Finance Matters*

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Abstract

We present a model in which the importance of financial intermediation for development can be measured. We generate financial differences by varying the degree to which contracts can be enforced. Economies where enforcement is poor employ less capital and less efficient technologies. Calibrated simulations reveal that both effects are important. Yet, accounting for all the observed dispersion in output requires a higher capital share or a lower elasticity of substitution between capital and labor than usually assumed. We find that the effects of changes in those technological parameters on output are markedly larger when financial frictions are present. Finance, that is, matters.

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1 Introduction

Under standard neoclassical assumptions, observed differences in human and physical capital per worker cannot account for differences in output per worker across nations.\footnote{See for example Chari, Kehoe, and McGrattan (1997), Prescott (1998), or Hall and Jones (1999).} In the language of development accounting, total factor productivity (TFP) varies greatly across countries. At the same time, financial development and economic development are highly correlated.\footnote{See for example Goldsmith (1969), McKinnon (1973), Shaw (1973), Rajan and Zingales (1998), or Beck, Levine, and Loayza (2000).} This strong empirical relationship is often presented as evidence that financial development causes economic development by promoting investment and making the allocation of resources more efficient. A number of models have established a theoretical link between finance and development.\footnote{See for example Boyd and Prescott (1986) and Greenwood and Jovanovic (1990), or Aghion, Howitt, and Mayer-Foulkes (2005), for a connection between finance and growth.} Our goal is to quantify the importance of finance for economic development in a dynamic general equilibrium model.\footnote{Using data from Thailand, Jeong and Townsend (2004) find that capital deepening is an important determinant of measured TFP growth. Models that quantify the importance of factors other than finance include Hall and Jones (1999), Acemoglu and Zilibotti (2001), Restuccia and Rogerson (2003), Herrendorf and Teixeira (2004), and Restuccia (2004).}

In our model economy agents live for three periods, but only work in the first two. When young, they supply labor inelastically. In the second period of their life they can choose to manage a technology that transforms capital and labor into the consumption good. As in the span of control model of Lucas (1978), some agents manage resources more efficiently than others. Managers finance part of their capital through their own savings, and borrow the rest from an intermediary. The market for loans is imperfect however, as agents can choose to default on the payment they owe the intermediary at an exogenous cost.\footnote{In order to understand a number of corporate finance regularities, Shleifer and Wolfenzon (2002) build a model economy that bears some similarities to ours. Not only their focus is different, but also their results are theoretical, not quantitative.}

This default cost is the basis for the quantitative exercise we carry out in this paper. In all our experiments, we compare economies that differ in one respect only: the degree to which financial contracts can be enforced. By generating financial differences via enforcement
frictions, we adopt and formalize the view that the quality of institutions is a key determinant of financial development. Economies where contract enforcement is poor emphasize self-financing and, therefore, production on a small scale. Less productive technologies need to be operated for labor markets to clear. Output is lower in economies with bad financial markets because less capital is used in production (the capital intensity channel), and because capital is not directed to its best uses (the allocation channel). In other words, our model incorporates the two channels emphasized by the financial development literature ever since the seminal work of Goldsmith (1969), McKinnon (1973), and Shaw (1973).

To quantify the importance of these effects, we begin by calibrating our model to match relevant features of the U.S. economy. In particular, we calibrate the distribution of managerial talent to match salient features of the organization of production in the United States, and the default cost to approximate the ratio of aggregate financial liabilities to output. Then we vary the default parameter to generate a sequence of economies with different levels of financial development.

We find that finance disrupts the organization of production greatly. As access to finance falls, more agents need to become managers for labor markets to clear (more inefficient technologies need to be activated), and the average size of establishments falls by magnitudes very similar to what one observes in the available cross-country data. We also find that financial differences have a large impact on capital-output ratios, and can generate output variations that far exceed what differences in these capital-output ratios alone would predict. That is, we find that both the allocation and the capital intensity channels emphasized by the literature on financial development are quantitatively important. Despite that, under standard technological assumptions, the resulting dispersion in output falls short of what one observes in the data.

These results are robust to even drastic changes in most parameters, with two important exceptions. Raising the capital share greatly increases the effect of finance on output, for obvious reasons. Similarly, reducing the degree to which labor can be substituted for capital magnifies the output effect without changing the impact of finance on the capital-output ratio.
much. In sum, we find that the extent to which finance matters depends on the importance of physical capital in production. Under standard technological assumptions, the capital share is too small and labor is too easy to substitute for capital for finance to explain all the output dispersion one sees in the data.

That varying those two parameters can magnify the effect of factor differences on output is well-known. For instance, Caselli (2003) argues that if the elasticity of substitution between capital and labor is small enough, the observed dispersion in factor endowments can account for the dispersion in output across countries. What we find is that financial differences provide a complementary magnification effect. The effect on output of given changes in the elasticity of substitution becomes much larger when one models financial frictions explicitly.

More generally, our findings confirm the importance of correctly measuring the parameters that describe technological opportunities. For now, those parameters remain the cause of much debate. The common motivation for assuming Cobb-Douglas technological opportunities is that factor shares are fairly constant over time in the United States, as originally emphasized by Kaldor (1961). After allocating proprietor’s income to capital and labor, Gollin (2002) finds that the capital share varies between 0.2 and 0.35 across countries. Jones (2003), using cross-country and cross-industry data, finds that capital shares exhibit substantial trends and fluctuations. Attempts to measure the elasticity of substitution directly yield vastly different results. The results of Restuccia and Urrutia (2001) provide support for the Cobb-Douglas specification, but a panel version of the same estimation exercise lead Pessoa, Pessoa, and Rob (2003) to reject the Cobb-Douglas specification in favor of a an elasticity of substitution estimate of around 0.7. Allowing for biased technical change Antràs (2004) also concludes that the U.S. economy is not well described by a Cobb-Douglas aggregate production function. Similarly, Duffy and Paggioergiou (2000) reject the Cobb-Douglas specification in a sample of 82 countries, and find in addition that the elasticity of substitution between capital and labor rises with the level of economic development. Our point in this paper is not to settle these empirical questions, but rather to stress their importance.

\footnote{A different motivation, based on the distribution of new ideas, is provided in Jones (2005).}
Whether financial disruptions can account for the bulk of the output variation we observe in the data hinges on the outcome of this debate over factor shares and elasticities.

In sum, our experiments suggest that financial development matters a great deal for economic development. Standard models cannot account for observed output differences given observed capital intensity differences. Contractual imperfections in the market for finance may be the needed magnification channel.

2 The economy

We consider a discrete-time model in which a mass one of 3-period-lived agents are born each period. Agents are endowed with managerial ability $z \in Z$ which is public information. The distribution $\mu$ of managerial ability is the same across generations and has finite support. In the first period of their life agents inelastically supply one unit of labor services. In the second period, they can once again supply labor services or, instead, can choose to become managers. Managers of ability $z$ transform inputs $l > 0$ of labor services and $k > 0$ of physical capital into the unique consumption good according to the following gross schedule:

$$F(l, k, z) = Az [\alpha k^{\rho} + (1 - \alpha)l^{\rho}]^{\frac{1}{\rho}} + (1 - \delta)k,$$

where $A > 0$, $\alpha \in (0, 1)$, $\nu < 1$ to allow for managerial profits, $\rho \in (-\infty, 1]$ measures the degree to which capital and labor can be substituted for each other, and $\delta \in (0, 1)$ is the rate of depreciation of physical capital. In the last period of their life, agents do not work. Agents’ preferences over lifetime consumption profiles $(c_1, c_2, c_3)$ are represented by the following utility function:

$$U(c_1, c_2, c_3) = \log c_1 + \beta \log c_2 + \beta^2 \log c_3,$$

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Even though the plant technology is strictly concave in labor services and capital, aggregate returns are constant in this economy. There are three inputs in our economy: managerial services, labor services and capital. Doubling population, all else equal, doubles the number of potential workers and managers. If population and the capital stock both double, so does output.
where $\beta \in (0, 1)$.

