-0.57	-0.42	3.31	1.42	0.95	0.70	56.6	59.7	59.5	58.4
200	-1.61	-0.85	-0.56	-0.41	2.69	1.25	0.79	0.58	49.1	47.6	46.7	46.2
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.22	-0.51	-0.28	-0.23	8.30	2.64	1.43	1.06	53.9	66.7	72.3	69.7
50	-1.23	-0.47	-0.31	-0.19	6.63	1.97	1.17	0.81	50.8	67.3	67.3	70.2
100	-1.07	-0.43	-0.31	-0.22	5.08	1.52	0.90	0.63	47.6	62.4	65.6	64.6
200	-0.89	-0.47	-0.31	-0.21	4.30	1.26	0.71	0.50	42.4	55.6	58.7	59.5
	MGMW estimator, $q = 5$											
30	-0.82	-0.40	-0.15	-0.13	5.38	3.06	2.01	1.56	80.1	79.1	78.9	78.3
50	-0.99	-0.51	-0.26	-0.13	4.34	2.32	1.60	1.21	79.0	80.9	78.3	79.1
100	-0.96	-0.33	-0.22	-0.12	3.32	1.66	1.17	0.87	76.5	79.1	79.8	78.0
200	-0.84	-0.39	-0.24	-0.15	2.58	1.28	0.85	0.62	73.1	77.3	78.0	78.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0$, and factor+SAR cross section dependence of errors. See notes to Table S1.

Table S5: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.95	-0.20	-0.10	-0.06	4.36	1.71	1.06	0.75	67.7	84.6	87.6	90.2	88.7	94.2	94.0	94.5
50	-1.11	-0.27	-0.10	-0.05	3.34	1.36	0.81	0.59	68.6	83.3	88.9	89.6	89.4	92.6	93.9	94.9
100	-0.96	-0.22	-0.08	-0.05	2.33	0.91	0.57	0.40	67.1	84.8	88.1	91.2	89.3	93.5	94.1	94.5
200	-1.06	-0.22	-0.09	-0.05	1.83	0.66	0.39	0.29	61.8	84.1	89.2	91.2	85.1	93.6	95.1	94.8
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-3.54	-1.27	-0.86	-0.48	9.29	3.92	4.57	3.11	66.4	78.6	80.5	80.3	64.6	77.1	79.8	79.2
50	-3.45	-1.34	-0.80	-0.65	5.27	3.17	2.81	2.68	60.3	74.5	76.8	76.9	58.2	73.8	76.4	77.1
100	-3.44	-1.34	-0.80	-0.57	4.53	2.75	1.97	2.01	45.2	66.6	72.6	72.9	44.2	65.6	72.1	72.4
200	-3.48	-1.25	-0.77	-0.58	4.02	2.03	1.96	1.37	26.8	58.6	66.5	68.1	25.5	57.4	65.7	68.0
	SPMG estimator								Standard				Robust bootstrapped			
30	0.23	0.08	0.02	0.01	5.10	1.81	1.09	0.77	59.1	80.1	86.4	88.3	86.0	93.2	93.9	94.3
50	0.00	-0.04	0.00	0.00	3.72	1.43	0.83	0.60	60.4	78.9	87.4	88.1	86.0	92.0	94.0	94.3
100	0.13	0.02	0.02	0.01	2.53	0.95	0.59	0.41	60.1	81.2	86.0	89.6	86.2	93.3	93.4	93.7
200	-0.01	0.02	0.01	0.00	1.76	0.67	0.40	0.29	61.9	81.0	87.3	88.6	87.0	92.5	94.2	94.9

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-6.01	-3.26	-2.29	-1.79	7.67	4.45	3.30	2.76	67.7	74.0	77.7	80.3
50	-6.01	-3.30	-2.34	-1.89	7.02	4.02	3.05	2.64	57.1	63.3	67.8	73.2
100	-5.98	-3.28	-2.33	-1.82	6.52	3.67	2.69	2.20	33.9	41.8	49.7	56.5
200	-5.97	-3.23	-2.29	-1.81	6.23	3.44	2.48	2.02	16.9	21.1	28.1	36.9
	PDOLS estimator, leads and lags order $p = 4$											
30	-4.03	-2.15	-1.54	-1.23	7.13	3.99	2.96	2.52	80.4	88.5	89.1	90.0
50	-3.95	-2.17	-1.60	-1.34	5.89	3.30	2.62	2.37	78.5	84.7	85.6	87.7
100	-4.01	-2.20	-1.59	-1.28	5.08	2.86	2.16	1.82	67.6	74.3	76.7	80.6
200	-4.02	-2.16	-1.56	-1.27	4.58	2.52	1.86	1.58	49.9	60.4	65.1	71.0
	PDOLS estimator, leads and lags order $p = 8$											
30	-3.19	-1.62	-1.19	-0.95	9.92	4.31	3.05	2.54	69.4	90.3	91.6	92.6
50	-2.99	-1.58	-1.22	-1.07	7.39	3.27	2.60	2.38	73.3	88.7	90.2	92.0
100	-3.01	-1.65	-1.23	-1.00	5.75	2.71	2.03	1.72	68.9	84.7	86.9	89.8
200	-2.95	-1.62	-1.20	-1.01	4.57	2.21	1.64	1.42	65.5	79.2	83.0	85.0
	MGMW estimator, $q = 5$											
30	-6.90	-2.87	-1.68	-1.15	10.58	5.95	4.55	4.04	81.0	89.8	92.0	93.5
50	-6.82	-3.00	-1.86	-1.45	9.00	4.95	3.75	3.11	74.7	85.9	90.6	92.9
100	-6.90	-2.96	-1.80	-1.29	8.16	4.13	2.91	2.39	60.0	77.7	86.5	89.4
200	-6.86	-2.93	-1.76	-1.28	7.49	3.50	2.41	1.93	34.2	63.6	76.4	83.1

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and no CS dependence of errors. See notes to Table S1.

Table S6: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-1.04	-0.24	-0.11	-0.07	5.04	2.16	1.36	1.00	63.0	75.1	79.3	80.3	88.8	93.9	94.4	94.1
50	-1.14	-0.26	-0.10	-0.06	3.88	1.65	1.02	0.78	62.1	74.3	79.3	79.5	88.3	92.6	94.5	93.6
100	-1.03	-0.22	-0.09	-0.06	2.78	1.13	0.70	0.51	59.3	75.6	80.6	82.0	87.5	93.1	95.1	95.5
200	-1.05	-0.21	-0.08	-0.04	2.08	0.82	0.51	0.37	54.9	73.9	79.1	80.9	86.5	92.8	95.1	94.7
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-3.42	-1.28	-0.78	-0.51	6.95	4.41	3.66	3.65	60.4	71.4	73.0	74.7	66.8	78.6	81.7	81.3
50	-3.65	-1.36	-0.86	-0.77	5.83	3.31	3.14	3.45	53.9	67.6	72.1	70.9	60.2	75.2	79.0	78.0
100	-3.63	-1.33	-0.83	-0.63	5.06	2.76	2.10	2.22	43.3	61.9	66.9	70.1	49.9	69.8	73.7	76.1
200	-3.46	-1.32	-0.82	-0.58	4.27	2.09	1.91	1.92	29.5	56.2	63.6	65.7	34.2	61.9	69.3	72.2
	SPMG estimator								Standard				Robust bootstrapped			
30	0.13	0.07	0.02	0.00	5.81	2.31	1.42	1.03	54.9	70.4	76.3	78.5	85.7	93.0	93.3	94.2
50	0.00	-0.01	0.00	-0.01	4.48	1.73	1.05	0.81	53.7	70.1	76.1	77.1	84.4	92.0	93.7	93.2
100	0.06	0.03	0.01	0.00	3.02	1.17	0.72	0.52	53.4	71.0	78.4	80.5	85.5	93.0	94.4	95.5
200	0.04	0.05	0.03	0.02	2.08	0.83	0.52	0.38	53.5	72.0	76.9	79.2	86.5	91.8	94.0	93.8

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-6.08	-3.36	-2.41	-1.92	8.16	4.81	3.57	3.02	64.5	69.8	72.5	75.4
50	-6.24	-3.40	-2.43	-2.00	7.52	4.27	3.24	2.82	53.5	60.4	65.2	68.2
100	-6.11	-3.34	-2.42	-1.94	6.80	3.83	2.85	2.36	34.2	41.5	47.0	55.1
200	-6.02	-3.29	-2.35	-1.88	6.38	3.54	2.57	2.11	18.4	23.5	30.1	37.5
	PDOLS estimator, leads and lags order $p = 4$											
30	-4.00	-2.20	-1.61	-1.32	7.87	4.40	3.23	2.77	74.3	82.1	82.2	85.1
50	-4.19	-2.23	-1.66	-1.41	6.60	3.57	2.83	2.55	72.0	78.5	80.0	82.5
100	-4.10	-2.23	-1.66	-1.37	5.47	3.03	2.30	1.97	64.7	70.5	72.8	77.7
200	-4.01	-2.19	-1.60	-1.32	4.78	2.63	1.93	1.65	50.2	58.5	63.8	67.7
	PDOLS estimator, leads and lags order $p = 8$											
30	-3.07	-1.63	-1.23	-1.02	11.02	4.80	3.35	2.80	66.2	82.9	85.5	87.5
50	-3.36	-1.61	-1.26	-1.12	8.67	3.57	2.82	2.57	67.1	83.4	86.0	86.7
100	-2.99	-1.65	-1.28	-1.08	6.38	2.90	2.17	1.87	64.6	80.6	82.9	86.3
200	-2.94	-1.65	-1.23	-1.04	4.90	2.37	1.72	1.49	62.9	75.8	78.2	82.5
	MGMW estimator, $q = 5$											
30	-6.95	-2.89	-1.79	-1.36	11.11	6.55	4.82	4.28	77.8	85.6	87.1	88.8
50	-7.00	-3.11	-1.85	-1.43	9.66	5.32	4.03	3.38	71.0	81.9	87.8	89.5
100	-7.11	-2.99	-1.90	-1.45	8.53	4.29	3.11	2.59	57.6	76.8	82.7	86.0
200	-6.88	-2.97	-1.80	-1.35	7.69	3.69	2.47	2.02	38.0	62.9	75.4	80.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and SAR CS dependence of errors. See notes to Table S1.

Table S7: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				RMSE (×100)				95% CI coverage rate (x100)							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.36	-0.08	0.00	-0.04	3.77	1.71	1.07	0.80	53.6	64.3	69.2	68.4	89.7	92.9	94.2	93.9
50	-0.38	-0.07	-0.03	0.02	3.03	1.31	0.84	0.63	51.7	63.6	65.4	68.3	89.6	94.4	94.2	94.2
100	-0.37	-0.02	-0.01	-0.01	2.19	1.00	0.62	0.45	48.7	58.8	64.6	66.2	89.2	94.0	95.1	94.7
200	-0.25	-0.05	-0.03	-0.01	1.63	0.75	0.46	0.34	47.8	55.1	61.1	60.8	90.7	94.7	95.9	95.5
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-1.68	-0.63	-0.39	-0.30	5.07	3.60	2.69	2.25	57.3	63.8	67.1	64.6	74.8	82.5	84.9	82.6
50	-1.71	-0.79	-0.56	-0.38	4.04	2.67	2.14	1.96	53.0	62.5	63.5	63.9	75.5	82.1	83.2	82.6
100	-1.77	-0.65	-0.44	-0.37	3.33	2.05	1.57	1.53	45.4	58.2	60.6	63.7	71.7	84.6	84.1	82.9
200	-1.59	-0.65	-0.40	-0.34	2.68	1.48	1.15	1.13	40.3	51.6	58.7	61.0	73.8	84.3	86.0	84.3
	SPMG estimator								Standard				Robust bootstrapped			
30	0.00	0.03	0.06	0.00	5.34	1.86	1.12	0.82	45.3	59.3	65.6	66.2	83.9	90.9	93.0	93.4
50	-0.10	0.02	0.01	0.04	4.67	1.41	0.89	0.66	44.0	59.1	61.4	64.8	84.3	92.4	93.2	93.7
100	-0.14	0.09	0.03	0.01	5.01	1.09	0.64	0.46	41.0	54.4	60.8	63.5	83.7	91.5	94.1	93.8
200	0.14	0.05	0.02	0.01	2.47	0.86	0.48	0.35	41.2	51.6	58.9	61.2	83.5	91.6	94.5	94.5

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				RMSE (×100)				95% CI cov.r. (x100)			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-3.09	-1.79	-1.36	-1.07	5.31	3.10	2.35	1.96	68.6	71.8	73.0	75.5
50	-3.32	-1.86	-1.42	-1.12	4.64	2.73	2.17	1.75	62.0	67.3	67.2	72.7
100	-3.24	-1.82	-1.40	-1.11	4.08	2.33	1.85	1.50	50.3	56.7	59.0	65.4
200	-3.12	-1.85	-1.32	-1.07	3.69	2.21	1.57	1.31	36.9	40.1	43.3	51.4
	PDOLS estimator, leads and lags order $p = 4$											
30	-2.39	-1.30	-0.99	-0.78	5.78	3.09	2.22	1.86	70.0	75.6	78.9	79.3
50	-2.60	-1.34	-1.05	-0.83	4.89	2.53	1.99	1.60	66.0	75.9	76.2	80.9
100	-2.48	-1.31	-1.03	-0.83	3.97	2.00	1.58	1.30	60.6	71.7	73.3	76.8
200	-2.35	-1.36	-0.97	-0.79	3.38	1.83	1.28	1.07	53.5	61.6	65.4	71.2
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.97	-0.97	-0.77	-0.62	8.96	3.55	2.37	1.93	55.0	73.8	80.3	79.2
50	-2.07	-1.02	-0.81	-0.64	7.27	2.75	2.03	1.62	52.3	75.8	79.6	83.7
100	-1.85	-0.99	-0.81	-0.65	5.58	2.08	1.56	1.26	50.7	75.1	79.0	83.4
200	-1.70	-1.05	-0.76	-0.62	4.71	1.82	1.19	0.99	43.8	69.2	73.7	79.0
	MGMW estimator, $q = 5$											
30	-0.94	-0.44	-0.30	-0.24	5.90	3.84	2.79	2.39	82.4	83.1	84.4	85.2
50	-1.19	-0.61	-0.45	-0.26	4.88	2.98	2.30	1.93	81.6	84.4	84.2	86.9
100	-1.10	-0.48	-0.36	-0.25	3.70	2.10	1.62	1.53	79.7	84.0	86.2	87.0
200	-1.00	-0.52	-0.34	-0.24	2.93	1.67	1.20	1.03	76.8	81.3	84.9	86.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S8: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.2$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				RMSE (×100)				95% CI coverage rate (x100)							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-1.28	-0.25	-0.12	-0.07	4.92	1.86	1.16	0.83	67.2	84.9	88.1	90.0	88.0	94.3	94.5	94.1
50	-1.38	-0.31	-0.12	-0.07	3.76	1.48	0.89	0.64	66.0	82.6	88.2	90.1	88.0	92.4	93.7	94.2
100	-1.24	-0.28	-0.10	-0.05	2.64	1.01	0.62	0.43	63.9	83.8	88.2	91.0	88.4	93.3	94.3	95.0
200	-1.29	-0.26	-0.10	-0.06	2.08	0.73	0.44	0.32	57.7	82.9	88.1	89.9	84.5	93.4	94.7	94.5
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-6.92	-3.03	-2.26	-1.52	13.29	8.25	7.10	6.43	50.9	66.8	69.2	70.7	49.2	66.5	68.4	70.8
50	-7.05	-3.30	-2.35	-1.94	10.15	6.46	5.69	5.65	40.9	60.2	63.4	66.1	39.5	59.3	62.7	65.6
100	-7.00	-3.33	-2.09	-1.90	8.61	5.73	4.79	4.81	26.2	51.9	60.9	63.9	25.9	51.2	60.4	63.5
200	-7.12	-3.22	-2.22	-1.96	8.06	4.39	3.84	7.77	12.3	38.8	50.0	57.4	11.8	38.8	50.0	57.4
	SPMG estimator								Standard				Robust bootstrapped			
30	0.27	0.11	0.04	0.02	5.80	1.99	1.20	0.85	57.5	80.7	86.5	87.8	85.3	93.6	93.9	94.5
50	0.05	0.00	0.01	0.00	4.18	1.55	0.92	0.66	58.8	79.8	86.7	89.0	85.2	92.0	93.4	93.9
100	0.13	0.03	0.03	0.02	2.80	1.05	0.64	0.44	58.6	81.0	87.0	89.6	86.7	92.9	93.5	95.0
200	0.03	0.05	0.03	0.02	1.91	0.73	0.45	0.32	60.1	80.4	86.3	88.6	88.4	92.7	93.9	94.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				RMSE (×100)				95% CI cov.r. (x100)			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-9.92	-5.89	-4.53	-3.79	12.06	7.61	6.18	5.45	64.3	77.1	81.5	85.6
50	-10.19	-6.17	-4.84	-4.14	11.49	7.27	5.94	5.30	53.8	66.3	72.7	75.7
100	-10.10	-6.14	-4.69	-3.93	10.84	6.77	5.28	4.51	35.4	46.8	54.4	61.6
200	-10.10	-6.10	-4.67	-3.95	10.48	6.43	4.97	4.27	24.8	26.3	32.0	39.6
	PDOLS estimator, leads and lags order $p = 4$											
30	-8.45	-5.03	-3.93	-3.33	11.88	7.40	6.02	5.33	72.5	85.3	86.3	88.4
50	-8.70	-5.29	-4.27	-3.71	10.85	6.80	5.66	5.11	67.6	80.3	80.1	82.0
100	-8.71	-5.34	-4.12	-3.50	9.92	6.21	4.88	4.22	52.9	66.4	68.9	73.0
200	-8.72	-5.31	-4.11	-3.52	9.35	5.77	4.50	3.92	35.7	49.9	52.0	57.8
	PDOLS estimator, leads and lags order $p = 8$											
30	-7.91	-4.74	-3.72	-3.16	14.59	7.94	6.29	5.49	63.7	85.2	88.4	89.8
50	-8.26	-4.93	-4.06	-3.55	12.51	6.96	5.77	5.20	61.2	82.1	83.7	85.9
100	-8.18	-5.05	-3.92	-3.34	10.64	6.26	4.87	4.20	50.5	71.3	76.9	79.6
200	-8.15	-5.02	-3.91	-3.36	9.51	5.65	4.41	3.85	37.7	57.9	64.7	66.8
	MGMW estimator, $q = 5$											
30	-10.59	-5.62	-4.21	-3.38	15.01	9.85	8.12	7.58	76.3	85.9	89.5	91.4
50	-10.73	-5.97	-4.44	-3.92	13.27	8.54	7.06	6.30	66.4	79.4	83.8	87.2
100	-10.76	-5.81	-4.20	-3.48	12.19	7.35	5.76	5.00	48.3	68.0	75.7	79.1
200	-10.74	-5.89	-4.28	-3.52	11.47	6.65	5.06	4.33	21.9	45.8	58.7	66.5

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.2$, and no CS dependence of errors. See notes to Table S1.

Table S9: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.2$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-1.34	-0.30	-0.13	-0.09	5.48	2.32	1.46	1.07	61.9	76.2	78.9	81.0	89.3	94.5	94.6	94.5
50	-1.41	-0.29	-0.12	-0.08	4.24	1.78	1.10	0.84	62.2	75.1	80.9	79.8	88.7	92.7	94.2	93.8
100	-1.32	-0.27	-0.11	-0.06	3.06	1.22	0.75	0.54	58.5	76.2	81.4	83.5	87.4	93.7	95.0	95.3
200	-1.29	-0.25	-0.10	-0.05	2.32	0.88	0.55	0.40	53.7	74.4	78.9	82.7	85.9	93.0	94.7	93.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-6.85	-3.18	-2.35	-1.77	11.67	9.88	7.15	8.68	48.7	61.9	66.0	68.0	53.5	67.0	71.1	71.8
50	-7.21	-3.45	-2.61	-2.15	10.42	6.44	6.43	6.27	40.2	57.9	62.8	63.0	44.4	63.1	66.5	67.2
100	-7.22	-3.33	-2.33	-1.94	9.18	5.48	4.63	4.42	26.7	51.2	58.5	60.3	29.7	54.4	62.4	63.8
200	-7.00	-3.36	-2.35	-1.87	8.28	4.53	3.86	3.44	14.3	38.6	49.8	54.5	16.3	42.1	53.5	57.3
	SPMG estimator								Standard				Robust bootstrapped			
30	0.21	0.10	0.04	0.01	6.37	2.48	1.52	1.10	52.9	70.8	76.9	79.3	85.8	93.3	93.6	94.3
50	0.06	0.05	0.02	0.00	4.85	1.86	1.14	0.86	53.4	71.8	78.1	77.8	85.4	92.4	93.6	93.2
100	0.03	0.04	0.02	0.01	3.22	1.25	0.77	0.56	53.3	72.8	79.7	81.3	86.4	93.3	94.6	95.2
200	0.07	0.07	0.04	0.03	2.23	0.90	0.56	0.41	53.7	71.2	76.5	79.4	85.9	92.0	93.5	93.8

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-10.13	-6.12	-4.80	-4.10	12.72	8.01	6.60	5.85	63.2	73.2	78.0	81.8
50	-10.51	-6.39	-5.07	-4.37	12.07	7.66	6.35	5.69	51.6	61.9	68.5	74.1
100	-10.33	-6.29	-4.91	-4.19	11.19	6.99	5.55	4.82	35.6	45.5	52.5	57.9
200	-10.13	-6.22	-4.83	-4.14	10.57	6.57	5.17	4.48	25.5	26.6	33.0	38.5
	PDOLS estimator, leads and lags order $p = 4$											
30	-8.57	-5.22	-4.17	-3.61	12.67	7.81	6.45	5.72	70.3	82.2	83.9	86.4
50	-9.06	-5.50	-4.48	-3.91	11.58	7.22	6.10	5.51	63.8	75.2	76.9	80.1
100	-8.91	-5.46	-4.32	-3.73	10.31	6.43	5.14	4.52	51.1	63.7	66.8	69.6
200	-8.70	-5.41	-4.26	-3.69	9.44	5.91	4.69	4.13	37.4	48.6	50.2	54.4
	PDOLS estimator, leads and lags order $p = 8$											
30	-8.07	-4.89	-3.95	-3.43	15.66	8.41	6.76	5.90	61.3	81.6	85.2	87.8
50	-8.71	-5.12	-4.26	-3.73	13.73	7.43	6.23	5.61	58.6	79.0	82.4	83.1
100	-8.24	-5.15	-4.11	-3.57	11.07	6.48	5.14	4.51	50.0	69.6	74.7	76.5
200	-8.16	-5.13	-4.06	-3.53	9.68	5.82	4.61	4.06	37.9	57.2	62.3	64.7
	MGMW estimator, $q = 5$											
30	-10.90	-6.05	-4.47	-3.98	15.81	10.70	8.56	8.14	72.6	83.0	87.3	88.6
50	-11.08	-6.32	-4.62	-3.96	14.11	9.21	7.65	6.73	63.3	76.9	81.2	84.0
100	-10.96	-5.88	-4.40	-3.75	12.59	7.55	6.01	5.35	46.3	65.7	73.5	77.8
200	-10.82	-5.99	-4.40	-3.71	11.69	6.82	5.25	4.61	24.0	46.9	57.8	63.2

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.2$, and SAR CS dependence of errors. See notes to Table S1.

Table S10: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.2$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.47	-0.09	0.00	-0.04	4.13	1.80	1.12	0.84	54.9	65.9	71.4	70.2	90.1	93.4	94.8	94.2
50	-0.45	-0.09	-0.03	0.01	3.20	1.37	0.89	0.66	52.7	65.7	68.0	69.6	89.8	94.5	94.3	94.0
100	-0.46	-0.04	-0.01	-0.01	2.33	1.05	0.64	0.47	48.2	61.3	66.2	68.7	89.5	94.3	94.9	95.0
200	-0.32	-0.06	-0.03	-0.01	1.75	0.79	0.48	0.36	48.7	56.8	63.6	62.3	90.1	94.6	95.9	95.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-3.23	-1.65	-1.16	-1.20	7.84	6.85	5.40	5.93	50.2	59.4	61.7	61.4	65.0	74.0	73.9	73.3
50	-3.28	-1.78	-1.53	-1.19	8.53	4.72	4.29	3.81	45.3	56.3	59.8	61.8	62.5	71.2	72.0	72.8
100	-3.50	-1.64	-1.30	-1.13	5.55	3.58	3.45	2.97	37.6	54.3	57.1	59.5	54.0	71.1	71.3	72.0
200	-3.25	-1.68	-1.25	-1.16	4.68	2.91	2.75	2.44	31.2	46.7	52.4	54.5	50.4	67.2	72.3	70.5
	SPMG estimator								Standard				Robust bootstrapped			
30	0.10	0.05	0.06	0.00	6.79	1.97	1.18	0.86	44.2	59.8	67.1	67.8	84.4	91.6	94.3	93.6
50	-0.20	0.04	0.02	0.03	7.15	1.48	0.94	0.69	44.5	60.9	64.2	65.8	84.2	92.5	93.5	93.2
100	-0.15	0.09	0.04	0.02	5.59	1.15	0.67	0.48	41.6	56.0	62.4	65.3	84.1	92.1	94.8	94.2
200	0.14	0.06	0.02	0.02	3.37	0.92	0.50	0.36	40.7	53.6	62.1	61.5	84.2	91.9	94.0	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.71	-3.05	-2.46	-2.07	7.36	4.79	4.04	3.47	72.7	81.5	82.0	85.6
50	-5.11	-3.16	-2.60	-2.22	6.69	4.33	3.72	3.20	68.3	75.3	77.0	82.5
100	-5.04	-3.15	-2.60	-2.16	6.06	3.88	3.26	2.76	56.9	63.8	66.9	72.1
200	-4.83	-3.16	-2.44	-2.15	5.52	3.63	2.85	2.55	45.2	48.2	52.2	57.8
	PDOLS estimator, leads and lags order $p = 4$											
30	-4.33	-2.72	-2.19	-1.85	8.12	4.95	4.08	3.47	73.4	84.0	85.7	88.6
50	-4.74	-2.80	-2.34	-2.00	7.22	4.31	3.67	3.14	69.7	82.9	82.9	86.8
100	-4.64	-2.81	-2.34	-1.95	6.22	3.73	3.12	2.63	62.3	74.7	76.3	79.9
200	-4.37	-2.85	-2.19	-1.94	5.47	3.44	2.67	2.39	54.7	65.4	66.9	70.1
	PDOLS estimator, leads and lags order $p = 8$											
30	-4.13	-2.59	-2.09	-1.77	11.15	5.54	4.39	3.65	56.0	82.1	86.7	88.6
50	-4.45	-2.65	-2.23	-1.90	9.49	4.71	3.84	3.25	54.5	82.2	83.6	88.7
100	-4.26	-2.67	-2.25	-1.85	7.62	3.92	3.23	2.65	50.3	77.8	80.6	84.1
200	-3.92	-2.75	-2.09	-1.86	6.53	3.59	2.68	2.38	42.7	69.1	74.3	76.5
	MGMW estimator, $q = 5$											
30	-1.25	-0.83	-0.69	-0.61	7.62	5.62	4.78	4.31	85.0	87.4	87.4	91.2
50	-1.69	-0.92	-0.89	-0.58	6.40	4.49	3.83	3.44	83.5	87.3	88.1	90.5
100	-1.63	-0.88	-0.69	-0.52	4.99	3.29	2.81	2.61	80.3	86.2	88.3	89.2
200	-1.43	-0.90	-0.65	-0.55	4.12	2.74	2.17	2.02	76.6	81.6	84.9	85.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S11: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-5.99	-2.81	-1.77	-1.27	7.00	3.22	2.04	1.45	27.4	33.7	38.4	41.7	52.7	49.8	51.8	50.9
50	-5.84	-2.65	-1.70	-1.22	6.50	2.92	1.86	1.34	14.6	19.9	21.5	23.7	34.0	32.8	31.8	31.7
100	-5.68	-2.67	-1.69	-1.21	6.03	2.79	1.77	1.27	2.5	2.9	3.6	4.9	10.0	7.1	6.4	7.8
200	-5.59	-2.63	-1.66	-1.19	5.75	2.70	1.70	1.22	0.0	0.0	0.0	0.1	0.8	0.3	0.1	0.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-2.76	-0.81	-0.36	-0.21	4.15	1.65	1.00	0.70	67.2	85.2	88.5	91.4	65.6	83.9	88.4	90.9
50	-2.61	-0.68	-0.30	-0.17	3.50	1.29	0.76	0.54	62.6	84.4	89.7	91.8	59.9	83.3	89.1	91.2
100	-2.59	-0.73	-0.33	-0.18	3.08	1.06	0.59	0.40	44.7	75.8	86.5	90.0	43.3	74.5	85.5	89.5
200	-2.59	-0.72	-0.31	-0.17	2.83	0.91	0.47	0.31	22.3	63.8	80.5	86.5	21.0	61.6	79.6	85.6
	SPMG estimator								Standard				Robust bootstrapped			
30	-0.03	-0.04	-0.01	-0.01	3.62	1.43	0.89	0.62	66.9	83.6	88.2	90.1	88.1	92.6	93.8	94.7
50	0.00	0.04	0.01	0.01	2.77	1.08	0.64	0.48	67.2	83.6	89.6	90.5	88.2	93.0	95.1	94.4
100	0.01	-0.03	-0.01	0.00	1.91	0.77	0.47	0.32	66.5	83.5	88.3	91.6	90.3	93.1	93.9	95.6
200	-0.01	-0.01	0.00	0.00	1.35	0.55	0.33	0.23	68.1	82.4	88.6	90.3	88.5	92.6	94.0	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.73	-2.32	-1.47	-1.07	5.64	2.74	1.77	1.28	57.5	59.1	61.2	61.7
50	-4.57	-2.17	-1.40	-1.02	5.11	2.44	1.58	1.15	43.2	45.7	47.8	48.4
100	-4.51	-2.20	-1.41	-1.02	4.81	2.34	1.50	1.09	17.1	17.5	17.4	18.3
200	-4.53	-2.18	-1.40	-1.01	4.67	2.26	1.44	1.04	1.6	2.2	2.3	2.8
	PDOLS estimator, leads and lags order $p = 4$											
30	-1.69	-0.76	-0.46	-0.33	4.21	1.73	1.09	0.78	84.6	90.0	90.0	90.9
50	-1.56	-0.63	-0.40	-0.29	3.29	1.35	0.84	0.61	86.1	89.4	89.2	90.4
100	-1.50	-0.69	-0.43	-0.31	2.55	1.09	0.68	0.48	82.0	84.4	85.9	86.1
200	-1.59	-0.68	-0.42	-0.30	2.13	0.91	0.56	0.40	72.9	76.1	76.5	78.3
	PDOLS estimator, leads and lags order $p = 8$											
30	-0.51	-0.26	-0.14	-0.10	6.98	1.93	1.10	0.77	73.8	89.9	91.9	93.5
50	-0.60	-0.16	-0.10	-0.07	5.23	1.46	0.84	0.59	75.9	91.2	93.1	93.0
100	-0.52	-0.21	-0.13	-0.08	3.75	1.06	0.61	0.41	77.2	90.3	91.9	92.4
200	-0.71	-0.20	-0.11	-0.08	2.67	0.76	0.43	0.30	76.7	90.9	92.5	92.5
	MGMW estimator, $q = 5$											
30	-5.18	-1.66	-0.76	-0.43	7.24	3.16	1.97	1.43	79.7	88.7	92.1	93.3
50	-4.82	-1.45	-0.66	-0.38	6.17	2.50	1.53	1.14	74.4	89.9	92.0	93.7
100	-5.03	-1.50	-0.68	-0.39	5.75	2.12	1.22	0.84	53.8	81.0	89.5	90.9
200	-4.90	-1.51	-0.67	-0.37	5.30	1.84	0.98	0.64	30.7	68.9	83.7	88.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and no CS dependence of errors. See notes to Table S1.

Table S12: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-6.14	-2.94	-1.91	-1.37	7.56	3.55	2.32	1.67	28.9	36.9	38.3	41.0	61.6	63.7	64.6	64.8
50	-5.95	-2.75	-1.79	-1.29	6.91	3.18	2.06	1.48	18.7	24.6	25.8	27.0	46.6	49.6	49.1	51.0
100	-5.78	-2.73	-1.76	-1.27	6.27	2.93	1.89	1.37	5.4	6.0	8.0	8.6	21.6	20.6	20.8	22.5
200	-5.65	-2.71	-1.73	-1.25	5.88	2.82	1.79	1.30	0.4	0.5	0.9	0.7	3.2	3.3	2.9	3.6
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-2.88	-0.84	-0.40	-0.22	4.85	2.06	1.31	0.94	62.2	75.1	79.6	81.1	70.2	85.0	88.4	91.0
50	-2.59	-0.69	-0.31	-0.16	3.95	1.61	1.00	0.71	58.5	76.0	79.6	82.0	64.9	84.2	88.2	91.1
100	-2.66	-0.75	-0.34	-0.18	3.38	1.25	0.73	0.51	44.4	68.1	77.8	81.3	52.8	78.1	87.2	90.0
200	-2.64	-0.76	-0.34	-0.18	3.01	1.05	0.58	0.39	25.1	57.9	72.0	77.9	31.6	68.8	82.1	88.1
	SPMG estimator								Standard				Robust bootstrapped			
30	-0.06	0.00	-0.01	-0.01	4.45	1.88	1.17	0.84	55.8	73.4	77.5	78.7	88.5	92.5	93.6	94.1
50	0.13	0.06	0.02	0.02	3.39	1.41	0.89	0.65	58.8	72.9	77.0	79.0	87.6	92.4	93.9	93.8
100	0.02	0.00	0.00	0.01	2.31	0.98	0.62	0.45	58.9	71.9	77.1	79.1	88.6	92.1	93.3	94.4
200	-0.03	-0.02	0.00	0.00	1.62	0.71	0.44	0.31	60.1	72.5	77.9	79.4	87.1	91.5	93.4	93.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.88	-2.42	-1.59	-1.16	6.23	3.08	2.05	1.50	55.8	56.4	56.8	56.7
50	-4.62	-2.23	-1.46	-1.07	5.45	2.68	1.76	1.29	45.0	47.1	46.4	47.8
100	-4.63	-2.26	-1.47	-1.07	5.08	2.49	1.62	1.18	21.2	22.6	22.7	24.3
200	-4.59	-2.25	-1.46	-1.06	4.80	2.36	1.53	1.12	4.3	5.0	5.4	5.9
	PDOLS estimator, leads and lags order $p = 4$											
30	-1.71	-0.79	-0.50	-0.36	5.15	2.18	1.41	1.01	75.8	80.7	80.2	82.5
50	-1.50	-0.64	-0.41	-0.30	3.98	1.69	1.09	0.78	77.9	81.4	80.6	81.9
100	-1.56	-0.70	-0.44	-0.31	3.01	1.29	0.82	0.59	73.4	76.6	77.0	78.0
200	-1.62	-0.71	-0.45	-0.31	2.40	1.05	0.66	0.47	66.6	68.3	69.9	70.5
	PDOLS estimator, leads and lags order $p = 8$											
30	-0.43	-0.27	-0.16	-0.11	8.27	2.48	1.48	1.03	66.7	82.1	82.6	83.9
50	-0.59	-0.15	-0.10	-0.06	6.32	1.90	1.13	0.78	67.5	82.4	81.7	83.5
100	-0.55	-0.20	-0.13	-0.07	4.41	1.31	0.77	0.54	69.1	82.1	84.1	84.9
200	-0.72	-0.21	-0.12	-0.08	3.07	0.95	0.55	0.39	71.9	82.1	82.7	84.2
	MGMW estimator, $q = 5$											
30	-5.26	-1.59	-0.77	-0.45	7.84	3.58	2.32	1.70	75.4	83.7	86.0	86.3
50	-4.71	-1.43	-0.71	-0.42	6.46	2.81	1.76	1.31	72.6	84.2	87.3	88.3
100	-5.03	-1.52	-0.71	-0.39	5.96	2.32	1.35	0.96	54.1	76.9	84.1	86.8
200	-4.95	-1.52	-0.70	-0.39	5.42	1.96	1.07	0.73	32.5	66.5	79.8	84.4

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and SAR CS dependence of errors. See notes to Table S1.