Managers can self-finance part of their capital using savings from the first period of their life. We denote by $s$ the capital so invested by managers. They can also borrow $d \geq 0$ from an intermediary. In addition to extending loans to managers, the intermediary can store deposits on behalf of agents with exogenous net return $r > 0$, and can borrow without bound at that same rate. We will think of the resources invested in the storage technology as capital not used in production. In this broad sense, these resources include savings hoarded by households, and residential investment. The assumption that the intermediary can borrow at rate $r$ is made for simplicity. Investment in the storage technology is positive in all our simulations given the calibration choices we make in the quantitative section.\(^8\)

Let $w$ denote the price of labor services. In this paper we will only consider equilibria where this price is constant over time. Given quantity $k = s + d$ of physical capital, a manager of ability $z$ generates net income $\Pi(k, z; w)$ where

$$\Pi(k, z; w) = \max_l \left[ F(l, k, z) - wl - k(1 + r) \right].$$

The market for loans is imperfect. Specifically, managers can choose to default on their loan and economize on the payment they owe the intermediary, in which case they incur a default cost equal to fraction $\eta > 0$ of their income. We further assume that the intermediary behaves competitively, so that among the individually rational loans that cover the opportunity cost $r$ of funds, the most favorable to the manager prevails. Therefore, financial contracts for managers of ability $z \in Z$ with savings $a \geq 0$ solve:

$$\max_{s \leq a, d \geq 0} \quad \Pi(s + d, z; w)$$

s.t. $\Pi(s + d, z; w) + s(1 + r) \geq (1 - \eta) [\Pi(s + d, z; w) + (s + d)(1 + r)].$

The constraint states that it must be individually rational for the managers to repay their

\(^8\)Some aspects of our quantitative results are sensitive to how these activities are recorded in National Income and Product Accounts (NIPA), as we discuss in section 4.6.
loan. Let \( d(a, z; \eta, w), s(a, z; \eta, w) \) and \( l(a, z; \eta, w) \) denote the policy functions associated with the above problem, and let \( V(a, z; \eta, w) = \Pi(s(a, z; \eta, w) + d(a, z; \eta, w), z; w) \) be the resulting net income.

When the loan market is perfect (\( \eta = 1 \)), managers of ability \( z \) operate with the optimal quantity \( k^*(z; w) = \arg \max_{k \geq 0} \Pi(k, z; w) \) of physical capital, given the price of labor. When \( \eta < 1 \), managers are constrained to operate at a sub-optimal scale, unless their savings exceed \( a^*(z; \eta, w) = \inf\{a \geq 0 : s(a, z; \eta, w) + d(a, z; \eta, w) = k^*(z; w)\} \). In particular, \( V(a, z; \eta, w) = \Pi(k^*(z; w), z; w) \) for all \( a \geq a^*(z; \eta, w) \). The lemma below states that below threshold \( a^* \), the manager’s access to outside financing rises with her assets and her managerial ability. This occurs because increases in \( a \) or \( z \) weaken the individual rationality constraint in the manager’s problem. This result also states that it is optimal for borrowing constrained managers to invest all their savings in their establishment. This is because the marginal product of physical capital exceeds its opportunity cost \((1 + r)\) in establishments operated at a sub-optimal scale.

**Lemma 1.** Given \( w > 0 \) and \( \eta > 0 \),

(i) \( V(\cdot, z; \eta, w) \) is strictly increasing and concave on \([0, a^*(z; \eta, w)]\) for all \( z \in Z \);

(ii) \( V(a, \cdot; \eta, w) \) rises strictly for all \( a > 0 \);

(iii) \( s(a, z; \eta, w) = a \) on \([0, a^*(z; \eta, w)]\) for all \( z \in Z \);

(iv) \( d(\cdot, z; \eta, w) \) is strictly increasing and concave on \([0, a^*(z; \eta, w)]\) for all \( z \in Z \);

(v) \( d(a, \cdot; \eta, w) \) is increasing for all \( a \in [0, a^*(z; \eta, w)] \).

This result is established in the appendix. It implies that limited enforcement disrupts the allocation of resources in potentially two ways. First, establishments are generally operated below their optimal scale. Second, occupational choices depend not only on agents’ managerial ability, but also on their wealth. But we will now argue that even when contractual imperfections are present, occupational choices are monotonic in ability: agents whose ability
exceeds a certain threshold become managers in the second period, while agents below that
threshold remain workers.\textsuperscript{9} To that end, we need to state the problem solved by young agents:

\[
\begin{align*}
\max_{a_1, a_2 \geq 0} & \quad \log c_1 + \beta \log c_2 + \beta^2 \log c_3 \\
\text{s.t.} & \quad c_1 + a_1 = w \\
& \quad c_2 + a_2 = a_1 (1 + r) + \max \left( w, V(a_1, z; \eta, w) \right) \\
& \quad c_3 = a_2 (1 + r).
\end{align*}
\]

Solutions to this maximization problem need not be unique. Indeed, when \( w = V(a_1, z; \eta, w) \), agents are indifferent between the two possible occupations in the second period. But the level \( k(z; \eta, w) \) with which managers of ability \( z \) operate is unique. To see this, recall that by lemma 1, \( V \) is strictly concave in \( a \) below \( a^*(z; \eta, w) \). Therefore, agents of ability \( z \) who become managers are either unconstrained, or solve a problem that is strictly concave in \( a_1 \).

Also, note that agents are indifferent between occupations for at most one ability value. In fact, given \( w > 0 \) and \( \eta \geq 0 \), there exists a unique \( z(\eta, w) \) such that agents become managers in the second period of their life when \( z > z(\eta, w) \), remain workers when \( z < z(\eta, w) \), and are indifferent between the two occupations otherwise. If agents of a given managerial ability find that becoming a manager maximizes their lifetime utility, this remains true for agents of higher ability. Indeed, productivity and access to outside financing both rise with managerial ability, by lemma 1.