Table S13: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-1.95	-0.94	-0.61	-0.45	3.54	1.65	1.08	0.78	47.2	53.6	54.2	57.0	83.2	85.9	89.2	88.8
50	-1.84	-0.86	-0.59	-0.42	3.08	1.41	0.93	0.67	40.7	46.3	48.7	49.8	79.6	83.3	83.6	86.2
100	-1.75	-0.85	-0.57	-0.41	2.54	1.21	0.80	0.58	30.5	35.1	36.8	38.2	74.8	77.2	77.6	79.5
200	-1.68	-0.84	-0.56	-0.41	2.28	1.11	0.73	0.54	22.1	24.2	24.0	25.1	67.1	68.4	68.3	68.6
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-1.32	-0.39	-0.14	-0.11	3.30	1.46	0.91	0.67	57.5	66.7	71.0	71.1	77.2	86.6	90.9	91.3
50	-1.18	-0.31	-0.16	-0.08	2.70	1.13	0.71	0.51	53.5	65.3	68.5	70.2	75.0	87.0	90.5	91.3
100	-1.14	-0.29	-0.15	-0.07	2.06	0.84	0.51	0.37	49.5	62.6	67.4	69.9	73.1	86.5	90.3	92.2
200	-1.11	-0.31	-0.15	-0.08	1.77	0.68	0.39	0.28	43.5	57.9	64.0	65.8	66.3	83.4	89.4	90.9
	SPMG estimator								Standard				Robust bootstrapped			
30	-0.05	-0.04	0.01	-0.01	3.25	1.32	0.82	0.59	49.1	60.9	66.8	69.4	86.5	92.3	93.7	94.6
50	-0.01	0.02	-0.01	0.01	2.60	1.00	0.63	0.46	46.6	61.5	64.4	68.1	86.4	92.0	94.2	93.8
100	0.03	0.02	-0.01	0.01	1.77	0.70	0.44	0.33	49.5	61.4	65.0	67.3	87.8	92.1	93.7	93.9
200	0.04	0.00	0.00	0.00	1.42	0.52	0.31	0.23	46.2	59.9	64.6	67.0	87.4	91.4	93.6	93.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-2.24	-1.14	-0.73	-0.56	3.75	1.88	1.22	0.92	60.1	61.3	60.2	59.6
50	-2.21	-1.05	-0.72	-0.53	3.30	1.58	1.05	0.79	52.2	54.6	54.1	54.4
100	-2.08	-1.02	-0.71	-0.51	2.73	1.35	0.93	0.68	41.4	41.5	40.0	43.3
200	-2.07	-1.04	-0.69	-0.52	2.54	1.26	0.85	0.64	25.0	24.1	23.2	24.6
	PDOLS estimator, leads and lags order $p = 4$											
30	-1.20	-0.52	-0.29	-0.25	3.94	1.64	1.00	0.74	65.0	70.2	71.1	70.3
50	-1.16	-0.44	-0.30	-0.21	3.21	1.25	0.80	0.58	62.8	68.6	69.5	68.1
100	-1.05	-0.43	-0.30	-0.20	2.38	0.96	0.61	0.44	59.3	65.0	66.2	66.3
200	-1.05	-0.47	-0.29	-0.21	1.97	0.81	0.49	0.36	54.3	58.0	58.9	57.3
	PDOLS estimator, leads and lags order $p = 8$											
30	-0.60	-0.23	-0.10	-0.11	6.74	1.91	1.10	0.76	53.7	69.3	72.6	71.6
50	-0.52	-0.18	-0.11	-0.07	5.36	1.45	0.85	0.59	52.1	69.2	70.1	71.6
100	-0.46	-0.17	-0.12	-0.06	3.95	1.07	0.59	0.41	53.3	66.8	71.2	70.3
200	-0.64	-0.20	-0.11	-0.08	2.99	0.81	0.44	0.31	51.1	66.4	67.2	70.0
	MGMW estimator, $q = 5$											
30	-1.51	-0.49	-0.24	-0.17	4.22	2.08	1.44	1.03	75.5	76.2	77.1	78.3
50	-1.47	-0.43	-0.22	-0.13	3.51	1.63	1.08	0.79	73.1	77.0	79.6	78.9
100	-1.35	-0.44	-0.23	-0.11	2.63	1.23	0.79	0.57	70.0	75.3	78.7	78.8
200	-1.22	-0.43	-0.22	-0.13	2.16	0.94	0.59	0.44	65.6	72.9	76.3	77.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S14: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-6.11	-2.87	-1.79	-1.28	7.16	3.30	2.08	1.48	27.7	34.8	39.3	42.1	53.8	50.1	52.5	51.8
50	-5.91	-2.67	-1.72	-1.23	6.62	2.96	1.89	1.35	15.7	20.4	23.3	25.3	36.8	34.7	33.8	33.4
100	-5.77	-2.69	-1.70	-1.22	6.14	2.82	1.79	1.28	2.6	3.1	3.8	5.2	11.2	8.1	7.1	8.4
200	-5.66	-2.66	-1.67	-1.20	5.84	2.73	1.72	1.23	0.0	0.0	0.0	0.1	0.8	0.2	0.3	0.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-4.44	-1.88	-1.28	-1.12	6.97	6.08	3.62	3.27	58.7	70.8	74.0	74.9	57.2	70.1	73.7	74.9
50	-4.29	-1.87	-1.44	-1.10	6.01	3.58	3.64	3.01	48.0	66.7	69.9	72.1	46.4	66.0	69.9	71.5
100	-4.15	-1.79	-1.28	-1.04	5.31	3.85	2.22	2.08	31.2	53.7	63.0	63.4	30.2	53.7	62.8	62.6
200	-4.30	-1.99	-1.39	-1.07	4.82	2.57	2.12	1.70	12.4	36.0	47.5	53.0	11.8	36.1	47.9	53.4
	SPMG estimator								Standard				Robust bootstrapped			
30	-0.04	-0.06	-0.01	-0.01	3.72	1.47	0.91	0.64	67.5	83.8	88.4	90.4	88.6	92.9	94.0	95.1
50	0.02	0.04	0.01	0.01	2.85	1.11	0.66	0.49	66.9	83.2	89.9	90.8	88.4	93.2	95.5	94.1
100	0.00	-0.03	-0.01	0.00	1.97	0.79	0.48	0.33	67.3	83.1	88.3	91.2	90.2	93.1	94.1	95.5
200	-0.01	0.00	0.00	0.00	1.38	0.56	0.34	0.24	67.7	83.0	89.0	90.5	89.2	93.0	94.4	95.0

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-6.48	-3.73	-2.68	-2.19	8.09	5.05	3.96	3.39	59.2	65.7	70.1	74.1
50	-6.31	-3.54	-2.62	-2.11	7.33	4.39	3.45	2.92	45.1	56.8	61.4	67.2
100	-6.19	-3.49	-2.55	-2.06	6.71	3.94	3.00	2.50	25.9	34.8	39.7	49.7
200	-6.30	-3.59	-2.62	-2.11	6.58	3.84	2.87	2.36	12.8	16.0	22.9	29.9
	PDOLS estimator, leads and lags order $p = 4$											
30	-3.54	-2.25	-1.71	-1.48	6.84	4.36	3.53	3.09	81.4	87.8	89.8	91.8
50	-3.46	-2.09	-1.67	-1.41	5.63	3.54	2.92	2.56	79.2	86.2	87.0	89.2
100	-3.31	-2.05	-1.61	-1.37	4.55	2.85	2.34	2.03	72.8	76.9	81.4	83.2
200	-3.51	-2.17	-1.69	-1.43	4.18	2.63	2.10	1.81	55.3	63.0	65.3	70.5
	PDOLS estimator, leads and lags order $p = 8$											
30	-2.33	-1.75	-1.39	-1.25	9.33	4.53	3.63	3.16	71.4	88.5	90.8	93.0
50	-2.51	-1.63	-1.38	-1.20	7.10	3.62	2.96	2.58	73.6	88.5	90.7	92.5
100	-2.26	-1.58	-1.32	-1.16	5.14	2.72	2.26	1.97	73.7	84.6	87.4	88.1
200	-2.57	-1.70	-1.39	-1.22	4.25	2.39	1.93	1.69	66.8	77.7	80.0	80.9
	MGMW estimator, $q = 5$											
30	-6.60	-2.92	-1.78	-1.51	9.46	5.27	4.80	3.73	78.0	88.2	92.9	93.8
50	-6.24	-2.66	-1.81	-1.41	8.00	4.24	3.28	2.93	70.6	87.5	90.9	94.4
100	-6.45	-2.72	-1.68	-1.33	7.36	3.66	2.60	2.30	49.7	76.2	85.0	88.6
200	-6.36	-2.78	-1.80	-1.35	6.86	3.28	2.33	1.91	25.1	56.4	71.2	79.5

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and no CS dependence of errors. See notes to Table S1.

Table S15: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-6.25	-2.99	-1.93	-1.39	7.72	3.63	2.35	1.69	29.3	36.4	39.4	42.1	62.3	64.6	64.9	65.8
50	-6.04	-2.78	-1.81	-1.30	7.03	3.22	2.08	1.50	19.2	25.3	26.4	28.1	47.8	49.5	49.7	51.9
100	-5.86	-2.75	-1.77	-1.28	6.37	2.95	1.90	1.38	6.0	6.7	7.6	9.7	22.3	22.2	21.2	23.3
200	-5.73	-2.74	-1.74	-1.26	5.96	2.85	1.81	1.31	0.4	0.5	1.0	0.9	3.5	3.6	2.8	3.8
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-4.68	-2.21	-1.49	-1.27	7.67	4.74	4.29	3.62	54.4	65.4	68.7	70.2	60.3	72.9	76.1	77.0
50	-4.31	-1.90	-1.44	-1.15	6.39	3.71	3.21	3.01	48.2	63.5	66.6	68.5	53.0	69.3	73.2	74.2
100	-4.25	-1.99	-1.41	-1.11	5.42	3.04	2.49	2.16	32.0	52.0	59.0	60.9	38.8	58.0	65.7	67.3
200	-4.37	-2.09	-1.43	-1.12	5.08	2.78	2.45	2.04	13.5	36.3	45.1	52.3	16.5	41.7	50.9	56.7
	SPMG estimator								Standard				Robust bootstrapped			
30	-0.04	-0.01	-0.02	-0.01	4.54	1.92	1.19	0.85	57.2	73.7	77.5	80.1	88.5	92.9	94.0	94.5
50	0.15	0.06	0.02	0.03	3.52	1.44	0.90	0.66	57.6	72.9	76.7	79.4	86.9	92.2	93.4	94.0
100	0.02	0.00	0.00	0.01	2.36	1.00	0.63	0.46	59.7	73.0	77.3	79.8	88.5	92.4	93.6	94.4
200	-0.02	-0.02	-0.01	0.00	1.65	0.72	0.44	0.31	59.4	72.7	77.9	79.8	87.8	91.8	93.8	93.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-6.68	-3.94	-2.93	-2.39	8.57	5.42	4.34	3.71	57.6	62.6	66.6	71.1
50	-6.40	-3.60	-2.71	-2.22	7.63	4.54	3.61	3.12	46.6	55.7	59.0	64.0
100	-6.35	-3.61	-2.68	-2.17	7.01	4.13	3.18	2.67	26.8	36.6	41.5	48.7
200	-6.39	-3.69	-2.72	-2.21	6.73	3.99	2.99	2.48	13.7	16.3	21.9	30.3
	PDOLS estimator, leads and lags order $p = 4$											
30	-3.65	-2.40	-1.90	-1.63	7.53	4.73	3.89	3.39	76.1	82.8	84.2	87.4
50	-3.43	-2.08	-1.71	-1.48	6.09	3.65	3.05	2.72	75.0	81.7	82.5	85.5
100	-3.40	-2.12	-1.70	-1.45	4.90	3.06	2.49	2.18	69.4	74.0	77.6	80.8
200	-3.58	-2.25	-1.76	-1.50	4.37	2.78	2.20	1.91	55.1	61.7	63.2	67.8
	PDOLS estimator, leads and lags order $p = 8$											
30	-2.38	-1.90	-1.57	-1.39	10.49	4.97	4.01	3.46	67.2	82.2	85.2	88.6
50	-2.50	-1.59	-1.41	-1.26	7.97	3.73	3.08	2.74	66.1	82.6	85.7	87.9
100	-2.34	-1.63	-1.40	-1.23	5.72	2.95	2.42	2.12	66.8	80.8	83.5	86.0
200	-2.65	-1.77	-1.44	-1.28	4.53	2.56	2.02	1.78	63.6	73.7	76.5	78.6
	MGMW estimator, $q = 5$											
30	-6.72	-2.96	-1.93	-1.57	9.88	5.67	4.44	3.90	74.7	85.0	88.4	90.3
50	-6.14	-2.63	-1.89	-1.50	8.15	4.41	3.52	3.19	71.3	84.1	89.0	90.6
100	-6.46	-2.80	-1.76	-1.38	7.52	3.82	2.80	2.38	49.6	73.7	82.1	85.9
200	-6.43	-2.84	-1.86	-1.41	7.00	3.41	2.41	2.00	27.9	55.8	70.0	78.1

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and SAR CS dependence of errors. See notes to Table S1.

Table S16: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-1.97	-0.95	-0.62	-0.46	3.59	1.67	1.09	0.80	48.1	53.7	55.0	57.2	83.1	86.6	89.2	88.4
50	-1.85	-0.87	-0.60	-0.42	3.11	1.42	0.94	0.67	41.1	47.1	48.7	50.7	81.1	83.3	83.5	85.7
100	-1.77	-0.85	-0.58	-0.41	2.57	1.23	0.81	0.59	30.7	35.9	37.3	39.8	74.9	77.2	78.0	79.5
200	-1.69	-0.85	-0.57	-0.42	2.30	1.12	0.73	0.55	22.2	24.5	24.8	25.9	67.6	68.3	68.3	68.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-2.01	-0.91	-0.60	-0.62	4.56	3.10	2.40	2.28	54.5	61.8	66.0	66.0	71.9	78.0	80.3	79.0
50	-1.87	-0.86	-0.68	-0.60	3.89	2.33	2.09	1.92	50.3	61.8	63.0	63.7	67.6	77.4	77.4	76.4
100	-1.83	-0.84	-0.66	-0.53	3.03	1.76	1.51	1.60	43.2	55.7	59.4	60.8	61.1	71.6	74.2	73.1
200	-1.85	-0.88	-0.63	-0.54	2.70	1.51	1.29	1.15	35.3	49.2	55.4	57.6	53.4	65.3	68.5	68.9
	SPMG estimator								Standard				Robust bootstrapped			
30	-0.06	-0.04	0.01	-0.01	3.29	1.34	0.83	0.60	49.6	61.4	68.6	70.0	87.1	92.2	93.8	94.1
50	0.01	0.03	-0.01	0.01	2.65	1.02	0.64	0.46	47.6	62.2	65.5	68.5	86.8	91.7	93.8	94.0
100	0.03	0.02	-0.01	0.01	1.86	0.72	0.45	0.33	50.7	61.4	66.8	67.2	87.5	92.0	93.8	94.0
200	0.03	0.00	0.00	0.00	1.87	0.52	0.31	0.24	47.2	61.5	64.8	67.2	87.5	91.4	93.4	93.6

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-2.95	-1.76	-1.26	-1.07	4.77	2.93	2.35	2.01	65.5	70.6	74.7	75.8
50	-2.91	-1.62	-1.26	-1.05	4.19	2.41	1.94	1.72	59.4	69.1	70.9	74.9
100	-2.78	-1.59	-1.24	-1.00	3.57	2.13	1.68	1.43	49.4	60.2	62.3	69.0
200	-2.81	-1.64	-1.23	-1.02	3.37	1.99	1.52	1.30	37.4	44.3	48.0	54.8
	PDOLS estimator, leads and lags order $p = 4$											
30	-1.99	-1.20	-0.86	-0.78	5.00	2.81	2.27	1.93	70.1	78.3	80.7	83.4
50	-1.97	-1.06	-0.87	-0.76	4.14	2.16	1.77	1.60	69.8	79.5	81.8	83.4
100	-1.85	-1.06	-0.87	-0.72	3.26	1.82	1.44	1.26	65.7	77.9	79.8	83.8
200	-1.90	-1.14	-0.86	-0.75	2.83	1.60	1.23	1.08	61.6	70.3	73.4	76.9
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.49	-0.95	-0.69	-0.66	7.68	3.15	2.45	2.01	57.3	77.5	82.1	83.4
50	-1.38	-0.85	-0.72	-0.65	6.20	2.40	1.88	1.66	55.5	79.6	82.7	85.3
100	-1.28	-0.84	-0.72	-0.60	4.65	1.94	1.46	1.28	56.4	80.4	83.8	87.3
200	-1.52	-0.93	-0.72	-0.64	3.64	1.60	1.20	1.05	53.1	76.8	80.9	84.1
	MGMW estimator, $q = 5$											
30	-1.76	-0.77	-0.44	-0.42	5.07	2.94	2.46	2.17	79.3	81.8	84.8	87.1
50	-1.65	-0.64	-0.46	-0.41	4.11	2.40	1.92	1.69	78.2	83.0	86.3	88.0
100	-1.56	-0.70	-0.44	-0.30	3.13	1.86	1.41	1.24	76.0	82.1	87.6	88.3
200	-1.43	-0.68	-0.45	-0.35	2.61	1.43	1.11	0.96	73.2	79.6	83.2	85.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S17: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-6.47	-2.99	-1.86	-1.32	7.75	3.49	2.18	1.56	30.6	37.3	44.0	47.2	56.8	54.1	55.9	56.5
50	-6.24	-2.78	-1.79	-1.27	7.06	3.11	1.99	1.42	17.4	23.1	27.6	29.2	39.8	39.2	38.0	38.5
100	-6.06	-2.78	-1.75	-1.25	6.48	2.94	1.86	1.32	3.6	4.8	6.0	6.9	13.8	11.6	12.0	11.0
200	-5.92	-2.76	-1.72	-1.23	6.12	2.84	1.78	1.27	0.1	0.1	0.3	0.3	1.3	0.8	0.6	1.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-9.50	-5.27	-4.20	-3.94	13.76	10.15	8.37	7.99	41.3	54.1	57.5	58.1	39.5	53.1	57.6	58.1
50	-9.52	-5.49	-4.41	-3.69	12.26	8.07	7.26	6.34	27.4	45.7	48.4	53.5	27.4	45.7	48.6	53.4
100	-9.18	-5.55	-4.43	-3.85	11.00	8.06	5.95	5.43	14.2	30.0	36.8	40.9	14.7	30.0	36.6	40.5
200	-9.19	-5.56	-4.41	-3.71	9.91	6.29	5.27	4.57	4.4	14.2	22.2	26.7	4.1	14.5	22.4	27.5
	SPMG estimator								Standard				Robust bootstrapped			
30	-0.01	-0.06	-0.01	-0.01	4.13	1.63	0.99	0.70	65.6	83.5	87.7	89.7	88.5	93.4	94.2	94.6
50	0.04	0.05	0.01	0.02	3.18	1.21	0.74	0.54	65.3	84.5	88.7	90.1	88.1	93.5	94.6	94.1
100	0.01	-0.02	-0.01	0.00	2.18	0.87	0.53	0.36	66.3	82.1	87.4	90.8	89.2	93.4	93.9	95.5
200	0.00	-0.01	0.00	0.01	1.50	0.63	0.37	0.26	66.3	81.4	88.1	90.2	88.6	92.0	94.2	94.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-11.57	-7.67	-6.14	-5.39	13.84	9.60	7.98	7.22	58.1	66.6	71.7	75.1
50	-11.50	-7.61	-6.12	-5.26	13.00	8.99	7.40	6.49	45.5	55.2	59.7	66.0
100	-11.29	-7.66	-6.25	-5.43	12.06	8.34	6.89	6.09	30.9	33.4	36.7	43.4
200	-11.41	-7.61	-6.14	-5.35	11.82	7.98	6.50	5.70	20.5	16.2	19.6	22.0
	PDOLS estimator, leads and lags order $p = 4$											
30	-9.13	-6.39	-5.30	-4.76	12.94	9.04	7.61	6.94	70.4	77.9	80.8	83.6
50	-9.09	-6.41	-5.32	-4.67	11.58	8.31	6.94	6.15	64.2	73.3	73.1	76.6
100	-8.85	-6.49	-5.47	-4.84	10.18	7.43	6.30	5.65	50.7	54.3	55.5	59.7
200	-9.08	-6.43	-5.35	-4.76	9.78	6.96	5.80	5.19	29.9	35.3	36.6	37.5
	PDOLS estimator, leads and lags order $p = 8$											
30	-7.74	-5.89	-4.99	-4.54	15.04	9.33	7.75	7.02	62.7	78.1	82.8	84.4
50	-8.03	-6.01	-5.05	-4.48	12.61	8.41	6.99	6.20	58.5	73.6	75.3	79.7
100	-7.81	-6.10	-5.24	-4.67	10.40	7.33	6.24	5.60	52.6	59.5	60.9	65.3
200	-8.02	-5.99	-5.08	-4.58	9.47	6.69	5.63	5.09	35.5	42.1	45.4	45.7
	MGMW estimator, $q = 5$											
30	-11.14	-6.76	-5.07	-4.40	14.66	10.10	8.74	7.69	71.4	82.6	87.0	90.5
50	-10.58	-6.26	-4.99	-4.32	12.84	8.39	7.09	6.42	61.6	77.5	82.0	86.0
100	-10.85	-6.51	-5.07	-4.37	12.09	7.69	6.21	5.57	39.1	56.8	64.9	70.2
200	-10.56	-6.45	-5.02	-4.31	11.16	7.03	5.66	4.97	15.7	32.9	42.8	49.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and no CS dependence of errors. See notes to Table S1.