Given a wage rate \( w \geq 0 \), and an enforcement parameter \( \eta \), optimal agent policies are fully described by the threshold managerial ability \( z(\eta, w) \), the quantity \( k(z; \eta, w) \) of capital employed by managers of ability \( z \geq z(\eta, w) \), and the quantity

\[
l(z; \eta, w) = \arg \max_l F[l, k(z; \eta, w), z] - wl
\]

\textsuperscript{9}This simplifying result hinges on our assumption that agents are born identical. Sources of wealth inequality that are not monotonic in managerial ability, such as bequest inequality, would yield a different outcome.
of labor these agents choose to manage. A steady-state equilibrium is a constant wage rate \( w \) such that, given the associated policies, the market for labor clears.\(^\text{10}\) To make this formal, note that managers of ability \( z > z(\eta, w) \) have a net excess demand for labor of \( l(z; \eta, w) - 1 \) over their lifetime, since they supply one unit of labor when young. For their part, agents who do not become managers supply 2 units of labor during their lifetime. Agents of ability \( z(\eta, w) \) can be assigned to either occupations in the second period. Each possible fraction \( \theta \in [0, 1] \) of those agents assigned to management generates a different value of the aggregate excess demand for labor. These observations lead to the following definition of the excess demand for labor correspondence, for all \( w \geq 0 \) and \( \eta \geq 0 \):

\[
ED(w; \eta) = \left\{ \int_{z > z(\eta, w)} [l(z; \eta, w) - 1]d\mu - 2 \int_{z < z(\eta, w)} d\mu + \mu(z(\eta, w)) \left[ \theta \left(l(z(\eta, w); \eta, w) - 1\right) - 2(1 - \theta) \right] : \theta \in [0, 1] \right\}
\]

A steady state equilibrium is a value for \( w \) such that \( 0 \in ED(w; \eta) \). We will call a steady state equilibrium with \( w > 0 \), hence positive output, non-degenerate. The following result provides conditions under which non-degenerate steady-state equilibria exist:

**Proposition 1.** There exist \( \rho < 0 \) and \( \eta < 1 \) such that a non-degenerate steady state exists provided:

1. \( \rho > \rho^\text{p} \), or,
2. \( \eta \geq \eta^\text{p} \).

A non-degenerate equilibrium exists provided that labor and capital are substitutable enough, or that the degree to which contracts can be enforced is high enough, for any given set of other exogenous parameters. In the appendix, we argue that a non-degenerate steady-state equilibrium exists if \( ED(w; \eta) \cap \mathbb{R}_+ \neq \emptyset \) for \( w \) low enough, that is, if excess demand

\(^{10}\)Since by assumption capital is available perfectly elastically at price \( 1 + r \), the market for capital clears trivially.
becomes positive when \( w \) becomes small. We also argue that this condition is met when \( \rho \geq 0 \) (which includes the Cobb-Douglas case, \( \rho = 0 \)), but may fail to hold when \( \rho < 0 \).\(^{11}\)

Given the calibration choices we make in section 4, we find that the economy only collapses for extremely low values of \( \eta \) and \( \rho \). There, we use numerical simulations to compare economies that differ in one respect only: the degree \( \eta \) to which contracts can be enforced. Intuitively, we should expect steady state wages to rise with the quality of enforcement. Indeed, as \( \eta \) rises more contracts become enforceable and managers’ access to outside financing improves. Managers should then operate at a scale closer to the optimal one, with a positive effect on output and the marginal product of labor. One can in fact show that when \( \rho > \rho_c \), for each monotonic sequence of enforcement parameters there exists a corresponding sequence of monotonic steady state wages. The point of the remainder of this paper is to gauge the quantitative importance of enforcement, and therefore access to finance, on an economy’s output and capital intensity.

### 3 Data

In this section we establish the data benchmarks against which the results of the quantitative experiments will be compared. Our principal source of data is the Penn World Table, Mark 6.1, which is described in detail in Heston, Summers, and Aten (2002). Our sample consists of those countries that report output per worker in 1996 and report investment data for at least 15 years leading up to 1996. This gives us a sample of 120 countries.

As a measure of output per worker we use the series \( \text{RGDPW} \). Since the capital per worker data (KAPW) can be somewhat unreliable, and is still unavailable at this time for Mark 6.1, we constructed capital stock series for each country using the perpetual inventory method. Our investment series is the product of GDP per capita in 1996 international prices (RGDPL) and the corresponding investment share series (Ki). For the initial capital stock we

\(^{11}\)When \( \rho < 0 \), the marginal product of capital does not diverge to \( +\infty \) when \( k \) falls. As a result, for \( \eta \) small enough, managers may not be able to borrow any capital, even if \( w = 0 \).
consider two alternative methods. Method 1 sets the initial capital stock equal to the initial investment divided by the growth rate of investment (its geometric average over the sample) plus the depreciation rate (which we set to 10% a year.) This is tantamount to assuming that such an economy is on a balanced growth path in the initial period. Method 2 sets the capital-output ratio in the initial period equal to that in the last period (1996). This is a good approximation if the economy is close to a balanced growth path throughout the period considered. The two methods yield capital stocks (relative to the U.S) in 1996 that are very close to each other, so we choose the first one. Finally, we divide the capital stock in 1996, by GDP per capita in 1996 (as given by RGDPL.) to obtain capital-output ratios.

The first chart in figure 1 plots relative capital-output ratios against relative output per worker. To calculate dispersion measures for output per capita and capital-output ratios we divided the top decile by the bottom decile. The richest 10% are 35 times richer than the bottom 10%, and have 5 times the capital-output ratio of countries in the bottom decile.\footnote{Hsieh and Klenow (2004) find that if one uses domestic prices, the dispersion in investment rates across countries is even smaller.} These numbers have prompted the literature to conclude that the dispersion in capital-output ratios across countries cannot account for observed differences in output per worker.\footnote{This remains true even if one accounts for human capital. See for example Prescott (1998) and Hall and Jones (1999).} A standard development accounting exercise would lead one to conclude that total factor productivity varies greatly across countries. The standard approach would measure productivity as:

\[
\text{Measured Productivity} = \frac{Y}{K^{\alpha}L^{(1-\alpha)}},
\]

where $Y$ is GDP, $K$ is aggregate capital, $L$ is aggregate labor, and $\alpha$ is the capital share. Assuming $\alpha = 0.3$ for our sample of 120 countries, the richest 10% have a measured productivity 9.5 times higher than the bottom 10%. This statistic does not have the usual interpretation in our model economy, since aggregate technological opportunities are not well described by a Cobb-Douglas specification unless $\rho = 0$. Nonetheless, we report it in all quantitative
experiments so that our simulated numbers can be compared to the outcome of standard
development accounting exercises.

Measuring the quantity of financial intermediation in a given economy is a more compli-
cated issue. There is a large literature that studies the statistical relationship between proxies
for financial intermediation and growth or development.\textsuperscript{14} The ideal proxy would measure the
ability of a manager to obtain financing for a project. In our model economy, \textit{outside capital}
is the part of capital that does not come from managers’ savings, the variable we termed \(d\) in
the previous section. Given the set of financial statistics that are available for a reasonably
large cross-section of nations, we believe the best proxy to be the sum of: (i) the credit firms
obtain from banks and other financial institutions (ii) the credit they obtain from issuing
bonds, and, (iii) the funds they obtain from issuing stock. The database compiled by Beck,
Demirguc-Kunt, and Levine\textsuperscript{15} contains (i), and a market value measure of (ii) and (iii), which
we use for lack of historical value numbers. Whether historical valuations of outstanding
stocks are more appropriate than market valuations is not obvious, however. Stock values
reflect in part the value of retained earnings. These are funds which corporations effectively
borrow from their shareholders.

The average ratio of outside capital (so defined) to output in the United States during the
1980’s and 1990’s is roughly 2. This is the number we use in our calibrated simulations. Using
unrelated data and a different methodology, McGrattan and Prescott (2000) arrive at a ratio
of 1.8 for the year 2000. Their number is smaller in part because they ignore corporate debt
holdings and liabilities. Figure 1 shows relative output, relative capital-output ratios, and
relative measured productivity plotted against the outside capital to output ratio relative to
the U.S. for 25 countries.\textsuperscript{16} As is well known, the quantity of finance, the level of development,
capital-output ratios, and total factor productivity measures are all positively correlated.