Table S18: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-6.64	-3.14	-2.01	-1.43	8.28	3.84	2.47	1.77	31.1	39.0	42.4	44.9	63.4	66.7	67.0	68.0
50	-6.37	-2.89	-1.88	-1.33	7.50	3.37	2.17	1.55	20.6	27.3	28.7	30.1	49.3	52.0	52.3	55.0
100	-6.15	-2.85	-1.83	-1.32	6.71	3.08	1.98	1.43	6.3	8.2	9.6	10.8	24.1	25.4	24.3	25.9
200	-5.99	-2.84	-1.79	-1.30	6.26	2.96	1.87	1.35	0.5	0.4	1.3	1.8	4.5	4.5	4.6	5.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-9.84	-5.64	-4.68	-4.37	14.03	10.00	8.77	9.33	38.6	52.2	55.3	55.9	42.5	57.2	59.7	59.9
50	-9.57	-5.62	-4.57	-3.92	12.41	8.24	7.48	6.79	30.0	45.3	48.6	50.4	33.5	49.7	52.2	53.8
100	-9.26	-5.76	-4.72	-4.18	10.96	7.24	6.45	5.85	16.5	29.9	35.6	40.1	18.8	32.2	38.1	42.6
200	-9.44	-5.84	-4.61	-3.97	10.32	6.64	5.57	4.95	4.7	13.9	22.0	26.9	5.4	16.1	24.4	29.3
	SPMG estimator								Standard				Robust bootstrapped			
30	-0.02	-0.01	-0.01	0.00	4.94	2.05	1.27	0.91	57.2	74.1	78.5	81.3	89.3	93.2	94.2	94.1
50	0.16	0.08	0.03	0.03	3.86	1.53	0.95	0.70	58.4	75.2	78.2	80.7	86.3	92.9	93.9	93.8
100	0.05	0.01	0.00	0.01	2.55	1.07	0.67	0.48	59.5	73.0	77.5	81.7	88.9	92.5	93.9	94.9
200	-0.01	-0.01	0.00	0.00	1.78	0.77	0.47	0.33	58.6	72.1	76.9	82.0	88.1	91.6	93.0	93.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-11.83	-8.01	-6.66	-5.89	14.43	10.14	8.69	7.85	55.9	63.5	68.3	72.8
50	-11.73	-7.82	-6.40	-5.59	13.43	9.32	7.81	6.95	44.7	55.2	56.1	62.3
100	-11.62	-7.95	-6.57	-5.76	12.51	8.73	7.30	6.49	28.9	32.9	36.9	42.6
200	-11.60	-7.87	-6.43	-5.65	12.05	8.28	6.80	6.02	19.7	17.3	19.1	21.4
	PDOLS estimator, leads and lags order $p = 4$											
30	-9.34	-6.68	-5.76	-5.22	13.62	9.59	8.32	7.56	68.2	75.3	79.2	81.1
50	-9.23	-6.56	-5.54	-4.97	12.07	8.62	7.33	6.60	63.6	70.7	70.1	74.2
100	-9.14	-6.74	-5.76	-5.16	10.66	7.83	6.70	6.04	49.5	52.9	54.4	56.2
200	-9.24	-6.67	-5.61	-5.04	10.04	7.25	6.09	5.50	30.9	34.6	35.1	36.0
	PDOLS estimator, leads and lags order $p = 8$											
30	-8.10	-6.18	-5.46	-4.99	16.17	9.91	8.53	7.66	59.2	76.0	80.2	82.4
50	-8.25	-6.15	-5.28	-4.79	13.40	8.73	7.39	6.66	56.3	73.1	72.4	75.8
100	-8.13	-6.34	-5.53	-4.98	11.03	7.76	6.66	6.00	49.4	58.2	60.8	62.4
200	-8.23	-6.25	-5.34	-4.86	9.80	7.01	5.93	5.39	35.3	40.7	44.4	43.4
	MGMW estimator, $q = 5$											
30	-11.21	-6.89	-5.35	-4.81	15.17	10.47	9.06	8.06	70.0	81.1	85.9	87.4
50	-10.57	-6.36	-5.13	-4.55	13.07	8.76	7.49	6.78	61.0	75.0	80.2	82.0
100	-11.02	-6.74	-5.26	-4.57	12.38	7.96	6.47	5.79	38.3	53.4	62.7	67.4
200	-10.75	-6.65	-5.23	-4.52	11.42	7.27	5.88	5.22	16.2	30.6	39.8	50.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and SAR CS dependence of errors. See notes to Table S1.

Table S19: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-2.05	-0.99	-0.64	-0.47	3.78	1.75	1.14	0.83	49.5	55.9	57.6	59.9	84.0	86.9	89.4	88.1
50	-1.93	-0.89	-0.62	-0.43	3.27	1.48	0.97	0.69	42.4	50.2	51.0	53.5	81.4	84.7	84.6	87.0
100	-1.85	-0.87	-0.59	-0.42	2.70	1.27	0.84	0.60	33.3	37.7	38.7	41.8	75.3	78.9	78.5	80.4
200	-1.76	-0.88	-0.58	-0.43	2.40	1.16	0.75	0.57	23.3	26.6	26.3	27.3	69.4	69.3	69.0	69.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-4.38	-2.29	-2.04	-1.97	8.18	6.09	5.26	5.21	47.5	57.9	60.3	61.4	59.4	67.6	70.1	69.9
50	-4.10	-2.58	-2.08	-1.94	6.96	4.86	4.14	4.10	41.1	52.7	56.2	55.9	52.4	62.4	65.9	64.6
100	-4.02	-2.47	-2.21	-1.91	5.73	3.97	3.60	3.35	33.2	44.6	46.9	50.6	42.8	53.7	54.8	58.5
200	-4.08	-2.50	-2.06	-1.90	5.52	3.58	3.02	2.79	21.4	34.8	40.3	41.9	30.4	44.5	48.4	50.3
	SPMG estimator								Standard				Robust bootstrapped			
30	-0.05	-0.05	0.01	-0.01	3.63	1.40	0.87	0.63	49.7	63.8	71.0	71.6	87.5	92.4	93.9	94.0
50	0.00	0.04	-0.01	0.02	2.83	1.07	0.66	0.48	49.6	63.4	68.2	71.2	86.7	91.8	94.4	94.3
100	0.05	0.02	-0.01	0.01	2.32	0.75	0.47	0.34	49.8	63.9	67.1	69.8	87.9	92.8	93.9	94.4
200	-0.01	0.00	0.00	0.00	3.43	0.55	0.33	0.25	49.2	63.1	67.8	69.2	88.0	91.9	93.8	94.0

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-5.17	-3.47	-2.82	-2.52	7.44	5.19	4.53	4.01	71.1	78.8	82.8	84.4
50	-5.18	-3.41	-2.85	-2.50	6.83	4.66	3.95	3.56	64.7	74.4	75.8	78.8
100	-4.97	-3.37	-2.89	-2.54	5.99	4.18	3.58	3.19	55.9	63.3	62.1	67.4
200	-4.99	-3.36	-2.79	-2.52	5.76	3.89	3.25	2.97	42.2	45.3	46.3	50.0
	PDOLS estimator, leads and lags order $p = 4$											
30	-4.56	-3.06	-2.52	-2.30	8.07	5.28	4.57	4.03	72.5	83.1	86.4	88.0
50	-4.59	-3.03	-2.57	-2.29	7.10	4.63	3.89	3.51	70.2	80.6	82.9	85.3
100	-4.34	-3.03	-2.63	-2.34	5.92	4.06	3.45	3.10	63.6	74.4	74.0	76.5
200	-4.43	-3.03	-2.54	-2.33	5.59	3.68	3.06	2.82	54.6	60.2	63.2	62.1
	PDOLS estimator, leads and lags order $p = 8$											
30	-4.15	-2.91	-2.44	-2.24	10.81	5.78	4.91	4.24	58.0	81.6	86.0	88.0
50	-4.21	-2.96	-2.49	-2.23	9.08	5.07	4.09	3.65	55.6	79.0	84.0	85.8
100	-3.87	-2.95	-2.59	-2.30	7.10	4.36	3.57	3.18	54.1	74.6	77.2	79.4
200	-4.17	-2.95	-2.48	-2.29	6.41	3.82	3.11	2.86	45.5	65.2	69.5	69.3
	MGMW estimator, $q = 5$											
30	-2.64	-1.46	-1.00	-1.10	7.07	5.09	4.48	4.10	82.1	87.1	89.4	90.9
50	-2.47	-1.46	-1.12	-0.95	5.96	4.05	3.72	3.38	80.7	85.1	86.9	89.8
100	-2.25	-1.46	-1.19	-1.01	4.70	3.38	2.84	2.61	76.7	81.8	85.3	86.5
200	-2.11	-1.38	-1.12	-1.04	4.08	2.69	2.33	2.21	71.8	76.2	79.0	79.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S20: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.71	-0.12	-0.08	-0.06	3.74	1.60	1.02	0.73	71.5	85.2	87.4	90.3	91.4	94.4	93.9	94.2
50	-0.55	-0.17	-0.09	-0.04	2.84	1.23	0.74	0.54	71.7	85.8	89.8	91.7	91.4	93.8	95.2	95.5
100	-0.61	-0.14	-0.06	-0.04	2.09	0.87	0.55	0.39	68.0	84.9	88.2	90.4	89.8	93.4	93.9	94.6
200	-0.68	-0.16	-0.07	-0.04	1.52	0.61	0.37	0.27	67.7	84.2	89.6	90.6	89.3	93.8	94.7	95.7
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-2.28	-0.51	-0.24	-0.15	4.42	1.88	1.20	0.87	75.6	87.4	90.4	91.2	72.7	86.5	90.0	90.4
50	-2.02	-0.61	-0.28	-0.14	3.56	1.53	0.93	0.65	70.9	84.8	89.2	92.1	69.4	83.5	88.7	91.4
100	-2.02	-0.53	-0.24	-0.13	2.89	1.12	0.69	0.49	63.5	83.5	87.5	90.1	61.3	82.5	86.9	89.9
200	-2.06	-0.56	-0.24	-0.14	2.51	0.89	0.49	0.34	47.3	76.9	86.6	89.3	46.0	75.1	85.7	88.9
	SPMG estimator								Standard				Robust bootstrapped			
30	0.29	0.12	0.02	-0.01	4.41	1.72	1.07	0.75	63.0	80.3	85.8	88.7	88.0	92.8	93.1	94.4
50	0.43	0.04	0.00	0.01	3.31	1.31	0.77	0.55	61.9	81.8	87.8	90.4	86.9	93.7	94.5	95.5
100	0.28	0.08	0.03	0.02	2.35	0.90	0.56	0.40	59.0	81.4	87.0	88.0	86.6	93.1	94.3	94.3
200	0.19	0.05	0.02	0.01	1.60	0.63	0.38	0.27	62.8	81.2	88.4	90.3	87.3	93.9	95.1	95.0

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.85	-2.27	-1.51	-1.13	6.01	2.91	1.95	1.44	68.4	72.8	70.9	72.3
50	-4.68	-2.39	-1.55	-1.12	5.48	2.79	1.81	1.31	55.6	55.7	56.8	58.6
100	-4.63	-2.29	-1.51	-1.11	5.06	2.51	1.66	1.22	34.8	32.8	34.4	34.5
200	-4.67	-2.32	-1.50	-1.10	4.87	2.42	1.57	1.15	11.2	9.2	8.6	9.2
	PDOLS estimator, leads and lags order $p = 4$											
30	-2.61	-1.07	-0.70	-0.53	5.09	2.22	1.42	1.03	83.3	87.7	87.9	87.5
50	-2.45	-1.19	-0.76	-0.54	4.28	1.91	1.19	0.85	78.8	82.8	83.4	84.9
100	-2.49	-1.12	-0.72	-0.52	3.48	1.54	0.98	0.71	72.8	76.2	76.1	77.6
200	-2.56	-1.16	-0.72	-0.52	3.06	1.37	0.85	0.61	59.3	58.1	60.5	61.7
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.37	-0.40	-0.28	-0.22	7.78	2.38	1.40	0.97	73.5	90.3	90.8	91.8
50	-1.25	-0.55	-0.34	-0.23	6.16	1.85	1.05	0.74	73.8	88.6	90.9	92.6
100	-1.34	-0.49	-0.31	-0.21	4.42	1.35	0.80	0.56	73.1	88.6	88.2	90.3
200	-1.38	-0.52	-0.30	-0.21	3.20	1.01	0.58	0.41	74.7	85.6	87.4	87.8
	MGMW estimator, $q = 5$											
30	-5.52	-1.74	-0.81	-0.58	9.06	4.15	2.80	2.01	84.4	92.2	94.1	93.8
50	-5.33	-1.96	-0.93	-0.58	7.76	3.63	2.22	1.59	79.9	88.5	92.4	93.2
100	-5.47	-1.77	-0.92	-0.56	6.77	2.83	1.75	1.24	67.9	85.0	89.1	91.0
200	-5.50	-1.89	-0.93	-0.54	6.16	2.41	1.38	0.94	46.5	74.0	83.8	88.4

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$, and no CS dependence of errors. See notes to Table S1.

Table S21: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.83	-0.20	-0.12	-0.09	4.68	2.06	1.33	0.98	64.0	74.9	78.2	81.5	89.6	93.3	94.5	94.0
50	-0.73	-0.20	-0.09	-0.05	3.54	1.56	0.98	0.74	62.5	74.7	79.1	80.7	90.2	94.0	95.2	94.9
100	-0.70	-0.17	-0.06	-0.04	2.54	1.11	0.70	0.51	61.4	73.4	79.6	80.5	88.2	93.2	93.7	94.4
200	-0.77	-0.18	-0.08	-0.04	1.86	0.77	0.48	0.35	58.7	75.9	79.4	82.3	88.1	93.0	94.7	94.7
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-2.39	-0.60	-0.31	-0.19	5.41	2.40	1.55	1.15	68.2	77.9	79.1	81.3	73.8	85.8	89.5	90.6
50	-2.19	-0.67	-0.31	-0.17	4.28	1.92	1.21	0.88	65.1	76.1	79.3	81.7	70.7	85.0	89.2	91.5
100	-2.13	-0.58	-0.25	-0.14	3.36	1.40	0.86	0.62	57.2	73.4	78.9	80.9	63.5	81.9	88.2	89.8
200	-2.19	-0.60	-0.26	-0.15	2.85	1.05	0.62	0.44	43.4	68.5	76.7	80.6	50.8	77.6	86.9	89.7
	SPMG estimator								Standard				Robust bootstrapped			
30	0.24	0.05	-0.02	-0.03	5.50	2.17	1.37	1.00	56.0	71.5	74.4	78.6	86.7	92.5	93.8	93.4
50	0.29	0.02	0.00	0.00	4.05	1.65	1.01	0.75	53.6	69.5	77.0	78.3	87.3	92.7	94.8	94.4
100	0.24	0.06	0.05	0.02	2.87	1.15	0.72	0.53	52.3	70.1	75.5	79.0	85.0	92.0	92.8	93.2
200	0.14	0.03	0.01	0.01	1.95	0.79	0.49	0.35	54.5	72.0	76.8	81.0	86.0	93.0	94.3	94.2

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.95	-2.38	-1.61	-1.22	6.76	3.36	2.26	1.71	63.5	65.7	65.0	65.5
50	-4.79	-2.48	-1.63	-1.19	5.96	3.08	2.03	1.50	53.7	54.1	53.5	55.6
100	-4.65	-2.35	-1.55	-1.15	5.29	2.68	1.77	1.32	36.5	35.7	37.4	36.8
200	-4.76	-2.37	-1.56	-1.14	5.06	2.53	1.66	1.22	14.4	11.6	12.7	12.9
	PDOLS estimator, leads and lags order $p = 4$											
30	-2.66	-1.12	-0.76	-0.58	6.28	2.74	1.77	1.31	75.3	79.7	78.1	79.5
50	-2.49	-1.23	-0.80	-0.57	5.05	2.29	1.44	1.06	73.5	74.7	75.5	76.6
100	-2.47	-1.15	-0.73	-0.54	3.94	1.79	1.13	0.83	67.8	69.2	69.7	70.6
200	-2.61	-1.18	-0.75	-0.54	3.35	1.49	0.95	0.68	56.2	56.8	56.0	57.7
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.38	-0.42	-0.30	-0.24	9.27	3.00	1.80	1.28	65.6	80.8	81.5	82.2
50	-1.12	-0.58	-0.35	-0.25	7.17	2.34	1.35	0.98	65.2	80.5	82.2	81.9
100	-1.31	-0.52	-0.31	-0.21	5.07	1.69	1.00	0.70	68.6	79.6	80.6	82.5
200	-1.44	-0.52	-0.32	-0.22	3.67	1.20	0.72	0.50	68.4	78.6	78.6	80.6
	MGMW estimator, $q = 5$											
30	-5.84	-1.89	-0.96	-0.64	9.87	4.72	3.18	2.36	79.5	85.7	87.0	86.9
50	-5.63	-1.99	-0.98	-0.62	8.47	3.92	2.48	1.88	75.2	82.9	86.8	87.4
100	-5.40	-1.83	-0.93	-0.55	6.92	3.08	1.91	1.36	65.4	79.8	85.4	86.8
200	-5.63	-1.97	-1.00	-0.58	6.41	2.57	1.51	1.06	44.0	70.7	78.9	84.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$, and SAR CS dependence of errors. See notes to Table S1.

Table S22: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.22	-0.02	0.00	-0.01	3.69	1.65	1.05	0.80	50.8	61.6	68.0	68.2	90.1	93.8	94.6	93.7
50	-0.35	-0.07	-0.01	-0.01	2.78	1.28	0.78	0.60	51.3	62.1	67.8	68.9	90.4	93.4	95.0	95.2
100	-0.25	-0.07	-0.01	-0.01	2.01	0.96	0.60	0.45	50.2	59.1	64.9	64.4	90.8	94.2	94.8	95.1
200	-0.19	-0.03	-0.03	-0.01	1.55	0.70	0.45	0.33	45.0	57.4	59.6	61.7	92.0	95.0	95.2	96.2
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-1.03	-0.30	-0.10	-0.06	3.99	1.94	1.25	0.94	58.5	64.0	67.4	68.9	80.4	89.0	92.2	91.8
50	-1.22	-0.32	-0.10	-0.08	3.26	1.52	0.95	0.70	54.2	63.4	68.4	67.9	79.0	88.8	93.6	94.3
100	-1.09	-0.30	-0.13	-0.06	2.54	1.17	0.72	0.54	50.2	61.6	65.2	65.3	80.6	90.8	94.3	94.5
200	-1.03	-0.27	-0.14	-0.07	2.08	0.90	0.56	0.41	44.7	53.9	59.0	62.1	81.8	93.6	96.0	96.4
	SPMG estimator								Standard				Robust bootstrapped			
30	0.10	0.06	0.05	0.01	5.53	1.79	1.10	0.83	43.6	57.7	64.7	65.5	84.8	92.3	94.0	93.7
50	-0.08	0.03	0.03	0.01	3.48	1.40	0.82	0.62	42.7	59.1	64.6	67.0	85.6	90.5	94.5	94.4
100	-0.05	0.03	0.04	0.02	5.27	1.03	0.62	0.46	41.1	55.9	61.2	63.8	85.3	92.2	94.0	94.0
200	0.03	0.05	0.01	0.01	3.56	0.78	0.46	0.34	38.6	53.9	57.5	59.5	84.9	92.2	93.8	94.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-2.50	-1.38	-0.94	-0.72	4.45	2.39	1.62	1.23	64.5	62.9	63.2	64.0
50	-2.76	-1.45	-0.96	-0.75	4.02	2.10	1.42	1.08	55.5	55.9	57.4	55.8
100	-2.58	-1.42	-0.98	-0.73	3.40	1.86	1.27	0.95	45.2	43.1	42.9	41.2
200	-2.56	-1.40	-0.95	-0.71	3.08	1.68	1.13	0.86	29.6	26.3	22.5	24.7
	PDOLS estimator, leads and lags order $p = 4$											
30	-1.61	-0.82	-0.51	-0.40	4.95	2.22	1.39	1.05	64.5	68.3	69.4	68.8
50	-1.91	-0.87	-0.54	-0.43	4.13	1.80	1.13	0.84	62.7	65.1	67.6	66.3
100	-1.74	-0.87	-0.56	-0.41	3.26	1.48	0.94	0.68	57.0	59.2	59.5	58.1
200	-1.70	-0.85	-0.57	-0.41	2.77	1.24	0.79	0.58	47.4	47.9	46.5	47.5
	PDOLS estimator, leads and lags order $p = 8$											
30	-0.91	-0.41	-0.24	-0.19	8.19	2.52	1.46	1.06	51.5	68.2	69.2	70.9
50	-1.29	-0.49	-0.27	-0.21	6.42	2.03	1.15	0.80	52.7	65.7	70.0	71.3
100	-1.16	-0.51	-0.31	-0.19	5.20	1.56	0.93	0.62	48.4	62.6	64.7	65.4
200	-1.04	-0.47	-0.30	-0.21	4.28	1.26	0.71	0.49	41.6	55.5	59.0	60.5
	MGMW estimator, $q = 5$											
30	-0.97	-0.45	-0.16	-0.08	5.67	2.95	2.02	1.54	79.6	78.8	79.5	79.0
50	-1.16	-0.41	-0.19	-0.15	4.36	2.30	1.54	1.18	77.8	78.4	81.2	80.6
100	-0.81	-0.38	-0.22	-0.14	3.17	1.71	1.16	0.85	78.0	79.2	80.6	79.6
200	-0.78	-0.38	-0.20	-0.12	2.45	1.27	0.82	0.63	75.6	77.6	79.4	78.8

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S23: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.76	-0.12	-0.08	-0.06	3.90	1.65	1.06	0.75	70.7	84.9	87.7	89.5	90.6	94.2	93.8	94.2
50	-0.60	-0.18	-0.09	-0.05	2.92	1.27	0.76	0.55	71.7	85.4	89.3	91.8	90.6	93.1	95.0	95.7
100	-0.69	-0.15	-0.07	-0.04	2.16	0.89	0.56	0.40	68.3	84.3	88.5	90.2	89.3	93.8	94.1	94.7
200	-0.74	-0.16	-0.07	-0.04	1.58	0.63	0.38	0.27	66.8	84.3	89.3	91.1	89.1	93.7	94.4	95.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-3.50	-1.19	-0.75	-0.79	6.47	4.30	3.61	8.74	67.5	78.6	80.8	80.9	65.0	78.0	80.8	80.9
50	-3.16	-1.29	-0.79	-0.53	5.21	3.54	2.93	2.24	62.2	73.9	77.2	79.2	61.1	73.2	77.0	78.9
100	-3.15	-1.19	-0.69	-0.51	5.36	2.70	3.60	2.01	49.3	69.4	72.3	73.5	48.8	68.9	71.7	73.5
200	-3.28	-1.22	-0.73	-0.58	4.13	2.00	1.68	1.44	31.4	58.8	67.7	70.9	30.6	58.4	67.4	70.6
	SPMG estimator								Standard				Robust bootstrapped			
30	0.29	0.14	0.03	0.00	4.60	1.79	1.11	0.77	62.6	80.3	85.9	88.2	88.2	92.8	93.2	94.0
50	0.45	0.06	0.00	0.01	3.41	1.35	0.79	0.57	61.7	80.6	87.9	90.9	86.6	93.5	94.3	95.3
100	0.26	0.08	0.04	0.02	2.42	0.93	0.57	0.41	59.0	81.0	86.5	87.9	86.6	92.9	94.5	94.3
200	0.20	0.06	0.02	0.01	1.65	0.65	0.39	0.28	62.5	81.2	87.9	90.3	86.9	94.1	94.8	95.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-6.22	-3.26	-2.33	-1.88	7.82	4.41	3.37	2.83	67.5	74.4	76.3	78.6
50	-5.98	-3.36	-2.36	-1.83	7.05	4.17	3.05	2.46	57.8	61.4	66.0	70.6
100	-6.02	-3.25	-2.33	-1.83	6.65	3.68	2.74	2.24	36.9	44.9	50.0	57.0
200	-6.03	-3.27	-2.32	-1.82	6.33	3.49	2.52	2.02	20.1	21.3	26.9	35.4
	PDOLS estimator, leads and lags order $p = 4$											
30	-4.14	-2.15	-1.58	-1.32	7.10	3.91	3.03	2.59	81.3	87.5	87.5	89.7
50	-3.98	-2.25	-1.63	-1.29	6.10	3.52	2.63	2.15	76.4	84.0	82.9	87.2
100	-4.09	-2.16	-1.59	-1.28	5.30	2.88	2.21	1.87	68.1	74.4	77.9	80.8
200	-4.11	-2.20	-1.59	-1.28	4.70	2.57	1.91	1.58	51.6	57.3	62.9	68.5
	PDOLS estimator, leads and lags order $p = 8$											
30	-3.00	-1.58	-1.21	-1.06	9.45	4.06	3.09	2.62	71.7	89.7	90.8	92.7
50	-2.91	-1.68	-1.25	-1.02	7.81	3.54	2.59	2.13	72.4	87.9	89.8	91.8
100	-3.13	-1.62	-1.22	-1.01	6.02	2.72	2.10	1.79	68.5	85.0	87.4	89.4
200	-3.06	-1.64	-1.23	-1.01	4.67	2.26	1.69	1.43	66.2	78.8	81.6	84.6
	MGMW estimator, $q = 5$											
30	-6.71	-2.78	-1.65	-1.34	10.90	5.80	4.82	3.99	84.0	91.6	93.9	94.5
50	-6.58	-3.03	-1.85	-1.41	9.32	5.08	3.72	3.18	78.2	86.4	91.4	93.7
100	-6.79	-2.79	-1.74	-1.30	8.26	4.16	2.99	2.54	63.3	82.8	86.3	88.4
200	-6.85	-2.88	-1.78	-1.27	7.56	3.52	2.44	1.97	40.0	66.0	78.7	83.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and no CS dependence of errors. See notes to Table S1.

Table S24: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.89	-0.21	-0.12	-0.08	4.78	2.09	1.35	0.99	63.9	75.4	78.8	81.2	90.5	93.4	94.7	94.3
50	-0.77	-0.21	-0.10	-0.05	3.60	1.59	1.00	0.75	62.5	75.3	79.5	80.9	89.9	93.6	95.3	94.5
100	-0.78	-0.19	-0.06	-0.04	2.59	1.13	0.70	0.52	61.8	74.1	80.7	81.2	87.9	93.2	93.8	94.1
200	-0.84	-0.19	-0.08	-0.05	1.92	0.78	0.49	0.35	58.1	75.6	80.0	82.9	87.6	93.3	94.2	94.4
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-3.63	-1.31	-0.64	-0.66	7.58	4.56	9.49	3.31	60.5	71.6	74.1	74.5	66.6	79.2	82.6	82.4
50	-3.37	-1.41	-0.90	-0.61	5.94	3.39	3.06	2.67	56.9	67.8	71.0	72.2	62.9	75.7	79.1	80.1
100	-3.34	-1.25	-0.86	-0.58	4.85	3.03	2.18	1.92	46.5	63.5	67.2	68.9	52.6	71.2	73.8	75.6
200	-3.27	-1.28	-0.77	-0.61	7.15	2.13	1.66	1.56	29.9	56.0	64.3	67.5	36.2	62.8	70.3	72.6
	SPMG estimator								Standard				Robust bootstrapped			
30	0.27	0.06	-0.01	-0.02	5.68	2.22	1.40	1.02	55.4	71.6	74.9	78.9	86.8	92.2	93.7	93.5
50	0.32	0.03	0.00	0.00	4.14	1.69	1.03	0.76	53.5	71.0	77.8	79.3	86.9	93.2	94.8	94.5
100	0.23	0.06	0.06	0.03	2.91	1.17	0.73	0.53	53.3	70.2	76.3	80.6	85.3	92.1	93.4	93.4
200	0.15	0.04	0.02	0.01	2.00	0.80	0.50	0.36	54.0	72.1	77.7	81.7	85.2	92.3	94.5	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-6.37	-3.39	-2.45	-1.99	8.45	4.71	3.57	3.06	64.7	69.1	71.7	73.3
50	-6.12	-3.49	-2.48	-1.95	7.49	4.44	3.25	2.62	54.3	59.1	61.9	66.7
100	-6.03	-3.34	-2.41	-1.93	6.81	3.87	2.89	2.37	38.3	44.9	49.4	55.7
200	-6.14	-3.35	-2.41	-1.90	6.50	3.61	2.64	2.14	19.8	21.5	27.6	34.8
	PDOLS estimator, leads and lags order $p = 4$											
30	-4.24	-2.23	-1.65	-1.40	8.01	4.19	3.20	2.80	74.1	81.5	81.4	83.8
50	-4.03	-2.34	-1.71	-1.37	6.69	3.79	2.81	2.29	71.2	76.8	77.9	81.6
100	-4.05	-2.23	-1.65	-1.36	5.56	3.08	2.36	1.99	65.2	71.6	73.9	76.9
200	-4.18	-2.25	-1.66	-1.34	4.92	2.68	2.02	1.69	50.4	56.7	59.5	66.3
	PDOLS estimator, leads and lags order $p = 8$											
30	-3.14	-1.61	-1.25	-1.10	10.92	4.40	3.27	2.83	64.6	83.1	84.6	87.0
50	-2.80	-1.77	-1.32	-1.09	8.53	3.88	2.78	2.25	66.2	81.9	84.1	85.8
100	-3.04	-1.69	-1.28	-1.08	6.50	2.96	2.26	1.91	64.7	79.2	82.1	85.1
200	-3.14	-1.68	-1.29	-1.07	5.06	2.40	1.82	1.53	61.1	74.7	78.2	81.0
	MGMW estimator, $q = 5$											
30	-7.15	-2.98	-1.96	-1.56	11.45	6.51	4.81	4.47	79.1	86.8	88.8	90.1
50	-7.00	-3.09	-1.94	-1.49	10.18	5.45	4.03	3.47	73.2	82.9	86.9	89.8
100	-6.70	-2.83	-1.82	-1.37	8.34	4.29	3.16	2.65	62.5	76.6	83.2	86.0
200	-6.98	-2.96	-1.87	-1.39	7.82	3.65	2.57	2.14	38.0	63.9	74.6	81.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and SAR CS dependence of errors. See notes to Table S1.

Table S25: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.25	-0.02	-0.01	-0.01	3.75	1.67	1.06	0.81	51.9	62.3	68.2	69.3	90.1	94.1	94.6	93.6
50	-0.38	-0.08	-0.01	-0.01	2.83	1.30	0.79	0.61	52.5	62.7	67.4	69.8	90.3	93.4	95.3	95.4
100	-0.28	-0.07	-0.01	-0.01	2.05	0.97	0.60	0.46	49.5	60.4	65.5	65.3	91.0	94.1	95.0	94.9
200	-0.21	-0.04	-0.04	-0.01	1.58	0.71	0.45	0.33	45.2	58.4	60.1	62.2	91.9	94.8	95.4	96.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-1.65	-0.59	-0.30	-0.30	5.01	3.54	2.47	2.26	55.5	63.6	66.3	65.9	75.0	82.9	85.0	83.1
50	-1.69	-0.72	-0.47	-0.31	4.08	2.42	1.95	1.79	53.5	60.8	64.2	64.3	74.1	82.1	84.6	83.2
100	-1.68	-0.61	-0.47	-0.30	3.29	2.22	2.05	1.50	47.6	57.2	60.3	62.6	72.2	82.5	83.0	82.4
200	-1.60	-0.60	-0.35	-0.32	2.82	1.44	2.62	0.98	40.2	54.6	56.4	62.2	73.6	85.2	85.1	85.4
	SPMG estimator								Standard				Robust bootstrapped			
30	0.06	0.07	0.04	0.01	5.93	1.81	1.11	0.84	43.6	58.1	65.6	65.9	85.3	92.4	94.1	93.9
50	-0.09	0.04	0.03	0.01	3.64	1.43	0.83	0.63	42.2	59.5	65.0	67.9	86.1	90.9	94.8	94.2
100	0.03	0.04	0.04	0.02	3.10	1.04	0.63	0.47	41.2	56.5	62.0	64.1	85.5	92.4	94.2	94.1
200	0.01	0.05	0.01	0.01	4.06	0.79	0.47	0.35	37.9	54.2	57.8	60.6	85.0	92.0	94.0	95.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-3.13	-1.81	-1.31	-1.07	5.23	3.10	2.27	2.02	67.6	70.5	73.1	75.7
50	-3.31	-1.91	-1.37	-1.12	4.64	2.75	2.08	1.68	60.6	66.0	68.9	71.0
100	-3.16	-1.87	-1.38	-1.09	4.06	2.41	1.82	1.46	51.7	56.0	59.6	64.3
200	-3.16	-1.85	-1.35	-1.07	3.71	2.18	1.61	1.31	36.7	39.6	42.7	50.0
	PDOLS estimator, leads and lags order $p = 4$											
30	-2.37	-1.31	-0.92	-0.78	5.76	3.02	2.11	1.93	68.4	76.6	78.1	79.8
50	-2.56	-1.38	-1.00	-0.83	4.81	2.54	1.88	1.50	66.8	75.8	77.3	79.8
100	-2.43	-1.37	-1.00	-0.80	3.97	2.10	1.56	1.26	61.6	69.1	73.7	77.2
200	-2.43	-1.35	-1.00	-0.79	3.44	1.79	1.31	1.07	52.5	60.7	62.9	70.4
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.80	-0.98	-0.68	-0.59	8.93	3.42	2.23	1.98	53.4	75.4	79.0	81.2
50	-1.99	-1.05	-0.76	-0.64	6.98	2.78	1.93	1.48	53.8	74.9	79.6	83.7
100	-1.96	-1.08	-0.79	-0.61	5.71	2.19	1.55	1.22	51.9	73.1	78.7	82.1
200	-1.86	-1.03	-0.78	-0.62	4.76	1.79	1.22	0.99	44.1	68.7	73.6	79.6
	MGMW estimator, $q = 5$											
30	-1.21	-0.57	-0.28	-0.20	6.18	3.59	2.85	2.60	82.1	83.2	85.0	85.5
50	-1.29	-0.57	-0.38	-0.33	4.89	3.00	2.14	1.92	81.6	82.4	85.6	86.9
100	-0.92	-0.54	-0.33	-0.24	3.55	2.19	1.71	1.37	80.4	82.2	86.6	86.9
200	-0.92	-0.52	-0.31	-0.18	2.77	1.62	1.22	1.02	78.4	81.7	83.7	88.5

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S26: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.2$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.99	-0.16	-0.10	-0.07	4.39	1.83	1.16	0.83	70.8	84.5	88.3	90.2	90.5	94.4	93.6	94.2
50	-0.79	-0.24	-0.11	-0.05	3.24	1.39	0.83	0.60	71.4	84.8	89.9	91.6	90.3	92.9	95.6	95.9
100	-0.90	-0.18	-0.08	-0.05	2.42	0.97	0.60	0.44	65.5	83.9	89.1	90.0	89.0	93.4	94.3	94.3
200	-0.94	-0.20	-0.09	-0.05	1.80	0.69	0.41	0.30	63.1	83.4	89.7	91.0	88.2	93.6	94.9	95.0
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-7.28	-3.16	-2.23	-2.01	11.94	8.51	6.89	10.95	51.3	64.5	68.0	70.2	50.1	64.4	67.5	70.4
50	-6.89	-3.24	-2.30	-1.75	10.02	6.21	5.85	5.00	44.3	61.8	65.5	69.0	43.7	62.2	65.1	69.1
100	-6.66	-3.07	-2.26	-1.84	8.90	5.50	5.43	4.85	29.1	52.5	58.8	63.7	29.5	53.3	58.9	63.9
200	-6.85	-3.17	-2.18	-1.93	8.70	4.35	3.76	4.02	13.8	42.1	52.8	58.3	14.2	42.0	53.3	59.0
	SPMG estimator								Standard				Robust bootstrapped			
30	0.42	0.19	0.05	0.01	5.22	1.98	1.22	0.85	59.4	79.3	85.9	88.2	87.7	93.4	93.1	93.8
50	0.55	0.07	0.02	0.02	3.82	1.47	0.86	0.62	59.1	80.0	88.0	90.7	87.1	93.1	95.4	95.5
100	0.32	0.13	0.06	0.03	2.70	1.03	0.62	0.45	57.2	79.8	87.3	88.3	86.0	92.6	94.5	94.8
200	0.24	0.09	0.03	0.02	1.80	0.70	0.42	0.30	61.7	81.2	88.1	89.8	87.1	92.7	94.8	94.9

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-10.38	-6.26	-4.79	-3.93	12.58	8.17	6.56	5.65	65.9	74.4	78.4	83.9
50	-10.29	-6.32	-4.76	-4.01	11.71	7.48	5.89	5.10	54.7	64.2	70.6	75.7
100	-10.08	-6.15	-4.76	-4.05	10.86	6.79	5.43	4.70	38.1	45.3	55.1	61.8
200	-10.21	-6.14	-4.73	-4.00	10.59	6.47	5.05	4.33	27.5	26.2	31.1	38.4
	PDOLS estimator, leads and lags order $p = 4$											
30	-8.87	-5.44	-4.20	-3.48	12.39	8.02	6.40	5.53	72.5	83.4	84.5	87.7
50	-8.91	-5.50	-4.18	-3.57	11.32	7.09	5.61	4.89	67.7	78.0	80.2	82.7
100	-8.73	-5.36	-4.19	-3.63	10.02	6.25	5.05	4.43	57.1	67.3	69.1	73.9
200	-8.88	-5.34	-4.17	-3.57	9.51	5.81	4.59	3.99	40.8	50.1	52.3	56.4
	PDOLS estimator, leads and lags order $p = 8$											
30	-8.07	-5.15	-3.97	-3.30	14.73	8.53	6.63	5.68	63.7	84.2	85.5	89.7
50	-8.30	-5.23	-3.95	-3.41	13.05	7.36	5.73	4.97	61.9	80.7	84.2	85.7
100	-8.11	-5.09	-3.99	-3.48	10.69	6.32	5.08	4.45	53.7	72.1	76.5	80.6
200	-8.23	-5.05	-3.96	-3.42	9.59	5.71	4.50	3.93	38.9	60.2	64.3	67.2
	MGMW estimator, $q = 5$											
30	-10.81	-5.75	-4.20	-3.51	15.78	10.00	9.05	7.84	77.0	87.3	90.5	92.5
50	-10.66	-5.96	-4.35	-3.56	13.83	8.92	7.19	6.15	69.7	79.9	86.4	88.6
100	-10.70	-5.87	-4.28	-3.67	12.28	7.48	5.94	5.27	51.4	70.2	74.3	78.6
200	-10.88	-5.90	-4.24	-3.55	11.68	6.73	5.14	4.47	25.7	49.7	62.3	66.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.2$, and no CS dependence of errors. See notes to Table S1.