\textsuperscript{14}See, for example, Beck, Levine, and Losyza (2000).
\textsuperscript{15}This database is available at http://legacy.csom.umn.edu/WWWPages/FACULTY/RLevine/Index.html
\textsuperscript{16}These are the 25 countries that report data for items (i), (ii), and (iii). The country with the smallest
outside capital to output ratio relative to the U.S in this sample is Argentina, at 0.1, while the one with the
highest is Japan at 1.24.
4 Quantitative Experiments

In our model economy, finance affects development in two ways. First, economies with better financial markets employ more capital. Henceforth, we refer to this effect as the *capital intensity channel*. Second, they direct capital to more efficient uses (more productive plants), an effect we refer to as the *allocation channel*. In this section we conduct a series of experiments to measure the quantitative importance of these two channels, and to find out whether financial differences can account for observed differences in income per worker across countries. We are particularly interested in asking whether the model can generate these sizable differences in output while keeping the differences in capital-output ratios small.

4.1 Calibration

We begin by setting parameters to match salient features of the U.S. economy. Assuming that an individual’s work life lasts 40 years, a period in the model economy corresponds to 20 years. We set the yearly interest rate to 4% \((r = 1.04^{20} - 1)\), and then set \(\beta = \frac{1}{1+r} = 0.4564\). We set \(\delta = 0.88\), which implies that yearly depreciation is 10%.

To calibrate the parameters governing the production function we first assume an elasticity of substitution of \(\sigma = \frac{1}{1-\rho} = 1\). In that case, \(F(l, k, z) = Az^\alpha \nu \lambda (1-\alpha) \nu + (1-\delta) k\). When there are no contractual imperfections \((\eta = 1)\), the share of Gross Domestic Product that accrues to capital is \(\frac{K(r+\delta)}{Y} = \alpha \nu\).

Here, \(K\) denotes the sum across plants of all capital, i.e. \(\int_Z [(s(a, z; \eta, w) + d(a, z; \eta, w)) d\mu]\). That is, we begin by assuming that the resources invested in the storage technology are not reported as investment in National Income and Product Accounts. Under that assumption, the sum of all capital used by managers is the proper counterpart in our model to the measure of the capital stock we introduced in the data section. In all our simulations, aggregate

\(^{17}\)Although below we calibrate \(\eta\) to a value lower than one in the U.S., the capital share does not fall much from its complete market benchmark. Specifically, we calibrate \(\eta\) to match a loans to yearly GDP ratio of 2. This means the capital to GDP ratio must then exceed 2, which in turn implies the capital share must exceed 28%, 2 times the yearly values of \(r + \delta\).
savings, $S \equiv \int (a_1(z; \eta, w) + a_2(z; \eta, w)) \, d\mu$, exceed $K$ so that some resources are invested in the storage technology. Subsection 4.6 discusses the impact of treating $S - K$ as part of capital formation.

Gollin (2002) finds that the share of capital income in GDP varies between 0.2 and 0.35 in most countries. We set $\alpha \nu = 0.3$. Another condition is necessary to identify $\alpha$ and $\nu$. Absent contractual imperfections, $1 - \nu$ measures the share of income that accrues to managerial services. One way to set this share could be to set it equal to the share of proprietors’ income in national income, but this measure would not be consistent with our theory. We think of $F$ as an establishment technology, and our model does not distinguish between forms of ownership. In that sense, aggregate returns to managerial services should include the income of managers who operate an establishment on behalf of a corporation. Since corporate data do not allow one to measure the share of income that accrues to managerial services, we assume that the repartition of income across factors is independent of the ownership type. Under that assumption, and under the assumption that in sole proprietorships the owner plays the role of the manager in our model, a good proxy for the managerial share is the ratio of net income to the sum of payments to physical capital, labor, and the owner in sole proprietorships, after correcting for the fact that part of net income rewards the owner’s capital input. Using Internal Revenue Service data available between 1989 and 1998, we calculated that among manufacturing sole proprietorships, in the United States, aggregate net income represents (on average) 22% of payments to labor, capital and the owner (see appendix B.1.) Assuming that fraction $\alpha$ of net income rewards the owner’s capital input, we obtain the following condition for $\nu$ and $\alpha$:

$$1 - \nu = 0.22(1 - \alpha)$$

The approximate solution to our two conditions on $\nu$ and $\alpha$ is $\alpha = 0.35$ and $\nu = 0.85$.\(^{18}\)

The distribution of managerial talent, $\mu$, and the default cost, $\eta$, are calibrated together to

\(^{18}\)Using very different arguments, Atkeson, Khan, and Ohanian (1996) arrive at the same value for the degree of strict concavity of the plant production function. This is also the value used by Jermann and Quadrini (2003) and Atkeson and Kehoe (2001).
match the ratio of outside capital to GDP (calculated using the data and method described in the previous section) and two moments of the size distribution of manufacturing establishments in the United States. Specifically, we assume that \( z \) is log-normally distributed with parameters \((\lambda_1, \lambda_2)\). We calibrate these two parameters to match 1) the percentage of manufacturing establishments with 9 employees or fewer, which is around 50\% according to County Business Patterns Survey data between 1988 and 1998, and 2) the average size of manufacturing establishments in the United States, which is around 50 employees according to the same source. This is meant to capture two salient features of the organization of production in the United States: the majority of establishments are small, but small establishments account for a low fraction of employment. We use manufacturing data to allow for international comparisons. Data on the organization of production outside of the manufacturing sector are seldom available for developing nations, and are unreliable when they exist, given the preponderance of undeclared, informal activities in services in those countries. Appendix B.2 describes the joint calibration of \( \lambda_1, \lambda_2, \eta \) in more detail. The exact values we used for all exogenous parameters in our experiments are in the appendix as well.

4.2 Results

Our experiments consist of comparing steady state equilibria in economies that differ in their degree of contractual imperfections and thus in their outside capital to output ratio. In these comparisons, we focus on what happens to output, capital-output ratios, and measured productivity, but we also highlight some subsidiary implications that lend credibility to the framework we propose. Figure 2 shows the results of such an experiment in the Cobb-Douglas case.

Not surprisingly, the model is qualitatively consistent with the empirical correlation between finance, output, capital intensity, and measured productivity. The model also correctly

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19 This implies a distribution of establishment sizes that is approximately log-normal as well, since labor is linear in \( z^{\frac{1}{\eta}} \) when \( \eta = 1 \).

20 Calibrating \( \lambda_1 \) and \( \lambda_2 \) to moments of the distribution of all establishments in the U.S. does not alter our main quantitative findings. Those results are available upon request.
predicts a positive correlation between average establishment size and the outside capital to output ratio.\textsuperscript{21} In fact, the model’s quantitative predictions for the organization of production are very reasonable given available data. For instance, we calculate Argentina’s outside capital to GDP ratio to be 10 times lower than in the United States. The model with $\sigma = 1$ predicts that in such an economy the average size of establishments should be roughly 4 times lower than in the United States (see figure 2), or around 12.5 employees. According to Argentina’s economic census, the average size of manufacturing establishments was around 11 employees in 1985 and 9 employees in 1993. Mexico’s outside capital to GDP ratio is similar to Argentina’s. According to its economic census, manufacturing establishments counted 19 employees on average in 1988.\textsuperscript{22} Finance disrupts the organization of production in our model by magnitudes similar to what one observes in the available data.