Table S27: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.2$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-1.17	-0.26	-0.14	-0.10	5.30	2.27	1.46	1.06	63.8	76.0	79.6	81.6	90.3	93.3	94.1	94.2
50	-1.00	-0.29	-0.12	-0.06	3.97	1.70	1.06	0.79	61.6	76.3	81.2	82.1	90.6	93.9	95.1	94.9
100	-1.03	-0.21	-0.07	-0.04	2.85	1.19	0.74	0.55	60.2	75.9	80.8	81.9	88.0	94.1	94.3	94.1
200	-1.06	-0.23	-0.10	-0.06	2.13	0.84	0.52	0.38	55.9	76.1	80.9	82.8	87.0	93.4	94.0	94.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-6.18	-3.36	-2.33	-1.84	50.67	8.29	11.62	6.90	48.6	62.0	64.7	66.4	53.9	68.2	70.8	71.2
50	-7.11	-3.58	-2.70	-2.24	10.34	6.47	6.36	7.40	41.8	56.9	62.7	65.3	45.8	62.5	68.0	69.4
100	-6.95	-3.32	-2.40	-1.87	9.25	5.62	4.66	6.12	30.1	50.2	56.3	60.9	34.1	55.7	60.7	64.5
200	-7.04	-3.30	-2.29	-2.04	10.11	4.58	3.75	3.73	14.0	38.6	50.4	55.9	16.1	42.9	54.7	59.0
	SPMG estimator								Standard				Robust bootstrapped			
30	0.35	0.10	0.01	-0.01	6.27	2.40	1.52	1.10	55.4	72.2	76.5	79.2	86.8	92.7	93.2	93.7
50	0.39	0.03	0.01	0.01	4.60	1.82	1.10	0.81	54.3	71.9	78.6	80.3	86.7	93.6	94.8	94.7
100	0.24	0.10	0.08	0.04	3.16	1.25	0.76	0.57	53.1	71.6	77.9	80.6	85.0	92.7	94.4	93.9
200	0.19	0.07	0.03	0.02	2.15	0.86	0.53	0.38	53.5	73.8	79.0	81.8	87.1	93.1	94.3	94.6

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-10.56	-6.45	-5.00	-4.21	13.05	8.52	6.90	6.10	62.6	72.3	76.1	80.7
50	-10.48	-6.57	-5.07	-4.29	12.10	7.86	6.27	5.43	53.6	61.7	67.7	73.8
100	-10.17	-6.31	-4.95	-4.29	11.06	7.04	5.66	4.97	37.7	46.8	53.2	59.8
200	-10.40	-6.29	-4.92	-4.19	10.84	6.65	5.27	4.55	26.6	25.5	31.3	37.3
	PDOLS estimator, leads and lags order $p = 4$											
30	-9.03	-5.59	-4.39	-3.74	13.01	8.36	6.74	5.97	70.2	79.7	81.9	85.3
50	-9.05	-5.72	-4.46	-3.83	11.76	7.47	5.99	5.21	63.8	73.4	77.7	79.8
100	-8.80	-5.50	-4.37	-3.85	10.28	6.51	5.28	4.69	56.2	65.2	67.1	70.6
200	-9.05	-5.47	-4.34	-3.75	9.78	5.97	4.80	4.20	39.7	47.7	50.1	54.3
	PDOLS estimator, leads and lags order $p = 8$											
30	-8.33	-5.27	-4.14	-3.56	15.81	8.89	7.00	6.15	59.7	80.5	83.4	86.5
50	-8.26	-5.43	-4.23	-3.66	13.63	7.76	6.13	5.29	59.0	78.3	80.9	83.3
100	-8.23	-5.24	-4.18	-3.71	11.18	6.63	5.32	4.72	51.6	70.2	74.4	76.6
200	-8.44	-5.17	-4.14	-3.60	9.95	5.87	4.71	4.14	38.3	57.3	61.3	64.5
	MGMW estimator, $q = 5$											
30	-11.19	-6.16	-4.71	-3.95	16.04	10.77	9.03	8.39	75.7	83.8	86.7	88.9
50	-10.97	-6.20	-4.47	-3.90	14.58	9.16	7.41	6.67	66.6	78.5	83.1	84.9
100	-10.65	-5.99	-4.48	-3.78	12.48	7.64	6.16	5.46	50.6	67.5	72.9	76.3
200	-10.98	-6.06	-4.53	-3.79	11.92	6.90	5.41	4.69	25.7	45.7	57.5	63.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.2$, and SAR CS dependence of errors. See notes to Table S1.

Table S28: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.2$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-0.35	-0.02	-0.03	-0.01	3.98	1.76	1.11	0.85	54.0	64.5	70.6	70.5	90.2	93.7	94.8	94.1
50	-0.47	-0.11	-0.02	-0.02	3.02	1.36	0.81	0.63	53.4	64.5	69.7	72.1	90.8	94.0	95.6	95.8
100	-0.36	-0.07	-0.01	-0.01	2.18	1.01	0.63	0.48	51.2	62.7	68.0	66.8	91.5	94.5	95.1	95.1
200	-0.28	-0.05	-0.04	-0.02	1.67	0.74	0.48	0.35	46.0	58.8	61.7	63.2	92.2	95.1	95.2	95.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-3.36	-1.65	-1.07	-1.03	7.59	5.94	4.81	4.87	51.7	61.4	62.7	63.4	66.1	75.9	75.9	74.6
50	-3.43	-1.85	-1.27	-1.10	7.55	4.72	3.79	3.54	43.4	56.4	60.1	60.9	59.6	72.2	73.4	73.4
100	-3.39	-1.68	-1.41	-1.12	5.26	3.77	3.48	3.08	38.7	52.0	56.8	60.6	55.8	69.4	69.9	72.4
200	-3.29	-1.66	-1.20	-1.14	4.78	3.01	3.49	2.25	29.9	45.7	50.4	54.0	50.2	68.0	68.7	69.7
	SPMG estimator								Standard				Robust bootstrapped			
30	0.02	0.10	0.04	0.02	7.15	1.91	1.16	0.89	44.2	61.4	67.7	67.6	85.0	92.5	94.7	93.8
50	-0.12	0.03	0.04	0.02	4.45	1.49	0.87	0.65	43.1	60.7	67.4	70.2	85.6	91.1	95.2	95.0
100	-0.12	0.07	0.05	0.03	7.04	1.10	0.66	0.49	41.4	58.7	64.7	65.6	85.2	92.6	94.0	94.4
200	-0.08	0.07	0.02	0.01	5.90	0.83	0.50	0.36	39.0	56.0	59.5	61.9	85.0	92.3	93.8	95.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.92	-3.12	-2.47	-2.03	7.38	4.91	3.98	3.57	73.1	79.3	81.1	86.9
50	-5.19	-3.25	-2.53	-2.22	6.79	4.41	3.59	3.16	65.3	74.9	77.9	80.8
100	-4.98	-3.18	-2.58	-2.20	6.01	3.90	3.25	2.80	56.0	65.4	66.0	71.5
200	-4.94	-3.17	-2.52	-2.15	5.60	3.62	2.92	2.55	45.5	46.2	50.7	58.2
	PDOLS estimator, leads and lags order $p = 4$											
30	-4.52	-2.79	-2.19	-1.81	8.21	5.06	3.99	3.60	73.3	83.5	85.2	89.0
50	-4.81	-2.91	-2.26	-2.00	7.20	4.41	3.51	3.08	69.3	81.5	84.4	86.7
100	-4.63	-2.87	-2.32	-1.99	6.18	3.77	3.11	2.67	62.9	76.4	77.1	80.2
200	-4.55	-2.85	-2.27	-1.94	5.62	3.42	2.74	2.39	52.4	64.3	66.0	70.4
	PDOLS estimator, leads and lags order $p = 8$											
30	-4.22	-2.65	-2.06	-1.70	11.34	5.67	4.25	3.81	56.5	81.3	86.6	89.7
50	-4.49	-2.77	-2.15	-1.90	9.38	4.83	3.70	3.16	53.2	80.5	86.1	88.1
100	-4.42	-2.79	-2.23	-1.90	7.76	4.03	3.20	2.71	50.4	76.5	80.6	84.9
200	-4.28	-2.73	-2.16	-1.85	6.71	3.54	2.74	2.38	41.2	67.3	72.6	77.7
	MGMW estimator, $q = 5$											
30	-1.78	-1.01	-0.75	-0.43	8.03	5.41	4.70	4.41	85.0	87.3	89.6	91.8
50	-1.92	-0.96	-0.75	-0.62	6.48	4.47	3.77	3.46	83.2	86.4	89.2	90.8
100	-1.38	-0.90	-0.72	-0.58	4.76	3.34	2.95	2.59	80.2	85.3	86.7	89.0
200	-1.26	-0.93	-0.60	-0.46	3.81	2.70	2.21	2.02	78.7	80.9	84.0	86.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S29: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-5.24	-2.67	-1.71	-1.24	6.37	3.14	1.99	1.43	32.7	36.3	41.0	43.0	57.4	51.8	54.2	53.0
50	-5.01	-2.48	-1.62	-1.18	5.67	2.74	1.79	1.30	19.1	22.1	24.0	27.1	41.6	37.5	34.9	35.5
100	-4.89	-2.50	-1.63	-1.18	5.26	2.63	1.71	1.24	5.0	3.5	4.3	5.2	16.5	8.6	8.0	8.6
200	-4.77	-2.46	-1.60	-1.17	4.96	2.53	1.64	1.20	0.3	0.0	0.1	0.0	2.3	0.6	0.4	0.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-2.49	-0.74	-0.34	-0.19	3.94	1.64	1.02	0.72	71.2	84.2	89.0	90.4	70.3	83.5	89.5	90.5
50	-2.50	-0.64	-0.28	-0.15	3.46	1.29	0.77	0.55	63.6	84.6	90.3	91.6	61.7	84.4	89.7	91.5
100	-2.41	-0.69	-0.30	-0.17	2.94	1.05	0.61	0.42	50.6	77.0	85.6	89.2	50.1	77.3	85.2	89.4
200	-2.36	-0.68	-0.30	-0.17	2.64	0.87	0.47	0.31	30.4	68.1	80.9	86.2	29.8	66.9	80.2	85.9
	SPMG estimator								Standard				Robust bootstrapped			
30	0.08	-0.03	-0.01	0.00	3.40	1.42	0.88	0.63	69.0	83.1	87.7	90.2	89.1	92.1	94.2	93.6
50	0.13	0.07	0.04	0.03	2.58	1.06	0.66	0.48	67.7	84.5	88.4	89.9	90.2	94.6	94.0	94.5
100	0.12	0.00	0.00	0.00	1.78	0.73	0.46	0.33	68.7	85.4	87.7	90.2	89.1	94.2	93.7	94.6
200	0.16	0.02	0.01	0.00	1.28	0.53	0.32	0.23	68.0	83.7	89.3	90.5	89.8	93.0	95.2	95.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.65	-2.29	-1.47	-1.06	5.53	2.74	1.77	1.28	59.3	62.6	62.5	63.5
50	-4.67	-2.17	-1.39	-1.02	5.24	2.45	1.58	1.16	43.5	48.1	47.0	50.6
100	-4.54	-2.20	-1.41	-1.02	4.85	2.35	1.51	1.09	20.0	19.7	20.1	22.6
200	-4.47	-2.19	-1.40	-1.02	4.62	2.26	1.45	1.05	4.7	2.5	2.6	2.6
	PDOLS estimator, leads and lags order $p = 4$											
30	-1.67	-0.74	-0.47	-0.32	4.18	1.77	1.11	0.79	84.5	89.1	89.6	89.8
50	-1.64	-0.65	-0.40	-0.29	3.38	1.38	0.85	0.62	83.7	90.0	90.4	90.1
100	-1.56	-0.68	-0.42	-0.30	2.63	1.11	0.70	0.50	80.6	84.3	83.8	85.5
200	-1.53	-0.68	-0.42	-0.30	2.12	0.91	0.56	0.41	75.0	76.2	77.4	76.1
	PDOLS estimator, leads and lags order $p = 8$											
30	-0.46	-0.26	-0.16	-0.10	7.11	1.99	1.13	0.79	73.0	90.6	92.0	91.8
50	-0.58	-0.17	-0.09	-0.07	5.44	1.54	0.86	0.60	74.8	90.4	92.8	94.1
100	-0.45	-0.20	-0.12	-0.08	3.85	1.09	0.64	0.44	76.3	90.3	90.9	92.1
200	-0.59	-0.21	-0.12	-0.09	2.75	0.78	0.44	0.31	77.7	90.4	91.2	91.9
	MGMW estimator, $q = 5$											
30	-5.11	-1.51	-0.67	-0.39	7.48	3.18	2.00	1.45	81.3	89.0	92.2	92.9
50	-5.00	-1.42	-0.64	-0.36	6.57	2.56	1.58	1.14	73.8	89.6	93.1	93.1
100	-4.92	-1.50	-0.69	-0.38	5.73	2.12	1.23	0.86	58.8	82.3	89.9	91.9
200	-4.87	-1.49	-0.71	-0.39	5.30	1.83	1.00	0.66	33.5	70.6	82.6	89.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and no CS dependence of errors. See notes to Table S1.

Table S30: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-5.57	-2.83	-1.84	-1.34	7.14	3.51	2.26	1.66	34.0	37.9	40.6	41.8	66.0	64.0	64.5	64.9
50	-5.36	-2.64	-1.74	-1.29	6.31	3.06	1.99	1.48	21.4	25.7	26.9	28.5	52.1	50.6	50.9	52.0
100	-5.14	-2.62	-1.72	-1.26	5.66	2.83	1.86	1.36	7.2	7.2	7.7	9.2	27.0	22.2	21.4	21.6
200	-5.09	-2.55	-1.67	-1.23	5.34	2.66	1.74	1.28	1.1	0.6	0.8	0.5	6.8	4.3	3.8	3.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-2.64	-0.79	-0.37	-0.20	4.73	2.08	1.34	0.98	65.0	75.7	78.7	80.1	72.4	85.7	89.5	91.0
50	-2.62	-0.67	-0.31	-0.18	4.01	1.64	1.01	0.74	57.0	74.6	80.1	81.5	65.2	84.9	90.0	90.8
100	-2.49	-0.75	-0.34	-0.19	3.27	1.28	0.77	0.55	48.8	68.8	76.2	79.1	56.8	79.4	86.1	88.8
200	-2.46	-0.72	-0.32	-0.19	2.87	1.01	0.57	0.39	30.9	62.3	72.7	76.3	38.2	72.7	83.8	87.9
	SPMG estimator								Standard				Robust bootstrapped			
30	0.10	0.00	0.00	0.01	4.24	1.80	1.18	0.87	58.7	72.8	76.3	78.1	88.7	93.3	93.7	92.9
50	0.18	0.09	0.04	0.02	3.28	1.39	0.86	0.64	57.9	72.3	78.1	80.1	89.3	93.2	94.4	95.0
100	0.13	-0.01	-0.01	-0.01	2.22	0.96	0.61	0.45	58.6	73.2	77.2	78.8	88.5	92.6	93.2	94.2
200	0.12	0.02	0.01	0.00	1.57	0.67	0.42	0.31	58.9	73.5	76.9	79.7	88.5	92.6	94.1	94.2

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.79	-2.39	-1.58	-1.14	6.18	3.11	2.06	1.51	56.7	58.1	56.6	57.8
50	-4.81	-2.26	-1.49	-1.10	5.68	2.73	1.79	1.33	41.5	46.8	45.7	46.7
100	-4.59	-2.28	-1.48	-1.08	5.06	2.52	1.64	1.21	24.2	23.9	23.9	24.5
200	-4.55	-2.25	-1.45	-1.07	4.78	2.37	1.53	1.13	6.8	4.8	5.8	6.0
	PDOLS estimator, leads and lags order $p = 4$											
30	-1.70	-0.75	-0.50	-0.34	5.13	2.22	1.44	1.05	76.3	80.9	80.4	81.1
50	-1.64	-0.66	-0.43	-0.32	4.04	1.74	1.10	0.81	77.7	80.1	81.1	79.7
100	-1.54	-0.71	-0.45	-0.33	3.08	1.34	0.86	0.63	73.5	75.3	75.0	76.2
200	-1.55	-0.71	-0.43	-0.32	2.38	1.05	0.66	0.48	70.4	69.4	70.0	69.7
	PDOLS estimator, leads and lags order $p = 8$											
30	-0.63	-0.25	-0.17	-0.09	8.23	2.53	1.50	1.07	67.1	81.7	81.5	82.7
50	-0.57	-0.15	-0.09	-0.08	6.47	1.98	1.14	0.80	68.8	81.7	83.9	82.7
100	-0.47	-0.20	-0.13	-0.09	4.55	1.39	0.83	0.59	69.1	81.1	80.9	82.1
200	-0.57	-0.23	-0.12	-0.09	3.16	0.98	0.58	0.40	71.2	81.2	82.4	82.2
	MGMW estimator, $q = 5$											
30	-5.01	-1.64	-0.71	-0.37	7.88	3.71	2.38	1.69	75.7	84.7	86.1	87.6
50	-5.12	-1.42	-0.64	-0.39	6.93	2.82	1.83	1.35	70.1	84.0	87.1	87.8
100	-4.92	-1.54	-0.71	-0.40	5.94	2.35	1.39	0.99	57.2	76.4	84.2	86.4
200	-4.84	-1.51	-0.71	-0.42	5.36	1.95	1.08	0.75	35.2	68.1	79.4	83.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and SAR CS dependence of errors. See notes to Table S1.