The model also predicts a large impact of finance on capital-output ratios. It predicts that capital-output ratios should vary proportionally to the outside capital to GDP ratio. In the data (bottom-left graph in figure 1), nations with one fifth the outside capital to GDP ratio of the United States have roughly two-thirds the capital-output ratio of the United States. In this sense, the experiment implies too much dispersion in capital-output ratios. In section 4.6, we will argue that this quantitative feature of the model is sensitive to the treatment of resources invested in the storage technology.\textsuperscript{23} Given this dispersion in capital-output ratios, and holding productivity constant, a standard neoclassical production function would predict that output should vary by a factor of one to $5^{\alpha}$, which is roughly 2, assuming capital share $\alpha = 0.3$. Instead, as shown in the first panel of figure 2, our model predicts that output should vary by a factor of 3.5. This additional variation owes to the resource allocation channel and

\textsuperscript{21}Average establishment size is a step function of the finance-to-output ratio because the distribution of managerial talent is discrete. Jumps occur where agents of a given managerial talent become indifferent between becoming workers and managers in the second period of their life.

\textsuperscript{22}See Tybout (2000) for more quantitative evidence on the organization of production in developing countries.

\textsuperscript{23}Recall also that, for lack of financial data, figure 1 excludes all the countries with less than half the U.S. capital-output ratio. As we calculated in section 3, capital-output ratios vary by a factor of one to five worldwide (see the top-left graph of figure 1), which is exactly the dispersion in capital-output ratios predicted by our model.
is reflected in the decline of measured productivity in figure 2. Not only do economies with little finance operate with less capital, they also direct that capital to less productive uses.

4.3 The importance of the allocation channel

While it is instructive to compare the output variation our model generates relative to what standard neoclassical accounting would suggest, these back-of-the-envelope calculations are not a very good measure of the true magnitude of the allocation channel. To better gauge its importance, consider a version of our economy in which agents all have the same managerial talent. In this environment as in the benchmark environment, economies with less finance operate with less capital and more managers. But it is no longer the case that they employ more inefficient technologies, as all managers are equally productive. The quantitative effects of finance in this economy with homogenous agents are compared to the results we obtained with heterogenous agents in the top two graphs of figure 3. First, note that capital-output ratios vary much less than before. The reason for this is somewhat subtle. Because agents are homogenous, they must be indifferent between working and managing in the second period of their life (unless all agents choose to become managers in the second period, which does not happen in any of our experiments given our calibration choices.) This means that all managers have a flat lifetime income profile. In economies with heterogenous agents on the other hand, only marginal managers have flat income profiles, while other managers earn rents and have steeper income profiles. Because steeper income profiles reduce the willingness of managers to transfer resources into the second period, the savings rate of managers is much higher in economies with homogenous agents than in our benchmark economy (at least twice

\footnote{Naturally, such an economy delivers very counterfactual predictions for the organization of production. First, all establishments now have the same size. Second, and less obviously, establishments must now be small, even when there are no contractual imperfections. To see this, note that if all agents become managers in the second period of their life, the size of all establishments in equilibrium is one employee. Bigger establishments arise in equilibrium if, and only if, some agents become workers in the second period of their life, while others become managers. In that case, agents must be indifferent between working and managing. Simple manipulations of first-order conditions for profit maximization then imply that the unique equilibrium size in that case is bounded above by the ratio of the labor share to the managerial share, which is roughly 4 employees given our calibration choices.}
as high in all simulations given our calibration choices.) As a result, a much greater fraction of capital is self-financed in the homogenous agent case, and finance has a smaller impact on capital intensity. Specifically, capital intensity now varies only by a factor of 1.65 as we go from self-financing to perfect markets. Holding everything else constant across economies, standard neoclassical accounting would then predict that output should vary by a factor of $1.65^{1-\alpha} \simeq 1.24$, given $\alpha = 0.3$. In our simulations, output turns out to vary by a factor of 1.15. In other words, without agent heterogeneity, our model generates no output dispersion beyond the standard capital intensity channel.

Our model of finance, therefore, improves upon standard development models by generating output dispersion beyond what capital intensity differences can explain. We find, in other words, that the capital intensity channel and the allocation channel are both quantitatively important. Nonetheless, the model’s predictions for output dispersion falls short of what one observes in the data. This is not surprising given that our model abstracts (among other things) from differences in human capital across economies. Moreover, the importance of finance for output, and our other quantitative findings, could be sensitive to our calibration choices. We now turn to evaluating this possibility.

4.4 Sensitivity analysis

Since the managerial share is not a standard parameter, it is important to verify that our results do not hinge on the value of $\nu$ we proposed in section 4.1. Figure 4 shows the effect of dropping the managerial share from 15% to 10%, and of raising it to 20%, holding the capital share constant. It can readily be seen that this has little effect on results. Another key parameter is the discount rate $\beta$, since it determines the willingness of agents to save for life-cycle reasons. In particular, lowering $\beta$ reduces the propensity of agents to save, which could give finance a greater role. Figure 5 shows the effect of lowering $\beta$ until investment in the storage technology in the benchmark economy is approximately zero. This drastic change
in value has almost no noticeable effect on our results.\footnote{The symmetric experiment, setting $r$ so that $S = K$ given our assumed $\beta$ makes no significant impact either. This experiment is of independent interest as well. It could be interpreted as a world in which the price of capital is set by a large economy with respect to which other economies are small and open. This is a reasonable description, for instance, of the relationship of several Latin American nations with the U.S.} We also verified that our results are not sensitive to even large changes in the target for the outside capital to output ratio in the U.S. Those results are available upon request.

Two calibration choices that, on the other hand, have a big impact on our results are the capital share and the elasticity of substitution between capital and labor. Gollin (2002) finds that cross-country capital shares vary between 0.2 and 0.35. Figure 6 shows the results for capital shares of $0.4 = 0.47 \times 0.85$ and $0.2 = 0.24 \times 0.85$ respectively (in the benchmark case we used $0.3 = 0.35 \times 0.85$), while managerial shares are kept constant, and labor shares are adjusted to make up for the difference. The high capital share generates a dispersion in output of magnitude 10, which is a significant improvement relative to the benchmark.

Lowering the degree of substitution between capital and labor also helps. Figure 7 shows the effect of lowering $\sigma$ to 0.75, 0.5, and 0, the Leontief case,\footnote{For low values of the relative outside capital-output ratio we lose some accuracy when $\sigma$ is small, as can be seen in the capital-output ratio graph.} which we include here for illustrative purposes. As the elasticity of substitution falls, the dispersion in output rises to become comparable to what we observe in the data. Yet, the capital-output ratio dispersion does not differ much from that in the Cobb-Douglas case. Consequently, most of the extra variation in output is reflected in (naively) measured productivity, as in the data. With elasticities of substitution between $\sigma = 0.75$ and $\sigma = 0.5$, the model can easily account for the observed dispersion of output and measured productivity across countries.

\section*{4.5 Finance matters}

That lowering the capital share and the degree to which capital and labor can be substituted for each other leads to more variation in output is well-known (see Caselli (2003) for an extensive discussion of this topic.) If one makes capital more important in production, a
given level of variation in capital intensity implies more variation in output for obvious reasons. What, then, does one learn from our experiments?

Finance changes the relationship between capital intensity and output for independent reasons. In our model, the allocation channel almost doubles the predicted ratio of output variation to capital intensity variation. This implies that the technological parameter changes one needs in order to generate all the observed dispersion of output across countries are much smaller. To illustrate this, consider once again an economy where all agents are homogenous, and lower the elasticity of substitution between capital and labor from 1 to 0.75. The results are shown in the bottom half of figure 3. Although the output effect is (marginally) higher than in the Cobb-Douglas case with homogenous agents, the ratio of output variation to capital-output variation is almost unchanged. By contrast, when agents are heterogeneous and the allocation channel is operative, lowering the elasticity of substitution to 0.75 doubles the ratio of output variation to capital-output variation (the ratio goes from 0.7 to 1.5). Put another way, a given change in the elasticity substitution raises output variation by a much larger amount when financial disruptions are explicitly modeled. Therefore, our experiments not only show that both channels emphasized by the financial development literature are quantitatively important, but they also show that the deviations one needs from standard technological assumptions in order to account for all the observed variation in output are smaller when one models financial disruptions explicitly.

4.6 Finance and capital intensity

The model with heterogenous agents appears to predict too much variation in capital-output ratios relative to financial differences, and correspondingly, given that the opportunity cost of capital is the same across economies, too much variation in capital income shares of GDP. In the data, capital income shares vary much less. But this finding depends on one’s interpretation of the part of aggregate savings, $S$, that is not used in production by managers.

\[27\text{See, again, Gollin (2002), Caselli (2003), or Cole, Ohanian, Riascos, and Schmitz (2004) for a comparison between the U.S. and Latin American countries.}\]
If \( S - K \) is interpreted as including residential investment and part of other non-equipment investment, parts of it should be treated as investment according to standard National Income and Product accounting, and the returns to these investment (possibly imputed returns in the case of residential investment) should be part of GDP. On the other hand, money hoarded by households or, more simply, unreported investments should not be treated as investment because they are not accounted as so by national accounts.\(^{28}\)

Assume that fraction \( \gamma \) of \( S - K \) qualifies as investment according to standard NIPA principles. In this case, the adequate counterpart in our model for capital as we measured it in the data section is \( K + \gamma(S - K) \), while the appropriate counterpart for GDP is \( Y + \gamma(S - K)r \). Figure 8 plots the quantitative impact of this redefinition of domestic capital and output assuming, by way of illustration that \( \gamma = 0.5 \), versus the benchmark (\( \gamma = 0 \)). The allocation channel now matters much more than before: output dispersion is almost unchanged, but capital-output ratios vary much less than before. Given the behavior of the capital-output ratio, capital shares of GDP now vary very little across countries. In the Cobb-Douglas case, the dispersion in output falls short of that in the data, but as before, this can be remedied by lowering the elasticity of substitution between capital and labor. This produces a world with as much dispersion in output as in the data, despite comparatively small differences in capital-output ratios and capital income shares. In short, we have a model of development driven by financial disruptions that quantitatively matches key development facts.

5 Conclusion

Our quantitative experiments attempt to fill what we think is a gap in the literature. Empirical economists have emphasized the strong statistical relationship between financial development and economic development. A number of theories have been developed that establish a

\(^{28}\)In that sense, the appropriate data counterpart for \( K \) may be equipment investment plus the part of non-equipment investment that contributes to productive activities. Then, the sensitivity of \( \frac{K}{Y} \) to financial differences is less problematic. DeLong and Summers (1991) calculate that equipment investment rates vary much more across countries than overall investment rates.
qualitative connection between finance and development. But neither approach can inform us on the quantitative importance of finance. We ask whether differences in the quantity of financial intermediation can account for cross-country differences in output and in capital intensity, and find that they can.

In our model, better financial markets raise output by increasing the capital used in production (the capital intensity channel) and by directing capital to its best uses (the allocation channel). Our calibrated exercises suggest that both channels are quantitatively important.

However, under a unitary factor elasticity of substitution, financial frictions alone cannot generate all the observed dispersion in output across countries and account for the fact that output varies much more than capital intensity across countries. Lowering the elasticity of substitution between capital and labor from 1 to 0.75, or 0.5, increases the impact of finance on output, without changing the role of capital intensity much. Raising the capital share above standard values has a similar impact.

We find, in fact, that financial disruptions make the impact of changes in the parameters that govern technological opportunities much greater. Put another way, when the role of finance is explicitly modeled, deviations from standard technological assumptions needed to account for the observed dispersion in output become much smaller.

Our model can also be extended to include other potential functions of the financial system. In particular, we ignore the role the financial system plays in alleviating informational frictions. Incorporating those features should help us better understand the role of finance in development. Whether doing this will alter our basic quantitative findings is unclear. Our quantitative analysis encompasses the two extreme cases of financial development: complete markets and pure self-financing. Moreover, in order to quantify the importance of these informational frictions, they must first be calibrated. We believe observed differences in the organization of production to be the natural disciplining benchmark when quantifying disruptions in the allocation of productive resources. Similar benchmarks will have to be provided for informational frictions.

29For such an exercise see Erosa and Hidalgo (2004).
A Proofs

A.1 Proof of lemma 1

Rewrite the managers’ problem as:

\[ V(a, z; \eta, w) = \max_{s \leq a, d \geq 0} \Pi(s + d, z; w) \]

s.t. \[ \Phi(s, d, z; \eta, w) \geq 0, \]

where \( \Phi(s, d; z; \eta, w) \equiv \eta [\Pi(s + d, z; w) + s(1 + r)] - (1 - \eta) d(1 + r) \). As in the text, let \( s(a, z; \eta, w) \) and \( d(a, z; \eta, w) \) be the solutions to the above problem and define \( a^*(z; \eta, w) = \inf \{ a : s(a, z; \eta, w) + d(a, z; \eta, w) = k^*(z; w) \} \). If \( a^*(z; \eta, w) = 0 \), the lemma holds trivially. So assume that \( a^*(z; \eta, w) > 0 \) and fix \( a < a^*(z; \eta, w) \). Start with item (ii). From the envelope theorem, \( V_2 = \Pi_2 + \lambda \Phi_2 > 0 \), where \( \lambda \geq 0 \) is the Lagrange multiplier associated with the incentive compatibility constraint. Next, necessary and sufficient conditions for a solution to the constrained maximization problem above are:

\[ \Pi_1(s + d, z; w) + \lambda \Phi_2(s, d, z; \eta, w) = 0 \]  
\[ \lambda \Phi(s, d, z; \eta, w) = 0 \]  
\[ \Pi_1(s + d, z; w) + \lambda \Phi_1(s, d, z; \eta, w) - \nu = 0 \]  
\[ \nu(a - s) = 0 \]

where \( \nu \geq 0 \) is the multiplier associated with the constraint \( s \leq a \). (A.1) and (A.3) imply \( \lambda (\Phi_1 - \Phi_2) = \nu \). Since \( a < a^*(z; \eta, w) \), the manager is borrowing constrained, i.e. \( \Pi_1(s + d, z; w) > 0 \) at the a solution. But A.1 then implies \( \lambda > 0 \) and \( \Phi_2 < 0 \). Differentiation of \( \Phi \) shows that \( \Phi_1 > 0 \) when \( \eta > 0 \). Since \( \Phi_2 < 0 \), it now follows that \( \nu > 0 \) for constrained agents, hence \( s = a \) by (A.4). This establishes item (iii) of the lemma.