Table S31: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-1.67	-0.83	-0.60	-0.44	3.24	1.62	1.07	0.80	48.8	54.6	55.2	55.9	85.4	87.8	88.0	87.7
50	-1.67	-0.82	-0.55	-0.41	2.83	1.36	0.89	0.67	42.9	48.1	49.8	51.7	83.3	85.2	85.8	86.0
100	-1.53	-0.83	-0.54	-0.40	2.30	1.18	0.77	0.58	33.8	35.2	36.8	39.9	78.0	78.5	79.3	80.0
200	-1.52	-0.80	-0.53	-0.40	2.08	1.07	0.70	0.52	24.8	25.9	25.7	26.0	70.9	70.5	71.0	70.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-1.08	-0.29	-0.17	-0.09	3.05	1.45	0.94	0.68	60.0	65.7	67.3	70.5	79.2	87.4	89.9	91.2
50	-1.20	-0.28	-0.12	-0.06	2.63	1.11	0.70	0.52	57.3	65.8	70.4	71.6	76.0	87.5	90.8	91.9
100	-1.07	-0.33	-0.15	-0.08	2.05	0.85	0.52	0.38	51.5	62.9	66.1	69.1	74.2	86.3	90.0	91.5
200	-1.07	-0.31	-0.13	-0.08	1.74	0.66	0.39	0.27	46.0	58.0	65.0	68.6	68.4	84.6	90.1	92.2
	SPMG estimator								Standard				Robust bootstrapped			
30	0.09	0.05	-0.02	-0.01	3.04	1.33	0.83	0.61	50.2	60.9	64.1	66.3	88.3	92.4	93.4	92.7
50	0.01	0.03	0.02	0.02	2.28	0.99	0.63	0.46	50.0	61.5	67.2	67.7	89.1	91.9	93.2	93.7
100	0.09	-0.02	0.00	0.00	1.66	0.69	0.44	0.32	49.2	59.9	64.8	66.3	87.0	92.5	93.5	94.4
200	0.04	0.01	0.01	0.01	1.37	0.49	0.31	0.23	48.4	61.4	65.6	66.7	89.1	92.3	93.2	94.3

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-2.10	-1.06	-0.75	-0.54	3.55	1.84	1.26	0.92	62.9	62.0	61.6	60.4
50	-2.24	-1.03	-0.69	-0.52	3.28	1.57	1.03	0.79	54.0	55.6	55.9	56.4
100	-2.06	-1.06	-0.70	-0.52	2.74	1.39	0.92	0.69	43.1	40.5	40.9	40.7
200	-2.05	-1.04	-0.68	-0.52	2.51	1.25	0.84	0.63	27.2	23.8	25.3	23.6
	PDOLS estimator, leads and lags order $p = 4$											
30	-0.98	-0.45	-0.33	-0.22	3.74	1.61	1.04	0.74	66.6	68.4	68.6	70.8
50	-1.17	-0.42	-0.27	-0.20	3.14	1.26	0.79	0.58	64.3	69.5	68.6	70.7
100	-1.01	-0.48	-0.30	-0.21	2.36	1.00	0.62	0.45	61.4	65.0	64.4	66.2
200	-1.06	-0.47	-0.28	-0.21	1.92	0.79	0.49	0.35	55.9	56.2	59.2	58.7
	PDOLS estimator, leads and lags order $p = 8$											
30	-0.24	-0.17	-0.16	-0.09	6.65	1.91	1.12	0.77	53.6	70.3	67.8	71.5
50	-0.49	-0.14	-0.07	-0.06	5.17	1.45	0.84	0.58	54.0	70.8	72.0	73.0
100	-0.36	-0.19	-0.14	-0.07	3.83	1.06	0.62	0.43	53.0	67.5	68.2	71.1
200	-0.57	-0.19	-0.10	-0.09	2.94	0.77	0.44	0.31	51.2	67.0	68.5	69.5
	MGMW estimator, $q = 5$											
30	-1.26	-0.36	-0.26	-0.17	4.01	2.09	1.47	1.06	76.8	78.3	74.9	76.5
50	-1.44	-0.40	-0.19	-0.09	3.43	1.65	1.08	0.80	73.8	77.0	78.3	78.5
100	-1.21	-0.46	-0.22	-0.11	2.54	1.22	0.80	0.57	72.6	74.9	76.6	78.6
200	-1.23	-0.45	-0.20	-0.14	2.13	0.96	0.58	0.43	66.9	72.0	78.3	78.1

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S32: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-5.34	-2.72	-1.74	-1.26	6.54	3.20	2.03	1.46	33.2	37.8	42.6	44.4	59.3	52.5	53.9	53.6
50	-5.09	-2.51	-1.64	-1.19	5.78	2.79	1.81	1.32	20.2	23.2	24.1	28.5	43.0	38.9	36.2	37.2
100	-4.97	-2.52	-1.64	-1.19	5.37	2.66	1.73	1.25	5.3	3.8	4.5	6.2	17.8	9.2	8.8	9.4
200	-4.84	-2.48	-1.62	-1.17	5.03	2.56	1.66	1.21	0.4	0.1	0.1	0.2	2.7	0.7	0.4	0.4
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-4.17	-1.99	-1.35	-1.11	6.82	4.61	4.50	3.20	61.2	69.7	74.8	76.8	60.5	69.0	74.7	77.0
50	-4.25	-1.80	-1.35	-1.06	6.30	3.54	3.30	2.76	49.2	65.5	69.6	70.9	48.5	65.6	69.7	70.5
100	-4.09	-1.86	-1.27	-1.01	5.14	2.96	2.31	2.14	36.4	54.9	62.3	64.3	36.0	55.1	63.1	64.7
200	-4.03	-1.94	-1.31	-1.05	4.65	2.59	2.15	1.86	17.6	37.5	49.8	53.8	17.7	37.4	49.0	54.4
	SPMG estimator								Standard				Robust bootstrapped			
30	0.10	-0.04	-0.02	-0.01	3.56	1.46	0.91	0.64	68.0	83.5	88.2	89.7	89.0	92.5	94.2	94.0
50	0.12	0.07	0.04	0.03	2.65	1.09	0.68	0.49	67.5	83.5	89.1	89.8	90.5	94.3	94.6	94.7
100	0.13	0.00	0.00	0.00	1.84	0.75	0.47	0.34	68.2	84.0	87.8	89.3	89.1	94.6	93.3	93.8
200	0.17	0.02	0.01	0.00	1.30	0.54	0.33	0.24	68.7	84.0	89.1	90.3	89.8	93.3	94.8	94.9

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-6.40	-3.74	-2.67	-2.19	8.06	5.07	3.86	3.42	61.1	67.6	70.9	73.0
50	-6.43	-3.51	-2.58	-2.12	7.50	4.38	3.38	2.99	46.3	56.3	60.9	66.2
100	-6.27	-3.52	-2.58	-2.11	6.87	3.99	3.08	2.60	26.7	34.5	41.3	49.9
200	-6.22	-3.57	-2.59	-2.10	6.54	3.83	2.84	2.33	15.7	16.7	23.6	30.6
	PDOLS estimator, leads and lags order $p = 4$											
30	-3.50	-2.29	-1.72	-1.48	6.93	4.46	3.42	3.12	81.5	86.5	89.2	90.7
50	-3.54	-2.06	-1.62	-1.42	5.83	3.54	2.85	2.63	78.5	86.0	88.9	89.3
100	-3.46	-2.09	-1.64	-1.42	4.83	2.92	2.42	2.14	73.5	78.8	79.2	84.3
200	-3.40	-2.15	-1.66	-1.41	4.16	2.63	2.07	1.78	61.1	63.1	66.4	68.5
	PDOLS estimator, leads and lags order $p = 8$											
30	-2.24	-1.83	-1.41	-1.25	9.45	4.80	3.50	3.15	72.1	88.2	91.1	91.6
50	-2.52	-1.59	-1.33	-1.20	7.48	3.66	2.88	2.65	72.5	87.4	90.9	91.6
100	-2.38	-1.63	-1.35	-1.22	5.54	2.86	2.34	2.09	74.0	86.2	84.9	88.7
200	-2.39	-1.70	-1.38	-1.20	4.30	2.42	1.92	1.66	69.4	77.9	79.2	80.7
	MGMW estimator, $q = 5$											
30	-6.58	-2.80	-1.75	-1.41	9.77	5.46	4.15	3.62	80.3	89.5	93.2	94.8
50	-6.50	-2.66	-1.78	-1.37	8.64	4.53	3.52	3.10	70.7	86.5	91.9	94.2
100	-6.42	-2.81	-1.80	-1.39	7.47	3.82	2.79	2.31	52.6	77.4	85.4	90.0
200	-6.32	-2.80	-1.79	-1.39	6.91	3.34	2.33	1.98	29.4	57.9	71.2	80.5

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and no CS dependence of errors. See notes to Table S1.

Table S33: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-5.71	-2.88	-1.87	-1.36	7.35	3.57	2.30	1.69	34.4	37.5	41.1	42.3	66.0	64.9	64.8	65.2
50	-5.45	-2.67	-1.76	-1.29	6.43	3.10	2.02	1.49	22.2	26.4	27.7	29.1	52.4	52.7	51.9	52.3
100	-5.22	-2.64	-1.74	-1.27	5.75	2.86	1.87	1.37	7.6	8.1	8.4	10.0	27.9	23.0	21.5	22.7
200	-5.16	-2.58	-1.69	-1.24	5.42	2.69	1.76	1.29	1.1	0.8	0.9	0.5	7.0	4.5	4.0	4.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-4.33	-2.07	-1.40	-1.12	7.36	5.41	4.97	4.81	57.9	63.9	68.9	71.6	63.9	71.9	76.3	78.5
50	-4.36	-1.88	-1.46	-1.20	6.42	3.72	3.47	3.09	49.0	62.5	64.6	67.4	53.9	70.6	71.0	73.6
100	-4.17	-1.94	-1.39	-1.14	5.46	3.08	2.53	2.32	36.4	53.1	58.4	60.5	42.5	60.1	65.2	66.3
200	-4.13	-1.97	-1.37	-1.12	5.03	2.72	2.04	1.73	20.0	40.1	48.3	52.9	24.8	45.6	53.8	57.6
	SPMG estimator								Standard				Robust bootstrapped			
30	0.11	-0.01	-0.01	0.01	4.38	1.83	1.20	0.89	58.4	73.1	76.7	79.0	89.1	93.1	93.5	93.2
50	0.18	0.09	0.03	0.02	3.35	1.42	0.87	0.65	58.8	72.3	78.8	79.8	89.3	93.3	94.6	94.7
100	0.14	-0.01	-0.01	0.00	2.27	0.98	0.62	0.46	58.9	72.8	77.5	79.4	88.3	93.1	93.3	93.6
200	0.13	0.02	0.01	0.00	1.60	0.68	0.43	0.32	59.6	74.4	77.7	79.7	87.9	92.7	94.4	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-6.59	-3.89	-2.85	-2.37	8.59	5.41	4.15	3.70	58.5	63.8	65.8	68.5
50	-6.62	-3.66	-2.76	-2.31	7.91	4.65	3.66	3.25	45.6	56.1	58.2	64.5
100	-6.32	-3.63	-2.69	-2.24	7.02	4.15	3.22	2.78	29.4	35.9	41.6	47.8
200	-6.36	-3.68	-2.71	-2.21	6.75	3.97	2.98	2.46	16.5	17.2	24.0	31.0
	PDOLS estimator, leads and lags order $p = 4$											
30	-3.61	-2.35	-1.82	-1.59	7.64	4.78	3.68	3.37	75.8	82.3	83.6	86.4
50	-3.60	-2.13	-1.75	-1.56	6.35	3.78	3.10	2.86	73.7	81.2	83.5	85.3
100	-3.44	-2.14	-1.71	-1.52	5.09	3.07	2.54	2.29	70.0	75.3	76.1	78.9
200	-3.51	-2.22	-1.74	-1.49	4.40	2.76	2.18	1.88	58.5	62.3	64.9	68.0
	PDOLS estimator, leads and lags order $p = 8$											
30	-2.54	-1.86	-1.51	-1.35	10.59	5.14	3.78	3.42	66.2	82.8	85.6	87.2
50	-2.60	-1.63	-1.44	-1.34	8.37	3.90	3.15	2.88	67.3	82.6	86.1	87.3
100	-2.39	-1.66	-1.40	-1.30	6.05	3.02	2.46	2.23	68.2	81.0	82.8	84.7
200	-2.49	-1.75	-1.44	-1.27	4.63	2.57	2.03	1.76	65.9	75.0	75.9	78.1
	MGMW estimator, $q = 5$											
30	-6.43	-2.95	-1.83	-1.50	9.83	5.68	4.57	3.83	75.5	85.5	88.7	91.8
50	-6.65	-2.70	-1.84	-1.51	8.82	4.62	3.82	3.25	68.0	83.2	88.8	90.4
100	-6.40	-2.81	-1.83	-1.41	7.58	3.86	2.85	2.36	52.7	74.6	82.2	86.3
200	-6.32	-2.77	-1.82	-1.45	6.92	3.39	2.42	2.02	30.3	59.6	71.2	76.8

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and SAR CS dependence of errors. See notes to Table S1.

Table S34: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-1.69	-0.84	-0.60	-0.45	3.28	1.64	1.08	0.80	50.1	56.1	55.4	56.6	86.0	88.3	88.1	87.8
50	-1.70	-0.83	-0.56	-0.41	2.87	1.37	0.90	0.67	43.2	49.3	50.7	52.5	83.3	85.0	85.2	86.3
100	-1.55	-0.83	-0.54	-0.41	2.32	1.19	0.78	0.58	34.6	36.7	38.2	40.6	77.9	78.6	79.0	80.3
200	-1.53	-0.80	-0.53	-0.40	2.10	1.08	0.71	0.52	25.5	26.3	25.9	26.2	71.2	70.8	71.0	70.0
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-1.82	-0.88	-0.74	-0.54	4.42	2.78	2.44	2.31	56.0	62.5	63.9	64.9	72.1	78.9	79.2	80.3
50	-1.96	-0.84	-0.65	-0.53	3.97	2.23	2.02	1.95	50.8	61.1	61.5	65.3	67.7	77.4	76.5	76.7
100	-1.80	-0.86	-0.64	-0.53	3.01	1.78	1.48	1.42	45.2	57.0	57.9	60.7	62.3	72.9	72.1	72.8
200	-1.83	-0.84	-0.61	-0.54	2.67	1.65	1.28	1.07	36.2	50.4	55.7	57.2	54.1	65.9	69.2	69.3
	SPMG estimator								Standard				Robust bootstrapped			
30	0.09	0.05	-0.02	-0.01	3.08	1.34	0.83	0.61	51.7	61.7	66.0	67.5	88.9	92.1	93.5	92.9
50	0.00	0.03	0.03	0.02	2.31	1.01	0.64	0.46	50.3	62.5	67.2	69.2	89.4	92.1	93.6	93.9
100	0.09	-0.02	0.00	0.00	1.68	0.70	0.44	0.32	49.4	61.6	66.4	67.8	87.7	92.2	93.6	94.1
200	0.03	0.01	0.01	0.01	1.45	0.50	0.31	0.23	48.4	62.0	66.3	67.1	89.2	92.1	93.3	94.2

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-2.80	-1.65	-1.33	-1.05	4.55	2.88	2.32	2.01	68.5	72.4	73.8	75.3
50	-2.99	-1.58	-1.23	-1.03	4.28	2.43	1.98	1.72	60.6	69.6	71.7	76.0
100	-2.77	-1.63	-1.24	-1.02	3.60	2.16	1.69	1.47	52.3	59.3	61.0	67.4
200	-2.78	-1.64	-1.22	-1.03	3.34	1.97	1.51	1.30	37.6	43.0	49.6	55.2
	PDOLS estimator, leads and lags order $p = 4$											
30	-1.77	-1.11	-0.94	-0.76	4.87	2.79	2.19	1.93	70.7	79.0	79.7	83.6
50	-2.05	-1.03	-0.84	-0.74	4.27	2.23	1.85	1.60	70.7	79.6	82.0	85.1
100	-1.84	-1.10	-0.88	-0.73	3.28	1.85	1.45	1.29	68.5	76.5	78.6	83.4
200	-1.90	-1.14	-0.86	-0.75	2.82	1.57	1.22	1.07	61.4	68.9	73.3	77.0
	PDOLS estimator, leads and lags order $p = 8$											
30	-1.08	-0.88	-0.80	-0.65	7.75	3.17	2.31	2.04	55.8	77.2	79.7	83.0
50	-1.40	-0.79	-0.68	-0.61	6.15	2.44	1.97	1.63	57.3	79.4	82.5	86.9
100	-1.20	-0.86	-0.75	-0.62	4.55	1.93	1.47	1.31	53.8	78.4	83.4	86.5
200	-1.46	-0.91	-0.72	-0.65	3.63	1.56	1.18	1.04	51.5	77.8	80.8	83.8
	MGMW estimator, $q = 5$											
30	-1.42	-0.61	-0.52	-0.40	4.79	2.99	2.52	2.45	80.8	84.0	82.8	85.9
50	-1.63	-0.64	-0.40	-0.31	4.12	2.33	1.98	1.75	78.5	84.3	85.2	88.5
100	-1.32	-0.70	-0.47	-0.30	3.04	1.88	1.41	1.26	77.8	82.0	85.1	88.1
200	-1.42	-0.67	-0.43	-0.36	2.59	1.42	1.10	0.99	71.5	79.9	83.8	85.4

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S35: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-5.57	-2.82	-1.80	-1.29	6.98	3.37	2.14	1.54	35.2	40.9	45.7	48.0	60.7	57.3	57.8	57.4
50	-5.32	-2.60	-1.70	-1.23	6.13	2.93	1.89	1.38	21.6	27.3	28.4	32.6	47.3	42.4	39.3	41.6
100	-5.23	-2.62	-1.70	-1.22	5.69	2.78	1.80	1.29	6.7	5.6	6.7	8.2	20.9	13.2	11.7	11.8
200	-5.07	-2.56	-1.66	-1.21	5.30	2.65	1.72	1.25	0.7	0.4	0.3	0.3	4.1	1.3	0.9	1.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-9.22	-5.68	-4.45	-3.88	13.82	9.64	9.21	7.47	41.5	52.5	57.3	60.3	40.7	52.9	57.3	60.3
50	-9.15	-5.24	-4.27	-3.72	11.99	7.62	7.09	6.97	31.0	47.3	50.8	55.3	30.9	46.7	51.2	55.7
100	-9.17	-5.36	-4.20	-3.75	10.74	7.42	5.77	5.37	16.6	34.1	43.0	44.1	17.2	34.2	42.7	44.9
200	-8.90	-5.50	-4.24	-3.72	9.67	6.38	6.33	4.65	5.8	16.1	24.1	28.3	6.4	16.8	25.2	28.8
	SPMG estimator								Standard				Robust bootstrapped			
30	0.16	-0.02	-0.01	0.00	4.02	1.59	1.00	0.71	66.7	83.3	87.8	89.7	89.2	93.0	94.1	94.1
50	0.18	0.09	0.05	0.03	2.89	1.20	0.74	0.53	67.2	83.6	88.8	89.7	90.9	94.4	95.3	94.5
100	0.13	0.00	-0.01	0.00	2.04	0.83	0.52	0.37	67.3	83.5	87.4	89.7	89.1	93.4	93.3	94.4
200	0.19	0.02	0.01	0.01	1.45	0.59	0.36	0.26	66.8	83.6	89.3	90.1	89.3	93.3	94.6	94.5