To show part (iv), note that for constrained agents \( \Phi(a, d(a, z; \eta, w), z; \eta, w) = 0 \) in a neighborhood of \( a \). Differentiating with respect to \( a \) yields \( \Phi_1 + \Phi_2 \frac{\partial d}{\partial a} = 0 \), hence \( \frac{\partial d}{\partial a} = -\frac{\Phi_1}{\Phi_2} > 0 \) because \( \Phi_1 > 0 \) and \( \Phi_2 < 0 \) for constrained agents. For strict concavity, differentiating with respect to \( a \) once more gives

\[
\frac{\partial^2 d}{\partial a^2} = -\frac{\Phi_2 (\Phi_{11} + \Phi_{12} \frac{\partial d}{\partial a}) - \Phi_1 (\Phi_{21} + \Phi_{22} \frac{\partial d}{\partial a})}{\Phi_2^2}
\]

But differentiation of \( \Phi \) shows that \( \Phi_{12} = \Phi_{11} = \Phi_{22} = \eta \Pi_{11} < 0 \) for constrained agents. So \( \frac{\partial^2 d}{\partial a^2} < 0 \), as claimed.

For item (i), use the envelope theorem to obtain \( V_1(a, z; \eta, w) = \nu > 0 \). To show strict

\[30\]These conditions are sufficient because \( \Phi \) is concave.
conavity, note that $V_{11}(a, z; \eta, w) = \frac{\partial \nu}{\partial a}$. But, from (A.3),

$$
\frac{\partial \nu}{\partial a} = \Pi_{11} \left( 1 + \frac{\partial d}{\partial a} \right) + \frac{\partial \lambda}{\partial a} \Phi_1 + \lambda \left( \Phi_{11} + \Phi_{12} \frac{\partial d}{\partial a} \right).
$$

All we need to insure is that $\frac{\partial \lambda}{\partial a} < 0$. By (A.1),

$$
\frac{\partial \lambda}{\partial a} = -\Pi_{11} \left( 1 + \frac{\partial d}{\partial a} \right) \Phi_2 - \left( \Phi_{21} + \Phi_{22} \frac{\partial d}{\partial a} \right) \Pi_2 < 0.
$$

Finally, to establish item (v), differentiating the incentive compatibility constraint w.r.t. $z$ yields $\Phi_3 + \Phi_2 \frac{\partial d}{\partial z} = 0$. Thus, $\frac{\partial d}{\partial z} = -\frac{\Phi_3}{\Phi_2} > 0$, as $\Phi_3 > 0$, and $\Phi_2 < 0$ for constrained agents.

### A.2 Proof of proposition 2

First note that $ED(\cdot; \eta)$ is upper-hemicontinuous non-empty and convex valued for all $\eta \in [0, 1]$. Indeed, by the theorem of the maximum, the set of optimal policies for agents of each managerial ability $z$ is non-empty and varies upper-hemicontinuously with $w$. Because $\mu$ has finite support, the integrals in the definition of $ED$ are finite sums. It follows that $ED$ is non-empty and upper-continuous. Since different agent of ability $z$ can be assigned to different occupations, $ED$ is also convex-valued. Now, for $w$ high enough $ED(w; \eta) \subset \mathbb{R}_-$ for all $\eta \in [0, 1]$ since $\mu$ has bounded support. A standard application of Kakutani’s fixed point theorem now implies that a positive steady state exists provided $ED(w; \eta) \cap \mathbb{R}^+ \neq \emptyset$ for $w$ low enough when $\rho \geq 0$. That this is true when $\rho = 1$ (the linear case) is obvious. So it suffices to show that when $\rho \in [0, 1)$, $\lim_{w \to 0} d(0, z; \eta, w) > 0$ for all $\eta > 0$ and $z > 0$. Indeed, this implies that $l(z; \eta, w)$ grows without bound while $\varrho(\eta, w)$ goes to zero as $w$ becomes small.

To show that $\lim_{w \to 0} d(0, z; \eta, w) > 0$ for all $\eta > 0$ and $z > 0$ when $\rho \in [0, 1)$, notice that for all $(z, \eta, w)$, $d(0, z; \eta, w)$ is the unique solution to:

$$
d(0, z; \eta, w) = \frac{\eta}{(1 - \eta)(1 + r)} \Pi(d, z; w).
$$

When $\rho \in [0, 1)$, simple algebra shows that $\frac{\Pi(d, z; w)}{d}$ diverges to $+\infty$ as $d$ becomes small for all $z \in Z$ and $w > 0$. (When $\rho < 0$, $\frac{\Pi(d, z; w)}{d}$ does not diverge as $d$ becomes small. Indeed, $\Pi_1(d, z; w)$ converges to a finite constant when $d$ converges to zero.) So, whenever $\eta > 0$ and $\rho \geq 0$, $\lim_{w \to 0} d(0, z; \eta, w) > 0$ and we are done.
B Calibration details

B.1 Managerial share

As explained in the text, when $\eta = \sigma = 1$, the managerial share of output is the ratio of managerial income to the sum of payments to physical capital, labor, and the managerial input. We obtain an empirical counterpart to this ratio using sole proprietorship tax data.

For sole proprietorships, gross payments to capital, labor and the sole proprietor are business receipts minus all payments to intermediate inputs plus taxes paid. We calculate payments to intermediate inputs as Cost of Sales and Operations net of Cost of Labor + Supplies + Travel + Utilities + Advertising. Taxes paid need to be allocated to the three factors. Sole proprietors may deduct state and local income taxes, sales taxes, employment taxes, one half of their self-employment tax, personal property, and real estate taxes. We assume that these taxes accrue to the various sectors in proportion to overall factor shares. Then the ratio of payments to the sole proprietor to payments to other inputs can be approximated by the ratio of net income to business receipts minus payments to intermediate inputs minus taxes paid. Manufacturing sole proprietorship data available from the IRS then yields:

<table>
<thead>
<tr>
<th>Year</th>
<th>Receipts</th>
<th>Payments to intermediate inputs</th>
<th>Taxes paid</th>
<th>Net Income</th>
<th>$(4)/(1-(2)-(3))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1989</td>
<td>25,400,029</td>
<td>10,499,808†</td>
<td>332,950</td>
<td>3,228,762</td>
<td>0.222</td>
</tr>
<tr>
<td>1990</td>
<td>21,839,350</td>
<td>7,957,673†</td>
<td>343,016</td>
<td>2,467,377</td>
<td>0.182</td>
</tr>
<tr>
<td>1991</td>
<td>23,354,542</td>
<td>8,920,086†</td>
<td>360,103</td>
<td>2,595,448</td>
<td>0.184</td>
</tr>
<tr>
<td>1992</td>
<td>27,243,502</td>
<td>10,934,325†</td>
<td>579,542</td>
<td>3,508,402</td>
<td>0.223</td>
</tr>
<tr>
<td>1993</td>
<td>27,157,994</td>
<td>11,228,291</td>
<td>559,901</td>
<td>3,216,585</td>
<td>0.209</td>
</tr>
<tr>
<td>1994</td>
<td>32,928,845</td>
<td>13,966,846</td>
<td>649,892</td>
<td>3,927,951</td>
<td>0.215</td>
</tr>
<tr>
<td>1995</td>
<td>32,101,683</td>
<td>13,662,657</td>
<td>617,056</td>
<td>4,143,519</td>
<td>0.232</td>
</tr>
<tr>
<td>1996</td>
<td>32,057,221</td>
<td>13,327,790</td>
<td>592,845</td>
<td>4,377,188</td>
<td>0.240</td>
</tr>
<tr>
<td>1997</td>
<td>32,057,221</td>
<td>13,425,659</td>
<td>593,115</td>
<td>3,941,135</td>
<td>0.218</td>
</tr>
<tr>
<td>1998</td>
<td>27,327,211</td>
<td>12,988,688</td>
<td>430,230</td>
<td>3,608,920</td>
<td>0.259</td>
</tr>
</tbody>
</table>

Note: All figures in thousands of dollars. Data are from Internal Revenue Service, Sole Proprietorship Tax Statistics - Manufacturing Sole Proprietorships - General (http://www.irs.gov/taxstats/article/0,,id=96754,00.html.)