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-11.49	-7.81	-6.22	-5.53	13.86	9.83	8.10	7.40	59.4	68.3	71.9	77.0
50	-11.36	-7.52	-6.13	-5.34	12.85	8.76	7.30	6.51	46.2	55.9	59.3	65.7
100	-11.45	-7.58	-6.17	-5.40	12.28	8.34	6.93	6.12	29.6	36.2	39.1	45.2
200	-11.27	-7.59	-6.16	-5.33	11.70	7.97	6.53	5.70	20.7	17.3	21.4	24.4
	PDOLS estimator, leads and lags order $p = 4$											
30	-9.07	-6.60	-5.40	-4.92	13.02	9.35	7.75	7.15	72.3	78.1	81.4	84.7
50	-8.95	-6.30	-5.32	-4.73	11.48	8.03	6.81	6.15	64.8	73.4	74.7	78.1
100	-9.12	-6.37	-5.38	-4.82	10.54	7.44	6.34	5.70	51.8	58.8	58.2	61.1
200	-8.90	-6.39	-5.38	-4.74	9.66	6.93	5.86	5.21	38.9	36.2	38.8	39.3
	PDOLS estimator, leads and lags order $p = 8$											
30	-7.80	-6.23	-5.13	-4.69	15.12	9.80	7.96	7.24	64.8	77.2	82.3	84.4
50	-7.99	-5.85	-5.07	-4.53	12.90	8.13	6.86	6.16	60.3	74.7	75.9	79.4
100	-7.96	-5.94	-5.11	-4.64	10.72	7.36	6.27	5.66	54.1	64.2	63.2	67.0
200	-7.83	-5.95	-5.14	-4.56	9.40	6.68	5.72	5.10	41.4	47.2	46.9	48.7
	MGMW estimator, $q = 5$											
30	-10.93	-6.53	-5.21	-4.51	15.06	10.32	9.14	7.86	73.9	84.3	87.0	89.9
50	-10.80	-6.52	-5.13	-4.38	13.50	9.00	7.51	6.64	63.1	76.9	82.8	85.2
100	-10.94	-6.62	-5.05	-4.35	12.24	7.88	6.30	5.64	41.9	57.9	65.8	70.4
200	-10.73	-6.58	-5.11	-4.42	11.44	7.25	5.75	5.14	19.2	34.2	44.4	51.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and no CS dependence of errors. See notes to Table S1.

Table S36: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-6.05	-2.99	-1.93	-1.40	7.88	3.76	2.41	1.76	35.3	39.7	43.5	45.3	67.2	66.2	67.3	66.3
50	-5.69	-2.76	-1.82	-1.34	6.77	3.23	2.12	1.56	23.3	28.9	30.6	32.0	55.3	55.3	53.8	54.9
100	-5.49	-2.73	-1.80	-1.31	6.09	2.99	1.95	1.42	8.2	9.4	10.1	10.6	31.1	26.0	25.6	25.5
200	-5.40	-2.66	-1.74	-1.27	5.70	2.79	1.81	1.33	1.3	1.1	1.1	1.2	9.1	5.8	5.2	6.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-9.61	-6.04	-4.81	-4.10	13.54	10.71	10.02	8.56	39.8	49.8	54.5	57.4	43.8	54.2	58.7	60.7
50	-9.28	-5.34	-4.36	-4.09	11.99	8.96	11.95	6.77	31.1	45.1	48.9	51.3	35.5	49.5	52.2	55.1
100	-9.21	-5.54	-4.51	-4.07	10.88	7.45	6.19	5.68	17.2	32.1	38.9	40.4	21.1	36.0	41.9	43.8
200	-9.16	-5.68	-4.51	-4.02	10.23	6.81	5.40	4.99	7.1	17.0	23.7	27.8	7.8	19.0	25.6	30.0
	SPMG estimator								Standard				Robust bootstrapped			
30	0.18	0.02	0.00	0.02	4.78	1.96	1.28	0.94	60.4	75.1	78.0	79.8	89.1	93.3	93.6	93.3
50	0.28	0.12	0.04	0.02	3.64	1.51	0.94	0.68	58.4	74.6	79.4	81.6	89.3	93.7	95.0	95.0
100	0.16	0.00	-0.01	0.00	2.46	1.04	0.66	0.48	60.2	74.3	78.5	80.5	88.4	93.3	94.0	94.0
200	0.14	0.03	0.01	0.00	1.74	0.73	0.45	0.33	60.3	75.6	78.7	81.4	88.0	92.7	94.4	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-11.90	-8.24	-6.70	-6.07	14.57	10.51	8.77	8.14	56.3	64.7	69.0	72.0
50	-11.69	-7.85	-6.51	-5.75	13.38	9.24	7.83	7.08	45.4	55.5	56.7	61.6
100	-11.55	-7.84	-6.48	-5.77	12.51	8.69	7.33	6.59	31.6	35.0	36.1	40.5
200	-11.54	-7.85	-6.44	-5.63	12.04	8.27	6.84	6.03	21.6	17.4	19.7	23.8
	PDOLS estimator, leads and lags order $p = 4$											
30	-9.44	-6.96	-5.83	-5.42	13.78	10.02	8.40	7.87	67.7	75.0	79.1	81.7
50	-9.14	-6.58	-5.66	-5.11	12.06	8.49	7.31	6.70	64.4	71.7	70.6	75.7
100	-9.17	-6.60	-5.66	-5.16	10.83	7.78	6.73	6.14	52.2	56.4	53.7	56.3
200	-9.14	-6.63	-5.63	-5.02	10.01	7.23	6.15	5.52	37.0	37.2	36.4	38.9
	PDOLS estimator, leads and lags order $p = 8$											
30	-8.35	-6.55	-5.58	-5.22	16.08	10.55	8.62	8.00	60.8	74.9	79.6	82.8
50	-8.24	-6.15	-5.40	-4.91	13.71	8.63	7.38	6.74	57.9	72.3	73.5	77.7
100	-8.15	-6.15	-5.39	-4.99	11.31	7.72	6.67	6.11	52.0	61.9	60.4	62.6
200	-8.05	-6.21	-5.38	-4.83	9.78	7.01	6.01	5.42	40.7	46.8	45.0	47.1
	MGMW estimator, $q = 5$											
30	-11.14	-7.01	-5.44	-4.87	15.23	10.84	9.28	8.36	71.9	81.4	85.4	87.8
50	-11.09	-6.57	-5.39	-4.64	13.85	9.08	7.97	7.05	60.2	74.3	80.3	82.9
100	-10.97	-6.67	-5.31	-4.52	12.35	8.01	6.66	5.88	42.4	56.2	63.7	66.7
200	-10.66	-6.63	-5.18	-4.59	11.43	7.31	5.89	5.30	19.1	34.6	44.8	47.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and SAR CS dependence of errors. See notes to Table S1.

Table S37: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and factor+SAR CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
30	-1.74	-0.87	-0.62	-0.45	3.40	1.72	1.12	0.83	50.4	57.8	58.0	59.7	87.6	87.9	88.2	88.1
50	-1.77	-0.85	-0.58	-0.42	3.01	1.43	0.93	0.70	45.2	50.7	52.9	55.0	83.9	86.1	86.4	87.3
100	-1.62	-0.85	-0.56	-0.41	2.44	1.23	0.80	0.60	35.9	38.7	39.7	43.4	78.3	78.8	79.7	81.2
200	-1.59	-0.82	-0.55	-0.41	2.20	1.11	0.73	0.54	26.9	27.6	27.5	28.8	71.8	71.5	71.8	71.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
30	-4.04	-2.69	-2.21	-1.94	8.42	6.28	5.31	4.90	49.0	54.2	58.8	60.9	60.5	65.9	67.6	69.6
50	-4.19	-2.53	-2.18	-1.83	7.14	5.13	4.76	4.28	41.0	52.6	55.8	59.5	52.0	62.8	63.6	66.3
100	-4.04	-2.47	-2.16	-1.99	6.13	4.56	3.78	4.20	33.2	45.5	48.1	49.9	43.1	55.2	55.7	57.7
200	-4.04	-2.51	-2.11	-1.85	5.28	3.69	3.10	2.82	22.9	35.6	38.0	41.6	32.5	45.2	47.5	49.4
	SPMG estimator								Standard				Robust bootstrapped			
30	0.12	0.05	-0.02	0.00	3.23	1.40	0.88	0.64	53.0	64.3	67.5	69.8	89.4	92.3	93.6	93.2
50	0.00	0.05	0.03	0.02	2.43	1.05	0.66	0.48	53.0	64.0	68.6	71.4	89.5	92.4	93.5	94.3
100	0.10	-0.01	0.00	0.01	1.85	0.74	0.46	0.34	51.2	63.2	69.9	71.4	87.5	92.4	94.0	94.1
200	0.05	0.01	0.01	0.01	1.24	0.53	0.33	0.24	50.1	63.0	67.5	70.3	88.7	92.4	93.6	94.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
30	-4.97	-3.61	-2.95	-2.59	7.31	5.37	4.64	4.15	74.5	79.3	81.2	84.6
50	-5.14	-3.39	-2.88	-2.51	6.81	4.64	3.98	3.62	65.2	75.0	74.2	78.7
100	-4.97	-3.33	-2.90	-2.54	6.03	4.10	3.62	3.24	56.4	62.3	62.7	64.4
200	-4.89	-3.35	-2.82	-2.51	5.65	3.85	3.29	2.95	44.8	44.2	45.0	50.0
	PDOLS estimator, leads and lags order $p = 4$											
30	-4.27	-3.27	-2.69	-2.37	7.91	5.51	4.68	4.16	73.3	83.0	84.8	88.2
50	-4.52	-3.02	-2.60	-2.30	7.04	4.59	3.93	3.57	70.9	81.5	82.0	84.5
100	-4.38	-2.96	-2.65	-2.33	5.99	3.94	3.50	3.14	63.7	74.5	75.1	74.1
200	-4.35	-3.01	-2.57	-2.30	5.48	3.64	3.11	2.79	56.9	62.1	60.7	63.9
	PDOLS estimator, leads and lags order $p = 8$											
30	-3.62	-3.23	-2.65	-2.32	10.63	6.17	5.00	4.38	56.5	81.0	84.2	88.0
50	-3.93	-2.92	-2.54	-2.23	8.74	4.95	4.15	3.69	57.5	80.7	82.7	86.1
100	-3.88	-2.84	-2.62	-2.29	7.12	4.14	3.64	3.22	52.9	75.5	77.2	77.4
200	-4.04	-2.91	-2.53	-2.27	6.24	3.77	3.18	2.83	48.2	66.7	67.1	69.0
	MGMW estimator, $q = 5$											
30	-2.04	-1.62	-1.23	-1.11	7.06	5.18	4.74	4.40	83.5	87.6	89.3	90.1
50	-2.41	-1.48	-1.21	-0.95	6.02	4.08	3.66	3.36	81.3	84.1	87.6	88.5
100	-1.95	-1.40	-1.21	-0.95	4.59	3.22	2.87	2.55	78.5	82.7	83.8	85.9
200	-2.04	-1.32	-1.09	-1.01	4.08	2.67	2.30	2.17	72.3	78.1	79.6	80.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S38: Coverage rate of robust asymptotic 95 percent confidence intervals of SPMG estimator of LR coefficient θ

A: Experiments 1-18 (summarized in Table S1)

$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	Experiment 1				**Experiment 2**				**Experiment 3**			
30	57.80	80.00	86.05	88.25	61.20	80.80	86.20	88.80	63.15	81.15	88.95	90.90
50	58.10	78.80	86.35	87.45	59.10	80.35	87.55	87.85	63.50	84.90	88.45	90.80
100	59.80	80.15	86.20	89.30	60.35	81.70	87.55	90.40	64.70	84.50	91.15	92.50
200	59.90	80.25	87.00	88.50	60.90	81.60	87.25	89.25	68.15	86.20	92.75	94.35
	Experiment 4				**Experiment 5**				**Experiment 6**			
30	57.75	80.00	86.00	88.10	61.10	80.20	86.00	88.55	62.90	81.00	88.55	90.45
50	58.35	78.35	87.10	87.40	59.55	80.55	87.80	88.30	63.25	84.70	88.50	90.80
100	58.75	80.55	85.55	89.50	60.20	82.40	87.60	90.00	63.15	84.60	90.70	92.35
200	60.20	79.70	87.05	88.55	61.80	81.10	87.10	88.95	68.20	85.65	92.75	94.25
	Experiment 7				**Experiment 8**				**Experiment 9**			
30	55.80	79.80	86.35	87.10	60.25	80.05	86.45	88.60	60.90	81.75	88.70	90.80
50	57.00	78.65	86.00	88.40	59.40	79.90	86.85	88.00	63.50	83.75	88.30	90.50
100	57.65	80.05	86.45	89.45	60.00	80.85	87.90	90.15	63.25	84.10	90.75	92.05
200	59.15	80.00	86.35	88.35	60.85	80.55	85.70	87.95	66.50	85.90	92.05	93.90
	Experiment 10				**Experiment 11**				**Experiment 12**			
30	65.70	83.40	87.60	89.65	65.15	83.40	88.50	90.15	68.90	84.65	89.15	91.30
50	65.65	82.15	88.95	90.15	66.60	81.65	87.15	90.00	68.15	83.50	89.45	90.55
100	64.95	82.85	88.10	91.60	66.90	82.35	87.80	90.90	70.10	84.60	89.25	90.35
200	66.05	81.55	87.85	90.05	66.80	82.55	87.65	89.55	68.40	83.65	89.70	90.35
	Experiment 13				**Experiment 14**				**Experiment 15**			
30	65.65	83.35	87.75	90.25	66.20	83.55	88.50	90.60	69.70	84.75	90.05	91.00
50	64.70	82.15	89.10	90.15	65.65	81.85	87.40	90.00	68.10	84.00	89.30	90.85
100	66.25	82.30	87.95	90.80	67.55	82.60	88.05	91.10	70.25	84.45	89.25	90.45
200	65.90	82.35	88.60	90.20	66.95	82.45	88.00	89.90	68.90	84.05	90.00	90.75
	Experiment 16				**Experiment 17**				**Experiment 18**			
30	64.45	82.85	87.25	89.30	65.45	83.25	88.50	90.65	70.05	84.80	89.70	91.05
50	63.95	82.75	88.00	89.35	65.10	82.80	88.05	90.10	67.30	83.95	89.45	91.40
100	64.95	81.65	87.20	91.00	65.90	81.35	88.25	91.20	69.20	84.60	89.65	90.80
200	64.85	80.05	87.45	89.65	65.25	82.00	86.65	89.45	70.15	84.30	89.40	91.40

Notes: This table reports coverage rate MC findings for 95 percent confidence intervals computed based on the robust variance matrix estimator (S.26), using asymptotic critical values. See Table S1 fof the list of individual experiments.

Table S38 (Continued): Coverage rate of robust asymptotic 95 percent confidence intervals of SPMG estimator of LR coefficient θ

B: Experiments 19-36 (summarized in Table S1)

$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	Experiment 19				**Experiment 20**				**Experiment 21**			
30	61.80	79.50	85.25	88.65	64.15	81.45	86.65	88.85	64.00	83.50	89.20	90.70
50	60.55	80.70	86.90	90.00	61.00	80.90	87.30	89.95	64.30	83.20	90.45	92.10
100	57.90	80.45	85.85	87.90	59.60	79.45	86.80	89.40	65.85	86.50	90.65	92.60
200	61.10	80.50	87.60	89.20	62.75	81.50	88.00	89.70	68.60	87.65	92.05	94.65
	Experiment 22				**Experiment 23**				**Experiment 24**			
30	60.90	79.35	85.55	88.25	62.90	81.10	86.55	89.20	64.55	83.65	89.25	90.55
50	60.65	79.35	87.35	90.30	60.65	80.55	87.35	89.80	64.45	82.95	90.45	91.75
100	57.85	80.15	86.20	87.60	60.70	80.25	87.05	89.10	65.10	86.10	90.50	92.35
200	60.50	80.10	87.25	89.60	61.80	81.90	87.45	89.60	67.65	87.25	92.20	94.75
	Experiment 25				**Experiment 26**				**Experiment 27**			
30	58.40	78.90	85.10	88.30	62.60	81.70	86.55	88.55	63.85	82.85	89.25	90.10
50	58.00	79.15	87.15	90.55	60.80	80.20	87.65	89.75	62.70	83.30	90.30	92.20
100	56.60	78.90	86.75	87.70	59.90	80.30	86.05	88.75	63.55	85.15	90.65	92.55
200	60.75	80.05	87.35	89.30	60.40	81.45	87.40	89.80	65.85	86.65	91.85	94.25
	Experiment 28				**Experiment 29**				**Experiment 30**			
30	67.30	81.85	87.35	89.50	67.20	84.60	87.70	89.05	70.75	84.05	88.95	89.15
50	66.35	83.40	87.90	89.25	67.35	83.30	88.90	90.35	70.45	84.30	88.45	90.45
100	67.35	84.30	87.25	89.35	66.60	83.90	87.60	90.30	70.05	85.05	90.00	91.35
200	67.00	83.05	88.65	90.10	67.15	83.60	88.70	90.65	71.80	84.60	89.05	91.65
	Experiment 31				**Experiment 32**				**Experiment 33**			
30	66.45	82.55	87.85	89.30	66.95	83.95	87.10	89.35	71.05	84.20	89.15	89.25
50	65.95	83.35	88.15	89.40	67.40	83.90	88.40	90.45	70.70	83.90	88.85	90.75
100	67.40	83.15	87.05	89.10	66.45	84.20	87.60	89.65	69.35	85.00	89.70	91.35
200	67.75	83.25	89.10	90.10	67.45	83.65	88.85	90.00	72.30	85.00	89.35	91.95
	Experiment 34				**Experiment 35**				**Experiment 36**			
30	66.20	82.50	87.10	89.05	67.45	83.50	87.40	89.30	71.20	83.90	88.60	89.35
50	66.45	83.75	88.40	89.10	66.85	84.40	88.60	91.10	70.50	84.45	88.80	90.85
100	66.10	82.25	87.00	88.95	66.50	83.85	88.45	89.75	69.05	84.95	89.70	91.50
200	65.55	82.70	88.55	89.65	66.65	83.90	87.95	90.30	71.65	84.45	89.20	92.00

Notes: This table reports coverage rate MC findings for 95 percent confidence intervals computed based on the robust variance matrix estimator (S.26), using asymptotic critical values. See Table S1 fof the list of individual experiments.

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