† The IRS did not provide the cost of supplies between 1989 and 1992. We impute it using its average ratio to receipts during the other six years.

B.2 Distribution of managerial talent

Given all parameters, steady state equilibria can be computed using standard techniques. For each set of exogenous parameters, we guess a wage, compute optimal policies for all agents by backward induction, and then update our wage guess until the labor market approximately clears. In all our quantitative experiments, the approximate equilibrium wage proved unique (specifically, the aggregate excess demand for labor function we computed proved monotonic). In this section we make more precise the procedure we use to calibrate the parameters of the distribution of managerial talent, and the degree $\eta$ to which contracts can be enforced.
We assume that managerial talent is log-normally distributed with location parameter \( \lambda_1 \) and dispersion parameter \( \lambda_2 > 0 \). Given \( \lambda_1 \) and \( \lambda_2 \), we use a discretized version of the resulting distribution. Specifically, we assign mass to the set \( \{ z_i = \left( \frac{i}{100} \right)^2 : i = 1, 2 \ldots 100 \} \).

In particular, the density of mass points is higher near the origin, as we found that doing this improved the precision of the algorithm. Letting \( F(\lambda_1, \lambda_2, \cdot) \) denote the cumulative distribution function of the log-normal distribution with parameters \( (\lambda_1, \lambda_2) \) we set

\[
\mu(z_i) = F \left( \lambda_1, \lambda_2, \frac{z_{i+1} - z_i}{2} \right) - F \left( \lambda_1, \lambda_2, \frac{z_i - z_{i-1}}{2} \right) \text{ for all } i,
\]

with the convention that \( z_0 = 0 \) and that \( z_{101} = +\infty \).

Holding all other parameters fixed, denote by \( \bar{n}(\lambda_1, \lambda_2, \eta) \) and \( \phi(\lambda_1, \lambda_2, \eta) \) the average size of establishments and the fraction of employment in establishments with fewer than 50 employees in equilibrium given \( \lambda_1, \lambda_2, \) and \( \eta \). Let \( \eta(\lambda_1, \lambda_2) \) be the value for \( \eta \) such that the equilibrium ratio of loans to gross yearly output is 2 given \( \lambda_1 \) and \( \lambda_2 \). Our calibration objective is to choose \( \lambda_1 \) and \( \lambda_2 \) so that:

\[
\bar{n}(\lambda_1, \lambda_2, \eta(\lambda_1, \lambda_2)) = 50 \quad \text{(B.1)}
\]

\[
\phi(\lambda_1, \lambda_2, \eta(\lambda_1, \lambda_2)) = 0.50 \quad \text{(B.2)}
\]

We meet those goals using a downhill simplex method\(^{31}\) to minimize:

\[
[50 - \bar{n}(\lambda_1, \lambda_2, \eta(\lambda_1, \lambda_2))]^2 + [50 - 100\phi(\lambda_1, \lambda_2, \eta(\lambda_1, \lambda_2))]^2
\]

In all our experiments, we were able to meet our two calibration targets almost exactly. Furthermore, even drastic changes in initial conditions (the coordinates of the initial simplex) led to the same results suggesting that despite the complexity of the mapping between parameters and \( \bar{n} \) and \( \phi \), solutions to (B.1-B.2) are unique. The following table shows the resulting parameters in all our experiments.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Low ( \beta )</th>
<th>Low ( \nu )</th>
<th>High ( \nu )</th>
<th>High ( \alpha )</th>
<th>Low ( \alpha )</th>
<th>( \sigma = 0.75 )</th>
<th>( \sigma = 0.50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>1.19</td>
<td>1.19</td>
<td>1.19</td>
<td>1.19</td>
<td>1.19</td>
<td>1.19</td>
<td>1.19</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( \frac{1}{10} )</td>
<td>0.675</td>
<td>1</td>
<td>1.19</td>
<td>1.19</td>
<td>1.19</td>
<td>1.19</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.35</td>
<td>0.35</td>
<td>0.33</td>
<td>0.37</td>
<td>0.47</td>
<td>0.24</td>
<td>0.35</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.85</td>
<td>0.85</td>
<td>0.80</td>
<td>0.90</td>
<td>0.85</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.475</td>
<td>0.475</td>
<td>0.419</td>
<td>0.531</td>
<td>0.391</td>
<td>0.587</td>
<td>0.475</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>-2.404</td>
<td>-2.148</td>
<td>-3.819</td>
<td>-3.917</td>
<td>-4.443</td>
<td>-2.194</td>
<td>-2.342</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>0.657</td>
<td>0.612</td>
<td>1.556</td>
<td>0.317</td>
<td>2.312</td>
<td>0.469</td>
<td>0.858</td>
</tr>
</tbody>
</table>

Note: We set the normalization parameter \( A \) to 30 in all experiments.

\(^{31}\)Our C routine uses Amoeba.c from Numerical Recipes.
References


Figure 1: Data

World dispersion
Output and outside capital
Capital–output ratios and outside capital
Measured productivity and outside capital
Figure 2: Cobb-Douglas

![Graphs showing the relationship between relative output, measured productivity, capital-output ratio, and average plant size with respect to the relative outside capital-output ratio.](image)
Figure 3: Homogenous agents

Relative output: Cobb–Douglas

Relative capital–output ratio: Cobb–Douglas

Relative output: $\sigma=0.75$

Relative capital–output ratio: $\sigma=0.75$
Figure 4: Different managerial shares

- Relative output
- Relative measured productivity
- Relative capital–output ratio
- Relative average plant size

\[ \nu = 0.15 \]
\[ \nu = 0.1 \]
\[ \nu = 0.2 \]
Figure 5: Low propensity to save

\[ \beta = \frac{1}{1+r} \]
\[ \beta = \frac{0.675}{1+r} \]
Figure 6: Different capital shares

- Relative output
- Relative measured productivity
- Relative capital–output ratio
- Relative average plant size

Graphs showing the relationships between relative output, measured productivity, capital–output ratio, and average plant size for different values of $\alpha$. The graphs illustrate how these variables change with respect to the relative outside capital–output ratio for $\alpha = 0.35$, $\alpha = 0.47$, and $\alpha = 0.24$.
Figure 7: Different elasticities of substitution

Relative output

Relative measured productivity

Relative capital–output ratio

Relative average plant size

\[
\sigma = 1 \\
\sigma = 0.75 \\
\sigma = 0.5 \\
\sigma = 0
\]
Figure 8: Broad definition of capital

[Diagrams showing relative output and capital-output ratio for different values of σ and γ.]